STRUCTURAL ANALYSIS

EIGHTH EDITION

SOLUTION MANUAL

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1–1. The floor of a heavy storage warehouse building is made of 6-in.-thick stone concrete. If the floor is a slab having a length of 15 ft and width of 10 ft, determine the resultant force caused by the dead load and the live load.

From Table 1–3

$$\text{DL} = [12\ \text{lb/ft}^2 \cdot \text{in.}(6\ \text{in.})]\ (15\ \text{ft})(10\ \text{ft}) = 10{,}800\ \text{lb}$$

From Table 1–4

$$\text{LL} = (250\ \text{lb/ft}^2)(15\ \text{ft})(10\ \text{ft}) = 37{,}500\ \text{lb}$$

Total Load

$$F = 48{,}300\ \text{lb} = 48.3\ \text{k}$$ **Ans.**

1–2. The floor of the office building is made of 4-in.-thick lightweight concrete. If the office floor is a slab having a length of 20 ft and width of 15 ft, determine the resultant force caused by the dead load and the live load.

From Table 1–3

$$\text{DL} = [8\ \text{lb/ft}^2 \cdot \text{in.}\ (4\ \text{in.})]\ (20\ \text{ft})(15\ \text{ft}) = 9600\ \text{lb}$$

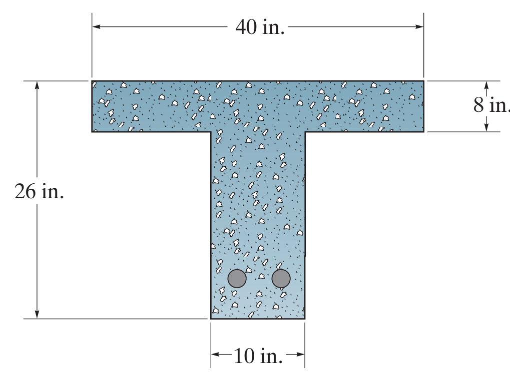
From Table 1–4

$$\text{LL} = (50\ \text{lb/ft}^2)(20\ \text{ft})(15\ \text{ft}) = 15{,}000\ \text{lb}$$

Total Load

$$F = 24{,}600\ \text{lb} = 24.6\ \text{k}$$ **Ans.**

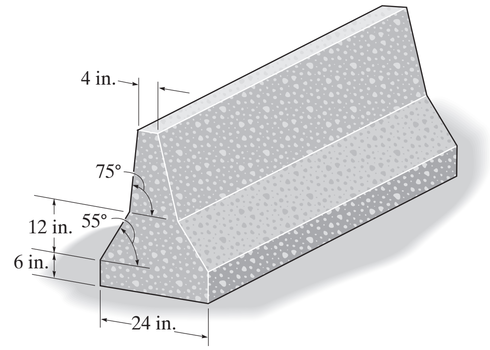
1–3. The T-beam is made from concrete having a specific weight of 150 lb/ft^3. Determine the dead load per foot length of beam. Neglect the weight of the steel reinforcement.



$$w = (150\ \text{lb/ft}^3)\ [(40\ \text{in.})(8\ \text{in.}) + (18\ \text{in.})\ (10\ \text{in.})] \left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right)$$

$$w = 521\ \text{lb/ft}$$ **Ans.**

***1–4.** The "New Jersey" barrier is commonly used during highway construction. Determine its weight per foot of length if it is made from plain stone concrete.

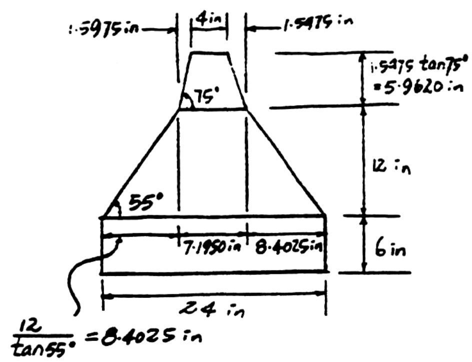


$$\text{Cross-sectional area} = 6(24) + \left(\frac{1}{2}\right)(24 + 7.1950)(12) + \left(\frac{1}{2}\right)(4 + 7.1950)(5.9620)$$

$$= 364.54 \text{ in}^2$$

Use Table 1–2.

$$w = 144 \text{ lb/ft}^3 \, (364.54 \text{ in}^2)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) = 365 \text{ lb/ft} \qquad \textbf{Ans.}$$



1–5. The floor of a light storage warehouse is made of 150-mm-thick lightweight plain concrete. If the floor is a slab having a length of 7 m and width of 3 m, determine the resultant force caused by the dead load and the live load.

From Table 1–3

$$\text{DL} = [0.015 \text{ kN/m}^2 \cdot \text{mm} \, (150 \text{ mm})] \, (7 \text{ m}) \, (3 \text{ m}) = 47.25 \text{ kN}$$

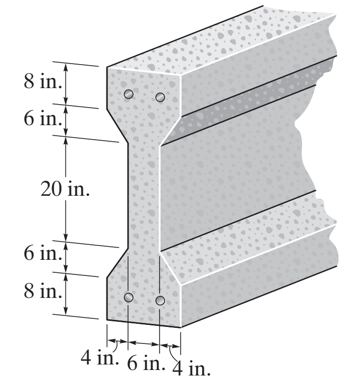
From Table 1–4

$$\text{LL} = (6.00 \text{ kN/m}^2) \, (7 \text{ m}) \, (3 \text{ m}) = 126 \text{ kN}$$

Total Load

$$F = 126 \text{ kN} + 47.25 \text{ kN} = 173 \text{ kN} \qquad \textbf{Ans.}$$

1–6. The prestressed concrete girder is made from plain stone concrete and four $\frac{3}{4}$-in. cold form steel reinforcing rods. Determine the dead weight of the girder per foot of its length.



$$\text{Area of concrete} = 48(6) + 4\left[\frac{1}{2}(14 + 8)(4)\right] - 4(\pi)\left(\frac{3}{8}\right)^2 = 462.23 \text{ in}^2$$

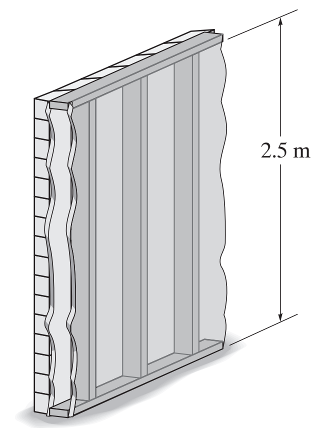
$$\text{Area of steel} = 4(\pi)\left(\frac{3}{8}\right)^2 = 1.767 \text{ in}^2$$

From Table 1–2,

$$w = (144 \text{ lb/ft}^3)(462.23 \text{ in}^2)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) + 492 \text{ lb/ft}^3(1.767 \text{ in}^2)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$= 468 \text{ lb/ft}$$ **Ans.**

1–7. The wall is 2.5 m high and consists of 51 mm × 102 mm studs plastered on one side. On the other side is 13 mm fiberboard, and 102 mm clay brick. Determine the average load in kN/m of length of wall that the wall exerts on the floor.



Use Table 1–3.

For studs

Weight = 0.57 kN/m² (2.5 m) = 1.425 kN/m

For fiberboard

Weight = 0.04 kN/m² (2.5 m) = 0.1 kN/m

For clay brick

Weight = 1.87 kN/m² (2.5 m) = 4.675 kN/m

Total weight = 6.20 kN/m **Ans.**

***1–8.** A building wall consists of exterior stud walls with brick veneer and 13 mm fiberboard on one side. If the wall is 4 m high, determine the load in kN/m that it exerts on the floor.

For stud wall with brick veneer.

$w = (2.30 \text{ kN/m}^2)(4 \text{ m}) = 9.20 \text{ kN/m}$

For Fiber board

$w = (0.04 \text{ kN/m}^2)(4 \text{ m}) = 0.16 \text{ kN/m}$

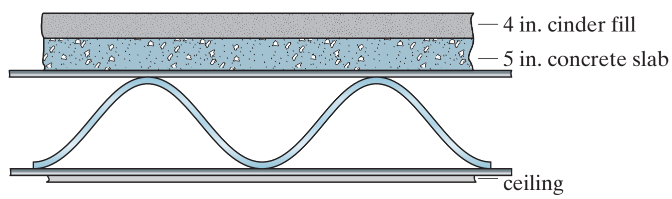
Total weight $= 9.2 + 0.16 = 9.36 \text{ kN/m}$ **Ans.**

1–9. The interior wall of a building is made from 2 × 4 wood studs, plastered on two sides. If the wall is 12 ft high, determine the load in lb/ft of length of wall that it exerts on the floor.

From Table 1–3

$w = (20 \text{ lb/ft}^2)(12 \text{ ft}) = 240 \text{ lb/ft}$ **Ans.**

1–10. The second floor of a light manufacturing building is constructed from a 5-in.-thick stone concrete slab with an added 4-in. cinder concrete fill as shown. If the suspended ceiling of the first floor consists of metal lath and gypsum plaster, determine the dead load for design in pounds per square foot of floor area.



From Table 1–3,

5-in. concrete slab	= (12)(5)	= 60.0
4-in. cinder fill	= (9)(4)	= 36.0
metal lath & plaster		= 10.0
Total dead load		= 106.0 lb/ft^2

Ans.

1–11. A four-story office building has interior columns spaced 30 ft apart in two perpendicular directions. If the flat-roof live loading is estimated to be 30 lb/ft^2, determine the reduced live load supported by a typical interior column located at ground level.

Floor load:

$L_o = 50 \text{ psf}$

$A_t = (30)(30) = 900 \text{ ft}^2 > 400 \text{ ft}^2$

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right)$$

$$L = 50\left(0.25 + \frac{15}{\sqrt{4(900)}}\right) = 25 \text{ psf}$$

$$\% \text{ reduction} = \frac{25}{50} = 50\% > 40\% \text{ (OK)}$$

$F_s = 3[(25 \text{ psf})(30 \text{ ft})(30 \text{ ft})] + 30 \text{ psf}(30 \text{ ft})(30 \text{ ft}) = 94.5 \text{ k}$ **Ans.**

***1–12.** A two-story light storage warehouse has interior columns that are spaced 12 ft apart in two perpendicular directions. If the live loading on the roof is estimated to be 25 lb/ft^2, determine the reduced live load supported by a typical interior column at (a) the ground-floor level, and (b) the second-floor level.

$A_t = (12)(12) = 144 \text{ ft}^2$

$F_R = (25)(144) = 3600 \text{ lb} = 3.6 \text{ k}$

Since $A_t = 4(144) \text{ ft}^2 > 400 \text{ ft}^2$

$$L = 12.5\left(0.25 + \frac{15}{\sqrt{(4)(144)}}\right) = 109.375 \text{ lb/ft}^2$$

(a) For ground floor column

$L = 109 \text{ psf} > 0.5\, L_o = 62.5 \text{ psf}$ OK

$F_F = (109.375)(144) = 15.75 \text{ k}$

$F = F_F + F_R = 15.75 \text{ k} + 3.6 \text{ k} = 19.4 \text{ k}$ **Ans.**

(b) For second floor column

$F = F_R = 3.60 \text{ k}$ **Ans.**

1–13. The office building has interior columns spaced 5 m apart in perpendicular directions. Determine the reduced live load supported by a typical interior column located on the first floor under the offices.

From Table 1–4

$L_o = 2.40\ \text{kN/m}^2$

$A_T = (5\ \text{m})(5\ \text{m}) = 25\ \text{m}^2$

$K_{LL} = 4$

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right)$$

$$L = 2.40\left(0.25 + \frac{4.57}{\sqrt{4(25)}}\right)$$

$L = 1.70\ \text{kN/m}^2$ **Ans.**

$1.70\ \text{kN/m}^2 > 0.4\,L_o = 0.96\ \text{kN/m}^2$ OK

1–14. A two-story hotel has interior columns for the rooms that are spaced 6 m apart in two perpendicular directions. Determine the reduced live load supported by a typical interior column on the first floor under the public rooms.

Table 1–4

$L_o = 4.79\ \text{kN/m}^2$

$A_T = (6\ \text{m})(6\ \text{m}) = 36\ \text{m}^2$

$K_{LL} = 4$

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}\,A_T}}\right)$$

$$L = 4.79\left(0.25 + \frac{4.57}{\sqrt{4(36)}}\right)$$

$L = 3.02\ \text{kN/m}^2$ **Ans.**

$3.02\ \text{kN/m}^2 > 0.4\,L_o = 1.916\ \text{kN/m}^2$ OK

1–15. Wind blows on the side of a fully enclosed hospital located on open flat terrain in Arizona. Determine the external pressure acting over the windward wall, which has a height of 30 ft. The roof is flat.

$V = 120 \text{ mi/h}$

$K_{zt} = 1.0$

$K_d = 1.0$

$q_z = 0.00256\, K_z K_{zt} K_d V^2$

$= 0.00256\, K_z\,(1.0)(1.0)(120)^2$

$= 36.86\, K_z$

From Table 1–5,

z	K_z	q_z
0–15	0.85	31.33
20	0.90	33.18
25	0.94	34.65
30	0.98	36.13

Thus,

$p = q\,G\,C_p - q_h\,(G\,C_{p_i})$

$= q\,(0.85)(0.8) - 36.13\,(\pm 0.18)$

$= 0.68q \mp 6.503$

$p_{0\text{–}15} = 0.68(31.33) \mp 6.503 = 14.8 \text{ psf or } 27.8 \text{ psf}$ **Ans.**

$p_{20} = 0.68(33.18) \mp 6.503 = 16.1 \text{ psf or } 29.1 \text{ psf}$ **Ans.**

$p_{25} = 0.68(34.65) \mp 6.503 = 17.1 \text{ psf or } 30.1 \text{ psf}$ **Ans.**

$p_{30} = 0.68(36.13) \mp 6.503 = 18.1 \text{ psf or } 31.1 \text{ psf}$ **Ans.**

***1–16.** Wind blows on the side of the fully enclosed hospital located on open flat terrain in Arizona. Determine the external pressure acting on the leeward wall, which has a length of 200 ft and a height of 30 ft.

$V = 120 \text{ mi/h}$

$K_{zt} = 1.0$

$K_d = 1.0$

$q_h = 0.00256\, K_z K_{zt} K_d V^2$

$= 0.00256\, K_z(1.0)(1.0)(120)^2$

$= 36.86\, K_z$

1–16. Continued

From Table 1–5, for $z = h = 30$ ft, $K_z = 0.98$

$$q_h = 36.86(0.98) = 36.13$$

From the text

$$\frac{L_o}{B} = \frac{200}{200} = 1 \text{ so that } C_p = -0.5$$

$$p = q\,GC_p - q_h(GC_{p_2})$$

$$p = 36.13(0.85)(-0.5) - 36.13(\pm 0.18)$$

$$p = -21.9 \text{ psf or } -8.85 \text{ psf}$$ **Ans.**

1–17. A closed storage building is located on open flat terrain in central Ohio. If the side wall of the building is 20 ft high, determine the external wind pressure acting on the windward and leeward walls. Each wall is 60 ft long. Assume the roof is essentially flat.

$$V = 105 \text{ mi/h}$$

$$K_{zt} = 1.0$$

$$K_d = 1.0$$

$$q = 0.00256\,K_z K_{zt} K_d V^2$$

$$= 0.00256\,K_z(1.0)(1.0)\,(105)^2$$

$$= 28.22\,K_z$$

From Table 1–5

z	K_z	q_z
0–15	0.85	23.99
20	0.90	25.40

Thus, for windward wall

$$p = qGC_p - q_h(GC_{p_i})$$

$$= q(0.85)(0.8) - 25.40(\pm 0.18)$$

$$= 0.68\,q \mp 4.572$$

$$p_{0-15} = 0.68\,(23.99) \mp 4.572 = 11.7 \text{ psf or } 20.9 \text{ psf}$$ **Ans.**

$$p_{20} = 0.68\,(25.40) \mp 4.572 = 12.7 \text{ psf or } 21.8 \text{ psf}$$ **Ans.**

Leeward wall

$$\frac{L}{B} = \frac{60}{60} = 1 \text{ so that } C_p = -0.5$$

$$p = q\,GC_p - q_h(GC_{p_i})$$

$$p = 25.40(0.85)(-0.5) - 25.40\,(\pm 0.18)$$

$$p = -15.4 \text{ psf or } -6.22 \text{ psf}$$ **Ans.**

1–18. The light metal storage building is on open flat terrain in central Oklahoma. If the side wall of the building is 14 ft high, what are the two values of the external wind pressure acting on this wall when the wind blows on the back of the building? The roof is essentially flat and the building is fully enclosed.

$V = 105 \text{ mi/h}$

$K_{zt} = 1.0$

$K_d = 1.0$

$q_z = 0.00256\, K_z K_{zt} K_d V^2$

$= 0.00256\, K_z\,(1.0)(1.0)(105)^2$

$= 28.22\, K_z$

From Table 1–5

For $0 \leq z \leq 15 \text{ ft} K_z = 0.85$

Thus,

$q_z = 28.22(0.85) = 23.99$

$p = q\, GC_p - q_h(GC_{p_i})$

$p = (23.99)(0.85)(0.7) - (23.99)(\pm 0.18)$

$p = -9.96 \text{ psf}$ or $p = -18.6 \text{ psf}$ **Ans.**

1–19. Determine the resultant force acting perpendicular to the face of the billboard and through its center if it is located in Michigan on open flat terrain. The sign is rigid and has a width of 12 m and a height of 3 m. Its top side is 15 m from the ground.

$q_h = 0.613\, K_z K_{zt} K_d V^2$

Since $z = h = 15 \text{ m}$ $K_z = 1.09$

$K_{zt} = 1.0$

$K_d = 1.0$

$V = 47 \text{ m/s}$

$q_h = 0.613(1.09)(1.0)(1.0)(47)^2$

$= 1476.0 \text{ N/m}^2$

$B/s = \dfrac{12 \text{ m}}{3 \text{ m}} = 4, s/h = \dfrac{3}{15} = 0.2$

From Table 1–6

$C_f = 1.80$

$F = q_h\, GC_f A_s$

$= (1476.0)(0.85)(1.80)(12)(3) = 81.3 \text{ kN}$ **Ans.**

***1–20.** A hospital located in central Illinois has a flat roof. Determine the snow load in kN/m^2 that is required to design the roof.

$p_f = 0.7\, C_c C_t I_s p_g$

$p_f = 0.7(0.8)(1.0)(1.20)(0.96)$

$= 0.6451 \text{ kN/m}^2$

Also

$p_f = I_s p_g = (1.20)(0.96) = 1.152 \text{ kN/m}^2$

use

$p_f = 1.15 \text{ kN/m}^2$ **Ans.**

1–21. The school building has a flat roof. It is located in an open area where the ground snow load is 0.68 kN/m^2. Determine the snow load that is required to design the roof.

$p_f = 0.7\, C_c C_t I_s p_g$

$p_f = 0.7(0.8)(1.0)(1.20)(0.68)$

$= 0.457 \text{ kN/m}^2$

Also

$p_f = p_f = I_s p_g = (1.20)(0.68) = 0.816 \text{ kN/m}^2$

use

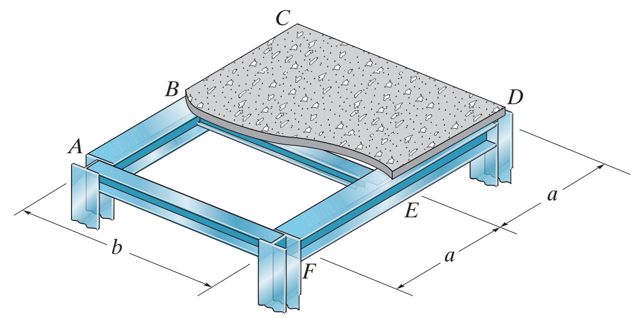
$p_f = 0.816 \text{ kN/m}^2$ **Ans.**

1–22. The hospital is located in an open area and has a flat roof and the ground snow load is 30 lb/ft^2. Determine the design snow load for the roof.

Since $p_q = 30 \text{ lb/ft}^2 > 20 \text{ lb/ft}^2$ then

$p_f = I_s\, p_g = 1.20(30) = 36 \text{ lb/ft}^2$ **Ans.**

2–1. The steel framework is used to support the reinforced stone concrete slab that is used for an office. The slab is 200 mm thick. Sketch the loading that acts along members BE and FED. Take $a = 2$ m, $b = 5$ m. *Hint*: See Tables 1–2 and 1–4.



Beam *BE*. Since $\frac{b}{a} = \frac{5\text{ m}}{2\text{ m}} = 2.5$, the concrete slab will behave as a one way slab.

Thus, the tributary area for this beam is rectangular shown in Fig. *a* and the intensity of the uniform distributed load is

200 mm thick reinforced stone concrete slab:
$(23.6\text{ kN/m}^3)(0.2\text{ m})(2\text{ m}) = 9.44\text{ kN/m}$

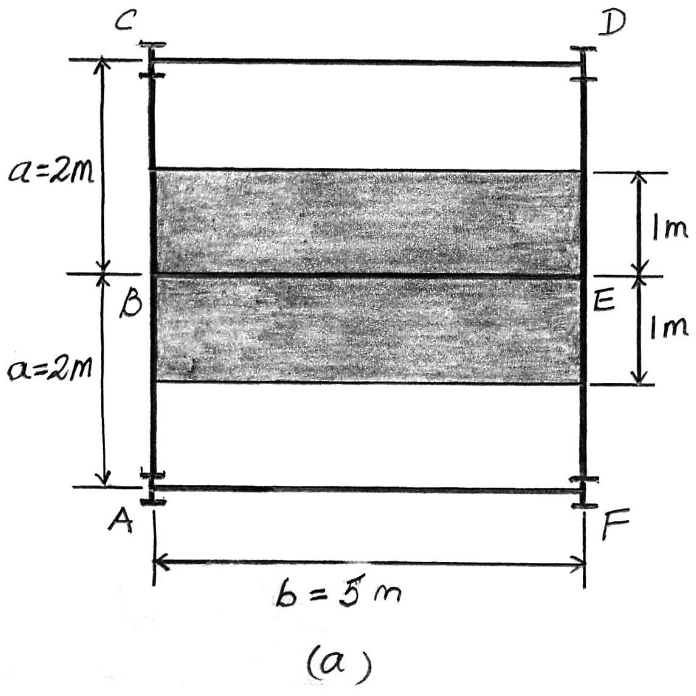
Live load for office: $(2.40\text{ kN/m}^2)(2\text{ m}) = \frac{480\text{ kN/m}}{14.24\text{ kN/m}}$ **Ans.**

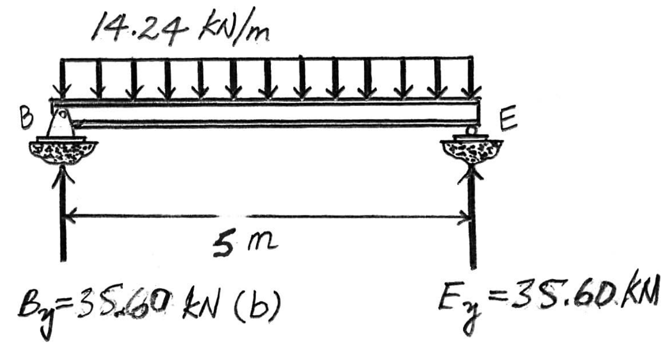
Due to symmetry the vertical reaction at B and E are

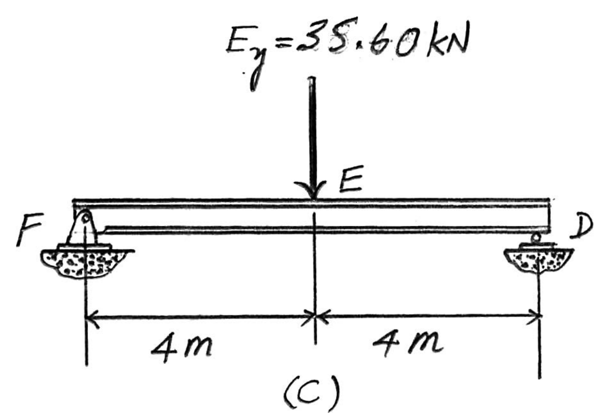
$$B_y = E_y = (14.24\text{ kN/m})(5)/2 = 35.6\text{ kN}$$

The loading diagram for beam BE is shown in Fig. *b*.

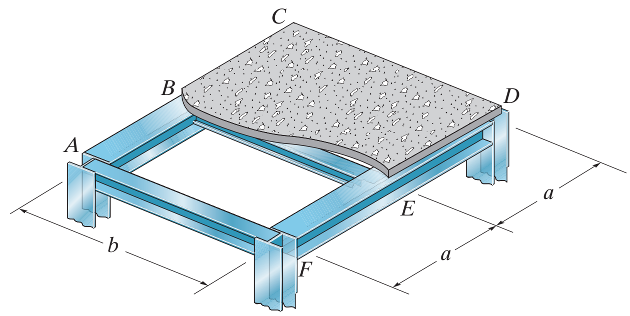
Beam *FED*. The only load this beam supports is the vertical reaction of beam BE at E which is $E_y = 35.6$ kN. The loading diagram for this beam is shown in Fig. *c*.







2–2. Solve Prob. 2–1 with $a = 3$ m, $b = 4$ m.



Beam *BE*. Since $\dfrac{b}{a} = \dfrac{4}{3} < 2$, the concrete slab will behave as a two way slab. Thus, the tributary area for this beam is the hexagonal area shown in Fig. *a* and the maximum intensity of the distributed load is

200 mm thick reinforced stone concrete slab: $(23.6\ \text{kN/m}^3)(0.2\ \text{m})(3\ \text{m}) = 14.16\ \text{kN/m}$

Live load for office: $(2.40\ \text{kN/m}^2)(3\ \text{m}) = 7.20\ \text{kN/m}$

$= 21.36\ \text{kN/m}$ **Ans.**

Due to symmetry, the vertical reactions at B and E are

$$B_y = E_y = \frac{2\left[\frac{1}{2}(21.36\ \text{kN/m})(1.5\ \text{m})\right] + (21.36\ \text{kN/m})(1\ \text{m})}{2}$$

$$= 26.70\ \text{kN}$$

The loading diagram for Beam *BE* is shown in Fig. *b*.

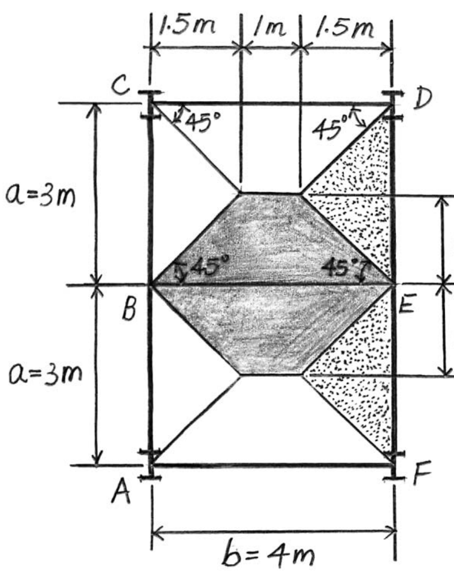
Beam *FED*. The loadings that are supported by this beam are the vertical reaction of beam *BE* at *E* which is $E_y = 26.70$ kN and the triangular distributed load of which its tributary area is the triangular area shown in Fig. *a*. Its maximum intensity is

200 mm thick reinforced stone concrete slab: $(23.6\ \text{kN/m}^3)(0.2\ \text{m})(1.5\ \text{m}) = 7.08\ \text{kN/m}$

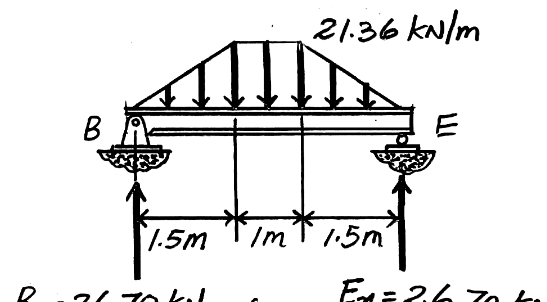
Live load for office: $(2.40\ \text{kN/m}^2)(1.5\ \text{m}) = 3.60\ \text{kN/m}$

$= 10.68\ \text{kN/m}$ **Ans.**

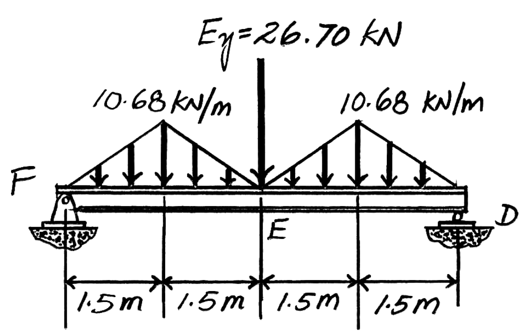
The loading diagram for Beam *FED* is shown in Fig. *c*.



(a)

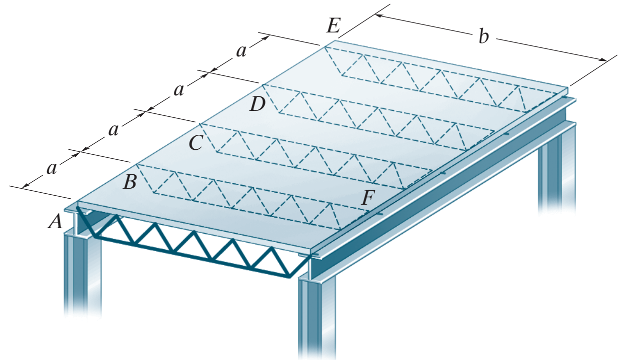


(b)



(c)

2–3. The floor system used in a school classroom consists of a 4-in. reinforced stone concrete slab. Sketch the loading that acts along the joist *BF* and side girder *ABCDE*. Set $a = 10$ ft, $b = 30$ ft. *Hint*: See Tables 1–2 and 1–4.



Joist *BF*. Since $\frac{b}{a} = \frac{30\text{ ft}}{10\text{ ft}} = 3$, the concrete slab will behave as a one way slab. Thus, the tributary area for this joist is the rectangular area shown in Fig. *a* and the intensity of the uniform distributed load is

4 in thick reinforced stone concrete slab: $(0.15\text{ k/ft}^3)\left(\frac{4}{12}\text{ ft}\right)(10\text{ ft}) = 0.5\text{ k/ft}$

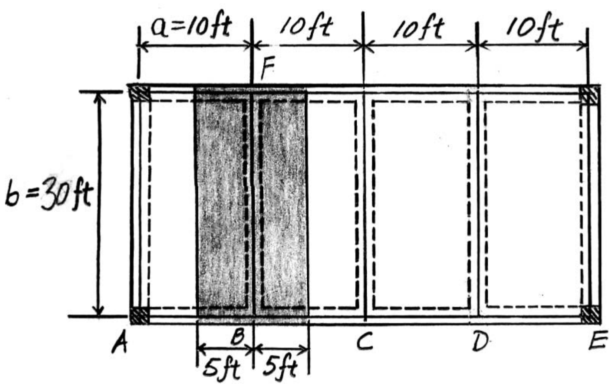
Live load for classroom: $(0.04\text{ k/ft}^2)(10\text{ ft}) = \frac{0.4\text{ k/ft}}{0.9\text{ k/ft}}$ **Ans.**

Due to symmetry, the vertical reactions at *B* and *F* are

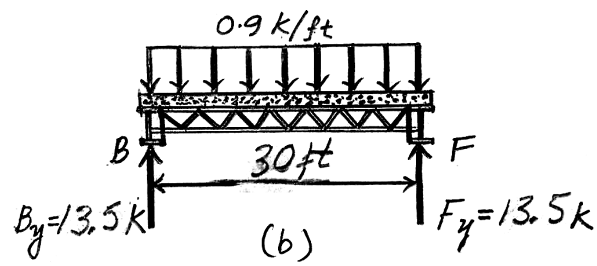
$$B_y = F_y = (0.9\text{ k/ft})(30\text{ ft})/2 = 13.5\text{ k}$$ **Ans.**

The loading diagram for joist *BF* is shown in Fig. *b*.

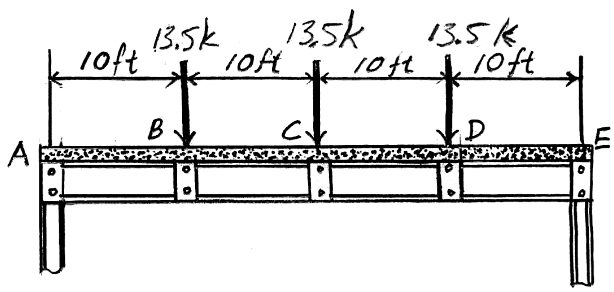
Girder *ABCDE*. The loads that act on this girder are the vertical reactions of the joists at *B*, *C*, and *D*, which are $B_y = C_y = D_y = 13.5$ k. The loading diagram for this girder is shown in Fig. *c*.



(a)

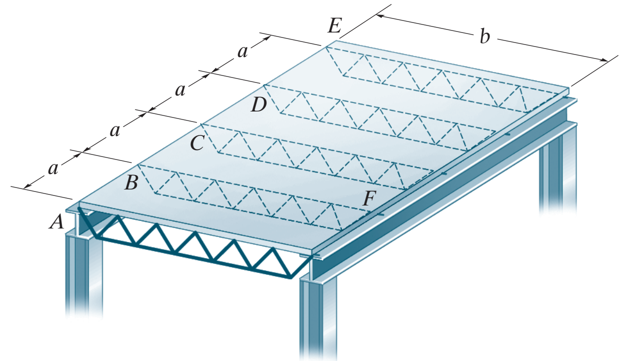


(b)



(c)

***2–4.** Solve Prob. 2–3 with $a = 10$ ft, $b = 15$ ft.



Joist *BF*. Since $\frac{b}{a} = \frac{15\text{ ft}}{10\text{ ft}} = 1.5 < 2$, the concrete slab will behave as a two way slab. Thus, the tributary area for the joist is the hexagonal area as shown in Fig. *a* and the maximum intensity of the distributed load is

4 in thick reinforced stone concrete slab: $(0.15\text{ k/ft}^3)\left(\frac{4}{12}\text{ ft}\right)(10\text{ ft}) = 0.5\text{ k/ft}$

Live load for classroom: $(0.04\text{ k/ft}^2)(10\text{ ft}) = \frac{0.4\text{ k/ft}}{0.9\text{ k/ft}}$ **Ans.**

Due to symmetry, the vertical reactions at B and G are

$$B_y = F_y = \frac{2\left[\frac{1}{2}(0.9\text{ k/ft})(5\text{ ft})\right] + (0.9\text{ k/ft})(5\text{ ft})}{2} = 4.50\text{ k} \quad \textbf{Ans.}$$

The loading diagram for beam BF is shown in Fig. *b*.

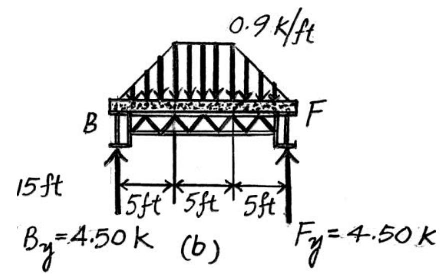
Girder *ABCDE*. The loadings that are supported by this girder are the vertical reactions of the joist at B, C and D which are $B_y = C_y = D_y = 4.50$ k and the triangular distributed load shown in Fig. *a*. Its maximum intensity is

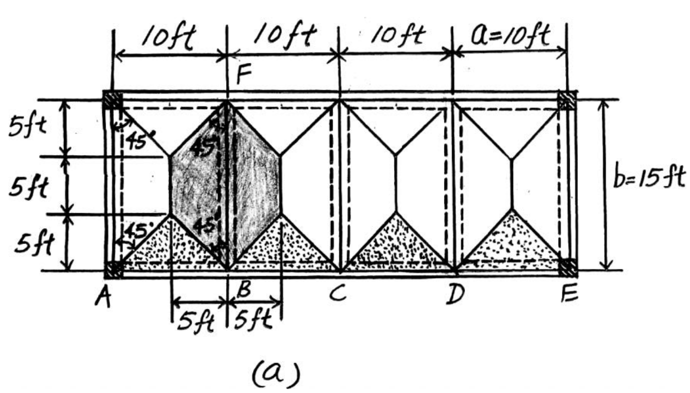
4 in thick reinforced stone concrete slab:

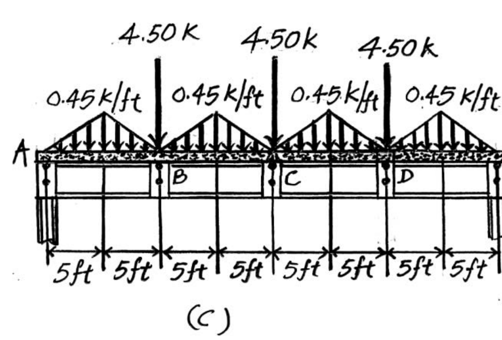
$$(0.15\text{ k/ft}^3)\left(\frac{4}{12}\text{ ft}\right)(5\text{ ft}) = 0.25\text{ k/ft}$$

Live load for classroom: $(0.04\text{ k/ft}^2)(5\text{ ft}) = \frac{0.20\text{ k/ft}}{0.45\text{ k/ft}}$ **Ans.**

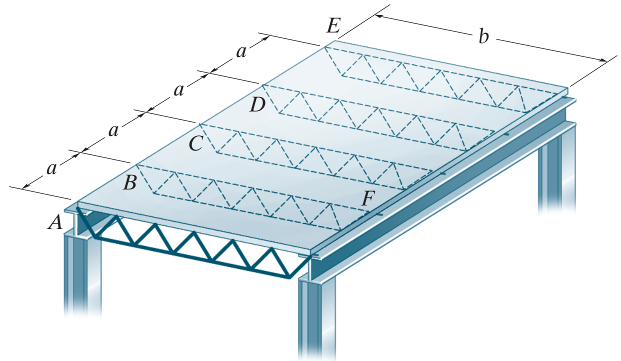
The loading diagram for the girder $ABCDE$ is shown in Fig. *c*.







2–5. Solve Prob. 2–3 with $a = 7.5$ ft, $b = 20$ ft.



Beam *BF*. Since $\frac{b}{a} = \frac{20\text{ ft}}{7.5\text{ ft}} = 2.7 > 2$, the concrete slab will behave as a one way slab. Thus, the tributary area for this beam is a rectangle shown in Fig. *a* and the intensity of the distributed load is

4 in thick reinforced stone concrete slab: $(0.15\text{ k/ft}^3)\left(\frac{4}{12}\text{ ft}\right)(7.5\text{ ft}) = 0.375\text{ k/ft}$

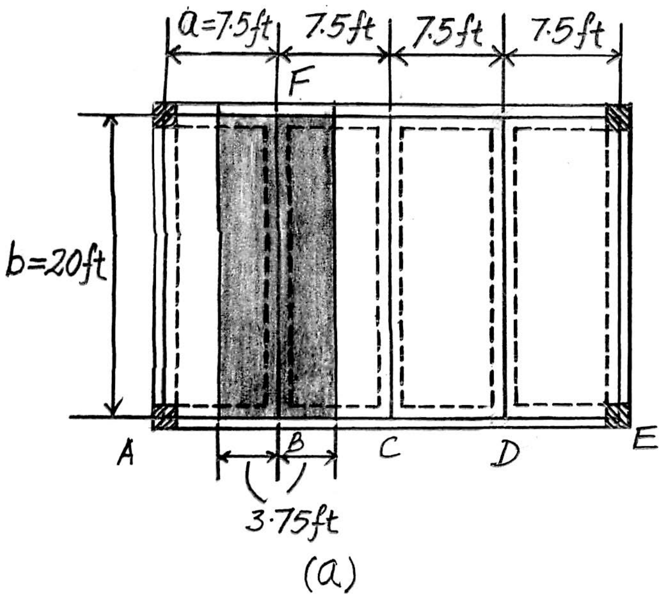
Live load from classroom: $(0.04\text{ k/ft}^2)(7.5\text{ ft}) \quad = \frac{0.300\text{ k/ft}}{0.675\text{ k/ft}}$ **Ans.**

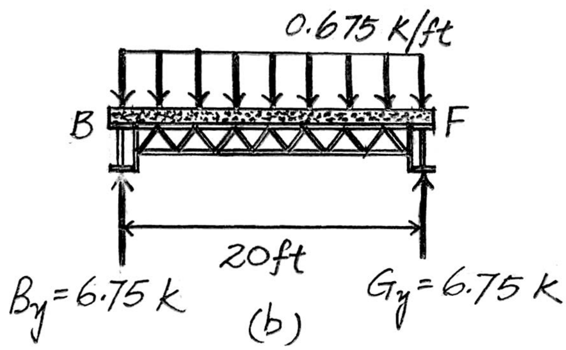
Due to symmetry, the vertical reactions at B and F are

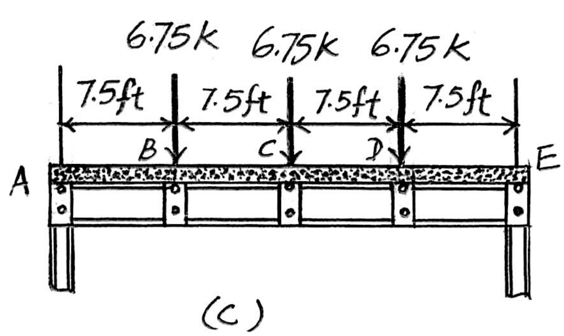
$$B_y = F_y = \frac{(0.675\text{ k/ft})(20\text{ ft})}{2} = 6.75\text{ k}$$ **Ans.**

The loading diagram for beam *BF* is shown in Fig. *b*.

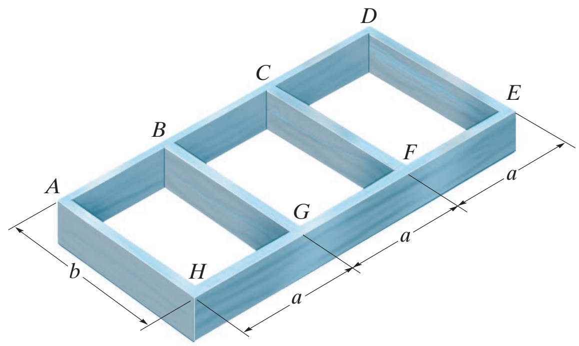
Beam *ABCD*. The loading diagram for this beam is shown in Fig. *c*.







2–6. The frame is used to support a 2-in.-thick plywood floor of a residential dwelling. Sketch the loading that acts along members *BG* and *ABCD*. Set $a = 5$ ft, $b = 15$ ft. *Hint*: See Tables 1–2 and 1–4.



Beam *BG*. Since $\frac{b}{a} = \frac{15 \text{ ft}}{5 \text{ ft}} = 3$, the plywood platform will behave as a one way slab. Thus, the tributary area for this beam is rectangular as shown in Fig. *a* and the intensity of the uniform distributed load is

2 in thick plywood platform: $\left(36 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{2}{12} \text{ ft}\right)(5\text{ft}) = 30 \text{ lb/ft}$

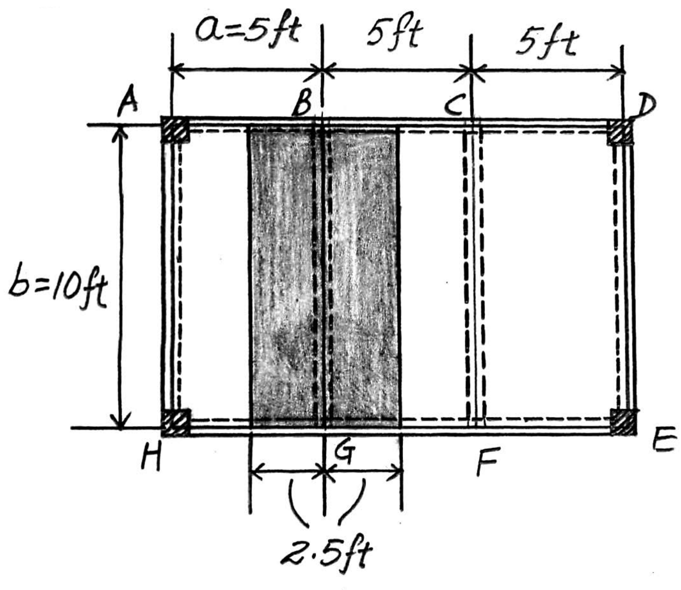
Line load for residential dweller: $\left(40 \frac{\text{lb}}{\text{ft}^2}\right)(5 \text{ ft}) = \frac{200 \text{ lb/ft}}{230 \text{ lb/ft}}$ **Ans.**

Due to symmetry, the vertical reactions at *B* and *G* are

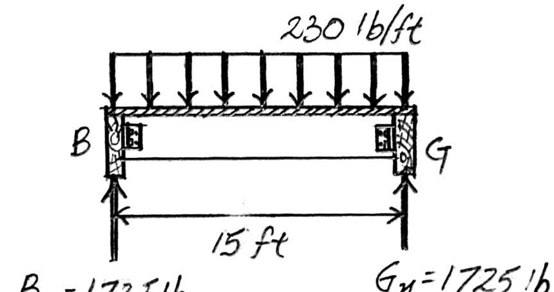
$B_y = G_y = \frac{(230 \text{ lb/ft})(15 \text{ ft})}{2} = 1725$ **Ans.**

The loading diagram for beam *BG* is shown in Fig. *a*.

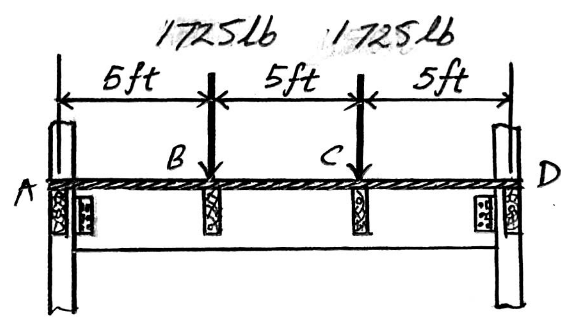
Beam *ABCD*. The loads that act on this beam are the vertical reactions of beams *BG* and *CF* at *B* and *C* which are $B_y = C_y = 1725$ lb. The loading diagram is shown in Fig. *c*.



(a)

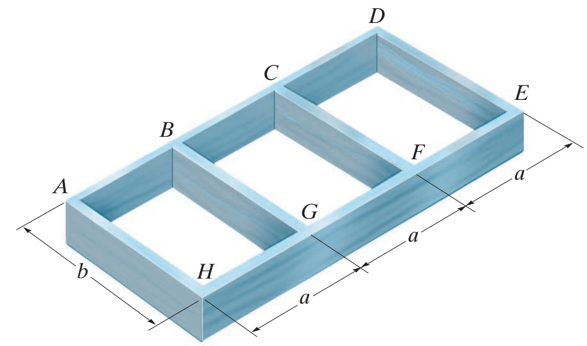


(b)



(c)

2–7. Solve Prob. 2–6, with $a = 8$ ft, $b = 8$ ft.



Beam *BG*. Since $\frac{b}{a} = \frac{8\text{ ft}}{8\text{ ft}} = 1 < 2$, the plywood platform will behave as a two way slab. Thus, the tributary area for this beam is the shaded square area shown in Fig. *a* and the maximum intensity of the distributed load is

2 in thick plywood platform: $(36\text{ lb/ft}^3)\left(\frac{2}{12}\text{ in}\right)(8\text{ ft}) = 48\text{ lb/ft}$

Live load for residential dwelling: $(40\text{ lb/ft})(8\text{ ft}) = \frac{320\text{ lb/ft}}{368\text{ lb/ft}}$ **Ans.**

Due to symmetry, the vertical reactions at B and G are

$$B_y = G_y = \frac{\frac{1}{2}(368\text{ lb/ft})\,(8\text{ ft})}{2} = 736\text{ lb}$$ **Ans.**

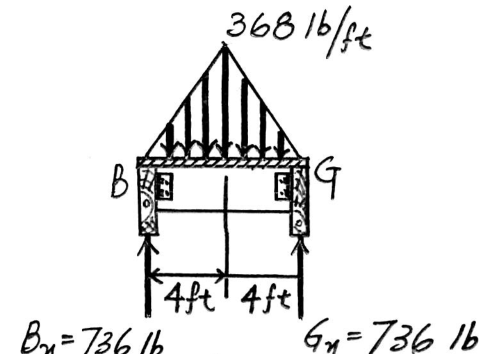
The loading diagram for the beam BG is shown in Fig. *b*

Beam *ABCD*. The loadings that are supported by this beam are the vertical reactions of beams BG and CF at B and C which are $B_y = C_y = 736$ lb and the distributed load which is the triangular area shown in Fig. *a*. Its maximum intensity is

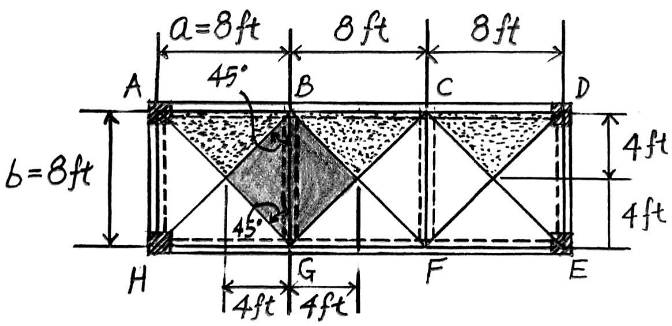
2 in thick plywood platform: $(36\text{ lb/ft}^3)\left(\frac{2}{12\text{ ft}}\right)(4\text{ ft}) = 24\text{ lb/ft}$

Live load for residential dwelling: $(40\text{ lb/ft}^2)(4\text{ lb/ft}) = \frac{160\text{ lb/ft}}{184\text{ lb/ft}}$ **Ans.**

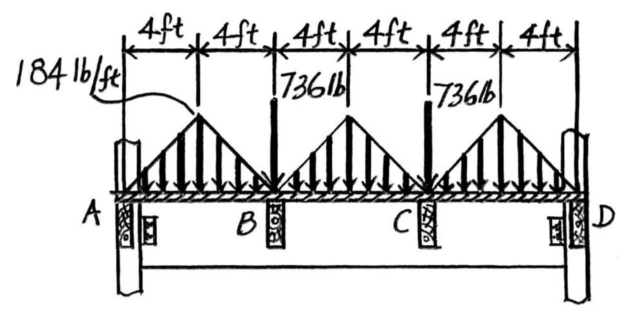
The loading diagram for beam $ABCD$ is shown in Fig. *c*.



(b)

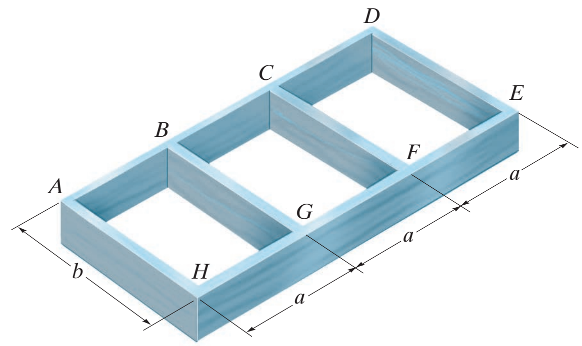


(a)



(c)

***2–8.** Solve Prob. 2–6, with $a = 9$ ft, $b = 15$ ft.



Beam *BG*. Since $\frac{b}{a} = \frac{15 \text{ ft}}{9 \text{ ft}} = 1.67 < 2$, the plywood platform will behave as a two way slab. Thus, the tributary area for this beam is the octagonal area shown in Fig. *a* and the maximum intensity of the distributed load is

$$\text{2 in thick plywood platform: } (36 \text{ lb/ft}^3)\left(\frac{2}{12} \text{ in}\right)(9 \text{ ft}) = 54 \text{ lb/ft}$$

$$\text{Live load for residential dwelling: } (40 \text{ lb/ft}^2)(9 \text{ ft}) = \frac{360 \text{ lb/ft}}{414 \text{ lb/ft}}$$ **Ans.**

Due to symmetry, the vertical reactions at B and G are

$$B_y = G_y = \frac{2\left[\frac{1}{2}(414 \text{ lb/ft})(4.5 \text{ ft})\right] + (414 \text{ lb/ft})(6 \text{ ft})}{2} = 2173.5 \text{ lb}$$

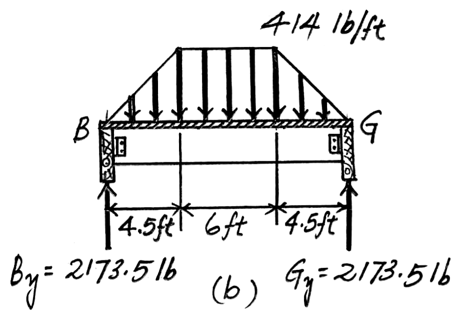
The loading diagram for beam BG is shown in Fig. *b*.

Beam *ABCD*. The loading that is supported by this beam are the vertical reactions of beams BG and CF at B and C which is $B_y = C_y = 2173.5$ lb and the triangular distributed load shown in Fig. *a*. Its maximum intensity is

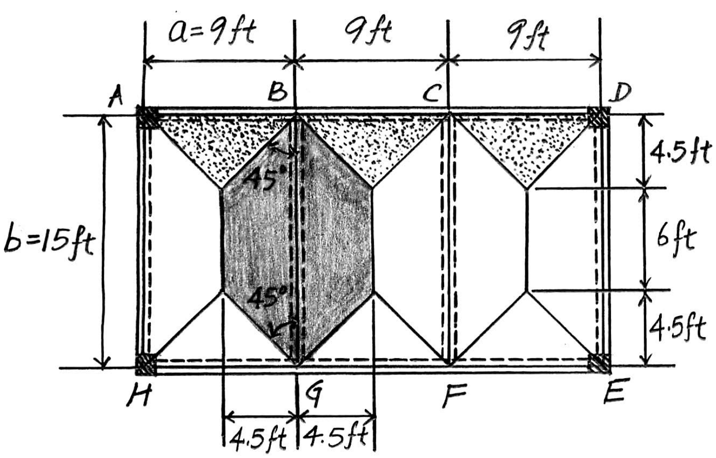
$$\text{2 in thick plywood platform: } (36 \text{ lb/ft}^3)\left(\frac{2}{12} \text{ ft}\right)(4.5 \text{ ft}) = 27 \text{ lb/ft}$$

$$\text{Live load for residential dwelling: } (40 \text{ lb/ft}^2)(4.5 \text{ ft}) = \frac{180 \text{ lb/ft}}{207 \text{ lb/ft}}$$ **Ans.**

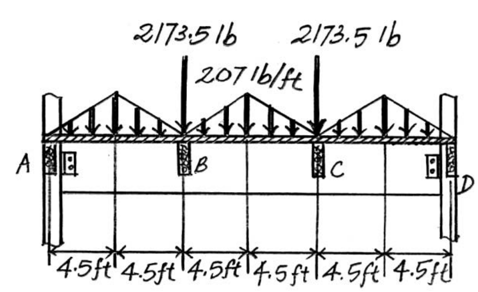
The loading diagram for beam $ABCD$ is shown in Fig. *c*.



(b)

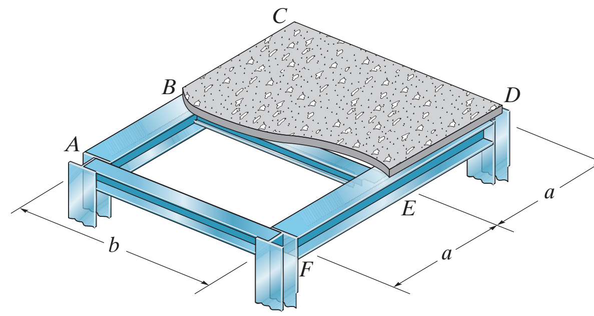


(a)



(c)

2–9. The steel framework is used to support the 4-in. reinforced stone concrete slab that carries a uniform live loading of 500 lb/ft^2. Sketch the loading that acts along members BE and FED. Set $b = 10$ ft, $a = 7.5$ ft. *Hint*: See Table 1–2.



Beam *BE*. Since $\frac{b}{a} = \frac{10}{7.5} < 2$, the concrete slab will behave as a two way slab. Thus, the tributary area for this beam is the octagonal area shown in Fig. *a* and the maximum intensity of the distributed load is

$$\text{4 in thick reinforced stone concrete slab: } (0.15\text{ k/ft}^3)\left(\frac{4}{12}\text{ ft}\right)(7.5\text{ ft}) = 0.375\text{ k/ft}$$

$$\text{Floor Live Load: } (0.5\text{ k/ft}^2)(7.5\text{ ft}) = \frac{3.75\text{ k/ft}}{4.125\text{ k/ft}} \qquad \textbf{Ans.}$$

Due to symmetry, the vertical reactions at B and E are

$$B_y = E_y = \frac{2\left[\frac{1}{2}(4.125\text{ k/ft})(3.75\text{ ft})\right] + (4.125\text{ k/ft})(2.5\text{ ft})}{2} = 12.89\text{ k}$$

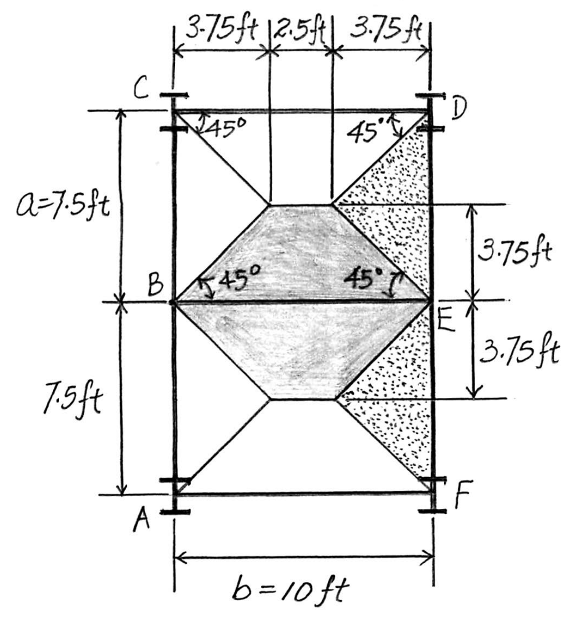
The loading diagram for this beam is shown in Fig. *b*.

Beam *FED*. The loadings that are supported by this beam are the vertical reaction of beam BE at E which is $E_y = 12.89$ k and the triangular distributed load shown in Fig. *a*. Its maximum intensity is

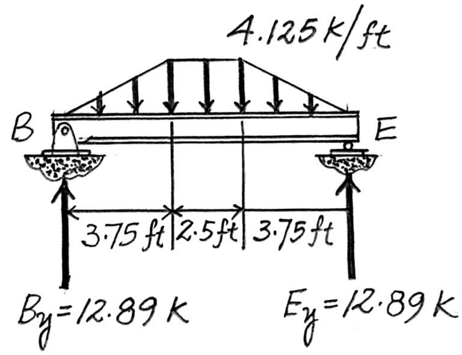
$$\text{4 in thick reinforced stone concrete slab: } (0.15\text{ k/ft}^3)\left(\frac{4}{12}\text{ ft}\right)(3.75\text{ ft}) = 0.1875\text{ k/ft}$$

$$\text{Floor live load: } (0.5\text{ k/ft}^2)(3.75\text{ ft}) = \frac{1.875\text{ k/ft}}{2.06\text{ k/ft}} \qquad \textbf{Ans.}$$

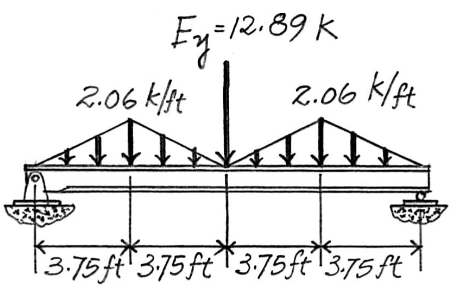
The loading diagram for this beam is shown in Fig. *c*.



(a)

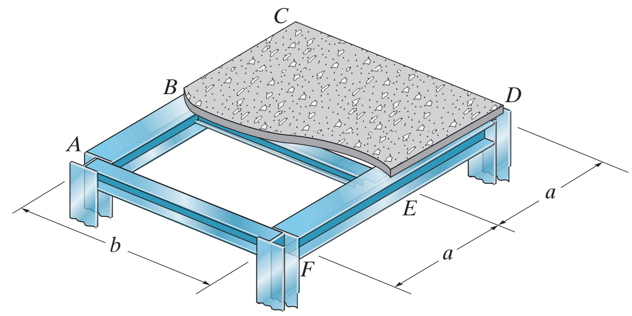


(b)



(c)

2–10. Solve Prob. 2–9, with $b = 12$ ft, $a = 4$ ft.



Beam *BE*. Since $\dfrac{b}{a} = \dfrac{12}{4} = 3 > 2$, the concrete slab will behave as a one way slab. Thus, the tributary area for this beam is the rectangular area shown in Fig. *a* and the intensity of the distributed load is

4 in thick reinforced stone concrete slab: $(0.15\text{ k/ft}^2)\left(\dfrac{4}{12}\text{ ft}\right)(4\text{ ft}) = 0.20\text{ k/ft}$

Floor Live load: $(0.5\text{ k/ft}^2)(4\text{ ft}) = \dfrac{2.00\text{ k/ft}}{2.20\text{ k/ft}}$ **Ans.**

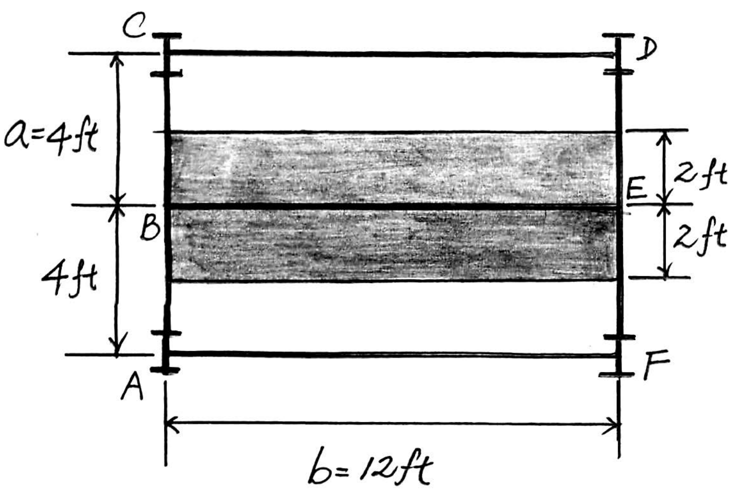
Due to symmetry, the vertical reactions at B and E are

$$B_y = E_y = \frac{(2.20\text{ k/ft})(12\text{ ft})}{2} = 13.2\text{ k}$$

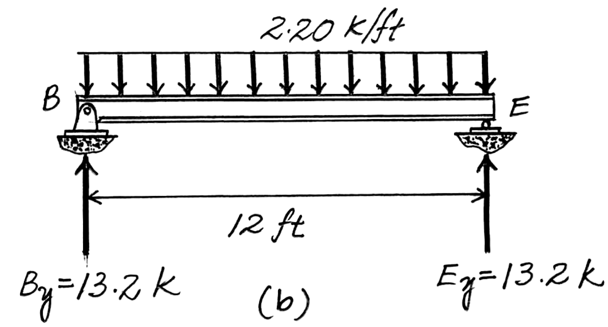
The loading diagram of this beam is shown in Fig. *b*.

Beam *FED*. The only load this beam supports is the vertical reaction of beam BE at E which is $E_y = 13.2$ k. **Ans.**

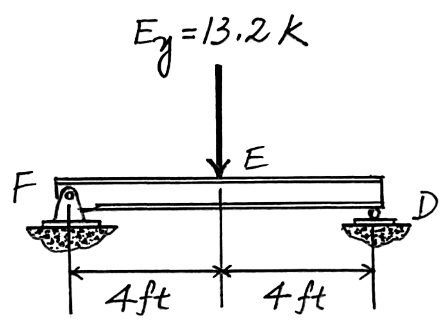
The loading diagram is shown in Fig. *c*.



(a)

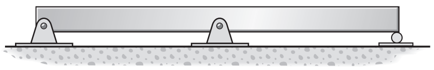


(b)

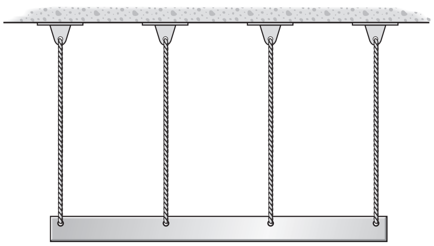


(c)

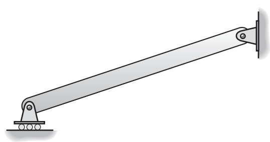
2–11. Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.



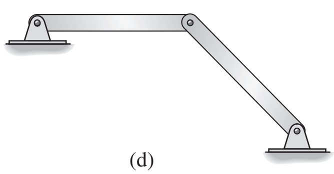
(a)



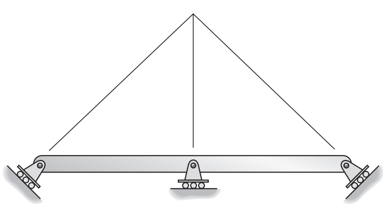
(b)



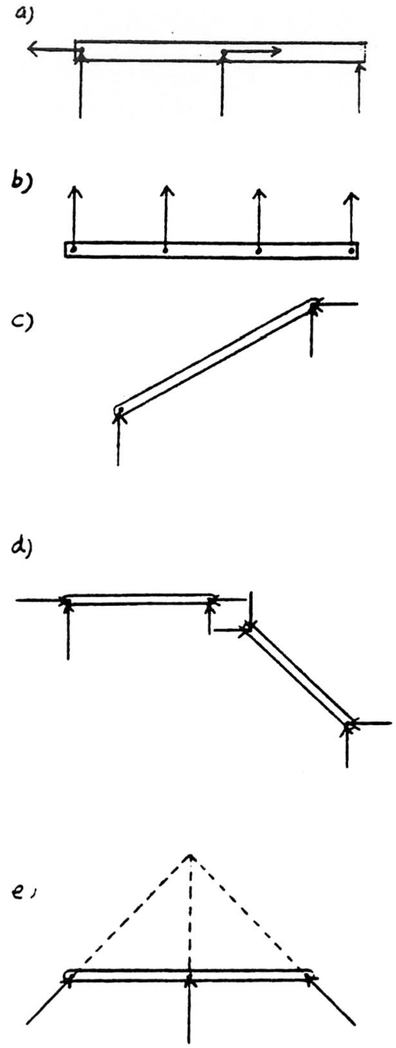
(c)



(d)



(e)



(a) $r = 5 \qquad 3n = 3(1) < 5$

Indeterminate to 2°. **Ans.**

(b) Parallel reactions

Unstable. **Ans.**

(c) $r = 3 \qquad 3n = 3(1) < 3$

Statically determinate. **Ans.**

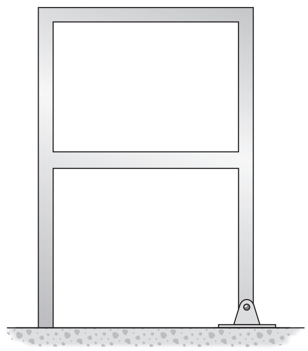
(d) $r = 6 \qquad 3n = 3(2) < 6$

Statically determinate. **Ans.**

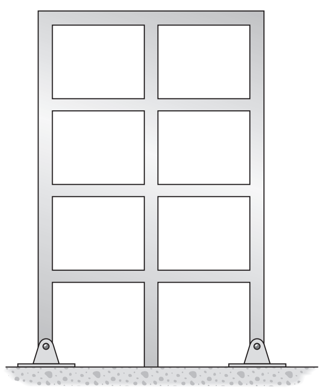
(e) Concurrent reactions

Unstable. **Ans.**

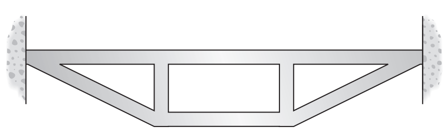
***2–12.** Classify each of the frames as statically determinate or indeterminate. If indeterminate, specify the degree of indeterminacy. All internal joints are fixed connected.



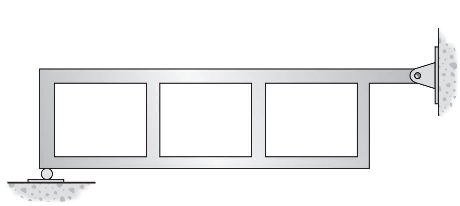
(a)



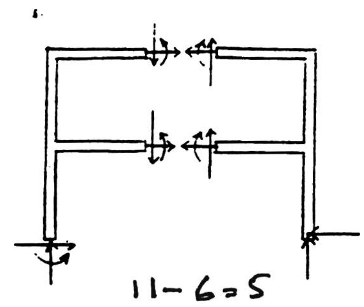
(b)

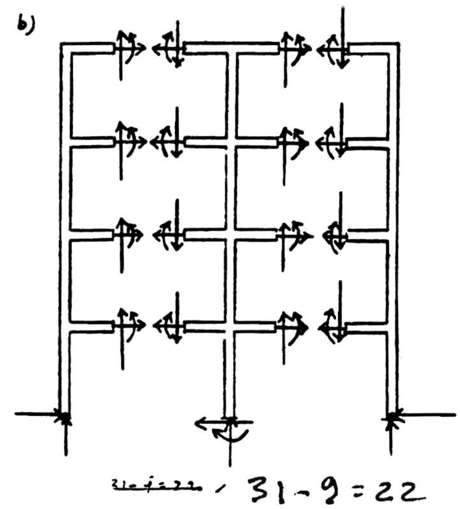


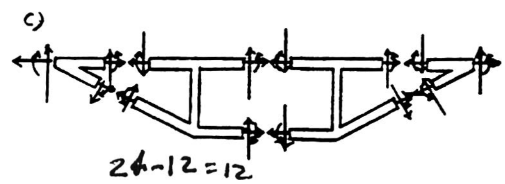
(c)

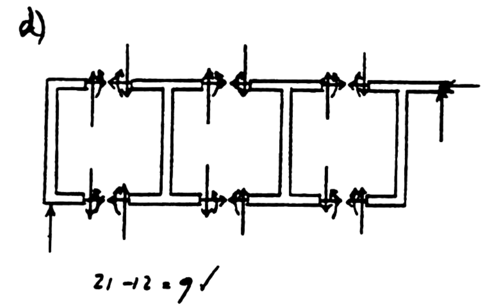


(d)









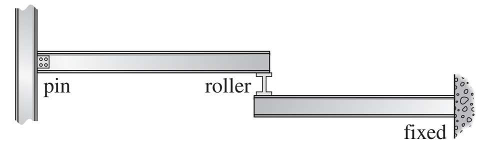
(a) Statically indeterminate to 5°. **Ans.**

(b) Statically indeterminate to 22°. **Ans.**

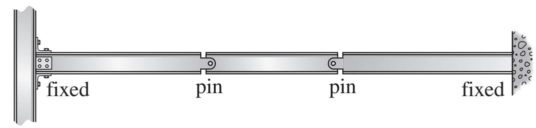
(c) Statically indeterminate to 12°. **Ans.**

(d) Statically indeterminate to 9°. **Ans.**

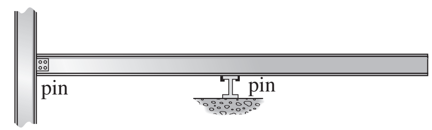
2–13. Classify each of the structures as statically determinate, statically indeterminate, stable, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.



(a)



(b)

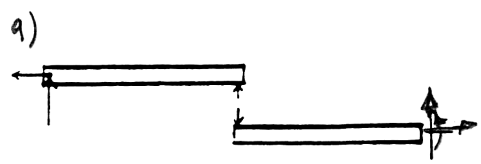


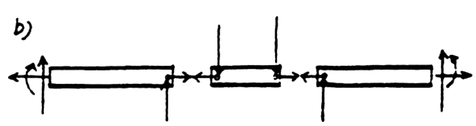
(c)

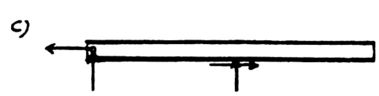
(a) $r = 6 \qquad 3n = 3(2) = 6$
Statically determinate. **Ans.**

(b) $r = 10 \qquad 3n = 3(3) < 10$
Statically indeterminate to 1°. **Ans.**

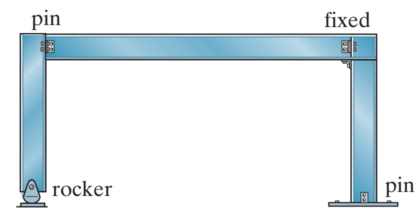
(c) $r = 4 \qquad 3n = 3(1) < 4$
Statically determinate to 1°. **Ans.**



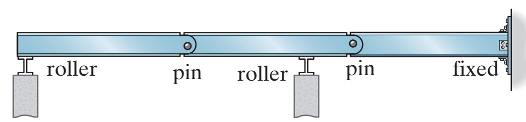




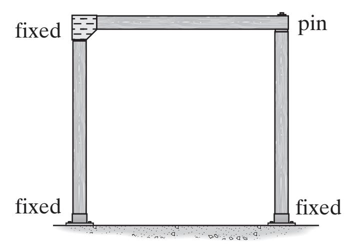
2–14. Classify each of the structures as statically determinate, statically indeterminate, stable, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.



(a)



(b)



(c)

(a) $r = 5 \qquad 3n = 3(2) = 6$

$r < 3n$

Unstable.

(b) $r = 9 \qquad 3n = 3(3) = 9$

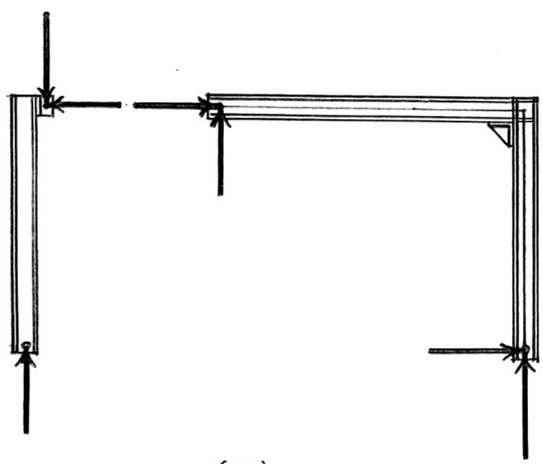
$r = 3n$

Stable and statically determinate.

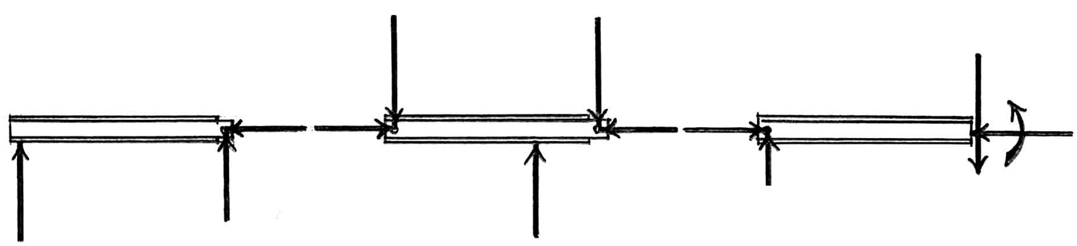
(c) $r = 8 \qquad 3n = 3(2) = 6$

$r - 3n = 8 - 6 = 2$

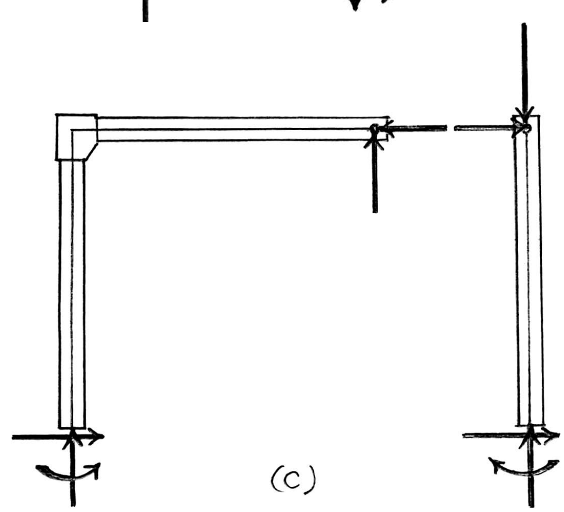
Stable and statically indeterminate to the second degree.



(a)



(b)



(c)

2–15. Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy.

(a) $r = 5 \qquad 3n = 3(2) = 6$

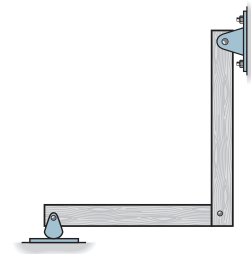
$r < 3\text{n}$

Unstable.

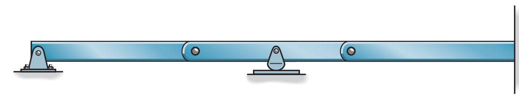
(b) $r = 10 \qquad 3n = 3(3) = 9$ and $r - 3n = 10 - 9 = 1$

Stable and statically indeterminate to first degree.

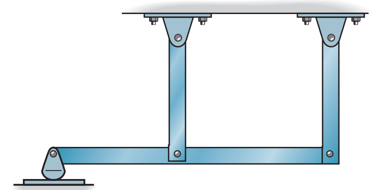
(c) Since the rocker on the horizontal member can not resist a horizontal force component, the structure is unstable.



(a)



(b)



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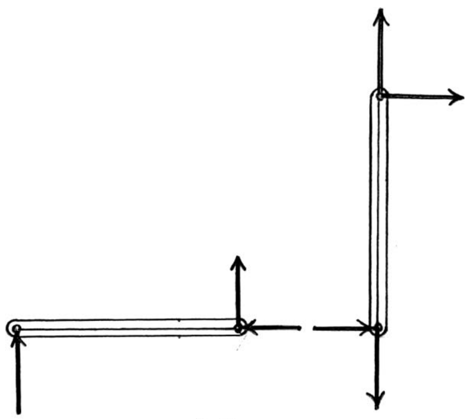
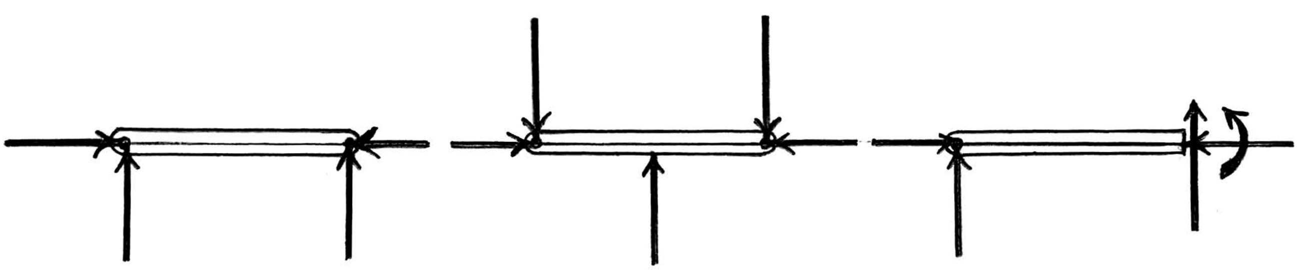
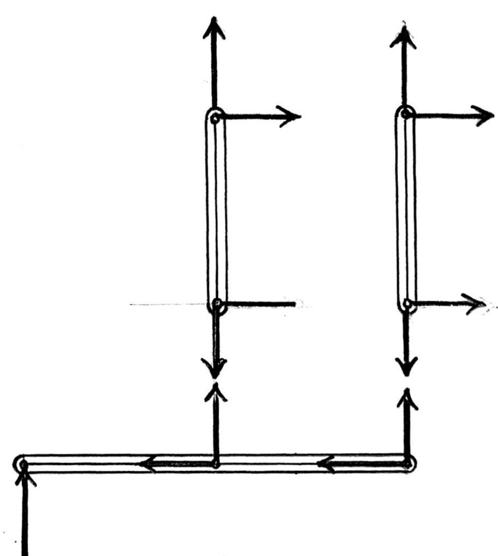


(a)



(b)



(c)

***2–16.** Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy.

(a) $r = 6 \qquad 3n = 3(1) = 3$

$r - 3n = 6 - 3 = 3$

Stable and statically indeterminate to the third degree.

(b) $r = 4 \qquad 3n = 3(1) = 3$

$r - 3n = 4 - 3 = 1$

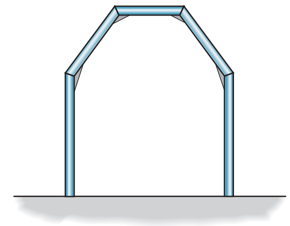
Stable and statically indeterminate to the first degree.

(c) $r = 3 \qquad 3n = 3(1) = 3 \qquad r = 3n$

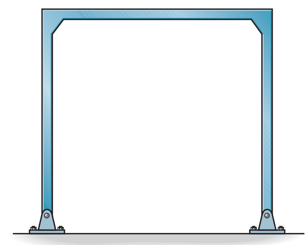
Stable and statically determinate.

(d) $r = 6 \qquad 3n = 3(2) = 6 \qquad r = 3n$

Stable and statically determinate.

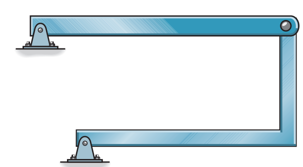


(a)

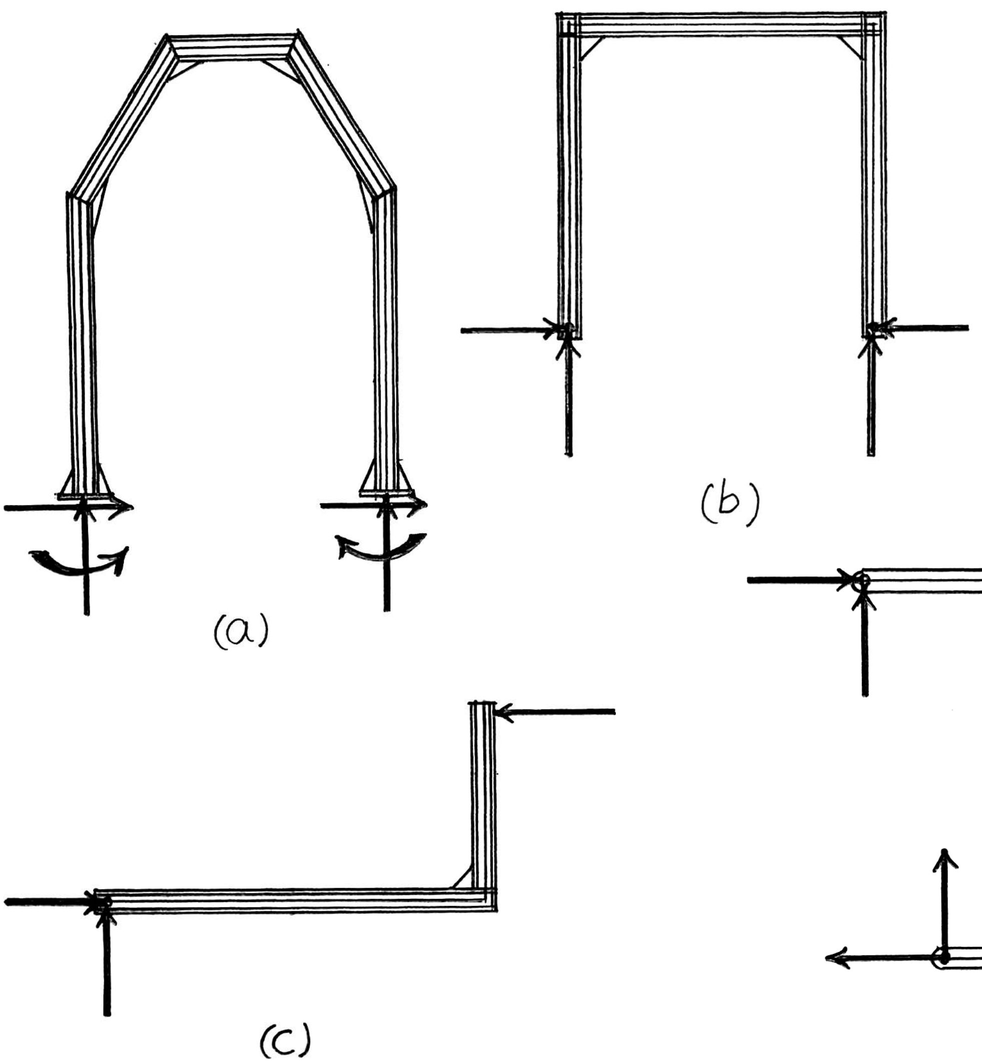


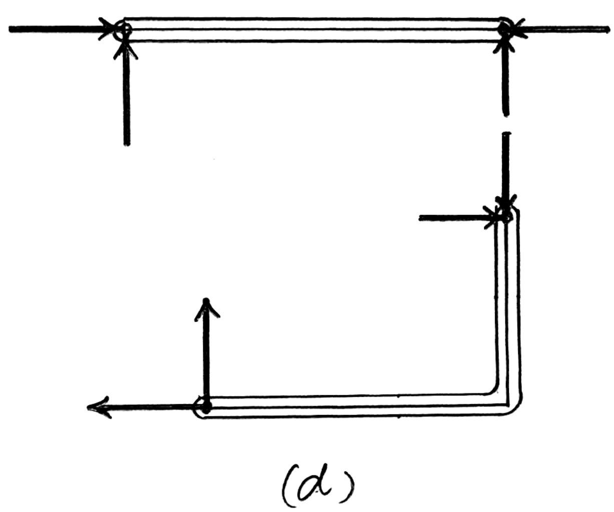
(b)

(c)



(d)



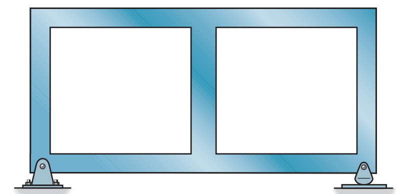


2–17. Classify each of the structures as statically determinate, statically indeterminate, stable, or unstable. If indeterminate, specify the degree of indeterminacy.

(a)

(a) $r = 2 \qquad 3n = 3(1) = 3 \qquad r < 3n$

Unstable.

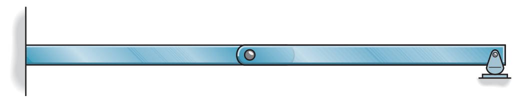


(b)

(b) $r = 12 \qquad 3n = 3(2) = 6 \qquad r > 3n$

$r - 3n = 12 - 6 = 6$

Stable and statically indeterminate to the sixth degree.



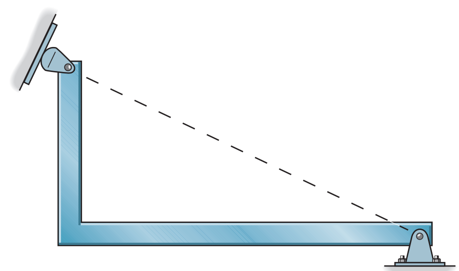
(c)

(c) $r = 6 \qquad 3n = 3(2) = 6$

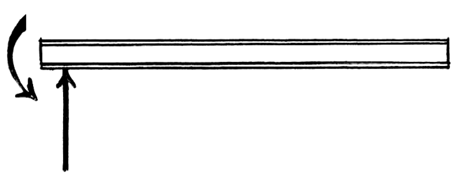
$r = 3n$

Stable and statically determinate.

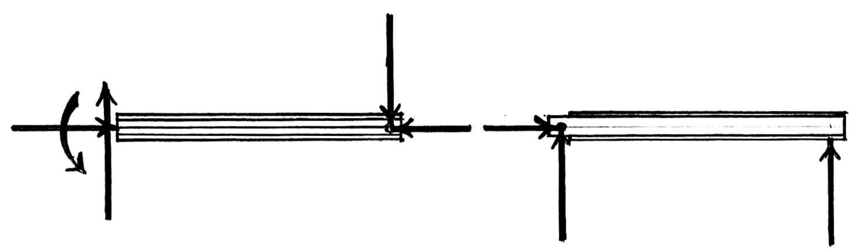
(d) Unstable since the lines of action of the reactive force components are concurrent.



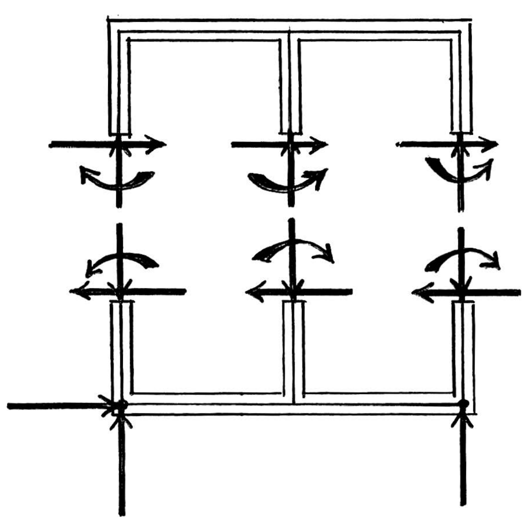
(d)



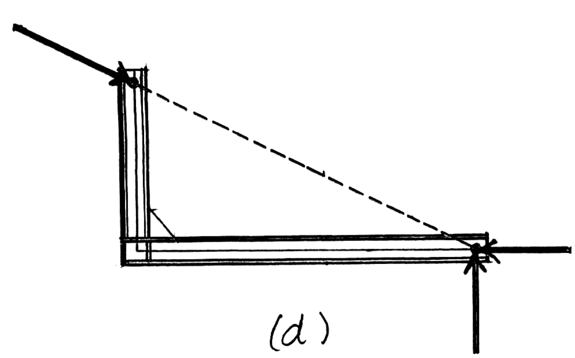
(a)



(c)

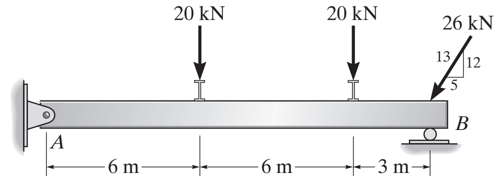


(b)



(d)

2–18. Determine the reactions on the beam. Neglect the thickness of the beam.

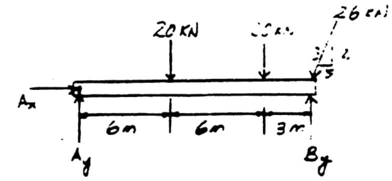


$\curvearrowleft + \sum M_A = 0; \quad B_y(15) - 20(6) - 20(12) - 26\left(\frac{12}{13}\right)(15) = 0$

$B_y = 48.0\text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y + 48.0 - 20 - 20 - \frac{12}{13}(26) = 0$

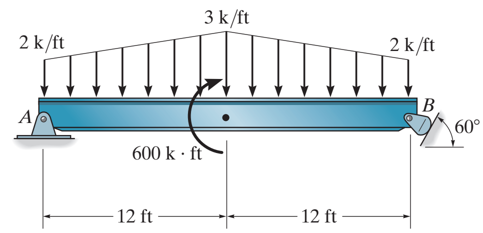
$A_y = 16.0\text{ kN}$ **Ans.**



$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - \left(\frac{5}{13}\right)26 = 0$

$A_x = 10.0\text{ kN}$ **Ans.**

2–19. Determine the reactions on the beam.



$\curvearrowleft + \sum M_A = 0; \quad -60(12) - 600 + F_B \cos 60°\,(24) = 0$

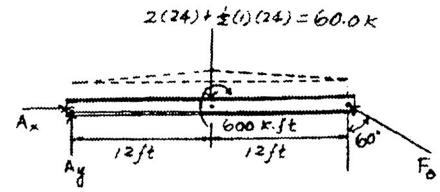
$F_B = 110.00\text{ k} = 110\text{ k}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 110.00 \sin 60° = 0$

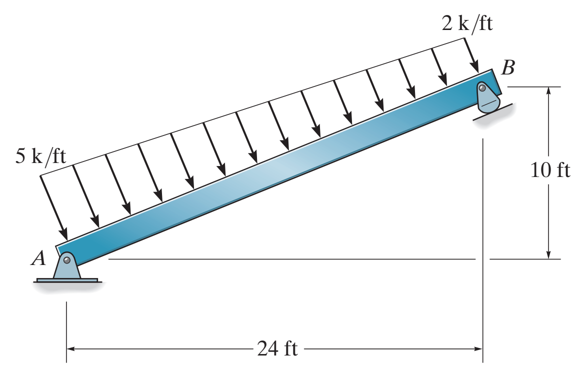
$A_x = 95.3\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y = 110.00 \cos 60° - 60 = 0$

$A_y = 5.00\text{ k}$ **Ans.**



***2–20.** Determine the reactions on the beam.

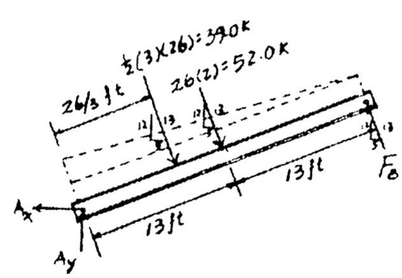


$\curvearrowleft + \sum M_A = 0; \quad F_B(26) - 52(13) - 39\left(\frac{1}{3}\right)(26) = 0$

$F_B = 39.0\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y - \frac{12}{13}(39) - \left(\frac{12}{13}\right)52 + \left(\frac{12}{13}\right)(39.0) = 0$

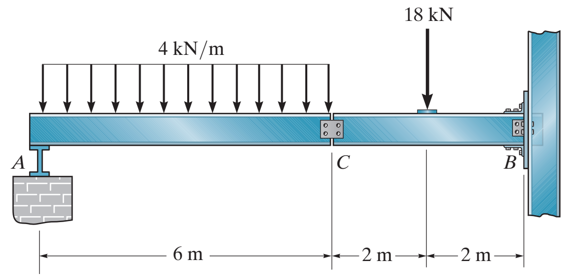
$A_y = 48.0\text{ k}$ **Ans.**



$\overset{+}{\rightarrow} \sum F_x = 0; \quad -A_x + \left(\frac{5}{13}\right)39 + \left(\frac{5}{13}\right)52 - \left(\frac{5}{13}\right)39.0 = 0$

$A_x = 20.0\text{ k}$ **Ans.**

2–21. Determine the reactions at the supports A and B of the compound beam. Assume there is a pin at C.



Equations of Equilibrium: First consider the FBD of segment AC in Fig. *a*. N_A and C_y can be determined directly by writing the moment equations of equilibrium about C and A respectively.

$\circlearrowleft + \sum M_C = 0; \quad 4(6)(3) - N_A(6) = 0 \quad N_A = 12 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad C_y(6) - 4(6)(3) = 0 \quad C_y = 12 \text{ kN}$ **Ans.**

Then,

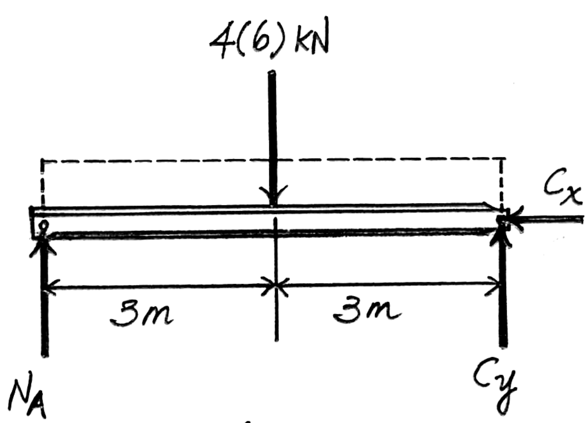
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 0 - C_x = 0 \quad C_x = 0$

Using the FBD of segment CB, Fig. *b*,

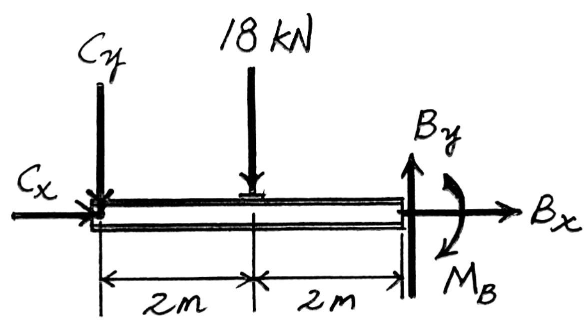
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 0 + B_x = 0 \quad B_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad B_y - 12 - 18 = 0 \quad B_y = 30 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_B = 0; \quad 12(4) + 18(2) - M_B = 0 \quad M_B = 84 \text{ kN} \cdot \text{m}$ **Ans.**

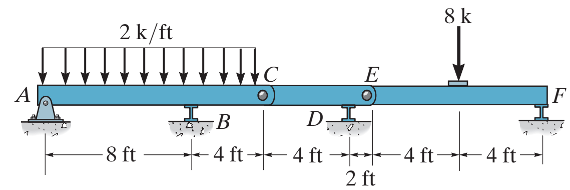


(a)



(b)

2–22. Determine the reactions at the supports A, B, D, and F.



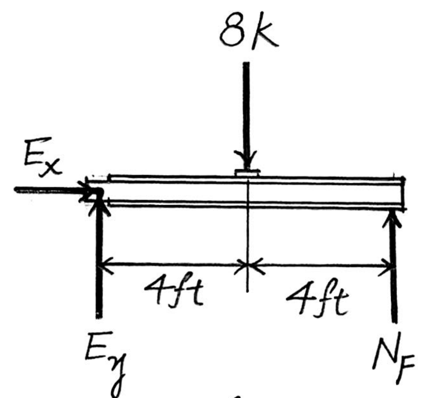
Equations of Equilibrium: First consider the FBD of segment EF in Fig. a. N_F and E_y can be determined directly by writing the moment equations of equilibrium about E and F respectively.

$\circlearrowleft + \sum M_E = 0; \quad N_F - (8) - 8(4) = 0 \quad N_F = 4.00 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_F = 0; \quad 8(4) - E_y(8) = 0 \quad E_y = 4.00 \text{ k}$

Then

$\overset{+}{\rightarrow} \sum F_x = 0; \quad E_x = 0$



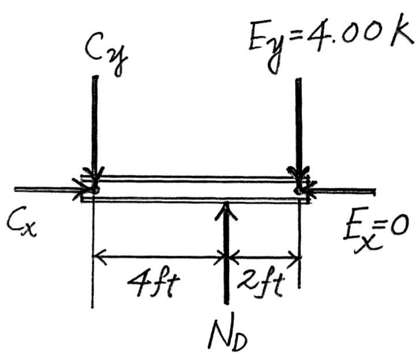
(a)

Consider the FBD of segment CDE, Fig. b,

$\overset{+}{\rightarrow} \sum F_x = 0; \quad C_x - 0 = 0 \quad C_x = 0$

$\circlearrowleft + \sum M_C = 0; \quad N_P(4) - 4.00(6) = 0 \quad N_D = 6.00 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_D = 0; \quad C_y(4) - 4.00(2) = 0 \quad C_y = 2.00 \text{ k}$



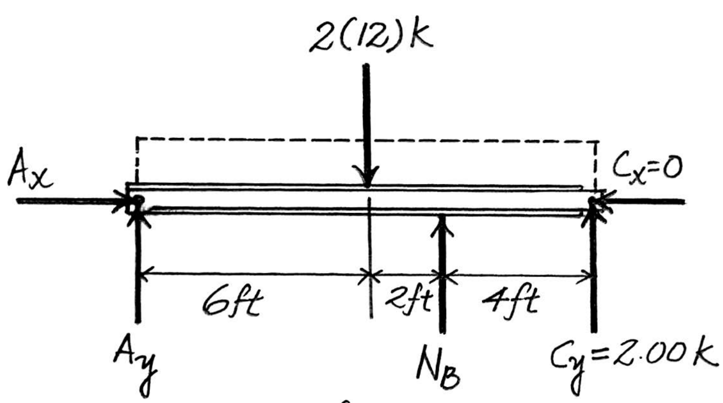
(b)

Now consider the FBD of segment ABC, Fig. c.

$\circlearrowleft + \sum M_A = 0; \quad N_B(8) + 2.00(12) - 2(12)(6) = 0 \quad N_B = 15.0 \text{ k}$ **Ans.**

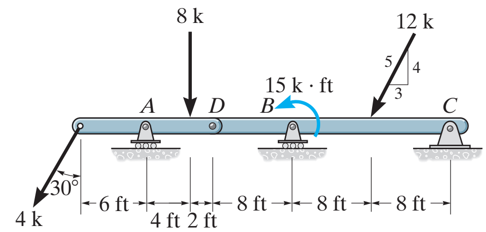
$\circlearrowleft + \sum M_B = 0; \quad 2(12)(2) + 2.00(4) - A_y(8) = 0 \quad A_y = 7.00 \text{ k}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 0 = 0 \quad A_x = 0$ **Ans.**



(c)

2–23. The compound beam is pin supported at C and supported by a roller at A and B. There is a hinge (pin) at D. Determine the reactions at the supports. Neglect the thickness of the beam.



Equations of Equilibrium: Consider the FBD of segment AD, Fig. a.

$\overset{+}{\rightarrow}\sum F_x = 0; \quad D_x - 4\sin 30° = 0 \qquad D_x = 2.00 \text{ k}$

$\circlearrowleft + \sum M_D = 0; \quad 8(2) + 4\cos 30°(12) - N_A(6) = 0 \qquad N_A = 9.59 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad D_y(6) + 4\cos 30°(6) - 8(4) = 0 \qquad D_y = 1.869 \text{ k}$

Now consider the FBD of segment DBC shown in Fig. b,

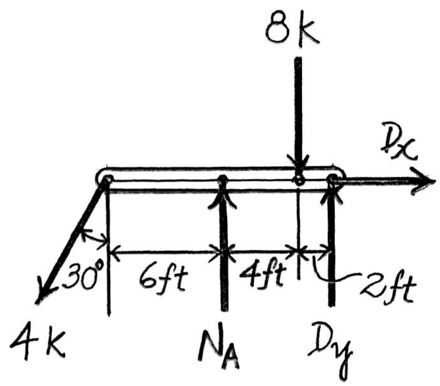
$\overset{+}{\rightarrow}\sum F_x = 0; \quad C_x - 2.00 - 12\left(\frac{3}{5}\right) = 0 \qquad C_x = 9.20 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_C = 0; \quad 1.869(24) + 15 + 12\left(\frac{4}{5}\right)(8) - N_B(16) = 0$

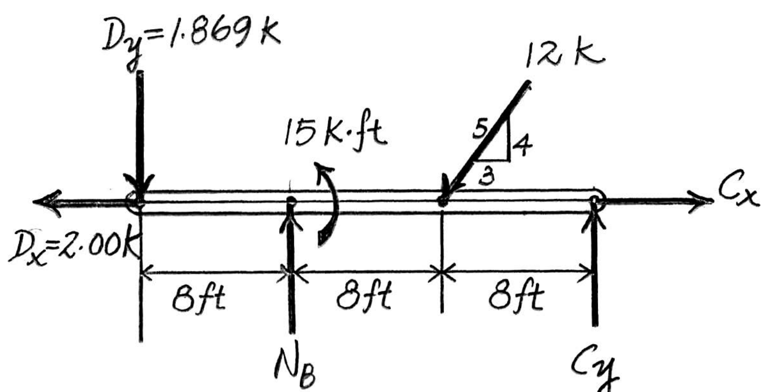
$N_B = 8.54 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_B = 0; \quad 1.869(8) + 15 - 12\left(\frac{4}{5}\right)(8) - C_y(16) = 0$

$C_y = 2.93 \text{ k}$ **Ans.**

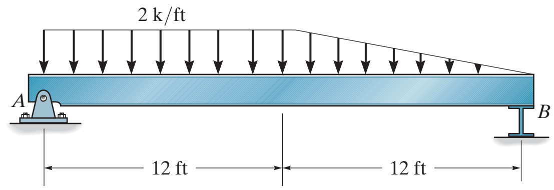


(a)



(b)

***2–24.** Determine the reactions on the beam. The support at B can be assumed to be a roller.

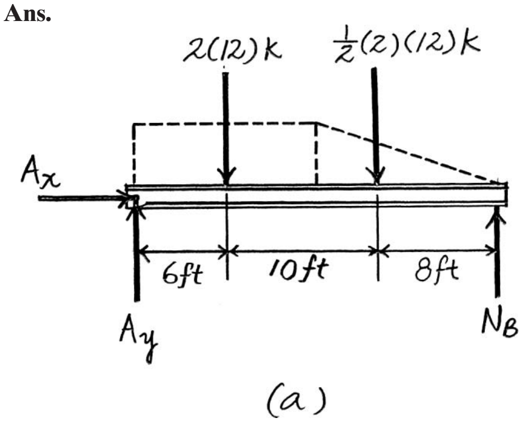


Equations of Equilibrium:

$$\circlearrowleft + \sum M_A = 0; \quad N_B(24) - 2(12)(6) - \frac{1}{2}(2)(12)(16) = 0 \qquad N_B = 14.0 \text{ k} \quad \textbf{Ans.}$$

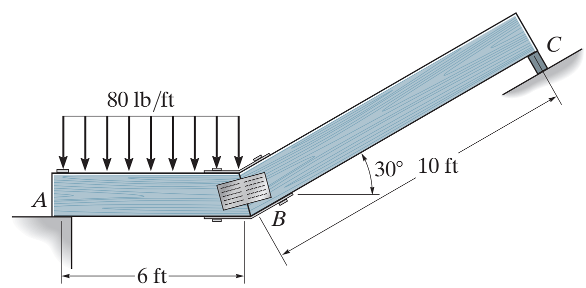
$$\circlearrowleft + \sum M_B = 0; \quad \frac{1}{2}(2)(12)(8) + 2(12)(18) - A_y(24) = 0 \qquad A_y = 22.0 \text{ k} \quad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0 \qquad \textbf{Ans.}$$



(a)

2–25. Determine the reactions at the smooth support C and pinned support A. Assume the connection at B is fixed connected.



$$\circlearrowleft + \sum M_A = 0; \quad C_y(10 + 6 \sin 60°) - 480(3) = 0$$

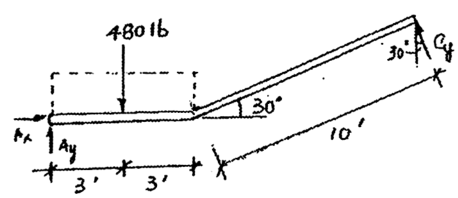
$$C_y = 94.76 \text{ lb} = 94.8 \text{ lb} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 94.76 \sin 30° = 0$$

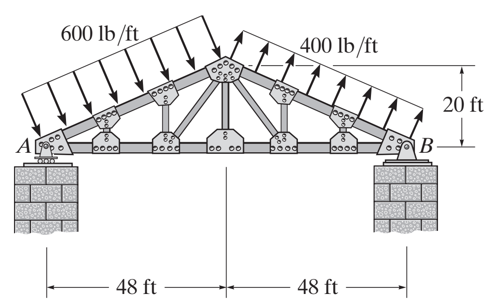
$$A_y = 47.4 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad A_y + 94.76 \cos 30° - 480 = 0$$

$$A_y = 398 \text{ lb} \qquad \textbf{Ans.}$$



2–26. Determine the reactions at the truss supports A and B. The distributed loading is caused by wind.



$$\circlearrowleft + \sum M_A = 0; \quad B_y(96) + \left(\frac{12}{13}\right)20.8(72) - \left(\frac{5}{13}\right)20.8(10)$$

$$-\left(\frac{12}{13}\right)31.2(24) - \left(\frac{5}{13}\right)31.2(10) = 0$$

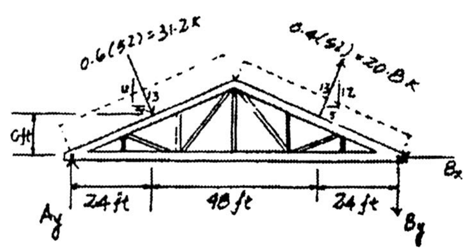
$B_y = 5.117 \text{ kN} = 5.12 \text{ kN}$ **Ans.**

$$+\uparrow \sum F_y = 0; \quad A_y - 5.117 + \left(\frac{12}{13}\right)20.8 - \left(\frac{12}{13}\right)31.2 = 0$$

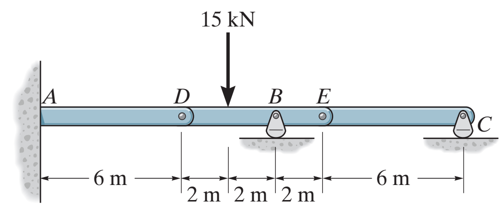
$A_y = 14.7 \text{ kN}$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad -B_x + \left(\frac{5}{13}\right)31.2 + \left(\frac{5}{13}\right)20.8 = 0$$

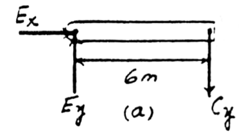
$B_x = 20.0 \text{ kN}$ **Ans.**



2–27. The compound beam is fixed at A and supported by a rocker at B and C. There are hinges pins at D and E. Determine the reactions at the supports.



Equations of Equilibrium: From FBD(a),

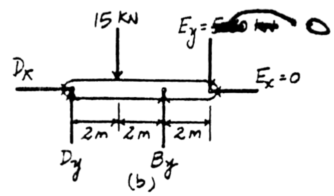


$\circlearrowleft + \sum M_E = 0; \quad C_y(6) = 0 \quad C_y = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad E_y - 0 = 0 \quad E_y = 0$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad E_x = 0$

From FBD (b),



$\circlearrowleft + \sum M_D = 0; \quad B_y(4) - 15(2) = 0$

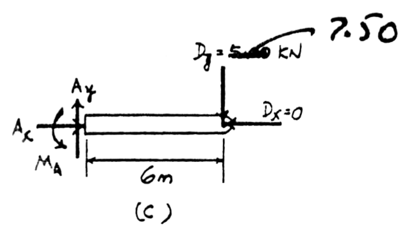
$B_y = 7.50 \text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad D_y + 7.50 - 15 = 0$

$D_y = 7.50 \text{ kN}$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad D_x = 0$

From FBD (c),



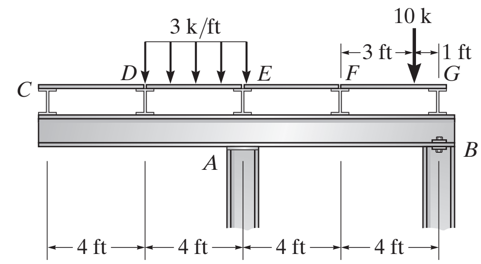
$\circlearrowleft + \sum M_A = 0; \quad M_A - 7.50(6) = 0$

$M_A = 45.0 \text{ kN} \cdot \text{m}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y - 7.50 = 0 \quad A_y = 7.50 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0$ **Ans.**

***2–28.** Determine the reactions at the supports A and B. The floor decks CD, DE, EF, and FG transmit their loads to the girder on smooth supports. Assume A is a roller and B is a pin.



Consider the entire system.

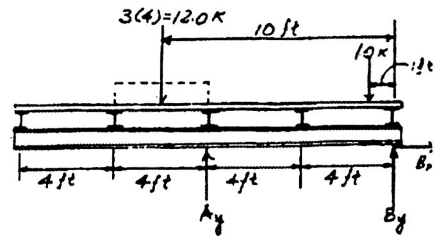
$\zeta + \sum M_B = 0; \quad 10(1) + 12(10) - A_y(8) = 0$

$A_y = 16.25 \text{ k} = 16.3 \text{ k}$ **Ans.**

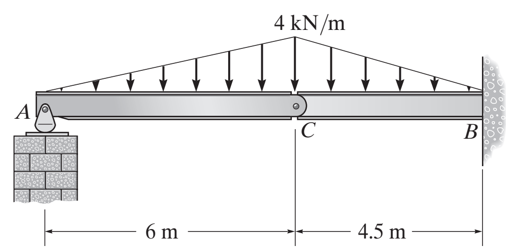
$\overset{+}{\rightarrow} \sum F_x = 0; \quad B_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 16.25 - 12 - 10 + B_y = 0$

$B_y = 5.75 \text{ k}$ **Ans.**



2–29. Determine the reactions at the supports A and B of the compound beam. There is a pin at C.



Member AC:

$\zeta + \sum M_C = 0; -A_y(6) + 12(2) = 0$

$A_y = 4.00 \text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad C_y + 4.00 - 12 = 0$

$C_y = 8.00 \text{ kN}$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad C_x = 0$

Member CB:

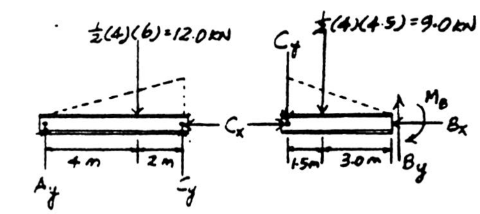
$\zeta + \sum M_B = 0; -M_B + 8.00(4.5) + 9(3) = 0$

$M_B = 63.0 \text{ kN} \cdot \text{m}$ **Ans.**

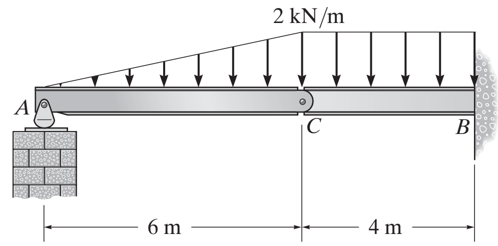
$+\uparrow \sum F_y = 0; \quad B_y - 8 - 9 = 0$

$B_y = 17.0 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad B_x = 0$ **Ans.**



2–30. Determine the reactions at the supports A and B of the compound beam. There is a pin at C.



Member AC:

$\circlearrowleft + \sum M_C = 0;\quad -A_y(6) + 6(2) = 0;\quad A_y = 2.00\text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;\quad C_x = 0$

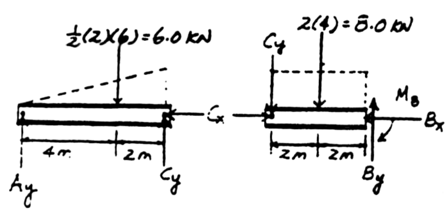
$+\uparrow \sum F_y = 0;\quad 2.00 - 6 + C_y = 0;\quad C_y = 4.00\text{ kN}$

Member BC:

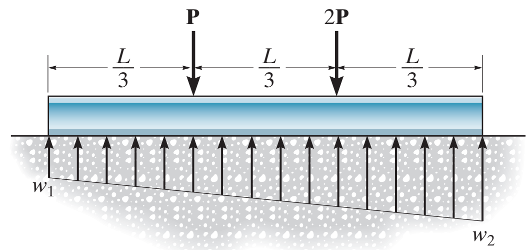
$+\uparrow \sum F_y = 0;\quad -4.00 - 8 + B_y = 0;\quad B_y = 12.0\text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 0 - B_x = 0;\quad B_x = 0$ **Ans.**

$\circlearrowleft + \sum M_B = 0;\quad -M_B + 8(2) + 4.00(4) = 0;\quad M_B = 32.0\text{ kN}\cdot\text{m}$ **Ans.**



2–31. The beam is subjected to the two concentrated loads as shown. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium (a) in terms of the parameters shown; (b) set $P = 500$ lb, $L = 12$ ft.



Equations of Equilibrium: The load intensity w_1 can be determined directly by summing moments about point A.

$$\circlearrowleft + \sum M_A = 0; \quad P\left(\frac{L}{3}\right) - w_1 L\left(\frac{L}{6}\right) = 0$$

$$w_1 = \frac{2P}{L} \qquad \textbf{Ans.}$$

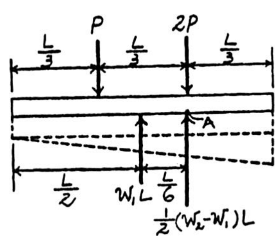
$$+\uparrow \sum F_y = 0; \quad \frac{1}{2}\left(w_2 - \frac{2P}{L}\right)L + \frac{2P}{L}(L) - 3P = 0$$

$$w_2 = \left(\frac{4P}{L}\right) \qquad \textbf{Ans.}$$

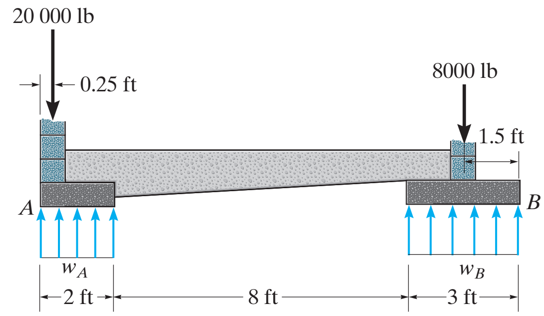
If $P = 500$ lb and $L = 12$ ft,

$$w_1 = \frac{2(500)}{12} = 83.3 \text{ lb/ft} \qquad \textbf{Ans.}$$

$$w_2 = \frac{4(500)}{12} = 167 \text{ lb/ft} \qquad \textbf{Ans.}$$



***2–32** The cantilever footing is used to support a wall near its edge A so that it causes a uniform soil pressure under the footing. Determine the uniform distribution loads, w_A and w_B, measured in lb/ft at pads A and B, necessary to support the wall forces of 8000 lb and 20 000 lb.

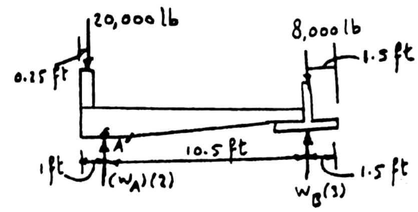


$\circlearrowleft + \sum M_A = 0; \quad -8000(10.5) + w_B(3)(10.5) + 20\,000(0.75) = 0$

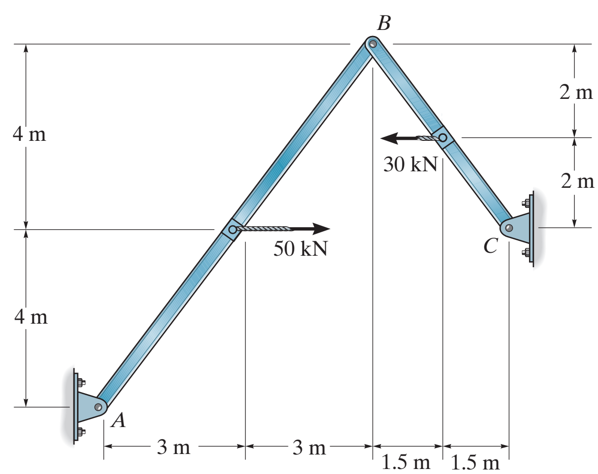
$w_B = 2190.5 \text{ lb/ft} = 2.19 \text{ k/ft}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 2190.5(3) - 28\,000 + w_A(2) = 0$

$w_A = 10.7 \text{ k/ft}$ **Ans.**



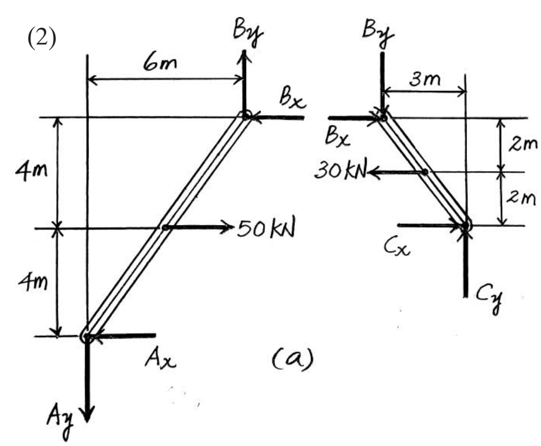
2–33. Determine the horizontal and vertical components of reaction acting at the supports A and C.



Equations of Equilibrium: Referring to the FBDs of segments AB and BC respectively shown in Fig. a,

$\circlearrowleft + \sum M_A = 0; \quad B_x(8) + B_y(6) - 50(4) = 0$ (1)

$\circlearrowleft + \sum M_C = 0; \quad B_y(3) - B_x(4) + 30(2) = 0$ (2)



2–33. Continued

Solving,

$B_y = 6.667 \text{ kN} \quad B_x = 20.0 \text{ kN}$

Segment AB,

$\xrightarrow{+} \sum F_x = 0; \quad 50 - 20.0 - A_x = 0 \quad A_x = 30.0 \text{ kN}$ **Ans.**

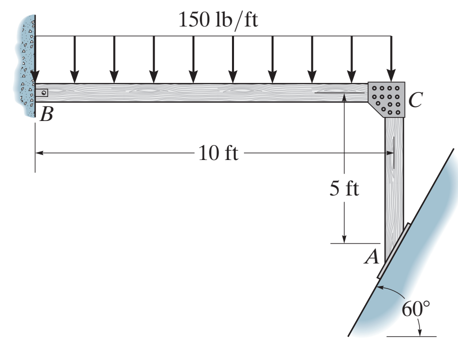
$+\uparrow \sum F_y = 0; \quad 6.667 - A_y = 0 \quad A_y = 6.67 \text{ kN}$ **Ans.**

Segment BC,

$\xrightarrow{+} \sum F_x = 0; \quad C_x + 20.0 - 30 = 0 \quad C_x - 10.0 \text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad C_y - 6.667 = 0 \quad Cy = 6.67 \text{ kN}$ **Ans.**

2–34. Determine the reactions at the smooth support A and the pin support B. The joint at C is fixed connected.



Equations of Equilibrium: Referring to the FBD in Fig. *a*.

$\circlearrowleft + \sum M_B = 0; \quad N_A \cos 60°(10) - N_A \sin 60°(5) - 150(10)(5) = 0$

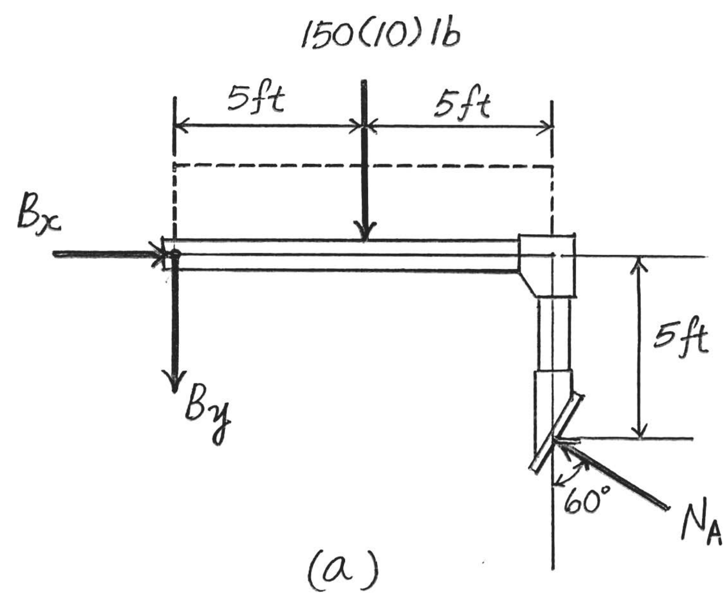
$N_A = 11196.15 \text{ lb} = 11.2 \text{ k}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad B_x - 11196.15 \sin 60° = 0$

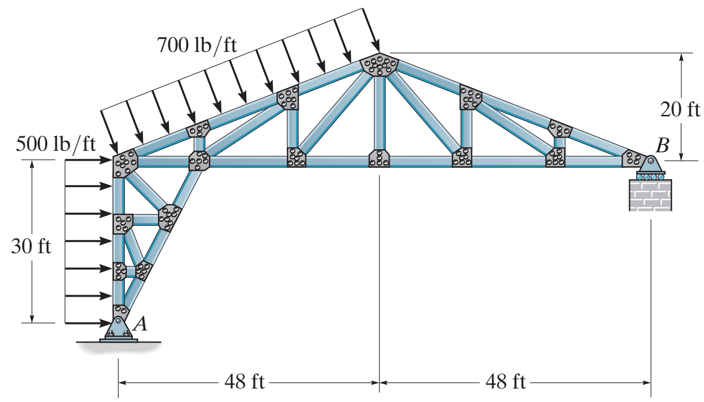
$B_x = 9696.15 \text{ lb} = 9.70 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 11196.15 \cos 60° - 150(10) - B_y = 0$ **Ans.**

$B_y = 4098.08 \text{ lb} = 4.10 \text{ k}$



2–35. Determine the reactions at the supports A and B.



700 lb/ft at 52 ft = 36,400 lb or 36.4 k

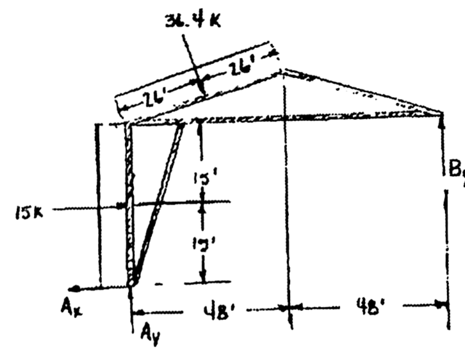
500 lb/ft at 30 ft = 15,000 lb or 15.0 k

$\circlearrowleft + \sum M_A = 0; \quad 96(B_y) - 24\left(\frac{48}{52}\right)(36.4) - 40\left(\frac{20}{52}\right)(36.4) - 15(15) = 0$

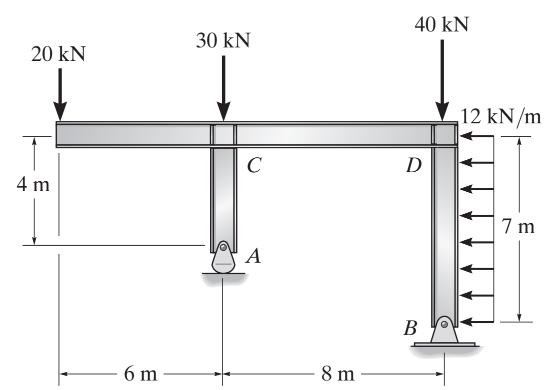
$B_y = 16.58 \text{ k} = 16.6 \text{ k}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad 15 + \frac{20}{52}(36.4) - A_x = 0; \quad A_x = 29.0 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y + B_y - \frac{48}{52}(36.4) = 0; \quad A_y = 17.0 \text{ k}$ **Ans.**



***2–36.** Determine the horizontal and vertical components of reaction at the supports A and B. Assume the joints at C and D are fixed connections.



$\circlearrowleft + \sum M_B = 0; \quad 20(14) + 30(8) + 84(3.5) - A_y(8) = 0$

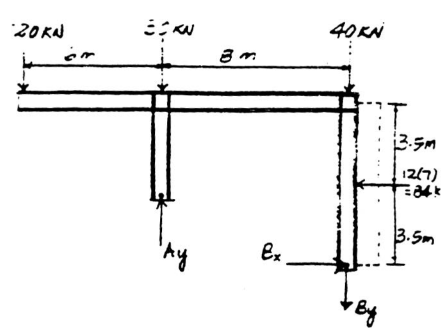
$A_y = 101.75 \text{ kN} = 102 \text{ kN}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad B_x - 84 = 0$

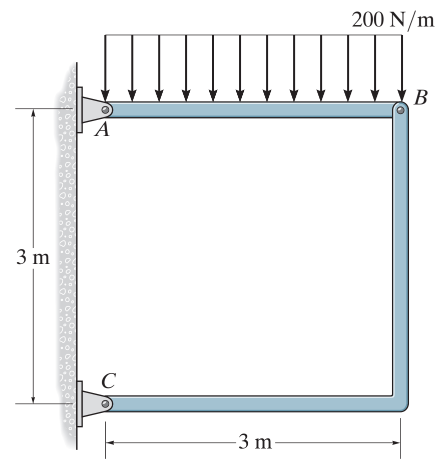
$B_x = 84.0 \text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 101.75 - 20 - 30 - 40 - B_y = 0$

$B_y = 11.8 \text{ kN}$ **Ans.**



2–37. Determine the horizontal and vertical components force at pins A and C of the two-member frame.



Free Body Diagram: The solution for this problem will be simplified if one realizes that member BC is a two force member.

Equations of Equilibrium:

$\circlearrowleft + \sum M_A = 0; \quad F_{BC} \cos 45° (3) - 600 (1.5) = 0$

$F_{BC} = 424.26 \text{ N}$

$+\uparrow \sum F_y = 0; \quad A_y + 424.26 \cos 45° - 600 = 0$

$A_y = 300 \text{ N}$ **Ans.**

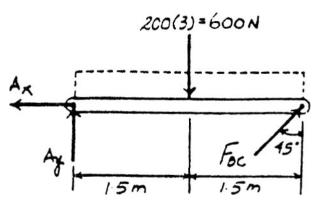
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 424.26 \sin 45° - A_x = 0$ **Ans.**

$A_x = 300 \text{ N}$

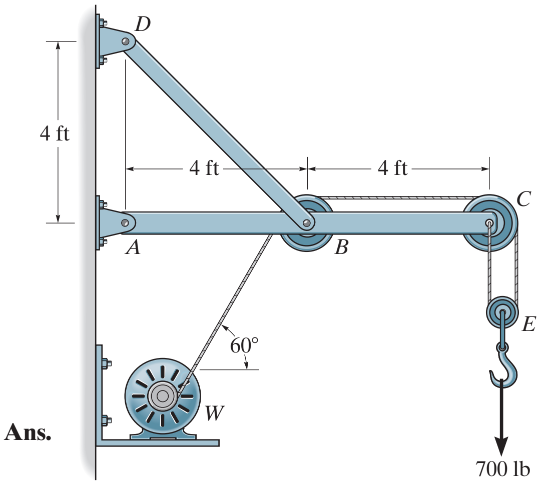
For pin C,

$C_x = F_{BC} \sin 45° = 424.26 \sin 45° = 300 \text{ N}$ **Ans.**

$C_y = F_{BC} \cos 45° = 424.26 \cos 45° = 300 \text{ N}$ **Ans.**



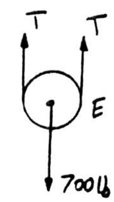
2–38. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins *A* and *D*. Also, what is the force in the cable at the winch *W*?



Pulley *E*:

$+\uparrow \Sigma F_y = 0; \quad 2T - 700 = 0$

$T = 350 \text{ lb}$ **Ans.**



Member *ABC*:

$\circlearrowleft + \sum M_A = 0; \quad T_{BD} \sin 45° (4) - 350 \sin 60°(4) - 700(8) = 0$

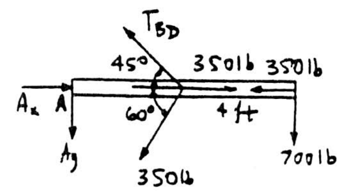
$T_{BD} = 2409 \text{ lb}$

$+\uparrow \sum F_y = 0; \quad A_y + 2409 \sin 45° - 350 \sin 60° - 700 = 0$

$A_y = 700 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 2409 \cos 45° - 350 \cos 60° + 350 - 350 = 0$

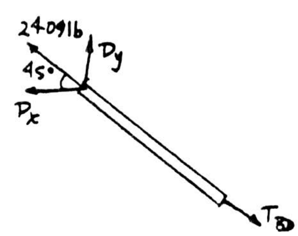
$A_x = 1.88 \text{ k}$ **Ans.**



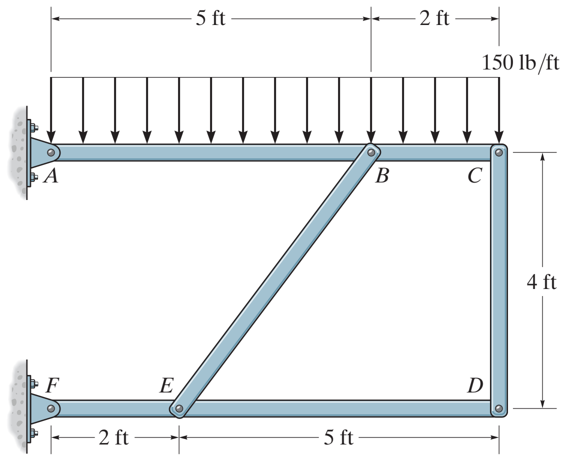
At *D*:

$D_x = 2409 \cos 45° = 1703.1 \text{ lb} = 1.70 \text{ k}$ **Ans.**

$D_y = 2409 \sin 45° = 1.70 \text{ k}$ **Ans.**



2–39. Determine the resultant forces at pins B and C on member ABC of the four-member frame.

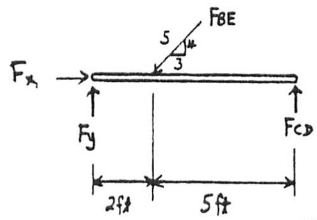


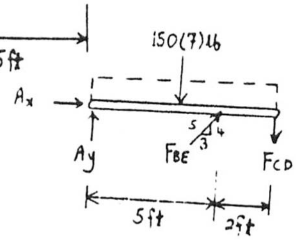
$$\zeta + \sum M_F = 0; \quad F_{CD}(7) - \frac{4}{5}F_{BE}(2) = 0$$

$$\zeta + \sum M_A = 0; \quad -150(7)(3.5) + \frac{4}{5}F_{BE}(5) - F_{CD}(7) = 0$$

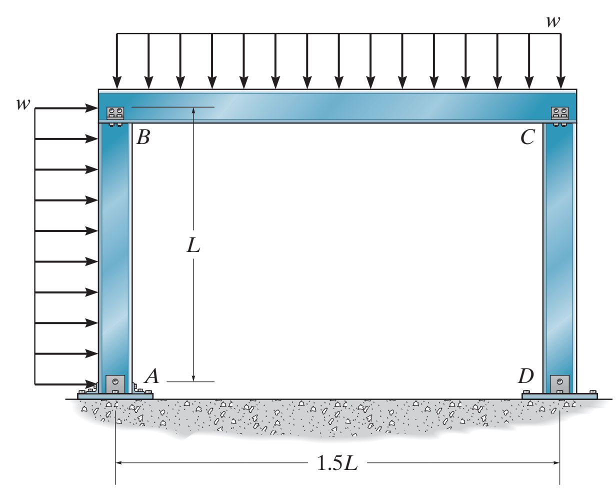
$$F_{BE} = 1531 \text{ lb} = 1.53 \text{ k}$$ **Ans.**

$$F_{CD} = 350 \text{ lb}$$ **Ans.**





***2–40.** Determine the reactions at the supports is A and D. Assume A is fixed and B and C and D are pins.



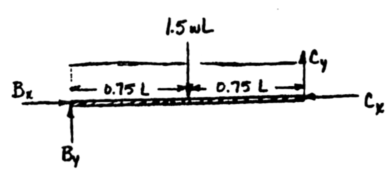
Member BC:

$$\zeta + \sum M_B = 0; \quad C_y(1.5L) - (1.5wL)\left(\frac{1.5L}{2}\right) = 0$$

$$C_y = 0.75\, wL$$

$$+\uparrow \sum F_y = 0; \quad B_y - 1.5wL + 0.75\, wL = 0$$

$$B_y = 0.75\, wL$$



Member CD:

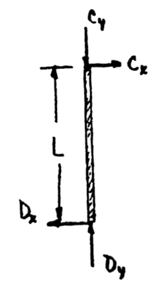
$$\zeta + \sum M_D = 0; \quad C_x = 0$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad D_x = 0$$ **Ans.**

$$+\uparrow \sum F_y = 0; \quad D_y - 0.75wL = 0$$

$$D_y = 0.75\, wL$$ **Ans.**

***2–40. Continued**



Member *BC*:

$$\overset{+}{\rightarrow}\sum F_x = 0;\quad B_x - 0 = 0; \qquad B_x = 0$$

Member *AB*:

$$\overset{+}{\rightarrow}\sum F_x = 0;\quad wL - A_x = 0$$ **Ans.**

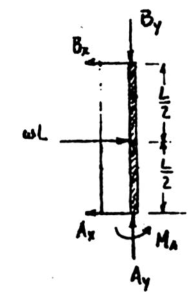
$$A_x = wL$$ **Ans.**

$$+\uparrow\sum F_y = 0;\quad A_y - 0.75\,wL = 0$$

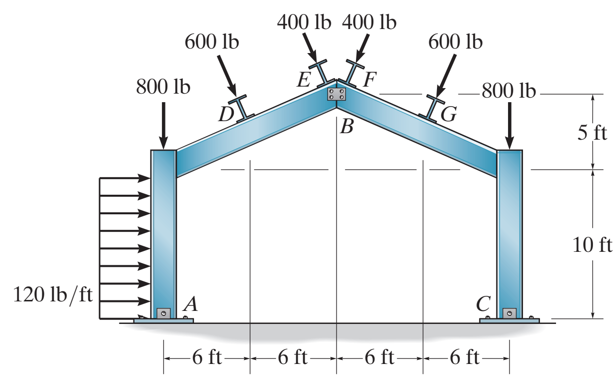
$$A_y = 0.75\,wL$$ **Ans.**

$$\circlearrowleft+\sum M_A = 0;\quad M_A - wL\left(\frac{L}{2}\right) = 0$$

$$M_A = \frac{wL^2}{2}$$ **Ans.**



2–41. Determine the horizontal and vertical reactions at the connections *A* and *C* of the gable frame. Assume that *A*, *B*, and *C* are pin connections. The purlin loads such as *D* and *E* are applied perpendicular to the center line of each girder.

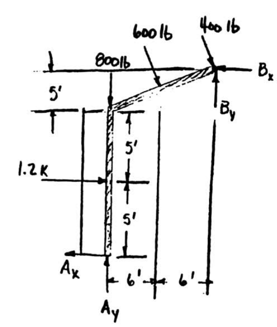


Member *AB*:

$$\circlearrowleft+\sum M_A = 0;\quad B_x(15) + B_y(12) - (1200)(5) - 600\left(\frac{12}{13}\right)(16) - 600\left(\frac{5}{13}\right)(12.5)$$

$$- 400\left(\frac{12}{13}\right)(12) - 400\left(\frac{5}{13}\right)(15) = 0$$

$$B_x(15) + B_y(12) = 18{,}946.154 \qquad (1)$$

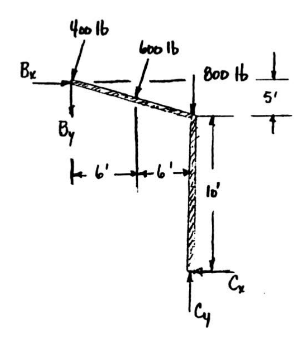


Member *BC*:

$$\circlearrowleft+\sum M_C = 0;\quad -(B_x)(15) + B_y(12) + (600)\left(\frac{12}{13}\right)(6) + 600\left(\frac{5}{13}\right)(12.5)$$

$$+ 400\left(\frac{12}{13}\right)(12) + 400\left(\frac{5}{13}\right)(15) = 0$$

$$B_x(15) - B_y(12) = 12.946.15 \qquad (2)$$



2–41. Continued

Solving Eqs. (1) and (2),

$$B_x = 1063.08 \text{ lb}, \qquad B_y = 250.0 \text{ lb}$$

Member *AB*:

$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad -A_x + 1200 + 1000\left(\frac{5}{13}\right) - 1063.08 = 0$$

$$A_x = 522 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow\sum F_y = 0; \qquad A_y - 800 - 1000\left(\frac{12}{13}\right) + 250 = 0$$

$$A_y = 1473 \text{ lb} = 1.47 \text{ k} \qquad \textbf{Ans.}$$

Member *BC*:

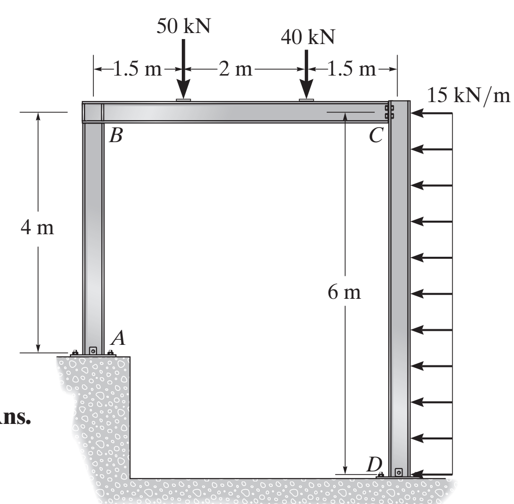
$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad -C_x - 1000\left(\frac{5}{13}\right) + 1063.08 = 0$$

$$C_x = 678 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow\sum F_y = 0; \qquad C_y - 800 - 1000\left(\frac{12}{13}\right) + 250.0 = 0$$

$$C_y = 1973 \text{ lb} = 1.97 \text{ k} \qquad \textbf{Ans.}$$

2–42. Determine the horizontal and vertical components of reaction at *A*, *C*, and *D*. Assume the frame is pin connected at *A*, *C*, and *D*, and there is a fixed-connected joint at *B*.



Member *CD*:

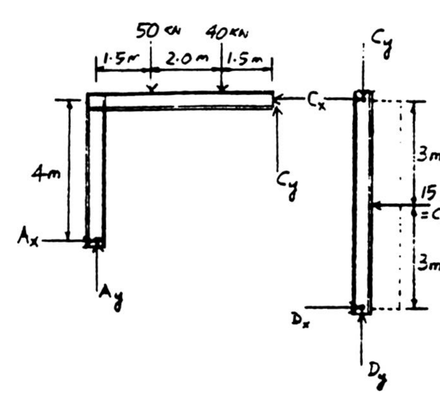
$$\circlearrowleft+\sum M_D = 0; \qquad -C_x(6) + 90(3) = 0$$

$$C_x = 45.0 \text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad D_x + 45 - 90 = 0$$

$$D_x = 45.0 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow\sum F_y = 0; \qquad D_y - C_y = 0 \qquad (1)$$



Member *ABC*:

$$\circlearrowleft+\sum M_A = 0; \qquad C_y(5) + 45.0(4) - 50(1.5) - 40(3.5) = 0$$

$$C_y = 7.00 \text{ kN} \qquad \textbf{Ans.}$$

2–42. Continued

$+\uparrow\sum F_y = 0;\quad A_y + 7.00 - 50 - 40 = 0$

$A_y = 83.0\text{ kN}$ **Ans.**

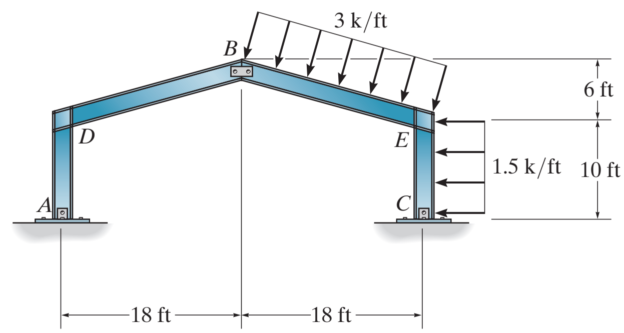
$\overset{+}{\rightarrow}\sum F_x = 0;\quad A_x - 45.0 = 0$

$A_x = 45.0\text{ kN}$ **Ans.**

From Eq. (1).

$D_y = 7.00\text{ kN}$ **Ans.**

2–43. Determine the horizontal and vertical components at A, B, and C. Assume the frame is pin connected at these points. The joints at D and E are fixed connected.



$\circlearrowleft+\sum M_A = 0;\quad -18\text{ ft}\,(B_y) + 16\text{ ft}\,(B_x) = 0$ (1)

$\circlearrowleft+\sum M_C = 0;\ 15\text{ k}\,(5\text{ft}) + 9\text{ ft}\,(56.92\text{ k}\,(\cos 18.43°)) + 13\text{ ft}\,(56.92\text{ k}\,(\sin 18.43°))$

$-16\text{ ft}\,(B_x) - 18\text{ ft}\,(B_x) = 0$ (2)

Solving Eq. 1 & 2

$B_x = 24.84\text{ k}$ **Ans.**

$B_y = 22.08\text{ k}$ **Ans.**

$\overset{+}{\rightarrow}\sum F_x = 0;\quad A_x - 24.84\text{ k} = 0$

$A_x = 24.84\text{ k}$

$+\uparrow\sum F_y = 0;\quad A_y - 22.08\text{ k} = 0$

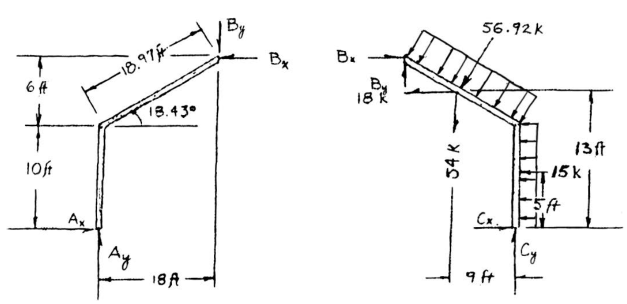
$A_y = 22.08\text{ k}$

$\overset{+}{\rightarrow}\sum F_x = 0;\quad C_x - 15\text{ k} - \sin(18.43°)\,(56.92\text{ k}) + 24.84\text{ k}$

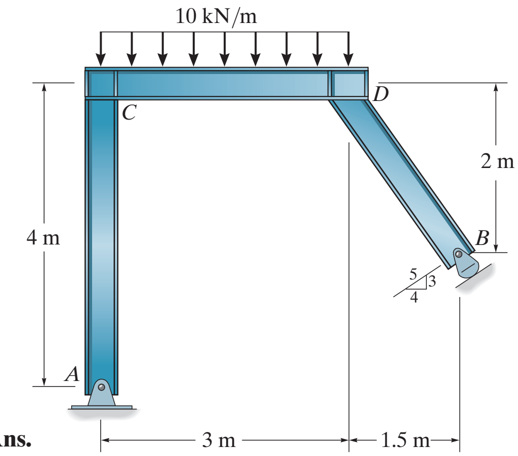
$C_x = 8.16\text{ k}$ **Ans.**

$+\uparrow\sum F_y = 0;\quad Cy + 22.08\text{ k} - \cos(18.43°)(56.92\text{ k}) = 0$

$Cy = 31.9\text{ k}$ **Ans.**

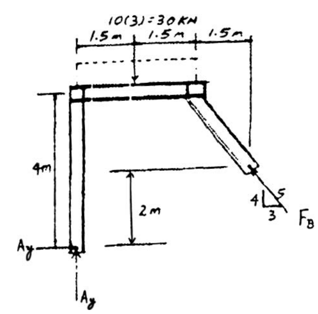


***2–44.** Determine the reactions at the supports A and B. The joints C and D are fixed connected.



$$\zeta + \sum M_A = 0; \quad \frac{4}{5}F_B(4.5) + \frac{3}{5}F_B(2) - 30(1.5) = 0$$

$$F_B = 9.375 \text{ kN} = 9.38 \text{ kN}$$ **Ans.**



$$+\uparrow \sum F_y = 0; \quad A_y + \frac{4}{5}(9.375) - 30 = 0$$

$$A_y = 22.5 \text{ kN}$$ **Ans.**

$$\xrightarrow{+} \sum F_x = 0; \quad A_x - \frac{3}{5}(9.375) = 0$$

$$A_x = 5.63 \text{ kN}$$ **Ans.**

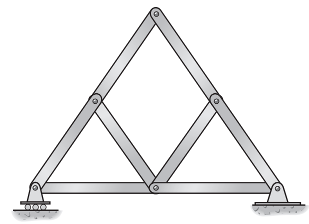
3–1. Classify each of the following trusses as statically determinate, statically indeterminate, or unstable. If indeterminate, state its degree.

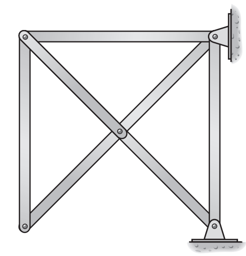
a) $b = 8,\ r = 3,\ j = 6$
$b + r = 2j$
$11 < 12$
Unstable. **Ans.**

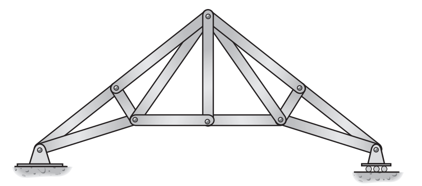
b) $b = 7,\ r = 4,\ j = 5$
$b + r = 2j$
$11 > 10$
Statically indeterminate to 1°. **Ans.**

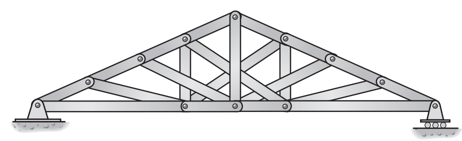
c) $b = 13,\ r = 3,\ j = 8$
$b + r = 2j$
$16 = 16$
Statically determinate. **Ans.**

d) $b = 21,\ r = 3,\ j = 12$
$b + r = 2j$
$24 = 24$
Statically determinate. **Ans.**


(a)


(b)


(c)


(d)

3–2. Classify each of the following trusses as stable, unstable, statically determinate, or statically indeterminate. If indeterminate, state its degree.

(a) $r = 3$
$b = 15$
$j = 9$

$3 + 15 = 9(2)$

Statically determinate. **Ans.**

(b) $r = 3$
$b = 11$
$j = 7$

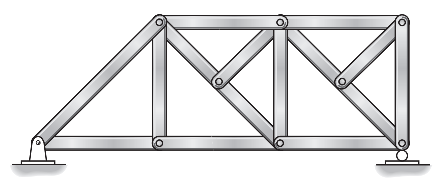
$3 + 11 = 7(2)$

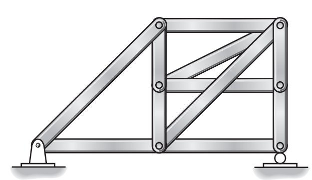
Statically determinate. **Ans.**

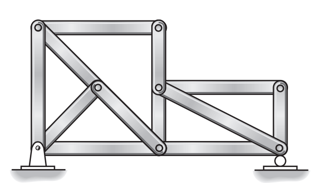
(c) $r = 3$
$b = 12$
$j = 8$

$3 + 12 < 8(2)$
$15 < 16$

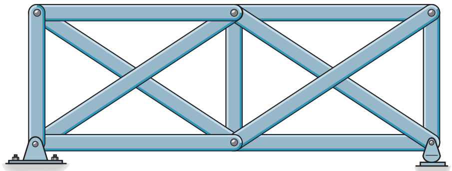
Unstable. **Ans.**


(a)

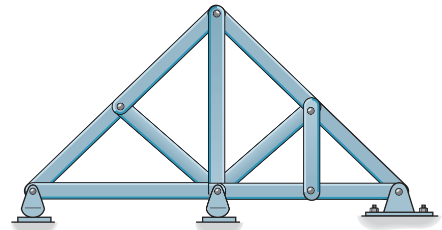

(b)


(c)

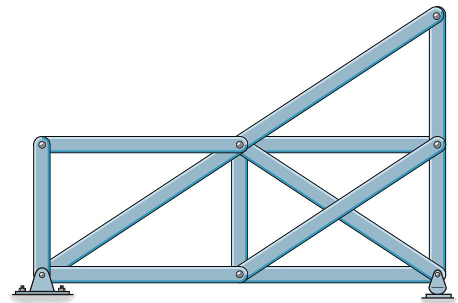
3–3. Classify each of the following trusses as statically determinate, indeterminate, or unstable. If indeterminate, state its degree.



(a)



(b)



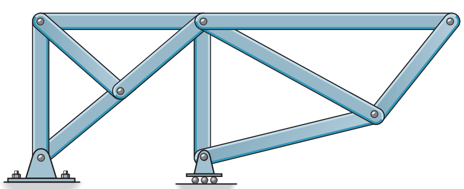
(c)

a) By inspection, the truss is **internally and externally stable**. Here, $b = 11$, $r = 3$ and $j = 6$. Since $b + r > 2j$ and $(b + r) - 2j = 14 - 12 = 2$, the truss is **statically indeterminate to the second degree**.

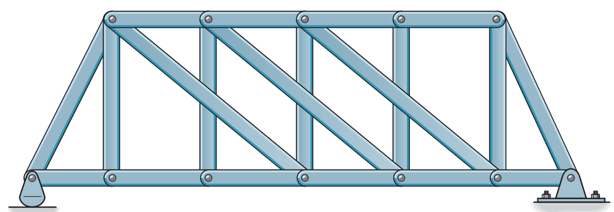
b) By inspection, the truss is **internally and externally stable**. Here, $b = 11$, $r = 4$ and $j = 7$. Since $b + r > 2j$ and $(b + r) - 2j = 15 - 14 = 1$, the truss is **statically indeterminate to the first degree**.

c) By inspection, the truss is **internally and externally stable**. Here, $b = 12$, $r = 3$ and $j = 7$. Since $b + r > 2j$ and $(b + r) - 2j = 15 - 14 = 1$, the truss is **statically indeterminate to the first degree**.

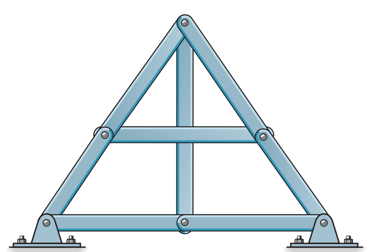
***3–4.** Classify each of the following trusses as statically determinate, statically indeterminate, or unstable. If indeterminate, state its degree.



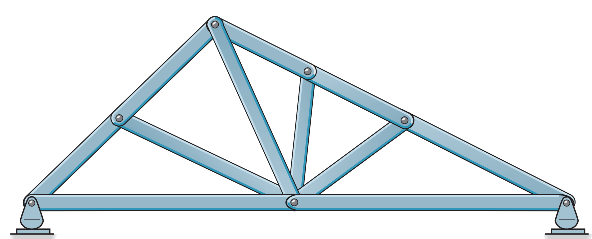
(a)



(b)



(c)



(d)

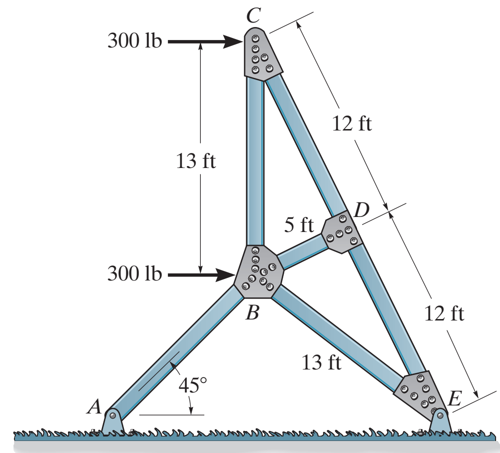
a) Here $b = 10$, $r = 3$ and $j = 7$. Since $b + r < 2j$, the truss is **unstable**.

b) Here $b = 20$, $r = 3$ and $j = 12$. Since $b + r < 2j$, the truss is **unstable**.

c) By inspection, the truss is **internally and externally stable**. Here, $b = 8$, $r = 4$ and $j = 6$. Since $b + r = 2j$, the truss is **statically determinate**.

d) By inspection, the truss is **unstable externally** since the line of action of all the support reactions are parallel.

