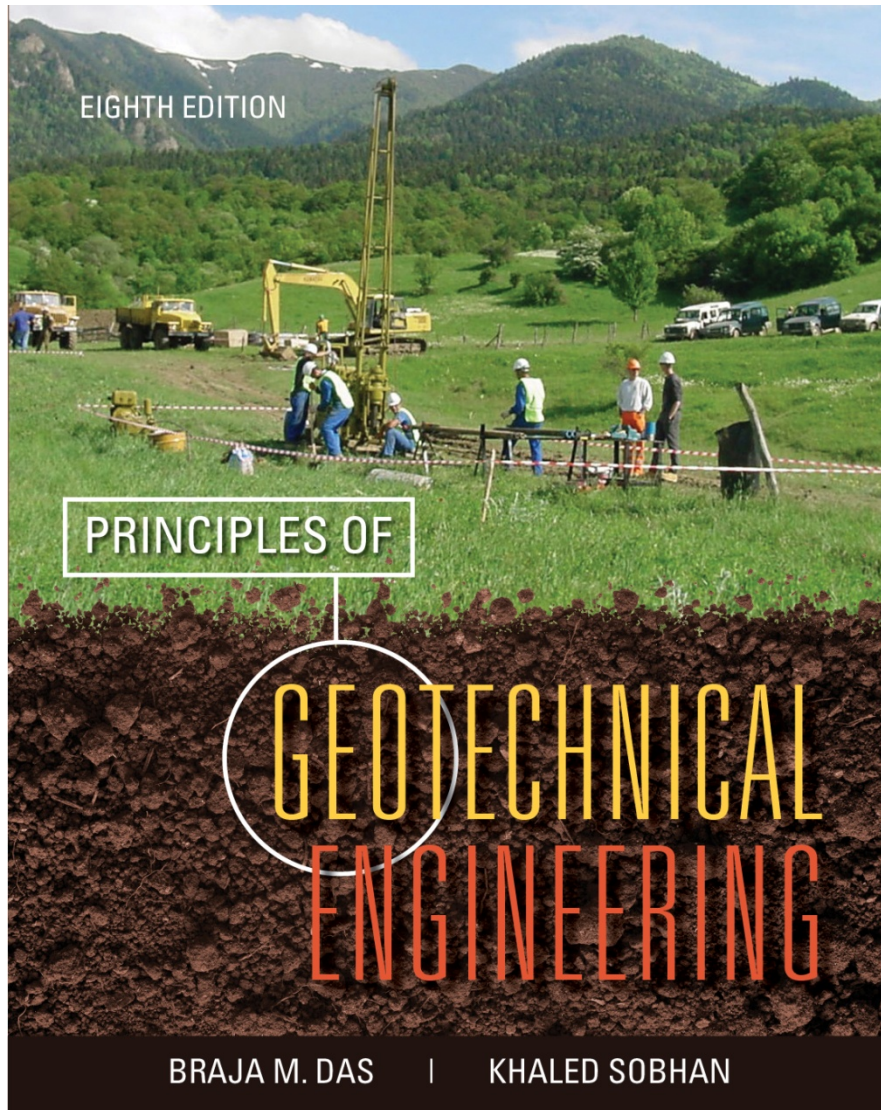


An Instructor's Solutions Manual to Accompany

PRINCIPLES OF GEOTECHNICAL ENGINEERING, 8TH EDITION

BRAJA M. DAS & KHALED SOBHAN



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ENGINEERING

Eighth Edition, SI

BRAJA M. DAS
KHALED SOBHAN

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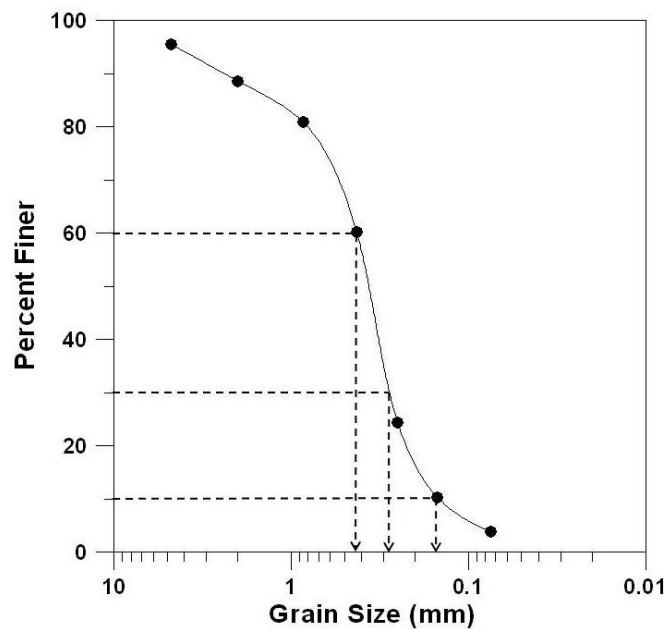
Chapter 2

$$2.1 \quad C_u = \frac{D_{60}}{D_{10}} = \frac{0.42}{0.16} = 2.625 \approx \mathbf{2.63}; \quad C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.21^2}{(0.42)(0.16)} = 0.656 \approx \mathbf{0.66}$$

$$2.2 \quad C_u = \frac{D_{60}}{D_{10}} = \frac{0.81}{0.27} = \mathbf{3.0}; \quad C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.41^2}{(0.81)(0.27)} = 0.768 \approx \mathbf{0.77}$$

2.3 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent finer
4	28	4.54	95.46
10	42	6.81	88.65
20	48	7.78	80.88
40	128	20.75	60.13
60	221	35.82	24.31
100	86	13.94	10.37
200	40	6.48	3.89
Pan	24	3.89	0.00
$\Sigma 617 \text{ g}$			



b. $D_{10} = 0.16 \text{ mm}$; $D_{30} = 0.29 \text{ mm}$; $D_{60} = 0.45 \text{ mm}$

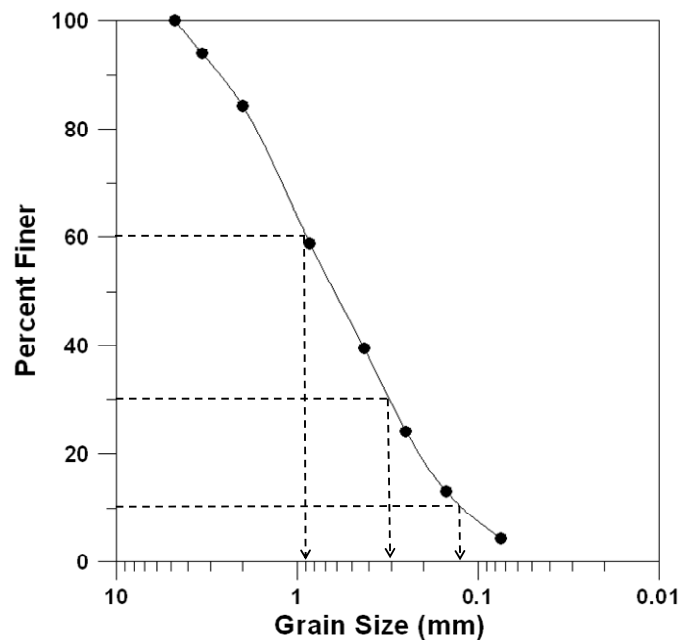
c. $C_u = \frac{D_{60}}{D_{10}} = \frac{0.45}{0.16} = 2.812 \approx 2.81$

d. $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.29^2}{(0.45)(0.16)} = 1.168 \approx 1.17$

2.4 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent Finer
4	0	0.0	100.00
6	30	6.0	94.0
10	48.7	9.74	84.26
20	127.3	25.46	58.80
40	96.8	19.36	39.44
60	76.6	15.32	24.12
100	55.2	11.04	13.08
200	43.4	8.68	4.40
Pan	22	4.40	0.00

$\Sigma 500 \text{ g}$



b. $D_{10} = 0.13 \text{ mm}$; $D_{30} = 0.3 \text{ mm}$; $D_{60} = 0.9 \text{ mm}$

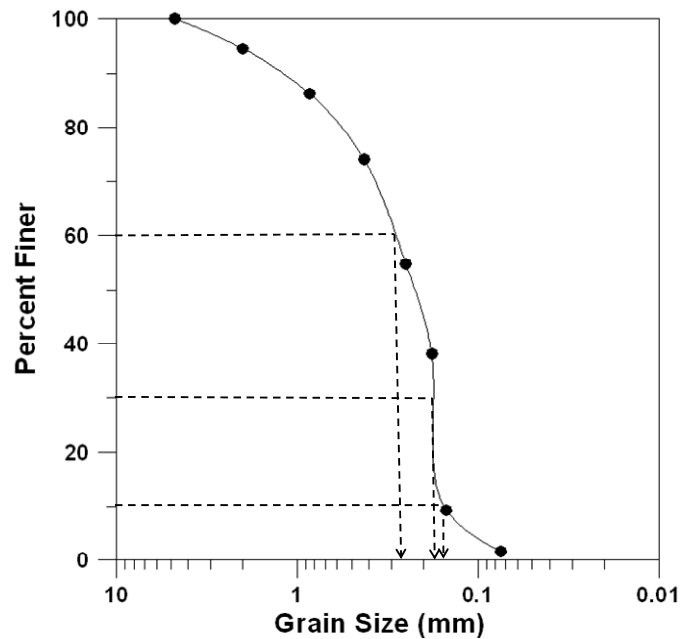
$$c. \quad C_u = \frac{D_{60}}{D_{10}} = \frac{0.9}{0.13} = 6.923 \approx \mathbf{6.92}$$

$$d. \quad C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.3^2}{(0.9)(0.13)} = 0.769 \approx \mathbf{0.77}$$

2.5 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent finer
4	0	0.0	100.00
10	40	5.49	94.51
20	60	8.23	86.28
40	89	12.21	74.07
60	140	19.20	54.87
80	122	16.74	38.13
100	210	28.81	9.33
200	56	7.68	1.65
Pan	12	1.65	0.00

$\Sigma 729 \text{ g}$



b. $D_{10} = \mathbf{0.17 \text{ mm}}$; $D_{30} = \mathbf{0.18 \text{ mm}}$; $D_{60} = \mathbf{0.28 \text{ mm}}$

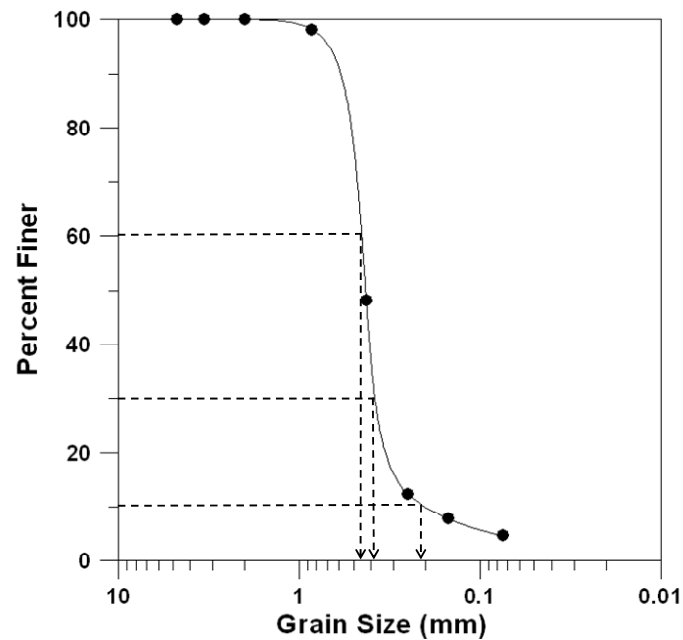
$$c. \quad C_u = \frac{D_{60}}{D_{10}} = \frac{0.28}{0.17} = 1.647 \approx \mathbf{1.65}$$

$$d. \quad C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.18^2}{(0.28)(0.17)} = \mathbf{0.68}$$

2.6 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent finer
4	0	0.0	100.00
6	0	0.0	100.00
10	0	0.0	100.00
20	9.1	1.82	98.18
40	249.4	49.88	48.3
60	179.8	35.96	12.34
100	22.7	4.54	7.8
200	15.5	3.1	4.7
Pan	23.5	4.7	0.00

Σ 500 g

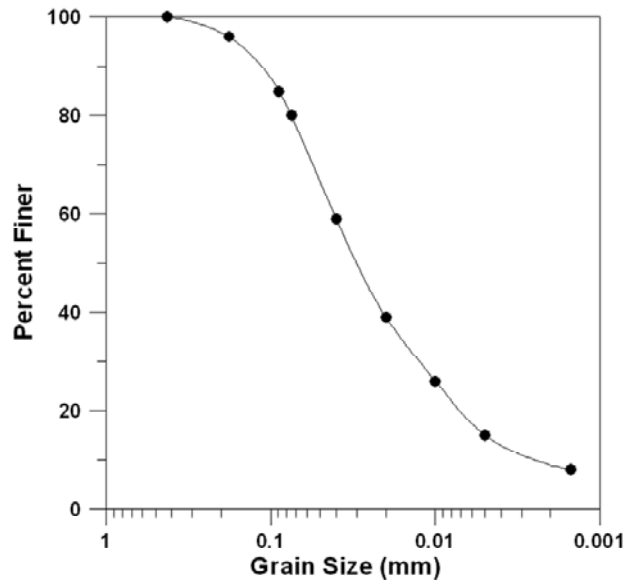


$$b. \quad D_{10} = \mathbf{0.21 \text{ mm}}; D_{30} = \mathbf{0.39 \text{ mm}}; D_{60} = \mathbf{0.45 \text{ mm}}$$

$$c. \quad C_u = \frac{D_{60}}{D_{10}} = \frac{0.45}{0.21} = 2.142 \approx \mathbf{2.14}$$

$$d. \quad C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.39^2}{(0.45)(0.21)} = 1.609 \approx \mathbf{1.61}$$

2.7 a.



- b. Percent passing 2 mm = 100
 Percent passing 0.06 mm = 73
 Percent passing 0.002 mm = 9

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 73 = 27\%$
 SILT: $73 - 9 = 64\%$
 CLAY: $9 - 0 = 9\%$

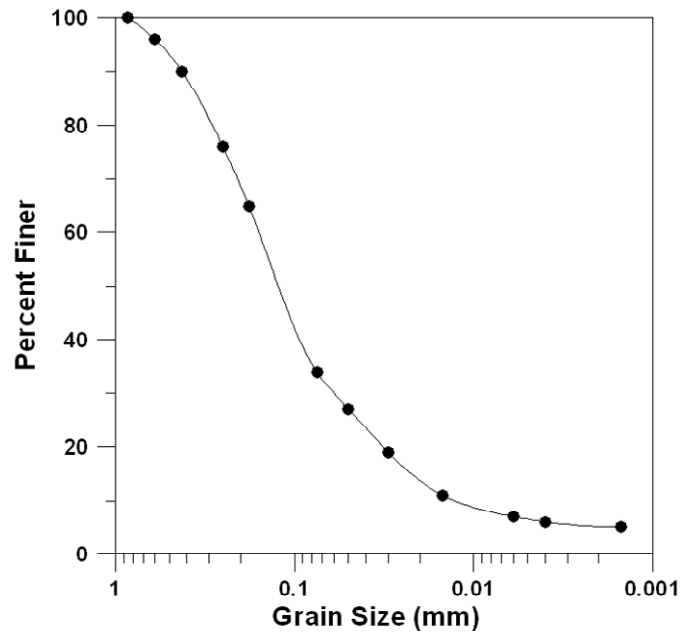
- c. Percent passing 2 mm = 100
 Percent passing 0.05 mm = 68
 Percent passing 0.002 mm = 9

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 68 = 32\%$
 SILT: $68 - 9 = 59\%$
 CLAY: $9 - 0 = 9\%$

- d. Percent passing 2 mm = 100
 Percent passing 0.075 mm = 80
 Percent passing 0.002 mm = 9

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 80 = 20\%$
 SILT: $80 - 9 = 71\%$
 CLAY: $9 - 0 = 9\%$

2.8 a.



- b. Percent passing 2 mm = 100
 Percent passing 0.06 mm = 30
 Percent passing 0.002 mm = 5

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 30 = 70\%$
 SILT: $70 - 5 = 65\%$
 CLAY: $5 - 0 = 5\%$

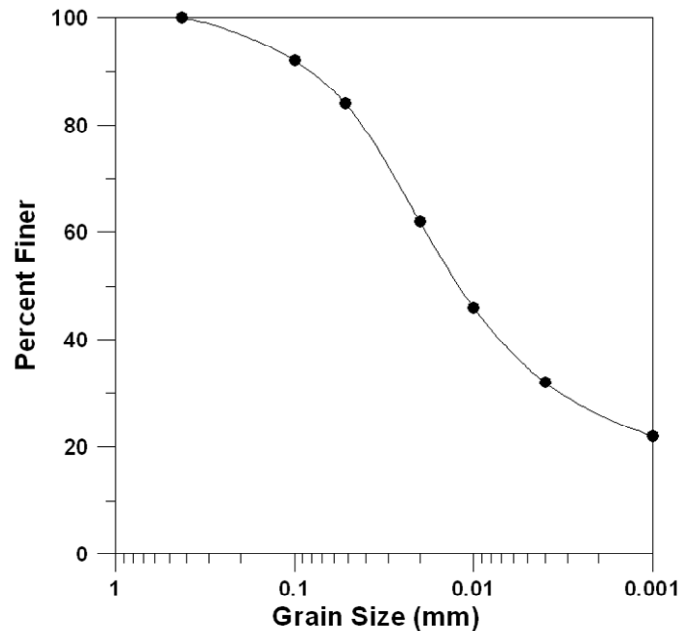
- c. Percent passing 2 mm = 100
 Percent passing 0.05 mm = 28
 Percent passing 0.002 mm = 5

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 28 = 72\%$
 SILT: $72 - 5 = 67\%$
 CLAY: $5 - 0 = 5\%$

- d. Percent passing 2 mm = 100
 Percent passing 0.075 mm = 34
 Percent passing 0.002 mm = 5

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 34 = 66\%$
 SILT: $66 - 5 = 61\%$
 CLAY: $5 - 0 = 5\%$

2.9 a.



- b. Percent passing 2 mm = 100
 Percent passing 0.06 mm = 84
 Percent passing 0.002 mm = 28

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 84 = 16\%$
 SILT: $84 - 28 = 56\%$
 CLAY: $28 - 0 = 28\%$

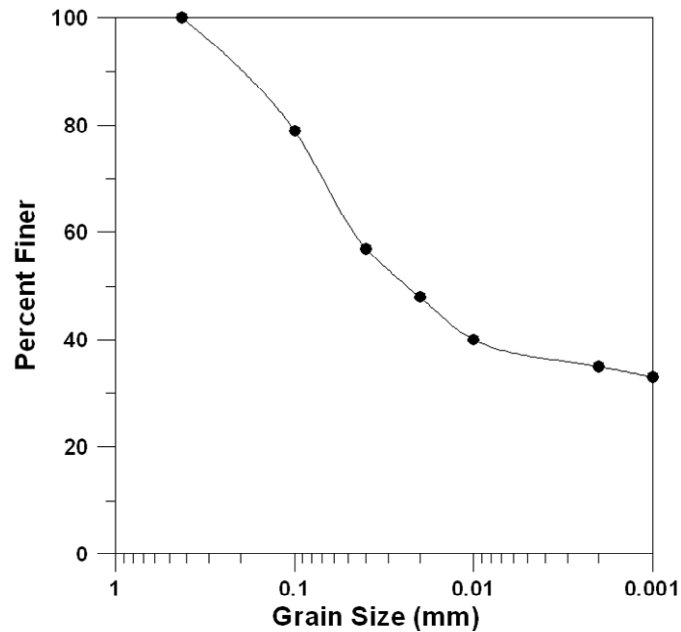
- c. Percent passing 2 mm = 100
 Percent passing 0.05 mm = 83
 Percent passing 0.002 mm = 28

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 83 = 17\%$
 SILT: $83 - 28 = 55\%$
 CLAY: $28 - 0 = 28\%$

- d. Percent passing 2 mm = 100
 Percent passing 0.075 mm = 90
 Percent passing 0.002 mm = 28

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 90 = 10\%$
 SILT: $90 - 28 = 62\%$
 CLAY: $28 - 0 = 28\%$

2.10 a.



- b. Percent passing 2 mm = 100
 Percent passing 0.06 mm = 65
 Percent passing 0.002 mm = 35

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 65 = 35\%$
 SILT: $65 - 35 = 30\%$
 CLAY: $35 - 0 = 35\%$

- c. Percent passing 2 mm = 100
 Percent passing 0.05 mm = 62
 Percent passing 0.002 mm = 35

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 62 = 38\%$
 SILT: $62 - 35 = 27\%$
 CLAY: $35 - 0 = 35\%$

- d. Percent passing 2 mm = 100
 Percent passing 0.075 mm = 70
 Percent passing 0.002 mm = 35

GRAVEL: $100 - 100 = 0\%$
 SAND: $100 - 70 = 30\%$
 SILT: $70 - 35 = 35\%$
 CLAY: $35 - 0 = 35\%$

2.11 $G_s = 2.7$; temperature = 24° ; time = 60 min; $L = 9.2$ cm

Eq. (2.5): $D \text{ (mm)} = K \sqrt{\frac{L \text{ (cm)}}{t \text{ (min)}}}$

From Table 2.6 for $G_s = 2.7$ and temperature = 24° , $K = 0.01282$

$$D = 0.01282 \sqrt{\frac{9.2}{60}} = \mathbf{0.005 \text{ mm}}$$

2.12 $G_s = 2.75$; temperature = 23°C ; time = 100 min; $L = 12.8 \text{ cm}$

$$\text{Eq. (2.5): } D (\text{mm}) = K \sqrt{\frac{L (\text{cm})}{t (\text{min})}}$$

From Table 2.6 for $G_s = 2.75$ and temperature = 23° , $K = 0.01279$

$$D = 0.01279 \sqrt{\frac{12.8}{100}} = \mathbf{0.0046 \text{ mm}}$$

CRITICAL THINKING PROBLEM

2.C.1 a. Soil A: $C_u = \frac{D_{60}}{D_{10}} = \frac{11}{0.6} = \mathbf{18.33}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{5^2}{(11)(0.6)} = \mathbf{3.78}$

Soil B: $C_u = \frac{D_{60}}{D_{10}} = \frac{7}{0.2} = \mathbf{35}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{2.1^2}{(7)(0.2)} = \mathbf{3.15}$

Soil C: $C_u = \frac{D_{60}}{D_{10}} = \frac{4.5}{0.15} = \mathbf{30}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{1^2}{(4.5)(0.15)} = \mathbf{1.48}$

- b. Soil A is coarser than Soil C. A higher percentage of soil C is finer than any given size compared to Soil A. For example, about 15% is finer than 1 mm for Soil A, whereas almost 30% is finer than 1 mm in case of soil C.
- c. Particle segregation may take place in aggregate stockpiles such that there is a separation of coarser and finer particles. This makes representative sampling difficult. Therefore Soils A, B, and C demonstrate quite different particle size distribution.

d. Soil A:

Percent passing 4.75 mm = 29

Percent passing 0.075 mm = 1

GRAVEL: $100 - 29 = 71\%$

SAND: $29 - 1 = 28\%$

FINES: $1 - 0 = 1\%$

Soil B:

Percent passing 4.75 mm = 45

Percent passing 0.075 mm = 2

GRAVEL: $100 - 45 = 55\%$

SAND: $45 - 2 = 43\%$

FINES: $2 - 0 = 2\%$

Soil C:

Percent passing 4.75 mm = 53

Percent passing 0.075 mm = 3

GRAVEL: $100 - 53 = 47\%$

SAND: $47 - 3 = 44\%$

FINES: $3 - 0 = 3\%$

Chapter 3

$$3.1 \quad \gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{G_s\gamma_w}{1 + e} + \frac{e\gamma_w}{1 + e} = \frac{e\gamma_w}{(1 + e)w_{\text{sat}}} + n\gamma_w = n\left(\frac{1 + w_{\text{sat}}}{w_{\text{sat}}}\right)\gamma_w$$

$$3.2 \quad \gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{G_s\gamma_w}{1 + e} + \frac{e\gamma_w}{1 + e} = \gamma_d + \left(\frac{e}{1 + e}\right)\gamma_w$$

Rearranging, $\gamma_{\text{sat}}(1 + e) = \gamma_d(1 + e) + e\gamma_w$

Therefore, $e = \frac{\gamma_{\text{sat}} - \gamma_d}{\gamma_d - \gamma_{\text{sat}} + \gamma_w}$

$$3.3 \quad \gamma_{\text{sat}} = \left(\frac{1 + w_{\text{sat}}}{1 + e}\right)G_s\gamma_w = \left(\frac{1 + w_{\text{sat}}}{1 + e}\right)\frac{e\gamma_w}{w_{\text{sat}}} = \frac{(1 + w_{\text{sat}})n\gamma_w}{w_{\text{sat}}}$$

Rearranging, $w_{\text{sat}}(\gamma_{\text{sat}} - n\gamma_w) = n\gamma_w$

Therefore, $w_{\text{sat}} = \frac{n\gamma_w}{\gamma_{\text{sat}} - n\gamma_w}$

$$3.4 \quad \text{a.} \quad \gamma = \frac{W}{V} = \frac{12.5}{0.1} = \mathbf{125 \text{ lb/ft}^3}$$

$$\text{b.} \quad \gamma_d = \frac{\gamma}{1 + w} = \frac{125}{1 + 0.14} = \mathbf{109.64 \text{ lb/ft}^3}$$

$$\text{c.} \quad e = \frac{G_s\gamma_w}{\gamma_d} - 1 = \frac{(2.71)(62.4)}{109.64} - 1 = \mathbf{0.54}$$

$$\text{d.} \quad n = \frac{e}{1 + e} = \frac{0.54}{1 + 0.54} = \mathbf{0.35}$$

$$e. \quad S = \frac{(w)(G_s)}{e} = \frac{(0.14)(2.71)}{0.54} = 0.702 = \mathbf{70.2\%}$$

$$f. \quad \text{Volume of water} = \frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{(125 - 109.64)(0.1)}{62.4} \approx \mathbf{0.024 \text{ ft}^3}$$

$$3.5 \quad a. \quad \gamma = \left(\frac{1+w}{1+e} \right) G_s \gamma_w; 19.2 = \frac{(1+0.098)(2.69)(9.81)}{1+e}; e = \mathbf{0.51}$$

$$b. \quad \gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{(2.69)(9.81)}{1+0.51} = \mathbf{17.48 \text{ kN/m}^3}$$

$$c. \quad S = \frac{(w)(G_s)}{e} = \frac{(0.098)(2.69)}{0.51} = 0.517 = \mathbf{51.7\%}$$

$$3.6 \quad a. \quad \gamma = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{(2.69)(9.81) + (0.9)(0.51)(9.81)}{1+0.51} = 20.45 \text{ kN/m}^3$$

$$\text{Water to be added} = 20.45 - 19.2 = \mathbf{1.25 \text{ kN/m}^3}$$

$$b. \quad \gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.69 + 0.51)(9.81)}{1+0.51} = 20.78 \text{ kN/m}^3$$

$$\text{Water to be added} = 20.78 - 19.2 = \mathbf{1.58 \text{ kN/m}^3}$$

$$3.7 \quad a. \quad V = \frac{\pi}{4} (2.8)^2 (22) \left(\frac{1}{12^3} \right) = 0.078 \text{ ft}^3; \gamma = \frac{W}{V} = \frac{9.56}{0.078} = \mathbf{122.56 \text{ lb/ft}^3}$$

$$b. \quad w = \frac{W - W_s}{W_s} = \frac{9.56 - 8.51}{8.51} = 0.1233 = \mathbf{12.33\%}$$

$$c. \quad \gamma_d = \frac{W_s}{V} = \frac{8.51}{0.078} = \mathbf{109.1 \text{ lb/ft}^3}$$

$$d. \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.69)(62.4)}{109.1} - 1 \approx \mathbf{0.54}$$

$$e. \quad S = \frac{wG_s}{e} = \frac{(0.1233)(2.69)}{0.54} = 0.614 = \mathbf{61.4\%}$$

$$3.8 \quad a. \quad \gamma = \frac{(1+w)G_s \gamma_w}{1+e} = \frac{(1+w)G_s \gamma_w}{1 + \frac{wG_s}{S}}; \quad 108 = \frac{(1+0.26)(G_s)(62.4)}{1 + \frac{(0.26)(G_s)}{0.72}}; \quad G_s = \mathbf{2.72}$$

$$b. \quad e = \frac{wG_s}{S} = \frac{(0.26)(2.72)}{0.72} = \mathbf{0.98}$$

$$c. \quad \gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.72 + 0.98)(62.4)}{1 + 0.98} = \mathbf{116.6 \text{ lb/ft}^3}$$

$$3.9 \quad a. \quad \gamma_d = \frac{\gamma}{1+w} = \frac{20.6}{1+0.166} = \mathbf{17.67 \text{ kN/m}^3}$$

$$b. \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.74)(9.81)}{17.67} - 1 \approx \mathbf{0.52}$$

$$c. \quad n = \frac{e}{1+e} = \frac{0.52}{1+0.52} = \mathbf{0.34}$$

$$d. \quad S = \frac{wG_s}{e} = \frac{(0.166)(2.74)}{0.52} = 0.874 = \mathbf{87.4\%}$$

$$3.10 \quad a. \quad \gamma = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{(2.74)(9.81) + (0.9)(0.52)(9.81)}{1+0.52} = 20.7 \text{ kN/m}^3$$

$$\text{Water to be added} = 20.7 - 20.6 = \mathbf{0.1 \text{ kN/m}^3}$$

$$b. \quad \gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.74 + 0.52)(9.81)}{1+0.52} = 21.04 \text{ kN/m}^3$$

$$\text{Water to be added} = 21.04 - 20.6 = \mathbf{0.44 \text{ kN/m}^3}$$

$$3.11 \quad a. \quad \rho_d = \frac{\rho}{1+w} = \frac{1750}{1+0.23} = \mathbf{1422.76 \text{ kg/m}^3}$$

$$b. \quad e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{(2.73)(1000)}{1422.76} - 1 = 0.92; \quad n = \frac{e}{1+e} = \frac{0.92}{1+0.92} = \mathbf{0.48}$$

$$c. \quad S = \frac{wG_s}{e} = \frac{(0.23)(2.73)}{0.92} = 0.682 = \mathbf{68.2\%}$$

$$d. \quad \rho_{\text{sat}} = \frac{(G_s + e)\rho_w}{1+e} = \frac{(2.73 + 0.92)(1000)}{1+0.23} \approx 2967 \text{ kg/m}^3$$

$$\text{Water to be added} = 2967 - 1750 = \mathbf{1217 \text{ kg/m}^3}$$

$$3.12 \quad a. \quad \gamma_d = \frac{\gamma}{1+w} = \frac{30.75}{0.25(1+0.098)} = \mathbf{112 \text{ lb/ft}^3}$$

$$b. \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.66)(62.4)}{112} - 1 \approx \mathbf{0.48}$$

$$c. \quad \text{Volume of water} = \frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{\left(\frac{30.75}{0.25} - 112\right)(0.25)}{62.4} \approx \mathbf{0.044 \text{ ft}^3}$$

$$3.13 \quad a. \quad e = \frac{n}{1-n} = \frac{0.3}{1-0.3} = \mathbf{0.43}$$

$$b. \quad \rho_d = \frac{G_s \rho_w}{1+e}; \quad G_s = \frac{\rho_d(1+e)}{\rho_w} = \frac{1800(1+0.43)}{1000} = \mathbf{2.57}$$

$$3.14 \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.69)(62.4)}{105} - 1 \approx \mathbf{0.598}$$

$$S = \frac{wG_s}{e} = \frac{(0.17)(2.69)}{0.598} = 0.764 = \mathbf{76.4\%}$$

$$3.15 \quad a. \quad \gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}} = \frac{(1+0.182)(2.67)(62.4)}{1+\frac{(0.182)(2.67)}{0.8}} = \mathbf{122.5 \text{ lb/ft}^3}$$

$$b. \quad \gamma_d = \frac{\gamma}{1+w} = \frac{122.5}{1+0.182} = 103.6$$

$$\text{Volume of water} = \frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{(122.5 - 103.6)(1)}{62.4} = \mathbf{0.302 \text{ ft}^3/\text{ft}^3 \text{ of soil}}$$

$$3.16 \quad a. \quad \gamma = \frac{(G_s + Se)\gamma_w}{1+e}; \quad 106 = \frac{(G_s + 0.55e)(62.4)}{1+e}$$

$$G_s = 1.148e + 1.698 \quad (i)$$

$$114 = \frac{(G_s + 0.822e)(62.4)}{1+e} \quad (ii)$$

From (i) and (ii): $G_s = \mathbf{2.73}$

b. Using $G_s = 2.73$ in Equation (i), we get $e = \mathbf{0.9}$

$$3.17 \quad a. \quad D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}; \quad 0.65 = \frac{0.75 - e}{0.75 - 0.52}; \quad e = \mathbf{0.6}$$

$$b. \quad \gamma_d = \frac{G_s\gamma_w}{1+e} = \frac{(2.67)(9.81)}{1+0.6} = \mathbf{16.37 \text{ kN/m}^3}$$

$$3.18 \quad D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}; \quad 0.82 = \frac{0.72 - e}{0.72 - 0.46}; \quad e \approx \mathbf{0.51}$$

$$\gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(1+0.11)(2.68)(9.81)}{1+0.51} = \mathbf{19.32 \text{ kN/m}^3}$$

$$3.19 \quad \gamma_d = \frac{\gamma}{1+w} = \frac{115}{1+0.08} = 106.48 \text{ lb/ft}^3$$

$$D_r = \frac{\left[\frac{1}{\gamma_{d(\min)}} \right] - \left[\frac{1}{\gamma_d} \right]}{\left[\frac{1}{\gamma_{d(\min)}} \right] - \left[\frac{1}{\gamma_{d(\max)}} \right]} = \frac{\left[\frac{1}{92} \right] - \left[\frac{1}{106.48} \right]}{\left[\frac{1}{92} \right] - \left[\frac{1}{108} \right]} = 0.918 = \mathbf{91.8\%}$$

CRITICAL THINKING PROBLEMS

$$3.C.1 \quad a. \quad e = \frac{V_v}{V_s}; \quad e_1 + 1 = \frac{V_1}{V_s}$$

$$1 + 0.92 = \frac{V_1}{V_s}; \quad V_1 = 1.92V_s$$

$$1 + 0.65 = \frac{V_2}{V_s}; \quad V_2 = 1.65V_s$$

$$\frac{\Delta V}{V} = \frac{V_1 - V_2}{V_1} = \frac{1.92 - 1.65}{1.92} = 0.14 = \mathbf{14\% \text{ (decrease)}}$$

$$b. \quad \gamma_{d(1)} = \frac{G_s\gamma_w}{1+e_1} = \frac{G_s\gamma_w}{1+0.92} = \frac{G_s\gamma_w}{1.92}$$

$$\gamma_{d(2)} = \frac{G_s\gamma_w}{1.65}$$

$$\frac{\Delta \gamma_d}{\gamma_{d(1)}} = \frac{\gamma_{d(2)} - \gamma_{d(1)}}{\gamma_{d(1)}} = \frac{\frac{1}{1.65} - \frac{1}{1.92}}{\frac{1}{1.92}} = 0.163 = \mathbf{16.3\% \text{ (increase)}}$$

$$\text{c. } S_1 = \frac{wG_s}{e_1} = \frac{wG_s}{0.92}; \quad S_2 = \frac{wG_s}{0.65}$$

$$\frac{\Delta S}{S_1} = \frac{S_2 - S_1}{S_1} = \frac{\frac{1}{0.65} - \frac{1}{0.92}}{\frac{1}{0.92}} = 0.415 = \mathbf{41.5\% \text{ (increase)}}$$

$$3.C.2 \quad \text{a. } D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

$$e_1 = e_{\max} - D_r(e_{\max} - e_{\min}) = 0.92 - 0.47(0.92 - 0.53) = 0.736$$

$$\gamma_d = \frac{G_s \gamma_w}{1 + e_1} = \frac{(2.65)(9.81)}{1 + 0.736} = \mathbf{14.97 \text{ kN/m}^3} \text{ (before compaction)}$$

$$e_2 = e_{\max} - D_r(e_{\max} - e_{\min}) = 0.92 - 0.8(0.92 - 0.53) = 0.608$$

$$\gamma_d = \frac{(2.65)(9.81)}{1 + 0.608} = \mathbf{16.17 \text{ kN/m}^3} \text{ (after compaction)}$$

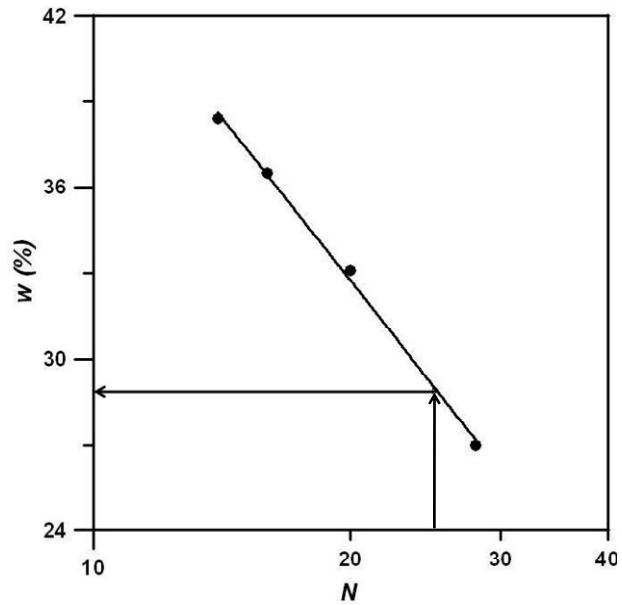
$$\text{b. } \frac{\Delta H}{H} = \frac{\Delta e}{1 + e_1} = \frac{0.736 - 0.608}{1 + 0.736} = 0.074; \quad \Delta H = 0.074H = (0.074)(2) = 0.148 \text{ m}$$

$$\text{Final Height} = 2 - 0.148 = \mathbf{1.852 \text{ m}}$$

Chapter 4

- 4.1 a. Refer to the plot
of w versus N .
 $LL = 29.0$

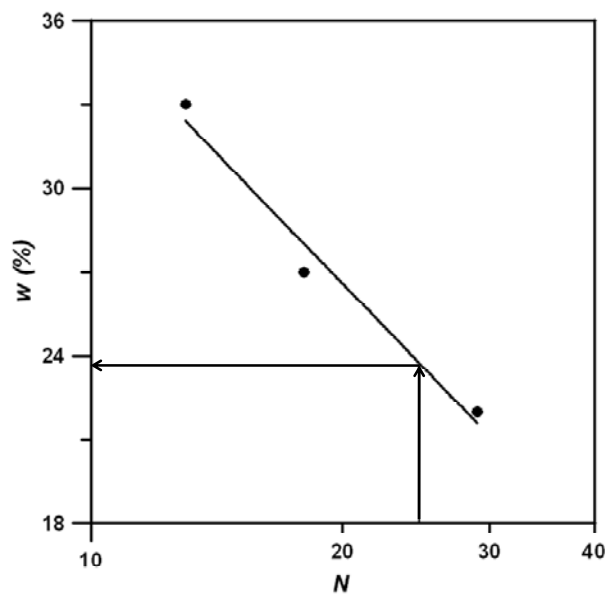
b. $PI = LL - PL$
 $= 29.0 - 13.4$
 $= 15.6$



4.2 $LI = \frac{w - PL}{LL - PL} = \frac{32 - 13.4}{15.6} = 1.19$

- 4.3 a. From the plot,
 $LL = 23.6$.

b. $PI = LL - PL$
 $= 23.6 - 19.1$
 $= 4.5$



$$4.4 \quad LI = \frac{w - PL}{LL - PL} = \frac{21 - 19.1}{4.5} = \mathbf{0.422}$$

$$4.5 \quad SL = \left(\frac{M_1 - M_2}{M_2} \right) (100) - \left(\frac{V_i - V_f}{M_2} \right) (\rho_w) (100)$$

$$= \left(\frac{34 - 24}{24} \right) (100) - \left(\frac{20.2 - 14.3}{24} \right) (1) (100) = \mathbf{17.08}$$

$$SR = \frac{M_2}{V_f \rho_w} = \frac{24}{(14.3)(1)} = \mathbf{1.68}$$

$$4.6 \quad SL = \left(\frac{M_1 - M_2}{M_2} \right) (100) - \left(\frac{V_i - V_f}{M_2} \right) (\rho_w) (100)$$

$$= \left(\frac{44.6 - 32.8}{32.8} \right) (100) - \left(\frac{16.2 - 10.8}{32.8} \right) (1) (100) = \mathbf{19.51}$$

$$SR = \frac{M_2}{V_f \rho_w} = \frac{32.8}{(10.8)(1)} = \mathbf{3.03}$$

CRITICAL THINKING PROBLEMS

$$4.C.1 \quad a. \quad \text{From Eq. (4.26): } A = \frac{PI}{(\% \text{ of clay - size fraction, by weight})}$$

The computed *PI* values are provided in the table on the following page.