3–5. A sign is subjected to a wind loading that exerts horizontal forces of 300 lb on joints *B* and *C* of one of the side supporting trusses. Determine the force in each member of the truss and state if the members are in tension or compression.



Joint *C*: Fig *a*.

$\overset{+}{\rightarrow} \sum F_x = 0;\ 300 - F_{CD}\left(\frac{5}{13}\right) = 0 \quad F_{CD} = 780 \text{ lb (C)}$ **Ans.**

$+\uparrow \sum F_y = 0;\ 780\left(\frac{12}{13}\right) - F_{CB} = 0 \quad F_{CB} = 720 \text{ lb (T)}$ **Ans.**

Joint *D*: Fig. *b*.

$+\nearrow \sum F_x = 0; \quad F_{DB} = 0$ **Ans.**

$+\nwarrow \sum F_y = 0; \quad F_{DE} - 780 = 0 \quad F_{DE} = 780 \text{ lb (C)}$ **Ans.**

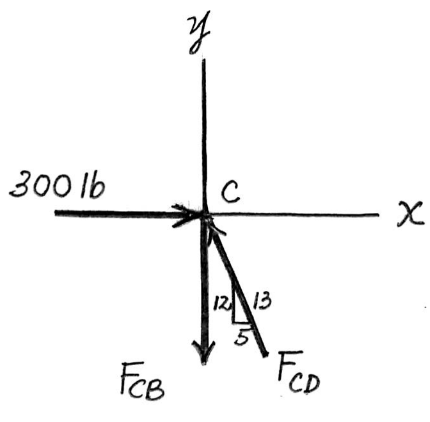
Joint *B*: Fig. *c*.

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 300 + F_{BE}\sin 45.24° - F_{BA}\cos 45° = 0$

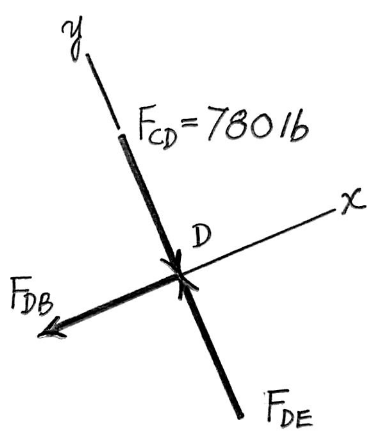
$+\uparrow \sum F_y = 0; \quad 720 - F_{BE}\cos 45.24° - F_{BA}\sin 45° = 0$

Solving

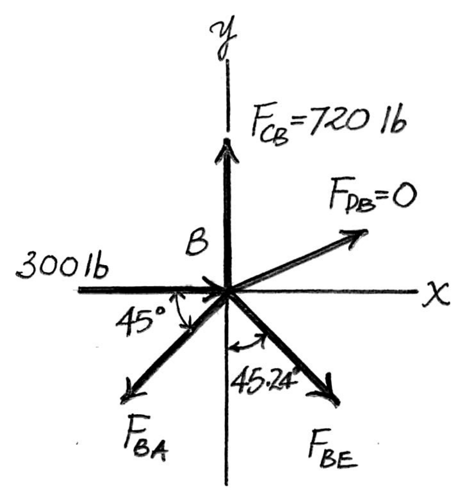
$F_{BE} = 296.99 \text{ lb} = 297 \text{ lb (T)} \quad F_{BA} = 722.49 \text{ lb (T)} = 722 \text{ lb (T)}$ **Ans.**



(a)

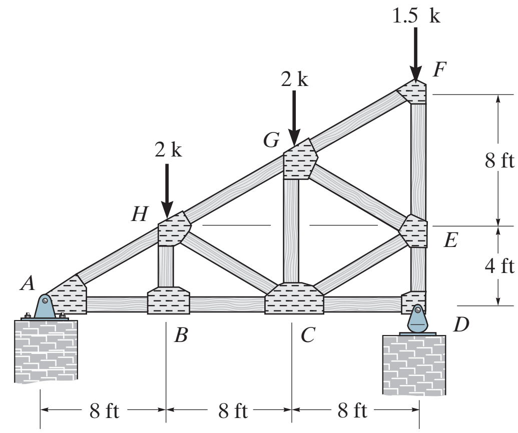


(b)



(c)

3–6. Determine the force in each member of the truss. Indicate if the members are in tension or compression. Assume all members are pin connected.



Support Reactions. Referring to the FBD of the entire truss, Fig. *a*

$$\circlearrowleft + \sum M_D = 0; \quad 2(8) + 2(16) - A_y(24) = 0 \quad A_y = 2.0\text{ k}$$

$$\xrightarrow{+} \sum F_x = 0; \quad A_x = 0$$

Method of Joint.

Joint *A*: Fig. *b*,

$$+\uparrow \sum F_y = 0; \quad 2.0 - F_{AH}\left(\frac{1}{\sqrt{5}}\right) = 0 \quad F_{AH} = 4.472\text{ k (C)} = 4.47\text{k (C)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} - 4.472\left(\frac{2}{\sqrt{5}}\right) = 0 \quad F_{AB} = 4.00\text{ k (T)} \qquad \textbf{Ans.}$$

Joint *B*: Fig. *c*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} - 4.00 = 0 \quad F_{BC} = 4.00\text{ k (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad F_{BH} = 0 \qquad \textbf{Ans.}$$

Joint *H*: Fig. *d*,

$$+\uparrow \sum F_y = 0; \quad F_{HC}\sin 53.13^\circ - 2\sin 63.43^\circ = 0 \quad F_{HC} = 2.236\text{ k (C)} = 2.24\text{ k (C)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \sum F_x = 0; \quad 4.472 - 2\cos 63.43^\circ - 2.236\cos 53.13^\circ - F_{HG} = 0$$

$$F_{HG} = 2.236\text{ k (C)} = 2.24\text{ k (C)} \qquad \textbf{Ans.}$$

Joint *F*: Fig. *e*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{FG} = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad F_{FE} - 1.5 = 0 \quad F_{FE} = 1.5\text{ k (C)} \qquad \textbf{Ans.}$$

Joint *G*: Fig. *f*,

$$\xrightarrow{+} \sum F_x = 0; \quad 2.236\left(\frac{2}{\sqrt{5}}\right) - F_{GE} = \left(\frac{2}{\sqrt{5}}\right) = 0 \quad F_{GE} = 2.236\text{ k (C)} = 2.24\text{ k (C)} \qquad \text{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad 2.236\left(\frac{1}{\sqrt{5}}\right) + 2.236\left(\frac{1}{\sqrt{5}}\right) - 2 - F_{GC} = 0 \quad F_{GC} = 0 \qquad \textbf{Ans.}$$

Joint *E*: Fig. *g*,

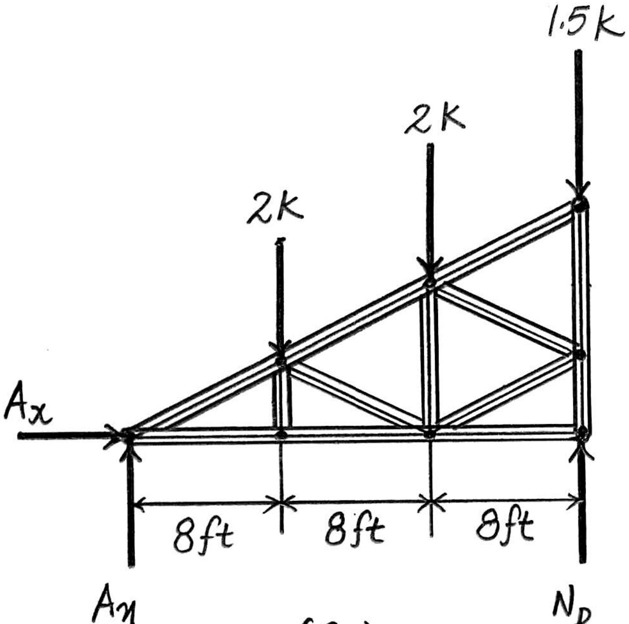
$$\xrightarrow{+} \sum F_x = 0; \quad 2.236\left(\frac{2}{\sqrt{5}}\right) - F_{EC}\left(\frac{2}{\sqrt{5}}\right) = 0 \quad F_{EC} = 2.236\text{ k (T)} = 2.24\text{ k (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad F_{ED} = 2.236\left(\frac{1}{\sqrt{5}}\right) - 2.236\left(\frac{1}{\sqrt{5}}\right) - 1.5 = 0 \quad F_{ED} = 3.5\text{ k (C)} \qquad \textbf{Ans.}$$

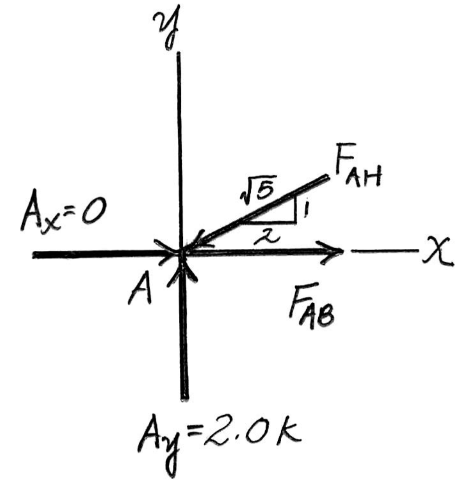
Joint *D*: Fig. *h*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{DC} = 0 \qquad \textbf{Ans.}$$

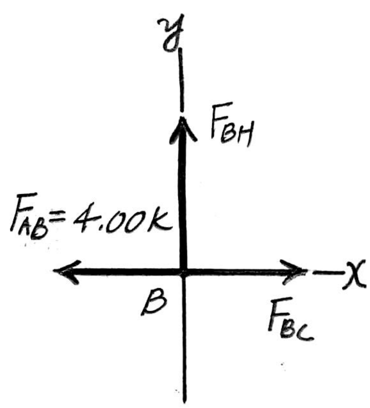
3–6. Continued



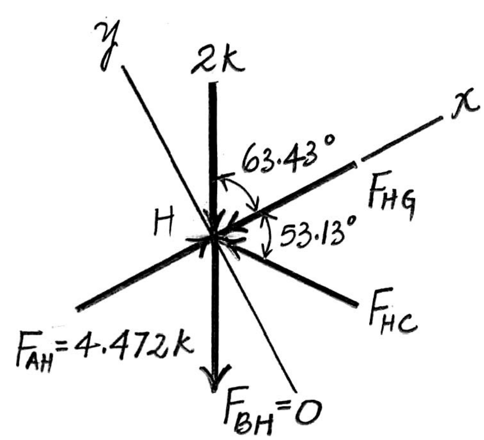
(a)



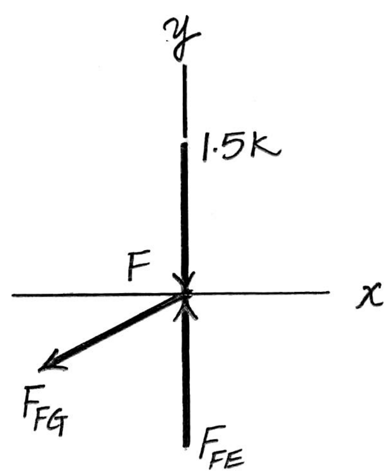
(b)



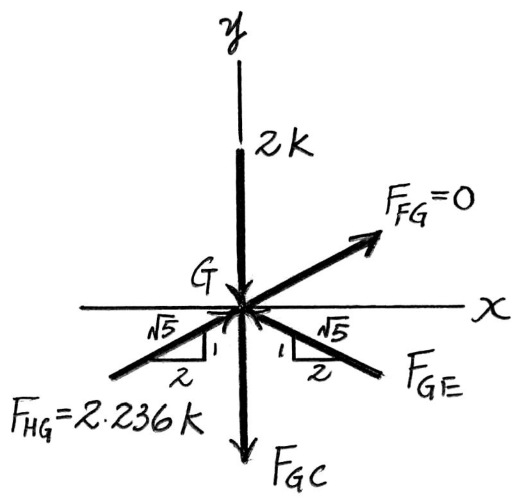
(c)



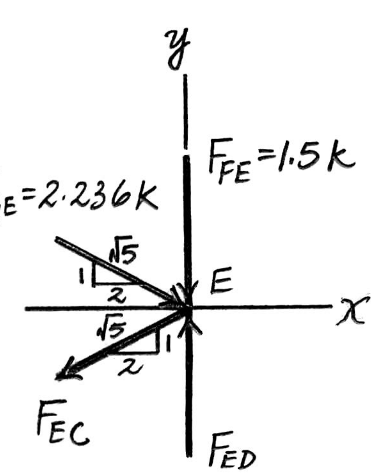
(d)



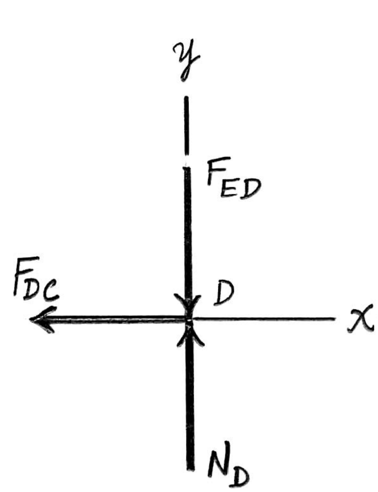
(e)



(f)

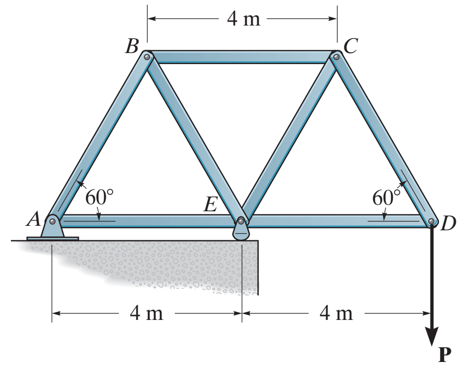


(g)



(h)

3–7. Determine the force in each member of the truss. State whether the members are in tension or compression. Set $P = 8$ kN.

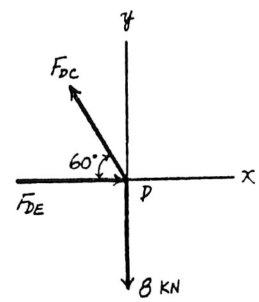


***Method of Joints*:** In this case, the support reactions are not required for determining the member forces.

Joint *D*:

$+\uparrow \sum F_y = 0; \quad F_{DC} \sin 60° - 8 = 0$

$F_{DC} = 9.238 \text{ kN (T)} = 9.24 \text{ kN (T)}$ **Ans.**



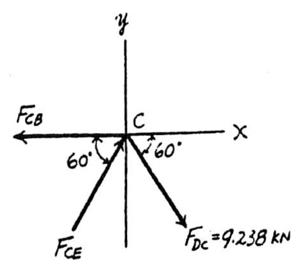
$\xrightarrow{+} \sum F_x = 0; \quad F_{DE} - 9.238 \cos 60° = 0$

$F_{DE} = 4.619 \text{ kN (C)} = 4.62 \text{ kN (C)}$ **Ans.**

Joint *C*:

$+\uparrow \sum F_y = 0; \quad F_{CE} \sin 60° - 9.328 \sin 60° = 0$

$F_{CE} = 9.238 \text{ kN (C)} = 9.24 \text{ kN (C)}$ **Ans.**



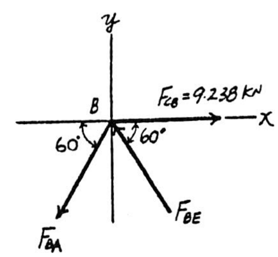
$\xrightarrow{+} \sum F_x = 0; \quad 2(9.238 \cos 60°) - F_{CB} = 0$

$F_{CB} = 9.238 \text{ kN (T)} = 9.24 \text{ kN (T)}$ **Ans.**

Joint *B*:

$+\uparrow \sum F_y = 0; \quad F_{BE} \sin 60° - F_{BA} \sin 60° = 0$

$F_{BE} = F_{BA} = F$



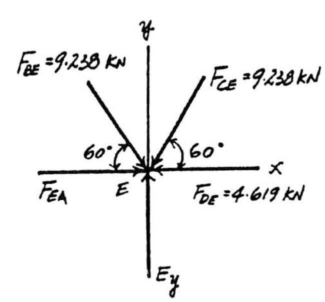
$\xrightarrow{+} \sum F_x = 0; \quad 9.238 - 2F \cos 60° = 0$

$F = 9.238$ kN

Thus, $F_{BE} = 9.24 \text{ kN (C)} \quad F_{BA} = 9.24 \text{ kN (T)}$ **Ans.**

Joint *E*:

$+\uparrow \sum F_y = 0; \quad E_y - 2(9.238 \sin 60°) = 0 \quad E_y = 16.0 \text{ kN}$

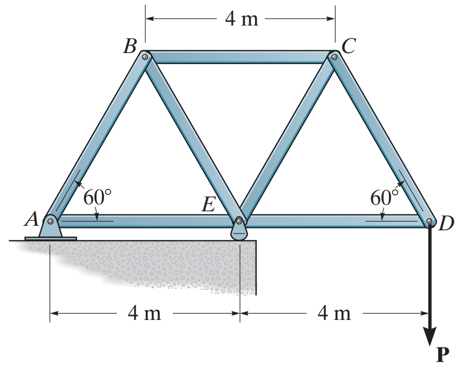


$\xrightarrow{+} \sum F_x = 0; \quad F_{BA} + 9.238 \cos 60° - 9.238 \cos 60° - 4.619 = 0$

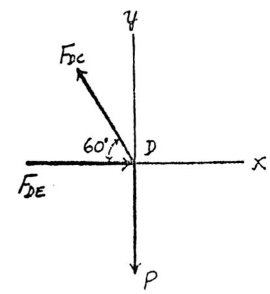
$F_{EA} = 4.62 \text{ kN (C)}$ **Ans.**

Note: The support reactions A_x and A_y can be determined by analyzing Joint *A* using the results obtained above.

***3–8.** If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.



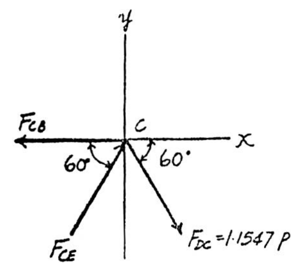
Method of Joints: In this case, the support reactions are not required for determining the member forces.



Joint *D*:

$+\uparrow \sum F_y = 0; \quad F_{DC} \sin 60° - P = 0 \quad F_{DC} = 1.1547P \text{ (T)}$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad F_{DE} - 1.1547P \cos 60° = 0 \quad F_{DE} = 0.57735P \text{ (C)}$

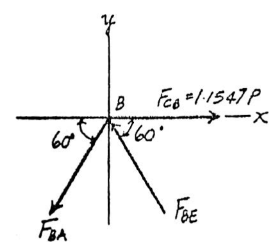


Joint *C*:

$+\uparrow \sum F_y = 0; \quad F_{CE} \sin 60° - 1.1547P \sin 60° = 0$

$$F_{CE} = 1.1547P \text{ (C)}$$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 2(1.1547P \cos 60° - F_{CB} = 0 \quad F_{CB} = 1.1547P \text{ (T)}$

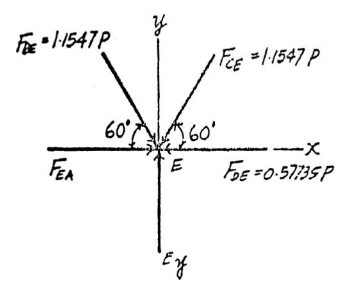


Joint *B*:

$+\uparrow \sum F_y = 0; \quad F_{BE} \sin 60° - F_{BE} \sin 60° = 0 \quad F_{BE} = F_{BA} = F$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 1.1547P - 2F \cos 60° = 0 \quad F = 1.1547P$

Thus, $\quad F_{BE} = 1.1547P\ (C) \quad F_{BA} = 1.1547P \text{ (T)}$



Joint *E*:

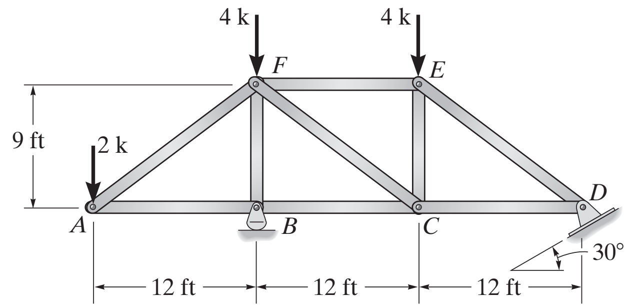
$\overset{+}{\rightarrow} \sum F_x = 0; \quad F_{EA} + 1.1547P \cos 60° - 1.1547P \cos 60° - 0.57735P = 0$

$$F_{EA} = 0.57735P \text{ (C)}$$

From the above analysis, the maximum compression and tension in the truss members is $1.1547P$. For this case, compression controls which requires

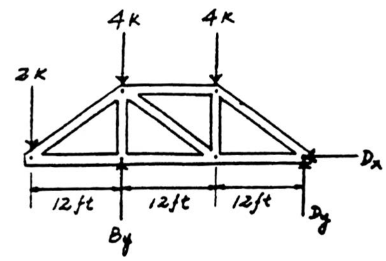
$$1.1547P = 6$$
$$P = 5.20 \text{ kN} \qquad \textbf{Ans.}$$

3–9. Determine the force in each member of the truss. State if the members are in tension or compression.



Reactions:

$$B_y = 9.00\text{ k}, \quad D_x = 0, \quad D_y = 1.00\text{ k}$$



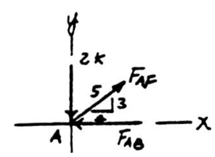
Joint *A*:

$$+\uparrow \sum F_y = 0; \quad \frac{3}{5}(F_{AF}) - 2 = 0$$

$$F_{AF} = 3.333\text{ k} = 3.33\text{ k (T)}$$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad -F_{AB} + \frac{4}{5}(3.333) = 0$$

$$F_{AB} = 2.667\text{ k} = 2.67\text{ k (C)}$$ **Ans.**



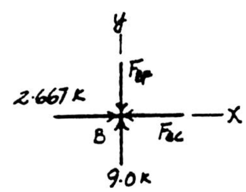
Joint *B*:

$$+\uparrow \sum F_y = 0; \quad 9.00 - (F_{BF}) = 0$$

$$F_{BF} = 9.00\text{ k (C)}$$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 2.667 - F_{BC} = 0$$

$$F_{BC} = 2.667\text{ k} = 2.67\text{ k (C)}$$ **Ans.**



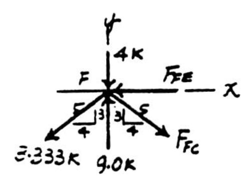
Joint *F*:

$$+\uparrow \sum F_y = 0; \quad -\frac{3}{5}(F_{FC}) - 4 - \frac{3}{5}(3.333) + 9 = 0$$

$$F_{FC} = 5.00\text{ k (T)}$$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad -F_{FE} - \frac{4}{5}(3.333) + \frac{4}{5}(5.00) = 0$$

$$F_{FE} = 1.333\text{ k} = 1.33\text{ k (C)}$$ **Ans.**



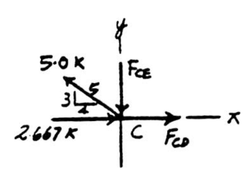
Joint *C*:

$$+\uparrow \sum F_y = 0; \quad -F_{CE} + \frac{3}{5}(5.00) = 0$$

$$F_{CE} = 3.00\text{ k (C)}$$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad F_{CD} + (2.667) - \frac{4}{5}(5.00) = 0$$

$$F_{CD} = 1.333\text{ k} = 1.33\text{ k (T)}$$ **Ans.**

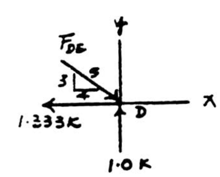


Joint *D*:

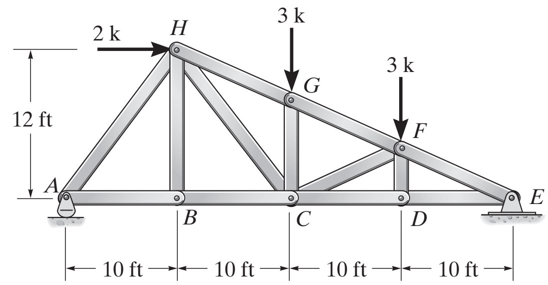
$$+\uparrow \sum F_y = 0; \quad -\frac{3}{5}(F_{DE}) + 1 = 0$$

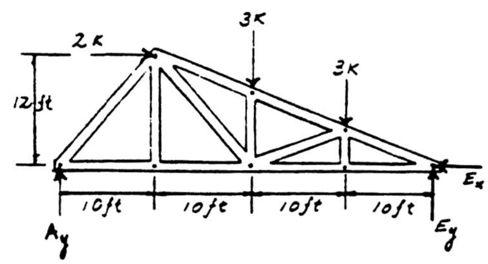
$$F_{DE} = 1.667\text{ k} = 1.67\text{ k (C)}$$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad \frac{4}{5}(1.667) - 1.333 = 0 \qquad \text{(Check)}$$



3–10. Determine the force in each member of the truss. State if the members are in tension or comprehension.





Reactions:

$A_y = 1.65 \text{ k}, \quad E_x = 2.00 \text{ k}, \quad E_y = 4.35 \text{ k}$

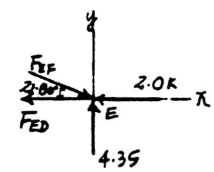
Joint *E*:

$+\uparrow \sum F_y = 0; \quad -(F_{EF}) \sin 21.80° + 4.35 = 0$

$F_{EF} = 11.71 \text{ k} = 11.7 \text{ k (C)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad -F_{ED} - 2 + 11.71 \cos 21.80° = 0$

$F_{ED} = 8.875 \text{ k (T)}$ **Ans.**

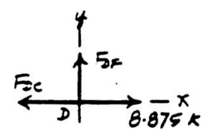


Joint *D*:

$+\uparrow \sum F_y = 0; \quad F_{DF} = 0$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad -F_{DC} + 8.875 = 0$

$F_{DC} = 8.875 \text{ k (T)}$ **Ans.**



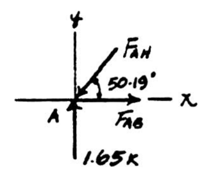
Joint *A*:

$+\uparrow \sum F_y = 0; \quad -F_{AH} \sin 50.19° + 1.65 = 0$

$F_{AH} = 2.148\, k = 2.15 \text{ k (C)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} - 2.148 (\cos 50.19°) = 0$

$F_{AB} = 1.375 \text{ k (T)}$ **Ans.**

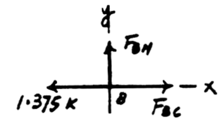


Joint *B*:

$+\uparrow \sum F_y = 0; \quad F_{BH} = 0$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} - 1.375 = 0$

$F_{BC} = 1.375 \text{ k (T)}$ **Ans.**



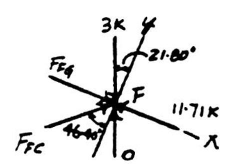
Joint *F*:

$+\nearrow \sum F_y = 0; \quad F_{FC} \cos 46.40° - 3 \cos 21.80° = 0$

$F_{FC} = 4.039 \text{ k} = 4.04 \text{ k (C)}$ **Ans.**

$+\searrow \sum F_x = 0; \quad F_{FG} + 3 \sin 21.80° + 4.039 \sin 46.40° - 11.71 = 0$

$F_{FG} = 7.671 \text{ k} = 7.67 \text{ k (C)}$ **Ans.**

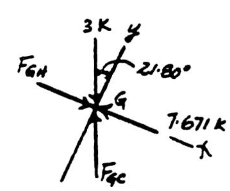


Joint *G*:

$+\nearrow \sum F_y = 0; \quad F_{GC} \cos 21.80° - 3 \cos 21.80° = 0 \quad F_{GC} = 3.00 \text{ k (C)}$ **Ans.**

$+\searrow \sum F_x = 0; \quad F_{GH} + 3 \sin 21.80° - 3 \sin 21.80° - 7.671 = 0;$

$F_{GH} = 7.671 \text{ k} = 7.67 \text{ k (C)}$ **Ans.**

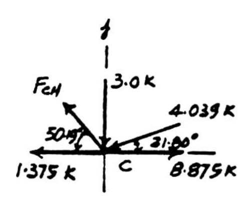


Joint *C*:

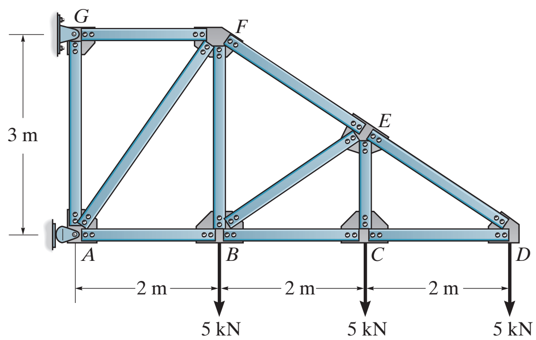
$+\uparrow \sum F_y = 0; \quad F_{CH} \sin 50.19° - 3.00 - 4.039 \sin 21.80° = 0$

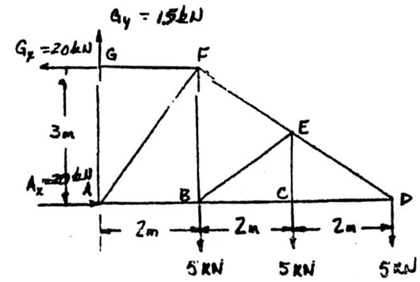
$F_{CH} = 5.858 \text{ k} = 5.86 \text{ k (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad -4.039 \cos 21.80° - 5.858 \cos 51.9° - 1.375 + 8.875 = 0$ (Check)

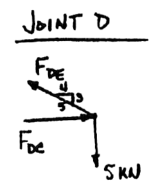


3–11. Determine the force in each member of the truss. State if the members are in tension or compression. Assume all members are pin connected.





Joint *D*:

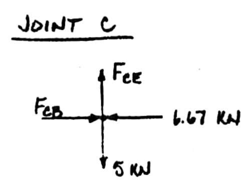


$+\uparrow \sum F_y = 0; \qquad F_{ED}\left(\frac{3}{5}\right) - 5 = 0; \qquad F_{ED} = 8.33 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \qquad F_{CD} - \frac{4}{5}(8.33) = 0;$

$F_{CD} = 6.67 \text{ kN (C)}$ **Ans.**

Joint *C*:



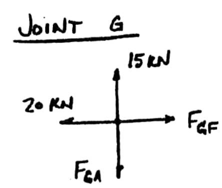
$\overset{+}{\rightarrow} \sum F_x = 0; \qquad F_{BC} - 6.67 = 0;$

$F_{BC} = 6.67 \text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad F_{CE} - 5 = 0;$

$F_{CE} = 5 \text{ kN (T)}$ **Ans.**

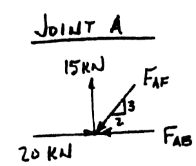
Joint *G*:



$\overset{+}{\rightarrow} \sum F_x = 0; \qquad F_{GF} - 20 = 0; \qquad F_{GF} = 20 \text{ kN (T)}$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad 15 - F_{GA} = 0; \qquad F_{GA} = 15 \text{ kN (T)}$ **Ans.**

Joint *A*:



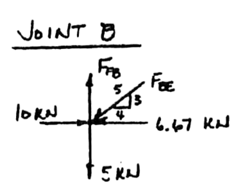
$+\uparrow \sum F_y = 0; \qquad 15 - F_{AF}(\sin 56.3°) = 0;$

$F_{AF} = 18.0 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \qquad -F_{AB} - 18.0(\cos 56.3°) + 20 = 0;$

$F_{AB} = 10.0 \text{ kN (C)}$ **Ans.**

Joint *B*:



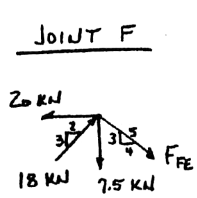
$\overset{+}{\rightarrow} \sum F_x = 0; \qquad -F_{BE}\left(\frac{4}{5}\right) + 10.0 - 6.67 = 0;$

$F_{BE} = 4.17 \text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad F_{FB} - 5 - 4.17\left(\frac{3}{5}\right) = 0;$

$F_{FB} = 7.50 \text{ kN (T)}$ **Ans.**

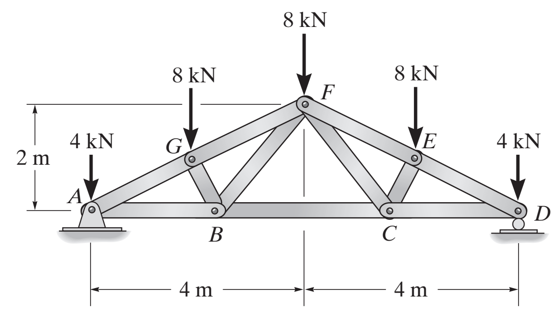
Joint *F*:



$+\uparrow \sum F_y = 0; \qquad 18(\sin 56.3°) - 7.5 - F_{FE}\left(\frac{3}{5}\right) = 0;$

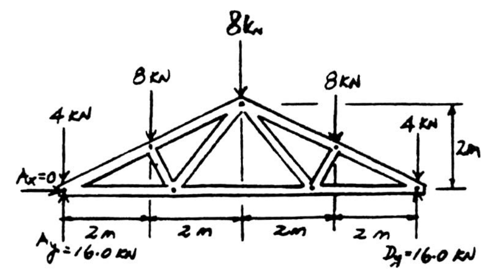
$F_{FE} = 12.5 \text{ kN (T)}$ **Ans.**

***3–12.** Determine the force in each member of the truss. State if the members are in tension or compression. Assume all members are pin connected. $AG = GF = FE = ED$.



Reactions:

$A_x = 0, \qquad A_y = 16.0\text{ kN}$



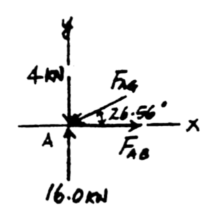
Joint A:

$+\uparrow \sum F_y = 0; \qquad 16 - 4 - F_{AG}\sin 26.565° = 0$

$F_{AG} = 26.83\text{ kN} = 26.8\text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \qquad -26.83\cos 26.565° + F_{AB} = 0$

$F_{AB} = 24.0\text{ kN (T)}$ **Ans.**



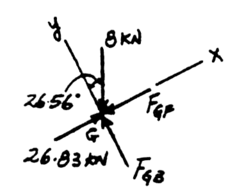
Joint G:

$+\nwarrow \sum F_y = 0; \qquad -8\cos 26.565° + F_{GB} = 0$

$F_{GB} = 7.155\text{ kN} = 7.16\text{ kN (C)}$ **Ans.**

$+\nearrow \sum F_x = 0; \qquad 26.83 - F_{GF} - 8\sin 26.56° = 0$

$F_{GF} = 23.36\text{ kN} = 23.3\text{ kN (C)}$ **Ans.**



Joint B:

$+\uparrow \sum F_y = 0; \qquad F_{BF}\sin 53.13° - 7.155\sin 63.43° = 0$

$F_{BF} = 8.00\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \qquad F_{BC} - 24.0 + 7.155\cos 63.43° + 8.00\cos 53.13° = 0$

$F_{BC} = 16.0\text{ kN (T)}$ **Ans.**

Due to symmetrical loading and geometry:

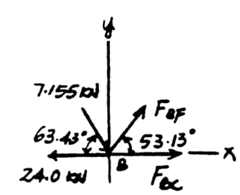
$F_{CD} = F_{AB} = 24.0\text{ kN (T)}$ **Ans.**

$F_{EF} = F_{GF} = 23.3\text{ kN (C)}$ **Ans.**

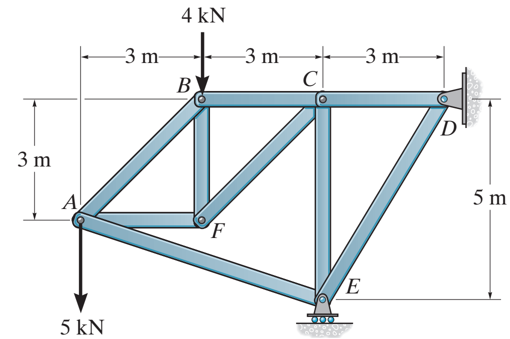
$F_{DE} = F_{AG} = 26.8\text{ kN (C)}$ **Ans.**

$F_{EC} = F_{GB} = 7.16\text{ kN (C)}$ **Ans.**

$F_{CF} = F_{BF} = 8.00\text{ kN (T)}$ **Ans.**



3–13. Determine the force in each member of the truss and state if the members are in tension or compression.

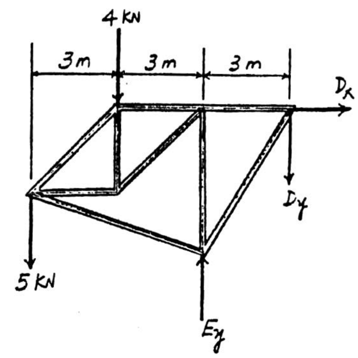


Support Reactions:

$\circlearrowleft + \sum M_D = 0;\quad 4(6) + 5(9) - E_y(3) = 0 \quad E_y = 23.0 \text{ kN}$

$+\uparrow \sum F_y = 0;\quad 23.0 - 4 - 5 - D_y = 0 \quad D_y = 14.0 \text{ kN}$

$\overset{+}{\rightarrow} \sum F_x = 0;\quad D_x = 0$



Method of Joints:

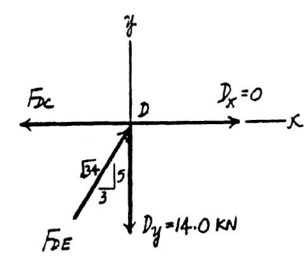
Joint *D*:

$+\uparrow \sum F_y = 0;\quad F_{DE}\left(\frac{5}{\sqrt{34}}\right) - 14.0 = 0$

$F_{DE} = 16.33 \text{ kN (C)} = 16.3 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 16.33\left(\frac{3}{\sqrt{34}}\right) - F_{DC} = 0$

$F_{DC} = 8.40 \text{ kN (T)}$ **Ans.**



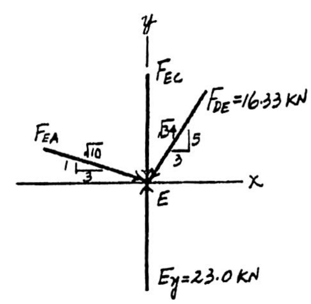
Joint *E*:

$\overset{+}{\rightarrow} \sum F_x = 0;\quad F_{EA}\left(\frac{3}{\sqrt{10}}\right) - 16.33\left(\frac{3}{\sqrt{34}}\right) = 0$

$F_{EA} = 8.854 \text{ kN (C)} = 8.85 \text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0;\quad 23.0 - 16.33\left(\frac{5}{\sqrt{34}}\right) - 8.854\left(\frac{1}{\sqrt{10}}\right) - F_{EC} = 0$

$F_{EC} = 6.20 \text{ kN (C)}$ **Ans.**



Joint *C*:

$+\uparrow \sum F_y = 0;\quad 6.20 - F_{CF} \sin 45° = 0$

$F_{CF} = 8.768 \text{ kN (T)} = 8.77 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 8.40 - 8.768 \cos 45° - F_{CB} = 0$

$F_{CB} = 2.20 \text{ kN (T)}$ **Ans.**

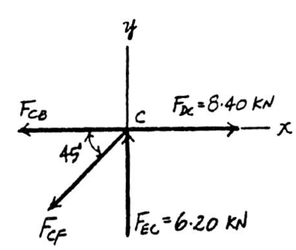
Joint *B*:

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 2.20 - F_{BA} \cos 45° = 0$

$F_{BA} = 3.111 \text{ kN (T)} = 3.11 \text{ kN (T)}$ **Ans.**

$+\uparrow \sum F_y = 0;\quad F_{BF} - 4 - 3.111 \sin 45° = 0$

$F_{BF} = 6.20 \text{ kN (C)}$ **Ans.**



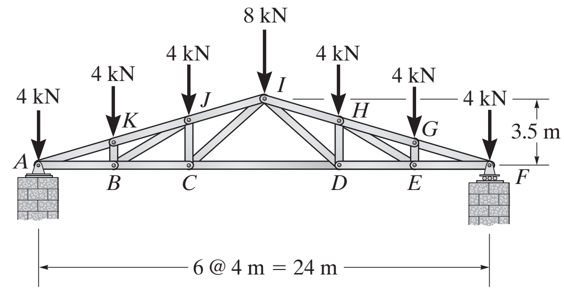
Joint *F*:

$+\uparrow \sum F_y = 0;\quad 8.768 \sin 45° - 6.20 = 0 \text{ (Check!)}$

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 8.768 \cos 45° - F_{FA} = 0$

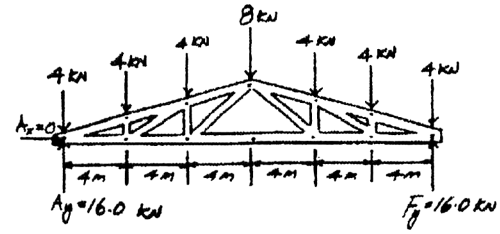
$F_{FA} = 6.20 \text{ kN (T)}$ **Ans.**

3–14. Determine the force in each member of the roof truss. State if the members are in tension or compression.



Reactions:

$A_y = 16.0\text{ kN}, \quad A_x = 0, \quad F_y = 16.0\text{ kN}$



Joint *A*:

$+\uparrow \sum F_y = 0; \quad -F_{AK} \sin 16.26° - 4 + 16 = 0$

$F_{AK} = 42.86\text{ kN} = 42.9\text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} - 42.86 \cos 16.26° = 0$

$F_{AB} = 41.14\text{ kN} = 41.1\text{ kN (T)}$ **Ans.**

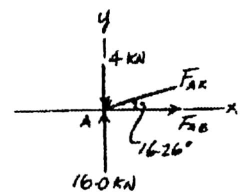
Joint *K*:

$+\nwarrow \sum F_y = 0; \quad -4 \cos 16.26° + F_{KB} \cos 16.26° = 0$

$F_{KB} = 4.00\text{ kN (C)}$ **Ans.**

$+\nearrow \sum F_x = 0; \quad 42.86 + 4.00 \sin 16.26° - 4.00 \sin 16.26° - F_{KJ} = 0$

$F_{KJ} = 42.86\text{ kN} = 42.9\text{ kN (C)}$ **Ans.**



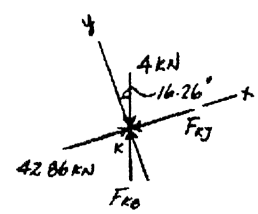
Joint *B*:

$+\uparrow \sum F_y = 0; \quad F_{BJ} \sin 30.26° - 4 = 0$

$F_{BJ} = 7.938\text{ kN} = 7.94\text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} + 7.938 \cos 30.26° - 41.14 = 0$

$F_{BC} = 34.29\text{ kN} = 34.3\text{ kN (T)}$ **Ans.**



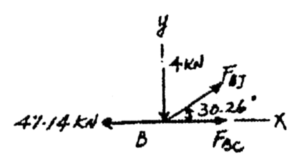
Joint *J*:

$\xrightarrow{+} \sum F_x = 0; \quad -F_{JI} \cos 16.26° - 7.939 \sin 59.74° + 42.86 \cos 16.26° = 0$

$F_{JI} = 35.71\text{ kN} = 35.7\text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad F_{JC} + 42.86 \sin 16.26° - 7.939 \cos 59.74° - 4 - 35.71 \sin 16.26° = 0$

$F_{JC} = 6.00\text{ kN (C)}$ **Ans.**



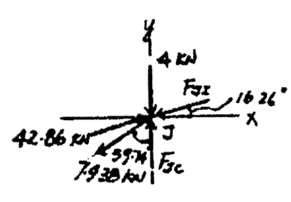
Joint *C*:

$+\uparrow \sum F_y = 0; \quad F_{CI} \sin 41.19° - 6.00 = 0$

$F_{CI} = 9.111\text{ kN} = 9.11\text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{CD} + 9.111 \cos 41.19° - 34.29 = 0$

$F_{CD} = 27.4\text{ kN (T)}$ **Ans.**



Due to symmetrical loading and geometry

$F_{IH} = 35.7\text{ kN (C)}$ **Ans.**

$F_{HD} = 6.00\text{ kN (C)}$ **Ans.**

$F_{HE} = 7.94\text{ kN (T)}$ **Ans.**

$F_{HG} = 42.9\text{ kN (C)}$ **Ans.**

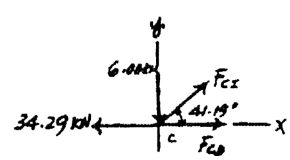
$F_{ED} = 34.3\text{ kN (T)}$ **Ans.**

$F_{ID} = 9.11\text{ kN (T)}$ **Ans.**

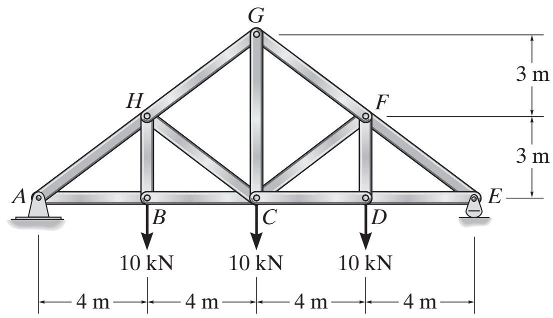
$F_{FG} = 42.9\text{ kN (C)}$ **Ans.**

$F_{GE} = 4.00\text{ kN (C)}$ **Ans.**

$F_{FE} = 41.1\text{ kN (T)}$ **Ans.**



3–15. Determine the force in each member of the roof truss. State if the members are in tension or compression. Assume all members are pin connected.



Joint *A*:

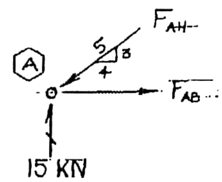
$$\sum F_y = 0; \quad -\frac{3}{5}F_{AH} + 15\text{ kN} = 0$$

$$F_{AH} = 25\text{ kN (C)} \qquad \textbf{Ans.}$$

$$\sum F_x = 0; \quad -\frac{4}{5}(25\text{ kN}) + F_{AB} = 0$$

$$F_{AB} = 20\text{ kN (T)} \qquad \textbf{Ans.}$$

JOINT A:

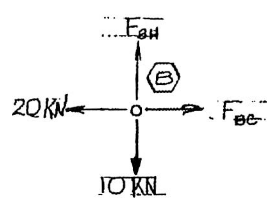


Joint *B*:

$$\sum F_x = 0; \quad F_{BC} = 20\text{ kN (T)} \qquad \textbf{Ans.}$$

$$\sum F_y = 0; \quad F_{BH} = 10\text{ kN (T)} \qquad \textbf{Ans.}$$

JOINT B:



Joint *H*:

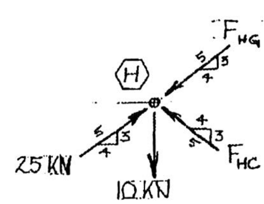
$$\sum F_y = 0; \quad \frac{3}{5}(25\text{ kN}) - 10\text{ kN} + \frac{3}{5}F_{HC} - \frac{3}{5}F_{HG} = 0$$

$$\sum F_x = 0; \quad \frac{4}{5}(25\text{ kN}) - \frac{4}{5}F_{HC} - \frac{4}{5}F_{HG} = 0$$

$$F_{HG} = 16.7\text{ kN (C)} \qquad \textbf{Ans.}$$

$$F_{HC} = 8.33\text{ kN (C)} \qquad \textbf{Ans.}$$

JOINT H



Joint *G*:

$$\sum F_x = 0; \quad \frac{4}{5}(16.67\text{ kN}) - \frac{4}{5}F_{GF} = 0$$

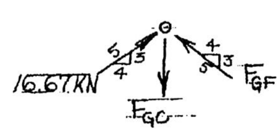
$$F_{GF} = 16.7\text{ kN (C)} \qquad \textbf{Ans.}$$

$$\sum F_x = 0; \quad \frac{3}{5}(16.67\text{ kN}) + \frac{3}{5}(16.67\text{ kN}) - F_{GC} = 0$$

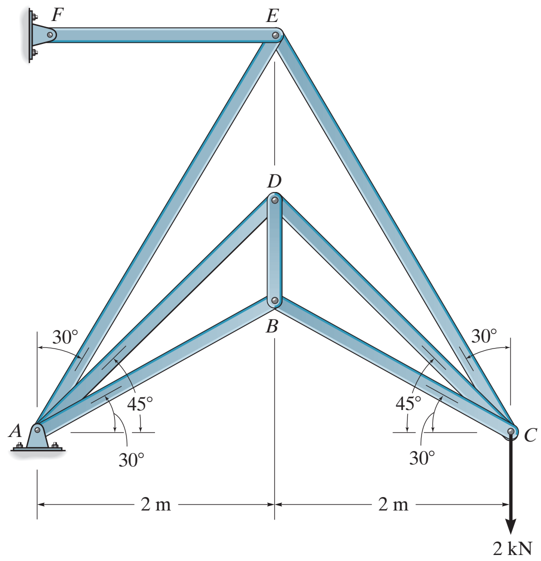
$$F_{GC} = 20\text{ kN (C)} \qquad \textbf{Ans.}$$

The other members are determined from symmetry.

JOINT G:



***3–16.** Determine the force in each member of the truss. State if the members are in tension or compression.



Joint *E*:

$+\uparrow \sum F_y = 0; \quad F_{EA} = F_{EC}$

$\xrightarrow{+} \sum F_x = 0; \quad 2.31 - 2\,F_{EA} \sin 30° = 0$

$F_{EA} = 2.31 \text{ kN (C)}$ **Ans.**

$F_{EC} = 2.31 \text{ kN (T)}$ **Ans.**

Joint *A*:

$\xrightarrow{+} \sum F_x = 0; \quad 2.31 - 2.31 \sin 30° - F_{AB} \cos 30° + F_{AD} \cos 45° = 0$

$+\uparrow \sum F_y = 0; \quad 2 - 2.31 \cos 30° + F_{AD} \sin 45° - F_{AB} \sin 30° = 0$

$F_{AD} = 2.24 \text{ kN (T)}$ **Ans.**

$F_{AB} = 3.16 \text{ kN (C)}$ **Ans.**

Joint *B*:

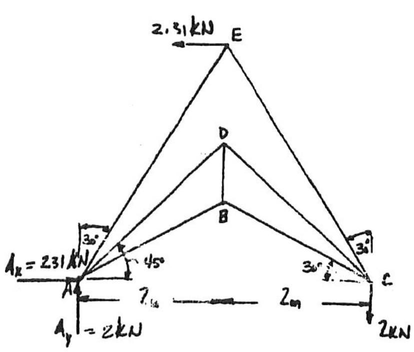
$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} = 3.16 \text{ kN (C)}$ **Ans.**

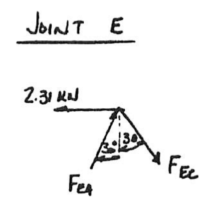
$+\uparrow \sum F_y = 0; \quad 2(3.16) \sin 30° - F_{BD} = 0$

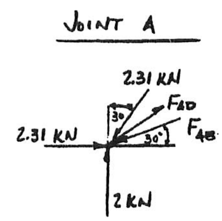
$F_{BD} = 3.16 \text{ kN (C)}$ **Ans.**

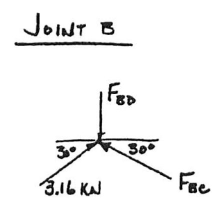
Joint *D*:

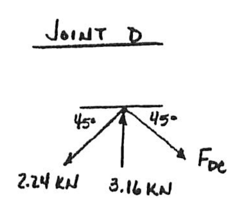
$F_{DC} = 2.24 \text{ kN (T)}$ **Ans.**



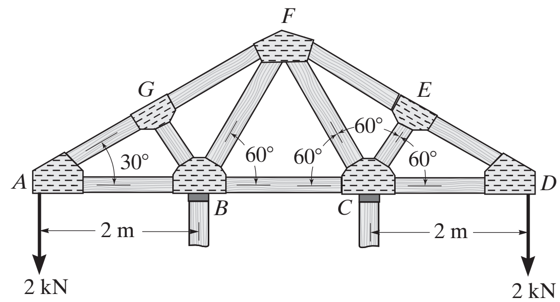








3–17. Determine the force in each member of the roof truss. State if the members are in tension or compression. Assume *B* is a pin and *C* is a roller support.



Support Reactions. Referring to the FBD of the entire truss, Fig. *a*,

$\circlearrowleft + \sum M_C = 0; \quad 2(4) - 2(2) - N_B(2) = 0 \quad N_B = 2.00 \text{ kN}$

Method of joint.

Joint *A*: Fig. *b*,

$+\uparrow \sum F_y = 0; \quad F_{AG} \sin 30° - 2 = 0 \quad F_{AG} = 4.00 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 4.00 \cos 30° - F_{AB} = 0 \quad F_{AB} = 3.464 \text{ kN (C)} = 3.46 \text{ kN (C)}$ **Ans.**

Joint *G*: Fig. *c*,

$+\nearrow \sum F_x = 0; \quad F_{GF} - 4.00 = 0 \quad F_{GF} = 4.00 \text{ kN (T)}$ **Ans.**

$+\nwarrow \sum F_y = 0; \quad F_{GB} = 0$ **Ans.**

Joint *B*: Fig. *d*,

$+\uparrow \sum F_y = 0; \quad 2 - F_{BF} \sin 60° = 0 \quad F_{BF} = 2.309 \text{ kN (C)} = 2.31 \text{ kN (C)}$ **Ans.**

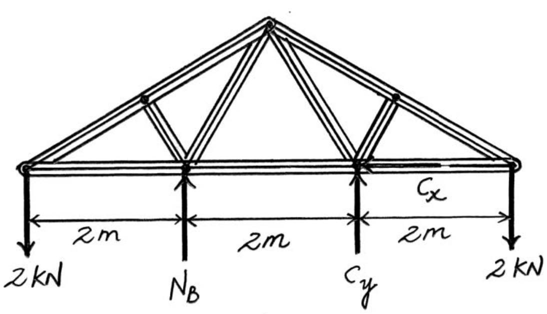
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 3.464 - 2.309 \cos 60° - F_{BC} = 0 \quad F_{BC} = 2.309 \text{ kN (C)} - 2.31 \text{ kN (C)}$ **Ans.**

Due to symmetry,

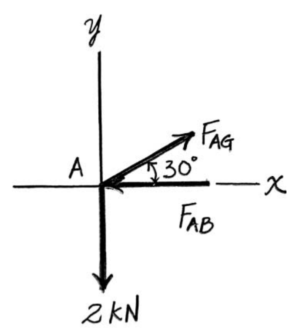
$F_{DE} = F_{AG} = 4.00 \text{ kN (T)} \qquad F_{DC} = F_{AB} = 3.46 \text{ kN (C)}$ **Ans.**

$F_{EF} = F_{GF} = 4.00 \text{ kN (T)} \qquad F_{EC} = F_{GB} = 0$ **Ans.**

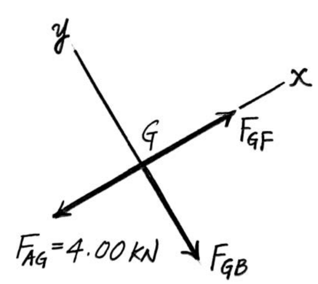
$F_{CF} = F_{BF} = 2.31 \text{ kN (C)}$



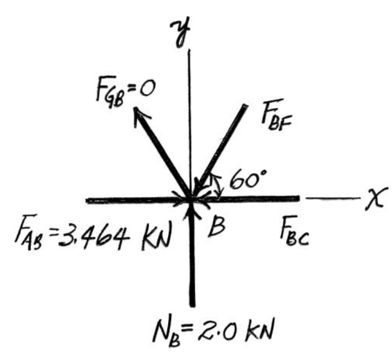
(a)



(b)

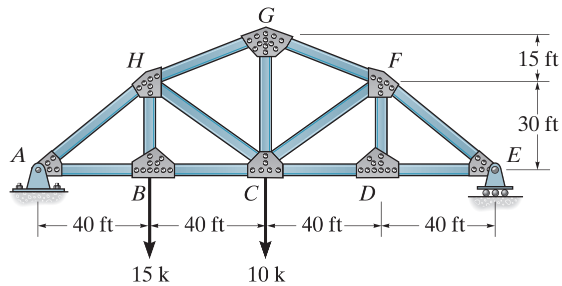


(c)



(d)

3–18. Determine the force in members *GF*, *FC*, and *CD* of the bridge truss. State if the members are in tension of compression. Assume all members are pin connected.



$$\circlearrowleft + \sum M_F = 0; \quad -F_{DC}(30) + 8.75\,(40) = 0$$

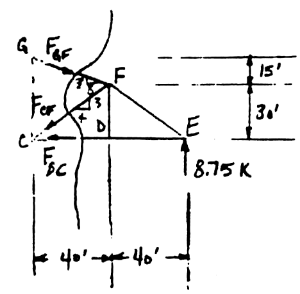
$$F_{DC} = 11.7 \text{ k (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_C = 0; \quad -F_{FC}\left(\frac{8}{\sqrt{73}}\right)(45) + 8.75\,(80) = 0$$

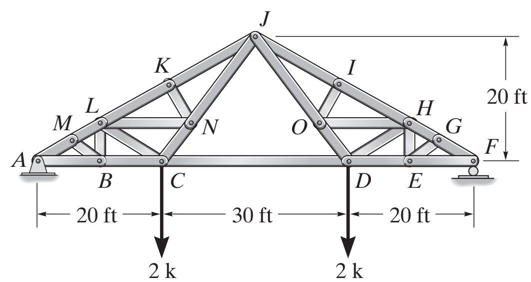
$$F_{FG} = 16.6 \text{ k (C)} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad 8.75 - 16.6\left(\frac{3}{\sqrt{73}}\right) \cdot F_{FC}\left(\frac{3}{5}\right) = 0$$

$$F_{FC} = 4.86 \text{ k (T)} \qquad \textbf{Ans.}$$



3–19. Determine the force in members *JK*, *JN*, and *CD*. State if the members are in tension of compression. Identify all the zero-force members.

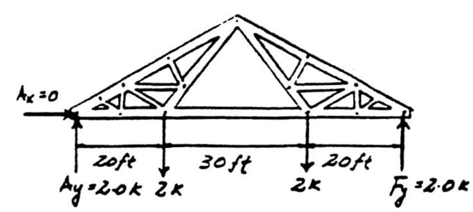


Reactions:

$$A_x = 0, \qquad A_y = 2.0 \text{ k}, \qquad F_y = 2.0 \text{ k}$$

$$\circlearrowleft + \sum M_J = 0; \quad F_{CD}(20) + 2(15) - 2(35) = 0$$

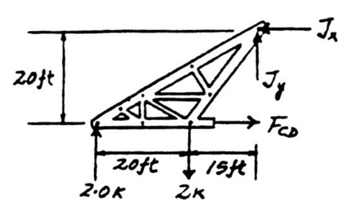
$$F_{CD} = 2.00 \text{ k (T)} \qquad \textbf{Ans.}$$



$$+\uparrow \sum F_y = 0; \quad J_y = 0$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad -J_x + 2.00 = 0; \quad J_x = 2.00 \text{ k}$$

Joint *J*:



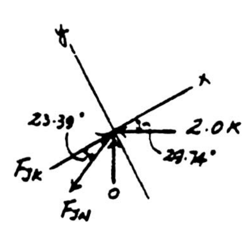
$$\nwarrow + \sum F_y = 0; \quad -F_{JN} \sin 23.39° + 2 \sin 29.74° = 0$$

$$F_{JN} = 2.50 \text{ k (T)} \qquad \textbf{Ans.}$$

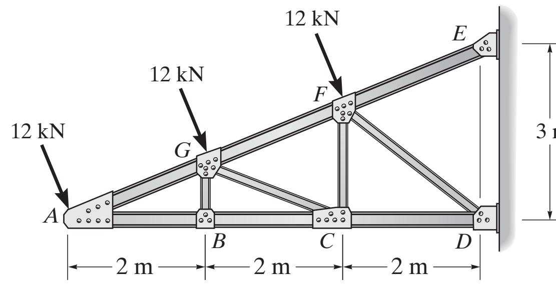
$$+\nearrow \sum F_x = 0; \quad F_{JK} \cos 29.74° - 2.50 \cos 23.39°$$

$$F_{JK} = 4.03 \text{ k (C)} \qquad \textbf{Ans.}$$

Members *KN*, *NL*, *MB*, *BL*, *CL*, *IO*, *OH*, *GE*, *EH*, *HD* are zero force members. **Ans.**



***3–20.** Determine the force in members *GF*, *FC*, and *CD* of the cantilever truss. State if the members are in tension of compression. Assume all members are pin connected.



$\circlearrowleft + \sum M_C = 0;$ $\quad 12 \text{ kN} (\cos 26.57°)(4 \text{ m}) + 12 \text{ kN} (\cos 26.57°)(2\text{m})$

$-12 \text{ kN} (\sin 26.57°)(1 \text{ m}) - F_{GF} \sin 26.57° \ (4 \text{ m}) = 0$

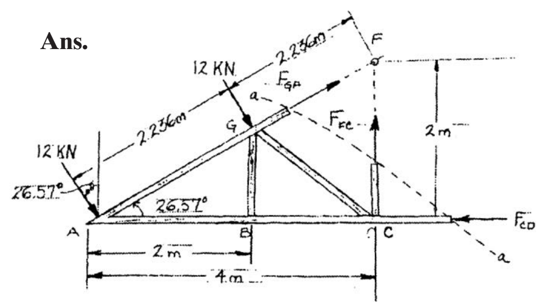
$F_{GF} = 33.0 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $\quad -12 \text{ kN} (2.236 \text{ m}) + F_{FC}(4 \text{ m}) = 0$

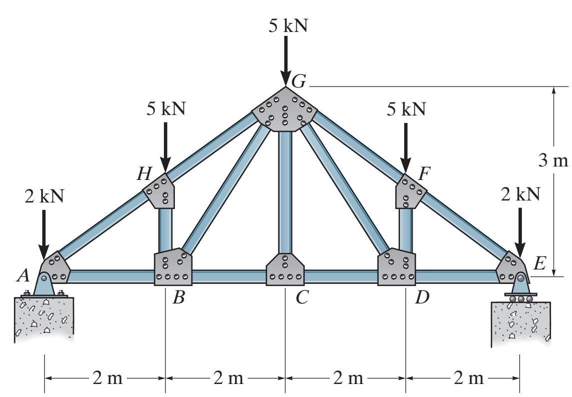
$F_{FC} = 6.71 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \sum M_F = 0;$ $\quad 12 \text{ kN} (2.236 \text{ m}) + 12 \text{ kN} (2)(2.236 \text{ m}) - F_{CD}(2 \text{ m}) = 0$

$F_{CD} = 40.2 \text{ kN (C)}$ **Ans.**



3–21. The *Howe* truss is subjected to the loading shown. Determine the forces in members *GF*, *CD*, and *GC*. State if the members are in tension or compression. Assume all members are pin connected.

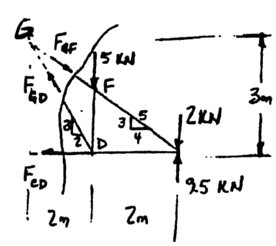


$\circlearrowleft + \sum M_G = 0;$ $\quad F_{CD}(3) - 9.5(4) + 5(2) + 2(4) = 0$

$F_{CD} = 6.67 \text{ kN (T)}$ **Ans.**

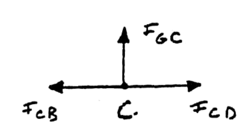
$\circlearrowleft + \sum M_D = 0;$ $\quad -9.5(2) + 2(2) + \frac{4}{5}(1.5)\,F_{GF} = 0$

$F_{GF} = 12.5 \text{ kN (C)}$ **Ans.**

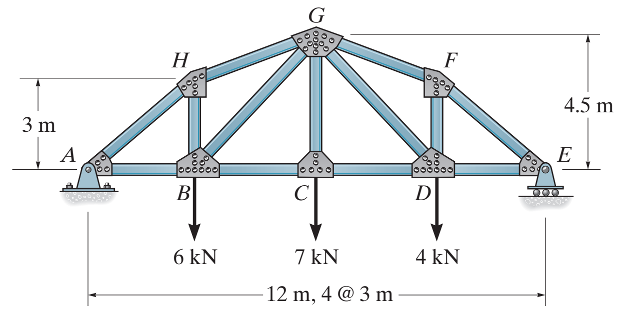


Joint *C*:

$F_{GC} = 0$ **Ans.**

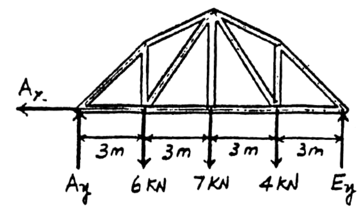


3–22. Determine the force in members BG, HG, and BC of the truss and state if the members are in tension or compression.



$\zeta + \sum M_E = 0; \quad 6(9) + 7(6) + 4(3) - A_y(12) = 0 \quad A_y = 9.00 \text{ kN}$

$\xrightarrow{+} \sum F_x = 0; \quad A_x = 0$



Method of Sections:

$\zeta + \sum M_G = 0; \quad F_{BC}(4.5) + 6(3) - 9(6) = 0$

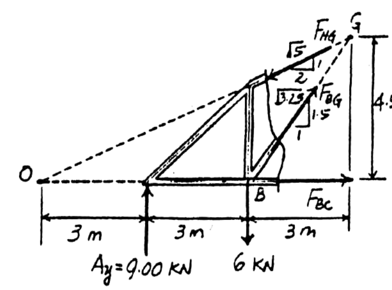
$F_{BC} = 8.00 \text{ kN (T)}$ **Ans.**

$\zeta + \sum M_B = 0; \quad F_{HG}\left(\frac{1}{\sqrt{5}}\right)(6) - 9(3) = 0$

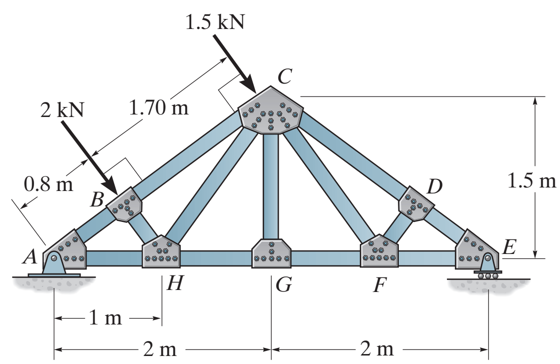
$F_{HG} = 10.1 \text{ kN (C)}$ **Ans.**

$\zeta + \sum M_O = 0; \quad F_{BG}\left(\frac{1.5}{\sqrt{3.25}}\right)(6) + 9(3) - 6(6) = 0$

$F_{BG} = 1.80 \text{ kN (T)}$ **Ans.**



3–23. Determine the force in members GF, CF, and CD of the roof truss and indicate if the members are in tension or compression.



$\zeta + \sum M_A = 0; \quad E_y(4) - 2(0.8) - 1.5(2.50) = 0 \quad E_y = 1.3375 \text{ kN}$

Method of Sections:

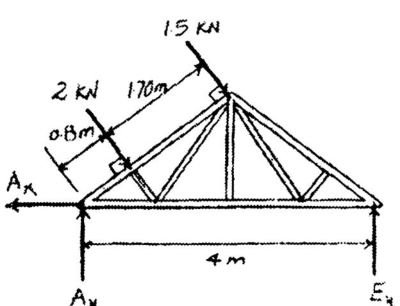
$\zeta + \sum M_C = 0; \quad 1.3375(2) - F_{GF}(1.5) = 0$

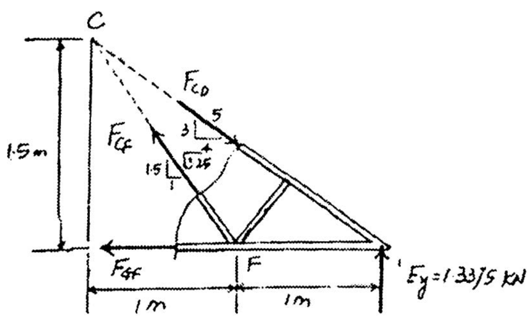
$F_{GF} = 1.78 \text{ kN(T)}$ **Ans.**

$\zeta + \sum M_F = 0; \quad 1.3375(1) - F_{CD}\left(\frac{3}{5}\right)(1) = 0$

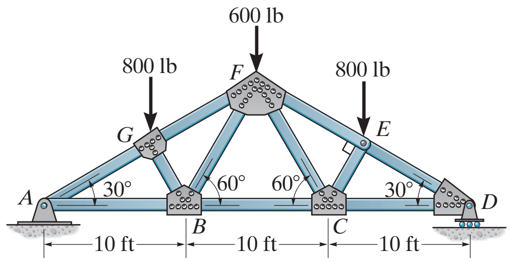
$F_{CD} = 2.23 \text{ kN (C)}$ **Ans.**

$\zeta + \sum M_E = 0 \quad F_{CF}\left(\frac{1.5}{\sqrt{3.25}}\right)(1) = 0 \quad F_{CF} = 0$ **Ans.**

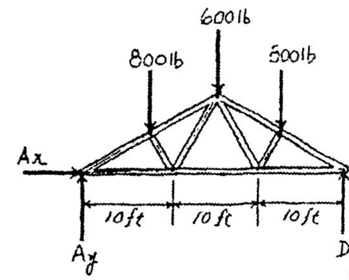




***3–24.** Determine the force in members *GF*, *FB*, and *BC* of the *Fink* truss and state if the members are in tension or compression.



Support Reactions: Due to symmetry. $D_y = A_y$.



$+\uparrow \sum F_y = 0; \quad 2A_y - 800 - 600 - 800 = 0 \quad A_y = 1100 \text{ lb}$

$\xrightarrow{+} \sum F_x = 0; \quad A_x = 0$

Method of Sections:

$\circlearrowleft + \sum M_B = 0; \quad F_{GF} \sin 30° (10) + 800(10 - 10\cos^2 30°) - 1100(10) = 0$

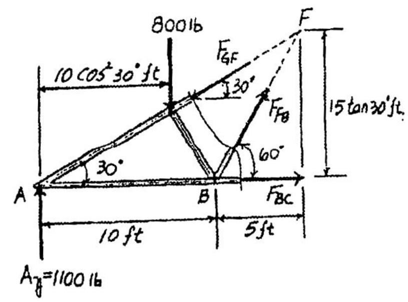
$F_{GF} = 1800 \text{ lb (C)} = 1.80 \text{ k (C)}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad F_{FB} \sin 60° (10) - 800(10\cos^2 30°) = 0$

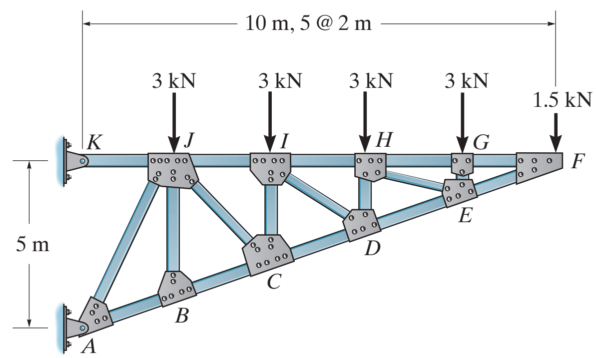
$F_{FB} = 692.82 \text{ lb (T)} = 693 \text{ lb (T)}$ **Ans.**

$\circlearrowleft + \sum M_F = 0; \quad F_{BC}(15 \tan 30°) + 800(15 - 10\cos^2 30°) - 1100(15) = 0$

$F_{BC} = 1212.43 \text{ lb (T)} = 1.21 \text{ k (T)}$ **Ans.**



3–25. Determine the force in members *IH*, *ID*, and *CD* of the truss. State if the members are in tension or compression. Assume all members are pin connected.



Referring to the FBD of the right segment of the truss sectioned through a–a, Fig. *a*,

$\circlearrowleft + \sum M_D = 0; \quad F_{IH}(2) - 3(2) - 1.5(4) = 0$

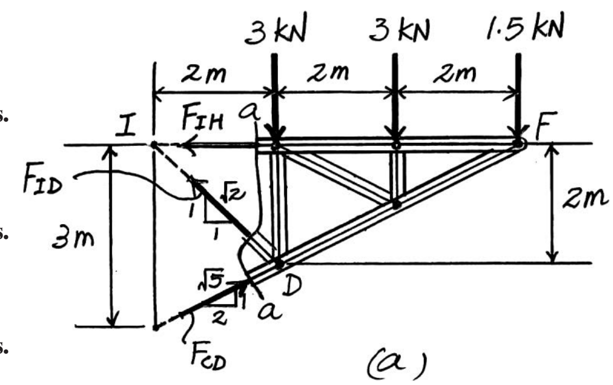
$F_{IH} = 6.00 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \sum M_F = 0; \quad 3(2) + 3(4) - F_{ID}\left(\frac{1}{\sqrt{2}}\right)(6) = 0$

$F_{ID} = 4.243 \text{ kN (T)} = 4.24 \text{ kN (T)}$ **Ans.**

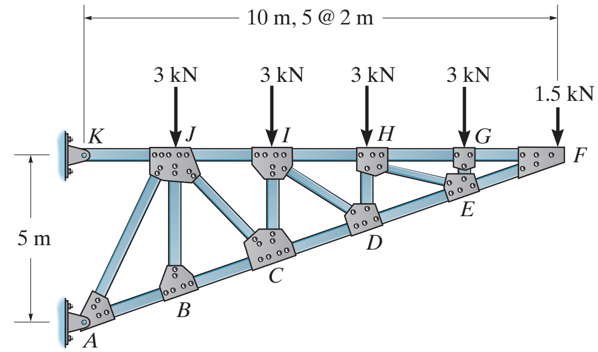
$\circlearrowleft + \sum M_I = 0; \quad F_{CD}\left(\frac{1}{\sqrt{5}}\right)(6) - 3(2) - 3(4) - 1.5(6) = 0$

$F_{CD} = 10.06 \text{ kN} = 10.1 \text{ kN (C)}$ **Ans.**



(a)

3–26. Determine the force in members *JI*, *IC*, and *CD* of the truss. State if the members are in tension or compression. Assume all members are pin connected.



Consider the FBD of the right segment of the truss sectioned through a–a, Fig. *a*,

$\curvearrowleft + \sum M_C = 0; \quad F_{JI}(3) - 3(2) - 3(4) - 1.5(6) = 0$

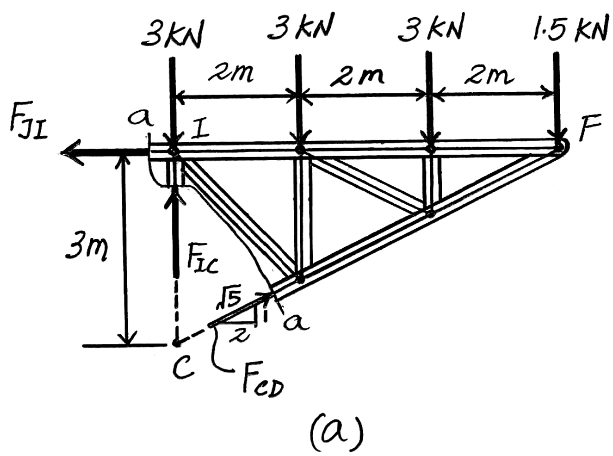
$F_{JI} = 9.00 \text{ kN (T)}$ **Ans.**

$\curvearrowleft + \sum M_F = 0; \quad 3(6) + 3(4) + 3(2) - F_{IC}(6) = 0$

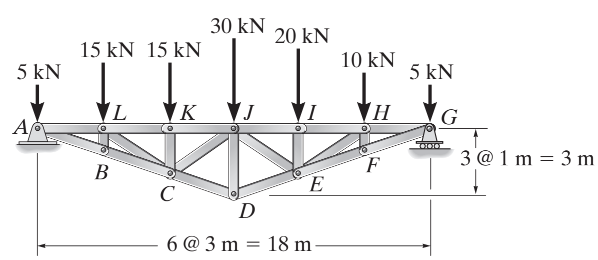
$F_{IC} = 6.00 \text{ kN (C)}$ **Ans.**

$\curvearrowleft + \sum M_I = 0; \quad F_{CD}\left(\frac{1}{\sqrt{5}}\right)(6) - 1.5(6) - 3(4) - 3(2) = 0$

$F_{CD} = 10.06 \text{ kN (C)} = 10.1 \text{ kN (C)}$ **Ans.**



3–27. Determine the forces in members *KJ*, *CD*, and *CJ* of the truss. State if the members are in tension or compression.

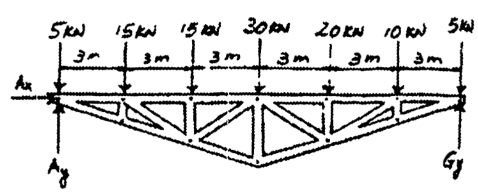


Entire truss:

$\xrightarrow{+} \sum F_x = 0; \quad A_x = 0$

$\curvearrowleft + \sum M_A = 0; \quad -15(3) - 15(6) - 30(9) - 20(12) - 10(15) - 5(18) + G_y(18) = 0$

$G_y = 49.17 \text{ kN}$



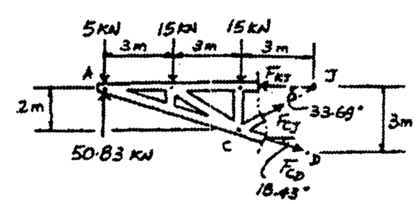
$+\uparrow \sum F_y = 0; \quad A_y - 5 - 15 - 15 - 30 - 20 - 10 - 5 + 49.167 = 0$

$A_y = 50.83 \text{ kN}$

Section:

$\curvearrowleft + \sum M_C = 0; \quad 15(3) + 5(6) - 50.83(6) + F_{KJ}(2) = 0$

$F_{KJ} = 115 \text{ kN (C)}$ **Ans.**



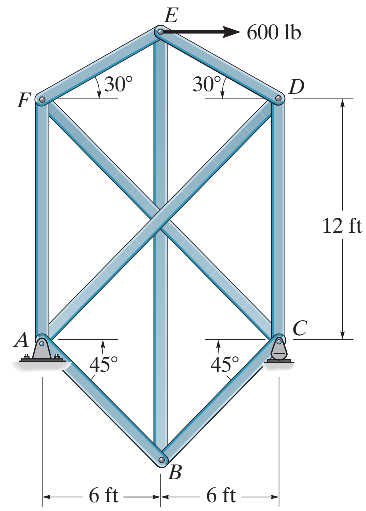
$\curvearrowleft + \sum M_A = 0; \quad -15(3) - 15(6) + F_{CJ} \sin 33.69° (9) = 0$

$F_{CJ} = 27.0 \text{ kN (T)}$ **Ans.**

$\curvearrowleft + \sum M_J = 0; \quad -50.83(9) + 5(9) + 15(6) + 15(3) + F_{CD} \cos 18.43° (3) = 0$

$F_{CD} = 97.5 \text{ kN (T)}$ **Ans.**

***3–28.** Determine the forces in all the members of the complex truss. State if the members are in tension or compression. *Hint:* Substitute member AD with one placed between E and C.



$S_i = S'_i + \chi(S_i)$

$F_{EC} = S'_{EC} + (x)\, S_{EC} = 0$

$747.9 + x(0.526) = 0$

$x = 1421.86$

Thus:

$F_{AF} = S_{AF} + (x)\, S_{AF}$

$= 1373.21 + (1421.86)(-1.41)$

$= -646.3 \text{ lb}$

$F_{AF} = 646 \text{ lb (C)}$ **Ans.**

In a similar manner:

$F_{AB} = 580 \text{ lb(C)}$ **Ans.**

$F_{EB} = 820 \text{ lb(T)}$ **Ans.**

$F_{BC} = 580 \text{ lb(C)}$ **Ans.**

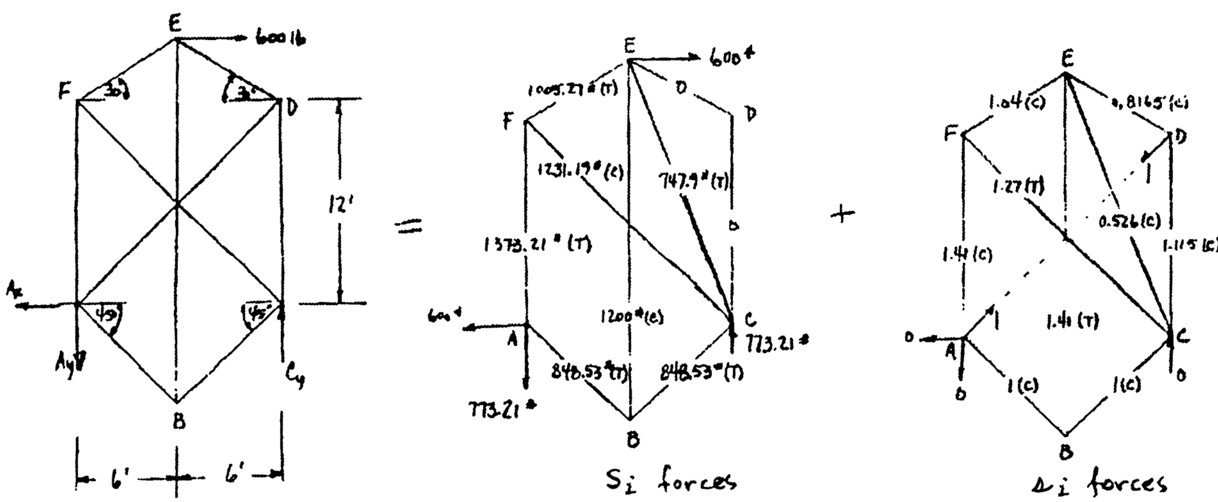
$F_{EF} = 473 \text{ lb(C)}$ **Ans.**

$F_{CF} = 580 \text{ lb(T)}$ **Ans.**

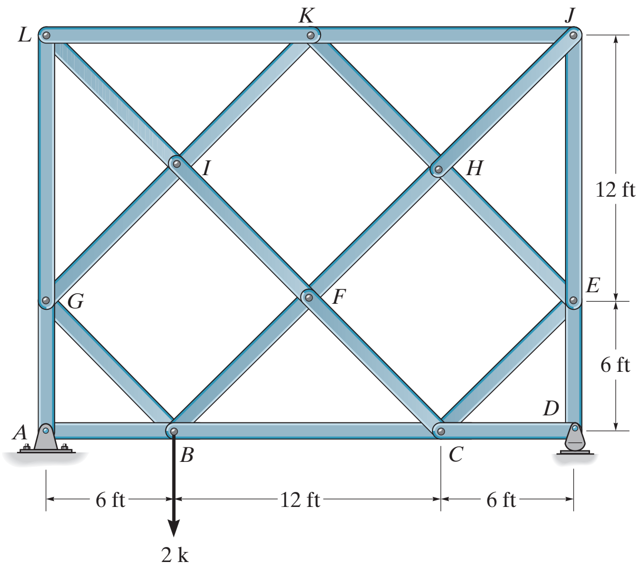
$F_{CD} = 1593 \text{ lb(C)}$ **Ans.**

$F_{ED} = 1166 \text{ lb(C)}$ **Ans.**

$F_{DA} = 1428 \text{ lb(T)}$ **Ans.**



3–29. Determine the forces in all the members of the lattice (complex) truss. State if the members are in tension or compression. *Hint:* Substitute member *JE* by one placed between *K* and *F*.



$S_i = S'_i + X\,(S_i)$

$F_{KF} = 1.5 + 1(x) = 0; \qquad x = -1.5$

Thus:

$F_{AB} = 0$ **Ans.**

$F_{AG} = 1.50$ k (C) **Ans.**

$F_{GB} = 0.707$ k (T) **Ans.**

$F_{GL} = 0.500$ k (C) **Ans.**

$F_{GI} = 0.707$ k (C) **Ans.**

$F_{LI} = 0.707$ k (T) **Ans.**

$F_{LK} = 0.500$ k (C) **Ans.**

$F_{IK} = 0.707$ k (C) **Ans.**

$F_{IF} = 0.707$ k (T) **Ans.**

$F_{BF} = 2.12$ k (T) **Ans.**

$F_{BC} = 1.00$ k (C) **Ans.**

$F_{FC} = 0.707$ k (T) **Ans.**

$F_{FH} = 2.12$ k (T) **Ans.**

$F_{KH} = 0.707$ k (T) **Ans.**

$F_{KJ} = 1.50$ k (C) **Ans.**

$F_{JH} = 2.12$ k (T) **Ans.**

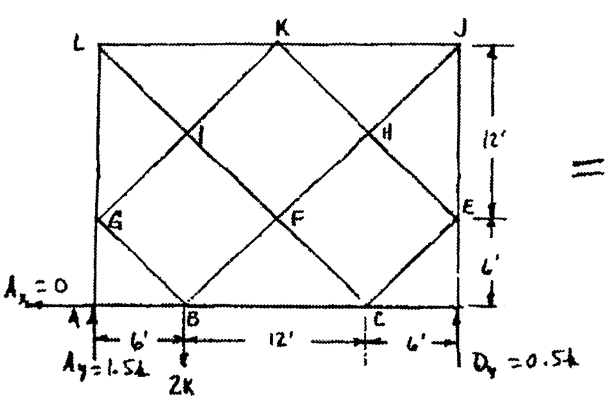
$F_{CD} = 0$ **Ans.**

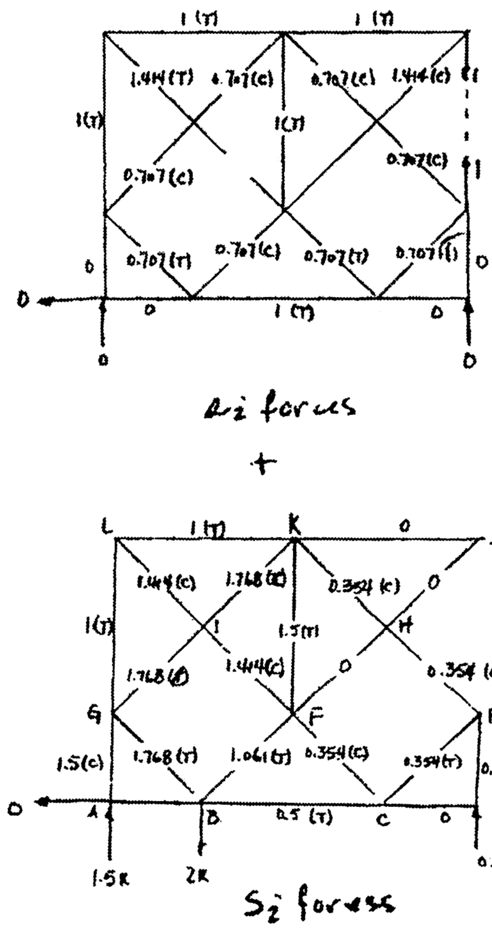
$F_{DE} = 0.500$ k (C) **Ans.**

$F_{CE} = 0.707$ k (C) **Ans.**

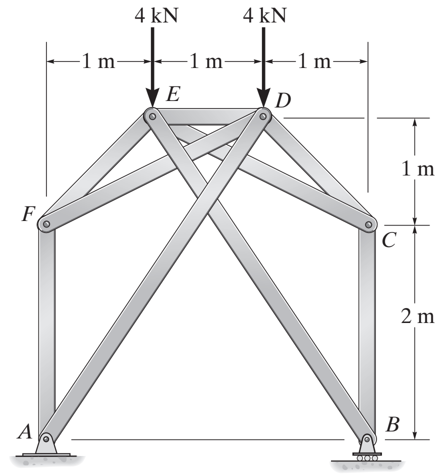
$F_{HE} = 0.707$ k (T) **Ans.**

$F_{JE} = 1.50$ k (C) **Ans.**





3–30. Determine the force in each member and state if the members are in tension or compression.



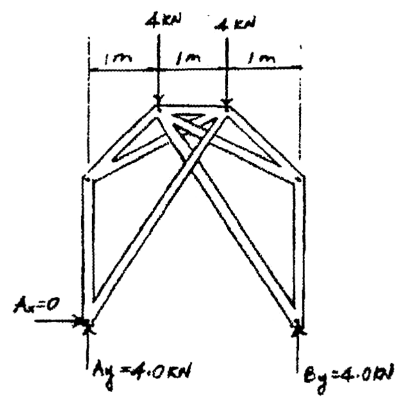
Reactions:

$A_x = 0, \quad A_y = 4.00\text{ kN}, \quad B_y = 4.00\text{ kN}$

Joint *A*:

$\xrightarrow{+} \sum F_x = 0; \quad F_{AD} = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 4.00 - F_{AF} = 0; \quad F_{AF} = 4.00\text{ kN (C)}$ **Ans.**



Joint *F*:

$\nwarrow + \sum F_y = 0; \quad 4.00 \sin 45° - F_{FD} \sin 18.43° = 0$

$F_{FD} = 8.944\text{ kN} = 8.94\text{ kN (T)}$ **Ans.**

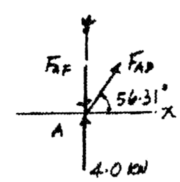
$+\nearrow \sum F_x = 0; \quad 4.00 \cos 45° + 8.94 \cos 18.43° - F_{FE} = 0$

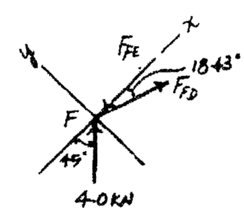
$F_{FE} = 11.313\text{ kN} = 11.3\text{ kN (C)}$ **Ans.**

Due to symmetrical loading and geometry

$F_{BC} = 4.00\text{ kN (C)} \quad F_{CE} = 8.94\text{ kN (T)}$ **Ans.**

$F_{BE} = 0 \quad F_{CD} = 11.3\text{ kN (C)}$ **Ans.**



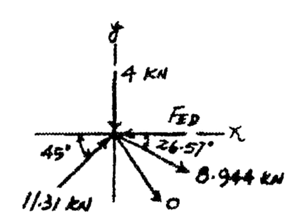


Joint *E*:

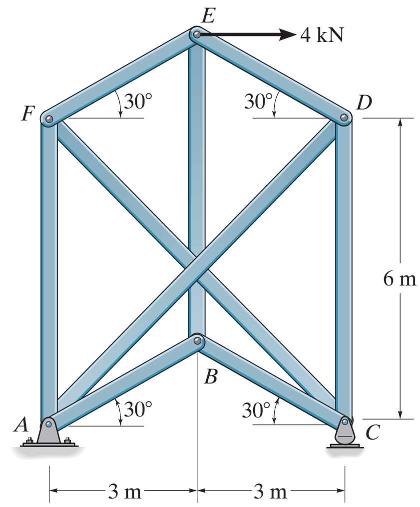
$\xrightarrow{+} \sum F_x = 0; \quad -F_{ED} + 8.944 \cos 26.56° + 11.31 \cos 45° = 0$

$F_{ED} = 16.0\text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad -4 - 8.944 \sin 26.56° + 11.31 \sin 45° = 0\text{ (Check)}$



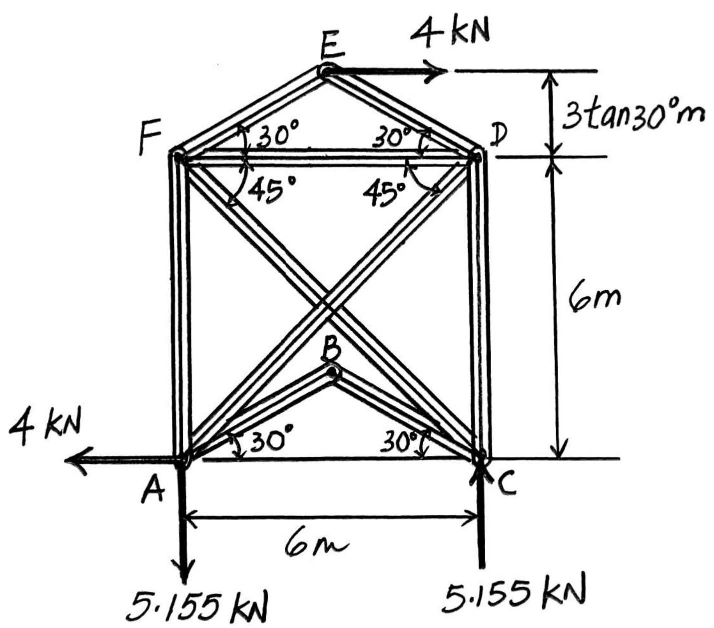
3–31. Determine the force in all the members of the complex truss. State if the members are in tension or compression.



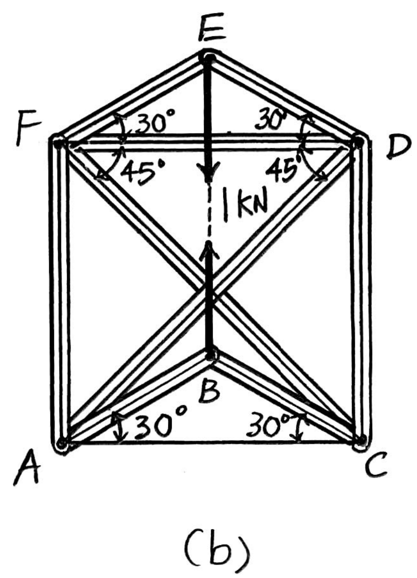
The member forces S_i' and S_i for each member of the reduced simple truss can be determined using method of joints by referring to Fig. *a* and *b*, respectively. Using the forces of the replacing member *DF*,

$$S_{DF} = S'_{DF} + XS_{DF}$$
$$0 = -2 + X(1.7320)$$
$$x = 1.1547$$

member	S_i' (kN)	S_i (kN)	XS_i (kN)	S_i (kN)
EF	2.3094	−1	−1.1547	1.15 (T)
ED	−2.3094	−1	−1.1547	3.46 (C)
BA	0	1	1.1547	1.15 (T)
BC	0	1	1.1547	1.15 (T)
AD	5.6569	−1.2247	−1.4142	4.24 (T)
AF	1.1547	0.3660	0.4226	1.58 (T)
CF	0	−1.2247	−1.4142	1.41 (C)
CD	−5.1547	0.3660	0.4226	4.73 (C)
BE	0	1	1.1547	1.15 (T)

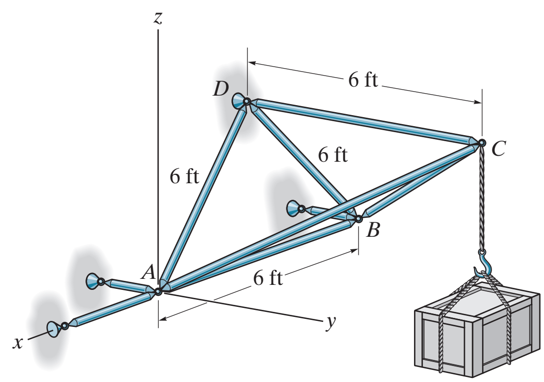


(a)



(b)

***3–32.** Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb.



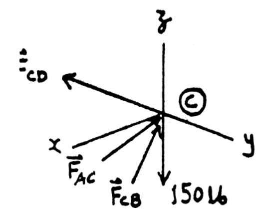
$$F_{CA} = F_{CA}\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\sin 60^\circ\,\mathbf{k}}{\sqrt{8}}\right]$$

$$= -0.354\,F_{CA}\mathbf{i} + 0.707\,F_{CA}\mathbf{j} + 0.612\,F_{CA}\mathbf{k}$$

$$F_{CB} = -0.354\,F_{CB}\mathbf{i} + 0.707\,F_{CB}\mathbf{j} + 0.612\,F_{CB}\mathbf{k}$$

$$F_{CD} = -F_{CD}\mathbf{j}$$

$$w = -150\,\mathbf{k}$$



$\sum F_x = 0;\quad -0.354F_{CA} + 0.354F_{CB} = 0$

$\sum F_y = 0;\quad 0.707F_{CA} + 0.707F_{CB} - F_{CD} = 0$

$\sum F_z = 0;\quad 0.612F_{CA} + 0.612F_{CB} - 150 = 0$

Solving:

$F_{CA} = F_{CB} = 122.5\text{ lb} = 122\text{ lb (C)}$ **Ans.**

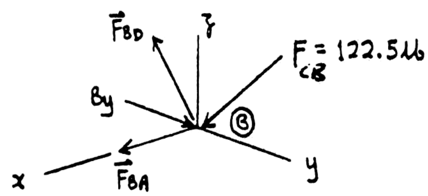
$F_{CD} = 173\text{ lb (T)}$ **Ans.**

$\mathbf{F}_{BA} = F_{BA}\mathbf{i}$

$\mathbf{F}_{BD} = F_{BD}\cos 60^\circ\,\mathbf{i} + F_{BD}\sin 60^\circ\,\mathbf{k}$

$\mathbf{F}_{CB} = 122.5\,(-0.354\mathbf{i} - 0.707\mathbf{j} - 0.612\,\mathbf{k})$

$= -43.3\text{i} - 86.6\text{j} - 75.0\text{ k}$



$\sum F_x = 0;\quad F_{BA} + F_{BD}\cos 60^\circ - 43.3 = 0$

$\sum F_z = 0;\quad F_{BD}\sin 60^\circ - 75 = 0$

Solving:

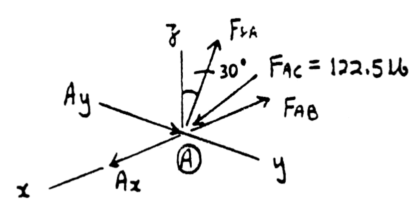
$F_{BD} = 86.6\text{ lb (T)}$ **Ans.**

$F_{BA} = 0$ **Ans.**

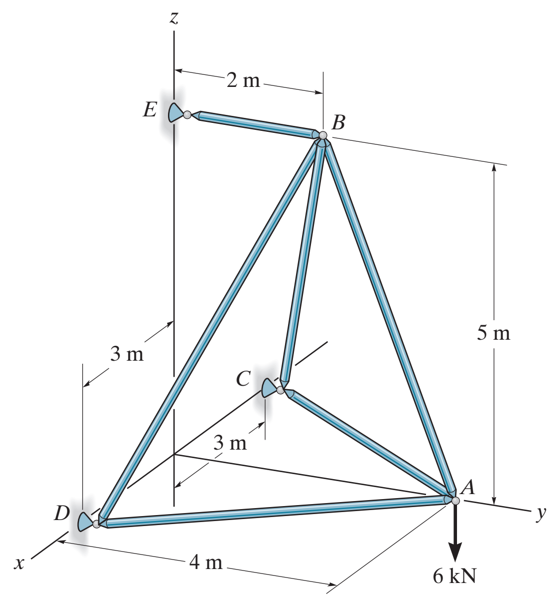
$F_{AC} = 122.5(0.354F_{AC}\mathbf{i} - 0.707F_{AC}\mathbf{j} - 0.612F_{AC}\mathbf{k})$

$\sum F_z = 0;\quad F_{DA}\cos 30^\circ - 0.612(122.5) = 0$

$F_{DA} = 86.6\text{ lb (T)}$ **Ans.**

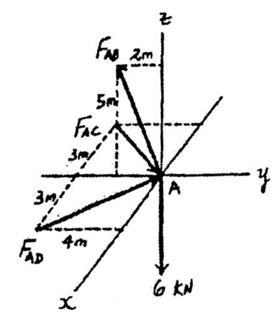


3–33. Determine the force in each member of the space truss and state if the members are in tension or compression. *Hint:* The support reaction at E acts along member EB. Why?



Method of Joints**:** In this case, the support reactions are not required for determining the member forces.

Joint A:



$$\sum F_x = 0; \qquad F_{AB}\left(\frac{5}{\sqrt{29}}\right) - 6 = 0$$

$$F_{AB} = 6.462 \text{ kN (T)} = 6.46 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\sum F_z = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AD}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AD} \qquad [1]$$

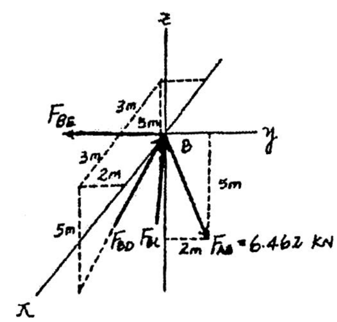
$$\sum F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AD}\left(\frac{4}{5}\right) - 6.462\left(\frac{2}{\sqrt{29}}\right) = 0$$

$$F_{AC} + F_{AD} = 3.00 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$F_{AC} = F_{AD} = 1.50 \text{ kN (C)} \qquad \textbf{Ans.}$$

Joint B:



$$\sum F_x = 0; \qquad F_{BC}\left(\frac{3}{\sqrt{38}}\right) - F_{BD}\left(\frac{3}{\sqrt{38}}\right) = 0 \qquad F_{BC} = F_{BD} \qquad [1]$$

$$\sum F_z = 0; \qquad F_{BC}\left(\frac{5}{\sqrt{38}}\right) + F_{BD}\left(\frac{5}{\sqrt{38}}\right) - 6.462\left(\frac{5}{\sqrt{29}}\right) = 0$$

$$F_{BC} + F_{BD} = 7.397 \qquad [2]$$

Solving Eqs. [1] and [2] yields

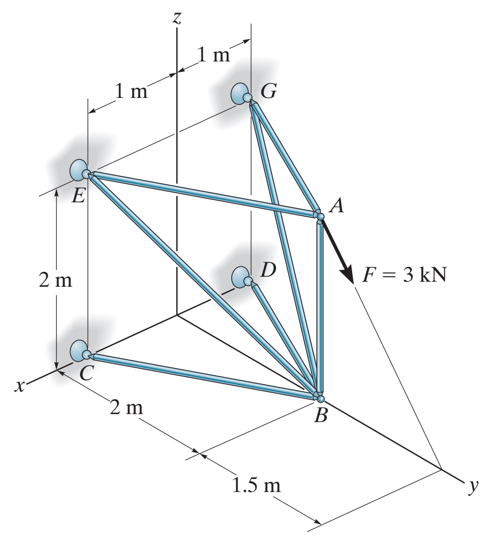
$$F_{BC} = F_{BD} = 3.699 \text{ kN (C)} = 3.70 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\sum F_y = 0; \qquad 2\left[3.699\left(\frac{2}{\sqrt{38}}\right)\right] + 6.462\left(\frac{2}{\sqrt{29}}\right) - F_{BE} = 0$$

$$F_{BE} = 4.80 \text{ kN (T)} \qquad \textbf{Ans.}$$

Note: The support reactions at supports C and D can be determined by analyzing joints C and D, respectively using the results oriented above.

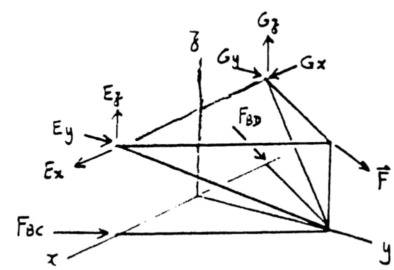
3–34. Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at C, D, E, and G. *Note:* Although this truss is indeterminate to the first degree, a solution is possible due to symmetry of geometry and loading.



$\sum (M_{EG})_x = 0;\qquad \frac{2}{\sqrt{5}}F_{BC}(2) + \frac{2}{\sqrt{5}}F_{BD}(2) - \frac{4}{5}(3)(2) = 0$

$F_{BC} + F_{BD} = 2.683 \text{ kN}$

Due to symmetry: $\quad F_{BC} = F_{BD} = 1.342 = 1.34 \text{ kN (C)}$ **Ans.**



Joint *A*:

$\sum F_z = 0;\qquad F_{AB} - \frac{4}{5}(3) = 0$

$F_{AB} = 2.4 \text{ kN (C)}$ **Ans.**

$\sum F_x = 0;\qquad F_{AG} = F_{AE}$

$\sum F_y = 0;\qquad \frac{3}{5}(3) - \frac{3}{\sqrt{5}}F_{AE} - \frac{3}{\sqrt{5}}F_{AG} = 0$

$F_{AG} = F_{AE} = 1.01 \text{ kN (T)}$ **Ans.**

Joint *B*:

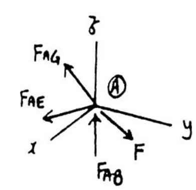
$\sum F_x = 0;\qquad \frac{1}{\sqrt{5}}(1.342) + \frac{1}{3}F_{BE} - \frac{1}{\sqrt{5}}(1.342) - \frac{1}{3}F_{BG} = 0$

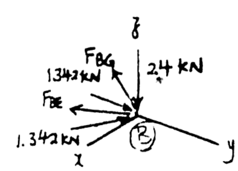
$\sum F_y = 0;\qquad \frac{2}{\sqrt{5}}(1.342) - \frac{2}{3}F_{BE} + \frac{2}{\sqrt{5}}(1.342) - \frac{2}{3}F_{BG} = 0$

$\sum F_z = 0;\qquad \frac{2}{3}F_{BE} + \frac{2}{3}F_{BG} - 2.4 = 0$

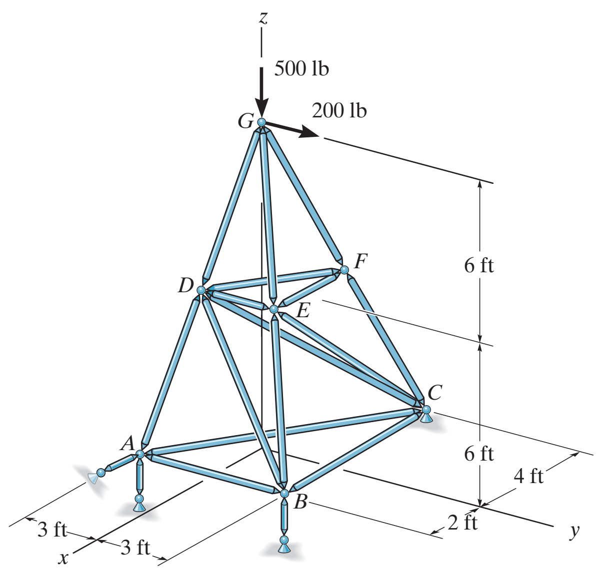
$F_{BG} = 1.80 \text{ kN (T)}$ **Ans.**

$F_{BE} = 1.80 \text{ kN (T)}$ **Ans.**





3–35. Determine the force in members *FE* and *ED* of the space truss and state if the members are in tension or compression. The truss is supported by a ball-and-socket joint at *C* and short links at *A* and *B*.

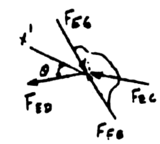


Joint *F*: F_{FG}, F_{FD}, and F_{FC} are lying in the same plane and x'axis is normal to that plane. Thus

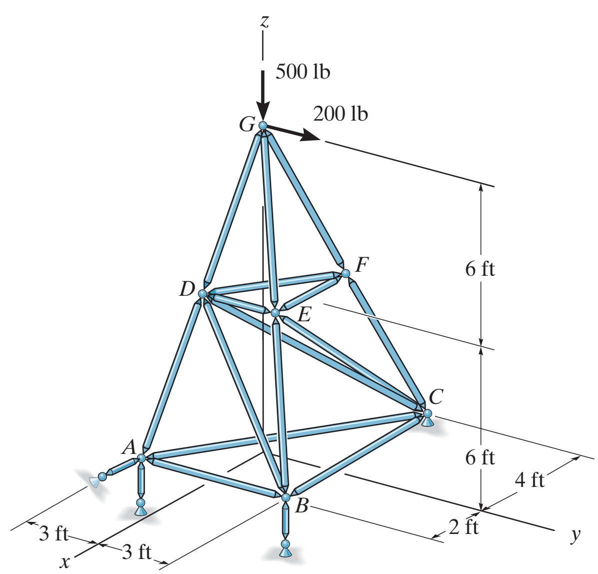
$$\sum F_{x'} = 0; \qquad F_{FE}\cos\theta = 0; \qquad F_{FE} = 0 \qquad \textbf{Ans.}$$

Joint *E*: F_{EG}, F_{BC}, and F_{EB} are lying in the same plane and x'axis is normal to that plane. Thus

$$\sum F_{x'} = 0; \qquad F_{ED}\cos\theta = 0; \qquad F_{ED} = 0 \qquad \textbf{Ans.}$$



***3–36.** Determine the force in members *GD*, *GE*, and *FD* of the space truss and state if the members are in tension or compression.



Joint *G*:

$$F_{GD} = F_{GD}\left(-\frac{2}{12.53}\mathbf{i} + \frac{3}{12.53}\mathbf{j} + \frac{12}{12.53}\mathbf{k}\right)$$

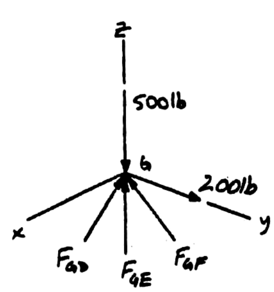
$$F_{GF} = F_{GF}\left(\frac{4}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} + \frac{12}{13}\mathbf{k}\right)$$

$$F_{GE} = F_{GE}\left(-\frac{2}{12.53}\mathbf{i} - \frac{3}{12.53}\mathbf{j} + \frac{12}{12.53}\mathbf{k}\right)$$

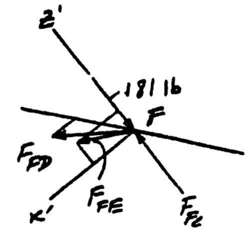
$$\sum F_x = 0; \qquad -F_{GD}\left(\frac{2}{12.53}\right) + F_{GF}\left(\frac{4}{13}\right) - F_{GE}\left(\frac{2}{12.53}\right) = 0$$

$$\sum F_y = 0; \qquad F_{GD}\left(\frac{3}{12.53}\right) + F_{GF}\left(\frac{3}{13}\right) - F_{GE}\left(\frac{3}{12.53}\right) + 200 = 0$$

$$\sum F_z = 0; \qquad F_{GD}\left(\frac{12}{12.53}\right) + F_{GF}\left(\frac{12}{13}\right) - F_{GE}\left(\frac{12}{12.53}\right) - 500 = 0$$



3–36. Continued



Solving,

$F_{GD} = -157 \text{ lb} = 157 \text{ lb (T)}$ **Ans.**

$F_{GF} = 181 \text{ lb (C)}$

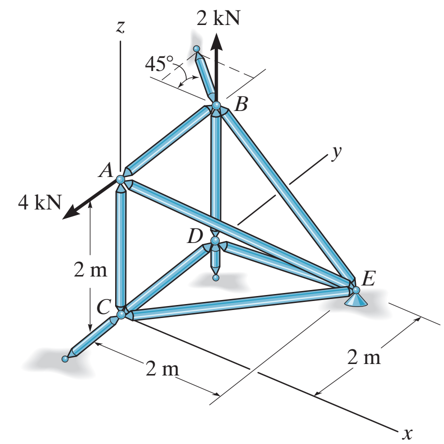
$F_{GE} = 505 \text{ lb (C)}$ **Ans.**

Joint *F*:

Orient the x', y', z' axes as shown.

$\sum F_{y'} = 0; \quad F_{FD} = 0$ **Ans.**

3–37. Determine the force in each member of the space truss. Indicate if the members are in tension or compression.



Joint *A*:

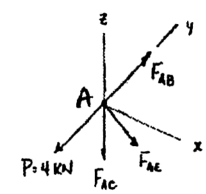
$\sum F_x = 0; \qquad 0.577F_{AE} = 0$

$\sum F_y = 0; \qquad -4 + F_{AB} + 0.577\,F_{AE} = 0$

$\sum F_z = 0; \qquad -F_{AC} - 0.577\,F_{AE} = 0$

$F_{AC} = F_{AE} = 0$ **Ans.**

$F_{AB} = 4 \text{ kN (T)}$ **Ans.**



Joint *B*:

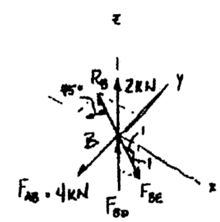
$\sum F_x = 0; \qquad -R_B(\cos 45°) + 0.707F_{BE} = 0$

$\sum F_y = 0; \qquad -4 + R_B(\sin 45°) = 0$

$\sum F_z = 0; \qquad 2 + F_{BD} - 0.707\,F_{BE} = 0$

$R_B = F_{BE} = 5.66 \text{ kN (T)}$ **Ans.**

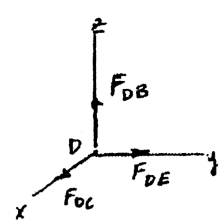
$F_{BD} = 2 \text{ kN (C)}$ **Ans.**



Joint *D*:

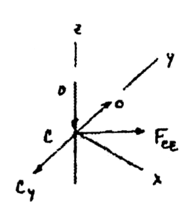
$\sum F_x = 0; \qquad F_{DE} = 0$ **Ans.**

$\sum F_y = 0; \qquad F_{DC} = 0$ **Ans.**

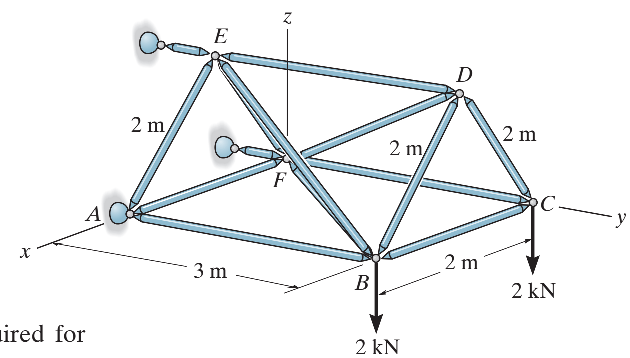


Joint *C*:

$\sum F_x = 0; \qquad F_{CE} = 0$ **Ans.**

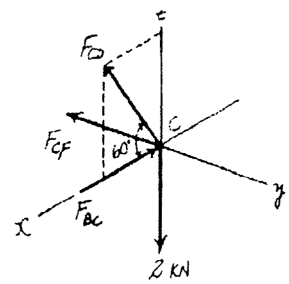


3–38. Determine the force in members BE, DF, and BC of the space truss and state if the members are in tension or compression.



***Method of Joints*:** In this case, the support reactions are not required for determining the member forces.

Joint *C*:

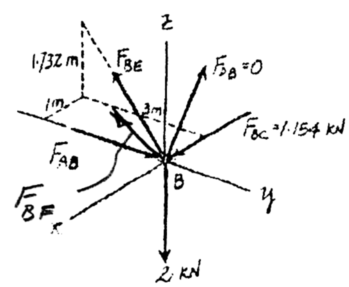


$$\sum F_z = 0; \qquad F_{CD} \sin 60° - 2 = 0 \qquad F_{CD} = 2.309 \text{ kN (T)}$$

$$\sum F_x = 0; \qquad 2.309 \cos 60° - F_{BC} = 0$$

$$F_{BC} = 1.154 \text{ kN (C)} = 1.15 \text{ kN (C)} \qquad \textbf{Ans.}$$

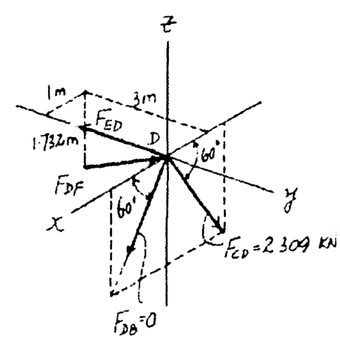
Joint *D*: Since F_{CD}, F_{DE} and F_{DF} lie within the same plane and F_{DE} is out of this plane, then $F_{DE} = 0$.



$$\sum F_x = 0; \qquad F_{DF}\left(\frac{1}{\sqrt{13}}\right) - 2.309 \cos 60° = 0$$

$$F_{DF} = 4.16 \text{ kN (C)} \qquad \textbf{Ans.}$$

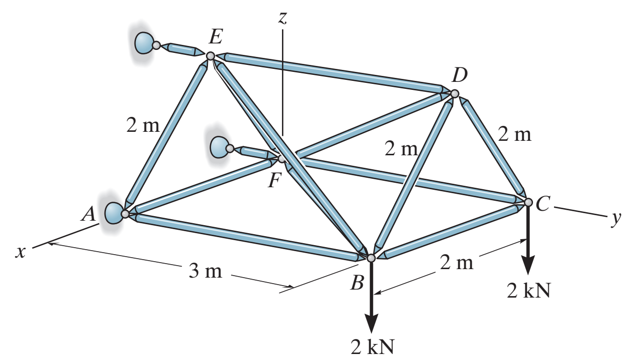
Joint *R*:



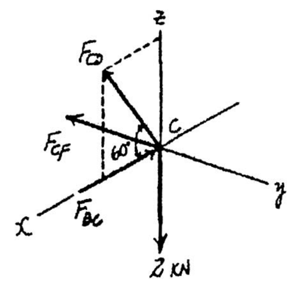
$$\sum F_z = 0; \qquad F_{BE}\left(\frac{1.732}{\sqrt{13}}\right) - 2 = 0$$

$$F_{BE} = 4.16 \text{ kN (T)} \qquad \textbf{Ans.}$$

3–39. Determine the force in members *CD*, *ED*, and *CF* of the space truss and state if the members are in tension or compression.



Method of Joints**:** In this case, the support reactions are not required for determining the member forces.



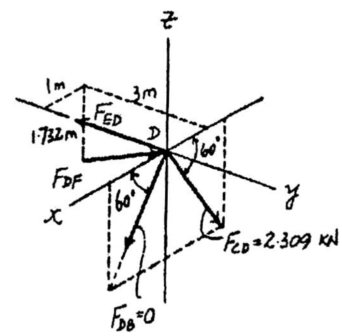
Joint *C*: Since F_{CD}, F_{BC} and 2 kN force lie within the same plane and F_{CF} is out of this plane, then

$$F_{CF} = 0 \quad \textbf{Ans.}$$

$$\sum F_t = 0; \qquad F_{CD} \sin 60° - 2 = 0$$

$$F_{CD} = 2.309 \text{ kN (T)} = 2.31 \text{ kN (T)} \quad \textbf{Ans.}$$

$$\sum Fx = 0; \qquad 2.309 \cos 60° - F_{BC} = 0 \qquad F_{BC} = 1.154 \text{ kN (C)}$$



Joint *D*: Since F_{CD}, F_{DE}, and F_{DE} lie within the same plane and F_{DE} is out of this plane, then $F_{DE} = 0$.

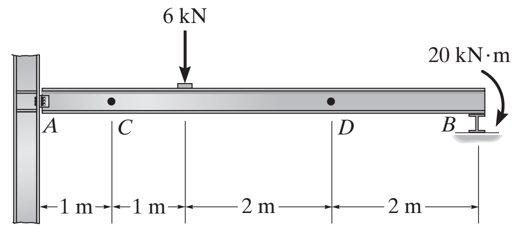
$$\sum F_x = 0; \qquad F_{DF}\left(\frac{1}{\sqrt{13}}\right) - 2.309 \cos 60° = 0$$

$$F_{DF} = 4.163 \text{ kN (C)}$$

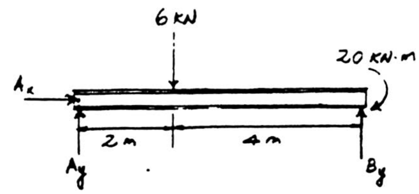
$$\sum F_y = 0; \qquad 4.163\left(\frac{3}{\sqrt{13}}\right) - F_{ED} = 0$$

$$F_{ED} = 3.46 \text{ kN (T)} \quad \textbf{Ans.}$$

4–1. Determine the internal normal force, shear force, and bending moment in the beam at points C and D. Assume the support at A is a pin and B is a roller.



Entire beam:



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0$

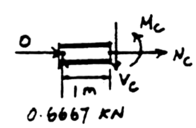
$\circlearrowleft + \sum M_A = 0;$ $\quad B_y(6) - 20 - 6(2) = 0$

$B_y = 5.333 \text{ kN}$

$+\uparrow \sum F_y = 0;$ $\quad A_y + 5.333 - 6 = 0$

$A_y = 0.6667 \text{ kN}$

Segment AC:



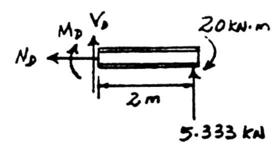
$\xrightarrow{+} \sum F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 0.6667 - V_C = 0$

$V_C = 0.667 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_C = 0;$ $\quad M_C - 0.6667(1) = 0$

$M_C = 0.667 \text{ kN} \cdot \text{m}$ **Ans.**



Segment DB:

$\xrightarrow{+} \sum F_x = 0;$ $\quad N_D = 0$ **Ans.**

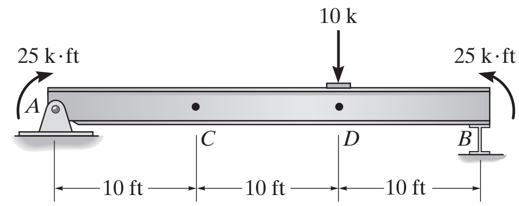
$+\uparrow \sum F_y = 0;$ $\quad V_D + 5.333 = 0$

$V_D = -5.33 \text{ kN}$ **Ans.**

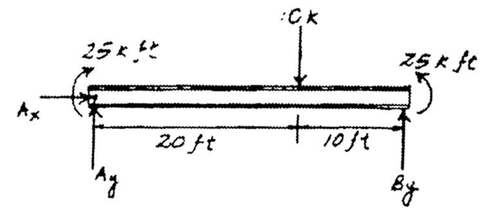
$\circlearrowleft + \sum M_D = 0;$ $\quad -M_D + 5.333(2) - 20 = 0$

$M_D = -9.33 \text{ kN} \cdot \text{m}$ **Ans.**

4–2. Determine the internal normal force, shear force, and bending moment in the beam at points C and D. Assume the support at B is a roller. Point D is located just to the right of the 10–k load.



Entire Beam:



$\curvearrowleft + \sum M_A = 0;$ $B_y(30) + 25 - 25 - 10(20) = 0$

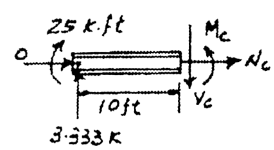
$B_y = 6.667 \text{ k}$

$+\uparrow \sum F_y = 0;$ $A_y + 6.667 - 10 = 0$

$A_y = 3.333 \text{ k}$

$\overset{+}{\rightarrow} \sum F_x = 0;$ $A_x = 0$

Segment AC:



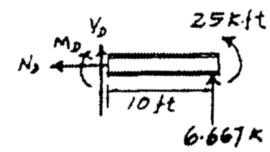
$\overset{+}{\rightarrow} \sum F_x = 0;$ $N_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $-V_C + 3.333 = 0$

$V_C = 3.33 \text{ k}$ **Ans.**

$\curvearrowleft + \sum M_C = 0;$ $M_C - 25 - 3.333(10) = 0$

$M_C = 58.3 \text{ k} \cdot \text{ft}$ **Ans.**



Segment DB:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $N_D = 0$ **Ans.**

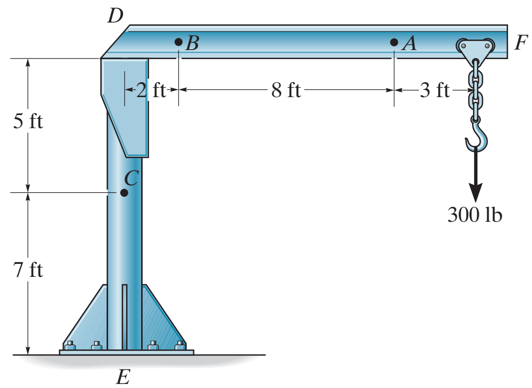
$+\uparrow \sum F_y = 0;$ $V_D + 6.6667 = 0$

$V_D = -6.67 \text{ k}$ **Ans.**

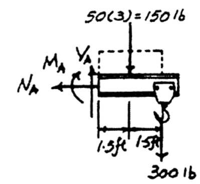
$\curvearrowleft + \sum M_D = 0;$ $-M_D + 25 + 6.667(10) = 0$

$M_D = 91.7 \text{ k} \cdot \text{ft}$ **Ans.**

4–3. The boom DF of the jib crane and the column DE have a uniform weight of 50 lb/ft. If the hoist and load weigh 300 lb, determine the internal normal force, shear force, and bending moment in the crane at points A, B, and C.



Equations of Equilibrium: For point A



$\overset{+}{\leftarrow} \sum F_x = 0;$ $\qquad N_A = 0$ **Ans.**

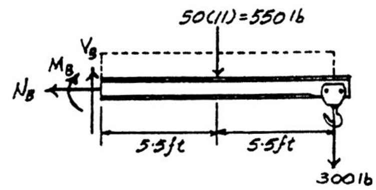
$+\uparrow \sum F_y = 0;$ $\qquad V_A - 150 - 300 = 0$

$V_A = 450 \text{ lb}$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $\qquad -M_A - 150(1.5) - 300(3) = 0$

$M_A = -1125 \text{ lb} \cdot \text{ft} = -1.125 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.

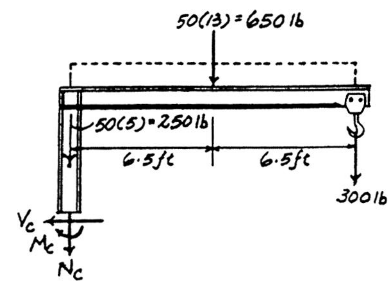


Equations of Equilibrium: For point B

$\overset{+}{\leftarrow} \sum F_x = 0;$ $\qquad N_B = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\qquad V_B - 550 - 300 = 0$

$V_B = 850 \text{ lb}$ **Ans.**



$\circlearrowleft + \sum M_B = 0;$ $\qquad -M_B - 550(5.5) - 300(11) = 0$

$M_B = -6325 \text{ lb} \cdot \text{ft} = -6.325 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_B acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point C

$\overset{+}{\leftarrow} \sum F_x = 0;$ $\qquad V_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\qquad -N_C - 250 - 650 - 300 = 0$

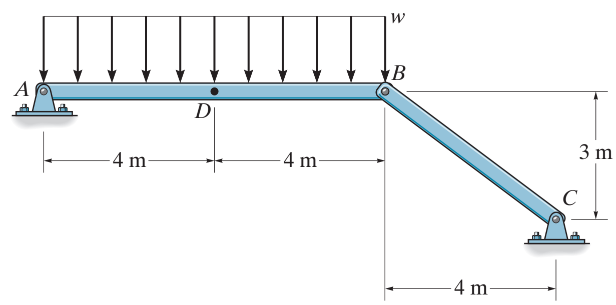
$N_C = -1200 \text{ lb} = -1.20 \text{ kip}$ **Ans.**

$\circlearrowleft + \sum M_C = 0;$ $\qquad -M_C - 650(6.5) - 300(13) = 0$

$M_C = -8125 \text{ lb} \cdot \text{ft} = -8.125 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicate that N_C and M_C act in the opposite direction to that shown on FBD.

***4–4.** Determine the internal normal force, shear force, and bending moment at point D. Take $w = 150$ N/m.



$\curvearrowleft + \sum M_A = 0; \quad -150(8)(4) + \frac{3}{5}F_{BC}(8) = 0$

$F_{BC} = 1000 \text{ N}$

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - \frac{4}{5}(1000) = 0$

$A_x = 800 \text{ N}$

$+\uparrow \sum F_y = 0; \quad A_y - 150(8) + \frac{3}{5}(1000) = 0$

$A_y = 600 \text{ N}$

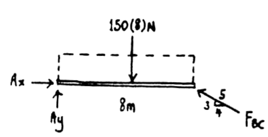
$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_D = -800 \text{ N}$ **Ans.**

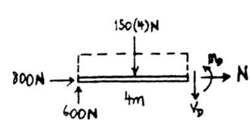
$+\uparrow \sum F_y = 0; \quad 600 - 150(4) - V_D = 0$

$V_D = 0$ **Ans.**

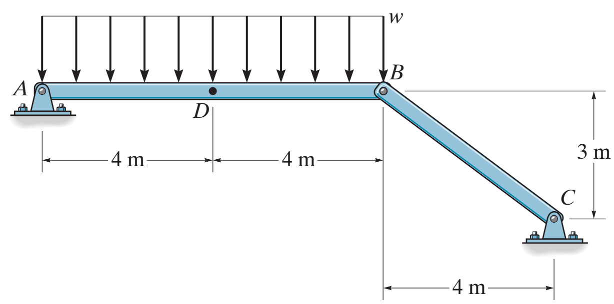
$\curvearrowleft + \sum M_D = 0; \quad -600(4) + 150(4)(2) + M_D = 0$

$M_D = 1200 \text{ N} \cdot \text{m} = 1.20 \text{ kN} \cdot \text{m}$ **Ans.**





4–5. The beam AB will fail if the maximum internal moment at D reaches 800 N·m or the normal force in member BC becomes 1500 N. Determine the largest load w it can support.



Assume maximum moment occurs at D;

$\circlearrowleft + \sum M_D = 0;$ $\quad M_D - \frac{8w}{2}(4) + 4w\,(2) = 0$

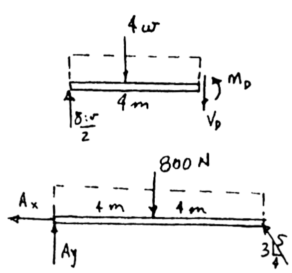
$800 = 8\,w$

$w = 100\ \text{N/m}$

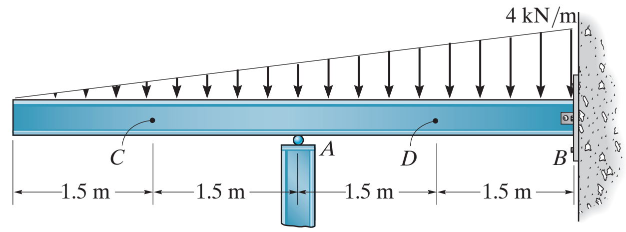
$\circlearrowleft + \sum M_A = 0;$ $\quad -800(4) + T_{BC}(0.6)(8) = 0$

$T_{BC} = 666.7\ \text{N} < 1500\ \text{N}$ **(O. K!)**

$w = 100\ \text{N/m}$ **Ans.**



4–6. Determine the internal normal force, shear force, and bending moment in the beam at points C and D. Assume the support at A is a roller and B is a pin.



Support Reactions. Referring to the FBD of the entire beam in Fig. a,

$\circlearrowleft + \sum M_B = 0; \quad \frac{1}{2}(4)(6)(2) - A_y(3) = 0 \quad A_y = 8 \text{ kN}$

Internal Loadings. Referring to the FBD of the left segment of the beam sectioned through point C, Fig. b,

$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad -\frac{1}{2}(1)(1.5) - V_C = 0 \quad V_C = -0.75 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_C = 0; \quad M_C + \frac{1}{2}(1)(1.5)(0.5) = 0 \quad M_C = -0.375 \text{ kN} \cdot \text{m}$ **Ans.**

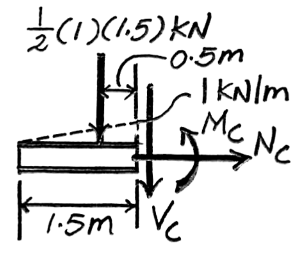
Referring to the FBD of the left segment of the beam sectioned through point D, Fig. c,

$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_D = 0$ **Ans.**

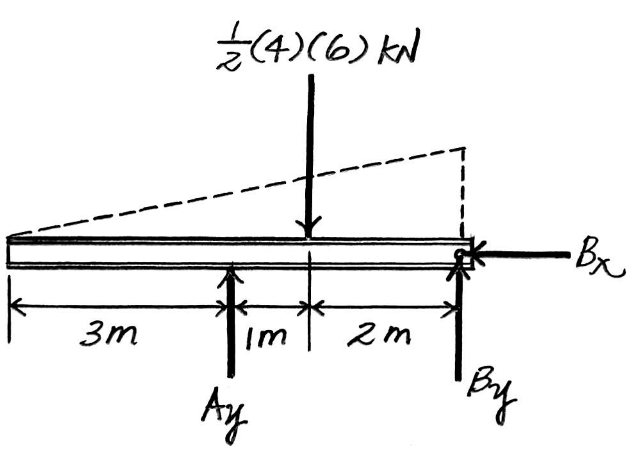
$+\uparrow \sum F_y = 0; \quad 8 - \frac{1}{2}(3)(4.5) - V_D = 0 \quad V_D = 1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_D = 0; \quad M_D + \frac{1}{2}(3)(-4.5)(1.5) - 8(1.5) = 0$

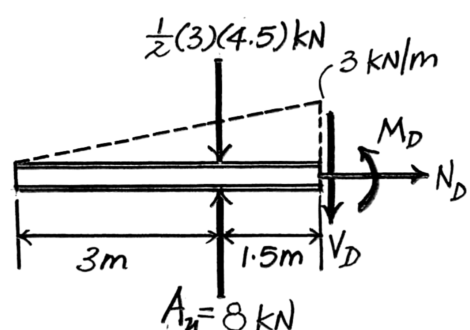
$M_D = 1.875 \text{ kN} \cdot \text{m}$ **Ans.**



(b)

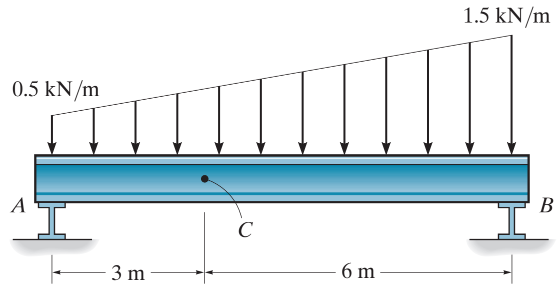


(a)

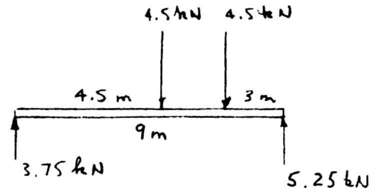


(c)

4–7. Determine the internal normal force, shear force, and bending moment at point C. Assume the reactions at the supports A and B are vertical.



$\xrightarrow{+} \sum F_x = 0;$ $N_C = 0$ **Ans.**

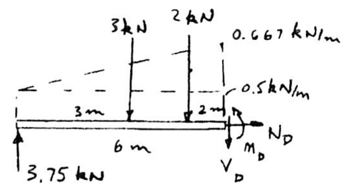


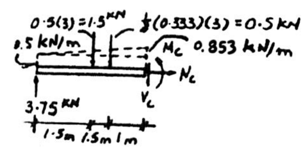
$+\downarrow \sum F_y = 0;$ $V_C + 0.5 + 1.5 - 3.75 = 0$

$V_C = 1.75 \text{ kN}$ **Ans.**

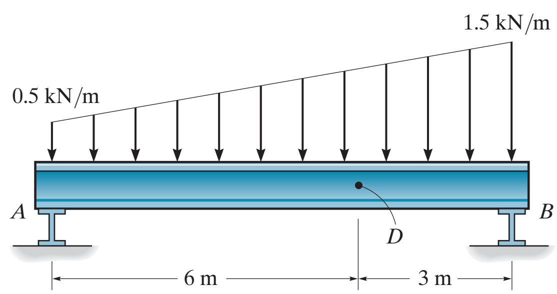
$\circlearrowleft + \sum M_C = 0;$ $M_C + 0.5(1) + 1.5(1.5) - 3.75(3) = 0$

$M_C = 8.50 \text{ kN} \cdot \text{m}$ **Ans.**

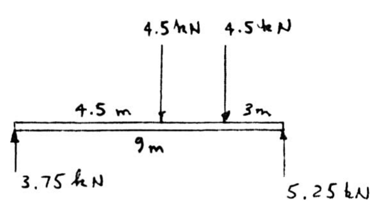




***4–8.** Determine the internal normal force, shear force, and bending moment at point D. Assume the reactions at the supports A and B are vertical.



$\xrightarrow{+} \sum F_x = 0;$ $N_D = 0$ **Ans.**

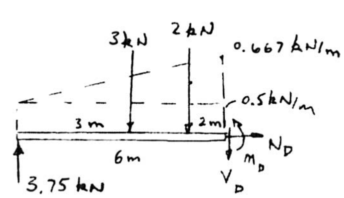


$+\uparrow \sum F_y = 0;$ $3.75 - 3 - 2 - V_D = 0$

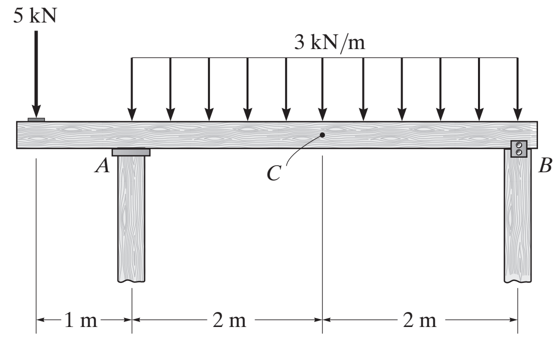
$V_D = -1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_D = 0;$ $M_D + 2(2) + 3(3) - 3.75(6) = 0$

$M_D = 9.50 \text{ kN} \cdot \text{m}$ **Ans.**



4–9. Determine the internal normal force, shear force, and bending moment in the beam at point *C*. The support at *A* is a roller and *B* is pinned.



Support Reactions. Referring to the FBD of the entire beam in Fig *a*,

$\circlearrowleft + \sum M_A = 0; \quad B_y(4) + 5(1) - 3(4)(2) = 0 \quad B_y = 4.75 \text{ kN}$

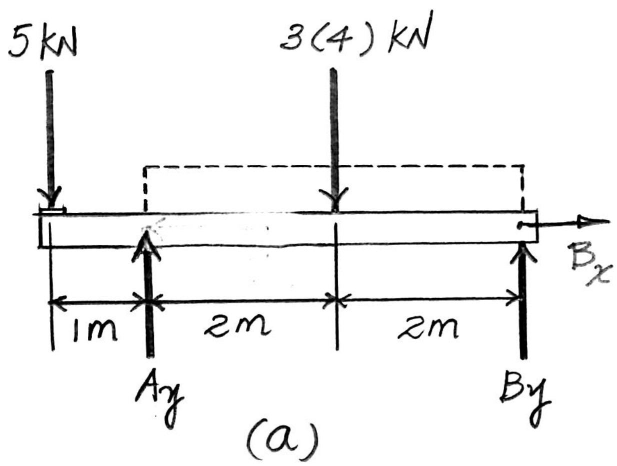
$\overset{+}{\rightarrow} \sum F_x = 0; \quad B_x = 0$

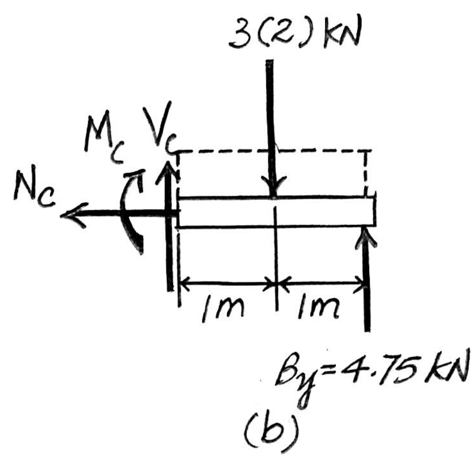
Internal Loadings. Referring to the FBD of the right segment of the beam sectioned through point *c*, Fig. *b*,

$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_C = 0$ **Ans.**

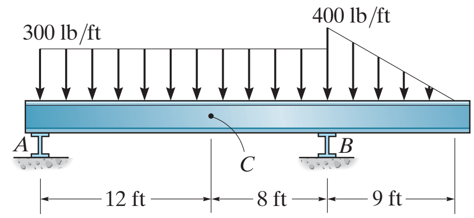
$+\uparrow \sum F_y = 0; \quad V_C + 4.75 - 3(2) = 0 \quad V_C = 1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_C = 0; \quad 4.75(2) - 3(2)(1) - M_C = 0 \quad M_C = 3.50 \text{ kN} \cdot \text{m}$ **Ans.**





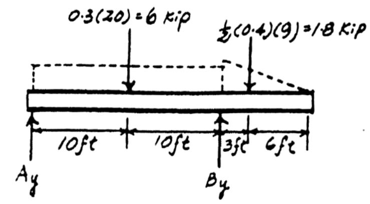
4–10. Determine the internal normal force, shear force, and bending moment at point C. Assume the reactions at the supports A and B are vertical.



Support Reactions:

$\circlearrowleft + \sum M_A = 0;$ $\quad B_y(20) - 6(10) - 1.8(23) = 0$

$B_y = 5.07 \text{ kip}$



$+\uparrow \sum F_y = 0;$ $\quad A_y + 5.07 - 6 - 1.8 = 0$

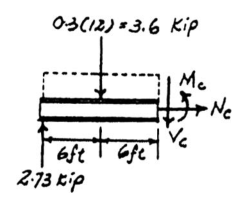
$A_y = 2.73 \text{ kip}$

Equations of Equilibrium: For point C

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 273 - 3.60 - V_C = 0$

$V_C = -0.870 \text{ kip}$ **Ans.**

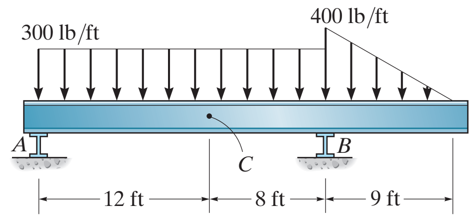


$\circlearrowleft + \sum M_C = 0;$ $\quad M_C + 3.60(6) - 2.73(12) = 0$

$M_C = 11.2 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that V_C acts in the opposite direction to that shown on FBD.

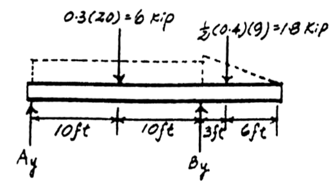
4–11. Determine the internal normal force, shear force, and bending moment at points D and E. Assume the reactions at the supports A and B are vertical.



Support Reactions:

$\circlearrowleft + \sum M_A = 0;$ $\quad B_y(20) - 6(10) - 1.8(23) = 0$

$B_y = 5.07 \text{ kip}$



$+\uparrow \sum F_y = 0;$ $\quad A_y + 5.07 - 6 - 1.8 = 0$

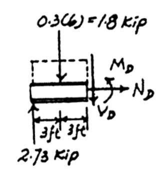
$A_y = 2.73 \text{ kip}$

Equations of Equilibrium: For point D

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad N_D = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 2.73 - 1.8 - V_D = 0$

$V_D = 0.930 \text{ kip}$ **Ans.**



$\circlearrowleft + \sum M_D = 0;$ $\quad M_D + 1.8(3) - 2.73(6) = 0$

$M_D = 11.0 \text{ kip} \cdot \text{ft}$ **Ans.**

Equations of Equilibrium: For point E

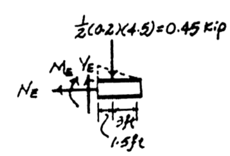
$\overset{+}{\leftarrow} \sum F_x = 0;$ $\quad N_E = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad V_E - 0.45 = 0$

$V_E = 0.450 \text{ kip}$ **Ans.**

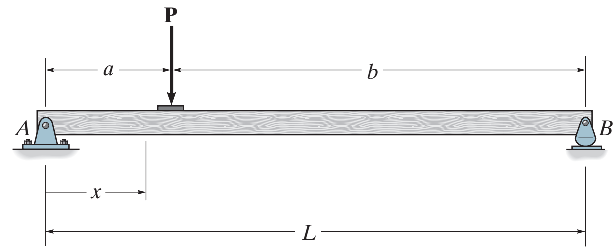
$\circlearrowleft + \sum M_E = 0;$ $\quad -M_E - 0.45(1.5) = 0$

$M_E = -0.675 \text{ kip} \cdot \text{ft}$ **Ans.**



Negative sign indicates that M_E acts in the opposite direction to that shown on FBD.

***4–12.** Determine the shear and moment throughout the beam as a function of x.



Support Reactions: Referring to the FBD of the entire beam in Fig. *a*,

$\zeta + \sum M_A = 0;\quad N_B(L) - Pa = 0\quad N_B = \dfrac{Pa}{L}$

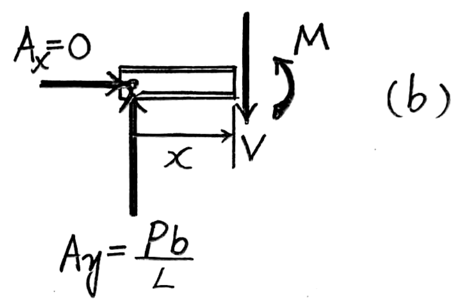
$\zeta + \sum M_B = 0;\quad Pb - A_y(L) = 0\quad A_y = \dfrac{Pb}{L}$

$\overset{+}{\rightarrow} \sum F_x = 0;\quad A_x = 0$

Internal Loading: For $0 \le x < a$, refer to the FBD of the left segment of the beam in Fig. *b*.

$+\uparrow \sum F_y = 0;\quad \dfrac{Pb}{L} - V = 0\quad V = \dfrac{Pb}{L}$ **Ans.**

$\zeta + \sum M_O = 0;\quad M - \dfrac{Pb}{L}x = 0\quad M = \dfrac{Pb}{L}x$ **Ans.**

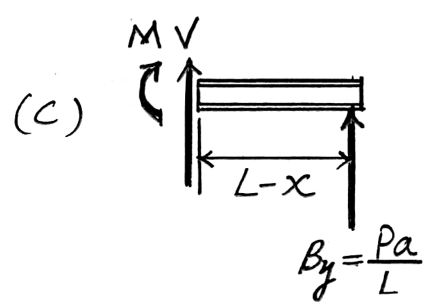


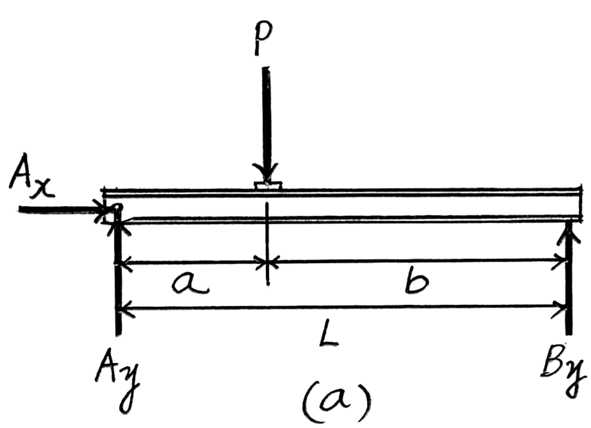
For $a < x \le L$, refer to the FBD of the right segment of the beam in Fig. *c*.

$+\uparrow \sum F_y = 0;\quad V + \dfrac{Pa}{L} = 0\quad V = -\dfrac{Pa}{L}$ **Ans.**

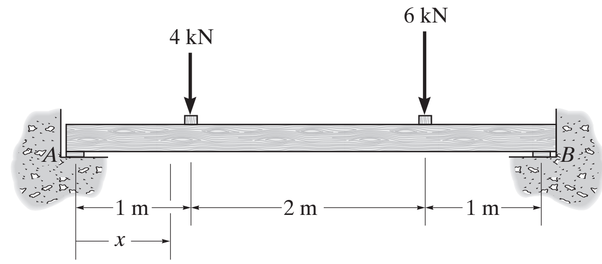
$\zeta + \sum M_O = 0;\quad \dfrac{Pa}{L}(L - x) - M = 0$

$M = \dfrac{Pa}{L}(L - x)$ **Ans.**





4–13. Determine the shear and moment in the floor girder as a function of x. Assume the support at A is a pin and B is a roller.



Support Reactions: Referring to the FBD of the entire beam in Fig. *a*.

$\circlearrowleft + \sum M_A = 0;\quad B_y(4) - 4(1) - 6(3) = 0$

$B_y = 5.50 \text{ kN}$

$\circlearrowleft + \sum M_B = 0;\quad 6(1) + 4(3) - A_y(4) = 0$

$A_y = 4.50 \text{ kN}$

$\overset{+}{\rightarrow} \sum F_x = 0;\quad A_x = 0$

Internal Loadings: For $0 \le x < 1$ m, Referring to the FBD of the left segment of the beam in Fig. *b*,

$+\uparrow \sum F_y = 0;\quad 4.50 - V = 0 \quad V = 4.50 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_O = 0;\quad M - 4.50x = 0$

$M = \{4.50x\} \text{ kN} \cdot \text{m}$ **Ans.**

For $1 \text{ m} < x < 3$ m, referring to the FBD of the left segment of the beam in Fig. *c*,

$+\uparrow \sum F_y = 0;\quad 4.50 - 4 - V = 0$

$V = 0.500 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_O = 0;\quad M + 4(x - 1) - 4.50x = 0$

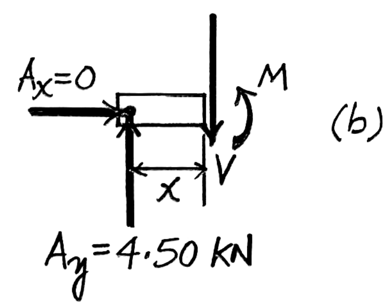
$M = \{0.5x + 4\} \text{ kN} \cdot \text{m}$ **Ans.**

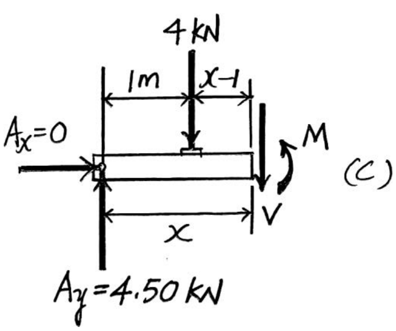
For $3 \text{ m} < x \le 4$ m, referring to the FBD of the right segment of the beam in Fig. *d*,

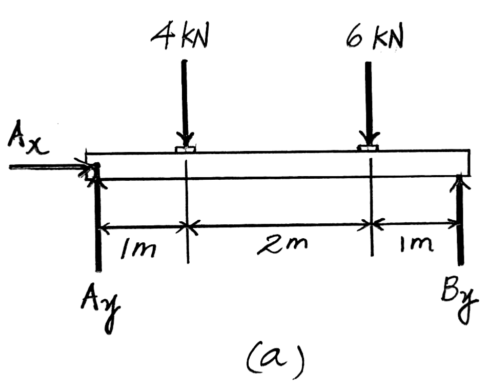
$+\uparrow \sum F_y = 0;\quad V + 5.50 = 0 \quad V = -5.50 \text{ kN}$ **Ans.**

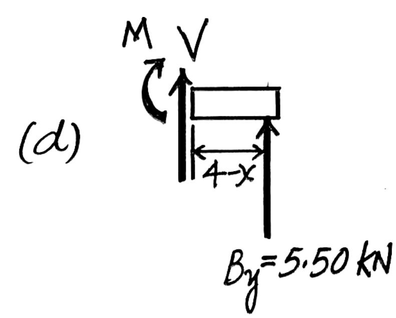
$\circlearrowleft + \sum M_O = 0;\quad 5.50(4 - x) - M = 0$

$M = \{-5.50x + 22\} \text{ kN} \cdot \text{m}$ **Ans.**

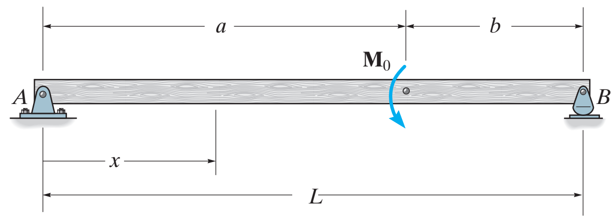








4–14. Determine the shear and moment throughout the beam as a function of x.



Support Reactions: Referring to the FBD of the entire beam in Fig. *a*

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad A_x = 0$$

$$\circlearrowleft + \sum M_A = 0; \qquad M_O - N_B(L) = 0 \qquad B_y = \frac{M_O}{L}$$

$$\circlearrowleft + \sum M_B = 0; \qquad M_O - A_y(L) = 0 \qquad A_y = \frac{M_O}{L}$$

Internal Loadings: For $0 \le x < a$, refer to the FBD of the left segment of the beam is Fig. *b*.

$$+\uparrow \sum F_y = 0; \qquad \frac{M_O}{L} - V = 0 \qquad V = \frac{M_O}{L} \qquad \textbf{Ans.}$$

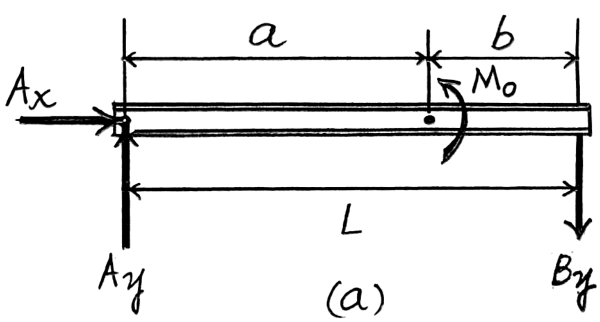
$$\circlearrowleft + \sum M_o = 0; \qquad M - \frac{M_o}{L}x = 0 \qquad M = \frac{M_O}{L}x \qquad \textbf{Ans.}$$

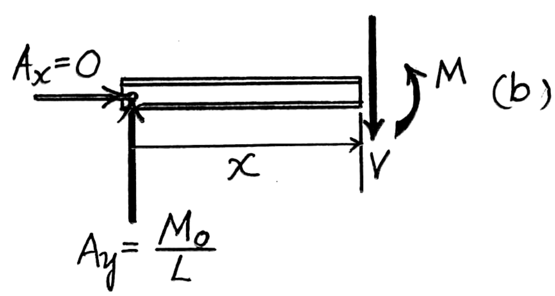
For $a < x \le L$, refer to the FBD of the right segment of the beam in Fig. *c*

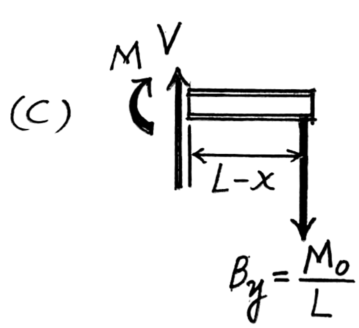
$$+\uparrow \sum F_y = 0; \qquad V - \frac{M_O}{L} = 0 \qquad V = \frac{M_O}{L} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_o = 0; \qquad -M - \frac{M_o}{L}(L - x) = 0 \qquad \textbf{Ans.}$$

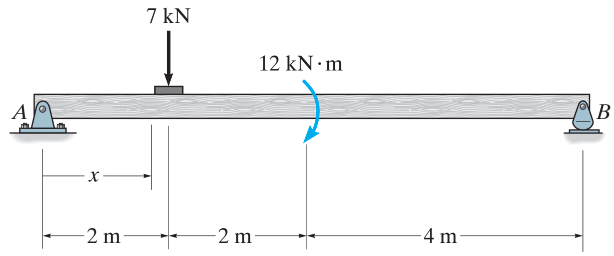
$$M = -\frac{M_o}{L}(L - x) \qquad \textbf{Ans.}$$



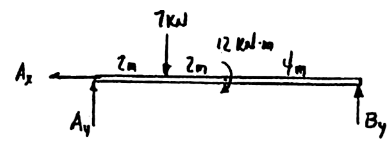




4–15. Determine the shear and moment throughout the beam as a function of x.