Soil	% clay (< 0.002 mm in size)	A	$\tau_{f\text{-undisturbed}}$ (kN/m^2)	S_t	PI	$\tau_{f\text{-remolded}}$ (kN/m^2)
Beauharnois	79	0.52	18	14	41.08	1.3
Detroit I	36	0.36	17	2.5	12.96	6.9
Horten	40	0.42	41	17	16.8	2.4
Gosport	55	0.89	29	2.2	48.95	13.0
Mexico City	90	4.5	46	5.3	405	8.7
Shellhaven	41	1.33	36	7.6	54.53	4.8
St. Thuribe	36	0.33	38	150	11.88	0.3

b. From Table 4.2:

Beauharnois

$A = 0.52$; $PI = 41.08$; Mineral: **Illite**

Detroit I

$A = 0.36$; $PI = 12.96$; Mineral: **Kaolinite**

Horten

$A = 0.42$; $PI = 16.8$; Mineral: **Kaolinite**

Gosport

$A = 0.89$; $PI = 48.95$; Mineral: **Illite**

Mexico City

$A = 4.5$; $PI = 405$; Mineral: **Montmorillonite**

Shellhaven

$A = 1.33$; $PI = 54.53$; Mineral: **Illite**

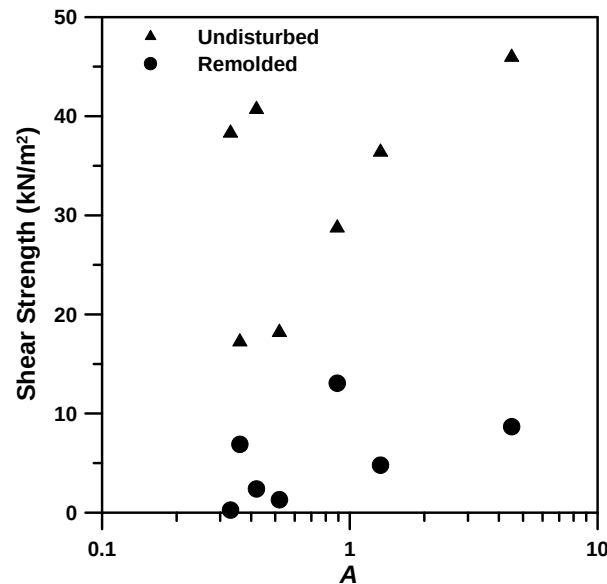
St. Thuribe

$A = 0.33$; $PI = 11.88$; Mineral: **Kaolinite**

c. Sensitivity, $S_t = \frac{\tau_{f\text{-undisturbed}}}{\tau_{f\text{-remolded}}}$

$\tau_{f\text{-remolded}}$ is calculated using the above equation and listed in the table (Part a).

d. The plots are shown below.



Explanation: The shear strength of clay comes from two components: the cohesion, which is the cementing force between particles, and the frictional resistance, which is mainly due to the movement of one particle over another. The greater the activity of clay, the greater is the contribution of cohesion to shear strength. Although no reliable correlation can be developed from the above plots, both the undisturbed and remolded shear strengths certainly show increasing trends as the activity increases.

4.C.2 a. The liquidity index is given by: $LI = \frac{w - PL}{LL - PL}$

The range of liquidity index corresponding to the range in natural water content is calculated and listed in the table.

Soil	% clay (< 0.002 mm in size)	w_n (%)	LL	PL	LI
1	34	59-67	49	26	1.43-1.78
2	44	18-36	37	21	-0.18-0.94
3	54	51-56	61	26	0.71-0.86
4	81	61-70	58	24	1.08-1.35
5	28	441-600	511	192	0.78-1.28
6	67	98-111	132	49	0.59-0.75
7	72	51-65	89	31	0.34-0.57

- b. **Soils 1 and 4:** Since LI range is greater than 1.0, the water content is greater than the liquid limit. From Figure 4.1, the soil behaves like a viscous fluid with practically no shearing resistance.

Soil 2: At a water content of 18%, the $LI < 0$, and the soil behaves like a brittle solid with high shear resistance. At water content of 36%, the soil is in a plastic state ($0 < LI < 1$) showing moderate shearing resistance and a ductile behavior.

Soils 3, 6, and 7: Since $0 < LI < 1$, the water content is less than the liquid limit. From Figure 4.1, the soil is in the plastic state showing moderate shearing resistance and a ductile behavior.

Soil 5: At a water content of 441%, the soil is in the plastic state ($0 < LI < 1$) with moderate shear resistance. At water content of 600%, the $LI > 1$, and the soil becomes a viscous fluid with practically no shearing resistance.

Chapter 5

5.1 Refer to Figure 5.1.

Soil	Classification
A	Clay
B	Sandy clay
C	Loam
D	Sandy clay and sandy clay loam (borderline)
E	Sandy loam

5.2 SOIL A: From Table 5.1, the soil is A-2-4. The GI for A-2-4 is zero.
Classification: **A-2-4(0)**.

SOIL B: From Table 5.1, the soil is A-3. $GI = 0$. Classification: **A-3(0)**.

SOIL C: From Table 5.1, the soil is A-2-6. Equation 5.2:

$$GI = 0.01(F_{200} - 15)(PI - 10) = 0.01(12 - 15)(13 - 10) = -0.09 \approx 0$$
 Classification: **A-2-6(0)**

SOIL D: From Table 5.1, the soil is A-2-7. Equation 5.2:

$$GI = 0.01(F_{200} - 15)(PI - 10) = 0.01(30 - 15)(18 - 10) = 1.2 \approx 1$$
 Classification: **A-2-7(1)**

SOIL E: From Table 5.1, the soil is A-1-b. $GI = 0$. Classification: **A-1-b(0)**.

5.3 SOIL A: From Table 5.1, the soil is A-7-5. Note: $PI = 21 < LL - 30 = 22$
 Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (72 - 35)[0.2 + 0.005(52 - 40)] + 0.01(72 - 15)(21 - 10)$$

$$= 15.89 \approx 16$$
 Classification: **A-7-5(16)**.

SOIL B: From Table 5.1, the soil is A-6. Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (58 - 35)[0.2 + 0.005(38 - 40)] + 0.01(58 - 15)(12 - 10) = 5.23 \approx 5$$
 Classification: **A-6(5)**

SOIL C: From Table 5.1, the soil is A-7-6. Note: $PI = 14 > LL - 30 = 11$
 Eq. (5.1):
 $GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$
 $GI = (64 - 35)[0.2 + 0.005(41 - 40)] + 0.01(64 - 15)(14 - 10) = 7.9 \approx 8$
 Classification: **A-7-6(8)**

SOIL D: From Table 5.1, the soil is A-6. Eq. (5.1):
 $GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$
 $GI = (82 - 35)[0.2 + 0.005(32 - 40)] + 0.01(82 - 15)(12 - 10) = 8.86 \approx 9$
 Classification: **A-6(9)**

SOIL E: From Table 5.1, the soil is A-6. Eq. (5.1):
 $GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$
 $GI = (48 - 35)[0.2 + 0.005(30 - 40)] + 0.01(48 - 15)(11 - 10) = 2.28 \approx 2$
 Classification: **A-6(2)**

5.4 SOIL 1: Fine fraction = % passing No. 200 sieve = 30%
 Coarse fraction = $100 - 30 = 70\%$
 Gravel fraction = $100 - 70 = 30\%$
 Sand fraction = $70 - 30 = 40\%$
 More than 50% of coarse fraction passing No. 4 sieve, so sandy soil.
 Table 5.2 and Figure 5.3: **SC**
 Figure 5.4: More than 15% gravel. **Clayey sand with gravel.**

SOIL 2: Coarse fraction = $100 - 20 = 80\%$
 Gravel fraction = $100 - 48 = 52\%$
 Sand fraction = $80 - 52 = 28\%$
 Table 5.2 and Figure 5.3: **GC**
 Figure 5.4: Greater than 15% sand. **Clayey gravel with sand**

SOIL 3: Coarse fraction = $100 - 30 = 70\%$
 Gravel fraction = $100 - 95 = 5\%$
 Sand fraction = $95 - 70 = 25\%$
 Table 5.2: fine-grained soil; $LL = 52$; $PI = 28$.
 Table 5.2 and Figure 5.3: **CH**
 Figure 5.5: $\geq 30\%$ plus 200, % sand $>$ % gravel, $< 15\%$ gravel,
 so **sandy fat clay**

- SOIL 4: Coarse fraction = $100 - 82 = 18\%$
 Gravel fraction = $100 - 100 = 0\%$
 Sand fraction = $18 - 0 = 18\%$
 Table 5.2: fine-grained soil; $LL = 30$; $PI = 19$.
 Table 5.2 and Figure 5.3: **CL**
 Figure 5.5: **lean clay with sand**
- SOIL 5: Coarse fraction = $100 - 74 = 26\%$
 Gravel fraction = $100 - 100 = 0\%$
 Sand fraction = $26 - 0 = 26\%$
 Table 5.2: fine-grained soil; $LL = 35$; $PI = 21$.
 Table 5.2 and Figure 5.3: **CL**
 Figure 5.5: **lean clay with sand**
- SOIL 6: Coarse fraction = $100 - 26 = 74\%$
 Gravel fraction = $100 - 87 = 13\%$
 Sand fraction = $74 - 13 = 61\%$
 Table 5.2: coarse-grained soil; $LL = 38$; $PI = 18$.
 Table 5.2 and Figure 5.3: **SC**
 Figure 5.4: $< 15\%$ gravel; **clayey sand**
- SOIL 7: Coarse fraction = $100 - 78 = 22\%$
 Gravel fraction = $100 - 88 = 12\%$
 Sand fraction = $22 - 12 = 10\%$
 Table 5.2: fine-grained soil; $LL = 52$; $PI = 28$.
 Table 5.2 and Figure 5.3: **CH**
 Figure 5.5: $< 30\%$ plus 200, % sand $<$ % gravel; **fat clay with gravel**
- SOIL 8: Coarse fraction = $100 - 57 = 43\%$
 Gravel fraction = $100 - 99 = 1\%$
 Sand fraction = $43 - 1 = 42\%$
 Table 5.2: fine-grained soil; $LL = 54$; $PI = 26$.
 Table 5.2 and Figure 5.3: **CH**
 Figure 5.5: $\geq 30\%$ plus 200, % sand $>$ % gravel; **sandy fat clay**
- SOIL 9: Coarse fraction = $100 - 11 = 89\%$
 Gravel fraction = $100 - 71 = 29\%$
 Sand fraction = $89 - 29 = 70\%$
 $LL = 32$; $PI = 16$; $C_u = 4.8$; $C_c = 2.9$. Table 5.2 and Figure 5.3: **SP-SC**
 Figure 5.4: **poorly graded sand with clay and gravel**

SOIL 10: Coarse fraction = $100 - 2 = 98\%$
 Gravel fraction = $100 - 100 = 0\%$
 Sand fraction = $98 - 0 = 98\%$
 $C_u = 7.2$; $C_c = 2.2$. Table 5.2: **SW**
 Figure 5.4: $<15\%$ gravel; **well graded sand**

SOIL 11: Coarse fraction = $100 - 65 = 35\%$
 Gravel fraction = $100 - 89 = 11\%$
 Sand fraction = $35 - 11 = 24\%$
 Table 5.2: fine-grained soil; $LL = 44$; $PI = 21$.
 Table 5.2 and Figure 5.3: **CL**
 Figure 5.5: **sandy lean clay**

SOIL 12: Coarse fraction = $100 - 8 = 92\%$
 Gravel fraction = $100 - 90 = 10\%$
 Sand fraction = $92 - 10 = 82\%$
 $LL = 39$; $PI = 31$; $C_u = 3.9$; $C_c = 2.1$. Table 5.2 and Figure 5.3: **SP-SC**
 Figure 5.4: **poorly graded sand with clay**

- 5.5 a. 13% passing No. 200 sieve; 38% passing No. 40 sieve; 90% passing No. 10 sieve. $PI = 23 - 19 = 4$. Referring to Table 5.1, the soil is A-1-b. $GI = 0$. So the soil is **A-1-b(0)**.
- b. Coarse fraction = $100 - 13 = 87\%$
 Gravel fraction = $100 - 100 = 0\%$
 Sand fraction = $87 - 0 = 87\%$
 $LL = 23$; $PI = 4$. From Table 5.2 and Figure 5.3, the group symbol is **SC**.
 From Figure 5.4, the group name is **clayey sand**.

CRITICAL THINKING PROBLEM

5.C.1 1. Stratum 2

18% passing No. 200 sieve; $PI = 5$. From Table 5.1, the soil is A-1-b.
 $GI = 0$; Soil classification: **A-1-b(0)**

Stratum 3

8% passing No. 200 sieve; NP. From Table 5.1, the soil is A-3.
 $GI = 0$; Soil classification: **A-3(0)**

Stratum 4

67% passing No. 200 sieve; $LL = 52$; $PI = 10$. From Table 5.1, the soil is A-5.

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (67 - 35)[0.2 + 0.005(52 - 40)] + 0.01(67 - 15)(10 - 10) = 8.3 \approx 8$$

Soil classification: **A-5(8)**

Stratum 5

52% passing No. 200 sieve; $LL = 36$; $PI = 9$. From Table 5.1, the soil is A-4.

$$GI = (52 - 35)[0.2 + 0.005(36 - 40)] + 0.01(52 - 15)(9 - 10) = 2.69 \approx 3$$

Soil classification: **A-4(3)**

2. Stratum 2

Coarse fraction: $100 - 18 = 82\%$; Table 5.2: coarse-grained soil. From Table 5.4, most probable soil classification (corresponding to A-1-b): SW, SP, GM, SM. Since it is a fine sand, and since C_c is not between 1 and 3, it is a poorly graded sand. Classification: **SP**

Stratum 3

Coarse fraction: $100 - 8 = 92\%$; Table 5.2: coarse-grained soil. From Table 5.4, most probable soil classification (corresponding to A-3): SP. Since it is a non-plastic fine sand, classification: **SP**

Stratum 4

Coarse fraction: $100 - 67 = 23\%$; Table 5.2: fine-grained soil. From Table 5.4, most probable soil classification (corresponding to A-5): OH, MH, ML, OL. Since the soil is an organic sandy silt, and since $LL = 52 > 50$, the classification is **OH**.

Stratum 5

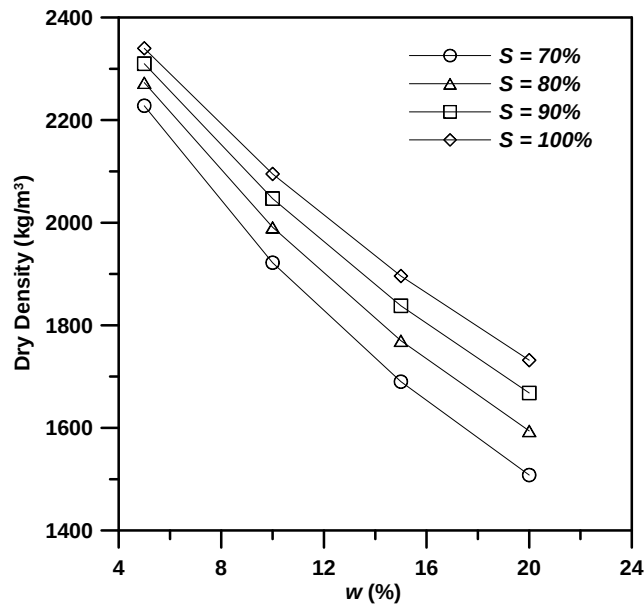
Coarse fraction: $100 - 52 = 48\%$; Table 5.2: fine-grained soil. From Table 5.4, most probable soil classification (corresponding to A-4): ML, OL. Since the soil is a sandy silt, and since $LL = 36 < 50$, the classification is **ML**.

Chapter 6

$$6.1 \quad \rho_d = \frac{G_s \rho_w}{1 + \frac{G_s w}{S}}$$

G_s	ρ_w (kg/m ³)	w (%)	$\rho_d @ S$ (kg/m ³)			
			70%	80%	90%	100%
2.65	1000	5	2228	2273	2310	2340
		10	1922	1991	2047	2095
		15	1690	1770	1838	1896
		20	1508	1594	1668	1732

The plot is shown below.



6.2 Eq. (6.4):

$$\gamma_{zav} = \frac{\gamma_w}{w + \frac{1}{G_s}} = \frac{9.81}{w + \frac{1}{2.68}} = \frac{9.81}{w + 0.3731}$$

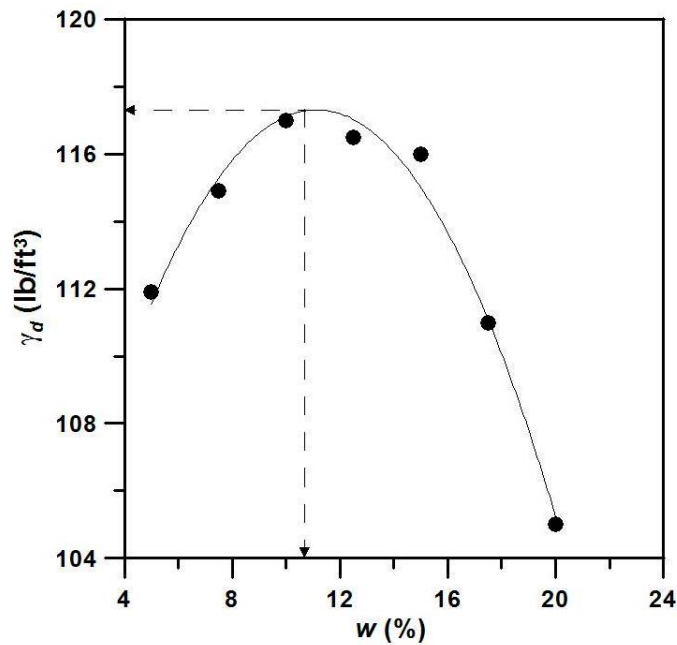
w (%)	γ_{zav} (kN/m ³)
5	23.18
10	20.73
15	18.75
20	17.11
25	15.74

The table can now be prepared.

6.3

Volume (ft ³)	Weight of soil mass, W (lb)	$\gamma = \frac{W}{V}$ (lb/ft ³)	w (%)	$\gamma_d = \frac{\gamma}{1 + \frac{w(\%)}{100}}$ (lb/ft ³)
$\frac{1}{30}$	3.92	117.6	5.0	111.9
$\frac{1}{30}$	4.12	123.6	7.5	114.9
$\frac{1}{30}$	4.29	128.7	10.0	117.0
$\frac{1}{30}$	4.37	131.1	12.5	116.5
$\frac{1}{30}$	4.45	133.5	15.0	116.0
$\frac{1}{30}$	4.35	130.5	17.5	111.0
$\frac{1}{30}$	4.20	126.0	20.0	105.0

- a. The plot of γ_d versus w is shown. $\gamma_{d(\max)} \approx 117.5 \text{ lb/ft}^3 @ w_{\text{opt}} = 10.8\%$



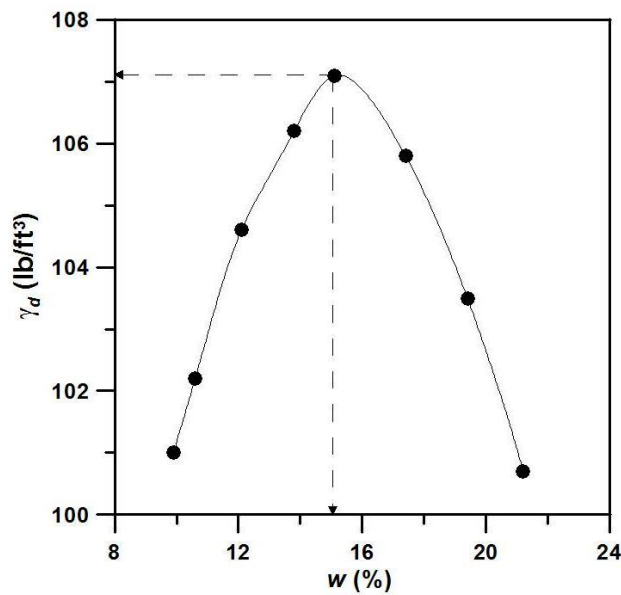
$$b. \quad \gamma_d = \frac{G_s \gamma_w}{1 + e}; \quad 117.5 = \frac{(2.68)(62.4)}{1 + e}; \quad e \approx 0.42$$

$$S = \frac{w G_s}{e} = \frac{(0.108)(2.68)}{0.423} = 0.684 = 68.4\%$$

6.4

Volume (ft ³)	Weight of soil mass, W (lb)	$\gamma = \frac{W}{V}$ (lb/ft ³)	w (%)	$\gamma_d = \frac{\gamma}{1 + \frac{w(\%)}{100}}$ (lb/ft ³)
$\frac{1}{30}$	3.70	111.0	9.9	101.0
$\frac{1}{30}$	3.77	113.1	10.6	102.2
$\frac{1}{30}$	3.91	117.3	12.1	104.6
$\frac{1}{30}$	4.03	120.9	13.8	106.2
$\frac{1}{30}$	4.11	123.3	15.1	107.1
$\frac{1}{30}$	4.14	124.2	17.4	105.8
$\frac{1}{30}$	4.12	123.6	19.4	103.5
$\frac{1}{30}$	4.07	122.1	21.2	100.7

- a. The plot of γ_d versus w is shown. $\gamma_{d(\max)} \approx 107.1 \text{ lb/ft}^3$ @ $w_{\text{opt}} = 15\%$



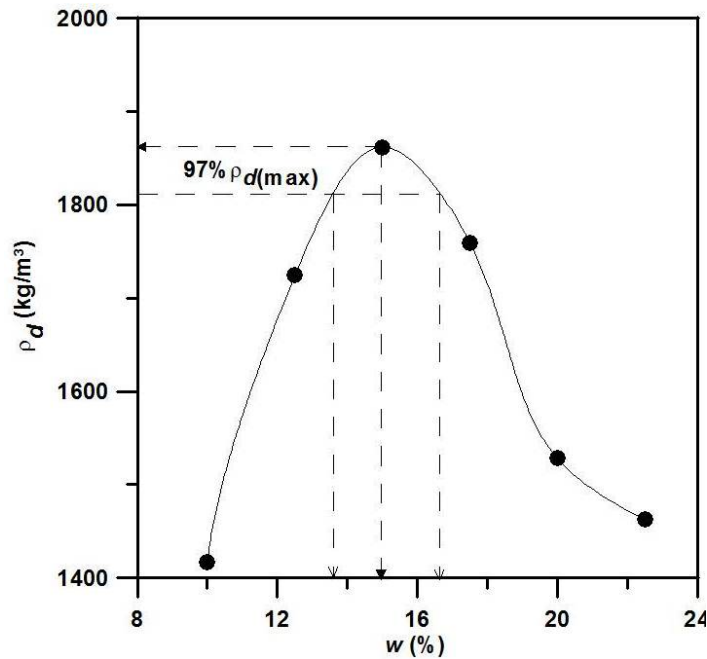
b. $\gamma_d = \frac{G_s \gamma_w}{1 + e}$; $107.1 = \frac{(2.7)(62.4)}{1 + e}$; $e \approx 0.57$

$$S = \frac{w G_s}{e} = \frac{(0.15)(2.7)}{0.57} = 0.71 = 71\%$$

6.5

Volume of mold V (cm^3)	Mass of wet soil M (kg)	$\rho = \frac{M}{V}$ (kg/m^3)	w (%)	ρ_d (kg/m^3)
943.3	1.47	1558.3	10.0	1416.6
943.3	1.83	1940.0	12.5	1724.4
943.3	2.02	2141.4	15.0	1862.0
943.3	1.95	2067.2	17.5	1759.3
943.3	1.73	1833.9	20.0	1528.2
943.3	1.69	1791.5	22.5	1462.4

- a. The plot of ρ_d versus w is shown. $\rho_{d(\max)} \approx 1870 \text{ kg/m}^3$ @ $w_{\text{opt}} = 15\%$



$$b. \quad R(\%) = \frac{\rho_{d(\text{field})}}{\rho_{d(\max)}} \times 100 = \left(\frac{\rho_{d(\text{field})}}{1870} \right) (100)$$

Therefore, $\rho_{d(\text{field})} = (0.97)(1870) \approx 1814 \text{ kg/m}^3$

From the graph, the acceptable range of moisture content is **13.5% – 16.5%**.

$$6.6 \quad \gamma_{(\text{in situ})} = 17.3 \text{ kN/m}^3; \gamma_{d(\text{in situ})} = \frac{17.3}{1 + \frac{16}{100}} = 14.91 \text{ kN/m}^3; \gamma_{d(\text{compacted})} = 18.1 \text{ kN/m}^3$$

$$\text{Volume of soil to be excavated} = (2000) \left(\frac{18.1}{14.91} \right) = \mathbf{2,428 \text{ m}^3}$$

$$\text{Weight of moist soil to be transported} = (2428 \times 17.3) = 42,004 \text{ kN}$$

$$\text{Number of truck loads} = \frac{(42004)(1000)}{(20)(2000)(4.45)} = 235.9 \approx \mathbf{236}$$

6.7 Dry weight of solids required at the embankment site:

$$W_s = 5000\gamma_d \text{ kN} = (5000) \left(\frac{G_s \times 9.81}{1.75} \right) = 28,029 G_s \text{ kN}$$

Borrow Pit	W_s (kN)	γ_d at borrow pit (kN/m ³)	Volume to be excavated from borrow pit = [W_s / γ_d (borrow pit)]	Cost/m ³ (\$)	Total cost (\$)
I	74276.8	$\frac{2.65 \times 9.81}{1 + 0.8} = 14.44$	5143.8 m³	8	41,150
II	75117.7	$\frac{2.68 \times 9.81}{1 + 0.9} = 13.83$	5431.5 m³	5	27,157
III	75958.6	$\frac{2.71 \times 9.81}{1 + 1.1} = 12.66$	5999.8 m³	9	53,998
IV	76799.4	$\frac{2.74 \times 9.81}{1 + 0.85} = 14.53$	5285.5 m³	12	63,427

a. Shown in table.

b. **Borrow Pit II**

$$6.8 \quad \text{From Eq. (6.20): } D_r = \left[\frac{\gamma_{d(\text{field})} - \gamma_{d(\text{min})}}{\gamma_{d(\text{max})} - \gamma_{d(\text{min})}} \right] \left[\frac{\gamma_{d(\text{max})}}{\gamma_{d(\text{field})}} \right]$$

$$0.75 = \left[\frac{\gamma_{d(\text{field})} - 15.5}{18.9 - 15.5} \right] \left[\frac{18.9}{\gamma_{d(\text{field})}} \right]; \gamma_{d(\text{field})} = 17.91 \text{ kN/m}^3$$

$$R(\%) = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{max})}}(100) = \left(\frac{17.91}{18.9}\right)(100) = \mathbf{94.8\%}$$

$$6.9 \quad R = 0.935 = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{max})}} = \frac{\gamma_{d(\text{field})}}{16.98}; \quad \gamma_{d(\text{field})} = \mathbf{15.87 \text{ kN/m}^3}$$

$$D_r = \left[\frac{\gamma_{d(\text{field})} - \gamma_{d(\text{min})}}{\gamma_{d(\text{max})} - \gamma_{d(\text{min})}} \right] \left[\frac{\gamma_{d(\text{max})}}{\gamma_{d(\text{field})}} \right] = \left(\frac{15.87 - 14.46}{16.98 - 14.46} \right) \left(\frac{16.98}{15.87} \right) = 0.598 = \mathbf{59.8\%}$$

$$6.10 \quad \text{a.} \quad R = 0.88 = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{max})}} = \frac{\gamma_{d(\text{field})}}{118}; \quad \gamma_{d(\text{field})} = \mathbf{103.84 \text{ lb/ft}^3}$$

$$\text{b.} \quad D_r = \left[\frac{\gamma_{d(\text{field})} - \gamma_{d(\text{min})}}{\gamma_{d(\text{max})} - \gamma_{d(\text{min})}} \right] \left[\frac{\gamma_{d(\text{max})}}{\gamma_{d(\text{field})}} \right] = \left(\frac{103.84 - 98}{118 - 98} \right) \left(\frac{118}{103.84} \right) = 0.331 = \mathbf{33.1\%}$$

$$\gamma = (1 + w)\gamma_{d(\text{field})} = (1 + 0.13)(103.84) = \mathbf{117.33 \text{ lb/ft}^3}$$

6.11 In the field:

Sand used to fill the hole and cone: $6.08 \text{ kg} - 2.86 \text{ kg} = 3.22 \text{ kg}$

Sand used to fill the hole: $3.22 \text{ kg} - 0.118 \text{ kg} = 3.102 \text{ kg}$

Volume of the hole: $\frac{3.102 \text{ kg}}{1731 \text{ kg/m}^3} = 0.001792 \text{ m}^3$

Moist density of compacted soil: $\frac{3.34}{0.001792} = 1863.84 \text{ kg/m}^3$

$$\gamma = \frac{(1863.84)(9.81)}{1000} = 18.28 \text{ kN/m}^3$$

$$\text{a.} \quad \gamma_d = \frac{\gamma}{1 + \frac{w(\%)}{100}} = \frac{18.28}{1 + \frac{12.1}{100}} = \mathbf{16.3 \text{ kN/m}^3}$$

$$\text{b.} \quad \text{From Problem 6.5: } \rho_{d(\text{max})} = 1870 \text{ kg/m}^3$$

$$\gamma_{d(\max)} = \frac{(1870)(9.81)}{1000} = 18.34 \text{ kN/m}^3$$

$$R = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\max)}} = \frac{16.30}{18.34} = 0.888 = \mathbf{88.8\%}$$

$$6.12 \quad S_N = (1.7) \sqrt{\frac{3}{(D_{50})^2} + \frac{1}{(D_{20})^2} + \frac{1}{(D_{10})^2}} = (1.7) \sqrt{\frac{3}{(1.98)^2} + \frac{1}{(0.31)^2} + \frac{1}{(0.18)^2}} = \mathbf{11.02}$$

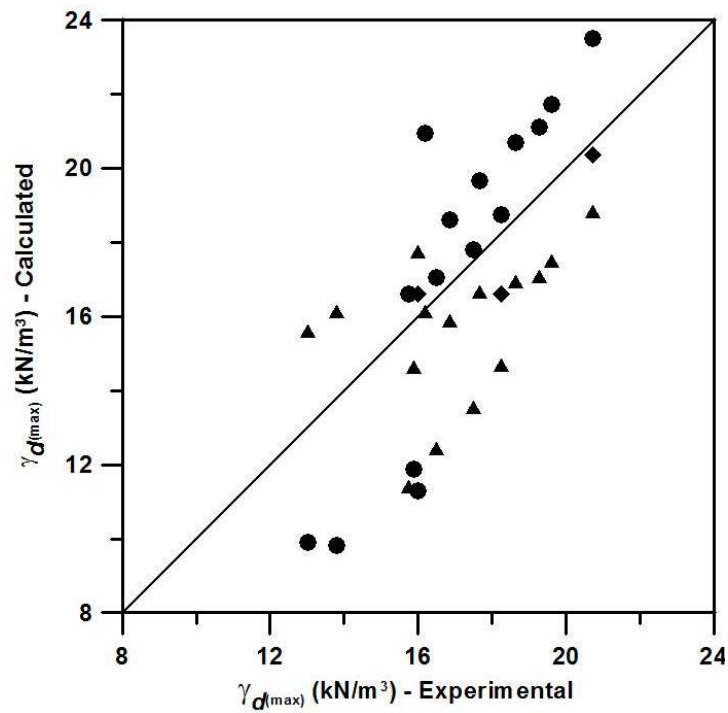
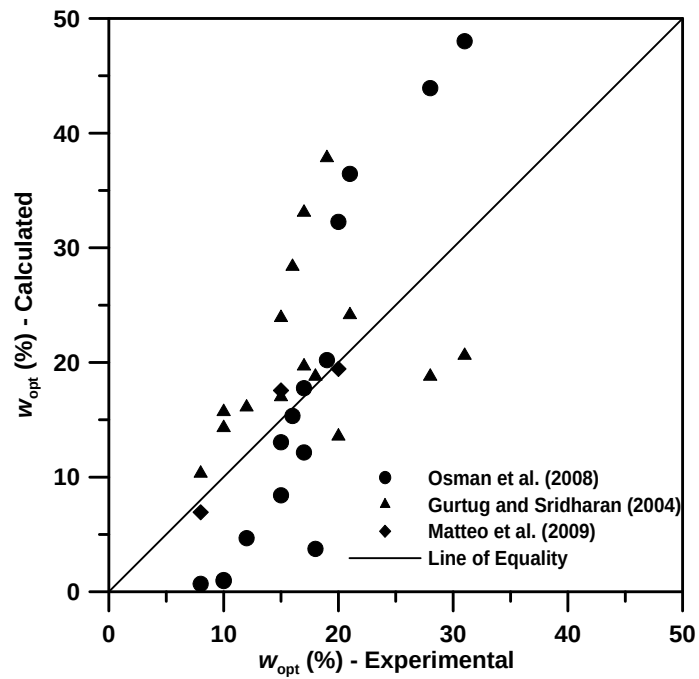
Rating: **GOOD**

CRITICAL THINKING PROBLEM

- 6.C.1 a. Osman et al. (2008) method: Eqs. (6.13) through (6.16) are used to calculate w_{opt} and $\gamma_{d(\max)}$. These values are listed in the following table.
- b. Gurtug and Sridharan (2004) method: Eqs. (6.11) and (6.12) are used to calculate w_{opt} and $\gamma_{d(\max)}$. These values are listed in the following table.
- c. Matteo et al. (2009) method: Eqs. (6.17) and (6.18) are used to calculate w_{opt} and $\gamma_{d(\max)}$, only for the modified Proctor tests. These values are listed in the following table.

Soil	E (kN-m/m ³)	w_{opt} (Exp.) (%)	$\gamma_{d(\max)}$ (Exp.) (kN/m ³)	Part a		Part b		Part c	
				w_{opt} (%)	$\gamma_{d(\max)}$ (kN/m ³)	w_{opt} (%)	$\gamma_{d(\max)}$ (kN/m ³)	w_{opt} (%)	$\gamma_{d(\max)}$ (kN/m ³)
1	2700	8	20.72	0.69	23.52	10.34	18.77	6.94	20.37
	600	10	19.62	0.93	21.73	14.31	17.46		
	354	10	19.29	1.02	21.11	15.70	17.02		
2	2700	20	16.00	32.26	11.31	13.57	17.69	19.44	16.6
	600	28	13.80	43.92	9.82	18.78	16.08		
	354	31	13.02	48.01	9.90	20.61	15.55		
3	2700	15	18.25	13.04	18.74	23.91	14.64	17.56	16.62
	1300	16	17.5	15.33	17.80	28.37	13.50		
	600	17	16.5	17.76	17.07	33.09	12.38		
	275	19	15.75	20.20	16.61	37.85	11.34		
4	600	21	15.89	36.45	11.89	24.15	14.58		
5	600	18	16.18	3.74	20.95	18.78	16.08		
6	600	17	16.87	12.15	18.62	19.67	15.82		
7	600	12	18.63	4.67	20.69	16.10	16.89		
8	600	15	17.65	8.41	19.65	16.99	16.62		

- d. The plots of calculated values against experimental results are shown in the figures below. The 45° line of equality is also drawn in both figures.



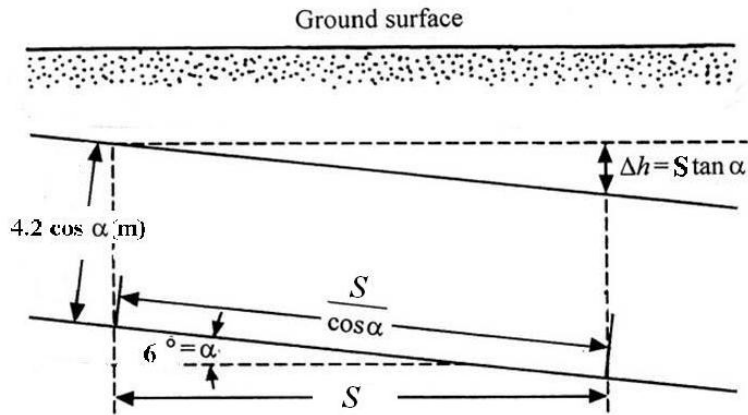
- e. Prediction of w_{opt} : For both the Osman et al. (2008) and Gurtug and Sridharan (2004) models, several data points are closely packed around the 45° line suggesting reasonable agreement between the calculated and experimental values. For the remaining soils, a poor agreement is observed. All 3 points for the Matteo (2009) model plot near the equality line showing close agreement.

Prediction of $\gamma_{d(max)}$: Most data points for all models show good agreement between the calculated and the experimental values.

Empirical models are often limited to the materials, test methods, and environmental conditions (among other factors) under which the experiments were conducted and the models developed. For new materials and conditions, the predicted values may not be reliable.