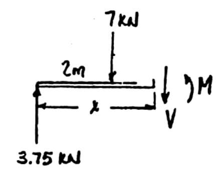
Reaction at A:



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\circlearrowleft + \sum M_B = 0;$ $\quad A_y(8) - 7(6) + 12 = 0; \quad A_y = 3.75\ \text{kN}$

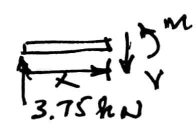
$0 \leq x < 2\ \text{m}$



$+\uparrow \sum F_y = 0;$ $\quad 3.75 - V = 0;$ $\quad V = 3.75\ \text{kN}$ **Ans.**

$\circlearrowleft + \sum M = 0;$ $\quad 3.75x - M = 0;$ $\quad M = 3.75x\ \text{kN}$ **Ans.**

$2\ \text{m} < x < 4\ \text{m}$

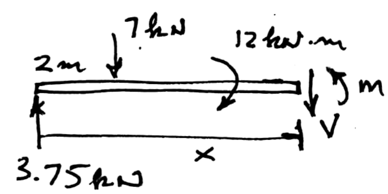


Segment:

$+\uparrow \sum F_y = 0;$ $\quad -V + 3.75 - 7 = 0;$ $\quad V = -3.25$ **Ans.**

$\circlearrowleft + \sum M = 0;$ $\quad -M + 3.75x - 7(x - 2) = 0;$

$M = -3.25x + 14$ **Ans.**



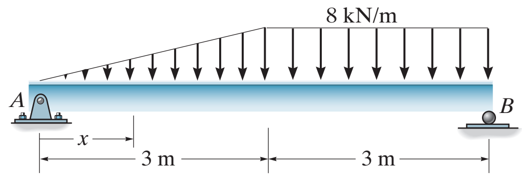
$4\ \text{m} < x \leq 8\ \text{m}$

$+\uparrow \sum F_y = 0;$ $\quad 3.75 - 7 - V = 0;$ $\quad V = -3.25\ \text{kN}$ **Ans.**

$\circlearrowleft + \sum M = 0;$ $\quad -3.75x + 7(x - 2) - 12 + M = 0;$

$M = 26 - 3.25x$ **Ans.**

***4–16.** Determine the shear and moment throughout the beam as a function of x.



Support Reactions. Referring to the FBD of the entire beam in Fig. a,

$$\zeta + \sum M_A = 0; \quad B_y(6) - 8(3)(4.5) - \frac{1}{2}(8)(3)(2) = 0$$

$$B_y = 22 \text{ kN}$$

$$\zeta + \sum M_B = 0; \quad 8(3)(1.5) + \frac{1}{2}(8)(3)(4) - A_y(6) = 0$$

$$A_y = 14 \text{ kN}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0$$

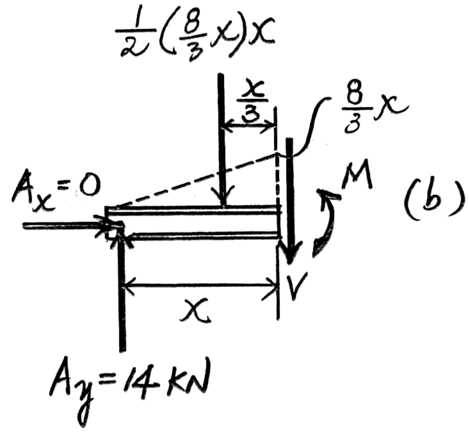
Internal Loadings: For $0 \le x < 3$ m, refer to the FBD of the left segment of the beam in Fig. b,

$$+\uparrow \sum F_y = 0; \quad 14 - \frac{1}{2}\left(\frac{8}{3}x\right)x - V = 0$$

$$V = \left\{-1.33x^2 + 14\right\} \text{ kN} \qquad \textbf{Ans.}$$

$$\zeta + \sum M_O = 0; \quad M + \frac{1}{2}\left(\frac{8}{3}x\right)(x)\left(\frac{x}{3}\right) - 14x = 0$$

$$M = \left\{-0.444\,x^3 + 14\,x\right\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



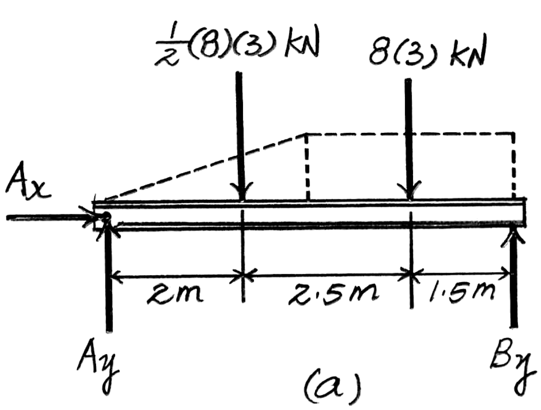
For 3 m $< x \le 6$ m, refer to the FBD of the right segment of the beam in Fig. c

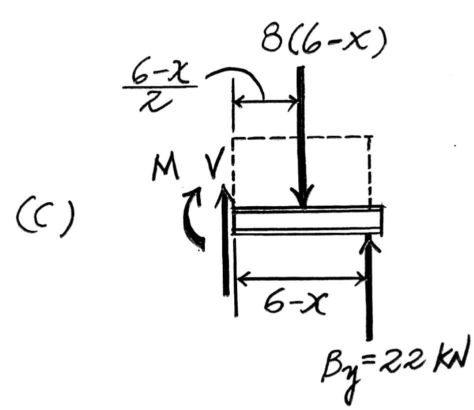
$$+\uparrow \sum F_y = 0; \quad V + 22 - 8(6 - x) = 0$$

$$V = \left\{-8x + 26\right\} \text{ kN} \qquad \textbf{Ans.}$$

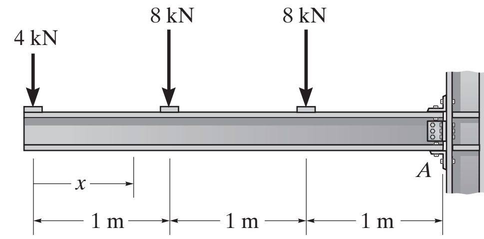
$$\zeta + \sum M_O = 0; \quad 22(6 - x) - 8(6 - x)\left(\frac{6 - x}{2}\right) - M = 0$$

$$M = \{-4x^2 + 26x - 12\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$





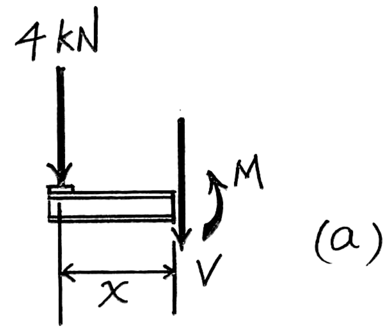
4–17. Determine the shear and moment throughout the beam as a function of x.



Internal Loadings. For $0 \le x \le 1$ m, referring to the FBD of the left segment of the beam in Fig. *a*,

$+\uparrow \sum F_y = 0;\quad -V - 4 = 0 \qquad V = -4 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_O = 0;\quad M + 4x = 0 \qquad M = \{-4x\} \text{ kN}\cdot\text{m}$ **Ans.**

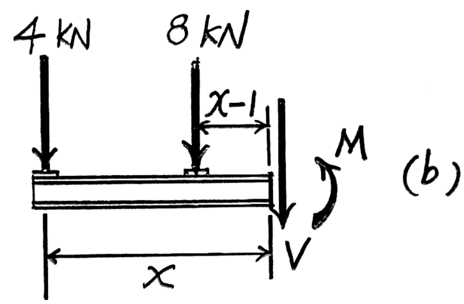


For $1 \text{ m} < x < 2$ m, referring to the FBD of the left segment of the beam in Fig. *b*,

$+\uparrow \sum F_y = 0;\quad -4 - 8 - V = 0 \qquad V = \{-12\} \text{ kN}\cdot\text{m}$ **Ans.**

$\circlearrowleft + \sum M_O = 0;\quad M + 8(x-1) + 4x = 0$

$M = \{-12x + 8\} \text{ kN}\cdot\text{m}$ **Ans.**

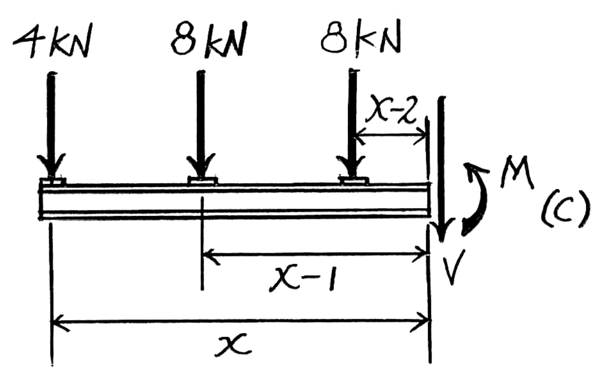


For $2 \text{ m} < x \le 3$ m, referring to the FBD of the left segment of the beam in Fig. *c*,

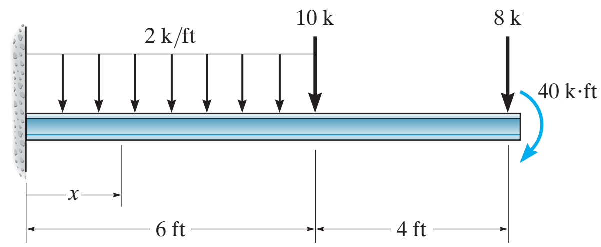
$+\uparrow \sum F_y = 0;\quad -4 - 8 - 8 - V = 0 \qquad V = \{-20\} \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_O = 0;\quad M + 4x + 8(x-1) + 8(x-2) = 0$

$M = \{-20x + 24\} \text{ kN}\cdot\text{m}$ **Ans.**



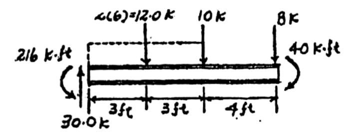
4–18. Determine the shear and moment throughout the beam as functions of x.



Support Reactions: As shown on FBD.

Shear and Moment Functions:

For $0 \le x < 6$ ft

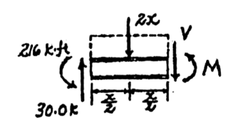


$$+\uparrow \sum F_y = 0; \quad 30.0 - 2x - V = 0$$

$$V = \{30.0 - 2x\} \text{ k}$$ **Ans.**

$$\circlearrowleft + \sum M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$$

$$M = \{-x^2 + 30.0x - 216\} \text{ k} \cdot \text{ft}$$ **Ans.**

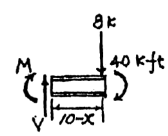


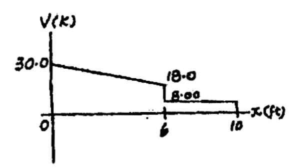
For $6 \text{ ft} < x \le 10$ ft

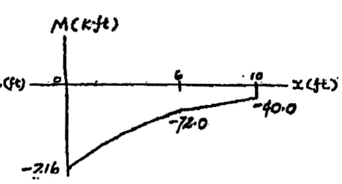
$$\xrightarrow{+} \sum F_y = 0; \quad V - 8 = 0 \qquad V = 8.00 \text{ k}$$ **Ans.**

$$\circlearrowleft + \sum M_{NA} = 0; \quad -M - 8\,(10 - x) - 40 = 0$$

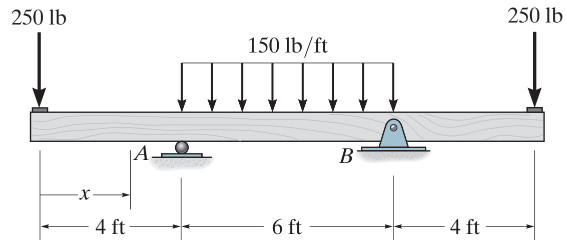
$$M = \{8.00x - 120\} \text{ k} \cdot \text{ft}$$ **Ans.**





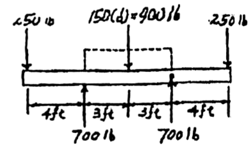


4–19. Determine the shear and moment throughout the beam as functions of x.



Support Reactions: As shown on FBD.

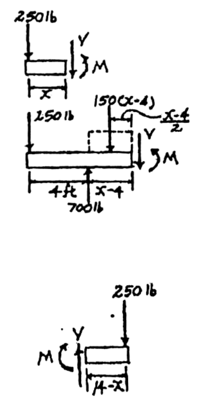
Shear and moment Functions:



For $0 \leq x < 4$ ft

$+\uparrow \sum F_y = 0;\quad -250 - V = 0 \qquad V = -250 \text{ lb}$ **Ans.**

$\circlearrowleft + \sum M_{NA} = 0;\quad M + 250x = 0 \quad M = \{-250x\} \text{ lb} \cdot \text{ft}$ **Ans.**



For $4 \text{ ft} < x < 10$ ft

$+\uparrow \sum F_y = 0;\quad -250 + 700 - 150(x - 4) - V = 0$

$V = \{1050 - 150x\} \text{ lb}$ **Ans.**

$\circlearrowleft + \sum M_{NA} = 0;\quad M + 150(x - 4)\left(\dfrac{x - 4}{2}\right) + 250x - 700(x - 4) = 0$

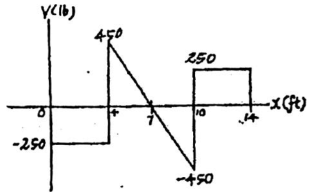
$M = \{-75x^2 + 1050x - 4000\} \text{lb} \cdot \text{ft}$ **Ans.**

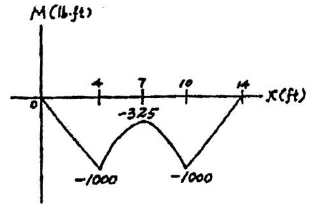
For $10 \text{ ft} < x \leq 14$ ft

$+\uparrow \sum F_y = 0;\quad V - 250 = 0 \qquad V = 250 \text{ lb}$ **Ans.**

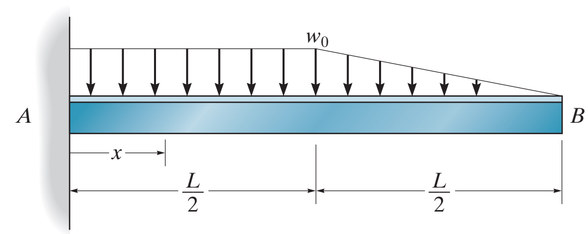
$\circlearrowleft + \sum M_{NA} = 0;\quad -M - 250(14 - x) = 0$

$M = \{250x - 3500\} \text{lb} \cdot \text{ft}$ **Ans.**



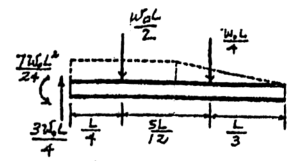


***4–20.** Determine the shear and moment in the beam as functions of x.



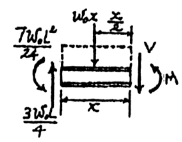
Support Reactions: As shown on FBD.

Shear and Moment Functions:



For $0 \le x < L/2$

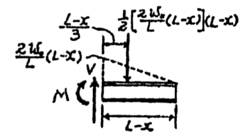
$$+\uparrow \sum F_y = 0; \quad \frac{3w_oL}{4} - w_o x - V = 0 \qquad V = \frac{w_o}{4}(3L - 4x) \qquad \textbf{Ans.}$$



$$\circlearrowleft + \sum M_{NA} = 0; \quad \frac{7w_oL^2}{24} - \frac{3w_oL}{4}x + w_o x\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w_o}{24}(-12x^2 + 18Lx - 7L^2) \qquad \textbf{Ans.}$$

For $L/2 < x \le L$

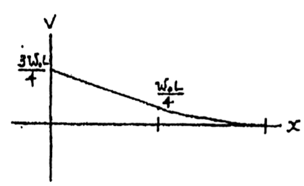


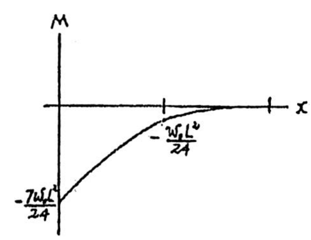
$$+\uparrow \sum F_y = 0; \quad V - \frac{1}{2}\left[\frac{2w_o}{L}(L-x)\right](L-x) = 0$$

$$V = \frac{w_o}{L}(L-x)^2 \qquad \textbf{Ans.}$$

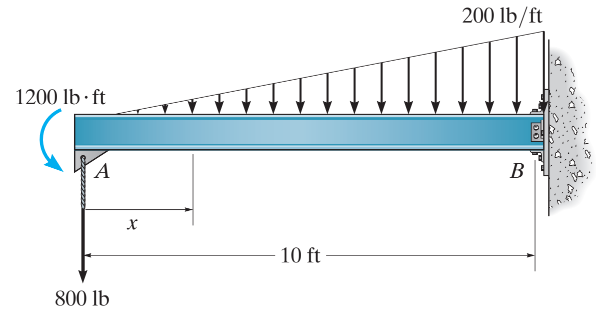
$$\circlearrowleft + \sum M_{NA} = 0; \quad -M - \frac{1}{2}\left[\frac{2w_o}{L}(L-x)\right](L-x)\left(\frac{L-x}{3}\right) = 0$$

$$M = \frac{w_o}{3L}(L-x)^2 \qquad \textbf{Ans.}$$





4–21. Determine the shear and moment in the beam as a function of x.



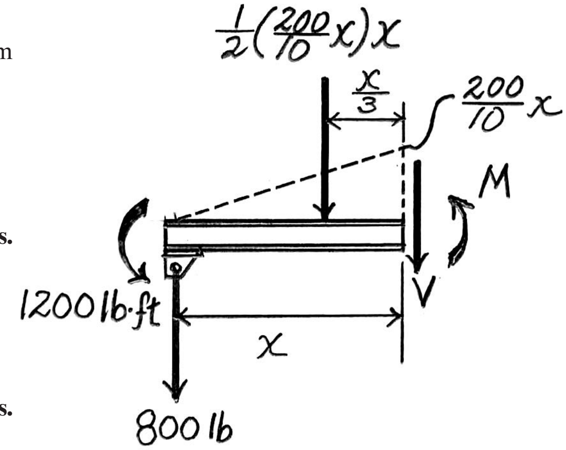
Internal Loadings: Referring to the FBD of the left segment of the beam in Fig. *a*,

$$+\uparrow \sum F_y = 0; \qquad -800 - \frac{1}{2}\left(\frac{200}{10}x\right)(x) - V = 0$$

$$V = \{-10x^2 - 800\}\text{lb} \qquad \textbf{Ans.}$$

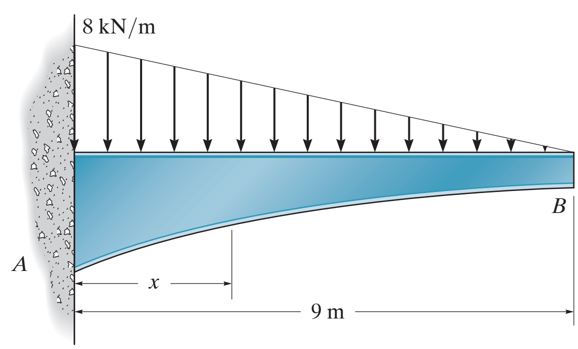
$$\circlearrowleft + \sum M_o = 0; \qquad M + \frac{1}{2}\left(\frac{200}{10}x\right)(x)\left(\frac{x}{3}\right) + 800x + 1200 = 0$$

$$M = \{-3.33x^3 - 800x - 1200\} \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



(a)

4–22 Determine the shear and moment throughout the tapered beam as a function of x.



$$\overset{+}{\rightarrow} \sum F_y = 0; \quad 36 - \frac{1}{2}\left(\frac{8}{9}x\right)(x) - \frac{8}{9}\left(8 - \frac{8}{9}x\right)x - V = 0$$

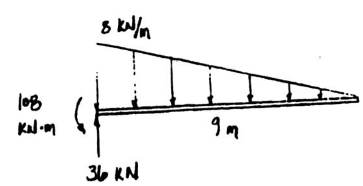
$$V = 36 - \frac{4}{9}x^2 - 8x + \frac{8}{9}x^2$$

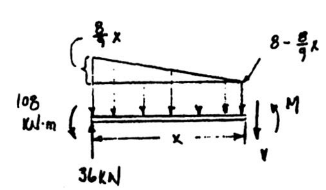
$$V = 0.444x^2 - 8x + 36$$

$$\circlearrowleft + \sum M = 0; \quad 108 + \frac{1}{2}\left(\frac{8}{9}x\right)(x)\left(\frac{2}{3}x\right) + \frac{8}{9}\left(8 - \frac{8}{9}x\right)(x)\left(\frac{x}{2}\right) - 36x + M = 0$$

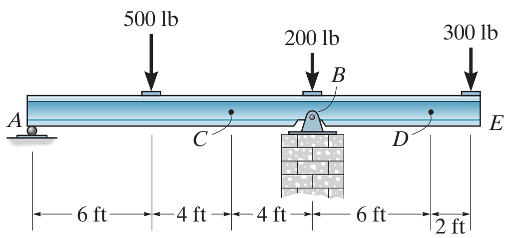
$$M = -108 - \frac{8}{27}x^3 - 4x^2 + \frac{8}{18}x^3 + 36x$$

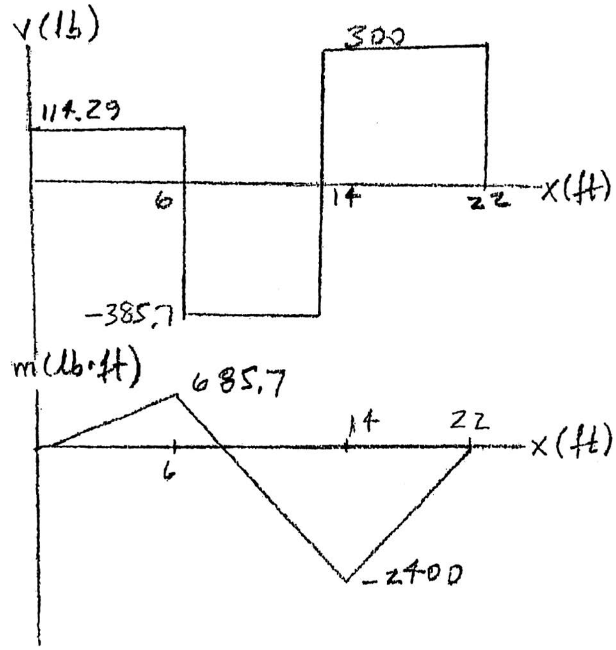
$$M = 0.148x^3 - 4x^2 + 36x - 108 \qquad \textbf{Ans.}$$



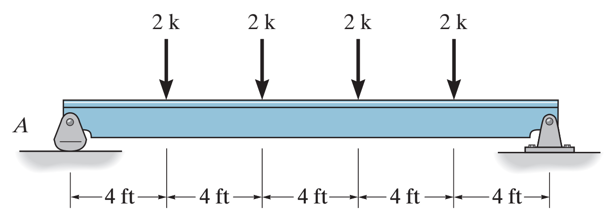


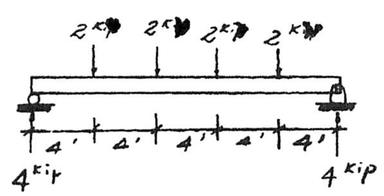
4–23. Draw the shear and moment diagrams for the beam.

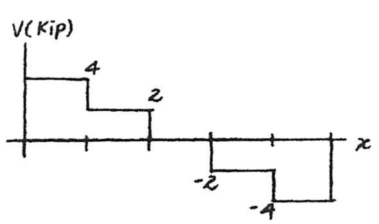


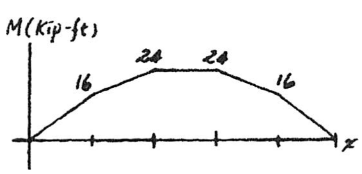


***4–24.** Draw the shear and moment diagrams for the beam.

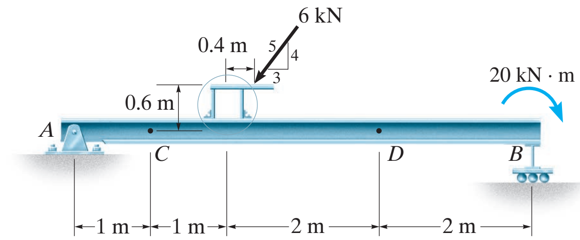






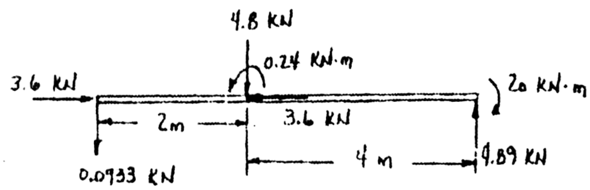


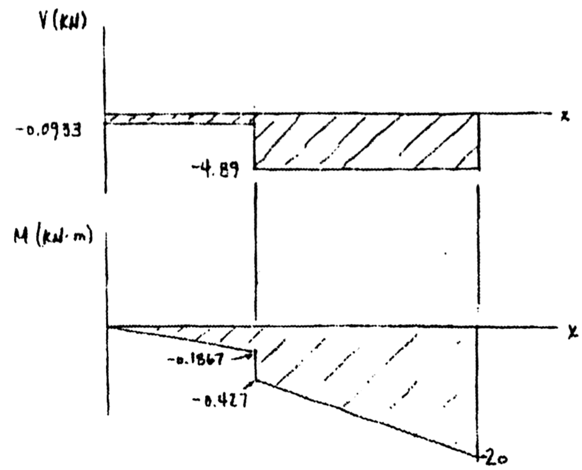
4–25. Draw the shear and moment diagrams for the beam.



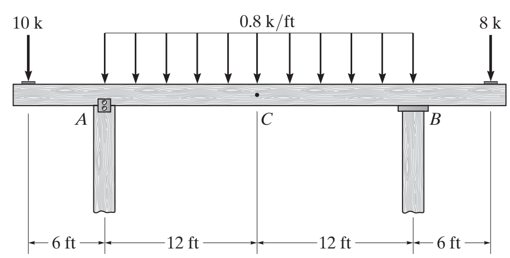
$V_{max} = -4.89$ kN **Ans.**

$M_{max} = -20$ kN · m **Ans.**



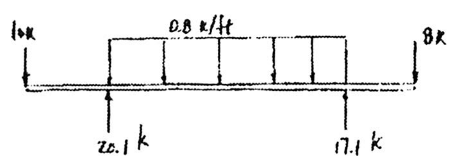


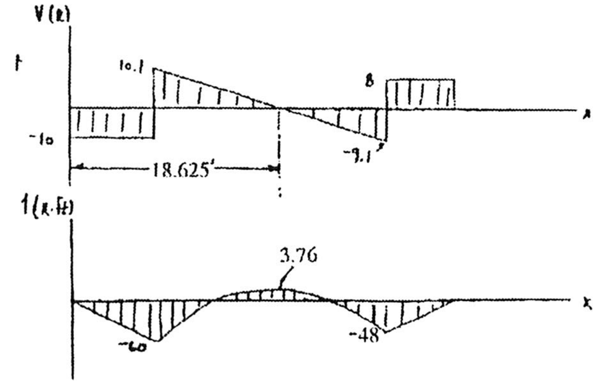
4–26. Draw the shear and moment diagrams of the beam.



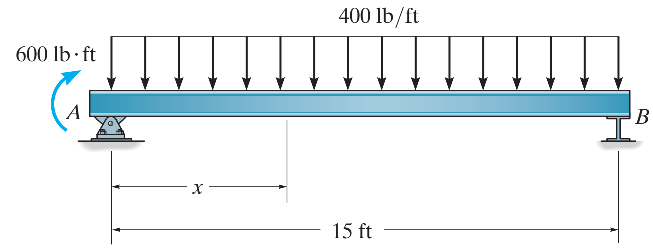
$V_{max} = -10.1$ k **Ans.**

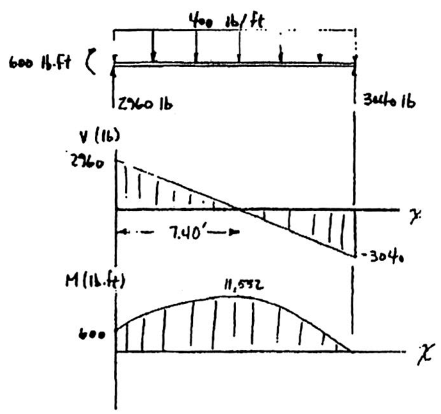
$M_{max} = -60$ k · ft **Ans.**



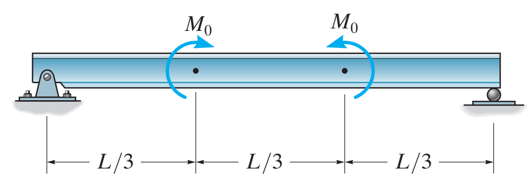


4–27. Draw the shear and moment diagrams for the beam.

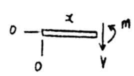




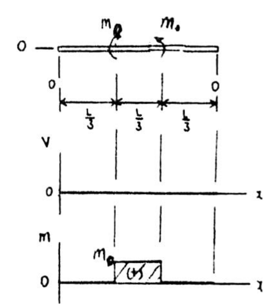
***4–28.** Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set M_O = 500 N · m, L = 8 m.



(a) For $0 \leq x \leq \dfrac{L}{3}$

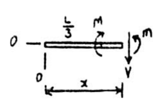


$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**



$\zeta + \sum M = 0; \quad M = 0$ **Ans.**

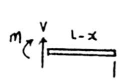
For $\dfrac{L}{3} < x < \dfrac{2L}{3}$



$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

$\zeta + \sum M = 0; \quad M = M_O$ **Ans.**

For $\dfrac{2L}{3} < x \leq L$

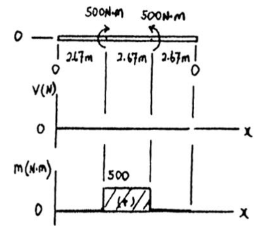


$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

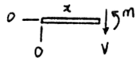
$\zeta + \sum M = 0; \quad M = 0$ **Ans.**

4–28. Continued

(b) Set $M_O = 500\ \text{N} \cdot \text{m},\ L = 8\ \text{m}$



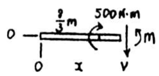
For $0 \le x < \dfrac{8}{3}\ \text{m}$



$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad M = 0$ **Ans.**

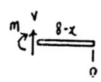
For $\dfrac{8}{3}\ \text{m} < x < \dfrac{16}{3}\ \text{m}$



$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad M = 500\ \text{N} \cdot \text{m}$ **Ans.**

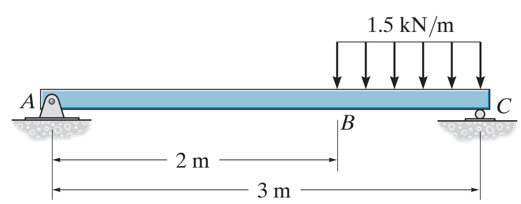
For $\dfrac{16}{3}\ \text{m} < x \le 8\ \text{m}$



$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad M = 0$ **Ans.**

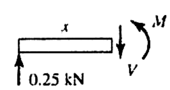
4–29. Draw the shear and moment diagrams for the beam.



Support Reactions:

$\circlearrowleft + \sum M_A = 0; \quad C_x(3) - 1.5(2.5) = 0 \quad C_x = 1.25\ \text{kN}$

$+\uparrow \sum F_y = 0; \quad A_y - 1.5 + 1.25 = 0 \quad A_y = 0.250\ \text{kN}$

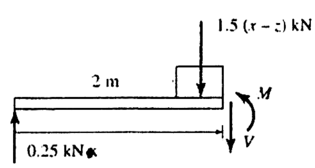


(a)

Shear and Moment Functions: For $0 \le x < 2$ m [FBD (a)],

$+\uparrow \sum F_y = 0; \quad 0.250 - V = 0 \quad V = 0.250\ \text{kN}$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad M - 0.250x = 0 \quad M = (0.250x)\ \text{kN} \cdot \text{m}$ **Ans.**



(b)

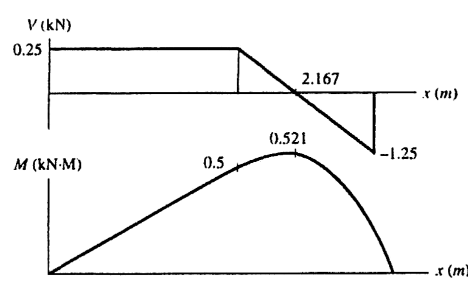
For $2\ \text{m} < x \le 3$ m [FBD (b)].

$+\uparrow \sum F_y = 0; \quad 0.25 - 1.5(x - 2) - V = 0$

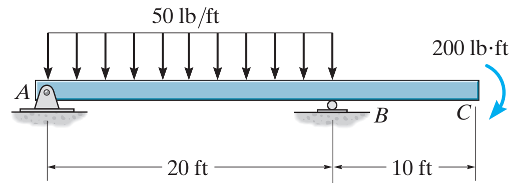
$V = (3.25 - 1.50x)\ \text{kN}$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad -0.25x + 1.5(x + 2)\left(\dfrac{x - 2}{2}\right) + M = 0$

$M = (-0.750x^2 + 3.25x - 3.00)\ \text{kN} \cdot \text{m}$ **Ans.**

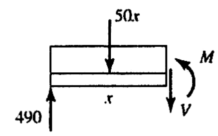


4–30. Draw the shear and bending-moment diagrams for the beam.



Support Reactions:

$\zeta + \sum M_B = 0;\quad 1000(10) - 200 - A_y(20) = 0 \quad A_y = 490 \text{ lb}$



(a)

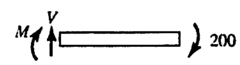
Shear and Moment Functions: For $0 \leq x < 20$ ft [FBD (a)].

$+\uparrow \sum F_y = 0;\quad 490 - 50x - V = 0$

$V = \{490 - 50.0x\}$ lb **Ans.**

$\zeta + \sum M = 0;\quad M + 50x\left(\dfrac{x}{2}\right) - 490x = 0$

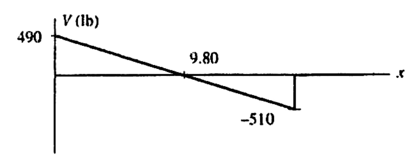
$M = (490x - 25.0x^2)$ lb · ft **Ans.**

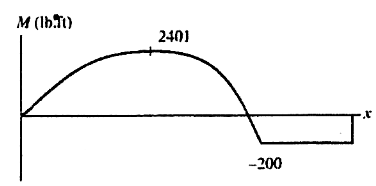


For 20 ft $< x \leq 30$ ft [FBD (b)],

$+\uparrow \sum F_y = 0;\quad V = 0$ **Ans.**

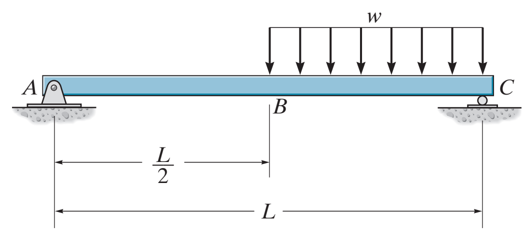
$\zeta + \sum M = 0;\quad -200 - M = 0 \quad M = -200$ lb · ft **Ans.**





(b)

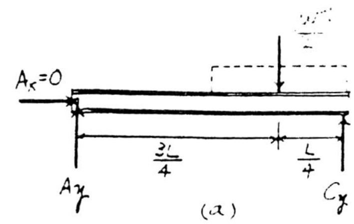
4–31. Draw the shear and moment diagrams for the beam.



Support Reactions: From FBD(a),

$$\zeta + \sum M_A = 0; \quad C_y(L) - \frac{wL}{2}\left(\frac{3L}{4}\right) = 0 \quad C_y = \frac{3wL}{8}$$

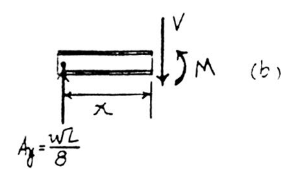
$$+\uparrow \sum F_y = 0; \quad A_y + \frac{3wL}{8} - \left(\frac{wL}{2}\right) = 0 \quad A_y = \frac{wL}{8}$$



Shear and Moment Functions: For $0 \leq x < \frac{L}{2}$ [FBD (b)],

$$+\uparrow \sum F_y = 0; \quad \frac{wL}{8} - V = 0 \quad V = \frac{wL}{8}$$ **Ans.**

$$\zeta + \sum M = 0; \quad M - \frac{wL}{8}(x) = 0 \quad M = \frac{wL}{8}(x)$$ **Ans.**



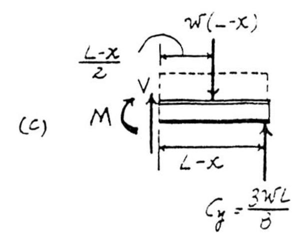
For $\frac{L}{2} < x \leq L$ [FBD (c)],

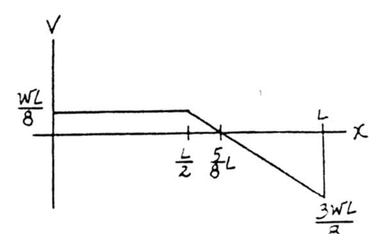
$$+\uparrow \sum F_y = 0; \quad V + \frac{3wL}{8} - w(L - x) = 0$$

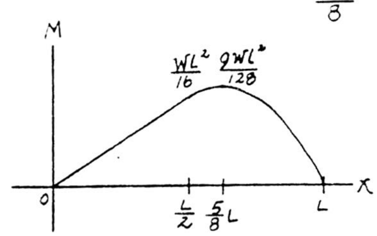
$$V = \frac{w}{8}(5L - 8x)$$

$$\zeta + \sum M_B = 0; \quad \frac{3wL}{8}(L - x) - w(L - x)\left(\frac{L - x}{2}\right) - M = 0$$

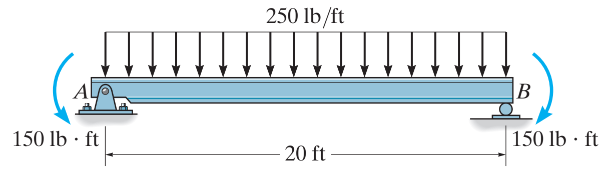
$$M = \frac{w}{8}(-L^2 + 5Lx - 4x^2)$$ **Ans.**





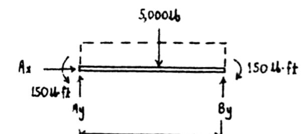


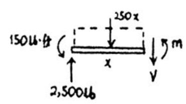
***4–32.** Draw the shear and moment diagrams for the beam.



$\circlearrowleft + \sum M_A = 0; \quad -5000(10) - 150 + B_y(20) = 0$

$B_y = 2500 \text{ lb}$

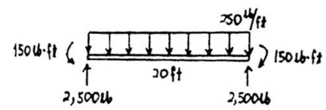




$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0$

$+\uparrow \sum F_y = 0; \quad A_y = 5000 + 2500 = 0$

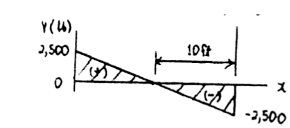
$A_y = 2500 \text{ lb}$



For $0 \leq x \leq 20$ ft

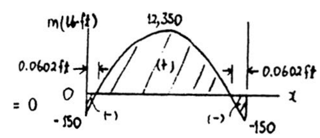
$+\uparrow \sum F_y = 0; \quad 2500 - 250x - V = 0$

$V = 250(10 - x)$ **Ans.**

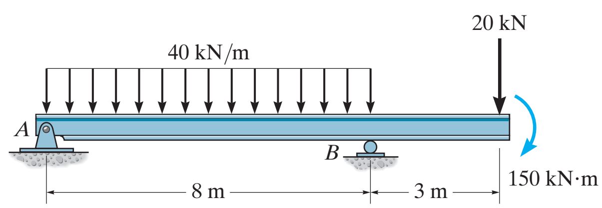


$\circlearrowleft + \sum M = 0; \quad -2500(x) + 150 + 250x\left(\frac{x}{2}\right) + M = 0$

$M = 25(100x - 5x^2 - 6)$ **Ans.**



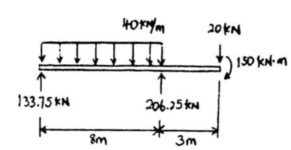
4–33. Draw the shear and moment diagrams for the beam.



$0 \leq x < 8$

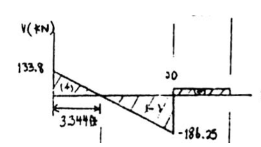
$+\uparrow \sum F_y = 0; \quad 133.75 - 40x - V = 0$

$V = 133.75 - 40x$ **Ans.**



$\circlearrowleft + \sum M = 0; \quad M + 40x\left(\frac{x}{2}\right) - 133.75x = 0$

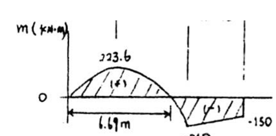
$M = 133.75x - 20x^2$ **Ans.**



$8 < x \leq 11$

$+\uparrow \sum F_y = 0; \quad V - 20 = 0$

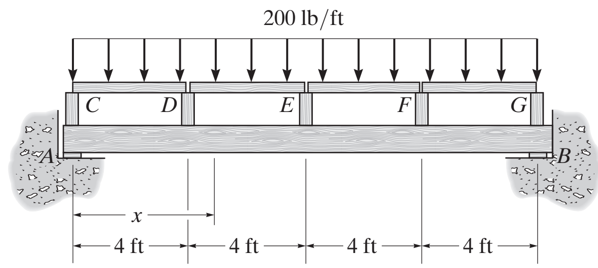
$V = 20$ **Ans.**



$\circlearrowright + \sum M = 0; \quad M + 20(11 - x) + 150 = 0$

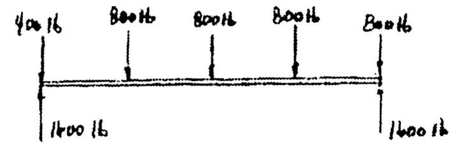
$M = 20x - 370$ **Ans.**

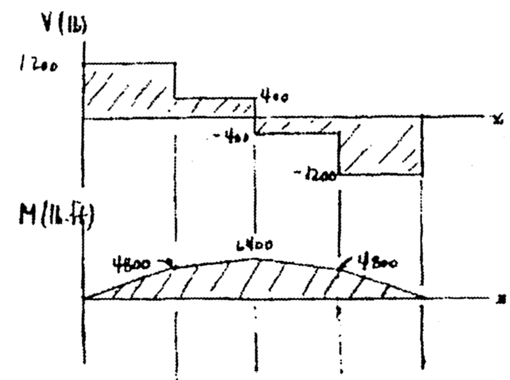
4–34. Draw the shear and moment diagrams for the beam.



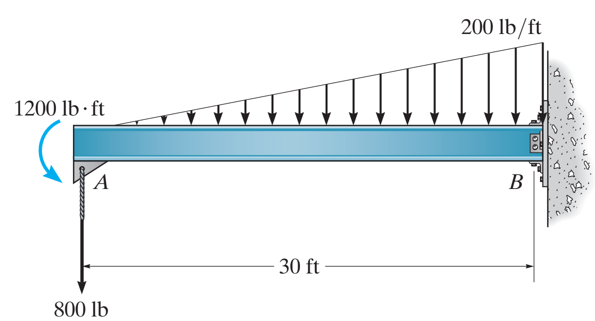
$V_{max} = \pm 1200$ lb **Ans.**

$M_{max} = 6400$ lb · ft **Ans.**



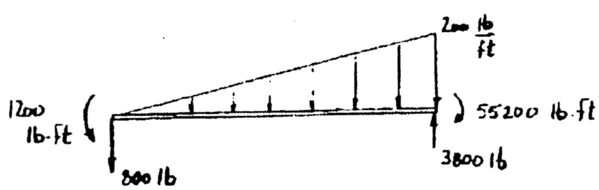


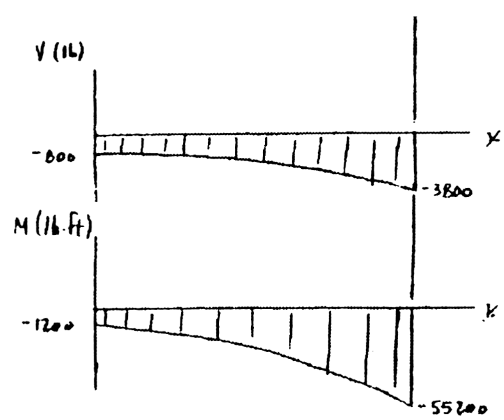
4–35. Draw the shear and moment diagrams for the beam.



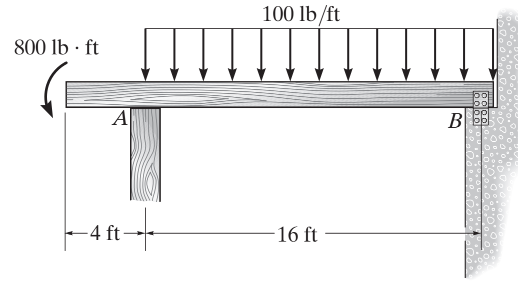
$V_{max} = -3.80$ k **Ans.**

$M_{max} = -55.2$ k · ft **Ans.**



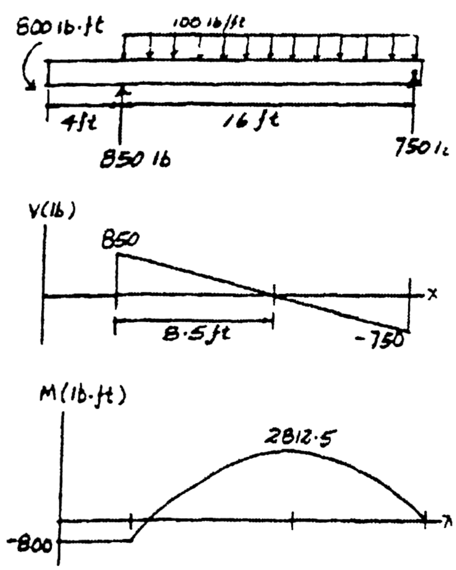


***4–36.** Draw the shear and moment diagrams of the beam. Assume the support at B is a pin and A is a roller.

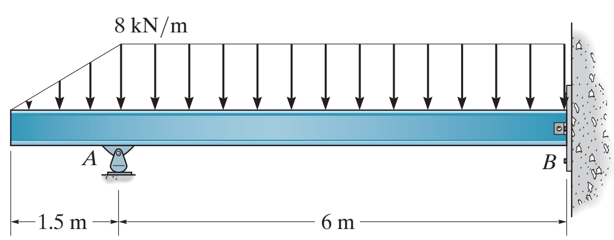


$V_{max} = 850$ lb **Ans.**

$M_{max} = -2.81$ K · ft **Ans.**

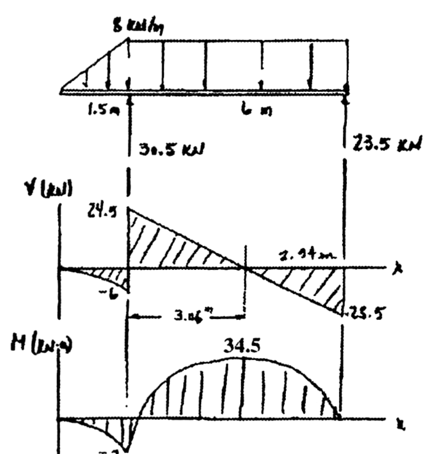


4–37. Draw the shear and moment diagrams for the beam. Assume the support at B is a pin.

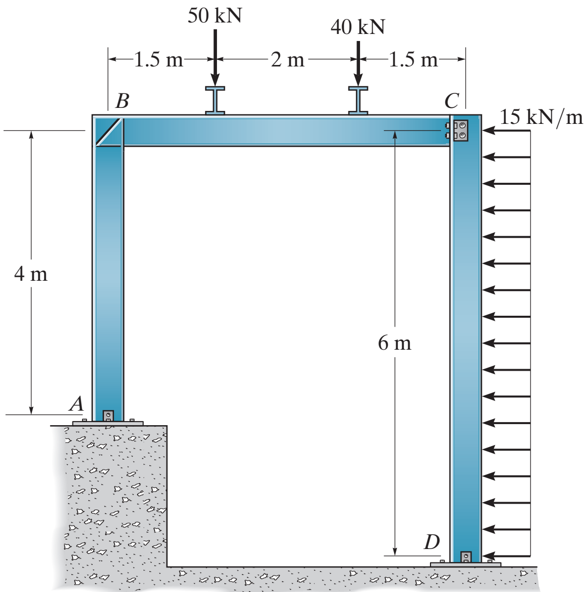


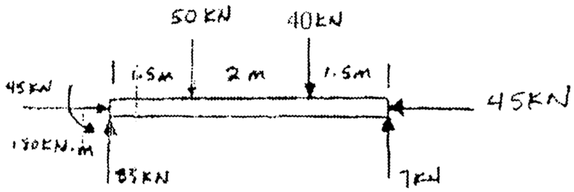
$V_{max} = 24.5$ kN **Ans.**

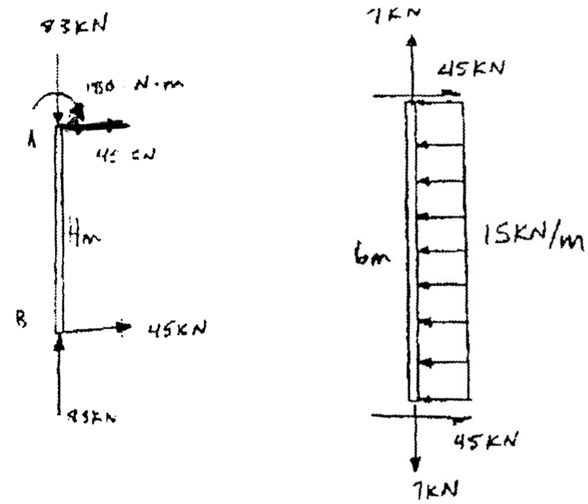
$M_{max} = 34.5$ kN · m **Ans.**

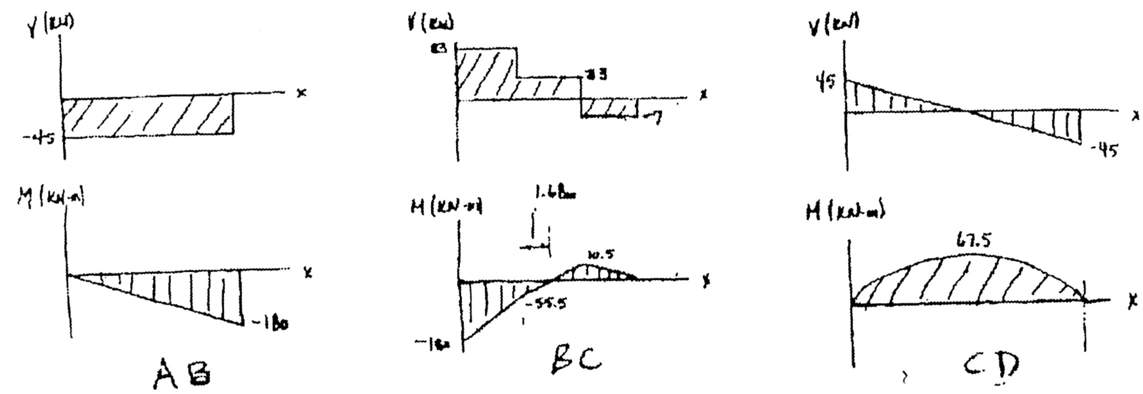


4–38. Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at A, C, and D and there is fixed joint at B.

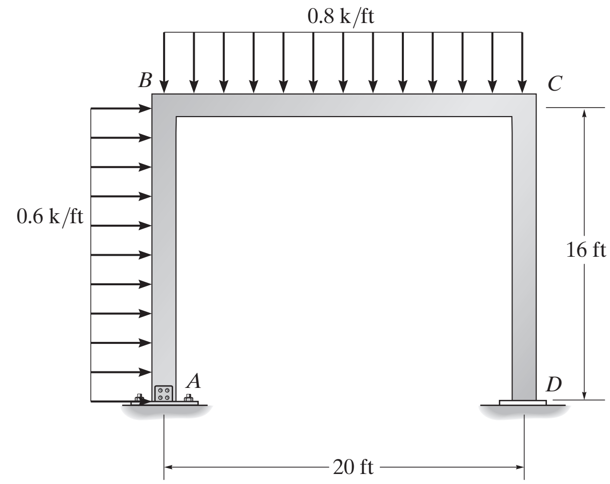






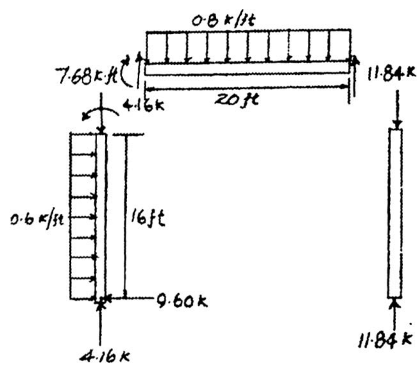


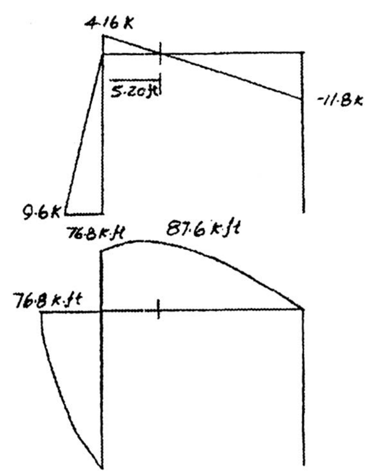
4–39. Draw the shear and moment diagrams for each member of the frame. Assume the support at A is a pin and D is a roller.



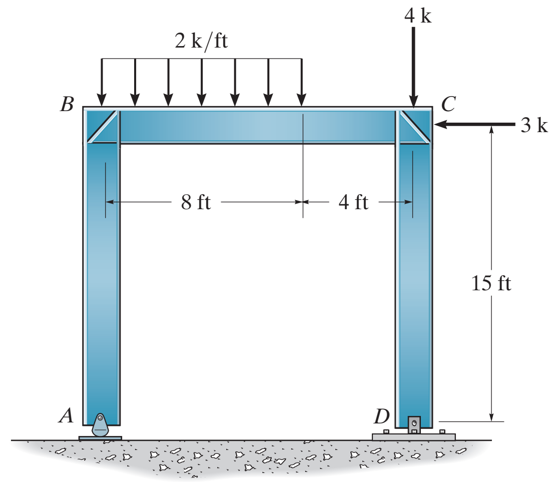
$V_{max} = -11.8$ k **Ans.**

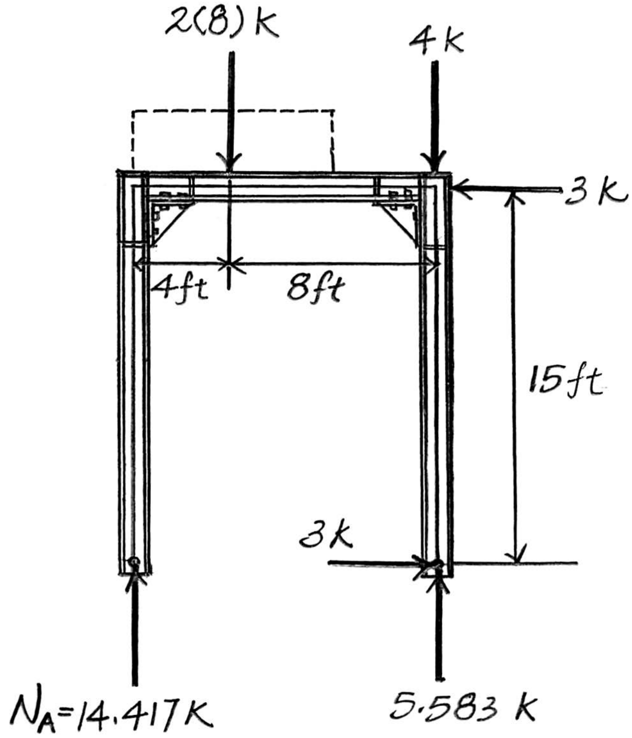
$M_{max} = -87.6$ k · ft **Ans.**

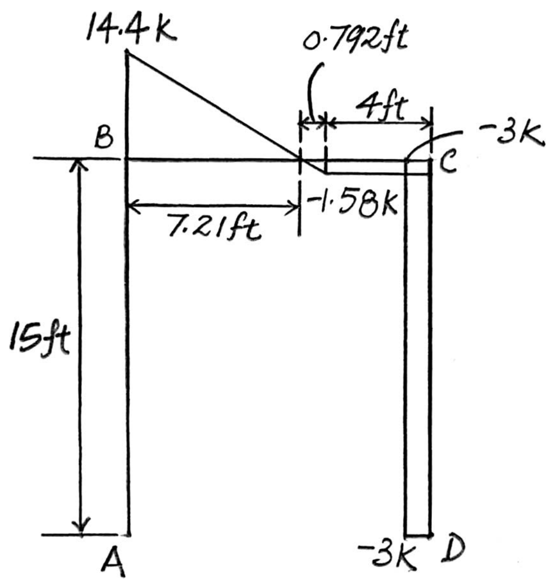


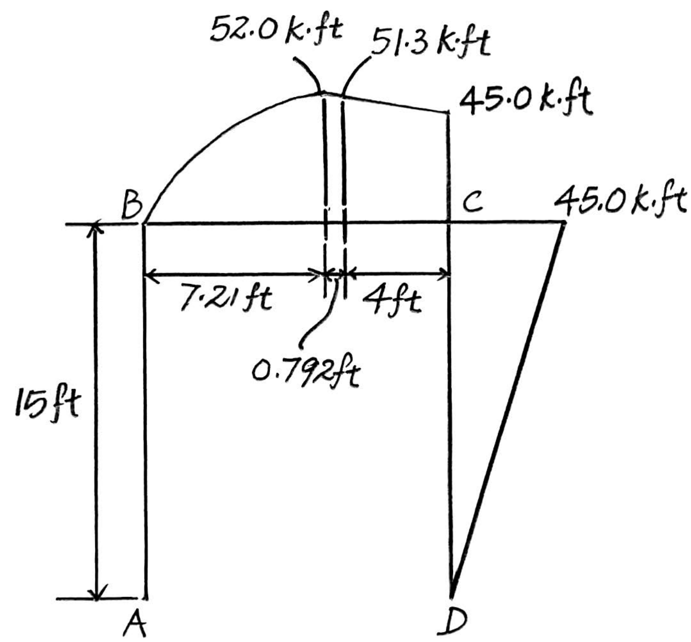


***4–40.** Draw the shear and moment diagrams for each member of the frame. Assume *A* is a rocker, and *D* is pinned.

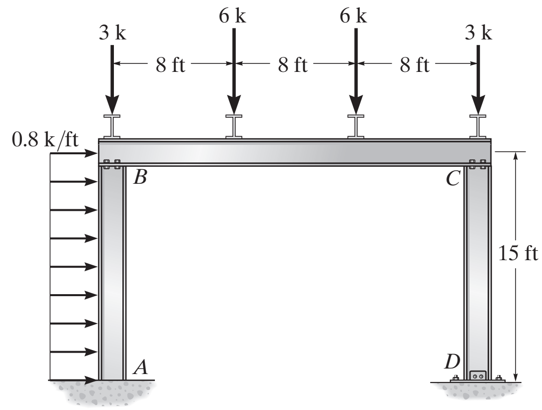


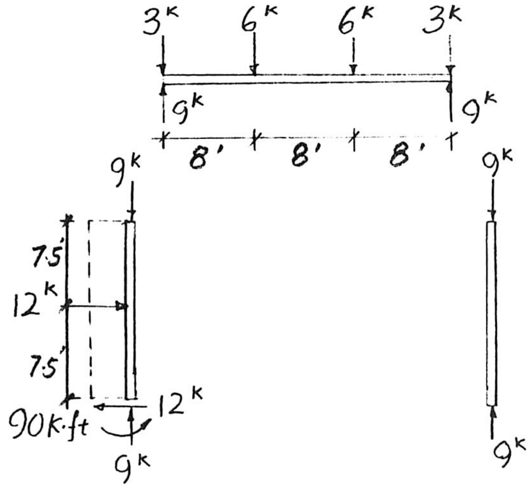


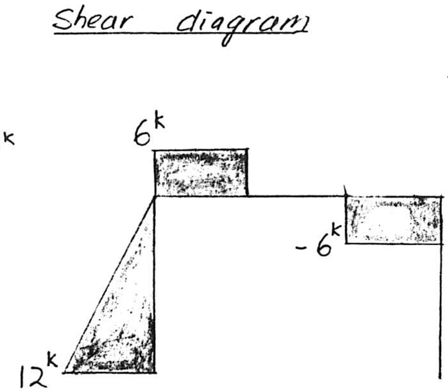


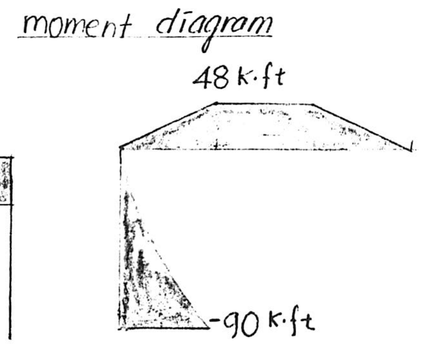


4–41. Draw the shear and moment diagrams for each member of the frame. Assume the frame is pin connected at B, C, and D and A is fixed.

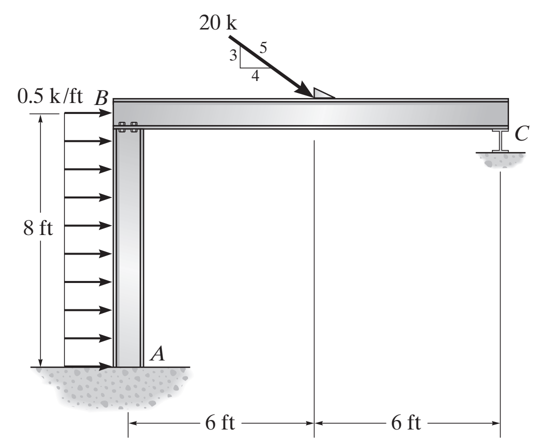






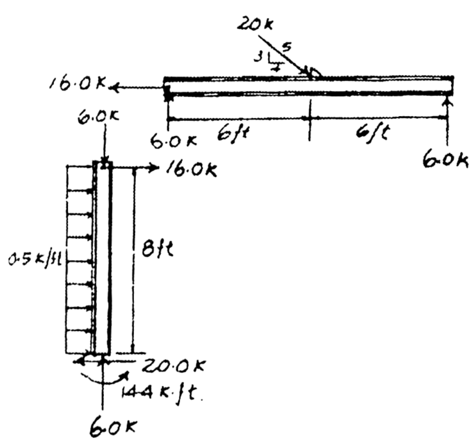


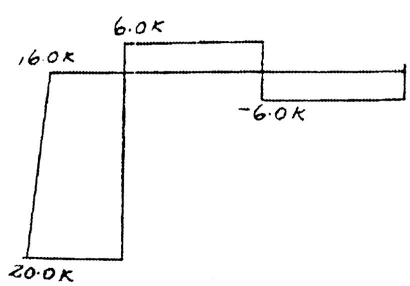
4–42. Draw the shear and moment diagrams for each member of the frame. Assume A is fixed, the joint at B is a pin, and support C is a roller.

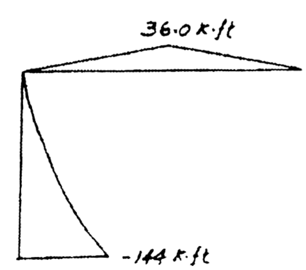


$V_{max} = 20.0$ k **Ans.**

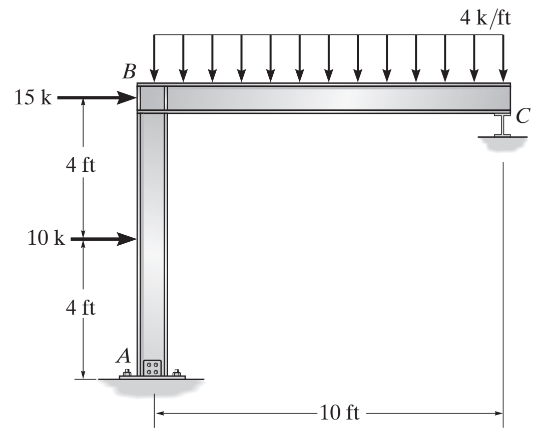
$M_{max} = -144$ k · ft **Ans.**

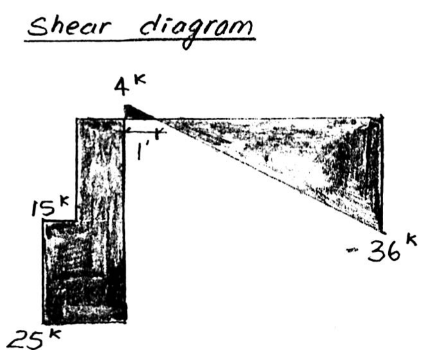


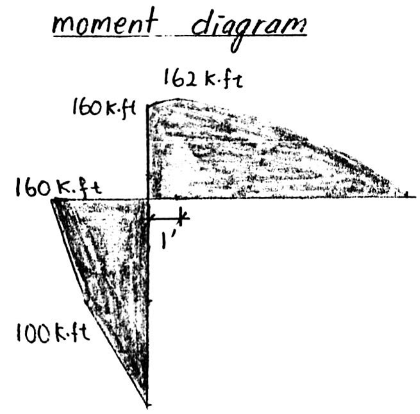


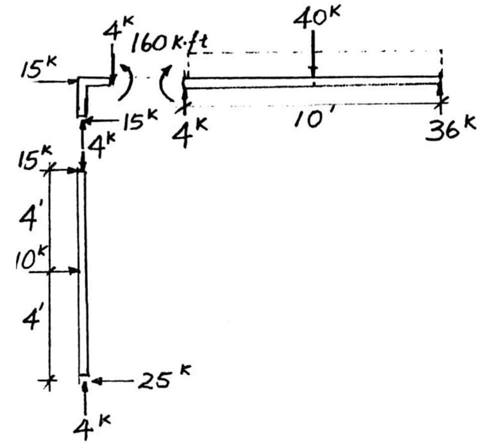


4–43. Draw the shear and moment diagrams for each member of the frame. Assume the frame is pin connected at A, and C is a roller.

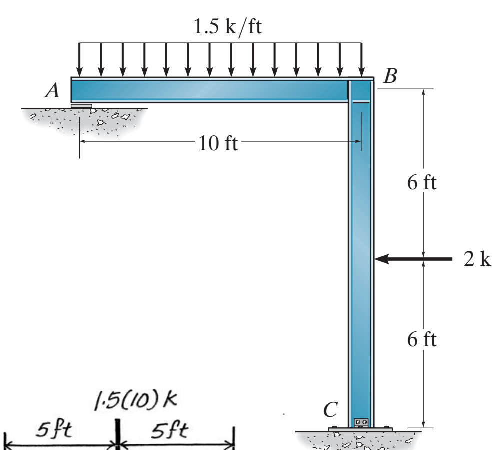


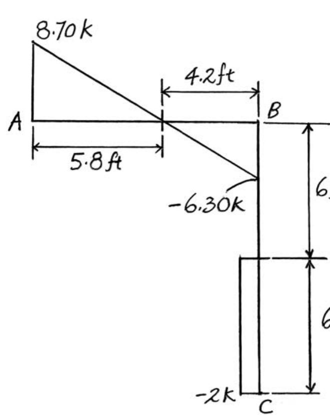


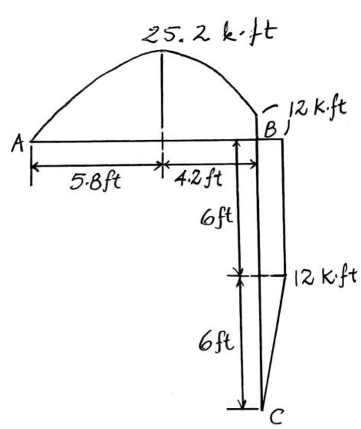


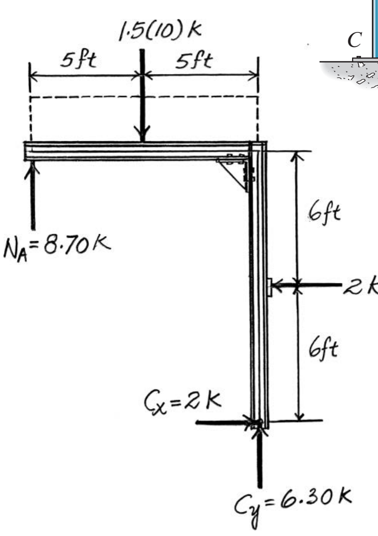


***4–44.** Draw the shear and moment diagrams for each member of the frame. Assume the frame is roller supported at A and pin supported at C.

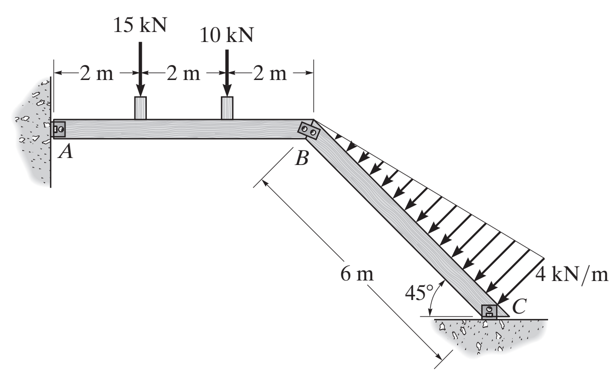








4–45. Draw the shear and moment diagrams for each member of the frame. The members are pin connected at A, B, and C.



Support Reactions:

$\zeta + \sum M_A = 0; \quad -15(2) - 10(4) + B_y(6) = 0$

$B_y = 11.667 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad A_y - 25 + 11.667 = 0$

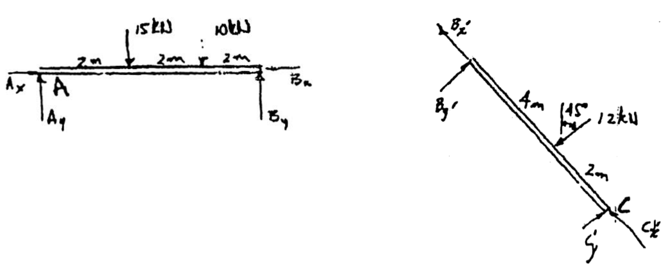
$A_y = 13.3 \text{ kN}$

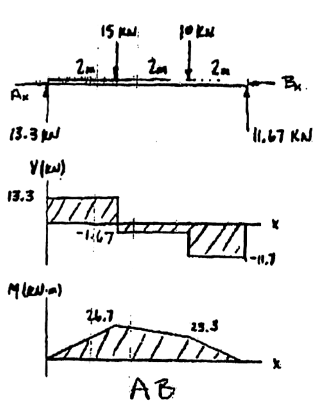
$\zeta + \sum M_c = 0; \quad 12(2) - B_{y'}(6) = 0$

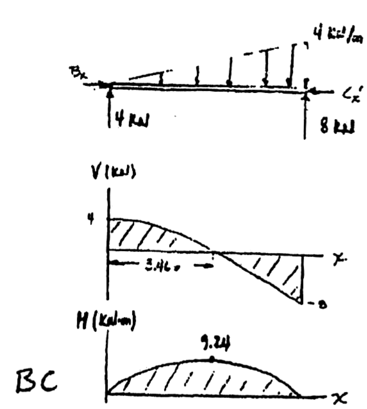
$B_{y'} = 4 \text{ kN}$

$+\nearrow \sum F_{y'} = 0; \quad 4 - 12 + C_{y'} = 0$

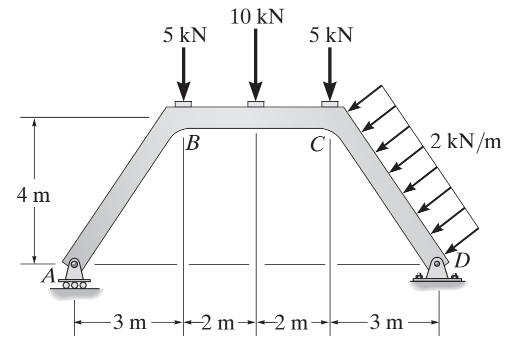
$C_{y'} = 8 \text{ kN}$







4–46. Draw the shear and moment diagrams for each member of the frame.



$\circlearrowleft + \sum M_D = 0; \quad 10(2.5) + 5(3) + 10(5) + 5(7) - A_y(10) = 0$

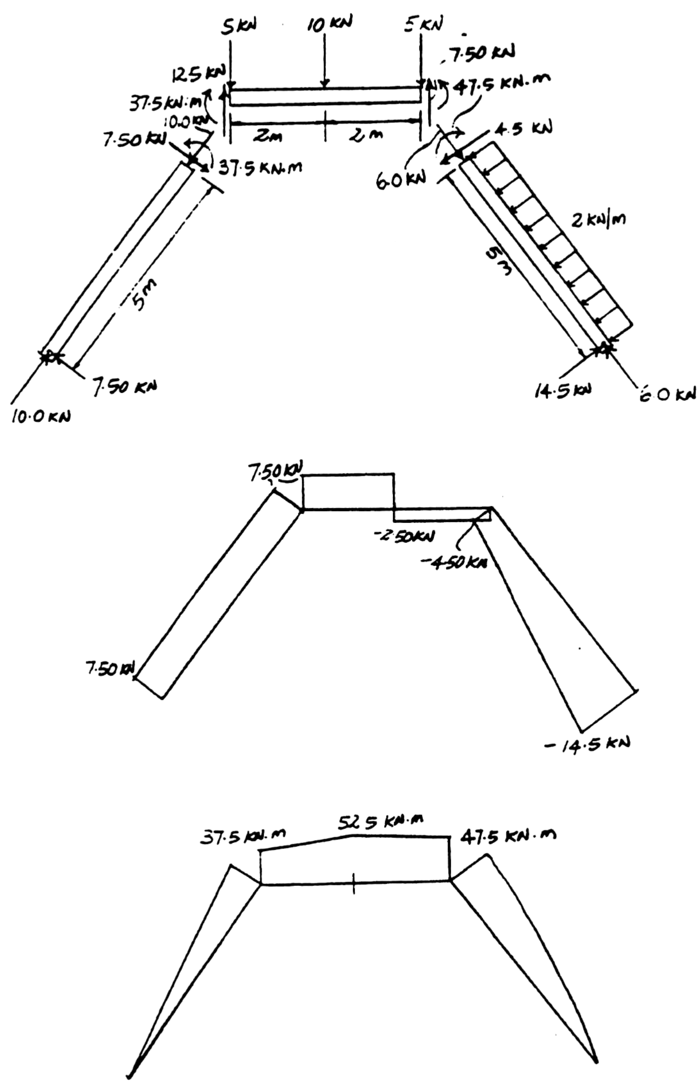
$A_y = 12.5 \text{ kN}$

$\xrightarrow{+} \sum F_x = 0; \quad -10\left(\frac{4}{5}\right) + D_x = 0$

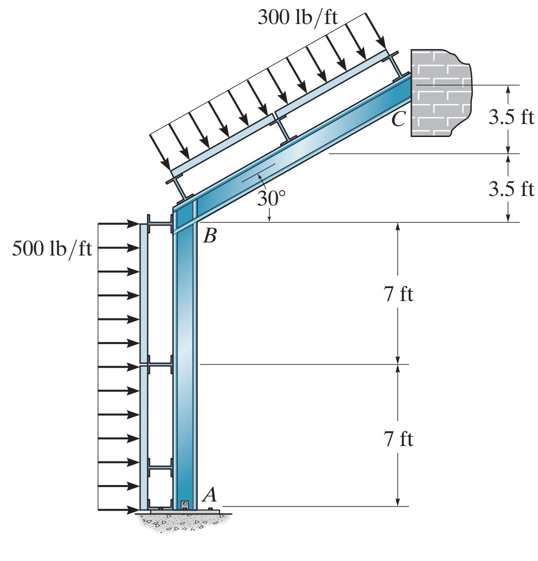
$D_x = 8 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad 12.5 - 5 - 10 - 5 - 10\left(\frac{3}{5}\right) + D_y = 0$

$D_y = 13.5 \text{ kN}$



4–47. Draw the shear and moment diagrams for each member of the frame. Assume the joint at A is a pin and support C is a roller. The joint at B is fixed. The wind load is transferred to the members at the girts and purlins from the simply supported wall and roof segments.



Support Reactions:

$$\circlearrowleft + \sum M_A = 0; \quad -3.5(7) - 1.75(14) - (4.20)(\sin 30°)(7 \cos 30°)$$

$$-4.20(\sin 30°)(14 + 3.5) + (21) = 0$$

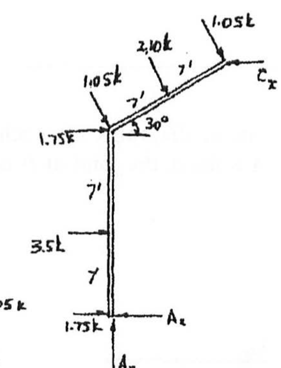
$$C_x = 5.133 \text{ kN}$$

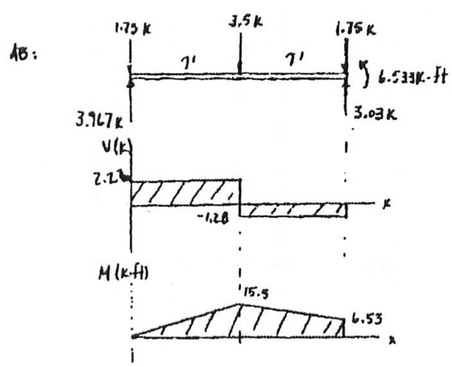
$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 1.75 + 3.5 + 1.75 + 4.20 \sin 30° - 5.133 - A_x = 0$$

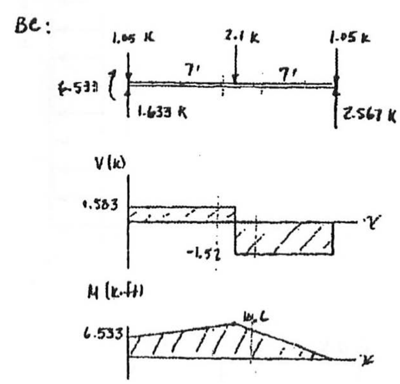
$$A_x = 3.967 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad A_y - 4.20 \cos 30° = 0$$

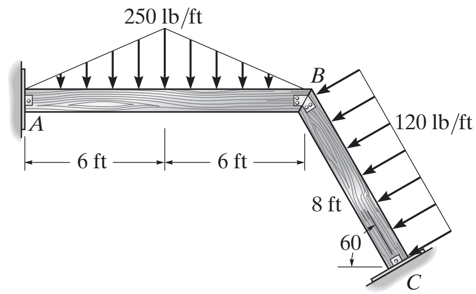
$$A_y = 3.64 \text{ kN}$$

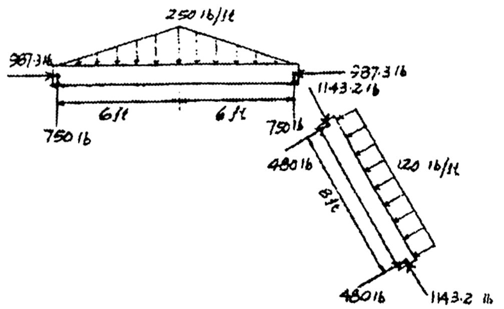


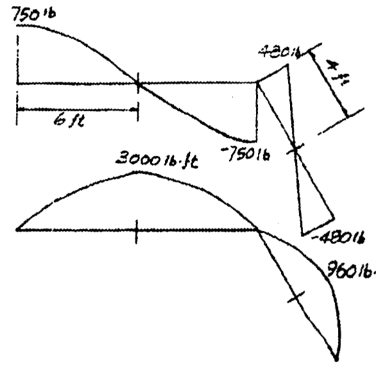




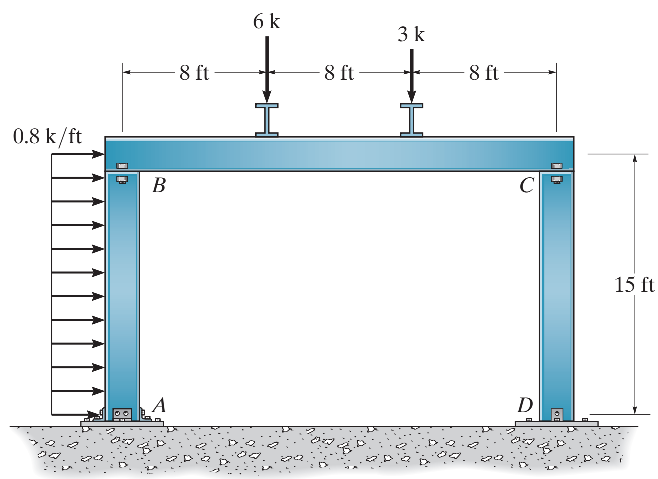
***4–48.** Draw the shear and moment diagrams for each member of the frame. The joints at *A*, *B* and *C* are pin connected.

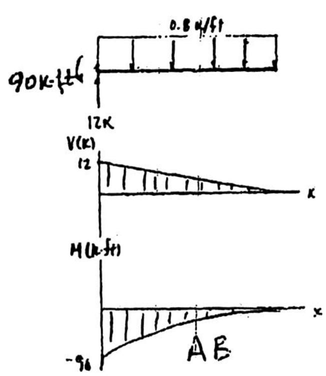


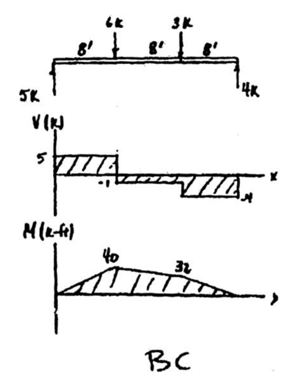


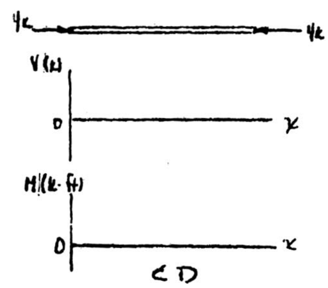


4–49. Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at *B*, *C* and *D* and *A* is fixed.

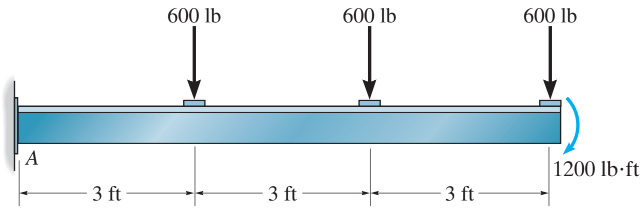


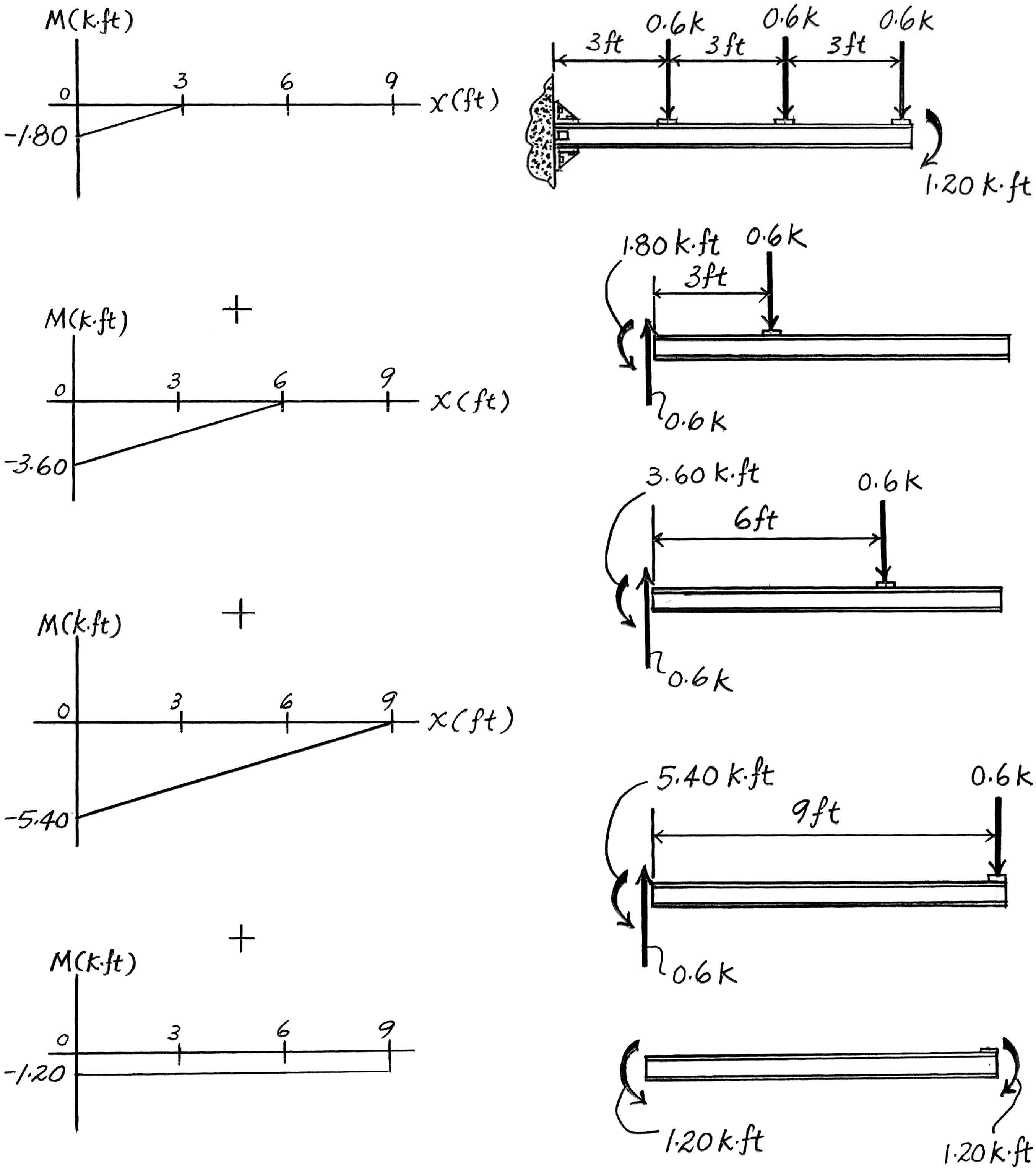




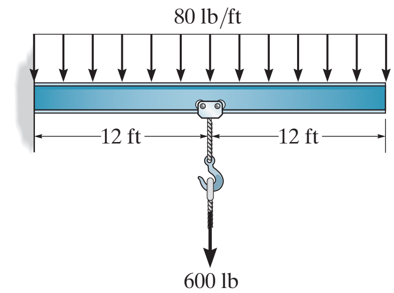


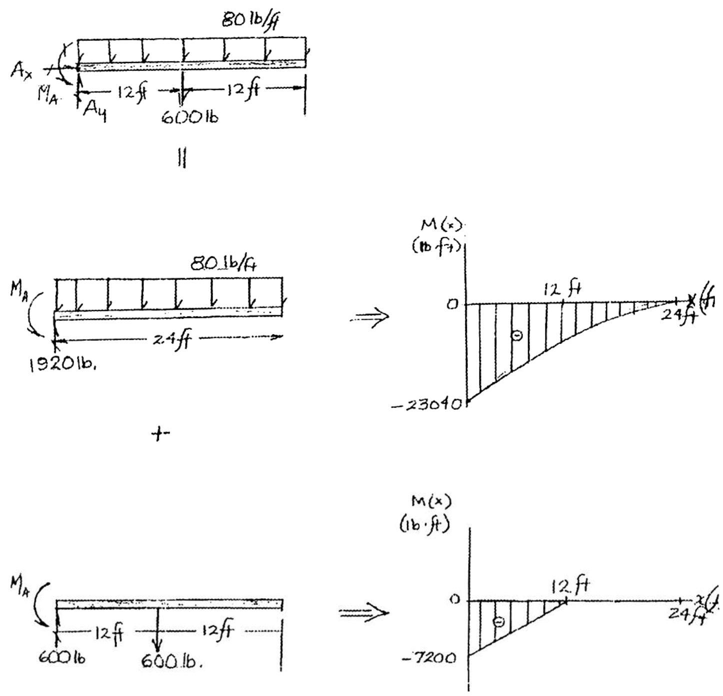
4–50. Draw the moment diagrams for the beam using the method of superposition. The beam is cantilevered from *A*.



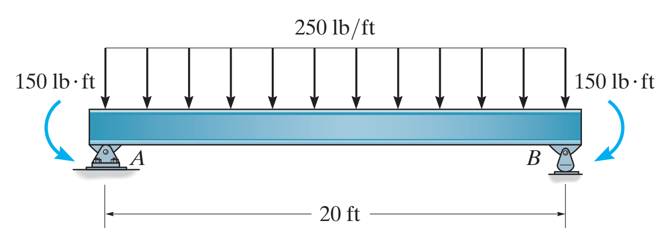


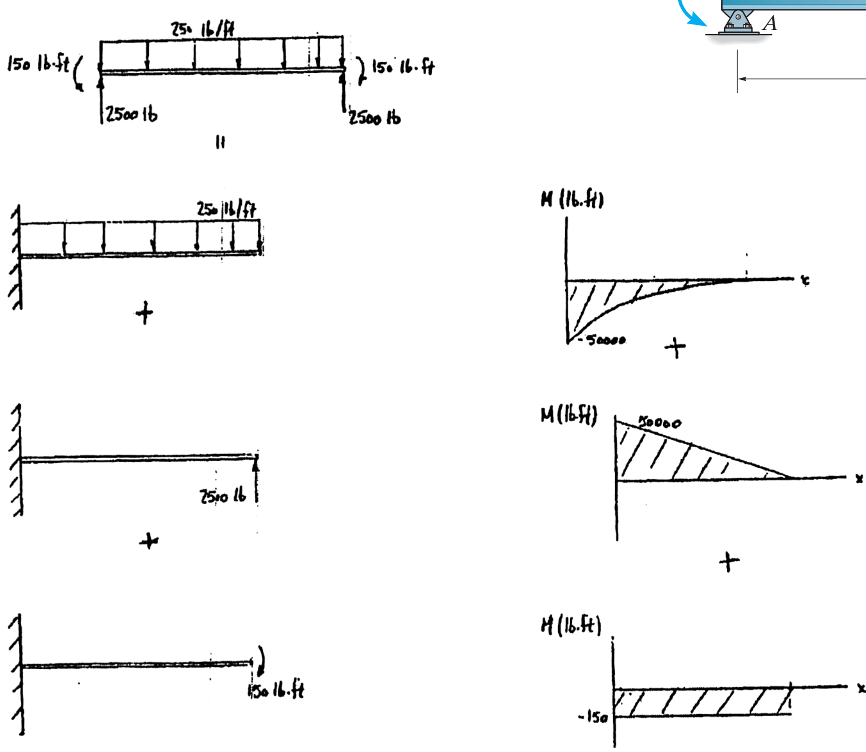
4–51. Draw the moment diagrams for the beam using the method of superposition.



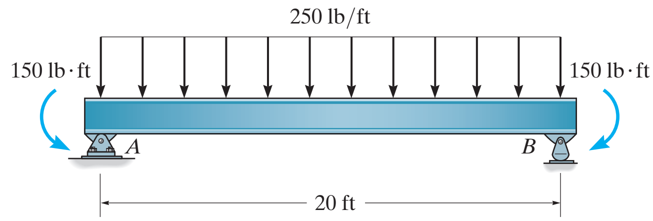


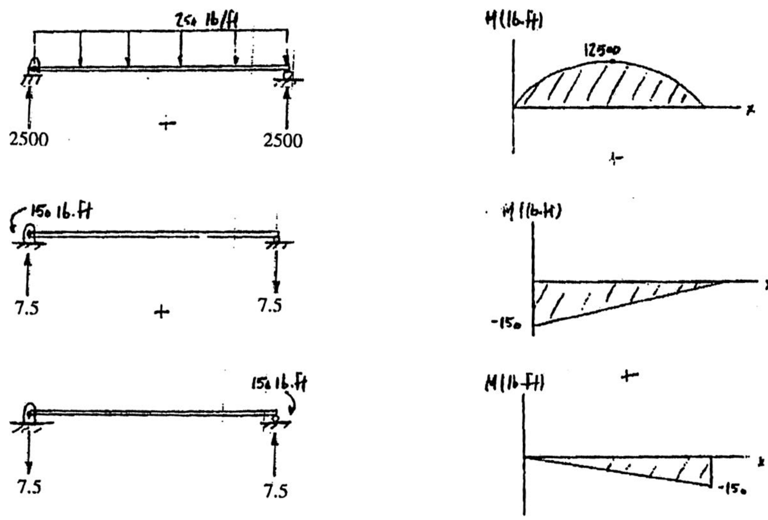
***4-52.** Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from end A.



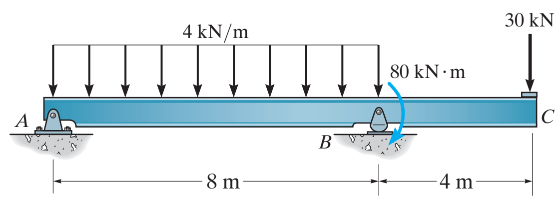


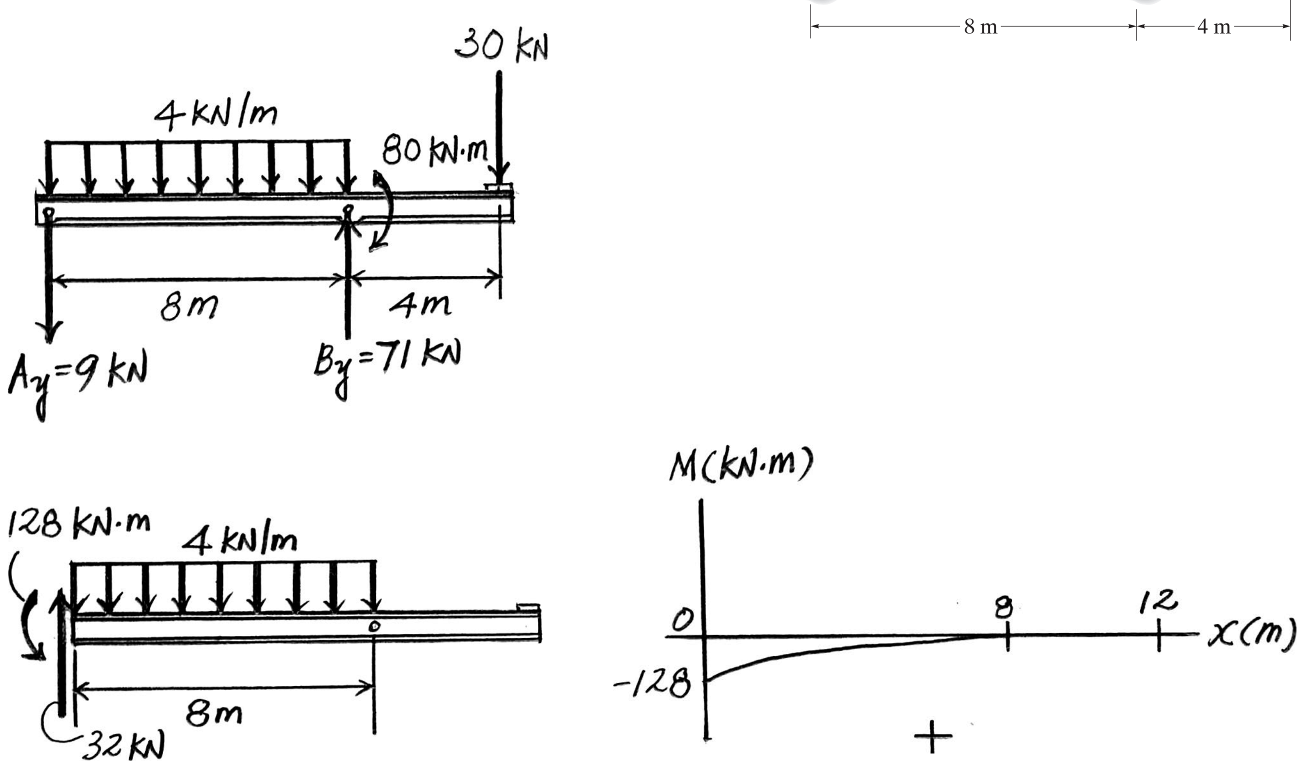
4–53. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be simply supported at *A* and *B* as shown.



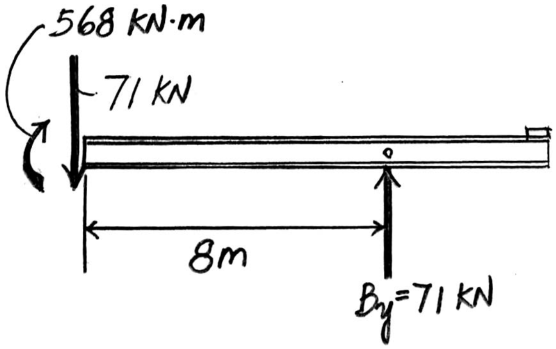


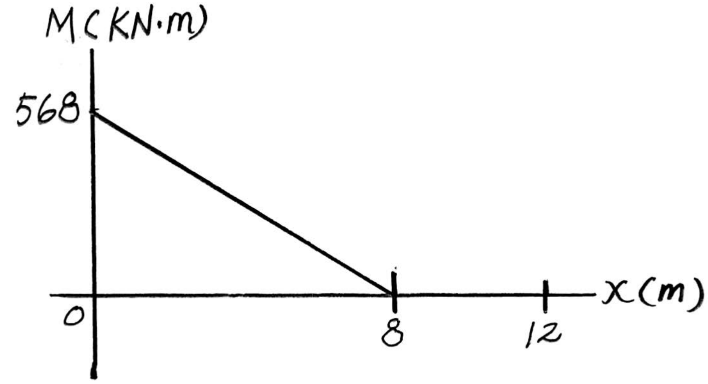
4–54. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from the pin support at *A*.





4–54. Continued





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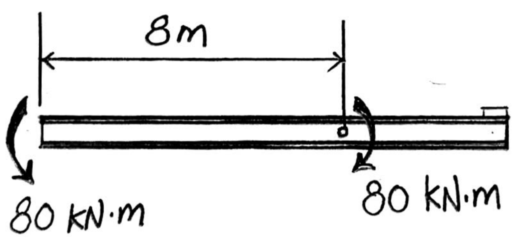

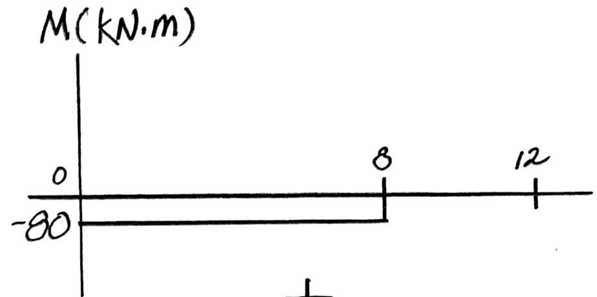

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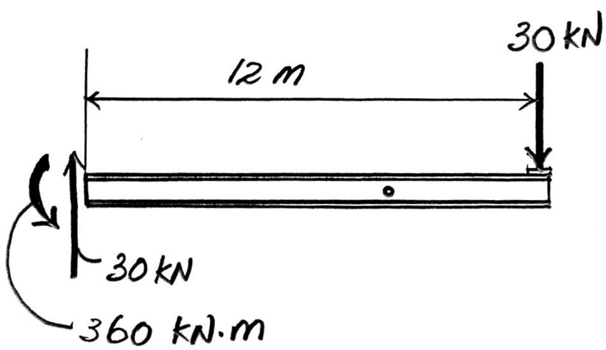
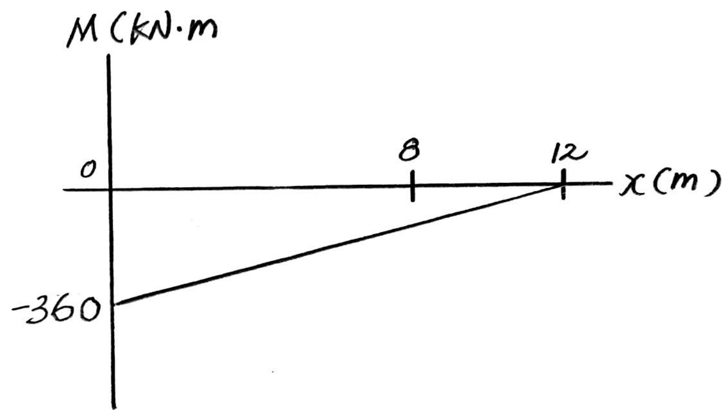
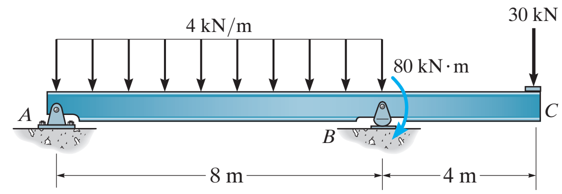
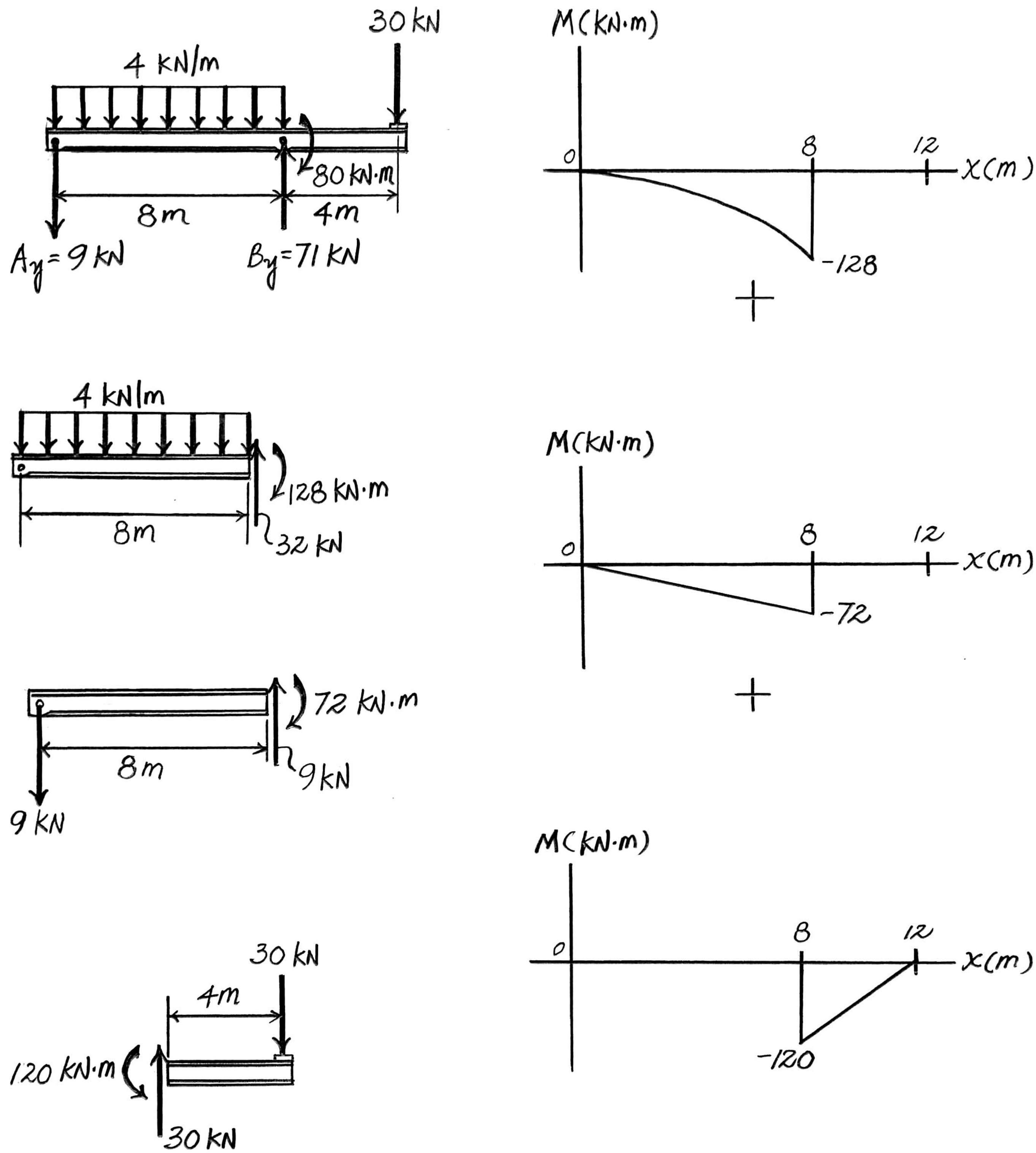


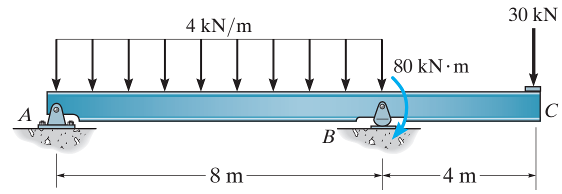


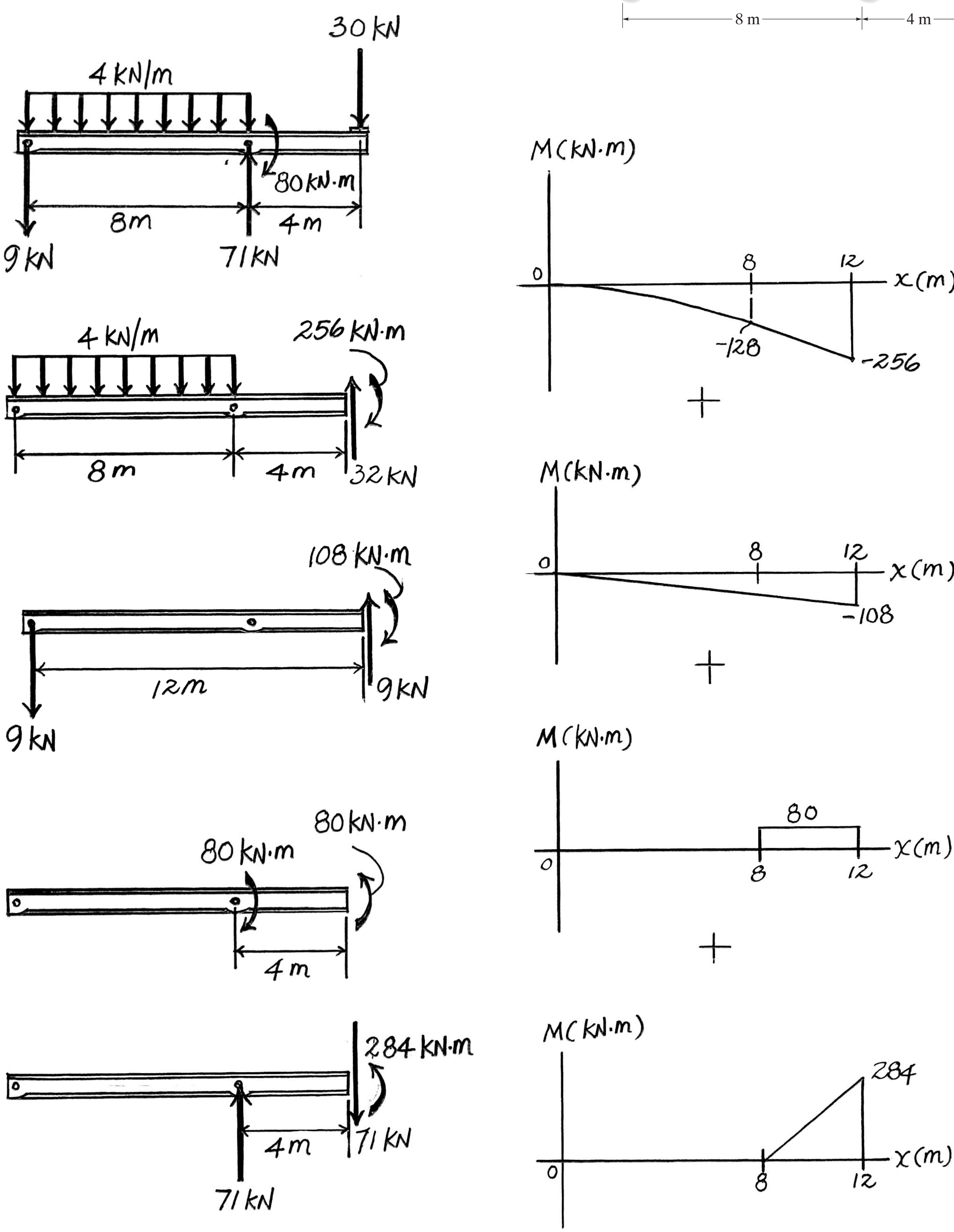
4–55. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from the rocker at B.



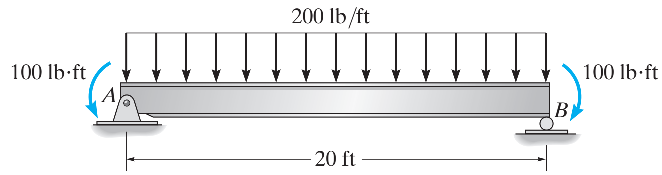


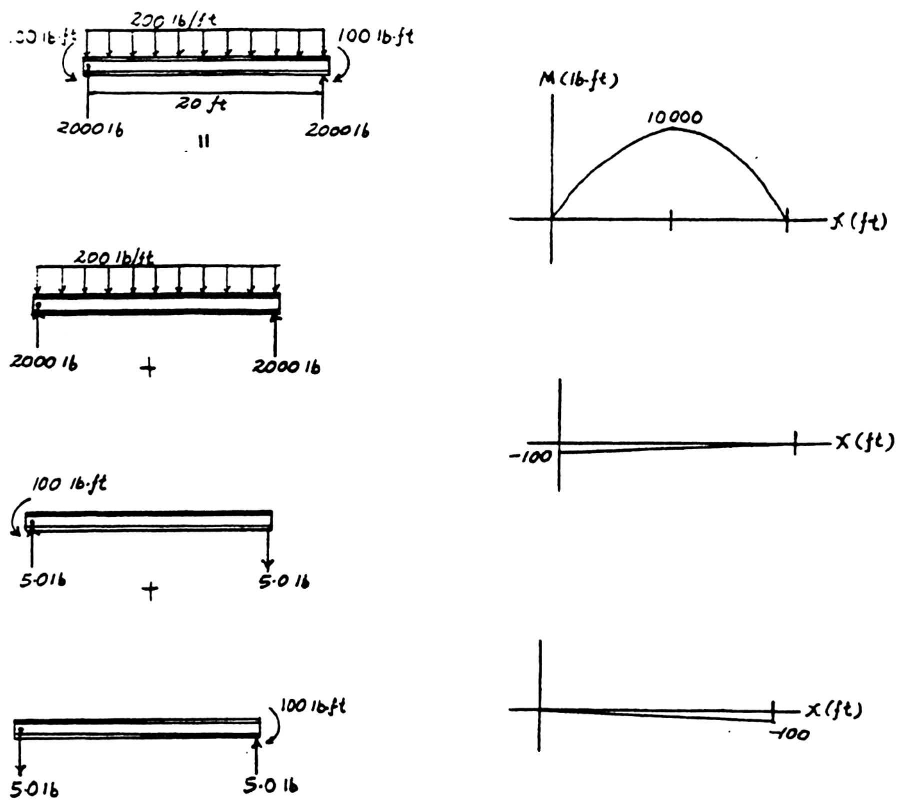
***4–56.** Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from end *C*.



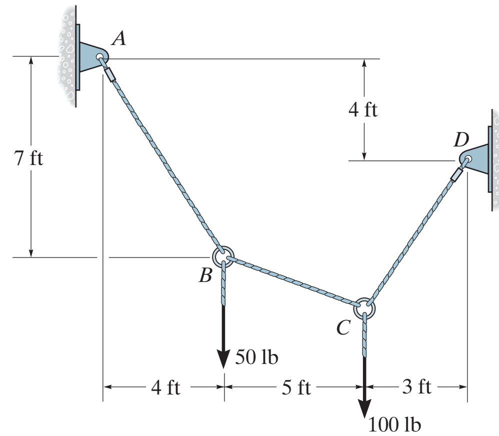


4–57. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be simply supported at A and B as shown.





5–1. Determine the tension in each segment of the cable and the cable's total length.

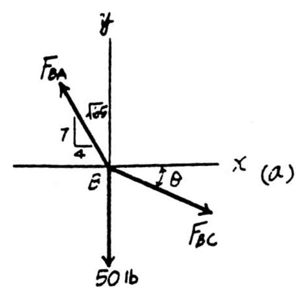


Equations of Equilibrium: Applying method of joints, we have

Joint *B*:

$$\xrightarrow{+} \sum F_x = 0; \qquad F_{BC}\cos\theta - F_{BA}\left(\frac{4}{\sqrt{65}}\right) = 0 \qquad [1]$$

$$+\uparrow \sum F_y = 0; \qquad F_{BA}\left(\frac{7}{\sqrt{65}}\right) - F_{BC}\sin\theta - 50 = 0 \qquad [2]$$



Joint *C*:

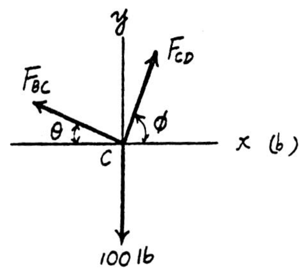
$$\xrightarrow{+} \sum F_x = 0; \qquad F_{CD}\cos\phi - F_{BC}\cos\theta = 0 \qquad [3]$$

$$+\uparrow \sum F_y = 0; \qquad F_{BC}\sin\theta + F_{CD}\sin\phi - 100 = 0 \qquad [4]$$

Geometry:

$$\sin\theta = \frac{y}{\sqrt{y^2 + 25}} \qquad \cos\theta = \frac{5}{\sqrt{y^2 + 25}}$$

$$\sin\phi = \frac{3 + y}{\sqrt{y^2 + 6y + 18}} \qquad \cos\phi = \frac{3}{\sqrt{y^2 + 6y + 18}}$$



Substitute the above results into Eqs. [1], [2], [3] and [4] and solve. We have

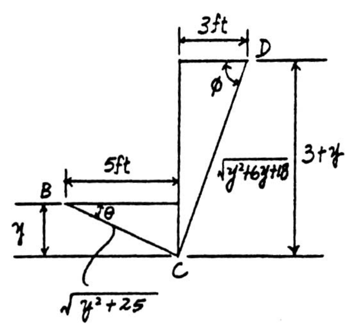
$F_{BC} = 46.7$ lb $\quad F_{BA} = 83.0$ lb $\quad F_{CD} = 88.1$ lb **Ans.**

$y = 2.679$ ft

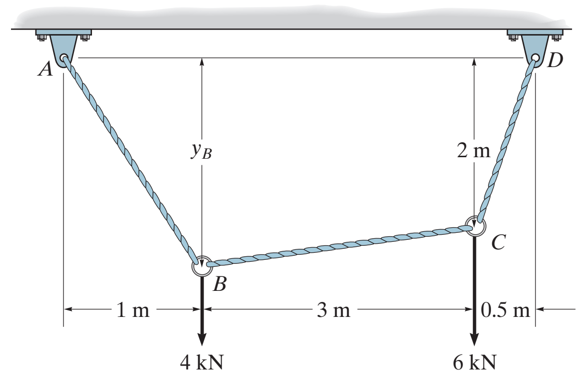
The total length of the cable is

$$l = \sqrt{7^2 + 4^2} + \sqrt{5^2 + 2.679^2} + \sqrt{3^2 + (2.679 + 3)^2}$$

$$= 20.2 \text{ ft}$$ **Ans.**



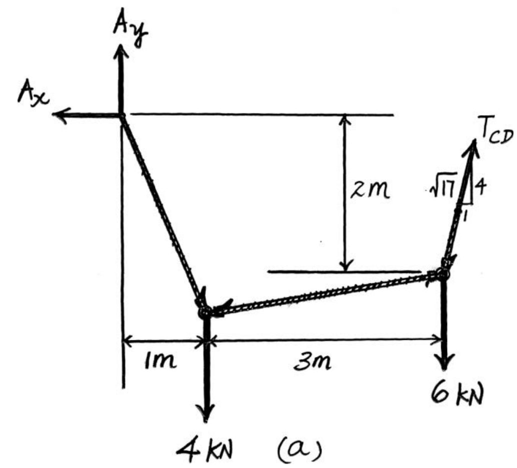
5–2. Cable *ABCD* supports the loading shown. Determine the maximum tension in the cable and the sag of point *B*.



Referring to the FBD in Fig. *a*,

$$\circlearrowleft + \sum M_A = 0; \quad T_{CD}\left(\frac{4}{\sqrt{17}}\right)(4) + T_{CD}\left(\frac{1}{\sqrt{17}}\right)(2) - 6(4) - 4(1) = 0$$

$$T_{CD} = 6.414 \text{ kN} = 6.41 \text{ kN(Max)}$$ **Ans.**



Joint *C*: Referring to the FBD in Fig. *b*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 6.414\left(\frac{1}{\sqrt{17}}\right) - T_{BC}\cos\theta = 0$$

$$+\uparrow \sum F_y = 0; \quad 6.414\left(\frac{4}{\sqrt{17}}\right) - 6 - T_{BC}\sin\theta = 0$$

Solving,

$$T_{BC} = 1.571 \text{ kN} = 1.57 \text{ kN} \qquad (<T_{CD})$$

$$\theta = 8.130°$$

Joint *B*: Referring to the FBD in Fig. *c*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 1.571\cos 8.130° - T_{AB}\cos\phi = 0$$

$$+\uparrow \sum F_y = 0; \quad T_{AB}\sin\phi + 1.571\sin 8.130° - 4 = 0$$

Solving,

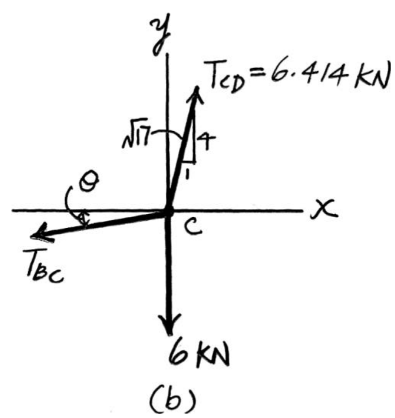
$$T_{AB} = 4.086 \text{ kN} = 4.09 \text{ kN} \qquad (<T_{CD})$$

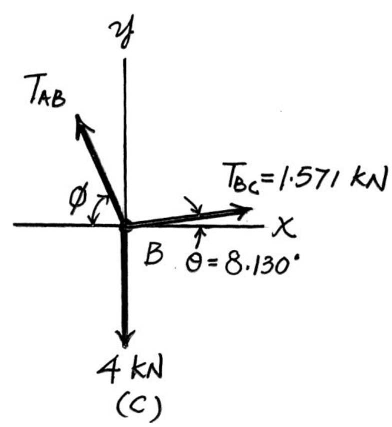
$$\phi = 67.62°$$

Then, from the geometry,

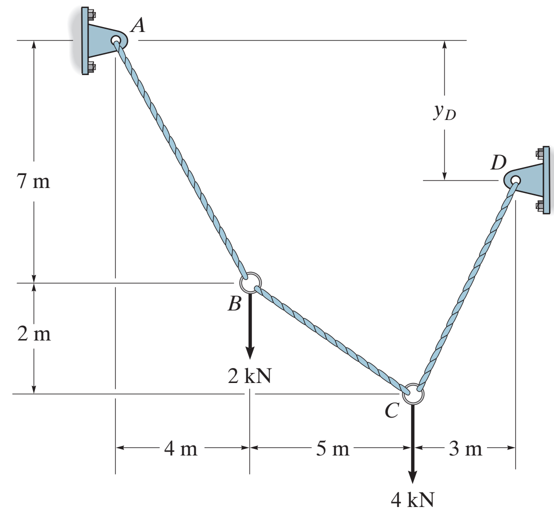
$$\frac{y_B}{1} = \tan\phi; \qquad y_B = 1\ \tan 67.62°$$

$$= 2.429 \text{ m} = 2.43 \text{ m}$$ **Ans.**





5–3. Determine the tension in each cable segment and the distance y_D.



Joint *B*: Referring to the FBD in Fig. *a*,

$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad T_{BC}\left(\frac{5}{\sqrt{29}}\right) - T_{AB}\left(\frac{4}{\sqrt{65}}\right) = 0$$

$$+\uparrow\sum F_y = 0; \qquad T_{AB}\left(\frac{7}{\sqrt{65}}\right) - T_{BC}\left(\frac{2}{\sqrt{29}}\right) - 2 = 0$$

Solving,

$$T_{AB} = 2.986 \text{ kN} = 2.99 \text{ kN} \qquad T_{BC} = 1.596 \text{ kN} = 1.60 \text{ kN} \qquad \textbf{Ans.}$$

Joint *C*: Referring to the FBD in Fig. *b*,

$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad T_{CD}\cos\theta - 1.596\left(\frac{5}{\sqrt{29}}\right) = 0$$

$$+\uparrow\sum F_y = 0; \qquad T_{CD}\sin\theta + 1.596\left(\frac{2}{\sqrt{29}}\right) - 4 = 0$$

Solving,

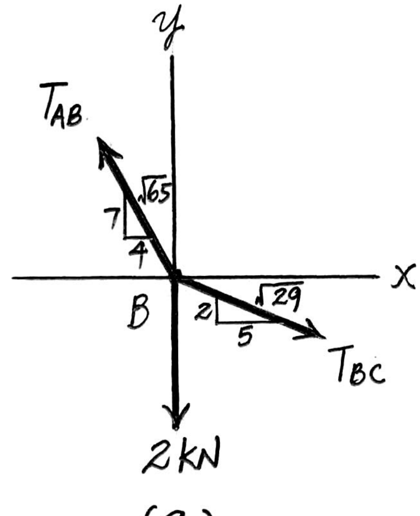
$$T_{CD} = 3.716 \text{ kN} = 3.72 \text{ kN} \qquad \textbf{Ans.}$$

$$\theta = 66.50^\circ$$

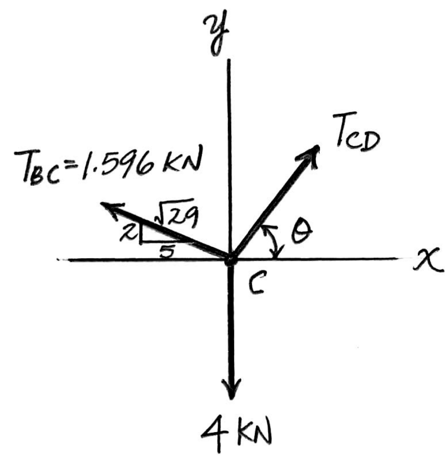
From the geometry,

$$y_D + 3\tan\theta = 9$$

$$y_D = 9 - 3\tan 66.50^\circ = 2.10 \text{ m} \qquad \textbf{Ans.}$$

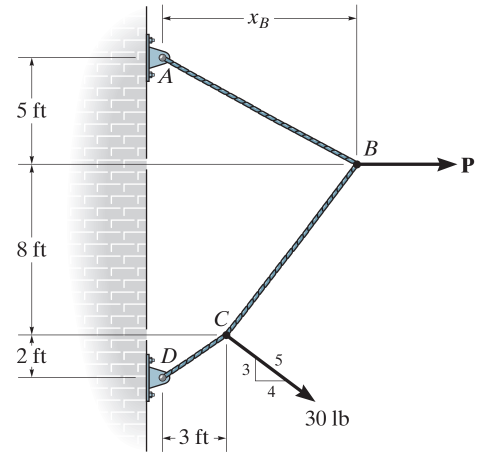


(a)



(b)

***5–4.** The cable supports the loading shown. Determine the distance x_B the force at point B acts from A. Set $P = 40$ lb.

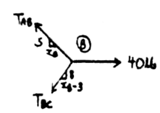


At B

$$\xrightarrow{+} \sum F_x = 0; \qquad 40 - \frac{x_B}{\sqrt{x_B^2 + 25}} T_{AB} - \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{5}{\sqrt{x_B^2 + 25}} T_{AB} - \frac{8}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 0$$

$$\frac{13x_B - 15}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 200 \qquad (1)$$

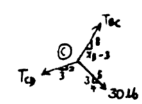


At C

$$\xrightarrow{+} \sum F_x = 0; \qquad \frac{4}{5}(30) + \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} - \frac{3}{\sqrt{13}} T_{CD} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{8}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} + \frac{2}{\sqrt{13}} T_{CD} - \frac{3}{5}(30) = 0$$

$$\frac{30 - 2x_B}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 102 \qquad (2)$$

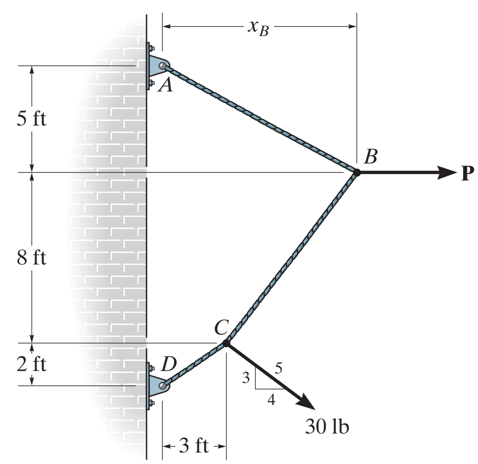


Solving Eqs. (1) & (2)

$$\frac{13x_B - 15}{30 - 2x_B} = \frac{200}{102}$$

$x_B = 4.36$ ft **Ans.**

5–5. The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.



At B

$$\xrightarrow{+} \sum F_x = 0; \qquad P - \frac{6}{\sqrt{61}} T_{AB} - \frac{3}{\sqrt{73}} T_{BC} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{5}{\sqrt{61}} T_{AB} - \frac{8}{\sqrt{73}} T_{BC} = 0$$

$$5P - \frac{63}{\sqrt{73}} T_{BC} = 0 \qquad (1)$$

At C

$$\xrightarrow{+} \sum F_x = 0; \qquad \frac{4}{5}(30) + \frac{3}{\sqrt{73}} T_{BC} - \frac{3}{\sqrt{13}} T_{CD} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{8}{\sqrt{73}} T_{BC} - \frac{2}{\sqrt{13}} T_{CD} - \frac{3}{5}(30) = 0$$

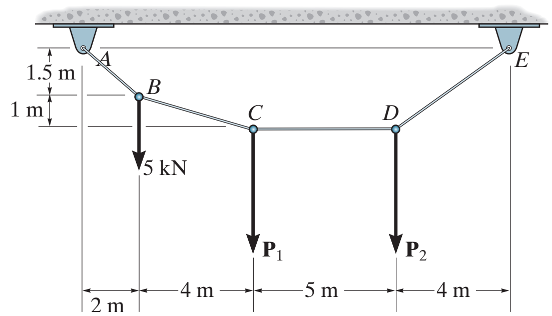
$$\frac{18}{\sqrt{73}} T_{BC} = 102 \qquad (2)$$

Solving Eqs. (1) & (2)

$$\frac{63}{18} = \frac{5P}{102}$$

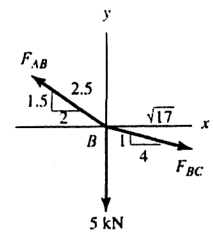
$P = 71.4$ lb **Ans.**

5–6. Determine the forces P_1 and P_2 needed to hold the cable in the position shown, i.e., so segment CD remains horizontal. Also find the maximum loading in the cable.



Method of Joints:

Joint *B*:



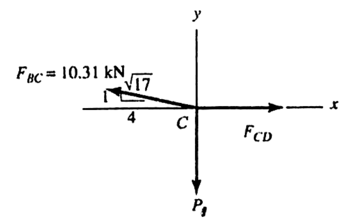
$\xrightarrow{+} \sum F_x = 0; \qquad F_{BC}\left(\frac{4}{\sqrt{17}}\right) - F_{AB}\left(\frac{2}{2.5}\right) = 0 \qquad [1]$

$+\uparrow \sum F_y = 0; \qquad F_{AB}\left(\frac{1.5}{2.5}\right) - F_{BC}\left(\frac{1}{\sqrt{17}}\right) - 5 = 0 \qquad [2]$

Solving Eqs. [1] and [2] yields

$F_{BC} = 10.31 \text{ kN} \qquad F_{AB} = 12.5 \text{ kN}$

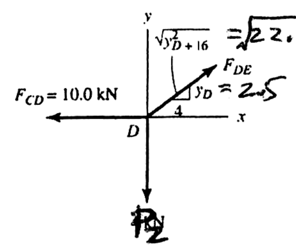
Joint *C*:



$\xrightarrow{+} \sum F_x = 0; \qquad F_{CD} - 10.31\left(\frac{4}{\sqrt{17}}\right) = 0 \quad F_{CD} = 10.00 \text{ kN}$

$+\uparrow \sum F_y = 0; \qquad 10.31\left(\frac{1}{\sqrt{17}}\right) - P_1 = 0 \quad P_1 = 2.50 \text{ kN}$ **Ans.**

Joint *D*:



$\xrightarrow{+} \sum F_x = 0; \qquad F_{DE}\left(\frac{4}{\sqrt{22.25}}\right) - 10 = 0 \qquad [1]$

$+\uparrow \sum F_y = 0; \qquad F_{DE}\left(\frac{25}{\sqrt{22.25}}\right) - P_2 = 0 \qquad [2]$

Solving Eqs. [1] and [2] yields

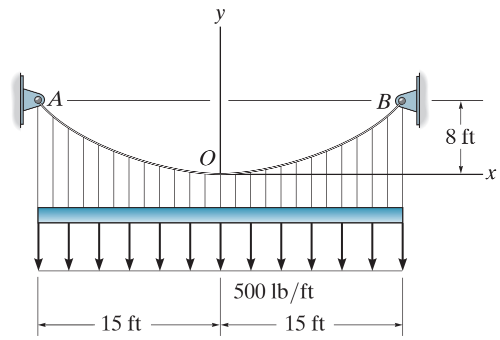
$P_2 = 6.25 \text{ kN}$ **Ans.**

$F_{DE} = 11.79 \text{ kN}$

Thus, the maximum tension in the cable is

$F_{max} = F_{AB} = 12.5 \text{ kN}$ **Ans.**

5–7. The cable is subjected to the uniform loading. If the slope of the cable at point O is zero, determine the equation of the curve and the force in the cable at O and B.



From Eq. 5–9.

$$y = \frac{h}{L^2}x^2 = \frac{8}{(15)^2}x^2$$

$$y = 0.0356x^2 \qquad \textbf{Ans.}$$

From Eq. 5–8

$$T_o = F_H = \frac{w_o L^2}{2h} = \frac{500(15)^2}{2(8)} = 7031.25 \text{ lb} = 7.03 \text{ k} \qquad \textbf{Ans.}$$

From Eq. 5–10.

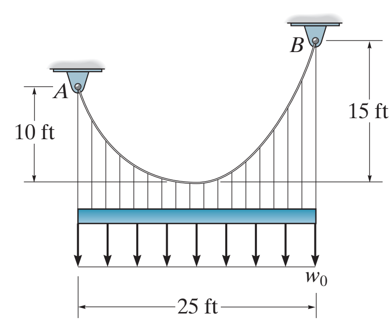
$$T_B = T_{\max} = \sqrt{(F_H)^2 + (w_o L)^2} = \sqrt{(7031.25)^2 + [(500)(15)]^2}$$

$$= 10\,280.5 \text{ lb} = 10.3 \text{ k} \qquad \textbf{Ans.}$$

Also, from Eq. 5–11

$$T_B = T_{\max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2} = 500(15)\sqrt{1 + \left(\frac{15}{2(8)}\right)^2} = 10\,280.5 \text{ lb} = 10.3 \text{ k} \qquad \textbf{Ans.}$$

***5–8.** The cable supports the uniform load of $w_0 = 600$ lb/ft. Determine the tension in the cable at each support A and B.



$$y = \frac{w_o}{2\,F_H}x^2$$

$$15 = \frac{600}{2\,F_H}x^2$$

$$10 = \frac{600}{2\,F_H}(25 - x)^2$$

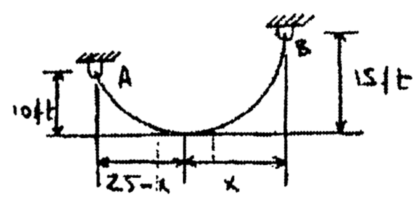
$$\frac{600}{2(15)}x^3 = \frac{600}{2(10)}(25 - x)^2$$

$$x^2 = 1.5(625 - 50x + x^2)$$

$$0.5x^2 - 75x + 937.50 = 0$$

Choose root < 25 ft

$$x = 13.76 \text{ ft}$$



$$F_H = \frac{w_o}{2y}x^2 = \frac{600}{2(15)}(13.76)^2 = 3788 \text{ lb}$$

5–8. Continued

At B:

$$y = \frac{w_o}{2\,F_H}\,x^2 = \frac{600}{2(3788)}\,x^2$$

$$\frac{dy}{dx} = \tan\theta_B = 0.15838\,x\Big|_{x=13.76} = 2.180$$

$$\theta_B = 65.36°$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{3788}{\cos\ 65.36°} = 9085\text{ lb} = 9.09\text{ kip}$$ **Ans.**

At A:

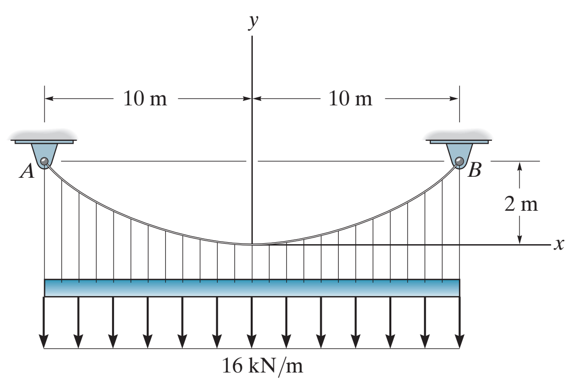
$$y = \frac{w_o}{2\,F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_A = 0.15838\,x\Big|_{x=(25-13.76)} = 1.780$$

$$\theta_{A=60.67°}$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{3788}{\cos 60.67°} = 7734\text{ lb} \ = 7.73\text{ kip}$$ **Ans.**

5–9. Determine the maximum and minimum tension in the cable.



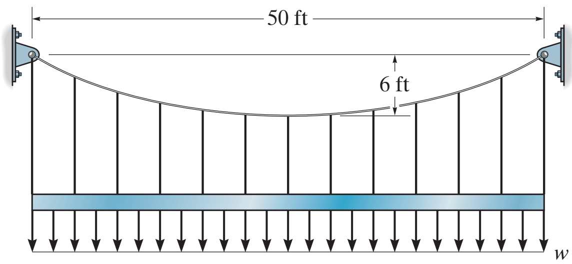
The minimum tension in the cable occurs when $\theta = 0°$. Thus, $T_{\min} = F_H$. With $w_o = 16$ kN/m, $L = 10$ m and $h = 2$ m,

$$T_{\min} = F_H = \frac{w_o L^2}{2\,h} = \frac{(16\text{ kN/m})(10\text{ m})^2}{2(2\text{ m})} = 400\text{ kN}$$ **Ans.**

And

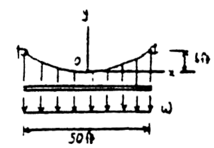
$$T_{\max} = \sqrt{F_{H^2} + (w_o L)^2}$$
$$= \sqrt{400^2 + [16(10)]^2}$$
$$= 430.81\text{ kN}$$
$$= 431\text{ kN}$$ **Ans.**

5–10. Determine the maximum uniform loading w, measured in lb/ft, that the cable can support if it is capable of sustaining a maximum tension of 3000 lb before it will break.



$$y = \frac{1}{F_H}\int\left(\int w dx\right)dx$$

At $x = 0$, $\quad \dfrac{dy}{dx} = 0$



At $x = 0$, $\quad y = 0$

$$C_1 = C_2 = 0$$

$$y = \frac{w}{2\,F_H}x^2$$

At $x = 25$ ft, $\quad y = 6$ ft $\qquad F_H = 52.08\,w$

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{w}{F_H}x\right|_{x=25\text{ ft}}$$

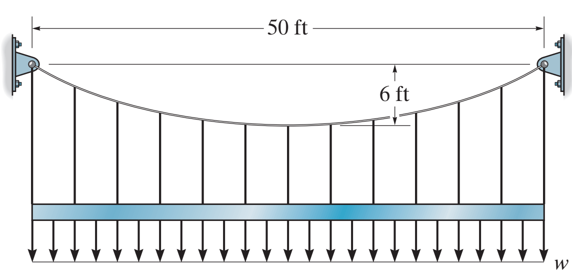
$$\theta_{max} = \tan^{-1}(0.48) = 25.64°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = 3000$$

$$F_H = 2705 \text{ lb}$$

$w = 51.9$ lb/ft **Ans.**

5–11. The cable is subjected to a uniform loading of $w = 250$ lb/ft. Determine the maximum and minimum tension in the cable.



$$F_H = \frac{w_o L^2}{8\,h} = \frac{250(50)^2}{8(6)} = 13\,021 \text{ lb}$$

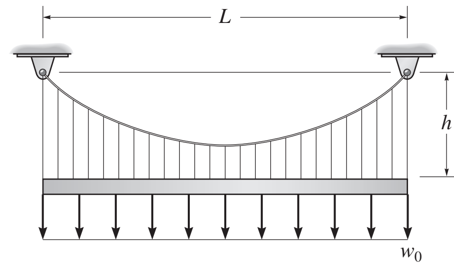
$$\theta_{max} = \tan^{-1}\left(\frac{w_o L}{2\,F_H}\right) = \tan^{-1}\left(\frac{250(50)}{2(13\,021)}\right) = 25.64°$$

$T_{max} = \dfrac{F_H}{\cos_{\theta_{max}}} = \dfrac{13\,021}{\cos 25.64°} = 14.4$ kip **Ans.**

The minimum tension occurs at $\theta = 0°$

$T_{min} = F_H = 13.0$ kip **Ans.**

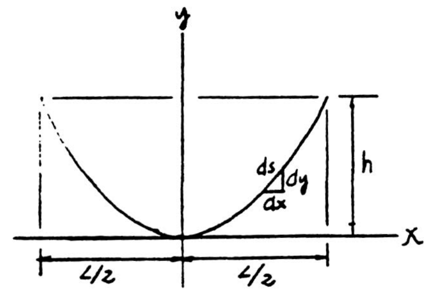
***5–12.** The cable shown is subjected to the uniform load w_0. Determine the ratio between the rise h and the span L that will result in using the minimum amount of material for the cable.



From Eq. 5–9,

$$y = \frac{h}{\left(\frac{L}{2}\right)^2}x^2 = \frac{4h}{L^2}x^2$$

$$\frac{dy}{dx} = \frac{8h}{L^2}x$$



From Eq. 5–8,

$$F_H = \frac{w_o\left(\frac{L}{2}\right)^2}{2h} = \frac{w_oL^2}{8h}$$

Since $\quad F_H = T\left(\frac{dx}{ds}\right), \quad$ then

$$T = \frac{w_oL^2}{8h}\left(\frac{ds}{dx}\right)$$

Let σ_{allow} be the allowable normal stress for the cable. Then

$$\frac{T}{A} = \sigma_{\text{allow}}$$

$$\frac{T}{\sigma_{\text{allow}}} = A$$

$$dV = A\,ds$$

$$dV = \frac{T}{\sigma_{\text{allow}}}ds$$

The volume of material is

$$V = \frac{2}{\sigma_{\text{allow}}}\int_0^{\frac{1}{2}} T\,ds = \frac{2}{\sigma_{\text{allow}}}\int_0^{\frac{1}{2}} \frac{w_oL^2}{8h}\left[\frac{(ds)^2}{dx}\right]$$

$$\frac{ds^2}{dx} = \frac{dx^2 + dy^2}{dx} = \left[\frac{dx^2 + dy^2}{dx^2}\right]dx = \left[1 + \left(\frac{dy}{dx}\right)^2\right]dx$$

$$= \int_0^{\frac{1}{2}} \frac{w_oL^2}{4h\sigma_{\text{allow}}}\left[1 + \left(\frac{dy}{dx}\right)^2\right]dx$$

$$= \frac{w_oL^2}{4h\sigma_{\text{allow}}}\int_0^{\frac{1}{2}}\left[1 + 64\left(\frac{h^2x^2}{L^4}\right)\right]dx$$

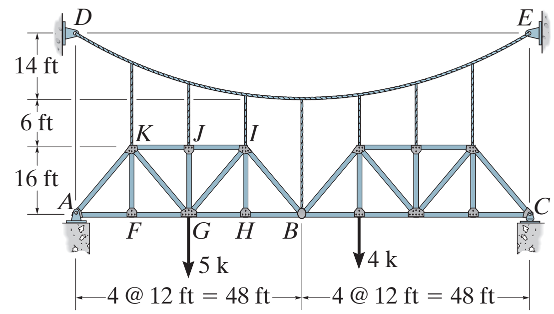
$$= \frac{w_oL^2}{4h\sigma_{\text{allow}}}\left[\frac{L}{2} + \frac{8h^2}{3L}\right] = \frac{w_oL^2}{8\,\sigma_{\text{allow}}}\left[\frac{L}{h} + \frac{16}{3}\left(\frac{h}{L}\right)\right]$$

Require,

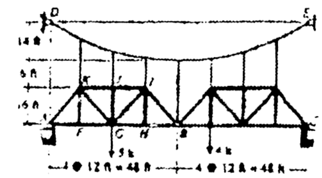
$$\frac{dV}{dh} = \frac{w_oL^2}{8\sigma_{\text{allow}}}\left[-\frac{L}{h^2} + \frac{16}{3L}\right] = 0$$

$$h = 0.433\,L \qquad \textbf{Ans.}$$

5–13. The trusses are pin connected and suspended from the parabolic cable. Determine the maximum force in the cable when the structure is subjected to the loading shown.



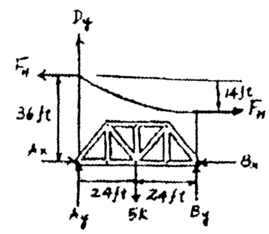
Entire structure:



$$\zeta + \sum M_C = 0; \quad 4(36) + 5(72) + F_H(36) - F_H(36) - (A_y + D_y(96) = 0$$

$$(A_y + D_y) = 5.25 \qquad (1)$$

Section *ABD*:



$$\zeta + \sum M_B = 0; \quad F_H(14) - (A_y + D_y)(48) + 5(24) = 0$$

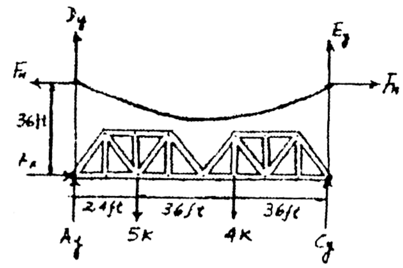
Using Eq. (1):

$$F_H = 9.42857 \text{ k}$$

From Eq. 5–8:

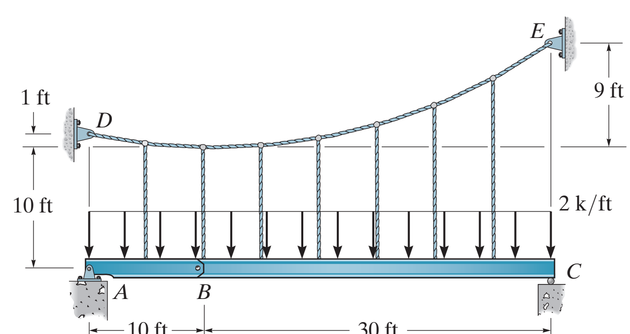
$$w_o = \frac{2F_H h}{L^2} = \frac{2(9.42857)(14)}{48^2} = 0.11458 \text{ k/ft}$$

From Eq. 5–11:



$$T_{\max} = w_o L \sqrt{1 + \left(\frac{L}{2h}\right)^2} = 0.11458(48)\sqrt{1 + \left[\frac{48}{2(14)}\right]^2} = 10.9 \text{ k} \qquad \textbf{Ans.}$$

5–14. Determine the maximum and minimum tension in the parabolic cable and the force in each of the hangers. The girder is subjected to the uniform load and is pin connected at *B*.



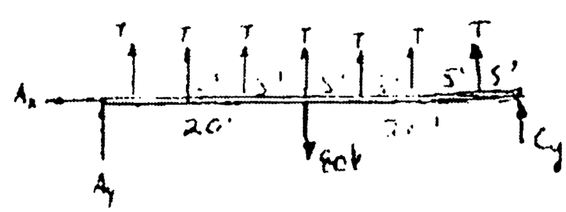
Member *BC*:

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad B_x = 0$$

Member *AB*:

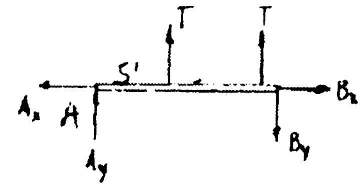
$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad A_x = 0$$

FBD 1:



$$\zeta + \sum M_A = 0; \qquad F_H(1) - B_y(10) - 20(5) = 0$$

5–14. Continued

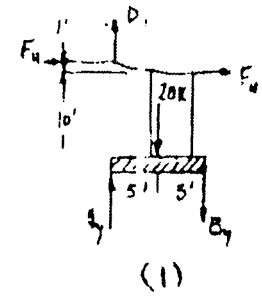


FBD 2:

$\zeta + \sum M_C = 0; \qquad -F_H(9) - B_y(30) + 60(15) = 0$

Solving,

$B_y = 0, \quad F_H = F_{min} = 100 \text{ k}$ **Ans.**



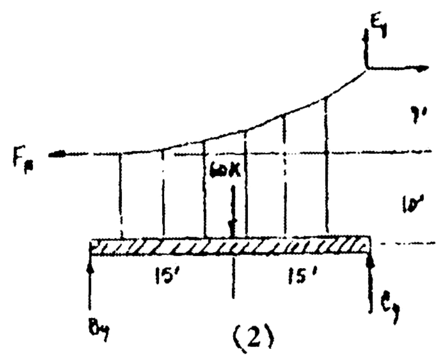
Max cable force occurs at E, where slope is the maximum.

From Eq. 5–8.

$$W_o = \frac{2F_H h}{L^2} = \frac{2(100)(9)}{30^2} = 2 \text{ k/ft}$$

From Eq. 5–11,

$$F_{max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2} = 2(30)\sqrt{1 + \left(\frac{30}{2(9)}\right)^2}$$

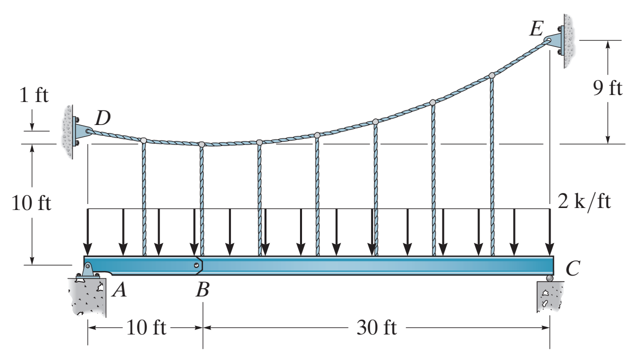


$F_{max} = 117 \text{ k}$ **Ans.**

Each hanger carries 5 ft of w_o.

$T = (2 \text{ k/ft})(5 \text{ ft}) = 10 \text{ k}$ **Ans.**

5–15. Draw the shear and moment diagrams for the pin-connected girders AB and BC. The cable has a parabolic shape.



$\zeta + \sum M_A = 0; \quad T(5) + T(10) + T(15) + T(20) + T(25)$

$+ T(30) + T(35) + C_y(40) - 80(20) = 0$

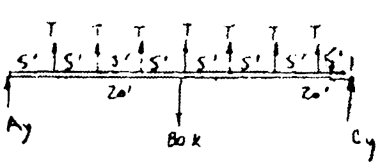
Set $T = 10$ k (See solution to Prob. 5–14)

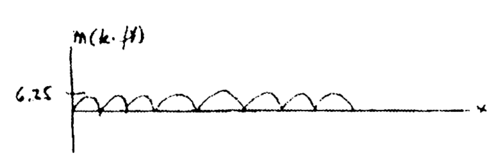
$C_y = 5 \text{ k}$

$+\uparrow \sum F_y = 0; \qquad 7(10) + 5 - 80 + A_y = 0$

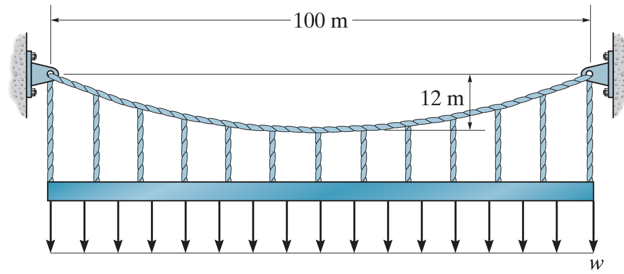
$A_y = 5 \text{ k}$

$M_{max} = 6.25 \text{ k} \cdot \text{ft}$ **Ans.**





***5–16.** The cable will break when the maximum tension reaches $T_{max} = 5000$ kN. Determine the maximum uniform distributed load w required to develop this maximum tension.



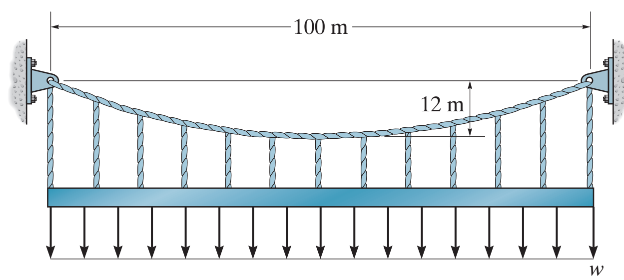
With $T_{max} = 80(10^3)$ kN, $L = 50$ m and $h = 12$ m,

$$T_{max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

$$8000 = w_o(50)\left[\sqrt{1 + \left(\frac{50}{24}\right)^2}\right]$$

$$w_o = 69.24 \text{ kN/m} = 69.2 \text{ kN/m}$$ **Ans.**

5–17. The cable is subjected to a uniform loading of $w = 60$ kN/m. Determine the maximum and minimum tension in cable.



The minimum tension in cable occurs when $\theta = 0°$. Thus, $T_{min} = F_H$.

$$T_{min} = F_H = \frac{w_o L^2}{2h} = \frac{(60 \text{ kN/m})(50 \text{ m})^2}{2(12 \text{ m})} = 6250 \text{ kN}$$

$$= 6.25 \text{ MN}$$ **Ans.**

And,

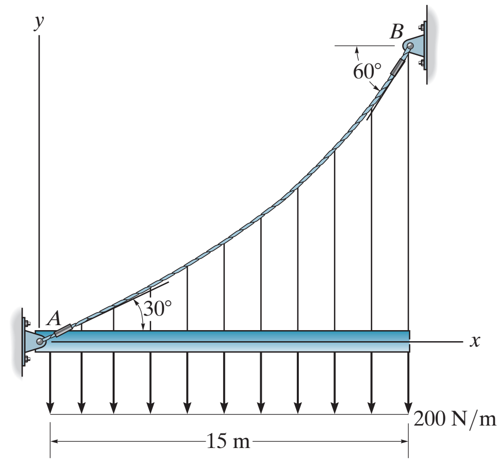
$$T_{max} = \sqrt{F_H^2 + (w_o L)^2}$$

$$= \sqrt{6250^2 + [60(50)]^2}$$

$$= 6932.71 \text{ kN}$$

$$= 6.93 \text{ MN}$$ **Ans.**

5–18. The cable AB is subjected to a uniform loading of 200 N/m. If the weight of the cable is neglected and the slope angles at points A and B are 30° and 60°, respectively, determine the curve that defines the cable shape and the maximum tension developed in the cable.



Here the boundary conditions are different from those in the text.

Integrate Eq. 5–2,

$T \sin\theta = 200x + C_1$

Divide by by Eq. 5–4, and use Eq. 5–3

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + C_1)$$

$$y = \frac{1}{F_H}(100x^2 + C_1 x + C_2)$$

At $x = 0$, $y = 0$; $C_2 = 0$

At $x = 0$, $\frac{dy}{dx} = \tan 30°$; $C_1 = F_H \tan 30°$

$$y = \frac{1}{F_H}(100x^2 + F_H \tan 30° x)$$

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + F_H \tan 30°)$$

At $x = 15$ m, $\frac{dy}{dx} = \tan 60°$; $F_H = 2598$ N

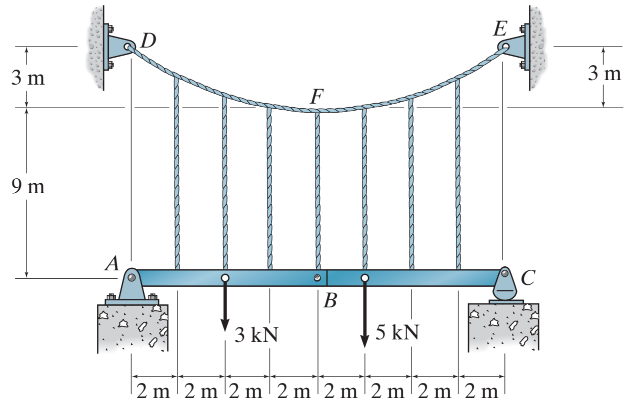
$y = (38.5x^2 + 577x)(10^{-3})$ m **Ans.**

$\theta_{max} = 60°$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2598}{\cos 60°} = 5196 \text{ N}$$

$T_{max} = 5.20$ kN **Ans.**

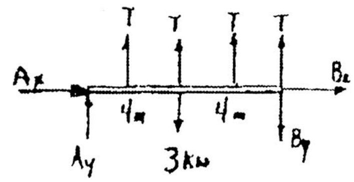
5–19. The beams AB and BC are supported by the cable that has a parabolic shape. Determine the tension in the cable at points D, F, and E, and the force in each of the equally spaced hangers.



$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad B_x = 0 \quad$ (Member BC)

$\circlearrowleft + \sum M_A = 0;$ $\quad F_F(12) - F_F(9) - B_y(8) - 3(4) = 0$

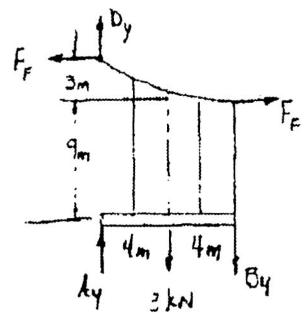
$$3F_F - B_y(8) = 12 \qquad (1)$$



$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad A_x = 0 \quad$ (Member AB)

$\circlearrowleft + \sum M_C = 0;$ $\quad -F_F(12) + F_F(9) - B_y(8) + 5(6) = 0$

$$-3F_F - B_y(8) = -30 \qquad (2)$$



Soving Eqs. (1) and (2),

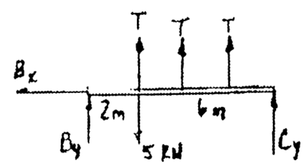
$$B_y = 1.125 \text{ kN}, \qquad F_F = 7.0 \text{ kN} \qquad \textbf{Ans.}$$

From Eq. 5–8.

$$w_o = \frac{2F_H h}{L^2} = \frac{2(7)(3)}{8^2} = 0.65625 \text{ kN/m}$$

From Eq. 5–11,

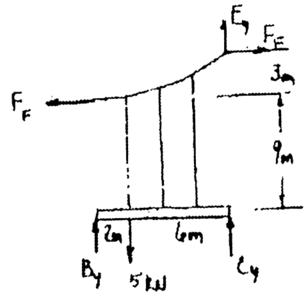
$$T_{\max} = w_o L \sqrt{1 + \left(\frac{L}{2h}\right)^2} = 0.65625(8)\sqrt{1 + \left(\frac{8}{2(3)}\right)^2}$$



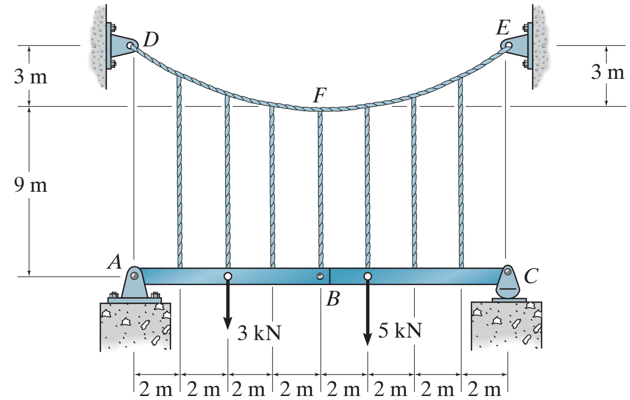
$$T_{\max} = T_E = T_D = 8.75 \text{ kN} \qquad \textbf{Ans.}$$

Load on each hanger,

$$T = 0.65625(2) = 1.3125 \text{ kN} = 1.31 \text{ kN} \qquad \textbf{Ans.}$$



***5–20.** Draw the shear and moment diagrams for beams *AB* and *BC*. The cable has a parabolic shape.



Member *ABC*:

$\circlearrowleft + \sum M_A = 0;$ $\quad T(2) + T(4) + T(6) + T(8) + T(10)$

$$+ T(12) + T(14) + C_y(16) - 3(4) - 5(10) = 0$$

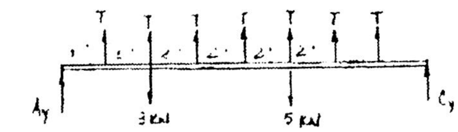
Set $T = 1.3125$ kN (See solution to Prob 5–19).

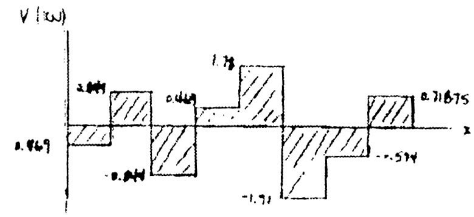
$$C_y = -0.71875 \text{ kN}$$

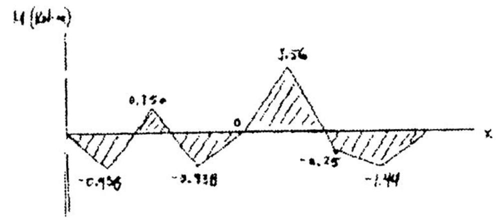
$+\uparrow \sum F_y = 0;$ $\quad 7(1.3125) - 8 - 0.71875 + A_y = 0$

$$A_y = -0.46875 \text{ kN}$$

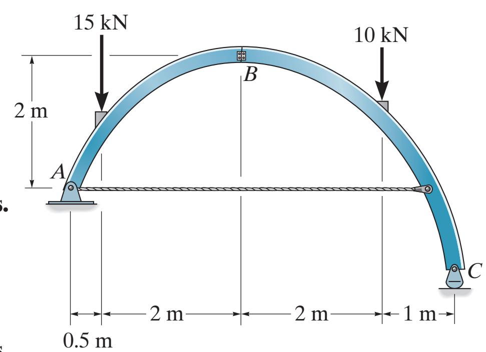
$$M_{max} = 3056 \text{ kN} \cdot \text{m}$$ **Ans.**







5–21. The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at *A* and *C* and the tension in the cable.



Entire arch:

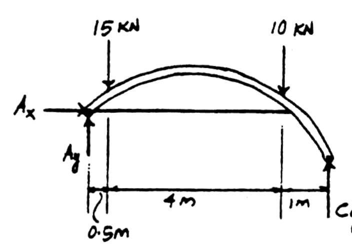
$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $\quad C_y(5.5) - 15(0.5) - 10(4.5) = 0$

$$C_y = 9.545 \text{ kN} = 9.55 \text{ kN}$$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 9.545 - 15 - 10 + A_y = 0$

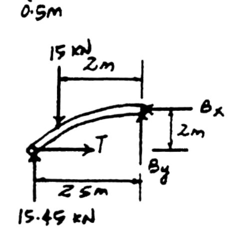
$$A_y = 15.45 \text{ kN} = 15.5 \text{ kN}$$ **Ans.**



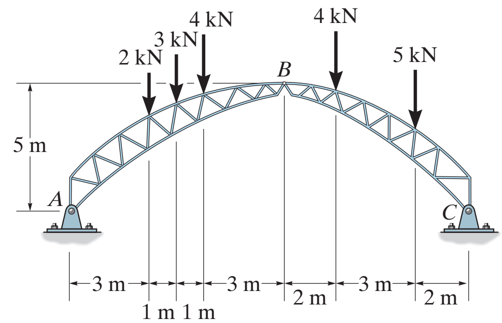
Section *AB*:

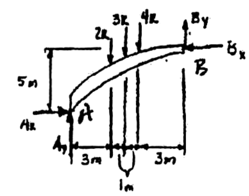
$\circlearrowleft + \sum M_B = 0;$ $\quad -15.45(2.5) + T(2) + 15(2) = 0$

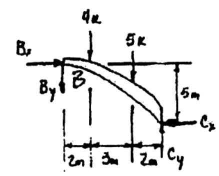
$$T = 4.32 \text{ kN}$$ **Ans.**



5–22. Determine the resultant forces at the pins A, B, and C of the three-hinged arched roof truss.







Member AB:

$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(8) - 2(3) - 3(4) - 4(5) = 0$

Member BC:

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(5) + B_y(7) + 5(2) + 4(5) = 0$

Soving,

$B_y = 0.533 \text{ k}, \qquad B_x = 6.7467 \text{ k}$

Member AB:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad A_x = 6.7467 \text{ k}$

$+\uparrow \sum F_y = 0;$ $\quad A_y - 9 + 0.533 = 0$

$A_y = 8.467 \text{ k}$

Member BC:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad C_x = 6.7467 \text{ k}$

$+\uparrow \sum F_y = 0;$ $\quad C_y - 9 + 0.533 = 0$

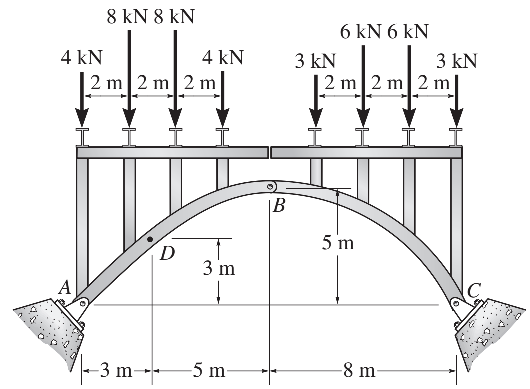
$C_y = 9.533 \text{ k}$

$F_B = \sqrt{(0.533)^2 + (6.7467)^2} = 6.77 \text{ k}$ **Ans.**

$F_A = \sqrt{(6.7467)^2 + (8.467)^2} = 10.8 \text{ k}$ **Ans.**

$F_C = \sqrt{(6.7467)^2 + (9.533)^2} = 11.7 \text{ k}$ **Ans.**

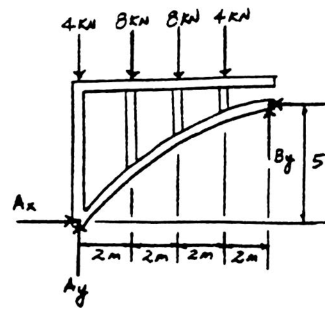
5–23. The three-hinged spandrel arch is subjected to the loading shown. Determine the internal moment in the arch at point D.



Member *AB*:

$\circlearrowleft + \sum M_A = 0; \qquad B_x(5) + B_y(8) - 8(2) - 8(4) - 4(6) = 0$

$$B_x + 1.6B_y = 14.4 \qquad (1)$$



Member *CB*:

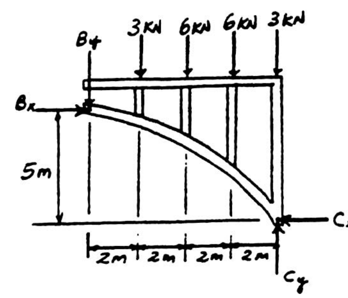
$\circlearrowleft + \sum M_C = 0; \qquad B_{(y)}(8) - B_x(5) + 6(2) + 6(4) + 3(6) = 0$

$$-B_x + 1.6B_y = -10.8 \qquad (2)$$

Soving Eqs. (1) and (2) yields:

$$B_y = 1.125 \text{ kN}$$

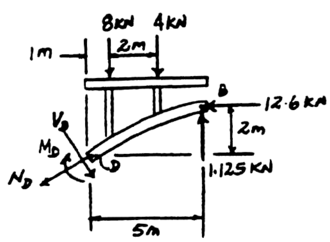
$$B_x = 12.6 \text{ kN}$$



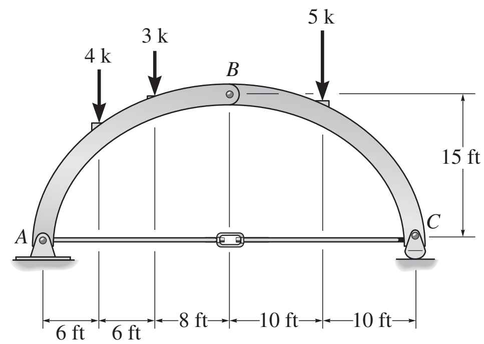
Segment *BD*:

$\circlearrowleft + \sum M_D = 0; \qquad -M_D + 12.6(2) + 1.125(5) - 8(1) - 4(3) = 0$

$M_D = 10.825 \text{ kN}\cdot\text{m} = 10.8 \text{ kN}\cdot\text{m}$ **Ans.**



***5–24.** The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at A and C, and the tension in the rod



Entire arch:

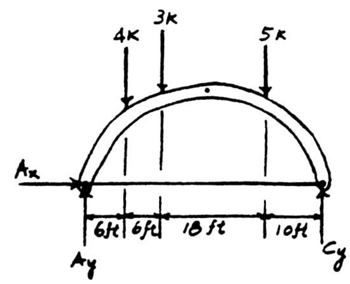
$\circlearrowleft + \sum M_A = 0;\quad -4(6) - 3(12) - 5(30) + C_y(40) = 0$

$C_y = 5.25 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;\quad A_y + 5.25 - 4 - 3 - 5 = 0$

$A_y = 6.75 \text{ k}$ **Ans.**

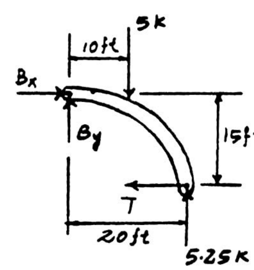
$\overset{+}{\rightarrow} \sum F_x = 0;\quad A_x = 0$ **Ans.**



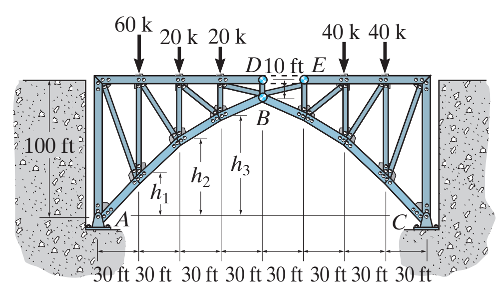
Section BC:

$\circlearrowleft + \sum M_B = 0;\quad -5(10) - T(15) + 5.25(20) = 0$

$T = 3.67 \text{ k}$ **Ans.**



5–25. The bridge is constructed as a *three-hinged trussed arch*. Determine the horizontal and vertical components of reaction at the hinges (pins) at A, B, and C. The dashed member DE is intended to carry *no* force.



Member AB:

$\circlearrowleft + \sum M_A = 0;\quad B_x(90) + B_y(120) - 20(90) - 20(90) - 60(30) = 0$

$9B_x + 12B_y = 480$ (1)

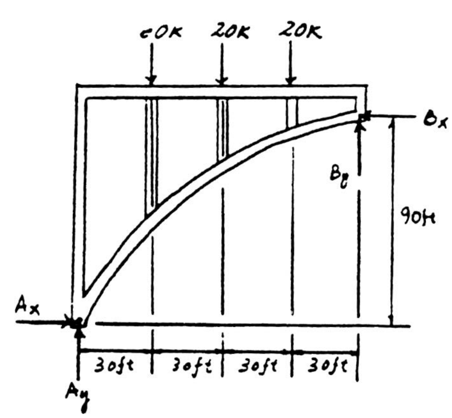
Member BC:

$\circlearrowleft + \sum M_C = 0;\quad -B_x(90) + B_y(120) + 40(30) + 40(60) = 0$

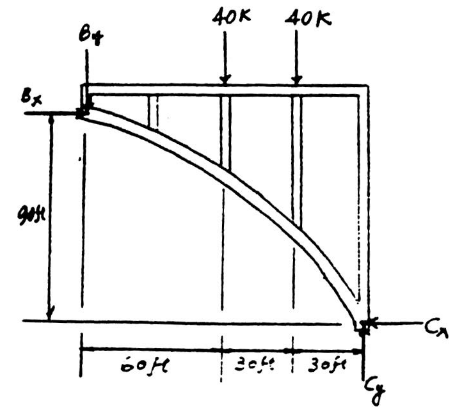
$-9B_x + 12B_y = -360$ (2)

Soving Eqs. (1) and (2) yields:

$B_x = 46.67 \text{ k} = 46.7 \text{ k} \qquad B_y = 5.00 \text{ k}$ **Ans.**



5–25. Continued



Member *AB*:

$\xrightarrow{+} \sum F_x = 0;$ $A_x - 46.67 = 0$

$A_x = 46.7 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $A_y - 60 - 20 - 20 + 5.00 = 0$

$A_y = 95.0 \text{ k}$ **Ans.**

Member *BC*:

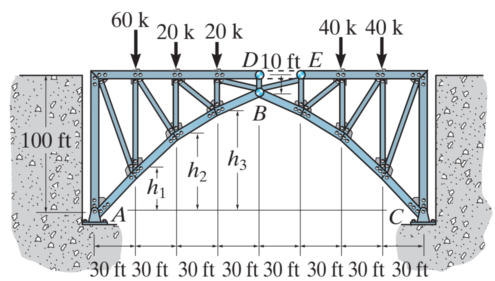
$\xrightarrow{+} \sum F_x = 0;$ $-C_x + 46.67 = 0$

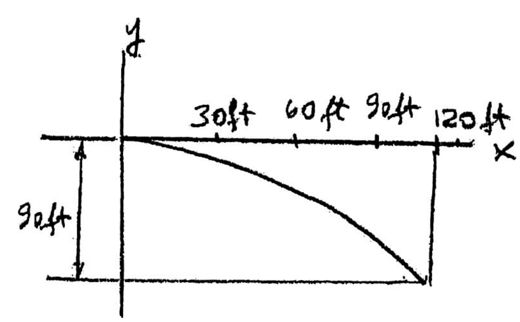
$C_x = 46.7 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $C_y - 5.00 - 40 - 40 = 0$

$C_y = 85 \text{ k}$ **Ans.**

5–26. Determine the design heights h_1, h_2, and h_3 of the bottom cord of the truss so the three-hinged trussed arch responds as a funicular arch.





$y = -Cx^2$

$-100 = -C(120)^2$

$C = 0.0069444$

Thus,

$y = -0.0069444x^2$

$y_1 = -0.0069444(90 \text{ ft})^2 = -56.25 \text{ ft}$

$y_2 = -0.0069444(60 \text{ ft})^2 = -25.00 \text{ ft}$

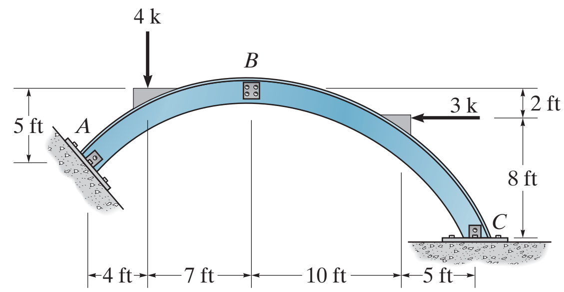
$y_3 = -0.0069444(30 \text{ ft})^2 = -6.25 \text{ ft}$

$h_1 = 100 \text{ ft} - 56.25 \text{ ft} = 43.75 \text{ ft}$ **Ans.**

$h_2 = 100 \text{ ft} - 25.00 \text{ ft} = 75.00 \text{ ft}$ **Ans.**

$h_3 = 100 \text{ ft} - 6.25 \text{ ft} = 93.75 \text{ ft}$ **Ans.**

5–27. Determine the horizontal and vertical components of reaction at A, B, and C of the three-hinged arch. Assume A, B, and C are pin connected.



Member AB:

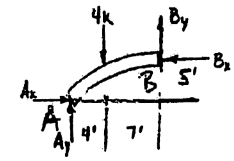
$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(11) - 4(4) = 0$

Member BC:

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(10) + B_y(15) + 3(8) = 0$

Soving,

$B_y = 0.216\text{ k}, \quad B_x = 2.72\text{ k}$ **Ans.**



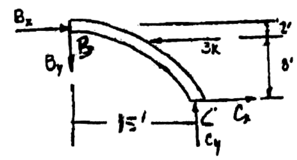
Member AB:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad A_x - 2.7243 = 0$

$A_x = 2.72\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad A_y - 4 + 0.216216 = 0$

$A_y = 3.78\text{ k}$ **Ans.**



Member BC:

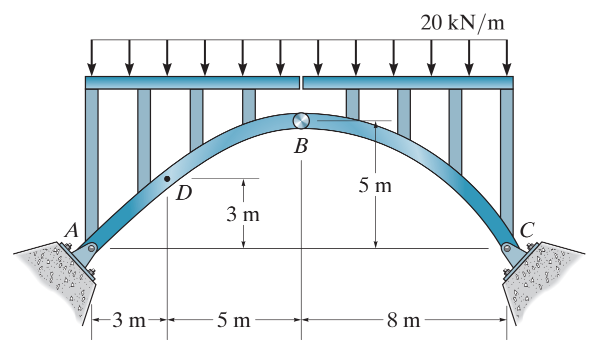
$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad C_x + 2.7243 - 3 = 0$

$C_x = 0.276\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad C_y - 0.216216 = 0$

$C_y = 0.216\text{ k}$ **Ans.**

***5–28.** The three-hinged spandrel arch is subjected to the uniform load of 20 kN/m. Determine the internal moment in the arch at point D.



Member AB:

$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(8) - 160(4) = 0$

Member BC:

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(5) + B_y(8) + 160(4) = 0$

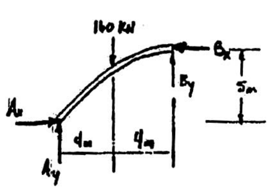
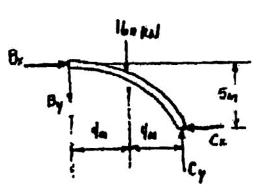
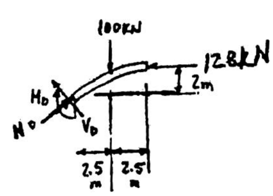
Solving,

$B_x = 128\text{ kN}, \qquad B_y = 0$

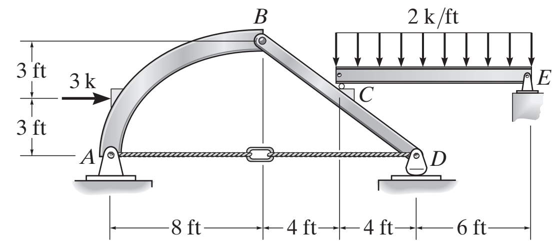
Segment DB:

$\circlearrowleft + \sum M_D = 0;$ $\quad 128(2) - 100(2.5) - M_D = 0$

$M_D = 6.00\text{ kN}\cdot\text{m}$ **Ans.**

5–29. The arch structure is subjected to the loading shown. Determine the horizontal and vertical components of reaction at A and D, and the tension in the rod AD.



$\overset{+}{\rightarrow} \sum F_x = 0; \qquad -A_x + 3\text{ k} = 0; \quad A_x = 3\text{ k}$ **Ans.**

$\zeta + \sum M_A = 0; \qquad -3\text{ k}\,(3\text{ ft}) - 10\text{ k}\,(12\text{ ft}) + D_y(16\text{ ft}) = 0$

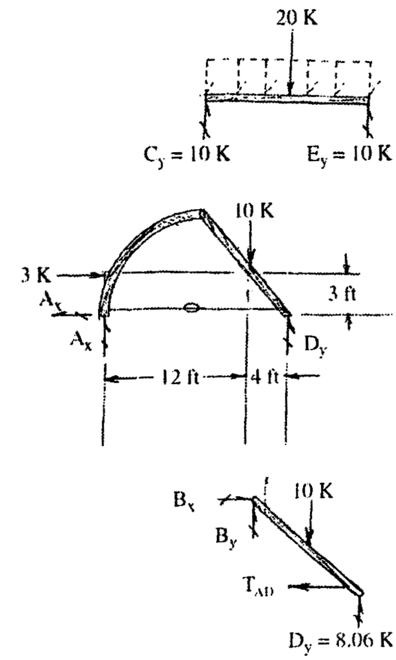
$D_y = 8.06\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y - 10\text{ k} + 8.06\text{ k} = 0$

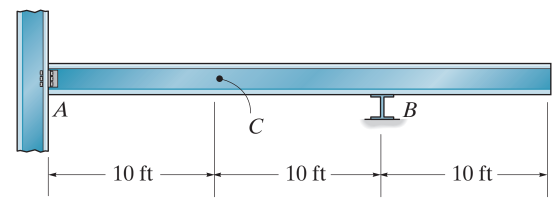
$A_y = 1.94\text{ k}$ **Ans.**

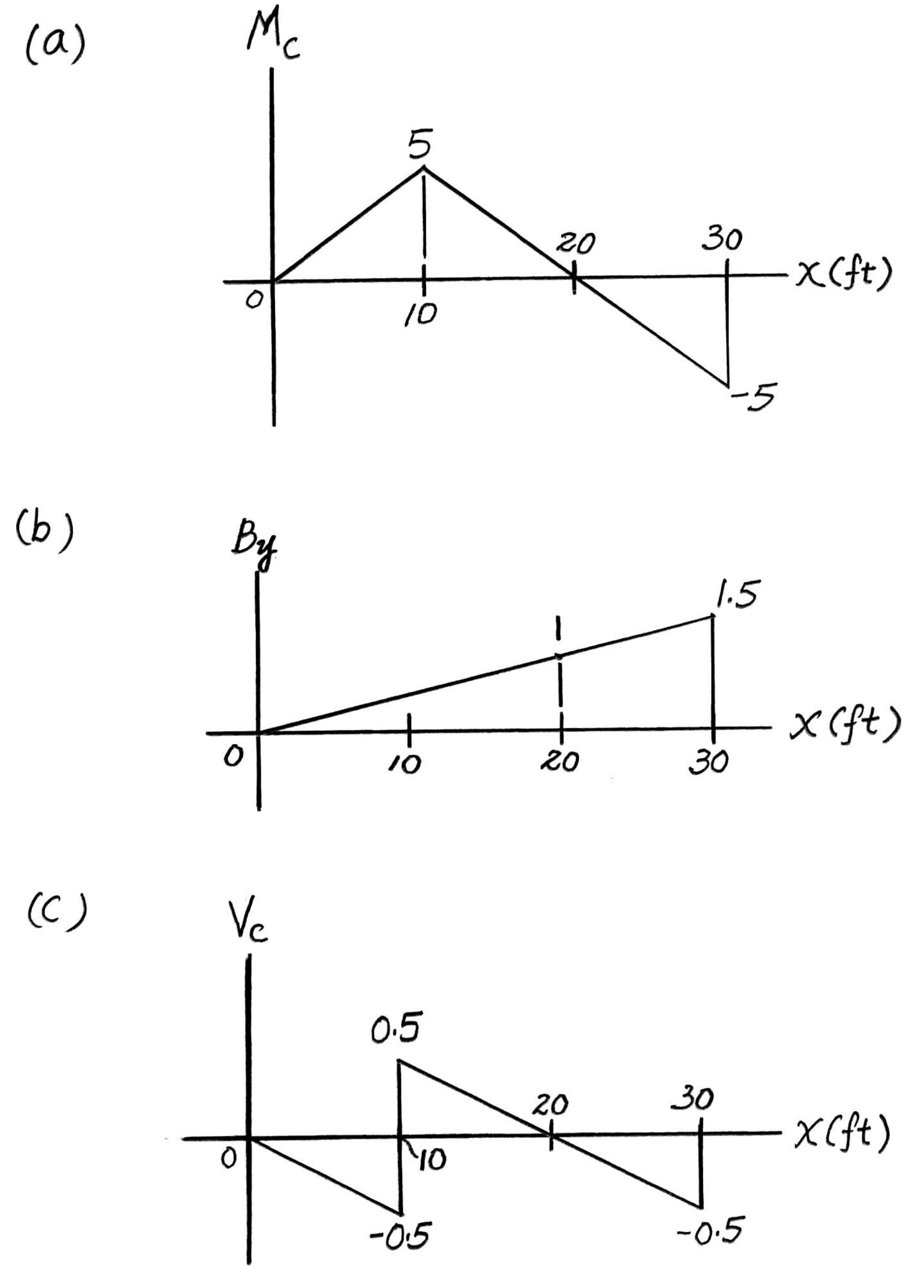
$\zeta + \sum M_B = 0; \qquad 8.06\text{ k}\,(8\text{ ft}) - 10\text{ k}\,(4\text{ ft}) - T_{AD}(6\text{ ft}) = 0$

$T_{AD} = 4.08\text{ k}$ **Ans.**

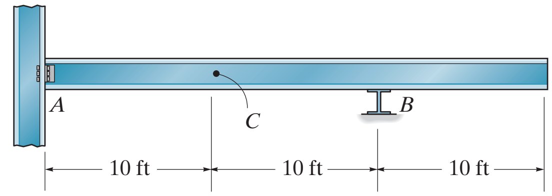


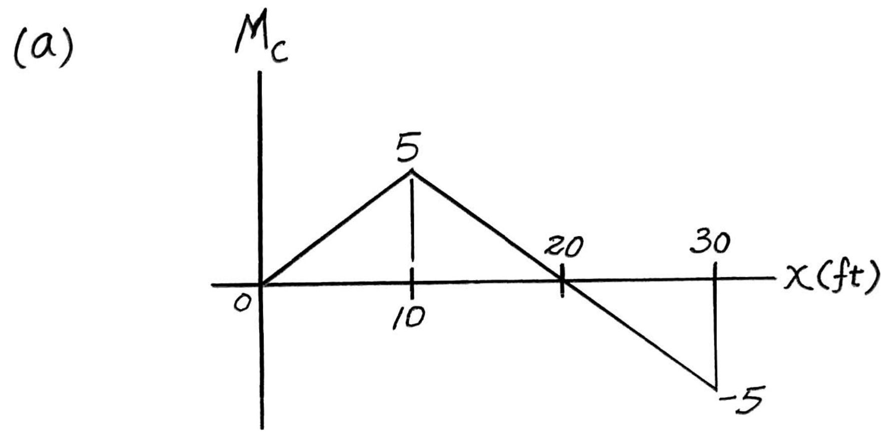
6–1. Draw the influence lines for (a) the moment at C, (b) the reaction at B, and (c) the shear at C. Assume A is pinned and B is a roller. Solve this problem using the basic method of Sec. 6–1.

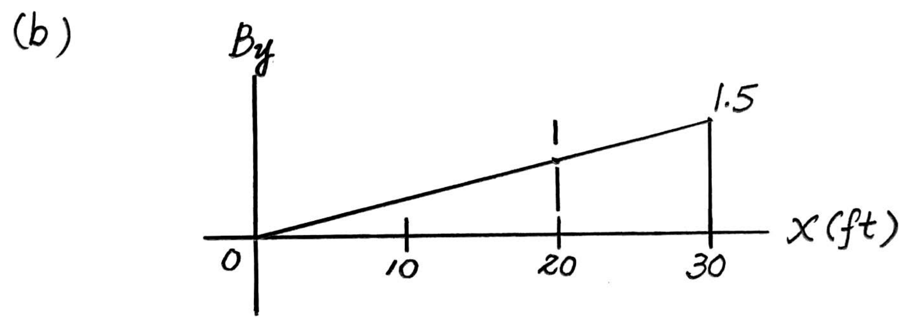


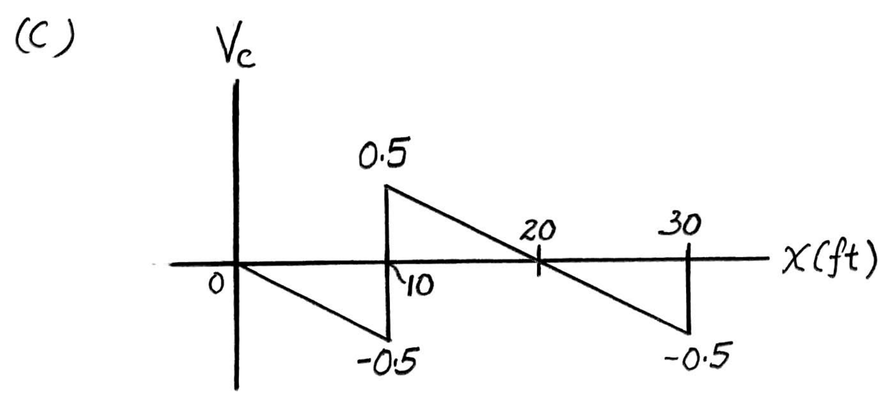


6–2. Solve Prob. 6–1 using the Müller-Breslau principle.

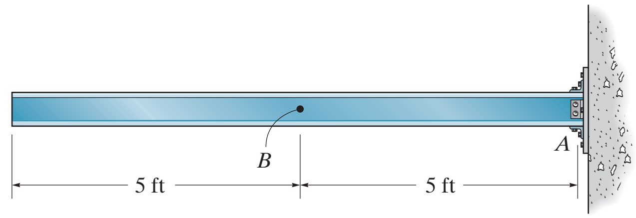




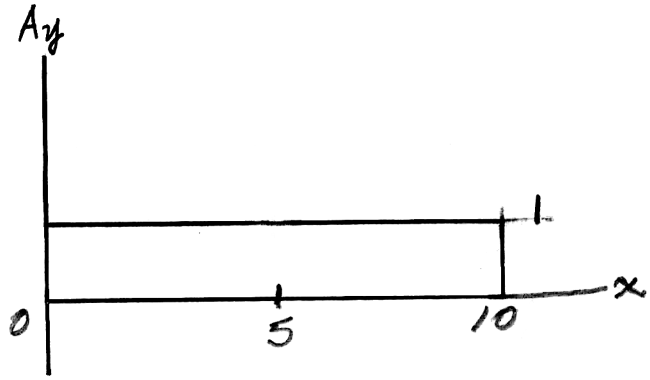




6–3. Draw the influence lines for (a) the vertical reaction at A, (b) the moment at A, and (c) the shear at B. Assume the support at A is fixed. Solve this problem using the basic method of Sec. 6–1.



(a)



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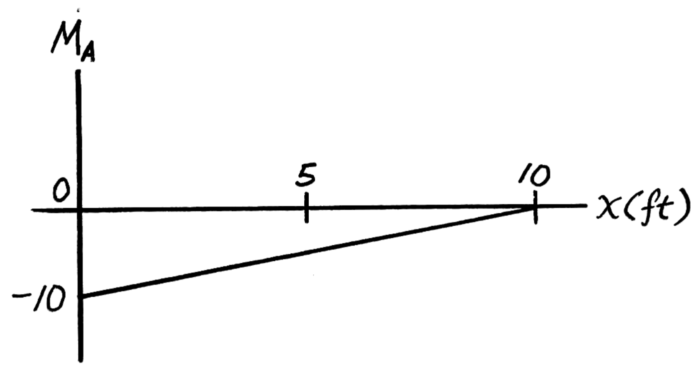
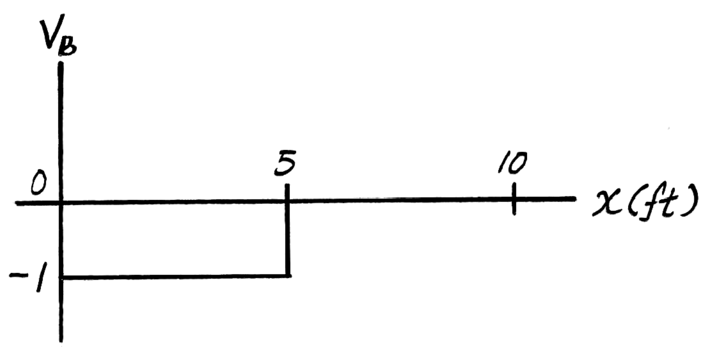
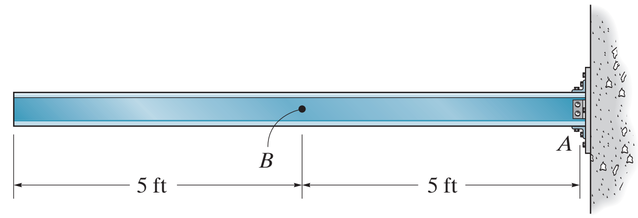


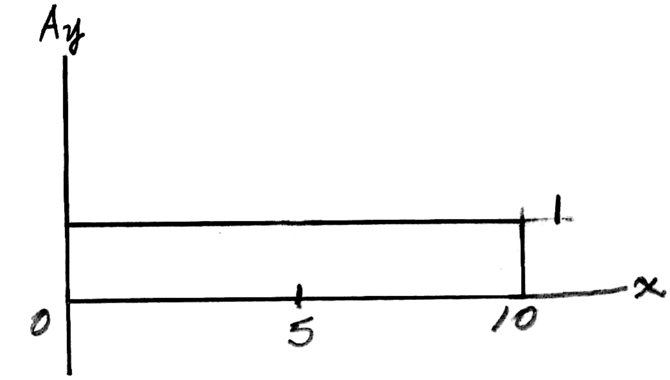
(c)



***6–4.** Solve Prob. 6–3 using the Müller-Breslau principle.



(a)



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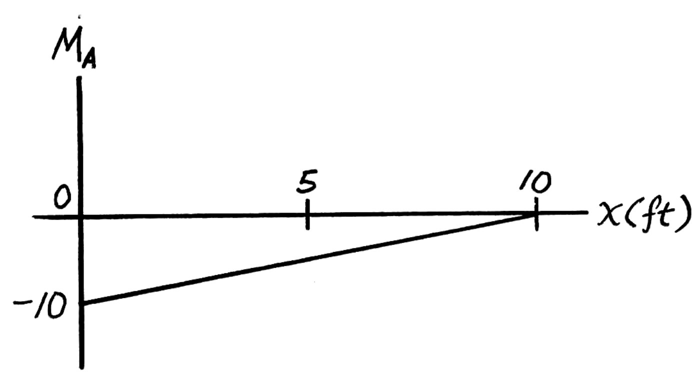

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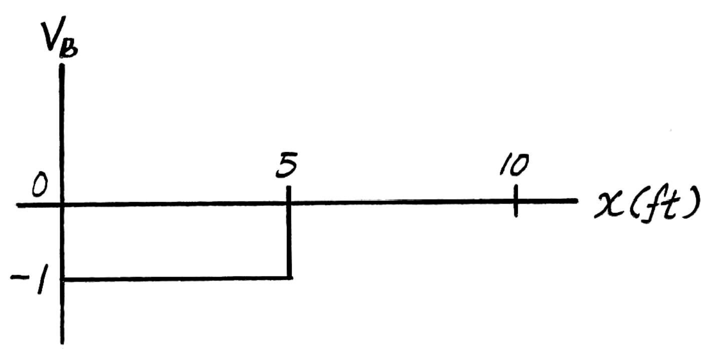
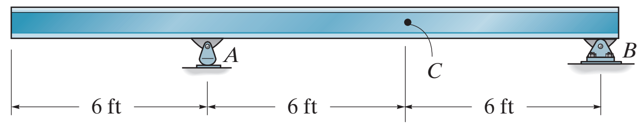
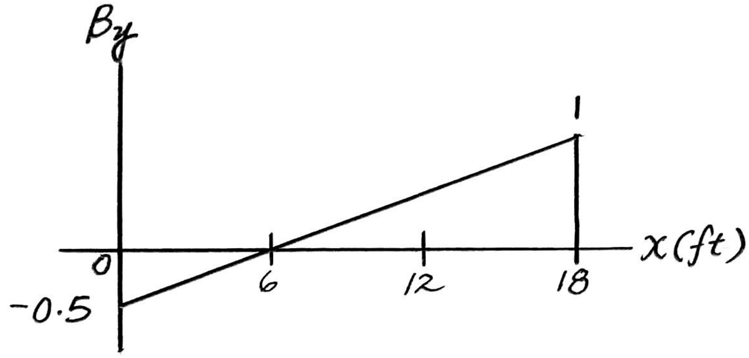


6–5. Draw the influence lines for (a) the vertical reaction at B, (b) the shear just to the right of the rocker at A, and (c) the moment at C. Solve this problem using the basic method of Sec. 6–1.



(a)



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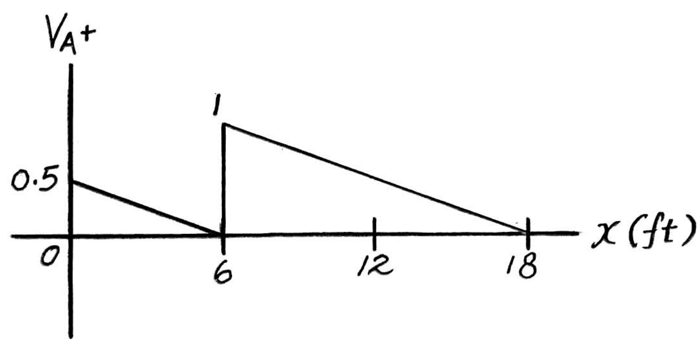
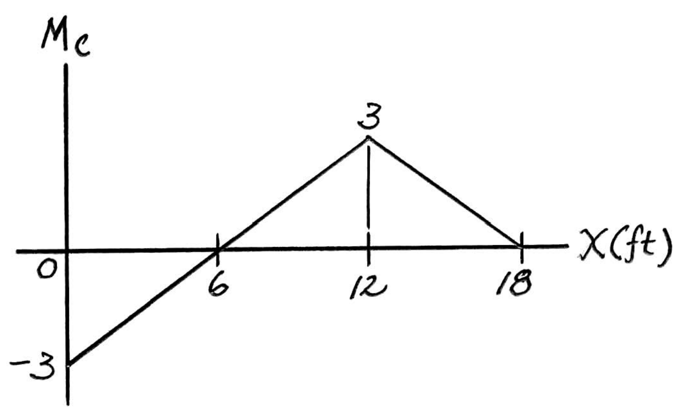
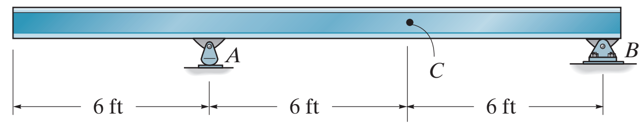


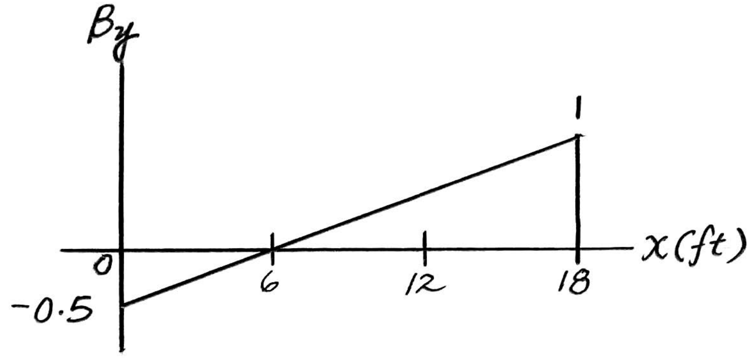
(c)



6–6. Solve Prob. 6–5 using Müller-Breslau's principle.



(a)



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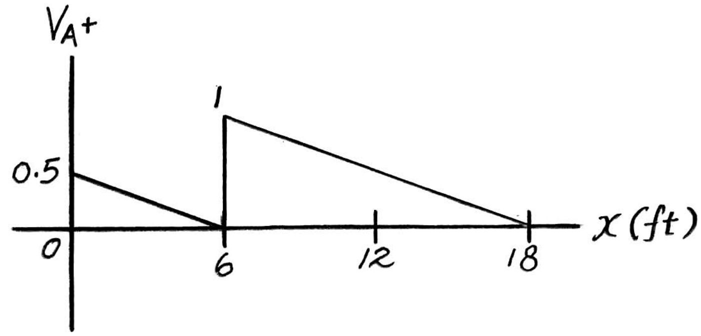

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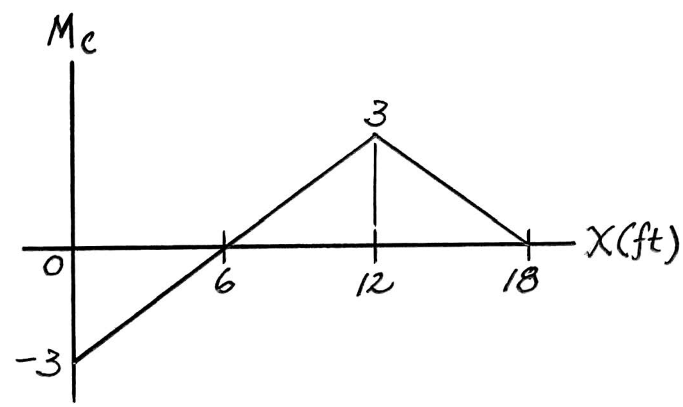
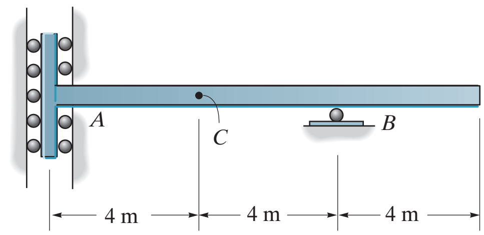
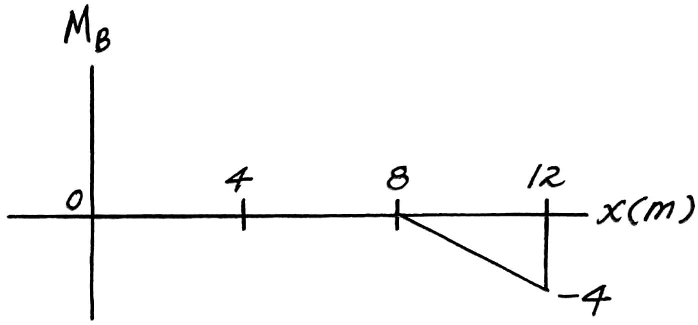


6–7. Draw the influence line for (a) the moment at *B*, (b) the shear at *C*, and (c) the vertical reaction at *B*. Solve this problem using the basic method of Sec. 6–1. *Hint*: The support at *A* resists only a horizontal force and a bending moment.



(a)



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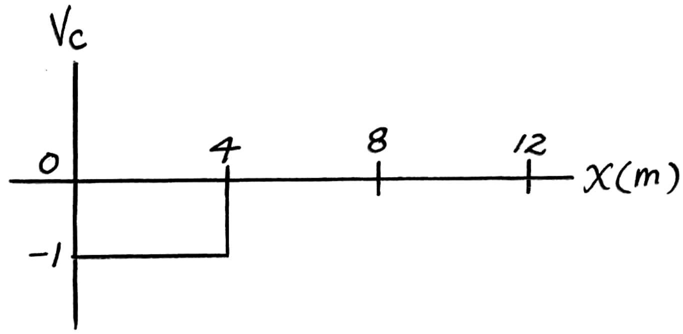

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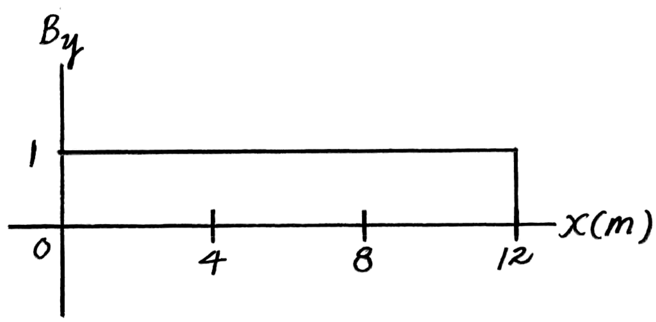
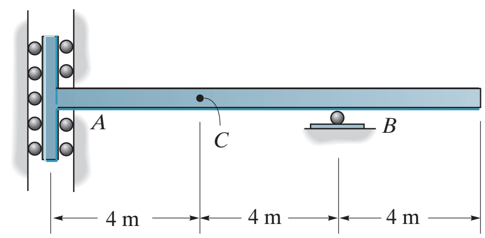
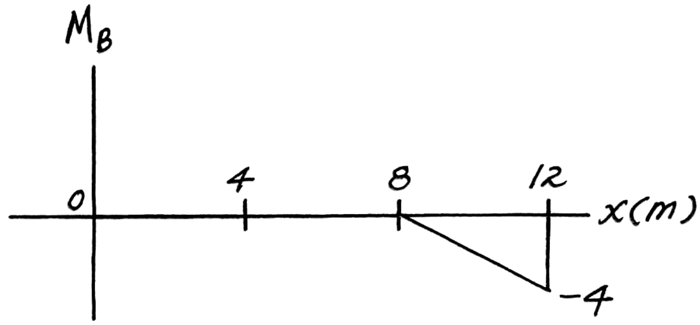


***6–8.** Solve Prob. 6–7 using the Müller-Breslau principle.



(a)



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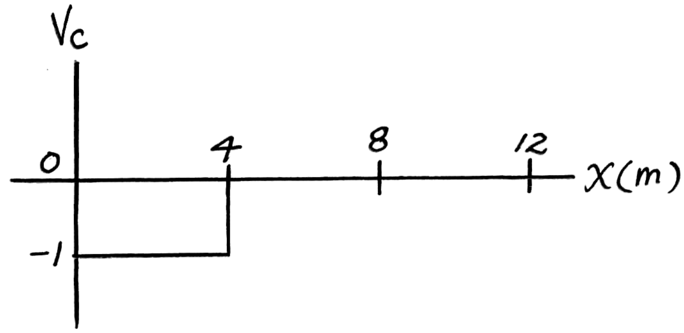

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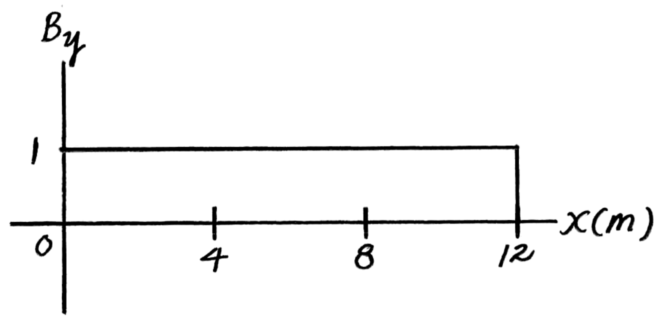
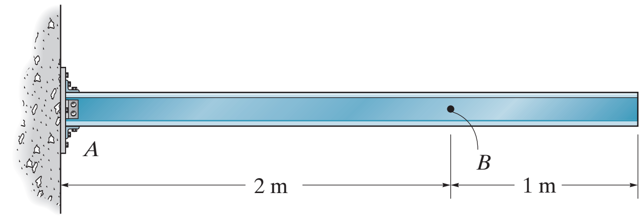
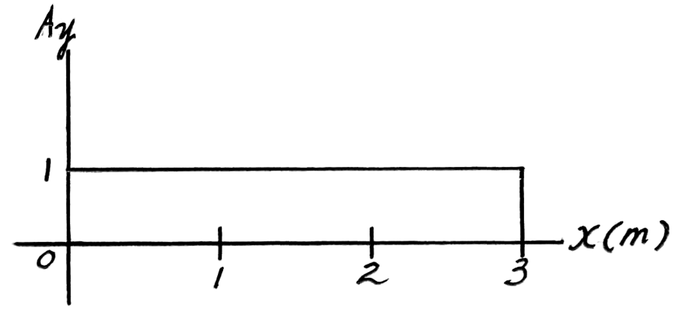


6–9. Draw the influence line for (a) the vertical reaction at A, (b) the shear at B, and (c) the moment at B. Assume A is fixed. Solve this problem using the basic method of Sec. 6–1.



(a)



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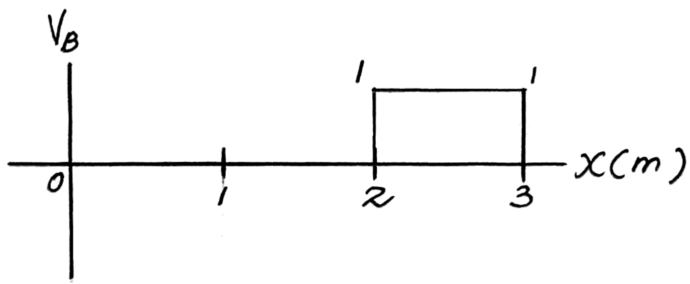

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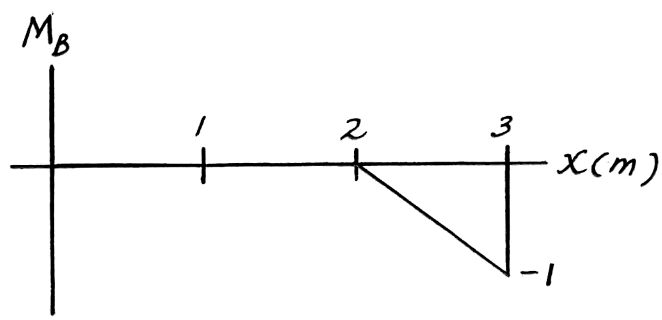
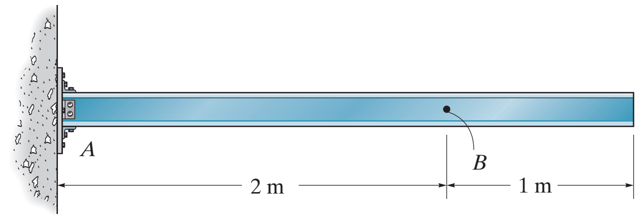
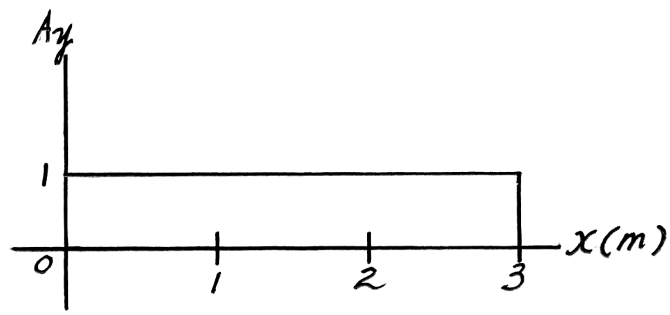


6–10. Solve Prob. 6–9 using the Müller-Breslau principle.



(a)



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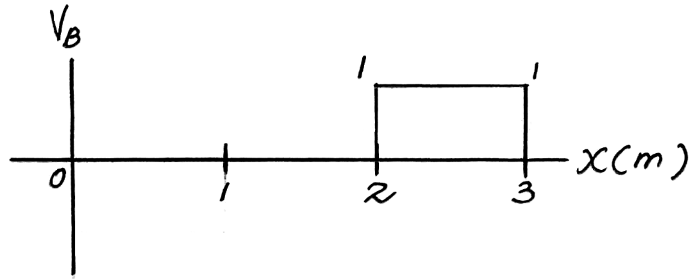

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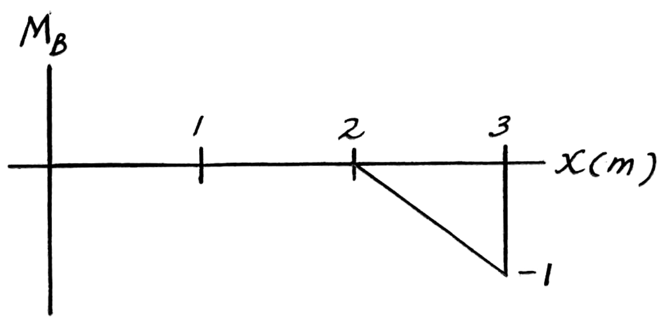
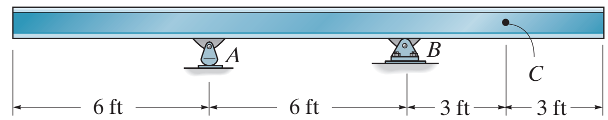
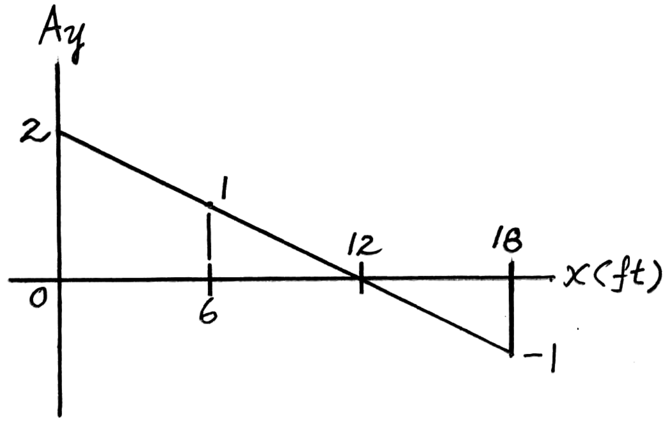


6–11. Draw the influence lines for (a) the vertical reaction at A, (b) the shear at C, and (c) the moment at C. Solve this problem using the basic method of Sec. 6–1.



(a)



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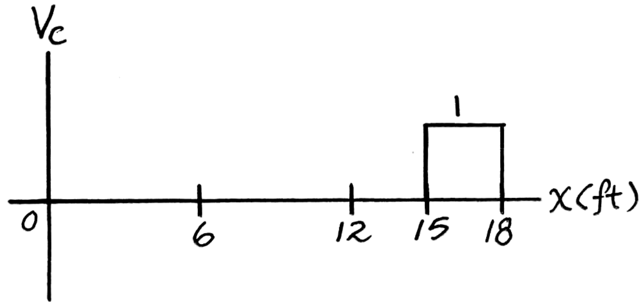
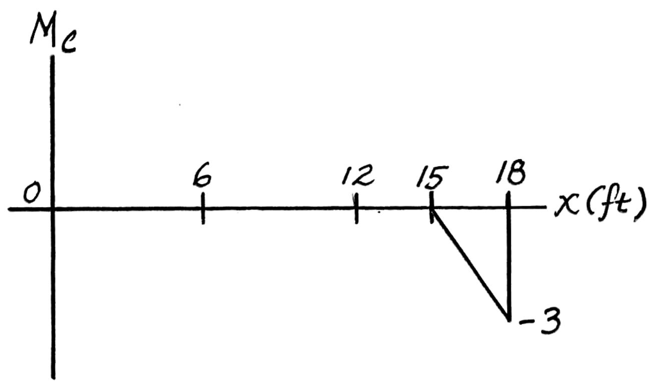
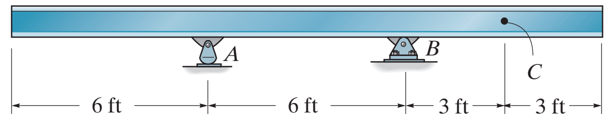


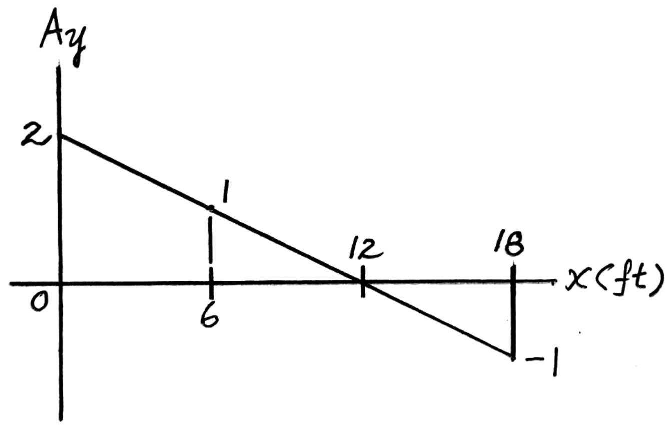
(c)



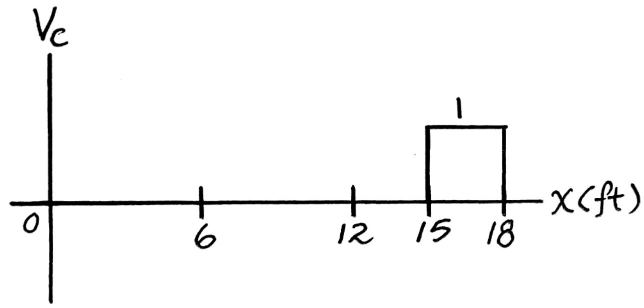
***6–12.** Solve Prob. 6–11 using Müller-Breslau's principle.



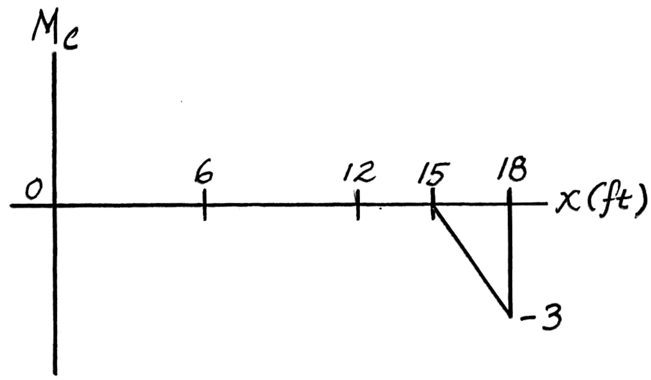
(a)



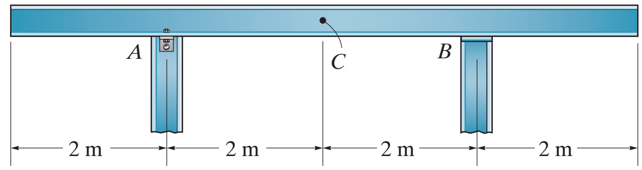
(b)

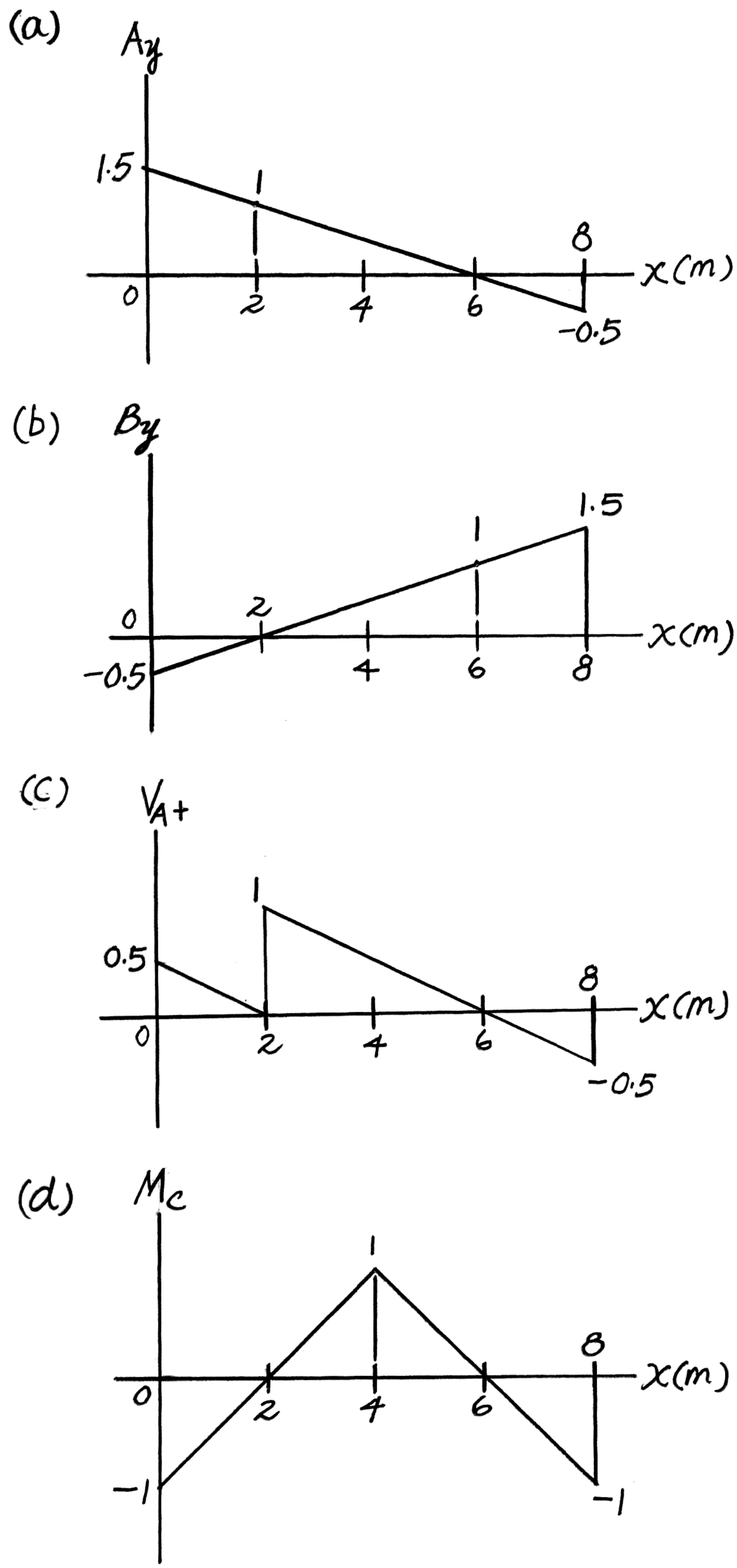


(c)

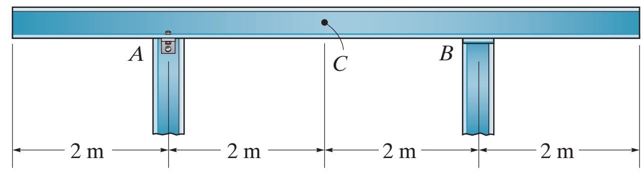


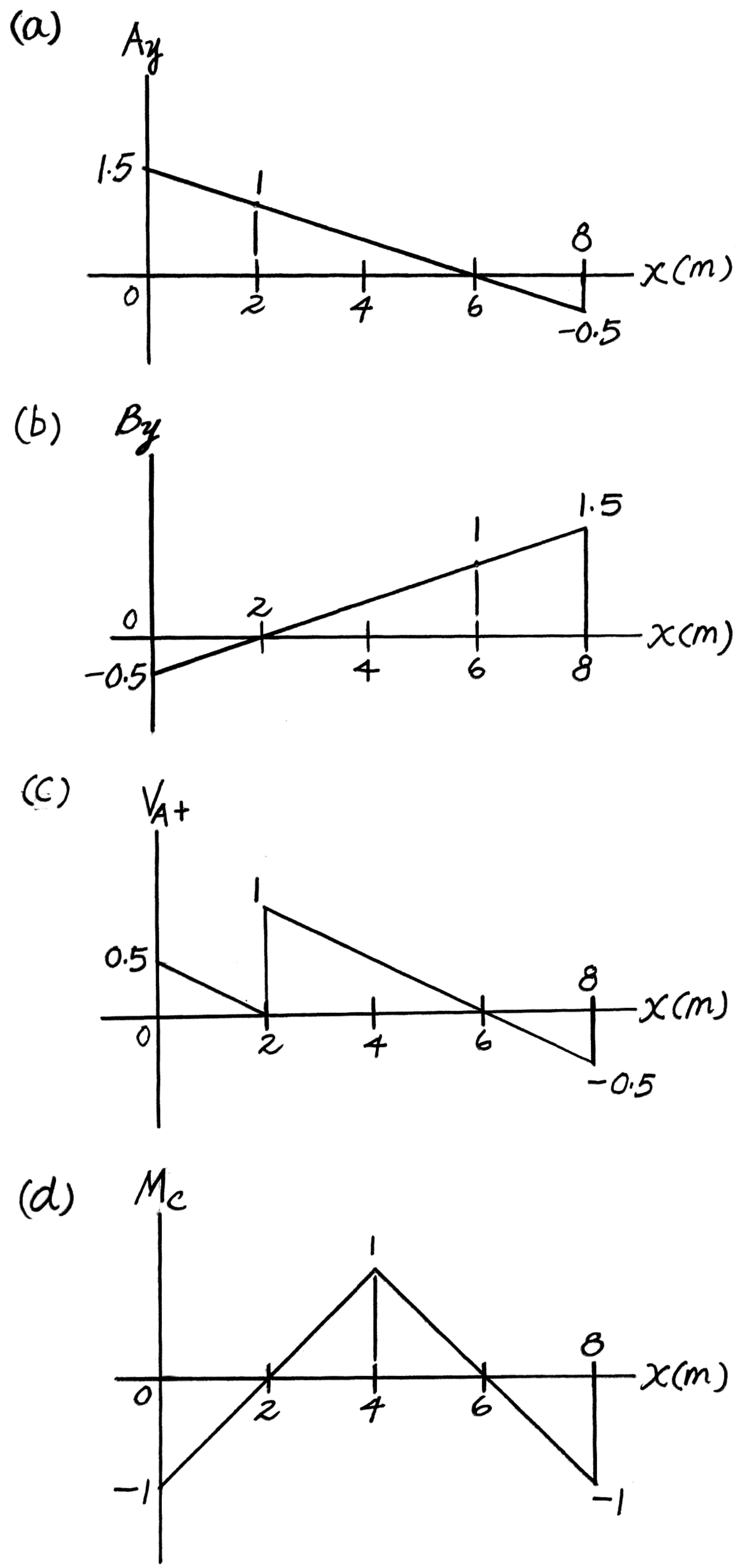
6–13. Draw the influence lines for (a) the vertical reaction at A, (b) the vertical reaction at B, (c) the shear just to the right of the support at A, and (d) the moment at C. Assume the support at A is a pin and B is a roller. Solve this problem using the basic method of Sec. 6–1.



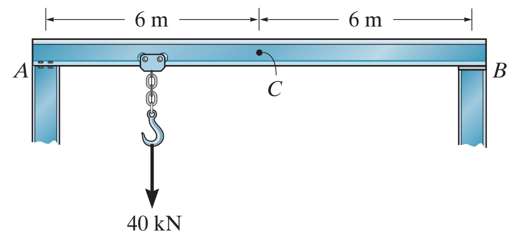


6–14. Solve Prob. 6–13 using the Müller-Breslau principle.

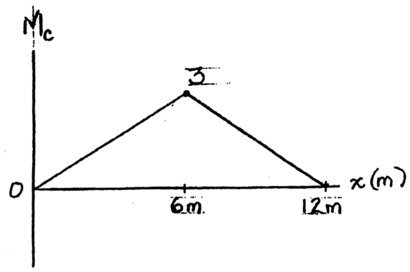




6–15. The beam is subjected to a uniform dead load of 1.2 kN/m and a single live load of 40 kN. Determine (a) the maximum moment created by these loads at C, and (b) the maximum positive shear at C. Assume A is a pin. and B is a roller.

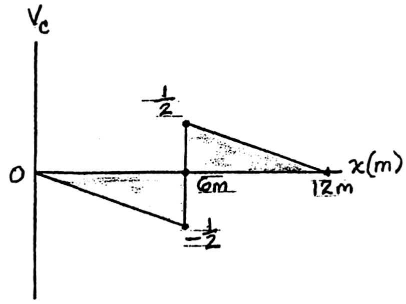


a.)



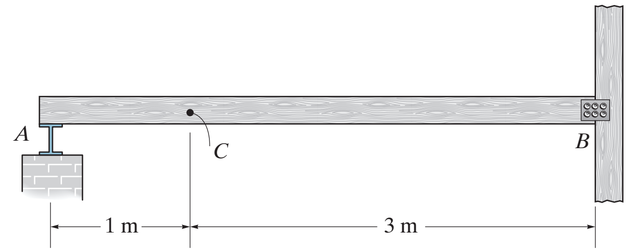
$$(M_C)_{\max} = 40\text{ kN }(3\text{ m}) + 1.2\text{ kN/m}\left(\frac{1}{2}\right)(12\text{ m})(3\text{ m}) = 141.6\text{ kN}\cdot\text{m}$$ **Ans.**

b.)



$$(V_C)_{\max} = 40\left(\frac{1}{2}\right) + 1.2\text{ kN/m}\left[\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(6) + \frac{1}{2}\left(\frac{1}{2}\right)(6)\right] = 20\text{ kN}$$ **Ans.**

***6–16.** The beam supports a uniform dead load of 500 N/m and a single live concentrated force of 3000 N. Determine (a) the maximum positive moment at C, and (b) the maximum positive shear at C. Assume the support at A is a roller and B is a pin.



Referring to the influence line for the moment at C shown in Fig. a, the maximum positive moment at C is

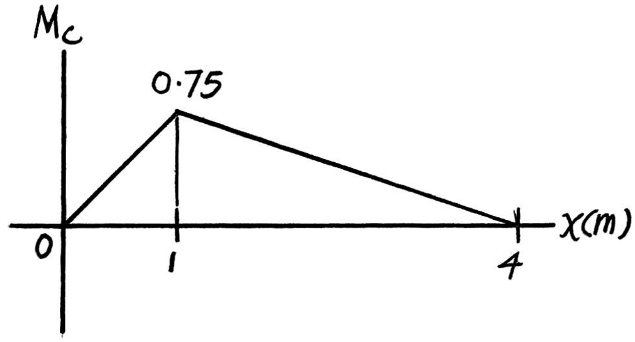
$$(M_c)_{\max(+)} = 0.75(3000) + \left[\frac{1}{2}(4 - 0)(0.75)\right](500)$$

$$= 3000\ \text{N}\cdot\text{m} = 3\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

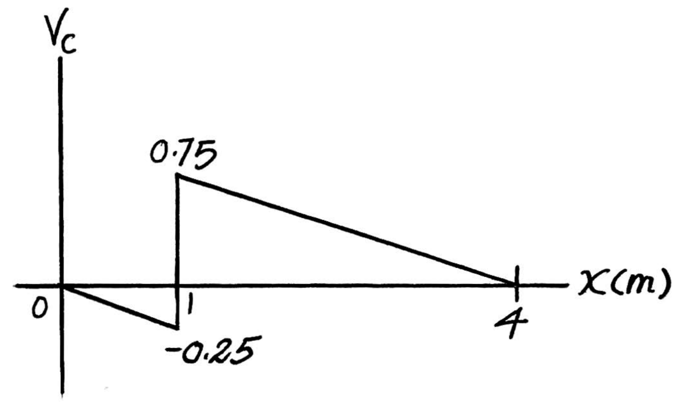
Referring to the influence line for the moment at C in Fig. b, the maximum positive shear at C is

$$(V_c)_{\max(+)} = 0.75(3000) + \left[\frac{1}{2}(1 - 0)(-0.25)\right](500) + \left[\frac{1}{2}(4 - 1)(0.75)\right](500)$$

$$= 2750\ \text{N} = 2.75\ \text{kN} \qquad \textbf{Ans.}$$

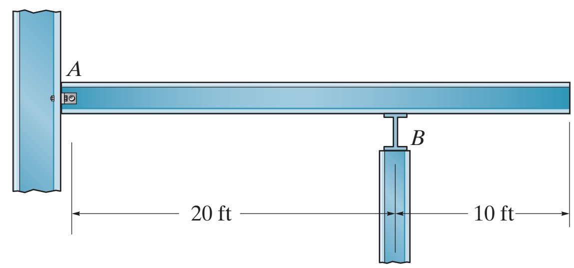


(a)



(b)

6–17. A uniform live load of 300 lb/ft and a single live concentrated force of 1500 lb are to be placed on the beam. The beam has a weight of 150 lb/ft. Determine (a) the maximum vertical reaction at support B, and (b) the maximum negative moment at point B. Assume the support at A is a pin and B is a roller.



Referring to the influence line for the vertical reaction at B shown in Fig. a, the maximum reaction that is

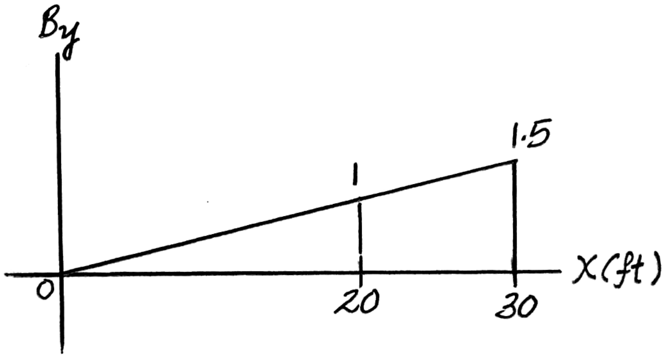
$$(B_y)_{\max(+)} = 1.5(1500) + \left[\frac{1}{2}(30 - 0)(1.5)\right](300 + 150)$$

$$= 12375 \text{ lb} = 12.4 \text{ k} \qquad \textbf{Ans.}$$

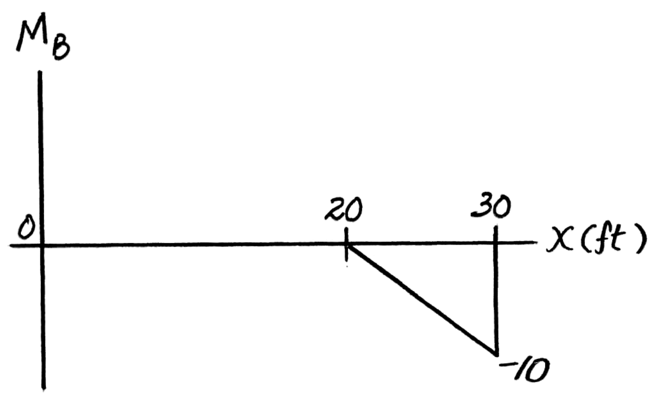
Referring to the influence line for the moment at B shown in Fig. b, the maximum negative moment is

$$(M_B)_{\max(-)} = -10(1500) + \left[\frac{1}{2}(30 - 20)(-10)\right](300 + 150)$$

$$= -37500 \text{ lb}\cdot\text{ft} = -37.5 \text{ k}\cdot\text{ft} \qquad \textbf{Ans.}$$

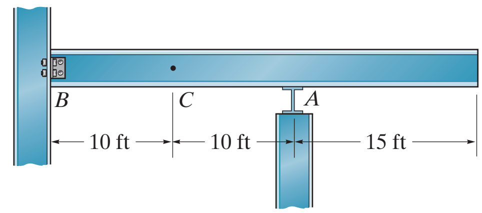


(a)



(b)

6–18. The beam supports a uniform dead load of 0.4 k/ft, a live load of 1.5 k/ft, and a single live concentrated force of 8 k. Determine (a) the maximum positive moment at C, and (b) the maximum positive vertical reaction at B. Assume A is a roller and B is a pin.



Referring to the influence line for the moment at C shown in Fig. *a*, the maximum positive moment is

$$(M_C)_{\max(+)} = 5(8) + \left[\frac{1}{2}(20 - 0)(5)\right](1.5) + \left[\frac{1}{2}(20 - 0)(5)\right](0.4)$$

$$+ \left[\frac{1}{2}(35 - 20)(-7.5)\right](0.4)$$

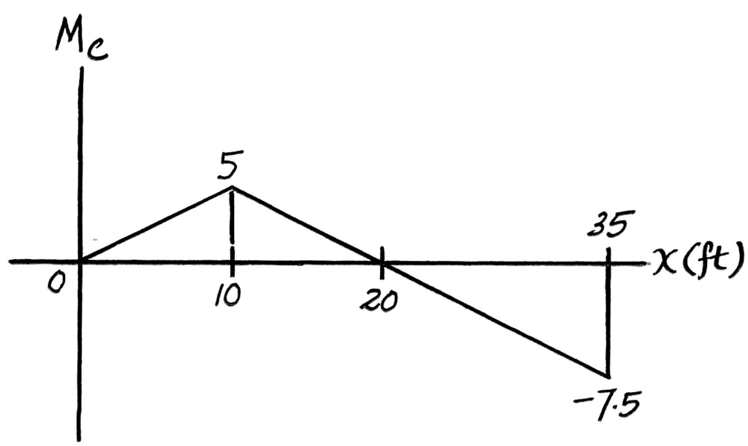
$$= 112.5 \text{ k} \cdot \text{ft}$$ **Ans.**

Referring to the influence line for the vertical reaction at B shown in Fig. *b*, the maximum positive reaction is

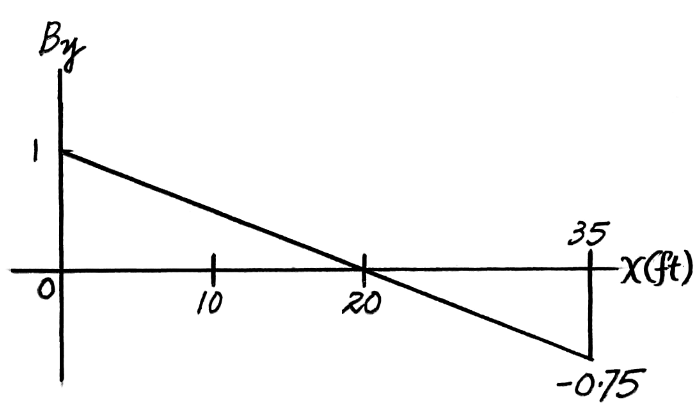
$$(B_y)_{\max(+)} = 1(8) + \left[\frac{1}{2}(20 - 0)(1)\right](1.5) + \left[\frac{1}{2}(20 - 0)(1)\right](0.4)$$

$$+ \left[\frac{1}{2}(35 - 20)(-0.75)\right](0.4)$$

$$= 24.75 \text{ k}$$ **Ans.**

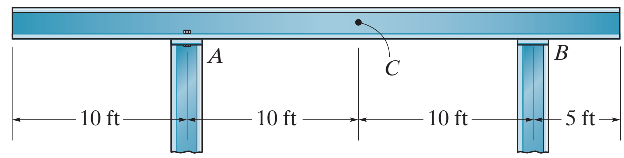


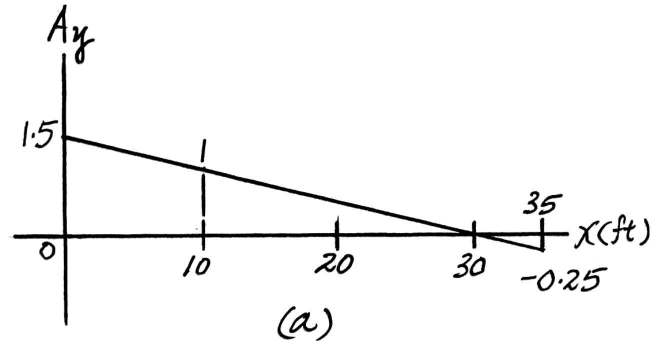
(a)



(b)

6–19. The beam is used to support a dead load of 0.6 k/ft, a live load of 2 k/ft and a concentrated live load of 8 k. Determine (a) the maximum positive (upward) reaction at A, (b) the maximum positive moment at C, and (c) the maximum positive shear just to the right of the support at A. Assume the support at A is a pin and B is a roller.





Referring to the influence line for the vertical reaction at A shown in Fig. a, the maximum positive vertical reaction is

$$(A_y)_{\max(+)} = 1.5(8) + \left[\frac{1}{2}(30 - 0)(1.5)\right](2)$$

$$+ \left[\frac{1}{2}(30 - 0)(1.5)\right](0.6) + \left[\frac{1}{2}(35 - 30)(-0.25)\right](0.6)$$

$$= 70.1 \text{ k}$$ **Ans.**

Referring to the influence line for the moment at C shown in Fig. b, the maximum positive moment is

$$(M_c)_{\max(+)} = 5(8) + \left[\frac{1}{2}(30 - 10)(5)\right](2) + \left[\frac{1}{2}(10 - 0)(-5)\right](0.6)$$

$$+ \left[\frac{1}{2}(30 - 10)(5)\right](0.6) + \left[\frac{1}{2}(35 - 30)(-2.5)\right](0.6)$$

$$= 151 \text{ k} \cdot \text{ft}$$ **Ans.**

Referring to the influence line for shear just to the right of A shown in Fig. c, the maximum positive shear is

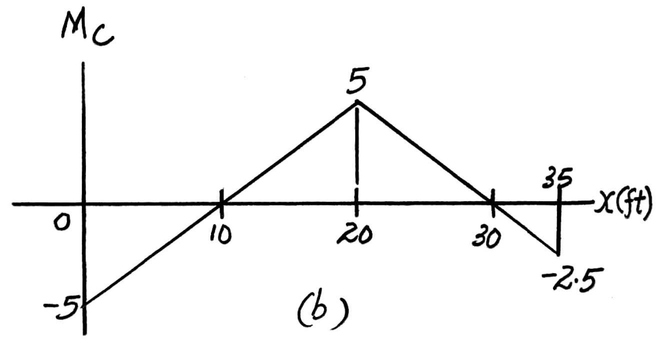
$$(V_{A^+})_{\max(+)} = 1(8) + \left[\frac{1}{2}(10 - 0)(0.5)\right](2)$$

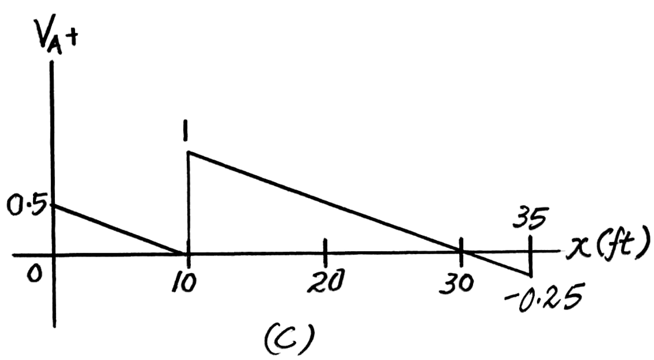
$$+ \left[\frac{1}{2}(30 - 10)(1)\right](2)$$

$$+ \left[\frac{1}{2}(10 - 0)(0.5)\right](0.6) + \left[\frac{1}{2}(30 - 10)(1)\right](0.6)$$

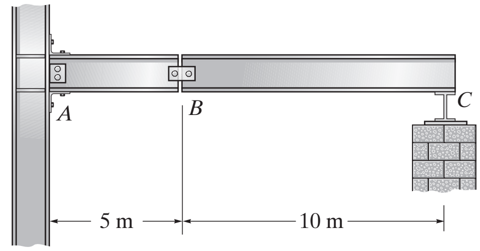
$$+ \left[\frac{1}{2}(35 - 30)(-0.25)\right](0.6)$$

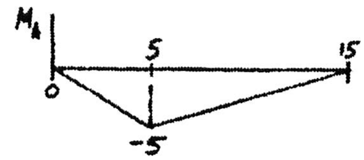
$$= 40.1 \text{ k}$$ **Ans.**





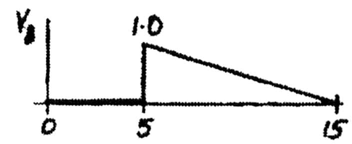
***6–20.** The compound beam is subjected to a uniform dead load of 1.5 kN/m and a single live load of 10 kN. Determine (a) the maximum negative moment created by these loads at A, and (b) the maximum positive shear at B. Assume A is a fixed support, B is a pin, and C is a roller.





$$(M_A)_{max} = 1.5\left(\frac{1}{2}\right)(15)(-5) + 10(-5)$$

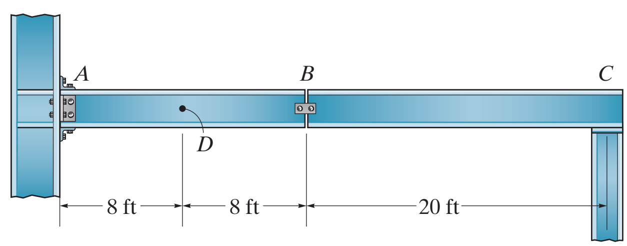
$$= -106 \text{ kN} \cdot \text{m}$$ **Ans.**



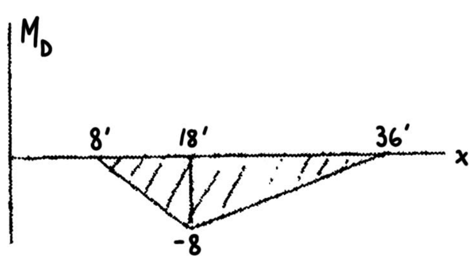
$$(V_B)_{max} = 1.5\left(\frac{1}{2}\right)(10)(1) + 10(1)$$

$$= 17.5 \text{ kN}$$ **Ans.**

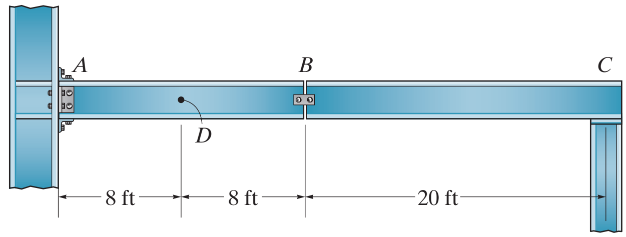
6–21. Where should a single 500-lb live load be placed on the beam so it causes the largest moment at D? What is this moment? Assume the support at A is fixed, B is pinned, and C is a roller.

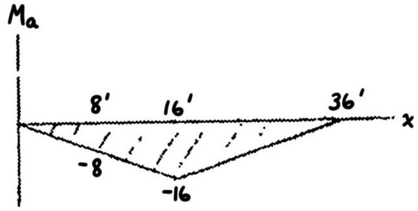


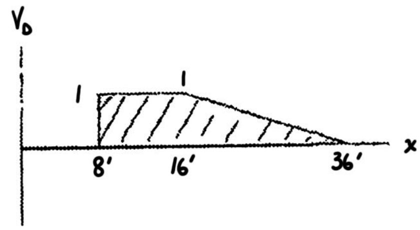
At point B: $(M_D)_{max} = 500(-8) = -4000 \text{ lb} \cdot \text{ft} = -4 \text{ k} \cdot \text{ft}$ **Ans.**



6–22. Where should the beam ABC be loaded with a 300 lb/ft uniform distributed live load so it causes (a) the largest moment at point A and (b) the largest shear at D? Calculate the values of the moment and shear. Assume the support at A is fixed, B is pinned and C is a roller.



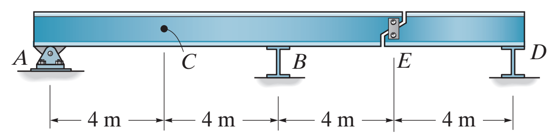




(a) $(M_A)_{max} = \frac{1}{2}(36)(-16)(0.3) = -86.4\text{ k}\cdot\text{ft}$ **Ans.**

(b) $(V_D)_{max} = \left[(1)(8) + \frac{1}{2}(1)(20)\right](0.3) = 5.40\text{ k}$ **Ans.**

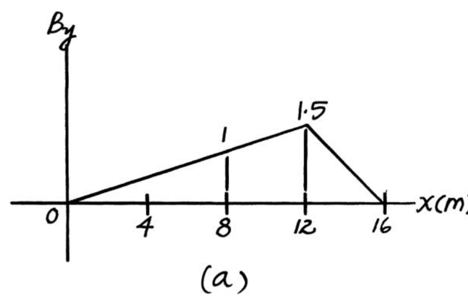
6–23. The beam is used to support a dead load of 800 N/m, a live load of 4 kN/m, and a concentrated live load of 20 kN. Determine (a) the maximum positive (upward) reaction at B, (b) the maximum positive moment at C, and (c) the maximum negative shear at C. Assume B and D are pins.



Referring to the influence line for the vertical reaction at B, the maximum positive reaction is

$$(B_y)_{max\,(+)} = 1.5(20) + \left[\frac{1}{2}(16 - 0)(1.5)\right](4) + \left[\frac{1}{2}(16 - 0)(1.5)\right](0.8)$$

$$= 87.6\text{ kN}$$ **Ans.**

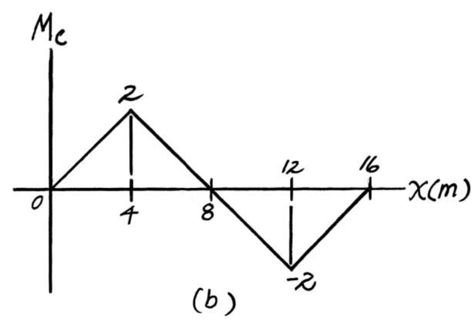


Referring to the influence line for the moment at C shown in Fig. b, the maximum positive moment is

$$(M_c)_{max\,(+)} = 2(20) + \left[\frac{1}{2}(8 - 0)(2)\right](4) + \left[\frac{1}{2}(8 - 0)(2)\right](0.8)$$

$$+ \left[\frac{1}{2}(16 - 8)(-2)\right](0.8)$$

$$= 72.0\text{ kN}\cdot\text{m}$$ **Ans.**



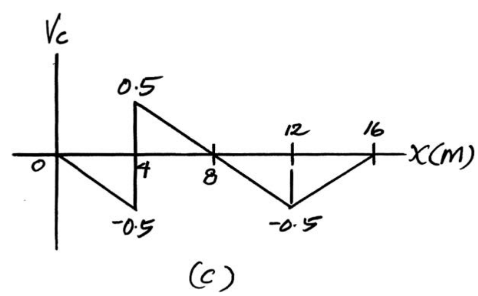
Referring to the influence line for the shear at C shown in, the maximum negative shear is

$$(V_C)_{max\,(-)} = -0.5(20) + \left[\frac{1}{2}(4 - 0)(-0.5)\right](4)$$

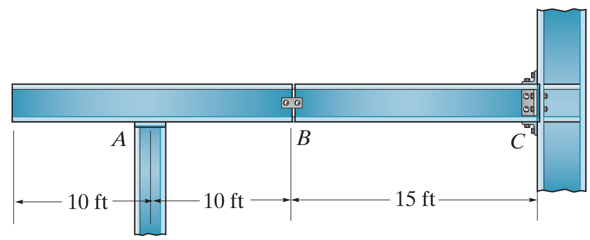
$$+ \left[\frac{1}{2}(16 - 8)(-0.5)\right](4) + \left[\frac{1}{2}(4 - 0)(-0.5)\right](0.8)$$

$$+ \left[\frac{1}{2}(8 - 4)(0.5)\right](0.8) + \left[\frac{1}{2}(16 - 8)(-0.5)\right](0.8)$$

$$= -23.6\text{ kN}$$ **Ans.**

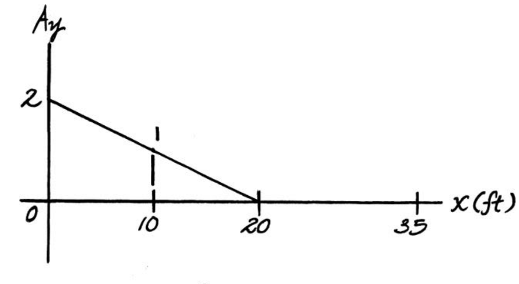


***6–24.** The beam is used to support a dead load of 400 lb/ft, a live load of 2 k/ft, and a concentrated live load of 8 k. Determine (a) the maximum positive vertical reaction at A, (b) the maximum positive shear just to the right of the support at A, and (c) the maximum negative moment at C. Assume A is a roller, C is fixed, and B is pinned.



Referring to the influence line for the vertical reaction at A shown in Fig. a, the maximum positive reaction is

$$(A_y)_{\max(+)} = 2(8) + \left[\frac{1}{2}(20 - 0)^7(2)\right](2 + 0.4) = 64.0 \text{ k} \qquad \textbf{Ans.}$$



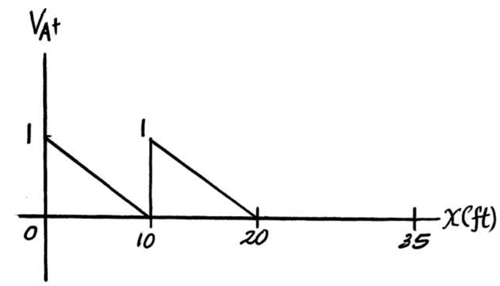
(a)

Referring to the influence line for the shear just to the right to the support at A shown in Fig. b, the maximum positive shear is

$$(V_{A^+})_{\max(+)} = 1(8) + \left[\frac{1}{2}(10 - 0)(1)\right](2 + 0.4)$$

$$+ \left[\frac{1}{2}(20 - 10)(1)\right](2 + 0.4)$$

$$= 32.0 \text{ k} \qquad \textbf{Ans.}$$



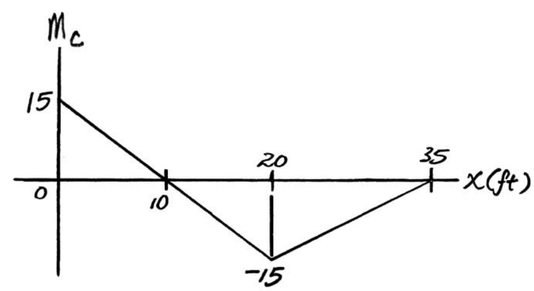
(b)

Referring to the influence line for the moment at C shown in Fig. c, the maximum negative moment is

$$(M_C)_{\max(-)} = -15(8) + \left[\frac{1}{2}(35 - 10)(-15)\right](2) + \left[\frac{1}{2}(10 - 0)(15)\right](0.4)$$

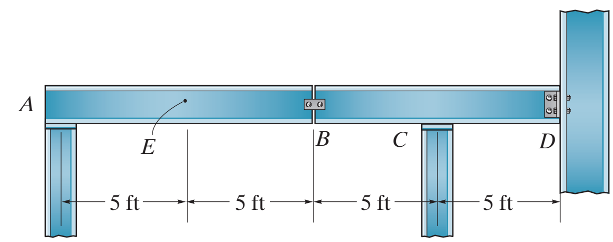
$$+ \left[\frac{1}{2}(35 - 10)(-15)\right](0.4)$$

$$= -540 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$



(c)

6–25. The beam is used to support a dead load of 500 lb/ft, a live load of 2 k/ft, and a concentrated live load of 8 k. Determine (a) the maximum positive (upward) reaction at A, (b) the maximum positive moment at E, and (c) the maximum positive shear just to the right of the support at C. Assume A and C are rollers and D is a pin.



Referring to the influence line for the vertical reaction at A shown in Fig. *a*, the maximum positive vertical reaction is

$$(A_y)_{\max(+)} = 1(8) + \left[\frac{1}{2}(10 - 0)(1)\right](2 + 0.5) = 20.5 \text{ k}$$ **Ans.**

Referring to the influence line for the moment at E shown in Fig. *b*, the maximum positive moment is

$$(M_E)_{\max(+)} = 2.5(8) + \left[\frac{1}{2}(10 - 0)(2.5)\right](2 + 0.5)$$

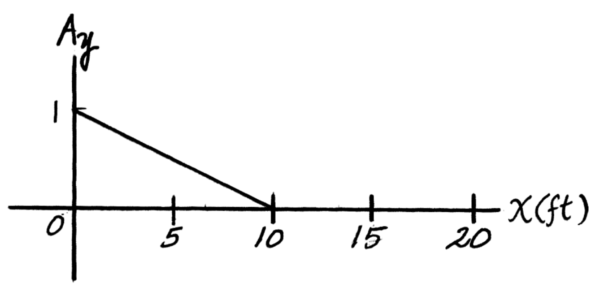
$$= 51.25 \text{ k} \cdot \text{ft}$$ **Ans.**

Referring to the influence line for the shear just to the right of support C, shown in Fig. *c*, the maximum positive shear is

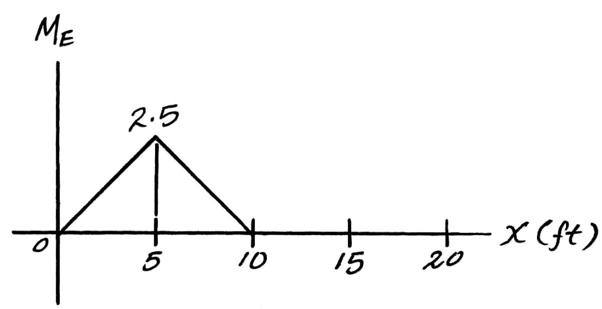
$$(V_{C^+})_{\max(+)} = 1(8) + \left[\frac{1}{2}(15 - 0)(1)\right](2 + 0.5)$$

$$+ \left[\frac{1}{2}(20 - 15)(1)\right](2 + 0.5)$$

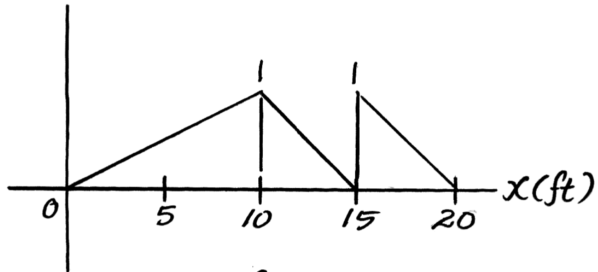
$$= 33.0 \text{ k}$$ **Ans.**



(a)

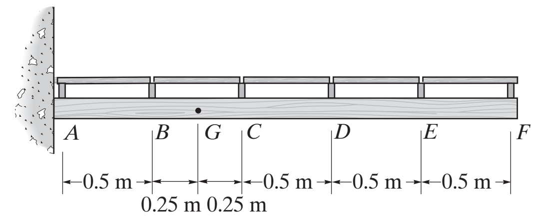


(b)



(c)

6–26. A uniform live load of 1.8 kN/m and a single concentrated live force of 4 kN are placed on the floor beams. Determine (a) the maximum positive shear in panel *BC* of the girder and (b) the maximum moment in the girder at *G*.



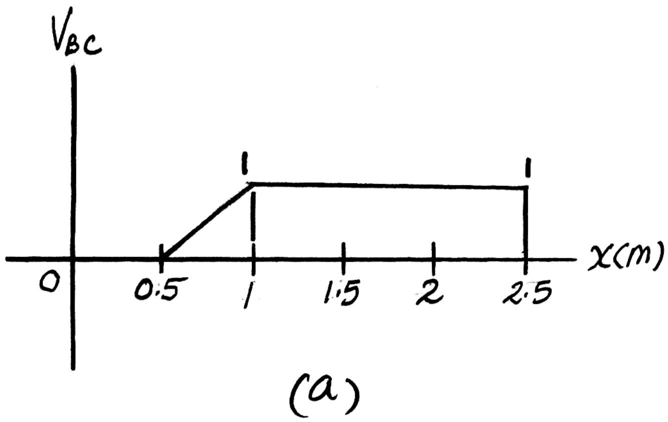
Referring to the influence line for the shear in panel *BC* shown in Fig. *a*, the maximum positive shear is

$$(V_{BC})_{\max(+)} = 1(4) + \left[\frac{1}{2}(1 - 0.5)(1)\right](1.8) + [(2.5 - 1)(1)](1.8) = 7.15 \text{ kN} \quad \textbf{Ans.}$$

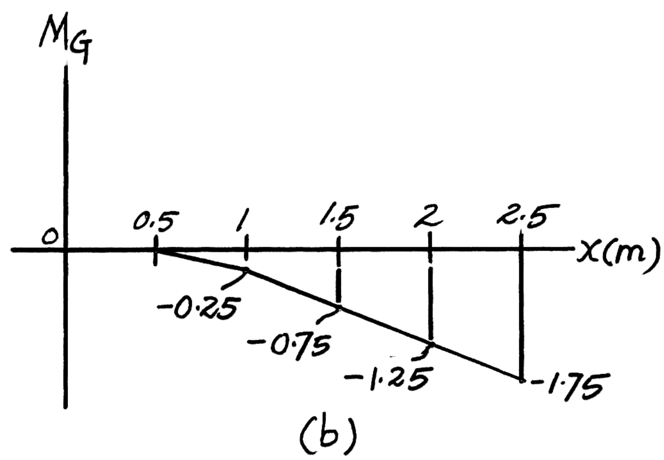
Referring to the influence line for the moment at *G* Fig. *b*, the maximum negative moment is

$$(M_G)_{\max(-)} = -1.75(4)\left[\frac{1}{2}(1 - 0.5)(-0.25)\right](1.8) + \left\{\frac{1}{2}(2.5 - 1)[-0.25 + (-1.75)]\right\}(1.8)$$

$$= -9.81 \text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$

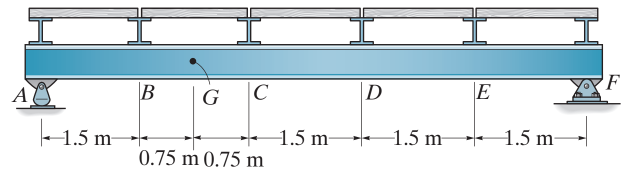


(a)



(b)

6–27. A uniform live load of 2.8 kN/m and a single concentrated live force of 20 kN are placed on the floor beams. If the beams also support a uniform dead load of 700 N/m, determine (a) the maximum positive shear in panel *BC* of the girder and (b) the maximum positive moment in the girder at *G*.



Referring to the influence line for the shear in panel *BC* as shown in Fig. *a*, the maximum position shear is

$$(V_{BC})_{\max(+)} = 0.6(20) + \left[\frac{1}{2}(7.5 - 1.875)(0.6)\right](2.8)$$

$$+ \left[\frac{1}{2}(1.875 - 0)(-0.2)\right](0.7) + \left[\frac{1}{2}(7.5 - 1.875)(0.6)\right](0.7)$$

$$= 17.8 \text{ kN}$$ **Ans.**

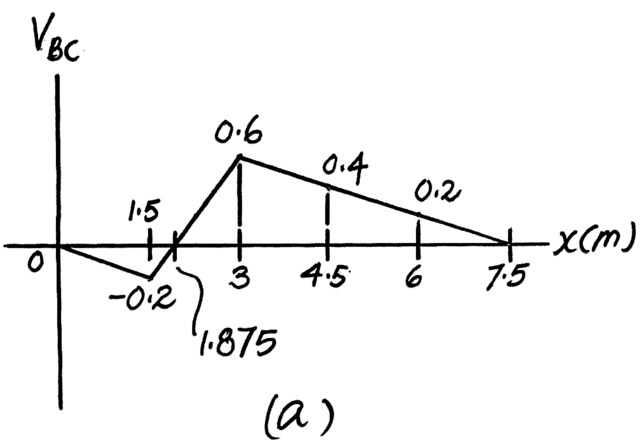
Referring to the influence line for the moment at *G* shown in Fig. *b*, the maximum positive moment is

$$(M_G)_{\max(+)} = 1.35(20)\left[\frac{1}{2}(1.5 - 0)(1.05)\right](2.8 + 0.7)$$

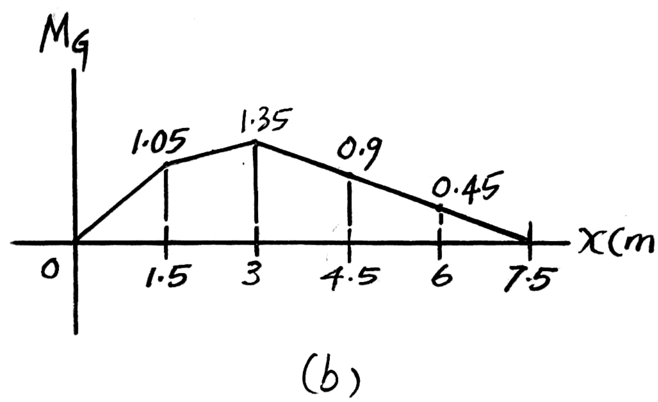
$$+ \left[\frac{1}{2}(3 - 1.5)(1.05 + 1.35)\right](2.8 + 0.7)$$

$$+ \left[\frac{1}{2}(7.5 - 3)(1.35)\right](2.8 + 0.7)$$

$$= 46.7 \text{ kN}\cdot\text{m}$$ **Ans.**

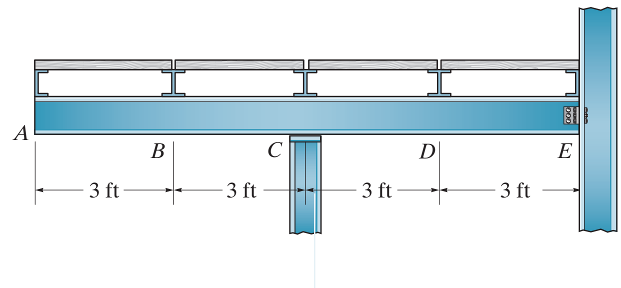


(a)



(b)

***6–28.** A uniform live load of 2 k/ft and a single concentrated live force of 6 k are placed on the floor beams. If the beams also support a uniform dead load of 350 lb/ft, determine (a) the maximum positive shear in panel *CD* of the girder and (b) the maximum negative moment in the girder at *D*. Assume the support at *C* is a roller and *E* is a pin.



Referring to the influence line for the shear in panel *CD* shown in Fig. *a*, the maximum positive shear is

$$(V_{CD})_{\max(+)} = 1(6) + \left[\frac{1}{2}(6-0)(1)\right](2+0.35) + \left[\frac{1}{2}(12-6)(0.5)\right](2+0.35)$$

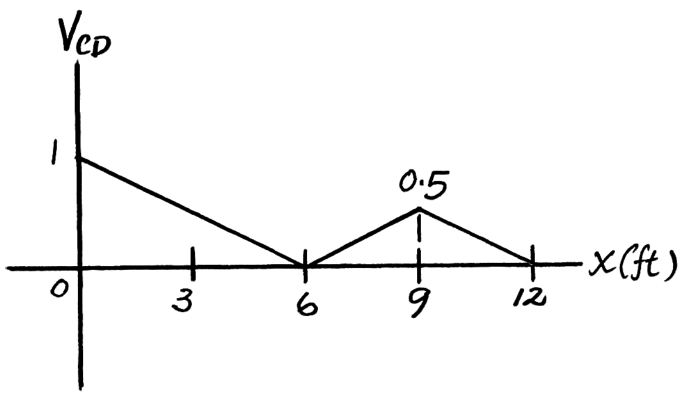
$$= 16.575\text{ k} = 16.6\text{ k} \qquad \textbf{Ans.}$$

Referring to the influence line for the moment at *D* shown in Fig. *b*, the maximum negative moment is

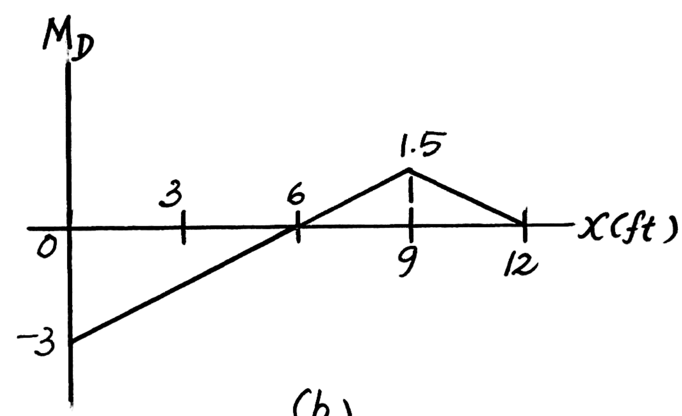
$$M_{D(\max)} = -3(6) + \left[\frac{1}{2}(6-0)(-3)\right](2) + \left[\frac{1}{2}(6-0)(-3)\right](0.35)$$

$$+ \left[\frac{1}{2}(12-6)(1.5)\right](0.35)$$

$$= -37.575\text{ k}\cdot\text{ft} = -37.6\text{ k}\cdot\text{ft} \qquad \textbf{Ans.}$$

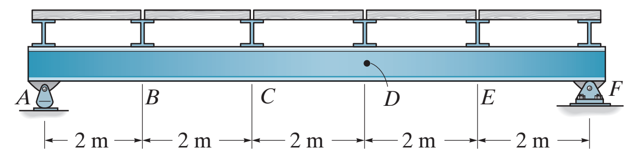


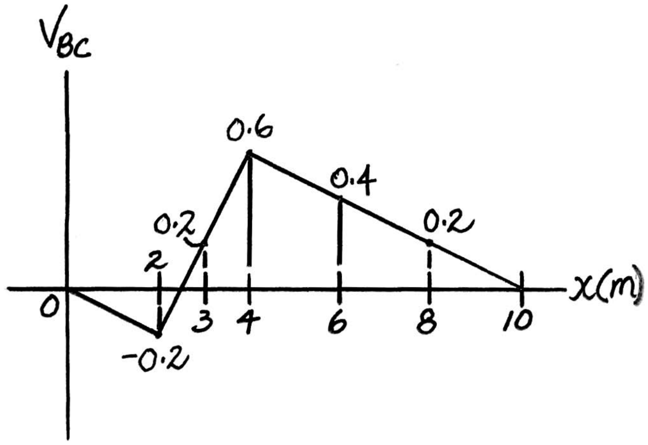
(a)

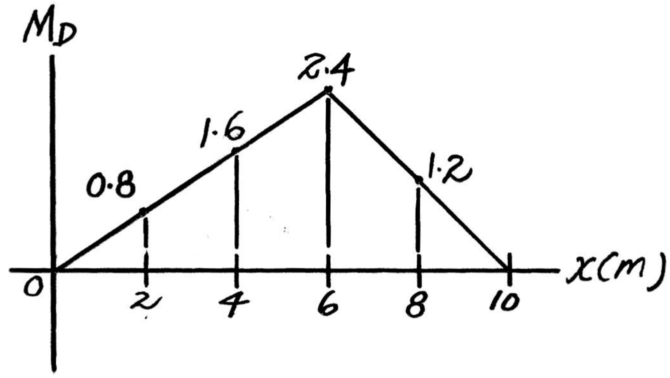


(b)

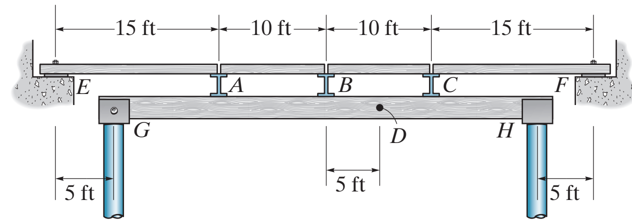
6–29. Draw the influence line for (a) the shear in panel *BC* of the girder, and (b) the moment at *D*.







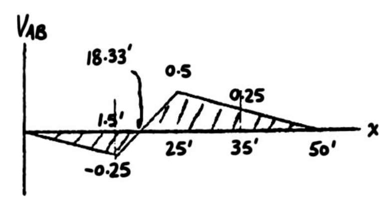
6–30. A uniform live load of 250 lb/ft and a single concentrated live force of 1.5 k are to be placed on the floor beams. Determine (a) the maximum positive shear in panel *AB*, and (b) the maximum moment at *D*. Assume only vertical reaction occur at the supports.

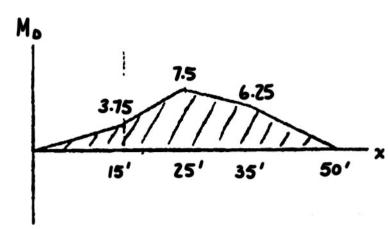


$$(V_{AB})_{\max} = \frac{1}{2}(50 - 18.33)(0.5)(0.250) + 0.5(1.5) = 2.73 \text{ k}$$ **Ans.**

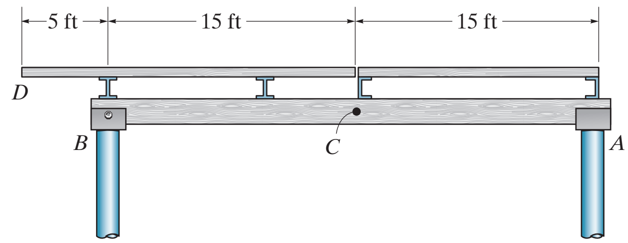
$$(M_D)_{\max} = \left[\frac{1}{2}(3.75)(15) + \frac{1}{2}(3.75 + 7.5)(10) + \frac{1}{2}(7.5 + 6.25)(10) + \frac{1}{2}(6.25)(15)\right](0.250) + 7.5(1.5)$$

$$= 61.25 \text{ k} \cdot \text{ft}$$ **Ans.**



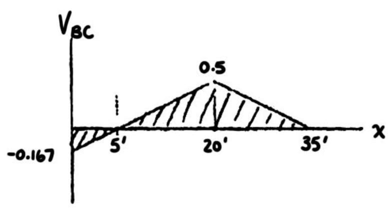


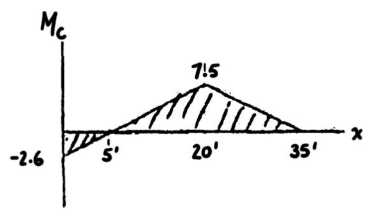
6–31. A uniform live load of 0.6 k/ft and a single concentrated live force of 5 k are to be placed on the top beams. Determine (a) the maximum positive shear in panel *BC* of the girder, and (b) the maximum positive moment at *C*. Assume the support at *B* is a roller and at *D* a pin.



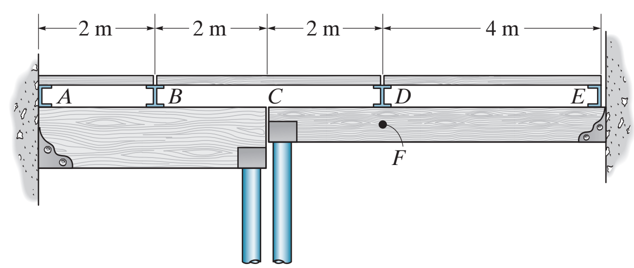
a) $(V_{BC})_{\max} = 5(0.5) + \frac{1}{2}(0.5)(30)(0.6) = 7\text{ k}$ **Ans.**

b) $(M_C)_{\max} = 7.5(5) + 0.6\left[\left(\frac{1}{2}\right)(30)(7.5)\right] = 105\text{ k}\cdot\text{ft}$ **Ans.**

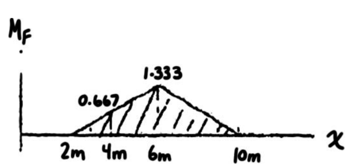




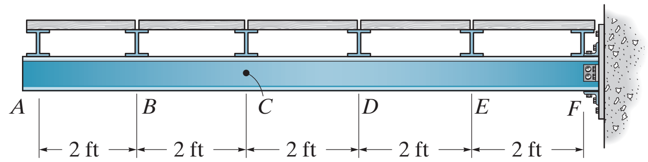
***6–32.** Draw the influence line for the moment at *F* in the girder. Determine the maximum positive live moment in the girder at *F* if a single concentrated live force of 8 kN moves across the top floor beams. Assume the supports for all members can only exert either upward or downward forces on the members.



$(M_F)_{\max} = 1.333(8) = 10.7\text{ kN}\cdot\text{m}$ **Ans.**



6–33. A uniform live load of 4 k/ft and a single concentrated live force of 20 k are placed on the floor beams. If the beams also support a uniform dead load of 700 lb/ft, determine (a) the maximum negative shear in panel DE of the girder and (b) the maximum negative moment in the girder at C.



By referring to the influence line for the shear in panel DE shown in Fig. a, the maximum negative shear is

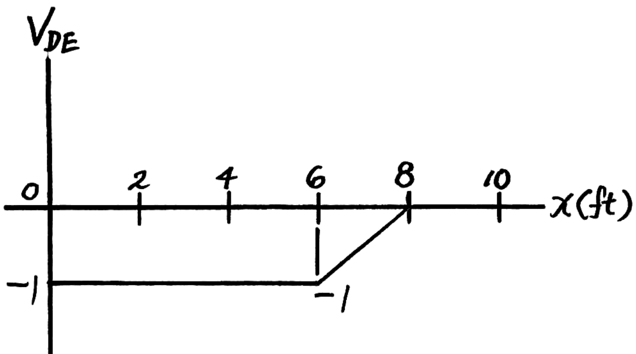
$$(V_{DE})_{\max(-)} = (-1)(20) + [(6-0)(-1)](4+0.7) + \left[\frac{1}{2}(8-6)(-1)\right](4+0.7)$$

$$= -52.9 \text{ k} \qquad \textbf{Ans.}$$

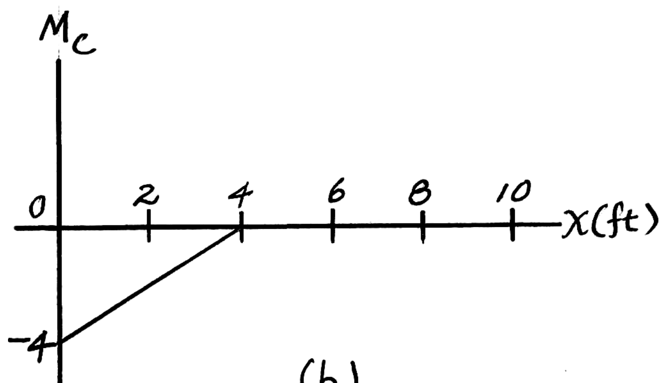
By referring to the influence line for the moment at C shown in Fig. b, the maximum negative moment is

$$(M_C)_{\max(-)} = -4(20) + \left[\frac{1}{2}(4-0)(-4)\right](4+0.7)$$

$$= -118 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$

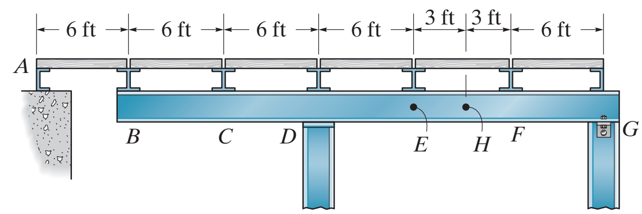


(a)



(b)

6–34. A uniform live load of 0.2 k/ft and a single concentrated live force of 4 k are placed on the floor beams. Determine (a) the maximum positive shear in panel DE of the girder, and (b) the maximum positive moment at H.



Referring to the influence line for the shear in panel DE shown in Fig. a, the maximum positive shear is

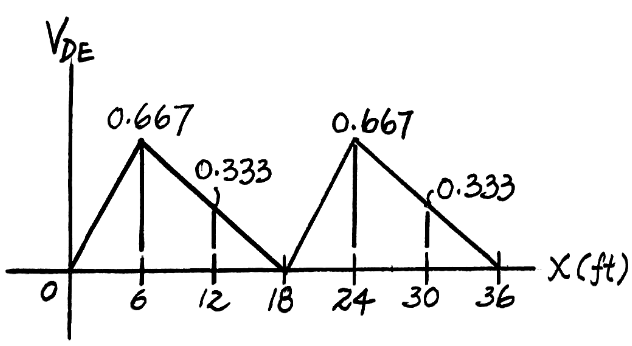
$$(V_{DE})_{\max(+)} = 0.6667(4) + \left[\frac{1}{2}(18 - 0)(0.6667)\right](0.2) + \left[\frac{1}{2}(36 - 18)(0.6667)\right](0.2)$$

$$= 5.07 \text{ k}$$ **Ans.**

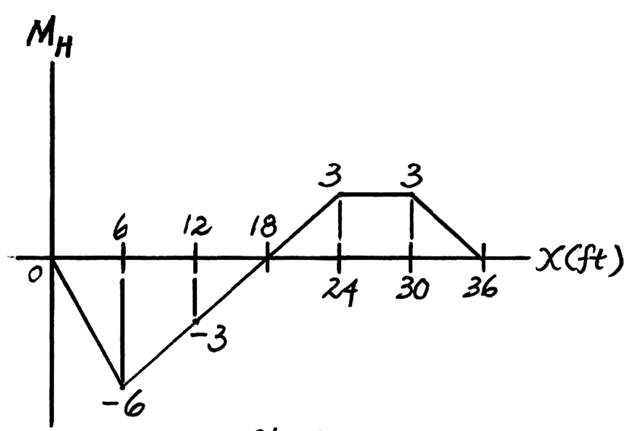
Referring to the influence line for the moment at H shown in Fig. b, the maximum positive moment is

$$(M_H)_{\max(+)} = 3(4) + \left[\frac{1}{2}(24 - 18)(3)\right](0.2) + [(30 - 24)(3)](0.2) + \left[\frac{1}{2}(36 - 30)(3)\right](0.2)$$

$$= 19.2 \text{ k} \cdot \text{ft}$$ **Ans.**

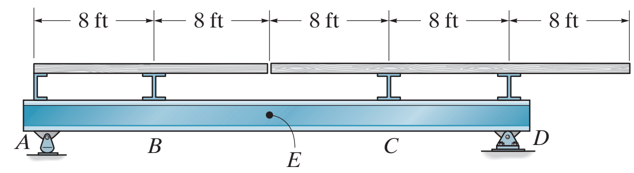


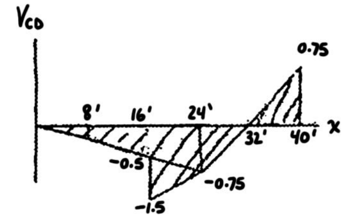
(a)



(b)

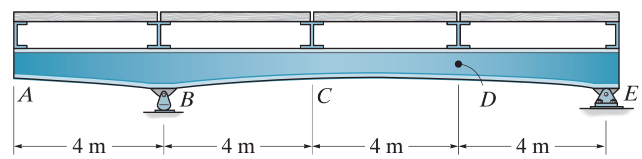
6–35. Draw the influence line for the shear in panel *CD* of the girder. Determine the maximum negative live shear in panel *CD* due to a uniform live load of 500 lb/ft acting on the top beams.





$$(V_{CD})_{\max(-)} = 500\left(\frac{1}{2}\right)(32)(-0.75) = -6\text{ k}$$ **Ans.**

***6–36.** A uniform live load of 6.5 kN/m and a single concentrated live force of 15 kN are placed on the floor beams. If the beams also support a uniform dead load of 600 N/m, determine (a) the maximum positive shear in panel *CD* of the girder and (b) the maximum positive moment in the girder at *D*.



Referring to the influence line for the shear in panel *CD* shown in Fig. *a*, the maximum positive shear is

$$(V_{CD})_{\max(+)} = (0.3333)(15) + \left[\frac{1}{2}(4-0)(0.3333)\right](6.5+0.6)$$

$$+ \left[\frac{1}{2}(16-10)(0.3333)\right](6.5+0.6)$$

$$+ \left[\frac{1}{2}(10-4)(-0.3333)\right](0.6)$$

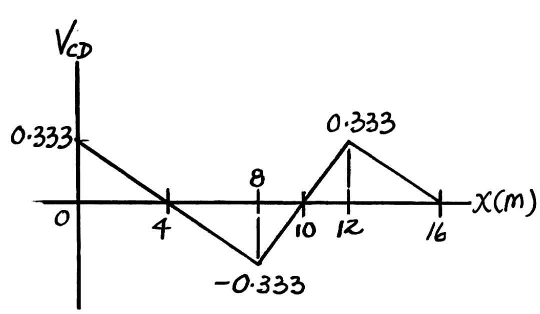
$$= 16.2\text{ kN}$$ **Ans.**

Referring to the influence line for the moment at *D* shown in Fig. *b*, the maximum positive moment is

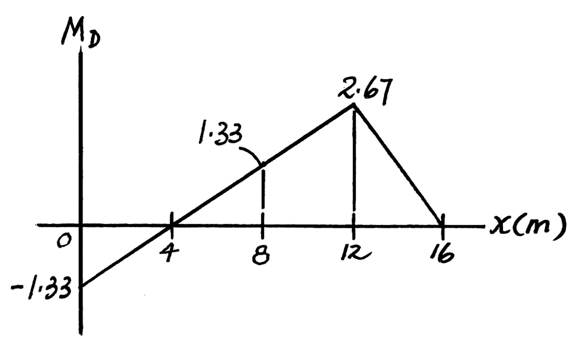
$$(M_D)_{\max(+)} = 2.6667(15) + \left[\frac{1}{2}(16-4)(2.6667)\right](6.5+0.6)$$

$$+ \left[\frac{1}{2}(4-0)(-1.3333)\right](0.6)$$

$$= 152\text{ kN}\cdot\text{m}$$ **Ans.**

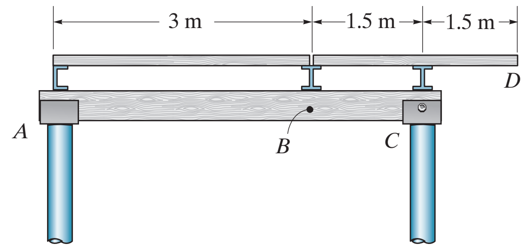


(a)



(b)

6–37. A uniform live load of 1.75 kN/m and a single concentrated live force of 8 kN are placed on the floor beams. If the beams also support a uniform dead load of 250 N/m, determine (a) the maximum negative shear in panel *BC* of the girder and (b) the maximum positive moment at *B*.



By referring to the influence line for the shear in panel *BC* shown in Fig. *a*, the maximum negative shear is

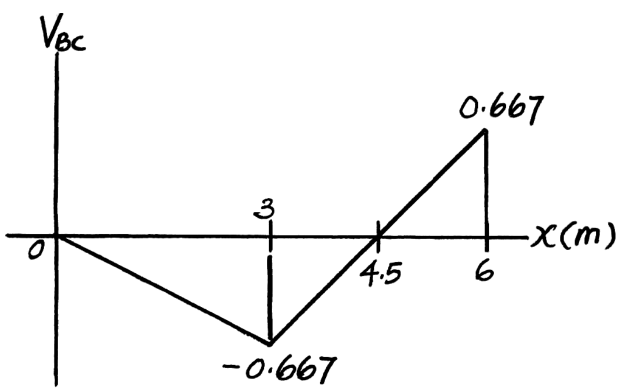
$$(V_{BC})_{\max(-)} = -0.6667(8) + \left[\frac{1}{2}(4.5 - 0)(-0.6667)\right](1.75 + 0.25) + \left[\frac{1}{2}(6 - 4.5)(0.6667)\right](0.25)$$

$$= -8.21 \text{ kN}$$ **Ans.**

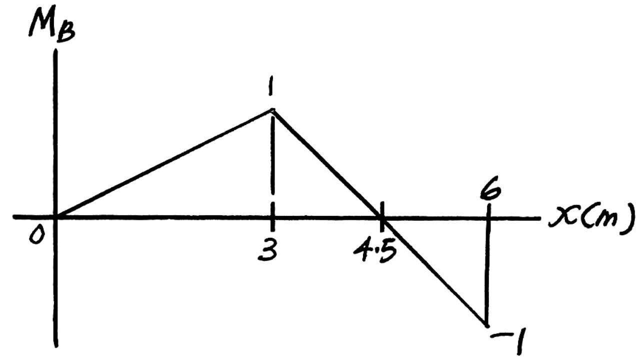
By referring to the influence line for the moment at *B* shown in Fig. *b*, the maximum positive moment is

$$(M_B)_{\max(+)} = 1(8) + \left[\frac{1}{2}(4.5 - 0)(1)\right](1.75 + 0.25) + \left[\frac{1}{2}(6 - 4.5)(-1)\right](0.25)$$

$$= 12.3 \text{ kN} \cdot \text{m}$$ **Ans.**

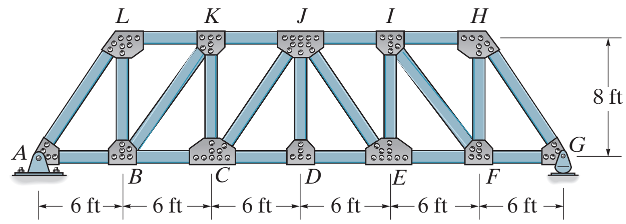


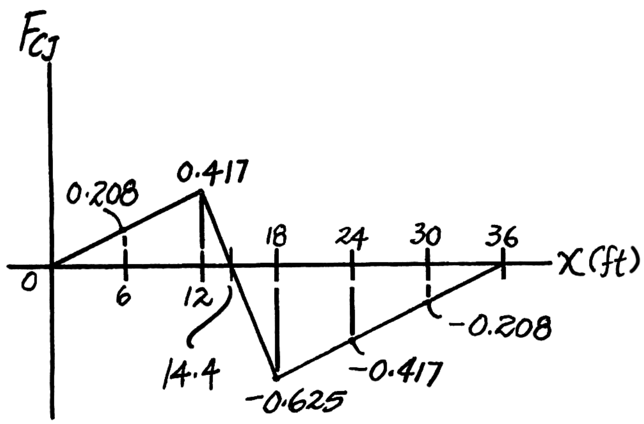
(a)

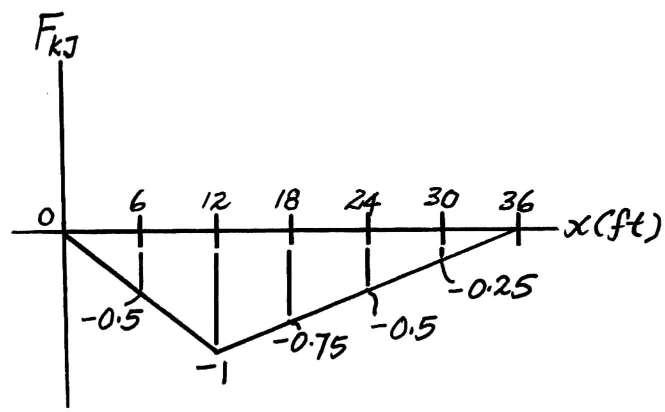


(b)

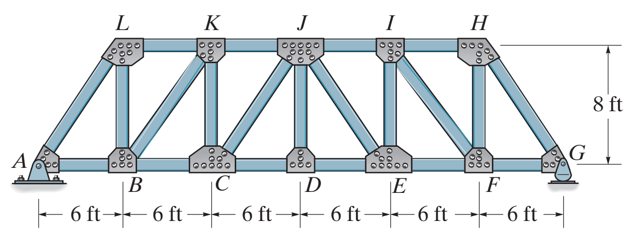
6–38. Draw the influence line for the force in (a) member *KJ* and (b) member *CJ*.

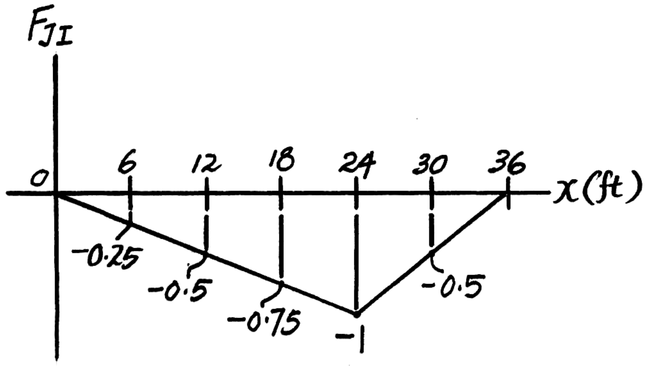


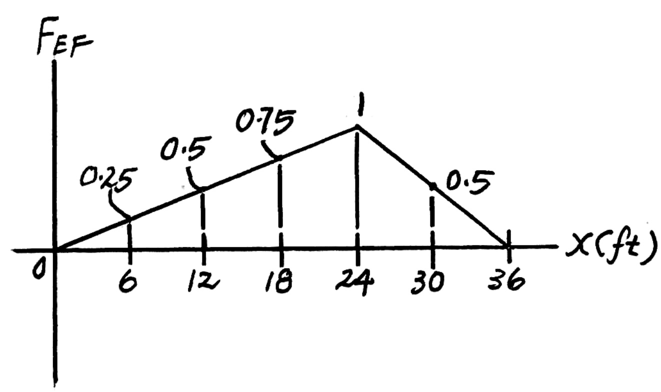


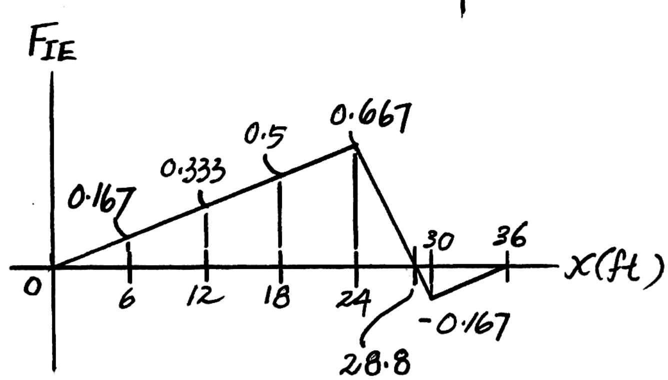


6–39. Draw the influence line for the force in (a) member *JI*, (b) member *IE*, and (c) member *EF*.

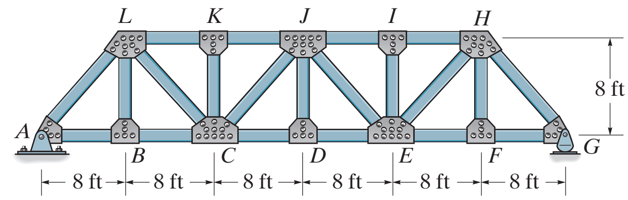


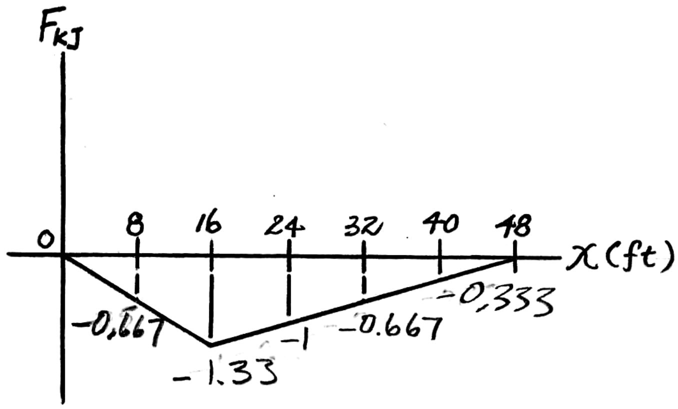




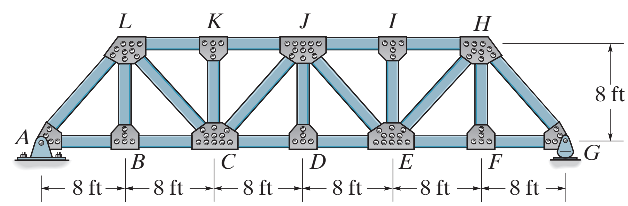


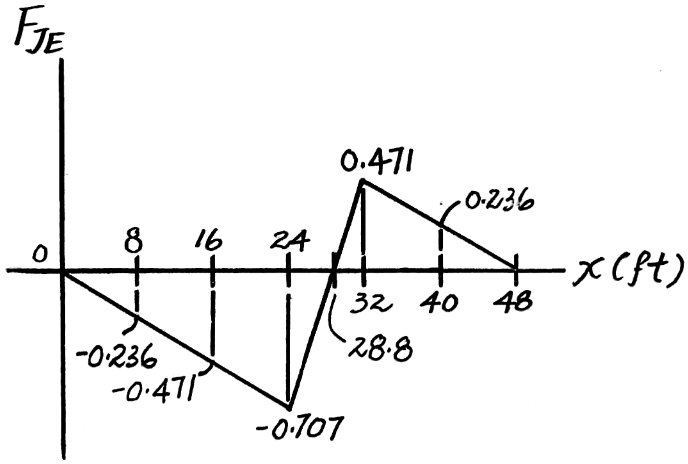
***6–40.** Draw the influence line for the force in member *KJ*.



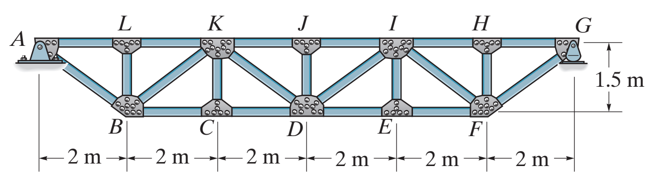


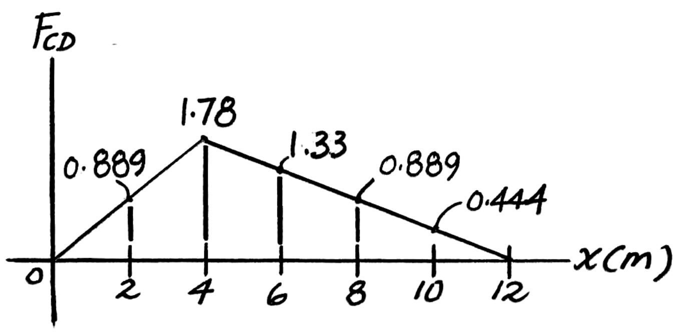
6–41. Draw the influence line for the force in member *JE*.



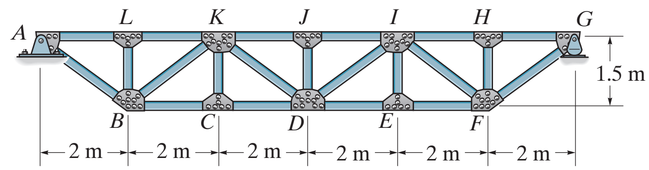


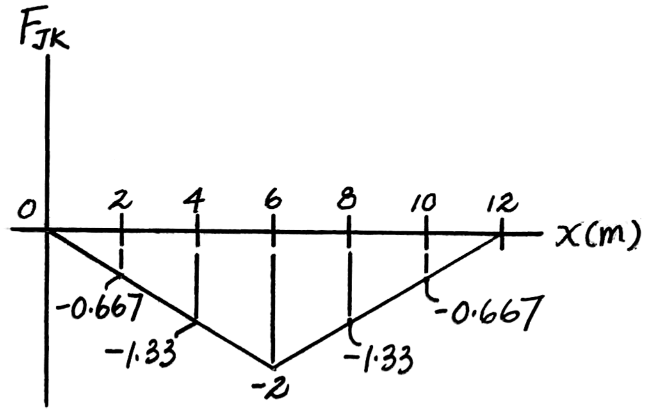
6–42. Draw the influence line for the force in member *CD*.



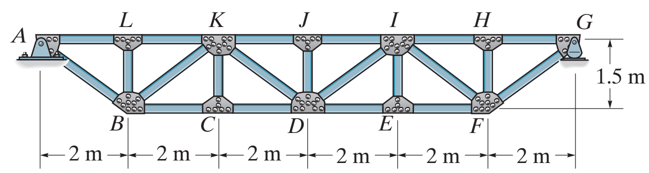


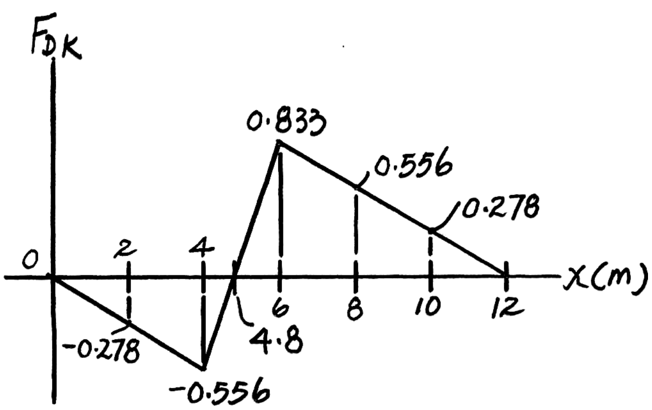
6–43. Draw the influence line for the force in member *JK*.



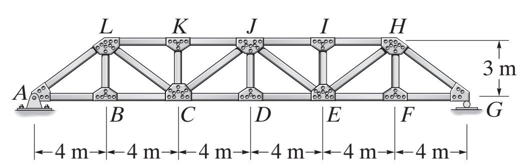


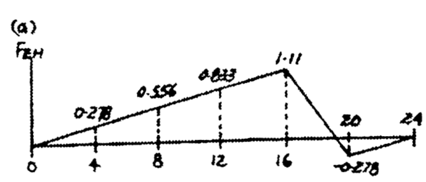
***6–44.** Draw the influence line for the force in member *DK*.

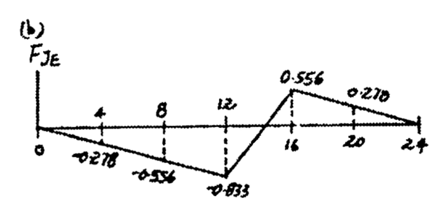




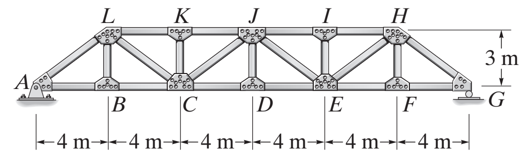
6–45. Draw the influence line for the force in (a) member *EH* and (b) member *JE*.

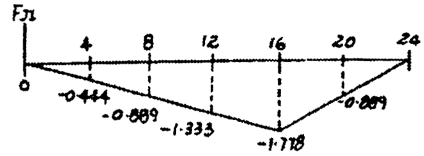




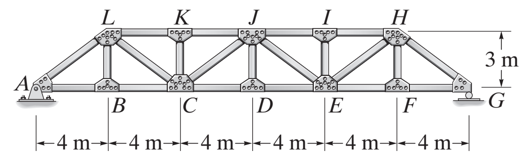


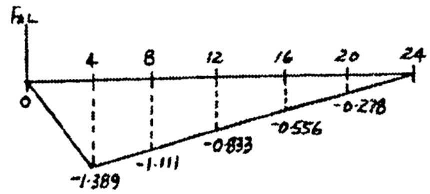
6–46. Draw the influence line for the force in member *JI*.



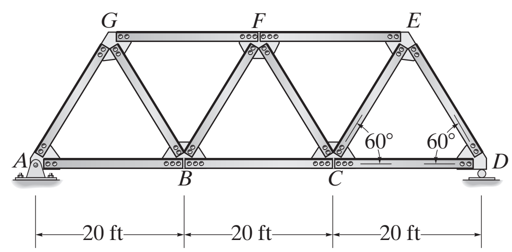


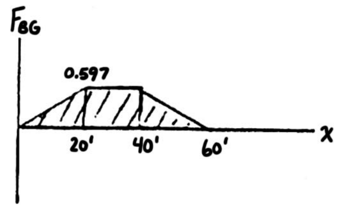
6–47. Draw the influence line for the force in member *AL*.



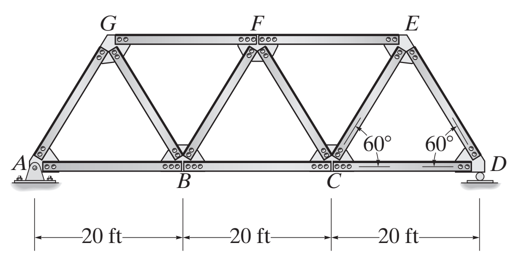


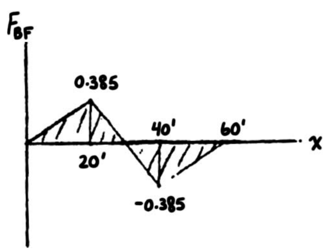
***6–48.** Draw the influence line for the force in member *BC* of the Warren truss. Indicate numerical values for the peaks. All members have the same length.



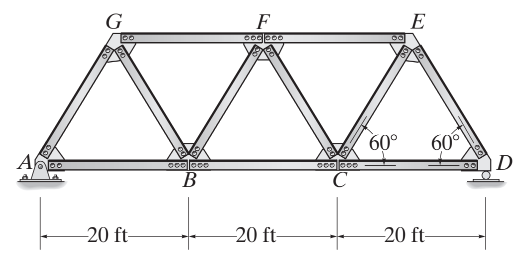


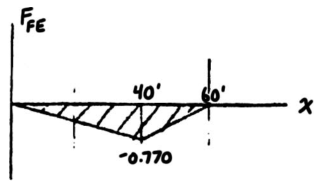
6–49. Draw the influence line for the force in member *BF* of the Warren truss. Indicate numerical values for the peaks. All members have the same length.



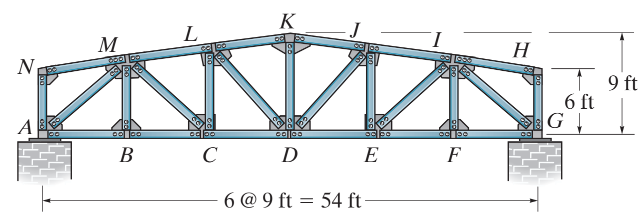


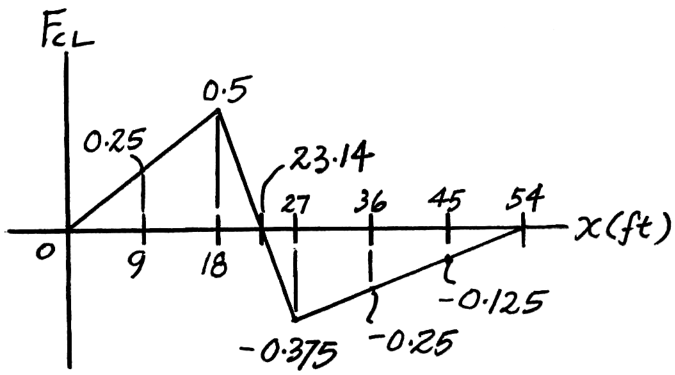
6–50. Draw the influence line for the force in member *FE* of the Warren truss. Indicate numerical values for the peaks. All members have the same length.



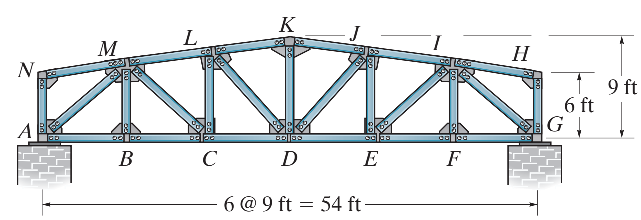


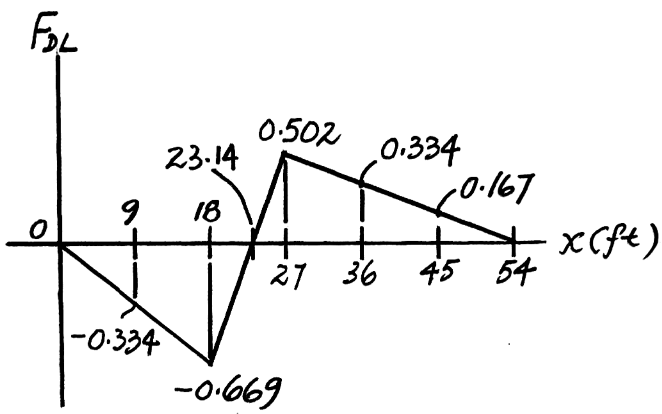
6–51. Draw the influence line for the force in member *CL*.



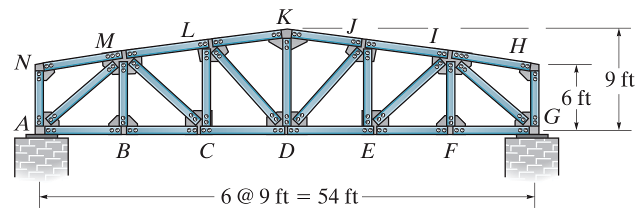


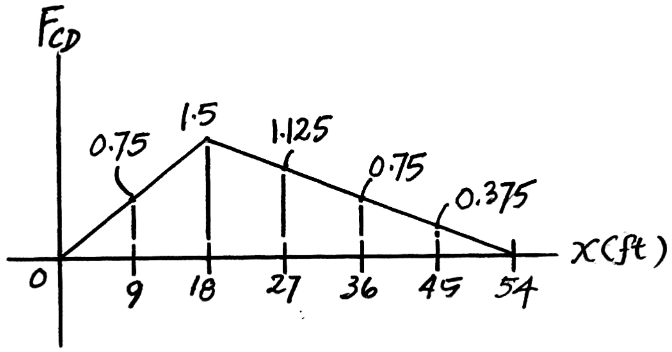
***6–52.** Draw the influence line for the force in member *DL*.



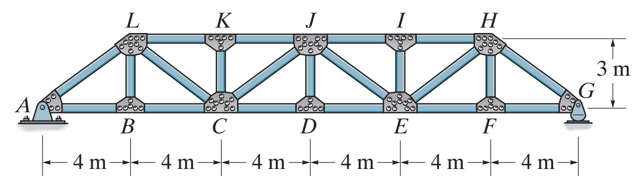


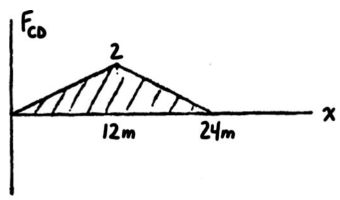
6–53. Draw the influence line for the force in member *CD*.



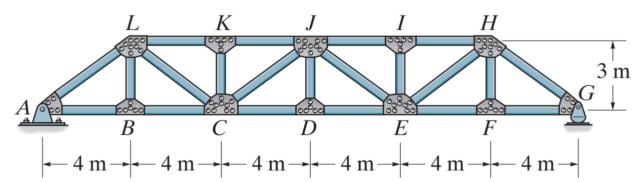


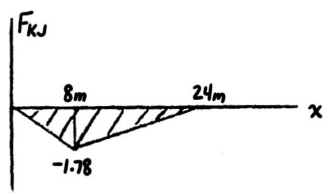
6–54. Draw the influence line for the force in member *CD*.



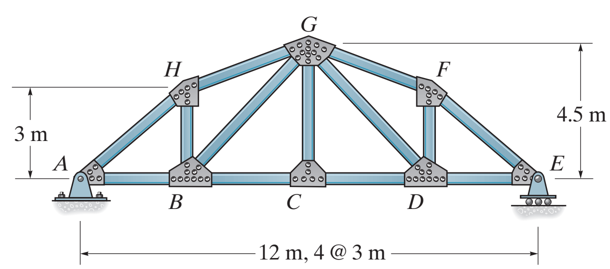


6–55. Draw the influence line for the force in member *KJ*.





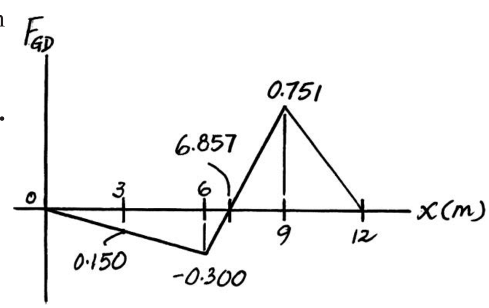
***6–56.** Draw the influence line for the force in member *GD*, then determine the maximum force (tension or compression) that can be developed in this member due to a uniform live load of 3 kN/m that acts on the bridge deck along the bottom cord of the truss.