Chapter 7

7.1



From the figure: $i = \frac{\text{head loss}}{\text{length}} = \frac{S \tan \alpha}{\left(\frac{S}{\cos \alpha}\right)} = \sin \alpha$

$$q = k i A = (k)(\sin \alpha)(4.2 \cos \alpha)(1)$$

$$k = 4.8 \times 10^{-3} \text{ cm/sec} = 4.8 \times 10^{-5} \text{ m/sec}$$

$$q = (4.8 \times 10^{-5})(\sin 6^\circ)(4.2 \cos 6^\circ)(\underbrace{3600}_{\text{to change to m/hr}}) = 0.0754 \text{ m}^3/\text{hr/m}$$

$$\approx 7.54 \times 10^{-2} \text{ m}^3/\text{hr/m}$$

7.2
$$i = \frac{h}{\left(\frac{S}{\cos \alpha}\right)}$$

$$q = k i A = k \left(\frac{h \cos \alpha}{S} \right) (H_1 \cos \alpha \times 1) = \left(\frac{0.075}{\underbrace{10^2}_{\text{m/sec}}} \right) \left(\frac{2.75 \cos 14}{30} \right) (2 \cos 14)$$

$$= 1.29 \times 10^{-4} \text{ m}^3/\text{sec/m}$$

$$7.3 \quad a. \quad k = \frac{QL}{Aht} = \frac{(350 \text{ cm}^2)(30 \text{ cm})}{(176.71 \text{ cm}^2)(50 \text{ cm})(300 \text{ sec})} = \mathbf{3.96 \times 10^{-3} \text{ cm/sec}}$$

$$b. \quad v_s = v \left(\frac{1+e}{e} \right); \quad v = ki$$

$$v_s = ki \left(\frac{1+e}{e} \right) = (0.00396) \left(\frac{50 \text{ cm}}{30 \text{ cm}} \right) \left(\frac{1+0.61}{0.61} \right) = \mathbf{0.0174 \text{ cm/sec}}$$

$$7.4 \quad k = \frac{QL}{Aht}; \quad 0.062 = \frac{(160)(15)}{(31.67)(h)(60)}$$

$$h = \mathbf{20.37 \text{ cm}}$$

$$7.5 \quad a. \quad k = 2.303 \left(\frac{aL}{At} \right) \log_{10} \left(\frac{h_1}{h_2} \right) = (2.303) \left[\frac{(0.25)(15)}{(19.64)(8 \times 60)} \right] \log_{10} \left(\frac{40}{20} \right)$$

$$= \mathbf{2.75 \times 10^{-4} \text{ cm/sec}}$$

$$b. \quad k = 2.303 \left(\frac{aL}{At} \right) \log_{10} \left(\frac{h_1}{h_2} \right)$$

$$0.000275 = (2.303) \left[\frac{(0.25)(15)}{(19.64)(6 \times 60)} \right] \log_{10} \left(\frac{40}{h_2} \right) = 0.00122 \log \left(\frac{40}{h_2} \right)$$

$$h_2 = \mathbf{23.82 \text{ cm}}$$

$$7.6 \quad a. \quad k = 2.303 \left(\frac{aL}{At} \right) \log_{10} \left(\frac{h_1}{h_2} \right)$$

$$= (2.303) \left(\frac{0.97 \times 50}{16 \times 10} \right) \log_{10} \left(\frac{41}{18.5} \right) = 0.241 \text{ cm/min} = 0.00402 \text{ cm/sec}$$

$$\bar{K} = \frac{k\eta}{\gamma_w} = \frac{(4.02 \times 10^{-5} \text{ m/sec})(1.005 \times 10^{-3} \text{ N} \cdot \text{s/m}^2)}{9.789 \times 10^3 \text{ N/m}^3} = \mathbf{4.13 \times 10^{-12} \text{ m}^2}$$

$$\text{b. } k = 2.303 \left(\frac{aL}{At} \right) \log_{10} \left(\frac{h_1}{h_2} \right)$$

$$0.241 \text{ cm/min} = (2.303) \left(\frac{0.97 \times 50}{16 \times 7} \right) \log_{10} \left(\frac{41}{h_2} \right)$$

$$h_2 = \mathbf{23.5 \text{ cm}}$$

7.7 From Eq. (7.15) and Table 7.2 for $T = 28^\circ \text{C}$:

$$k_{20^\circ \text{C}} = \left(\frac{\eta_{T^\circ \text{C}}}{\eta_{20^\circ \text{C}}} \right) k_{T^\circ \text{C}} = (0.832)(0.009) = \mathbf{7.49 \times 10^{-3} \text{ cm/sec}}$$

$$7.8 \quad \text{Eq. (7.31): } \frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}, \quad \text{or} \quad \frac{0.03}{k_2} = \frac{\left(\frac{0.62^3}{1+0.62} \right)}{\left(\frac{0.48^3}{1+0.48} \right)} = \frac{0.1471}{0.0747}$$

$$k_2 = \mathbf{0.015 \text{ cm/sec}}$$

$$7.9 \quad e = e_{\max} - (e_{\max} - e_{\min})D_r = 0.68 - (0.68 - 0.42)(0.52) = 0.544$$

$$\begin{aligned} \text{Eq. (7.32): } k \text{ (cm/sec)} &= 2.4622 \left[D_{10}^2 \frac{e^3}{1+e} \right]^{0.7825} \\ &= (2.4622) \left[(0.4)^2 \left(\frac{0.544^3}{1+0.544} \right) \right]^{0.7825} = \mathbf{0.1 \text{ cm/sec}} \end{aligned}$$

$$7.10 \quad e_1 = e_{\max} - (e_{\max} - e_{\min})D_r = 0.72 - (0.72 - 0.46)(0.8) = 0.512$$

$$e_2 = 0.72 - (0.72 - 0.46)(0.67) = 0.545$$

$$\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}, \quad \text{or} \quad \frac{0.006}{k_2} = \frac{\left(\frac{0.512^3}{1+0.512} \right)}{\left(\frac{0.545^3}{1+0.545} \right)} = \frac{0.0887}{0.1047}$$

$$k_2 = 7.08 \times 10^{-3} \text{ cm/sec}$$

$$7.11 \quad n_1 = 0.36; \quad e_1 = \frac{n_1}{1 - n_1} = \frac{0.36}{1 - 0.36} = 0.562$$

$$n_2 = 0.48; \quad e_2 = \frac{0.48}{1 - 0.48} = 0.923$$

$$\text{Eq. (7.31):} \quad k_2 = k_1 \left(\frac{e_2^3}{1 + e_2} \right) \left(\frac{1 + e_1}{e_1^3} \right) = k_1 \left(\frac{1 + e_1}{1 + e_2} \right) \left(\frac{e_2}{e_1} \right)^3 = (0.072) \left(\frac{1.562}{1.923} \right) \left(\frac{0.923}{0.562} \right)^3$$

$$k_2 = 0.259 \text{ cm/sec}$$

$$7.12 \quad \gamma_{d(\text{field})} = R\gamma_{d(\text{max})} = (0.9)(16) = 14.4 \text{ kN/m}^3$$

$$e = \frac{G_s \gamma_w}{\gamma_{d(\text{field})}} - 1 = \frac{(2.7)(9.81)}{14.4} - 1 = 0.839$$

$$\begin{aligned} \text{Eq. (7.34):} \quad k \text{ (cm/sec)} &= 35 \left(\frac{e^3}{1 + e} \right) (C_u^{0.6}) (D_{10} \text{ mm})^{2.32} \\ &= 35 \left(\frac{0.839^3}{1 + 0.839} \right) (3.1)^{0.6} (0.23)^{2.32} = 0.732 \text{ cm/sec} \end{aligned}$$

7.13

Sieve no.	Opening (cm)	Percent passing	Fraction between two consecutive sieves (%)
30	0.06	100	{----- 27
40	0.0425	73	{----- 14
60	0.02	59	{----- 36
100	0.015	23	{----- 23
200	0.0075	0	

$$\left. \begin{array}{l} \text{For fraction between} \\ \text{sieve Nos. 30 and 40} \end{array} \right\} \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{27}{0.06^{0.404} \times 0.0425^{0.595}} = 550.93$$

$$\left. \begin{array}{l} \text{For fraction between} \\ \text{sieve Nos. 40 and 60} \end{array} \right\} \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{14}{0.0425^{0.404} \times 0.02^{0.595}} = 514.21$$

$$\left. \begin{array}{l} \text{For fraction between} \\ \text{sieve Nos. 60 and 100} \end{array} \right\} \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{36}{0.02^{0.404} \times 0.015^{0.595}} = 2127.71$$

$$\left. \begin{array}{l} \text{For fraction between} \\ \text{sieve Nos. 100 and 200} \end{array} \right\} \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{23}{0.015^{0.404} \times 0.0075^{0.595}} = 2306.35$$

$$\Sigma \frac{\Sigma 100\%}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{100}{550.93 + 514.21 + 2127.71 + 2306.35} = 0.0182$$

$$k = (1.99 \times 10^4)(0.0182)^2 \left(\frac{1}{7.5} \right)^2 \left(\frac{0.68^3}{1 + 0.68} \right) = \mathbf{0.0219 \text{ cm/sec}}$$

$$7.14 \quad \frac{k_1}{k_2} = \left(\frac{e_1^n}{1 + e_1} \right) \left(\frac{1 + e_2}{e_2^n} \right) = \left(\frac{1 + e_2}{1 + e_1} \right) \left(\frac{e_1}{e_2} \right)^n$$

$$\frac{0.2 \times 10^{-6}}{0.91 \times 10^{-6}} = \left(\frac{2.6}{1.95} \right) \left(\frac{0.95}{1.6} \right)^n; 0.1648 = 0.593^n$$

$$n = \frac{\log 0.1648}{\log 0.593} = 3.45$$

$$k_1 = C \left(\frac{e_1^n}{1 + e_1} \right); \quad C = \frac{(0.2 \times 10^{-6})(1 + 0.95)}{0.95^{3.45}} = 4.655 \times 10^{-7} \text{ cm/sec}$$

$$k_3 = \left(\frac{1.1^{3.45}}{2.1} \right) (4.655 \times 10^{-7}) = \mathbf{3.08 \times 10^{-7} \text{ cm/sec}}$$

$$7.15 \quad \text{Eq. (7.37): } \log k = A' \log e + B'$$

$$A' = \frac{\log k_1 - \log k_2}{\log e_1 - \log e_2} = \frac{\log(0.2 \times 10^{-6}) - \log(0.91 \times 10^{-6})}{\log(0.95) - \log(1.6)} = 2.9$$

$$B' - \log k_1 - A' \log e_1 = \log(0.2 \times 10^{-6}) - 2.9 \log(0.95) = -6.634$$

$$\log(k_3) = (2.9) \log(1.1) - 6.634 = -6.514$$

$$k_3 = 3.062 \times 10^{-7} \text{ cm/sec}$$

$$7.16 \quad k_{H(\text{eq})} = \frac{1}{H}(k_1 H_1 + k_2 H_2 + \dots)$$

$$k_{H(\text{eq})} = \frac{1}{4}[(10^{-4})(1) + (2.8 \times 10^{-2})(1) + (3.5 \times 10^{-5})(2)] = 7.042 \times 10^{-3} \text{ cm/sec}$$

$$k_{V(\text{eq})} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \dots} = \frac{4}{\frac{1}{10^{-4}} + \frac{1}{2.8 \times 10^{-2}} + \frac{2}{3.5 \times 10^{-5}}} \\ = 5.95 \times 10^{-5} \text{ cm/sec}$$

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}} = \frac{7.042 \times 10^{-3}}{5.95 \times 10^{-5}} = \mathbf{118.35}$$

$$7.17 \quad q = kiA$$

$$i = \frac{160 - 150}{125} = 0.08$$

$$q = 250 \text{ m}^3/\text{day}$$

$$A = 2 \times 500 = 1000 \text{ m}^2$$

$$k = \frac{q}{iA} = \frac{250}{(0.08)(1000)} = \mathbf{3.125 \text{ m/day}}$$

CRITICAL THINKING PROBLEM

$$7.C.1 \quad a. \quad k_{v(\text{eq})} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}} = \frac{60}{\frac{20}{5 \times 10^{-3}} + \frac{20}{4.2 \times 10^{-2}} + \frac{20}{3.9 \times 10^{-4}}} = 1.076 \times 10^{-3} \text{ cm/sec}$$

$$q = k_{v(\text{eq})} i A = (0.001076) \left(\frac{47}{60} \right) \left(\frac{\pi}{4} \right) (15^2) = 0.149 \text{ cm}^3/\text{sec} = \mathbf{536.4 \text{ cm}^3/\text{hr}}$$

b. $x = 0 \text{ mm}$

$$Z = -220 \text{ mm};$$

$$\frac{u}{\gamma_w} = 470 + 220 = \mathbf{690 \text{ mm}}$$

$$h = \frac{u}{\gamma_w} + Z = 690 - 220 = \mathbf{470 \text{ mm}}$$

$x = 200 \text{ mm}$

$$Z = -220 \text{ mm}$$

$$k_{v(\text{eq})}i = k_1i_1$$

$$(0.001076)\left(\frac{47}{60}\right) = (0.005)\left(\frac{\Delta H}{20}\right)$$

$$\Delta H = 3.371 \text{ cm} = 33.71 \text{ mm}$$

$$\text{Therefore, } h = 470 - 33.71 = \mathbf{436.29 \text{ mm}}$$

$$\frac{u}{\gamma_w} = 436.29 - (-220) = \mathbf{656.29 \text{ mm}}$$

$x = 400 \text{ mm}$

$$Z = -220 \text{ mm}$$

$$k_{v(\text{eq})}i = k_2i_2$$

$$(0.001076)\left(\frac{47}{60}\right) = (0.042)\left(\frac{\Delta H}{20}\right)$$

$$\Delta H = 0.4 \text{ cm} = 4 \text{ mm}$$

$$\text{Therefore, } h = 436.29 - 4 = \mathbf{432.29 \text{ mm}}$$

$$\frac{u}{\gamma_w} = 432.29 - (-220) = \mathbf{652.29 \text{ mm}}$$

$$x = 600 \text{ mm}$$

$$Z = -220 \text{ mm}$$

$$k_{v(\text{eq})}i = k_3i_3$$

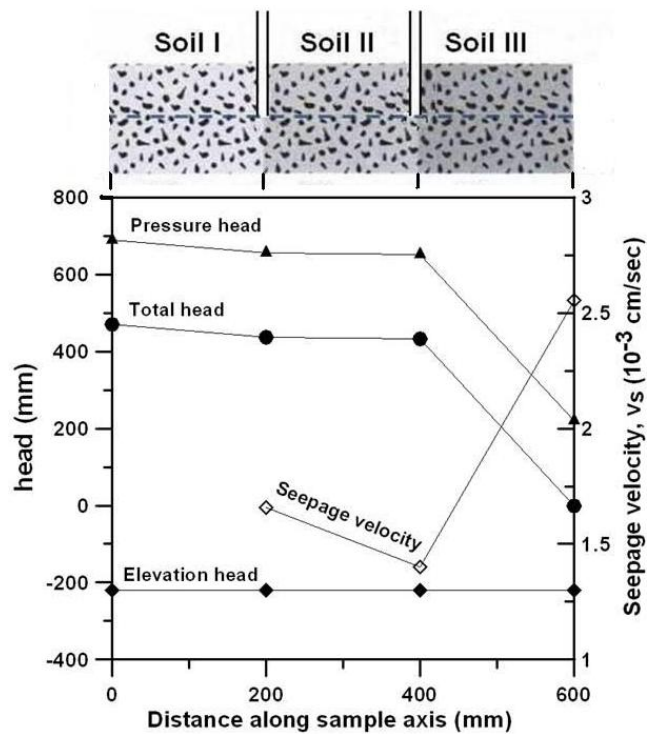
$$(0.001076)\left(\frac{47}{60}\right) = (0.00039)\left(\frac{\Delta H}{20}\right)$$

$$\Delta H = 43.224 \text{ cm} = 432.24 \text{ mm}$$

$$\text{Therefore, } h = 432.29 - 432.24 \approx 0 \text{ mm}$$

$$\frac{u}{\gamma_w} = 0 - (-220) = 220 \text{ mm}$$

- c. The variation of heads with distance is shown in the following figure.



$$d. \quad v = k_{v(\text{eq})}i = (0.001076)\left(\frac{47}{60}\right) = 0.000843 \text{ cm/sec}$$

$$v_s = \frac{v}{n}$$

$$\text{Soil I: } v_s = \frac{0.000843}{0.5} = \mathbf{0.00168 \text{ cm/sec}}$$

$$\text{Soil II: } v_s = \frac{0.000843}{0.6} = \mathbf{0.0014 \text{ cm/sec}}$$

$$\text{Soil III: } v_s = \frac{0.000843}{0.33} = \mathbf{0.00255 \text{ cm/sec}}$$

These values are plotted in the figure on the previous page.

- e. Height of water column is equal to the piezometric or pressure head at a point.
 Therefore: height of water in A = pressure head at $x = 200 \text{ mm} = \mathbf{656.29 \text{ mm}}$
 height of water in B = pressure head at $x = 400 \text{ mm} = \mathbf{652.29 \text{ mm}}$

Chapter 8

8.1 Eq. (8.14):
$$h_2 = \frac{h_1 k_1}{H_1 \left(\frac{k_1}{H_1} + \frac{k_2}{H_2} \right)}$$

$$8 \text{ cm} = \frac{(20 \text{ cm})(0.004 \text{ cm/sec})}{(10 \text{ cm}) \left(\frac{0.004 \text{ cm/sec}}{10 \text{ cm}} + \frac{k_2 \text{ cm/sec}}{15 \text{ cm}} \right)}$$

$k_2 = \mathbf{0.009 \text{ cm/sec}}$

8.2 The flow net is shown.

$k = 4 \times 10^{-4} \text{ cm/sec}$

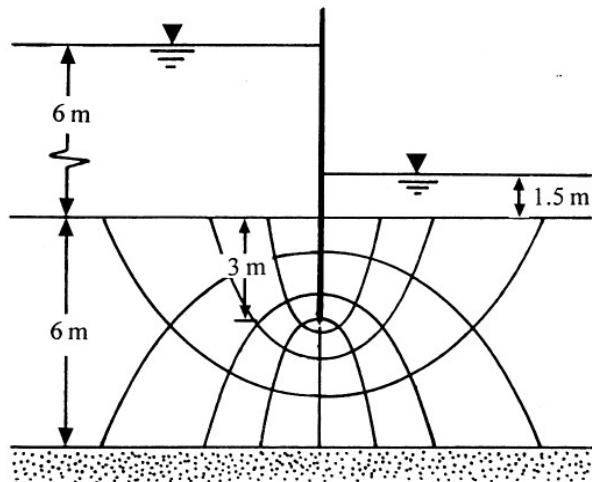
$H = H_1 - H_2$
 $= 6.0 - 1.5 = 4.5 \text{ m.}$

So

$$q = \left(\frac{4 \times 10^{-4}}{10^2} \right) \left(\frac{4.5 \times 4}{8} \right)$$

$$= 9 \times 10^{-6} \text{ m}^3/\text{m/sec}$$

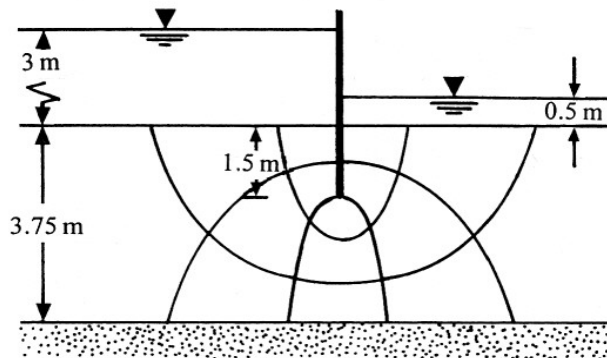
$$= \mathbf{77.76 \times 10^{-6} \text{ m}^3/\text{m/day}}$$



8.3 The flow net is shown.

$N_f = 3; N_d = 5$

$$q = kH \left(\frac{N_f}{N_d} \right)$$



$$q = \left(\frac{4 \times 10^{-4}}{10^2} \text{ m/sec} \right) (3 - 0.5) \left(\frac{3}{5} \right) = 6 \times 10^{-6} \text{ m}^3/\text{m}/\text{sec} = \mathbf{0.518 \text{ m}^3/\text{m}/\text{day}}$$

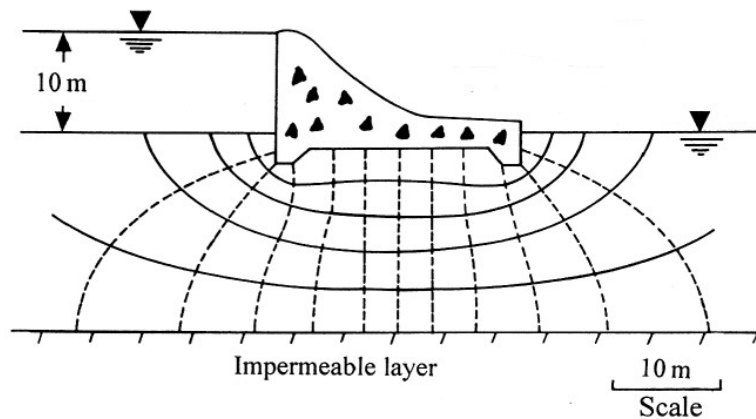
8.4 Based on the notations in Figure 8.10:

$$H = (4 - 1.5) \text{ m} = 2.5 \text{ m}; S = D = 3.6 \text{ m}; T' = D_1 = 6 \text{ m}; S/T' = 3.6/6 = 0.6$$

From the figure, $\frac{q}{kH} \approx 0.44$

$$q = (0.44)(2.5) \left(\frac{4 \times 10^{-4}}{10^2} \times 60 \times 60 \times 24 \text{ m/day} \right) = \mathbf{0.38 \text{ m}^3/\text{m}/\text{day}}$$

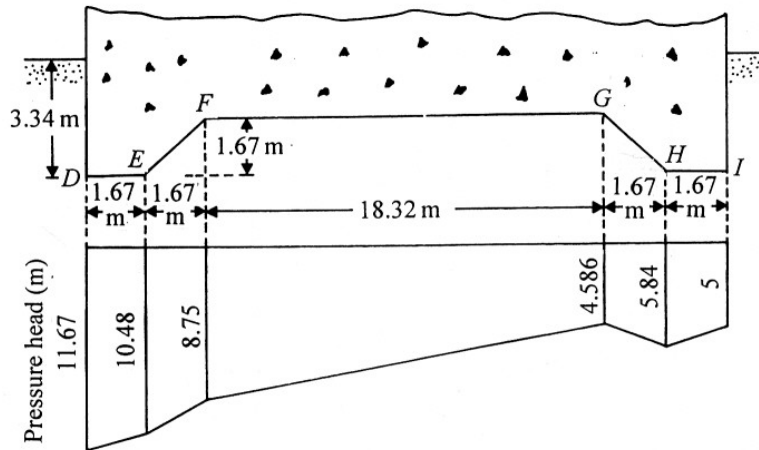
8.5 The flow net is shown.



$$q = kH \left(\frac{N_f}{N_d} \right) = \left(\frac{0.002}{10^2} \times 60 \times 60 \times 24 \text{ m/day} \right) (10) \left(\frac{5}{12} \right) = \mathbf{7.2 \text{ m}^3/\text{m}/\text{day}}$$

8.6 Refer to the flow net given in Problem 8.5 and the figure on the next page.

The flow net has 12 potential drops. Also, $H = 10 \text{ m}$. So the head loss for each drop = $(10/12) \text{ m}$. Thus,



$$\text{Pressure head at } D = (10 + 3.34) - (2)(10/12) = 11.67 \text{ m}$$

$$\text{Pressure head at } E = (10 + 3.34) - (3)(10/12) = 10.84 \text{ m}$$

$$\text{Pressure head at } F = (10 + 1.67) - (3.5)(10/12) = 8.75 \text{ m}$$

$$\text{Pressure head at } G = (10 + 1.67) - (8.5)(10/12) = 4.586 \text{ m}$$

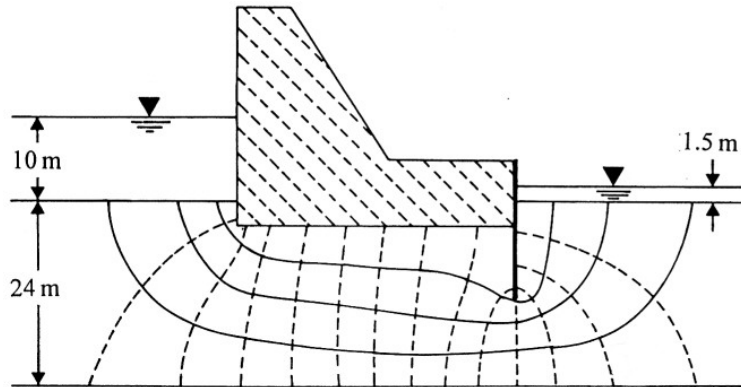
$$\text{Pressure head at } H = (10 + 3.34) - (9)(10/12) = 5.84 \text{ m}$$

$$\text{Pressure head at } I = (10 + 3.34) - (10)(10/12) = 5 \text{ m}$$

The pressure heads calculated are shown in the figure. The hydraulic uplift force per unit length of the structure can now be calculated to be

$$\begin{aligned}
 &= \gamma_w (\text{area of the pressure head diagram})(1) \\
 &= \left[\left(\frac{11.67 + 10.84}{2} \right) (1.67) + \left(\frac{10.84 + 8.754}{2} \right) (1.67) \right. \\
 &\quad \left. + \left(\frac{8.75 + 4.586}{2} \right) (18.32) + \left(\frac{4.586 + 5.84}{2} \right) (1.67) + \left(\frac{5.84 + 5}{2} \right) (1.67) \right] \\
 &= (9.81)(18.8 + 16.36 + 122.16 + 8.71 + 9.05) \\
 &= \mathbf{1717.5 \text{ kN/m}}
 \end{aligned}$$

8.7 The flow net is shown. $N_f = 3$; $N_d = 5$.



$$q = kH \left(\frac{N_f}{N_d} \right) = \left(\frac{10^{-3}}{10^2} \right) (10 - 1.5) \left(\frac{4}{14} \right) = (10^{-5}) (8.5) \left(\frac{4}{14} \right)$$

$$= 2.429 \times 10^{-5} \text{ m}^3/\text{m}/\text{sec} \approx \mathbf{2.1 \text{ m}^3/\text{m}/\text{day}}$$

8.8 For this case, $T' = 8 \text{ m}$; $S = 4 \text{ m}$; $H = H_1 - H_2 = 6 \text{ m}$; $B = 8 \text{ m}$; $b = B/2 = 4 \text{ m}$.

a. $\frac{S}{T'} = \frac{4}{8} = 0.5$; $x = b - x' = 4 - 1 = 3 \text{ m}$; $\frac{x}{b} = \frac{3}{4} = 0.75$; $\frac{b}{T'} = \frac{4}{8} = 0.5$

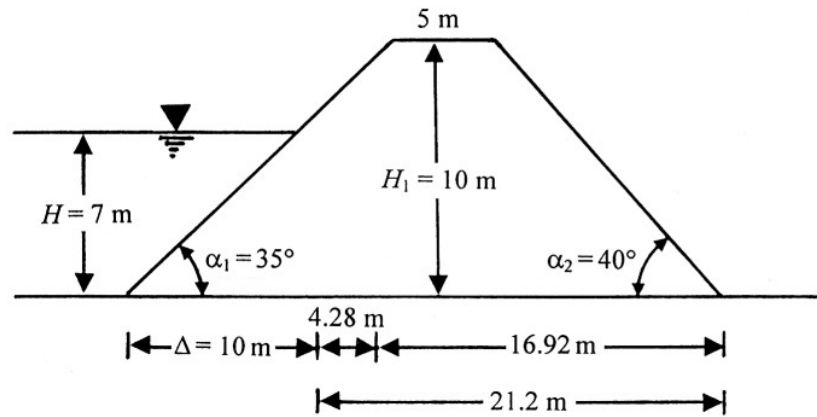
From Figure 8.11, $q/kH = 0.37$.

$$q = (0.37) \left(\frac{0.001}{10^2} \times 60 \times 60 \times 24 \right) (6) \approx \mathbf{1.92 \text{ m}^3/\text{m}/\text{day}}$$

b. $\frac{S}{T'} = 0.5$; $\frac{b}{T'} = 0.5$; $x = b - x' = 4 - 2 = 2 \text{ m}$; $\frac{x}{b} = \frac{2}{4} = 0.5$. So $q/kH = 0.4$.

$$q = (0.4) \left(\frac{0.001}{10^2} \times 60 \times 60 \times 24 \right) (6) \approx \mathbf{2.07 \text{ m}^3/\text{m}/\text{day}}$$

8.9



$$\alpha_1 = 35^\circ; \alpha_2 = 40^\circ; H = 7 \text{ m}; \Delta = 7 \cot 35 = 10 \text{ m}. 0.3\Delta = 3 \text{ m}.$$

$$\begin{aligned} d &= H_1 \cot \alpha_2 + L_1 + (H_1 - H) \cot \alpha_1 + 0.3\Delta \\ &= (10)(\cot 40) + 5 + (10 - 7) \cot 34 + 3 = 24.2 \text{ m} \end{aligned}$$

$$\begin{aligned} L &= \frac{d}{\cos \alpha_2} - \sqrt{\frac{d^2}{\cos^2 \alpha_2} - \frac{H^2}{\sin^2 \alpha_2}} = \frac{24.2}{\cos 40} - \sqrt{\left(\frac{24.2}{\cos 40}\right)^2 - \left(\frac{7}{\sin 40}\right)^2} \\ &= 1.94 \text{ m} \end{aligned}$$

$$\begin{aligned} q &= kL \tan \alpha_2 \sin \alpha_2 = \left[\left(\frac{3 \times 10^{-4}}{10^2} \right) (1.94) \right] (\tan 40)(\sin 40) \\ &= 3.139 \times 10^{-6} \text{ m}^3/\text{sec}/\text{m} \approx \mathbf{0.271 \text{ m}^3/\text{m}/\text{day}} \end{aligned}$$

8.10 From Problem 8.9, $d = 24.2 \text{ m}$; $H = 7 \text{ m}$; $\alpha_2 = 40^\circ$

$$\frac{d}{H} = \frac{24.2}{7} = 3.46; m \approx 0.25 \text{ (Figure 8.14)}$$

$$L = \frac{mH}{\sin \alpha_2} = \frac{(0.25)(7)}{\sin 40} = 2.72 \text{ m}$$

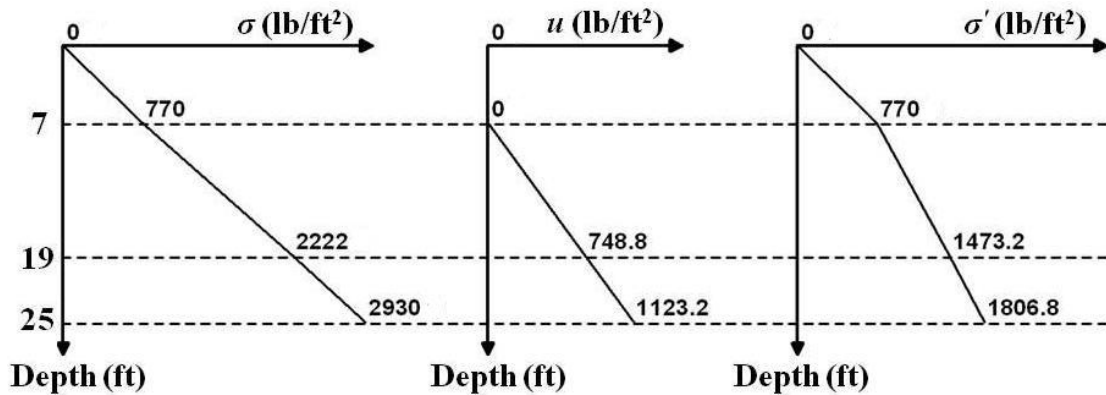
$$\begin{aligned} q &= kL \sin^2 \alpha_2 = \left(\frac{3 \times 10^{-4}}{10^2} \right) (2.72)(\sin^2 40) \\ &= 3.37 \times 10^{-6} \text{ m}^3/\text{sec}/\text{m} \approx \mathbf{0.291 \text{ m}^3/\text{m}/\text{day}} \end{aligned}$$

Chapter 9

9.1

Point	lb/ft ²		
	σ	u	σ'
A	0	0	0
B	$(7)(110) = \mathbf{770}$	0	770
C	$770 + (12)(121) = \mathbf{2222}$	$(62.4)(12) = \mathbf{748.8}$	1473.2
D	$2222 + (6)(118) = \mathbf{2930}$	$748.8 + (62.4)(6) = \mathbf{1123.2}$	1806.8

The plot is given below.



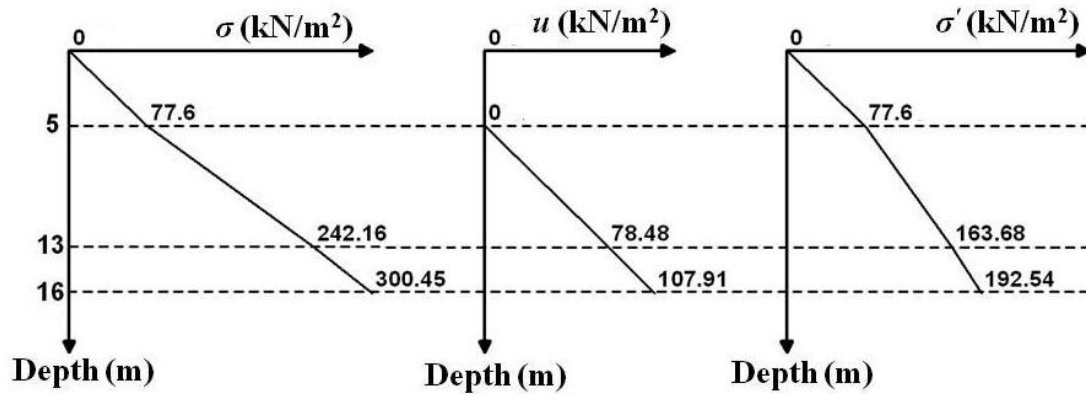
$$9.2 \quad \gamma_{d(\text{layer } 1)} = \frac{G_s \gamma_w}{1 + e} = \frac{(2.69)(9.81)}{1 + 0.7} = 15.52 \text{ kN/m}^3$$

$$\gamma_{\text{sat}(\text{layer } 2)} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{(9.81)(2.7 + 0.55)}{1 + 0.55} = 20.57 \text{ kN/m}^3$$

$$\gamma_{\text{sat}(\text{layer } 3)} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{(9.81) \left(\frac{1.2}{0.38} + 1.2 \right)}{1 + 1.2} = 19.43 \text{ kN/m}^3$$

Point	kN/m ²		
	σ	u	σ'
A	0	0	0
B	$(5)(15.52) = \mathbf{77.6}$	0	77.6
C	$77.6 + (8)(20.57) = \mathbf{242.16}$	$(9.81)(8) = \mathbf{78.48}$	163.68
D	$242.16 + (3)(19.43) = \mathbf{300.45}$	$78.48 + (9.81)(3) = \mathbf{107.9}$	192.54

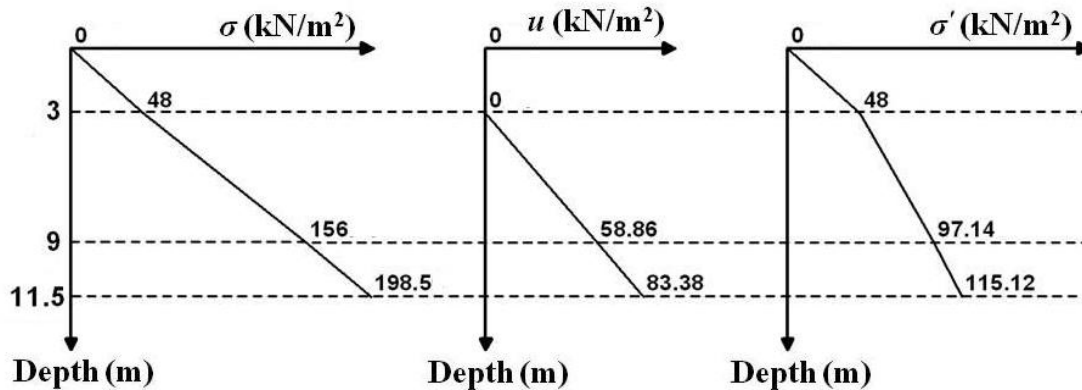
The plot is shown below.



9.3

Point	kN/m ²		
	σ	u	σ'
A	0	0	0
B	$(3)(16) = 48$	0	48
C	$48 + (6)(18) = 156$	$(9.81)(6) = 58.86$	97.14
D	$156 + (2.5)(17) = 198.5$	$58.86 + (9.81)(2.5) = 83.38$	115.12

The plot is shown below.