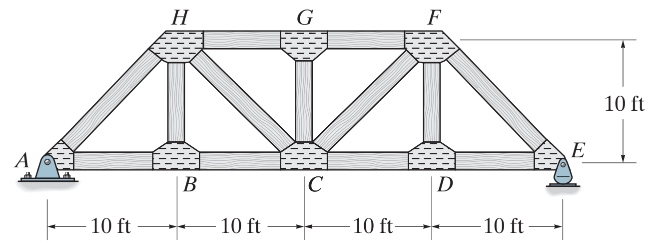
Referring to the influence line for the member force of number *GD*, the maximum tensile and compressive force is

$$(F_{GD})_{\max(+)} = \left[\frac{1}{2}(12 - 6.857)(0.751)\right](3) = 5.79 \text{ kN(T) (Max.)} \quad \textbf{Ans.}$$

$$(F_{GD})_{\min(-)} = \left[\frac{1}{2}(6.857 - 0)(-0.300)\right](3) = -3.09 \text{ kN (C)}$$

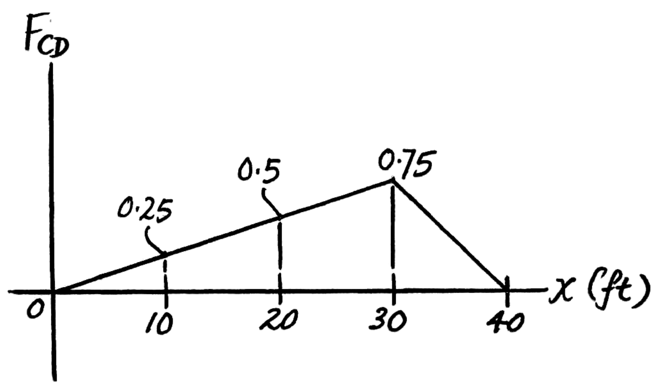


6–57. Draw the influence line for the force in member *CD*, and then determine the maximum force (tension or compression) that can be developed in this member due to a uniform live load of 800 lb/ft which acts along the bottom cord of the truss.

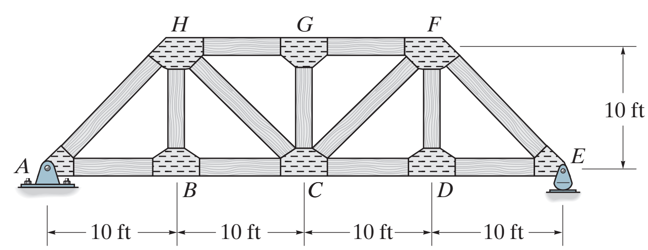


Referring to the influence line for the force of member *CD*, the maximum tensile force is

$$(F_{CD})_{\max(+)} = \left[\frac{1}{2}(40 - 0)(0.75)\right](0.8) = 12.0\text{ k (T)}$$ **Ans.**



6–58. Draw the influence line for the force in member *CF*, and then determine the maximum force (tension or compression) that can be developed in this member due to a uniform live load of 800 lb/ft which is transmitted to the truss along the bottom cord.

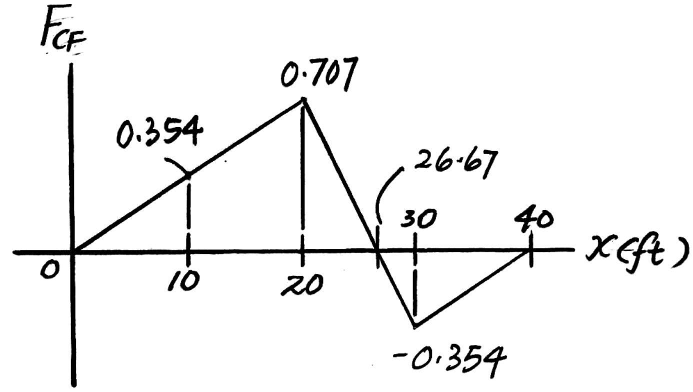


Referring to the influence line for the force in member *CF*, the maximum tensile and compressive force are

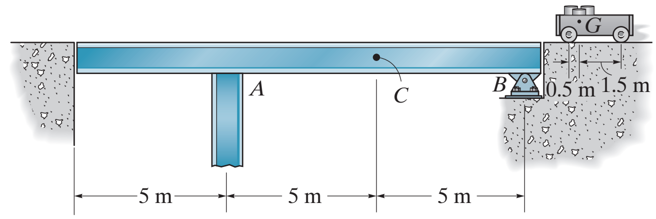
$$(F_{CF})_{\max(+)} = \left[\frac{1}{2}(26.67 - 0)(0.7071)\right](0.8) = 7.54\text{ k (T)}$$ **Ans.**

$$(F_{CF})_{\max(-)} = \left[\frac{1}{2}(40 - 26.67)(-0.3536)\right](0.8)$$

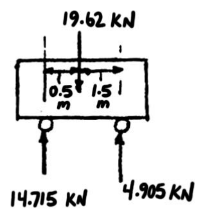
$$= -1.89\text{ k} = 1.89\text{ k (C)}$$

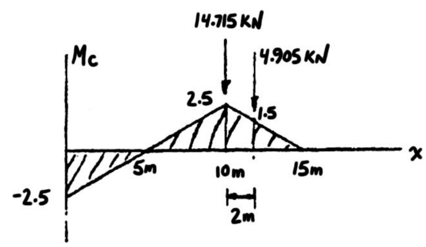


6–59. Determine the maximum live moment at point C on the single girder caused by the moving dolly that has a mass of 2 Mg and a mass center at G. Assume A is a roller.

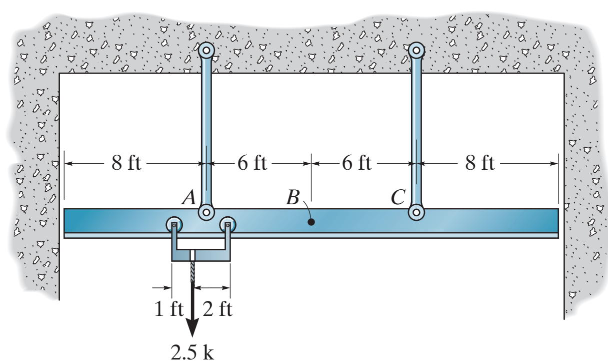


$(M_C)_{max} = 14.715(2.5) + 4.905(1.5) = 44.1 \text{ kN} \cdot \text{m}$ **Ans.**





***6–60.** Determine the maximum live moment in the suspended rail at point B if the rail supports the load of 2.5 k on the trolley.



Check maximum positive moment:

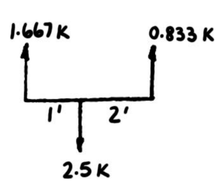
$$\frac{h}{3} = \frac{3}{6}; \quad h = 1.5 \text{ ft}$$

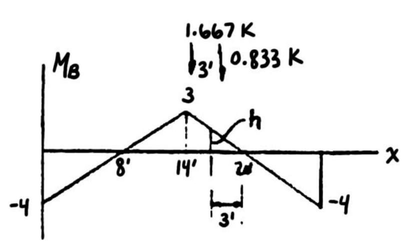
$(M_B)_{max} = 1.667(3) + (0.833)(1.5) = 6.25 \text{ k} \cdot \text{ft}$

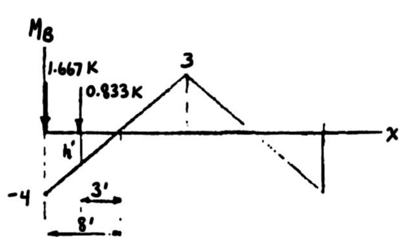
Check maximum negative moment:

$$\frac{h}{5} = \frac{4}{8}; \quad h = 2.5 \text{ ft}$$

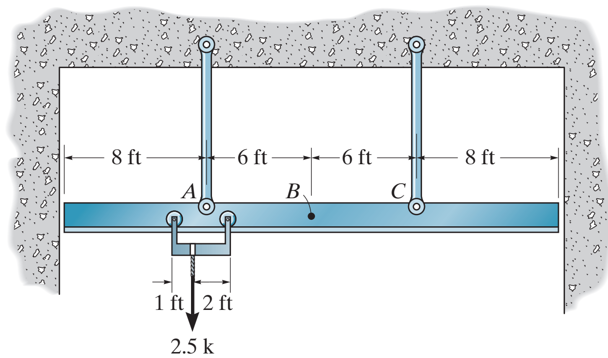
$(M_B)_{max} = 1.667(-4) + (0.833)(-2.5) = -8.75 \text{ k} \cdot \text{ft}$ **Ans.**







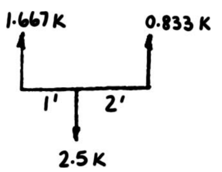
6–61. Determine the maximum positive shear at point B if the rail supports the load of 2.5 k on the trolley.

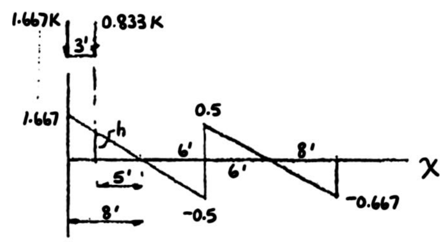


The position for maximum positive shear is shown.

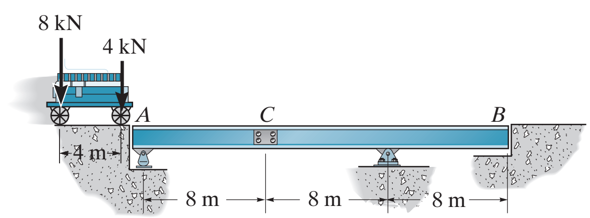
$$\frac{h}{5} = \frac{0.667}{8}; \quad h = 0.41667$$

$$(V_B)_{max} = 1.667(0.667) + 0.833(0.41667) = 1.46 \text{ k} \qquad \textbf{Ans.}$$



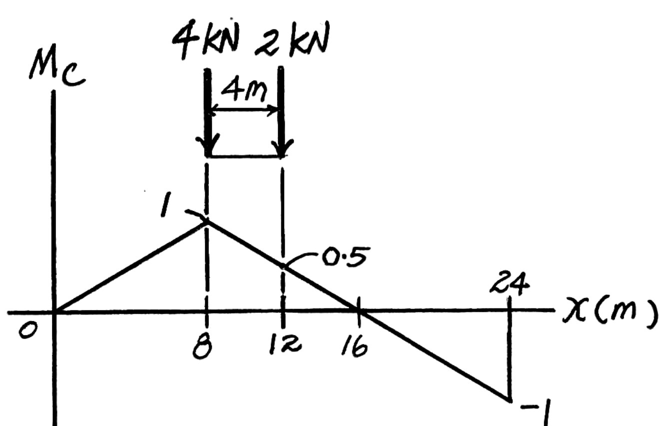


6–62. Determine the maximum positive moment at the splice C on the side girder caused by the moving load which travels along the center of the bridge.



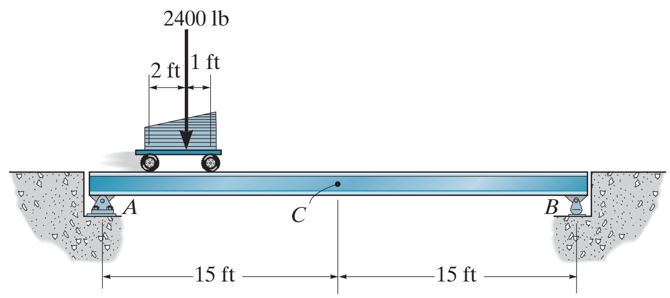
The maximum positive moment at point C occurs when the moving loads are at the position shown in Fig. *a*.

$$(M_C)_{max\,(+)} = 4(4) + 2(2) = 20.0 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



(a)

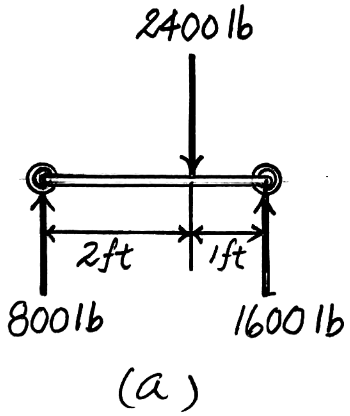
6–63. Determine the maximum moment at *C* caused by the moving load.

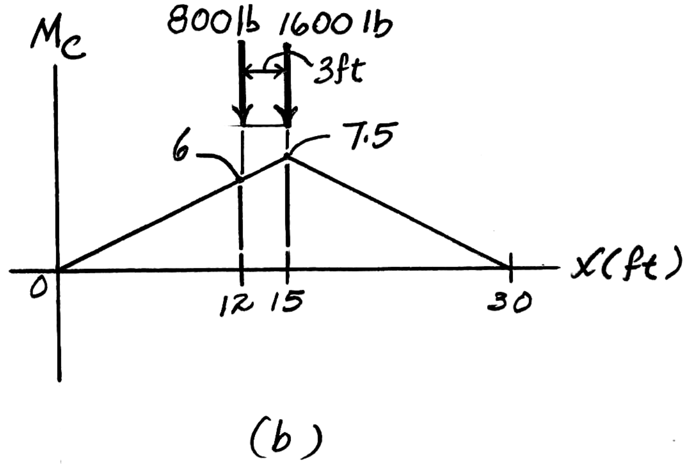


The vertical reactions of the wheels on the girder are as shown in Fig. *a*. The maximum positive moment at point *C* occurs when the moving loads are at the position shown in Fig. *b*.

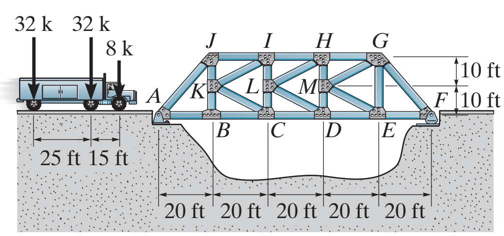
$$(M_C)_{\max(+)} = 7.5(1600) + 6(800) = 16800 \text{ lb} \cdot \text{ft}$$

$$= 16.8 \text{ k} \cdot \text{ft}$$ **Ans.**

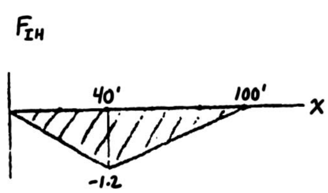


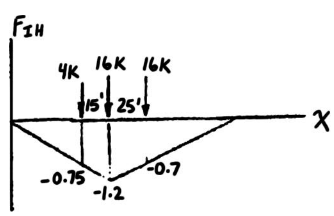


***6–64.** Draw the influence line for the force in member *IH* of the bridge truss. Determine the maximum force (tension or compression) that can be developed in this member due to a 72-k truck having the wheel loads shown. Assume the truck can travel in *either direction* along the *center* of the deck, so that half its load is transferred to each of the two side trusses. Also assume the members are pin-connected at the gusset plates.

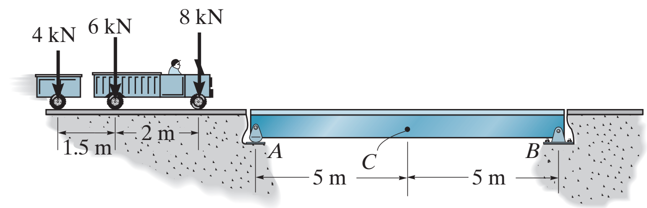


$$(F_{IH})_{\max} = 0.75(4) + 16(0.7) + 16(1.2) = 33.4 \text{ k (C)}$$ **Ans.**





6–65. Determine the maximum positive moment at point C on the single girder caused by the moving load.



Move the 8-kN force 2 m to the right of C. The change in moment is

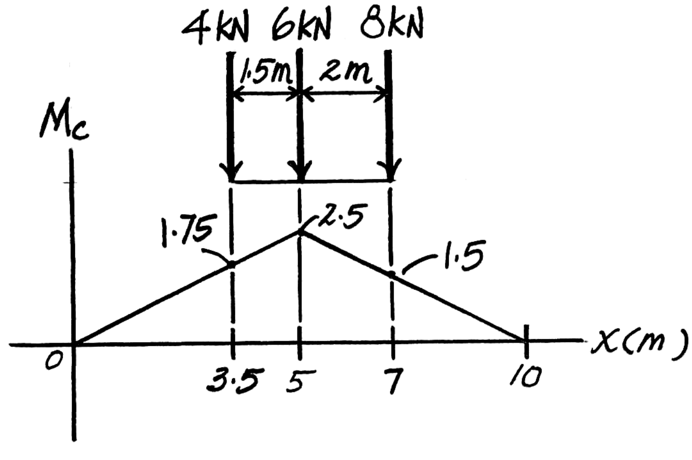
$$\Delta M = 8\left(-\frac{2.5}{5}\right)(2\text{ m}) + 6\left(\frac{2.5}{5}\right)(2) + 4\left(\frac{2.5}{5}\right)(2) = 2\text{ kN}\cdot\text{m}$$

Since ΔM is positive, we must investigate further. Next move the 6 kN force 1.5 m to the right of C, the change in moment is

$$\Delta M = 8\left(-\frac{2.5}{5}\right)(1.5) + 6\left(-\frac{2.5}{5}\right)(1.5) + 4\left(\frac{2.5}{5}\right)(1.5) = -7.5\text{ kN}\cdot\text{m}$$

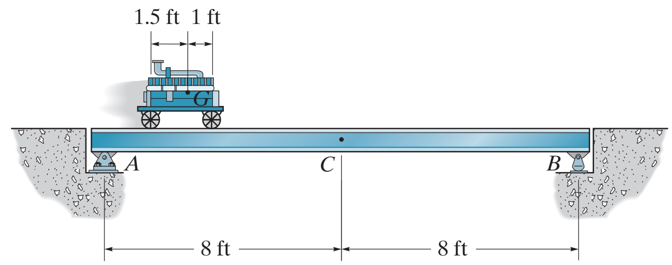
Since ΔM is negative, the case where the 6 kN force is at C will generate the maximum positive moment, Fig. *a*.

$$(M_C)_{\max(+)} = 1.75(4) + 6(2.5) + 8(1.5) = 34.0\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



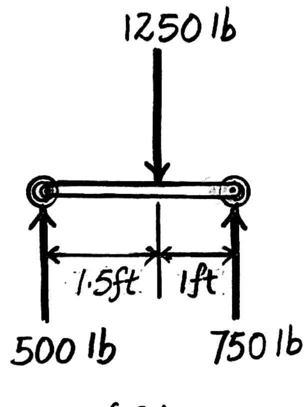
(a)

6–66. The cart has a weight of 2500 lb and a center of gravity at G. Determine the maximum positive moment created in the side girder at C as it crosses the bridge. Assume the car can travel in either direction along the *center* of the deck, so that *half* its load is transferred to each of the two side girders.

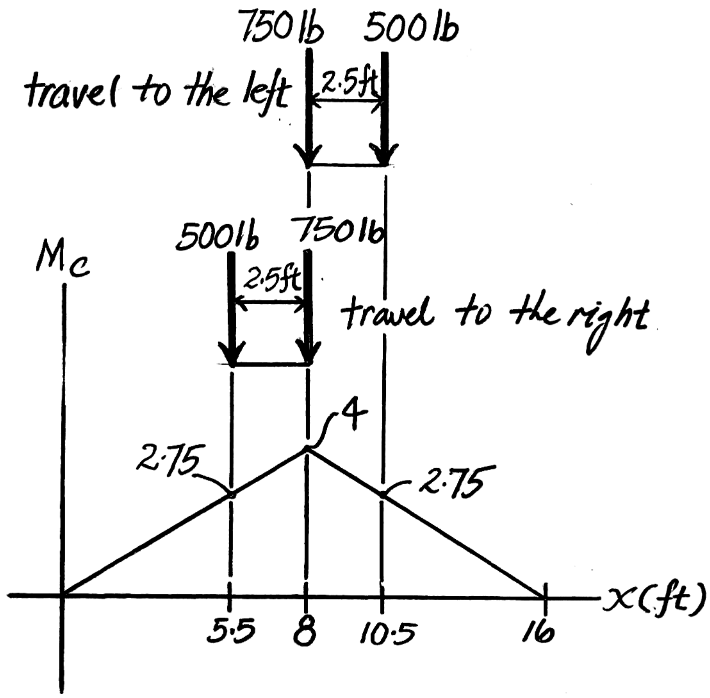


The vertical reaction of wheels on the girder are indicated in Fig. *a*. The maximum positive moment at point C occurs when the moving loads are in the positions shown in Fig. *b*. Due to the symmetry of the influence line about C, the maximum positive moment for both directions are the same.

$(M_C)_{\max(+)} = 4(750) + 2.75(500) = 4375 \text{ lb} \cdot \text{ft} = 4.375 \text{ k} \cdot \text{ft}$ **Ans.**

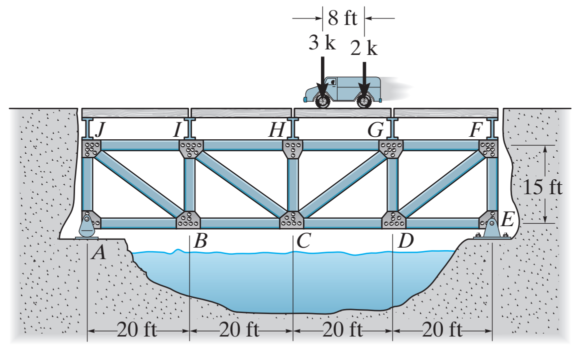


(a)

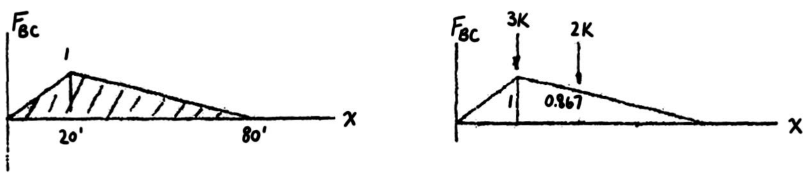


(b)

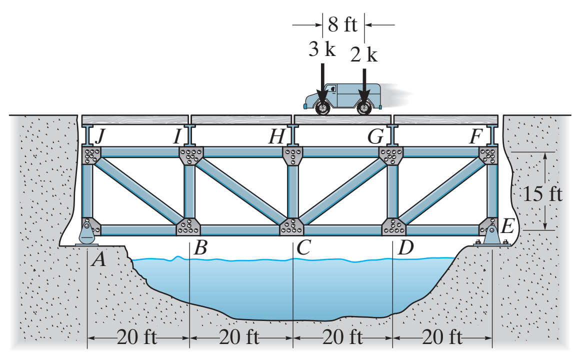
6–67. Draw the influence line for the force in member *BC* of the bridge truss. Determine the maximum force (tension or compression) that can be developed in the member due to a 5-k truck having the wheel loads shown. Assume the truck can travel in *either direction* along the *center* of the deck, so that *half* the load shown is transferred to each of the two side trusses. Also assume the members are pin connected at the gusset plates.



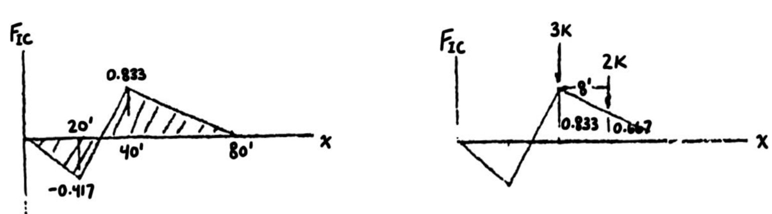
$$(F_{BC})_{max} = \frac{3(1) + 2(0.867)}{2} = 2.37 \text{ k (T)}$$ **Ans.**



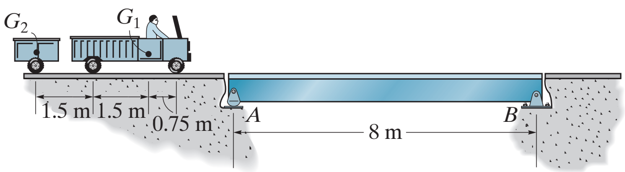
***6–68.** Draw the influence line for the force in member *IC* of the bridge truss. Determine the maximum force (tension or compression) that can be developed in the member due to a 5-k truck having the wheel loads shown. Assume the truck can travel in *either direction* along the *center* of the deck, so that half the load shown is transferred to each of the two side trusses. Also assume the members are pin connected at the gusset plates.



$$(F_{IC})_{max} = \frac{3(0.833) + 2(0.667)}{2} = 1.92 \text{ k (T)}$$ **Ans.**



6–69. The truck has a mass of 4 Mg and mass center at G_1, and the trailer has a mass of 1 Mg and mass center at G_2. Determine the absolute maximum live moment developed in the bridge.



Loading Resultant Location

$$\bar{x} = \frac{9810(0) + 39\,240(3)}{49\,050} = 2.4 \text{ m}$$

One possible placement on bridge is shown in FBD (1),

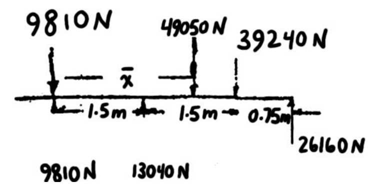
From the segment (2):

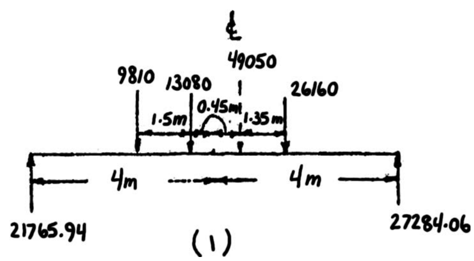
$$M_{\max} = 27\,284(4.45) - 26\,160(2.25) = 62.6 \text{ kN} \cdot \text{m}$$

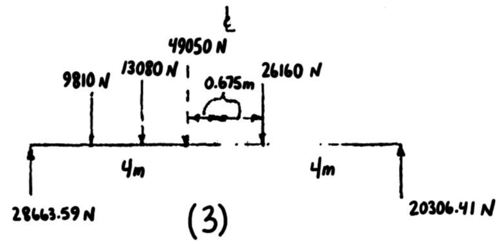
Another possible placement on bridge is shown in Fig. (3),

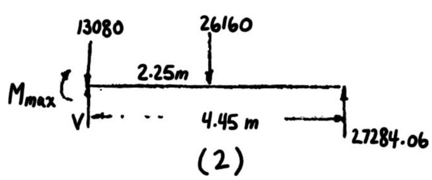
From the segment (4):

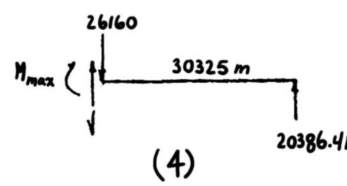
$$M_{\max} = 20\,386.41(3.325) = 67.8 \text{ kN} \cdot \text{m}$$ **Ans.**



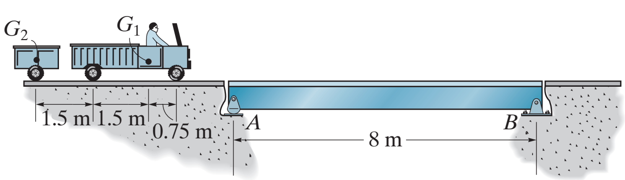






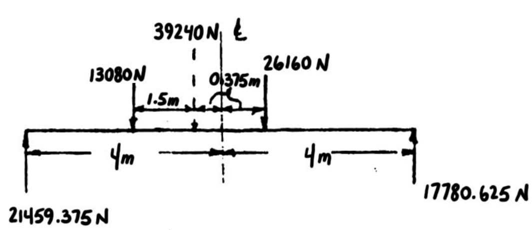


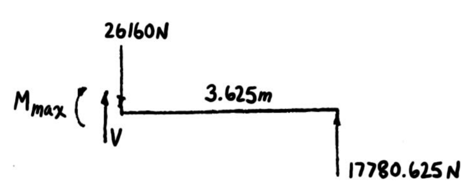
6–70. Determine the absolute maximum live moment in the bridge in Problem 6–69 if the trailer is removed.



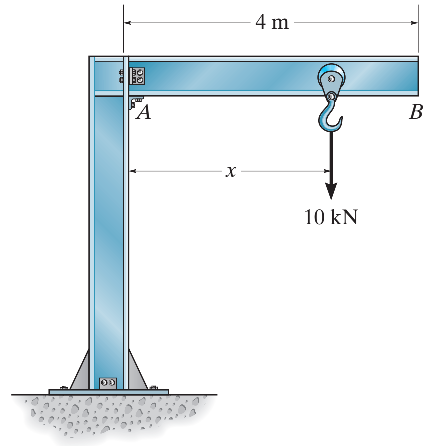
Placement is shown in FBD (1). Using segment (2):

$$M_{\max} = 17\,780.625(3.625) = 64.5 \text{ kN} \cdot \text{m}$$ **Ans.**





6–71. Determine the absolute maximum live shear and absolute maximum moment in the jib beam *AB* due to the 10-kN loading. The end constraints require $0.1 \text{ m} \leq x \leq 3.9 \text{ m}$.



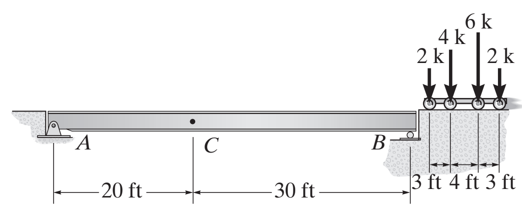
Abs. max. shear occurs when $0.1 \leq x \leq 3.9$ m

$V_{max} = 10$ kN **Ans.**

Abs. max. moment occurs when $x = 3.9$ m

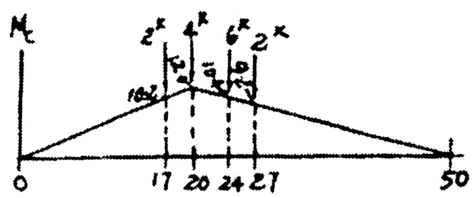
$M_{max} = -10(3.9) = -39 \text{ kN} \cdot \text{m}$ **Ans.**

***6–72.** Determine the maximum live moment at *C* caused by the moving loads.

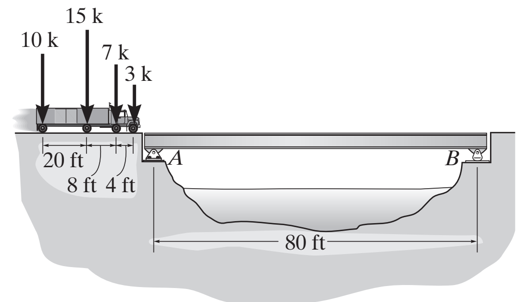


The worst case is

$$(M_C)_{max} = 2(10.2) + 4(12.0) + 6(10.4) + 2(9.2) = 149 \text{ k} \cdot \text{ft}$$ **Ans.**



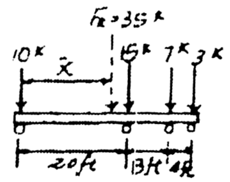
6–73. Determine the absolute maximum moment in the girder bridge due to the truck loading shown. The load is applied directly to the girder.

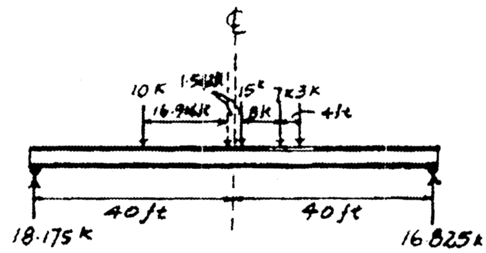


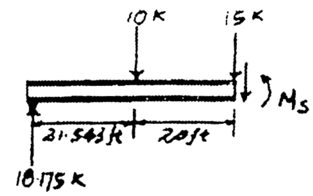
$$\bar{x} = \frac{15(20) + 7(28) + 3(32)}{35} = 16.914 \text{ ft}$$

$$\circlearrowleft + \sum M_{max} = 0; \qquad M_{max} + 10(20) - 18.175(41.543) = 0$$

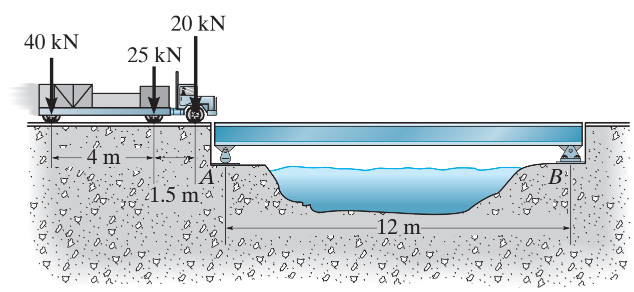
$$M_{max} = 555 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$







6–74. Determine the absolute maximum shear in the beam due to the loading shown.



The maximum shear occurs when the moving loads are positioned either with the 40 kN force just to the right of the support at A, Fig. a, or with the 20 kN force just to of the support it B, Fig. b. Referring to Fig. a,

$$\circlearrowleft + \sum M_B = 0; \quad 40(12) + 25(8) + 20(6.5) - A_y(12) = 0$$

$$A_y = 67.5 \text{ kN}$$

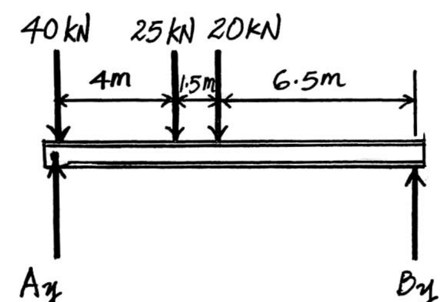
Referring to Fig. b,

$$\circlearrowleft + \sum M_A = 0; \quad B_y(12) - 20(12) - 25(10.5) - 40(6.5) = 0$$

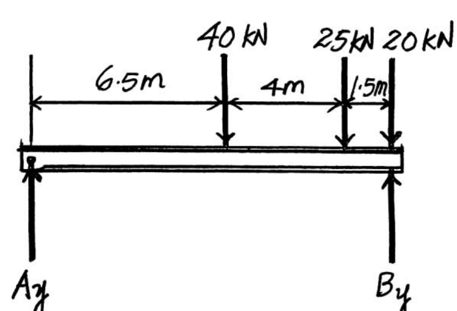
$$B_y = 63.54 \text{ kN}$$

Therefore, the absolute maximum shear occurs for the case in Fig. a,

$$V_{\substack{abs \\ max}} = A_y = 67.5 \text{ kN} \qquad \textbf{Ans.}$$

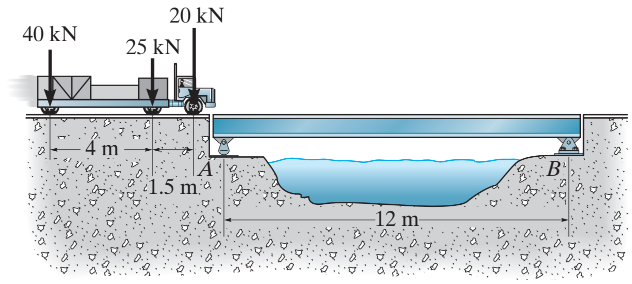


(a)



(b)

6–75. Determine the absolute maximum moment in the beam due to the loading shown.

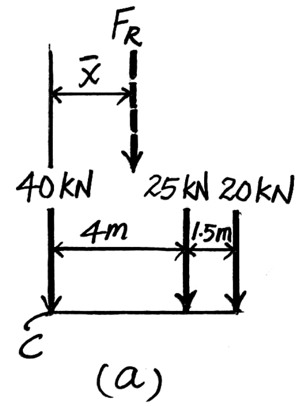


Referring to Fig. *a*, the location of F_R for the moving load is

$$+\downarrow F_R = \sum F_y; \qquad F_R = 40 + 25 + 20 = 85 \text{ kN}$$

$$\circlearrowleft + F_R\bar{x} = \sum M_C; \qquad -85\bar{x} = -25(4) - 20(5.5)$$

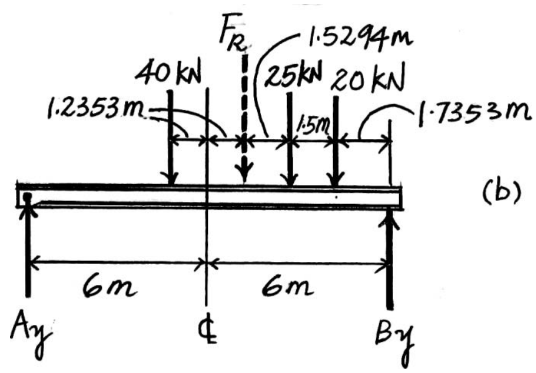
$$\bar{x} = 2.4706 \text{ m}$$



Assuming that the absolute maximum moment occurs under 40 kN force, Fig. *b*.

$$\circlearrowleft + \sum M_B = 0; \quad 20(1.7353) + 25(3.2353) + 40(7.2353) - A_y(12) = 0$$

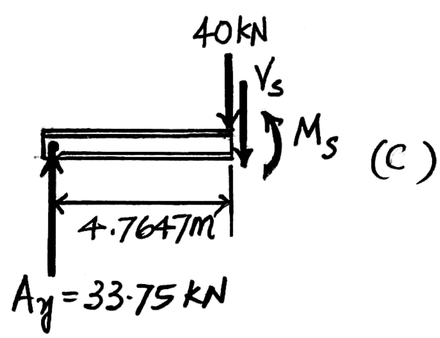
$$A_y = 33.75 \text{ kN}$$



Referring to Fig. *c*,

$$\circlearrowleft + \sum M_S = 0; \qquad M_S - 33.75(4.7647) = 0$$

$$M_S = 160.81 \text{ kN}\cdot\text{m}$$



Assuming that the absolute moment occurs under 25 kN force, Fig. *d*.

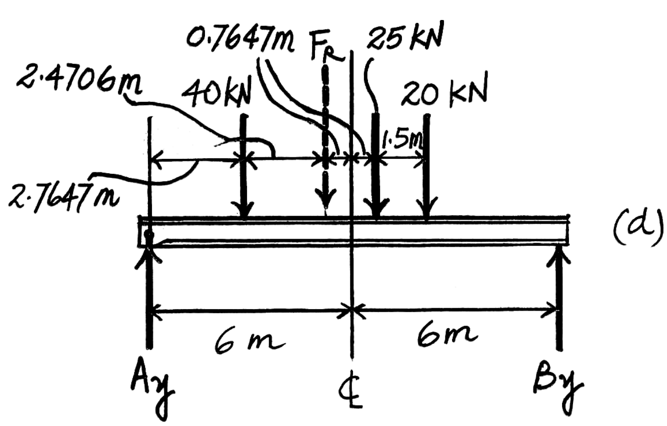
$$\circlearrowleft + \sum M_A = 0; \quad B_y(12) - 40(2.7647) - 25(6.7647) - 20(8.2647) = 0$$

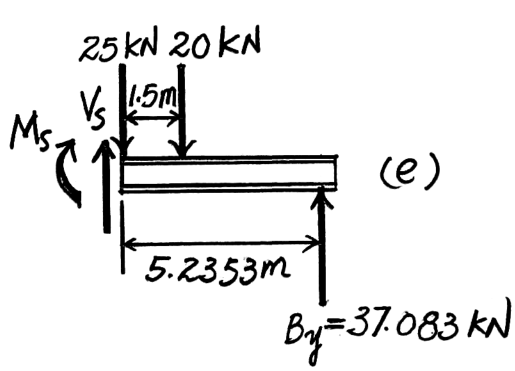
$$B_y = 37.083 \text{ kN}$$

Referring to Fig. *e*,

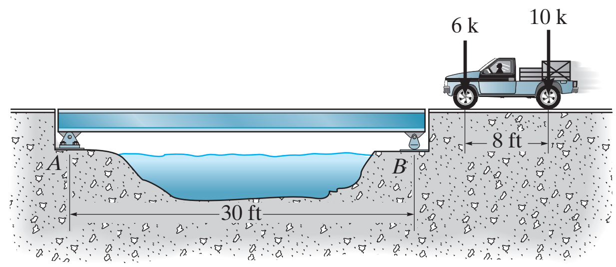
$$\circlearrowleft + \sum M_S = 0; \qquad 37.083(5.2353) - 20(1.5) - M_S = 0$$

$$M_S = 164.14 \text{ kN}\cdot\text{m} = 164 \text{ kN}\cdot\text{m (Abs. Max.)} \qquad \textbf{Ans.}$$

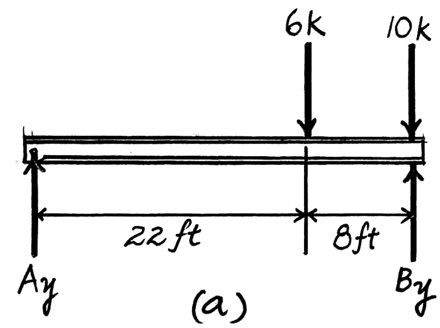




***6–76.** Determine the absolute maximum shear in the bridge girder due to the loading shown.



By inspection the maximum shear occurs when the moving loads are position with the 10 k force just to the left of the support at B, Fig. b.



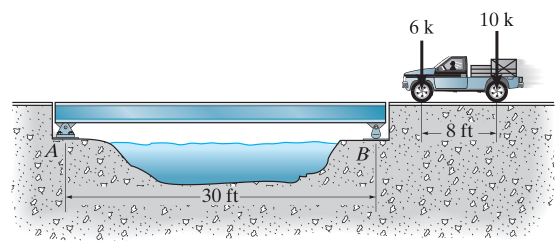
$\zeta + \sum M_A = 0; \quad B_y(30) - 6(22) - 10(30) = 0$

$B_y = 14.4 \text{ k}$

Therefore, the absolute maximum shear is

$V_{\substack{\text{abs}\\\text{max}}} = B_y = 14.4 \text{ k}$ **Ans.**

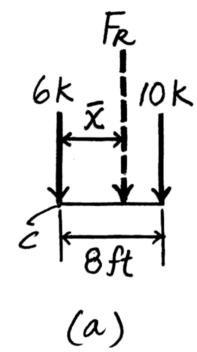
6–77. Determine the absolute maximum moment in the bridge girder due to the loading shown.



Referring to Fig. a, the location of F_R for the moving load is

$+\downarrow F_R = \sum F_y; \quad -F_R = -6 - 10 \quad F_R = 16 \text{ k}$

$\zeta + F_R\bar{x} = \sum M_C; \quad -16\bar{x} = -10(8) \quad \bar{x} = 5 \text{ ft}$

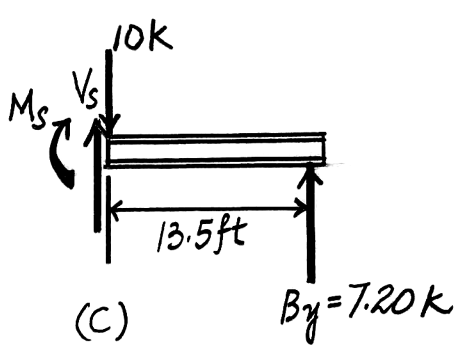


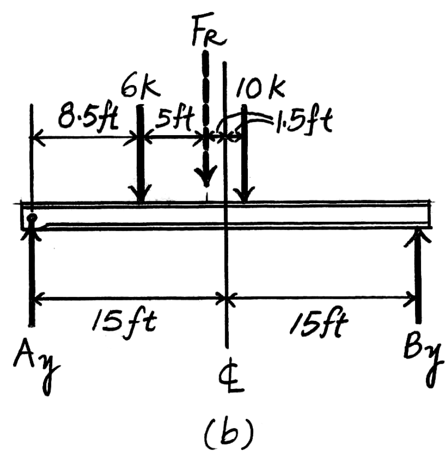
By observation, the absolute maximum moment occurs under the 10-k force, Fig. b,

$\zeta + \sum M_A = 0; \quad B_y(30) - 6(8.5) - 10(16.5) = 0 \quad B_y = 7.20 \text{ k}$

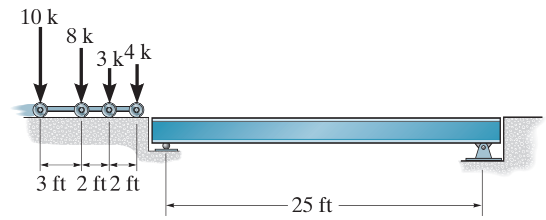
Referring to Fig. c,

$\zeta + \sum M_S = 0; \quad 7.20(13.5) - M_S = 0 \quad M_S = 97.2 \text{ k} \cdot \text{ft (Abs. Max.)}$ **Ans.**





6–78. Determine the absolute maximum moment in the girder due to the loading shown.

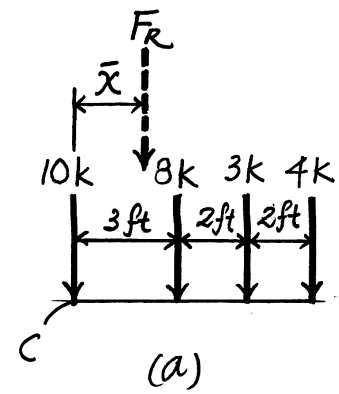


Referring to Fig. *a*, the location of F_R for the moving load is

$+\downarrow F_R = \sum F_y; \qquad F_R = 10 + 8 + 3 + 4 = 25 \text{ k}$

$\circlearrowleft + F_R\bar{x} = \sum M_C; \qquad -25\bar{x} = -8(3) - 3(5) - 4(7)$

$$\bar{x} = 2.68 \text{ ft.}$$



Assuming that the absolute maximum moment occurs under 10 k load, Fig. *b*,

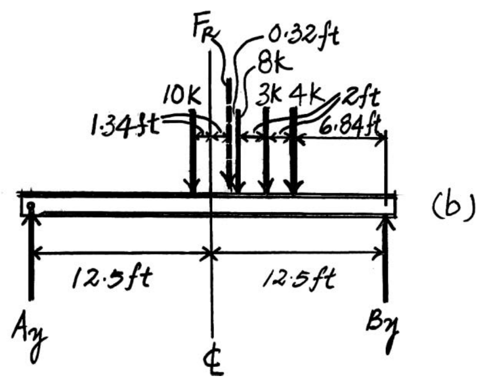
$\circlearrowleft + \sum M_B = 0; \qquad 4(6.84) + 3(8.84) + 8(10.84) + 10(13.84) - A_y(25) = 0$

$$A_y = 11.16 \text{ k}$$

Referring to Fig. *c*,

$\circlearrowleft + \sum M_S = 0; \qquad M_S - 11.16(11.16) = 0$

$$M_S = 124.55 \text{ k} \cdot \text{ft}$$



Assuming that the absolute maximum moment occurs under the 8-k force, Fig. *d*,

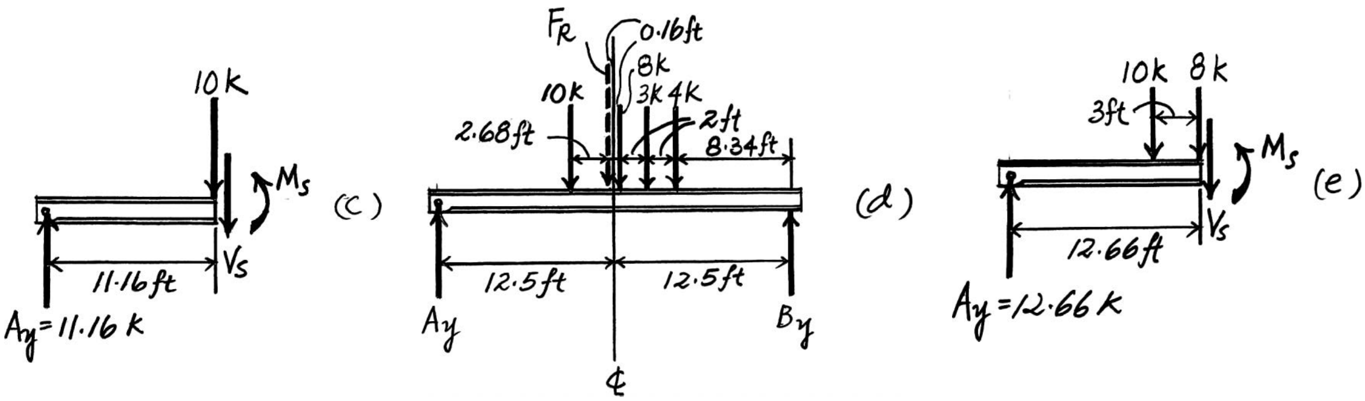
$\circlearrowleft + \sum M_B = 0; \qquad 4(8.34) + 3(10.34) + 8(12.34) + 10(15.34) - A_y(25) = 0$

$$A_y = 12.66 \text{ k}$$

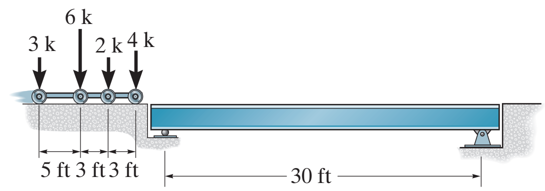
Referring to Fig. *e*,

$\circlearrowleft + \sum M_S = 0; \qquad M_S + 10(3) - 12.66(12.66) = 0$

$$M_S = 130.28 \text{ k} \cdot \text{ft} = 130 \text{ k} \cdot \text{ft (Abs. Max.)}$$ **Ans.**



6–79. Determine the absolute maximum shear in the beam due to the loading shown.



The maximum shear occurs when the moving loads are positioned either with the 3-k force just to the right of the support at A, Fig. a, or with the 4 k force just to the left of the support at B. Referring to Fig. a

$$\curvearrowleft + \sum M_B = 0; \qquad 4(19) + 2(22) + 6(25) + 3(30) - A_y(30) = 0$$

$$A_y = 12.0 \text{ k}$$

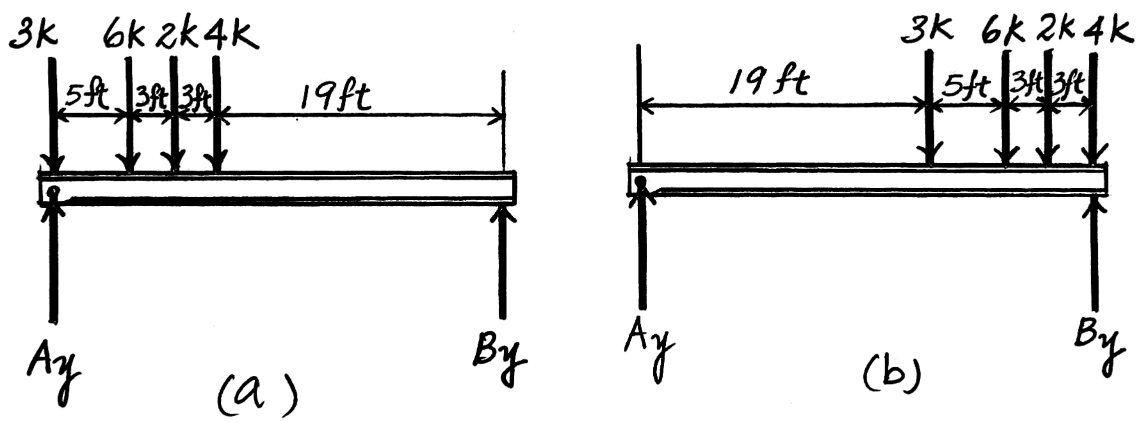
Referring to Fig. b

$$\curvearrowleft + \sum M_A = 0; \qquad B_y(30) - 3(19) - 6(24) - 2(27) - 4(30) = 0$$

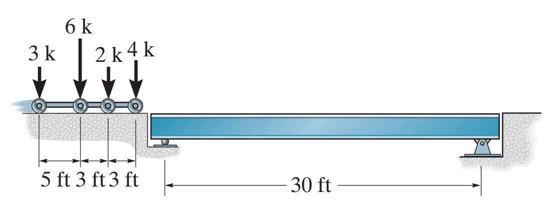
$$B_y = 12.5 \text{ k}$$

Therefore, the absolute maximum shear occurs for the case in Fig. b

$$V_{\substack{\text{abs}\\ \text{max}}} = B_y = 12.5 \text{ k} \qquad \textbf{Ans.}$$



***6–80.** Determine the absolute maximum moment in the bridge due to the loading shown.



Referring to Fig. a, the location of the F_R for the moving loads is

$$+\downarrow F_R = \sum F_y; \qquad F_R = 3 + 6 + 2 + 4 + 15 \text{ k}$$

$$\curvearrowleft + F_R\bar{x} = \sum M_C; \quad -15\bar{x} = -6(5) - 2(8) - 4(11)$$

$$\bar{x} = 6 \text{ ft}$$

***6–80. Continued**

Assuming that the absolute maximum moment occurs under the 6 k force, Fig. *b*,

$\circlearrowleft + \sum M_B = 0;\quad 4(9.5) + 2(12.5) + 6(15.5) + 3(20.5) - A_y(30) = 0$

$A_y = 7.25\text{ k}$

Referring to Fig. *c*,

$\circlearrowleft + \sum M_S = 0;\quad M_S + 3(5) - 7.25(14.5) = 0$

$M_S = 90.1\text{ k}\cdot\text{ft}$ **Ans.**

Assuming that the absolute maximum moment occurs under the 2 k force, Fig. *d*,

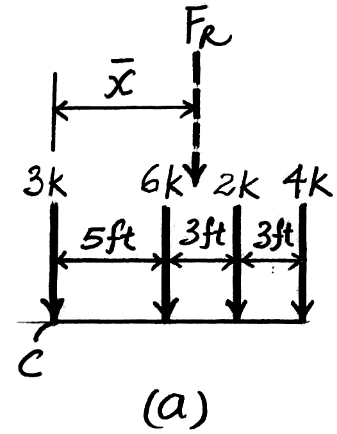
$\circlearrowleft + \sum M_A = 0;\quad B_y(30) - 3(8) - 6(13) - 2(16) - 4(19) = 0$

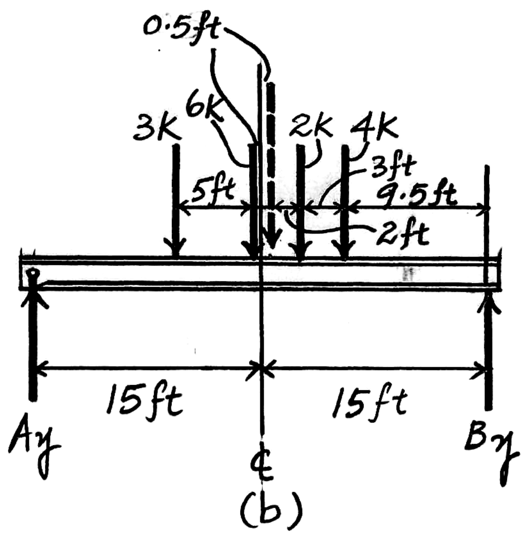
$B_y = 7.00\text{ k}$

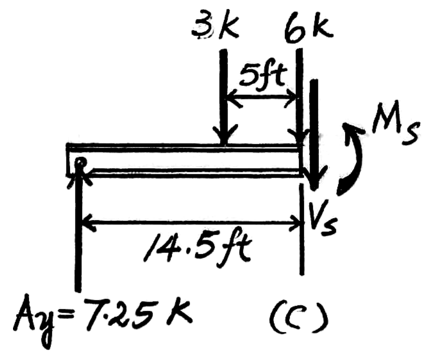
Referring to Fig. *e*,

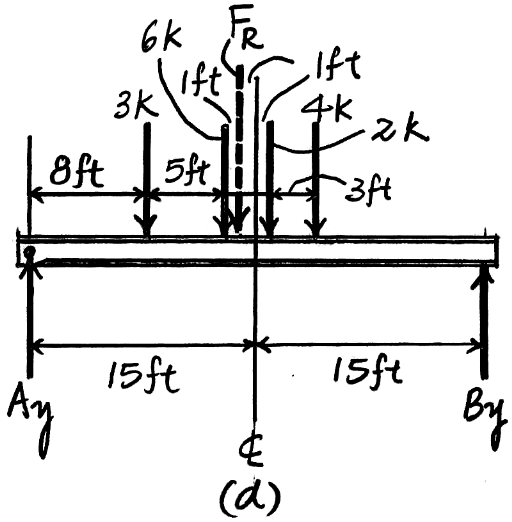
$\circlearrowleft + \sum M_S = 0;\quad 7.00(14) - 4(3) - M_S = 0$

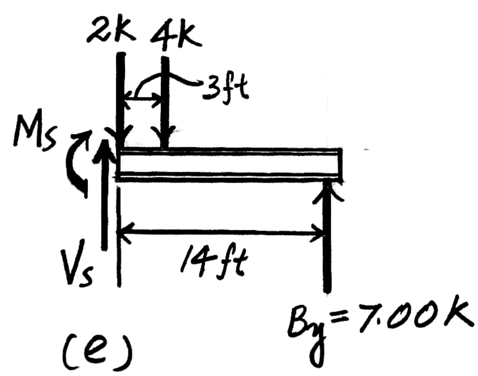
$M_S = 86.0\text{ k}\cdot\text{ft (Abs. Max.)}$



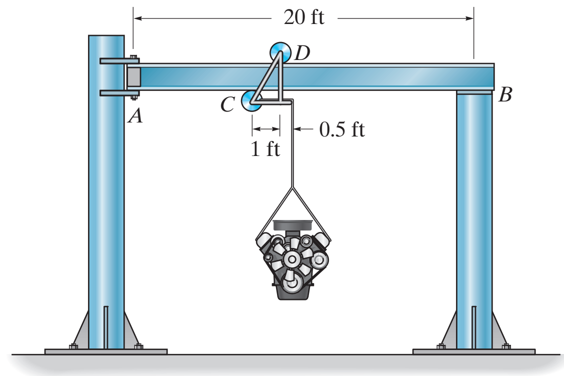








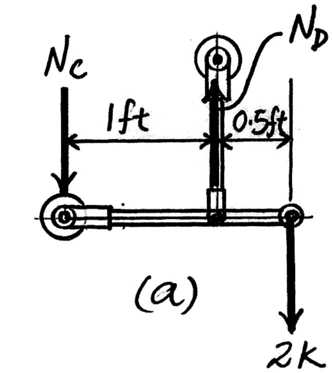
6–81. The trolley rolls at C and D along the bottom and top flange of beam AB. Determine the absolute maximum moment developed in the beam if the load supported by the trolley is 2 k. Assume the support at A is a pin and at B a roller.



Referring to the FBD of the trolley in Fig. *a*,

$\zeta + \sum M_C = 0;$ $\quad N_D(1) - 2(1.5) = 0$ $\quad N_D = 3.00\text{ k}$

$\zeta + \sum M_D = 0;$ $\quad N_C(1) - 2(0.5) = 0$ $\quad N_C = 1.00\text{ k}$



Referring to Fig. *b*, the location of F_R

$+\downarrow F_R = \sum F_Y;$ $\quad F_R = 3.00 - 1.00 = 2.00\text{ k}\downarrow$

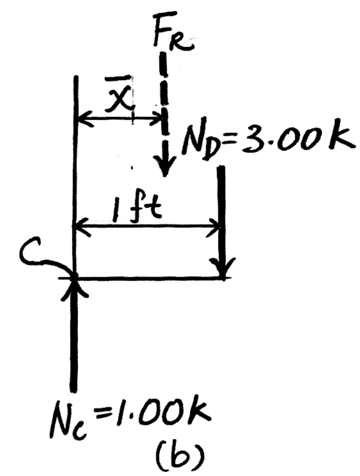
$\zeta + F_R\bar{x} = \sum M_C;$ $\quad -2.00(\bar{x}) = -3.00(1)$

$\bar{x} = 1.5\text{ ft}$

The absolute maximium moment occurs under the 3.00 k force, Fig. *c*.

$\zeta + \sum M_A = 0;$ $\quad B_y(20) + 1.00(8.75) - 3.00(9.75) = 0$

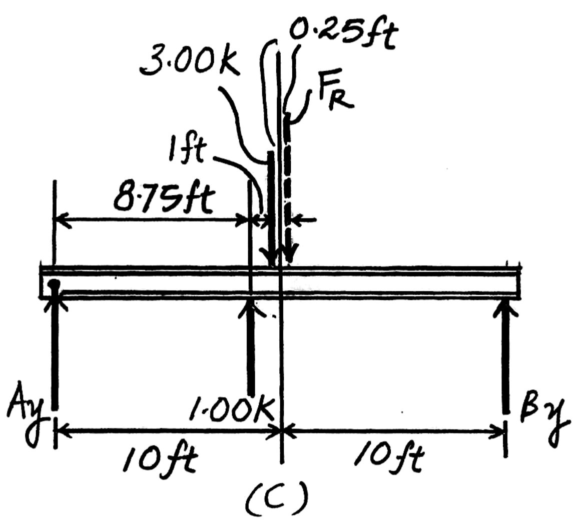
$B_y = 1.025\text{ k}$

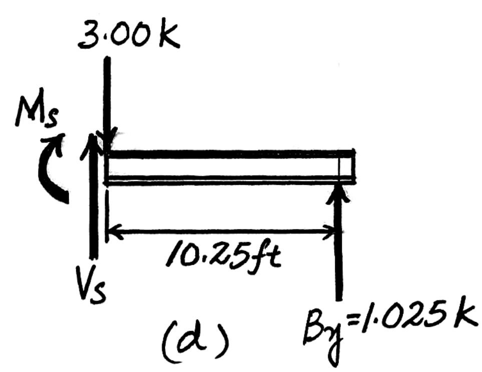


Referring to Fig. *d*,

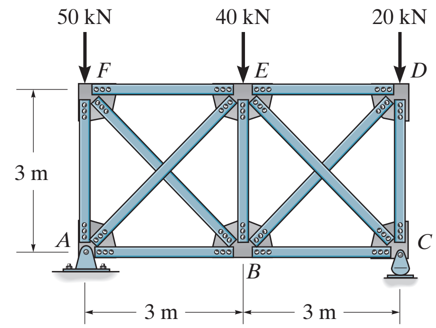
$\zeta + \sum M_S = 0;$ $\quad 1.025(10.25) - M_S = 0$

$M_S = 10.5\text{ k}\cdot\text{ft (Abs. Max.)}$ **Ans.**





7–1. Determine (approximately) the force in each member of the truss. Assume the diagonals can support either a tensile or a compressive force.

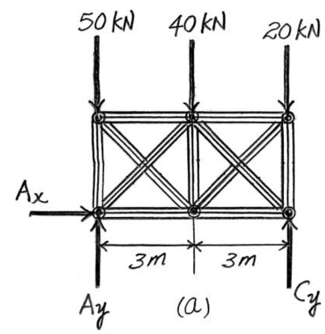


Support Reactions. Referring to Fig. *a*,

$\zeta+\sum M_A = 0;\quad C_y(6) - 40(3) - 20(6) = 0 \quad C_y = 40 \text{ kN}$

$\zeta+\sum M_C = 0;\quad 40(3) + 50(6) - A_y(6) = 0 \quad A_y = 70 \text{ kN}$

$\overset{+}{\rightarrow}\sum F_x = 0;\quad A_x = 0$



Method of Sections. It is required that $F_{BF} = F_{AE} = F_1$. Referring to Fig. *b*,

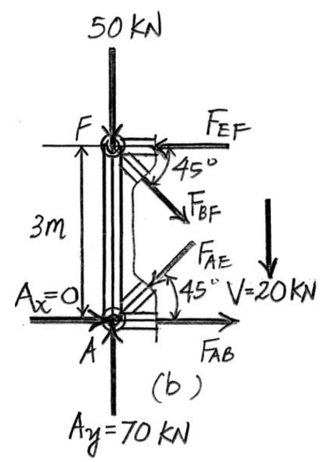
$+\uparrow\sum F_y = 0;\quad 70 - 50 - 2F_1 \sin 45° = 0 \quad F_1 = 14.14 \text{ kN}$

Therefore,

$F_{BF} = 14.1 \text{ kN (T)} \quad F_{AE} = 14.1 \text{ kN (C)}$ **Ans.**

$\zeta+\sum M_A = 0;\quad F_{EF}(3) - 14.14 \cos 45°(3) = 0 \quad F_{EF} = 10.0 \text{ kN (C)}$ **Ans.**

$\zeta+\sum M_F = 0;\quad F_{AB}(3) - 14.14 \cos 45°(3) = 0 \quad F_{AB} = 10.0 \text{ kN (T)}$ **Ans.**



Also, $F_{BD} = F_{CE} = F_2$. Referring to Fig. *c*,

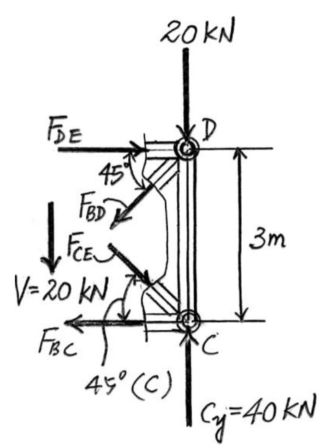
$+\uparrow\sum F_y = 0;\quad 40 - 20 - 2F_2 \sin 45° = 0 \quad F_2 = 14.14 \text{ kN}$

Therefore,

$F_{BD} = 14.1 \text{ kN (T)} \quad F_{CE} = 14.1 \text{ kN (C)}$ **Ans.**

$\zeta+\sum M_C = 0;\quad 14.14 \cos 45°(3) - F_{DE}(3) = 0 \quad F_{DE} = 10.0 \text{ kN (C)}$ **Ans.**

$\zeta+\sum M_D = 0;\quad 14.14 \cos 45°(3) - F_{BC}(3) = 0 \quad F_{BC} = 10.0 \text{ kN (T)}$ **Ans.**



Method of Joints.

Joint *A*: Referring to Fig. *d*,

$+\uparrow\sum F_y = 0;\quad 70 - 14.14 \sin 45° - F_{AF} = 0 \quad F_{AF} = 60.0 \text{ kN (C)}$ **Ans.**

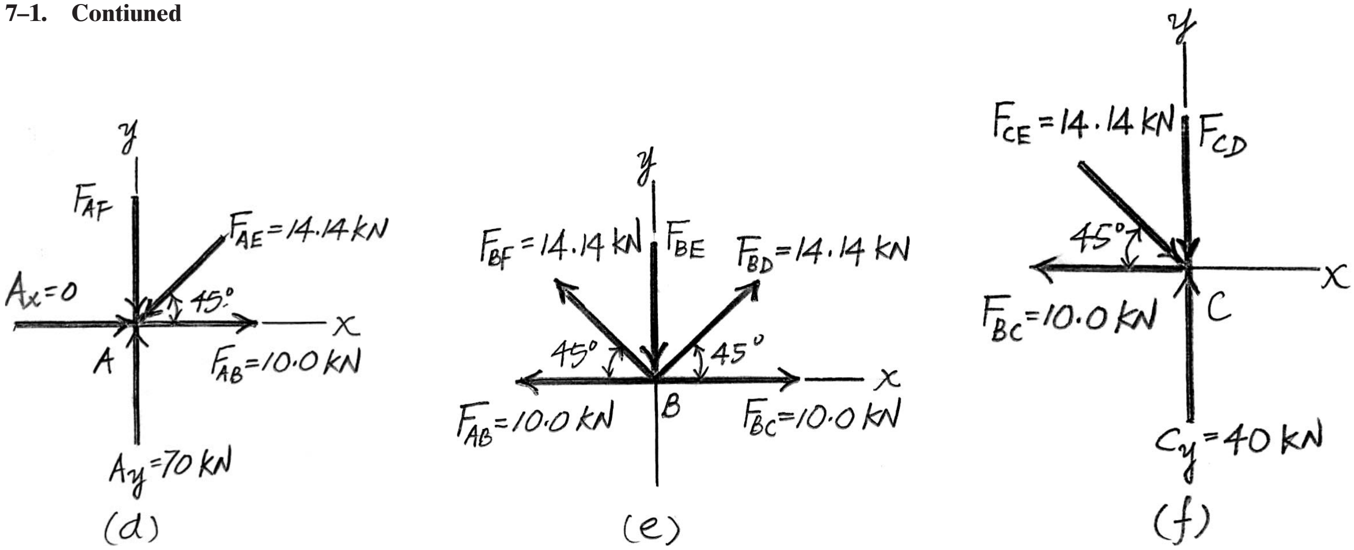
Joint *B*: Referring to Fig. *e*,

$+\uparrow\sum F_y = 0;\quad 14.14 \sin 45° + 14.14 \sin 45° - F_{BE} = 0 \quad F_{BE} = 20.0 \text{ kN (C)}$ **Ans.**

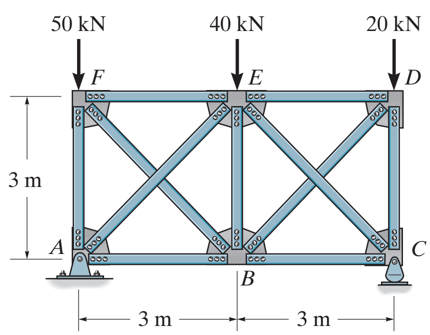
Joint *C*:

$+\uparrow\sum F_y = 0;\quad 40 - 14.14 \sin 45° - F_{CD} = 0 \quad F_{CD} = 30.0 \text{ kN (C)}$ **Ans.**

7–1. Contiuned



7–2. Solve Prob. 7–1 assuming that the diagonals cannot support a compressive force.



Support Reactions. Referring to Fig. *a*,

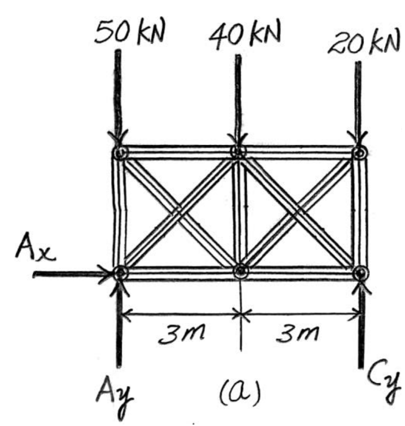
$$\zeta+\sum M_A = 0; \quad C_y(6) - 40(3) - 20(6) = 0 \quad C_y = 40 \text{ kN}$$

$$\zeta+\sum M_C = 0; \quad 40(3) + 50(6) - A_y(6) = 0 \quad A_y = 70 \text{ kN}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0$$

Method of Sections. It is required that

$$F_{AE} = F_{CE} = 0$$ **Ans.**



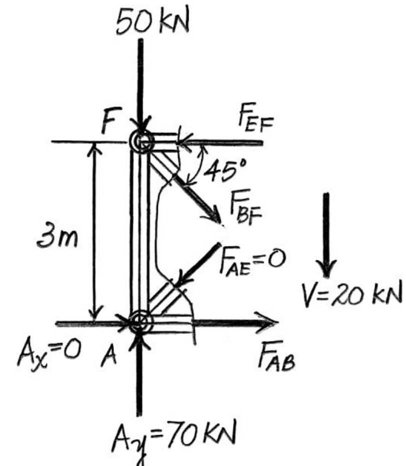
7–2. Continued

Referring to Fig. *b*,

$+\uparrow\sum F_y = 0;\ 70 - 50 - F_{BF}\sin 45^\circ = 0\ \ F_{BF} = 28.28\text{ kN (T)} = 28.3\text{ kN (T)}$ **Ans.**

$\circlearrowleft+\sum M_A = 0;\quad F_{EF}(3) - 28.28\cos 45^\circ(3) = 0\quad F_{EF} = 20.0\text{ kN (C)}$ **Ans.**

$\circlearrowleft+\sum M_F = 0\quad F_{AB}(3) = 0\quad F_{AB} = 0$ **Ans.**



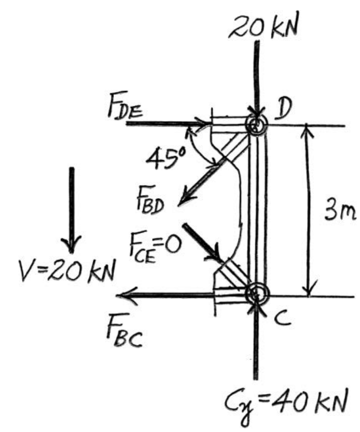
(b)

Referring to Fig. *c*,

$+\uparrow\sum F_y = 0;\ 40 - 20 - F_{BD}\sin 45^\circ = 0\ \ F_{BD} = 28.28\text{ kN (T)} = 28.3\text{ kN (T)}$ **Ans.**

$\circlearrowleft+\sum M_C = 0;\quad 28.28\cos 45^\circ(3) - F_{DE}(3) = 0\quad F_{DE} = 20.0\text{ kN (C)}$ **Ans.**

$\circlearrowleft+\sum M_D = 0;\quad -F_{BC}(3) = 0\quad F_{BC} = 0$ **Ans.**



(c)

Method of Joints.

Joint *A*: Referring to Fig. *d*,

$+\uparrow\sum F_y = 0;\quad 70 - F_{AF} = 0\quad F_{AF} = 70.0\text{ kN (C)}$ **Ans.**

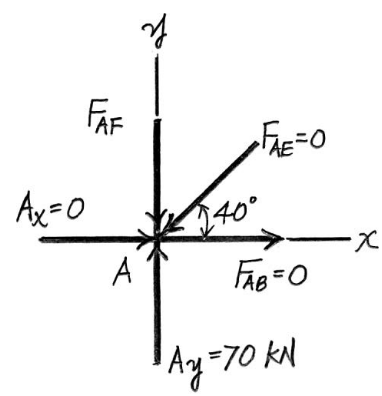
Joint *B*: Referring to Fig. *e*,

$+\uparrow\sum F_y = 0;\quad 28.28\sin 45^\circ + 28.28\sin 45^\circ - F_{BE} = 0$

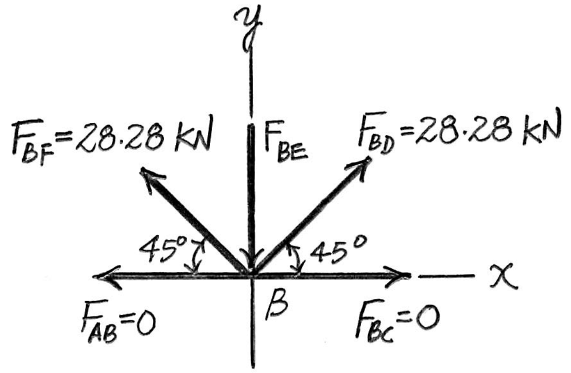
$F_{BE} = 40.0\text{ kN (C)}$ **Ans.**

Joint *C*: Referring to Fig. *f*,

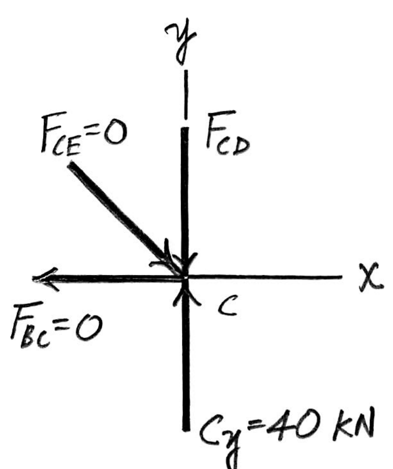
$+\uparrow\sum F_y = 0;\quad 40 - F_{CD} = 0\quad F_{CD} = 40.0\text{ kN (C)}$ **Ans.**



(d)

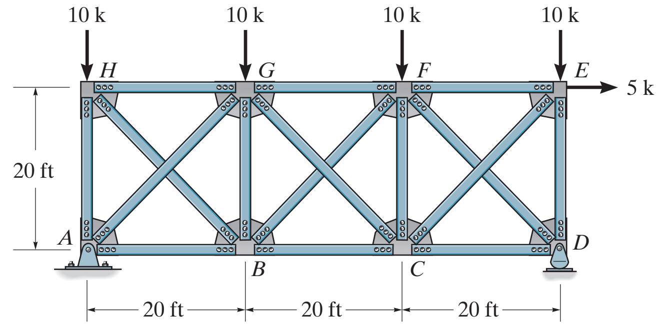


(e)



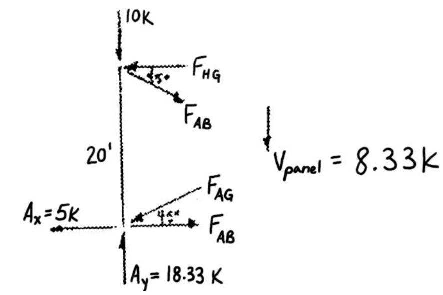
(f)

7–3. Determine (approximately) the force in each member of the truss. Assume the diagonals can support either a tensile or a compressive force.



$V_{\text{Panel}} = 8.33\text{ k}$

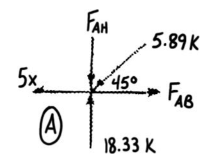
Assume V_{Panel} is carried equally by F_{HB} and F_{AG}, so



$$F_{HB} = \frac{\frac{8.33}{2}}{\cos 45°} = 5.89\text{ k (T)}$$ **Ans.**

$$F_{AG} = \frac{\frac{8.33}{2}}{\cos 45°} = 5.89\text{ k (C)}$$ **Ans.**

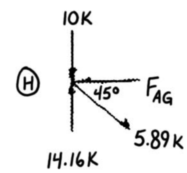
Joint *A*:



$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} - 5 - 5.89\ \cos 45° = 0; \quad F_{AB} = 9.17\text{ k (T)}$ **Ans.**

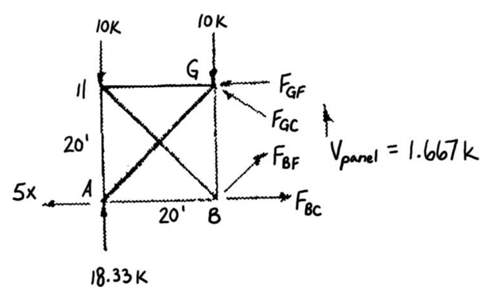
$+\uparrow \sum F_y = 0; \quad -F_{AH} + 18.33 - 5.89\ \sin 45° = 0; \quad F_{AH} = 14.16\text{ k (C)}$ **Ans.**

Joint *H*:



$\xrightarrow{+} \sum F_x = 0; \quad -F_{HG} + 5.89\ \cos 45° = 0; \quad F_{HG} = 4.17\text{ k (C)}$ **Ans.**

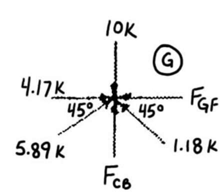
$V_{\text{Panel}} = 1.667\text{ k}$



$$F_{GC} = \frac{\frac{1.667}{2}}{\cos 45°} = 1.18\text{ k (C)}$$ **Ans.**

$$F_{BF} = \frac{\frac{1.667}{2}}{\cos 45°} = 1.18\text{ k (T)}$$ **Ans.**

Joint *G*:

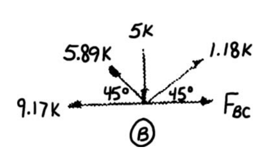


$\xrightarrow{+} \sum F_x = 0; \quad 4.17 + 5.89\ \cos 45° - 1.18\ \cos 45° - F_{GF} = 0$

$F_{GF} = 7.5\text{ k (C)}$ **Ans.**

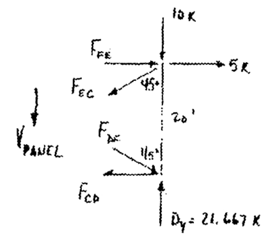
$+\uparrow \sum F_y = 0; \quad -10 + F_{GB} + 5.89\ \sin 45° + 1.18\ \sin 45° = 0$

$F_{GB} = 5.0\text{ k (C)}$ **Ans.**



7–3. Continued

Joint *B*:

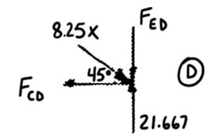


$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} + 1.18 \cos 45° - 9.17 - 5.89 \cos 45° = 0$

$F_{BC} = 12.5 \text{ k (T)}$ **Ans.**

$V_{Panel} = 21.667 - 10 = 11.667 \text{ k}$

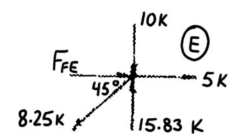
$$F_{EC} = \frac{\frac{11.667}{2}}{\cos 45°} = 8.25 \text{ k (T)}$$ **Ans.**



$$F_{DF} = \frac{\frac{11.567}{2}}{\cos 45°} = 8.25 \text{ k (C)}$$ **Ans.**

Joint *D*:

$\xrightarrow{+} \sum F_x = 0; \quad F_{CD} = 8.25 \cos 45° = 5.83 \text{ k (T)}$ **Ans.**

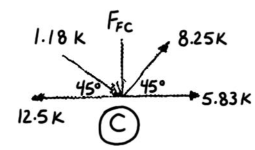


$+\uparrow \sum F_y = 0; \quad 21.667 - 8.25 \sin 45° - F_{ED} = 0$

$F_{ED} = 15.83 \text{ k (C)}$ **Ans.**

Joint *E*:

$\xrightarrow{+} \sum F_x = 0; \quad 5 + F_{FE} - 8.25 \cos 45° = 0$



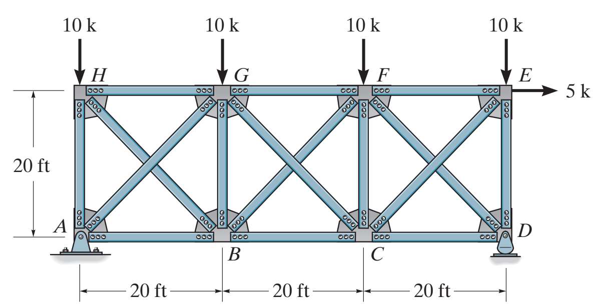
$F_{FE} = 0.833 \text{ k (C)}$ **Ans.**

Joint *C*:

$+\uparrow \sum F_y = 0; \quad -F_{FC} + 8.25 \sin 45° - 1.18 \sin 45° = 0$

$F_{FC} = 5.0 \text{ k (C)}$ **Ans.**

***7–4.** Solve Prob. 7–3 assuming that the diagonals cannot support a compressive force.



$V_{Panel} = 8.33 \text{ k}$

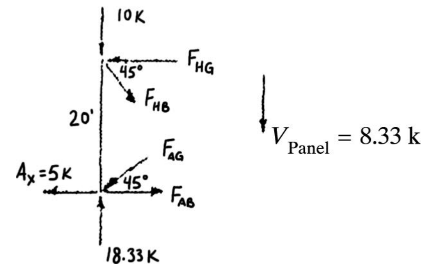
$F_{AG} = 0$ **Ans.**

$$F_{HB} = \frac{8.33}{\sin 45°} = 11.785 = 11.8 \text{ k}$$ **Ans.**

Joint *A*:

$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} = 5 \text{ k (T)}$ **Ans.**

7–4. Continued

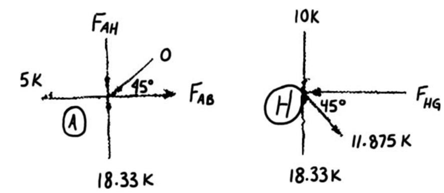


$+\uparrow \sum F_y = 0; \quad F_{AN} = 18.3 \text{ k (C)}$ **Ans.**

Joint *H*:

$\xrightarrow{+} \sum F_x = 0; \quad 11.785 \cos 45° - F_{HG} = 0$

$F_{HG} = 8.33 \text{ k (C)}$ **Ans.**

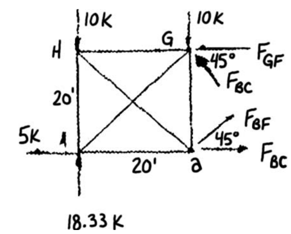


$V_{\text{Panel}} = 1.667 \text{ k}$

$F_{GC} = 0$ **Ans.**

$F_{BF} = \dfrac{1.667}{\sin 45°} = 2.36 \text{ k (T)}$ **Ans.**

Joint *B*:



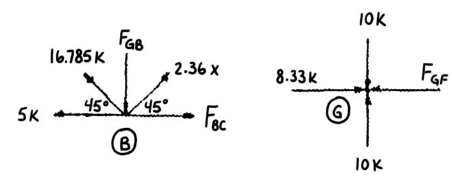
$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} + 2.36 \cos 45° - 11.785 \cos 45° - 5 = 0$

$F_{BC} = 11.7 \text{ k (T)}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad -F_{GB} + 11.785 \sin 45° + 2.36 \sin 45° = 0$

$F_{GB} = 10 \text{ k (C)}$ **Ans.**

Joint *G*:



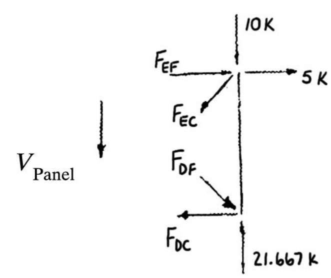
$\xrightarrow{+} \sum F_x = 0; \quad F_{GF} = 8.33 \text{ k (C)}$ **Ans.**

$V_{\text{Panel}} = 11.667 \text{ k}$

$F_{DF} = 0$ **Ans.**

$F_{EC} = \dfrac{11.667}{\sin 45°} = 16.5 \text{ k (T)}$ **Ans.**

Joint *D*:

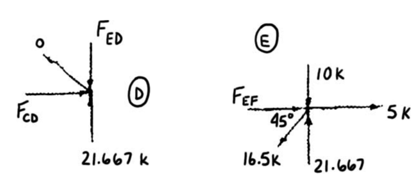


$\xrightarrow{+} \sum F_x = 0; \quad F_{CD} = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad F_{ED} = 21.7 \text{ k (C)}$ **Ans.**

Joint *E*:

$\xrightarrow{+} \sum F_x = 0; \quad F_{EF} + 5 - 16.5 \cos 45° = 0$

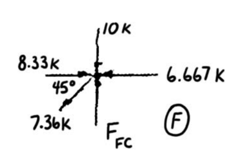


$F_{EF} = 6.67 \text{ k (C)}$ **Ans.**

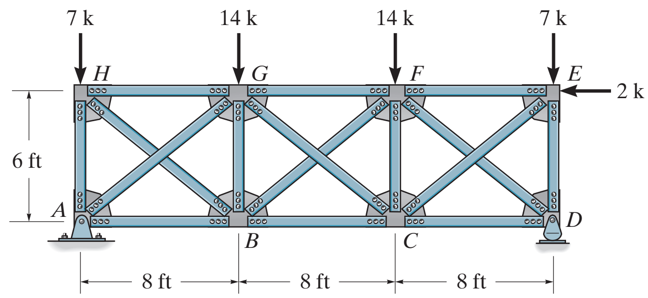
Joint *F*:

$+\uparrow \sum F_y = 0; \quad F_{FC} - 10 - 2.36 \sin 45° = 0$

$F_{FC} = 11.7 \text{ k (C)}$ **Ans.**



7–5. Determine (approximately) the force in each member of the truss. Assume the diagonals can support either a tensile or a compressive force.



Support Reactions. Referring to, Fig. *a*

$\xrightarrow{+} \sum F_x = 0; \quad A_x - 2 = 0 \quad A_x = 2\text{ k}$

$\circlearrowleft + \sum M_A = 0; \quad D_y(24) + 2(6) - 7(24) - 14(16) - 14(8) = 0 \quad D_y = 20.5\text{ k}$

$\circlearrowleft + \sum M_D = 0; \quad 14(8) + 14(16) + 7(24) + 2(6) - A_y(24) = 0 \quad A_y = 21.5\text{ k}$

Method of Sections. It is required that $F_{BH} = F_{AG} = F_1$. Referring to Fig. *b*,

$+\uparrow \sum F_y = 0; \quad 21.5 - 7 - 2F_1\left(\frac{3}{5}\right) = 0 \quad F_1 = 12.08\text{ k}$

Therefore,

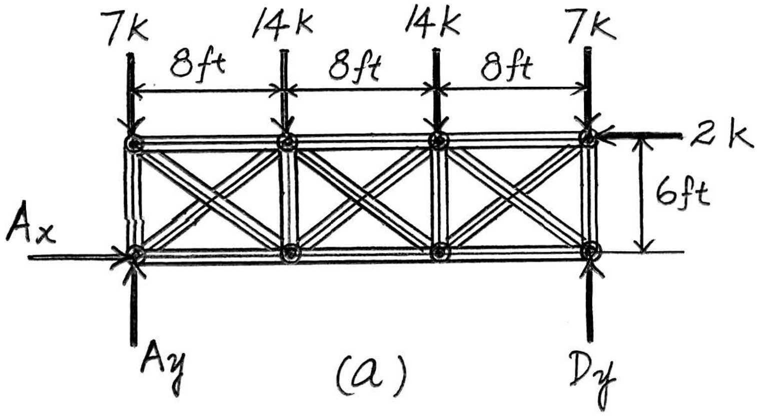
$F_{BH} = 12.1\text{ k (T)} \quad F_{AG} = 12.1\text{ k (C)}$ **Ans.**

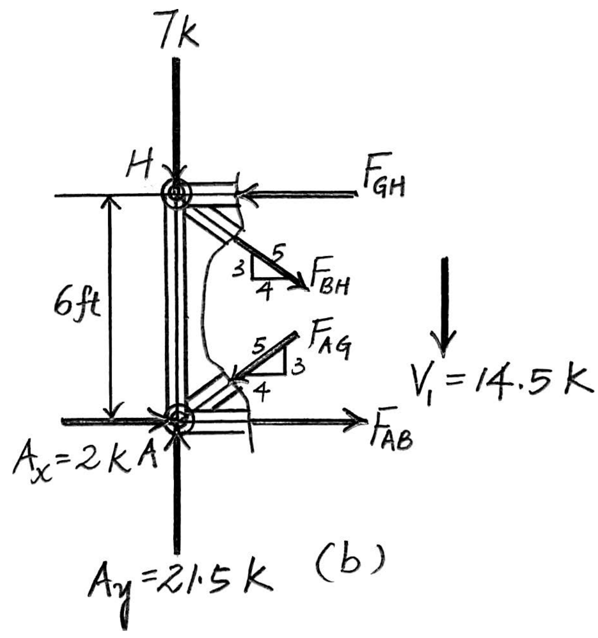
$\circlearrowleft + \sum M_H = 0; \quad F_{AB}(6) + 2(6) - 12.08\left(\frac{4}{5}\right)(6) = 0 \quad F_{AB} = 7.667\text{ k (T)} = 7.67\text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad F_{GH}(6) - 12.08\left(\frac{4}{5}\right)(6) = 0 \quad F_{GH} = 9.667\text{ k (C)} = 9.67\text{ k (C)}$ **Ans.**

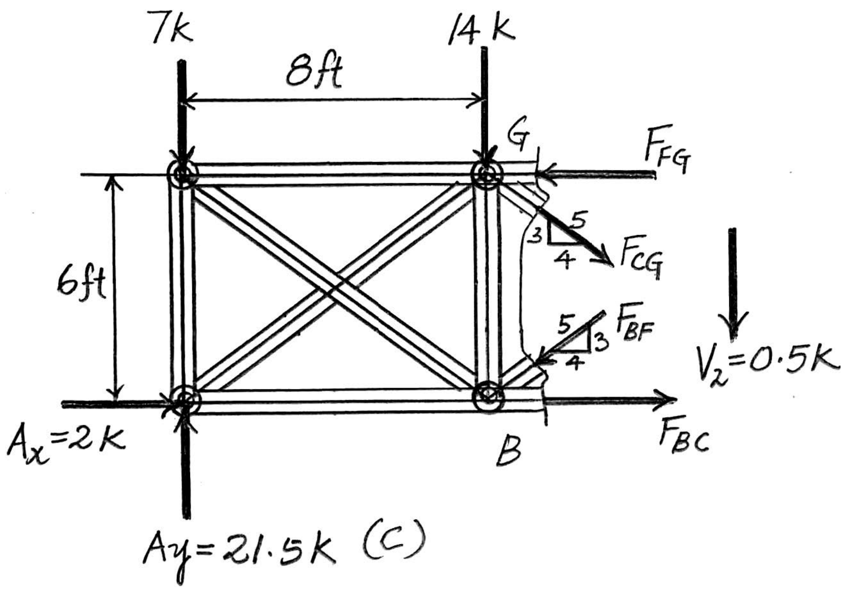
It is required that $F_{CG} = F_{BF} = F_2$. Referring to Fig. *c*,

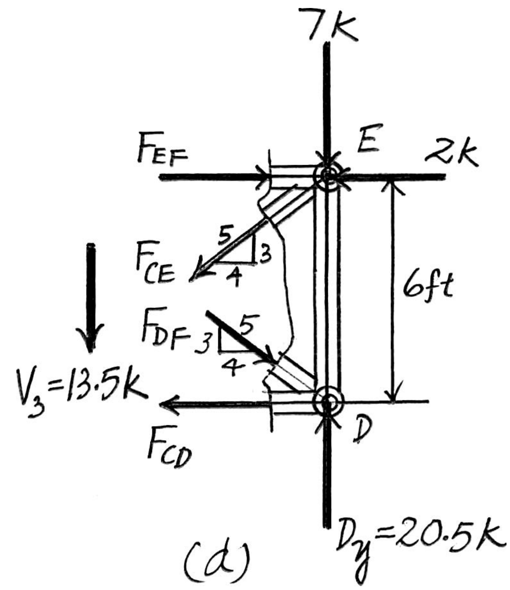
$+\uparrow \sum F_y = 0; \quad 21.5 - 7 - 14 - 2F_2\left(\frac{3}{5}\right) = 0 \quad F_2 = 0.4167\text{ k}$





7–5. Continued





Therefore,

$F_{CG} = 0.417$ k (T) $\quad F_{BF} = 0.417$ k (C) **Ans.**

$\circlearrowleft + \sum M_B = 0; \quad F_{FG}(6) - 0.4167\left(\frac{4}{5}\right)(6) + 7(8) - 21.5(8) = 0$

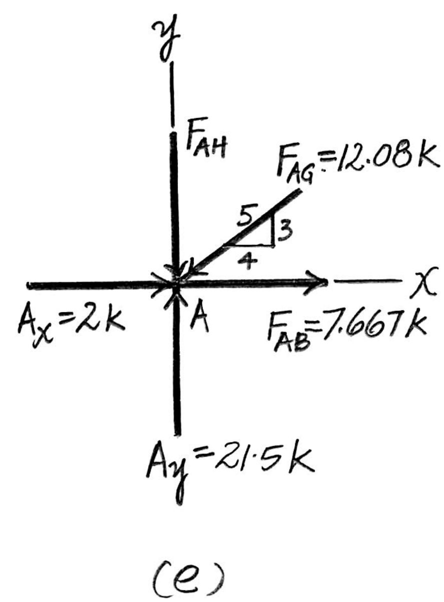
$F_{FG} = 19.67$ k (C) $= 19.7$ k (C) **Ans.**

$\circlearrowleft + \sum M_G = 0; \quad F_{BC}(6) + 7(8) + 2(6) - 21.5(8) - 0.4167\left(\frac{4}{5}\right)(6) = 0$

$F_{BC} = 17.67$ k (T) $= 17.7$ k (T) **Ans.**

It is required that $F_{CE} = F_{DF} = F_3$. Referring to Fig. *d*

$+\uparrow \sum F_y = 0; \quad 20.5 - 7 - 2F_3\left(\frac{3}{5}\right) = 0 \quad F_3 = 11.25$ k



Therefore,

$F_{CE} = 11.25$ k (T) $\quad F_{DF} = 11.25$ k (C) **Ans.**

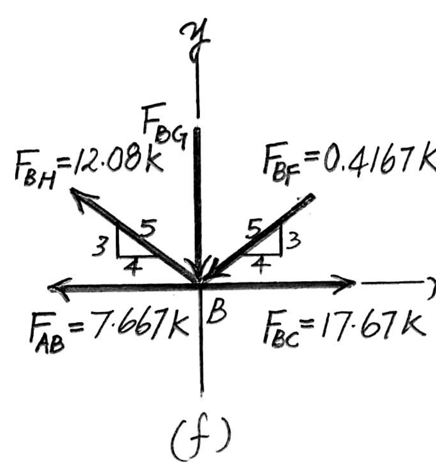
$\circlearrowleft + \sum M_D = 0; \quad 2(6) + 11.25\left(\frac{4}{5}\right)(6) - F_{EF}(6) = 0 \quad F_{EF} = 11.0$ k (C) **Ans.**

$\circlearrowleft + \sum M_E = 0; \quad 11.25(0.8)(6) - F_{CD}(6) = 0 \quad F_{CD} = 9.00$ k (T) **Ans.**

Method of Joints.

Joint *A*: Referring to Fig. *e*,

$+\uparrow \sum F_y = 0; \quad 21.5 - 12.08\left(\frac{3}{5}\right) - F_{AH} = 0 \quad F_{AH} = 14.25$ k (C) **Ans.**



Joint *B*: Referring to Fig. *f*,

$+\uparrow \sum F_y = 0; \quad 12.08\left(\frac{3}{5}\right) - 0.4167\left(\frac{3}{5}\right) - F_{BG} = 0 \quad F_{BG} = 7.00$ k (C) **Ans.**

Joint *C*: Referring Fig. *g*,

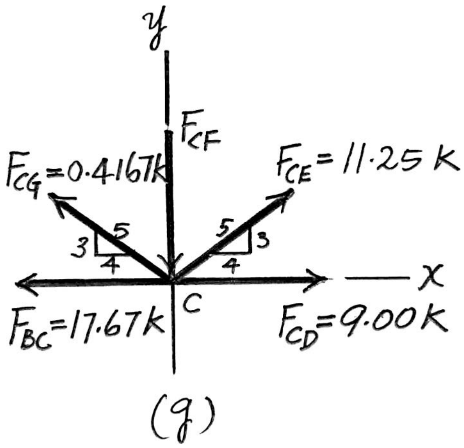
$+\uparrow \sum F_y = 0; \quad 11.25\left(\frac{3}{5}\right) + 0.4167\left(\frac{3}{5}\right) - F_{CF} = 0 \quad F_{CF} = 7.00$ k (C)

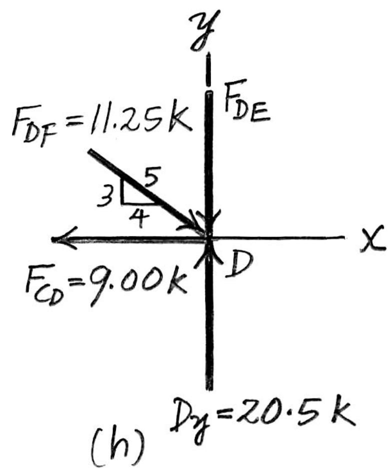
7–5. Continued

Joint *D*: Referring to Fig. *h*,

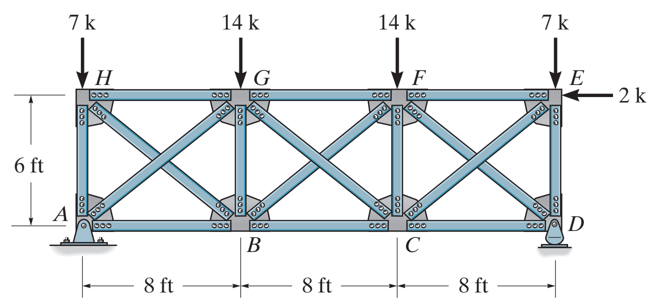
$+\uparrow \sum F_y = 0; \quad 20.5 - 11.25\left(\frac{3}{5}\right) - F_{DE} = 0$

$F_{DE} = 13.75 \text{ k}$ **Ans.**





7–6. Solve Prob. 7–5 assuming that the diagonals cannot support a compressive force.



Support Reactions. Referring to Fig. *a*,

$\xrightarrow{+} \sum F_x = 0; \quad A_x - 2 = 0 \quad A_x = 2 \text{ k}$

$\circlearrowleft + \sum M_A = 0; \quad D_y(24) + 2(6) - 7(24) - 14(16) - 14(8) = 0 \quad D_y = 20.5 \text{ k}$

$\circlearrowleft + \sum M_D = 0; \quad 14(8) + 14(16) + 7(24) + 2(6) - A_y(24) = 0 \quad A_y = 21.5 \text{ k}$

Method of Sections. It is required that

$F_{AG} = F_{BF} = F_{DF} = 0$ **Ans.**

Referring to Fig. *b*,

$+\uparrow \sum F_y = 0; \quad 21.5 - 7 - F_{BH}\left(\frac{3}{5}\right) = 0 \quad F_{BH} = 24.17 \text{ k (T)} = 24.2 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad F_{GH}(6) - 24.17\left(\frac{4}{5}\right)(6) = 0 \quad F_{GH} = 19.33 \text{ k (C)} = 19.3 \text{ k (C)}$

Ans.

$\circlearrowleft + \sum M_H = 0; \quad 2(6) - F_{AB}(6) = 0 \quad F_{AB} = 2.00 \text{ k (C)}$ **Ans.**

7–6. Continued

Referring to Fig. *c*

$+\uparrow \sum F_y = 0; \quad 21.5 - 7 - 14 - F_{CG}\left(\frac{3}{5}\right) = 0 \quad F_{CG} = 0.8333 \text{ k (T)} = 0.833 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_B = 0; \quad F_{FG}(6) + 7(8) - 21.5(8) - 0.8333\left(\frac{4}{5}\right)(6) = 0$

$F_{FG} = 20.0\, k\ (C)$ **Ans.**

$\circlearrowleft + \sum M_G = 0; \quad F_{BC}(6) + 7(8) + 2(6) - 21.5(8) = 0 \quad F_{BC} = 17.33 \text{ k (T)} = 17.3 \text{ k (T)}$ **Ans.**

Referring to Fig. *d*,

$+\uparrow \sum F_y = 0; \quad 20.5 - 7 - F_{CE}\left(\frac{3}{5}\right) = 0 \quad F_{CE} = 22.5 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_D = 0; \quad 2(6) + 22.5\left(\frac{4}{5}\right)(6) - F_{EF}(6) = 0 \quad F_{EF} = 20.0 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \sum M_E = 0; \quad -F_{CD}(6) = 0 \quad F_{CD} = 0$ **Ans.**

Method of Joints.

Joint *A*: Referring to Fig. *e*,

$+\uparrow \sum F_y = 0; \quad 21.5 - F_{AH} = 0 \quad F_{AH} = 21.5 \text{ k (C)}$ **Ans.**

Joint *B*: Referring to Fig. *f*,

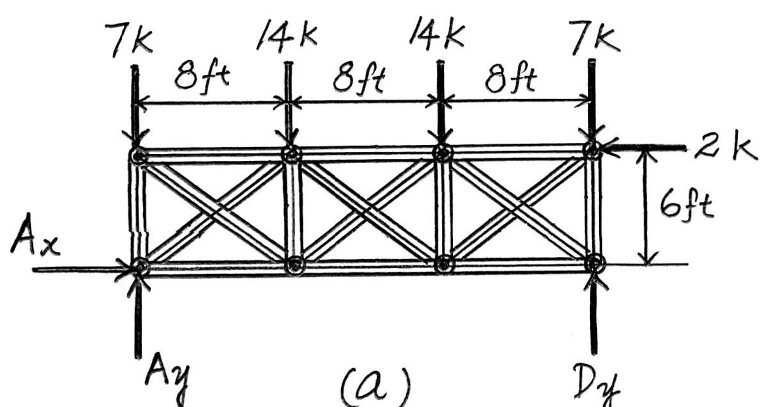
$+\uparrow \sum F_y = 0; \quad 24.17\left(\frac{3}{5}\right) - F_{BG} = 0 \quad F_{BG} = 14.5 \text{ k (C)}$ **Ans.**

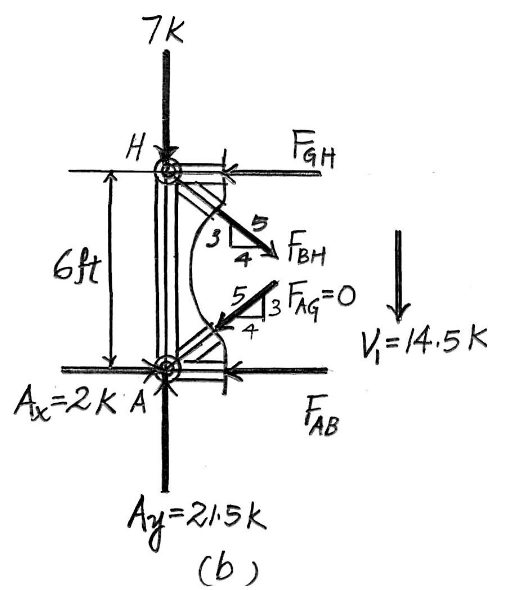
Joint *C*: Referring to Fig. *g*,

$+\uparrow \sum F_y = 0; \quad 0.8333\left(\frac{3}{5}\right) + 22.5\left(\frac{3}{5}\right) - F_{CF} = 0 \quad F_{CF} = 14.0 \text{ k (C)}$ **Ans.**

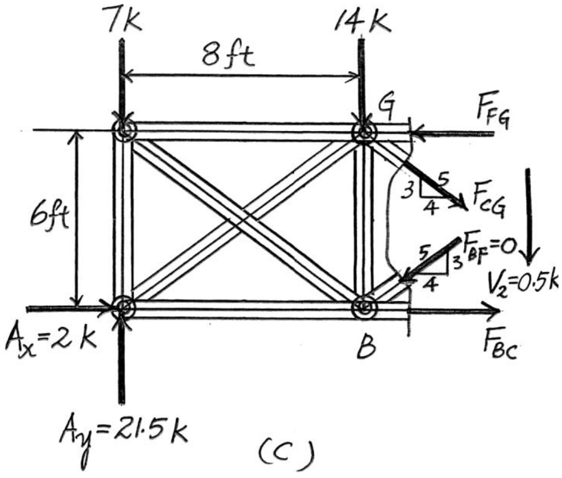
Joint *D*: Referring to Fig. *h*,

$+\uparrow \sum F_y = 0; \quad 20.5 - F_{DE} = 0 \quad F_{DE} = 20.5 \text{ k (C)}$ **Ans.**

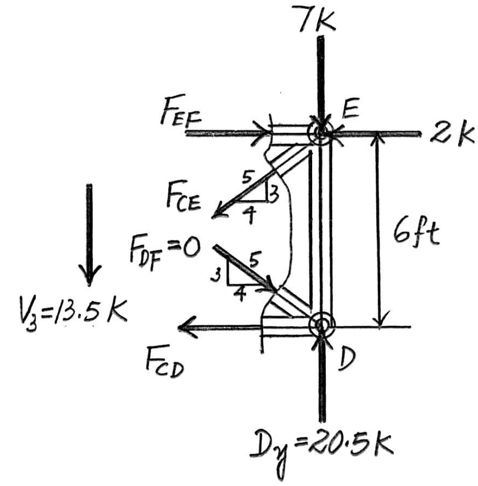




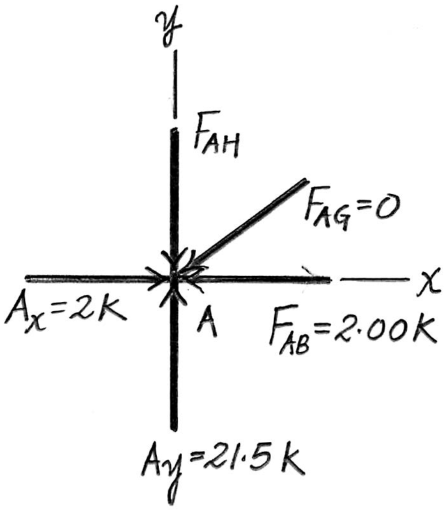
7–6. Continued



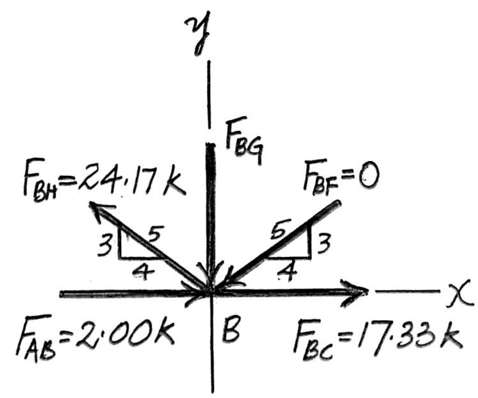
(c)



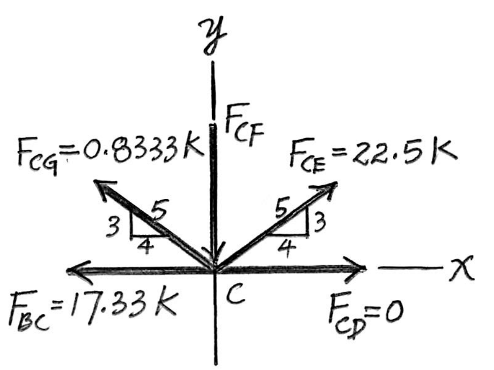
(d)



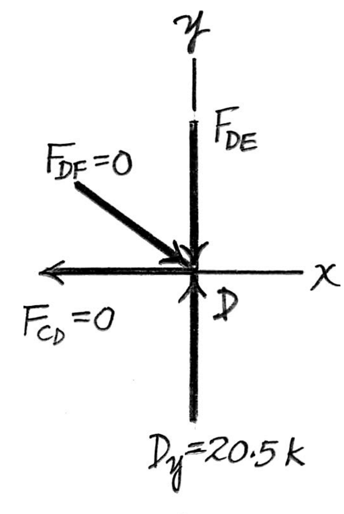
(e)



(f)

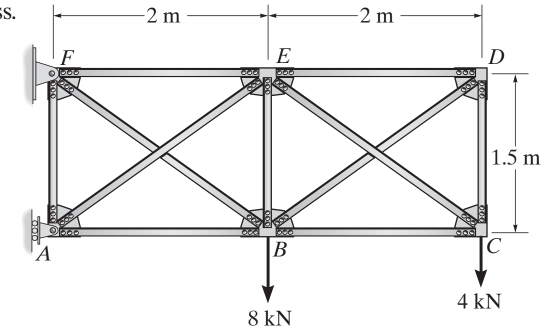


(g)

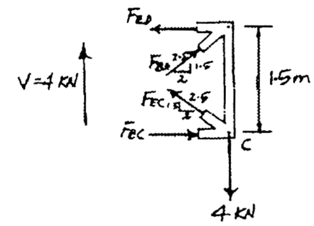


(h)

7–7. Determine (approximately) the force in each member of the truss. Assume the diagonals can support either a tensile or compressive force.



Assume $F_{BD} = F_{EC}$



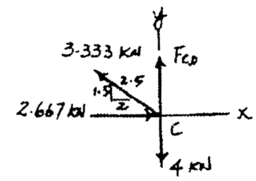
$$+\uparrow \sum F_y = 0; \quad 2F_{EC}\left(\frac{1.5}{2.5}\right) - 4 = 0$$

$F_{EC} = 3.333 \text{ kN} = 3.33 \text{ kN (T)}$ **Ans.**

$F_{BD} = 3.333 \text{ kN} = 3.33 \text{ kN (C)}$ **Ans.**

$$\zeta + \sum M_C = 0; \quad F_{ED}(1.5) - \left(\frac{2}{2.5}\right)(3.333)(1.5) = 0$$

$F_{ED} = 2.67 \text{ kN (T)}$ **Ans.**

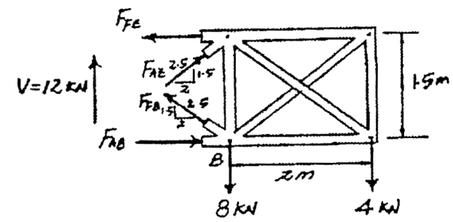


$$\overset{+}{\rightarrow} \sum F_x = 0; \quad F_{BC} = 2.67 \text{ kN (C)}$$ **Ans.**

Joint *C*:

$$+\uparrow \sum F_y = 0; \quad F_{CD} + 3.333\left(\frac{1.5}{2.5}\right) - 4 = 0$$

$F_{CD} = 2.00 \text{ kN (T)}$ **Ans.**

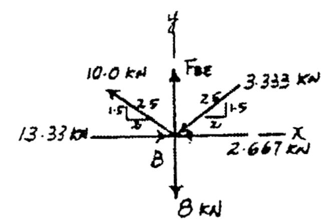


Assume $F_{FB} = F_{AE}$

$$+\uparrow \sum F_y = 0; \quad 2F_{FB}\left(\frac{1.5}{2.5}\right) - 8 - 4 = 0$$

$F_{FB} = 10.0 \text{ kN (T)}$ **Ans.**

$F_{AE} = 10.0 \text{ kN (C)}$ **Ans.**

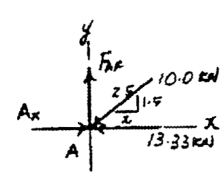


$$\zeta + \sum M_B = 0; \quad F_{FE}(1.5) - 10.0\left(\frac{2}{2.5}\right)(1.5) - 4(2) = 0$$

$F_{FE} = 13.3 \text{ kN (T)}$ **Ans.**

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad F_{AB} = 13.3 \text{ kN (C)}$$ **Ans.**

Joint *B*:



$$+\uparrow \sum F_y = 0; \quad F_{BE} + 10.0\left(\frac{1.5}{2.5}\right) - 3.333\left(\frac{1.5}{2.5}\right) - 8 = 0$$

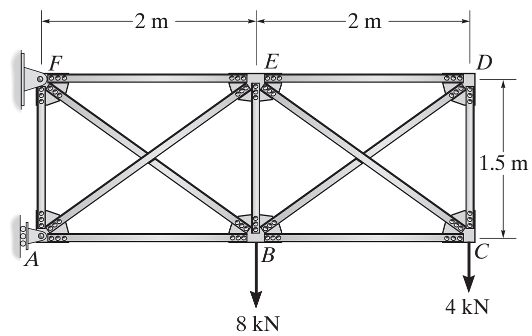
$F_{BE} = 4.00 \text{ kN (T)}$ **Ans.**

Joint *A*:

$$+\uparrow \sum F_y = 0; \quad F_{AF} - 10.0\left(\frac{1.5}{2.5}\right) = 0$$

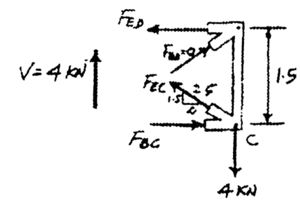
$F_{AF} = 6.00 \text{ kN (T)}$ **Ans.**

***7–8.** Solve Prob. 7–7 assuming that the diagonals cannot support a compressive force.



Assume $F_{BD} = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad F_{EC}\left(\frac{1.5}{2.5}\right) - 4 = 0$

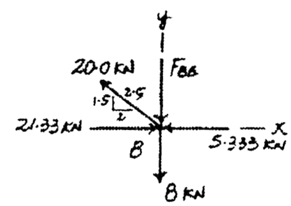


$F_{EC} = 6.667 \text{ kN} = 6.67 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \sum M_C = 0; \quad F_{ED} = 0$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{BC} - 6.667\left(\frac{2}{2.5}\right) = 0$

$F_{BC} = 5.33 \text{ kN (C)}$ **Ans.**



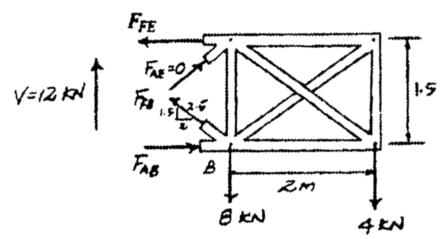
Joint *D*:

From Inspection:

$F_{CD} = 0$ **Ans.**

Assume $F_{AE} = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \quad F_{FB}\left(\frac{1.5}{2.5}\right) - 8 - 4 = 0$



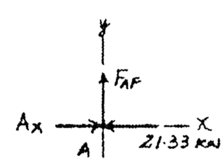
$F_{FB} = 20.0 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \sum M_B = 0; \quad F_{FE}(1.5) - 4(2) = 0$

$F_{FE} = 5.333 \text{ kN} = 5.33 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} - 5.333 - 20.0\left(\frac{2}{2.5}\right) = 0$

$F_{AB} = 21.3 \text{ kN (C)}$ **Ans.**



Joint *B*:

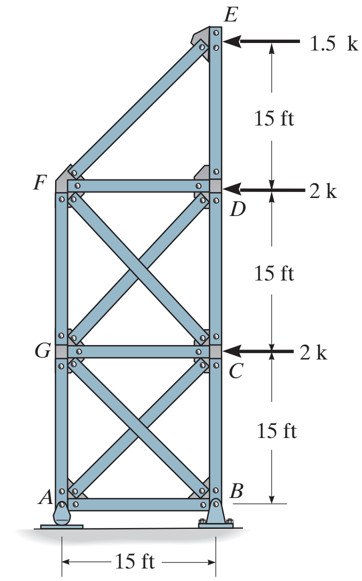
$+\uparrow \sum F_y = 0; \quad -F_{BE} - 8 + 20.0\left(\frac{1.5}{2.5}\right) = 0$

$F_{BE} = 4.00 \text{ kN (T)}$ **Ans.**

Joint *A*:

$+\uparrow \sum F_y = 0; \quad F_{AF} = 0$ **Ans.**

7–9. Determine (approximately) the force in each member of the truss. Assume the diagonals can support both tensile and compressive forces.



Method of Sections. It is required that $F_{CF} = F_{DG} = F_1$. Referring to Fig. *a*,

$\xrightarrow{+} \sum F_x = 0; \quad 2F_1 \sin 45° - 2 - 1.5 = 0 \quad F_1 = 2.475 \text{ k}$

Therefore,

$F_{CF} = 2.48 \text{ k (T)} \quad F_{DG} = 2.48 \text{ k (C)}$ **Ans.**

$\circlearrowleft + \sum M_D = 0; \quad 1.5(15) + 2.475 \cos 45° (15) - F_{FG}(15) = 0$

$F_{FG} = 3.25 \text{ k (C)}$ **Ans.**

$\circlearrowleft + \sum M_F = 0; \quad 1.5(15) + 2.475 \cos 45° (15) - F_{CD}(15) = 0$

$F_{CD} = 3.25 \text{ k (T)}$ **Ans.**

It is required that $F_{BG} = F_{AC} = F_2$. Referring to Fig. *b*,

$\xrightarrow{+} \sum F_x = 0; \quad 2F_2 \sin 45° - 2 - 2 - 1.5 = 0 \quad F_2 = 3.889 \text{ k}$

Therefore,

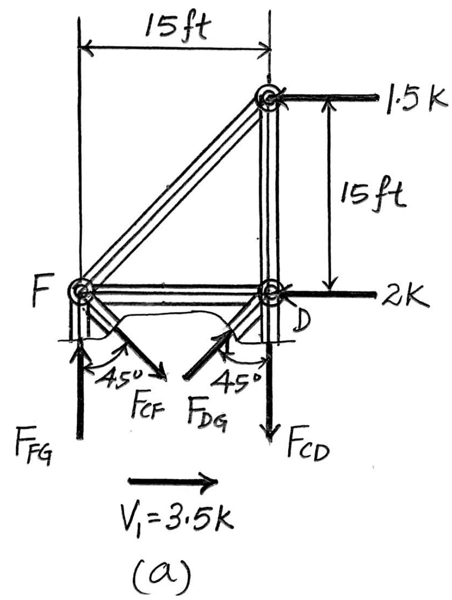
$F_{BG} = 3.89 \text{ k (T)} \quad F_{AC} = 3.89 \text{ k (C)}$ **Ans.**

$\circlearrowleft + \sum M_G = 0; \quad 1.5(30) + 2(15) + 3.889 \cos 45° (15) - F_{BC}(15) = 0$

$F_{BC} = 7.75 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_C = 0; \quad 1.5(30) + 2(15) + 3.889 \cos 45° (15) - F_{AG}(15) = 0$

$F_{AG} = 7.75 \text{ k (C)}$ **Ans.**



Method of Joints.

Joint *E*: Referring to Fig. *c*,

$\xrightarrow{+} \sum F_x = 0; \quad F_{EF} \cos 45° - 1.5 = 0 \quad F_{EF} = 2.121 \text{ k (C)} = 2.12 \text{ k (C)}$ **Ans.**

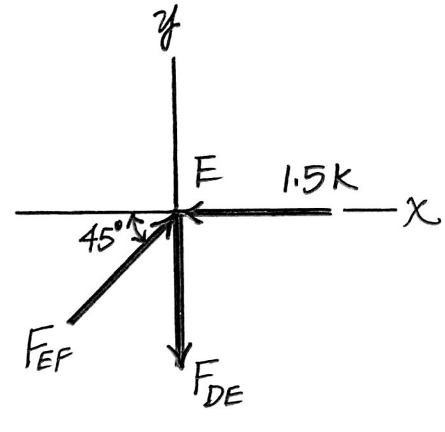
$+\uparrow \sum F_y = 0; \quad 2.121 \sin 45° - F_{DE} = 0 \quad F_{DE} = 1.50 \text{ k (T)}$ **Ans.**

Joint *F*: Referring to Fig. *d*,

$\xrightarrow{+} \sum F_x = 0; \quad 2.475 \sin 45° - 2.121 \cos 45° - F_{DF} = 0$

$F_{DF} = 0.250 \text{ k (C)}$ **Ans.**

7–9. Continued



(c)

Joint *G*: Referring to Fig. *e*,

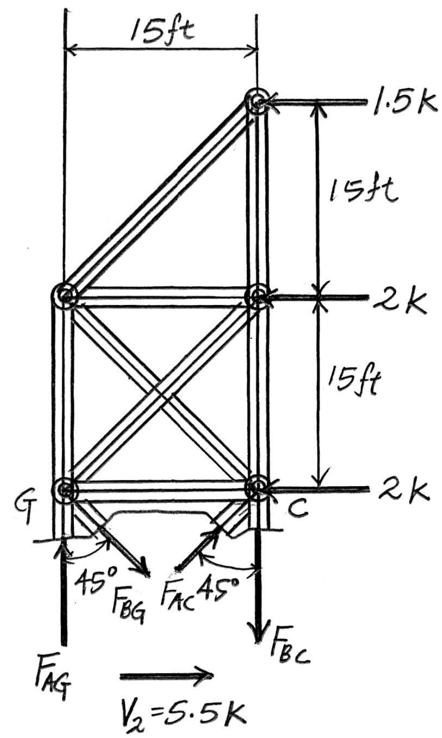
$$\xrightarrow{+} \sum F_x = 0; \quad 3.889 \sin 45° - 2.475 \cos 45° - F_{CG} = 0$$

$$F_{CG} = 1.00 \text{ k (C)}$$ **Ans.**

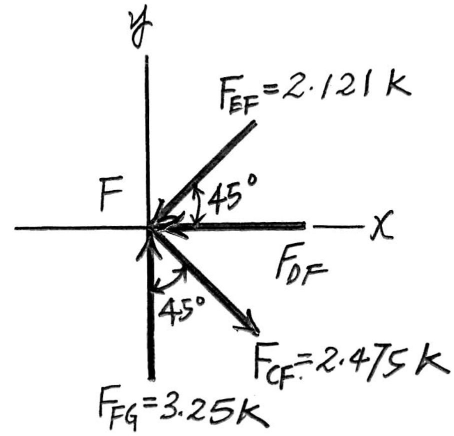
Joint *A*: Referring to Fig. *f*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} - 3.889 \cos 45° = 0$$

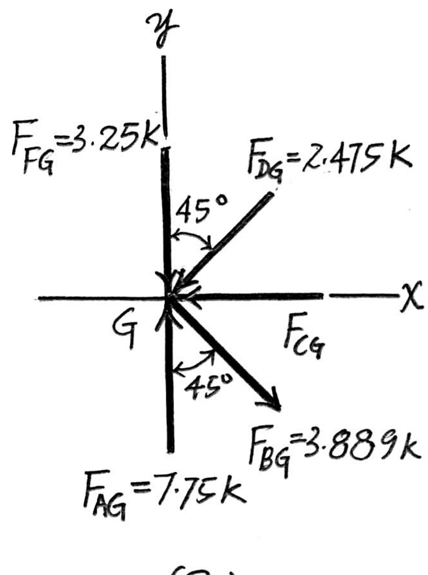
$$F_{AB} = 2.75 \text{ k}$$ **Ans.**



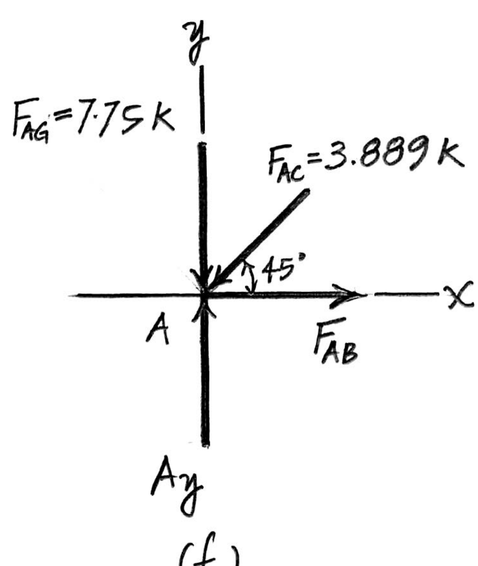
(b)



(d)

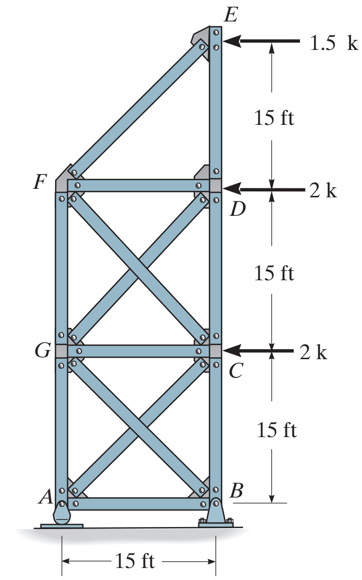


(e)



(f)

7–10. Determine (approximately) the force in each member of the truss. Assume the diagonals DG and AC cannot support a compressive force.



Method of Sections. It is required that

$$F_{DG} = F_{AC} = 0 \qquad \textbf{Ans.}$$

Referring to Fig. a,

$\overset{+}{\rightarrow} \sum F_x = 0;\quad F_{CF} \sin 45° - 1.5 - 2 = 0 \quad F_{CF} = 4.950 \text{ k (T)} = 4.95 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_F = 0;\quad 1.5(15) - F_{CD}(15) = 0 \quad F_{CD} = 1.50 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_D = 0;\quad 1.5(15) + 4.950 \cos 45°(15) - F_{FG}(15) = 0$

$F_{FG} = 5.00 \text{ k (C)}$ **Ans.**

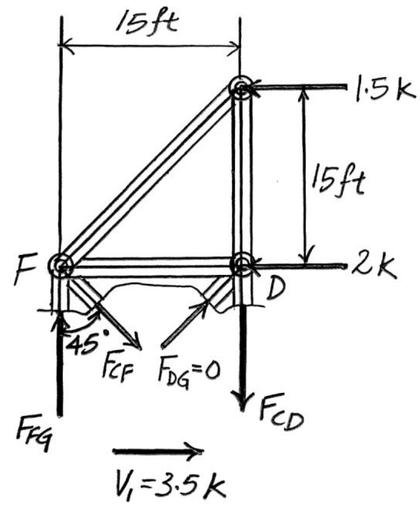
Referring to Fig. b,

$\overset{+}{\rightarrow} \sum F_x = 0;\quad F_{BG} \sin 45° - 2 - 2 - 1.5 = 0 \quad F_{BG} = 7.778 \text{ k (T)} = 7.78 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_G = 0;\quad 1.5(30) + 2(15) - F_{BC}(15) = 0 \quad F_{BC} = 5.00 \text{ k (T)}$ **Ans.**

$\circlearrowleft + \sum M_C = 0;\quad 1.5(30) + 2(15) + 7.778 \cos 45° - F_{AG}(15) = 0$

$F_{AG} = 10.5 \text{ k (C)}$ **Ans.**



(a)

Method of Joints.

Joint *E*: Referring to Fig. c,

$\overset{+}{\rightarrow} \sum F_x = 0;\quad F_{EF} \cos 45° - 1.5 = 0 \quad F_{EF} = 2.121 \text{ k (C)} = 2.12 \text{ k (C)}$ **Ans.**

$+\uparrow \sum F_y = 0;\quad 2.121 \sin 45° - F_{DE} = 0 \quad F_{DE} = 1.50 \text{ k (T)}$ **Ans.**

Joint *F*: Referring to Fig. d,

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 4.950 \sin 45° - 2.121 \cos 45° - F_{DF} = 0 \quad F_{DF} = 2.00 \text{ k (C)}$ **Ans.**

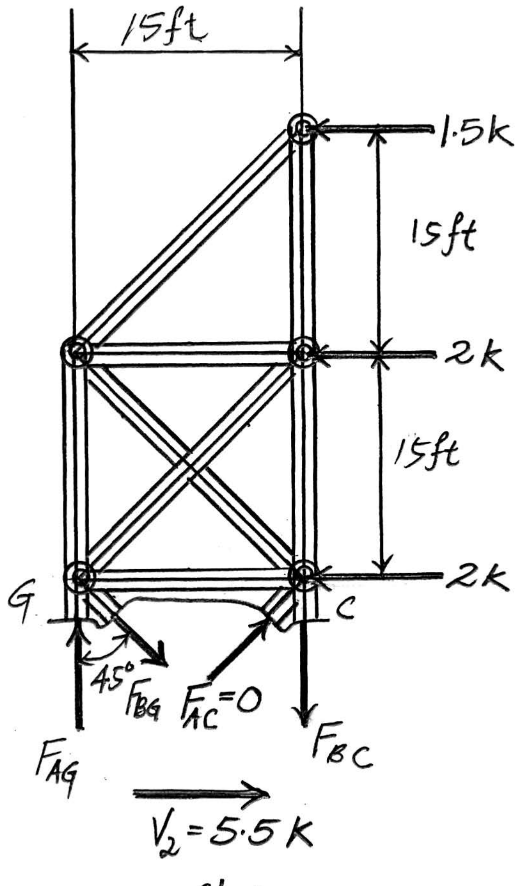
Joint *G*: Referring to Fig. e,

$\overset{+}{\rightarrow} \sum F_x = 0;\quad 7.778 \sin 45° - F_{CG} = 0 \quad F_{CG} = 5.50 \text{ k (C)}$ **Ans.**

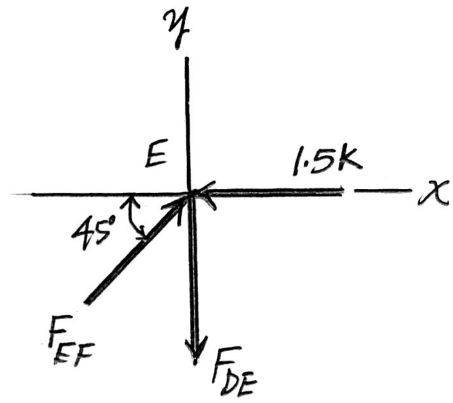
Joint *A*: Referring to Fig. f,

$\overset{+}{\rightarrow} \sum F_x = 0;\quad F_{AB} = 0$ **Ans.**

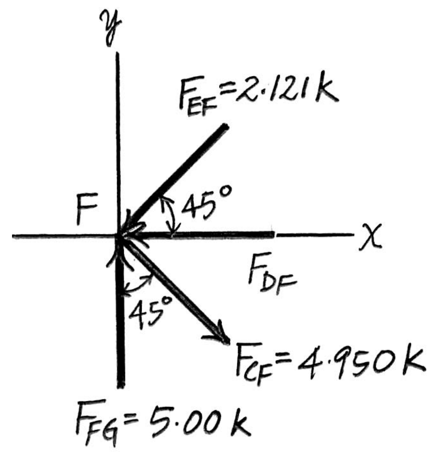
7–10. Continued



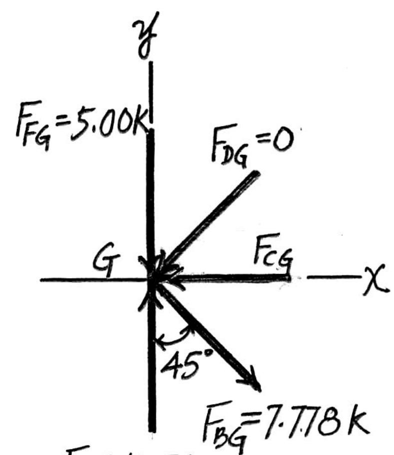
(b)



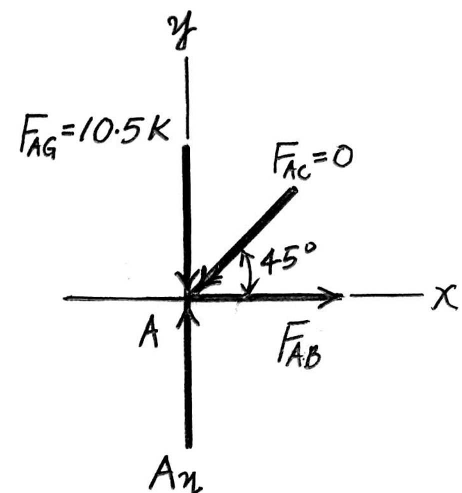
(c)



(d)

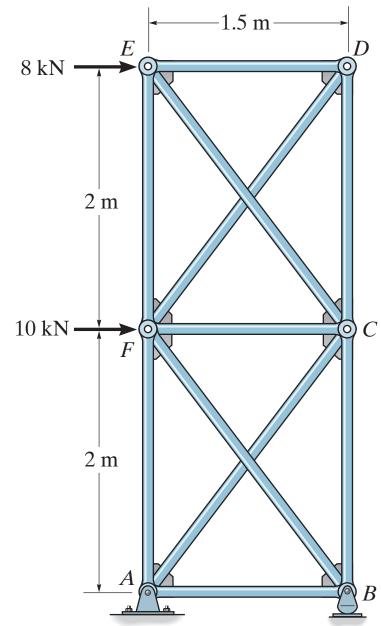


(e)



(f)

7–11. Determine (approximately) the force in each member of the truss. Assume the diagonals can support either a tensile or compressive force.



Method of Sections. It is required that $F_{CE} = F_{DF} = F_1$. Referring to Fig. *a*,

$$\xrightarrow{+} \sum F_x = 0; \quad 8 - 2F_1\left(\frac{3}{5}\right) = 0 \quad F_1 = 6.667 \text{ kN}$$

Therefore,

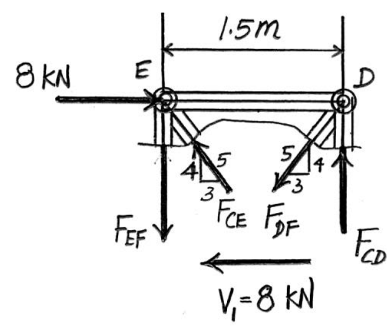
$F_{CE} = 6.67$ kN (C) $\quad F_{DF} = 6.67$ kN (T) **Ans.**

$$\circlearrowleft + \sum M_E = 0; \quad F_{CD}(1.5) - 6.667\left(\frac{4}{5}\right)(1.5) = 0 \quad F_{CD} = 5.333 \text{ kN (C)} = 5.33 \text{ kN (C)}$$

Ans.

$$\circlearrowleft + \sum M_D = 0; \quad F_{EF}(1.5) - 6.667\left(\frac{4}{5}\right)(1.5) = 0 \quad F_{EF} = 5.333 \text{ kN (T)} = 5.33 \text{ kN (T)}$$

Ans.



(a)

It is required that $F_{BF} = F_{AC} = F_2$ Referring to Fig. *b*,

$$\xrightarrow{+} \sum F_x = 0; \quad 8 + 10 - 2F_2\left(\frac{3}{5}\right) = 0 \quad F_2 = 15.0 \text{ kN}$$

Therefore,

$F_{BF} = 15.0$ kN (C) $\quad F_{AC} = 15.0$ kN (T) **Ans.**

$$\circlearrowleft + \sum M_F = 0; \quad F_{BC}(1.5) - 15.0\left(\frac{4}{5}\right)(1.5) - 8(2) = 0$$

$F_{BC} = 22.67$ kN (C) $= 22.7$ kN (C) **Ans.**

$$\circlearrowleft + \sum M_C = 0; \quad F_{AF}(1.5) - 15.0\left(\frac{4}{5}\right)(1.5) - 8(2) = 0$$

$F_{AF} = 22.67$ kN (T) $= 22.7$ kN (T) **Ans.**

Method of Joints.

Joint *D*: Referring to Fig. *c*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{DE} - 6.667\left(\frac{3}{5}\right) = 0 \quad F_{DE} = 4.00 \text{ kN (C)}$$ **Ans.**

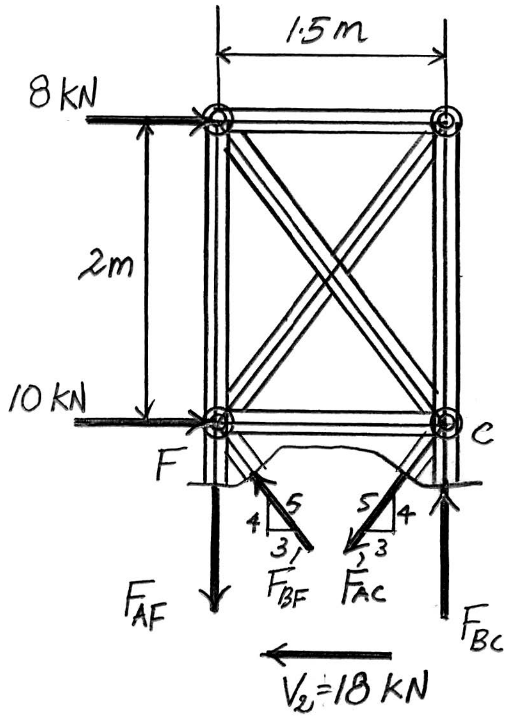
Joint *C*: Referring to Fig. *d*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{CF} + 6.667\left(\frac{3}{5}\right) - 15.0\left(\frac{3}{5}\right) = 0 \quad F_{CF} = 5.00 \text{ kN (C)}$$ **Ans.**

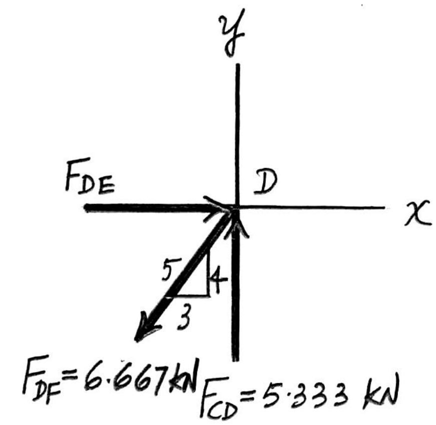
Joint *B*: Referring to Fig. *e*,

$$\xrightarrow{+} \sum F_x = 0; \quad 15.0\left(\frac{3}{5}\right) - F_{AB} = 9.00 \text{ kN (T)}$$ **Ans.**

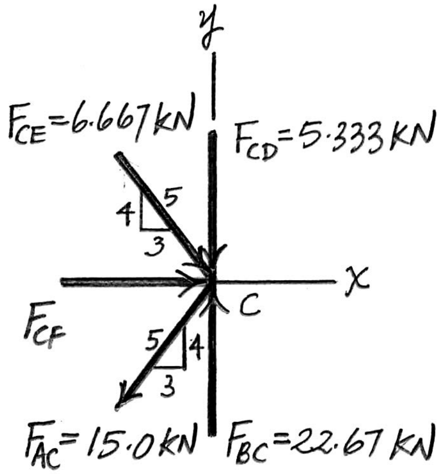
7–11. Continued



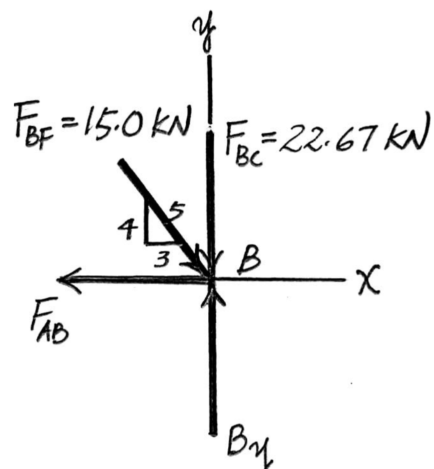
(b)



(c)

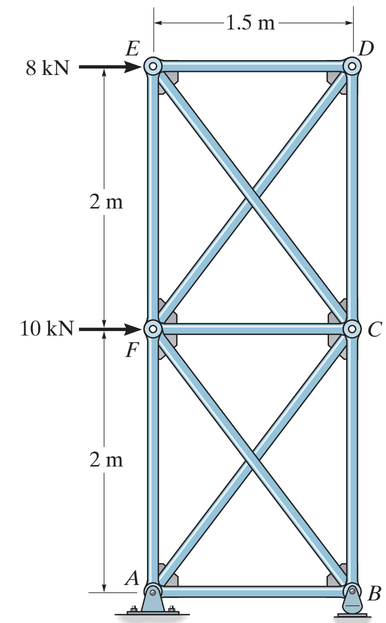


(d)



(e)

***7–12.** Determine (approximately) the force in each member of the truss. Assume the diagonals cannot support a compressive force.



Method of Sections. It is required that

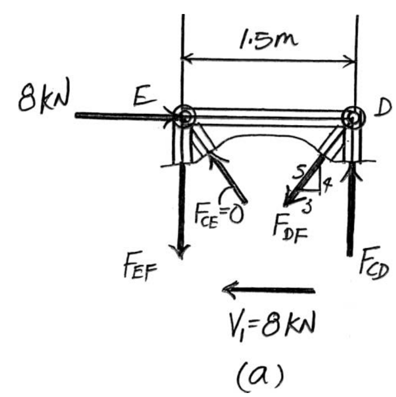
$$F_{CE} = F_{BF} = 0 \qquad \textbf{Ans.}$$

Referring to Fig. *a*,

$$\xrightarrow{+} \sum F_x = 0; \quad 8 - F_{DF}\left(\frac{3}{5}\right) = 0 \quad F_{DF} = 13.33 \text{ kN (T)} = 13.3 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_E = 0; \quad F_{CD}(1.5) - 13.33\left(\frac{4}{5}\right)(1.5) = 0 \quad F_{CD} = 10.67 \text{ kN (C)} = 10.7 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_D = 0; \quad F_{EF}(1.5) = 0 \quad F_{EF} = 0 \qquad \textbf{Ans.}$$



Referring to Fig. *b*,

$$\xrightarrow{+} \sum F_x = 0; \quad 8 + 10 - F_{AC}\left(\frac{3}{5}\right) = 0 \quad F_{AC} = 30.0 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_C = 0; \quad F_{AF}(1.5) - 8(2) = 0 \quad F_{AF} = 10.67 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_F = 0; \quad F_{BC}(1.5) - 30.0\left(\frac{4}{5}\right)(1.5) - 8(2) = 0$$

$$F_{BC} = 34.67 \text{ kN (C)} = 34.7 \text{ kN (C)} \qquad \textbf{Ans.}$$

Method of Joints.

Joint *E*: Referring to Fig. *c*,

$$\xrightarrow{+} \sum F_x = 0; \quad 8 - F_{DE} = 0 \quad F_{DE} = 8.00 \text{ kN (C)} \qquad \textbf{Ans.}$$

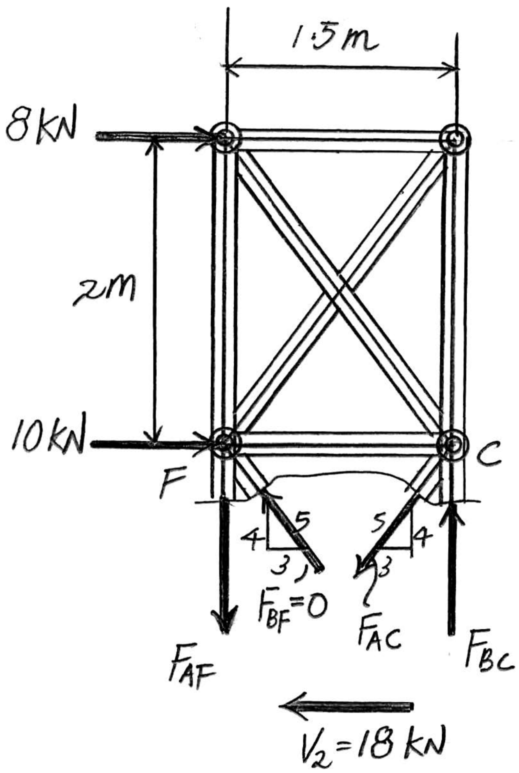
Joint *C*: Referring to Fig. *d*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{CF} - 30.0\left(\frac{3}{5}\right) = 0 \quad F_{CF} = 18.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

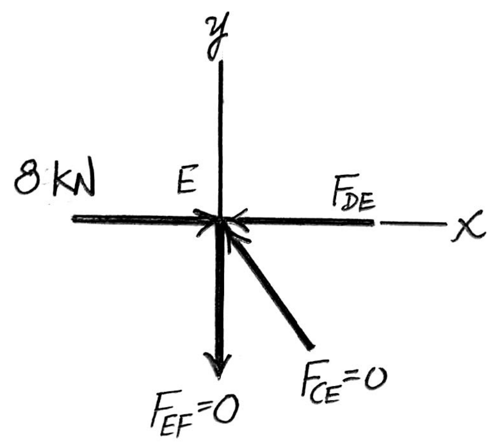
Joint *B*: Referring to Fig. *e*,

$$\xrightarrow{+} \sum F_x = 0; \quad F_{AB} = 0 \qquad \textbf{Ans.}$$

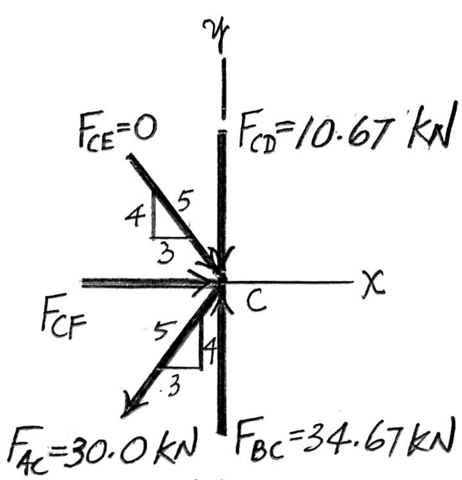
7–12. Continued



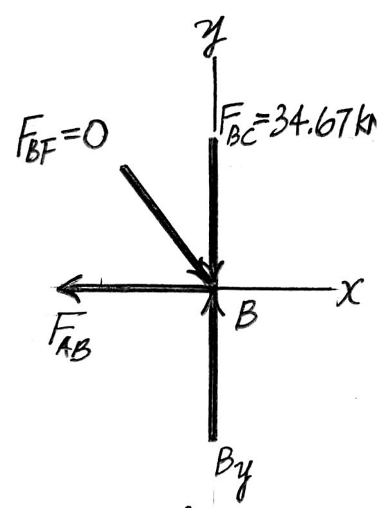
(b)



(c)

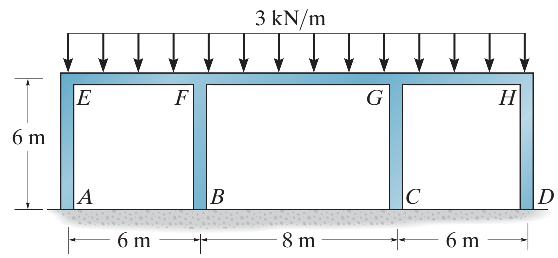


(d)



(e)

7–13. Determine (approximately) the internal moments at joints A and B of the frame.



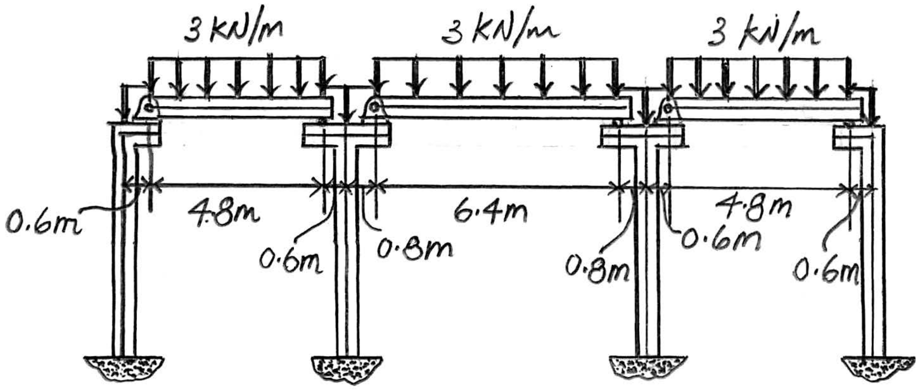
The frame can be simplified to that shown in Fig. *a*, referring to Fig. *b*,

$\circlearrowleft + \sum M_A = 0; \quad M_A - 7.2(0.6) - 3(0.6)(0.3) = 0 \quad M_A = 4.86 \text{ kN} \cdot \text{m}$ **Ans.**

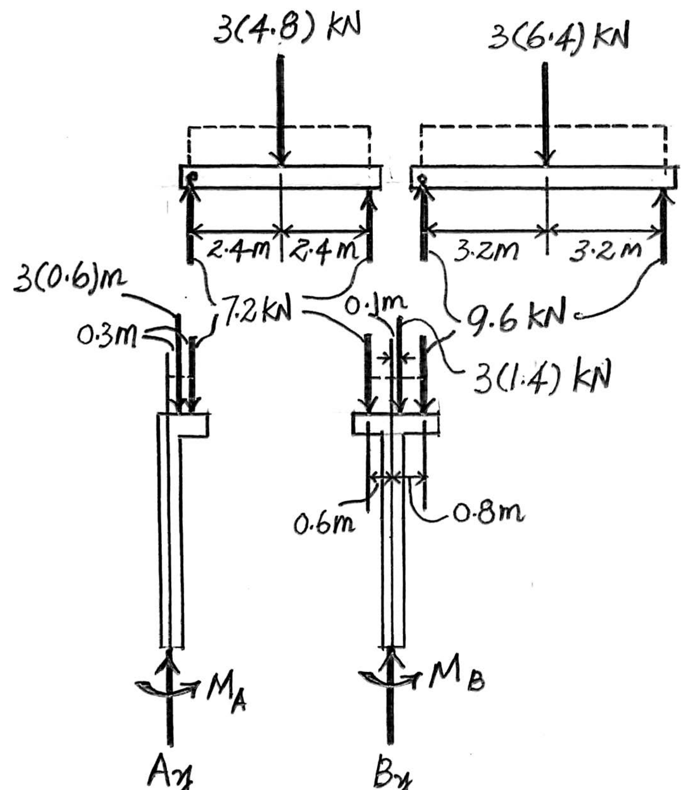
Referring to Fig. *c*,

$\circlearrowleft + \sum M_B = 0; \quad M_B - 9.6(0.8) - 3(1.4)(0.1) + 7.2(0.6) = 0$

$M_B = 3.78 \text{ kN} \cdot \text{m}$ **Ans.**

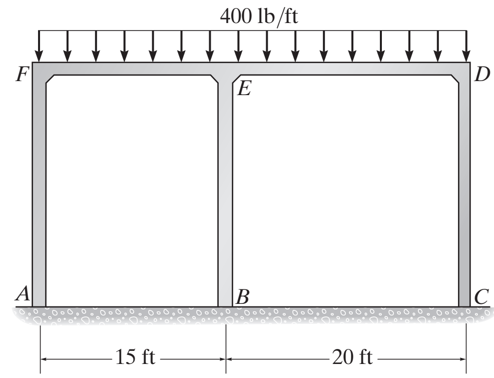


(a)



(b) (c)

7–14. Determine (approximately) the internal moments at joints F and D of the frame.

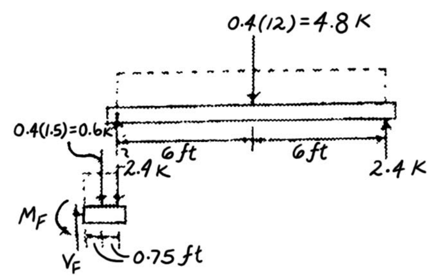


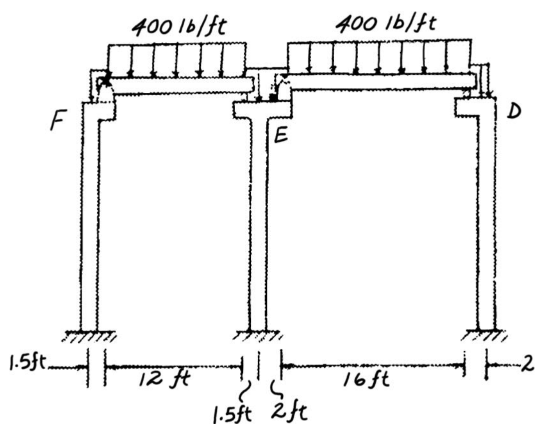
$\circlearrowleft + \sum M_F = 0; \quad M_F - 0.6(0.75) - 2.4(1.5) = 0$

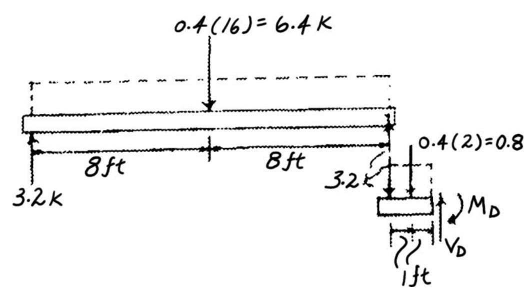
$M_F = 4.05 \text{ k} \cdot \text{ft}$ **Ans.**

$\circlearrowleft + \sum M_D = 0; \quad -M_D + 0.8(1) + 3.2(2) = 0$

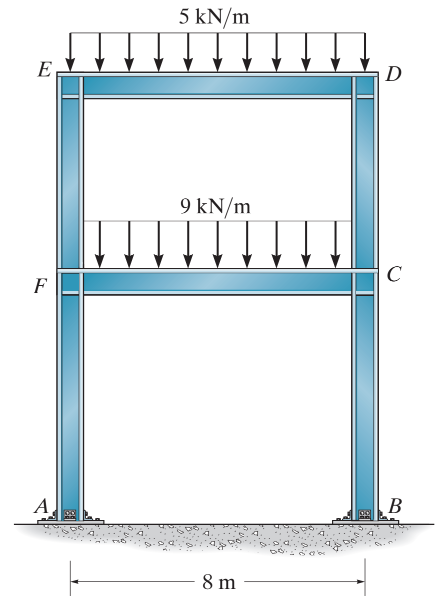
$M_D = 7.20 \text{ k} \cdot \text{ft}$ **Ans.**







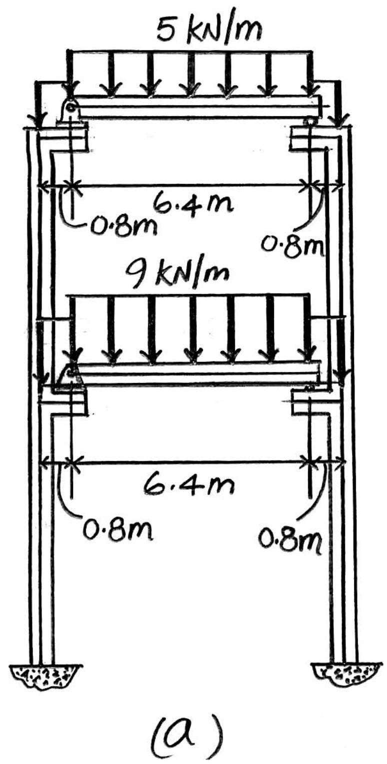
7–15. Determine (approximately) the internal moment at A caused by the vertical loading.