9.4 a. Water table drops 2 m within layer 2. Assuming dry condition for 2 m:

$$\gamma_{d(\text{layer 2})} = \frac{G_s \gamma_w}{1 + e} = \frac{(2.7)(9.81)}{1 + 0.55} = 17.08 \text{ kN/m}^3$$

$$\begin{aligned}\sigma'(\text{at point C}) &= (5)(15.52) + (2)(17.08) + (6)(20.57) - (6)(9.81) \\ &= 176.32 \text{ kN/m}^2\end{aligned}$$

$$\text{Increase in } \sigma': 176.32 - 163.68 = \mathbf{12.64 \text{ kN/m}^2}$$

- b. Water table rises to the surface. Layer 1 is saturated.

$$\gamma_{\text{sat}(\text{layer1})} = \frac{\gamma_w(G_s + e)}{1 + e} = \frac{(9.81)(2.69 + 0.7)}{1 + 0.7} = 19.56 \text{ kN/m}^3$$

$$\sigma'(\text{at point C}) = (5)(19.56) + (8)(20.57) - (13)(9.81) = 134.83 \text{ kN/m}^2$$

$$\text{Decrease in } \sigma': 163.68 - 134.83 = \mathbf{28.85 \text{ kN/m}^2}$$

- c. Water level rises 3 m above ground. All layers are saturated

$$\begin{aligned}\sigma'(\text{at point C}) &= (3)(9.81) + (5)(19.56) + (8)(20.57) - (16)(9.81) \\ &= 134.83 \text{ kN/m}^2\end{aligned}$$

$$\text{Decrease in } \sigma': 163.68 - 134.83 = \mathbf{28.85 \text{ kN/m}^2} \text{ (same as Part b)}$$

$$9.5 \quad \text{a. } \gamma_{d(\text{sand})} = \frac{G_s \gamma_w}{1 + e} = \frac{(2.66)(9.81)}{1 + 0.61} = 16.2 \text{ kN/m}^3$$

$$\gamma_{\text{sat}(\text{sand})} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.67 + 0.48)(9.81)}{1 + 0.48} = 20.88 \text{ kN/m}^3$$

Point	kN/m ²		
	σ	u	σ'
A	0	0	0
B	$(16.2)(4) = \mathbf{64.8}$	0	64.8
C	$64.8 + (20.88)(5) = \mathbf{169.2}$	$(9.81)(5) = \mathbf{49.05}$	120.15

- b. Let the height of rise be h . Portions of the top sand layer will be saturated.

$$\gamma_{\text{sat(top sand)}} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.66 + 0.61)(9.81)}{1 + 0.61} = 19.92 \text{ kN/m}^3$$

So, at any time, the stresses at C are:

$$\sigma = (4 - h)(16.2) + (h)(19.92) + (5)(20.88) = 169.2 + 3.72h$$

$$u = (5 + h)(9.81) = 49.05 + 9.81h$$

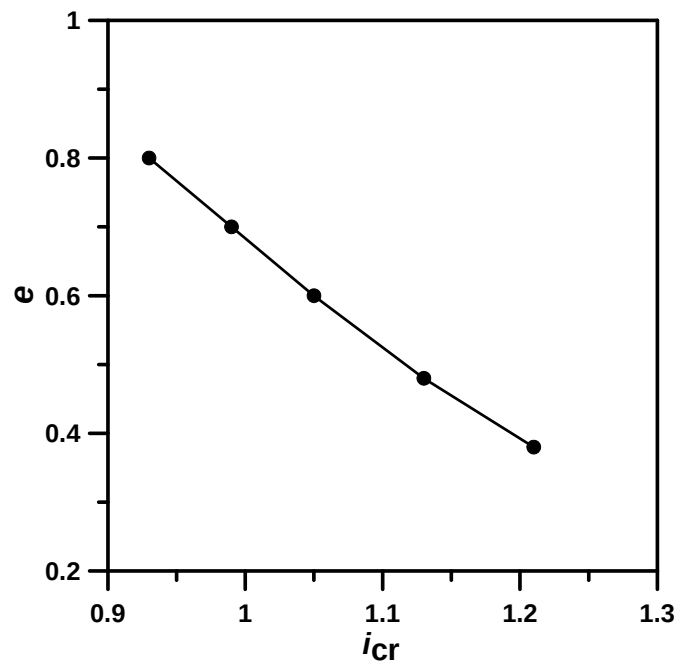
$$\sigma' = (169.2 + 3.72h) - (49.05 + 9.81h) = 120.15 - 6.09h$$

$$\text{New } \sigma' \text{ at } C: 111 = 120.15 - 6.09h; \quad h = \mathbf{1.5 \text{ m}}$$

$$9.6 \quad i_{\text{cr}} = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e} = \frac{2.68 - 1}{1 + e} = \frac{1.68}{1 + e}$$

e	i_{cr}
0.38	1.21
0.48	1.13
0.6	1.05
0.7	0.99
0.8	0.93

The plot is shown below.



$$9.7 \quad \gamma_{\text{sat(clay)}} = \frac{(1+w)G_s \gamma_w}{1+wG_s} = \frac{(1+0.29)(2.68)(9.81)}{1+(0.29)(2.68)} = 19.08 \text{ kN/m}^3$$

Let the depth of the excavation be H .

$$\text{So, } (10 - H)(19.08) - (6)(9.81) = 0 = \sigma'$$

$$H \approx \mathbf{6.91 \text{ m}}$$

9.8 Consider the stability of point A in terms of heaving.

$$\gamma_{\text{sat(clay)}} = \frac{(1925)(9.81)}{1000} = 18.88 \text{ kN/m}^3$$

$$\sigma_A = (10 - 5.75)(18.88) = 80.24 \text{ kN/m}^2$$

$$u_A = (6)(9.81) = 58.86 \text{ kN/m}^2$$

For heaving to occur, $\sigma' = 0$; or $\sigma = u$

$$\text{Therefore, factor of safety} = \frac{\sigma_A}{u_A} = \frac{80.24}{58.86} = \mathbf{1.36}$$

9.9 Let the maximum permissible depth of cut be H .

$$\sigma_A = (10 - H)(18.88)$$

$$u_A = (6)(9.81) = 58.86 \text{ kN/m}^2$$

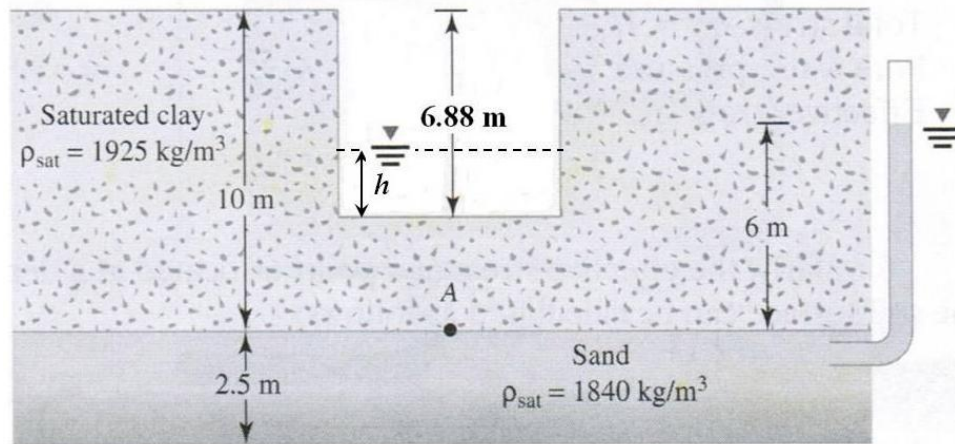
For heaving to occur, $\sigma' = 0$; or $\sigma_A - u_A = 0$

$$(10 - H)(18.88) - 58.86 = 0; \quad H = \mathbf{6.88 \text{ m}}$$

9.10 Let the height of water inside the cut be h (see figure on following page)

$$\sigma_A = (10 - 6.88)(18.88) + (h)(9.81) = 58.9 + 9.81h$$

$$u_A = (6)(9.81) = 58.86 \text{ kN/m}^2$$



Factor of safety: $\frac{\sigma_A}{u_A} = \frac{58.9 + 9.81h}{58.86} = 1.5$

$h = 3.0 \text{ m}$

9.11 a. $i = \frac{h}{H_2} = \frac{1.5}{2.5} = 0.6$

$q = kiA = (0.21)(0.6)(0.62 \times 100^2 \text{ cm}^2) = 781.2 \text{ cm}^3/\text{sec}$

b. $i_{cr} = \frac{\gamma'}{\gamma} = \frac{G_s - 1}{1 + e} = \frac{2.66 - 1}{1 + 0.49} = 1.11$

Since $i < i_{cr}$, **no boiling**.

c. $i = i_{cr} = \frac{h}{H_2}; \quad 1.11 = \frac{h}{2.5}$

$h = 2.77 \text{ m}$

9.12 a. $i = \frac{h}{H_2} = \frac{1.5}{4.5} = 0.33$

$q = kiA = (0.31)(0.33)(6.2) = 0.634 \text{ ft}^3/\text{min}$

b. Refer to Figure 9.4 (a). Since C is located at the middle of the soil layer,

$$z = H_2 / 2 = 4.5 / 2 = 2.25 \text{ ft}$$

$$\text{Eq. (9.7): } \sigma'_c = z\gamma' - iz\gamma_w = (2.25)(119-62.4) - (0.33)(2.25)(62.4) = \mathbf{81 \text{ lb/ft}^2}$$

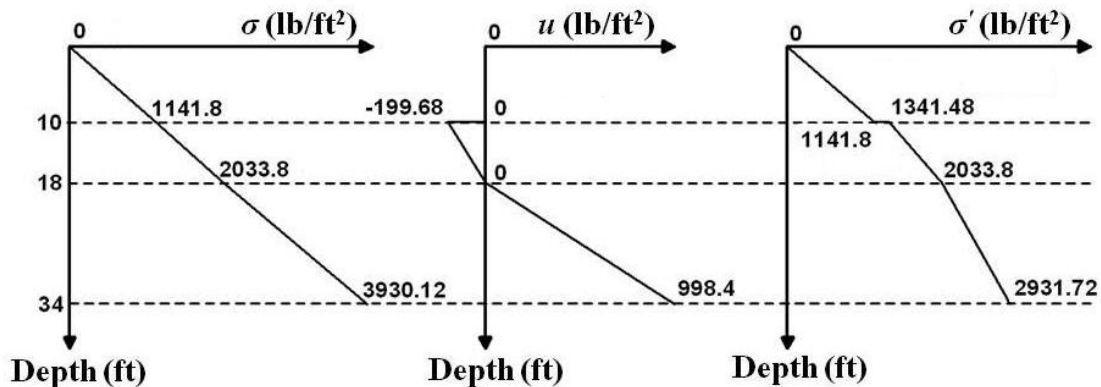
$$9.13 \quad \gamma_{d(\text{sand})} = \frac{G_s \gamma_w}{1 + e} = \frac{(2.69)(62.4)}{1 + 0.47} = 114.18 \text{ lb/ft}^3$$

$$\gamma_{\text{sat}(\text{clay; capillary zone})} = \frac{\gamma_w (G_s + Se)}{1 + e} = \frac{(62.4)[2.73 + (0.4)(0.68)]}{1 + 0.68} = 111.5 \text{ lb/ft}^3$$

$$\gamma_{\text{sat}(\text{clay})} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{(62.4)(2.7 + 0.89)}{1 + 0.89} = 118.52 \text{ lb/ft}^3$$

	lb/ft ²		
Depth (ft)	σ	u	σ'
0	0	0	0
10	(114.18)(10) = 1141.8	0	1141.8
		$(-0.4)(62.4)(8) = \mathbf{-199.68}$	1341.48
10+8=18	1141.8 + (111.5)(8) = 2033.8	0	2033.8
10+8+16=34	2033.8 + (118.52)(16) = 3930.12	$(16)(62.4) = \mathbf{998.4}$	2931.72

The plot is given below.



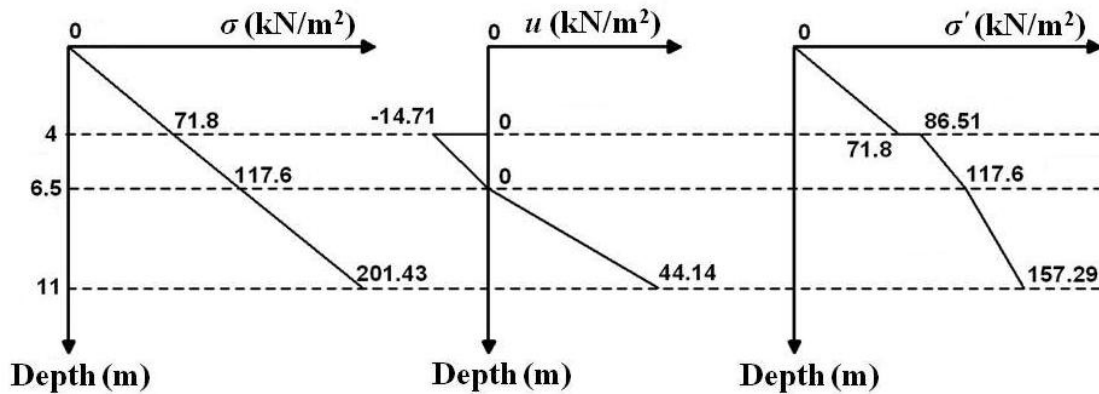
$$9.14 \quad \gamma_{d(\text{sand})} = \frac{(2.69)(9.81)}{1 + 0.47} = 17.95 \text{ kN/m}^3$$

$$\gamma_{\text{sat}(\text{clay: capillary zone})} = \frac{(9.81)[2.73 + (0.6)(0.68)]}{1 + 0.68} = 18.32 \text{ kN/m}^3$$

$$\gamma_{\text{sat}(\text{clay})} = \frac{(9.81)(2.7 + 0.89)}{1 + 0.89} = 18.63 \text{ kN/m}^3$$

Depth (m)	kN/m ²		
	σ	u	σ'
0	0	0	0
4	(17.95)(4) = 71.8	0	71.8
		(-0.6)(9.81)(2.5) = -14.71	86.51
4+2.5=6.5	71.8 + (18.32)(2.5) = 117.6	0	117.6
4+2.5+4.5=11	117.6 + (18.63)(4.5) = 201.43	(4.5)(9.81) = 44.14	157.29

The plot is given.



$$9.15 \quad \text{From Eq. (9.22), } FS = \frac{D\gamma'}{C_o\gamma_w(H_1 - H_2)}$$

$$D = 4.5 \text{ m; } \gamma' = 17 - 9.81 = 7.19 \text{ kN/m}^3; H_1 - H_2 = 7 - 3 = 4 \text{ m;}$$

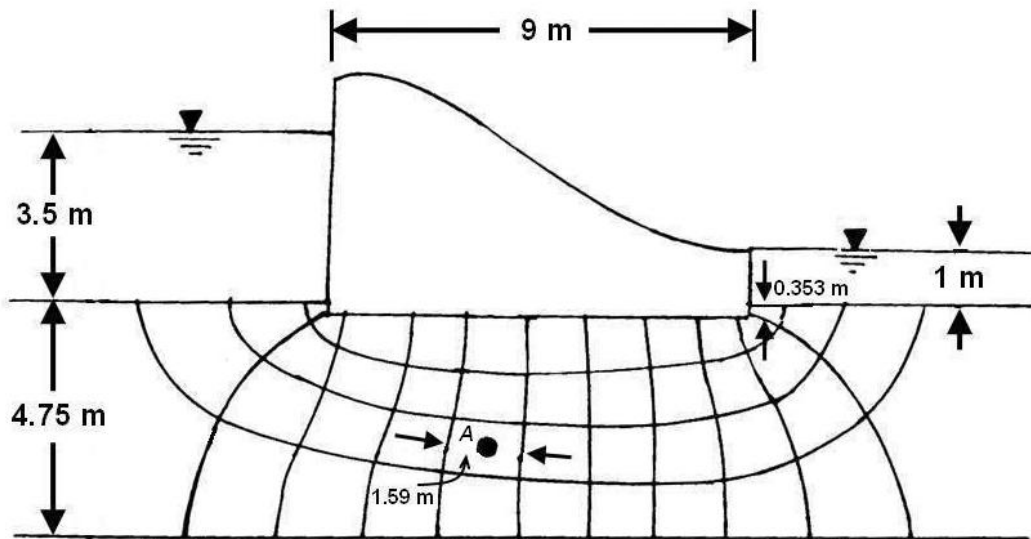
$$D/T = 4.5/12 = 0.375. \text{ From Table 9.1, } C_o = 0.354 \text{ (by linear interpolation).}$$

$$FS = \frac{(4.5)(7.19)}{(0.354)(9.81)(4)} = \mathbf{2.33}$$

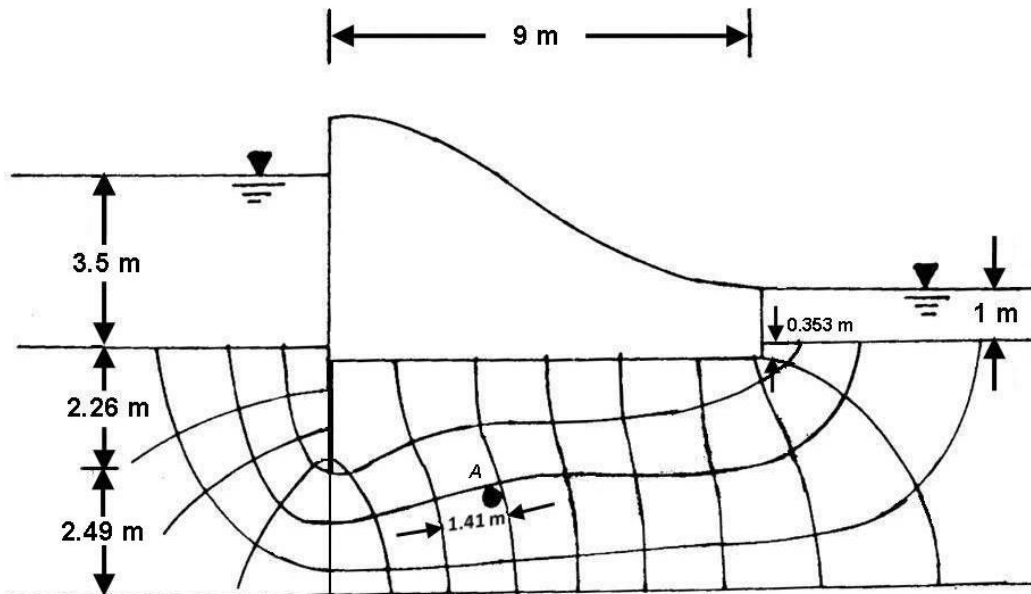
CRITICAL THINKING PROBLEM

9.C.1 a. The flow nets for both cases are given below:

Case 1



Case 2



b. Determination of $\frac{q}{k}$:

$$\text{From Eq. (8.21): } q = kH \frac{N_f}{N_d}$$

$$\text{Case 1: } N_f = 4, N_d = 11, H = 3.5 - 1 = 2.5 \text{ m}$$

$$\frac{q}{k} = (2.5) \left(\frac{4}{11} \right) = \mathbf{0.909 \text{ m}}$$

$$\text{Case 2: } N_f = 3.5, N_d = 13, H = 2.5 \text{ m}$$

$$\frac{q}{k} = (2.5) \left(\frac{3.5}{13} \right) = \mathbf{0.673 \text{ m}}$$

c. $FS = \frac{i_{cr}}{i_{exit}}$

$$i_{cr} = \frac{G_s - 1}{1 + e} = \frac{2.66 - 1}{1.55} = 1.071$$

Case 1: Refer to the flow net and Eq. (9.24a):

$$i_{exit} = \frac{H}{N_d l} = \frac{2.5}{(11)(0.353)} = 0.643$$

$$FS = \frac{1.071}{0.643} \approx \mathbf{1.67}$$

Case 2: Refer to the flow net and Eq. (9.24a):

$$i_{exit} = \frac{H}{N_d l} = \frac{2.5}{(13)(0.353)} = 0.545$$

$$FS = \frac{1.071}{0.545} \approx \mathbf{1.97}$$

- d. From Eq. (9.18), seepage force per unit volume is $\gamma_w i$

Case 1: Refer to the flow net. At A,

$$i = \frac{\Delta H}{l} = \frac{(2.5/11)}{1.59} = 0.143$$

$$\text{Seepage force} = \gamma_w i = (9.81)(0.143) = \mathbf{1.4 \text{ kN/m}^3}$$

Case 2: Refer to the flow net. At A,

$$i = \frac{\Delta H}{l} = \frac{(2.5/13)}{1.41} = 0.136$$

$$\text{Seepage force} = \gamma_w i = (9.81)(0.136) = \mathbf{1.33 \text{ kN/m}^3}$$

Installation of the sheet pile cut-off wall reduced the exit gradient and increased the factor of safety against heaving. Accordingly, at any point A, the seepage force also decreased due to a drop in the hydraulic gradient.

Chapter 10

$$10.1 \quad a. \quad \left. \begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix} \right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau_{xy}^2}$$

$$\sigma_x = 162 \text{ kN/m}^2; \sigma_y = 128 \text{ kN/m}^2; \tau_{xy} = +32 \text{ kN/m}^2$$

$$\left. \begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix} \right\} = \frac{128 + 162}{2} \pm \sqrt{\left(\frac{128 - 162}{2} \right)^2 + (32)^2}$$

$$\sigma_1 = \mathbf{181.23 \text{ kN/m}^2}; \sigma_3 = \mathbf{108.76 \text{ kN/m}^2}$$

$$b. \quad \sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta + \tau_{xy} \sin 2\theta; \quad \theta = 35^\circ$$

$$\sigma_n = \frac{128 + 162}{2} + \frac{128 - 162}{2} \cos[(2)(35)] + 32 \sin[(2)(35)] = \mathbf{169.25 \text{ kN/m}^2}$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{128 - 162}{2} \sin[(2)(35)] - 32 \cos[(2)(35)] = \mathbf{-26.92 \text{ kN/m}^2}$$

$$10.2 \quad a. \quad \sigma_x = 72 \text{ kN/m}^2; \sigma_y = 121 \text{ kN/m}^2; \tau_{xy} = 39 \text{ kN/m}^2; \theta = 147^\circ$$

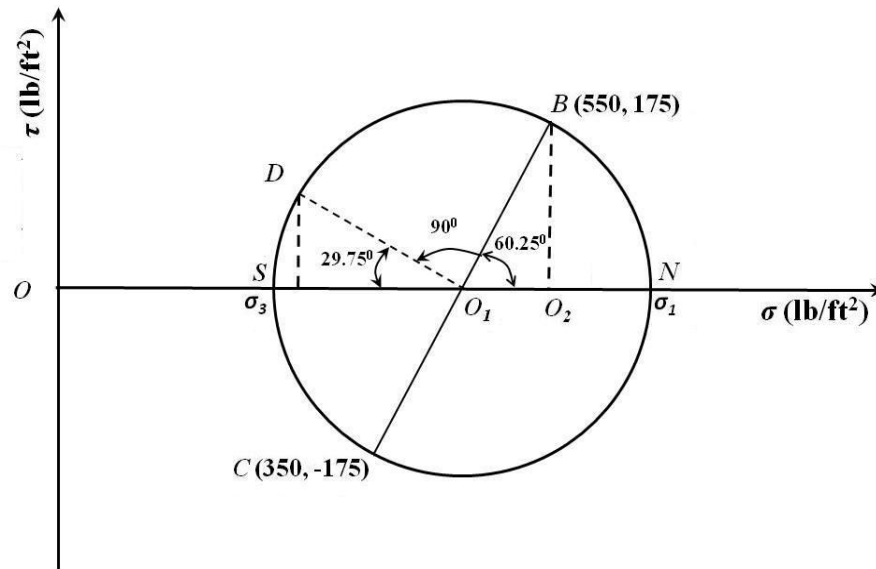
$$\left. \begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix} \right\} = \frac{121 + 72}{2} \pm \sqrt{\left(\frac{121 - 72}{2} \right)^2 + (39)^2}$$

$$\sigma_1 = \mathbf{142.55 \text{ kN/m}^2}; \sigma_3 = \mathbf{50.45 \text{ kN/m}^2}$$

$$b. \quad \sigma_n = \frac{121 + 72}{2} + \frac{121 - 72}{2} \cos[(2)(147)] + 39 \sin[(2)(147)] = \mathbf{131.33 \text{ kN/m}^2}$$

$$\tau_n = \frac{121 - 72}{2} \sin[(2)(147)] - (39) \cos[(2)(147)] = -38.24 \text{ kN/m}^2$$

10.3 a. The Mohr's circle is shown below.



$$\overline{OO_1} = \frac{550 + 350}{2} = 450 \text{ lb/ft}^2; \quad O_1O_2 = 550 - 450 = 100 \text{ lb/ft}^2$$

$$\overline{O_1B} = \sqrt{\left(\frac{350 - 550}{2}\right)^2 + (-175)^2} = 201.5 \text{ lb/ft}^2$$

$$\sigma_3 = \overline{OS} = 450 - 201.5 = 248.4 \text{ lb/ft}^2 (+)$$

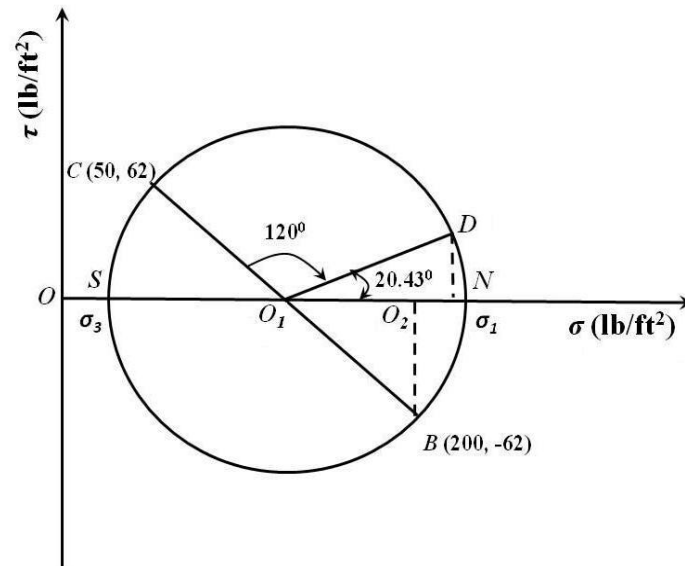
$$\sigma_1 = \overline{ON} = 450 + 201.5 = 651.5 \text{ lb/ft}^2 (+)$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{175}{100}\right) = 60.25^\circ$$

b. $\sigma_n = \overline{OO_1} - \overline{O_1D} \cos(29.75) = 450 - 201.5 \cos(29.75) = 275.1 \text{ lb/ft}^2 (+)$

$$\tau_n = \overline{O_1D} \sin(29.75) = 99.98 \text{ lb/ft}^2 (+)$$

10.4 a. The Mohr's circle is shown below.



$$\overline{OO_1} = \frac{200 + 50}{2} = 125 \text{ lb/ft}^2; \quad O_1O_2 = 200 - 125 = 75 \text{ lb/ft}^2$$

$$\overline{O_1B} = \sqrt{(75)^2 + (62)^2} = 97.3 \text{ lb/ft}^2$$

$$\sigma_1 = \overline{ON} = 125 + 97.3 = \mathbf{222.3 \text{ lb/ft}^2}$$

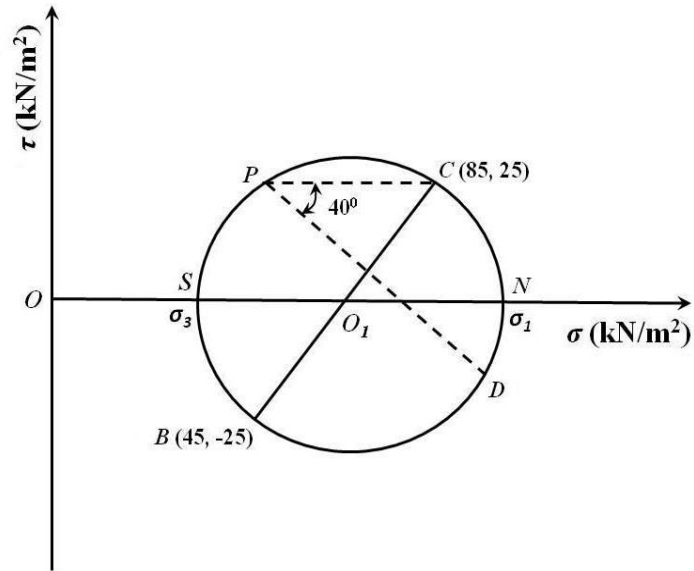
$$\sigma_3 = \overline{OS} = 125 - 97.3 = \mathbf{27.7 \text{ lb/ft}^2}$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{62}{75}\right) = 39.57^\circ$$

b. $\sigma_n = \overline{OO_1} + \overline{O_1D} \cos(20.43) = 125 + 97.3 \cos(20.43) = \mathbf{216 \text{ lb/ft}^2}$

$$\tau_n = 97.13 \sin(20.43) = \mathbf{33.9 \text{ lb/ft}^2}$$

10.5 a. The Mohr's circle is shown below.

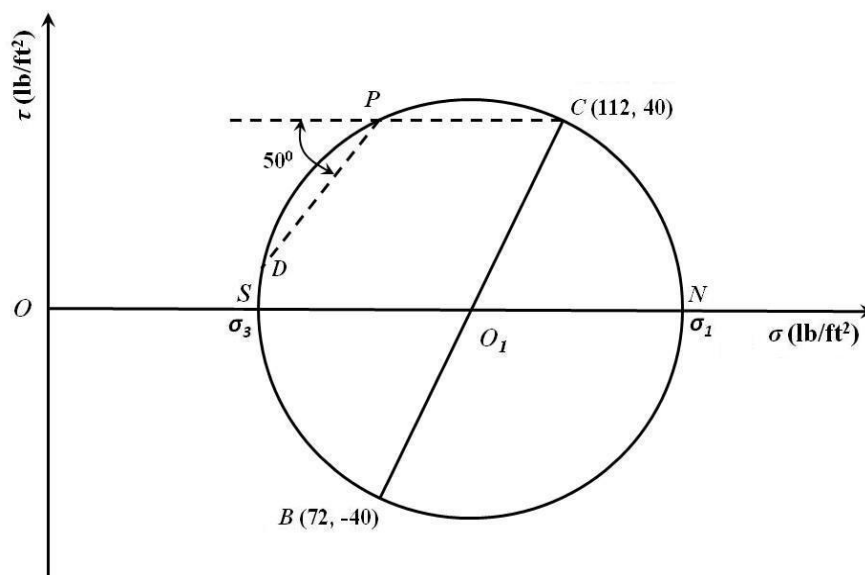


$$\sigma_1 = \overline{ON} = 99 \text{ kN/m}^2; \quad \sigma_3 = \overline{OS} = 33 \text{ kN/m}^2$$

b. σ_n and τ_n are coordinates of D . So

$$\sigma_n \approx 96 \text{ kN/m}^2; \quad \tau_n \approx 15.8 \text{ kN/m}^2 (-)$$

10.6 a. The Mohr's circle is shown below.



$$\sigma_1 = \overline{ON} = 136.5 \text{ lb/ft}^2; \quad \sigma_3 = \overline{OS} = 47.3 \text{ lb/ft}^2$$

b. σ_n and τ_n are coordinates of D . So

$$\sigma_n \approx 48 \text{ lb/ft}^2; \quad \tau_n \approx 10 \text{ lb/ft}^2$$

10.7

Load @	P (kN)	r (m)	z (m)	$\frac{r}{z}$	I_1 (Table 10.1)	$\Delta\sigma_z = \frac{P}{z^2} I_1$ (kN/m ²)
B	100	6	6	1.0	0.0844	0.234
C	200	$(6^2 + 6^2)^{0.5} = 8.48$	6	1.41	0.0311	0.173
D	400	$(6^2 + 3^2) = 6.708$	6	1.118	0.0648	0.72
						$\Delta\sigma_z = \Sigma 1.127 \text{ kN/m}^2$

10.8 Eq. (10.15):

$$\begin{aligned} \Delta\sigma_z &= \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2} = \frac{(2)(90)(3)^3}{\pi[(6.5)^2 + (3)^2]^2} + \frac{(2)(325)(3)^3}{\pi[2.5^2 + 3^2]^2} \\ &= 24.6 \text{ kN/m}^2 \end{aligned}$$

10.9 Eq. (10.15): In this case, $x_2 = 0$

$$\begin{aligned} \Delta\sigma_z &= \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2} \\ &= \frac{(2)(90)(3)^3}{\pi[(4 + 0)^2 + (3)^2]^2} + \frac{(2)(325)(3)^3}{\pi[0^2 + 3^2]^2} = 71.44 \text{ kN/m}^2 \end{aligned}$$

10.10
$$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2}$$

$$48 = \frac{(2)(q_1)(6)^3}{\pi[19^2 + 6^2]^2} + \frac{2(930)(6)^3}{\pi[5^2 + 6^2]^2} = 34.36 + 0.00087q_1$$

$$q_1 = 15,678 \text{ lb/ft}$$

$$10.11 \quad \Delta\sigma_z \text{ at } A \text{ due to } q_1 = \frac{2q_1 z^3}{\pi[x^2 + z^2]^2}, \text{ or } (\Delta\sigma_z)_1 = \frac{(2)(292)(3)^3}{\pi[(3)^2 + (3)^2]^2} = 15.49 \text{ kN/m}^2$$

Vertical component of $q_2 = q_2 \sin 45^\circ$

$$(\Delta\sigma_z)_2 = \frac{2q_2(\sin 45)(3)^3}{\pi[(7.5)^2 + (3)^2]^2}; (\Delta\sigma_z)_2 = 0.00285q_2$$

Horizontal component of $q_2 = q_2 \cos 45^\circ$

$$\text{From Eq. (10.17): } (\Delta\sigma_z)_3 = \frac{2q_2 xz^2}{\pi(x^2 + z^2)^2} = \frac{2q_2(\cos 45)(7.5)(3)^2}{\pi[7.5^2 + 3^2]^2} = 0.007136q_2$$

Total vertical stress,

$$\Delta\sigma_z = 42 \text{ kN/m}^2 = (\Delta\sigma_z)_1 + (\Delta\sigma_z)_2 + (\Delta\sigma_z)_3$$

$$42 = 15.49 + 0.00285q_2 + 0.007136q_2$$

$$q_2 = \mathbf{2656.3 \text{ kN/m}}$$

$$10.12 \quad B = 36 \text{ ft}; q = 900 \text{ lb/ft}^2; x = 21 \text{ ft}; z = 15 \text{ ft}$$

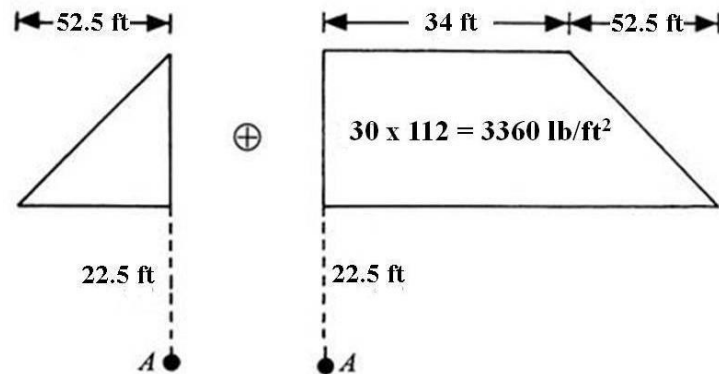
$$\frac{2x}{B} = \frac{(2)(27)}{36} = 1.5; \frac{2z}{B} = \frac{(2)(15)}{36} = 0.833. \text{ From Table 10.4, } \frac{\Delta\sigma_z}{q} = 0.18$$

$$\Delta\sigma_z = (0.18)(900) = \mathbf{162 \text{ lb/ft}^2}$$

$$10.13 \quad \frac{2x}{B} = \frac{(2)(0)}{6} = 0; \frac{2z}{B} = \frac{(2)(5)}{6} = 1.67. \text{ From Table 10.4, } \frac{\Delta\sigma_z}{q} = 0.61$$

$$\Delta\sigma_z = (120)(0.61) = \mathbf{73.2 \text{ kN/m}^2}$$

10.14 Refer to the figure.



For the left side (with the notations given in Figure 10.19):

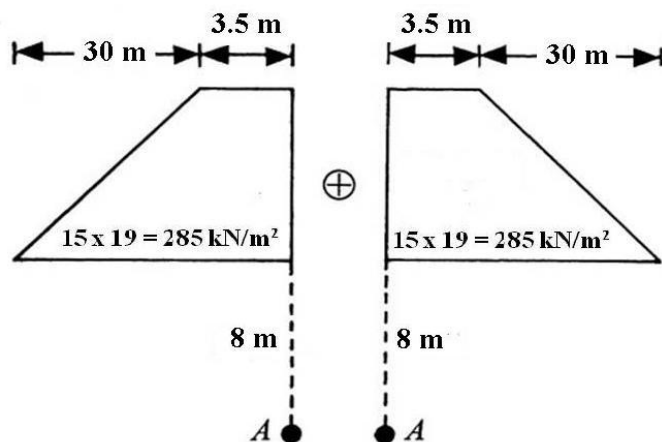
$$\frac{B_1}{z} = \frac{0}{22.5} = 0; \quad \frac{B_2}{z} = \frac{52.5}{22.5} = 2.33. \quad \text{From Figure 10.20, } I_{2(L)} = 0.375$$

For the right side:

$$\frac{B_1}{z} = \frac{34}{22.5} = 1.51; \quad \frac{B_2}{z} = \frac{52.5}{22.5} = 2.33. \quad \text{From Figure 10.20, } I_{2(R)} = 0.48$$

$$\Delta\sigma_z = q[I_{2(L)} + I_{2(R)}] = (3360)(0.375 + 0.48) = \mathbf{2872.8 \text{ lb/ft}^2}$$

10.15 At A:



For the
left side:

$$\frac{B_1}{z} = \frac{3.5}{8} = 0.437$$

$$\frac{B_2}{z} = \frac{30}{8} = 3.75$$

$$I_2 = 0.46$$

For the
right side:

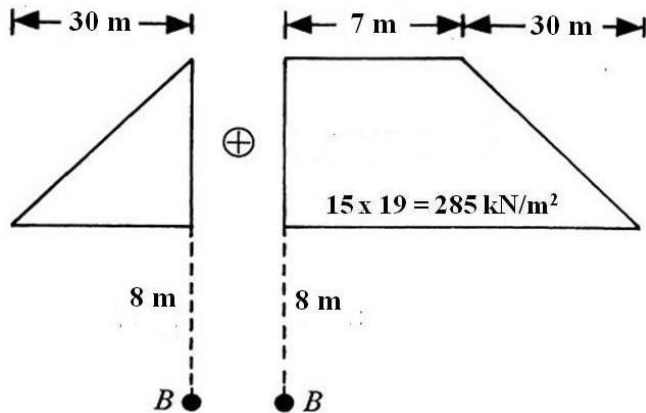
$$\frac{B_1}{z} = 0.437$$

$$\frac{B_2}{z} = 3.75$$

$$I_2 = 0.46$$

$$\Delta\sigma_z = (15)(19)(0.46 + 0.46) = \mathbf{262.2 \text{ kN/m}^2}$$

At B:



For the
left side:

$$\frac{B_1}{z} = \frac{0}{8} = 0$$

$$\frac{B_2}{z} = \frac{30}{8} = 3.75$$

$$I_2 = 0.41$$

For the
right side:

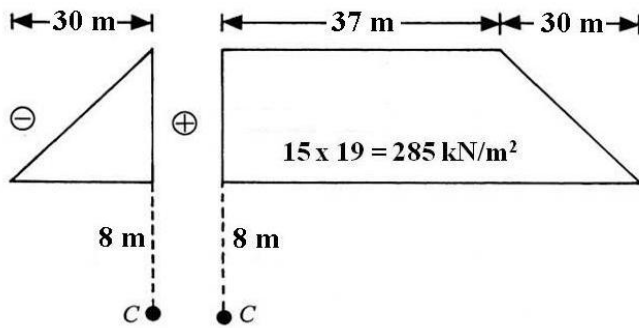
$$\frac{B_1}{z} = \frac{7}{8} = 0.875$$

$$\frac{B_2}{z} = \frac{30}{8} = 3.75$$

$$I_2 = 0.48$$

$$\Delta\sigma_z = (15)(19)(0.41 + 0.48) = 253.65 \text{ kN/m}^2$$

At C:



For the
left side:

$$\frac{B_1}{z} = 0$$

$$\frac{B_2}{z} = \frac{30}{8} = 3.75$$

$$I_2 = 0.41$$

For the
right side:

$$\frac{B_1}{z} = \frac{37}{8} = 4.625$$

$$\frac{B_2}{z} = \frac{30}{8} = 3.75$$

$$I_2 = 0.5$$

$$\Delta\sigma_z = (15)(19)(0.5 - 0.41) = 25.65 \text{ kN/m}^2$$

10.16 Eq. (10.26) and Table 10.6: $q = 2200 \text{ lb/ft}^2$

R (ft)	z (ft)	$\frac{z}{R}$	$\frac{\Delta\sigma_z}{q}$	$\Delta\sigma_z$ (lb/ft ²)
12	0	0	1	2200
12	4	0.333	0.9634	2119.5
12	8	0.666	0.8251	1815.2
12	16	1.333	0.4983	1096.2
12	32	2.667	0.1809	397.9

10.17 Eq. (10.27) and Tables 10.7 and 10.8: $q = 380 \text{ kN/m}^2$

$z \text{ (m)}$	$r \text{ (m)}$	$R \text{ (m)}$	$\frac{z}{R}$	$\frac{r}{R}$	A'	B'	$\Delta\sigma_z \text{ (kN/m}^2\text{)}$
3	0	5	0.6	0	0.48550	0.37831	328.24
3	1	5	0.6	0.2	0.47691	0.37531	323.84
3	3	5	0.6	0.6	0.40427	0.32822	278.34
3	5	5	0.6	1	0.25588	0.14440	152.1
3	7	5	0.6	1.4	0.12657	0.00085	48.42

10.18 Refer to the Newmark's chart.

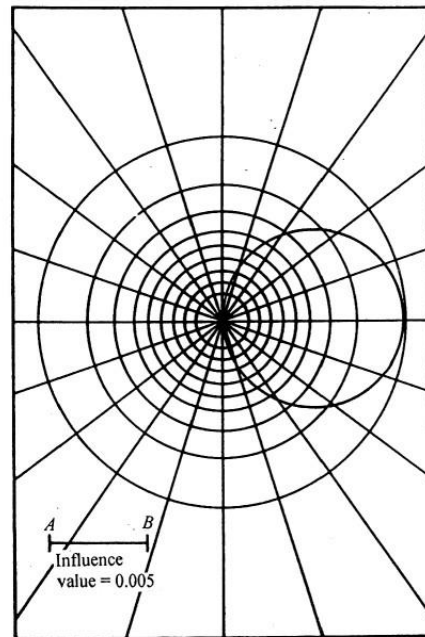
The plan is drawn to scale.

$$\overline{AB} = 6 \text{ m. } M \approx 65.$$

$$\Delta\sigma_z = (IV) q M$$

$$= (0.005)(450)(65)$$

$$= \mathbf{146.25 \text{ kN/m}^2}$$



10.19 Point A:

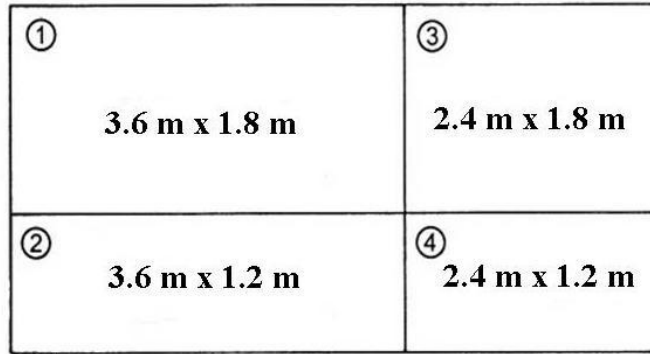
$$\text{Eqs. (10.32) and (10.33): } n = \frac{L}{z} = \frac{6}{3} = 2; \quad m = \frac{B}{z} = \frac{3}{3} = 1$$

$$\text{Eq. (10.30): } \Delta\sigma_z = q I_3; \quad \text{Table 10.9: } I_3 = 0.1999$$

$$\Delta\sigma_z = (225)(0.1999) = 44.97 \text{ kN/m}^2 \approx \mathbf{45 \text{ kN/m}^2}$$

Point B:

Refer to the figure on the next page.