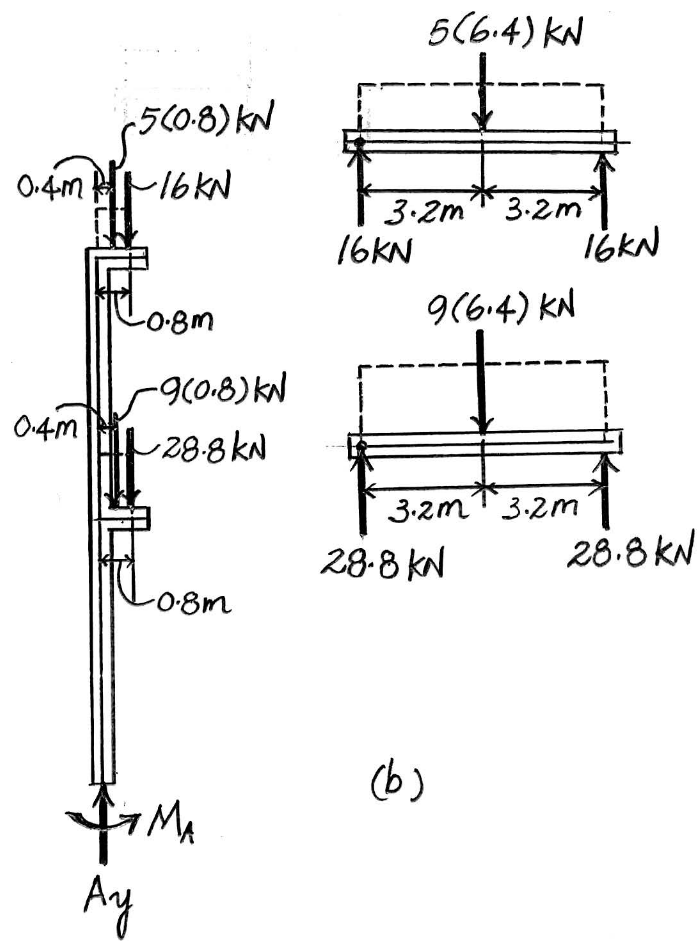


The frame can be simplified to that shown in Fig. a, Referring to Fig. b,

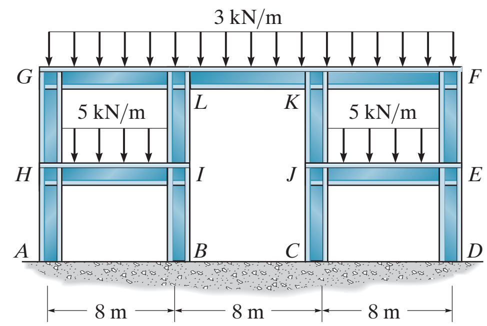
$$\circlearrowleft + \sum M_A = 0; \quad M_A - 5(0.8)(0.4) - 16(0.8) - 9(0.8)(0.4) - 28.8(0.8) = 0$$

$$M_A = 40.32 \text{ kN} \cdot \text{m} = 40.3 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$





***7–16.** Determine (approximately) the internal moments at A and B caused by the vertical loading.



The frame can be simplified to that shown in Fig. *a*. The reactions of the 3 kN/m and 5 kN/m uniform distributed loads are shown in Fig. *b* and *c* respectively. Referring to Fig. *d*,

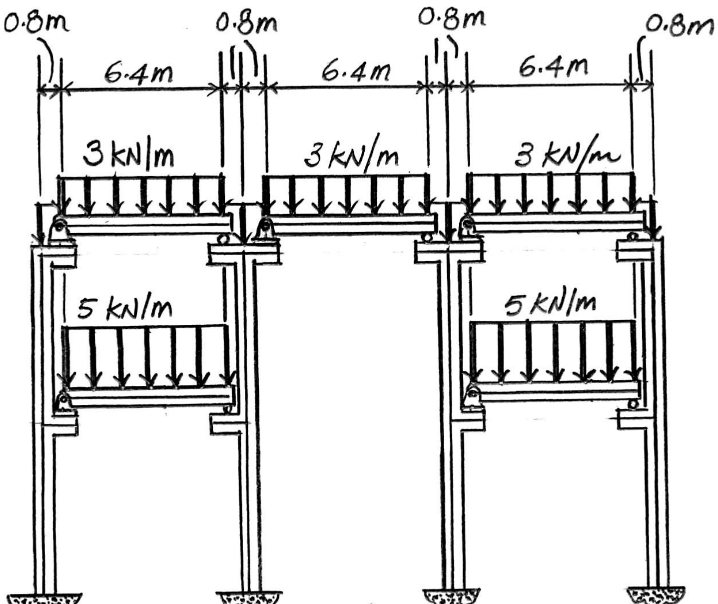
$$\circlearrowleft + \sum M_A = 0; \; M_A - 3(0.8)(0.4) - 9.6(0.8) - 5(0.8)(0.4) - 16(0.8) = 0$$

$$M_A = 23.04 \text{ kN} \cdot \text{m} = 23.0 \text{ kN} \cdot \text{m}$$ **Ans.**

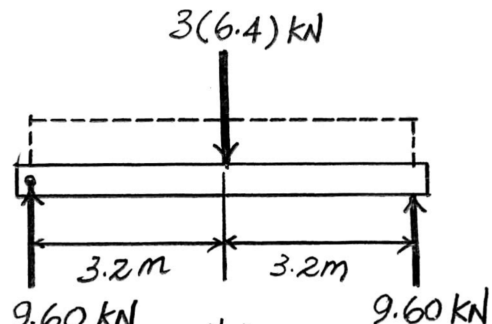
Referring to Fig. *e*,

$$\circlearrowleft + \sum M_B = 0; \; 9.60(0.8) - 9.60(0.8) + 5(0.8)(0.4) + 16(0.8) - M_B = 0$$

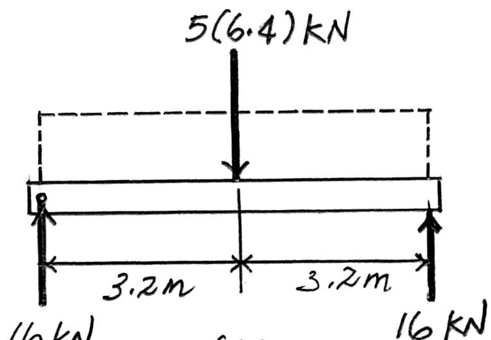
$$M_B = 14.4 \text{ kN} \cdot \text{m}$$ **Ans.**



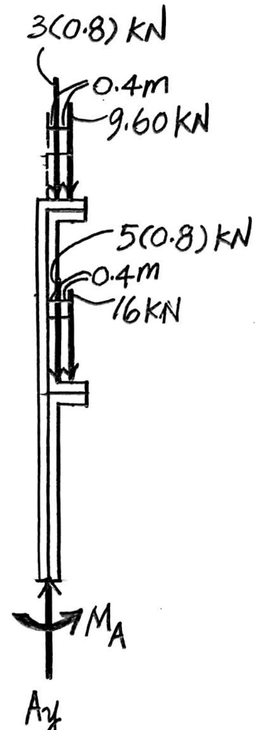
(a)



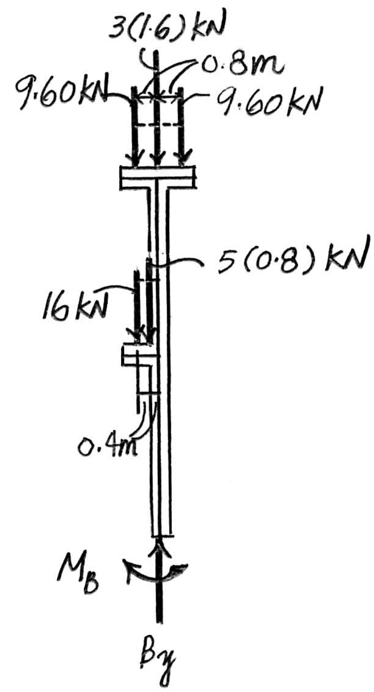
(b)



(c)

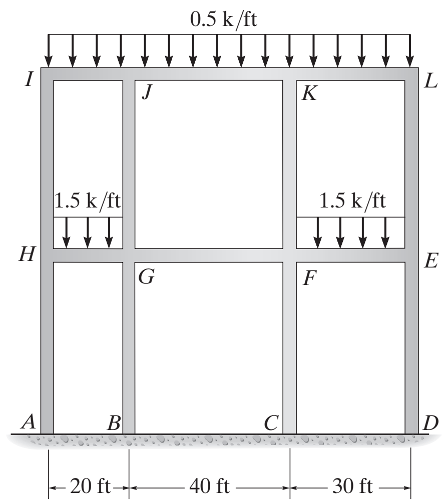


(d)



(e)

7–17. Determine (approximately) the internal moments at joints I and L. Also, what is the internal moment at joint H caused by member HG?



Joint *I*:

$\circlearrowleft + \sum M_I = 0; \qquad M_I - 1.0(1) - 4.0(2) = 0$

$M_I = 9.00 \text{ k} \cdot \text{ft}$ **Ans.**

Joint *L*:

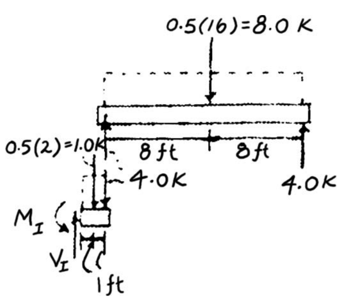
$\circlearrowleft + \sum M_L = 0; \qquad M_L - 6.0(3) - 1.5(1.5) = 0$

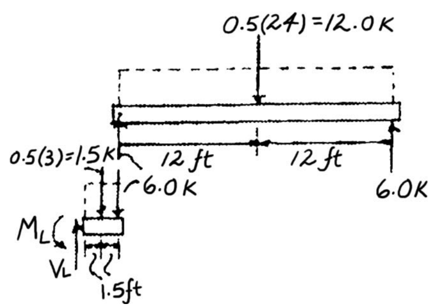
$M_L = 20.25 \text{ k} \cdot \text{ft}$ **Ans.**

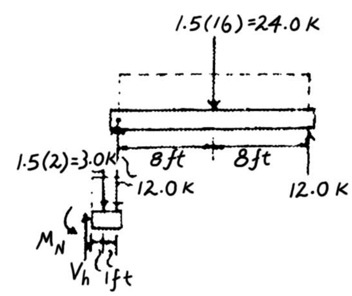
Joint *H*:

$\circlearrowleft + \sum M_H = 0; \qquad M_H - 3.0(1) - 12.0(2) = 0$

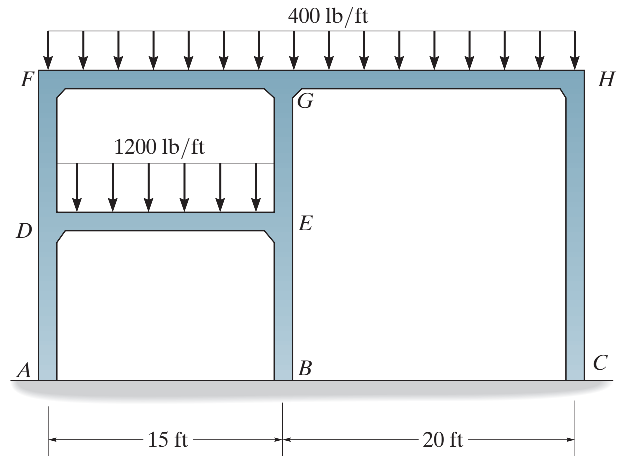
$M_H = 27.0 \text{ k} \cdot \text{ft}$ **Ans.**







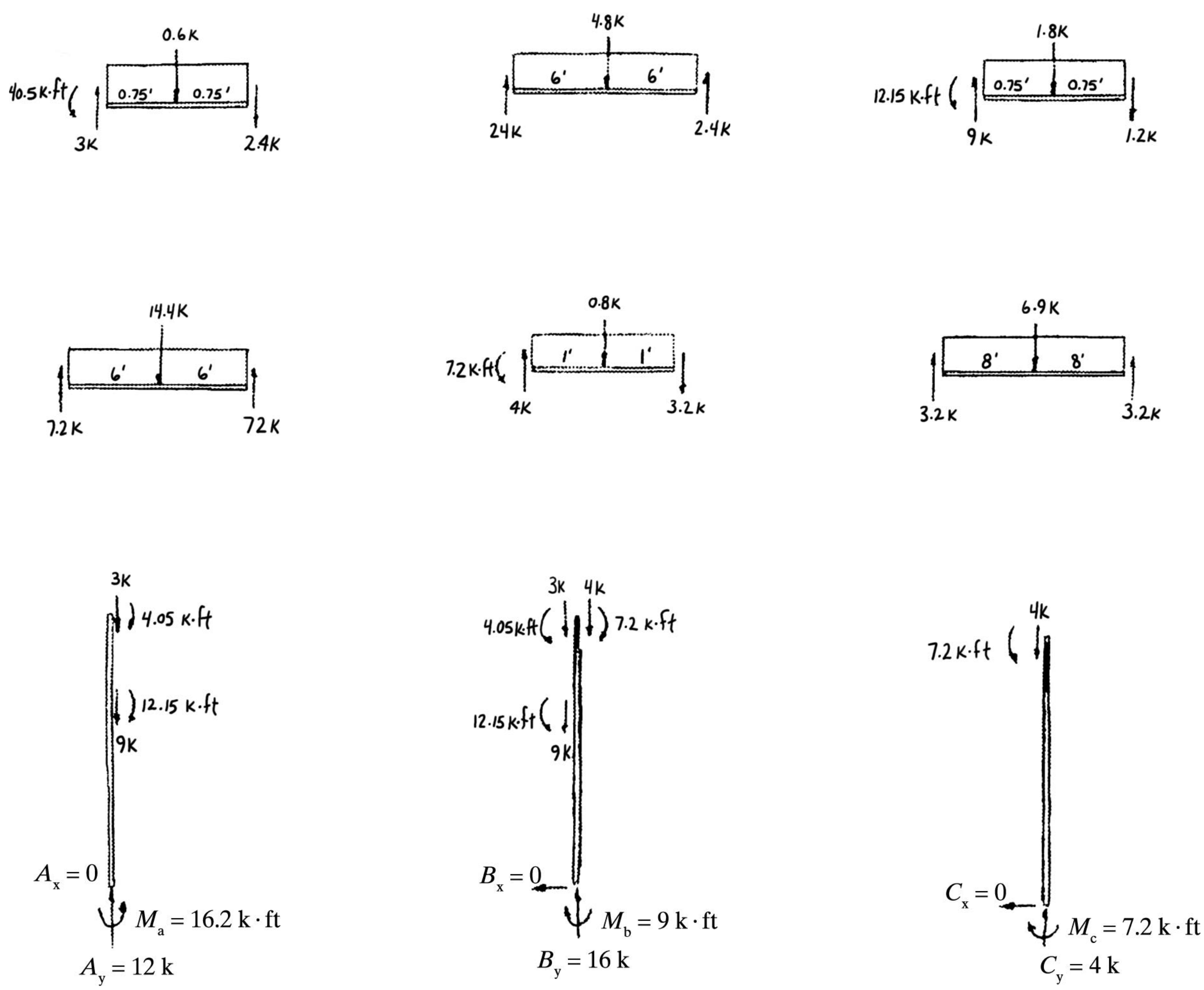
7–18. Determine (approximately) the support actions at A, B, and C of the frame.



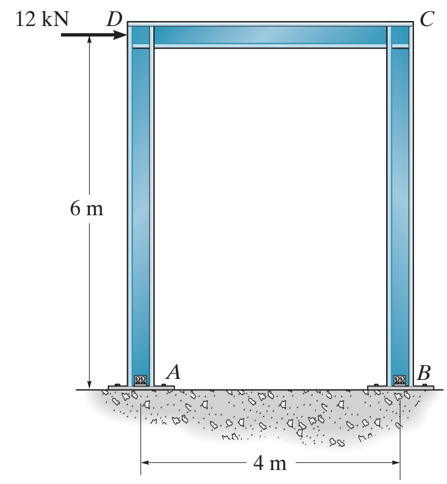
$A_x = 0 \qquad B_x = 0 \qquad C_x = 0$ **Ans.**

$A_y = 12\text{ k} \qquad B_y = 16\text{ k} \qquad C_y = 4\text{ k}$ **Ans.**

$M_A = 16.2\text{ k}\cdot\text{ft} \qquad M_B = 9\text{ k}\cdot\text{ft} \qquad M_C = 7.2\text{ k}\cdot\text{ft}$ **Ans.**



7–19. Determine (approximately) the support reactions at A and B of the portal frame. Assume the supports are (a) pinned, and (b) fixed.



For printed base, referring to Fig. a and b,

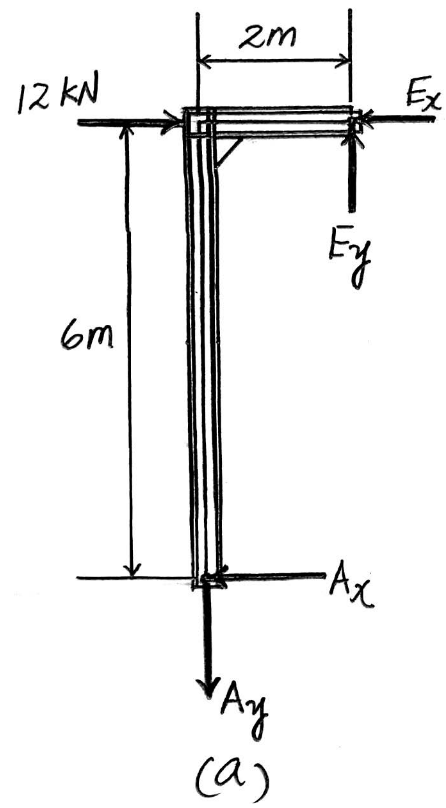
$\circlearrowleft + \sum M_A = 0; \quad E_x(6) + E_y(2) - 12(6) = 0 \quad (1)$

$\circlearrowleft + \sum M_B = 0; \quad E_y(6) - E_x(6) = 0 \quad (2)$

Solving Eqs. (1) and (2) yield

$E_y = 18.0 \text{ kN} \qquad E_x = 6.00 \text{ kN}$

Referring to Fig. a,



$\xrightarrow{+} \sum F_x = 0; \quad 12 - 6.00 - A_x = 0 \quad A_x = 6.00 \text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 18.0 - A_y = 0 \quad A_y = 18.0 \text{ kN}$ **Ans.**

Referring to Fig. b,

$\xrightarrow{+} \sum F_x = 0; \quad 6.00 - B_x = 0 \quad B_x = 6.00 \text{ kN}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad B_y - 18.0 = 0 \quad B_y = 18.0 \text{ kN}$ **Ans.**

For the fixed base, referring to Fig. c and d,

$\circlearrowleft + \sum M_E = 0; \quad F_x(3) + F_y(2) - 12(3) = 0 \quad (1)$

$\circlearrowleft + \sum M_G = 0; \quad F_y(2) - F_x(3) = 0 \quad (2)$

Solving Eqs (1) and (2) yields,

$F_y = 9.00 \text{ kN} \qquad F_x = 6.00 \text{ kN}$

Referring to Fig. c,

$\xrightarrow{+} \sum F_x = 0; \quad 12 - 6.00 - E_x = 0 \quad E_x = 6.00 \text{ kN}$

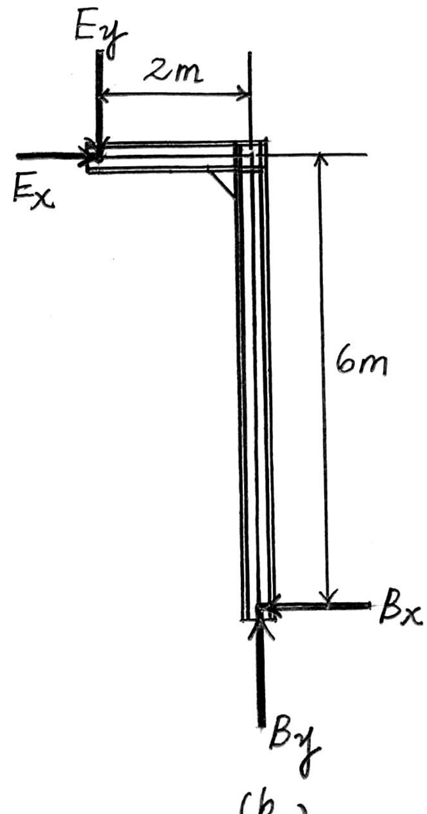
$+\uparrow \sum F_y = 0; \quad 9.00 - E_y = 0 \quad E_y = 9.00 \text{ kN}$

Referring to Fig. d,

$\xrightarrow{+} \sum F_x = 0; \quad 6.00 - G_x = 0 \quad G_x = 6.00 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad G_y - 9.00 = 0 \quad G_y = 9.00 \text{ kN}$

7–19. Continued

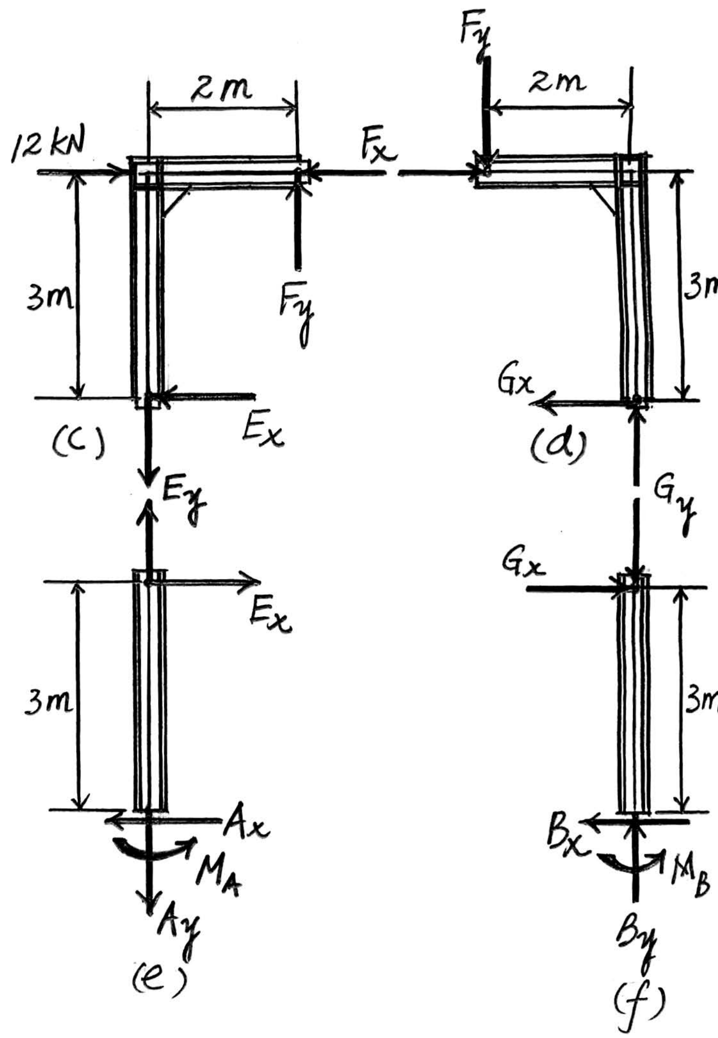


Referring to Fig. *e*,

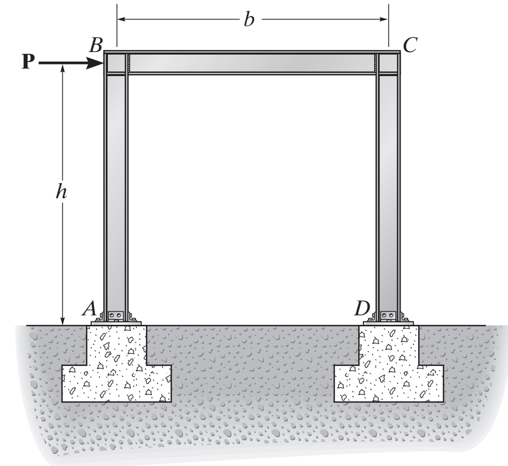
$\xrightarrow{+} \sum F_x = 0;$	$6.00 - A_x = 0$	$A_x = 6.00 \text{ kN}$	**Ans.**
$+\uparrow \sum F_y = 0;$	$9.00 - A_y = 0$	$A_y = 9.00 \text{ kN}$	**Ans.**
$\circlearrowleft + \sum M_A = 0;$	$M_A - 6.00(3) = 0$	$M_A = 18.0 \text{ kN} \cdot \text{m}$	**Ans.**

Referring to Fig. *f*,

$\xrightarrow{+} \sum F_x = 0;$	$6.00 - B_x = 0$	$B_x = 6.00 \text{ kN}$	**Ans.**
$+\uparrow \sum F_y = 0;$	$B_y - 9.00 = 0$	$B_y = 9.00 \text{ kN}$	**Ans.**
$\circlearrowleft + \sum M_B = 0;$	$M_B - 6.00(3) = 0$	$M_B = 18.0 \text{ kN} \cdot \text{m}$	**Ans.**



***7–20.** Determine (approximately) the internal moment and shear at the ends of each member of the portal frame. Assume the supports at A and D are partially fixed, such that an inflection point is located at $h/3$ from the bottom of each column.



$$\circlearrowleft + \sum M_B = 0; \qquad G_y(b) - P\left(\frac{2h}{3}\right) = 0$$

$$G_y = P\left(\frac{2h}{3b}\right)$$

$$+\uparrow \sum F_y = 0; \qquad E_y = \frac{2Ph}{3b} = 0$$

$$E_y = \frac{2Ph}{3b}$$

$$M_A = M_D = \frac{P}{2}\left(\frac{h}{3}\right) = \frac{Ph}{6} \qquad \textbf{Ans.}$$

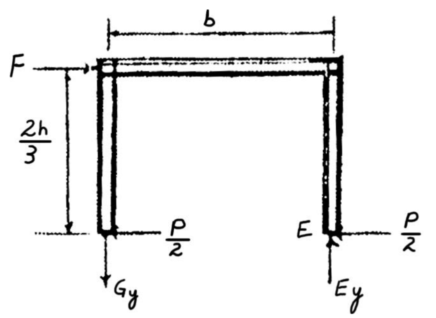
$$M_B = M_C = \frac{P}{2}\left(\frac{2h}{3}\right) = \frac{Ph}{3} \qquad \textbf{Ans.}$$

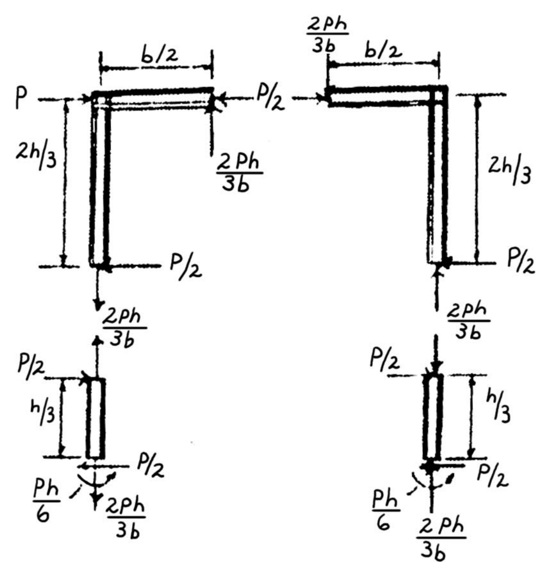
Member BC:

$$V_B = V_C = \frac{2Ph}{3b} \qquad \textbf{Ans.}$$

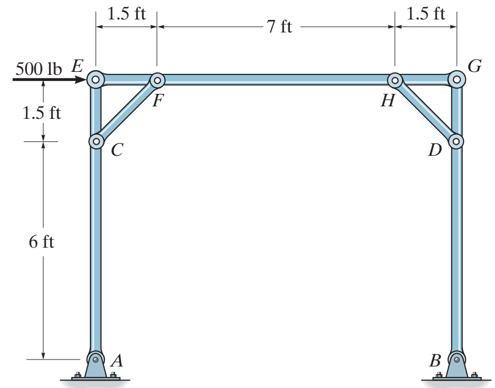
Members AB and CD:

$$V_A = V_B = V_C = V_D = \frac{P}{2} \qquad \textbf{Ans.}$$





7–21. Draw (approximately) the moment diagram for member *ACE* of the portal constructed with a *rigid* member *EG* and knee braces *CF* and *DH*. Assume that all points of connection are pins. Also determine the force in the knee brace *CF*.



Inflection points are at *A* and *B*.

From FBD (1):

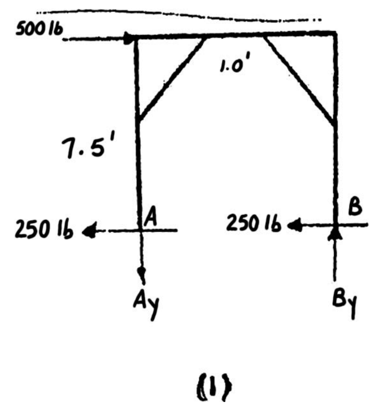
$\circlearrowleft + \sum M_B = 0; \quad A_y(10) - 500(7.5) = 0; \quad A_y = 375 \text{ lb}$

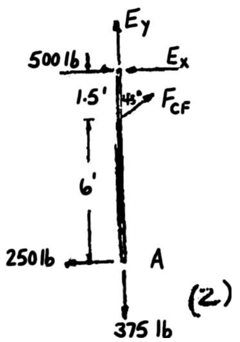
From FBD (2):

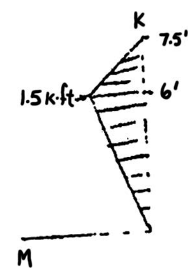
$\circlearrowleft + \sum M_E = 0; \quad 250(7.5) - F_{CF}(\sin 45°)(1.5) = 0; \quad F_{CF} = 1.77 \text{ k(T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad -250 + 1767.8(\sin 45°) + 500 - E_x = 0$

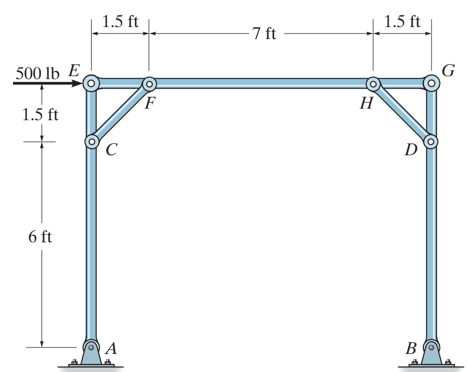
$E_x = 1500 \text{ lb}$







***7–22.** Solve Prob. 7–21 if the supports at *A* and *B* are fixed instead of pinned.



Inflection points are as mid-points of columns

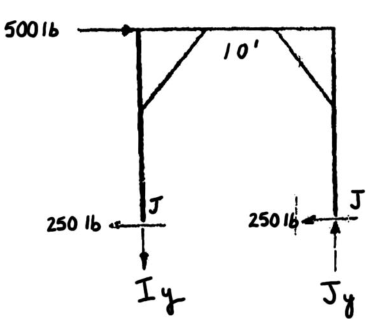
$\circlearrowleft + \sum M_I = 0; \quad J_y(10) - 500(3.5) = 0; \quad J_y = 175 \text{ lb}$

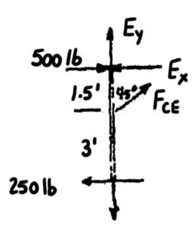
$+\uparrow \sum F_y = 0; \quad -I_y + 175 = 0; \quad I_y = 175 \text{ lb}$

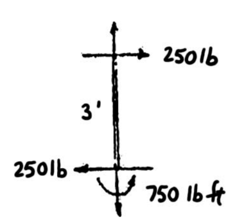
$\circlearrowleft + \sum M_E = 0; \quad 250(4.5) - F_{CE}(\sin 45°)(1.5) = 0; \quad F_{CE} = 1.06 \text{ k(T)}$ **Ans.**

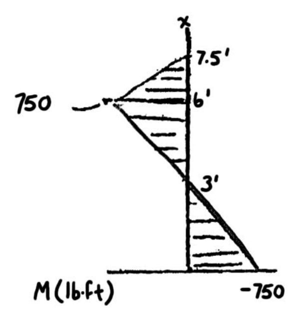
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 500 + 1060.66(\sin 45°) - 250 - E_x = 0;$

$E_x = 1.00 \text{ k}$

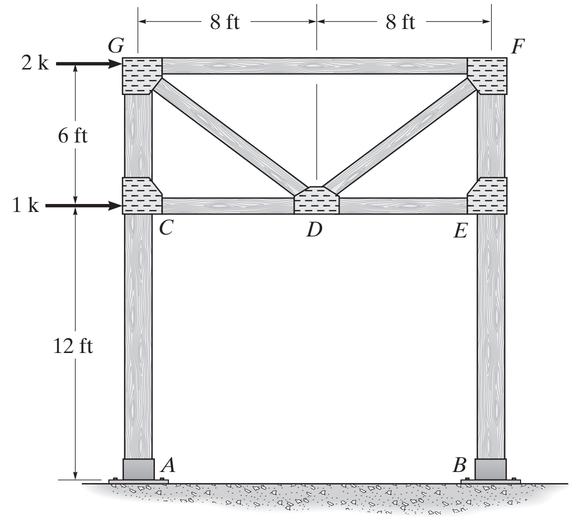








7–23. Determine (approximately) the force in each truss member of the portal frame. Also find the reactions at the fixed column supports A and B. Assume all members of the truss to be pin connected at their ends.



Assume that the horizontal reactive force component at fixed supports A and B are equal. Thus

$$A_x = B_x = \frac{2 + 1}{2} = 1.50 \text{ k} \qquad \textbf{Ans.}$$

Also, the points of inflection H and I are at 6 ft above A and B respectively. Referring to Fig. a,

$$\circlearrowleft + \sum M_I = 0; \quad H_y(16) - 1(6) - 2(12) = 0 \qquad H_y = 1.875 \text{ k}$$

$$+\uparrow \sum F_y = 0; \quad I_y - 1.875 = 0 \qquad I_y = 1.875 \text{ k}$$

Referring to Fig. b,

$$\xrightarrow{+} \sum F_x = 0; \quad H_x - 1.50 = 0 \qquad H_x = 1.50 \text{ k}$$

$$+\uparrow \sum F_y = 0; \quad 1.875 - A_y = 0 \qquad A_y = 1.875 \text{ k} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_A = 0; \quad M_A - 1.50(6) = 0 \qquad M_A = 9.00 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$

Referring to Fig. c,

$$\xrightarrow{+} \sum F_x = 0; \quad 1.50 - B_x = 0 \qquad B_x = 1.50 \text{ k}$$

$$+\uparrow \sum F_y = 0; \quad B_y - 1.875 = 0 \qquad B_y = 1.875 \text{ k} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_B = 0; \quad M_B - 1.50(6) = 0 \qquad M_B = 9.00 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$

Using the method of sections, Fig. d,

$$+\uparrow \sum F_y = 0; \quad F_{DG}\left(\frac{3}{5}\right) - 1.875 = 0 \qquad F_{DG} = 3.125 \text{ k (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_G = 0; \quad F_{CD}(6) + 1(6) - 1.50(12) = 0 \qquad F_{CD} = 2.00 \text{ k (C)} \qquad \textbf{Ans.}$$

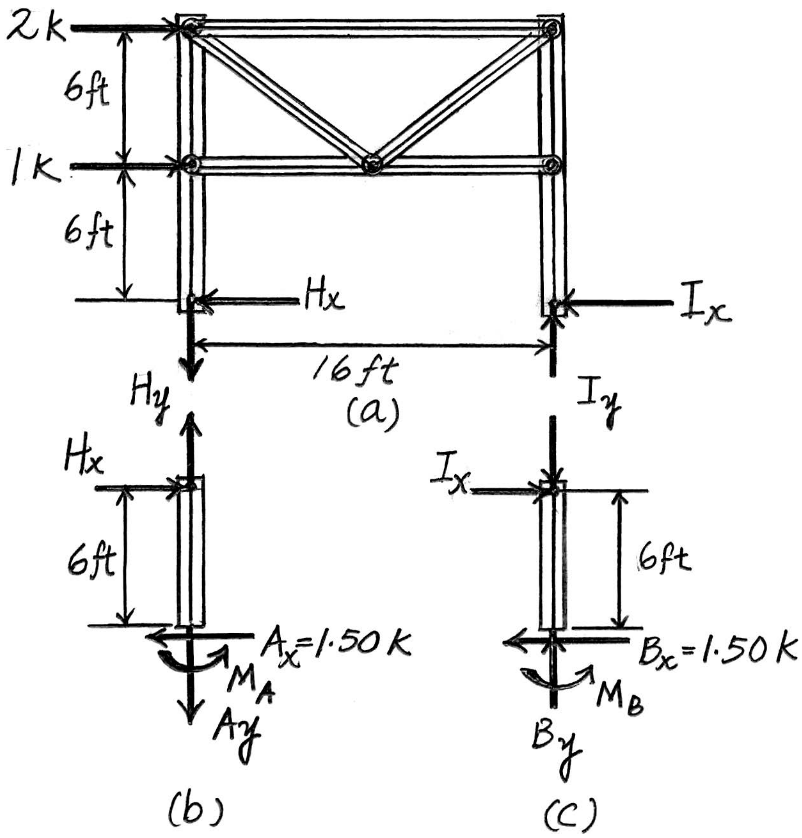
$$\circlearrowleft + \sum M_D = 0; \quad F_{FG}(6) - 2(6) + 1.5(6) + 1.875(8) = 0 \qquad F_{FG} = 1.00 \text{ k (C)} \qquad \textbf{Ans.}$$

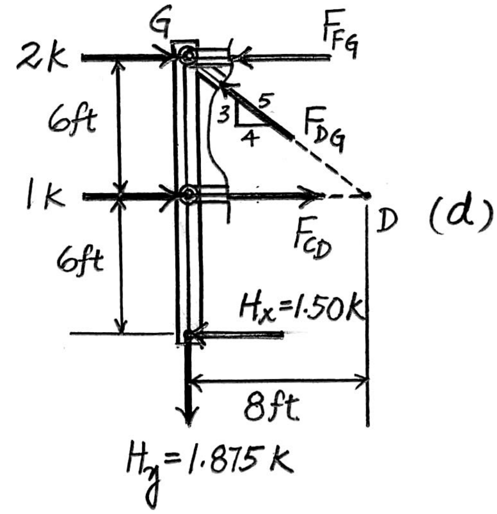
Using the method of Joints, Fig. e,

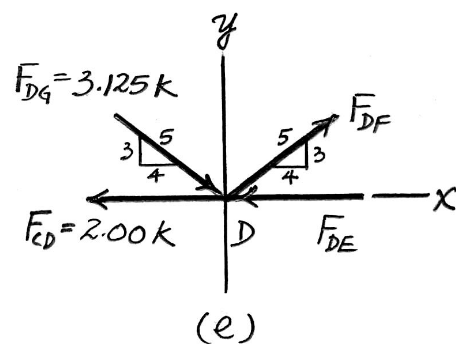
$$+\uparrow \sum F_y = 0; \quad F_{DF}\left(\frac{3}{5}\right) - 3.125\left(\frac{3}{5}\right) = 0 \qquad F_{DF} = 3.125 \text{ k (T)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \sum F_x = 0; \quad 3.125\left(\frac{4}{5}\right) + 3.125\left(\frac{4}{5}\right) - 2.00 - F_{DE} = 0 \qquad F_{DE} = 3.00 \text{ k (C)} \qquad \textbf{Ans.}$$

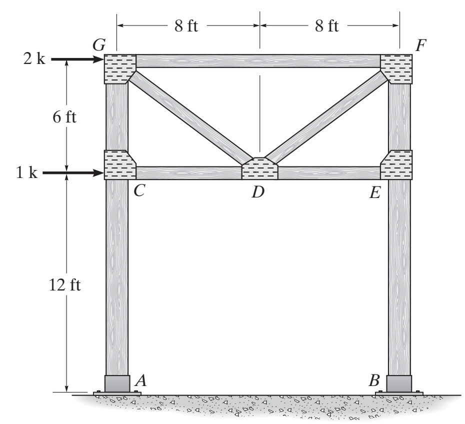
7–23. Continued







***7–24.** Solve Prob. 7–23 if the supports at *A* and *B* are pinned instead of fixed.



Assume that the horizontal reactive force component at pinal supports *A* and *B* are equal. Thus,

$$A_x = B_x = \frac{H2}{2} = 1.50 \text{ k} \qquad \textbf{Ans.}$$

Referring to Fig. *a*,

$$\circlearrowleft + \sum M_B = 0; \quad A_y(16) - 1(12) - 2(18) = 0 \quad A_y = 3.00 \text{ k} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad B_y - 3.00 = 0 \quad B_y = 3.00 \text{ k} \qquad \textbf{Ans.}$$

7–24. Continued

Using the method of sections and referring to Fig. *b*,

$+\uparrow \sum F_y = 0; \quad F_{DG}\left(\frac{3}{5}\right) - 3.00 = 0 \qquad F_{DG} = 5.00 \text{ k (C)}$ **Ans.**

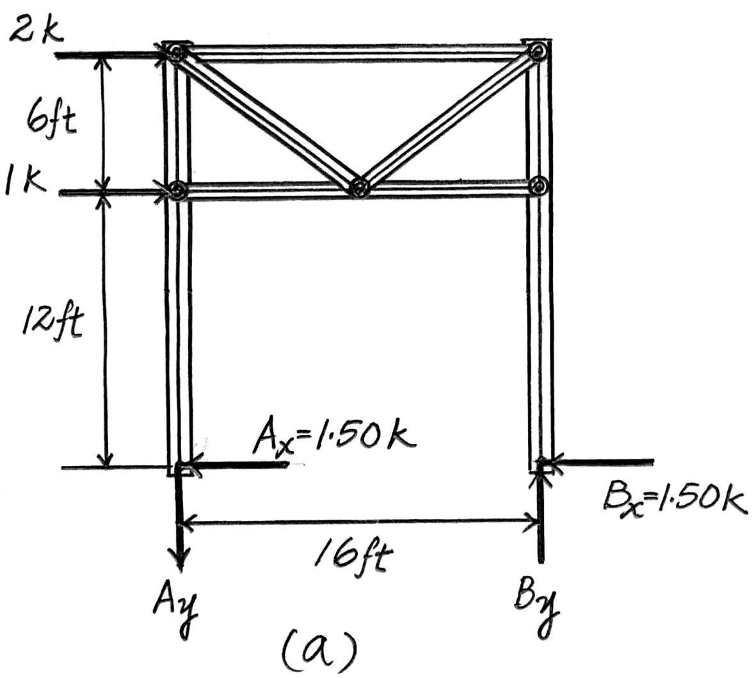
$\circlearrowleft + \sum M_D = 0; \quad F_{GF}(6) - 2(6) - 1.5(12) + 3(8) = 0 \qquad F_{GF} = 1.00 \text{ k (C)}$ **Ans.**

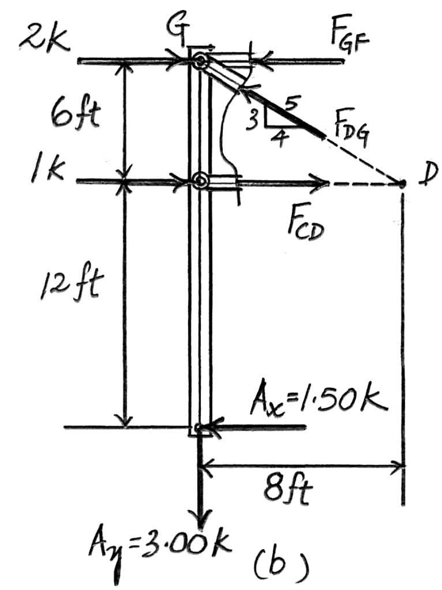
$\circlearrowleft + \sum M_G = 0; \quad F_{CD}(6) + 1(6) - 1.50(18) = 0 \qquad F_{CD} = 3.50 \text{ k (T)}$ **Ans.**

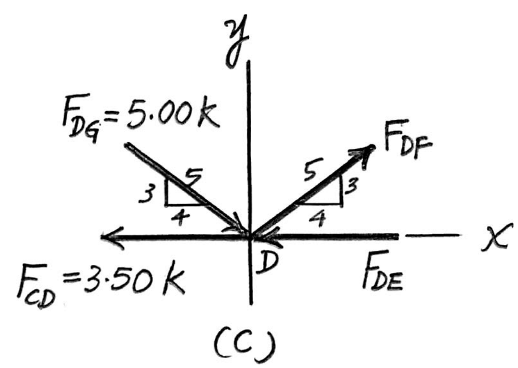
Using the method of joints, Fig. *c*,

$+\uparrow \sum F_y = 0; \quad F_{DF}\left(\frac{3}{5}\right) - 5.00\left(\frac{3}{5}\right) = 0 \qquad F_{DF} = 5.00 \text{ k (T)}$ **Ans.**

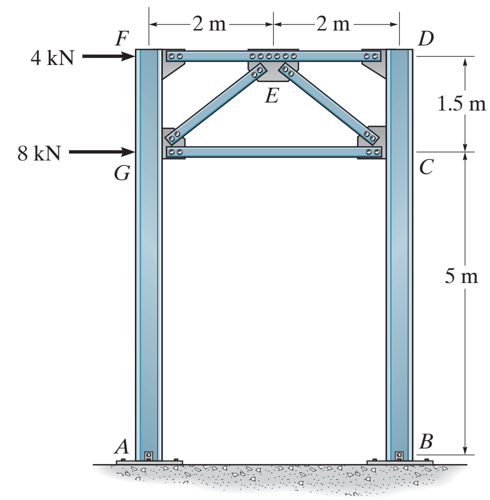
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 5.00\left(\frac{4}{5}\right) + 5.00\left(\frac{4}{5}\right) - 3.50 - F_{DE} = 0 \qquad F_{DE} = 4.50 \text{ k (C)}$ **Ans.**







7–25. Draw (approximately) the moment diagram for column AGF of the portal. Assume all truss members and the columns to be pin connected at their ends. Also determine the force in all the truss members.



Assume that the horizontal force components at pin supports A and B are equal.

Thus,

$$A_x = B_x = \frac{4 + 8}{2} = 6.00 \text{ kN}$$

Referring to Fig. *a*,

$$\circlearrowleft + \sum M_A = 0; \quad B_y(4) - 8(5) - 4(6.5) = 0 \quad B_y = 16.5 \text{ kN}$$

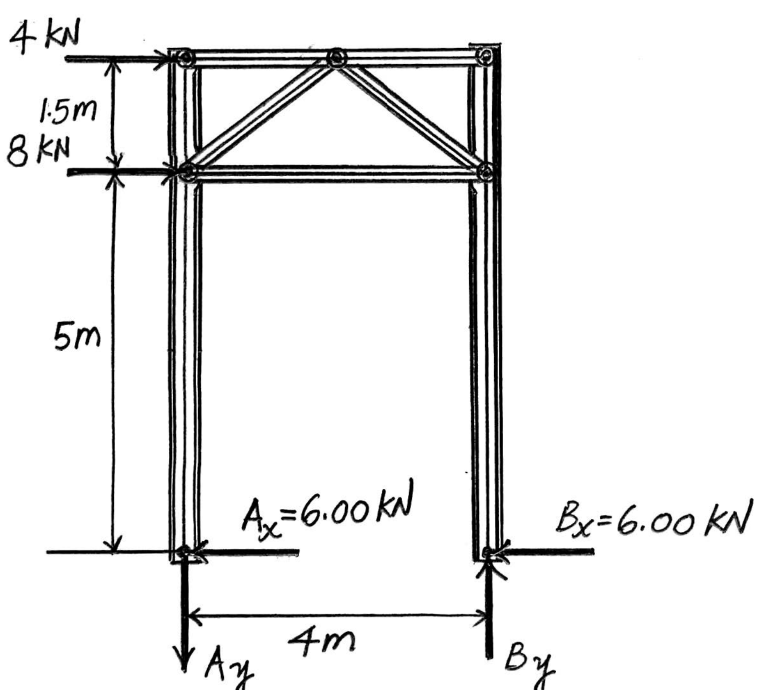
$$+\uparrow \sum F_y = 0; \quad 16.5 - A_y = 0 \quad A_y = 16.5 \text{ kN}$$

Using the method of sections, Fig. *b*,

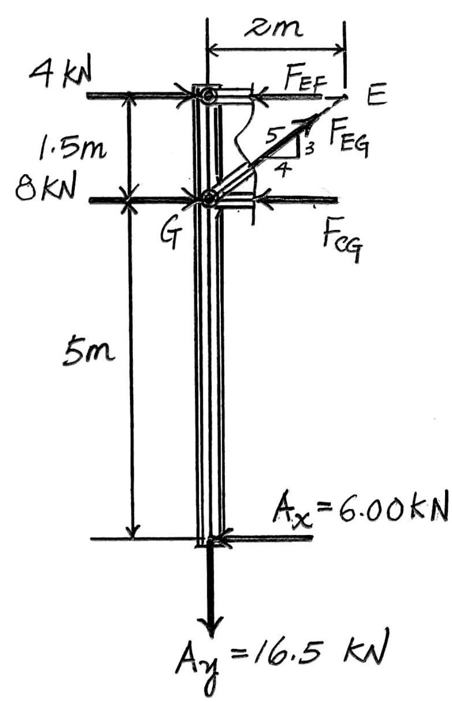
$$+\uparrow \sum F_y = 0; \quad F_{EG}\left(\frac{3}{5}\right) - 16.5 = 0 \quad F_{EG} = 27.5 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_G = 0; \quad F_{EF}(1.5) - 4(1.5) - 6.00(5) = 0 \quad F_{EF} = 24.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_E = 0; \quad 8(1.5) + 16.5(2) - 6(6.5) - F_{CG}(1.5) = 0 \quad F_{CG} = 4.00 \text{ kN (C)} \qquad \textbf{Ans.}$$



(a)



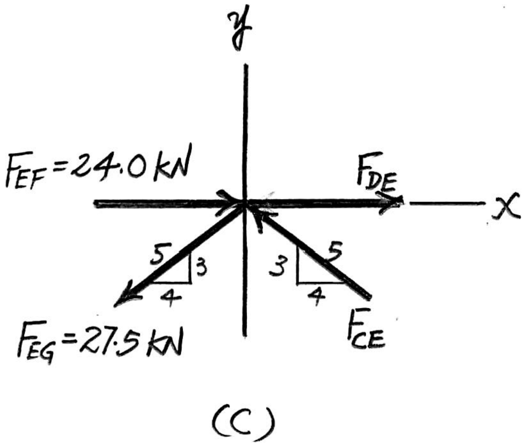
(b)

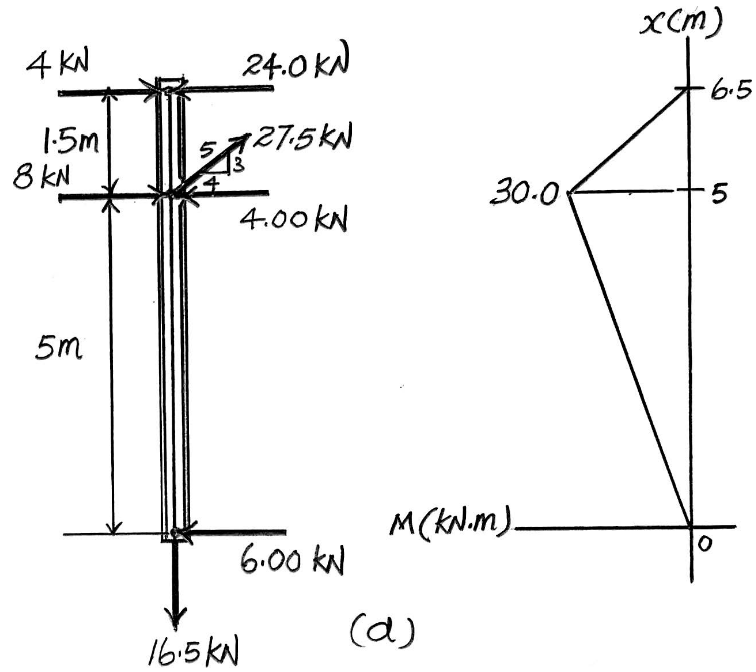
7–25. Continued

Using the method of joints, Fig. *c*,

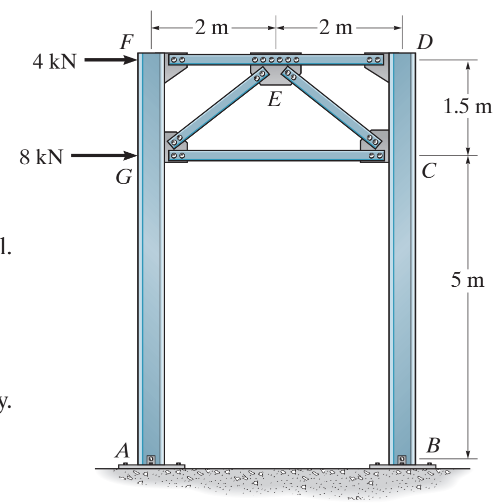
$+\uparrow \sum F_y = 0; \quad F_{CE}\left(\frac{3}{5}\right) - 27.5\left(\frac{3}{5}\right) = 0 \quad F_{CE} = 27.5 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad 24 - 27.5\left(\frac{4}{5}\right) - 27.5\left(\frac{4}{5}\right) + F_{DE} = 0 \quad F_{DE} = 20.0 \text{ kN (T)}$ **Ans.**





7–26. Draw (approximately) the moment diagram for column *AGF* of the portal. Assume all the members of the truss to be pin connected at their ends. The columns are fixed at *A* and *B*. Also determine the force in all the truss members.



Assume that the horizontal force components at fixed supports *A* and *B* are equal. Thus,

$$A_x = B_x = \frac{4 + 8}{2} = 6.00 \text{ kN}$$

Also, the points of inflection *H* and *I* are 2.5 m above *A* and *B*, respectively. Referring to Fig. *a*,

$\circlearrowleft + \sum M_I = 0; \quad H_y(4) - 8(2.5) - 4(4) = 0 \qquad H_y = 9.00 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad I_y - 9.00 = 0 \qquad I_y = 9.00 \text{ kN}$

7–26. Continued

Referring to Fig. *b*,

$\xrightarrow{+} \sum F_x = 0; \quad H_x - 6.00 = 0 \qquad H_x = 6.00 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad 9.00 - A_y = 0 \qquad A_y = 9.00 \text{ kN}$

$\circlearrowleft + \sum M_A = 0; \quad M_A - 6.00(2.5) = 0 \qquad M_A = 15.0 \text{ kN} \cdot \text{m}$

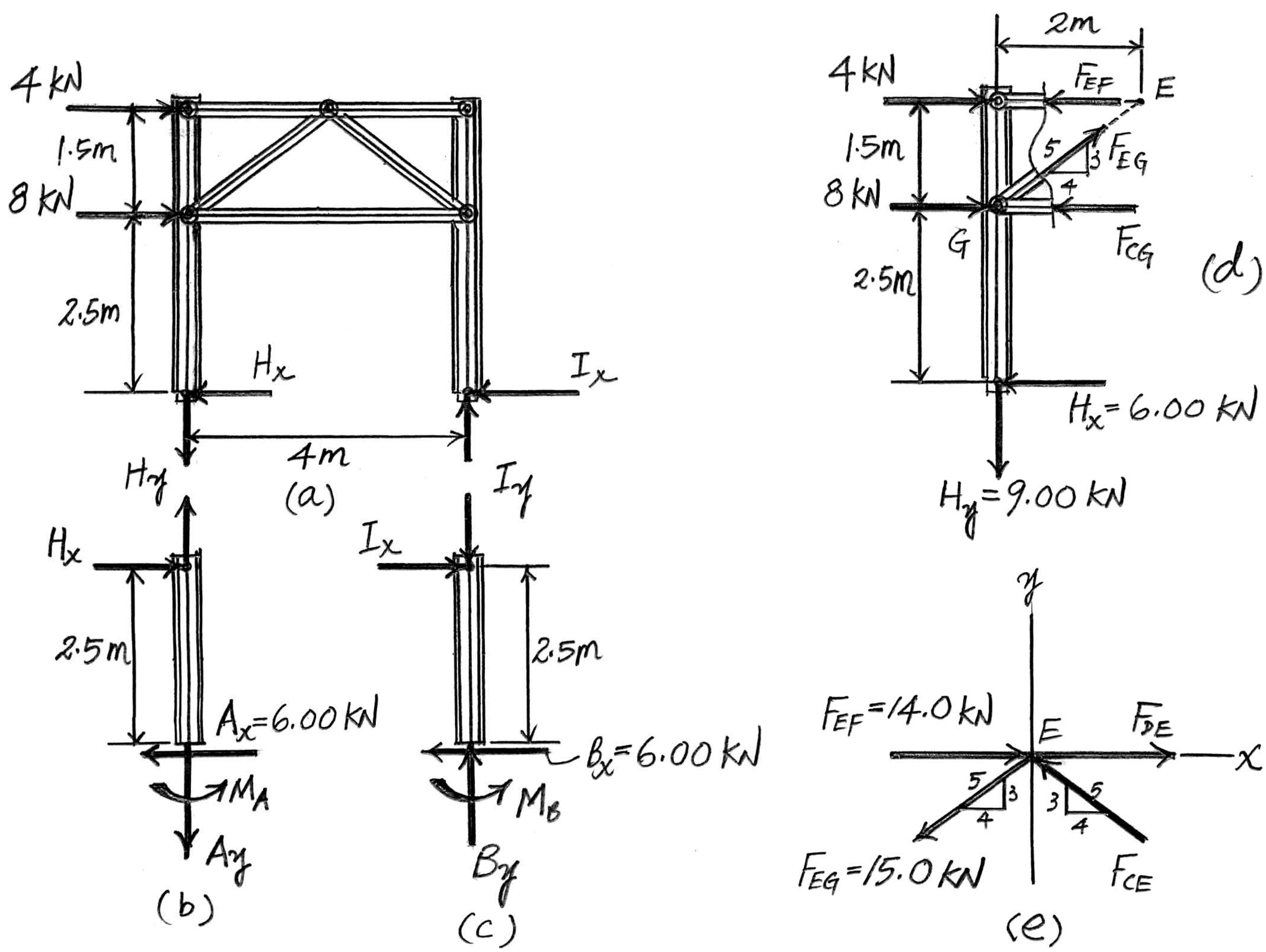
Using the method of sections, Fig. *d*,

$+\uparrow \sum F_y = 0; \quad F_{EG}\left(\frac{3}{5}\right) - 9.00 = 0 \qquad F_{EG} = 15.0 \text{ kN(T)}$ **Ans.**

$\circlearrowleft + \sum M_E = 0; \quad 8(1.5) + 9.00(2) - 6.00(4) - F_{CG}(1.5) = 0$

$F_{CG} = 4.00 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \sum M_G = 0; \quad F_{EF}(1.5) - 4(1.5) - 6(2.5) = 0 \qquad F_{EF} = 14.0 \text{ kN (C)}$ **Ans.**



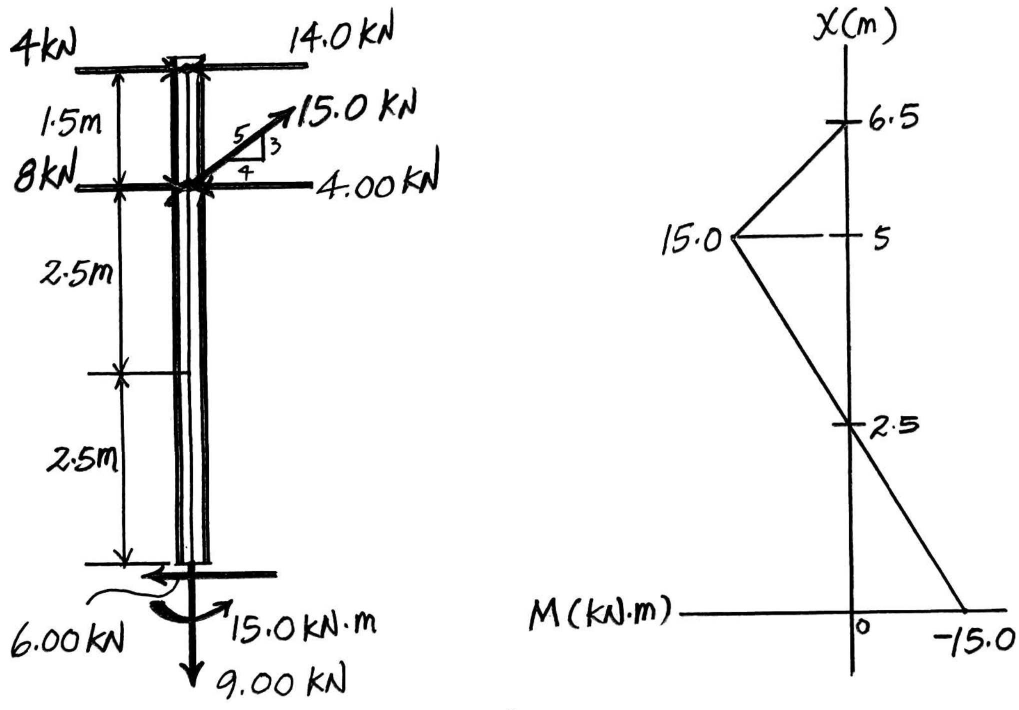
7–26. Continued

Using the method of joints, Fig. *e*,

$+\uparrow \sum F_y = 0; \quad F_{CE}\left(\frac{3}{5}\right) - 15.0\left(\frac{3}{5}\right) = 0 \quad F_{CE} = 15.0 \text{ kN (C)}$ **Ans.**

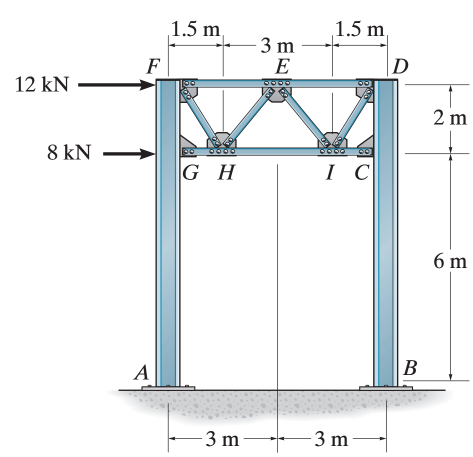
$\xrightarrow{+} \sum F_x = 0; \quad F_{DE} + 14.0 - 15.0\left(\frac{4}{5}\right) - 15.0\left(\frac{4}{5}\right) = 0$

$F_{DE} = 10.0 \text{ kN (T)}$ **Ans.**



(f)

7–27. Determine (approximately) the force in each truss member of the portal frame. Also find the reactions at the fixed column supports *A* and *B*. Assume all members of the truss to be pin connected at their ends.



Assume that the horizontal force components at fixed supports *A* and *B* are equal. Thus,

$A_x = B_x = \frac{12 + 8}{2} = 10.0 \text{ kN}$ **Ans.**

Also, the points of inflection *J* and *K* are 3 m above *A* and *B* respectively. Referring to Fig. *a*,

$\circlearrowleft + \sum M_k = 0; \quad J_y(6) - 8(3) - 12(5) = 0 \quad J_y = 14.0 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad K_y - 14.0 = 0 \quad K_y = 14.0 \text{ kN}$

7–27. Continued

Referring to Fig. *b*,

$\xrightarrow{+} \sum F_x = 0; \quad J_x - 10.0 = 0 \quad J_x = 10.0 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad 14.0 - A_y = 0 \quad A_y = 14.0 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad M_A - 10.0(3) = 0 \quad M_A = 30.0 \text{ kN} \cdot \text{m}$ **Ans.**

Referring to Fig. *c*,

$\xrightarrow{+} \sum F_x = 0; \quad B_x - 10.0 = 0 \quad B_x = 10.0 \text{ kN}$

$+\uparrow \sum F_y = 0; \quad B_y - 14.0 = 0 \quad B_y = 14.0 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_B = 0; \quad M_B - 10.0(3) = 0 \quad M_B = 30.0 \text{ kN} \cdot \text{m}$ **Ans.**

Using the metod of sections, Fig. *d*,

$+\uparrow \sum F_y = 0; \quad F_{FH}\left(\frac{4}{5}\right) - 14.0 = 0 \quad F_{FH} = 17.5 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \sum M_H = 0; \quad F_{EF}(2) + 14.0(1.5) - 12(2) - 10.0(3) = 0 \quad F_{EF} = 16.5 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \sum M_F = 0; \quad F_{GH}(2) + 8(2) - 10.0(5) = 0 \quad F_{GH} = 17.0 \text{ kN (T)}$ **Ans.**

Using the method of joints, Fig. *e* (Joint *H*),

$+\uparrow \sum F_y = 0; \quad F_{EH}\left(\frac{4}{5}\right) - 17.5\left(\frac{4}{5}\right) = 0 \quad F_{EH} = 17.5 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \quad 17.5\left(\frac{3}{5}\right) + 17.5\left(\frac{3}{5}\right) - 17.0 - F_{HI} = 0 \quad F_{HI} = 4.00 \text{ kN (C)}$ **Ans.**

Referring Fig. *f* (Joint *E*),

$+\uparrow \sum F_y = 0; \quad F_{EI}\left(\frac{4}{5}\right) - 17.5\left(\frac{4}{5}\right) = 0 \quad F_{EI} = 17.5 \text{ kN (C)}$ **Ans.**

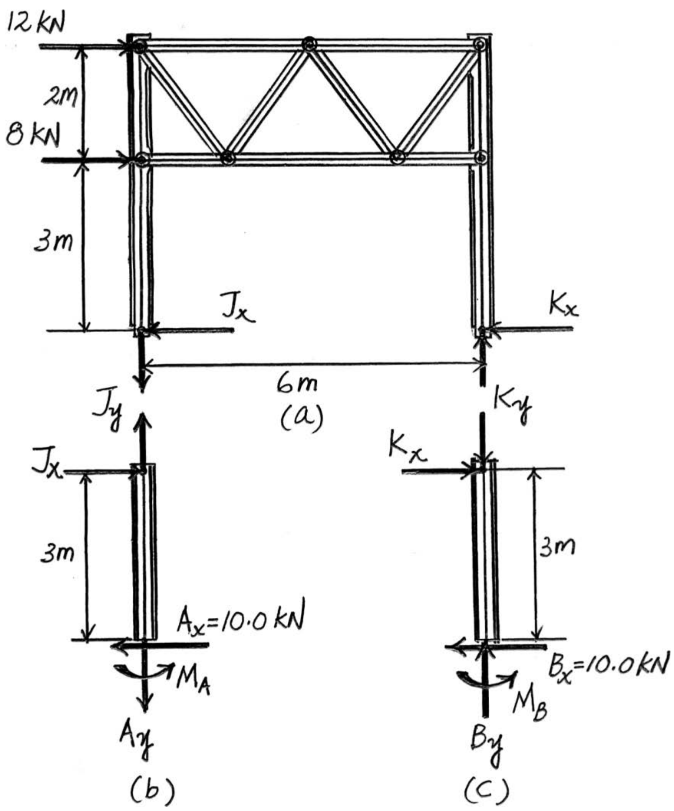
$\xrightarrow{+} \sum F_x = 0 \quad F_{DE} + 16.5 - 17.5\left(\frac{3}{5}\right) - 17.5\left(\frac{3}{5}\right) = 0 \quad F_{DE} = 4.50 \text{ kN (T)}$ **Ans.**

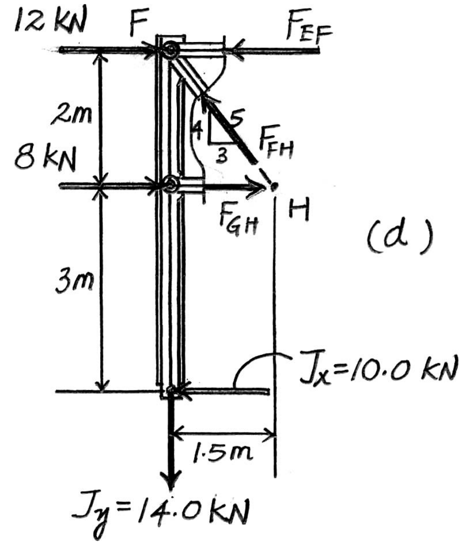
Referring to Fig. *g* (Joint *I*),

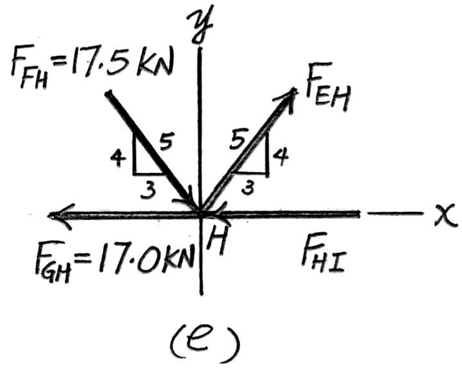
$+\uparrow \sum F_y = 0; \quad F_{DI}\left(\frac{4}{5}\right) - 17.5\left(\frac{4}{5}\right) = 0 \quad F_{DI} = 17.5 \text{ kN (T)}$ **Ans.**

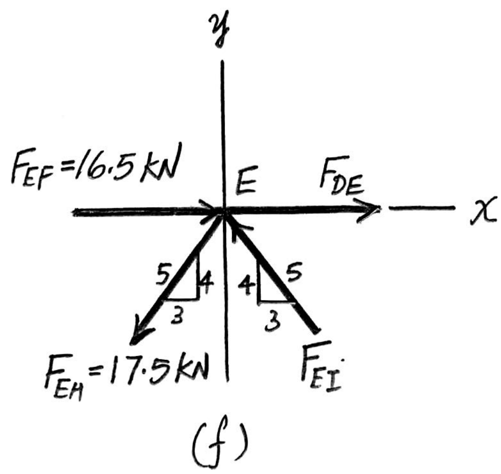
$\xrightarrow{+} \sum F_x = 0; \quad 17.5\left(\frac{3}{5}\right) + 17.5\left(\frac{3}{5}\right) + 4.00 - F_{CI} = 0 \quad F_{CI} = 25.0 \text{ kN (C)}$ **Ans.**

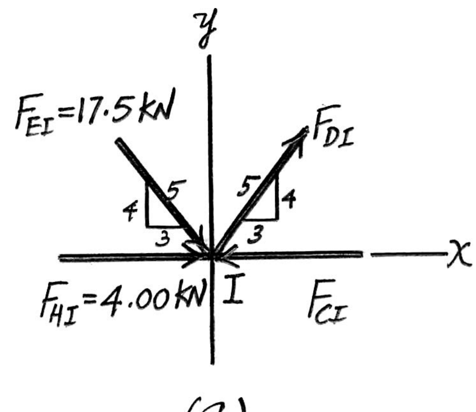
7–27. Continued



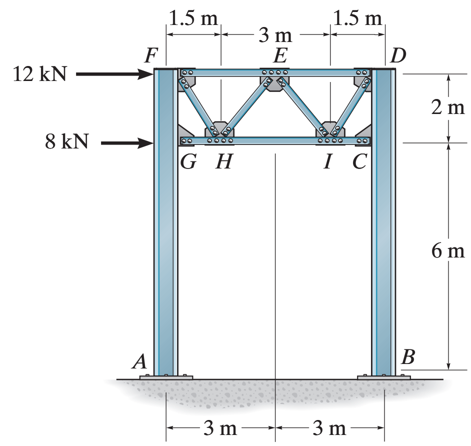








***7–28.** Solve Prob. 7–27 if the supports at A and B are pinned instead of fixed.



Assume that the horizontal force components at pin supports A and B are equal. Thus,

$$A_x = B_x = \frac{12 + 8}{2} = 10.0 \text{ kN} \qquad \textbf{Ans.}$$

Referring to Fig. a,

$$\circlearrowleft + \sum M_B = 0; \quad A_y(6) - 8(6) - 12(8) = 0 \quad A_y = 24.0 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad B_y - 24.0 = 0 \quad B_y = 24.0 \text{ kN} \qquad \textbf{Ans.}$$

Using the method of sections, Fig. b,

$$+\uparrow \sum F_y = 0; \quad F_{FH}\left(\frac{4}{5}\right) - 24.0 = 0 \quad F_{FH} = 30.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_H = 0; \quad F_{EF}(2) + 24.0(1.5) - 12(2) - 10.0(6) = 0$$

$$F_{EF} = 24.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_F = 0; \quad F_{GH}(2) + 8(2) - 10.0(8) = 0 \quad F_{GH} = 32.0 \text{ kN (T)} \qquad \textbf{Ans.}$$

Using method of joints, Fig. c (Joint H),

$$+\uparrow \sum F_y = 0; \quad F_{EH}\left(\frac{4}{5}\right) - 30.0\left(\frac{4}{5}\right) = 0 \quad F_{EH} = 30.0 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \sum F_x = 0; \quad 30.0\left(\frac{3}{5}\right) + 30.0\left(\frac{3}{5}\right) - 32.0 - F_{HI} = 0 \quad F_{HI} = 4.00 \text{ kN (C)} \qquad \textbf{Ans.}$$

Referring to Fig. d (Joint E),

$$+\uparrow \sum F_y = 0; \quad F_{EI}\left(\frac{4}{5}\right) - 30.0\left(\frac{4}{5}\right) = 0 \quad F_{EI} = 30.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \sum F_x = 0; \quad F_{DE} + 24.0 - 30.0\left(\frac{3}{5}\right) - 30.0\left(\frac{3}{5}\right) = 0 \quad F_{DE} = 12.0 \text{ kN (T)} \qquad \textbf{Ans.}$$

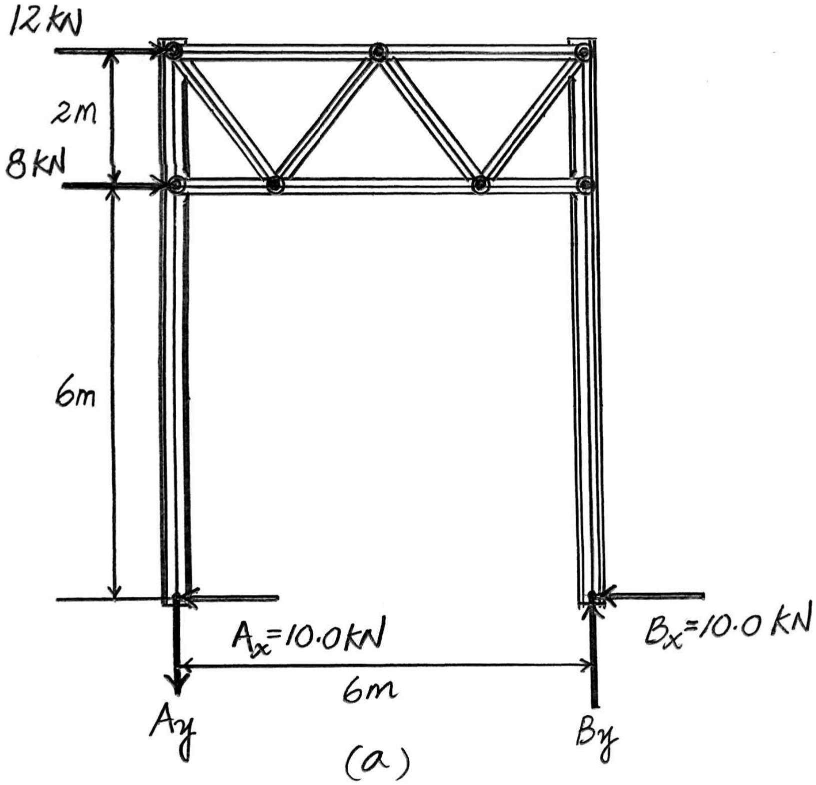
Referring to Fig. e (Joint I),

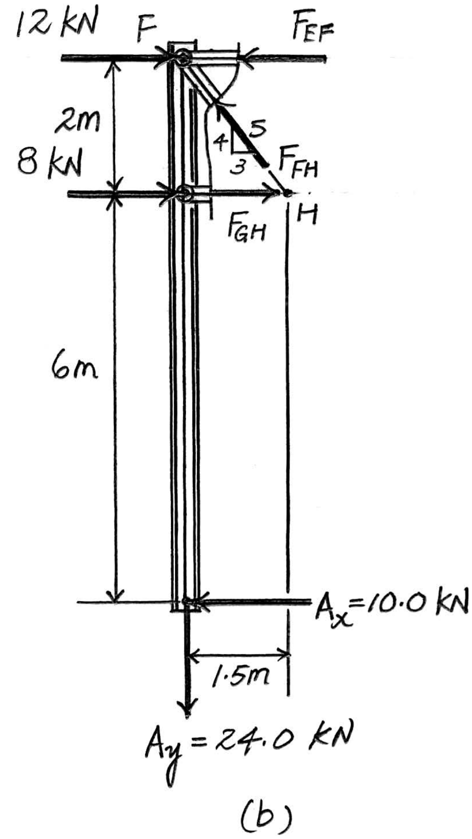
$$+\uparrow \sum F_y = 0; \quad F_{DI}\left(\frac{4}{5}\right) - 30.0\left(\frac{4}{5}\right) = 0 \quad F_{DI} = 30.0 \text{ kN (T)} \qquad \textbf{Ans.}$$

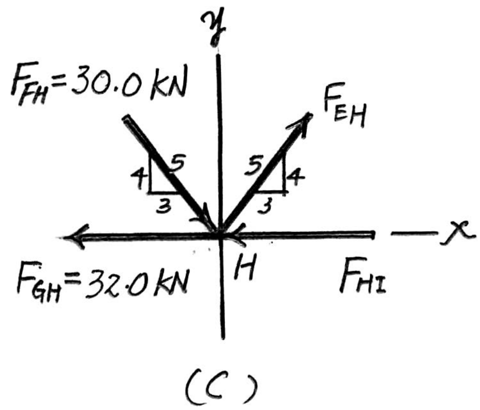
$$\xrightarrow{+} \sum F_x = 0; \quad 30.0\left(\frac{3}{5}\right) + 30.0\left(\frac{3}{5}\right) + 4.00 - F_{CI} = 0$$

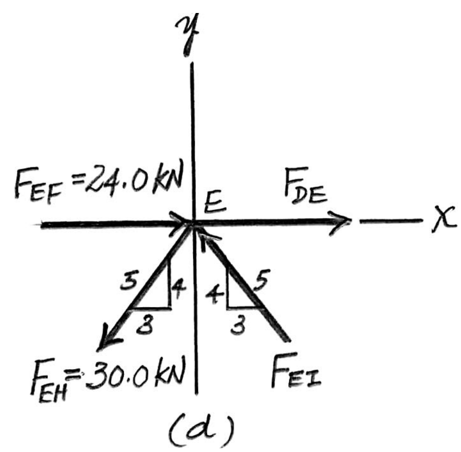
$$F_{CI} = 40.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

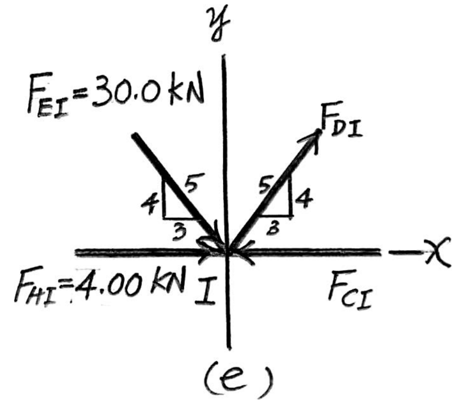
7–28. Continued



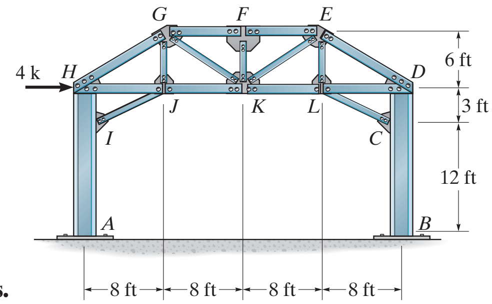








7–29. Determine (approximately) the force in members GF, GK, and JK of the portal frame. Also find the reactions at the fixed column supports A and B. Assume all members of the truss to be connected at their ends.



Assume that the horizontal force components at fixed supports A and B are equal. Thus,

$$A_x = B_x = \frac{4}{2} = 2.00 \text{ k} \qquad \textbf{Ans.}$$

Also, the points of inflection N and O are at 6 ft above A and B respectively. Referring to Fig. a,

$$\circlearrowleft + \sum M_B = 0; \quad N_y(32) - 4(9) = 0 \quad N_y = 1.125 \text{ k}$$

$$\circlearrowleft + \sum M_N = 0; \quad O_y(32) - 4(9) = 0 \quad O_y = 1.125 \text{ k}$$

Referring to Fig. b,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_x - 2.00 = 0 \quad N_x = 2.00 \text{ k}$$

$$+\uparrow \sum F_y = 0; \quad 1.125 - A_y = 0 \quad A_y = 1.125 \text{ k} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_A = 0; \quad M_A - 2.00(6) = 0 \quad M_A = 12.0 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$

Referring to Fig. c,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad B_x - 2.00 = 0 \quad B_x = 2.00 \text{ k}$$

$$+\uparrow \sum F_y = 0; \quad B_y - 1.125 = 0 \quad B_y = 1.125 \text{ k} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_B = 0; \quad M_B - 2.00(6) = 0 \quad M_B = 12.0 \text{ k} \cdot \text{ft} \qquad \textbf{Ans.}$$

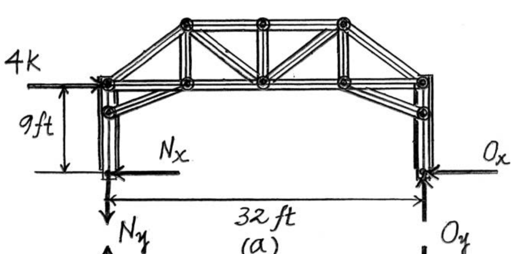
Using the method of sections, Fig. d,

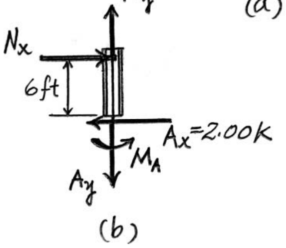
$$+\uparrow \sum F_y = 0; \quad F_{GK}\left(\frac{3}{5}\right) - 1.125 = 0 \quad F_{GK} = 1.875 \text{ k (C)} \qquad \textbf{Ans.}$$

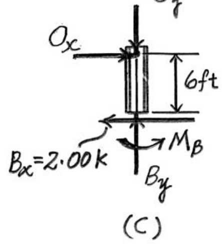
$$\circlearrowleft + \sum M_K = 0; \quad F_{GF}(6) + 1.125(16) - 2(9) = 0 \quad F_{GF} = 0 \qquad \textbf{Ans.}$$

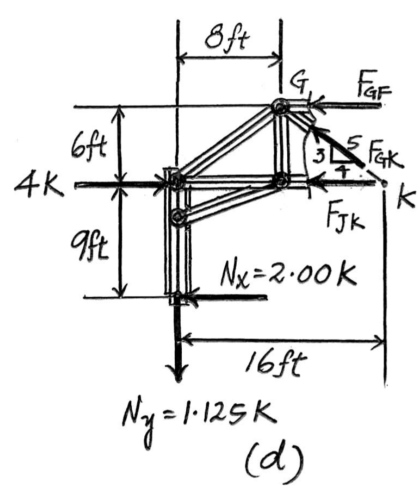
$$\circlearrowleft + \sum M_G = 0; \quad -F_{JK}(6) + 4(6) + 1.125(8) - 2.00(15) = 0$$

$$F_{JK} = 0.500 \text{ k (C)} \qquad \textbf{Ans.}$$

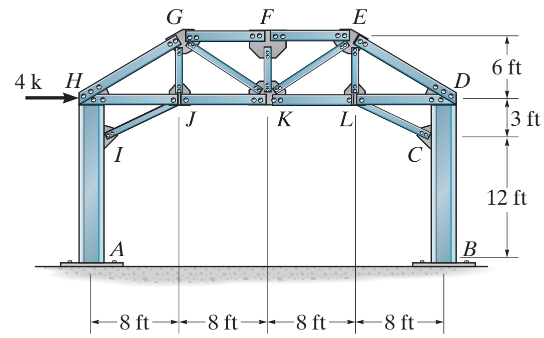








7–30. Solve Prob. 7–29 if the supports at A and B are pin connected instead of fixed.



Assume that the horizontal force components at pin supports A and B are equal. Thus,

$$A_x = B_x = \frac{4}{2} = 2.00 \text{ k}$$ **Ans.**

Referring to Fig. *a*,

$$\circlearrowleft + \sum M_A = 0; \quad B_y(32) - 4(15) = 0 \quad B_y = 1.875 \text{ k}$$ **Ans.**

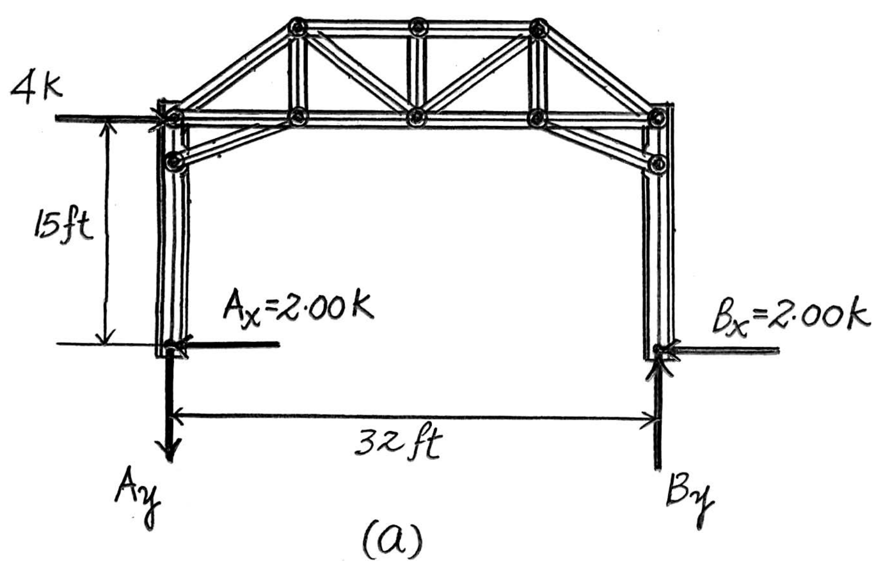
$$+\uparrow \sum F_y = 0; \quad 1.875 - A_y = 0 \quad A_y = 1.875 \text{ k}$$ **Ans.**

Using the method of sections, Fig. *b*,

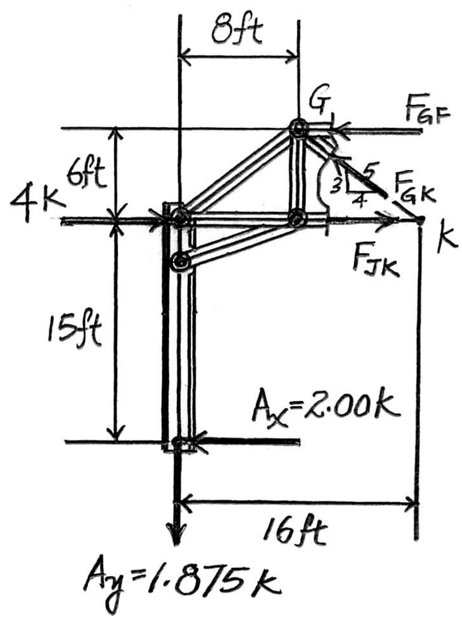
$$+\uparrow \sum F_y = 0; \quad F_{GK}\left(\frac{3}{5}\right) - 1.875 = 0 \quad F_{GK} = 3.125 \text{ k (C)}$$ **Ans.**

$$\circlearrowleft + \sum M_x = 0; \quad F_{GF}(6) + 1.875(16) - 2.00(15) = 0 \quad F_{GF} = 0$$ **Ans.**

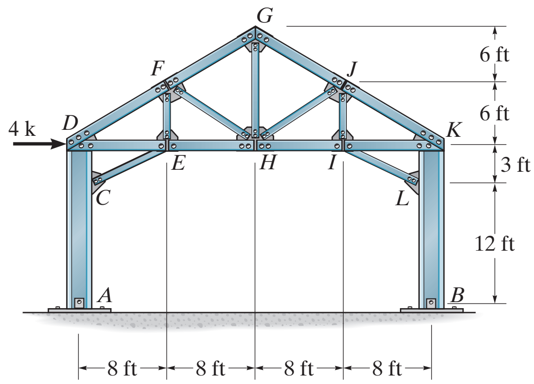
$$\circlearrowleft + \sum M_G = 0; \quad 4(6) + 1.875(8) - 2.00(21) + F_{JK}(6) = 0 \quad F_{JK} = 0.500 \text{ k (T)}$$ **Ans.**



(a)



7–31. Draw (approximately) the moment diagram for column *ACD* of the portal. Assume all truss members and the columns to be pin connected at their ends. Also determine the force in members *FG*, *FH*, and *EH*.



Assume that the horizontal force components at pin supports *A* and *B* are equal. Thus,

$$A_x = B_x = \frac{4}{2} = 2.00 \text{ k}$$

Referring to Fig. *a*,

$$\circlearrowleft + \sum M_B = 0; \quad A_y(32) - 4(15) = 0 \quad A_y = 1.875 \text{ k}$$

Using the method of sections, Fig. *b*,

$$\circlearrowleft + \sum M_H = 0; \quad F_{FG}\left(\frac{3}{5}\right)(16) + 1.875(16) - 2.00(15) = 0 \quad F_{FG} = 0$$ **Ans.**

$$\circlearrowleft + \sum M_F = 0; \quad 4(6) + 1.875(8) - 2.00(21) + F_{EH}(6) = 0$$

$$F_{EH} = 0.500 \text{ k (T)}$$ **Ans.**

$$\circlearrowleft + \sum M_D = 0; \quad F_{FH}\left(\frac{3}{5}\right)(16) - 2.00(15) = 0 \quad F_{FH} = 3.125 \text{ k (C)}$$ **Ans.**

Also, referring to Fig. *c*,

$$\circlearrowleft + \sum M_E = 0; \quad F_{DF}\left(\frac{3}{5}\right)(8) + 1.875(8) - 2.00(15) = 0$$

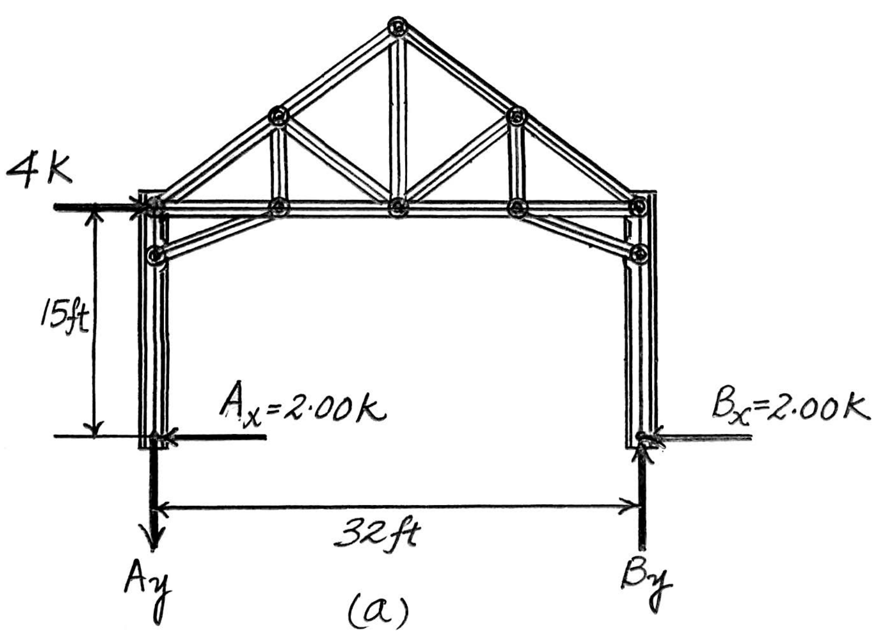
$$F_{DF} = 3.125 \text{ k (C)}$$

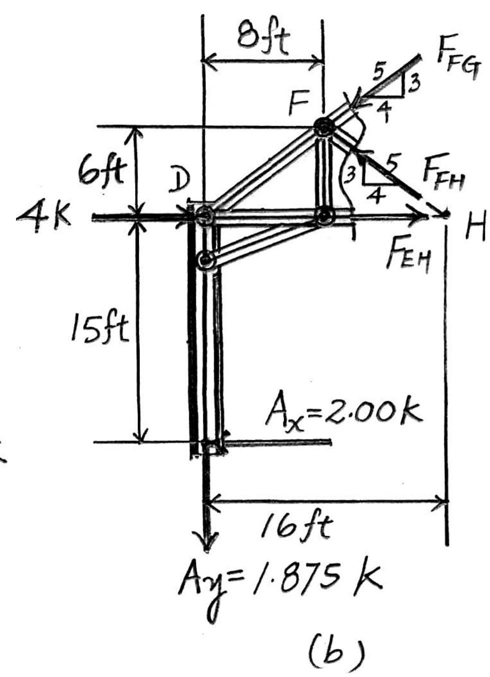
$$\circlearrowleft + \sum M_D = 0; \quad F_{CE}\left(\frac{3}{\sqrt{73}}\right)(8) - 2.00(15) = 0$$

$$F_{CE} = 10.68 \text{ k (T)}$$

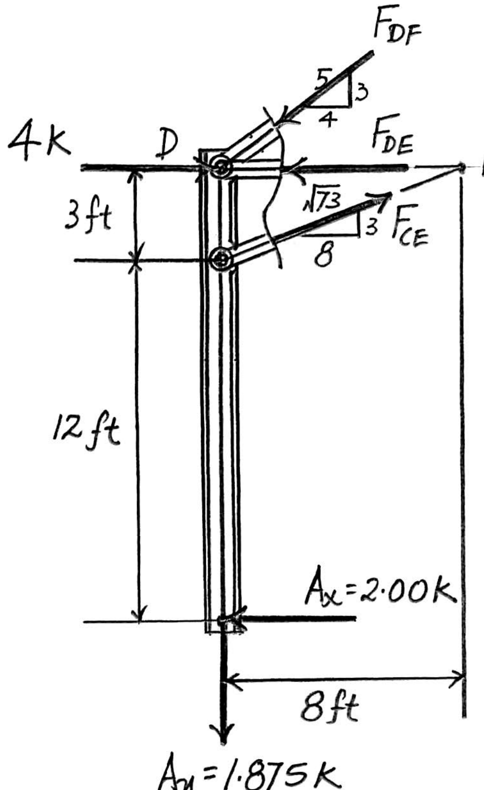
$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 4 + 10.68\left(\frac{8}{\sqrt{73}}\right) - 3.125\left(\frac{4}{5}\right) - 2.00 - F_{DE} = 0$$

$$F_{DE} = 9.50 \text{ k (C)}$$

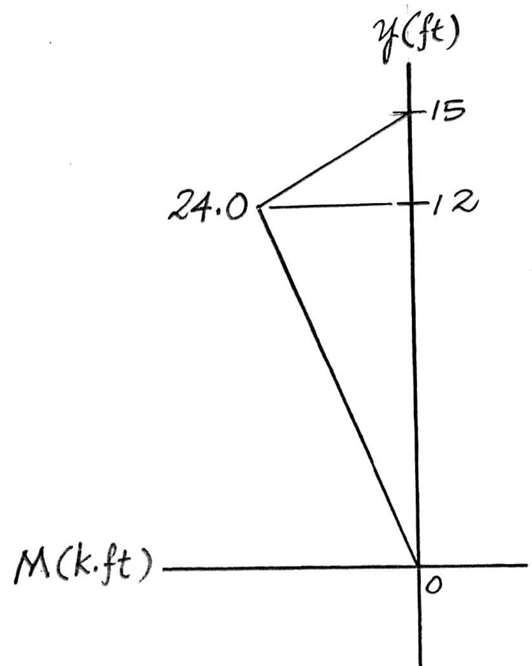




7–31. Continued

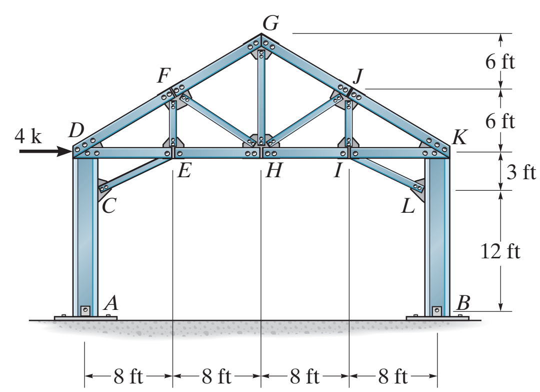


(c)



(d)

***7–32.** Solve Prob. 7–31 if the supports at A and B are fixed instead of pinned.



Assume that the horizontal force components at fixed supports A and B are equal. Thus,

$$A_x = B_x = \frac{4}{2} = 2.00 \text{ k}$$

Also, the points of inflection N and O are 6 ft above A and B respectively. Referring to Fig. a,

$$\circlearrowleft + \sum M_O = 0; \quad N_y(32) - 4(9) = 0 \quad N_y = 1.125 \text{ k}$$

Referring to Fig. b,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_x - 2.00 = 0 \quad N_x = 2.00 \text{ k}$$

$$\circlearrowleft + \sum M_A = 0; \quad M_A - 2.00(6) = 0 \quad M_A = 12.0 \text{ k ft}$$

$$+\uparrow \sum F_y = 0; \quad 1.125 - A_y = 0 \quad A_y = 1.125 \text{ k}$$

7–32. Continued

Using the method of sections, Fig. *d*,

$\zeta + \sum M_H = 0;\quad F_{FG}\left(\frac{3}{5}\right)(16) + 1.125(16) - 2.00(9) = 0 \quad F_{FG} = 0$ **Ans.**

$\zeta + \sum M_F = 0;\quad -F_{EH}(6) + 4(6) + 1.125(8) - 2.00(15) = 0 \quad F_{EH} = 0.500 \text{ k (C)}$ **Ans.**

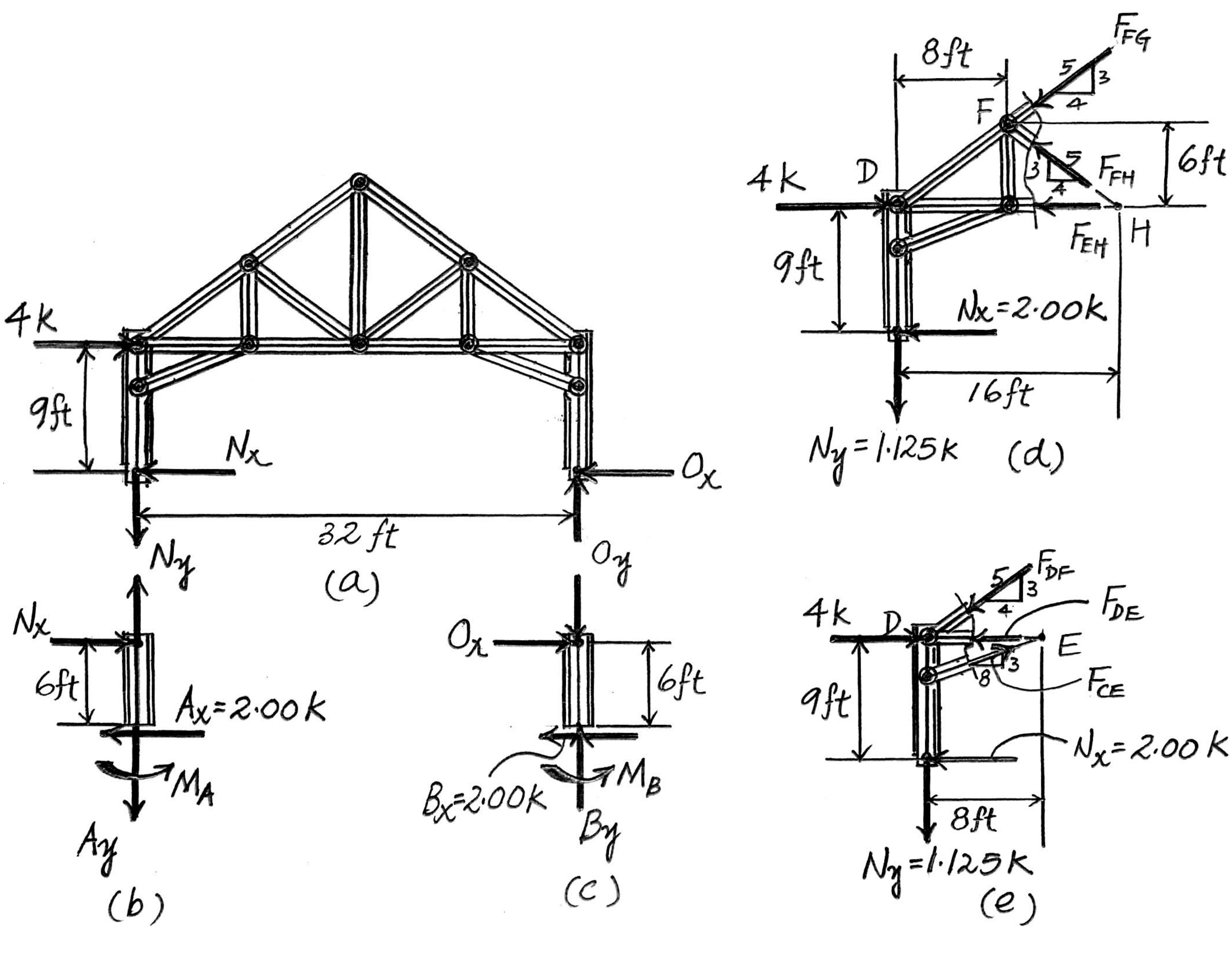
$\zeta + \sum M_D = 0;\quad F_{FH}\left(\frac{3}{5}\right)(16) - 2.00(9) = 0 \quad F_{FH} = 1.875 \text{ k (C)}$ **Ans.**

Also, referring to Fig *e*,

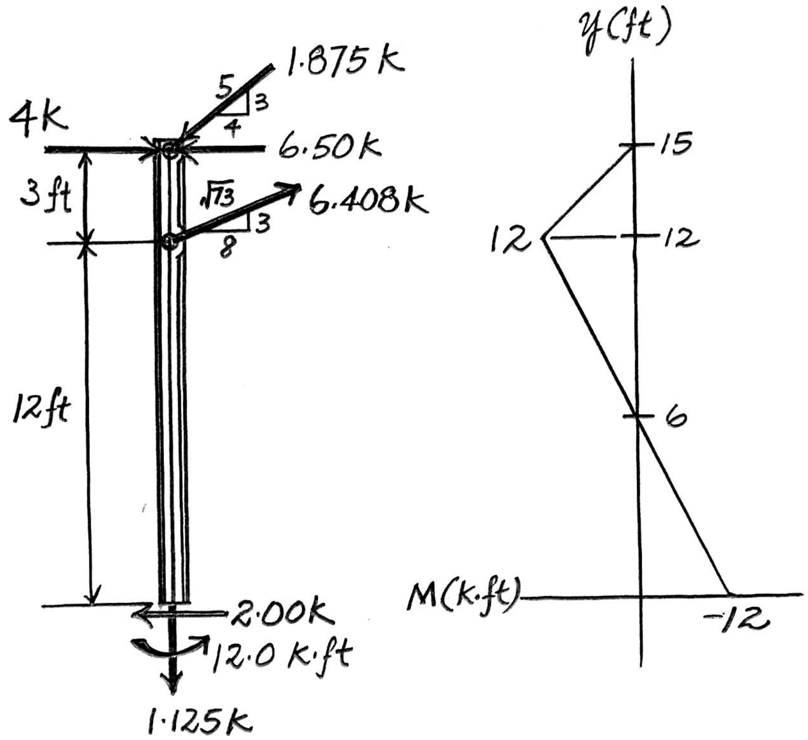
$\zeta + \sum M_E = 0;\quad F_{DF}\left(\frac{3}{5}\right)(8) + 1.125(8) - 2.00(9) = 0 \quad F_{DF} = 1.875 \text{ k (C)}$

$\zeta + \sum M_D = 0;\quad F_{CE}\left(\frac{3}{\sqrt{73}}\right)(8) - 2.00(9) = 0 \quad F_{CE} = 6.408 \text{ k (T)}$

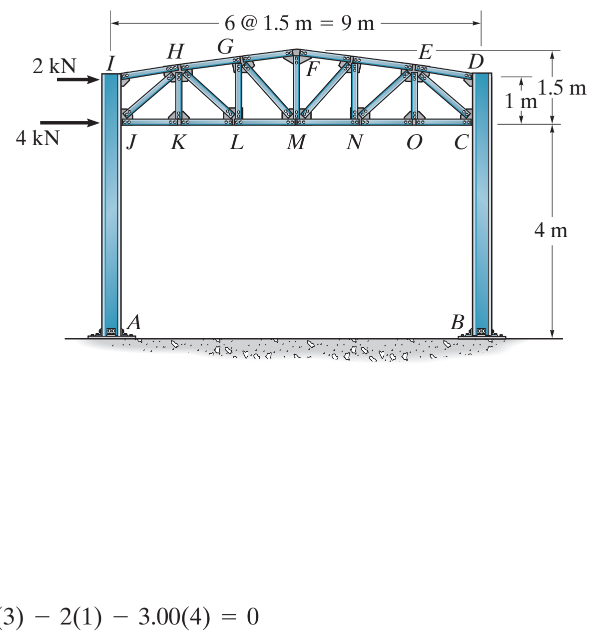
$\overset{+}{\rightarrow} \sum F_x = 0;\quad 4 + 6.408\left(\frac{8}{\sqrt{73}}\right) - 1.875\left(\frac{4}{5}\right) - 2.00 - F_{DE} = 0 \quad F_{DE} = 6.50 \text{ k (C)}$



7–32. Continued



7–33. Draw (approximately) the moment diagram for column AJI of the portal. Assume all truss members and the columns to be pin connected at their ends. Also determine the force in members HG, HL, and KL.



Assume the horizontal force components at pin supports A and B to be equal. Thus,

$$A_x = B_x = \frac{2 + 4}{2} = 3.00 \text{ kN}$$

Referring to Fig. a,

$$\zeta + \sum M_B = 0; \quad A_y(9) - 4(4) - 2(5) = 0 \quad A_y = 2.889 \text{ kN}$$

Using the method of sections, Fig. b,

$$\zeta + \sum M_L = 0; \quad F_{HG} \cos 6.340° (1.167) + F_{HG} \sin 6.340° (1.5) + 2.889(3) - 2(1) - 3.00(4) = 0$$

$$F_{HG} = 4.025 \text{ kN (C)} = 4.02 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\zeta + \sum M_H = 0; \quad F_{KL}(1.167) + 2(0.167) + 4(1.167) + 2.889(1.5) - 3.00(5.167) = 0$$

$$F_{KL} = 5.286 \text{ kN (T)} = 5.29 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad F_{HL} \cos 52.13° - 4.025 \sin 6.340° - 2.889 = 0$$

$$F_{HL} = 5.429 \text{ kN (C)} = 5.43 \text{ kN (C)} \qquad \textbf{Ans.}$$

7–33. Continued

Also, referring to Fig. *c*,

$\zeta + \sum M_H = 0; \quad F_{JK}(1.167) + 2(0.167) + 4(1.167) + 2.889(1.5) - 3.00(5.167) = 0$

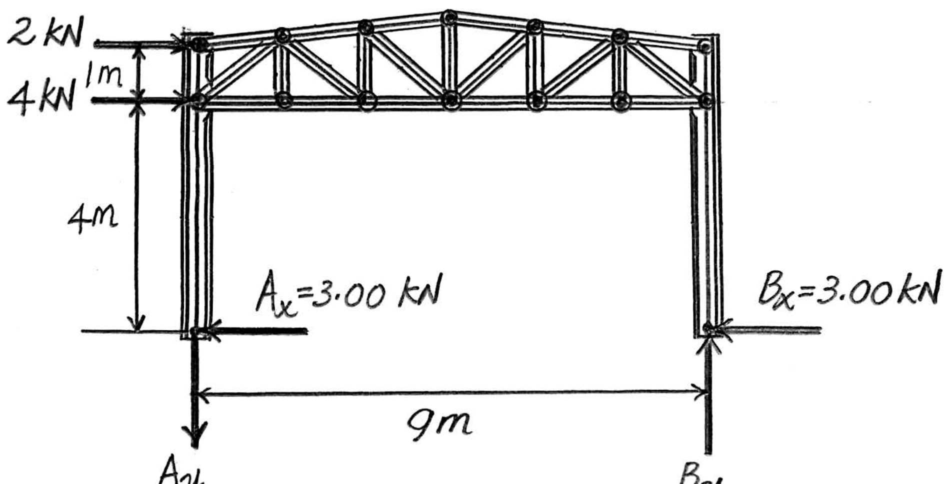
$F_{JK} = 5.286 \text{ kN (T)}$

$\zeta + \sum M_J = 0; \quad F_{IH} \cos 6.340° \ (1) - 2(1) - 3.00(4) = 0$

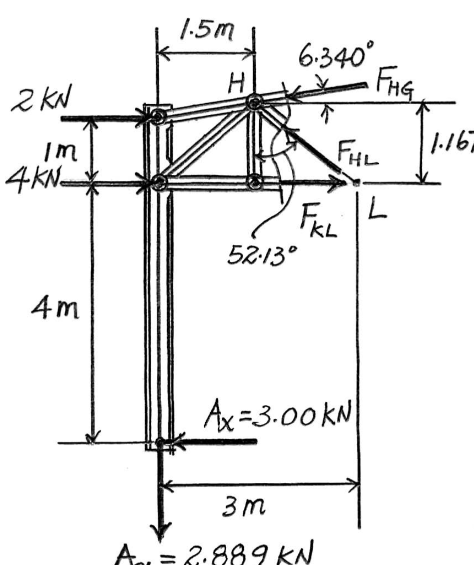
$F_{IH} = 14.09 \text{ kN (C)}$

$+\uparrow \sum F_y = 0; \quad F_{JH} \sin 37.87° - 14.09 \sin 6.340° - 2.889 = 0$

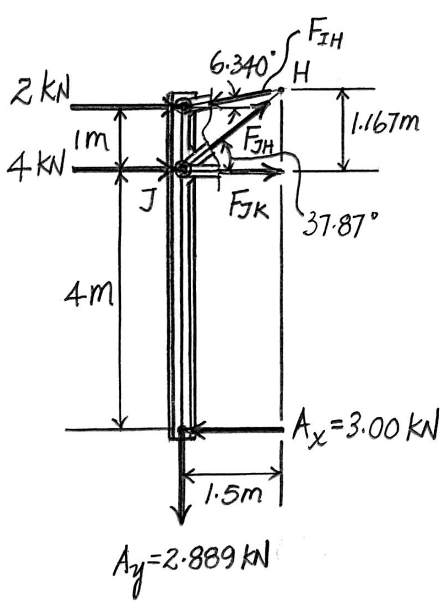
$F_{JH} = 7.239 \text{ kN (T)}$



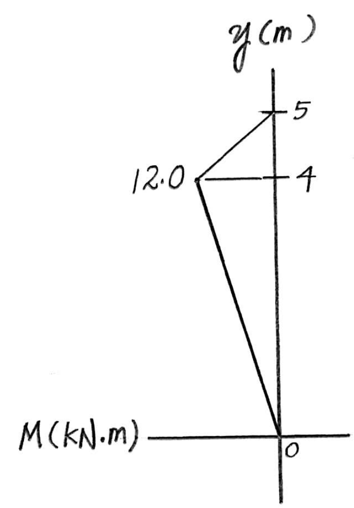
(a)



(b)

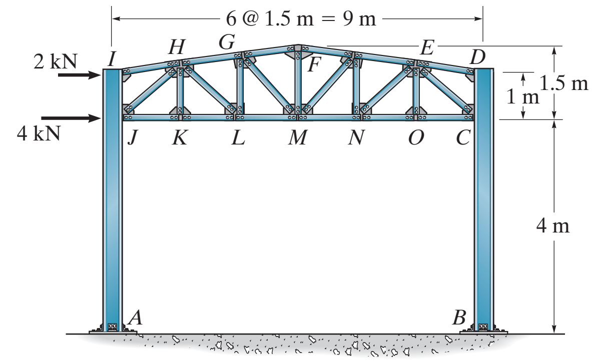


(c)



(d)

7–34. Solve Prob. 7–33 if the supports at *A* and *B* are fixed instead of pinned.



Assume that the horizontal force components at fixed supports *A* and *B* are equal. Therefore,

$$A_x = B_x = \frac{2 + 4}{2} = 3.00 \text{ kN}$$

Also, the reflection points *P* and *R* are located 2 m above *A* and *B* respectively. Referring to Fig. *a*

$$\zeta + \sum M_R = 0; \quad P_y(9) - 4(2) - 2(3) = 0 \quad P_y = 1.556 \text{ kN}$$

Referring to Fig. *b*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad P_x - 3.00 = 0 \quad P_x = 3.00 \text{ kN}$$

$$\zeta + \sum M_A = 0; \quad M_A - 3.00(2) = 0 \quad M_A = 6.00 \text{ kN}\cdot\text{m}$$

$$+\uparrow \sum F_y = 0; \quad 1.556 - A_y = 0 \quad A_y = 1.556 \text{ kN}$$

Using the method of sections, Fig. *d*,

$$\zeta + \sum M_L = 0; \quad F_{HG} \cos 6.340° (1.167) + F_{HG} \sin 6.340° (1.5) + 1.556(3) - 3.00(2) - 2(1) = 0$$

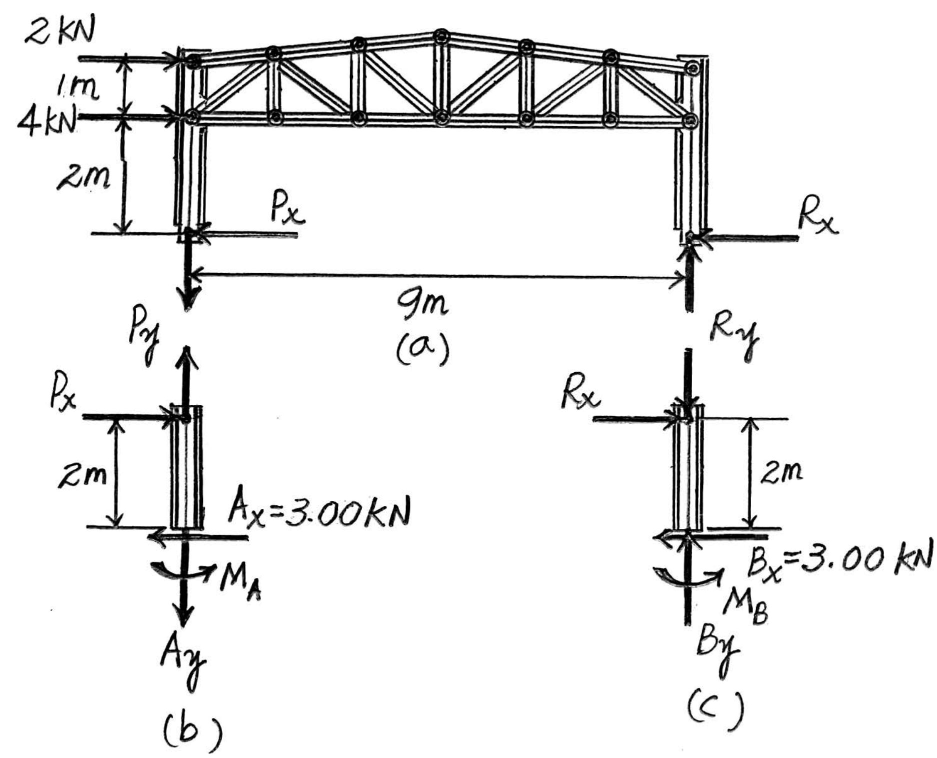
$$F_{HG} = 2.515 \text{ kN (C)} = 2.52 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\zeta + \sum M_H = 0; \quad F_{KL}(1.167) + 2(0.167) + 4(1.167) + 1.556(1.5) - 3.00(3.167) = 0$$

$$F_{KL} = 1.857 \text{ kN (T)} = 1.86 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad F_{HL} \cos 52.13° - 2.515 \sin 6.340° - 1.556 = 0$$

$$F_{HL} = 2.986 \text{ kN (C)} = 2.99 \text{ kN (C)} \qquad \textbf{Ans.}$$



7–34. Continued

Also referring to Fig. *e*,

$\circlearrowleft + \sum M_H = 0; \quad F_{JK}(1.167) + 4(1.167) + 2(0.167) + 1.556(1.5) - 3.00(3.167) = 0$

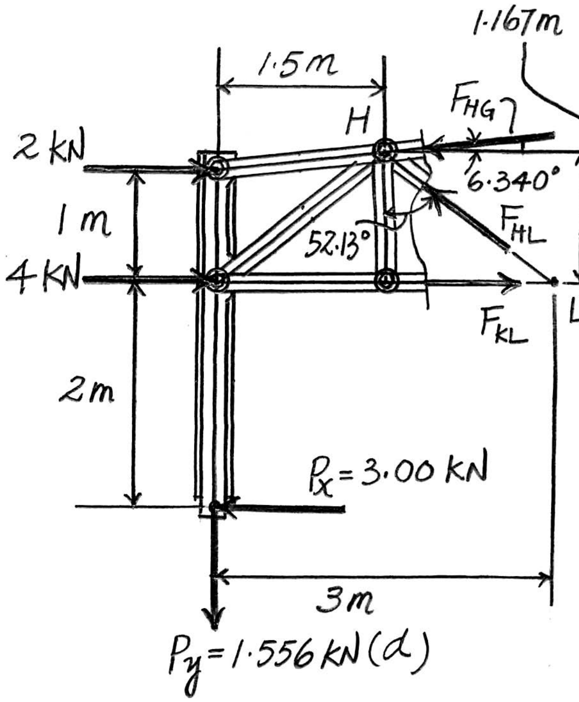
$F_{JK} = 1.857 \text{ kN (T)}$

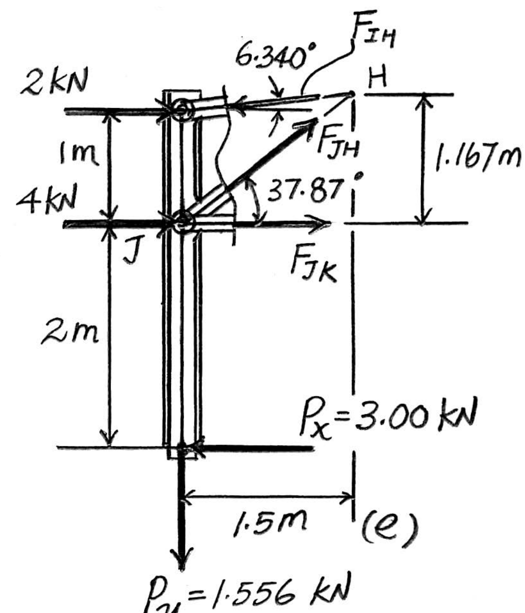
$\circlearrowleft + \sum M_J = 0; \quad F_{IH} \cos 6.340° (1) - 2(1) - 3.00(2) = 0$

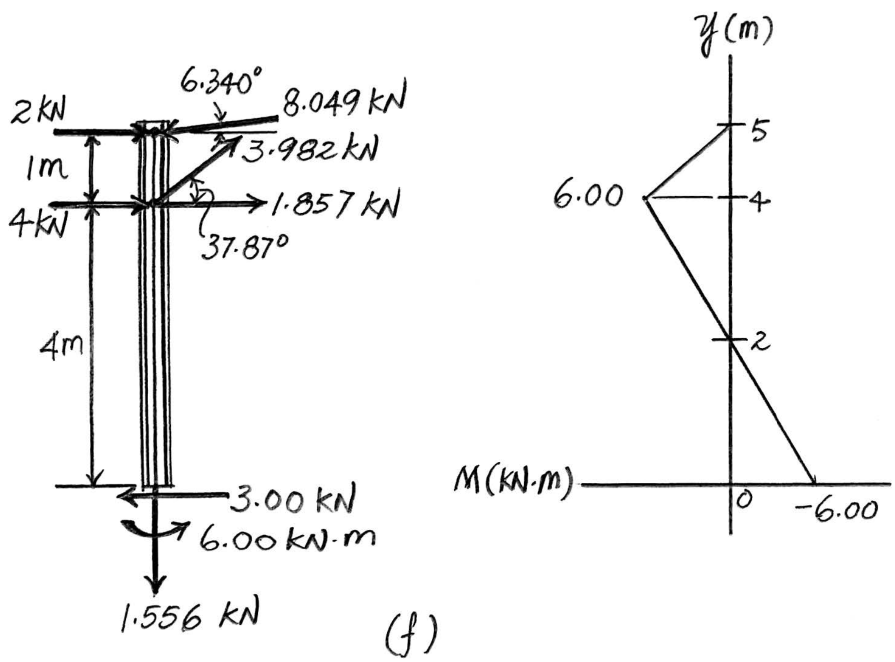
$F_{IH} = 8.049 \text{ kN (C)}$

$+\uparrow \sum F_y = 0; \quad F_{JH} \sin 37.87° - 8.049 \sin 6.340° - 1.556 = 0$

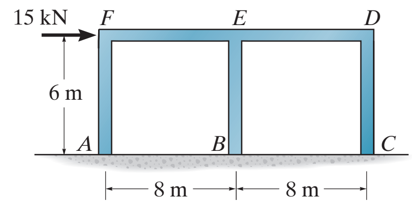
$F_{JH} = 3.982 \text{ kN (T)}$

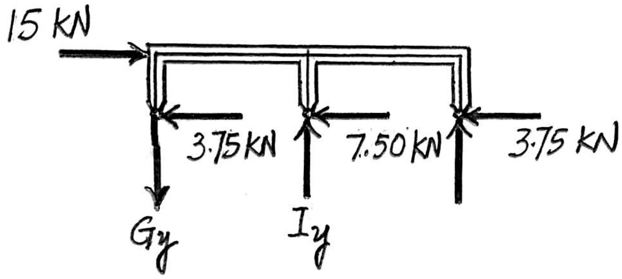


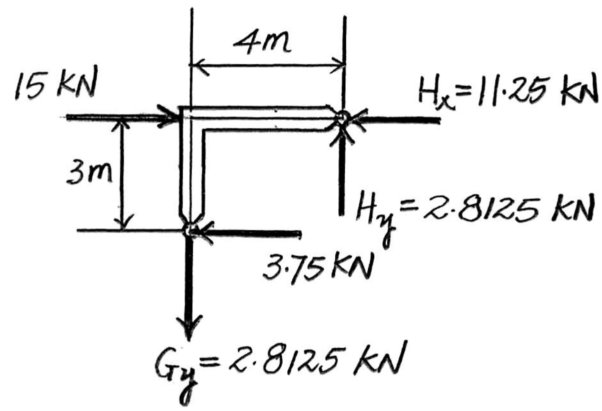


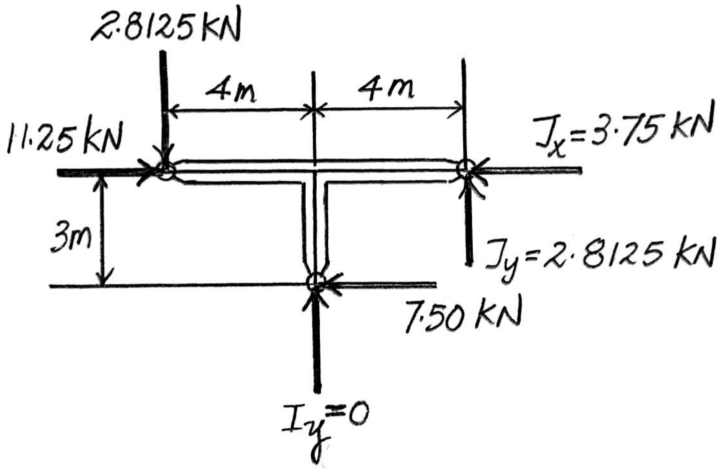


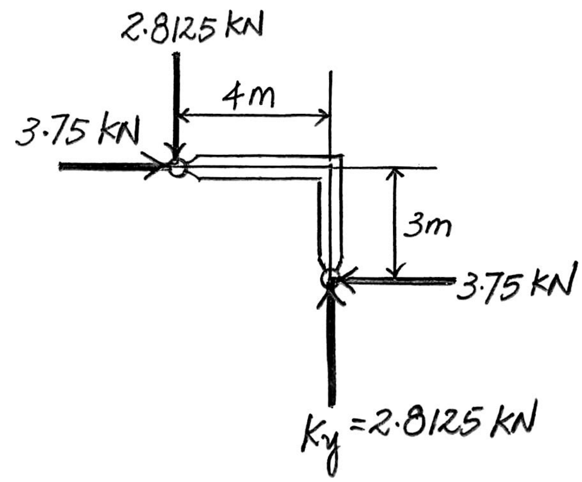
7–35. Use the portal method of analysis and draw the moment diagram for girder *FED*.

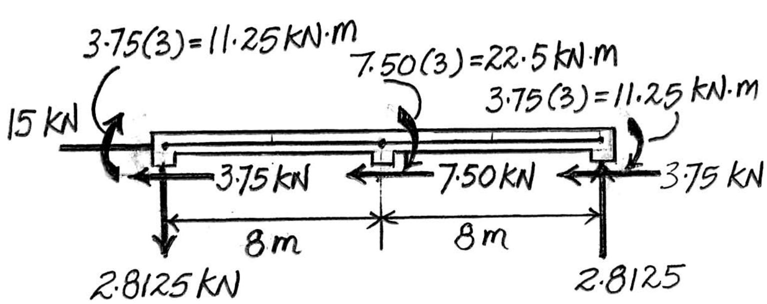


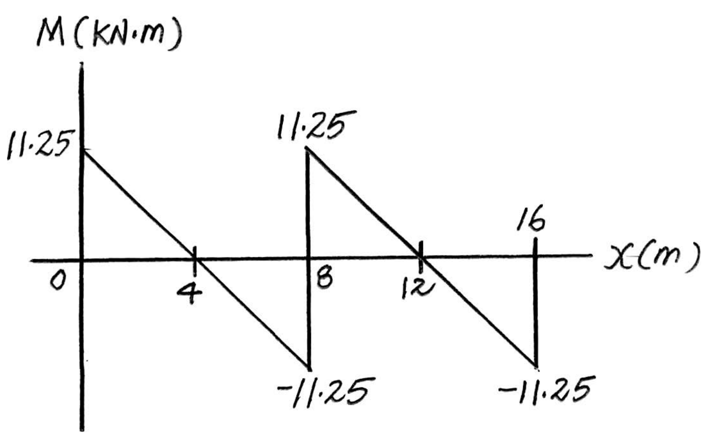




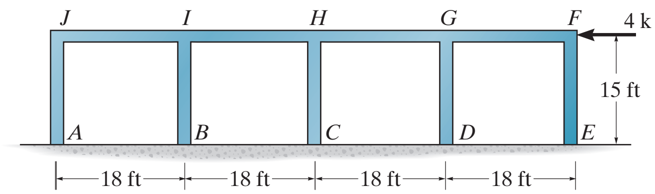


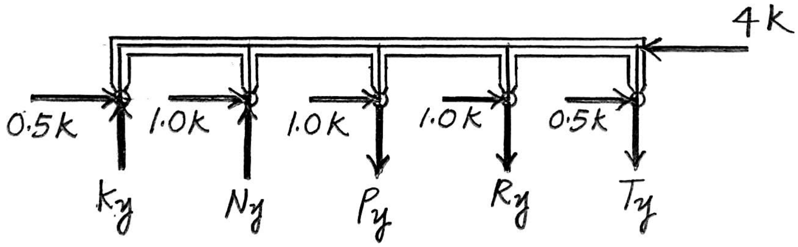


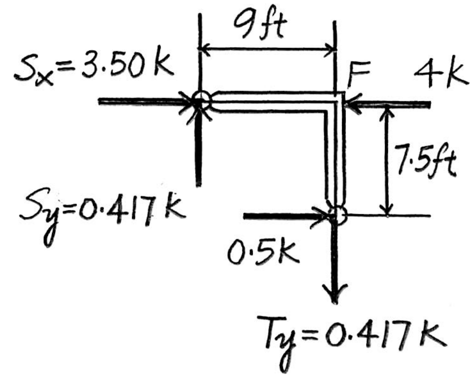


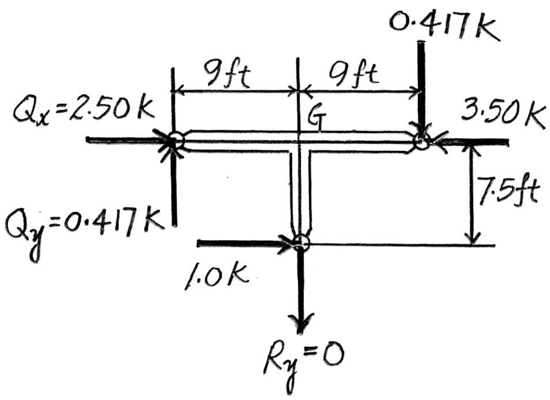


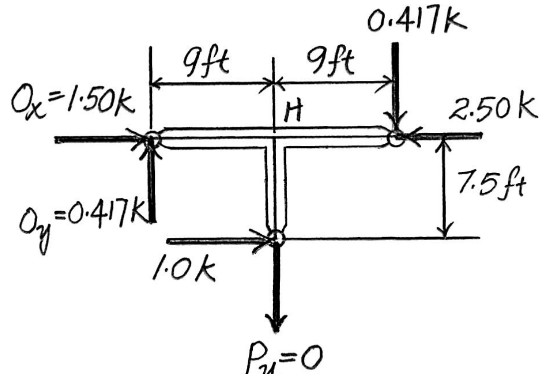
***7–36.** Use the portal method of analysis and draw the moment diagram for girder *JIHGF*.

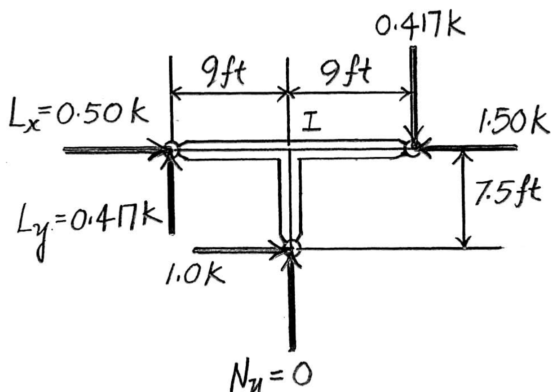


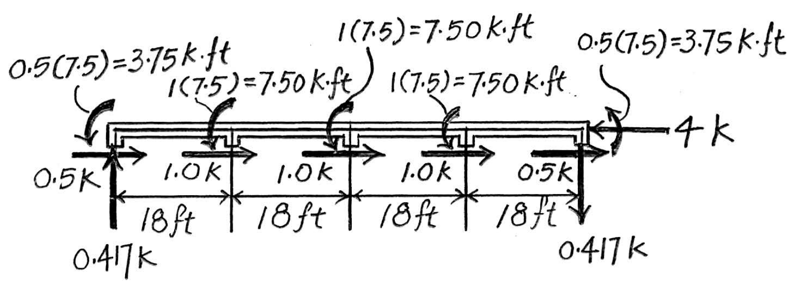


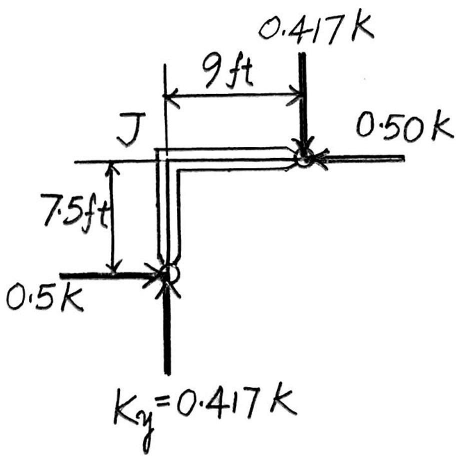


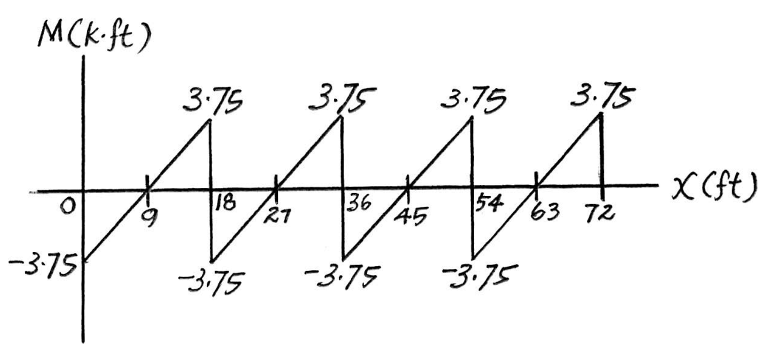




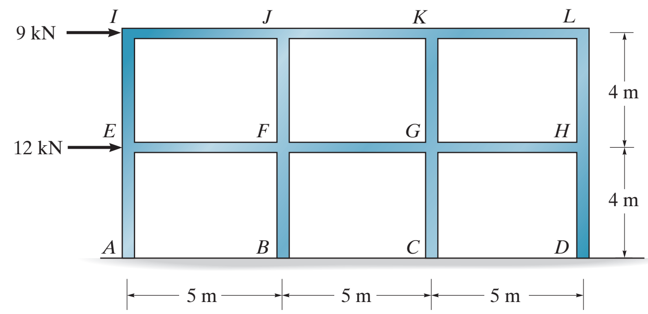


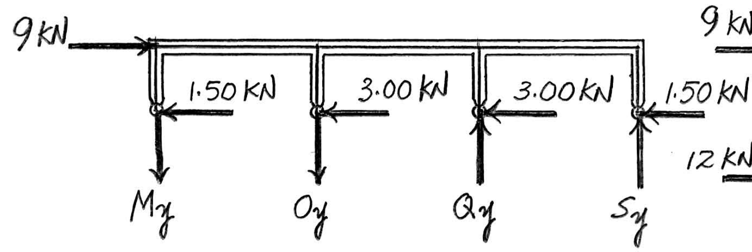


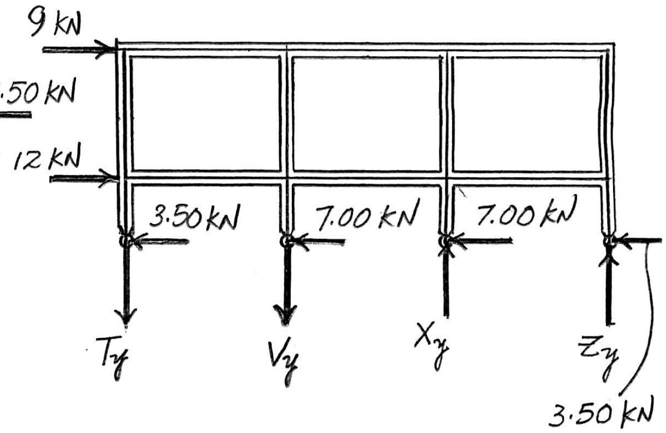


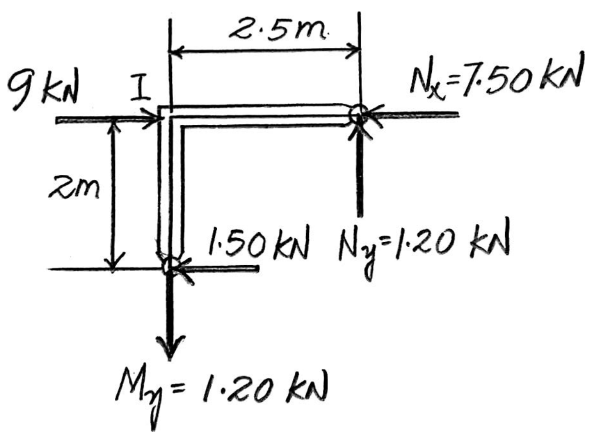


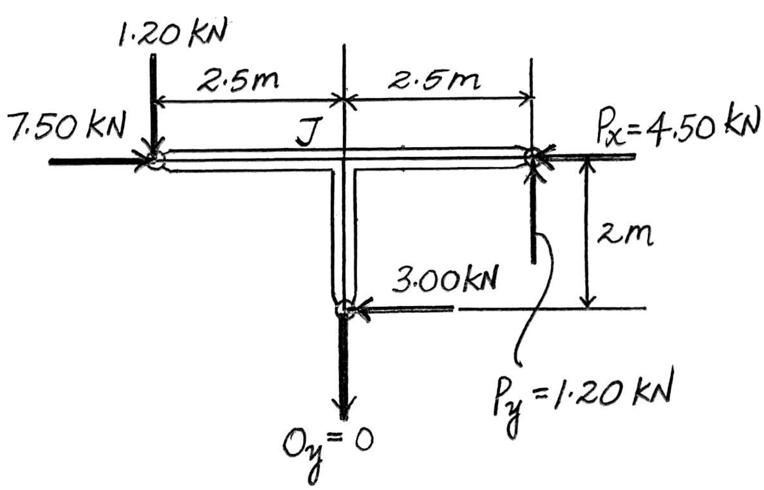
7–37. Use the portal method and determine (approximately) the reactions at supports A, B, C, and D.



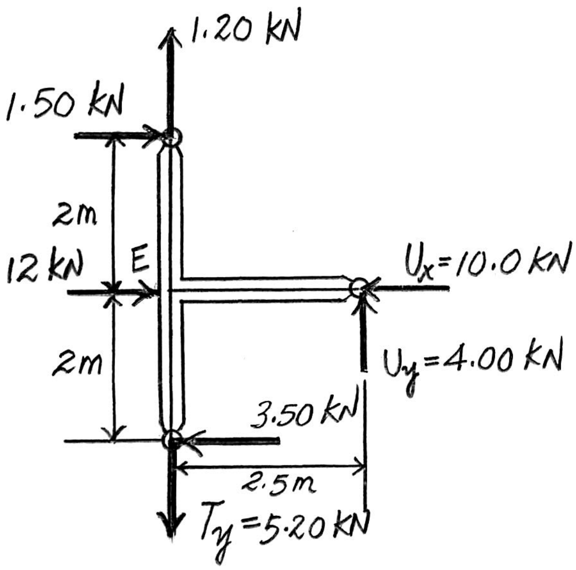


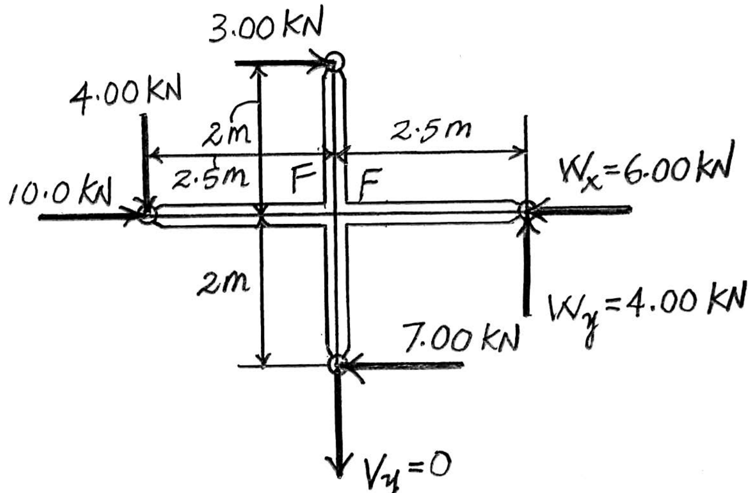


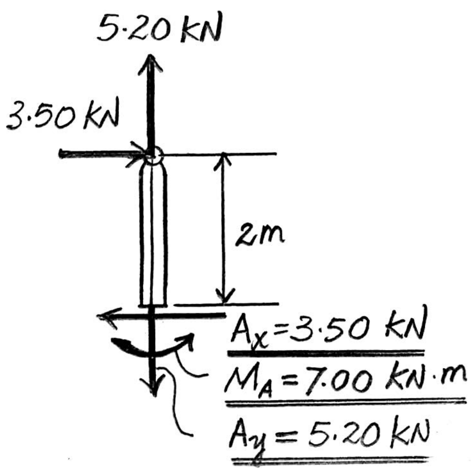


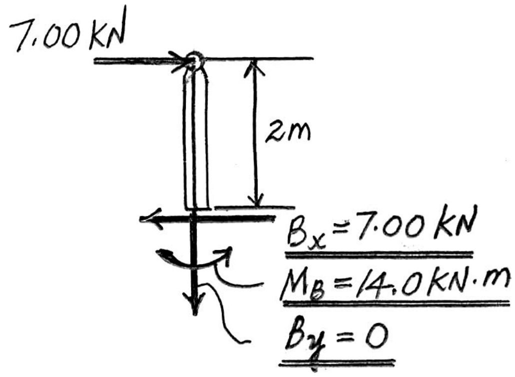


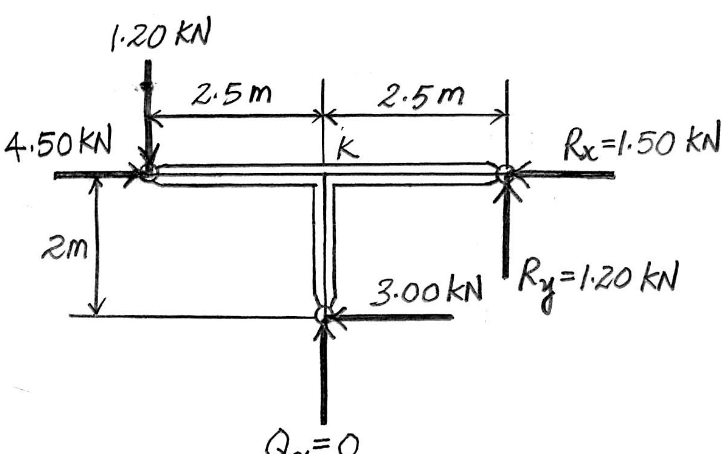
7–37. Continued

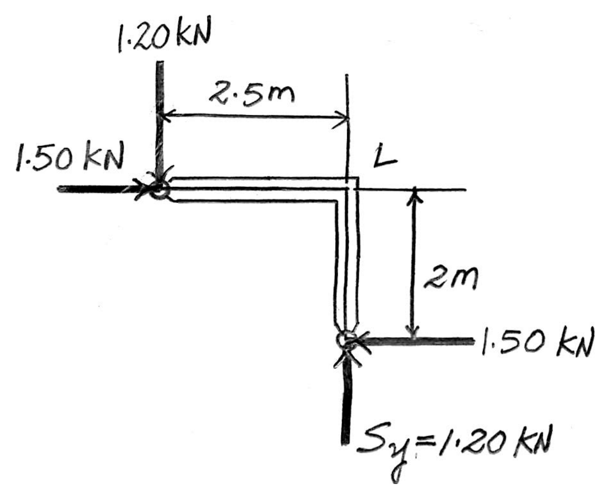












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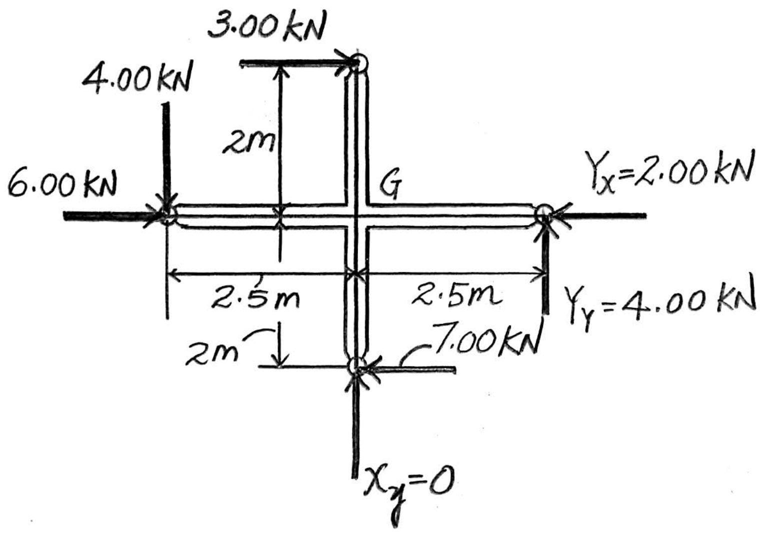
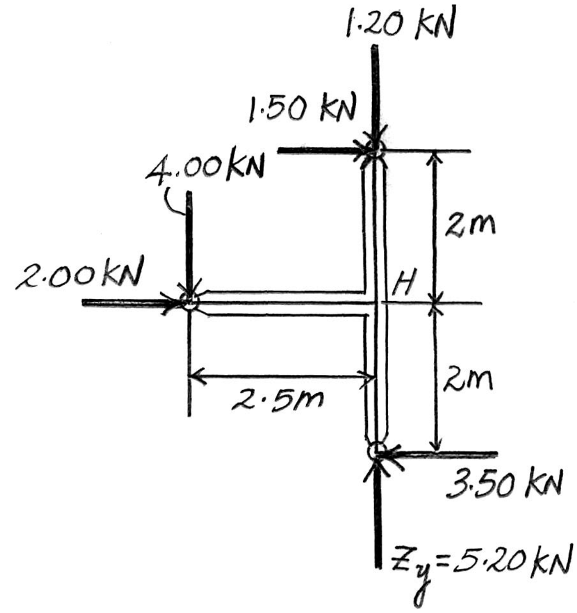
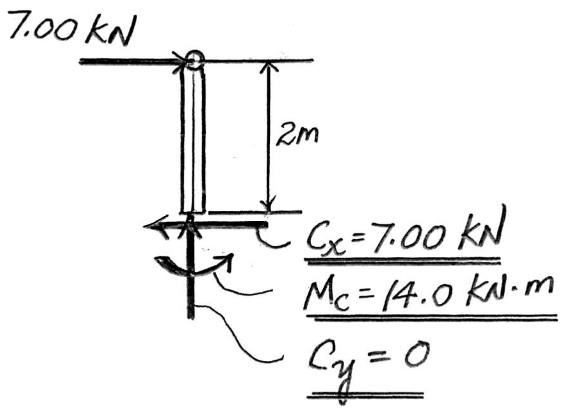
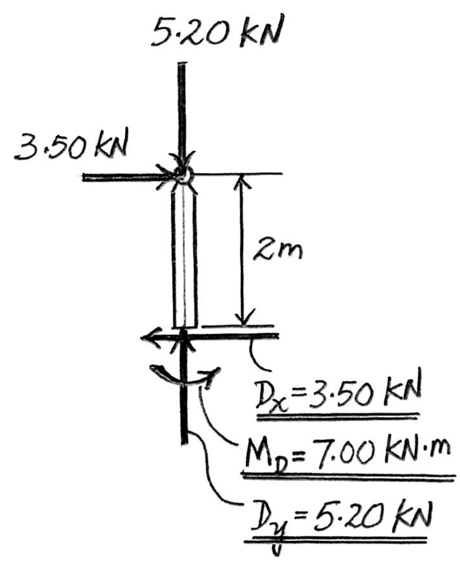
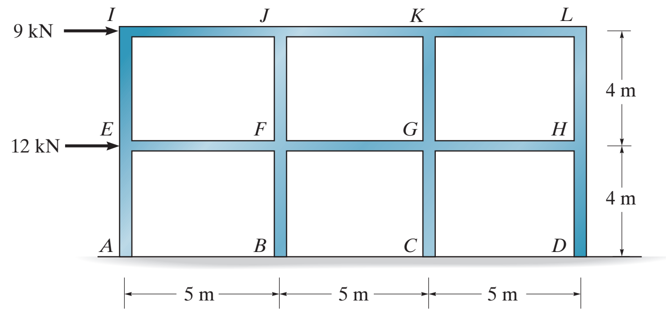


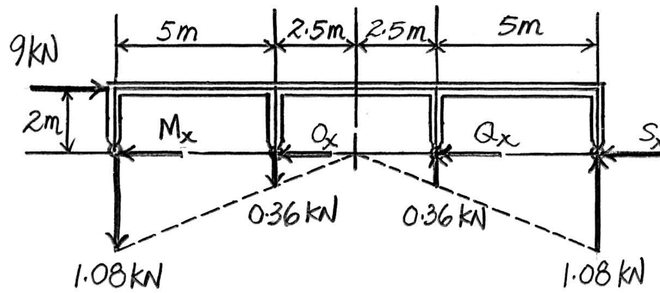


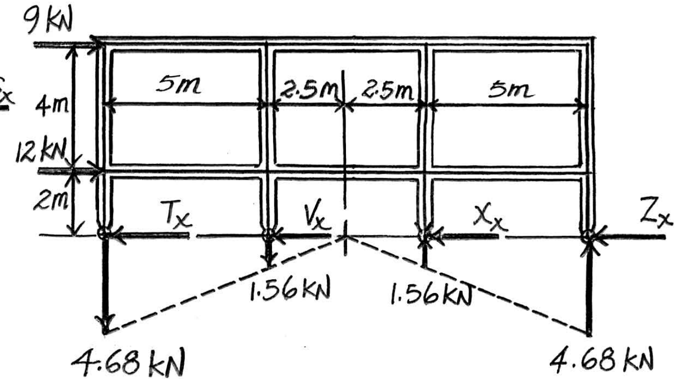


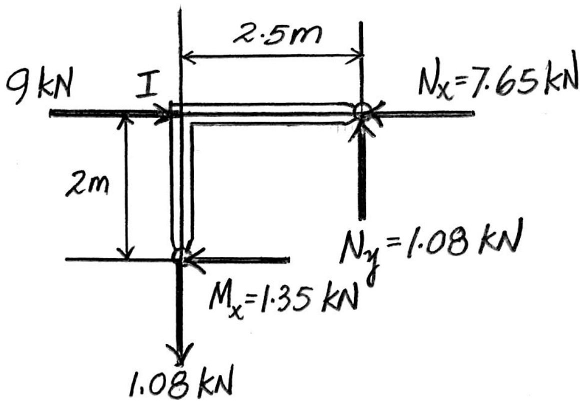


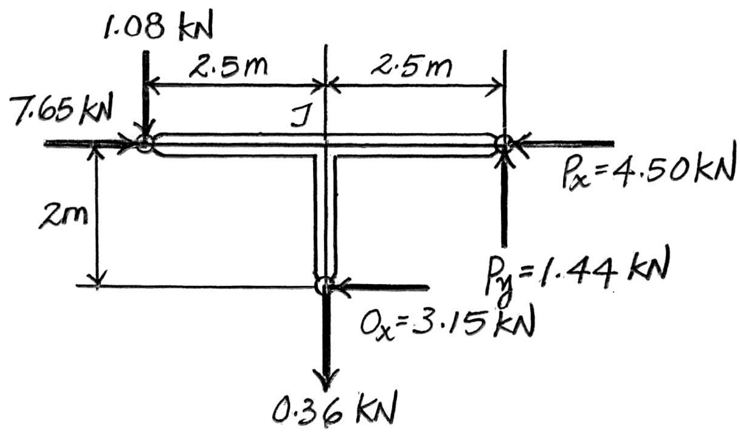
7–38. Use the cantilever method and determine (approximately) the reactions at supports A, B, C, and D. All columns have the same cross-sectional area.