For rectangle 1: $m = \frac{3.6}{3} = 1.2$; $n = \frac{1.8}{3} = 0.6$; $I_3 = 0.1431$

For rectangle 2: $m = \frac{3.6}{3} = 1.2$; $n = \frac{1.2}{3} = 0.4$; $I_3 = 0.1063$

For rectangle 3: $m = \frac{2.4}{3} = 0.8$; $n = \frac{1.8}{3} = 0.6$; $I_3 = 0.1247$

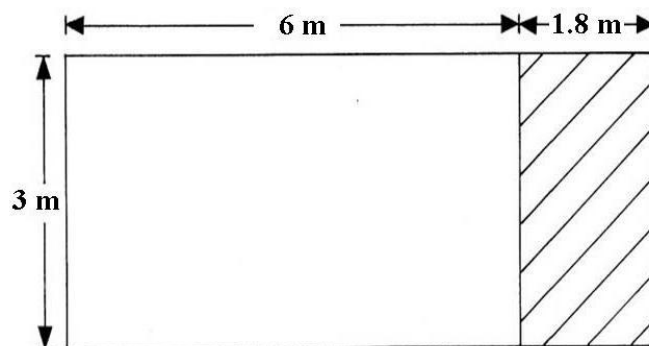
For rectangle 4: $m = \frac{2.4}{3} = 0.8$; $n = \frac{1.2}{3} = 0.4$; $I_3 = 0.0931$

$$\Delta\sigma_z = q[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}] = (225)(0.1431 + 0.1063 + 0.1247 + 0.0931)$$

$$= \mathbf{105.12 \text{ kN/m}^2}$$

Point C:

Refer to the figure.



$$\Delta\sigma_z = \left(\begin{array}{c} \text{stress at } C \text{ due to} \\ \text{area } 7.8 \text{ m} \times 3 \text{ m} \end{array} \right) - \left(\begin{array}{c} \text{stress at } C \text{ due to} \\ \text{area } 1.8 \text{ m} \times 3 \text{ m} \end{array} \right)$$

For rectangular area $7.8 \text{ m} \times 3 \text{ m}$: $m = \frac{7.8}{3} = 2.6$; $n = \frac{3}{3} = 1$; $I_3 = 0.2026$

For rectangular area $1.8 \text{ m} \times 3 \text{ m}$: $m = \frac{1.8}{3} = 0.6$; $n = \frac{3}{3} = 1$; $I_3 = 0.1361$

$$\Delta\sigma_z = q(0.2026 - 0.1361) = (225)(0.2026 - 0.1361) = \mathbf{14.96 \text{ kN/m}^2}$$

10.20 Eqs. (10.35), (10.37), (10.38), and (10.39):

$$b = \frac{B}{2} = \frac{3}{2} = 1.5 \text{ m}$$

$$m_1 = \frac{L}{B} = \frac{6}{3} = 2$$

	$z \text{ (m)}$				
	2	4	6	8	10
$n_1 = \frac{z}{b}$	1.33	2.66	4	5.33	6.66
I_4 (Table 10.10)	0.682	0.356	0.190	0.119	0.079
$\Delta\sigma_z = qI_4$ (kN/m^2)	153.4	80.1	42.7	26.8	17.8

CRITICAL THINKING PROBLEM

10.C.1

1. Vertical stress increase due to wheel load:

$$y = 0.305 \text{ m}; R = 0.15 \text{ m}; q = 565 \text{ kN/m}^2$$

Element	$r \text{ (m)}$	$\frac{y}{R}$	$\frac{r}{R}$	A'	B'	$\Delta\sigma_y \text{ (kN/m}^2\text{)}$
A	0.457	2.03	3.05	0.02221	0.00028	12.7
B	0.267	2.03	1.78	0.05278	0.04391	54.63
C	0	2.03	0	0.10557	0.17889	160.71

Overburden pressure at the middle of the layer = $0.305 \times 19.4 = 5.92 \text{ kN/m}^2$

Total vertical pressure, $\Delta\sigma_y$:

At A: $\sigma_{y-A} = 12.7 + 5.92 = \mathbf{18.62 \text{ kN/m}^2}$

At B: $\sigma_{y-B} = 54.63 + 5.92 = \mathbf{60.55 \text{ kN/m}^2}$

At C: $\sigma_{y-C} = 160.71 + 5.92 = \mathbf{166.63 \text{ kN/m}^2}$

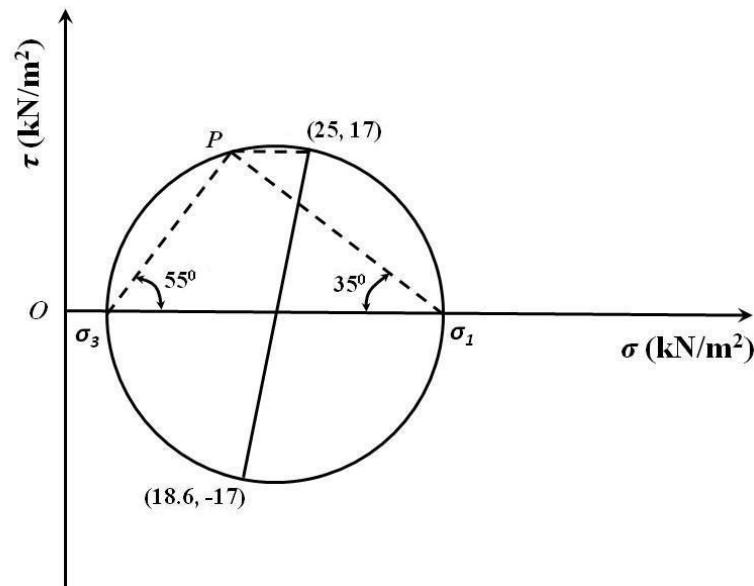
These values are entered into the following table.

Element at	Horizontal stress, σ_x (kN/m^2)	Shear stress, τ (kN/m^2)	Vertical stress, σ_y (kN/m^2)	σ_1 (kN/m^2)	σ_3 (kN/m^2)	α_i (deg)
A	25	17	18.62	39	4.5	55
B	32	45	60.55	93	1	48
C	7	0	166.63	167	7	0

2. Element at A:

The Mohr's circle is shown. $\sigma_1 \approx 39 \text{ kN/m}^2$; $\sigma_3 \approx 4.5 \text{ kN/m}^2$

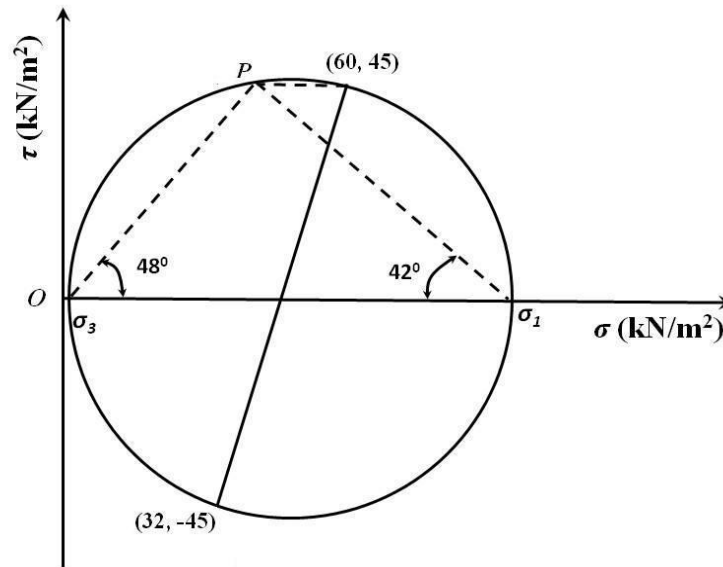
These values are entered in the above table. The pole is located at point P . The maximum principal stress acts on a plane which is inclined at 35° with the horizontal. Therefore, $\alpha_A = 90 - 35 = 55^\circ$.



Element at *B*:

The Mohr's circle is shown. $\sigma_1 \approx 93 \text{ kN/m}^2$; $\sigma_3 \approx 1 \text{ kN/m}^2$

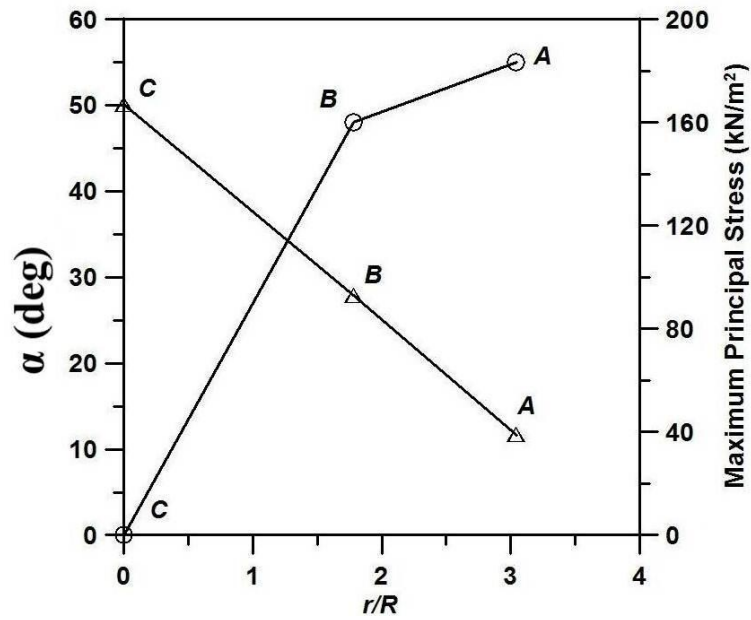
These values are entered in the table on the previous page. The pole is located at point *P*. The maximum principal stress acts on a plane which is inclined at 42° with the horizontal. Therefore, $\alpha_B = 90 - 42 = 48^\circ$.



Element at *C*:

Since there is no shear stress, the horizontal and vertical stresses are principal stresses. Therefore, $\sigma_1 \approx 167 \text{ kN/m}^2$; $\sigma_3 \approx 7 \text{ kN/m}^2$; $\alpha_C = 0^\circ$. These values are entered in the table on the previous page.

3. The plot is shown in the figure.



Chapter 11

$$11.1 \quad S_{e(\text{flexible, center})} = \Delta\sigma(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

$$\Delta\sigma = \frac{355}{(2)(3)} = 59.16 \text{ kN/m}^2$$

$$\text{Given: } \alpha = 4; B' = \frac{2}{2} = 1; \mu_s = 0.35; E_s = 13,500 \text{ kN/m}^2$$

$$I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

$$m' = \frac{L}{B} = \frac{3}{2} = 1.5; \quad n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{4}{\left(\frac{2}{2}\right)} = 4$$

From Table 11.1, $F_1 = 0.454$; from Table 11.2, $F_2 = 0.054$.

$$I_s = 0.454 + \frac{1 - (2)(0.4)}{1 - 0.4} (0.054) = 0.472$$

Also, with $\frac{D_f}{B} = \frac{1.5}{2} = 0.75$ and $\frac{L}{B} = 1.5$, Table 11.3 gives $I_f = 0.765$. Hence,

$$S_{e(\text{flexible, center})} = (59.16)[(4)(1)] \left(\frac{1 - 0.4^2}{13500} \right) (0.472)(0.765) = 0.0053 \text{ m} = 5.3 \text{ mm}$$

$$S_{e(\text{rigid})} = (0.93)(5.3) \approx \mathbf{4.93 \text{ mm}}$$

$$11.2 \quad \text{As in Problem 11.1, } S_{e(\text{rigid})} = 0.93 \Delta\sigma(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

$$E_s = \frac{\sum E_{s(i)} \Delta z}{\bar{z}} = \frac{(3000)(6) + (1100)(8) + (8500)(10)}{24} = 4658 \text{ lb/in}^2 = 670,752 \text{ lb/ft}^2$$

$$\text{Given: } B = L = 6 \text{ ft; } \mu_s = 0.3; \alpha = 4$$

$$\Delta\sigma = \frac{100000}{(6)(6)} = 2778 \text{ lb/ft}^2$$

$$B' = \frac{B}{2} = \frac{6}{2} = 3 \text{ ft.}$$

$$I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

$$m' = \frac{L}{B} = 1; \quad n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{24}{\left(\frac{6}{2}\right)} = 8$$

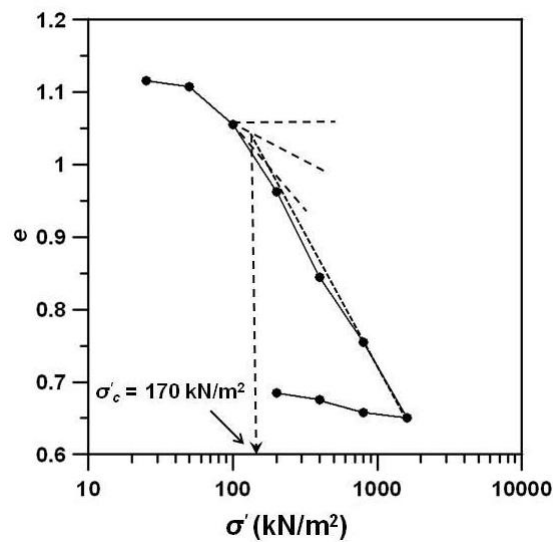
From Table 11.1, $F_1 = 0.482$; from Table 11.2, $F_2 = 0.02$

$$I_s = 0.482 + \frac{1 - (2)(0.3)}{1 - 0.3} (0.02) = 0.493$$

Also, $\frac{D_f}{B} = \frac{3}{6} = 0.5$. From Table 11.3, $I_f \approx 0.77$. So,

$$S_{e(\text{rigid})} = (0.93)(2778)(4 \times 3) \left(\frac{1 - 0.3^2}{670752} \right) (0.493)(0.77) = 0.016 \text{ ft} \approx \mathbf{0.2 \text{ in}}$$

11.3 a. The plot of e vs. σ' is shown.



b. $\sigma'_c = 170 \text{ kN/m}^2$

c. $C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.755 - 0.65}{\log\left(\frac{16}{8}\right)} \approx 0.35$

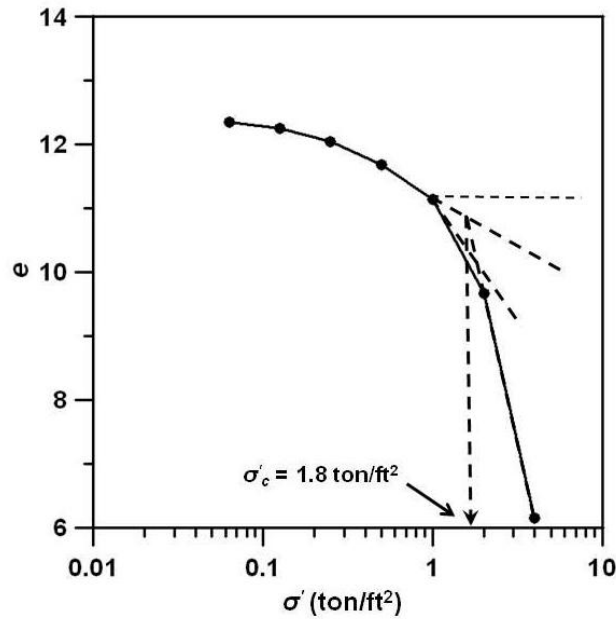
$$C_s = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.658 - 0.65}{\log\left(\frac{16}{8}\right)} = 0.026$$

$$\frac{C_s}{C_c} = \frac{0.026}{0.35} = \mathbf{0.074}$$

11.4 a. Height of solids: $H_s = \frac{W_s}{AG_s \gamma_w} = \frac{12 \text{ g}}{(4.91)(2.54)^2(2.49)(1)} = 0.152 \text{ cm} \approx 0.06 \text{ in.}$

σ' (ton/ft ²)	Change in dial reading (in.)	Final height, H (in.)	H_s (in.)	$H_v = H - H_s$ (in.)	$e = \frac{H_v}{H_s}$
0.063	0.0112	0.8013	0.06	0.7413	12.35
0.125	0.0059	0.7954	0.06	0.7354	12.25
0.250	0.0124	0.7830	0.06	0.723	12.05
0.500	0.0222	0.7608	0.06	0.7008	11.68
1.000	0.0324	0.7284	0.06	0.6684	11.14
2.000	0.0886	0.6398	0.06	0.5798	9.66
4.000	0.2105	0.4293	0.06	0.3693	6.15

The e -log σ' graph is plotted on the following page.



b. From the graph, $\sigma'_c = 1.8 \text{ ton/ft}^2$

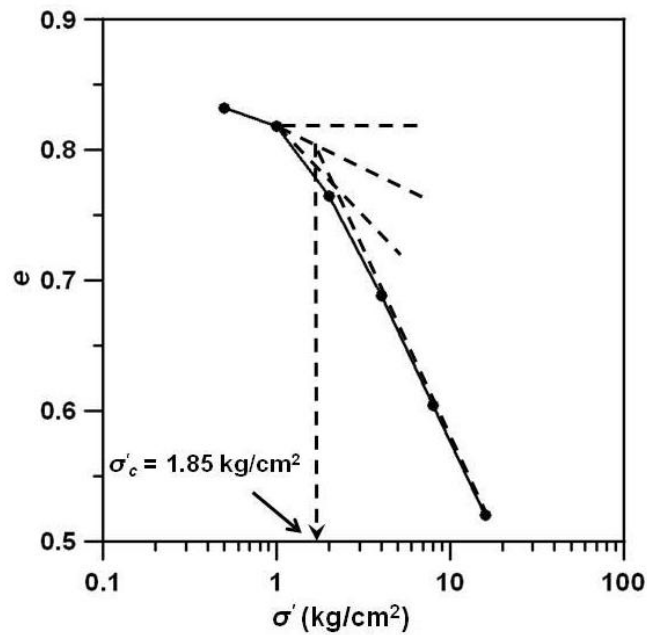
c.
$$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{9.66 - 6.15}{\log\left(\frac{4}{2}\right)} = 11.66$$

11.5
$$W_s = \frac{W}{1 + w} = \frac{140}{1 + 0.19} = 117.6 \text{ g}$$

$$H_s = \frac{W_s}{AG_s \gamma_w} = \frac{117.6}{(\pi)(3.175)^2 (2.7)(1)} = 1.375 \text{ cm}$$

σ' (kg/cm ²)	Final height, H (cm)	H_s (cm)	$H_v = H - H_s$ (cm)	$e = \frac{H_v}{H_s}$
0.5	2.519	1.375	1.144	0.832
1.0	2.5	1.375	1.125	0.818
2.0	2.428	1.375	1.053	0.765
4.0	2.322	1.375	0.947	0.688
8.0	2.206	1.375	0.831	0.604
16.0	2.09	1.375	0.715	0.576

The e -log σ' graph is plotted.



From the graph, $\sigma'_c = 1.85 \text{ kg/cm}^2$

$$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.688 - 0.604}{\log\left(\frac{8}{4}\right)} = 0.28$$

$$11.6 \quad a. \quad S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

$$C_c = 0.009(LL - 10) = (0.009)(42 - 10) = 0.288$$

$$\Delta\sigma = \frac{150}{(3)(3)} = 16.67 \text{ kN/m}^2$$

$$\gamma_d = \frac{(2.72)(9.81)}{1 + 0.7} = 15.7 \text{ kN/m}^3$$

$$\gamma_{\text{sat}} = \frac{(2.72 + 0.7)(9.81)}{1 + 0.7} = 19.73 \text{ kN/m}^3$$

$$\sigma'_o = (15.7)(1.5) + (19.73 - 9.81)\left(\frac{8}{2}\right) = 63.23 \text{ kN/m}^2$$

$$S_c = \frac{(0.288)(8)}{1 + 0.7} \log\left(\frac{63.23 + 16.67}{63.23}\right) = \mathbf{0.137 \text{ m}}$$

11.7 Eq. (11.68): $\Delta\sigma'_{\text{av}} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$

Eq. (10.35): $\Delta\sigma' = qI_4$

$$m_1 = \frac{L}{B} = \frac{3}{3} = 1; \quad b = \frac{B}{2} = 1.5 \text{ m}; \quad n_1 = \frac{z}{b}$$

$$q = \frac{150 \text{ kN}}{(3)(3)} = 16.67 \text{ kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m ²)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m ²)
1	0.0	1.5	0	16.67	≈ 1.0	16.67
1	4.0	1.5	2.67	16.67	0.231	3.85
1	8.0	1.5	5.33	16.67	0.065	1.08

$$\Delta\sigma'_{\text{av}} = \frac{16.67 + (4 \times 3.85) + 1.08}{6} = 5.52 \text{ kN/m}^2$$

$$S_c = \frac{(0.288)(8)}{1 + 0.7} \log\left(\frac{63.23 + 5.52}{63.23}\right) = \mathbf{0.049 \text{ m}}$$

11.8 a. $\sigma'_o = (6)(114) + (12)(118) + \left(\frac{18}{2}\right)(117) - (12 + 9)(62.4) = 1842.6 \text{ lb/ft}^2$

$$C_c = 0.009(LL - 10) = (0.009)(38 - 10) = 0.252$$

$$S_c = \frac{(0.252)(18)}{1+0.73} \log\left(\frac{1842.6+550}{1842.6}\right) = \mathbf{0.297 \text{ ft.} \approx 3.5 \text{ in.}}$$

$$\begin{aligned} \text{b. } S_c &= \frac{C_s H}{1+e_o} \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right) \\ &= \frac{\left(\frac{0.252}{5}\right)(18)}{1.73} \log\left(\frac{2200}{1842.6}\right) + \frac{(0.252)(18)}{1.73} \log\left(\frac{1842.6+550}{2200}\right) \\ &= 0.135 \text{ ft} \approx \mathbf{1.63 \text{ in.}} \end{aligned}$$

$$11.9 \quad \gamma_{d(\text{sand})} = \frac{G_s \gamma_w}{1+e} = \frac{(2.66)(9.81)}{1+0.65} = 15.81 \text{ kN/m}^3$$

$$\gamma'_{\text{sand}} = \frac{(G_s - 1)\gamma_w}{1+e} = \frac{(2.66-1)(9.81)}{1.65} = 9.87 \text{ kN/m}^3$$

$$\gamma'_{\text{clay}} = \frac{(G_s - 1)\gamma_w}{1+e} = \frac{(2.74-1)(9.81)}{1+0.98} = 8.62 \text{ kN/m}^3$$

$$\sigma'_o = (2)(15.81) + (4)(9.87) + \left(\frac{6}{2}\right)(8.62) = 96.96 \text{ kN/m}^2$$

$$C_c = 0.009(LL - 10) = (0.009)(54 - 10) = 0.396$$

$$\begin{aligned} S_c &= \frac{C_s H}{1+e_o} \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right) \\ &= \frac{\left(\frac{0.396}{6}\right)(6)}{1.98} \log\left(\frac{150}{96.96}\right) + \frac{(0.396)(6)}{1.98} \log\left(\frac{96.96+85}{150}\right) \\ &= 0.138 \text{ m} \approx \mathbf{13.8 \text{ cm}} \end{aligned}$$

$$11.10 \quad C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{1.22 - 0.97}{\log\left(\frac{225}{108}\right)} = 0.784$$

$$C_c = \frac{e_1 - e_3}{\log\left(\frac{\sigma'_3}{\sigma'_1}\right)}$$

$$e_3 = e_1 - C_c \log\left(\frac{\sigma'_3}{\sigma'_1}\right) = 1.22 - 0.784 \log\left(\frac{300}{108}\right) = \mathbf{0.872}$$

$$11.11 \quad C_c = \frac{0.92 - 0.77}{\log\left(\frac{3}{1.5}\right)} = 0.498$$

$$e_3 = 0.92 - 0.498 \log\left(\frac{4.5}{1.5}\right) = \mathbf{0.682}$$

11.12 The plot of e - $\log \sigma'$ is shown.

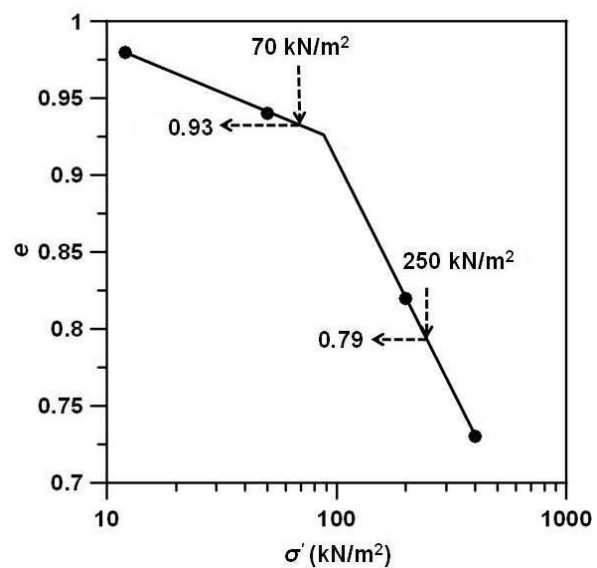
$$\sigma'_o = 70 \text{ kN/m}^2; e_1 = 0.93;$$

$$\sigma'_o + \Delta\sigma' = 250 \text{ kN/m}^2; e = 0.79$$

$$\Delta e = 0.93 - 0.79 = 0.14$$

$$S_c = \frac{H\Delta e}{1 + e_o} = \frac{(2)(0.14)}{1 + 0.93}$$

$$= \mathbf{0.145 \text{ m}}$$



$$11.13 \quad T_v = \frac{c_v t}{H_{dr}^2}; U = 75\%; T_v = 0.477 \text{ (Table 11.7)}$$

$$0.477 = \frac{(0.24 \text{ cm}^2/\text{min})(t)}{\left(\frac{600}{2} \text{ cm}\right)^2}; t = 178,875 \text{ min} = \mathbf{124.2 \text{ days}}$$

$$11.14 \quad a. \quad m_v = \frac{a_v}{1 + e_{av}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'} \right)}{1 + e_{av}}$$

$$\Delta e = e_1 - e_2 = 0.92 - 0.77 = 0.15$$

$$\Delta \sigma' = \sigma'_2 - \sigma'_1 = 3 - 1.5 = 1.5 \text{ ton/ft}^2$$

$$e_{av} = \frac{0.92 + 0.77}{2} = 0.845$$

$$m_v = \frac{a_v}{1 + e_{av}} = \frac{\left(\frac{0.15}{1.5} \right)}{1 + 0.845} = \mathbf{0.054 \text{ ft}^2/\text{ton}}$$

$$b. \quad c_v = \frac{k}{m_v \gamma_w} = 0.001085 \text{ in.}^2/\text{sec} = 7.534 \times 10^{-6} \text{ ft}^2/\text{sec}$$

$$7.534 \times 10^{-6} \text{ ft}^2/\text{sec} = \frac{k}{0.054 \text{ ft}^2/\text{ton} \left(\frac{62.4}{2000} \right) \text{ ton/ft}^3}$$

$$k = \mathbf{1.27 \times 10^{-8} \text{ ft/sec}}$$

11.15 In the laboratory:

$$T_{65} = \frac{c_v t_{65}}{H_{dr}^2}$$

$$0.304 = \frac{(c_v)(10 \text{ min})}{\left(\frac{0.019}{2} \text{ m} \right)^2}; \quad c_v = 2.74 \times 10^{-6} \text{ m}^2/\text{min}$$

$$U = 40\%; \quad T_{40} = 0.126 \text{ (Table 11.7)}$$

In the field:

$$T_{40} = \frac{c_v t_{40}}{H_{dr}^2}$$

$$0.126 = \frac{(2.74 \times 10^{-6} \text{ m}^2/\text{min})(t_{40})}{(4 \text{ m})^2}$$

$$t_{40} = 735,766 \text{ min} = \mathbf{511 \text{ days}}$$

$$11.16 \quad \text{a.} \quad m_v = \frac{a_v}{1 + e_{av}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'} \right)}{1 + e_{av}}$$

$$\Delta e = 0.9 - 0.75 = 0.15$$

$$\Delta \sigma' = 1.5 - 0.5 = 1 \text{ ton/ft}^2$$

$$e_{av} = \frac{0.9 + 0.75}{2} = 0.825$$

$$m_v = \frac{\left(\frac{0.15}{1} \right)}{1 + 0.825} = 0.082 \text{ ft}^2/\text{ton}$$

$$c_v = \frac{k}{m_v \gamma_w} = \frac{(6 \times 10^{-7})(0.0328) \text{ ft/sec}}{(0.082 \text{ ft}^2/\text{ton}) \left(\frac{62.4}{2000} \text{ ton/ft}^3 \right)}$$

$$= 7.69 \times 10^{-6} \text{ ft}^2/\text{sec} = 0.664 \text{ ft}^2/\text{day}$$

$$t_{50} = \frac{T_v H_{dr}^2}{c_v} = \frac{(0.197)(12)^2}{0.644} = \mathbf{44 \text{ days}}$$

$$\text{b.} \quad C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.9 - 0.75}{\log\left(\frac{1.5}{0.5}\right)} = 0.31$$

$$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta \sigma'}{\sigma'_o}\right) = \frac{(0.31)(12)}{1 + 0.9} \log\left(\frac{1.5}{0.5}\right) = 0.934 \text{ ft}$$

$$S_c \text{ at } 50\% = (0.5)(0.934) = 0.467 \text{ ft} \approx \mathbf{5.6 \text{ in.}}$$

$$11.17 \quad m_v = \frac{a_v}{1 + e_{av}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'}\right)}{1 + e_{av}} = \frac{\left(\frac{0.85 - 0.71}{250 - 125}\right)}{1 + \left(\frac{0.85 + 0.71}{2}\right)} = 6.29 \times 10^{-4} \text{ m}^2/\text{kN}$$

$$c_v = \frac{T_{70} H_{dr}^2}{t_{70}} = \frac{(0.403) \left(\frac{0.025}{2}\right)^2}{4.8} = 1.31 \times 10^{-5} \text{ m}^2/\text{min}$$

$$k = c_v m_v \gamma_w = (6.29 \times 10^{-4} \text{ m}^2/\text{kN})(1.31 \times 10^{-5} \text{ m}^2/\text{min})(9.81 \text{ kN/m}^3) \\ = \mathbf{8.08 \times 10^{-8} \text{ m/min}}$$

$$11.18 \quad \text{a.} \quad T_{90} = \frac{c_v t_{90}}{H_{dr}^2}; \quad 0.848 = \frac{c_v (180)}{\left(\frac{15}{2}\right)^2}$$

$$c_v = \mathbf{0.265 \text{ ft}^2/\text{day}}$$

$$\text{b.} \quad T_{65} = \frac{c_v t_{65}}{H_{dr}^2}$$

From Table 11.7, $U = 65\%$; $T_{65} = 0.119$

$$t_{65} = \frac{T_{65} H_{dr}^2}{c_v} = \frac{(0.119) \left(\frac{0.75}{(2)(12)} \text{ ft}\right)^2}{0.265} = \mathbf{4.38 \times 10^{-4} \text{ day} \approx 38 \text{ sec}}$$

$$11.19 \quad \text{a.} \quad \text{Eq. (11.68): } \Delta \sigma'_{av} = \frac{\Delta \sigma'_t + 4\Delta \sigma'_m + \Delta \sigma'_b}{6}$$

$$\text{Eq. (10.35): } \Delta \sigma' = q I_4$$

$$m_1 = \frac{L}{B} = \frac{2}{2} = 1; \quad b = \frac{B}{2} = 1.0 \text{ m}; \quad n_1 = \frac{z}{b}$$

$$q = \frac{300 \text{ kN}}{(2)(2)} = 75 \text{ kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m ²)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m ²)
1	1.0	1.0	1.0	75	0.701	52.57
1	2.0	1.0	2.0	75	0.336	25.2
1	3.0	1.0	3.0	75	0.179	13.42

$$\Delta\sigma'_{av} = \frac{52.57 + (4 \times 25.2) + 13.42}{6} = \mathbf{27.8 \text{ kN/m}^2}$$

$$\text{b. } \gamma_{\text{sat-clay}} = \frac{(1+w)\gamma_w G_s}{1+wG_s} = \frac{(1+0.24)(9.81)(2.74)}{1+(0.24)(2.74)} = 20.1 \text{ kN/m}^3$$

$$\sigma'_o = (1)(14) + (1)(17 - 9.81) + \left(\frac{2}{2}\right)(20.1 - 9.81) = 31.48 \text{ kN/m}^2$$

$$C_c = 0.009(LL - 10) = (0.009)(46 - 10) = 0.324$$

Since $\sigma'_o \leq \sigma'_c$, the clay is overconsolidated

$$\begin{aligned} S_c &= \frac{C_s H}{1+e_o} \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right) \\ &= \frac{\left(\frac{0.324}{5}\right)(2)}{1+0.657} \log\left(\frac{40}{31.48}\right) + \frac{(0.324)(2)}{1+0.657} \log\left(\frac{31.48 + 27.8}{40}\right) \\ &= 0.0749 \text{ m} \approx \mathbf{75 \text{ mm}} \end{aligned}$$

$$\text{c. } U(\%) = \left(\frac{19}{75}\right)(100) = \mathbf{25.3\%}$$

$$\text{d. } T_v = \frac{c_v t}{H_{dr}^2}; U = 25.3\%; T_v = 0.0503 \text{ (Table 11.7)}$$

$$0.0503 = \frac{(c_v)(365)}{(1 \text{ m})^2}$$

$$c_v = \mathbf{1.378 \times 10^{-4} \text{ m}^2/\text{day}}$$

$$\text{e. } T_v = \frac{(0.0001378)(2)(365)}{\left(\frac{2}{2}\right)^2} = 0.1$$

Table (11.7): $U(\%) \approx 36\%$

$$U(\%) = \left(\frac{S_{c(t)}}{S_c} \right) (100); S_{c(t)} = (0.36)(75) = \mathbf{27 \text{ mm}}$$

CRITICAL THINKING PROBLEM

11.C.1 a. For the clay layer:

$$\text{Eq. (11.68): } \Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

$$\text{Eq. (10.35): } \Delta\sigma' = qI_4$$

$$m_1 = \frac{L}{B} = \frac{10}{10} = 1; b = \frac{B}{2} = 5 \text{ m}; n_1 = \frac{z}{b}$$

$$q = (2)(19) = 38 \text{ kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m ²)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m ²)
1	4	5	0.8	38.0	0.8	30.4
1	6	5	1.2	38.0	0.606	23.02
1	8	5	1.6	38.0	0.449	17.06

$$\Delta\sigma'_{av} = \frac{30.4 + (4 \times 23.02) + 17.06}{6} = 23.25 \text{ kN/m}^2$$

$$\sigma'_o = (2)(15) + (2)(17) + \left(\frac{4}{2}\right)(18) - (2 + 2)(9.81) = 60.76 \text{ kN/m}^2$$

$$S_{c-\text{clay}} = \frac{(0.36)(4)}{1 + 1.1} \log\left(\frac{60.76 + 23.25}{60.76}\right) = \mathbf{0.096 \text{ m}}$$

For the peat layer:

$$m_1 = \frac{L}{B} = \frac{10}{10} = 1; \quad b = \frac{B}{2} = 5 \text{ m}; \quad n_1 = \frac{z}{b}$$

$$q = (2)(19) = 38 \text{ kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m ²)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m ²)
1	8	5	1.6	38.0	0.449	17.06
1	9	5	1.8	38.0	0.388	14.74
1	10	5	2.0	38.0	0.336	12.76

$$\Delta\sigma'_{av} = \frac{17.06 + (4 \times 14.74) + 12.76}{6} = 14.8 \text{ kN/m}^2$$

$$\sigma'_o = (2)(15) + (2)(17) + (4)(18) + (1)(16) - (7)(9.81) = 83.33 \text{ kN/m}^2$$

$$S_{c-\text{peat}} = \frac{(6.6)(2)}{1 + 5.9} \log\left(\frac{83.33 + 14.8}{83.33}\right) = \mathbf{0.136 \text{ m}}$$

$$\text{Total consolidation settlement, } S_c = 0.096 + 0.136 = \mathbf{0.232 \text{ m}}$$

- b. For the clay layer, a double drainage condition is assumed since the bottom peat layer has high void ratio and considered permeable.

$$t_{99-\text{clay}} = \frac{(H_{dr}^2)(T_{99})}{c_v} = \frac{(200^2)(1.781)}{0.003} = 23,746,666 \text{ sec} \approx \mathbf{275 \text{ days}}$$

For the peat layer, a single drainage is assumed since the top layer is considered to have relatively low permeability.

$$t_{99-\text{peat}} = \frac{(H_{dr}^2)(T_{99})}{c_v} = \frac{(200^2)(1.781)}{0.025} = 2,849,600 \text{ sec} \approx \mathbf{33 \text{ days}}$$

- c. Secondary compression in clay:

$$\Delta e_{\text{primary}} = C_c \log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right) = 0.36 \log\left(\frac{60.76 + 23.25}{60.76}\right) = 0.0506$$

$$e_p = e_0 - \Delta e_{\text{primary}} = 1.1 - 0.0506 = 1.049$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p} = \frac{0.03}{1 + 1.049} = 0.0146$$

$$S_{s\text{-clay}} = C'_\alpha H \log\left(\frac{t_2}{t_1}\right) = (0.0146)(4) \log\left(\frac{(2)(365)}{275}\right) = \mathbf{0.0247 \text{ m}}$$

Secondary compression in peat:

$$\Delta e_{\text{primary}} = C_c \log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right) = 6.6 \log\left(\frac{83.33 + 14.8}{83.33}\right) = 0.468$$

$$e_p = e_0 - \Delta e_{\text{primary}} = 5.9 - 0.468 = 5.432$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p} = \frac{0.263}{1 + 5.432} = 0.0408$$

$$S_{s\text{-peat}} = C'_\alpha H \log\left(\frac{t_2}{t_1}\right) = (0.0408)(2) \log\left(\frac{(2)(365)}{33}\right) = \mathbf{0.109 \text{ m}}$$

d. Total settlement:

$$S_c + S_{s\text{-clay}} + S_{s\text{-peat}} = 0.232 + 0.0247 + 0.109 = \mathbf{0.365 \text{ m}}$$

e. Time factor in 3 months:

$$T_v = \frac{c_v t}{H_{\text{dr}}^2} = \frac{(0.003)(3)(30)(24)(3600)}{200^2} = 0.583$$

Determine the degree of consolidation, U_z from Figure 11.25:

$$\frac{z}{H_{\text{dr}}} = \frac{3}{2} = 1.5; \quad U_z \approx 0.85$$

$$\text{Eq. 11.57: } U_z = 1 - \frac{u_z}{u_0}$$

Initial excess pore water pressure, $u_0 \approx \Delta\sigma' = 23.25 \text{ kN/m}^2$

$u_z = (1 - U_z)u_0 = (1 - 0.85)(23.25) = 3.487 \text{ kN/m}^2 = \text{remaining excess pore}$
water pressure at point A after 3 months.

The increase in effective stress after 3 months $= 23.25 - 3.487 = 19.76 \text{ kN/m}^2$

$$\sigma'_o = (2)(15) + (2)(17) + (3)(18) - (2 + 3)(9.81) = 68.95 \text{ kN/m}^2$$

Therefore, the final effective stress after 3 months $= 68.95 + 19.76$
 $= \mathbf{88.71 \text{ kN/m}^2}$

Chapter 12

- 12.1 a. $c' = 0$. From Eq. (12.3): $\tau_f = \sigma' \tan \phi'$

$$\tau = \frac{300}{(1000)(0.063)^2} = 75 \text{ kN/m}^2$$

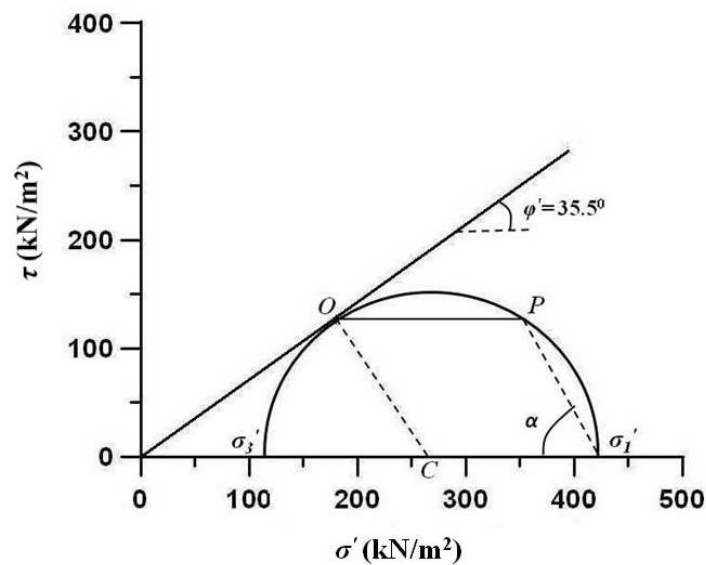
So, $75 = 105 \tan \phi'$

$$\phi' = \tan^{-1}\left(\frac{75}{105}\right) = 35.5^\circ$$

- b. For $\sigma' = 180 \text{ kN/m}^2$, $\tau_f = 180 \tan 35.5^\circ = 128.39 \text{ kN/m}^2$

Shear force, $S = (128.39)(1000)(0.063)^2 = 509.5 \text{ N}$

- 12.2 The point O (180, 128.4) represents the failure stress conditions on the Mohr-Coulomb failure envelope. The perpendicular line OC to the failure envelope determines the center, C , of the Mohr's circle. With the center at C , and the radius as OC , the Mohr's circle is drawn by trial and error such that the circle is tangent to the failure envelope at O . From the graph,



a. $\sigma'_3 \approx 115 \text{ kN/m}^2$; $\sigma'_1 \approx 420 \text{ kN/m}^2$

- b. The horizontal line OP drawn from O determines the pole P . Therefore, the orientation or the major principal plane with the horizontal is given by the angle $\alpha \approx 65^\circ$.

12.3 For $\sigma' = 28 \text{ lb/in}^2$, $\tau_f = 28 \tan 33^\circ = 18.18 \text{ lb/in}^2$

Shear force, $S = (18.18)(2.5)^2 = 113.65 \text{ lb}$

12.4 Area of specimen $A = \left(\frac{\pi}{4}\right)(2)^2 = 3.14 \text{ in.}^2$

Test No.	Normal force N (lb)	$\sigma' = \frac{N}{A}$ (lb/in. ²)	Shear force S (lb)	$\tau_f = \frac{S}{A}$ (lb/in. ²)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	15	4.77	5.25	1.67	19.29
2	30	9.55	10.5	3.34	19.27
3	48	15.28	16.8	5.35	19.29
4	83	26.43	29.8	9.5	19.77

A graph of τ_f vs. σ' will yield $\phi' = 19.5^\circ$.

12.5 Area of specimen $A = \left(\frac{\pi}{4}\right)(0.05)^2 = 0.00196 \text{ m}^2$

Test No.	Normal force N (N)	$\sigma' = \frac{N}{A}$ (N/m ²)	Shear force S (N)	$\tau_f = \frac{S}{A}$ (N/m ²)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	250	79.6	139	44.26	29.07
2	375	119.4	209	66.56	29.13
3	450	143.3	250	79.61	29.05
4	540	171.9	300	95.54	29.06

A graph of τ_f vs. σ' will yield $\phi' \approx 29^\circ$.

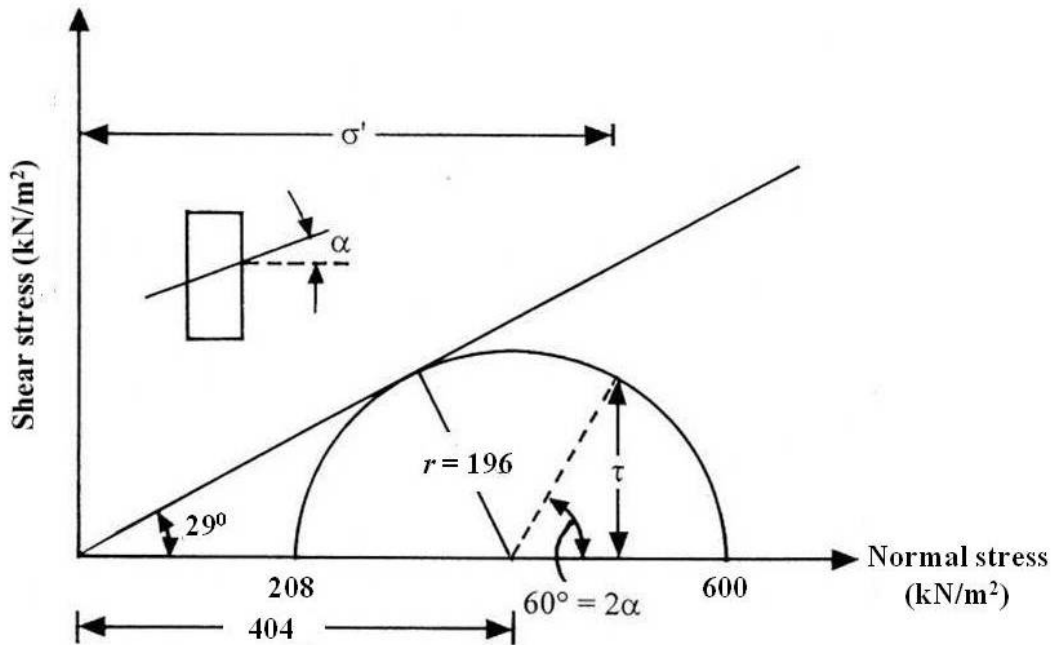
12.6 $c' = 0$. From Eq. (12.8): $\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = 30^\circ$

$$\sigma'_1 = 208 \tan^2 \left(45 + \frac{29}{2} \right) \approx 600 \text{ kN/m}^2$$

$$\Delta \sigma_{d(\text{failure})} = \sigma'_1 - \sigma'_3 = 600 - 208 = \mathbf{392 \text{ kN/m}^2}$$

12.7 a. From Eq. (12.4): $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{29}{2} = \mathbf{59.5^\circ}$

b. Refer to the figure.



$$\tau = 196 \sin 60^\circ = \mathbf{169.7 \text{ kN/m}^2}$$

$$\sigma' = 404 + r \cos 60 = 404 + 196 \cos 60 = \mathbf{502 \text{ kN/m}^2}$$

For failure, $\tau_f = \sigma' \tan \phi' = 502 \tan 29 = 278.26 \text{ kN/m}^2$. Since the developed shear stress = 169.5 kN/m^2 (which is less than 278.26 kN/m^2), the specimen did not fail along this plane.

12.8 $\phi' = 28 + 0.18D_r = 28 + (0.18)(68) = 40.24^\circ$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = 150 \tan^2 \left(45 + \frac{40.24}{2} \right) = \mathbf{697.43 \text{ kN/m}^2}$$

$$12.9 \quad \sigma'_3 = 125 \text{ kN/m}^2; \quad \sigma'_1 = \sigma'_3 + \Delta\sigma_{d(f)} = 125 + 175 = 300 \text{ kN/m}^2$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right); \quad 300 = 125 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\phi' \approx \mathbf{24.3^\circ}$$

$$12.10 \quad \sigma'_3 + \Delta\sigma'_3 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = \sigma'_3 \tan^2 (60.5^\circ) = 3.12 \sigma'_3$$

$$1 + \frac{\Delta\sigma'_3}{\sigma'_3} = 3.12; \quad \frac{\Delta\sigma'_3}{\sigma'_3} = 2.12$$

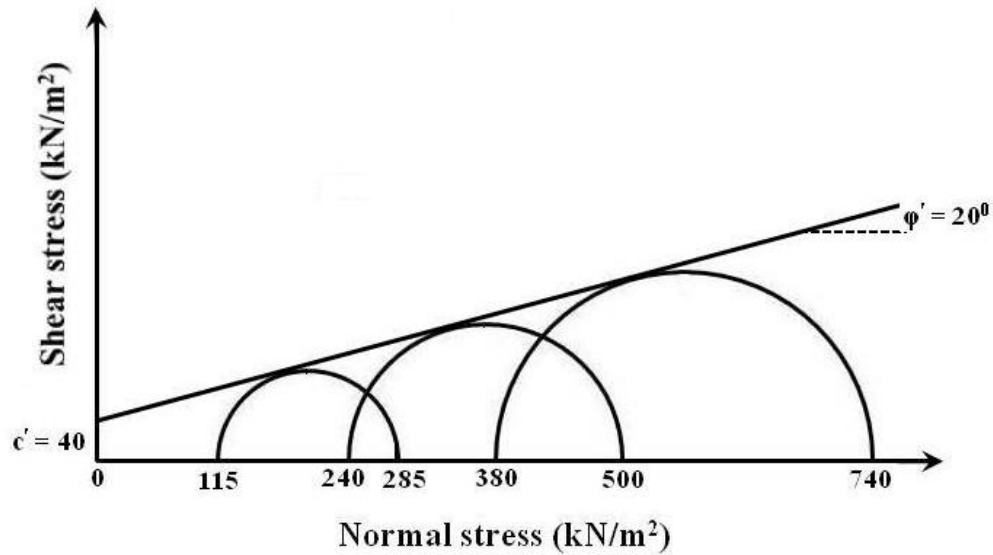
$$\sigma'_3 = \frac{18}{2.12} = \mathbf{8.49 \text{ lb/in.}^2}$$

$$12.11 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = (\sigma_3 - \Delta u_{d(f)}) \tan^2 \left(45 + \frac{31}{2} \right)^2$$

$$\sigma'_1 = (15 - 4.8) \tan^2 (60.5^\circ) = \mathbf{31.86 \text{ lb/in.}^2}$$

- 12.12 a. The effective principal stresses at failure are calculated as follows and the Mohr-Coulomb failure envelope is drawn from the Mohr's circles on the following page.

Test no.	σ_3 (kN/m ²)	$(\Delta\sigma_d)_f$ (kN/m ²)	$(\Delta u_d)_f$ (kN/m ²)	$\sigma'_3 = \sigma_3 - (\Delta u_d)_f$ (kN/m ²)	$\sigma'_1 = \sigma'_3 + (\Delta\sigma_d)_f$ (kN/m ²)
1	100	170	-15	115	285
2	200	260	-40	240	500
3	300	360	-80	380	740



From the graph: $c' \approx 40 \text{ kN/m}^2$ and $\phi' \approx 20^\circ$

- b. Effective stress in the middle of the clay layer:

$$\sigma'_0 = (2)(19 - 9.8) = 18.4 \text{ kN/m}^2$$

$$\tau = c' + \sigma' \tan \phi' = 40 + 18.4 \tan(20) = 46.7 \text{ kN/m}^2$$

12.13 a. $\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right); (25 + 33) = 25 \tan^2 \left(45 + \frac{\phi'}{2} \right)$

$$\phi' \approx 23.4^\circ$$

b. $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{23.4}{2} = 56.7^\circ$

- c. From Eqs. (10.8) and (10.9):

$$\begin{aligned} \sigma'_f &= \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta = \frac{58 + 25}{2} + \frac{58 - 25}{2} \cos(2 \times 56.7) \\ &= 34.94 \text{ lb/in.}^2 \end{aligned}$$

$$\tau_f = \sigma' \tan \phi' = 34.94 \tan 23.4 = 15.12 \text{ lb/in.}^2$$

$$12.14 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right)$$

$$\text{Specimen I:} \quad (105 + 220) = 325 = 105 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right) \quad (\text{a})$$

$$\text{Specimen II:} \quad (210 + 400) = 610 = 210 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right) \quad (\text{b})$$

$$\text{Subtracting Eq. (a) from Eq. (b):} \quad 610 - 325 = 105 \tan^2 \left(45 + \frac{\phi'}{2} \right); \quad \phi' = \mathbf{27.48^\circ}$$

From Eq. (b):

$$c' = \frac{610 - 210 \tan^2 \left(45 + \frac{27.48}{2} \right)}{2 \tan \left(45 + \frac{27.48}{2} \right)} = \mathbf{12.48 \text{ kN/m}^2}$$

12.15 a. From Eqs. (10.8) and (10.9): $\theta = 40^\circ$

$$\begin{aligned} \sigma' &= \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta = \frac{105 + 325}{2} + \frac{325 - 105}{2} \cos(2 \times 40) \\ &= \mathbf{234.1 \text{ kN/m}^2} \end{aligned}$$

$$\tau = \frac{\sigma'_1 - \sigma'_3}{2} \sin 2\theta = \frac{325 - 105}{2} \sin(2 \times 40) = \mathbf{108.32 \text{ kN/m}^2}$$

b. The angle of inclination of the failure plane:

$$\theta = 45 + \frac{\phi'}{2} = 45 + \frac{27.48}{2} = \mathbf{58.73^\circ}$$

$$\begin{aligned} \sigma' &= \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta = \frac{610 + 210}{2} + \frac{610 - 210}{2} \cos(2 \times 58.73) \\ &= \mathbf{317.74 \text{ kN/m}^2} \end{aligned}$$

$$\tau = \frac{\sigma'_1 - \sigma'_3}{2} \sin 2\theta = \frac{610 - 210}{2} \sin(2 \times 58.73) = \mathbf{177.46 \text{ kN/m}^2}$$

$$12.16 \quad \sigma_1 = \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right); \quad \phi = 2 \left[\tan^{-1} \left(\frac{22+28}{22} \right)^{0.5} - 45 \right] = \mathbf{22.9^\circ}$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right); \quad \phi' = 2 \left[\tan^{-1} \left(\frac{22+28+4}{22+4} \right)^{0.5} - 45 \right] = \mathbf{20.48^\circ}$$

$$12.17 \quad \text{a.} \quad \sigma_3 = 150 \text{ kN/m}^2; \quad \sigma_1 = 150 + 120 = 270 \text{ kN/m}^2$$

$$\sigma_1 = \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right); \quad \frac{270}{150} = \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$\phi = \mathbf{16.6^\circ}$$

$$\text{b.} \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\frac{\sigma_1 - \Delta u_{d(f)}}{\sigma_3 - \Delta u_{d(f)}} = \tan^2 \left(45 + \frac{27}{2} \right) = 2.662$$

$$\frac{270 - \Delta u_{d(f)}}{150 - \Delta u_{d(f)}} = 2.662$$

$$\text{Or, } 270 - \Delta u_{d(f)} = 399.3 - 2.662 \Delta u_{d(f)}$$

$$\Delta u_{d(f)} = \mathbf{77.8 \text{ kN/m}^2}$$

$$12.18 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) = 150 \tan^2 (45 + 27/2) = 399.4 \text{ kN/m}^2$$

$$\Delta \sigma_{d(f)} = \sigma'_1 - \sigma'_3 = 399.4 - 150 = \mathbf{249.4 \text{ kN/m}^2}$$

$$12.19 \quad \sigma_1 = \sigma_3 \tan^2 \left(45 + \frac{\phi}{2} \right) = 20 \tan^2 \left(45 + \frac{31}{2} \right) = 62.48 \text{ lb/in.}^2$$

$$\Delta \sigma_{d(f)} = 62.48 - 20 = \mathbf{42.48 \text{ lb/in.}^2}$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\frac{62.48 - \Delta u_{d(f)}}{20 - \Delta u_{d(f)}} = \tan^2 \left(45 + \frac{24}{2} \right)$$

$$\Delta u_{d(f)} = -10.98 \text{ lb/in.}^2$$

A dense sand tends to expand during shear. Due to undrained condition, it creates a negative pore water pressure. A loose sand tends to contract during shear, and a positive pore water pressure is developed in undrained conditions. Therefore, for loose sand, $\sigma' < \sigma$, and so, $\phi' > \phi$.

$$12.20 \quad \sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$\frac{148 - \Delta u_{d(f)}}{0 - \Delta u_{d(f)}} = \tan^2 \left(45 + \frac{28}{2} \right)$$

$$\Delta u_{d(f)} = -83.6 \text{ kN/m}^2$$

12.21 a.	Test no.	$\frac{\sigma'_1 + \sigma'_3}{2} = p' \text{ (kN/m}^2\text{)}$	$\frac{\sigma'_1 - \sigma'_3}{2} = q' \text{ (lb/in.}^2\text{)}$
	1	212	108
	2	362	155

$$q' = m + p' \tan \alpha$$

$$108 = m + 212 \tan \alpha \quad (a)$$

$$155 = m + 362 \tan \alpha \quad (b)$$

$$m = 41.56 \text{ kN/m}^2, \quad \alpha = 17.4^\circ$$

$$b. \quad \phi' = \sin^{-1}(\tan \alpha) = \sin^{-1}(\tan 17.4) = 18.26^\circ$$

$$c' = \frac{m}{\cos \alpha} = \frac{41.56}{\cos(17.4)} = 43.55 \text{ kN/m}^2$$

$$12.22 \quad \frac{c_{u(VST)}}{\sigma'_o} = 0.11 + 0.0037PI$$

$$\sigma' = (2)(18) + (7)(19.5 - 9.81) = 103.83 \text{ kN/m}^2$$

$$c_{u(VST)} = [0.11 + (0.0037)(23)](103.83) = \mathbf{20.25 \text{ kN/m}^2}$$

CRITICAL THINKING PROBLEM

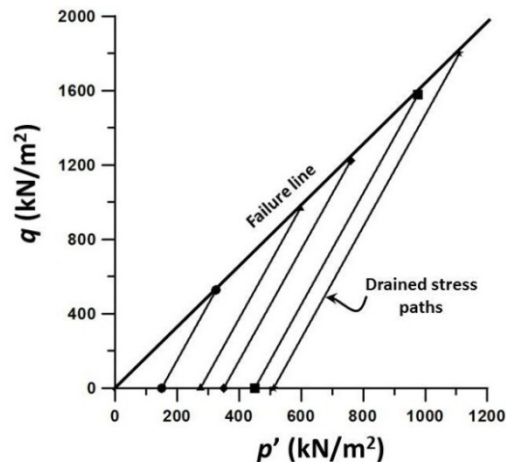
12.C.1 Task 1

$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) = \sigma'_3 + \frac{1}{3}\Delta\sigma_d; q = \Delta\sigma_d$$

At the end of consolidation, $p'_0 = \sigma'_3$; $q_0 = 0$

σ_3 (kN/m ²)	p'_0 (kN/m ²)	$(\Delta\sigma_d)_f = q_f$ (kN/m ²)	p'_f (kN/m ²)
150	150	527	325.67
275	275	965	596.67
350	350	1225	758.33
450	450	1580	976.67
510	510	1800	1110

The drained stress paths and the failure line are shown below.



Task 2

At point O : $p' = 0$; $q = 0$

At point P : $p' = \sigma'_3 = 250 \text{ kN/m}^2$; $q = 0$

At point A : $q = 1000 = \Delta\sigma_d$

$$p' = 675 = \sigma'_3 + \frac{1}{3} \Delta\sigma_d = \sigma'_3 + \frac{1}{3}(1000)$$

Therefore, $\sigma'_3 = 341.67 \text{ kN/m}^2$

Stress path O to P : Increase confining pressure from 0 to 250 kN/m^2 under drained condition (effective stress).

Stress path P to A : Increase confining pressure from 250 kN/m^2 to 341.67 kN/m^2 under drained condition (effective stress). Simultaneously increase axial stress from 0 to 1000 kN/m^2 .

Chapter 13

13.1 — 13.4 $K_o = (1 - \sin \phi')(\text{OCR})^{\sin \phi'}$

Problem	ϕ' (deg)	K_o	$P_o = \frac{1}{2} K_o \gamma H^2$	$\bar{z} = H / 3$
13.1	35	0.634	143.44 kN/m	1.67 m
13.2	33	0.627	8,607.1 lb/ft	5.67 ft
13.3	29	0.515	176.13 kN/m	2 m
13.4	40	0.463	8,625.7 lb/ft	6 ft

13.5 — 13.8 $K_a = \tan^2(45 - \phi'/2)$

Problem	ϕ' (deg)	K_a	$\sigma'_{a(z=H)} = K_a \gamma H$	$P_a = \frac{1}{2} K_a \gamma H^2$	$\bar{z} = H / 3$
13.5	32	0.307	$(0.307)(110)(14)$ = 472.7 lb/ft²	$(\frac{1}{2})(0.307)(110)(14)^2$ = 3309.4 lb/ft	4.66 ft
13.6	28	0.361	$(0.361)(99)(22)$ = 786.2 lb/ft²	$(\frac{1}{2})(0.361)(99)(22)^2$ = 8648.8 lb/ft	7.33 ft
13.7	37	0.248	$(0.248)(17.6)(5)$ = 21.8 kN/m²	$(\frac{1}{2})(0.248)(17.6)(5)^2$ = 54.56 kN/m	1.67 m
13.8	41	0.207	$(0.207)(19.5)(9)$ = 36.32 kN/m²	$(\frac{1}{2})(0.207)(19.5)(9)^2$ = 163.47 kN/m	3 m

Note: 1. Pressure distribution is similar to that shown in Figure 13.11a., i.e.,

$$\sigma'_a = 0 \text{ at } z = 0 \text{ and } \sigma'_a = K_a \gamma H \text{ at } z = H$$

2. $\bar{z}$ = distance measured from the bottom of the wall

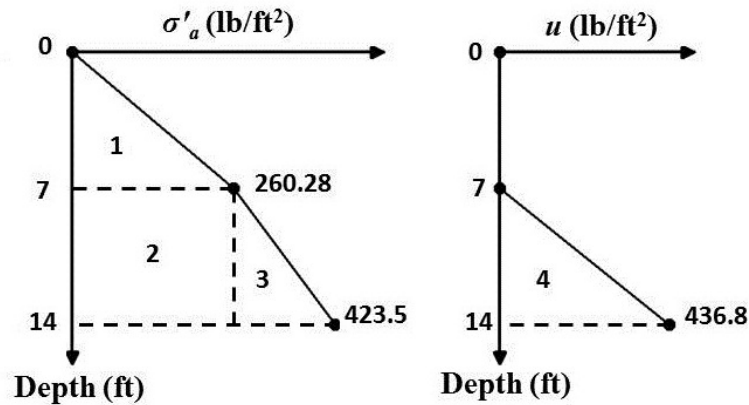
13.9 — 13.12 $K_p = \tan^2(45 + \phi'/2)$

Problem	ϕ' (deg)	K_p	$\sigma'_{p(z=H)} = K_p \gamma H$	$P_p = \frac{1}{2} K_p \gamma H^2$	$\bar{z} = H / 3$
13.9	32	3.254	$(3.254)(117)(11)$ = 4187.9 lb/ft²	$(\frac{1}{2})(3.254)(117)(11)^2$ = 23,033 lb/ft	3.67 ft
13.10	38	4.203	$(4.203)(101)(16)$ = 6792 lb/ft²	$(\frac{1}{2})(4.203)(101)(16)^2$ = 54,336 lb/ft	5.33 ft
13.11	30	3.0	$(3)(16.6)(7)$ = 348.6 kN/m²	$(\frac{1}{2})(3)(16.6)(7)^2$ = 1220.1 kN/m	2.33 m
13.12	27	2.662	$(2.662)(20.5)(12)$ = 654.8 kN/m²	$(\frac{1}{2})(2.662)(20.5)(12)^2$ = 3929.1 kN/m	4 m

Note: 1. $\sigma'_{p(z=0)} = 0$; triangular pressure distribution

2. $\bar{z}$ = distance measured from the bottom of the wall

13.13 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{28}{2}\right) = 0.361$. Refer to the figure.



$$z = 0 \text{ ft: } \sigma'_a = \sigma'_a K_a = 0; \quad u = 0$$

$$z = 7 \text{ ft: } \sigma'_a = (103)(7)(0.361) = 260.28 \text{ lb/ft}^2; \quad u = 0$$

$$z = 14 \text{ ft: } \sigma'_a = [(103)(7) + (127 - 62.4)(7)](0.361) = 423.5 \text{ lb/ft}^2$$

$$u = (62.4)(7) = 436.8 \text{ lb/ft}^2$$

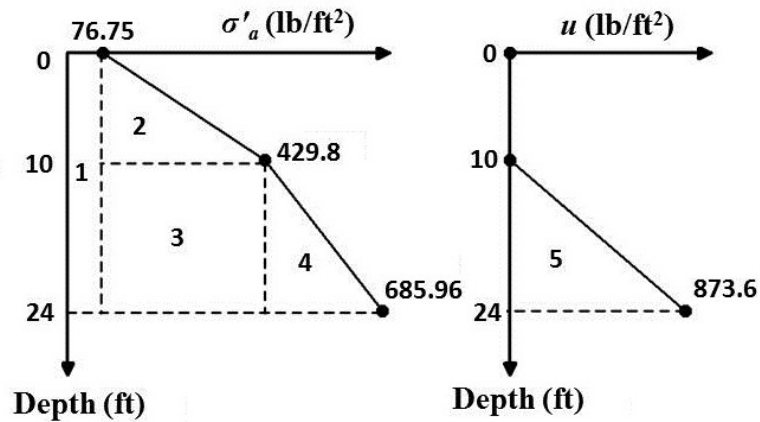
Area No.	Area
1	$\left(\frac{1}{2}\right)(7)(260.28) = 910.98 = 910.98$
2	$(260.28)(7) = 1,821.96$
3	$\left(\frac{1}{2}\right)(7)(423.5 - 260.28) = 571.27$
4	$\left(\frac{1}{2}\right)(7)(436.8) = 1,528.8$
$\Sigma 4,833 \text{ lb/ft}$	

Resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(910.98) \left(7 + \frac{7}{3} \right) + (1821.96) \left(\frac{7}{2} \right) + (571.27) \left(\frac{7}{3} \right) + (1528.8) \left(\frac{7}{3} \right) \right]}{4833}$$

$$= 4.09 \text{ ft}$$

13.14 $K_a = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$. Refer to the figure.



$$z = 0 \text{ ft: } \sigma'_a = q K_a = (250)(0.307) = 76.75 \text{ lb/ft}^2; u = 0$$

$$z = 10 \text{ ft: } \sigma'_a = [250 + (10)(115)](0.307) = 429.8 \text{ lb/ft}^2; u = 0$$

$$z = 24 \text{ ft: } \sigma'_a = [250 + (10)(115) + (122 - 62.4)(14)](0.307) = 685.96 \text{ lb/ft}^2$$

$$u = (62.4)(14) = 873.6 \text{ lb/ft}^2$$

Area No.	Area
1	$(76.75)(24) = 1,842$
2	$\left(\frac{1}{2}\right)(10)(429.8 - 76.75) = 1,765.25$
3	$(14)(429.8 - 76.75) = 4,942.7$
4	$\left(\frac{1}{2}\right)(14)(685.96 - 429.8) = 1,793.12$
5	$\left(\frac{1}{2}\right)(14)(873.6) = 6,115.20$

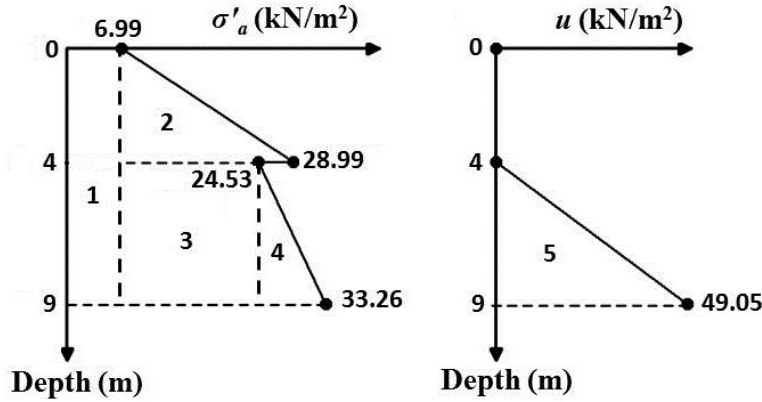
$$P_a = \Sigma 16,458 \text{ lb/ft}$$

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(1842)\left(\frac{24}{2}\right) + (1765.25)\left(14 + \frac{10}{3}\right) + (4942.7)\left(\frac{14}{2}\right) + (1793.12)\left(\frac{14}{3}\right) + (6115.2)\left(\frac{14}{3}\right) \right]}{16458} = 7.55 \text{ ft}$$

$$13.15 \quad K_{a(1)} = \tan^2 \left(45 - \frac{30}{2} \right) = 0.333; \quad K_{a(2)} = \tan^2 \left(45 - \frac{34}{2} \right) = 0.282.$$

Refer to the figure.



$$z = 0 \text{ m:} \quad \sigma'_a = q K_{a(1)} = (21)(0.333) = 6.99 \text{ kN/m}^2; \quad u = 0$$

$$z = 4 \text{ m:} \quad \sigma'_a = [(16.5)(4) + 21](0.333) = 28.99 \text{ kN/m}^2$$

$$\sigma'_a = [(16.5)(4) + 21](0.282) = 24.53 \text{ kN/m}^2$$

$$u = 0$$

$$z = 9 \text{ m:} \quad \sigma'_a = [(16.5)(4) + (20.2 - 9.81)(5)](0.282) = 33.26 \text{ kN/m}^2$$

$$u = (9.81)(5) = 49.05 \text{ kN/m}^2$$

Area No.	Area
1	$(6.99)(9) = 62.91$
2	$\left(\frac{1}{2}\right)(4)(28.99 - 6.99) = 44$
3	$(5)(24.53 - 6.99) = 87.7$
4	$\left(\frac{1}{2}\right)(5)(33.26 - 24.53) = 21.82$
5	$\left(\frac{1}{2}\right)(5)(49.05) = 122.62$

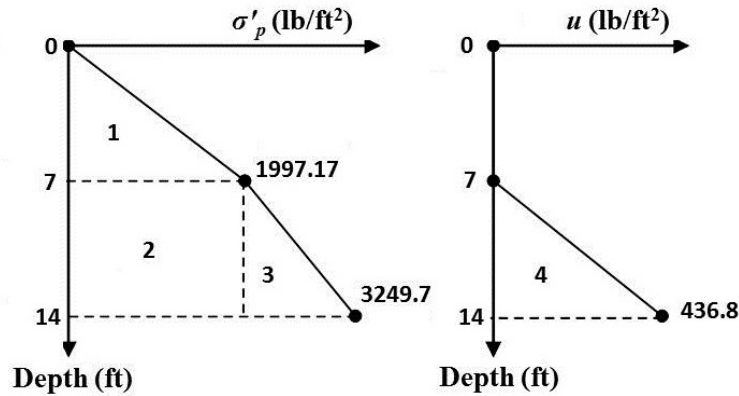
$$P_a = \Sigma 339.05 \text{ kN/m}$$

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(62.91) \left(\frac{9}{2} \right) + (44) \left(5 + \frac{4}{3} \right) + (87.7) \left(\frac{5}{2} \right) + (21.82) \left(\frac{5}{3} \right) + (122.62) \left(\frac{5}{3} \right) \right]}{339.05}$$

$$= 3.01 \text{ m}$$

13.16 $K_p = \tan^2\left(45 + \frac{28}{2}\right) = 2.77$. Refer to the figure.



$z = 0$ ft: $\sigma'_p = 0$; $u = 0$

$z = 7$ ft: $\sigma'_p = \gamma_1 z K_p = (103)(7)(2.77) = 1997.17 \text{ lb/ft}^2$; $u = 0$

$z = 14$ ft: $\sigma'_p = [(103)(7) + (127 - 62.4)(7)](2.77) = 3249.7 \text{ lb/ft}^2$
 $u = (62.4)(7) = 436.8 \text{ lb/ft}^2$

Area No.	Area
1	$(\frac{1}{2})(7)(1997.17) = 6,990.09$
2	$(7)(1997.17) = 13,980.19$
3	$(\frac{1}{2})(7)(3249.7 - 1997.17) = 4,383.8$
4	$(\frac{1}{2})(7)(436.8) = 1,528.8$

$P_a = \Sigma 26,883 \text{ lb/ft}$

Location of the resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(6990.09) \left(7 + \frac{7}{3} \right) + (13980.19) \left(\frac{7}{2} \right) + (4383.8) \left(\frac{7}{3} \right) + (1528.8) \left(\frac{7}{3} \right) \right]}{26883} = 4.76 \text{ ft}$$

13.17 a. Use Table 13.2: For $\alpha = 10^\circ$ and $\phi' = 36^\circ$, $K_a = 0.270$

$\sigma'_a = \gamma z K_a = (19)(4)(0.270) = 20.52 \text{ kN/m}^2$

b. Equation (13.24):

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} (0.27)(19)(4)^2 = \mathbf{41.04 \text{ kN/m}}$$

Location and Direction of Resultant: At a distance of $H/3 = 4/3 = 1.33 \text{ m}$ above the bottom of the wall inclined at an angle $\alpha = 10^\circ$ to the horizontal.

13.18 a. Use Table 13.3: For $\alpha = 10^\circ$ and $\phi' = 36^\circ$, $K_p = 3.598$

$$\sigma'_a = \gamma z K_p = (19)(4)(3.598) = 273.44 \text{ kN/m}^2$$

b. Equation (13.25):

$$P_p = \frac{1}{2} \gamma H^2 K_p = \frac{1}{2} (19)(4)^2 (3.598) = \mathbf{546.89 \text{ kN/m}}$$

Location and Direction of Resultant: At a distance of $H/3 = 4/3 = 1.33 \text{ m}$ above the bottom of the wall inclined at an angle $\alpha = 10^\circ$ to the horizontal.