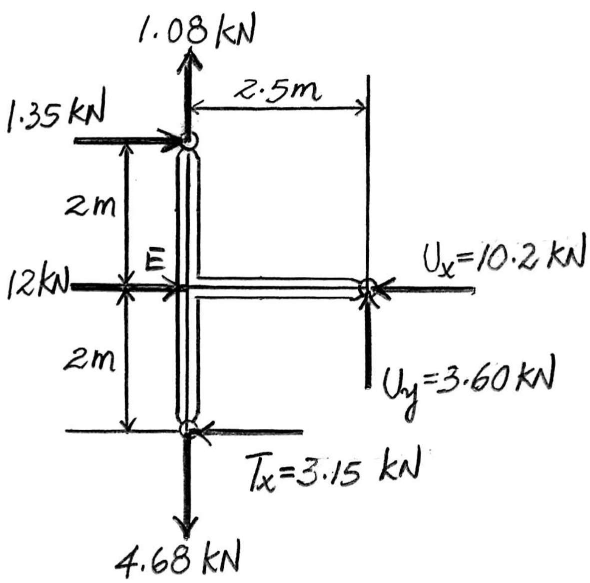


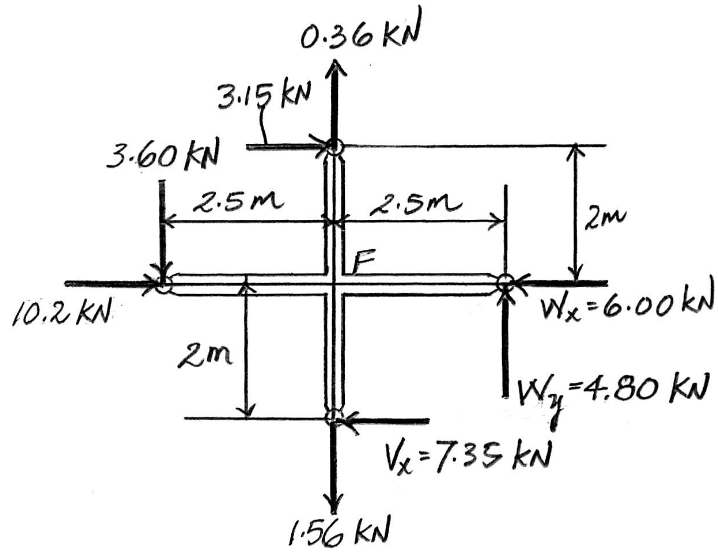




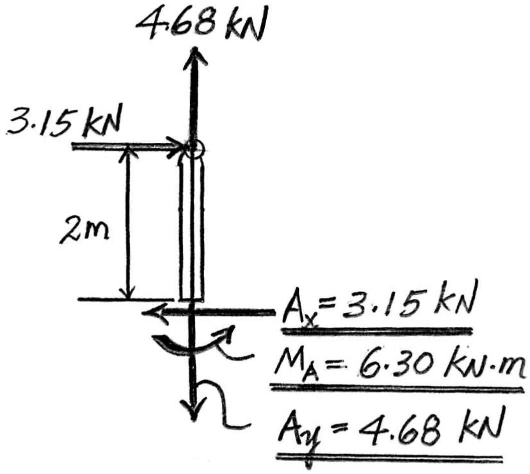


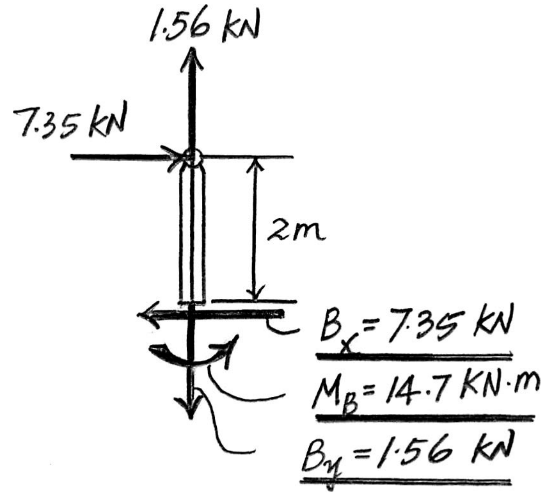


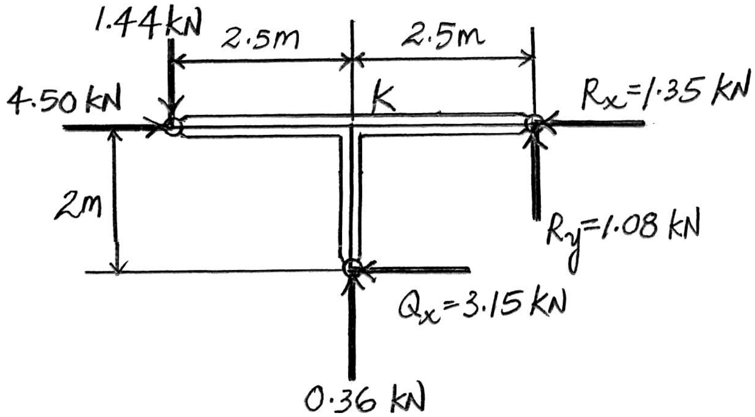


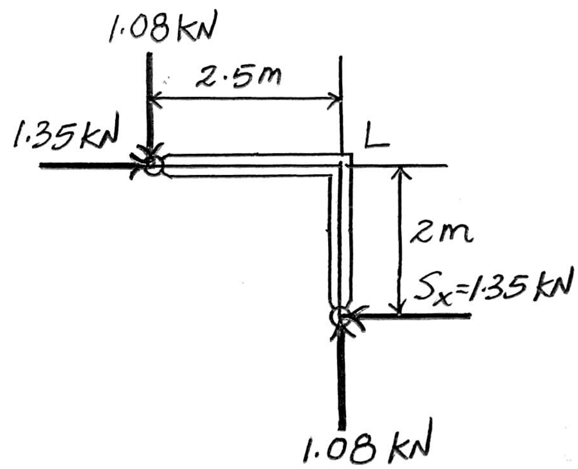


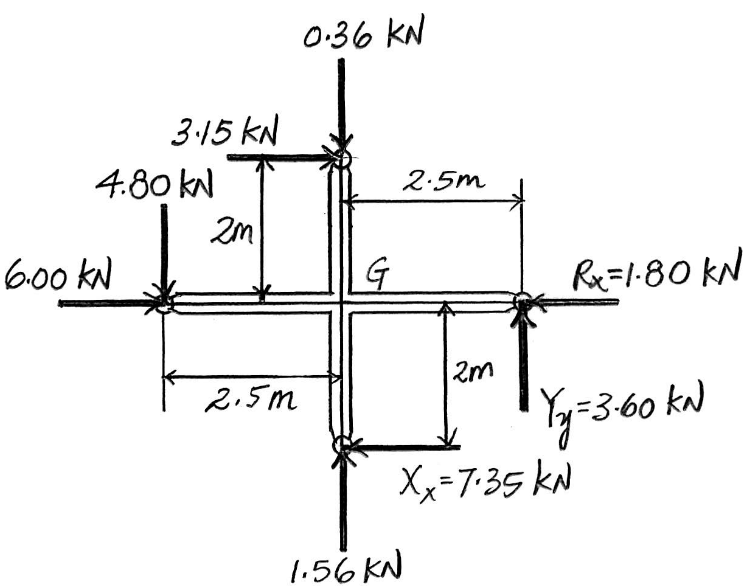
7–38. Continued

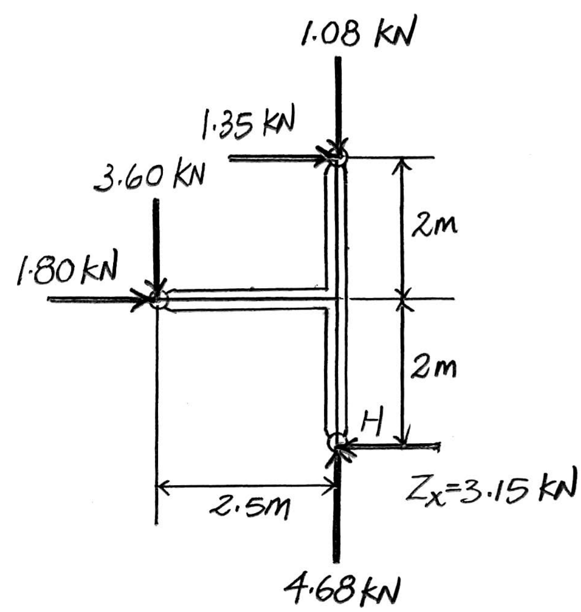












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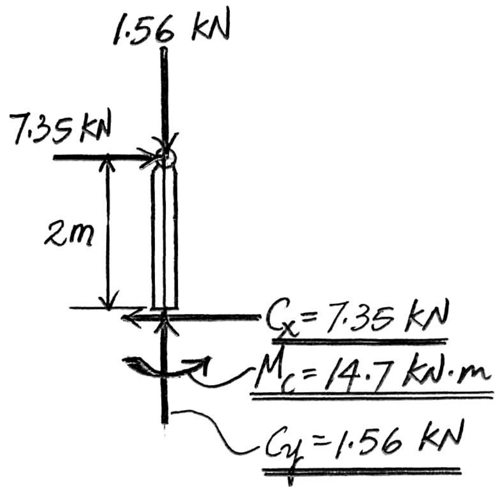
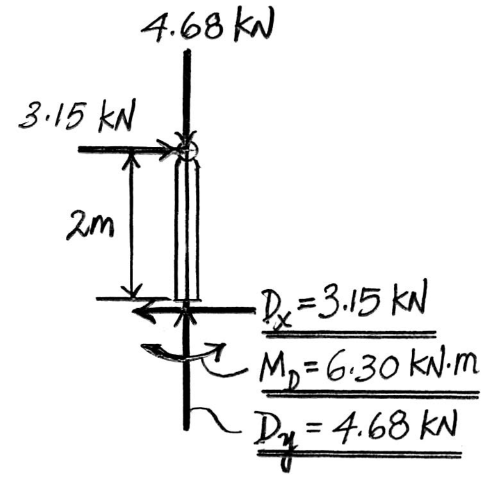
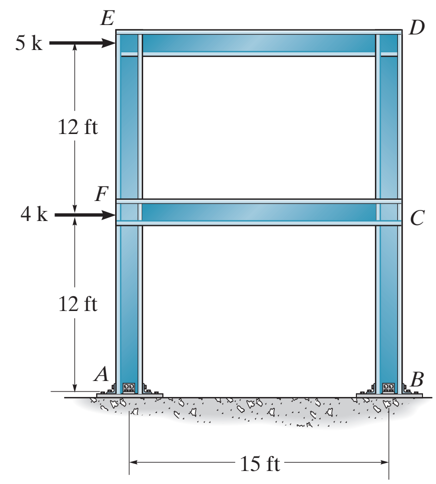
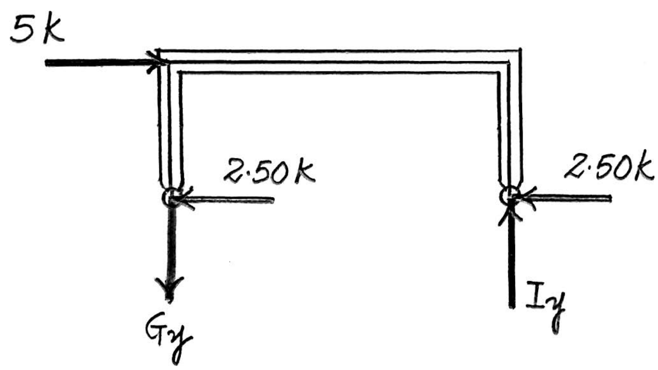
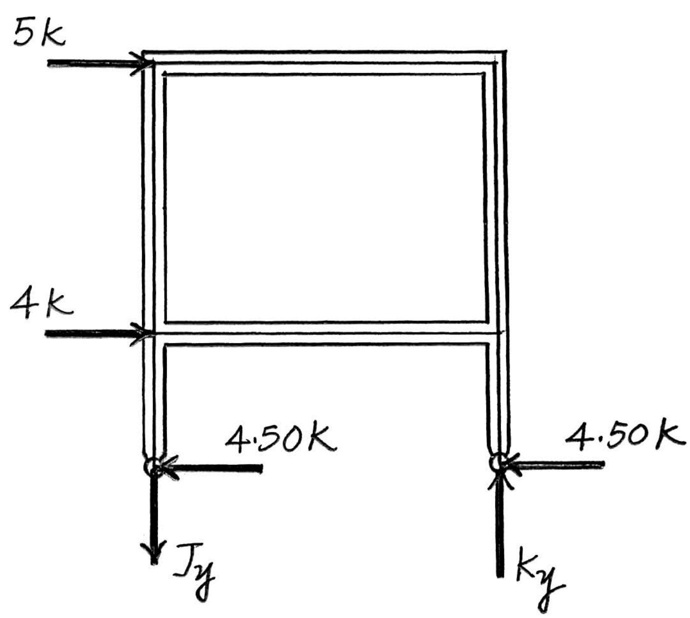


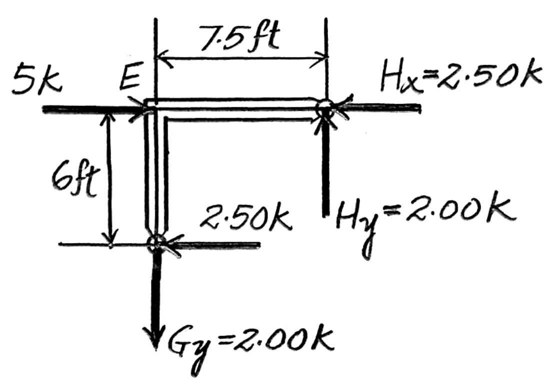


7–39. Use the portal method of analysis and draw the moment diagram for column *AFE*.

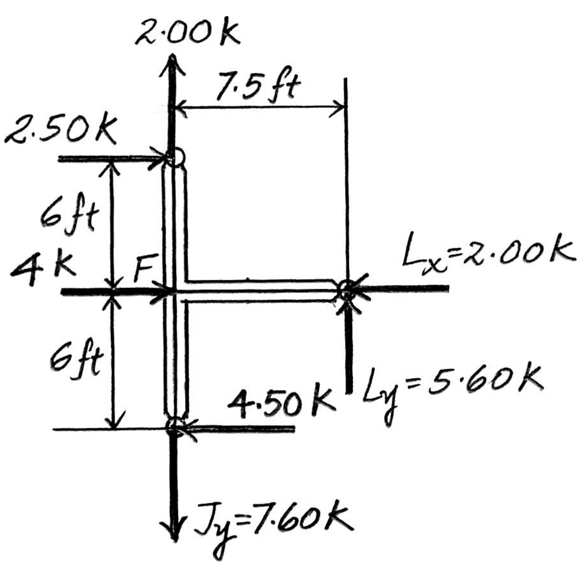


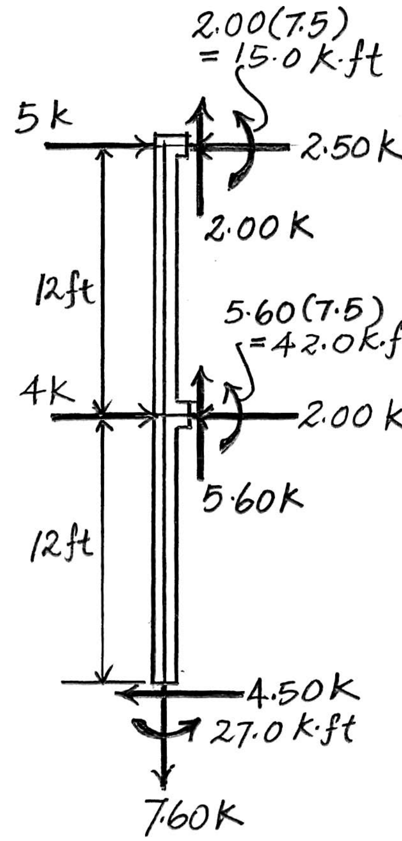


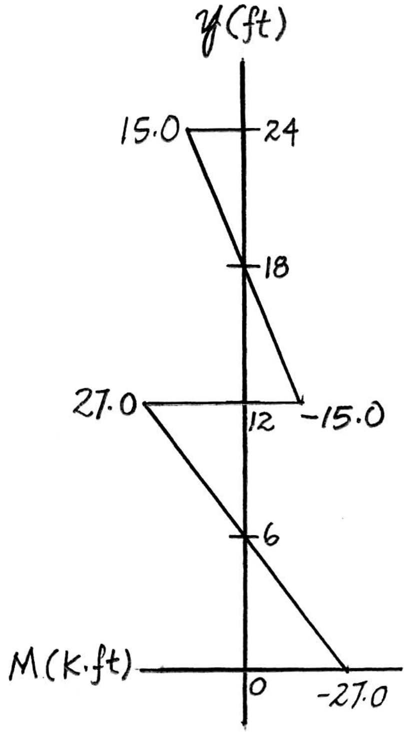


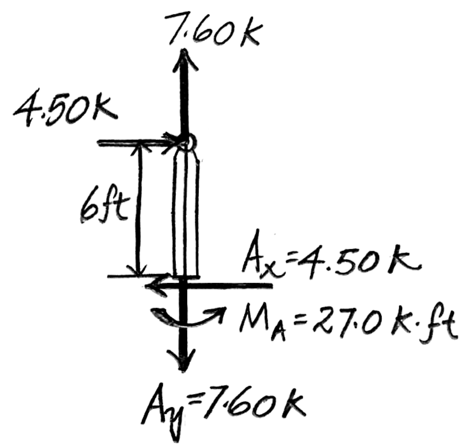


7–39. Continued

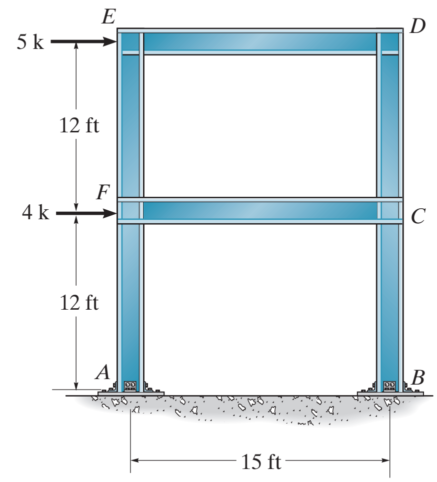


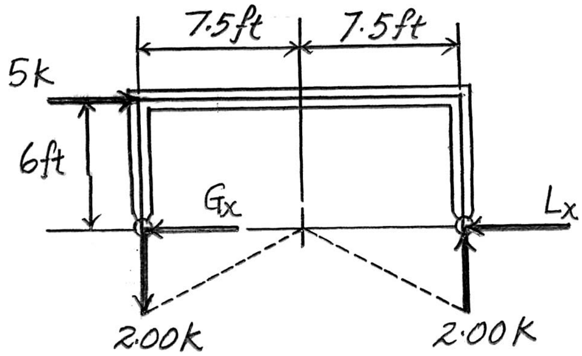


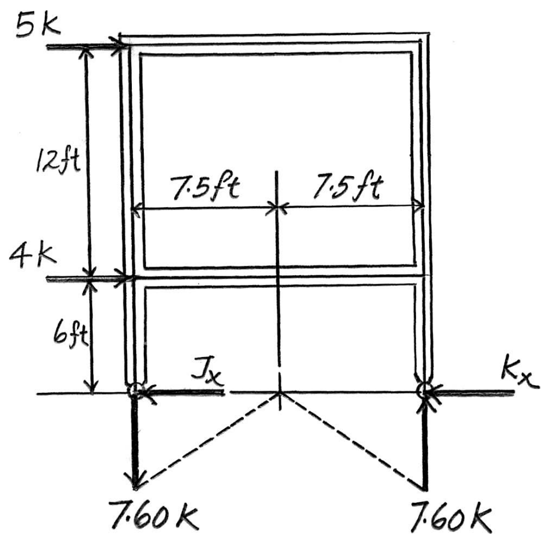


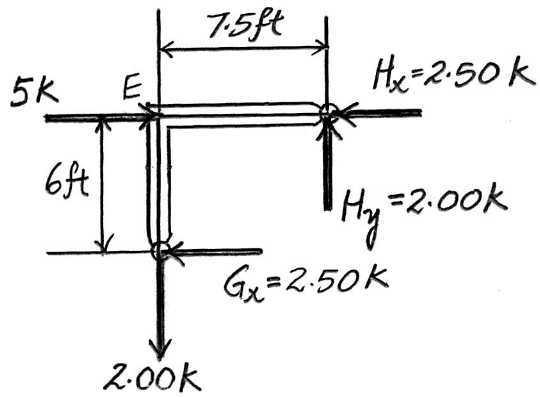


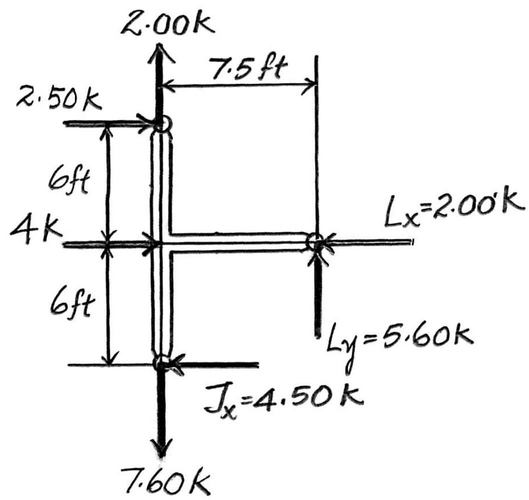
***7–40.** Solve Prob. 7–39 using the cantilever method of analysis. All the columns have the same cross-sectional area.

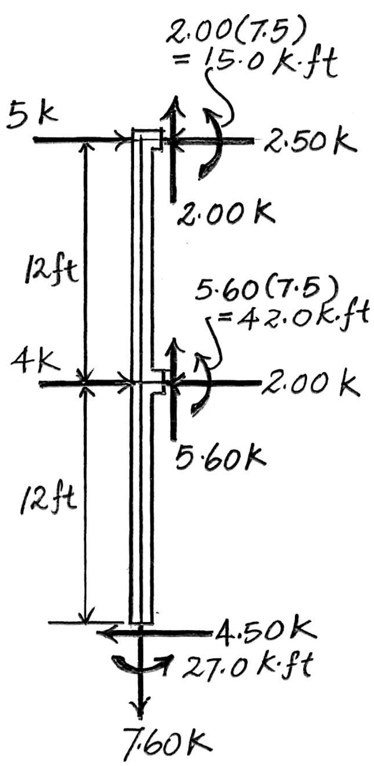


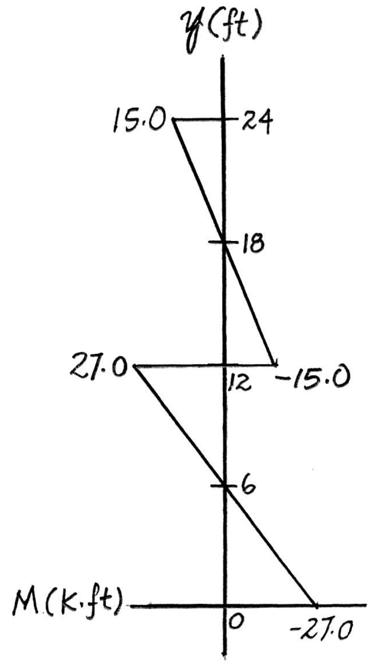


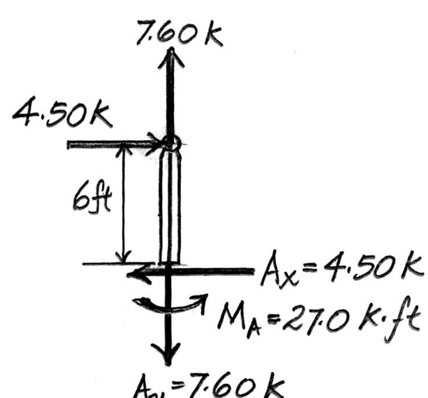




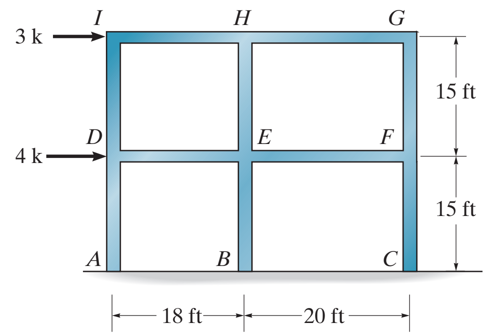


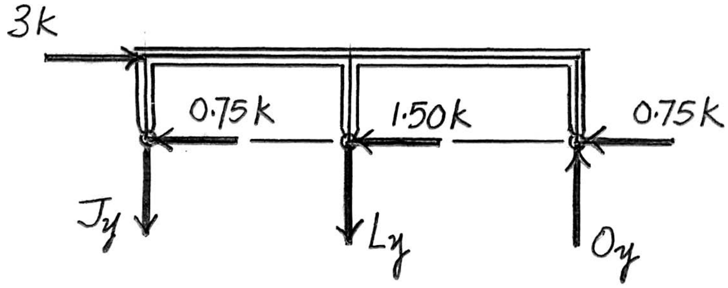


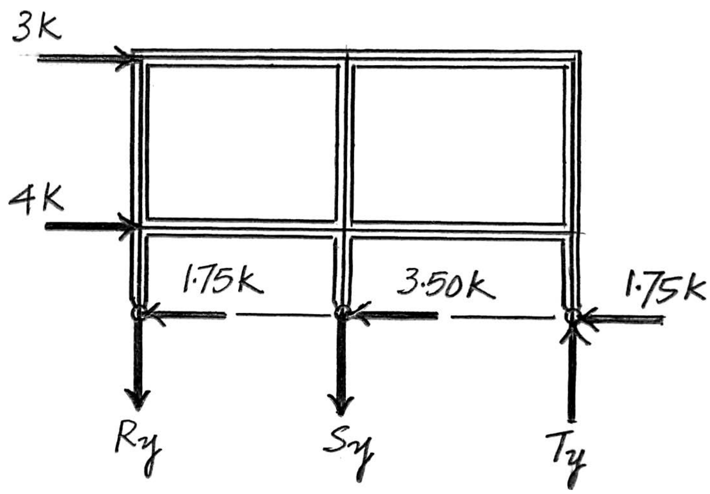


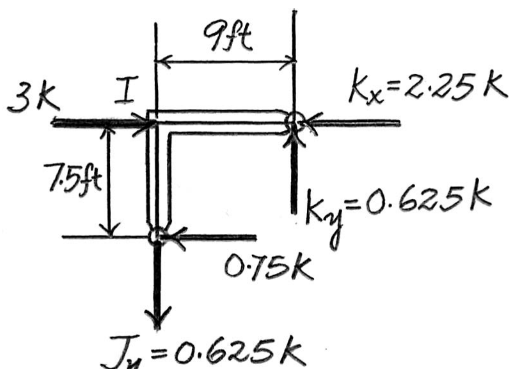


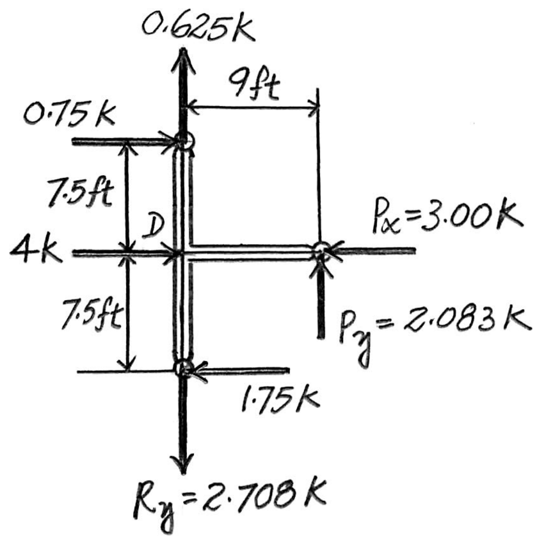
7–41. Use the portal method and determine (approximately) the reactions at A.

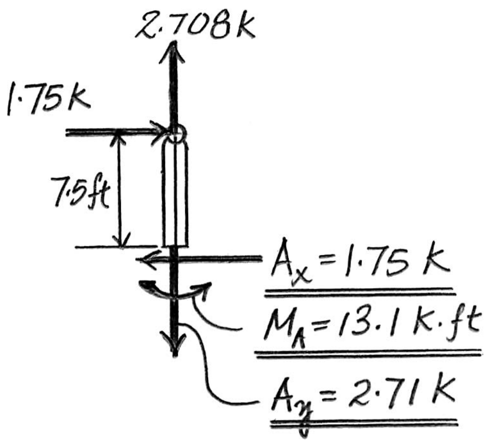




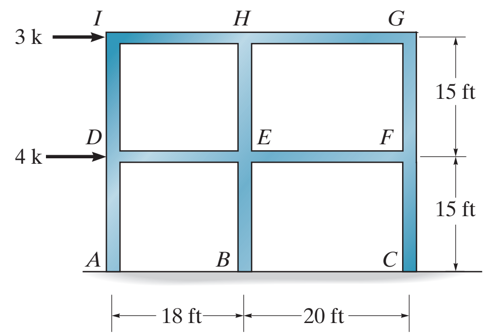


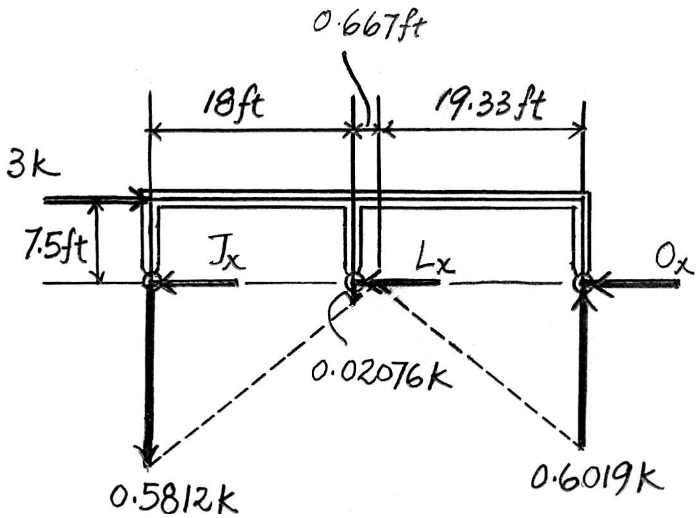


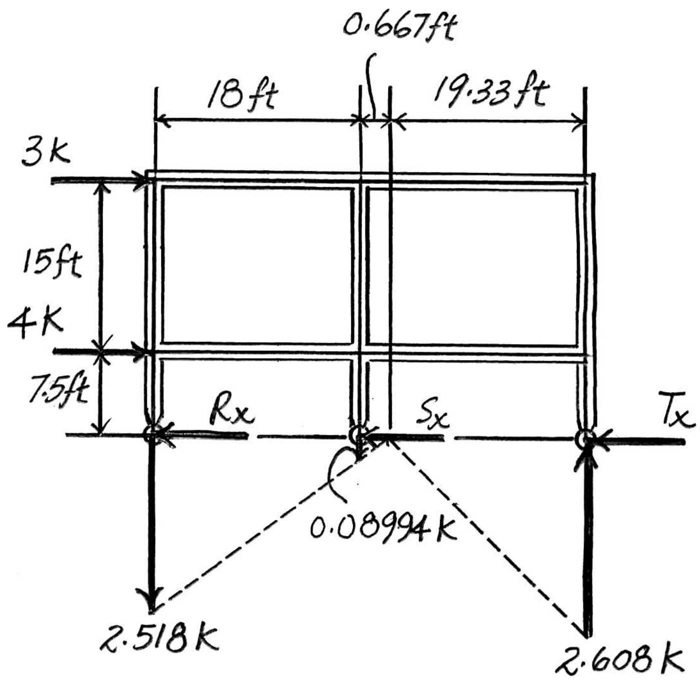


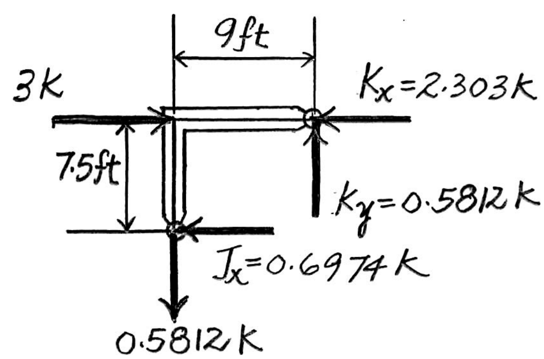


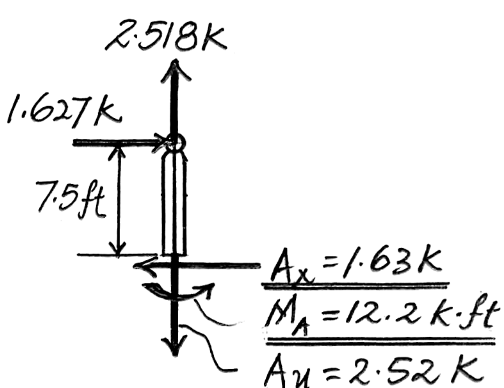
7–42. Use the cantilever method and determine (approximately) the reactions at A. All of the columns have the same cross-sectional area.

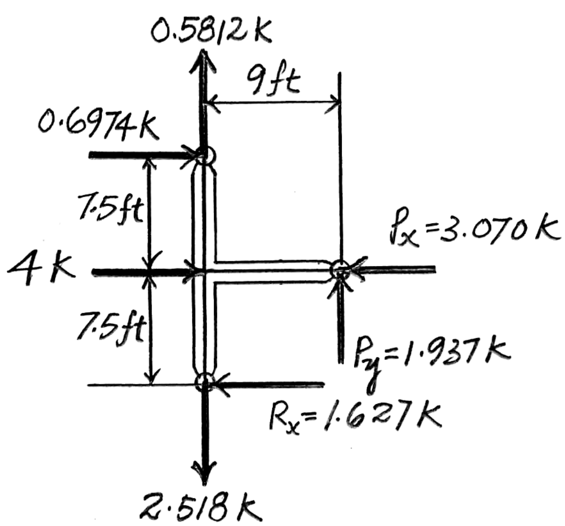




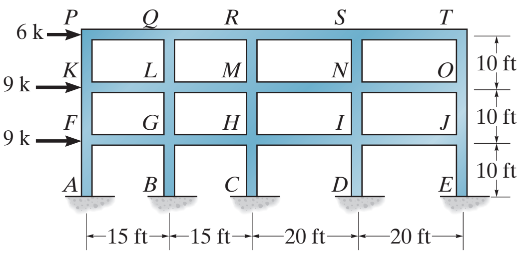


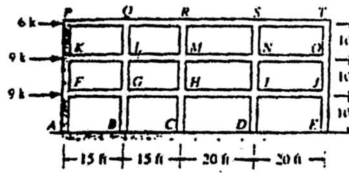






7–43. Draw (approximately) the moment diagram for girder *PQRST* and column *BGLQ* of the building frame. Use the portal method.





Top story

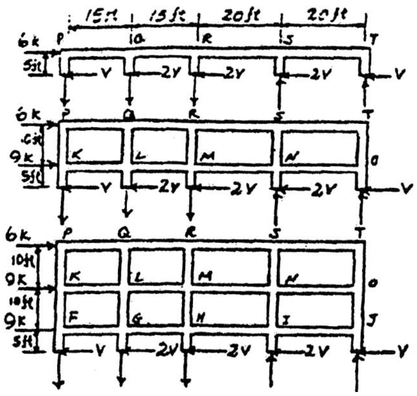
$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 6 - 8V = 0; \qquad\qquad V = 0.75 \text{ k}$

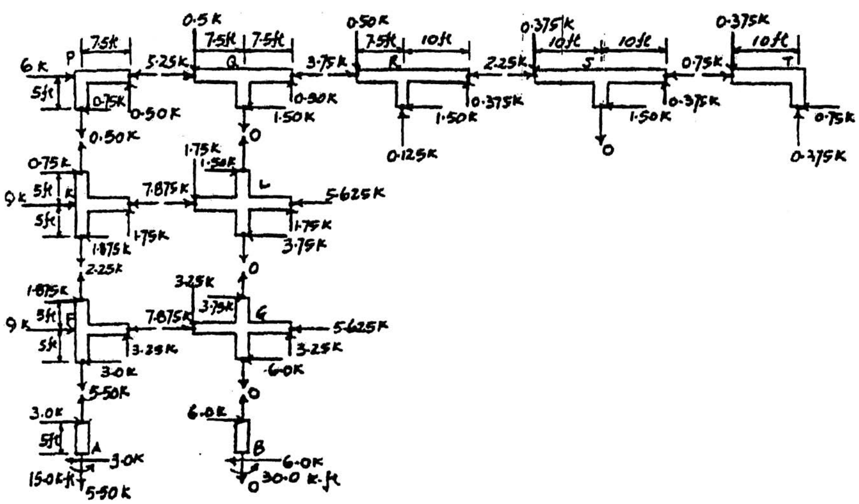
Second story

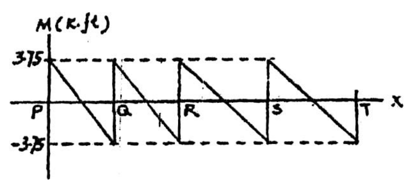
$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 6 + 9 - 8V = 0; \qquad\qquad V = 1.875 \text{ k}$

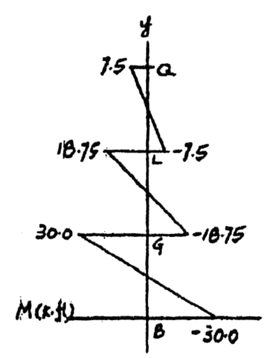
Bottom story

$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 6 + 9 + 9 - 8V = 0; \qquad\qquad V = 3.0 \text{ k}$

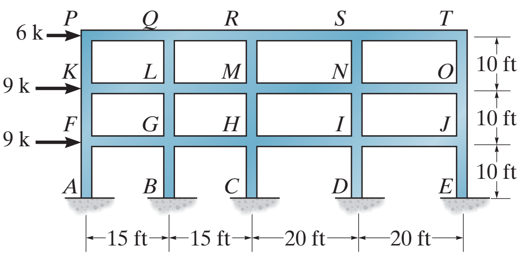


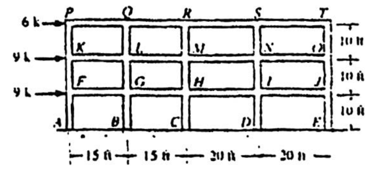




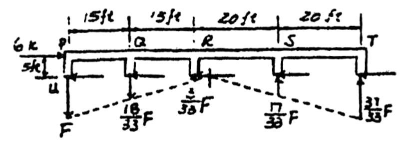


***7–44.** Draw (approximately) the moment diagram for girder *PQRST* and column *BGLQ* of the building frame. All columns have the same cross-sectional area. Use the cantilever method.



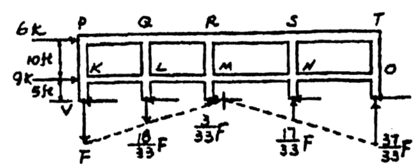


$$\bar{x} = \frac{15 + 30 + 50 + 70}{5} = 33 \text{ ft}$$



$$\circlearrowleft + \sum M_U = 0; \qquad -6(5) - \frac{18}{33}F(15) - \frac{3}{33}F(30) + \frac{17}{33}F(50) + \frac{37}{33}F(70) = 0$$

$$F = 0.3214 \text{ k}$$



$$\circlearrowleft + \sum M_V = 0;$$

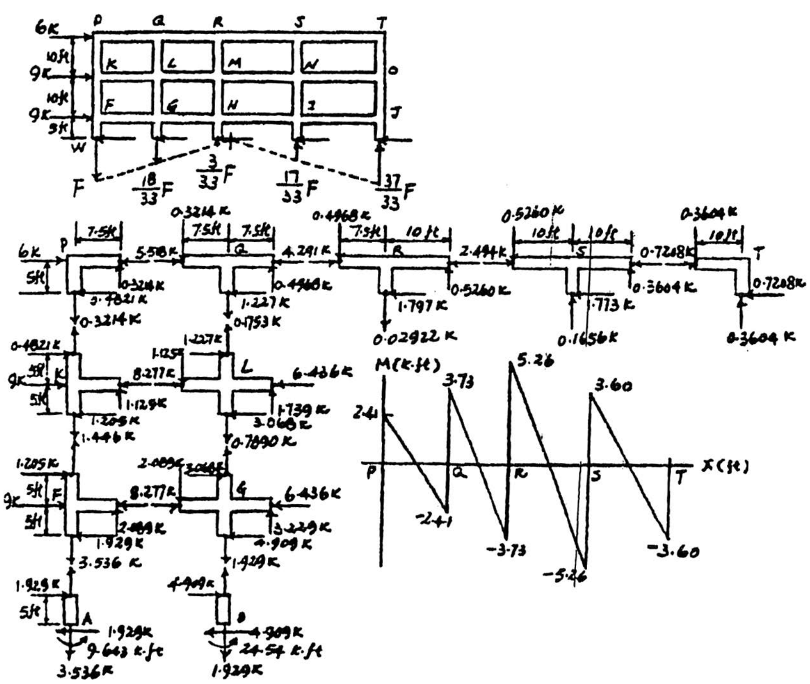
$$-6(15) - 9(5) - \frac{18}{33}F(15) - \frac{3}{33}F(30) + \frac{17}{33}F(50) + \frac{37}{33}F(70) = 0$$

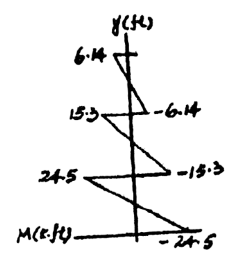
$$F = 1.446 \text{ k}$$

$$\circlearrowleft + \sum M_W = 0;$$

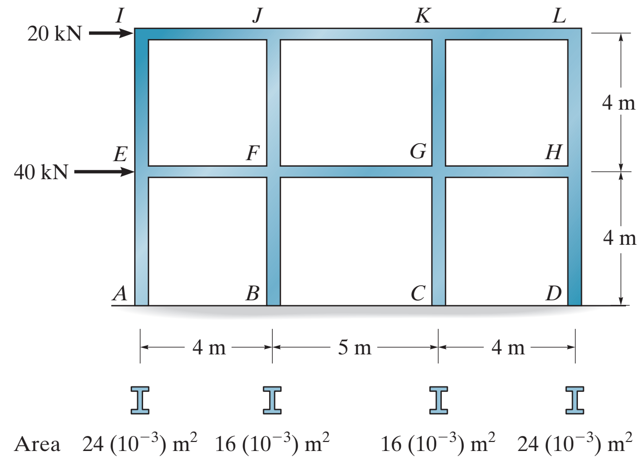
$$-6(25) - 9(15) - 9(5) - \frac{18}{33}F(15) - \frac{3}{33}F(30) + \frac{17}{33}F(50) + \frac{37}{33}F(70) = 0$$

$$F = 3.536 \text{ k}$$



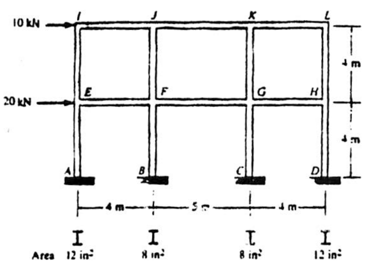


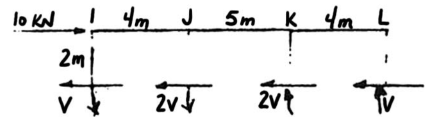
7–45. Draw the moment diagram for girder *IJKL* of the building frame. Use the portal method of analysis.

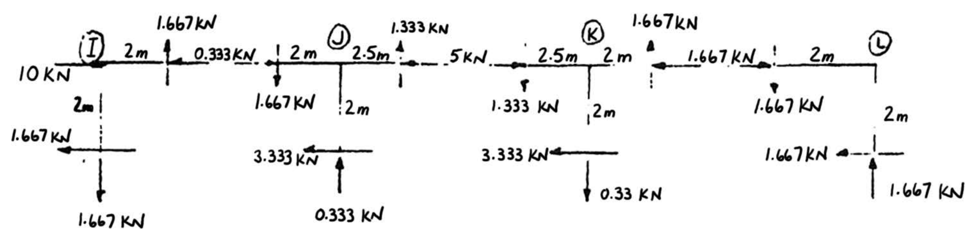


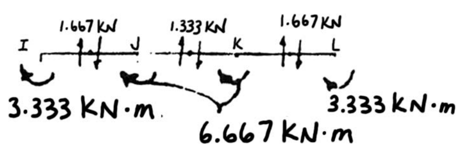
$\xrightarrow{+} \sum F_x = 0; \qquad 10 - 6V = 0; \qquad V = 1.667 \text{ kN}$

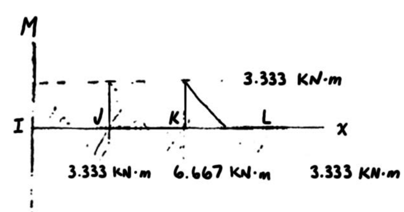
The equilibrium of each segment is shown on the FBDs.



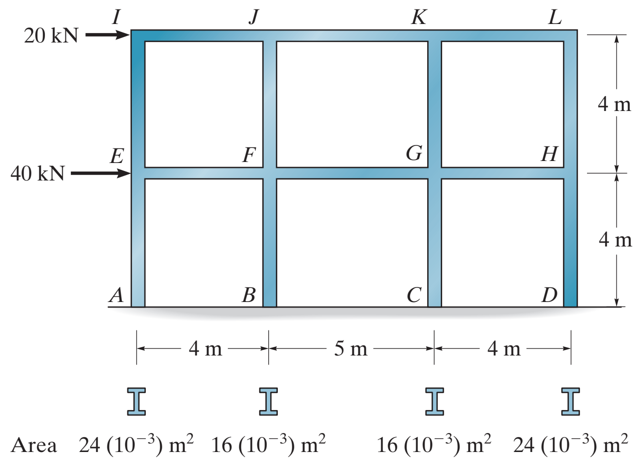








***7–46.** Solve Prob. 7–45 using the cantilever method of analysis. Each column has the cross-sectional area indicated.



The centroid of column area is in center of framework.

Since $\sigma = \dfrac{F}{A}$, then

$$\sigma_1 = \left(\frac{6.5}{2.5}\right)\sigma_2; \qquad \frac{F_1}{12} = \frac{6.5}{2.5}\left(\frac{F_2}{8}\right); \qquad F_1 = 3.90\,F_2$$

$$\sigma_4 = \sigma_1; \qquad F_4 = F_1$$

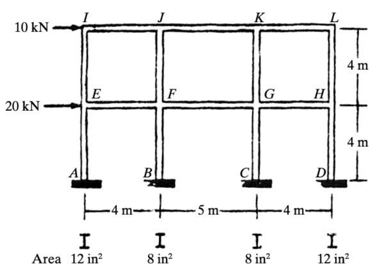
$$\sigma_2 = \sigma_3; \qquad F_2 = F_3$$

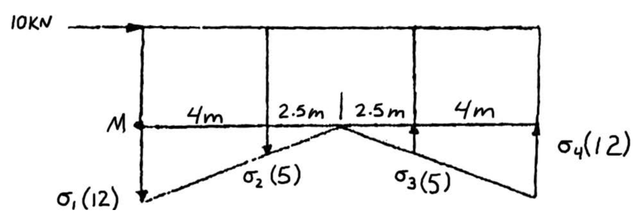
$$\zeta + \sum M_M = 0; \qquad -2(10) - 4(F_2) + 9(F_2) + 13(3.90\,F_2) = 0$$

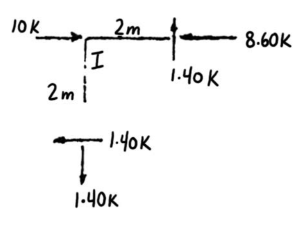
$$F_2 = 0.359 \text{ k}$$

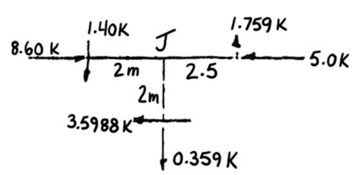
$$F_1 = 1.400 \text{ k}$$

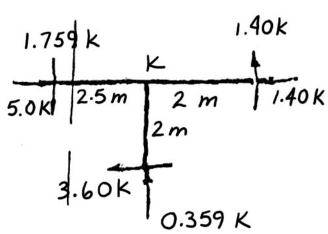
The equilibrium of each segment is shown on the FBDs.

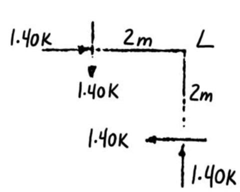


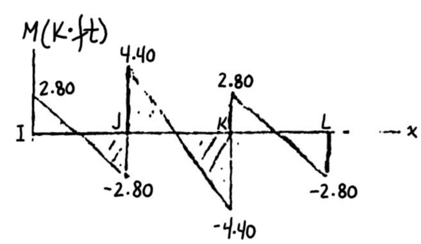




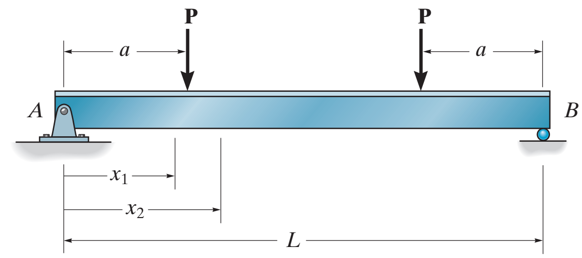








***8–1.** Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant.



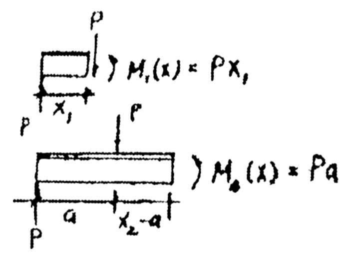
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^2}{6}C_1x_1 + C_1 \qquad (2)$$



For $M_1(x) = Pa$

$$EI\frac{d^2v_1}{dx_1^2} = Pa$$

$$EI\frac{dv_1}{dx_1} = Pax_1 + C_1 \qquad (3)$$

$$EIv_1 = \frac{Pax_1^2}{2} = C_3x_1 + C_4 \qquad (4)$$

Boundary conditions:

$$v_1 = 0 \quad \text{at} \quad x = 0$$

From Eq. (2)

$$C_2 = 0$$

Due to symmetry:

$$\frac{dv_1}{dx_1} = 0 \quad \text{at} \quad x_1 = \frac{L}{2}$$

From Eq. (3)

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = \frac{PaL}{2}$$

Continuity conditions:

$$v_1 = v_2 \quad \text{at} \quad x_1 = x_2 = a$$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} - \frac{Pa^3L}{2} + C_4$$

$$C_1a \cdot C_4 = \frac{Pa^3}{2} - \frac{Pa^3L}{2} \qquad (5)$$

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = a$$

8–1. Continued

$$\frac{Pa^3}{2} + C_1 = Pa^3 - \frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

Substitute C_1 into Eq. (5)

$$C_a = \frac{Pa^3}{6}$$

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}(x_1^2 + a^2 - aL)$$

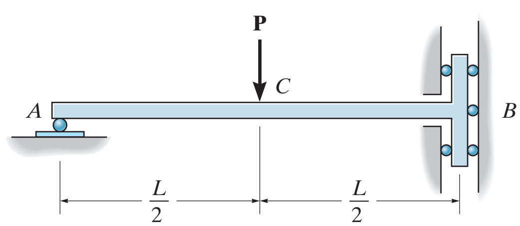
$$\theta_A = \frac{d_{v1}}{d_{x1}}\bigg|_{x_1=0} = \frac{Pa(a-L)}{2EI}$$ **Ans.**

$$v_1 = \frac{Px_1}{6EI}[x_1^2 + 3a(a-L)]$$ **Ans.**

$$v_2 = \frac{Pa}{6EI} + (3x_2(x_2 - L) + a^2)$$ **Ans.**

$$V_{x=1} = V_2\bigg|_{x=\frac{1}{2}} = \frac{Pa}{24EI}(4a^2 - 3L^2)$$ **Ans.**

8–2. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_2} = \frac{Px_1^2}{2} + C_1$$

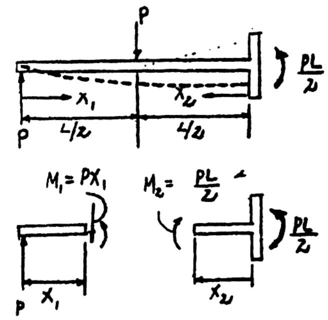
$$EIv_1 = \frac{Px_1^2}{6} + C_1x_1 + C_1$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_3 + C_4$$

8–2. Continued



Boundary conditions:

At $x_1 = 0, \quad v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, \quad v_1 = v_2, \quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{L}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{L}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{L}{2}\right)^2}{2} + C_1 = -\frac{P\left(\frac{L}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^3$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$

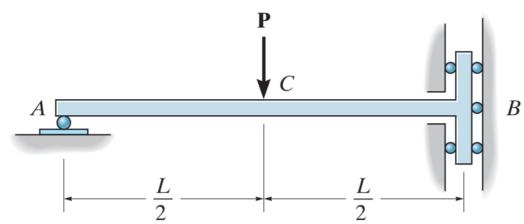
$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI}$$ **Ans.**

At $x_1 = \dfrac{L}{2}$

$$v_c = \frac{P\left(\frac{L}{2}\right)^3}{6EI} - \left(\frac{3}{8}PL^2\right)\left(\frac{L}{2}\right) + 0$$

$$v_c = \frac{-PL^3}{6EI}$$ **Ans.**

8–3. Determine the deflection at B of the bar in Prob. 8–2.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_2}{dx_1} = \frac{Px_1^2}{2} + C_1$$

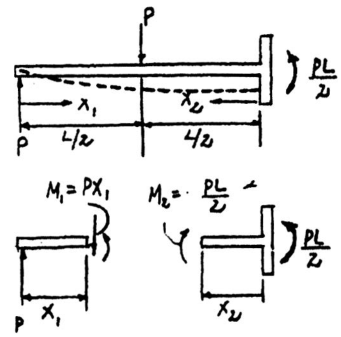
$$EIv_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2}\frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

8–3. Continued



Boundary conditions:

At $x_1 = 0, \quad v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, \quad v_1 = v_2, \quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{L}{2}\right)^3}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{L}{2}\right)^2}{4} + C_4$$

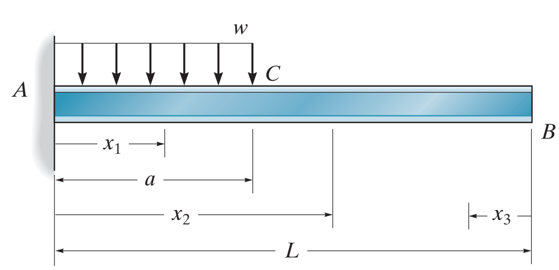
$$\frac{P\left(\frac{L}{2}\right)^3}{2} + C_1 = -\frac{P\left(\frac{L}{2}\right)}{2}; \qquad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = \frac{11}{48}PL^3$$

At $x_2 = 0$,

$$v_B = -\frac{11PL^3}{48EI}$$ **Ans.**

***8–4.** Determine the equations of the elastic curve using the coordinates x_1 and x_2, specify the slope and deflection at B. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\dfrac{w}{2}x_1^2 + wax_1 - \dfrac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

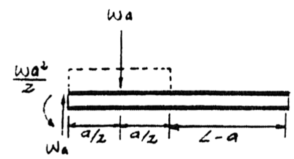
$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

8–4. Continued

$$EIv_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \quad (2)$$

For $M_2(x) = 0$; $EI\dfrac{d^2v_2}{dx_2^3} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \quad (3)$$

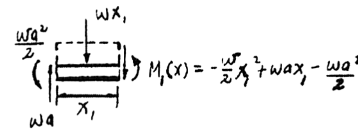


$$EI\, v_2 = C_3x_2 + C_4 \quad (4)$$

Boundary conditions:

At $x_1 = 0$, $\dfrac{dv_1}{dx_1} = 0$

From Eq. (1), $C_1 = 0$

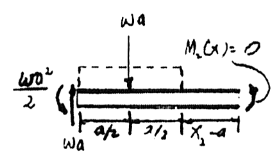


At $x_1 = 0$, $v_1 = 0$

From Eq. (2): $C_2 = 0$

Continuity conditions:

At $x_1 = a$, $x_2 = a$; $\dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$



From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \quad C_3 = -\frac{wa^3}{6}$$

From Eqs. (2) and (4),

At $x_1 = a$, $x_2 = a$ $v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \quad C_4 = \frac{wa^4}{24}$$

The slope, from Eq. (3),

$$\theta_B = \frac{dv_2}{dx_2} = \frac{wa^3}{6EI} \quad \textbf{Ans.}$$

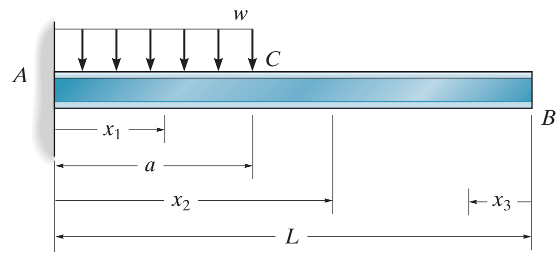
The elastic curve:

$$v_1 = \frac{w}{24EI}\left(-x_1^4 + 4ax_1^3 - 6a^2x_1^2\right) \quad \textbf{Ans.}$$

$$v_2 = \frac{wa^3}{24EI}\left(-4x_2 + a\right) \quad \textbf{Ans.}$$

$$v_1 = v_2\Big|_{x_3 = L} = \frac{wa^3}{24EI}\left(-4L + a\right) \quad \textbf{Ans.}$$

8–5. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope and deflection at point B. EI is constant.



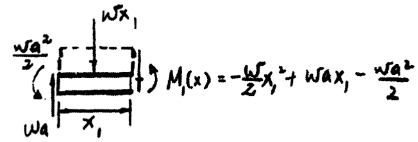
$$EI\frac{d^2v}{dx_2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

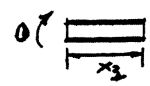
$$EIv_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = 0$; $\quad EI\frac{d^2v_3}{dx_3^2} = 0$

$$EI\frac{dv_3}{dx_3} = C_3 \qquad (3)$$

$$EI\,v_3 = C_3x_3 + C_4 \qquad (4)$$



Boundary conditions:

At $x_1 = 0$, $\quad \frac{dv_1}{dx_1} = 0$

From Eq. (1),

$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0$, $\quad v_1 = 0$

From Eq. (2),

$0 = -0 - 0 - 0 + 0 + C_2; \quad C_2 = 0$

Continuity conditions:

At $x_1 = a$, $\quad x_3 = L - a$; $\quad \frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{-wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

At $x_1 = a$, $\quad x_3 = L - a \quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L - a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3L}{6}$$

The slope

$$\frac{dv_3}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \frac{d_{v3}}{d_{x3}}\bigg|_{x_3 = 0} = \frac{wa^3}{6EI} \qquad \textbf{Ans.}$$

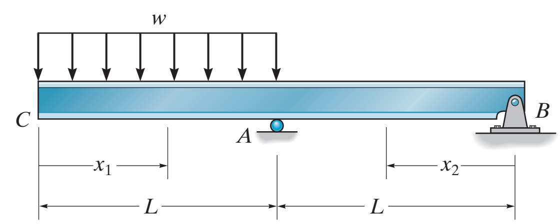
The elastic curve:

$$v_1 = \frac{wx_1^2}{24EI}\left(-x_1^2 + 4ax_1 - 6a^2\right) \qquad \textbf{Ans.}$$

$$v_3 = \frac{wa^3}{24EI}\left(4x_3 + a - 4L\right) \qquad \textbf{Ans.}$$

$$V_2 = V_3\bigg|_{x_3 = 0} = \frac{wa^3}{24EI}\left(a - 4L\right) \qquad \textbf{Ans.}$$

8–6. Determine the maximum deflection between the supports A and B. EI is constant. Use the method of integration.



Elastic curve and slope:

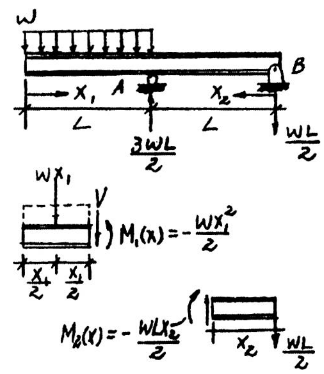
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^2}{2}$

$$EI\frac{d^1v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad (1)$$

$$EIv_1 = -\frac{wx_1^4}{24} + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_3^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-wLx_2^3}{412} + C_3x_3 + C_4 \qquad (4)$$

Boundary conditions:

$v_2 = 0$ at $x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad (5)$$

8–6. Continued

Continuity conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \quad \text{at} \quad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = \frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad (6)$$

$$\theta_A = \frac{d_{v1}}{d_{x1}}\bigg|_{x_1 = L} = -\frac{dv_2}{dv_3}\bigg|_{x_3 = L} = \frac{wL^3}{6EI}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4)$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI}(x_1 = 0)$$

The negative sign indicates downward displacement

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad (7)$$

$(v_2)_{\max}$ occurs when $\frac{dv_2}{dx_2} = 0$

From Eq. (6)

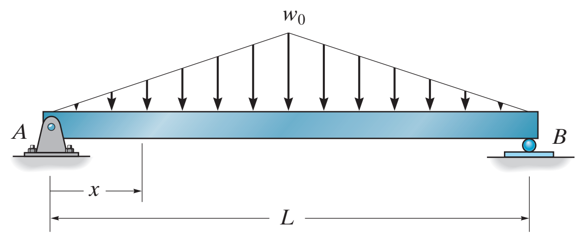
$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

$$(v_2)_{\max} = \frac{wL_4}{18\sqrt{3EI}} \qquad \textbf{Ans.}$$

8–7. Determine the elastic curve for the simply supported beam using the x coordinate $0 \le x \le L/2$. Also, determine the slope at A and the maximum deflection of the beam. EI is constant.

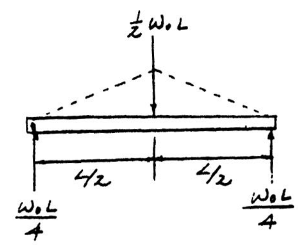


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_oL}{4}x - \frac{w_o}{3L}x^3$$

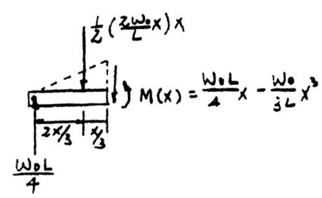
$$EI\frac{dv}{dx} = \frac{w_oL}{8}x^2 - \frac{w_o}{12L}x^4 + C_1 \quad (1)$$

$$EIv = \frac{w_oL}{24}x^3 - \frac{w_o}{60L}x^5 + C_1x + C_2 \quad (2)$$



Boundary conditions:

Due to symmetry, at $x = \frac{L}{2}, \frac{dv}{dx} = 0$



From Eq. (1),

$$0 = \frac{w_oL}{8}\left(\frac{L^2}{4}\right) - \frac{w_o}{12L}\left(\frac{L^4}{16}\right) + C_1; \qquad C_1 = -\frac{5w_oL^3}{192}$$

At $x = 0, \quad v = 0$

From Eq. (2),

$$0 = 0 - 0 + 0 + C_2; \quad C_2 = 0$$

From Eq. (1),

$$\frac{dv}{dx} = \frac{w_o}{192EIL}(24L^2x^2 - 16x^4 - 5L^4)$$

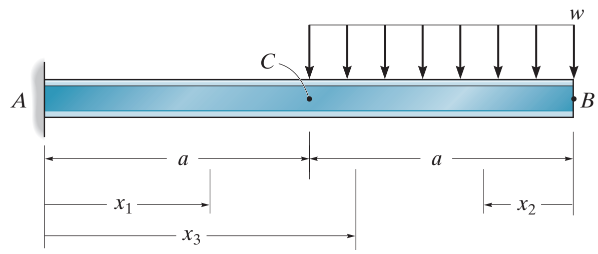
$$\theta_A = \frac{d_v}{d_x}\bigg|_{x=0} = -\frac{5w_oL^3}{192EI} = \frac{5w_oL^3}{192EI} \quad \textbf{Ans.}$$

From Eq. (2),

$$v = \frac{w_ox}{960EIL}(40L^2x^2 - 16x^4 - 25L^4) \quad \textbf{Ans.}$$

$$v_{max} = v\bigg|_{x=\frac{L}{3}} = -\frac{w_oL^4}{120EI} = \frac{w_oL^4}{120EI} \quad \textbf{Ans.}$$

***8–8.** Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope at C and displacement at B. EI is constant.



***Support Reactions and Elastic Curve*:** As shown on FBD(a).

***Moment Function*:** As shown on FBD(c) and (c).

***Slope and Elastic Curve*:**

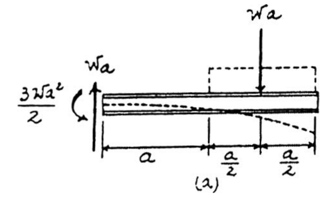
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \dfrac{3wa^2}{2}$,

$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \quad (1)$$

$$EIv_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \quad (2)$$

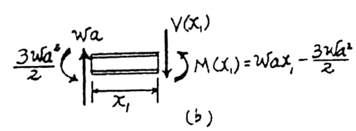


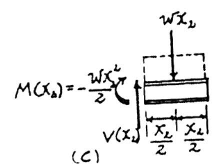
For $M(x_2) = -\dfrac{w}{2}x_2^2$,

$$EI\frac{d^2v_2}{dx_2^2} = -\frac{w}{2}x_2^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^3 + C_3 \quad (3)$$

$$EIv_2 = \frac{w}{24}x_2^4 + C_3x_2 + C_4 \quad (4)$$





***Boundary Conditions*:**

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$, From Eq. [1], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$ From Eq. [2], $C_2 = 0$

***Continuity Conditions*:**

At $x_1 = a$ and $x_2 = a$, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$ From Eqs. [1] and [3],

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = -\left(-\frac{wa^3}{6} + C_3\right) \qquad C_3 = \frac{7wa^3}{6}$$

At $x_1 = a$ and $x_2 = a$, $v_1 = v_2$. From Eqs. [2] and [4],

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = -\frac{wa^4}{24} + \frac{5wa^4}{6} + C_4 \qquad C_4 = -\frac{41wa^4}{8}$$

***The Slope*:** Substituting into Eq. [1],

$$\frac{dv_1}{dx_1} = \frac{wax_1}{2EI}(x_1 - 3a)$$

$$\theta_C = \frac{dv_2}{dx_2}\bigg|_{x_1 = a} = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

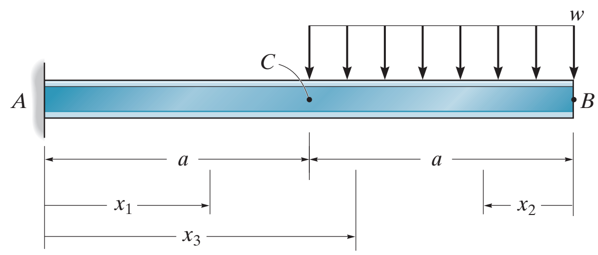
***The Elastic Curve*:** Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively

$$v_1 = \frac{wax_1}{12EI}(2x_1^2 - 9ax_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{w}{24EI}(-x_2^4 + 28a^3x_2 - 41a^4) \qquad \textbf{Ans.}$$

$$v_B = v_2\bigg|_{x_2 = 0} = -\frac{41wa^4}{24EI} \qquad \textbf{Ans.}$$

8–9. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope at B and deflection at C. EI is constant.



***Support Reactions and Elastic Curve*:** As shown on FBD(a).
***Moment Function*:** As shown on FBD(b) and (c).
***Slope and Elastic Curve*:**

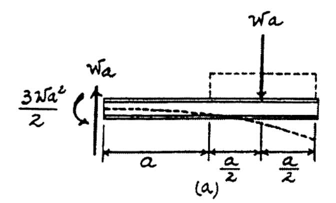
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \frac{3wa^2}{2}$,

$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EIv_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

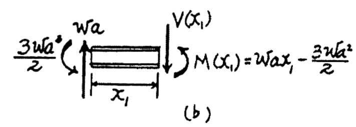


For $M(x_3) = 2wax_3 - \frac{w}{2}x_2^3 - 2wa^2$,

$$EI\frac{d^2v_3}{dx_3^2} = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$$

$$EI\frac{dv_3}{dx_3} = wax_3^2 - \frac{w}{6}x_3^3 - 2wa^2x_3 + C_3 \qquad (3)$$

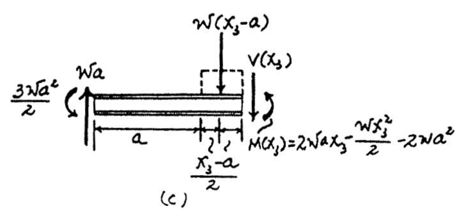
$$EIv_3 = \frac{wa}{3}x_3^3 - \frac{w}{24}x_3^4 - wa^2x_3^2 + C_3x_3 + C_4 \qquad (4)$$



***Boundary Conditions*:**

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$, From Eq. [1], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$, From Eq. [2], $C_2 = 0$



***Continuity Conditions*:**

At $x_1 = a$ and $x_3 = a$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$ From Eqs. [1] and [3],

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = wa^3 - \frac{wa^3}{6} - 2wa^3 + C_3 \qquad C_3 = \frac{wa^3}{6}$$

At $x_1 = a$ and $x_3 = a$, $v_1 = v_3$, From Eqs.[2] and [4],

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = \frac{wa^4}{3} - \frac{wa^4}{24} - wa^4 + \frac{wa^4}{6} + C_4 \qquad C_4 = -\frac{wa^4}{24}$$

***The Slope*:** Substituting the value of C_3 into Eq. [3],

$$\frac{dv_3}{dx_3} = \frac{w}{2EI}(6ax_3^2 - x_3^3 - 12a^2x_3 + a^3)$$

$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3 = 2a} = -\frac{7wa^3}{6EI} \qquad \textbf{Ans.}$$

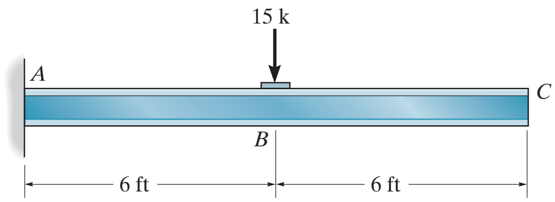
***The Elastic Curve*:** Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively,

$$v_1 = \frac{wax_1}{12EI}(2x_1^2 - 9ax_1) \qquad \textbf{Ans.}$$

$$v_C = \left.v_1\right|_{x_1 = a} = -\frac{7wa^4}{12EI} \qquad \textbf{Ans.}$$

$$v_3 = \frac{w}{24EI}(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4) \qquad \textbf{Ans.}$$

8–10. Determine the slope at B and the maximum displacement of the beam. Use the moment-area theorems. Take $E = 29(10^3)$ ksi, $I = 500$ in^4.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_B = |\theta_{B/A}| = \frac{1}{2}\left(\frac{90\text{ k}\cdot\text{ft}}{EI}\right)(6\text{ ft})$$

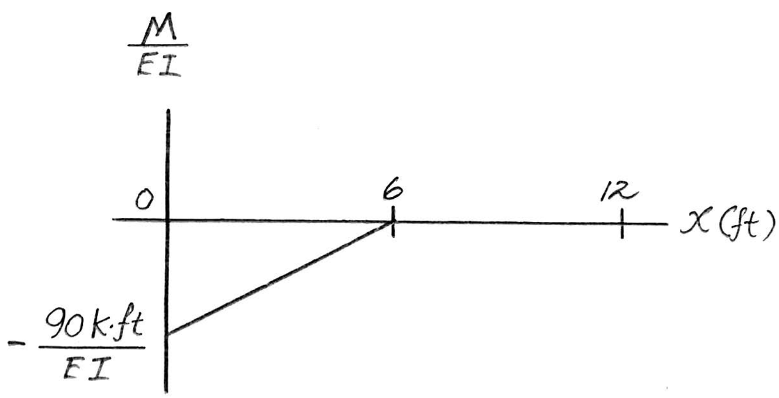
$$= \frac{270\text{ k}\cdot\text{ft}^2}{EI} = \frac{270\,(144)\text{ k}\cdot\text{in}^2}{\left[29(10^3)\frac{\text{k}}{\text{in}^2}\right](500\text{ in}^4)} = 0.00268\text{ rad}$$ **Ans.**

$$\Delta_{max} = \Delta_C = |t_{B/A}| = \left[\frac{1}{2}\left(\frac{90\text{ k}\cdot\text{ft}}{EI}\right)(6\text{ ft})\right]\left[6\text{ ft} + \frac{2}{3}(6\text{ ft})\right]$$

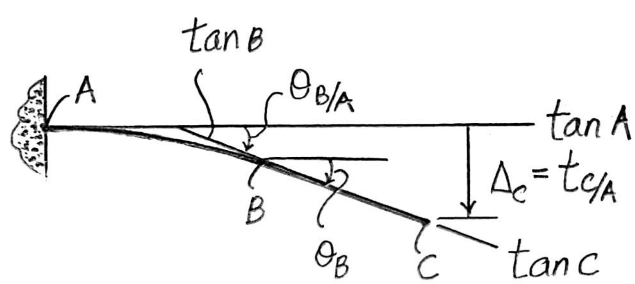
$$= \frac{2700\text{ k}\cdot\text{ft}^3}{EI}$$

$$= \frac{2700\,(1728)\text{ k}\cdot\text{in}^3}{\left[29(10^3)\frac{\text{k}}{\text{in}^2}\right](500\text{ in}^4)}$$

$$= 0.322\text{ in}\downarrow$$ **Ans.**

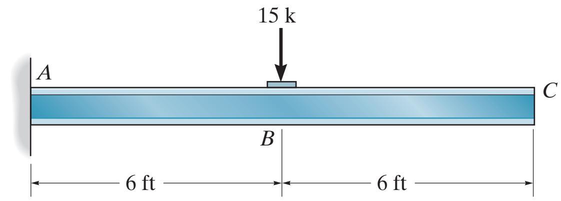


(a)



(b)

8–11. Solve Prob. 8–10 using the conjugate-beam method.

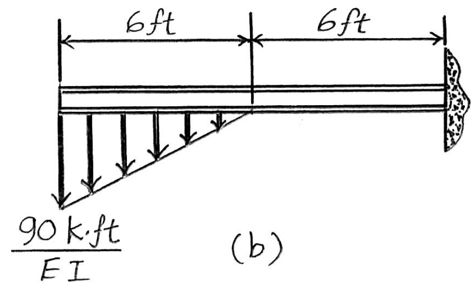


The real beam and conjugate beam are shown in Fig. *b* and *c*, respectively. Referring to Fig. *c*,

$$+\uparrow \sum F_y = 0; \qquad -V'_B - \frac{1}{2}\left(\frac{90\text{ k}\cdot\text{ft}}{EI}\right)(6\text{ ft}) = 0$$

$$\theta_B = V'_B = -\frac{270\text{ k}\cdot\text{ft}^2}{EI}$$

$$= \frac{270\,(12^2)\text{ k}\cdot\text{in}^2}{\left[29(10^3)\dfrac{\text{k}}{\text{in}^2}\right](500\text{ in}^4)} = 0.00268\text{ rad} \qquad \textbf{Ans.}$$

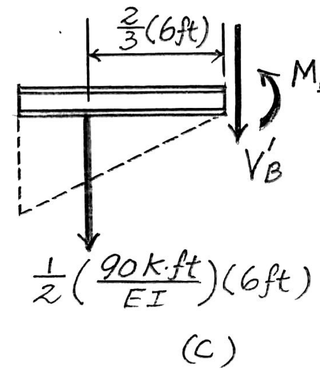


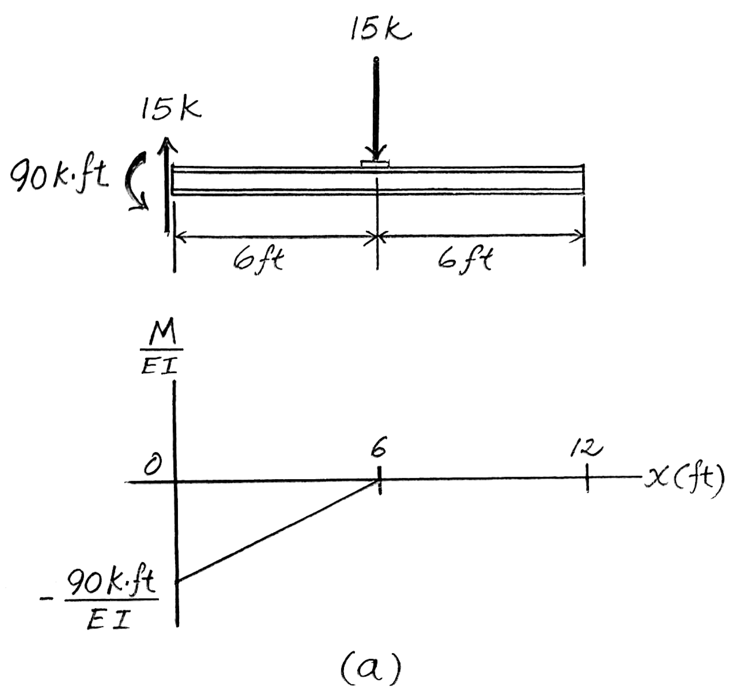
Referring to Fig. *d*,

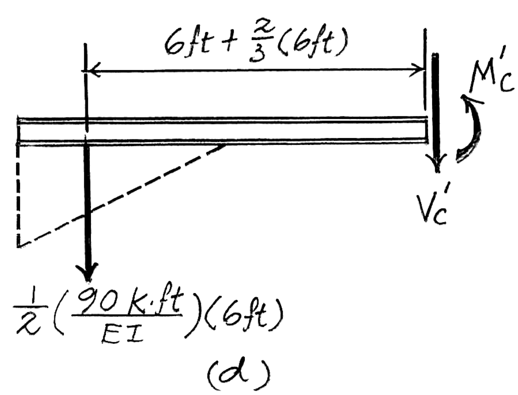
$$\circlearrowleft + \sum M_C = 0; \qquad M'_C + \left[\frac{1}{2}\left(\frac{90\text{ k}\cdot\text{ft}}{EI}\right)(6\text{ ft})\right]\left[6\text{ ft} + \frac{2}{3}(6\text{ ft})\right] = 0$$

$$\Delta_{\max} = \Delta_C = M'_C = -\frac{2700\text{ k}\cdot\text{ft}^3}{EI}$$

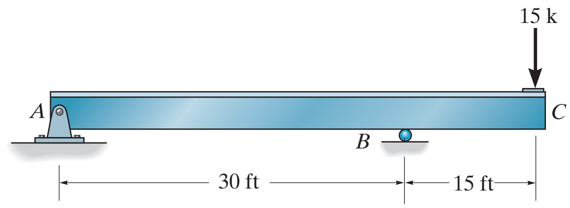
$$= \frac{2700\,(12^3)\text{ k}\cdot\text{in}^3}{\left[29(10^3)\dfrac{\text{k}}{\text{in}^2}\right](500\text{ in}^4)} = 0.322\text{ in} \downarrow \qquad \textbf{Ans.}$$







***8–12.** Determine the slope and displacement at *C*. *EI* is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

$$\theta_{C/A} = \frac{1}{2}\left(-\frac{225\ \text{k}\cdot\text{ft}}{EI}\right)(45\ \text{ft}) = -\frac{5062.5\ \text{k}\cdot\text{ft}^2}{EI} = \frac{5062.5\ \text{k}\cdot\text{ft}^2}{EI}$$

$$|t_{B/A}| = \left[\frac{1}{2}\left(\frac{225\ \text{k}\cdot\text{ft}}{EI}\right)(30\ \text{ft})\right]\left[\frac{1}{3}(30\ \text{ft})\right] = \frac{33750\ \text{k}\cdot\text{ft}^3}{EI}$$

$$|t_{C/A}| = \left[\frac{1}{2}\left(\frac{225\ \text{k}\cdot\text{ft}}{EI}\right)(30\ \text{ft})\right]\left[15\ \text{ft} + \frac{1}{3}(30\ \text{ft})\right] + \left[\frac{1}{2}\left(\frac{225\ \text{k}\cdot\text{ft}}{EI}\right)(15\ \text{ft})\right]\left[\frac{2}{3}(15\ \text{ft})\right]$$

$$= \frac{101250\ \text{k}\cdot\text{ft}^3}{EI}$$

Then,

$$\Delta' = \frac{45}{30}(t_{B/A}) = \frac{45}{30}\left(\frac{33750\ \text{k}\cdot\text{ft}^3}{EI}\right) = \frac{50625\ \text{k}\cdot\text{ft}^3}{EI}$$

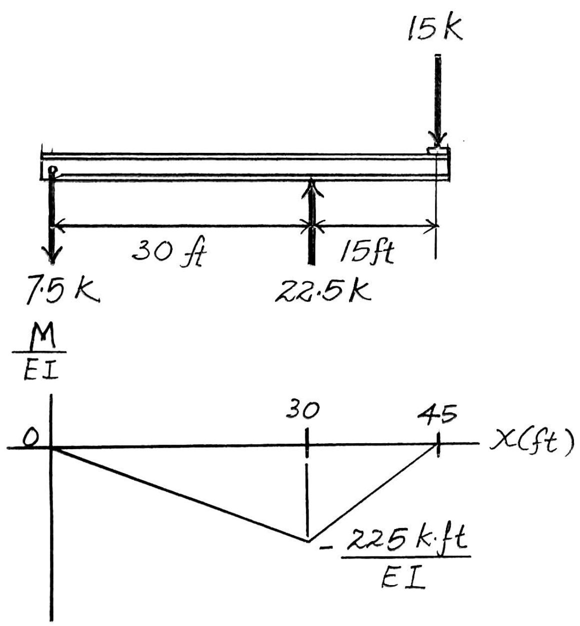
$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{33750\ \text{k}\cdot\text{ft}^3/EI}{30\ \text{ft}} = \frac{1125\ \text{k}\cdot\text{ft}^2}{EI}$$

$$+\circlearrowleft\ \theta_C = \theta_A + \theta_{C/A}$$

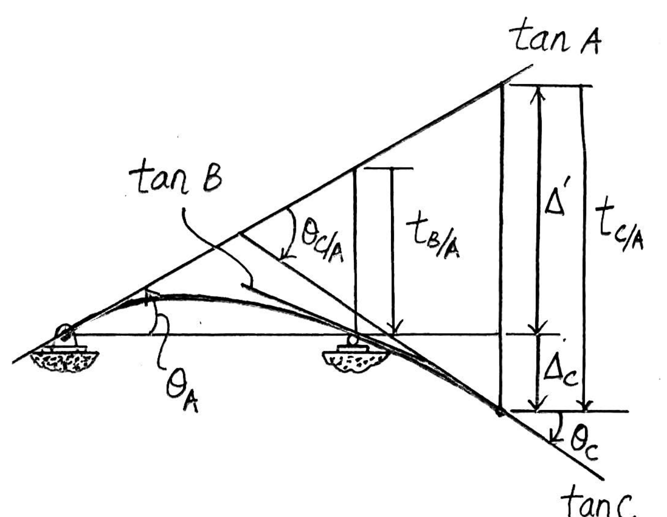
$$\theta_C = \frac{-1125\ \text{k}\cdot\text{ft}^2}{EI} + \frac{5062.5\ \text{k}\cdot\text{ft}^2}{EI} = \frac{3937.5\ \text{k}\cdot\text{ft}^2}{EI}$$ **Ans.**

$$\Delta_C = \left|t_{C/A}\right| - \Delta' = \frac{101250\ \text{k}\cdot\text{ft}^3}{EI} - \frac{50625\ \text{k}\cdot\text{ft}^3}{EI}$$

$$= \frac{50625\ \text{k}\cdot\text{ft}^3}{EI}\ \downarrow$$ **Ans.**

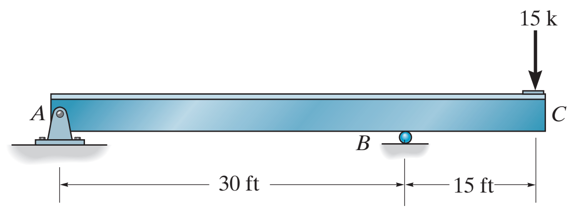


(a)



(b)

8–13. Solve Prob. 8–12 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\circlearrowleft + \sum M_A = 0; \quad B_y'(30\text{ ft}) - \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(30\text{ ft})\right](20\text{ ft})$$

$$B_y' = \frac{2250\text{ k}\cdot\text{ft}^2}{EI}$$

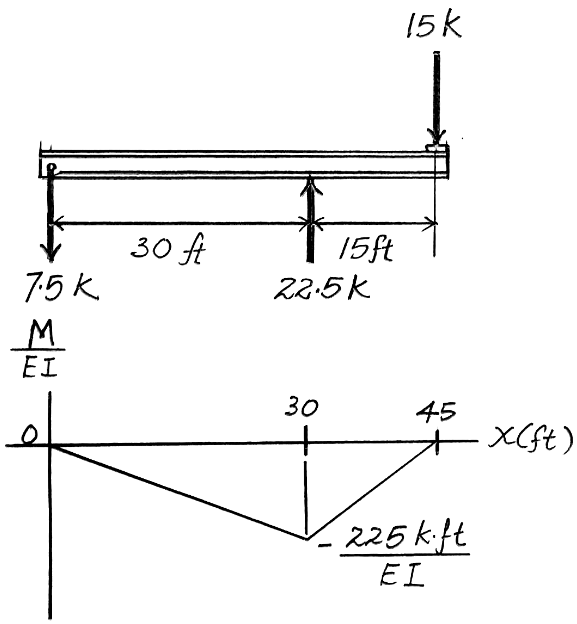
Referring to Fig. *d*,

$$+\uparrow \sum F_y = 0; \quad -V_C' - \frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(15\text{ ft}) - \frac{2250\text{ k}\cdot\text{ft}}{EI}$$

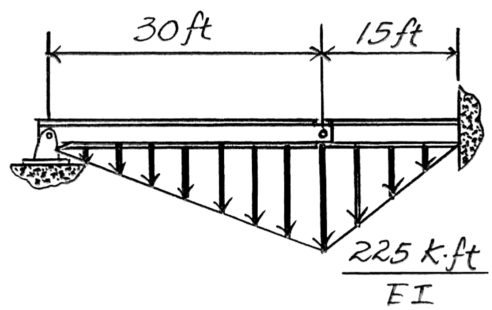
$$\theta_C = V_C' = -\frac{3937.5\text{ k}\cdot\text{ft}^2}{EI} = \frac{3937.5\text{ k}\cdot\text{ft}^2}{EI} \quad \text{Ans.}$$

$$\circlearrowleft + \sum M_C = 0; \quad M_C' + \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(15\text{ ft})\right](10\text{ ft}) + \left(\frac{2250\text{ k}\cdot\text{ft}^2}{EI}\right)(15\text{ ft})$$

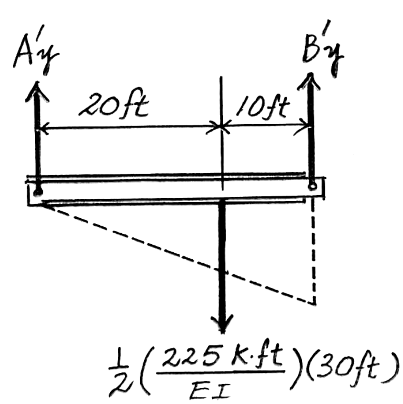
$$\Delta_C = M_C' = \frac{50625\text{ k}\cdot\text{ft}^3}{EI} = \frac{50625\text{ k}\cdot\text{ft}^3}{EI} \downarrow \quad \text{Ans.}$$



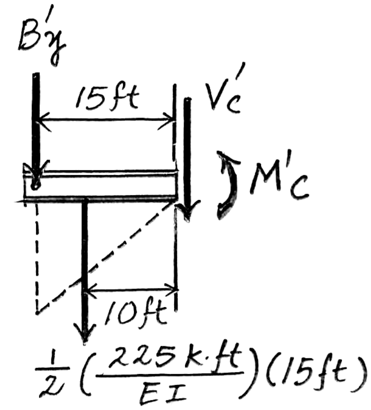
(a)



(b)

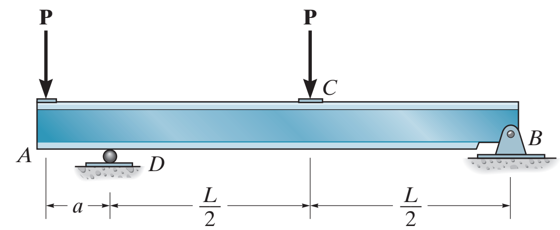


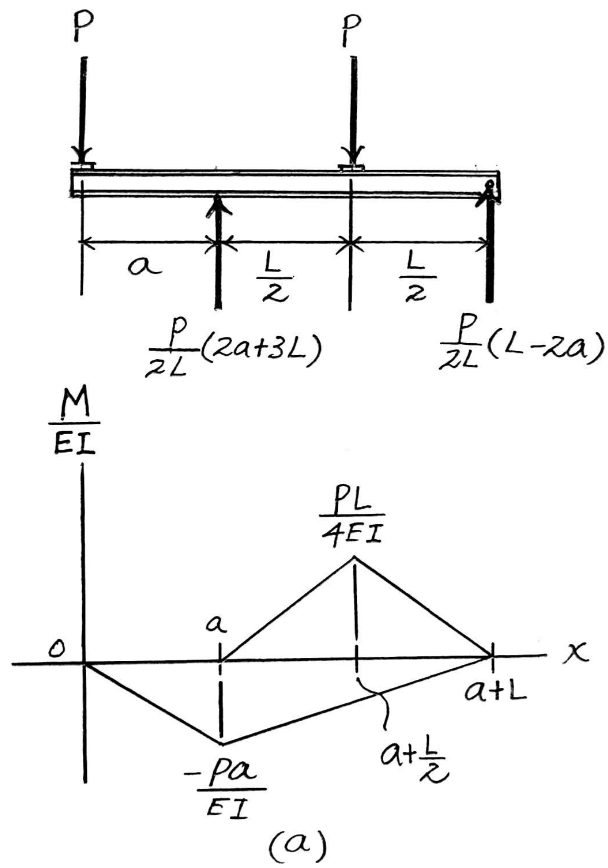
(c)



(d)

8–14. Determine the value of a so that the slope at A is equal to zero. EI is constant. Use the moment-area theorems.





Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{A/B} = \frac{1}{2}\left(\frac{PL}{4EI}\right)(L) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a + L)$$

$$= \frac{PL^2}{8EI} - \frac{Pa^2}{2EI} - \frac{PaL}{2EI}$$

$$t_{D/B} = \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{L}{2}\right) + \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\right]\left(\frac{L}{3}\right)$$

$$= \frac{PL^3}{16EI} - \frac{PaL^2}{6EI}$$

Then

$$\theta_B = \frac{t_{D/B}}{L} = \frac{PL^2}{16EI} - \frac{PaL}{6EI}$$

Here, it is required that

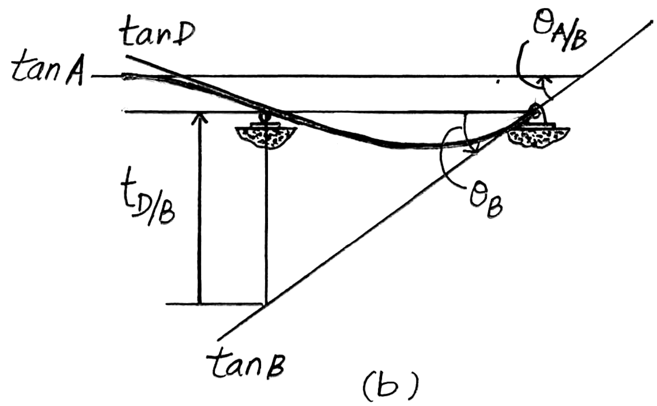
$$\theta_B = \theta_{A/B}$$

$$\frac{PL^2}{16EI} - \frac{PaL}{6EI} = \frac{PL^2}{8EI} - \frac{Pa^2}{2EI} - \frac{PaL}{2EI}$$

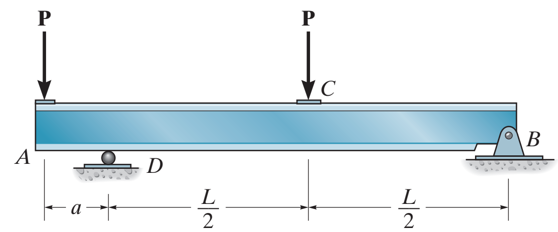
$$24a^2 + 16La - 3L^2 = 0$$

Choose the position root,

$$a = 0.153\,L$$ **Ans.**



8–15. Solve Prob. 8–14 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *d*,

$$\circlearrowleft + \sum M_B = 0; \quad D_y'(L) + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(L)\right]\left(\frac{2}{3}L\right) - \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{1}{2}\right) = 0$$

$$D_y' = \frac{PL^2}{16EI} - \frac{PaL}{3EI}$$

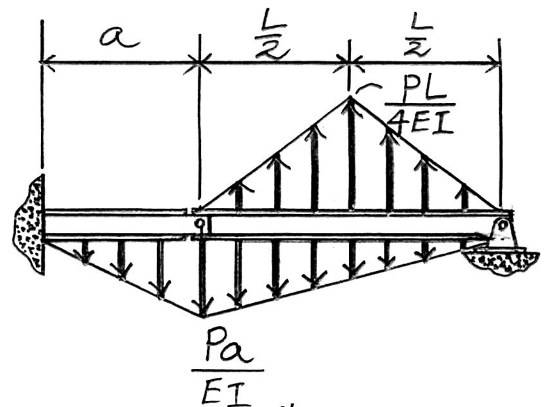
It is required that $V_A' = \theta_A = 0$, Referring to Fig. *c*,

$$+\uparrow \sum F_y = 0; \quad \frac{PL^2}{16EI} - \frac{PaL}{3EI} - \frac{Pa^2}{2EI} = 0$$

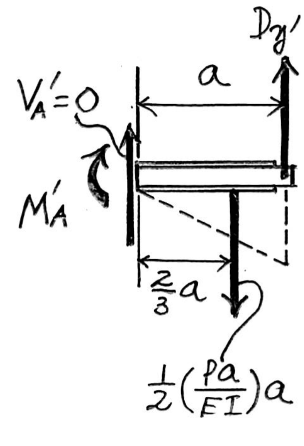
$$24a^2 + 16La - 3L^2 = 0$$

Choose the position root,

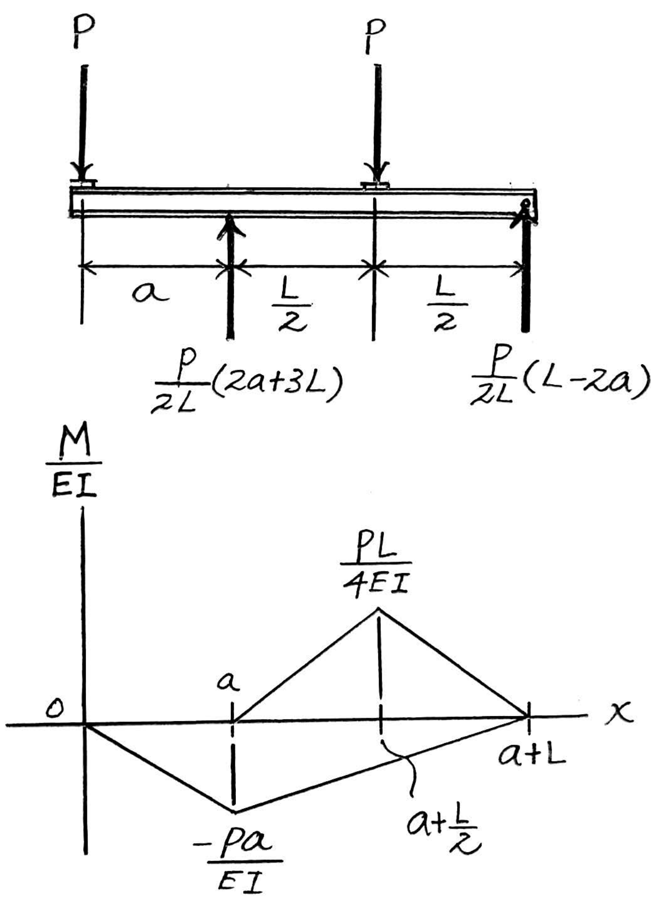
$$a = 0.153\ \text{L}$$ **Ans.**



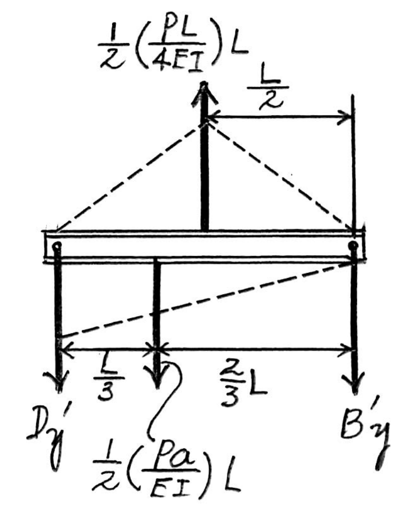
(b)



(c)

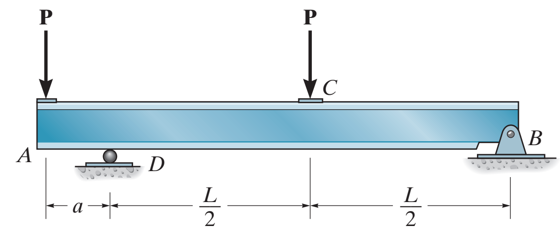


(a)



(d)

***8–16.** Determine the value of a so that the displacement at C is equal to zero. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 2 gives

$$t_{D/B} = \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{L}{2}\right) + \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\right]\left(\frac{L}{3}\right)$$

$$= \frac{PL^3}{16EI} - \frac{PaL^2}{6EI}$$

$$T_{C/B} = \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right] + \left[\frac{1}{2}\left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right]$$

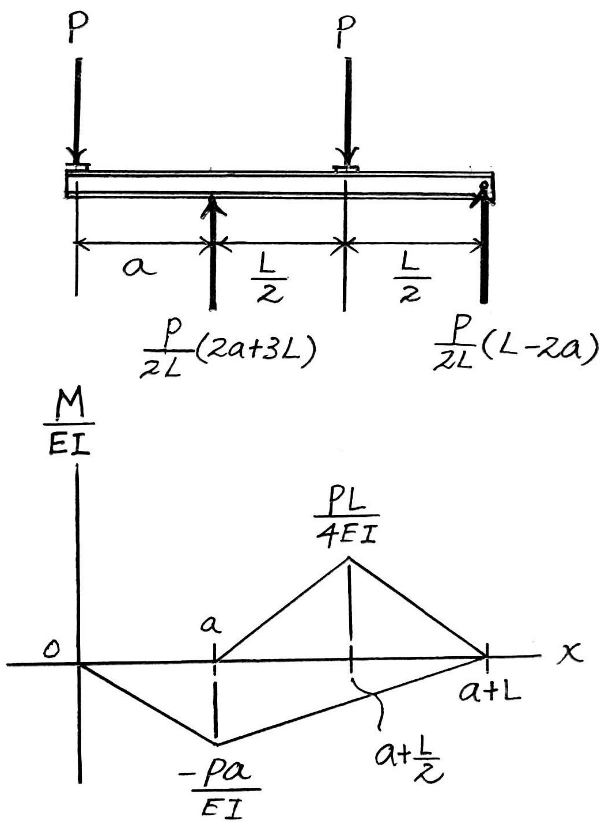
$$= \frac{PL^3}{96EI} - \frac{PaL^2}{48EI}$$

It is required that

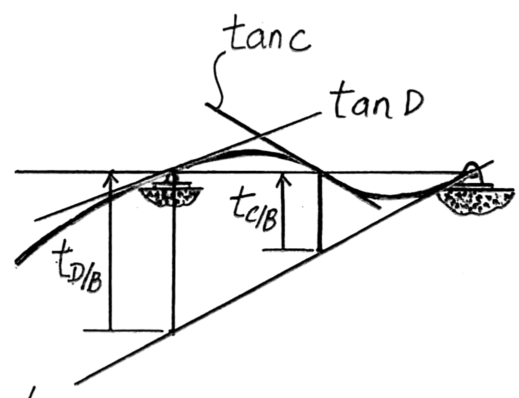
$$t_{C/B} = \frac{1}{2}t_{D/B}$$

$$\frac{PL^3}{96EI} - \frac{PaL^2}{48EI} = \frac{1}{2}\left[\frac{PL^3}{16EI} - \frac{PaL^2}{6EI}\right]$$

$$a = \frac{L}{3}$$ **Ans.**

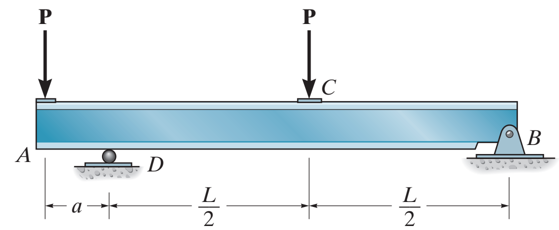


(a)



(b)

8–17. Solve Prob. 8–16 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\circlearrowleft + \sum M_D = 0; \qquad \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{L}{2}\right) - \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(L)\right]\left(\frac{L}{3}\right) - B'_y(L) = 0$$

$$-B'_y = \frac{PL^2}{16EI} - \frac{PaL}{6EI}$$

Here, it is required that $M'_C = \Delta_C = 0$. Referring to Fig. *d*,

$$\circlearrowleft + \sum M_C = 0; \qquad \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right]$$

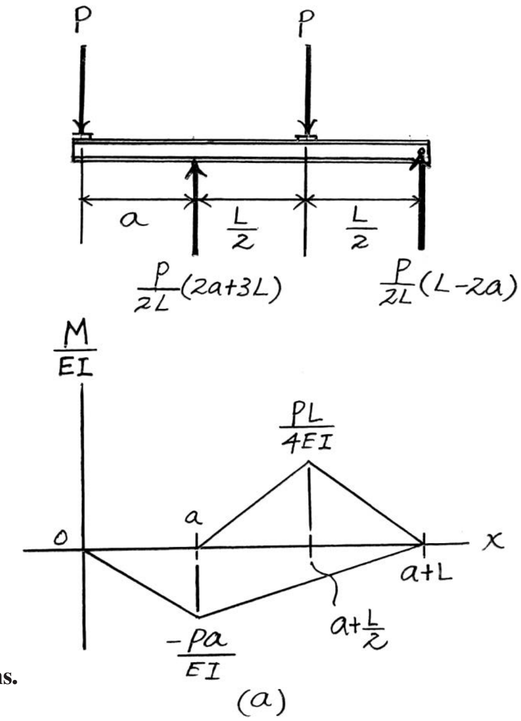
$$-\left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right]$$

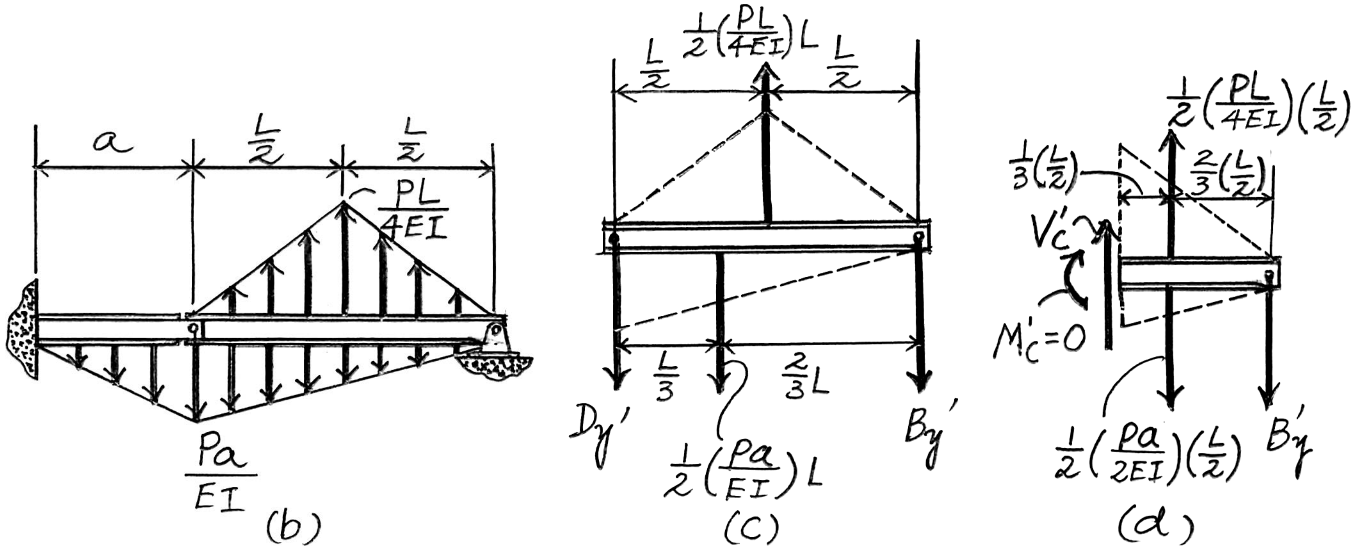
$$-\left[\frac{PL^2}{16EI} - \frac{PaL}{6EI}\right]\left(\frac{L}{2}\right) = 0$$

$$\frac{PL^3}{96EI} - \frac{PaL^2}{48EI} - \frac{PL^3}{32EI} + \frac{PaL^2}{12EI} = 0$$

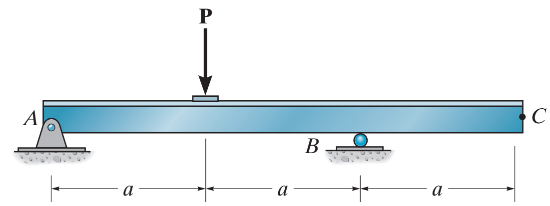
$$\frac{L}{96} - \frac{a}{48} - \frac{L}{32} + \frac{a}{12} = 0$$

$$a = \frac{L}{3}$$ **Ans.**





8–18. Determine the slope and the displacement at *C*. *EI* is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

$$t_{B/D} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{2}{3}a\right) = \frac{Pa^3}{6EI}$$

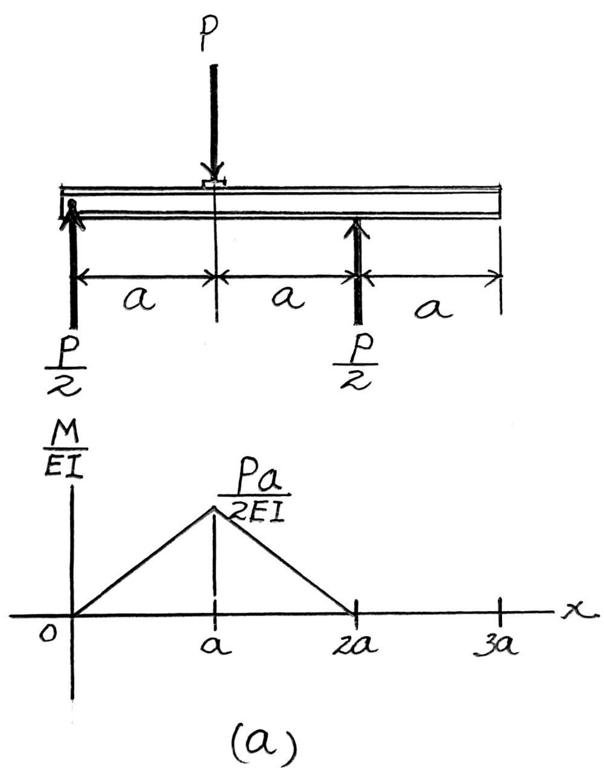
$$t_{C/D} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) = \frac{5Pa^3}{12EI}$$

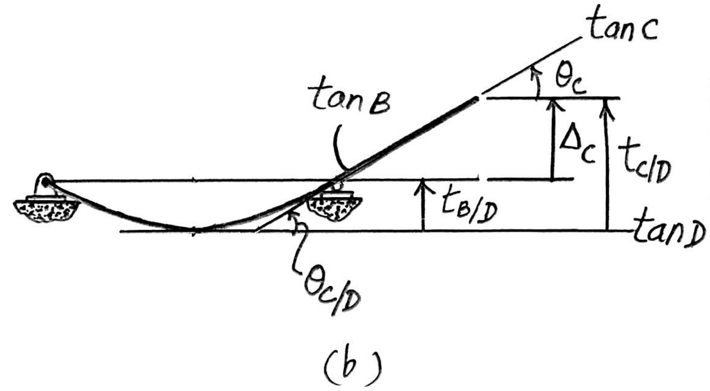
$$\theta_{C/D} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$

Then,

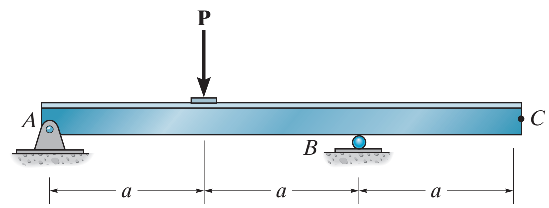
$$\theta_C = \theta_{C/D} = \frac{Pa^2}{4EI} \quad \measuredangle \qquad \textbf{Ans.}$$

$$\Delta_C = t_{C/D} - t_{B/D} = \frac{5Pa^3}{12EI} - \frac{Pa^3}{6EI} = \frac{Pa^3}{4EI} \quad \uparrow \qquad \textbf{Ans.}$$





8–19. Solve Prob. 8–18 using the conjugate-beam method.



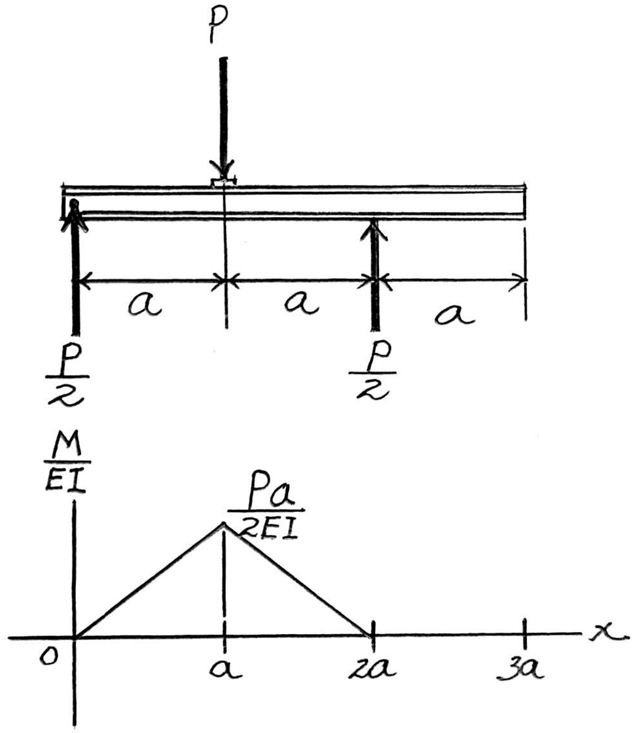
The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$\circlearrowleft + \sum M_A = 0; \qquad \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](a) - B'_y(2a) = 0 \quad B'_y = \frac{Pa^2}{4EI}$

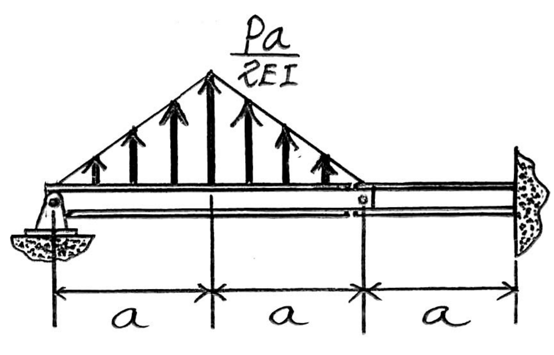
Referring to Fig. *d*

$+\uparrow \sum F_y = 0; \qquad \frac{Pa^2}{4EI} - V'_C = 0 \quad \theta_C = V'_C = \frac{Pa^2}{4EI} \quad \measuredangle$ **Ans.**

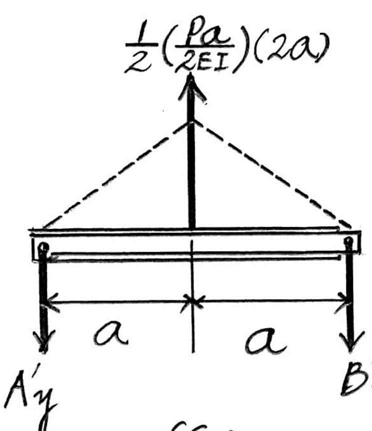
$\circlearrowleft + \sum M_C = 0; \quad M'_C - \frac{Pa^2}{4EI}(a) = 0 \quad \Delta_C = M'_C = \frac{Pa^3}{4EI} \quad \uparrow$ **Ans.**



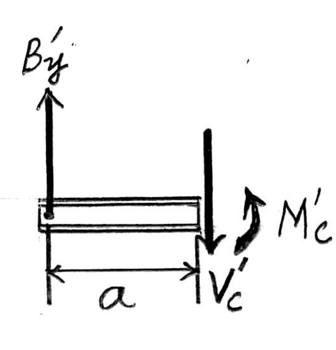
(a)



(b)

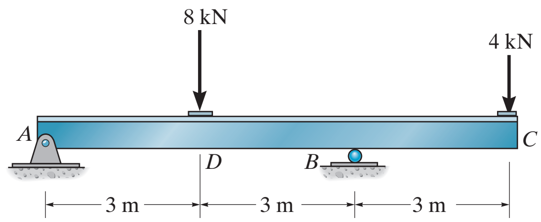


(c)



(d)

***8–20.** Determine the slope and the displacement at the end C of the beam. $E = 200$ GPa, $I = 70(10^6)$ mm^4. Use the moment-area theorems.



Using the $\dfrac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{C/A} = \frac{1}{2}\left(\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(6 \text{ m}) + \frac{1}{2}\left(-\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(9 \text{ m})$$

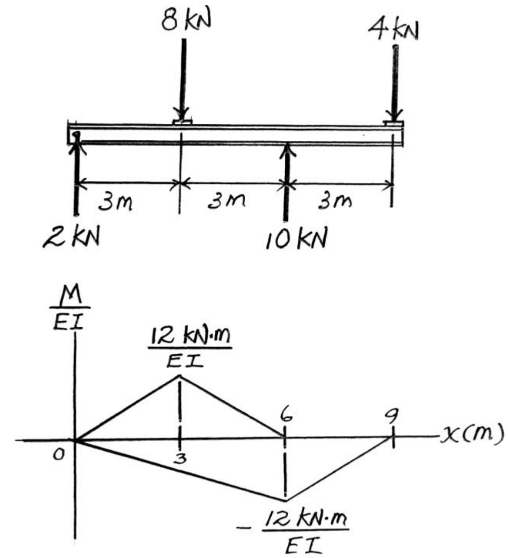
$$= -\frac{18 \text{ kN}\cdot\text{m}}{EI} = \frac{18 \text{ kN}\cdot\text{m}}{EI}$$

$$t_{B/A} = \left[\frac{1}{2}\left(\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(6 \text{ m})\right](3 \text{ m}) + \left[\frac{1}{2}\left(-\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(6 \text{ m})\right]\left[\frac{1}{3}(6 \text{ m})\right]$$

$$= \frac{36 \text{ kN}\cdot\text{m}^3}{EI}$$

$$t_{C/A} = \left[\frac{1}{2}\left(\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(6 \text{ m})\right](6 \text{ m}) + \left[\frac{1}{2}\left(-\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(6 \text{ m})\right]\left[3 \text{ m} + \frac{1}{3}(6 \text{ m})\right]$$
$$+ \left[\frac{1}{2}\left(-\frac{12 \text{ kN}\cdot\text{m}}{EI}\right)(3 \text{ m})\right]\left[\frac{2}{3}(3 \text{ m})\right]$$

$$= 0$$



(a)

Then

$$\theta_A = \frac{t_{B/A}}{L_{AB}} = \frac{36 \text{ kN}\cdot\text{m}^3/EI}{6 \text{ m}} = \frac{6 \text{ kN}\cdot\text{m}^2}{EI}$$

$$\Delta' = \frac{9}{6}t_{B/A} = \frac{9}{6}\left(\frac{36 \text{ kN}\cdot\text{m}^3}{EI}\right) = \frac{54 \text{ kN}\cdot\text{m}^3}{EI}$$

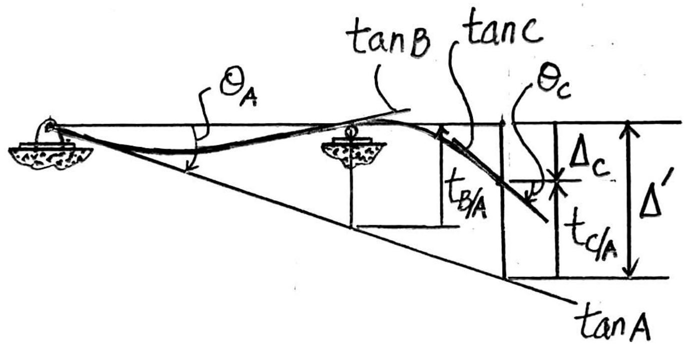
$$+\circlearrowleft\ \theta_C = \theta_A + \theta_{C/A} = \frac{6 \text{ kN}\cdot\text{m}^2}{EI} + \frac{18 \text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{24 \text{ kN}\cdot\text{m}^2}{EI} = \frac{24(10^3) \text{ N}\cdot\text{m}^2}{[200(10^9) \text{ N/m}^2][70(10^{-6}) \text{ m}^4]} = 0.00171 \text{ rad}$$ **Ans.**

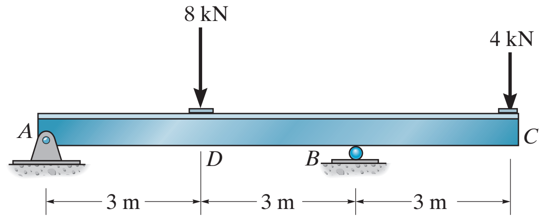
$$\Delta_C = \Delta' - t_{C/A} = \frac{54 \text{ kN}\cdot\text{m}^3}{EI} - 0$$

$$= \frac{54 \text{ kN}\cdot\text{m}^3}{EI} = \frac{54(10^3)\text{N}\cdot\text{m}^3}{[200(10^9) \text{ N/m}^2][70(10^{-6}) \text{ m}^4]} = 0.00386 \text{ m}$$

$$= 3.86 \text{ mm} \downarrow$$ **Ans.**



8–21. Solve Prob. 8–20 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*

$$\zeta+ \sum M_A = 0; \quad B'_y(6\text{ m}) + \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) - \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right]\left[\frac{2}{3}(6\text{ m})\right] = 0$$

$$B'_y = \frac{6\text{ kN}\cdot\text{m}^2}{EI}$$

Referring to Fig. *d*,

$$\zeta+ \sum F_y = 0; \quad -V'_C - \frac{6\text{ kN}\cdot\text{m}^2}{EI} - \frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m}) = 0$$

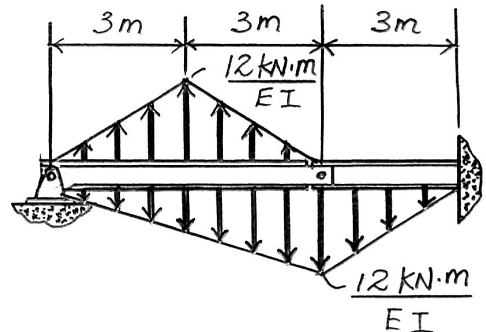
$$\theta_C = V'_C = -\frac{24\text{ kN}\cdot\text{m}^2}{EI} = \frac{24(10^3)\text{ N}\cdot\text{m}^2}{[(200(10^9)\text{ N/m}^2)][(70(10^{-6})\text{ m}^4]}$$

$$= 0.00171\text{ rad} \quad \textbf{Ans.}$$

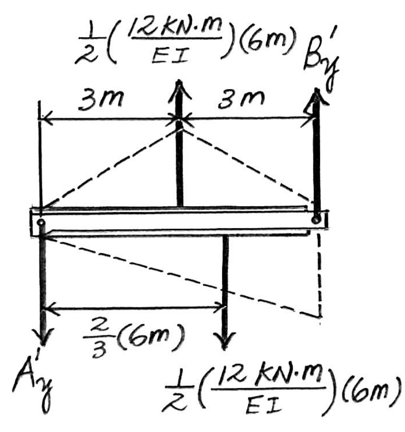
$$\zeta+ \sum M_C = 0; \quad M'_C + \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right] + \left(\frac{6\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m}) = 0$$

$$\Delta_C = M'_C = -\frac{54\text{ kN}\cdot\text{m}^3}{EI} = \frac{54\,(10^3)\text{ N}\cdot\text{m}^3}{[200(10^9)\text{N/m}^2]\,[70(10^{-6})\text{m}^4]}$$

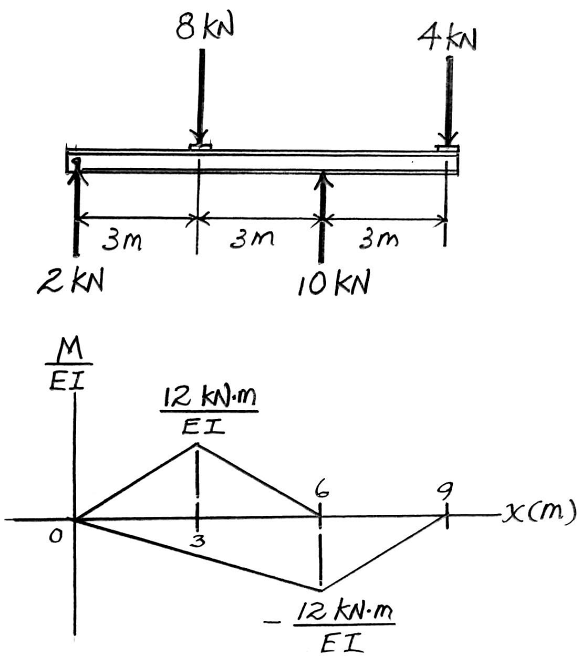
$$= 0.00386\text{ m} = 3.86\text{ mm} \downarrow \quad \textbf{Ans.}$$



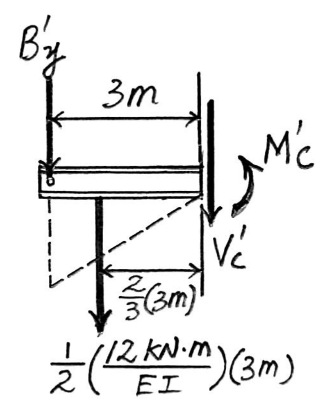
(b)



(c)

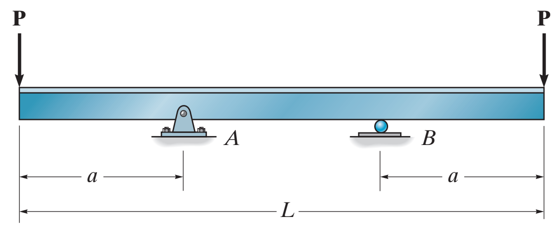


(a)



(d)

8–22. At what distance a should the bearing supports at A and B be placed so that the displacement at the center of the shaft is equal to the deflection at its ends? The bearings exert only vertical reactions on the shaft. EI is constant. Use the moment-area theorems.



Using the $\dfrac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively. Theorem 2 gives

$$t_{B/C} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(\frac{L-2a}{4}\right) = -\frac{Pa}{8EI}(L-2a)^2$$

$$t_{D/C} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a+\frac{L-2a}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

$$= -\left[\frac{Pa}{8EI}(L^2-4a^2) + \frac{Pa^3}{3EI}\right]$$

It is required that

$$t_{D/C} = 2\,t_{B/C}$$

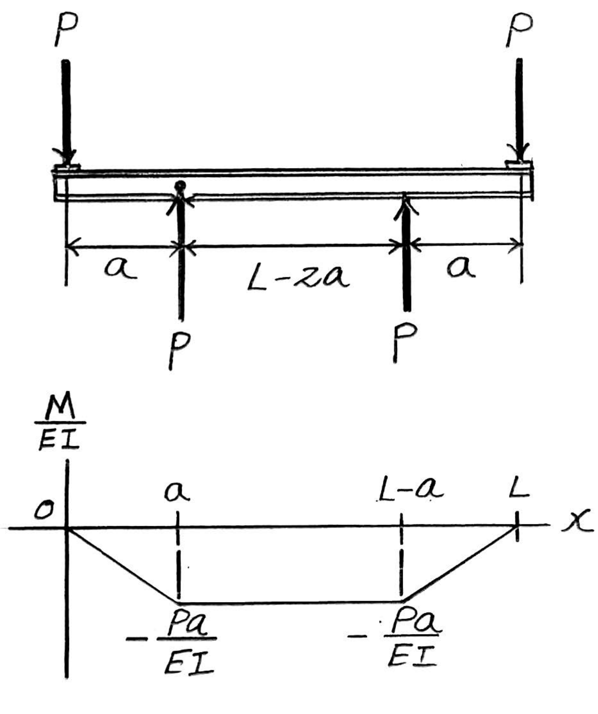
$$\frac{Pa}{8EI}(L^2-4a^2) + \frac{Pa^3}{3EI} = 2\left[\frac{Pa}{8EI} - (L-2a)^2\right]$$

$$\frac{7Pa^3}{6EI} - \frac{Pa^2L}{EI} + \frac{PaL^2}{8EI} = 0$$

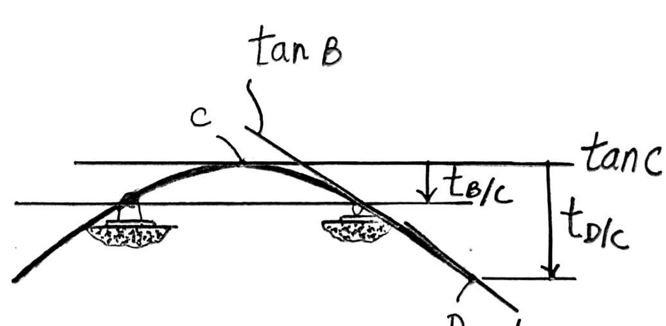
$$56a^2 - 48La + 6L^2 = 0$$

Choose

$$a = 0.152\ L$$ **Ans.**

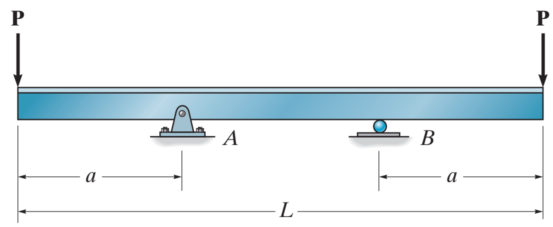


(a)



(b)

8–23. Solve Prob. 8–22 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\circlearrowleft + \sum M_A = 0; \quad B'_y(L - 2a) - \left[\frac{Pa}{EI}(L-2a)\right]\left(\frac{L-2a}{2}\right) = 0$$

$$B'_y = \frac{Pa}{2EI}(L-2a)$$

Referring to Fig. *d*,

$$M'_D + \frac{Pa}{2EI}(L - 2a)(a) + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) = 0$$

$$\Delta_D = M'_D = -\left[\frac{Pa^2}{2EI}(L - 2a) + \frac{Pa^3}{3EI}\right]$$

Referring to Fig. *e*,

$$\frac{Pa}{2EI}(L - 2a)\left(\frac{L - 2a}{2}\right) - \frac{Pa}{EI}\left(\frac{L - 2a}{2}\right)\left(\frac{L - 2a}{4}\right) - M'_C = 0$$

$$\Delta_C = M'_C = \frac{Pa}{8EI}(L-2a)^2$$

It is required that

$$|\Delta_D| = \Delta_C$$

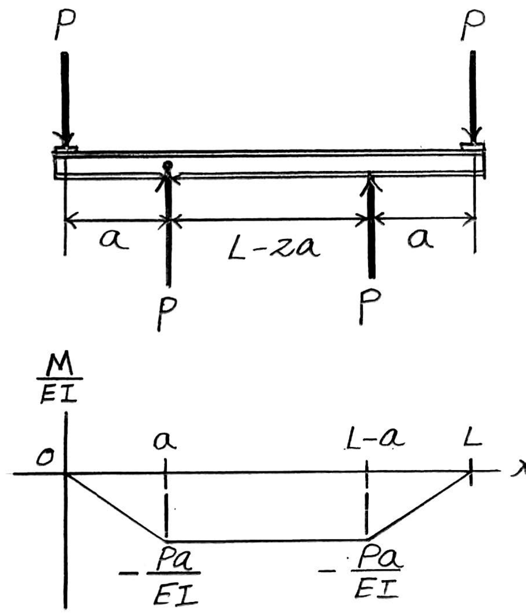
$$\frac{Pa^2}{2EI}(L-2a) + \frac{Pa^3}{3EI} = \frac{Pa}{8EI}(L-2a)^2$$

$$\frac{7Pa^3}{6EI} - \frac{Pa^2L}{EI} + \frac{PaL^2}{8EI} = 0$$

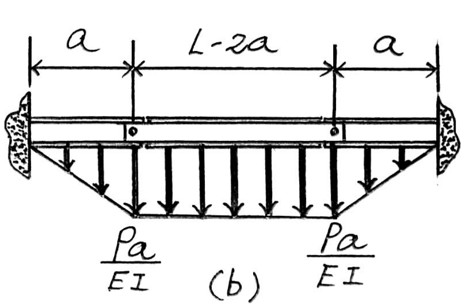
$$56a^2 - 48La + 6L^2 = 0$$

Choose

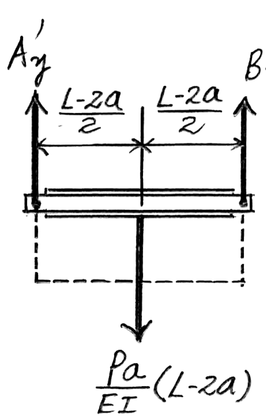
$a = 0.152\,L$ **Ans.**



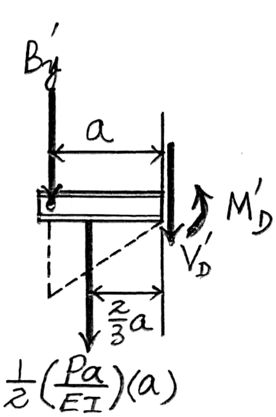
(a)



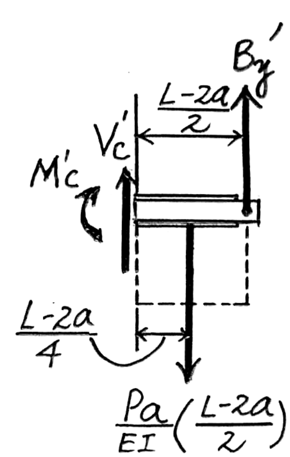
(b)



(c)

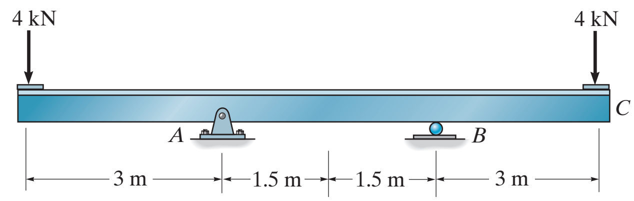


(d)



(e)

***8–24.** Determine the displacement at C and the slope at B. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

$$\theta_{B/D} = \left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(1.5\text{ m}) = -\frac{18\text{ kN}\cdot\text{m}^2}{EI}$$

$$t_{B/D} = \left[\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(1.5\text{ m})\right]\left[\frac{1}{2}(1.5\text{ m})\right] = \frac{13.5\text{ kN}\cdot\text{m}^3}{EI}$$

$$t_{C/D} = \left[\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(1.5\text{ m})\right]\left[\frac{1}{2}(1.5\text{ m}) + 3\text{ m}\right] + \left[\frac{1}{2}\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right]$$

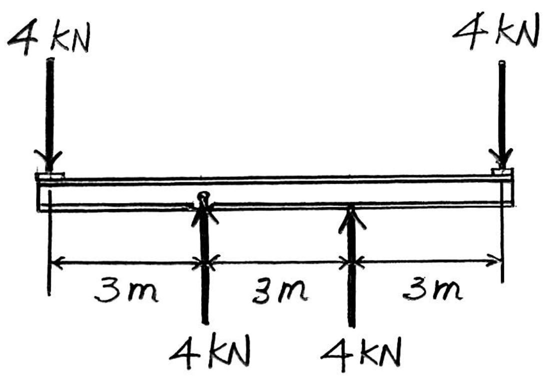
$$= \frac{103.5\text{ kN}\cdot\text{m}^3}{EI}$$

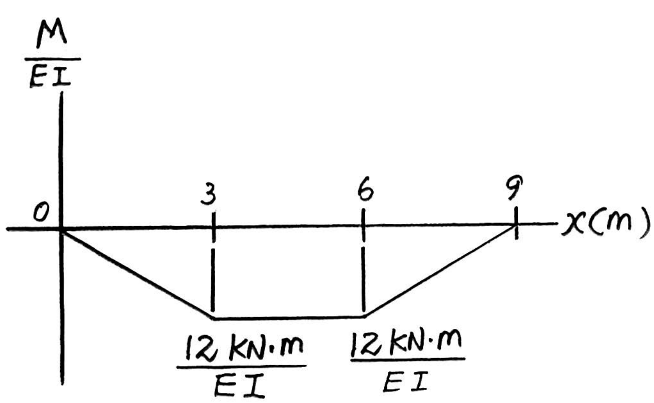
Then,

$$\theta_B = |\theta_{B/D}| = \frac{18\text{ kN}\cdot\text{m}^2}{EI}$$ **Ans.**

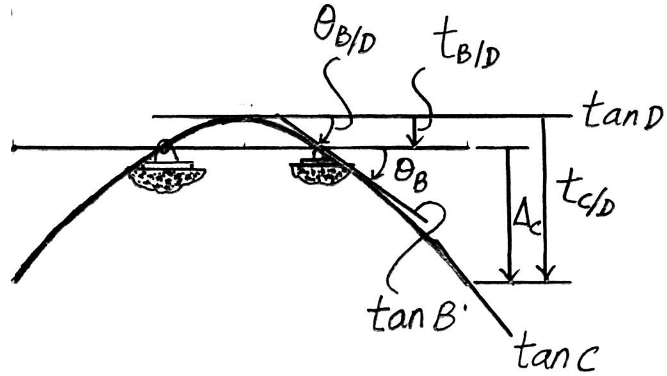
$$\Delta_C = |t_{C/D}| - |t_{B/D}| = \frac{103.5\text{ kN}\cdot\text{m}^3}{EI} - \frac{13.5\text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{90\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$ **Ans.**



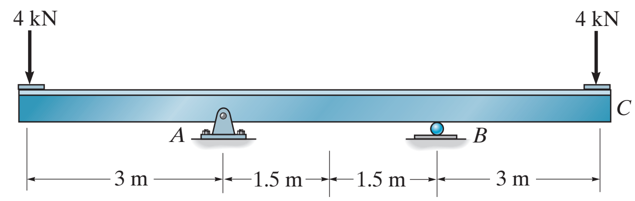


(a)



(b)

8–25. Solve Prob. 8–24 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

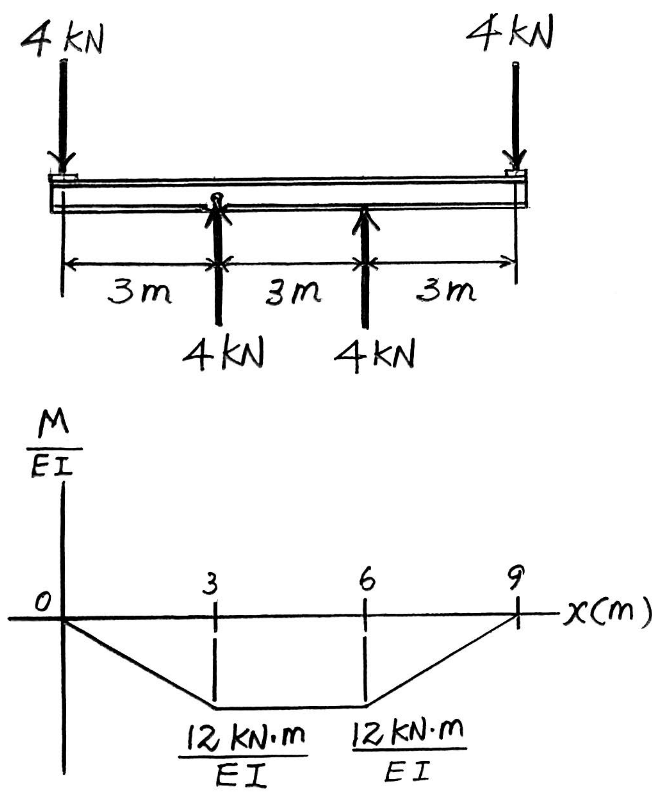
$$\circlearrowleft + \sum M_A = 0; \qquad B'_y\,(3\text{ m}) - \left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})(1.5\text{ m}) = 0$$

$$B'_y = \theta_B = \frac{18\text{ kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

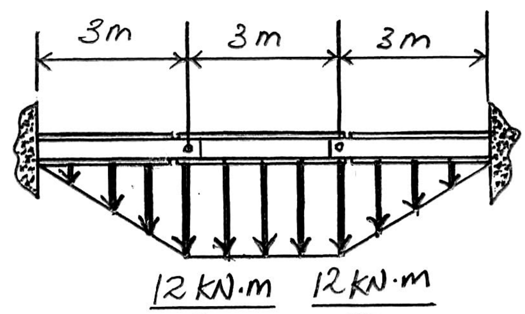
Referring to Fig. *d*,

$$\circlearrowleft + \sum M_C = 0; \quad M'_C + \left(\frac{18\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m}) + \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right] = 0$$

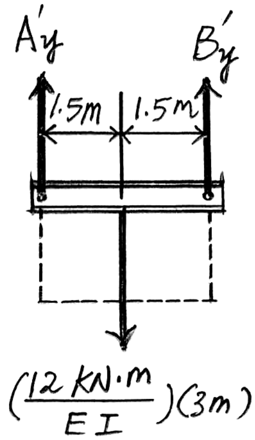
$$\Delta_C = M'_C = -\frac{90\text{ kN}\cdot\text{m}^3}{EI} = \frac{90\text{ kN}\cdot\text{m}^3}{EI} \downarrow \qquad \textbf{Ans.}$$



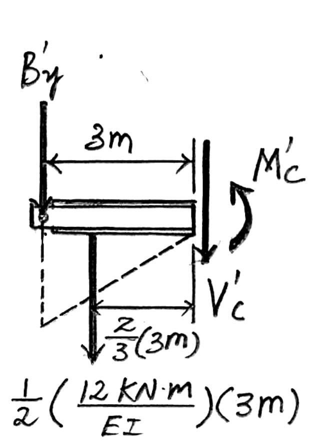
(a)



(b)

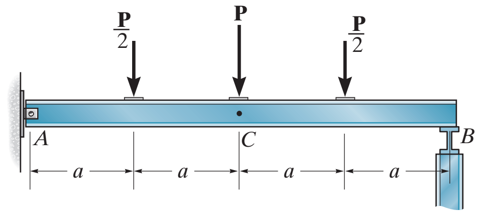


(c)



(d)

8–26. Determine the displacement at C and the slope at B. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{B/C} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{Pa}{EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{7Pa^2}{4EI}$$

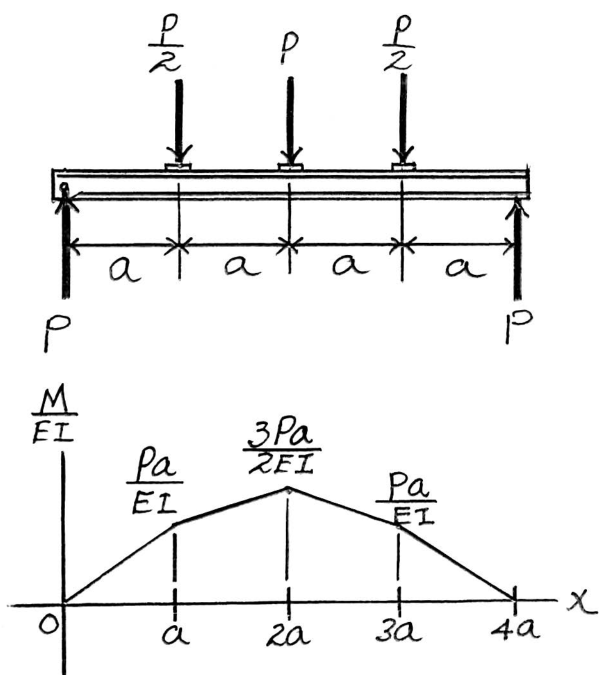
$$t_{B/C} = \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) + \left[\frac{Pa}{EI}(a)\right]\left(a + \frac{1}{2}a\right) + \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(a + \frac{2}{3}a\right)$$

$$= \frac{9Pa^3}{4EI}$$

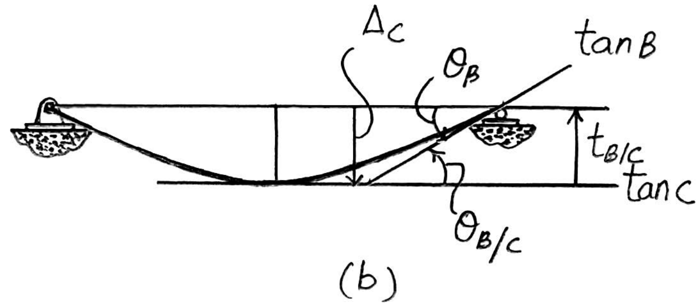
Then

$$\theta_B = \theta_{B/C} = \frac{7Pa^2}{4EI} \measuredangle$$ **Ans.**

$$\Delta_C = t_{B/C} = \frac{9Pa^3}{4EI} \downarrow$$ **Ans.**

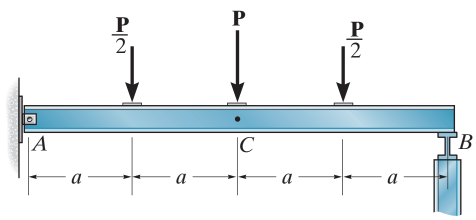


(a)



(b)

8–27. Determine the displacement at C and the slope at B. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. a and b, respectively. Referring to Fig. c,

$$\zeta+ \sum M_A = 0; \quad \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) + \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{10}{3}a\right)$$

$$+ \left[\left(\frac{Pa}{EI}\right)(2a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](2a)$$

$$-B'_y = (4a) = 0$$

$$\theta_B = B'_y = \frac{7Pa^2}{4EI} \quad \triangledown \qquad \textbf{Ans.}$$

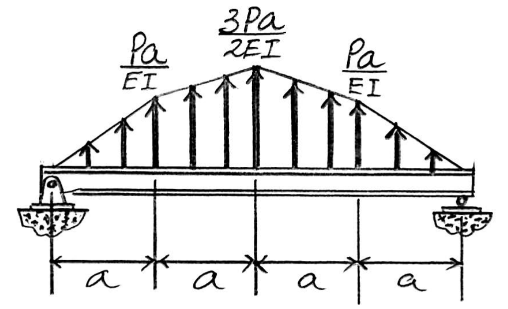
Referring to Fig. d,

$$\zeta+ \sum M_C = 0; \quad \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{4}{3}a\right) + \left[\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{a}{2}\right)$$

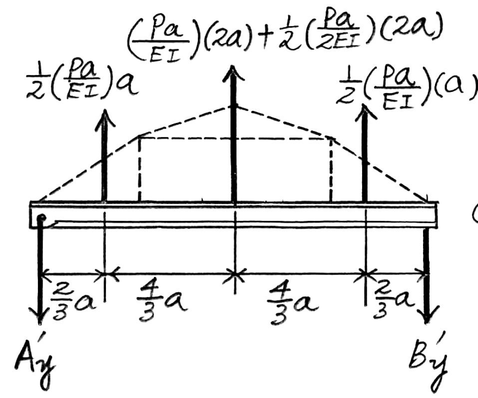
$$+ \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{a}{3}\right) - \frac{7Pa^2}{4EI}(2a)$$

$$-M'_C = 0$$

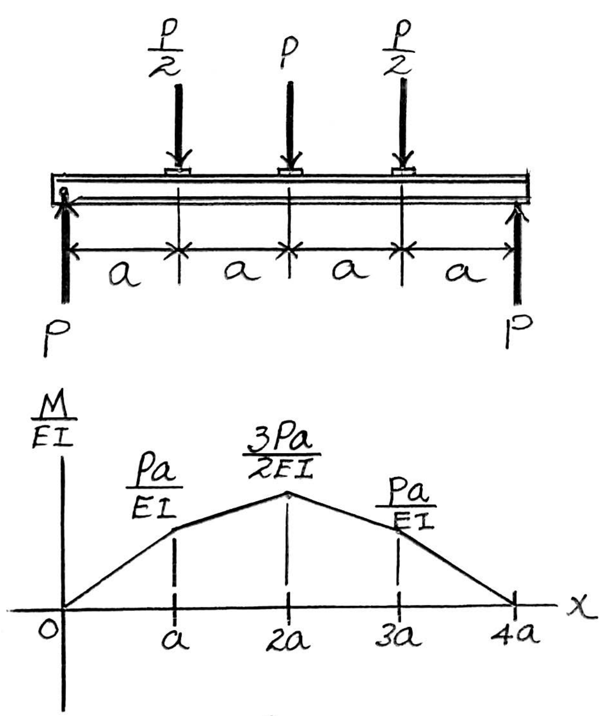
$$\Delta_C = M'_C = -\frac{9Pa^3}{4EI} = \frac{9Pa^3}{4EI} \quad \downarrow \qquad \textbf{Ans.}$$



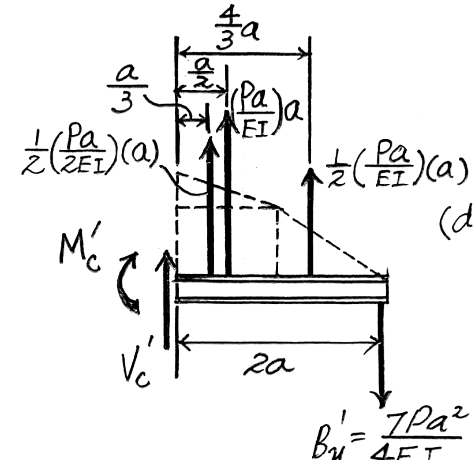
(b)



(c)

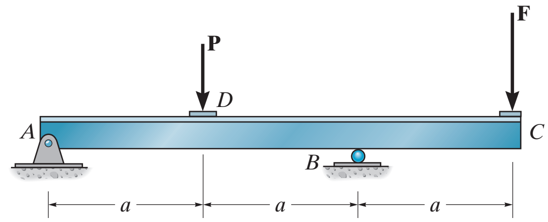


(a)



(d)

***8–28.** Determine the force **F** at the end of the beam C so that the displacement at C is zero. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. a and b, respectively, Theorem 2 gives

$$t_{B/A} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](a) + \left[\frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\right]\left[\frac{1}{3}(2a)\right] = \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

$$t_{C/A} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](2a) + \left[\frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\right]\left[\frac{1}{3}(2a) + a\right]$$

$$+ \left[\frac{1}{2}\left(-\frac{Fa}{EI}\right)(a)\right]\left[\frac{2}{3}(a)\right]$$

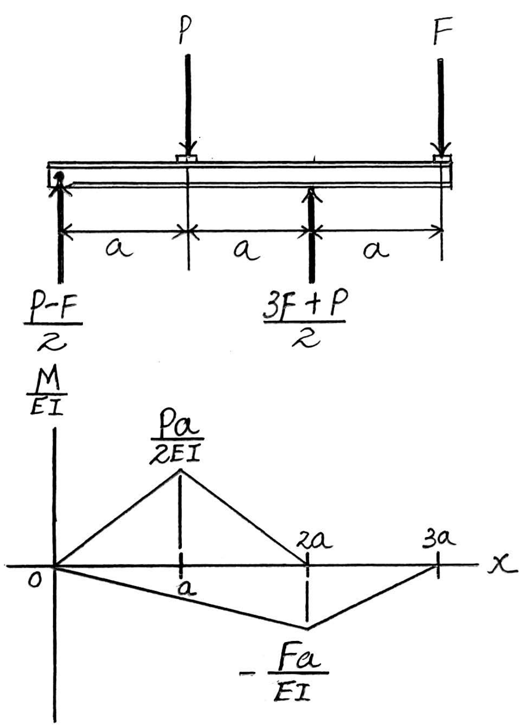
$$= \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

It is required that

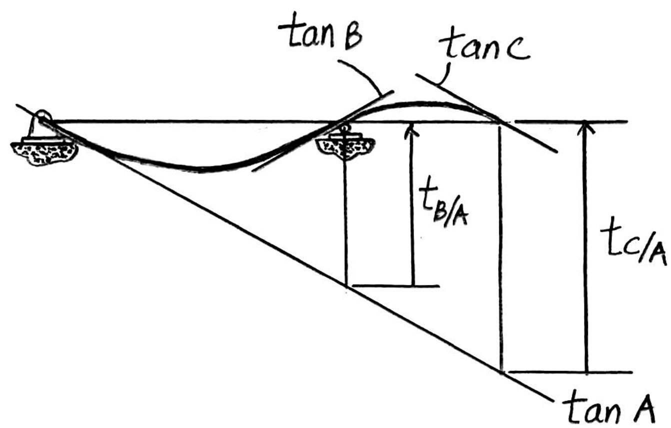
$$t_{C/A} = \frac{3}{2}t_{B/A}$$

$$\frac{Pa^3}{EI} - \frac{2Fa^3}{EI} = \frac{3}{2}\left[\frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}\right]$$

$$F = \frac{P}{4}$$ **Ans.**

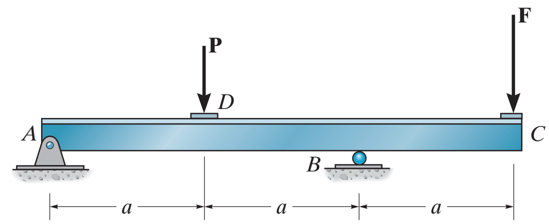


(a)



(b)

8–29. Determine the force **F** at the end of the beam C so that the displacement at C is zero. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. a and b, respectively. Referring to Fig. c,

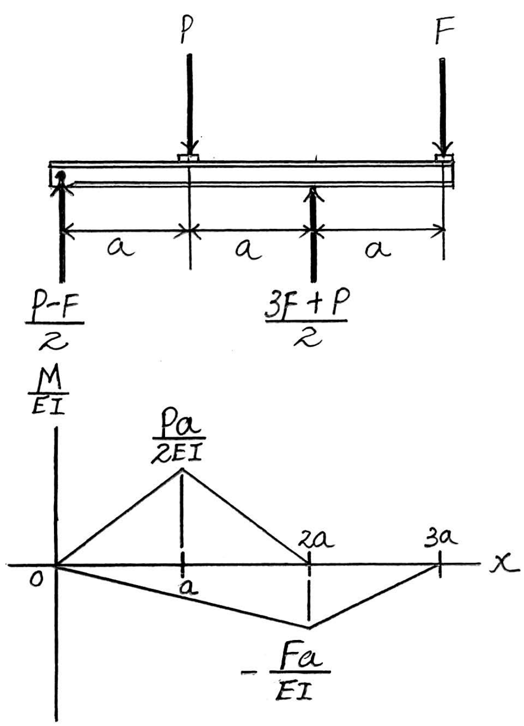
$$\zeta+\sum M_A = 0; \quad \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](a) - \left[\frac{1}{2}\left(\frac{Fa}{EI}\right)(2a)\right]\left[\frac{2}{3}(2a)\right] - B'_y(2a) = 0$$

$$B'_y = \frac{Pa^2}{4EI} - \frac{2Fa^2}{3EI}$$

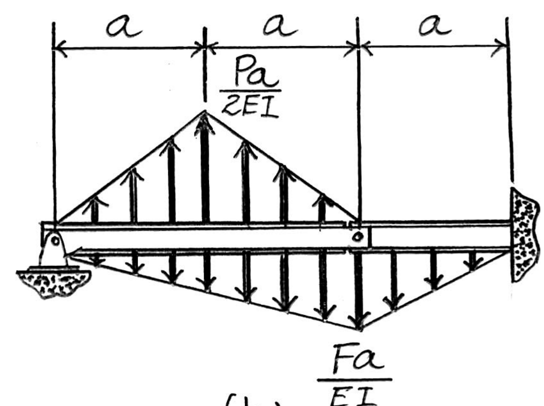
Here, it is required that $\Delta_C = M'_C = 0$. Referring to Fig. d,

$$\zeta+\sum M_C = 0; \quad \left[\frac{1}{2}\left(\frac{Fa}{EI}\right)(a)\right]\left[\frac{2}{3}(a)\right] - \left(\frac{Pa^2}{4EI} - \frac{2Fa^2}{3EI}\right)(a) = 0$$

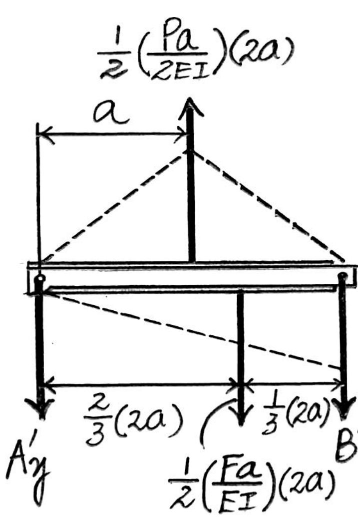
$$F = \frac{P}{4}$$ **Ans.**



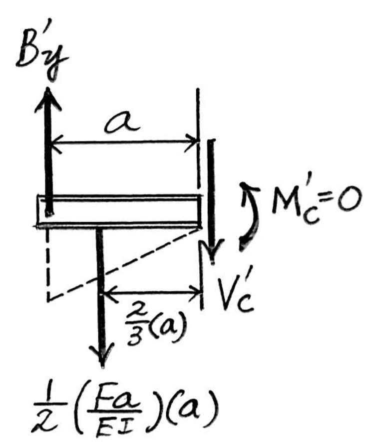
(a)



(b)

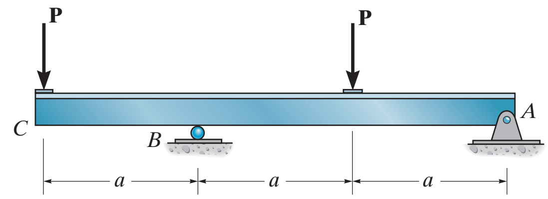


(c)



(d)

8–30. Determine the slope at *B* and the displacement at *C*. *EI* is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. *a* and *b*, Theorem 1 and 2 give

$$\theta_{B/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{Pa^2}{2EI} = \frac{Pa^2}{2EI}$$

$$t_{B/A} = \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\right]\left[\frac{1}{3}(a)\right] = -\frac{Pa^3}{6EI}$$

$$t_{C/A} = \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(2a)\right](a) = -\frac{Pa^3}{EI}$$

Then

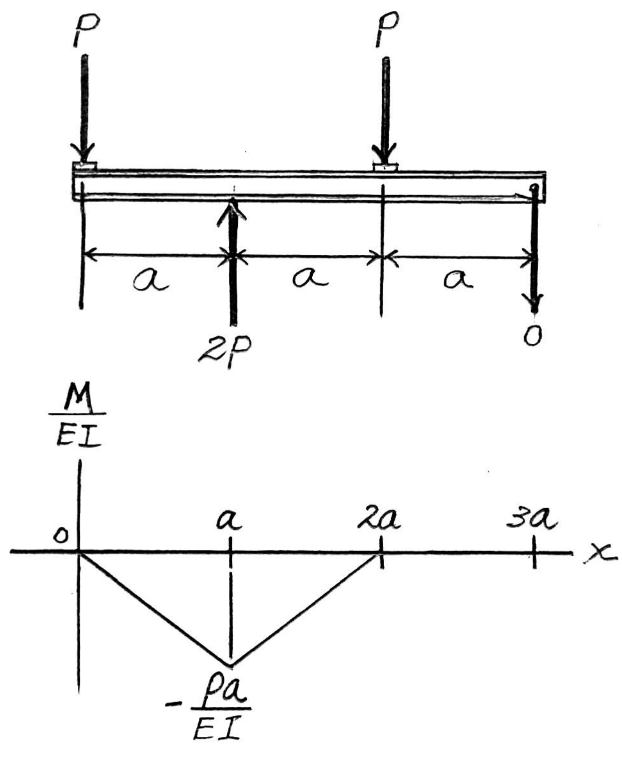
$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{Pa^3/6EI}{2a} = \frac{Pa^2}{12EI}$$

$$\Delta' = \frac{3}{2}|t_{B/A}| = \frac{3}{2}\left(\frac{Pa^3}{6EI}\right) = \frac{Pa^3}{4EI}$$

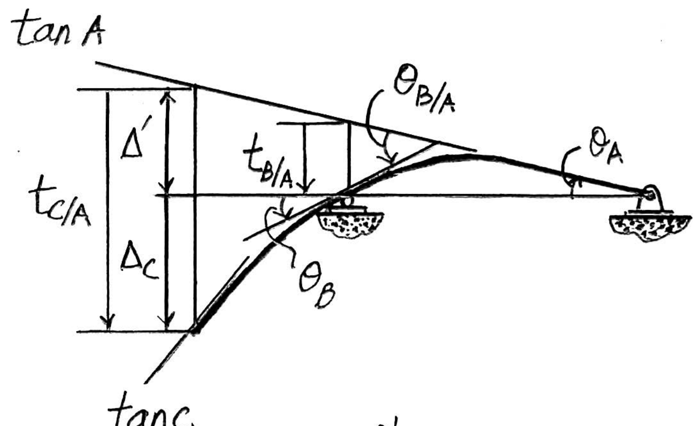
$$\theta_B = \theta_A + \theta_{B/A}$$

$$\circlearrowleft + \ \theta_B = -\frac{Pa^2}{12EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{12EI} \qquad \textbf{Ans.}$$

$$\Delta_C = |t_{C/A}| - \Delta' = \frac{Pa^3}{EI} - \frac{Pa^3}{4EI} = \frac{3Pa^3}{4EI} \downarrow \qquad \textbf{Ans.}$$

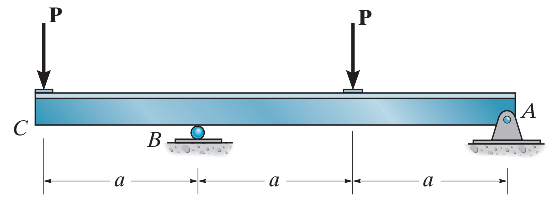


(a)



(b)

8–31. Determine the slope at B and the displacement at C. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. c and d, respectively. Referring to Fig. d,

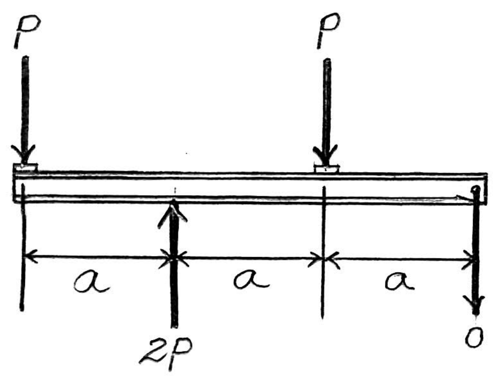
$$\zeta+ \sum M_A = 0; \quad \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) - B'_y\,(2a) = 0$$

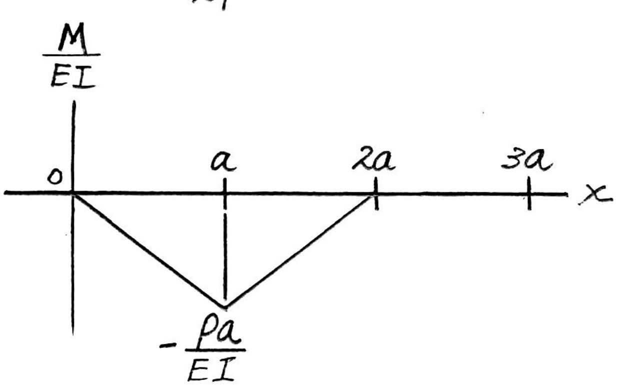
$$\theta_B = B'_y = \frac{5Pa^2}{12EI} \quad \text{Ans.}$$

Referring to Fig. c,

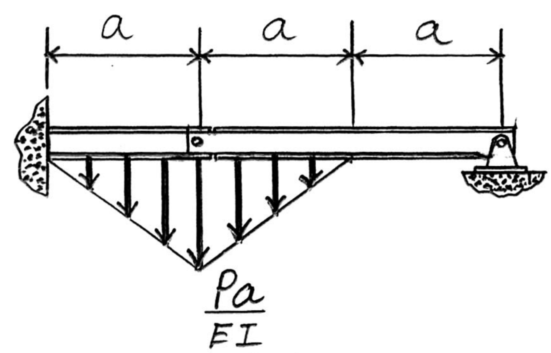
$$\zeta+ \sum M_C = 0; \quad -M'_C - \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) - \left(\frac{5Pa^2}{12EI}\right)(a) = 0$$

$$\Delta_C = M'_C = -\frac{3Pa^3}{4EI} = \frac{3Pa^3}{4EI} \downarrow \quad \text{Ans.}$$

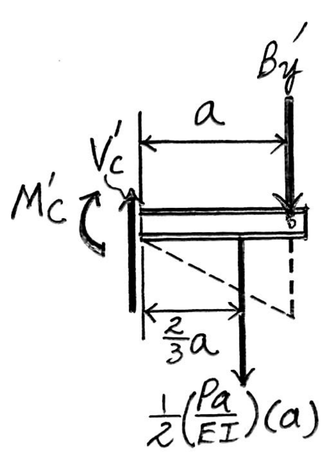




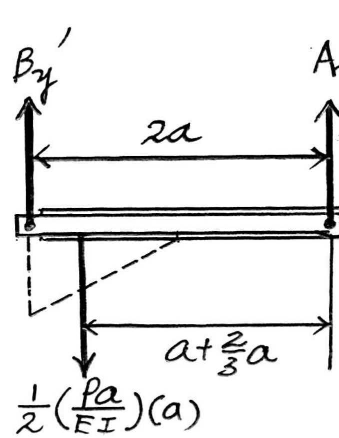
(a)



(b)

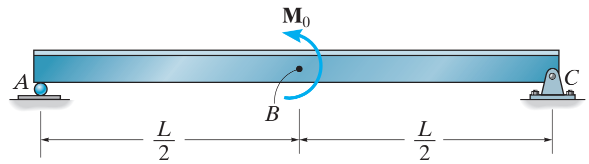


(c)



(d)

***8–32.** Determine the maximum displacement and the slope at A. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{D/A} = \frac{1}{2}\left(\frac{M_0}{EIL}x\right)(x) = \frac{M_0}{2EIL}x^2 \quad \measuredangle$$

$$t_{B/A} = \left[\frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right] = \frac{M_0L^2}{48EI}$$

Then,

$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{M_0L^2/48EI}{L/2} = \frac{M_0L}{24EI} \quad \triangledown \qquad \textbf{Ans.}$$

Here $\theta_D = 0$. Thus,

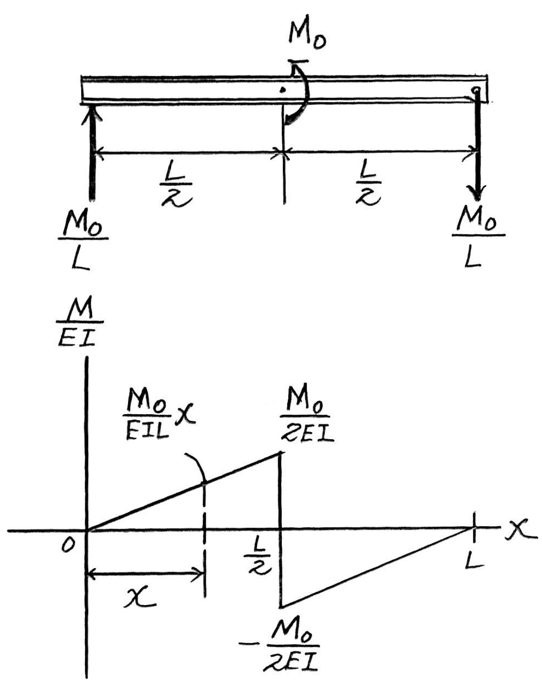
$$\theta_D = \theta_A + \theta_{D/A}$$

$$\circlearrowleft + \quad 0 = -\frac{M_0L}{24EI} + \frac{M_0}{2EIL}x^2 \qquad x = \frac{L}{\sqrt{12}} = 0.2887L$$

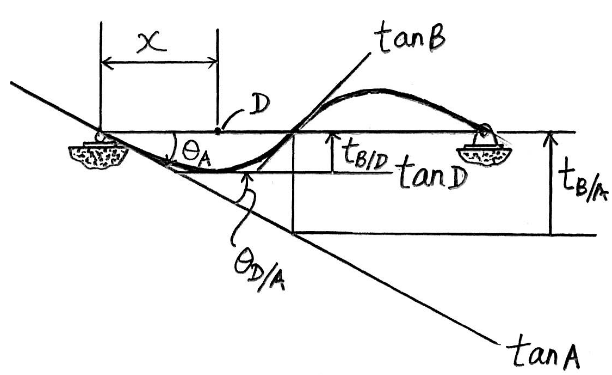
$$\Delta_{max} = \Delta_D = t_{B/D} = \left[\frac{1}{2}\left(\frac{0.2113M_0}{EI}\right)(0.2113L)\right]\left[\frac{1}{3}(0.2113L)\right]$$

$$+\left[\left(\frac{0.2887M_0}{EI}\right)(0.2113L)\right]\left[\frac{1}{2}(0.2113L)\right]$$

$$= \frac{0.00802M_0L^2}{EI} \downarrow \qquad \textbf{Ans.}$$

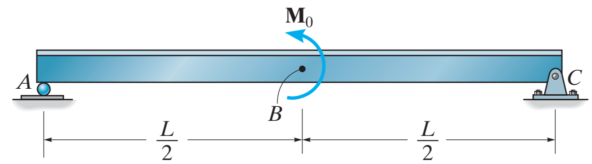


(a)



(b)

8–33. Determine the maximum displacement at B and the slope at A. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. a and b, respectively. Referring to Fig. c

$$\zeta+ \sum M_B = 0; \quad A'_y(L) - \left[\frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right)\right]\left(\frac{L}{3}\right) = 0$$

$$A'_y = \theta_A = \frac{M_0 L}{24EI} \qquad \textbf{Ans.}$$

Here it is required that $\theta_D = V'_D = 0$. Referring to Fig. d,

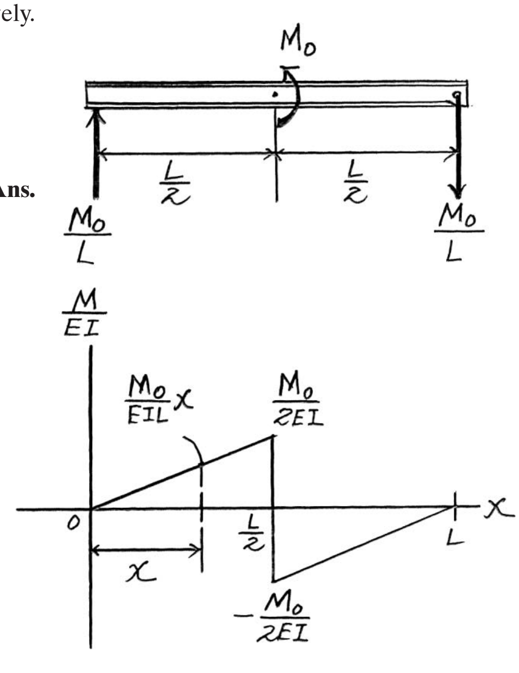
$$\uparrow \sum F_y = 0; \quad \frac{1}{2}\left(\frac{M_0}{EIL}x\right)(x) - \frac{M_0 L}{24EI} = 0$$

$$x = \frac{L}{\sqrt{12}}$$

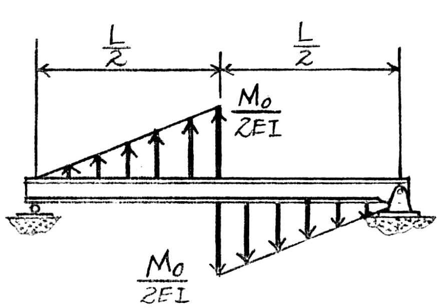
$$\zeta+ \sum M_D = 0; \quad M'_D + \left(\frac{M_0 L}{24EI}\right)\left(\frac{L}{\sqrt{12}}\right) - \frac{1}{2}\left(\frac{M_0}{EIL}\right)\left(\frac{L}{\sqrt{12}}\right)\left(\frac{L}{\sqrt{12}}\right)\left[\frac{1}{3}\left(\frac{L}{\sqrt{12}}\right)\right] = 0$$

$$\Delta_{max} = \Delta_D = M'_D = -\frac{0.00802 M_0 L^2}{EI}$$

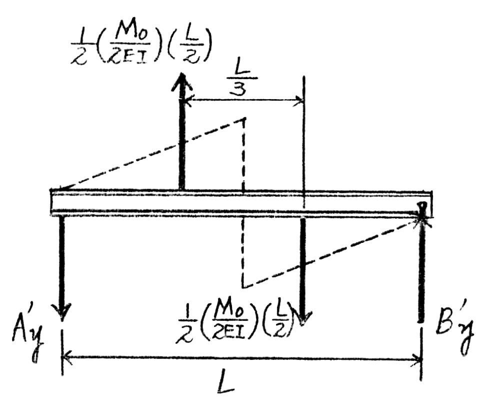
$$= \frac{0.00802 M_0 L^2}{EI} \downarrow \qquad \textbf{Ans.}$$



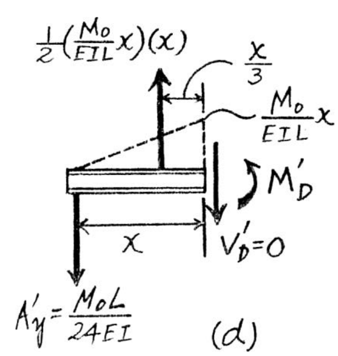
(a)



(b)

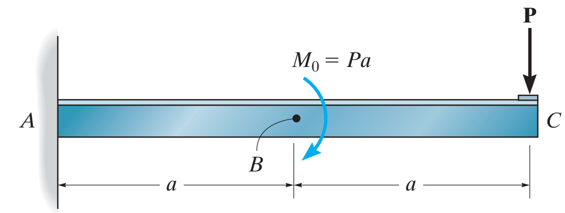


(c)



(d)

8–34. Determine the slope and displacement at C. EI is constant. Use the moment-area theorems.