13.19 a. $H = 5 \text{ m}$; $c_u = 17 \text{ kN/m}^2$; $\gamma = 21 \text{ kN/m}^2$; $\phi = 0$

$$K_a = \tan^2 \left(45 - \frac{\phi}{2} \right) = 1; \sigma'_a = \gamma z - 2c_u$$

At the top ($z = 0 \text{ m}$):

$$\sigma'_a = -2c_u = (-2)(17)$$

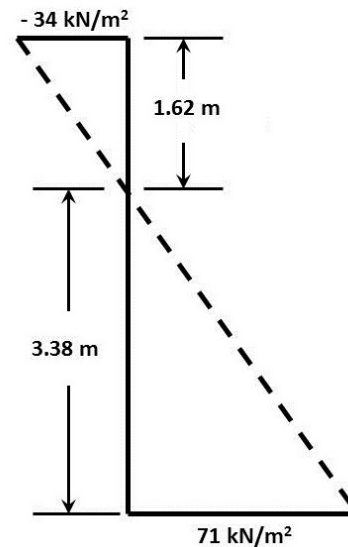
$$= -34 \text{ kN/m}^2$$

At the bottom ($z = 5 \text{ m}$):

$$\sigma'_a = (21)(5) - (2)(17)$$

$$= 71 \text{ kN/m}^2$$

The pressure diagram is shown.



b. Eq. (13.41): $z_o = \frac{2c_u}{\gamma} = \frac{(2)(17)}{21} = \mathbf{1.62 \text{ m}}$

c. Eq. (13.43): $P_a = \frac{1}{2}\gamma H^2 - 2c_u H = \frac{1}{2}(21)(5)^2 - (2)(17)(5) = \mathbf{92.5 \text{ kN/m}}$

d. Eq. (13.45):

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H + \frac{2c_u^2}{\gamma}$$

$$= \frac{1}{2}(21)(5)^2 - (2)(17)(5) + \frac{(2)(17)^2}{21} = \mathbf{120 \text{ kN/m}}$$

Resultant measured from the bottom:

$$\bar{z} = \frac{H - z_o}{3} = \frac{5 - 1.62}{3} = 1.126 \text{ m} \approx \mathbf{1.13 \text{ m}}$$

13.20 a. $\sigma_a = \sigma_o K_a - 2c\sqrt{K_a}$

$$\sigma_o = \gamma z + q; K_a = 1.$$

At $z = 0$ ft:

$$\sigma_o = 11 \text{ kN/m}^2$$

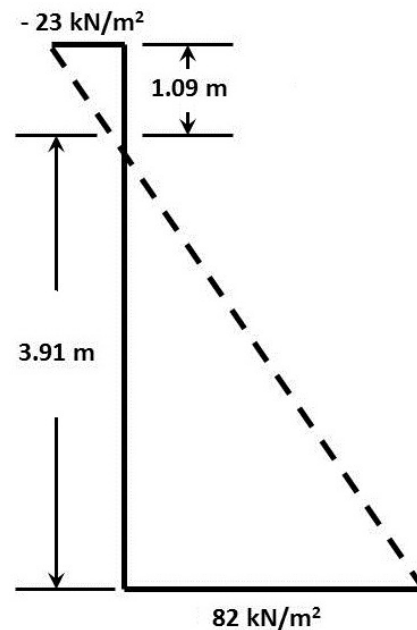
$$\sigma_a = 11 - (2)(17) = -23 \text{ kN/m}^2$$

At $z = 5$ m:

$$\sigma_o = (21)(5) + 11 = 116 \text{ kN/m}^2$$

$$\sigma_a = 116 - (2)(17) = 82 \text{ kN/m}^2$$

The pressure diagram is shown.



b. $\sigma_a = 0; (\gamma z_o + q) - 2c = 0$

$$z_o = \frac{2c_u - q}{\gamma} = \frac{34 - 11}{21} = \mathbf{1.09 \text{ m}}$$

c. Referring to the diagram in Part a,

$$P_a = \frac{1}{2}(3.91)(82) - \frac{1}{2}(23)(1.09) = \mathbf{147.8 \text{ kN/m}}$$

d. $P_a = \frac{1}{2}(3.91)(82) = \mathbf{160.3 \text{ kN/m}}$

Location of the resultant from the bottom of the wall: $\frac{3.91}{3} = \mathbf{1.3 \text{ m}}$

13.21 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{28}{2}\right) = 0.361; \sqrt{K_a} = 0.6.$ Eq. (13.44):

$$P_a = \frac{1}{2}K_a\gamma H^2 - 2\sqrt{K_a}c'H + \frac{2c'^2}{\gamma}$$

$$= \frac{1}{2}(0.361)(122)(33)^2 - (2)(0.6)(750)(33) + \frac{(2)(750)^2}{122} = \mathbf{3502.18 \text{ lb/ft}}$$

13.22 Use Eqs. (13.53) and (13.54). $\alpha = 0; \theta = 12^\circ; \phi' = 34^\circ; \gamma = 119 \text{ lb/ft}^3; H = 32 \text{ ft}$

Part	δ' (deg)	K_a [Eq. (13.54)]	$P_a = \frac{1}{2}K_a\gamma H^2$ [Eq. (13.53)]
1	14	0.3511	21,392 lb/ft
2	21	0.3509	21,381 lb/ft

P_a is located at a vertical distance of $32/3 = 10.67 \text{ ft}$ above the bottom of the wall and is inclined at an angle δ' to the normal drawn to the back face of the wall.

13.23 a. $\phi' = 38^\circ; \psi = 90 - \theta - \delta' = 90 - 5 - 20 = 65^\circ$

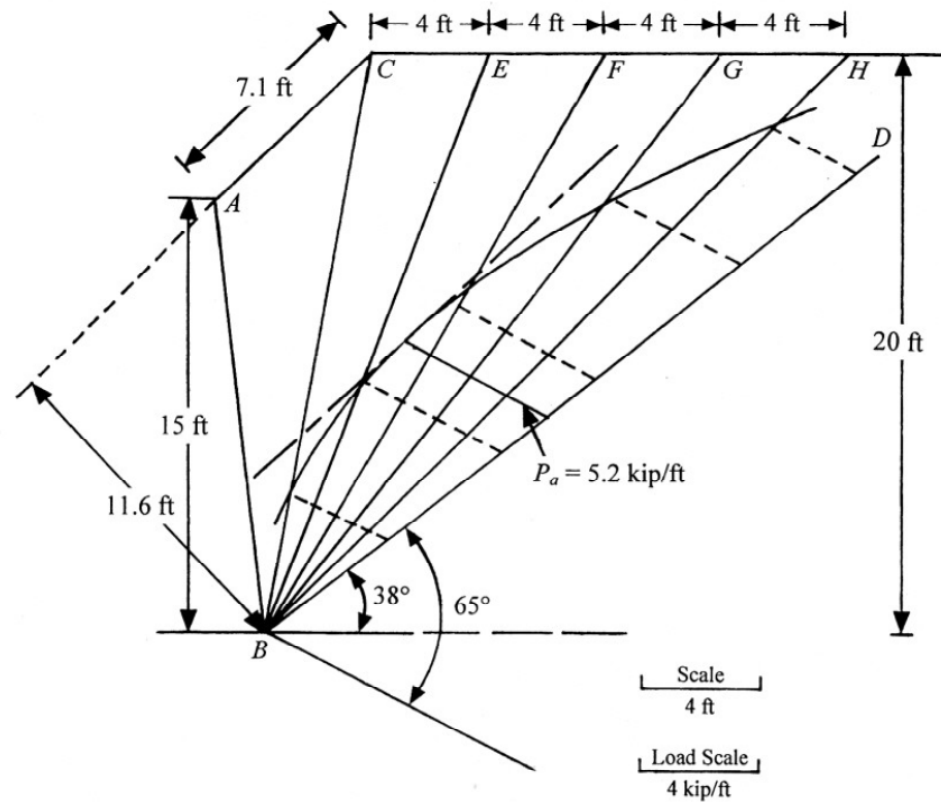
$$\text{Weight of wedge } ABC = \frac{1}{2}(11.6)(7.1)(\underbrace{128}_{\gamma}) = 5271 \text{ lb/ft} = 5.271 \text{ kip/ft}$$

The weight of each of the wedges

$$CBE, EBF, FBG, GBH = \frac{1}{2}(20)(4)(128) = 5120 \text{ lb/ft} = 5.12 \text{ kip/ft}$$

Wedge	Weight (kip/ft)
<i>ABC</i>	5.271
<i>ABE</i>	$5.271 + 5.12 = 10.391$
<i>ABF</i>	$10.391 + 5.12 = 15.511$
<i>ABG</i>	$15.511 + 5.12 = 20.631$
<i>ABH</i>	$20.631 + 5.12 = 25.751$

The graphical construction is shown. $P_a = 5.2 \text{ kip/ft}$



b. $\gamma = \frac{(1680)(9.81)}{1000} = 16.48 \text{ kN/m}^3; \phi' = 30^\circ; \psi = 90 - 10 - 30 = 50^\circ$

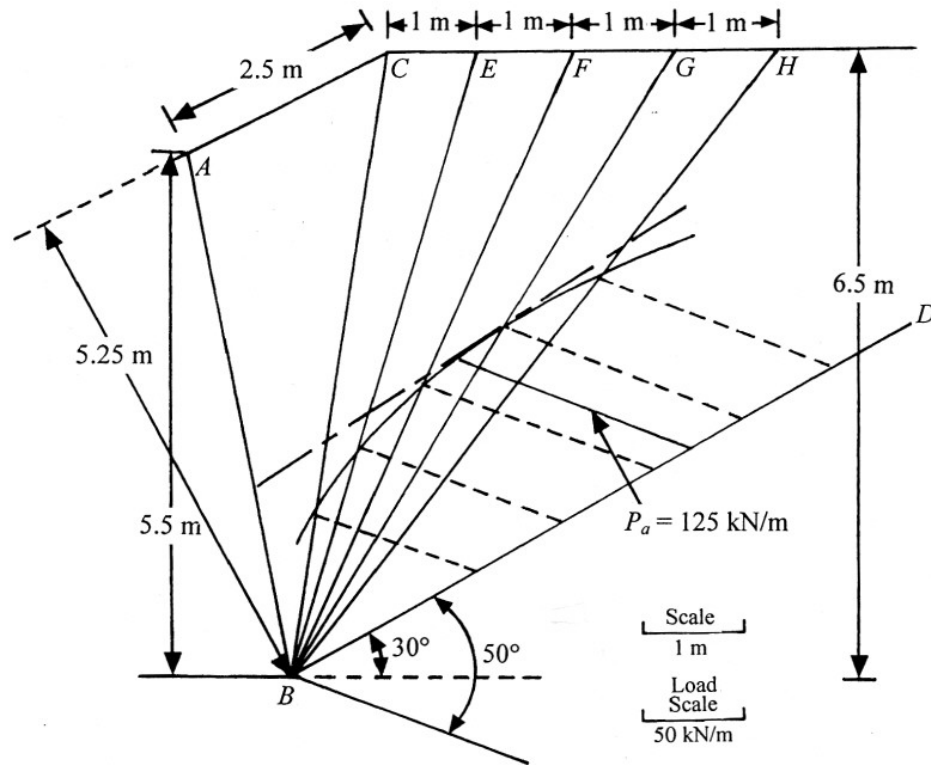
$$\text{Weight of wedge } ABC = \frac{1}{2}(5.25)(2.5)(16.48) = 108.15 \text{ kN/m}$$

The weight of each of the wedges

$$CBE, EBF, FBG, GBH = \frac{1}{2}(1)(6.5)(16.48) = 53.56 \text{ kN/m}$$

Wedge	Weight (kip/ft)
<i>ABC</i>	108.15
<i>ABE</i>	$108.15 + 53.56 = 161.71$
<i>ABF</i>	$161.71 + 53.56 = 215.27$
<i>ABG</i>	$215.27 + 53.56 = 268.83$
<i>ABH</i>	$268.83 + 53.56 = 322.39$

The graphical construction is shown. $P_a = 125 \text{ kN/m}$



13.24 From Eqs. (13.66) and (13.67), $\theta^* = \theta + \bar{\beta}$ and $\alpha^* = \alpha + \bar{\beta}$.

$$\bar{\beta} = \tan^{-1} \left(\frac{k_h}{1 - k_v} \right) = \tan^{-1} \left(\frac{0.1}{1 - 0} \right) = 5.71^\circ$$

$$\theta^* = 9^\circ + 5.71^\circ = 14.71^\circ$$

$$\alpha^* = 12^\circ + 5.71^\circ = 17.71^\circ$$

$$P_a(\theta^*, \alpha^*) = \frac{1}{2} \gamma H^2 K_a$$

$$\frac{\delta'}{\phi'} = \frac{2}{3}$$

From Table 13.5, for $\theta^* = 14.71^\circ$ and $\alpha^* = 17.71^\circ$, the value of $K_a \approx 0.582$.

From Eq. (13.70):

$$P_{ae} = P_a(\theta^*, \alpha^*)(1 - k_v) \left[\frac{\cos^2(\theta + \bar{\beta})}{\cos \theta \cos^2 \bar{\beta}} \right]$$

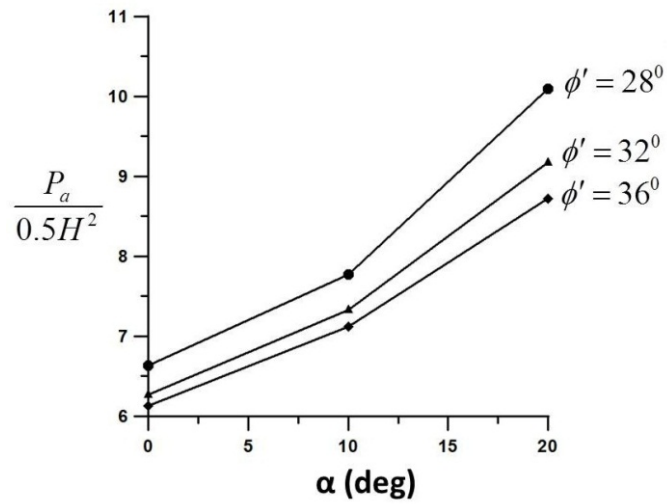
$$= \left[\left(\frac{1}{2} \right) (0.582) (19) (6)^2 \right] (1 - 0) \left[\frac{\cos^2(9 + 5.71)}{\cos(9) \cos^2(5.71)} \right] = \mathbf{190.41 \text{ kN/m}}$$

CRITICAL THINKING PROBLEM

13.C.1 Refer to Table A.1 to prepare the following table:

$\theta = 10^\circ$	$\gamma \text{ (kN/m}^3\text{)}$	ϕ'	$K_{a(R)}$	$K_{a(R)}\gamma$
$\alpha = 0^\circ$	16.5	28°	0.402	6.633
	17.7	32°	0.354	6.265
	19.5	36°	0.314	6.123
$\alpha = 10^\circ$	16.5	28°	0.471	7.771
	17.7	32°	0.414	7.327
	19.5	36°	0.365	7.117
$\alpha = 20^\circ$	16.5	28°	0.612	10.09
	17.7	32°	0.518	9.168
	19.5	36°	0.447	8.716

The graph of $\frac{P_a}{0.5H^2}$ versus backfill inclination angle, α , is shown on the next page.



The above chart shows that for any backfill inclination α and any wall height H , the active force P_a per unit length of the wall decreases as the soil friction angle (or the compacted unit weight) increases. For a desired level of P_a (at a given α), a compaction unit weight could be estimated from the chart for field specification.

Chapter 14

- 14.1 $P_p = \frac{1}{2}K_p\gamma H^2$; $K_p = K_{p(\delta'=0)}R$. With $\phi' = 30^\circ$, $\theta = 15^\circ$, and $\alpha = 0$, the value of $K_{p(\delta'=0)} = 2.34$ (Table 14.2). With $\theta = 15^\circ$, $\delta' = 18^\circ$, $\delta'/\phi' = 18/30 = 0.6$, the value of R is 1.55 (Table 14.3). So,

$$P_p = \frac{1}{2}(2.34 \times 1.55)(17.8)(5)^2 \approx \mathbf{807 \text{ kN/m}}$$

- 14.2 $P_p = \frac{1}{2}K_p\gamma H^2$. From Figure 14.5, for $\phi' = 35^\circ$ and $\delta' = 23.33^\circ$, $\delta'/\phi' = 0.666$, the value of K_p is about 6.2. So,

$$P_p = \frac{1}{2}(6.2)(112)(14)^2 = \mathbf{68,051 \text{ lb/ft}}$$

- 14.3 $P_p = \frac{1}{2}K_p\gamma H^2$. From Figure 14.4, for $\phi' = 30^\circ$ and $\delta' = 20^\circ$, the value of K_p is about 5.5. So,

$$P_p = \frac{1}{2}(5.5)(19)(4)^2 = \mathbf{836 \text{ kN/m}}$$

- 14.4 From Table 14.2, for $\phi' = 30^\circ$ and $\theta = 0$, the value of $K_{p(\delta'=0)} = 3.0$. For $\theta = 0$ and $\delta'/\phi' = 20/30 = 0.667$, the value of R is about 1.75. So

$$P_p = \frac{1}{2}(3 \times 1.75)(19)(4)^2 \approx \mathbf{798 \text{ kN/m}}$$

- 14.5 From Equation (14.15): $P_p = \frac{1}{2}\gamma H^2 K_p$

$$\text{Step 1: } \frac{\alpha}{\phi'} = \frac{10}{30} = 0.333$$

Step 2: From Figure 14.7: $K_{p(\delta'=\phi')} \approx 8.5$

Step 3: $\frac{\delta'}{\phi'} = \frac{18}{30} = 0.6$

Step 4: From Table 14.4: $R' = 0.811$

Step 5: $K_p = (R')[K_{p(\delta'=\phi')}] = (0.811)(8.5) = 6.893$

$$P_p = \frac{1}{2}(16.8)(4.75)^2(6.893) \approx \mathbf{1306 \text{ kN/m}}$$

14.6 Eq. (14.16): $P_{pe} = \left[\frac{1}{2} \gamma H^2 K_{p\gamma(e)} \right] \frac{1}{\cos \delta'}$

For $k_v = 0$, $k_h = 0.25$, $\delta'/\phi' = 15/30 = 0.5$, the value of $K_{p\gamma(e)} \approx 3.95$.

$$P_{pe} = \left[(0.5)(118)(15)^2(3.95) \right] \frac{1}{\cos 15} = \mathbf{54,286 \text{ lb/ft}}$$

14.7 $n_a = \frac{2.75 \text{ m}}{5.5 \text{ m}} = 0.5$. $\phi' = 40^\circ$; $\delta' = 15^\circ$. Table 14.5: $\frac{P_a}{0.5\gamma H^2} = 0.216$

$$P_a = (0.216)(0.5)(15.8)(5.5)^2 = \mathbf{51.61 \text{ kN/m}}$$

14.8 $n_a = \frac{6.5 \text{ ft}}{21 \text{ ft}} \approx 0.3$; $\frac{c'}{\gamma H} = \frac{255}{(121)(21)} = 0.1$

From Table 14.6 for $\phi' = 25^\circ$ and $\delta' = 15^\circ$, $\frac{P_a}{0.5\gamma H^2} = 0.085$.

$$P_a = (0.085)(0.5)(121)(21)^2 \approx \mathbf{2268 \text{ lb/ft}}$$

$$\begin{aligned}
 14.9 \quad \sigma_a &= 0.65\gamma H \tan^2\left(45 - \frac{\phi'}{2}\right) \\
 &= (0.65)(115)(27) \tan^2\left(45 - \frac{32}{2}\right) \\
 &= \mathbf{620.1 \text{ lb/ft}^2}
 \end{aligned}$$

$$\Sigma M_{B_1} = 0$$

$$A = \left(\frac{1}{6}\right) \left[(620.1)(9) \left(\frac{9}{2}\right) \right] = 4185.6 \text{ lb/ft}$$

$$B_1 = (620.1)(9) - 4185.6 = 1395.3 \text{ lb/ft}$$

$$B_2 = C_1 = \frac{(620.1)(6)}{2} = 1860.3 \text{ lb/ft}$$

$$\Sigma M_{C_2} = 0$$

$$D = \left(\frac{1}{6}\right) \left[(620.1)(12) \left(\frac{12}{2}\right) \right] = 7441.2 \text{ lb/ft}$$

$$C_2 = (620.1)(12) - 7441.2 = 0 \text{ lb/ft}$$

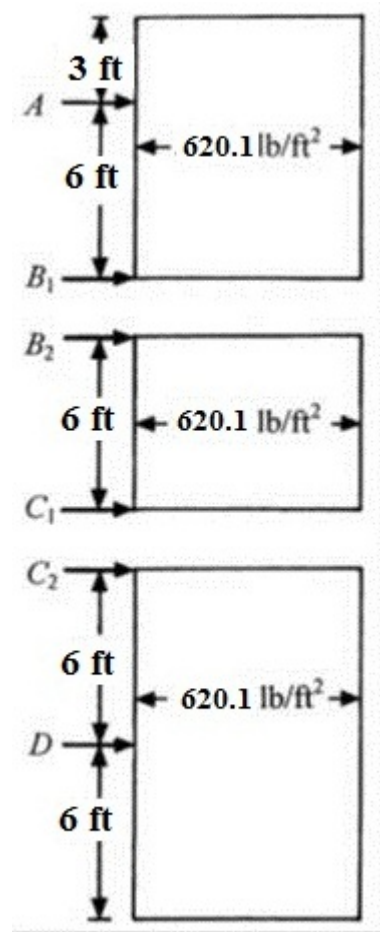
Strut Loads:

$$A = (4185.6)(10) \approx \mathbf{41,856 \text{ lb}}$$

$$\begin{aligned}
 B &= (B_1 + B_2)(10) \\
 &= (1395.3 + 1860.3)(10) \approx \mathbf{32,556 \text{ lb}}
 \end{aligned}$$

$$\begin{aligned}
 C &= (C_1 + C_2)(10) \\
 &= (1860.3 + 0)(10) = \mathbf{18,603 \text{ lb}}
 \end{aligned}$$

$$D = (7441.2)(10) = \mathbf{74,412 \text{ lb}}$$

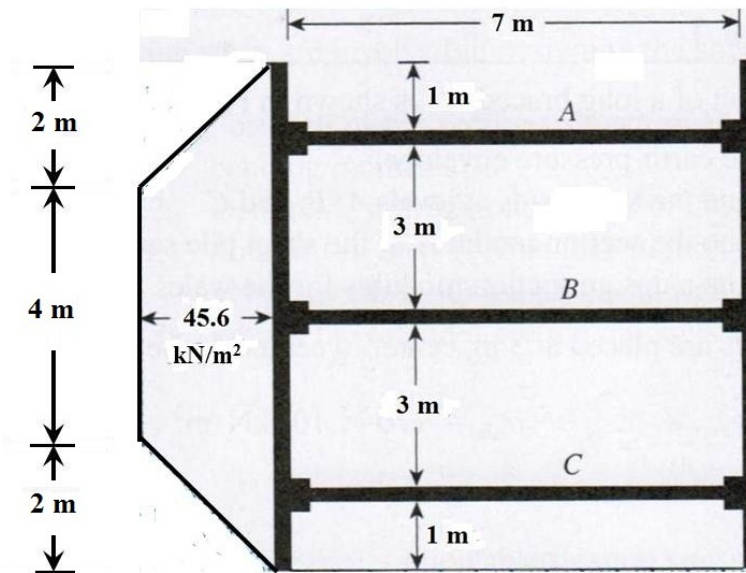


14.10 a. $\frac{\gamma H}{c} = \frac{(19)(8)}{42} = 3.61 < 4$

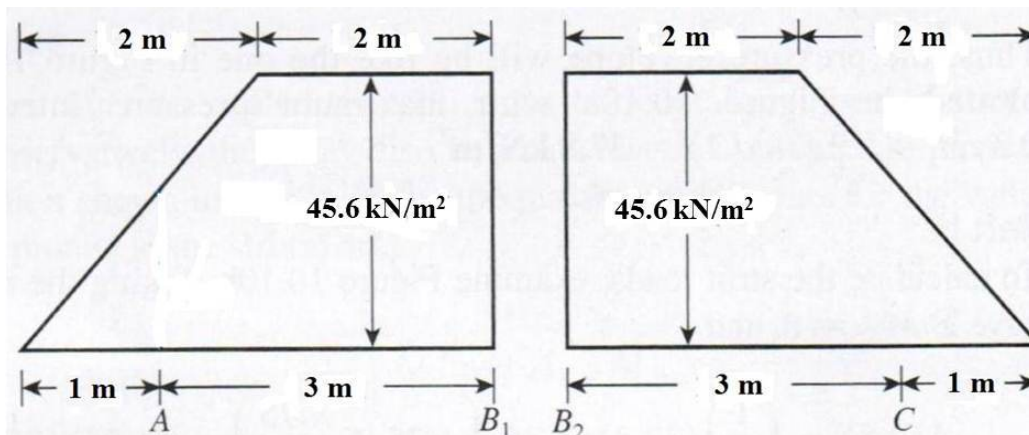
Use Figure 14.14(c) to determine the earth pressure envelope.

Maximum pressure intensity: $\sigma_a = 0.3\gamma H = (0.3)(19)(8) = 45.6 \text{ kN/m}^2$

The pressure envelope is shown below.



b. To determine the strut loads, refer to the following diagram.



$$\sum M_{B_1} = 0$$

$$A(3) - (0.5)(45.6)(2)\left(2 + \frac{2}{3}\right) - (2)(45.6)\left(\frac{2}{2}\right) = 0$$

Therefore, $A = 70.93 \text{ kN/m}$

Also, sum of vertical forces, $\sum V = 0$

$$A + B_1 = (0.5)(45.6)(2) + (45.6)(2)$$

or

$$70.93 + B_1 = 136.8$$

Therefore, $B_1 = 65.87 \text{ kN/m}$

Due to symmetry, $B_2 = 65.87 \text{ kN/m}$

and

$$C = 70.93 \text{ kN/m}$$

$$\text{Strut load at } A = (70.93)(s) = (70.93)(3.5) = \mathbf{248.25 \text{ kN}}$$

$$\text{Strut load at } B = (B_1 + B_2)(s) = (65.87 + 65.87)(3.5) = \mathbf{461.09 \text{ kN}}$$

$$\text{Strut load at } C = (70.93)(3.5) = \mathbf{248.25 \text{ kN}}$$

Chapter 15

15.1 Eq. (15.15):

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$2.75 = \frac{31}{(17.8)(H)(\cos^2 25)(\tan 25)} + \frac{\tan 28}{\tan 25}$$

$$2.75 = \frac{4.546}{H} + 1.14$$

$$H = \mathbf{2.82 \text{ m}}$$

15.2 Eq. (15.16):

$$H_{cr} = \frac{c'}{\gamma \cos^2 \beta (\tan \beta - \tan \phi')} = \frac{300}{115} \frac{1}{(\cos^2 30)(\tan 30 - \tan 21)} = \mathbf{17.97 \text{ ft}}$$

15.3 $\gamma' = 19.2 - 9.81 = 9.39 \text{ kN/m}^3$

Eq. (15.28):

$$\begin{aligned} F_s &= \frac{c'}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi'}{\gamma_{sat} \tan \beta} = \frac{46}{(19.2)(11)(\cos 18)^2 (\tan 18)} + \frac{9.39 \tan 22}{19.2 \tan 18} \\ &= 0.741 + 0.608 \approx \mathbf{1.35} \end{aligned}$$

15.4 $\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.73 + 0.69)(62.4)}{1 + 0.69} = 126.27 \text{ lb/ft}^3$

$$\gamma' = 126.27 - 62.4 = 63.87 \text{ lb/ft}^3$$

$$\begin{aligned}
 F_s &= \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi'}{\gamma_{\text{sat}} \tan \beta} \\
 &= \frac{1000}{(126.27)(27)(\cos^2 28)(\tan 28)} + \frac{63.87 \tan 18}{126.27 \tan 28} \\
 &= 0.707 + 0.309 \approx \mathbf{1.016}
 \end{aligned}$$

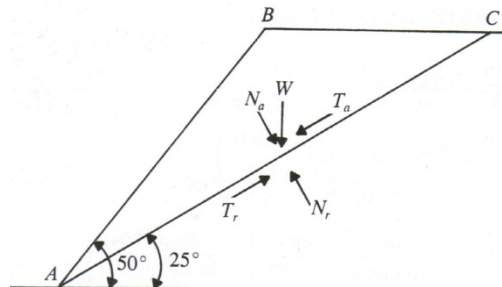
15.5 a. Eq. (15.15):

$$\begin{aligned}
 F_s &= \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta} \\
 F_s &= \frac{21}{\left[\frac{(1950)(9.81)}{1000} \right] (5)(\cos^2 18)(\tan 18)} + \frac{\tan 26}{\tan 18} \\
 &\approx \mathbf{2.25}
 \end{aligned}$$

b. Eq. (15.15):

$$\begin{aligned}
 F_s &= \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta} \\
 1.75 &= \frac{21}{\left[\frac{(1950)(9.81)}{1000} \right] (H)(\cos^2 27)(\tan 27)} + \frac{\tan 26}{\tan 27} \\
 1.75 &= \frac{2.713}{H} + 0.957 \\
 H &= \mathbf{3.42 \text{ m}}
 \end{aligned}$$

15.6 Consider a 1-ft length of the wedge ABC



Eq. (15.29):

$$W = \frac{1}{2} H^2 \gamma (\cot \theta - \cot \beta) = (0.5)(25^2)(115)(\cot 25 - \cot 50) = 46,913 \text{ lb}$$

$$T_a = 46913 \sin 25 = 19826.3 \text{ lb}$$

$$N_a = 46913 \cos 25 = 42517.6 \text{ lb}$$

$$T_r = \frac{1}{F_s} (\overline{AC} c' + N_a \tan \phi') = \frac{1}{F_s} \left[\left(\frac{25}{\sin 25} \right) (400) + (42517.6) \tan 25 \right] = \frac{43488.3}{F_s}$$

Since $T_r = T_a$,

$$\frac{43488.3}{F_s} = 19826.3$$

Therefore, $F_s = \mathbf{2.19}$

15.7 Eq. (15.42):

$$H_{cr} = \frac{4c'}{\gamma} \left[\frac{\sin \beta \cos \phi'}{1 - \cos(\beta - \phi')} \right] = \left[\frac{(4)(28)}{16.5} \right] \left[\frac{(\sin 58)(\cos 14)}{1 - \cos(58 - 14)} \right] = \mathbf{19.9 \text{ m}}$$

$$15.8 \quad F_s = 2.5; \quad c'_d = \frac{c'}{F_s} = \frac{28}{2.5} = 11.2 \text{ kN/m}^2; \quad \phi'_d = \tan^{-1} \left(\frac{\tan 14}{2.5} \right) = 5.69^\circ$$

$$H = \frac{4c'_d}{\gamma} \left[\frac{\sin \beta \cos \phi'_d}{1 - \cos(\beta - \phi'_d)} \right] = \left[\frac{(4)(11.2)}{16.5} \right] \left[\frac{(\sin 58)(\cos 5.69)}{1 - \cos(58 - 5.69)} \right] = \mathbf{5.89 \text{ m}}$$

$$15.9 \quad H = \frac{4c'_d}{\gamma} \left[\frac{\sin \beta \cos \phi'_d}{1 - \cos(\beta - \phi'_d)} \right]; \quad \gamma = 118 \text{ lb/ft}^3$$

$F_{s(\text{assumed})}$	$\phi'_d = \tan^{-1}\left(\frac{\tan 22}{F_s}\right)$ (deg)	$c'_d = \frac{c'}{F_s}$ (lb/ft ²)	β (deg)	H (ft)
2.0	11.42	350	45	49.27
2.5	9.18	280	45	35.02
3.0	7.67	233.3	45	27
2.75	8.36	254.5	45	30.54

$F_s \approx 2.75$

15.10 $\rho = 1800 \text{ kg/m}^3$; $\gamma = \frac{(1800)(9.81)}{1000} = 17.65 \text{ kN/m}^3$; $c' = 20 \text{ kN/m}^2$; $\phi' = 17^\circ$

$$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^\circ; F_s = 2.3; c'_d = \frac{c'}{F_s} = \frac{20}{2.3} = 8.69 \text{ kN/m}^2$$

$$\phi'_d = \tan^{-1}\left(\frac{\tan \phi'}{F_s}\right) = \tan^{-1}\left(\frac{\tan 17}{2.3}\right) = 7.57^\circ$$

$$H = \frac{4c'_d}{\gamma} \left[\frac{\sin \beta \cos \phi'_d}{1 - \cos(\beta - \phi'_d)} \right] = \left[\frac{(4)(8.69)}{17.65} \right] \left[\frac{(\sin 26.57)(\cos 7.57)}{1 - \cos(26.57 - 7.57)} \right] \approx \mathbf{16 \text{ m}}$$

15.11 $m \approx 0.185$ (From Figure 15.13). Eq. (15.48):

$$H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{26}{(18.5)(0.185)} = \mathbf{7.59 \text{ m} - \text{Toe circle}}$$

15.12 $m \approx 0.185$ for $\beta = 55^\circ$ (Figure 15.13)

$$c_d = \frac{c_u}{F_s} = \frac{26}{2.5} = 10.4 \text{ kN/m}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{10.4}{(18.5)(0.185)} = \mathbf{3.04 \text{ m}}$$

15.13 $\beta = \tan^{-1}\left[\frac{1}{2}\right] = 26.56^\circ$. For $\beta = 26.56^\circ$ and $D = 1.5$, $m = 0.16$ (Figure 15.13).

$$c_d = \frac{c_u}{F_s} = \frac{800}{2.5} = 320 \text{ lb/ft}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{320}{(119)(0.16)} = \mathbf{16.8 \text{ ft}}$$

15.14 $H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{800}{(119)(0.16)} = \mathbf{42 \text{ ft}}$. It is a **midpoint circle**.

15.15 a. $D = \frac{14}{10} = 1.4$; $\gamma_{\text{sat}} = 17 \text{ kN/m}^3$. For $\beta = 48^\circ$ and $D = 1.4$, $m = 0.18$.

$$H_{\text{cr}} = \frac{c_u}{\gamma m}; \quad c_u = (10)(17)(0.18) = \mathbf{30.6 \text{ kN/m}^2}$$

b. From Figure 15.13, **midpoint circle**

c. From Figure 15.15, $n \approx 0.85$.

$$\text{Distance} = nH = (0.85)(10) = \mathbf{8.5 \text{ m}}$$

15.16 a. $\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^\circ$

$$\frac{F_s}{\tan \phi'} = \frac{1}{\tan 12} = 4.7$$

From Figure 15.25

$$\frac{c'}{\gamma H_{\text{cr}} \tan \phi'} \approx 0.2$$

Or,

$$H_{\text{cr}} = \frac{750}{(118)(0.2)(\tan 12)} = \mathbf{149.5 \text{ ft}}$$

b. $\frac{F_s}{\tan \phi'} = \frac{1}{\tan 18} = 3.07$. $\beta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$.

Figure 15.25:

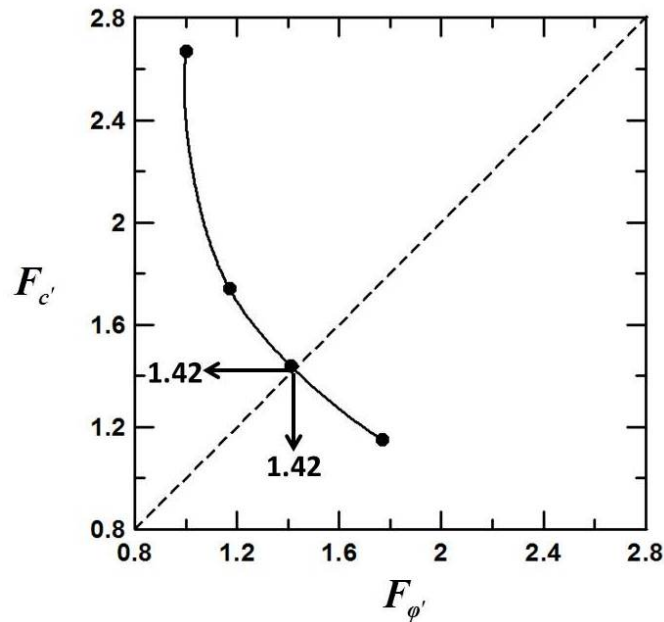
$$\frac{c'}{\gamma H_{\text{cr}} \tan \phi'} \approx 0.22; \quad \frac{30}{(17)(H_{\text{cr}})(\tan 18)} = 0.22$$

$$H_{\text{cr}} = \mathbf{24.7 \text{ m}}$$

$$15.17 \quad \beta = \tan^{-1}\left(\frac{1}{2.5}\right) = 21.8^\circ; \quad \phi' = 14^\circ; \quad c' = 500 \text{ lb/ft}^2; \quad \gamma = 120 \text{ lb/ft}^3.$$

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$	m	$c'_d = m\gamma H$ (lb/ft ²)	$F_{c'} = \frac{c'}{c'_d}$
8	1.77	0.06	432	1.15
10	1.41	0.048	346	1.44
12	1.17	0.04	288	1.74
14	1.00	0.026	187	2.67

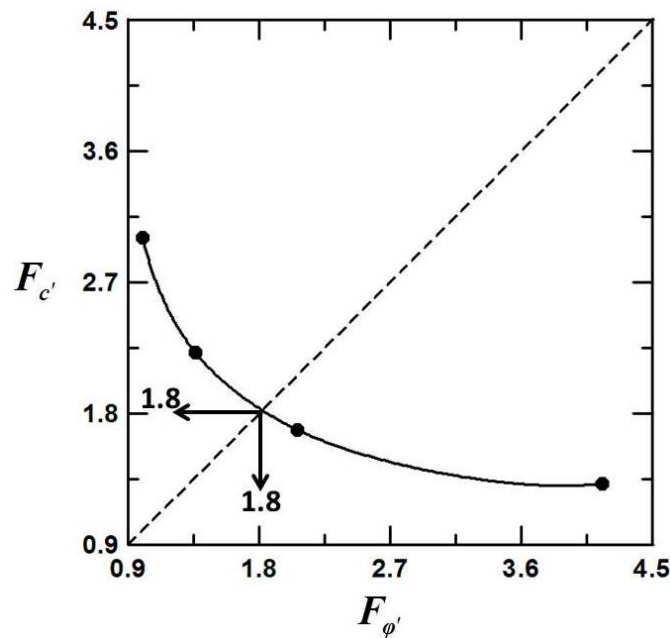
The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown. From the figure, $F_{c'} = F_{\phi'} = F_s = \mathbf{1.42}$.



15.18 $n' = 1$; $\beta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$; $\phi' = 20^\circ$; $c' = 32 \text{ kN/m}^2$; $\gamma = 18 \text{ kN/m}^3$;
 $H = 10 \text{ m}$

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$	m	$c'_d = m\gamma H$ (kN/m ²)	$F_{c'} = \frac{c'}{c'_d}$
5	4.16	0.134	24.12	1.32
10	2.06	0.105	18.9	1.69
15	1.36	0.08	14.4	2.22
20	1.00	0.059	10.62	3.01

The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown below. From the figure, $F_{\phi'} = F_{c'} = F_s = 1.8$.

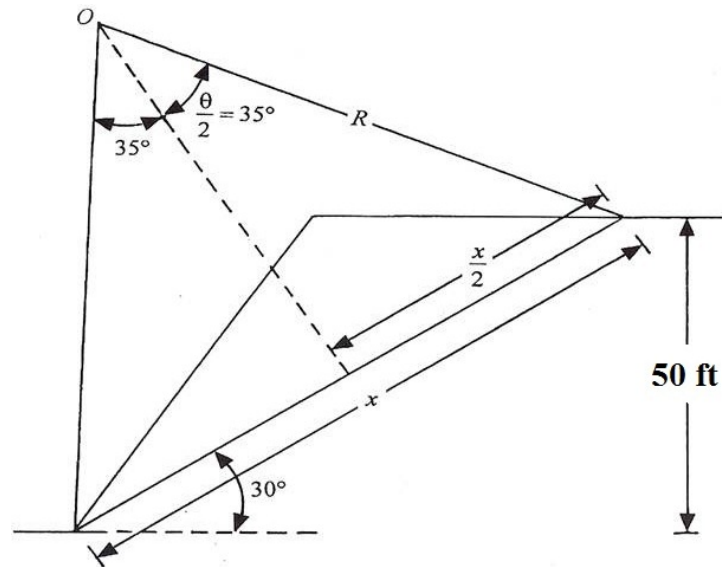


15.19 For $\beta = 21.8^\circ$ and $\frac{c'}{\gamma H \tan \phi'} = \frac{500}{(120)(60) \tan 14} = 0.278$, the value of $\frac{\tan \phi}{F_s}$

obtained from Figure (15.27) is about 0.15. Therefore,

$$F_s = \frac{\tan 14}{0.15} = 1.66$$

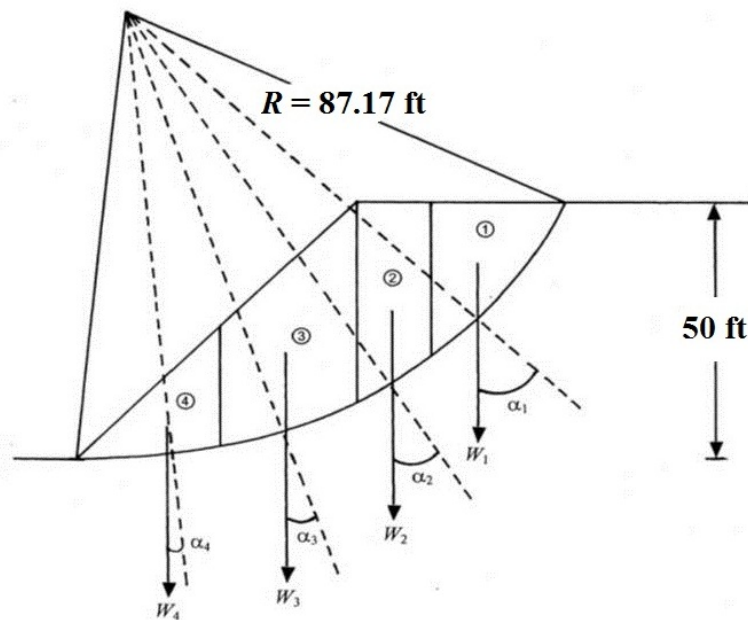
15.20 a. Refer to the figure.



$$\frac{50}{x} = \sin 30^\circ; x = 100 \text{ ft}$$

$$\frac{50}{\text{Radius}, R} = \sin 35^\circ; R = \frac{50}{\sin 35} = 87.17 \text{ ft}$$

With radius $R = 87.17 \text{ ft}$, the trial surface circle has been drawn.



Now the following table can be prepared.

Slice	Area of slices (ft ²)	Weight of slice			
		$W_n = A \times \gamma$ (kip/ft)	α_n (deg)	$W_n \cos \alpha_n$ (kip/ft)	$W_n \sin \alpha_n$ (kip/ft)
1	$\frac{(30)(23)}{2} = 345$	41.75	47	28.47	30.52
2	$\frac{(13)(30 + 41)}{2} = 462$	55.9	32	47.4	29.62
3	$\frac{(25)(41 + 25)}{2} = 825$	99.82	20	93.8	34.14
4	$\frac{(25)(25)}{2} = 312$	37.75	5	37.6	3.29
				$\Sigma 207.27$	$\Sigma 97.57$

$$F_s = \frac{R\theta c' + (\Sigma W_n \cos \alpha_n) \tan \phi'}{\Sigma W_n \sin \alpha_n}$$

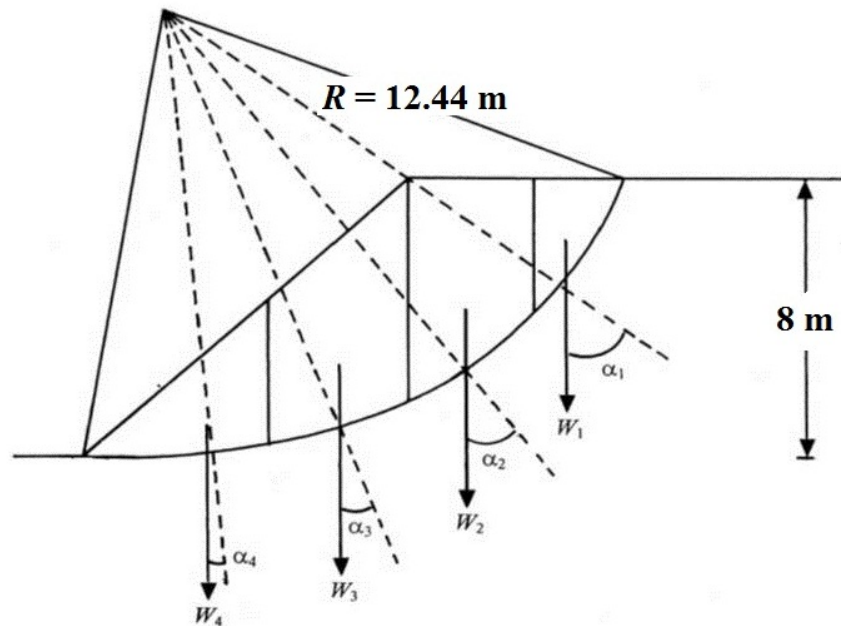
$$= \frac{(87.17) \left[\left(\frac{\pi}{180} \right) (70) \right] (0.65) + (207.27)(\tan 18)}{97.57} = \mathbf{1.4}$$

Note: The accuracy can be increased by increasing the number of slices.

b. As in Part a, $\frac{H}{x} = \sin \alpha$; $x = \frac{H}{\sin \alpha} = \frac{8}{\sin 30} = 16 \text{ m}$

$$\frac{\left(\frac{x}{2} \right)}{R} = \sin \frac{\theta}{2}, \text{ or } \frac{8}{\sin 40} = R = 12.44 \text{ m}$$

With a radius $R = 12.44 \text{ m}$, the trial surface has been drawn on the next page.



The following table can now be prepared.

Slice	Area of slices (m^2)	Weight of slice $W_n = A \times \gamma$ (kN/m)	α_n (deg)	$W_n \cos \alpha_n$ (kN/m)	$W_n \sin \alpha_n$ (kN/m)
1	$\frac{(2.36)(3.85)}{2} = 4.54$	77.18	54	45.36	62.44
2	$\frac{(3.85 + 6.51)(3.25)}{2} = 16.83$	286.11	38	225.45	176.14
3	$\frac{(6.51 + 4.14)(3.55)}{2} = 18.9$	321.3	20	301.92	109.89
4	$\frac{(4.14)(4.74)}{2} = 9.81$	166.77	6	165.85	17.43
				$\Sigma 738.58$	$\Sigma 365.9$

$$F_s = \frac{R\theta c' + (\Sigma W_n \cos \alpha_n) \tan \phi'}{\Sigma W_n \sin \alpha_n}$$

$$= \frac{(12.44) \left[\left(\frac{\pi}{180} \right) (80) \right] (27) + (738.58) (\tan 20)}{365.9} = 2.01$$

Note: The accuracy will improve with smaller slices.

$$15.21 \quad \phi' = 25^\circ; \beta = 26.56^\circ; r_u = 0.5; \frac{c'}{\gamma H} = \frac{20}{(19)(14)} \approx 0.075.$$

Using Table 15.3, the following table can be prepared.

D	m'	n'	$F_s = m' - n'r_u$
Toe circle	1.853	1.430	1.138
1.00	1.872	1.386	1.179
1.25	2.004	1.641	1.183
1.50	2.308	1.914	1.351

$$F_s \approx \mathbf{1.14}.$$

$$15.22 \quad \phi' = 20^\circ; \beta = 18.43^\circ; r_u = 0.5; \frac{c'}{\gamma H} = \frac{475}{(118)(40)} = 0.1$$

Using Table 15.3, the following table can be prepared.

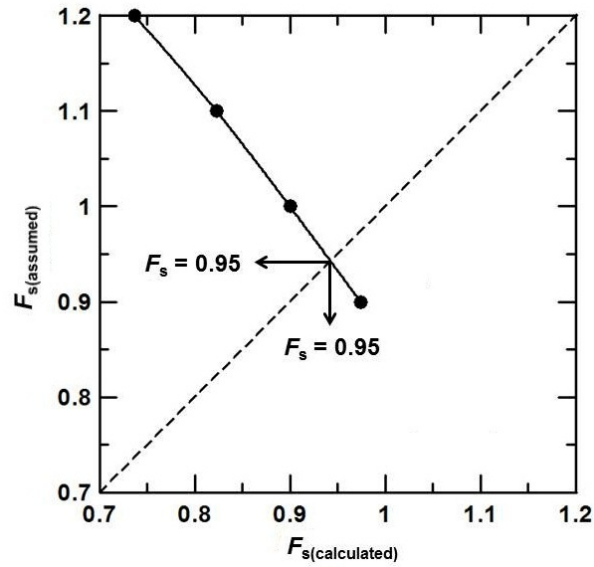
D	m'	n'	$F_s = m' - n'r_u$
Toe circle	2.286	1.588	1.49
1.00	2.421	1.472	1.68
1.25	2.283	1.558	1.50
1.50	2.387	1.742	1.51

$$F_s \approx \mathbf{1.49}.$$

$$15.23 \quad \beta = 26^\circ; \phi' = 21^\circ; r_u = 0.5; \gamma = 19 \text{ kN/m}^3; c' = 21 \text{ kN/m}^2; H = 17 \text{ m}$$

$F_{s(\text{assumed})}$	$\frac{c'}{\gamma H F_s}$	$\phi'_d \text{ (deg)}$	$F_{s(\text{calculated})} = \frac{\tan \phi'}{\tan \phi'_d}$
1.2	0.0541	27.5	0.737
1.1	0.0591	25	0.823
1.0	0.065	23	0.9
0.9	0.0722	21.5	0.974

From the plot on the next page, $F_s \approx \mathbf{0.95}$.



15.24 $\beta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ$; $\phi' = 24^\circ$; $c' = 27 \text{ kN/m}^2$; $\gamma = 17.5 \text{ kN/m}^3$; $r_u = 0.25$;

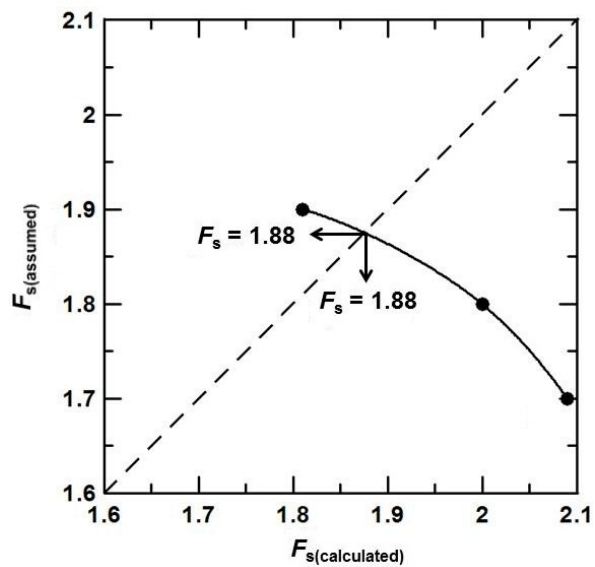
$H = 18 \text{ m}$

a. Spencer's solution (Figure 15.35):

$F_{s(\text{assumed})}$	$\frac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \frac{\tan \phi'}{\tan \phi'_d}$
1.7	0.0504	12	2.09
1.8	0.0476	12.5	2.0
1.9	0.045	13.8	1.81

From the graph,

$F_s \approx 1.88$.



b. Michalowski's solution (Figure 15.36)

$$\beta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ; \phi' = 24^\circ; c' = 27 \text{ kN/m}^2; \gamma = 17.5 \text{ kN/m}^3; r_u = 0.25;$$

$$H = 18 \text{ m}$$

$$\frac{c'}{\gamma H \tan \phi'} = \frac{27}{(17.5)(18) \tan 24} 0.192$$

$$\text{Figure (15.36): } \frac{F_s}{\tan 24} \approx 4.5; \text{ Therefore, } F_s \approx \mathbf{2.0}$$

Chapter 16

16.1 $\phi' = 32^\circ$; $N_c = 44.14$; $N_q = 28.52$; $N_\gamma = 26.87$ (Table 16.1)

$$\begin{aligned} q_{\text{all}} &= \frac{q_u}{F_s} = \frac{1}{3} \left(c'N_c + qN_q + \frac{1}{2} \gamma B N_\gamma \right) \\ &= \frac{1}{3} \left[(21)(44.14) + (1)(17.5)(28.52) + \frac{1}{2} (17.5)(1.5)(26.87) \right] \\ &= \mathbf{593 \text{ kN/m}^2} \end{aligned}$$

16.2 $\phi' = 24^\circ$; $N_c = 23.36$; $N_q = 11.40$; $N_\gamma = 7.08$ (Table 16.1)

$$\begin{aligned} q_{\text{all}} &= \frac{q_u}{F_s} = \frac{1}{4} \left(c'N_c + qN_q + \frac{1}{2} \gamma B N_\gamma \right) \\ &= \frac{1}{4} \left[(1500)(23.36) + (118)(4)(11.40) + \frac{1}{2} (6)(118)(7.08) \right] \\ &= \mathbf{10,732 \text{ lb/ft}^2} \end{aligned}$$

16.3 $\phi' = 0^\circ$; $N_c = 5.7$; $N_q = 1$; $N_\gamma = 0$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{6} (c_u N_c + q N_q) = \frac{1}{6} [(37)(5.7) + (19.5)(0.75)(1)] = \mathbf{37.58 \text{ kN/m}^2}$$

16.4 For a continuous foundation with vertical loading, all inclination factors and shape factors are equal to one. So,

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{F_s} \left(c'N_c \lambda_{cd} + qN_q \lambda_{qd} + \frac{1}{2} \gamma B N_\gamma \lambda_{\gamma d} \right)$$

$\phi' = 32^\circ$; $N_c = 35.49$; $N_q = 23.18$; $N_\gamma = 30.22$ (Table 16.2)

$$\lambda_{cd} = 1 + 0.4 \left(\frac{D_f}{B} \right) = 1 + 0.4 \left(\frac{1}{1.5} \right) = 1.266$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B} = 1 + 2 \tan 32 (1 - \sin 32)^2 \left(\frac{1}{1.5} \right) = 1.184$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{3} \left[(21)(35.49)(1.266) + (17.5)(1)(23.18)(1.184) \right] + \frac{1}{2} (17.5)(1.5)(30.22)(1) = \mathbf{606.8 \text{ kN/m}^2}$$

16.5 $\phi' = 24^\circ$; $N_c = 19.32$; $N_q = 9.60$; $N_\gamma = 9.44$

$$\lambda_{cd} = 1 + 0.4 \left(\frac{4}{6} \right) = 1.266$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B} = 1 + 2 \tan 24 (1 - \sin 24)^2 \left(\frac{4}{6} \right) = 1.209$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{4} \left[(1500)(19.32)(1.266) + (4)(118)(9.60)(1.209) \right] + \frac{1}{2} (118)(6)(9.44)(1) = \mathbf{11,377 \text{ lb/ft}^2}$$

16.6 $q_{\text{all}} = \frac{1}{F_s} \left(c' N_c \lambda_{cd} + q N_q \lambda_{qd} + \frac{1}{2} \gamma B N_\gamma \lambda_{\gamma d} \right)$

$\phi' = 0^\circ$; $N_c = 5.14$; $N_q = 1.0$; $N_\gamma = 0$

$$\lambda_{cd} = 1 + 0.4 \left(\frac{D_f}{B} \right) = 1 + 0.4 \left(\frac{0.75}{2.5} \right) = 1.12$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B} = 1$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{6} [(37)(5.14)(1.12) + (0.75)(19.5) + 0] = \mathbf{37.94 \text{ kN/m}^2}$$

16.7 Eq. (16.12): $q_u = 1.3c'N_c + qN_q + 0.4\gamma'BN_\gamma$

$$\phi' = 32^\circ; N_c = 44.04; N_q = 28.52; N_\gamma = 26.87$$

$$q = \gamma h + \gamma'(D_f - h) = (16 \times 0.9) + (18.9 - 9.81)(1.2 - 0.9) = 17.12 \text{ kN/m}^2$$

$$Q_{\text{all}} = \frac{q_u B^2}{F_s} = \frac{B^2}{F_s} (1.3c'N_c + qN_q + 0.4\gamma'BN_\gamma)$$

$$Q_{\text{all}} = \frac{1.75^2}{3.5} [(1.3)(17)(44.04) + (17.12)(28.52) + (0.4)(18.9 - 9.81)(1.75)(26.87)]$$

$$= \mathbf{1428 \text{ kN}}$$

16.8 $q = \gamma D_f = (16 \times 1.2) = 19.2 \text{ kN/m}^2$

$$D = h - D_f = (1.2 + 0.5) - 1.2 = 0.5 \text{ m}$$

Eq. (16.7):

$$\gamma_{\text{av}} = \frac{1}{B} [\gamma D + \gamma'(B - D)] = \frac{1}{1.75} [(16)(0.5) + (18.9 - 9.81)(1.75 - 0.5)] = 11.06 \text{ kN/m}^3$$

$$F_s = \frac{q_u B^2}{Q_{\text{all}}} = \frac{B^2}{Q_{\text{all}}} (1.3c'N_c + qN_q + 0.4\gamma'BN_\gamma)$$

$$F_s = \frac{1.75^2}{1428} [(1.3)(17)(44.04) + (17.12)(28.52) + (0.4)(11.06)(1.75)(26.87)] = \mathbf{3.58}$$

16.9 $\phi' = 22^\circ$. From Table 16.1, $N_c = 20.27$; $N_q = 9.19$; $N_\gamma = 5.09$

$$\gamma = \frac{(1750)(9.81)}{1000} = 17.16 \text{ kN/m}^3$$

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma_{\text{av}}BN_\gamma$$

$$q = \gamma D_f = (1.5)(17.16) = 25.74 \text{ kN/m}^2$$

$$\gamma_{\text{av}} = \frac{1}{B} [\gamma D + \gamma'(B - D)]$$

$$D = h - D_f = 2.5 - 1.5 = 1 \text{ m}$$

$$\gamma_{\text{sat}} = \frac{(1950)(9.81)}{1000} = 19.13 \text{ kN/m}^3$$

$$\gamma_{\text{av}} = \frac{1}{2}[(17.16)(1) + (19.13 - 9.81)(2 - 1)] = 13.24 \text{ kN/m}^3$$

$$\begin{aligned} q_u &= (1.3)(28)(20.27) + (25.74)(9.19) + (0.4)(13.24)(2)(5.09) \\ &= 1028.3 \text{ kN/m}^2 \end{aligned}$$

$$Q_{\text{all}} = \frac{(q_u)B^2}{F_s} = \frac{(1028.3)(2)^2}{3.5} = \mathbf{1175 \text{ kN}}$$

16.10 From Eq. (16.12): $q_{\text{all}} = \frac{1}{F_s}(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma)$

$$\phi' = 29^\circ; N_c = 34.24; N_q = 19.98; N_\gamma = 16.18 \text{ (Table 16.1)}$$

$$\begin{aligned} q_{\text{all}} &= \frac{1}{4}[(1.3)(900)(34.24) + (4.5)(116)(19.98) + (0.4)(116)(B)(16.18)] \\ &= 12622.6 + 187.7B \end{aligned} \tag{a}$$

$$q_{\text{all}} = \frac{250000}{B^2} \tag{b}$$

From Eqs. (a) and (b), $\frac{250,000}{B^2} = 12,622.6 + 187.7B$. By trial and error,

$$B \approx \mathbf{4.31 \text{ ft}}$$

16.11 $\phi' = 25^\circ$. From Table 16.1, $N_q = 12.72$; $N_\gamma = 8.34$.

$$q_{\text{all}} = \frac{1}{2.5}[(2.1 \times 19)(12.72) + (0.4)(19)(B)(8.34)] = 203 + 25.35B$$

$$q_{\text{all}} = \frac{550}{B^2} = 203 + 25.35B$$

$$B \approx \mathbf{1.5 \text{ m}}$$

$$16.12 \quad q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{F_s} \left(c'N_c \lambda_{cs} \lambda_{cd} + qN_q \lambda_{qs} \lambda_{qd} + \frac{1}{2} \gamma B N_\gamma \lambda_{\gamma s} \lambda_{\gamma d} \right)$$

$$\phi' = 32^\circ; N_c = 35.49; N_q = 23.18; N_\gamma = 30.22 \text{ (Table 16.2)}$$

$$\lambda_{cs} = 1 + \left(\frac{B}{L} \right) \left(\frac{N_q}{N_c} \right) = 1 + \left(\frac{23.18}{35.49} \right) = 1.653$$

$$\lambda_{cd} = 1 + 0.4 \left(\frac{D_f}{B} \right) = 1 + 0.4 \left(\frac{1.2}{1.75} \right) = 1.274$$

$$\lambda_{qs} = 1 + \left(\frac{B}{L} \right) \tan \phi' = 1 + \tan 32 = 1.624$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B} = 1 + 2 \tan 32 (1 - \sin 32)^2 \left(\frac{1}{1.75} \right) = 1.157$$

$$\lambda_{\gamma s} = 1 - 0.4 \left(\frac{B}{L} \right) = 0.6$$

$$\lambda_{\gamma d} = 1$$

$$q = \gamma h + \gamma' (D_f - h) = (16 \times 0.9) + (18.9 - 9.81)(1.2 - 0.9) = 17.12 \text{ kN/m}^2$$

$$Q_{\text{all}} = \frac{(1.75)^2}{3.5} \left[(17)(35.49)(1.653)(1.274) + (17.12)(23.18)(1.624)(1.157) \right. \\ \left. + \frac{1}{2} (18.9 - 9.81)(1.75)(30.22)(0.6)(1) \right]$$

$$\approx \mathbf{1890 \text{ kN/m}^2}$$

$$16.13 \quad \text{a. For vertical load, Eq. (16.44): } q_u = qN_q \lambda_{qd} \lambda_{qs} + \frac{1}{2} \gamma B' N_\gamma \lambda_{\gamma d} \lambda_{\gamma s}$$

$$c' = 0, \phi' = 31^\circ. \text{ Table 16.2: } N_q = 20.63; N_\gamma = 25.99.$$

$$B' = B - 2x = 2.5 - (2)(0.2) = 2.1 \text{ m}; L' = 2.5 \text{ m}$$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'} \right) \tan \phi' = 1 + \left(\frac{2.1}{2.5} \right) \tan 31 = 1.504$$

$$\lambda_{\gamma s} = 1 - 0.4 \left(\frac{B'}{L'} \right) = 1 - 0.4 \left(\frac{2.1}{2.5} \right) = 0.664$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B'} = 1 + 2 \tan 31 (1 - \sin 31)^2 \left(\frac{1}{2.1} \right) = 1.134$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (19)(1)(1.504)(1.134)(20.63) + \frac{1}{2}(19)(2.1)(25.99)(1)(0.664) \\ = 1012.8 \text{ kN/m}^2$$

$$Q_{\text{all}} = \frac{q_u B' L'}{F_s} = \frac{(1012.8)(2.1)(2.5)}{(5)} = \mathbf{1063.4 \text{ kN}}$$

b. $B' = 6 - (2)(0.9) = 4.2 \text{ ft}; L' = 6 \text{ ft}. \phi' = 26^\circ$

$$N_c = 22.25; N_q = 11.85; N_\gamma = 12.54$$

$$q_u = c' N_c \lambda_{cs} \lambda_{cd} + q N_q \lambda_{qs} \lambda_{qd} + \frac{1}{2} \gamma B' N_\gamma \lambda_{\gamma s} \lambda_{\gamma d}$$

$$\lambda_{cs} = 1 + \left(\frac{B'}{L'} \right) \left(\frac{N_q}{N_c} \right) = 1 + \left(\frac{4.2}{6} \right) \left(\frac{11.85}{22.25} \right) = 1.373$$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'} \right) \tan \phi' = 1 + \left(\frac{4.2}{6} \right) \tan 26 = 1.341$$

$$\lambda_{\gamma s} = 1 - 0.4 \left(\frac{B'}{L'} \right) = 1 - 0.4 \left(\frac{4.2}{6} \right) = 0.72$$

$$\lambda_{cd} = 1 + 0.4 \tan^{-1} \left(\frac{D_f}{B'} \right) = 1 + 0.4 \tan^{-1} \left(\frac{4}{4.2} \right) = 1.006$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B'} = 1 + 2 \tan 26 (1 - \sin 26)^2 \left(\frac{4}{4.2} \right) = 1.293$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (900)(22.25)(1.373)(1.006) + (115)(4)(11.85)(1.341)(1.293) \\ + (0.5)(115)(4.2)(12.54)(0.72)(1) \\ = 39,291 \text{ lb/ft}^2 = 39.29 \text{ kip/ft}^2$$

$$Q_{\text{all}} = \frac{(39.29)(4.2)(6)}{5} = \mathbf{198 \text{ kip}}$$

c. $\phi' = 38^\circ$; $c' = 0$. $N_q = 48.93$; $N_\gamma = 78.03$.

$$B' = 1.5 - (2)(0.1) = 1.3 \text{ m}; L' = 1.5 \text{ m}$$

$$\gamma = \frac{(1800)(9.81)}{1000} = 17.66 \text{ kN/m}^3$$

$$\text{For vertical load, Eq. (16.44): } q_u = qN_q \lambda_{qd} \lambda_{qs} + \frac{1}{2} \gamma B' N_\gamma \lambda_{\gamma d} \lambda_{\gamma s}$$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'} \right) \tan \phi' = 1 + \left(\frac{1.3}{1.5} \right) \tan 38 = 1.677$$

$$\lambda_{\gamma s} = 1 - 0.4 \left(\frac{B'}{L'} \right) = 1 - 0.4 \left(\frac{1.3}{1.5} \right) = 0.653$$

$$\lambda_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B'} = 1 + 2 \tan 38 (1 - \sin 38)^2 \left(\frac{1.5}{1.3} \right) = 1.266$$

$$\lambda_{\gamma d} = 1$$

$$\begin{aligned} q_u &= (17.66)(1.5)(1.266)(1.677)(48.93) + \frac{1}{2}(17.66)(1.3)(78.03)(1)(0.653) \\ &= 3336.7 \text{ kN/m}^2 \end{aligned}$$

$$Q_{\text{all}} = \frac{q_u B' L'}{F_s} = \frac{(3336.7)(1.3)(1.5)}{(5)} = \mathbf{1301.3 \text{ kN}}$$

16.14 Eq. (16.57): $q_{u(F)} = q_{u(P)} \left(\frac{B_F}{B_P} \right) = (6800) \left(\frac{5}{2} \right) = 170,000 \text{ lb/ft}^2$

$$Q_{\text{all}} = \frac{A q_{u(F)}}{5} = \left(\frac{170,000}{5} \right) (5)^2 = 85,000 \text{ lb} = \mathbf{85 \text{ kip}}$$

16.15 $q_{u(P)} = 320 \text{ kN/m}^2$. Eq. (16.56): $q_{u(F)} = q_{u(P)}$; $q_{u(F)} = 320 \text{ kN/m}^2$

$$Q_{\text{all}} = \frac{Aq_{u(F)}}{F_s} = \frac{\left(\frac{\pi}{4}\right)(2.5)^2(320)}{4} = 392.7 \text{ kN}$$

CRITICAL THINKING PROBLEM

C.16.1 The footing is placed at a depth of 1.5 m.

Part (a)

$$\underline{B = 1 \text{ m}}$$

$$\text{Eq. (16.49): } q_{\text{net}} = \frac{N_{60}}{0.05} F_d \left[\frac{S_e(\text{mm})}{25} \right]$$

$$2 \times B = 2 \times 1 = 2 \text{ m}$$

N_{60} should be averaged up to a distance of 2 m below the foundation or up to a depth = $1.5 + 2 = 3.5 \text{ m}$

Therefore, $N_{60\text{-avg}} = (12 + 7)/2 \approx 10$; $S_e = 20 \text{ mm}$

$$F_d = 1 + 0.33 \left(\frac{D_f}{B} \right) = 1 + 0.33 \left(\frac{1.5}{1} \right) = 1.495 \leq 1.33 \text{ So, } F_d = 1.33$$

$$q_{\text{net}} = \frac{10}{0.05} (1.33) \left[\frac{20}{25} \right] = 212.8 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(212.8)(1)^2}{3} \approx 71 \text{ kN}$$

$$\underline{B = 1.5 \text{ m}}$$

$$\text{Eq. (16.50): } q_{\text{net}} = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B} \right)^2 F_d \left[\frac{S_e(\text{mm})}{25} \right]$$

$$2 \times B = 2 \times 1.5 = 3 \text{ m}$$

N_{60} should be averaged up to a distance of 3 m below the foundation or up to a depth = $1.5 + 3 = 4.5$ m

Therefore, $N_{60\text{-avg}} = (12 + 7)/2 \approx 10$; $S_e = 20$ mm

$$F_d = 1 + 0.33 \left(\frac{D_f}{B} \right) = 1 + 0.33 \left(\frac{1.5}{1.5} \right) \approx 1.33$$

$$q_{\text{net}} = \frac{8}{0.08} \left(\frac{1.5 + 0.3}{1.5} \right)^2 (1.33) \left[\frac{20}{25} \right] = 153.2 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(153.2)(1.5)^2}{3} \approx 115 \text{ kN}$$

$B = 2$ m

$$\text{Eq. (16.50): } q_{\text{net}} = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B} \right)^2 F_d \left[\frac{S_e(\text{mm})}{25} \right]$$

$$2 \times B = 2 \times 2 = 4 \text{ m}$$

N_{60} should be averaged up to a distance of 4 m below the foundation or up to a depth = $1.5 + 4 = 5.5$ m

Therefore, $N_{60\text{-avg}} = (12 + 7 + 8)/3 = 9$; $S_e = 20$ mm

$$F_d = 1 + 0.33 \left(\frac{D_f}{B} \right) = 1 + 0.33 \left(\frac{1.5}{2} \right) = 1.247$$

$$q_{\text{net}} = \frac{8}{0.08} \left(\frac{2 + 0.3}{2} \right)^2 (1.247) \left[\frac{20}{25} \right] = 114.7 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(114.7)(2)^2}{3} \approx 229 \text{ kN}$$

$B = 3$ m

$$\text{Eq. (16.50): } q_{\text{net}} = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B} \right)^2 F_d \left[\frac{S_e(\text{mm})}{25} \right]$$

$$2 \times B = 2 \times 3 = 6 \text{ m}$$

N_{60} should be averaged up to a distance of 6 m below the foundation or up to a depth $= 1.5 + 6 = 7.5 \text{ m}$

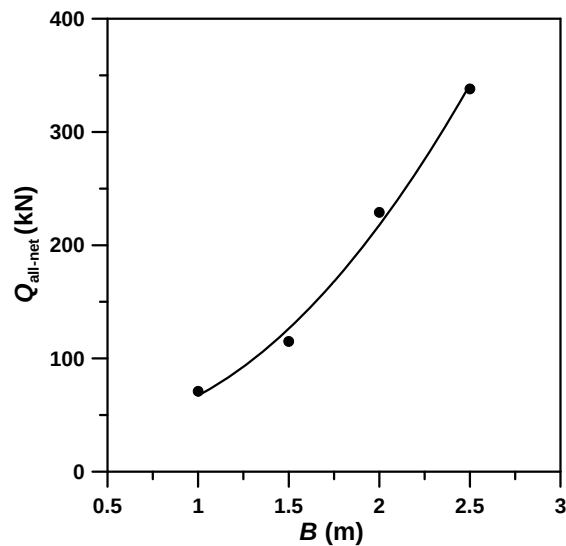
Therefore, $N_{60\text{-avg}} = (12 + 7 + 8 + 19)/4 \approx 12$; $S_e = 20 \text{ mm}$

$$F_d = 1 + 0.33 \left(\frac{D_f}{B} \right) = 1 + 0.33 \left(\frac{1.5}{3} \right) = 1.165$$

$$q_{\text{net}} = \frac{8}{0.08} \left(\frac{3 + 0.3}{3} \right)^2 (1.165) \left[\frac{20}{25} \right] = 112.77 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(112.77)(3)^2}{3} \approx 338 \text{ kN}$$

The design chart is shown below.



Part (b)

For any design footing size B ,

$$Q_{\text{all-net}} \geq Q_{\text{applied}} \dots\dots\dots (1)$$

$$Q_{\text{applied}} = 250 \text{ kN}$$

From the chart it is found that condition (1) is satisfied when $B \approx 2.25 \text{ m}$

Part (c)

$\phi' = 33^\circ$; $c' = 0$; $N_q = 32.33$; $N_\gamma = 31.94$ (Table 16.1); $B = 2.25$ m

$$\begin{aligned} q_{\text{all-net}} &= \frac{q_{u-\text{net}}}{F_s} = \frac{q_u - q}{3} = \frac{1}{3} (1.3c'N_c + qN_q + 0.4\gamma BN_\gamma - q) \\ &= \frac{1}{3} [(1.5)(17)(32.33) + 0.4(17)(2.25)(31.94) - (1.5)(17)] \\ &= 429.2 \text{ kN/m}^2 \end{aligned}$$

$$Q_{\text{all-net}} = q_{\text{all-net}} (B)^2 = (429.2)(2.25)^2 \approx \mathbf{2173 \text{ kN}}$$

Part (d)

Net allowable column load calculated by the Terzaghi's bearing capacity equation (Part c) is significantly higher than that calculated by the method based on N_{60} and limiting settlement value (Part b). In actual design of foundations, both bearing capacity (based on shear strength) and settlement criteria need to be satisfied. It is possible that the allowable column load calculated in Part (c) will change (decrease) if a settlement limit is imposed. Considering the uncertainty in sampling and testing of granular materials for the determination of shear strength, the method based on N_{60} and settlement criteria may be preferable for these conditions.

Chapter 17

17.1 Shelby tube A:

$$\text{Eq. (17.6): } A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(76.2)^2 - (73)^2}{(73)^2} \times 100 = \mathbf{8.96\% \leq 10\%}$$

Samples can be considered undisturbed. Suitable for grain size distribution, Atterberg limits, consolidation and unconfined compression tests.

Shelby tube B:

$$\text{Eq. (17.6): } A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(3.5)^2 - (3.375)^2}{(3.375)^2} \times 100 = \mathbf{7.54\% \leq 10\%}$$

Samples can be considered undisturbed. Suitable for grain size distribution, Atterberg limits, consolidation and unconfined compression tests.

Split spoon sampler:

$$\text{Eq. (17.6): } A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(50.8)^2 - (35)^2}{(35)^2} \times 100 = \mathbf{110\% \geq 10\%}$$

Samples can be considered as highly disturbed. Suitable for grain size distribution and Atterberg limits tests, but not for consolidation and unconfined compression tests.

17.2

Depth (m)	σ'_0 (kN/m ²)	$C_N = \left[\frac{\sigma'_0}{p_a} \right]^{-0.5}$	N_{60}	$(N_1)_{60} = C_N N_{60}$
2	34	1.71	7	$\approx \mathbf{12}$
4	68	1.21	10	$\approx \mathbf{12}$
6	102	0.99	11	$\approx \mathbf{11}$
8	136	0.857	14	$\approx \mathbf{12}$
10	170	0.767	9	$\approx \mathbf{7}$

$$17.3 \quad \phi' = \tan^{-1} \left[\frac{N_{60}}{12.2 + 20.3 \left(\frac{\sigma'_0}{p_a} \right)} \right]; \quad p_a \approx 100 \text{ kN/m}^2$$

Depth (m)	σ'_o (kN/m ²)	p_o (kN/m ²)	N_{60}	ϕ' (deg) [Eq. (17.20)]
2	34	100	7	20.1
4	68	100	10	21.0
6	102	100	11	18.48
8	136	100	14	19.37
10	170	100	9	10.9

Average $\phi' \approx 18^\circ$

17.4 Eq. (17.18): $D_r (\%) = \left[\frac{N_{60} \left(0.23 + \frac{0.06}{D_{50}} \right)^{1.7}}{9} \left(\frac{98}{\sigma'_o} \right) \right]^{0.5} \quad (100)$

Given $\gamma = 15.7 \text{ kN/m}^3$. The following table can now be prepared.

Depth z (m)	$\sigma'_o = \gamma z$ (kN/m ²)	D_{50} (mm)	N_{60}	D_r (%)
1.5	23.55	0.3	9	99.5 $\approx$ 100
3.0	47.1	0.3	10	74.2 $\approx$ 74
4.5	70.65	0.3	14	71.7 $\approx$ 72
6.0	94.2	0.3	18	70.4 $\approx$ 70
7.5	117.75	0.3	20	66.4 $\approx$ 66

17.5 $(N_1)_{60} = C_N N_{60} = \left(\frac{\sigma'_o}{p_a} \right)^{-0.5} N_{60}$; $\phi' = 27.1 + 0.3(N_1)_{60} - 0.00054(N_1)_{60}^2$

Depth (m)	σ'_o (kN/m ²)	N_{60}	C_N	$(N_1)_{60}$	ϕ' (deg)
1.5	$1.5 \times 18 = 27$	8	1.924	$15.4 \approx 15$	31.4
3	$3 \times 18 = 54$	9	1.36	$12.24 \approx 12$	30.6
4.5	$4.5 \times 18 = 81$	11	1.11	$12.2 \approx 12$	30.6
6	$\left[\frac{(5.3 \times 18)}{+ 0.7(18.8 - 9.8)} \right] = 101.7$	12	0.99	$11.89 \approx 12$	30.6
7.5	$101.7 + 1.5(18.8 - 9.8) = 115.2$	15	0.931	$13.96 \approx 14$	31.2
9	$115.2 + 1.5(18.8 - 9.8) = 128.7$	17	0.881	$14.97 \approx 15$	31.4

Average $\phi' \approx 31^\circ$

- 17.6 a. The properties should be averaged over a distance of $2B$ or 4 m below the footing, i.e. up to a depth of 5.5 m.

Therefore, design $N_{60} = (8 + 9 + 11) / 3 = \mathbf{9.33 \approx 9}$

and design ϕ' (deg) = $(31.4 + 30.6 + 30.6) / 3 = \mathbf{30.8}$

b. $F_d = 1 + 0.33 \left(\frac{D_f}{B} \right) = 1 + (0.33) \left(\frac{1.5}{2} \right) = 1.247$

$$q_{\text{net}} = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B} \right)^2 F_d \left(\frac{S_e}{25} \right) = \frac{9}{0.08} \left(\frac{2 + 0.3}{2} \right)^2 (1.247) \left(\frac{25}{25} \right) = 185.53 \text{ kN/m}^2$$

$$Q_{\text{net}} = q_{\text{net}} \times B^2 = 185.53 \times 4 = \mathbf{742 \text{ kN}}$$

17.7 Eq. (17.39): $\frac{\left(\frac{q_c}{p_a} \right)}{N_{60}} = 7.64 D_{50}^{0.26}$. Use $p_a \approx 100 \text{ kN/m}^2$.

Depth (m)	N_{60}	D_{50} (mm)	q_c (kN/m ²)
1.5	8	0.28	4,390
3	9	0.28	4,938
4.5	11	0.28	6,036
6	12	0.28	6,585
7.5	15	0.28	8,231
9	17	0.28	9,328

17.8 Eq. (17.33): $E_s = 3q_c$

Using q_c from Problem 17.7:

Depth (m)	q_c (kN/m ²)	E_s (kN/m ²)
1.5	4,390	13,170
3	4,938	14,814
4.5	6,036	18,108
6	6,585	19,755
7.5	8,231	24,693
9	9,328	27,984

17.9 Eq. (17.35): $c_u = \frac{q_c - \sigma_c}{N_k}; N_k \approx 18.3$

$$c_u = \frac{26000 - (22)(118)}{18.3} = \mathbf{1278.9 \text{ lb/ft}^2}$$

17.10 From Eq. (17.43):

$$\text{Recovery ratio, } R = \left(\frac{4.5}{8} \right) (100) = \mathbf{56.25\%}$$