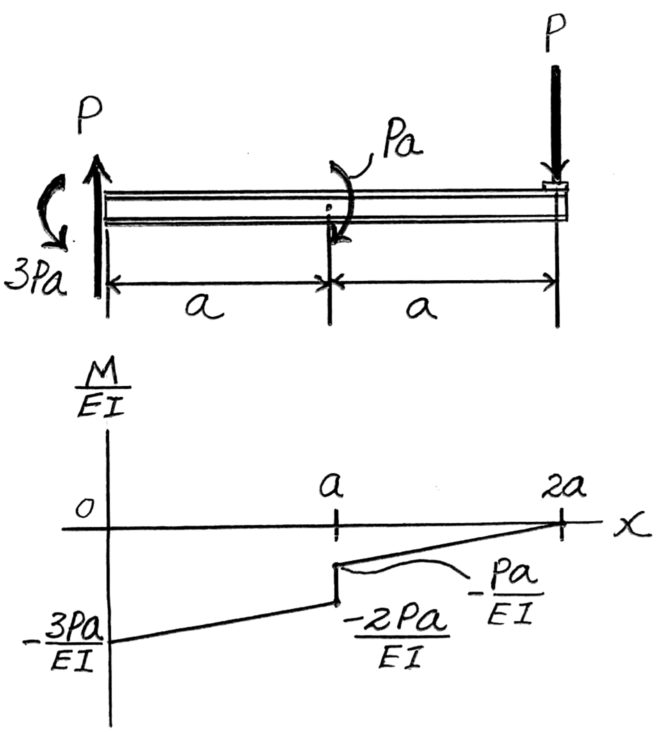
Using the $\dfrac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_C = |\theta_{C/A}| = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{2Pa}{EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) = \frac{3Pa^2}{EI}$$ **Ans.**

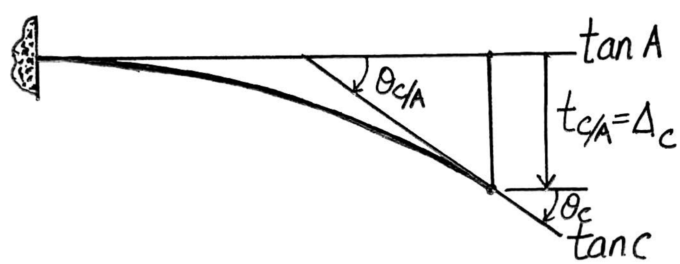
$$\Delta_C = |t_{C/A}| = \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) + \left[\left(\frac{2Pa}{EI}\right)(a)\right]\left(a + \frac{a}{2}\right)$$

$$+ \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) = 0$$

$$= \frac{25Pa^3}{6EI} \downarrow$$ **Ans.**

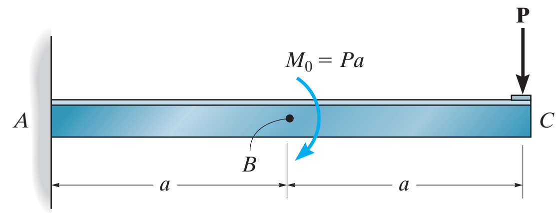


(a)



(b)

8–35. Determine the slope and displacement at *C*. *EI* is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

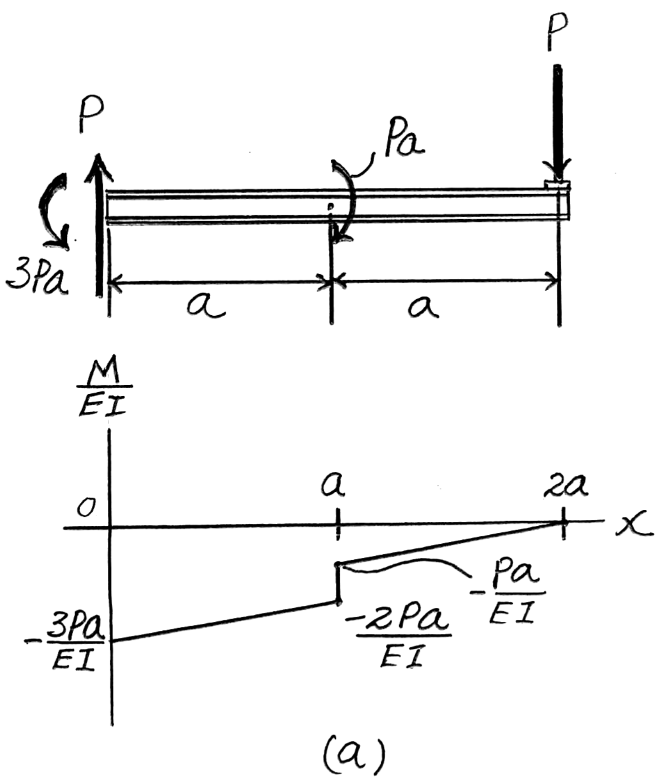
$$+\uparrow \sum F_y = 0; \quad -V'_C - \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) - \left(\frac{2Pa}{EI}\right)(a) - \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) = 0$$

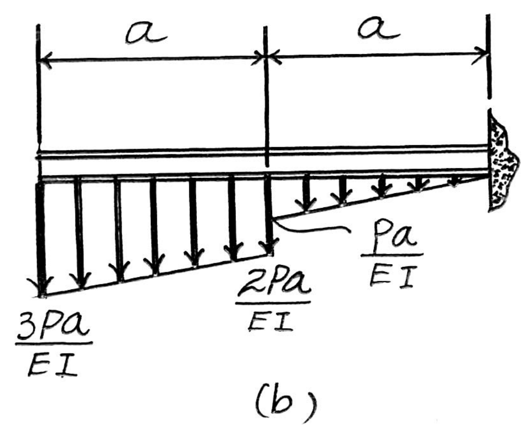
$$\theta_C = V'_C = -\frac{3Pa^2}{EI} = \frac{3Pa^2}{EI} \quad \text{Ans.}$$

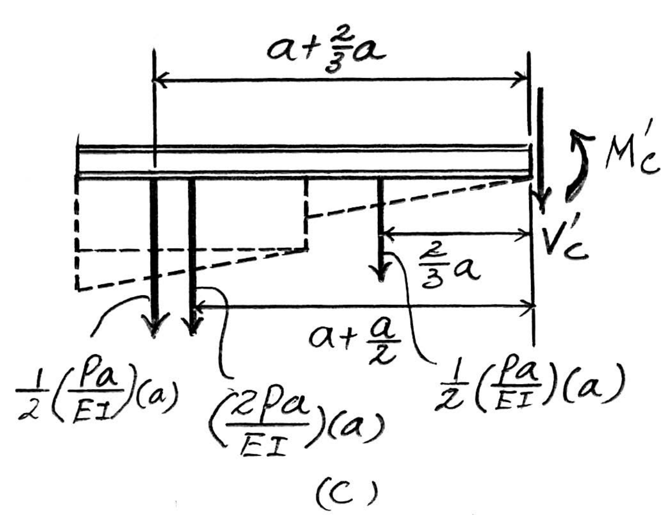
$$\circlearrowleft + \sum M_C = 0; \quad M'_C + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) + \left[\left(\frac{2Pa}{EI}\right)(a)\right]\left(a + \frac{a}{2}\right)$$

$$+\left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) = 0$$

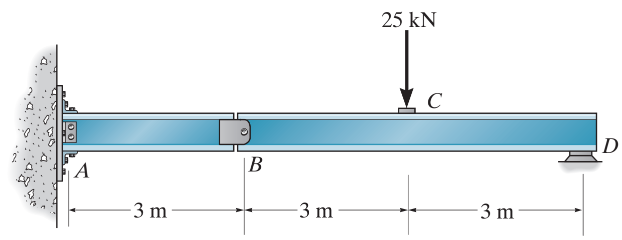
$$\Delta_C = M'_C = -\frac{25Pa^3}{6EI} = \frac{25Pa^3}{6EI} \downarrow \quad \text{Ans.}$$







***8–36.** Determine the displacement at C. Assume A is a fixed support, B is a pin, and D is a roller. EI is constant. Use the moment-area theorems.



Using the $\dfrac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\Delta_B = |t_{B/A}| = \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right] = \frac{112.5\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$t_{C/D} = \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{1}{3}(3\text{ m})\right] = \frac{56.25\text{ kN}\cdot\text{m}^3}{EI}$$

$$t_{B/D} = \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) = \frac{337.5\text{ kN}\cdot\text{m}^3}{EI}$$

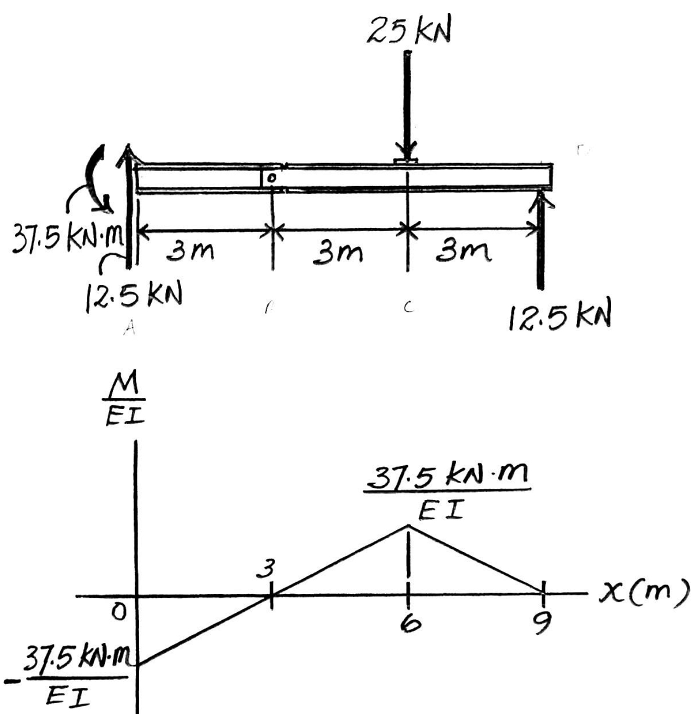
Then

$$\theta_D = \frac{\Delta_B + t_{B/D}}{L_{B/D}} = \frac{112.5\text{ kN}\cdot\text{m}^3/EI + 337.5\text{ kN}\cdot\text{m}^3/EI}{6\text{ m}} = \frac{75\text{ kN}\cdot\text{m}^2}{EI} \quad \text{Ans.}$$

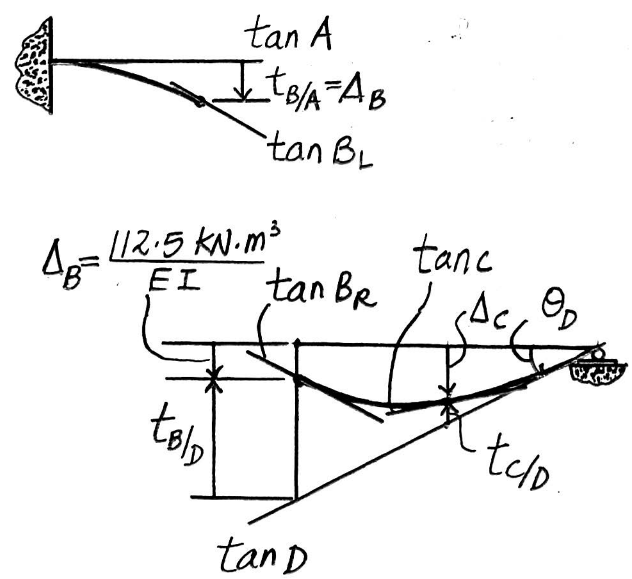
$$\Delta_C + t_{C/D} = \frac{1}{2}(\Delta_B + t_{B/D})$$

$$\Delta_C + \frac{56.25\text{ kN}\cdot\text{m}^3}{EI} = \frac{1}{2}\left(\frac{112.5\text{ kN}\cdot\text{m}^3}{EI} + \frac{337.5\text{ kN}\cdot\text{m}^3}{EI}\right)$$

$$\Delta_C = \frac{169\text{ kN}\cdot\text{m}^3}{EI} \downarrow \quad \text{Ans.}$$

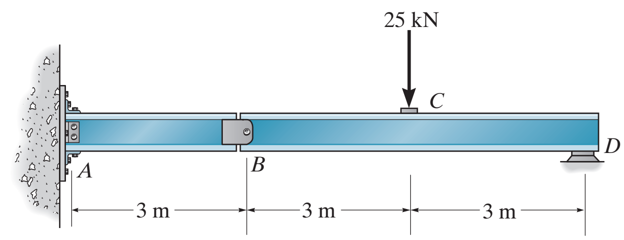


(a)



(b)

8–37. Determine the displacement at C. Assume A is a fixed support, B is a pin, and D is a roller. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

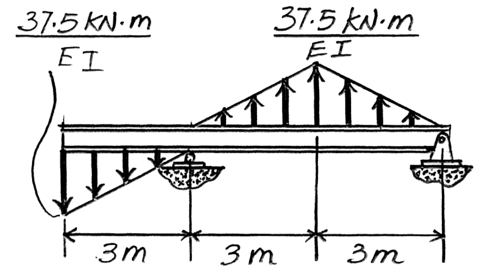
$$\circlearrowleft + \sum M_B = 0; \quad \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) + \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right](2\text{ m}) - D'_y(6\text{ m}) = 0$$

$$\theta_D = D'_y = \frac{75\text{ kN}\cdot\text{m}^2}{EI} \quad \text{Ans.}$$

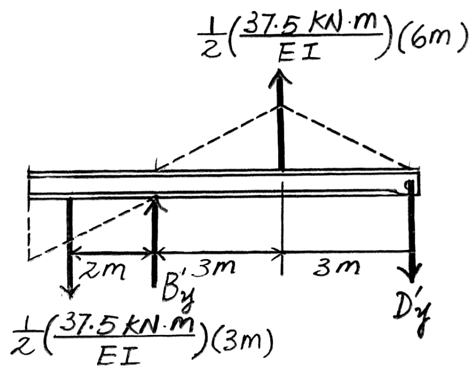
Referring to Fig. *d*,

$$\circlearrowleft + \sum M_C = 0; \quad \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right](1\text{ m}) - \left(\frac{75\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m}) - M'_C = 0$$

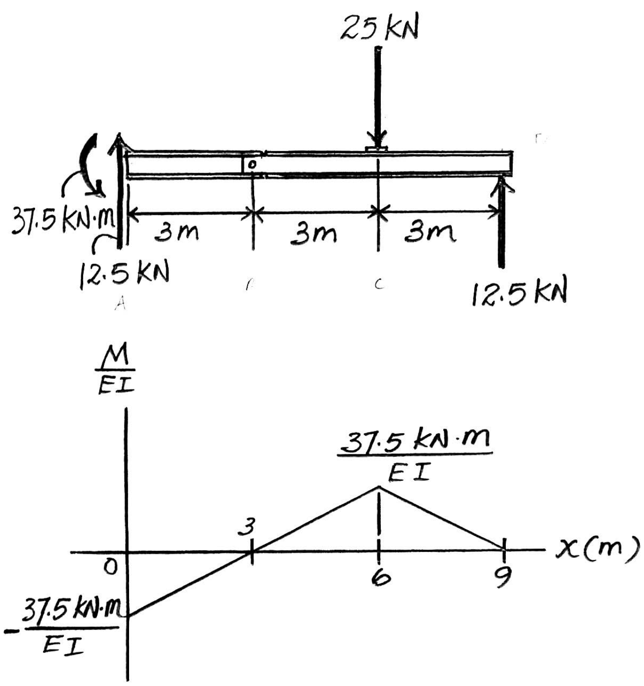
$$\Delta_C = -\frac{168.75\text{ kN}\cdot\text{m}^3}{EI} = \frac{168.75\text{ kN}\cdot\text{m}^3}{EI}\downarrow \quad \text{Ans.}$$



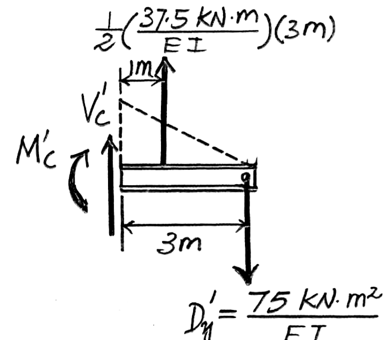
(b)



(c)

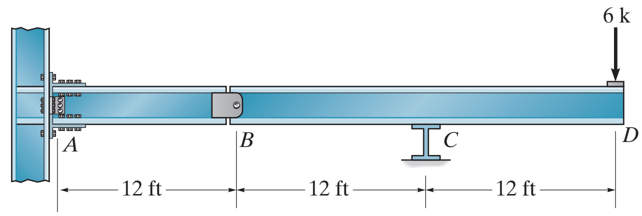


(a)



(d)

8–38. Determine the displacement at D and the slope at D. Assume A is a fixed support, B is a pin, and C is a roller. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\Delta_B = t_{B/A} = \left[\frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right]\left[\frac{2}{3}(12\text{ ft})\right] = \frac{3456\text{ k}\cdot\text{ft}^3}{EI}\uparrow$$

$$\theta_{D/B} = \frac{1}{2}\left(-\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(24\text{ ft}) = -\frac{864\text{ k}\cdot\text{ft}^2}{EI} = \frac{864\text{ k}\cdot\text{ft}^2}{EI}$$

$$t_{C/B} = \left[\frac{1}{2}\left(-\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right]\left[\frac{1}{3}(12\text{ ft})\right] = -\frac{1728\text{ k}\cdot\text{ft}^3}{EI}$$

$$t_{D/B} = \left[\frac{1}{2}\left(-\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(24\text{ ft})\right](12\text{ ft}) = -\frac{10368\text{ k}\cdot\text{ft}^3}{EI}$$

Then,

$$\Delta' = 2(\Delta_B - |t_{C/B}| = 2\left(\frac{3456\text{ k}\cdot\text{ft}^3}{EI} - \frac{1728\text{ k}\cdot\text{ft}^3}{EI}\right) = \frac{3456\text{ k}\cdot\text{ft}^3}{EI}$$

$$\theta_{BR} = \frac{\Delta'}{L_{BD}} = \frac{3456\text{ k}\cdot\text{ft}^3/EI}{24\text{ ft}} = \frac{144\text{ k}\cdot\text{ft}^2}{EI}$$

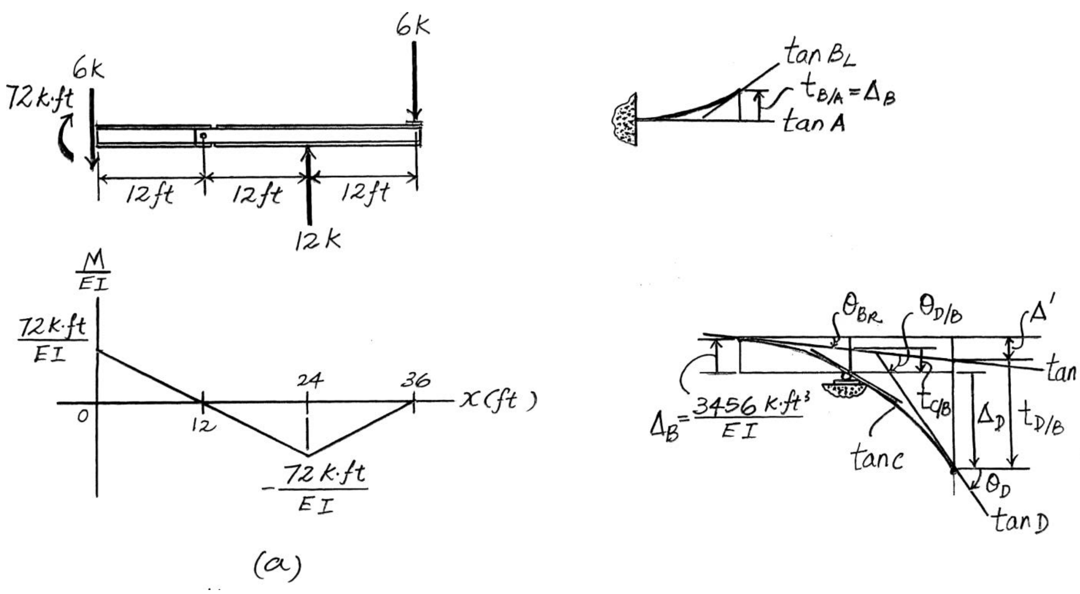
$$\theta_D = \theta_{BR} + \theta_{D/B}$$

$$+\circlearrowleft\ \theta_D = \frac{144\text{ k}\cdot\text{ft}^2}{EI} + \frac{864\text{ k}\cdot\text{ft}^2}{EI} = \frac{1008\text{ k}\cdot\text{ft}^2}{EI}$$ **Ans.**

$$\Delta_D = |t_{D/B}| + \Delta' - \Delta_B$$

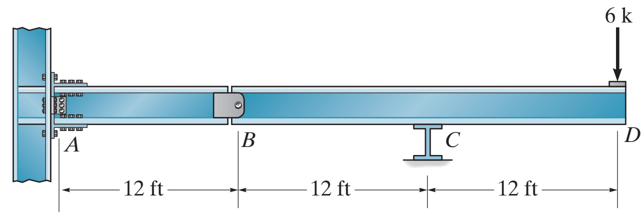
$$= \frac{10368\text{ k}\cdot\text{ft}^3}{EI} + \frac{3456\text{ k}\cdot\text{ft}^3}{EI} - \frac{3456\text{ k}\cdot\text{ft}^3}{EI}$$

$$= \frac{10{,}368\text{ k}\cdot\text{ft}^3}{EI}\downarrow$$ **Ans.**



(a)

8–39. Determine the displacement at D and the slope at D. Assume A is a fixed support, B is a pin, and C is a roller. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. a and b, respectively. Referring to Fig. c,

$$\zeta + \sum M_B = 0; \qquad C_y'(12\text{ ft}) - \left[\frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right](16\text{ ft}) = 0$$

$$C_y' = \frac{576\text{ k}\cdot\text{ft}^2}{EI}$$

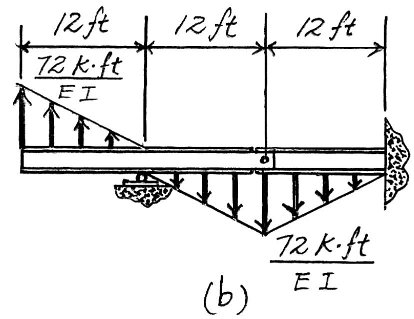
Referring to Fig. d,

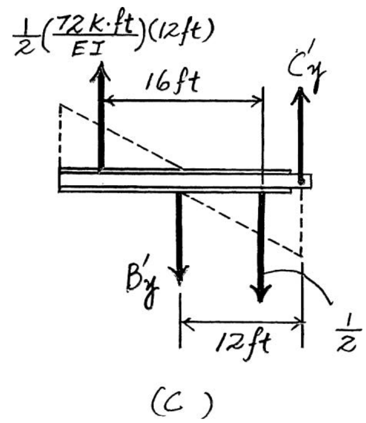
$$+\uparrow \sum F_y = 0; \quad -V_D' - \frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft}) - \frac{576\text{ k}\cdot\text{ft}^2}{EI} = 0$$

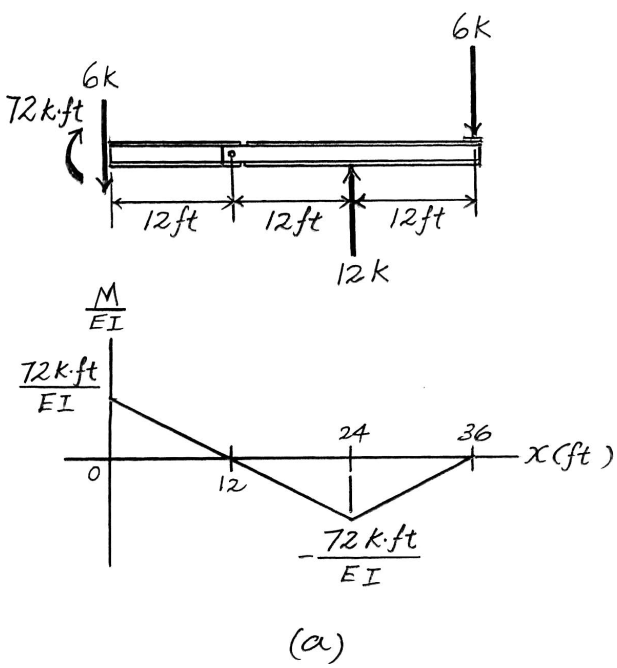
$$\theta_D = V_D' = -\frac{1008\text{ k}\cdot\text{ft}^2}{EI} = \frac{1008\text{ k}\cdot\text{ft}^2}{EI} \quad \textbf{Ans.}$$

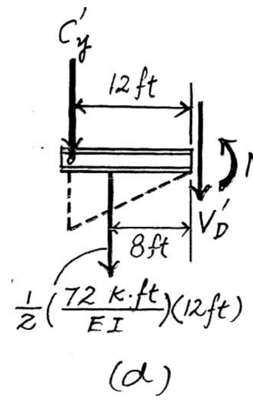
$$\zeta + \sum M_C = 0; \quad M_D' + \left[\frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right](8\text{ ft}) + \left(\frac{576\text{ k}\cdot\text{ft}^2}{EI}\right)(12\text{ ft}) = 0$$

$$M_D' = \Delta_D = -\frac{10368\text{ k}\cdot\text{ft}^3}{EI} = \frac{10{,}368\text{ k}\cdot\text{ft}^3}{EI}\downarrow \quad \textbf{Ans.}$$

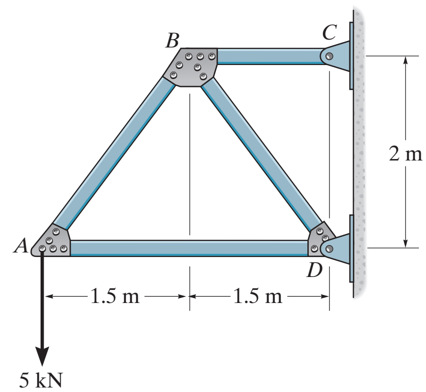








9–1. Determine the vertical displacement of joint A. Each bar is made of steel and has a cross-sectional area of 600 mm^2. Take $E = 200$ GPa. Use the method of virtual work.



The virtual forces and real forces in each member are shown in Fig. *a* and *b*, respectively.

Member	n(kN)	N(kN)	L(m)	nNL (kN^2 • m)
AB	1.25	6.25	2.50	19.531
AD	−0.75	−3.75	3	8.437
BD	−1.25	−6.25	2.50	19.531
BC	1.50	7.50	1.50	16.875
			Σ	64.375

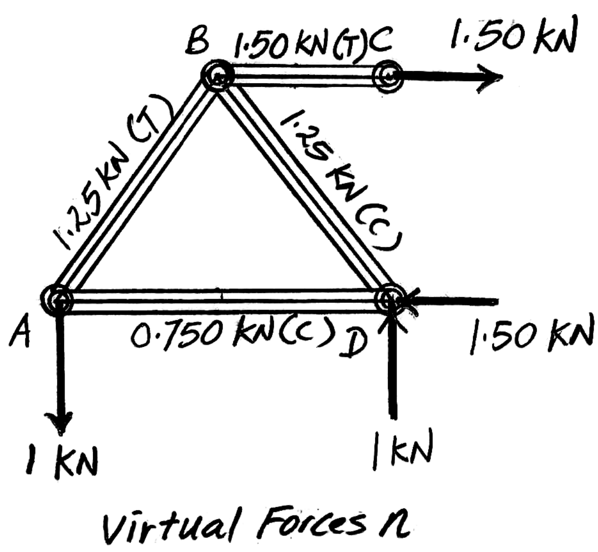
$$1\text{ kN}\cdot\Delta_{A_v} = \sum\frac{nNL}{AE} = \frac{64.375\text{ kN}^2\cdot\text{m}}{AE}$$

$$\Delta_{A_v} = \frac{64.375\text{ kN}\cdot\text{m}}{AE}$$

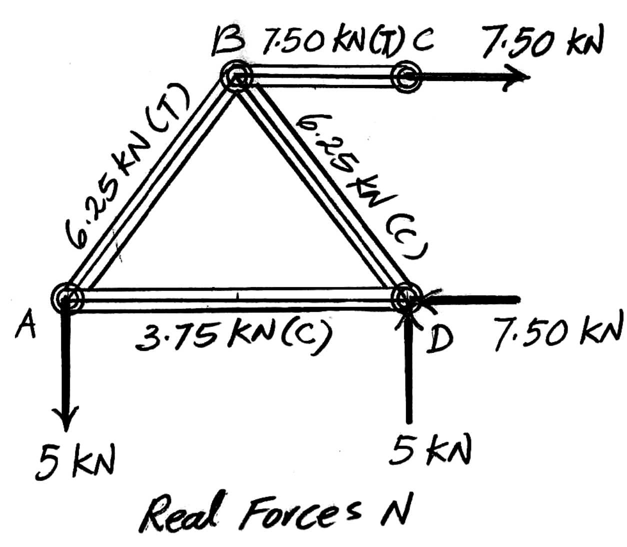
$$= \frac{64.375(10^3)\text{ N}\cdot\text{m}}{\left[0.6(10^{-3})\text{ m}^2\right]\left[200(10^9)\text{ N/m}^2\right]}$$

$$= 0.53646\,(10^{-3})\text{ m}$$

$$= 0.536\text{ mm}\downarrow$$ **Ans.**

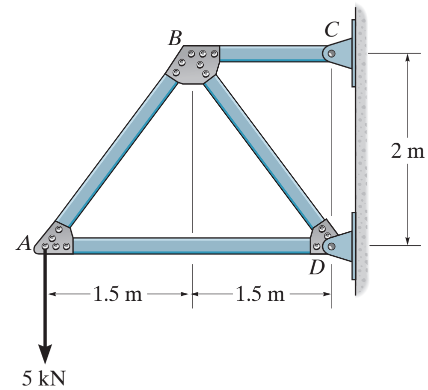


(a)



(b)

9–2. Solve Prob. 9–1 using Castigliano's theorem.



Member	$N(kN)$	$\dfrac{\partial N}{\partial P}$	$N(P = 5\text{kN})$	$L(\text{m})$	$N\left(\dfrac{\partial N}{\partial P}\right)L(\text{kN}\cdot\text{m})$
AB	1.25 P	1.25	6.25	2.5	19.531
AD	−0.750 P	−0.75	−3.75	3	8.437
BD	−1.25 P	−1.25	−6.25	2.5	19.531
BC	1.50 P	1.50	7.50	1.5	16.875
				Σ	64.375

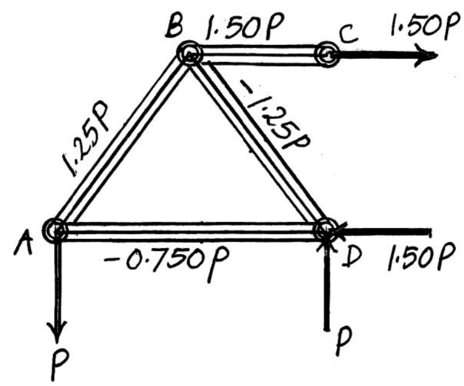
$$\Delta_{A_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$= \frac{64.375\ \text{kN}\cdot\text{m}}{AE}$$

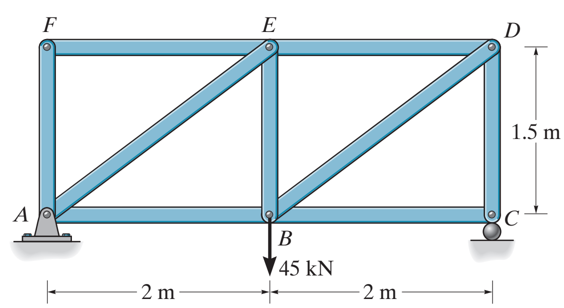
$$= \frac{64\cdot 375(10^3)\ \text{N}\cdot\text{m}}{[0.6(10^{-3})\,\text{m}^2][200(10^9)\,\text{N/m}^2]}$$

$$= 0.53646\ (10^{-3})\ \text{m}$$

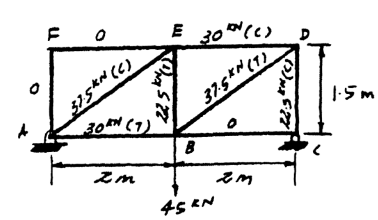
$$= 0.536\ \text{mm} \downarrow$$ **Ans.**



***9–3.** Determine the vertical displacement of joint B. For each member $A = 400\ \text{mm}^2$, $E = 200$ GPa. Use the method of virtual work.

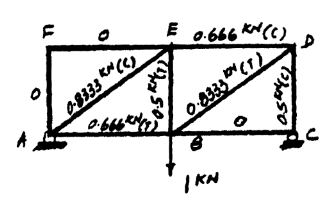


Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	0.50	22.5	1.5	16.875
ED	−0.6667	−30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				Σ = 270

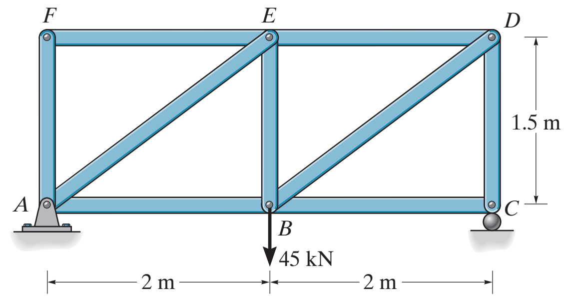


$$1\cdot\Delta_{B_v} = \sum\frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\,\text{m} = 3.38\ \text{mm}\downarrow$$ **Ans.**

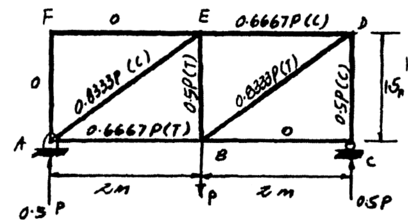


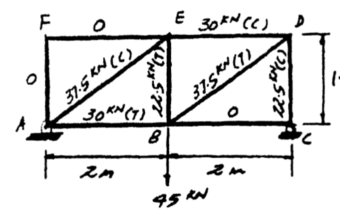
***9–4.** Solve Prob. 9–3 using Castigliano's theorem.



Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 45)$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AF	0	0	0	1.5	0
AE	$-0.8333P$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P$	0.6667	30.0	2.0	40.00
BE	$0.5P$	0.5	22.5	1.5	16.875
BD	$0.8333P$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-0.5P$	-0.5	-22.5	1.5	16.875
DE	$0.6667P$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0

$\Sigma = 270$

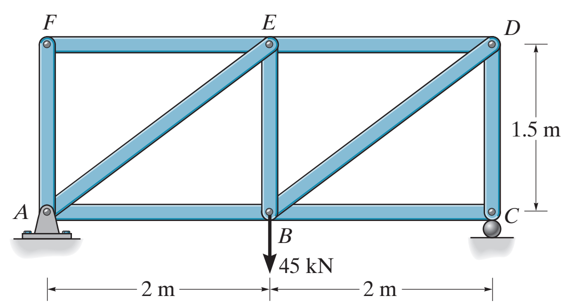




$$\delta_{B_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{270}{AE}$$

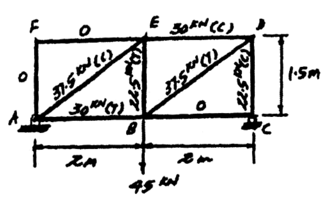
$$= \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\,\text{m} = 3.38\text{ mm} \qquad \textbf{Ans.}$$

9–5. Determine the vertical displacement of joint E. For each member $A = 400\text{ mm}^2$, $E = 200$ GPa. Use the method of virtual work.



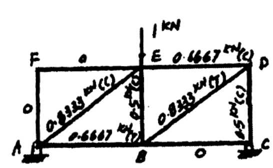
Member	n	N	L	nNL
AF	0	0	1.5	0
AE	-0.8333	-37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	0.50	22.5	1.5	-16.875
ED	-0.6667	30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	-0.5	-22.5	1.5	16.875

$\Sigma = 236.25$

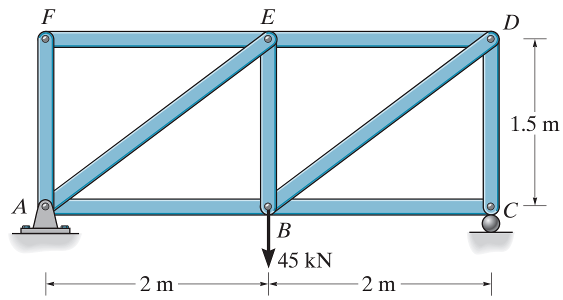


$$1 \cdot \Delta E_v = \sum \frac{nNL}{AE}$$

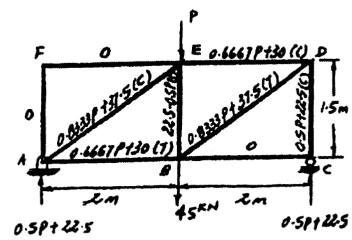
$$\Delta E_v = \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\,\text{m} = 2.95\text{ mm}\downarrow \qquad \textbf{Ans.}$$



9–6. Solve Prob. 9–5 using Castigliano's theorem.



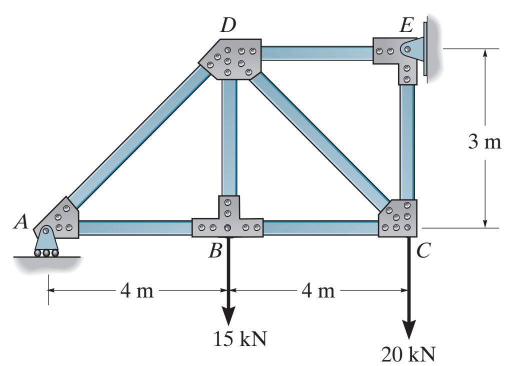
Member	N	$\frac{\partial N}{\partial P}$	$N(P = 45)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P + 37.5)$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P + 30$	0.6667	30.0	2.0	40.00
BE	$22.5 - 0.5P$	-0.5	22.5	1.5	-16.875
BD	$0.8333P + 37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P + 22.5)$	-0.5	-22.5	1.5	16.875
DE	$-(0.6667P + 30)$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$



$$\Delta_{E_v} = \sum N \frac{\partial N}{\partial P} \frac{L}{AE} = \frac{236.25}{AE}$$

$$= \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\,\text{m} = 2.95\text{ mm} \downarrow$$ **Ans.**

9–7. Determine the vertical displacement of joint D. Use the method of virtual work. AE is constant. Assume the members are pin connected at their ends.



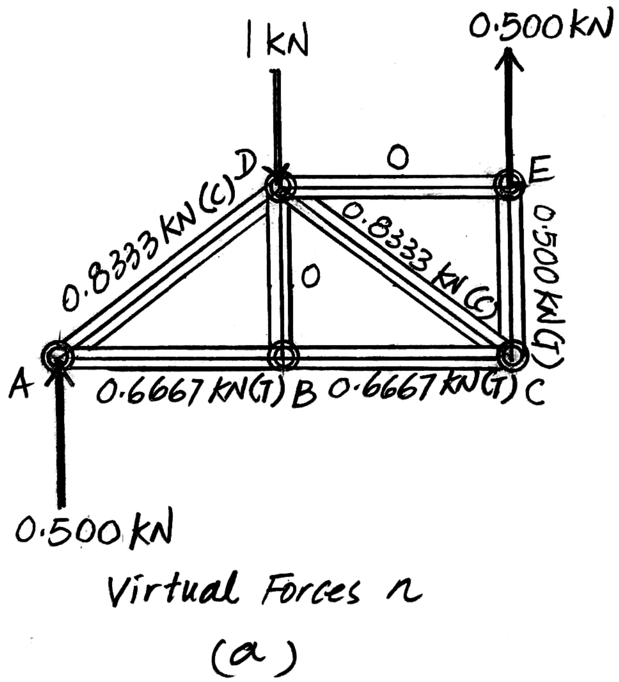
The virtual and real forces in each member are shown in Fig. *a* and *b*, respectively

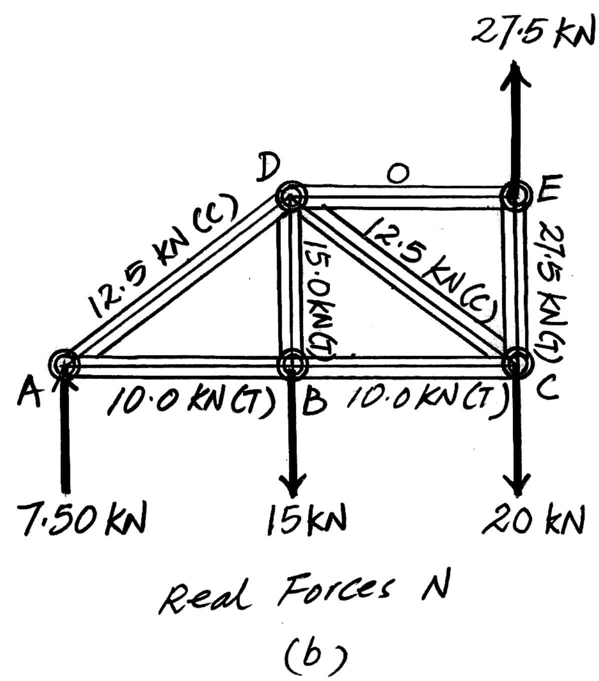
Member	$n(\text{kN})$	$N(\text{kN})$	$L(\text{m})$	$nNL(\text{kN}^2 \cdot \text{m})$
AB	0.6667	10.0	4	26.667
BC	0.6667	10.0	4	26.667
AD	-0.8333	-12.5	5	52.083
BD	0	15.0	3	0
CD	-0.8333	-12.5	5	52.083
CE	0.500	27.5	3	41.25
DE	0	0	4	0
			Σ	198.75

$$1\text{ kN} \cdot \Delta_{D_v} = \sum \frac{nNL}{AE} = \frac{198.75\text{ kN}^2 \cdot \text{m}}{AE}$$

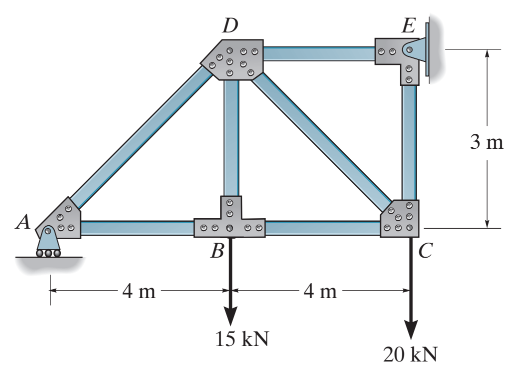
$$\Delta_{D_v} = \frac{198.75\text{ kN} \cdot \text{m}}{AE} = \frac{199\text{ kN} \cdot \text{m}}{AE} \downarrow$$ **Ans.**

9–7. Continued





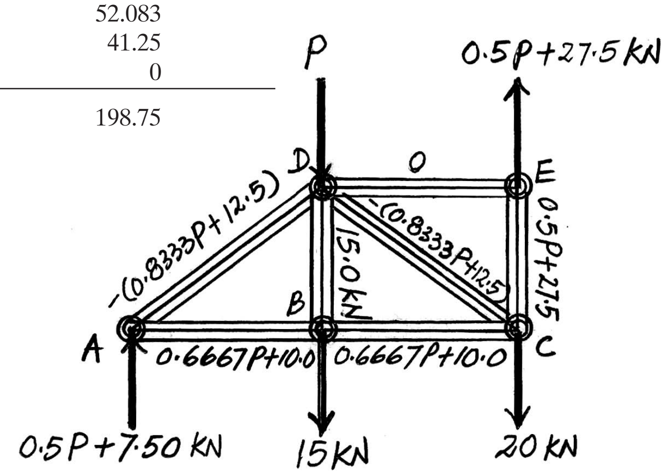
*9–8. Solve Prob. 9–7 using Castigliano's theorem.



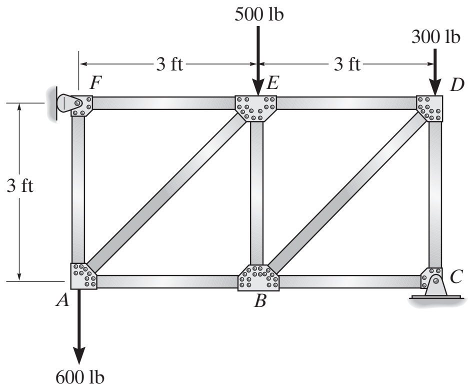
Member	N(kN)	$\frac{\partial N}{\partial P}$	$N(P = 0)$ kN	L(m)	$N\left(\frac{\partial N}{\partial P}\right)L$ (kN · m)
AB	$0.6667P + 10.0$	0.6667	10.0	4	26.667
BC	$0.6667P + 10.0$	0.6667	10.0	4	26.667
AD	$-(0.8333P + 12.5)$	−0.8333	−12.5	5	52.083
BD	15.0	0	15.0	3	0
CD	$-(0.8333P + 12.5)$	−0.8333	−12.5	5	52.083
CE	$0.5P + 27.5$	0.5	27.5	3	41.25
DE	0	0	0	4	0
				Σ	198.75

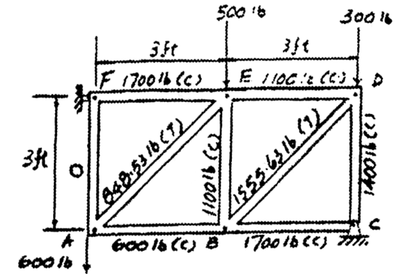
$$\Delta_{D_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

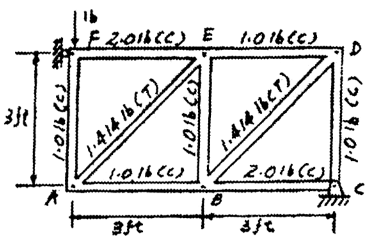
$$= \frac{198.75 \text{ kN} \cdot \text{m}}{AE} = \frac{199 \text{ kN} \cdot \text{m}}{AE} \downarrow \qquad \textbf{Ans.}$$



9–9. Determine the vertical displacement of the truss at joint F. Assume all members are pin connected at their end points. Take $A = 0.5\ \text{in}^2$ and $E = 29(10^3)$ ksi for each member. Use the method of virtual work.



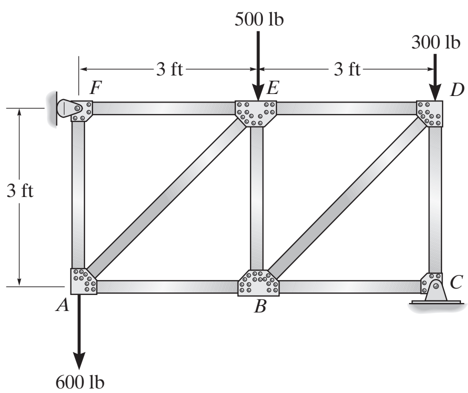




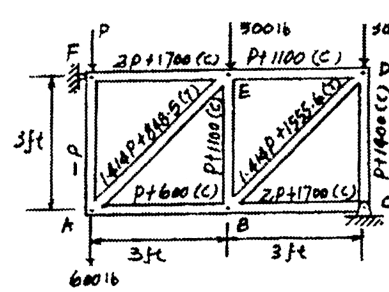
$$\Delta_{F_v} = \sum \frac{nNL}{AE} = \frac{L}{AE}[(-1.00)(-600)(3) + (1.414)(848.5)(4.243) + (-1.00)(0)(3)$$
$$+ (-1.00)(-1100)(3) + (1.414)(1555.6)(4.243) + (-2.00)(-1700)(3)$$
$$+ (-1.00)(-1400)(3) + (-1.00)(-1100)(3) + (-2.00)(-1700)(3)](12)$$

$$= \frac{47425.0(12)}{0.5(29)(10^6)} = 0.0392 \text{ in.} \downarrow$$ **Ans.**

9–10. Solve Prob. 9–9 using Castigliano's theorem.



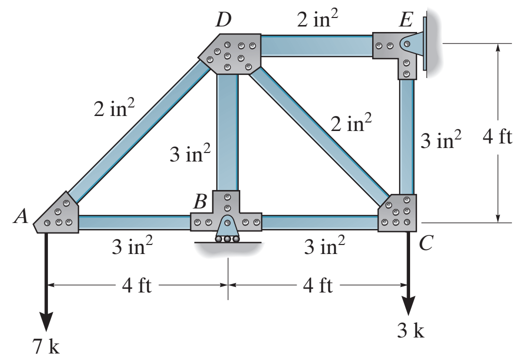
$$\Delta_{F_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{1}{AE}\ [-(P + 600)](-1)(3) + (1.414P + 848.5)(1.414)(4.243)$$
$$+ (-P)(-1)(3) + (-(P + 1100))(-1)(3)$$
$$+ (1.414P + 1555.6)(1.414)(4.243) + (-(2P + 1700))\ (-2)(3)$$
$$+ (-(P + 1400)(-1)(3) + (-(P + 1100))(-1)(3)$$
$$+ (-(2P + 1700))(-2)(3)](12) = \frac{(55.97P + 47.425.0)(12)}{(0.5(29(10)^6)}$$



Set $P = 0$ and evaluate

$$\Delta_{F_v} = 0.0392 \text{ in.} \downarrow$$ **Ans.**

9–11. Determine the vertical displacement of joint A. The cross-sectional area of each member is indicated in the figure. Assume the members are pin connected at their end points. $E = 29\,(10)^3$ ksi. Use the method of virtual work.



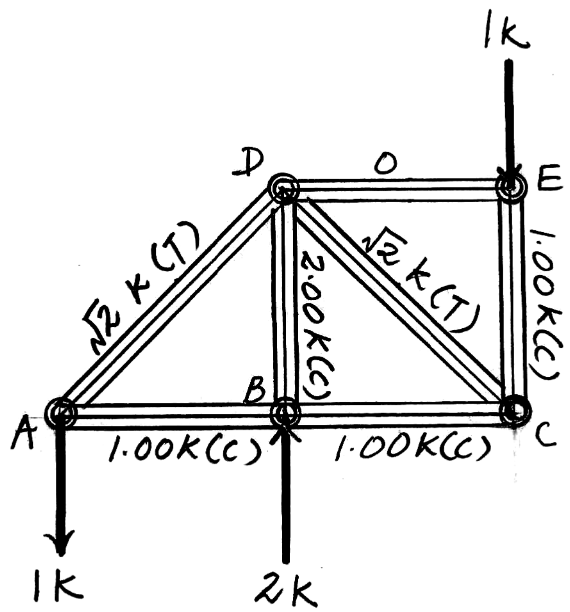
The virtual force and real force in each member are shown in Fig. *a* and *b*, respectively.

Member	$n(k)$	$N(k)$	L(ft)	$nNL(\text{k}^2 \cdot \text{ft})$
AB	−1.00	−7.00	4	28
BC	−1.00	−7.00	4	28
AD	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
BD	−2.00	−14.00	4	112
CD	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
CE	−1.00	−4.00	4	16
DE	0	0	4	0

$$1\,k \cdot \Delta_{A_v} = \sum \frac{nNL}{AE}$$

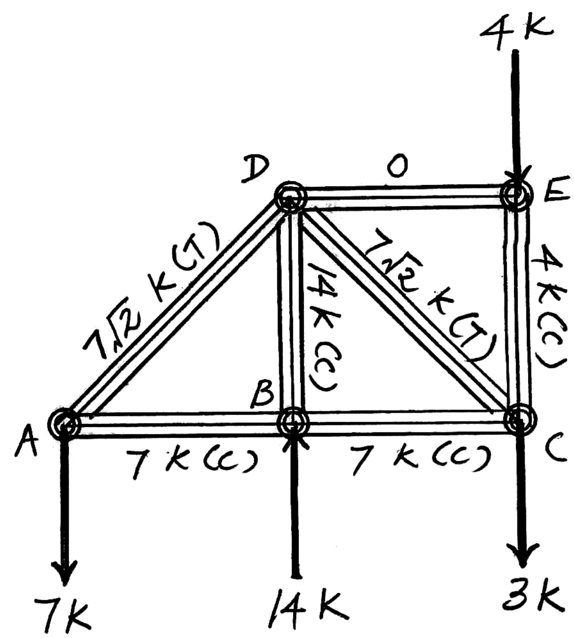
$$1\,k \cdot \Delta_{A_v} = \frac{(29 + 28 + 112 + 16)\text{k}^2 \cdot \text{ft}}{(3\text{in}^2)[29(10^3)\,\text{k/in}^2]} + \frac{(56\sqrt{2} + 56\sqrt{2})\,\text{k}^2 \cdot \text{ft}}{(2\text{in}^2)[29(10^3)\,\text{k/in}^2]}$$

$$\Delta_{A_v} = 0.004846 \text{ ft}\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 0.0582 \text{ in.} \downarrow \qquad \textbf{Ans.}$$



Virtual Forces n

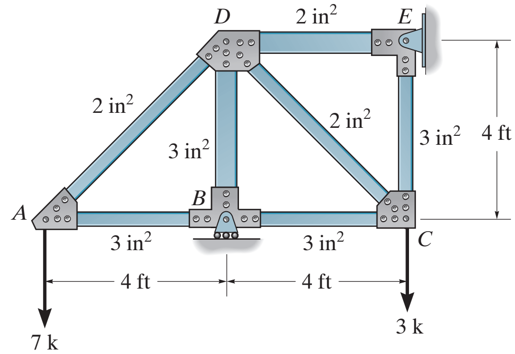
(a)



Real Forces N

(b)

***9–12.** Solve Prob. 9–11 using Castigliano's theorem.

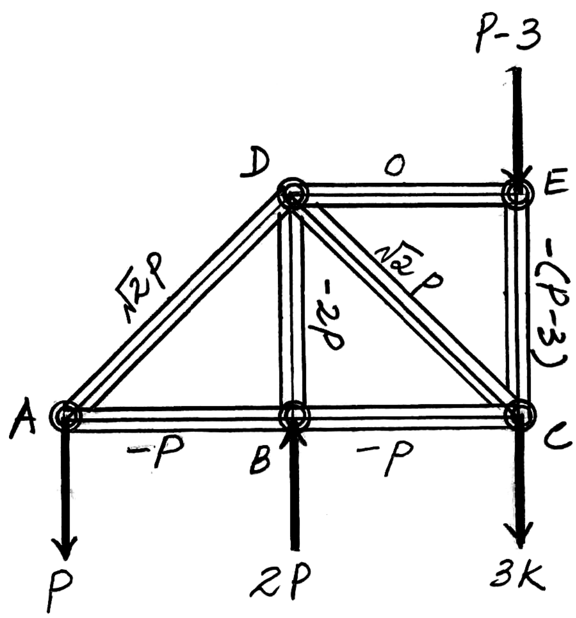


Member	$N(k)$	$\frac{\partial N}{\partial P}$	$N(P = 7k)$	L(ft)	$N\left(\frac{\partial N}{\partial P}\right)L(\text{k}\cdot\text{ft})$
AB	$-P$	-1	-7	4	28
BC	$-P$	-1	-7	4	28
AD	$\sqrt{2}P$	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
BD	$-2P$	-2	-14	4	112
CD	$\sqrt{2}P$	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
CE	$-(P-3)$	-1	-4	4	16
DE	0	0	0	4	0

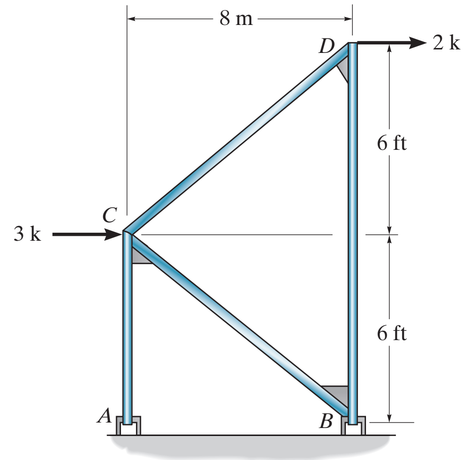
$$\Delta_{A_v} = \sum N\left(\frac{\delta N}{\delta P}\right)\frac{L}{AE}$$

$$= \frac{(28 + 28 + 112 + 16)\ \text{k}\cdot\text{ft}}{(3\ \text{in}^2)[29(10^3)\text{k/m}^2]} + \frac{56\sqrt{2} + 56\sqrt{2}\ \text{k}^2\cdot\text{ft}}{(2\ \text{in}^2)[29(10^3)\text{k/in}^2]}$$

$$= 0.004846\ \text{ft}\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = 0.0582\ \text{in}\downarrow$$ **Ans.**



9–13. Determine the horizontal displacement of joint D. Assume the members are pin connected at their end points. AE is constant. Use the method of virtual work.



The virtual force and real force in each member are shown in Fig. a and b, respectively.

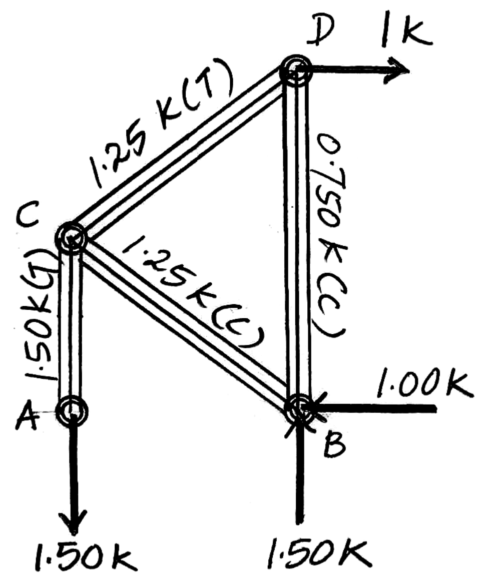
Member	$n(k)$	$N(k)$	L(ft)	$nNL(\text{k}^2 \cdot \text{ft})$
AC	1.50	5.25	6	47.25
BC	−1.25	−6.25	10	78.125
BD	−0.75	−1.50	12	13.50
CD	1.25	2.50	10	31.25
			Σ	170.125

$$1\text{k} \cdot \Delta_{D_h} = \sum \frac{nNL}{AE}$$

$$1\text{k} \cdot \Delta_{D_h} = \frac{170.125 \text{ k}^2 \cdot \text{ft}}{AE}$$

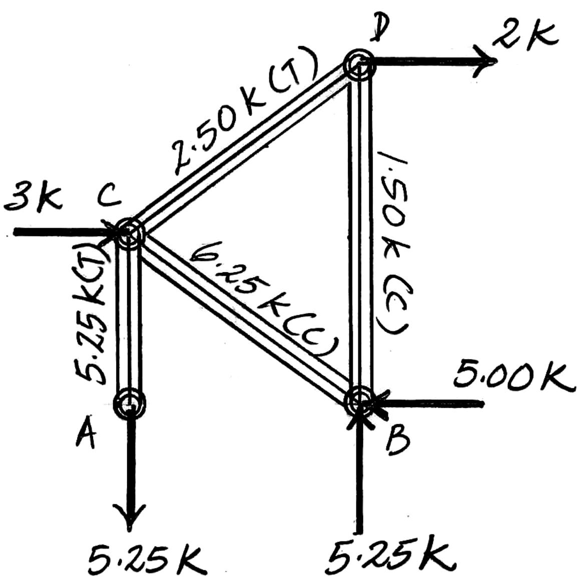
$$\Delta_{D_h} = \frac{170 \text{ k} \cdot \text{ft}}{AE} \rightarrow$$

Ans.



Virtual Forces n

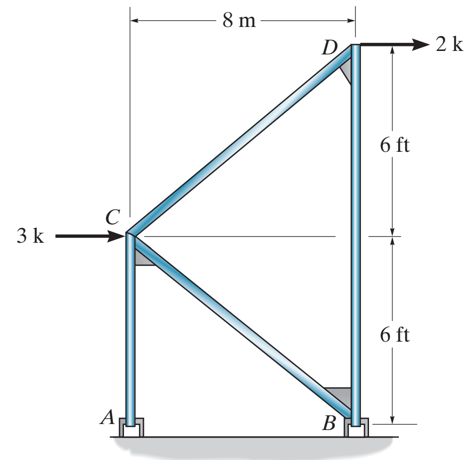
(a)



Real Forces N

(b)

9–14. Solve Prob. 9–13 using Castigliano's theorem.

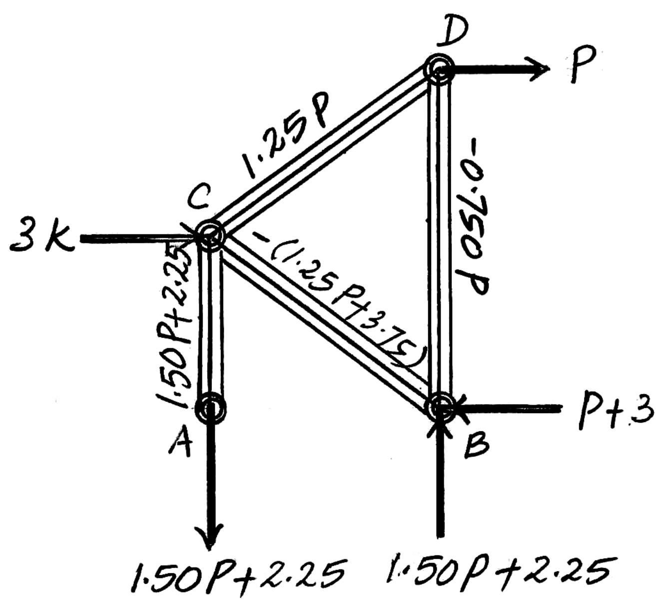


Member	$N(k)$	$\frac{\partial N}{\partial P}$	$N\ (P = 2\text{k})$	$L(\text{ft})$	$N\left(\frac{\partial N}{\partial P}\right)L(\text{k}\cdot\text{ft})$
AC	$1.50P + 2.25$	1.50	5.25	6	47.25
BC	$-(1.25P + 3.75)$	−1.25	−6.25	10	78.125
BD	$-0.750P$	−0.750	−1.50	12	13.5
CD	$1.25P$	1.25	2.50	10	31.25
				Σ	170.125

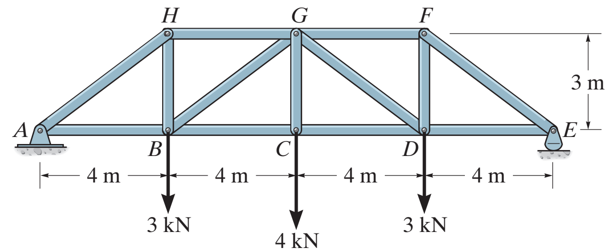
$$\Delta_{D_h} = \sum N\left(\frac{\delta N}{\delta P}\right)\frac{L}{AE}$$

$$= \frac{170.125\ \text{k}\cdot\text{ft}}{AE}$$

$$= \frac{170\ \text{k}\cdot\text{ft}}{AE} \rightarrow$$ **Ans.**



9–15. Determine the vertical displacement of joint C of the truss. Each member has a cross-sectional area of $A = 300\ \text{mm}^2$. $E = 200$ GPa. Use the method of virtual work.



The virtual and real forces in each member are shown in Fig. *a* and *b* respectively.

Member	$n(kN)$	$N(kN)$	L(m)	$nNL(\text{kN}^2 \cdot \text{m})$
AB	0.6667	6.667	4	17.78
DE	0.6667	6.667	4	17.78
BC	1.333	9.333	4	49.78
CD	1.333	9.333	4	49.78
AH	−0.8333	−8.333	5	34.72
EF	−0.8333	−8.333	5	34.72
BH	0.5	5	3	7.50
DF	0.5	5	3	7.50
BG	−0.8333	−3.333	5	13.89
DG	−0.8333	−3.333	5	13.89
GH	−0.6667	−6.6667	4	17.78
FG	−0.6667	−6.6667	4	17.78
CG	1	4	3	12.00
				$\Sigma = 294.89$

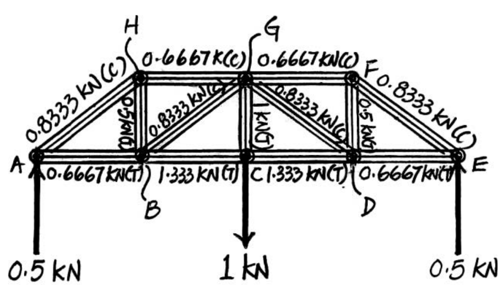
$$1\text{kN} \cdot \Delta_{C_v} = \sum \frac{nNL}{AE} = \frac{294.89\ \text{kN}^2 \cdot \text{m}}{AE}$$

$$\Delta_{C_v} = \frac{294.89\ \text{kN} \cdot \text{m}}{AE}$$

$$= \frac{294.89(10^3)\ \text{N} \cdot \text{m}}{[0.3(10^{-3})\ \text{m}^2][200(10^9)\ \text{N/m}^2]}$$

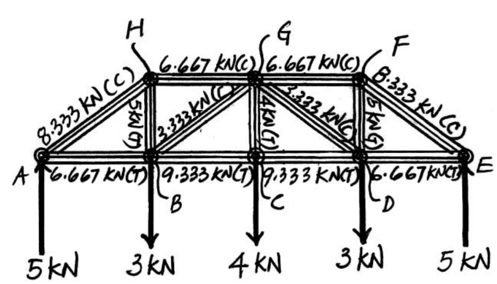
$$= 0.004914\,\text{m} = 4.91\,\text{mm} \quad \downarrow$$

Ans.



Virtual Forces n

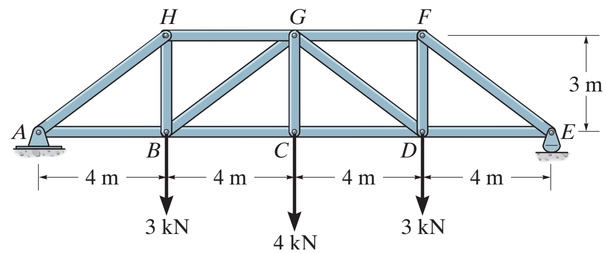
(a)



Real Forces N

(b)

***9–16.** Solve Prob. 9–15 using Castigliano's theorem.



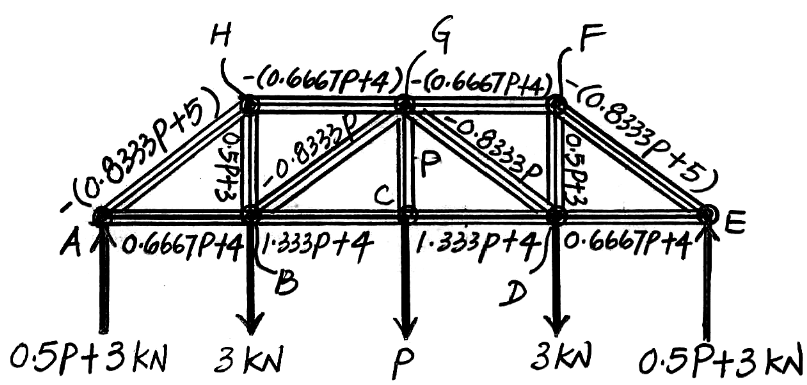
Member	$N(kN)$	$\frac{\partial N}{\partial P}$	$N(P = 4\text{ kN})$	L(m)	$N\left(\frac{\partial N}{\partial P}\right)L(\text{k}\cdot\text{m})$
AB	$0.6667P + 4$	0.6667	6.667	4	17.78
DE	$0.6667P + 4$	0.6667	6.667	4	17.78
BC	$1.333P + 4$	1.333	9.333	4	49.78
CD	$1.333P + 4$	1.333	9.333	4	49.78
AH	$-(0.8333P + 5)$	−0.8333	−8.333	5	34.72
EF	$-(0.8333P + 5)$	−0.8333	−8.333	5	34.72
BH	$0.5P + 3$	0.5	5	3	7.50
DF	$0.5P + 3$	0.5	5	3	7.50
BG	$-0.8333P$	−0.8333	−3.333	5	13.89
DG	$-0.8333P$	−0.8333	−3.333	5	13.89
GH	$-(0.6667P + 4)$	−0.6667	−6.667	4	17.78
FG	$-(0.6667P + 4)$	−0.6667	−6.667	4	17.78
CG	P	1	4	3	12.00
				Σ	294.89

$$\Delta_{C_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{294.89\text{ kN}\cdot\text{m}}{AE}$$

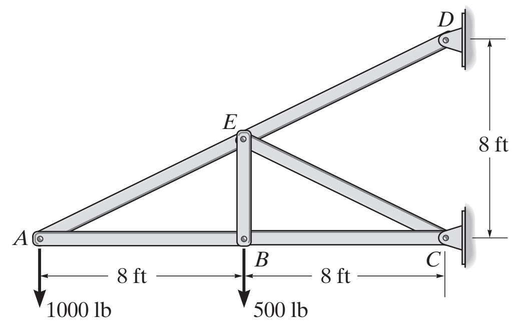
$$= \frac{294.89(10^3)\text{ N}\cdot\text{m}}{[0.3(10^{-3})\text{ m}^2][200(10^9)\text{ N/m}^2]}$$

$$= 0.004914\text{ m}$$

$$= 4.91\text{ mm} \quad \downarrow$$ **Ans.**

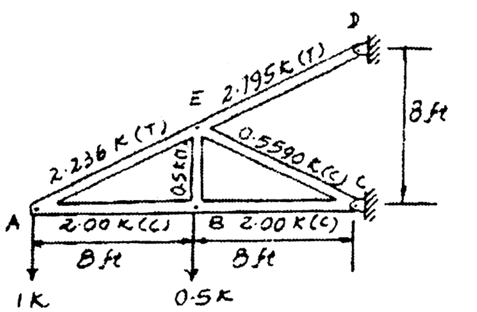


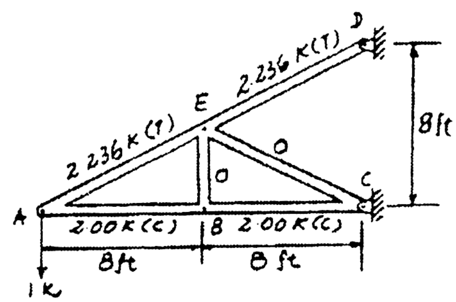
9–17. Determine the vertical displacement of joint A. Assume the members are pin connected at their end points. Take $A = 2\ \text{in}^2$ and $E = 29\ (10^3)$ for each member. Use the method of virtual work.



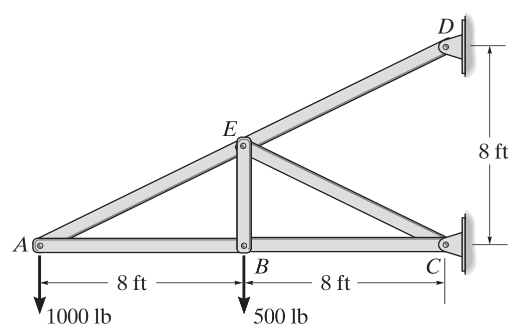
$$\Delta_{A_v} = \sum \frac{nNL}{AE} = \frac{1}{AE}[2(-2.00)(-2.00)(8) + (2.236)(2.236)(8.944) + (2.236)(2.795)(8.944)]$$

$$= \frac{164.62(12)}{(2)(29)(10^3)} = 0.0341 \text{ in.} \downarrow$$ **Ans.**





9–18. Solve Prob. 9–17 using Castigliano's theorem.

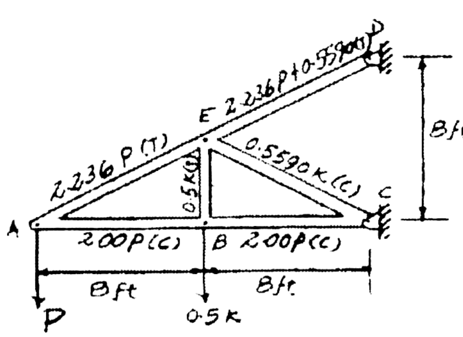


$$\Delta_{A_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{1}{AE}[-2P(-2)(8) + (2.236P)(2.236)(8.944)$$

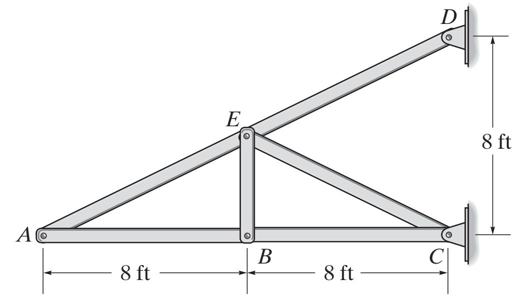
$$+ (-2P)(-2)(8) + (2.236P + 0.5590)(2.236)(8.944)](12)$$

Set $P = 1$ and evaluate

$$\Delta_{A_v} = \frac{164.62(12)}{(2)(29)(10^3)} = 0.0341 \text{ in.} \downarrow$$ **Ans.**

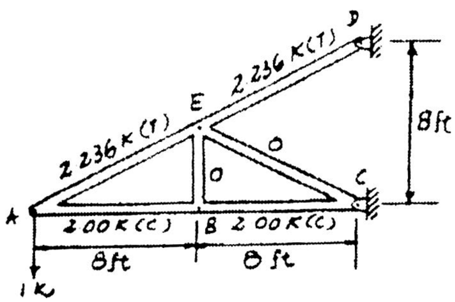


9–19. Determine the vertical displacement of joint A if members AB and BC experience a temperature increase of $\Delta T = 200°\text{F}$. Take $A = 2\ \text{in}^2$ and $E = 29(10^3)$ ksi. Also, $\alpha = 6.60\ (10^{-6})/°\text{F}$.

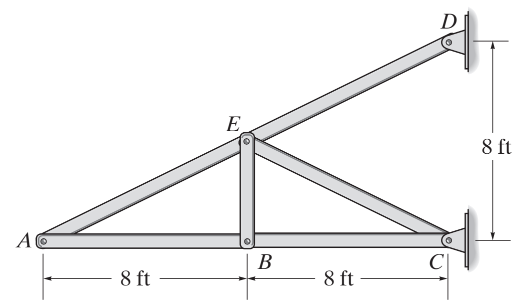


$$\Delta_{A_v} = \sum n\alpha\Delta T L = (-2)(6.60)(10^{-6})(200)(8)(12) + (-2)(6.60)(10^{-6})(200)(8)(12)$$

$$= -0.507\ \text{in.} = 0.507\ \text{in.}\ \uparrow$$ **Ans.**

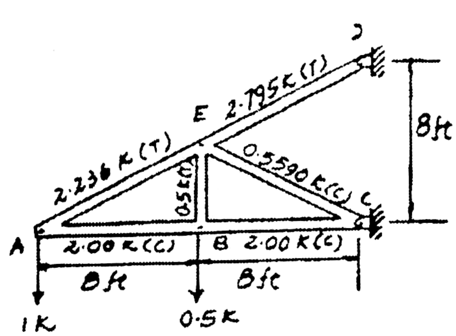


***9–20.** Determine the vertical displacement of joint A if member AE is fabricated 0.5 in. too short.

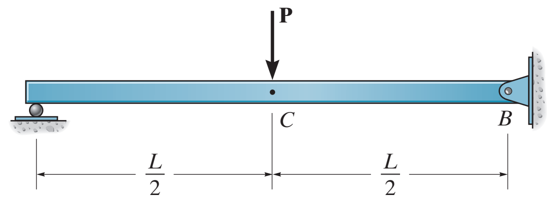


$$\Delta_{A_v} = \sum n\Delta L = (2.236)(-0.5)$$

$$= -1.12\ \text{in} = 1.12\ \text{in.}\ \uparrow$$ **Ans.**



9–21. Determine the displacement of point C and the slope at point B. EI is constant. Use the principle of virtual work.



Real Moment function M(x): As shown on figure (*a*).

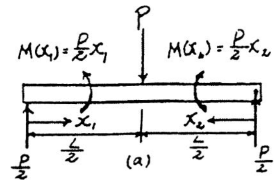
Virtual Moment Functions m(x) and $\mathbf{m}_\theta$***(x):*** As shown on figure (*b*) and (*c*).

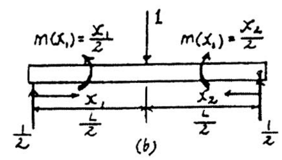
Virtual Work Equation: For the displacement at point C,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{1}} \left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right)dx_1\right]$$

$$\Delta_C = \frac{PL^3}{48\,EI} \quad \downarrow$$ **Ans.**



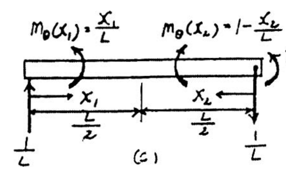


For the slope at B,

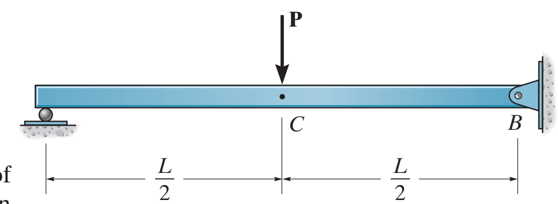
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

$$1 \cdot \theta_B = \frac{1}{EI}\left[\int_0^{\frac{L}{1}} \left(\frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right)dx_1 + \int_0^{\frac{L}{2}} \left(1 - \frac{x_2}{L}\right)\left(\frac{P}{2}x_2\right)dx_2\right]$$

$$\theta_B = \frac{PL^2}{16EI} \quad \measuredangle$$ **Ans.**



9–22. Solve Prob. 9–21 using Castigliano's theorem.



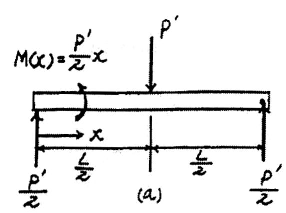
Internal Moment Function M(x): The internal moment function in terms of the load **P**' and couple moment **M**' and externally applied load are shown on figures (*a*) and (*b*), respectively.

Castigliano's Second Theorem: The displacement at C can be determined with $\frac{\partial M(x)}{\partial P'} = = \frac{x}{2}$ and set $P' = P$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{dx}{EI}$$

$$\Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{1}} \left(\frac{P}{2}x\right)\left(\frac{x}{2}\right)dx\right]$$

$$= \frac{PL^3}{48EI} \quad \downarrow$$ **Ans.**

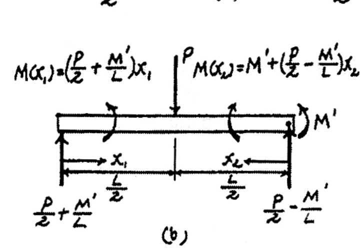


To determine the slope at B, with $\frac{\partial M(x_1)}{\partial M'} = \frac{x_1}{L}, \frac{\partial M(x_2)}{\partial M'} = 1 - \frac{x_2}{L}$ and setting $M' = 0$.

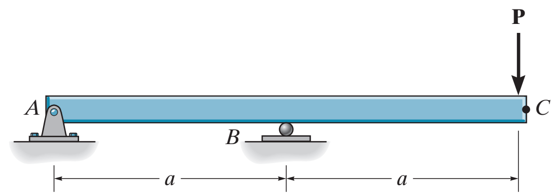
$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

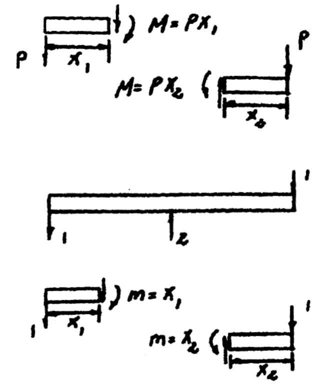
$$\theta_B = \frac{1}{EI}\int_0^{\frac{L}{1}} \left(\frac{P}{2}x_1\right)\left(\frac{x_1}{L}\right)dx_1 + \frac{1}{EI}\int_0^{\frac{L}{2}} \left(\frac{P}{2}x_2\right)\left(1 - \frac{x_2}{L}\right)dx_2$$

$$= \frac{PL^2}{16EI} \quad \measuredangle$$ **Ans.**



9–23. Determine the displacement at point C. EI is constant. Use the method of virtual work.



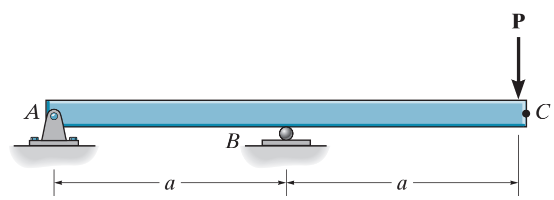


$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}\left[\int_o^a (x_1)(Px_1)dx_1 + \int_0^a (x_2)(Px_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI} \downarrow \qquad \textbf{Ans.}$$

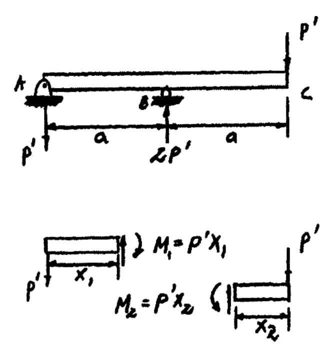
***9–24.** Solve Prob. 9–23 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P'} = x_1 \qquad \frac{\partial M_2}{\partial P'} = x_2$$

Set $P = P'$

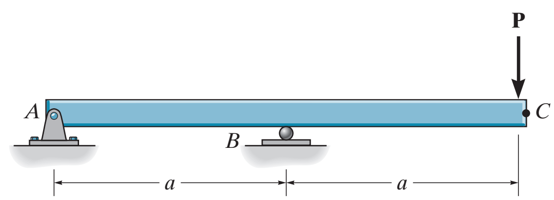
$$M_1 = Px_1 \qquad M_2 = Px_2$$

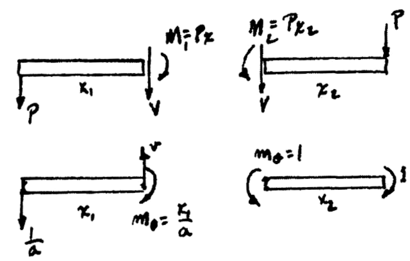


$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)dx = \frac{1}{EI}\left[\int_0^a (Px_1)(x_1)dx_1 + \int_0^a (Px_2)(x_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI} \qquad \textbf{Ans.}$$

9–25. Determine the slope at point C. EI is constant. Use the method of virtual work.



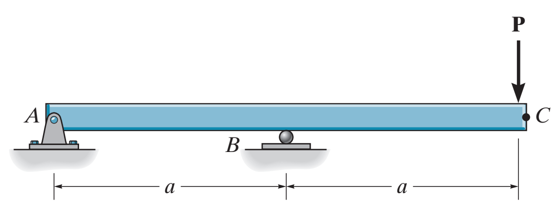


$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

$$\theta_C = \int_0^a \frac{(x_1/a)Px_1 dx_1}{EI} + \int_0^a \frac{(1)Px_2 dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$ **Ans.**

9–26. Solve Prob. 9–25 using Castigliano's theorem.

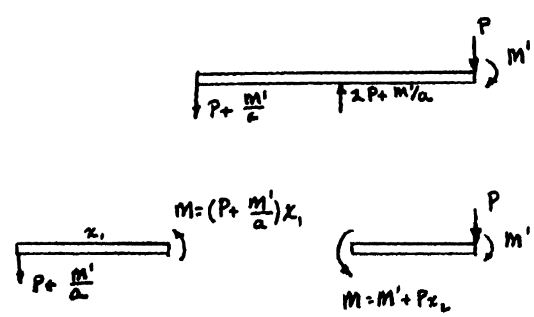


Set $M' = 0$

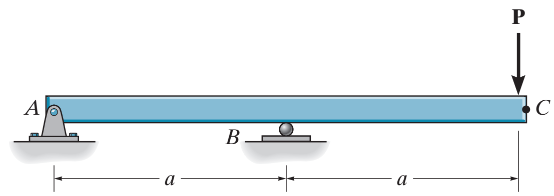
$$\theta_C = \int_0^L M\left(\frac{\delta M}{\delta M'}\right)\frac{dx}{EI}$$

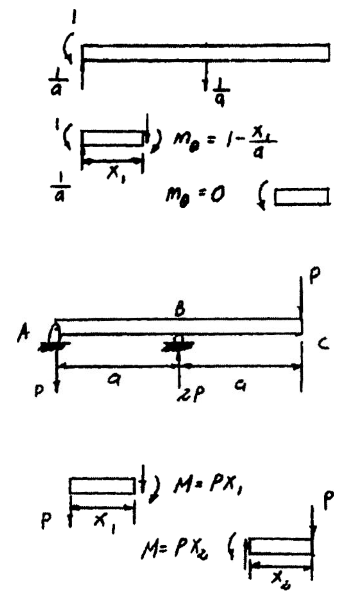
$$= \int_0^a \frac{(Px_1)(\frac{1}{a}x_1)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$ **Ans.**



9–27. Determine the slope at point A. EI is constant. Use the method of virtual work.



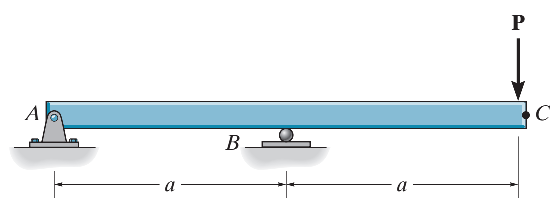


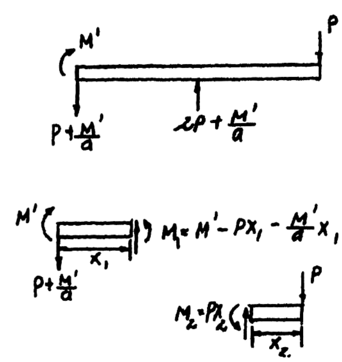
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)dx_1 + \int_0^a (0)(Px_2)dx_2\right] = \frac{Pa^2}{6EI} \quad \triangledown$$

Ans.

***9–28.** Solve Prob. 9–27 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \qquad \frac{\partial M_2}{\partial M'} = 0$$

Set $M' = 0$

$$M_1 = -Px_1 \qquad M_2 = Px_2$$

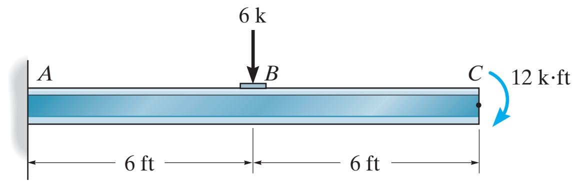
$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right]$$

$$= \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI}$$

Ans.

9–29. Determine the slope and displacement at point C. Use the method of virtual work. $E = 29(10^3)$ ksi, $I = 800\text{ in}^4$.



Referring to the virtual moment functions indicated in Fig. *a* and *b* and the real moment function in Fig. *c*, we have

$$1\text{k}\cdot\text{ft}\cdot\theta_c = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{6\text{ ft}} \frac{(-1)(-12)}{EI}dx_1 + \int_0^{6\text{ ft}} \frac{(-1)[-(6x_2+12)]}{EI}dx_2$$

$$= \frac{252\text{ k}^2\cdot\text{ft}^3}{EI}$$

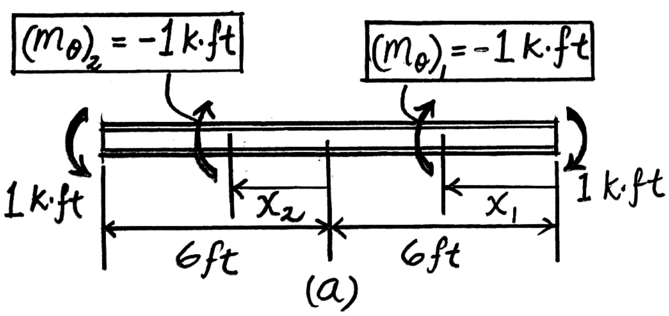
$$\theta_c = \frac{252\text{ k}\cdot\text{ft}^2}{EI} = \frac{252(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](800\text{ in}^4)} = 0.00156\text{ rad}$$ **Ans.**

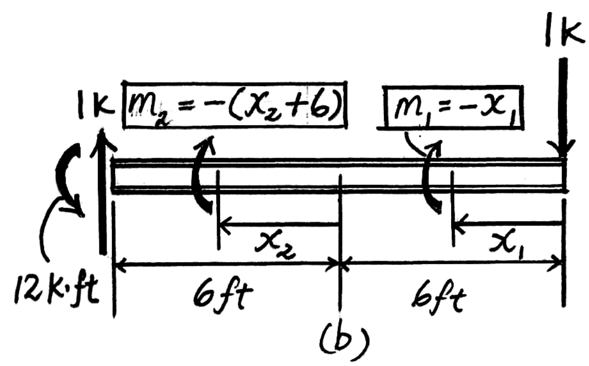
and

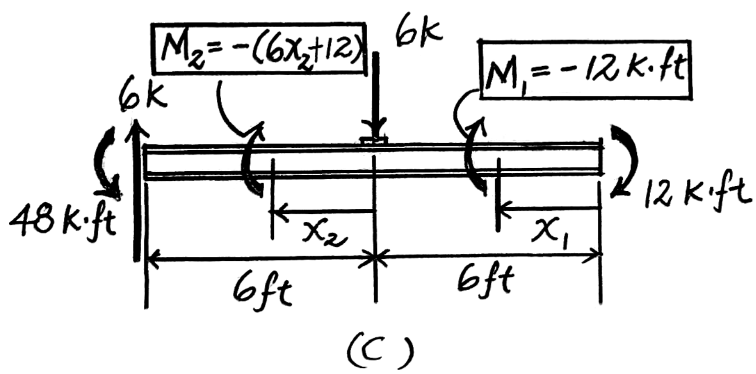
$$1\text{k}\cdot\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^{6\text{ ft}} \frac{(-x_1)(-12)}{EI}dx_1 + \int_0^{6\text{ ft}} \frac{([-(x_2+6)][-(6x_2+12)]}{EI}dx_2$$

$$= \frac{1944\text{ k}^2\cdot\text{ft}^3}{EI}$$

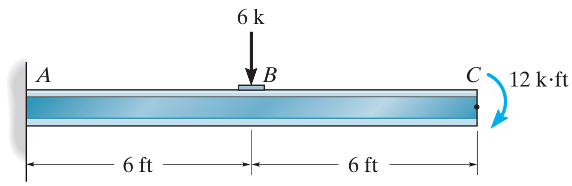
$$\Delta_C = \frac{1944\text{ k}\cdot\text{ft}^3}{EI} = \frac{1944(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](800\text{ in}^4)} = 0.415\text{ in} \downarrow$$ **Ans.**







9–30. Solve Prob. 9–29 using Castigliano's theorem.



For the slope, the moment functions are shown in Fig. *a*. Here, $\frac{\partial M_1}{\partial M'} = -1$ and $\frac{\partial M_2}{\partial M'} = -1$. Also, set $M' = 12$ kft, then $M_1 = -12 \text{ k} \cdot \text{ ft}$ and $M_2 = -(6x_2 + 12) \text{ k} \cdot \text{ft}$. Thus,

$$\theta_c = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{6 \text{ ft}} \frac{(-12)(-1)}{EI}dx_2 + \int_0^{6 \text{ ft}} \frac{-(6x_2 + 12(-1)}{EI}dx_2$$

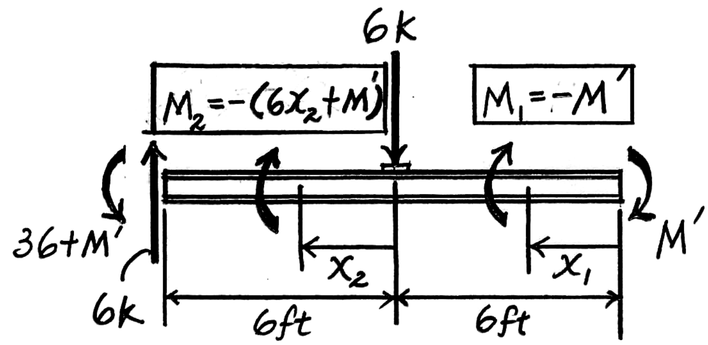
$$\theta_c = \frac{252 \text{ k} \cdot \text{ft}^2}{EI} = \frac{252(12^2) \text{ k} \cdot \text{in}^2}{[29(10^3 \text{ k/in}^2](800 \text{ in}^4)} = 0.00156 \text{ rad} \quad \text{Ans.}$$

For the displacement, the moment functions are shown in Fig. *b*. Here, $\frac{\partial M_1}{\partial P} = -x_1$ and $\frac{\partial M_2}{\partial P} = -(x_2 + 6)$. Also set, $P = 0$, then $M_1 = -12 \text{ k} \cdot \text{ft}$ and $M_2 = -(6x_2 + 12) \text{ k} \cdot \text{ft}$. Thus,

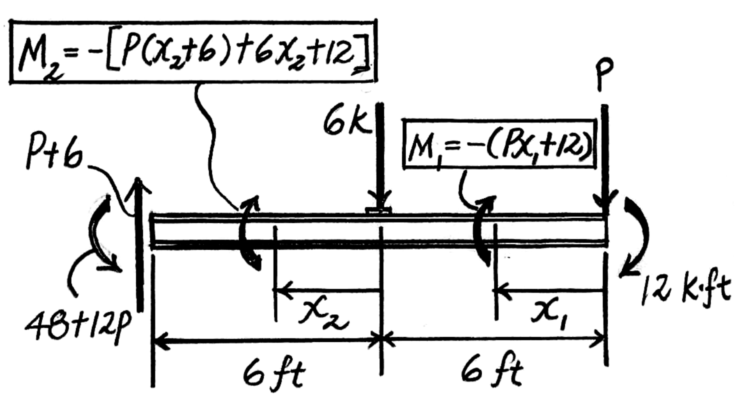
$$\Delta_C = \int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{6 \text{ ft}} \frac{(-12)(-x_1)}{EI}dx_1 + \int_0^{6 \text{ ft}} \frac{-(6x_2 + 12)[-(x_2 + 6)]}{EI}dx_2$$

$$= \frac{1944 \text{ k} \cdot \text{ft}^3}{EI}$$

$$= \frac{1944(12^3) \text{ k} \cdot \text{in}^3}{[29(10^3) \text{ k/in}^2](800 \text{ in}^4)} = 0.145 \text{ in} \downarrow \quad \text{Ans.}$$

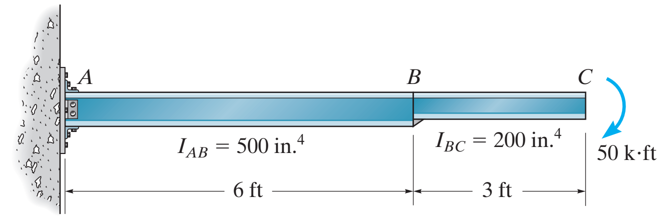


(a)



(b)

9–31. Determine the displacement and slope at point C of the cantilever beam. The moment of inertia of each segment is indicated in the figure. Take $E = 29(10^3)$ ksi. Use the principle of virtual work.



Referring to the virtual moment functions indicated in Fig. a and b and the real moment function in Fig. c, we have

$$1\text{k}\cdot\text{ft}\cdot\theta_c = \int_0^L \frac{m_0 M}{EI}dx = \int_0^{3\text{ ft}} \frac{(-1)(-50)}{EI_{BC}}dx_1 + \int_0^{6\text{ ft}} \frac{(-1)(-50)}{EI_{AB}}dx_2$$

$$1\text{k}\cdot\text{ft}\cdot\theta_c = \frac{150\text{ k}^2\cdot\text{ft}^3}{EI_{BC}} + \frac{300\text{ k}^2\cdot\text{ft}^3}{EI_{AB}}$$

$$\theta_c = \frac{150\text{ k}\cdot\text{ft}^2}{EI_{BC}} + \frac{300\text{ k}\cdot\text{ft}^2}{EI_{AB}}$$

$$= \frac{150(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](200\text{ in}^4)} + \frac{300(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)}$$

$$= 0.00670\text{ rad}$$ **Ans.**

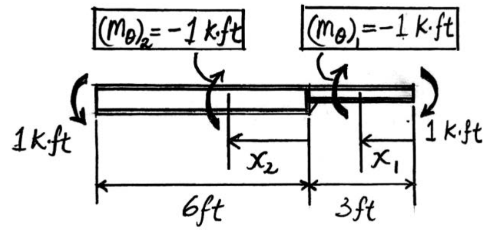
And

$$1\text{ k}\cdot\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^{3\text{ ft}} \frac{-x_1(-50)}{EI_{BC}}dx_1 + \int_0^{6\text{ ft}} \frac{-(x_2+3)(-50)}{EI_{AB}}dx_2$$

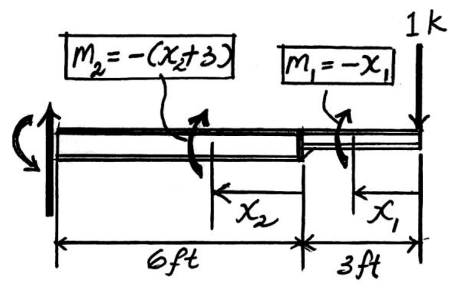
$$1\text{ k}\cdot\Delta_C = \frac{225\text{ k}^2\cdot\text{ft}^3}{EI_{BC}} + \frac{1800\text{ k}^2\cdot\text{ft}^3}{EI_{AB}}$$

$$\Delta_C = \frac{225\text{ k}\cdot\text{ft}^3}{EI_{BC}} + \frac{1800\text{ k}^2\cdot\text{ft}^3}{EI_{AB}}$$

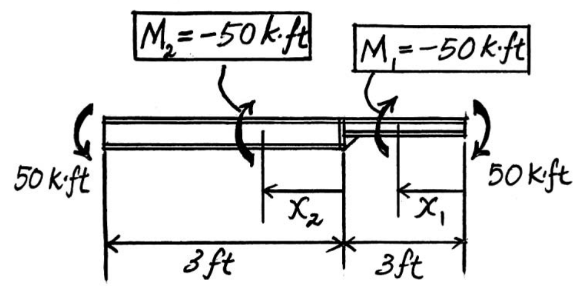
$$= \frac{225(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](200\text{ in}^4)} + \frac{1800(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)} = 0.282\text{ in}\downarrow$$ **Ans.**



(a)

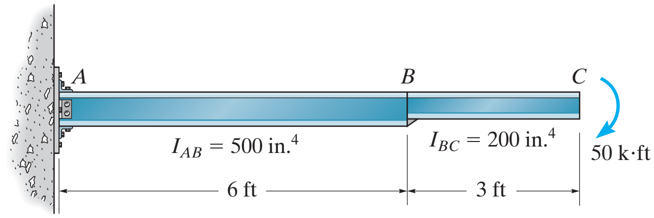


(b)



(c)

***9–32.** Solve Prob. 9–31 using Castigliano's theorem.



For the slope, the moment functions are shown in Fig. *a*. Here, $\dfrac{\partial M_1}{\partial M'} = \dfrac{\partial M_2}{\partial M'} = -1.$

Also, set $M' = 50\text{ k}\cdot\text{ft}$, then $M_1 = M_2 = -50\text{ k}\cdot\text{ft}$.

Thus,

$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{3\text{ ft}}\frac{-50(-1)dx}{EI_{BC}} + \int_0^{6\text{ ft}}\frac{-50(-1)dx}{EI_{AB}}$$

$$= \frac{150\text{ k}\cdot\text{ft}^2}{EI_{BC}} + \frac{300\text{ k}\cdot\text{ft}^2}{EI_{AB}}$$

$$= \frac{150(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3\text{ k/in}^2)](200\text{ in}^4)} + \frac{300(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)}$$

$$= 0.00670 \;\; \triangledown$$ **Ans.**

For the displacement, the moment functions are shown in Fig, *b*. Here, $\dfrac{\partial M_1}{\partial P} = -x_1$

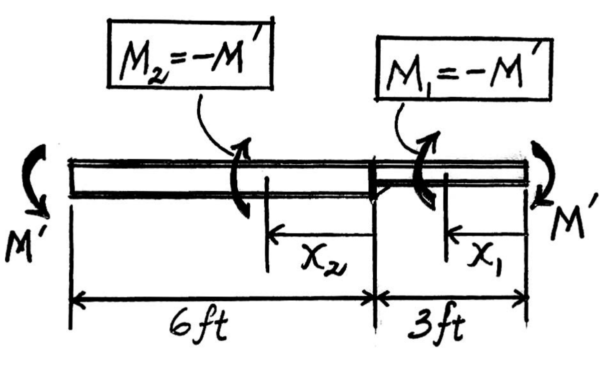
and $\dfrac{\partial M_2}{\partial P} = -(x_2 + 3)$. Also, set $P = 0$, then $M_1 = M_2 = -50\text{ k}\cdot\text{ft}$. Thus,

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{3\text{ ft}}\frac{(-50)(-x)dx}{EI_{BC}} + \int_0^{6\text{ ft}}\frac{(-50)[-(x_2 + 3)]dx}{EI_{AB}}$$

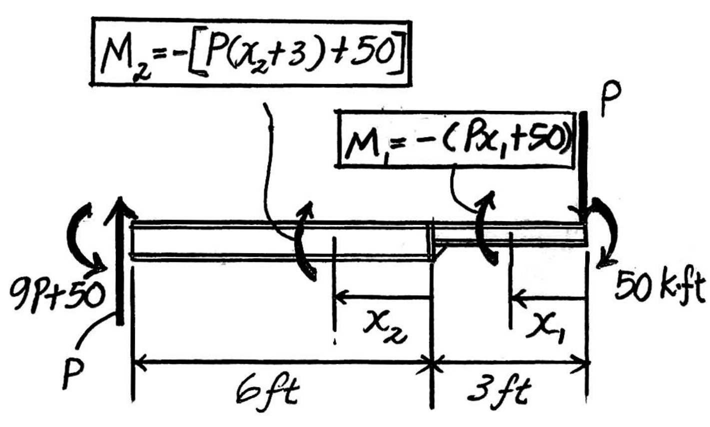
$$= \frac{225\text{ k}\cdot\text{ft}^3}{EI_{BC}} + \frac{1800\text{ k}\cdot\text{ft}^3}{EI_{AB}}$$

$$= \frac{225(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](200\text{ in}^4)} + \frac{1800(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)}$$

$$= 0.282\text{ in}\downarrow$$ **Ans.**

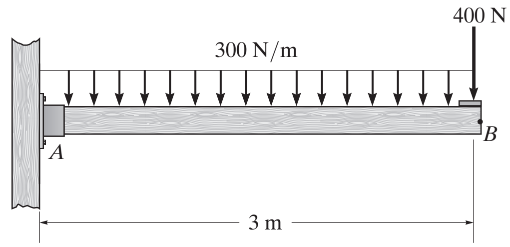


(a)



(b)

9–33. Determine the slope and displacement at point B. EI is constant. Use the method of virtual work.



Referring to the virtual moment function indicated in Fig. *a* and *b*, and real moment function in Fig. *c*, we have

$$1\text{ N}\cdot\text{m}\cdot\theta_B = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{3\text{ m}} \frac{(-1)[-(150x^2 + 400x)]}{EI}dx$$

$$1\text{ N}\cdot\text{m}\cdot\theta_B = \frac{3150\text{ N}^2\cdot\text{m}^3}{EI}$$

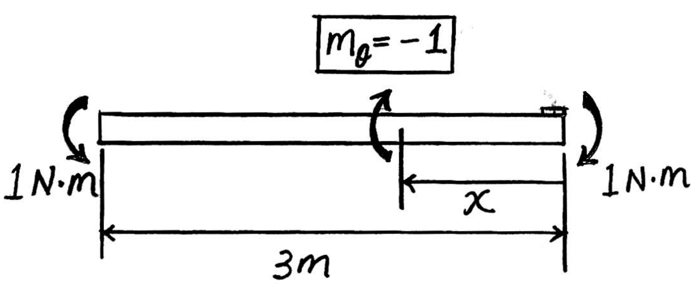
$$\theta_B = \frac{3150\ N\cdot\text{m}^2}{EI} \quad \triangledown \qquad \textbf{Ans.}$$

And

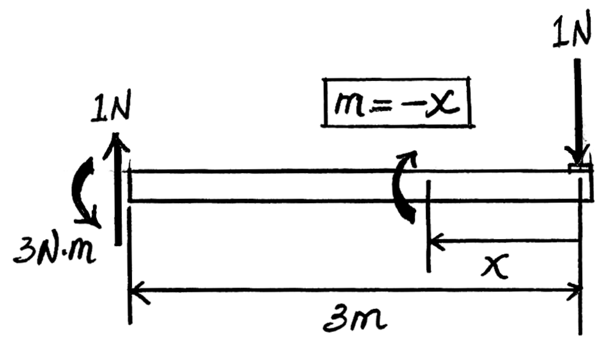
$$1\text{ N}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{3\text{ m}} \frac{(-x)[-(150x^2 + 400x)]}{EI}$$

$$1\text{ N}\cdot\Delta_B = \frac{6637.5\text{ N}^2\cdot\text{m}^3}{EI}$$

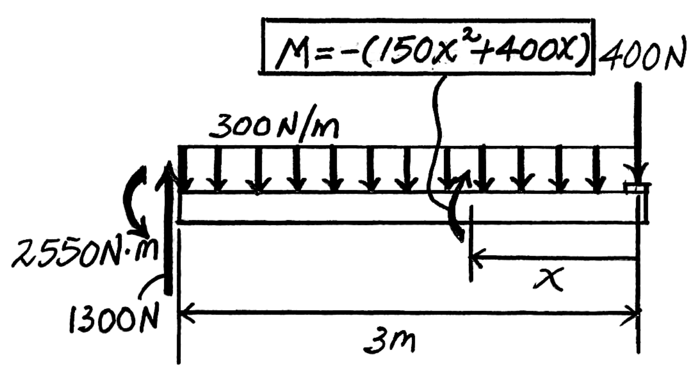
$$\Delta_B = \frac{6637.5\text{ N}\cdot\text{m}^3}{EI} \quad \downarrow \qquad \textbf{Ans.}$$



(a)

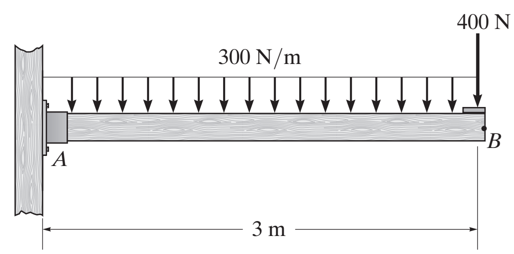


(b)



(c)

9–34. Solve Prob. 9–33 using Castigliano's theorem.



For the slope, the moment function is shown in Fig. *a*. Here, $\frac{\partial M}{\partial M'} = -1$.

Also, set $M' = 0$, then $M = -(150x^2 + 400x)$ N · m. Thus,

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{-(150x^2 + 400x)(-1)}{EI}dx$$

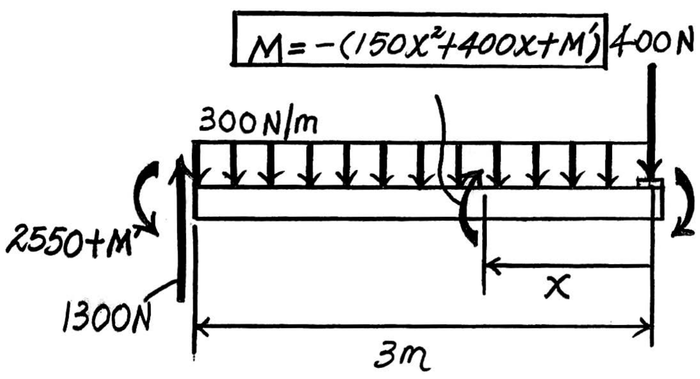
$$= \frac{3150 \text{ N}\cdot\text{m}^2}{EI} \quad \text{(cw)}$$ **Ans.**

For the displacement, the moment function is shown in Fig. *b*. Here, $\frac{\partial M}{\partial P} = -x$.

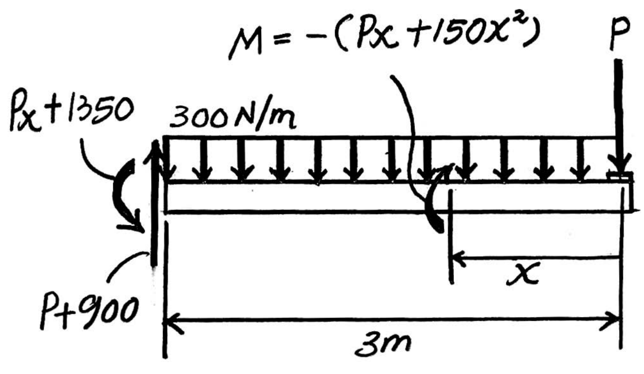
Also, set $P = 400$ N, then $M = (400x + 150x^2)$ N · m. Thus,

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{-(400x + 150x^2)(-x)}{EI}dx$$

$$= \frac{6637.5 \text{ N}\cdot\text{m}^3}{EI}\downarrow$$ **Ans.**

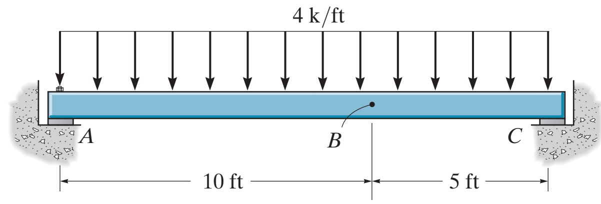


(a)



(b)

9–35. Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take $E = 29(10^3)$ ksi, $I = 300\ \text{in}^4$. Use the method of virtual work.

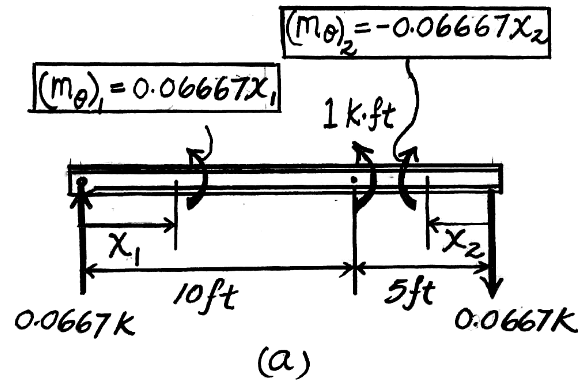


Referring to the virtual moment functions shown in Fig. a and b and the real moment function shown in Fig. c,

$$1\ \text{k}\cdot\text{ft}\cdot\theta_B = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{10\ \text{ft}} \frac{(0.06667x_1)(30x_1 - 2x_1^2)dx_1}{EI} + \int_0^{5\ \text{ft}} \frac{(-0.06667x_2)(30x_2 - 2x_2^2)}{EI}dx_2$$

$$1\ \text{k}\cdot\text{ft}\cdot\theta_B = \frac{270.83\ \text{k}^2\cdot\text{ft}^3}{EI}$$

$$\theta_B = \frac{270.83\ \text{k}\cdot\text{ft}^2}{EI} = \frac{270.83(12^2)\ \text{k}\cdot\text{in}^2}{[29(10^3)\ \text{k/in}^2](300\ \text{in}^4)} = 0.00448\ \text{rad} \quad \measuredangle \quad \textbf{Ans.}$$

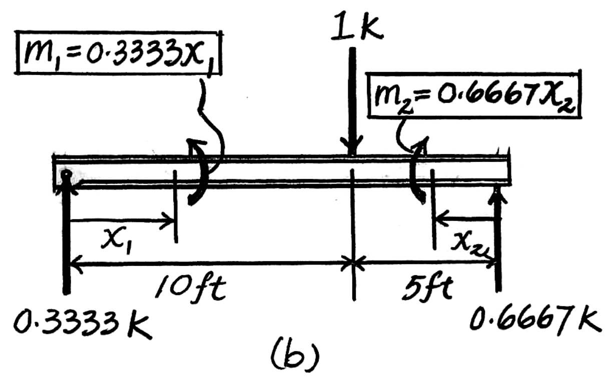


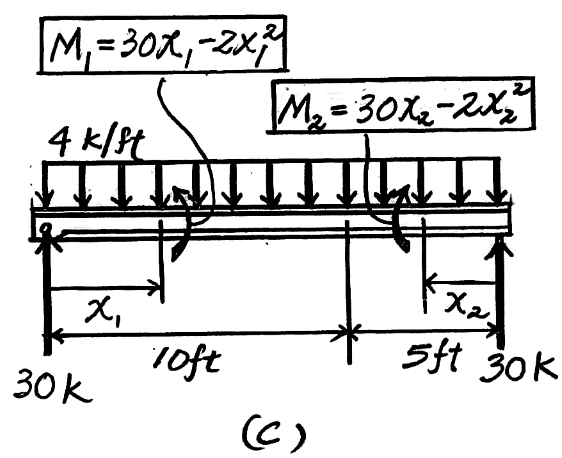
And

$$1\ \text{k}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{10\ \text{ft}} \frac{(0.3333x_1)(30x_1 - 2x_1^2)dx_1}{EI} + \int_0^{5\ \text{ft}} \frac{(0.6667x_2)(30x_2 - 2x_2^2)dx_2}{EI}$$

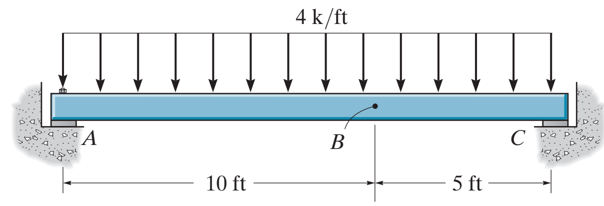
$$1\ \text{k}\cdot\Delta_B = \frac{2291.67\ \text{k}\cdot\text{ft}^3}{EI}$$

$$\Delta_B = \frac{2291.67\ \text{k}\cdot\text{ft}^3}{EI} = \frac{2291.67(12^3)\ \text{k}\cdot\text{in}^3}{[29(10^3)\ \text{k/in}^2](300\ \text{in}^4)} = 0\cdot 455\ \text{in} \downarrow \quad \textbf{Ans.}$$





***9–36.** Solve Prob. 9–35 using Castigliano's theorem.



For the slope, the moment functions are shown in Fig. *a*. Here,

$\frac{\partial M_1}{\partial M'} = 0.06667x_1$ and $\frac{\partial M_2}{\partial M'} = 0.06667x_2$. Also, set $M' = 0$, then

$M_1 = (30x_1 - 2x_1^2)$ k·ft and $M_2 = (30x_2 - 2x_2^2)$ k·ft. Thus,

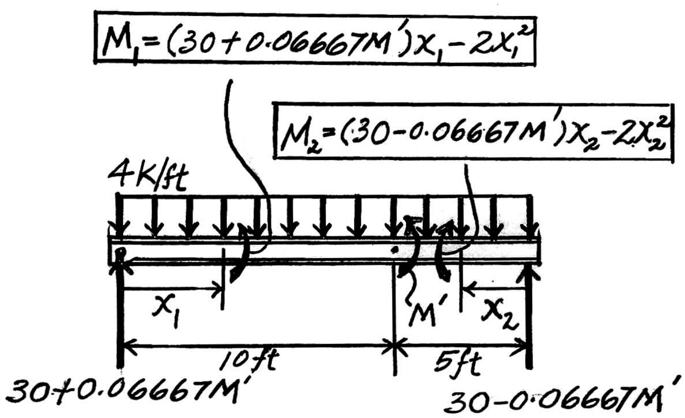
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{10\text{ ft}} \frac{(30x_1 - 2x_1^2)(0.06667x_1)dx_1}{EI} + \int_0^{5\text{ ft}} \frac{(30x_2 - 2x_2^2)(0.06667x_2)dx_2}{EI}$$

$$= \frac{270.83\text{ k}\cdot\text{ft}^2}{EI} = \frac{270.83(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{k/in}^2](300\text{ in}^4)} = 0.00448\text{ rad} \quad \measuredangle$$ **Ans.**

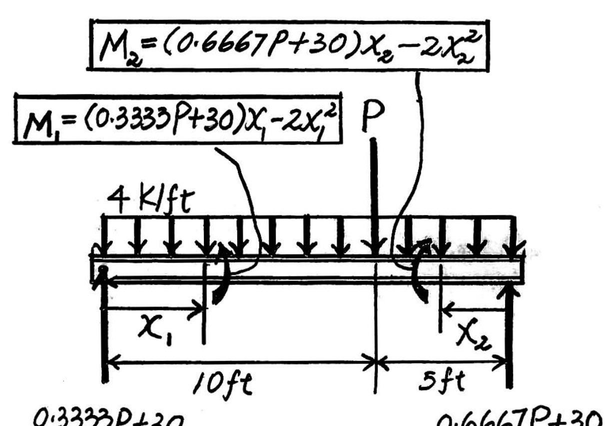
For the displacement, the moment fractions are shown in Fig. *b*. Here,
$\frac{\partial M_1}{\partial p} = 0.3333x_1$ and $\frac{\partial M_2}{\partial P} = 0.6667x_2$. Also, set $P = 0$, then
$M_1 = (30x_1 - 2x_1^2)$ k·ft and $M_2 = (30x_2 - 2x_2^2)$ k·ft. Thus

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{10\text{ ft}} \frac{30x_1 - 2x_1^2(0.3333x_1)dx_1}{EI} + \int_0^{5\text{ ft}} \frac{(30x_2 - 2x_2^2)(0.6667x_2)dx_2}{EI}$$

$$= \frac{2291.67\text{ k}\cdot\text{ft}^3}{EI} = \frac{2291.67(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{k/in}^2](300\text{ in}^4)} = 0.455\text{ in} \downarrow$$ **Ans.**

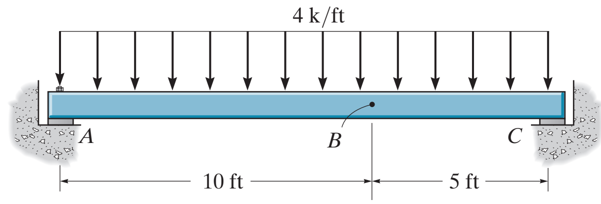


(a)



(b)

9–37. Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Account for the additional strain energy due to shear. Take $E = 29(10^3)$ ksi, $I = 300\text{ in}^4$, $G = 12(10^3)$ ksi, and assume AB has a cross-sectional area of $A = 7.50\text{ in}^2$. Use the method of virtual work.



The virtual shear and moment functions are shown in Fig. *a* and *b* and the real shear and moment functions are shown in Fig. *c*.

$$1\text{ k}\cdot\text{ft}\cdot\theta_B = \int_0^L \frac{m_\theta M}{EI}dx + \int_0^L \text{k}\left(\frac{\nu V}{GA}\right)dx$$

$$= \int_0^{10\text{ ft}} \frac{0.06667x_1(30x_1 - 2x_1^2)}{EI}dx_1 + \int_0^{10\text{ ft}} 1\left[\frac{0.06667(30 - 4x_1)}{GA}\right]dx_1$$

$$+ \int_0^{5\text{ ft}} \frac{(-0.06667x_2(30x_2 - 2x_2^2)}{EI}dx_2 + \int_0^{5\text{ ft}} 1\left[\frac{0.06667(4x_2 - 30)}{GA}\right]dx_2$$

$$= \frac{270.83\text{ k}^2\cdot\text{ft}^3}{EI} + 0$$

$$\theta_B = \frac{270.83\text{ k}\cdot\text{ft}^2}{EI} = \frac{270.83(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{k/in}^2(300\text{ in}^4)]} = 0.00448\text{ rad} \quad \measuredangle$$ **Ans.**

And

$$1\text{ k}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx + \int_0^L \text{k}\left(\frac{\nu V}{GA}\right)dx$$

$$= \int_0^{10\text{ ft}} \frac{(0.3333x_1)(30x_1 - 2x_1^2)}{EI}dx_1 + \int_0^{10\text{ ft}} 1\left[\frac{0.3333(30 - 4x_1)}{GA}\right]dx_1$$

$$+ \int_0^{5\text{ ft}} \frac{(0.6667x_2)(30x_2 - 2x_2^2)}{EI}dx_2 + \int_0^{5\text{ ft}} 1\left[\frac{(-0.6667)(4x_2 - 30)}{GA}\right]dx_2$$

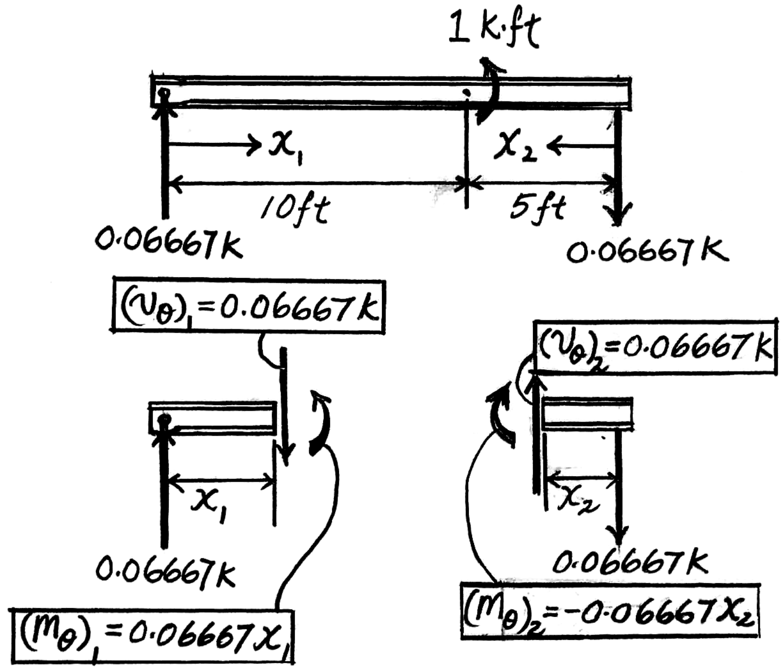
$$= \frac{2291.67\text{ k}^2\cdot\text{ft}^3}{EI} + \frac{100\text{ k}^2\cdot\text{ft}}{GA}$$

$$\Delta_B = \frac{2291.67\text{ k}\cdot\text{ft}^3}{EI} + \frac{100\text{ k}\cdot\text{ft}}{GA}$$

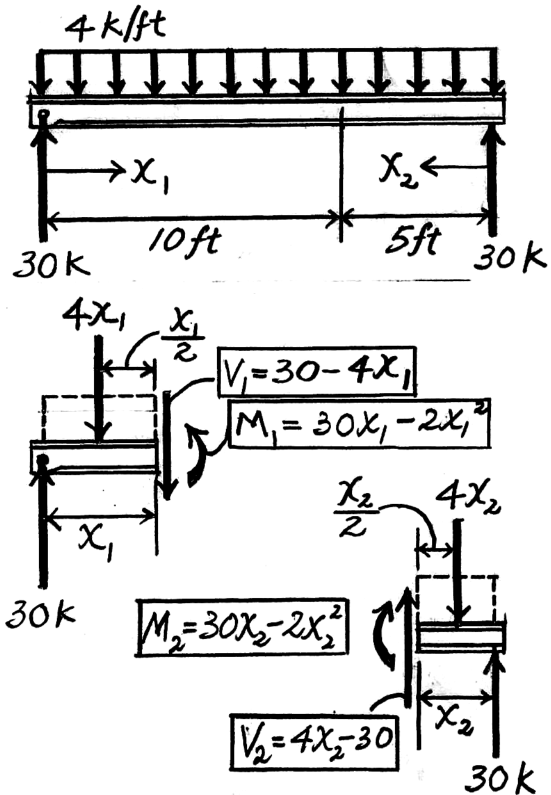
$$= \frac{2291.67(12^3)\text{k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](300\text{ in}^4)} + \frac{100(12)\text{ k}\cdot\text{in}}{[12(10^3)\text{ k/in}^2](7.50\text{ in}^2)}$$

$$= 0.469\text{ in}\downarrow$$ **Ans.**

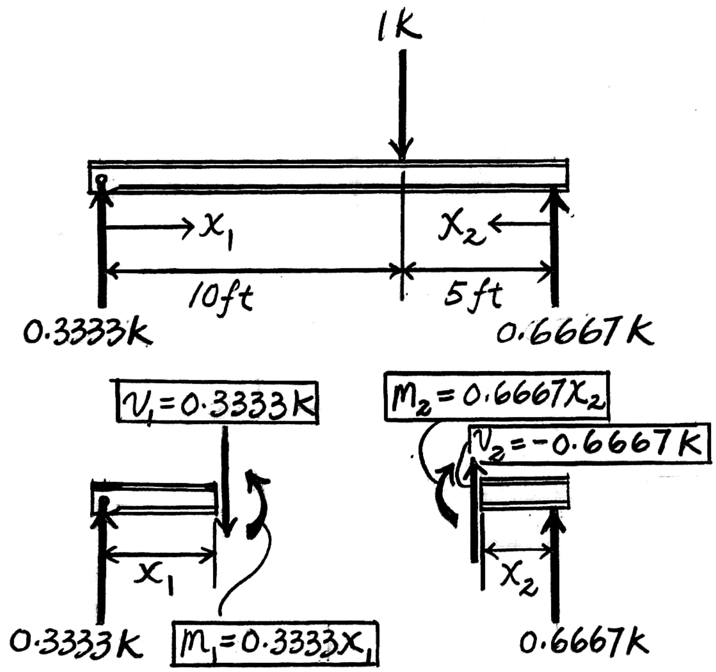
9–37. Continued



(a)

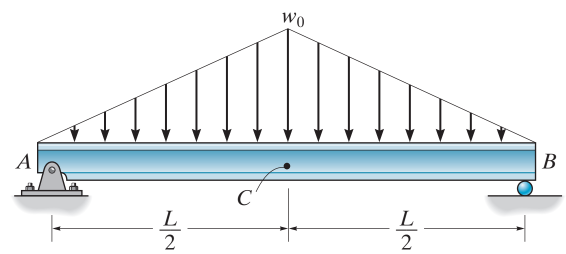


(c)



(b)

9–38. Determine the displacement of point C. Use the method of virtual work. EI is constant.

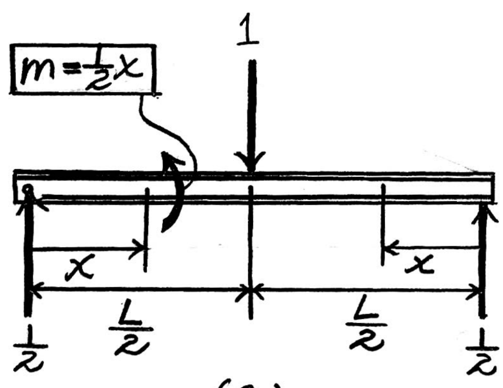


Referring to the virtual and real moment functions shown in Fig. *a* and *b*, respectively,

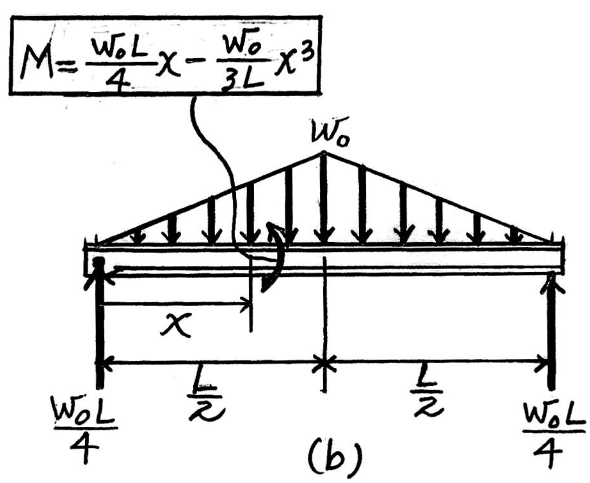
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}dx = 2\int_0^{\frac{L}{2}} \frac{\left(\frac{1}{2}\right)\left(\frac{w_0 L}{4}x - \frac{w_0}{3L}x^3\right)}{EI}dx$$

$$\Delta_C = \frac{w_0 L^4}{120EI} \downarrow$$

Ans.

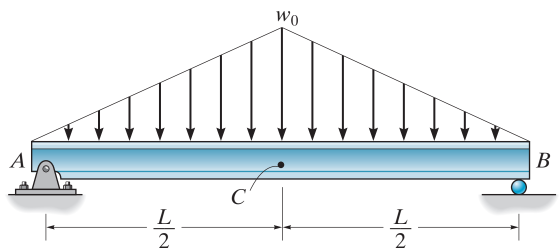


(a)



(b)

9–39. Solve Prob. 9–38 using Castigliano's theorem.

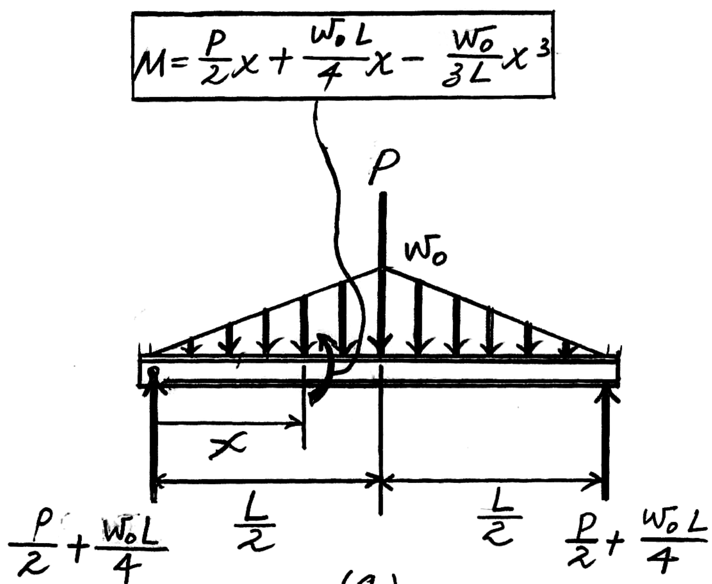


The moment function is shown in Fig. *a*. Here $\frac{\partial M}{\partial P} = \frac{1}{2}x$. Also, set $P = 0$, then $M = \frac{w_0 L}{4}x - \frac{w_0}{3L}x^3$. Thus

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = 2\int_0^{\frac{L}{2}} \frac{\left(\frac{w_0 L}{4}x - \frac{w_0}{3L}x^3\right)\left(\frac{1}{2}x\right)}{EI}\,dx$$

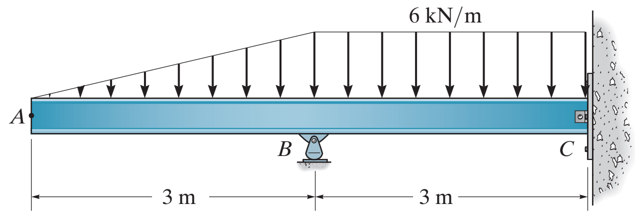
$$= \frac{w_0 L^4}{120EI} \downarrow$$

Ans.



(a)

***9–40.** Determine the slope and displacement at point A. Assume C is pinned. Use the principle of virtual work. EI is constant.



Referring to the virtual moment functions shown in Fig. a and b and the real moment functions in Fig. c, we have

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{3\text{ m}} \frac{(-1)(-0.3333x_1^3)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(0.3333x_2)(6x_2 - 3x_2^2)}{EI}dx_2$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{9\text{ kN}^2\cdot\text{m}^3}{EI}$$

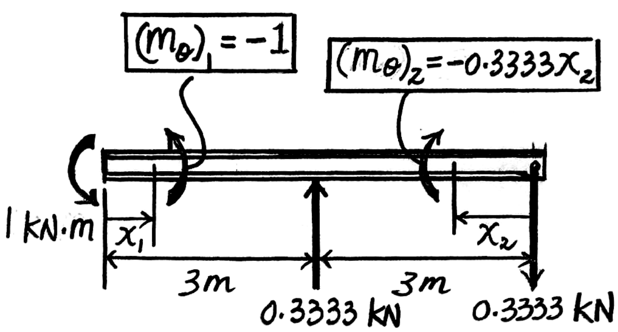
$$\theta_A = \frac{9\text{ kN}\cdot\text{m}^2}{EI}$$ **Ans.**

And

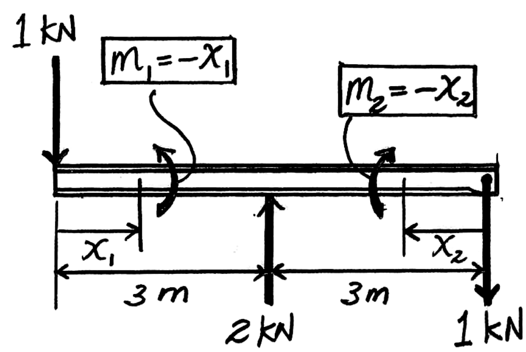
$$1\text{ kN}\cdot\Delta_A = \int_0^L \frac{mM}{EI}dx = \int_0^{3\text{ m}} \frac{(-x_1)(-0.3333x_1^3)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(-x_2)(6x_2 - 3x_2^2)}{EI}dx_2$$

$$1\text{ kN}\cdot\Delta_A = \frac{22.95\text{ kN}^2\cdot\text{m}^3}{EI}$$

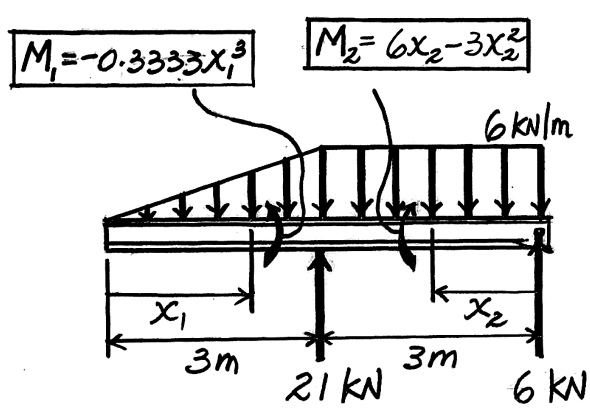
$$\Delta_A = \frac{22.95\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$ **Ans.**



(a)

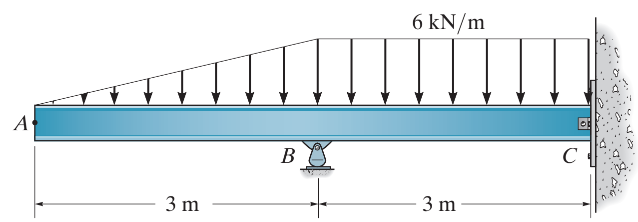


(b)



(c)

9–41. Solve Prob. 9–40 using Castigliano's theorem.



The slope, the moment functions are shown in Fig. *a*. Here, $\dfrac{\partial M_1}{\partial M'} = -1$ and $\dfrac{\partial M_2}{\partial M'} = -0.3333x_2$. Also, set $M' = 0$, then $M_1 = -0.3333x_1^3$ and $M_2 = 6x_2 - 3x_2^2$. Thus

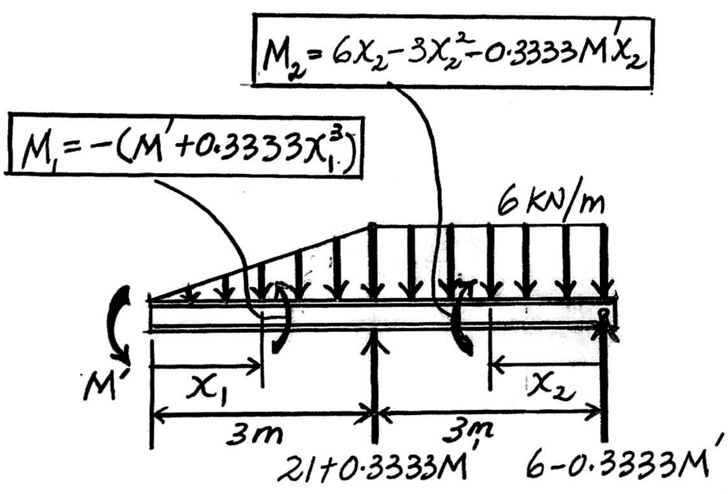
$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{(-0.3333x_1^3)(-1)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(6x_2 - 3x_2^2)(0.3333x_2)}{EI}dx_2$$

$$\theta_A = \frac{9\text{ kN}\cdot\text{m}^2}{EI} \quad \text{Ans.}$$

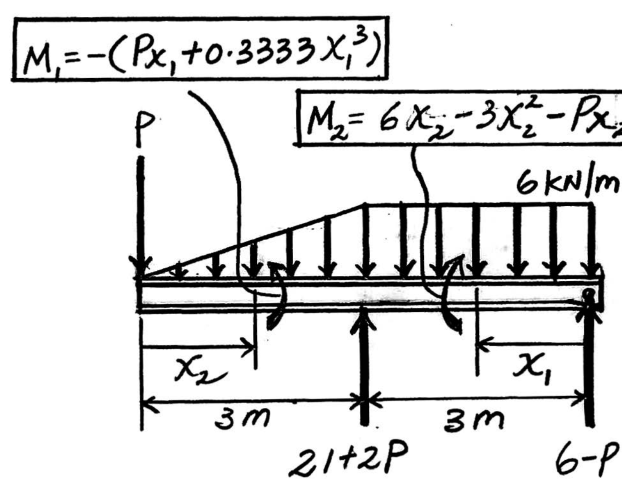
The displacement, the moment functions are shown in Fig. *b*. Here, $\dfrac{\partial M_1}{\partial P} = -x_1$ and $\dfrac{\partial M_2}{\partial P} = -x_2$. Also, set $P = 0$, then $M_1 = -0.3333x_1^3$ and $M_2 = 6x_2 - 3x_2^2$. Thus

$$\Delta_A = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{(-0.3333x_1^3)(-x_1)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(6x_2 - 3x_2^2)(-x_2)}{EI}dx_2$$

$$= \frac{22.95\text{ kN}\cdot\text{m}^3}{EI} \downarrow \quad \text{Ans.}$$

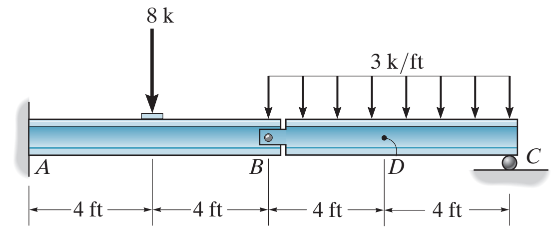


(a)



(b)

9–42. Determine the displacement at point D. Use the principle of virtual work. EI is constant.



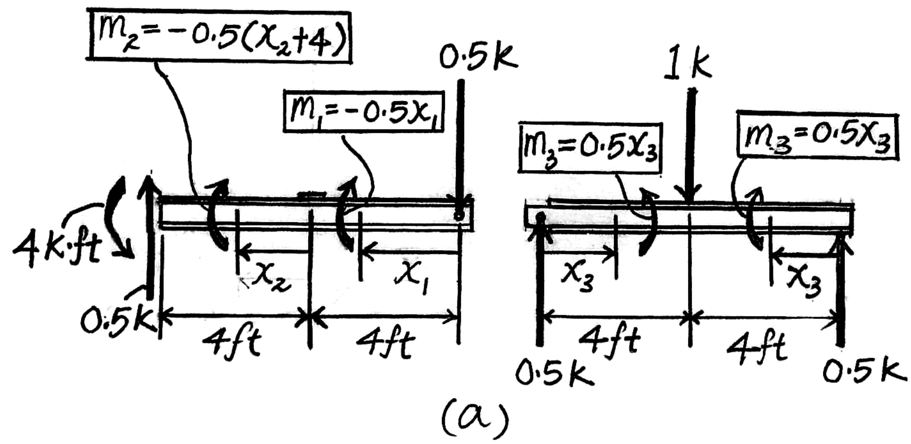
Referring to the virtual and real moment functions shown in Fig. *a* and *b*, respectively,

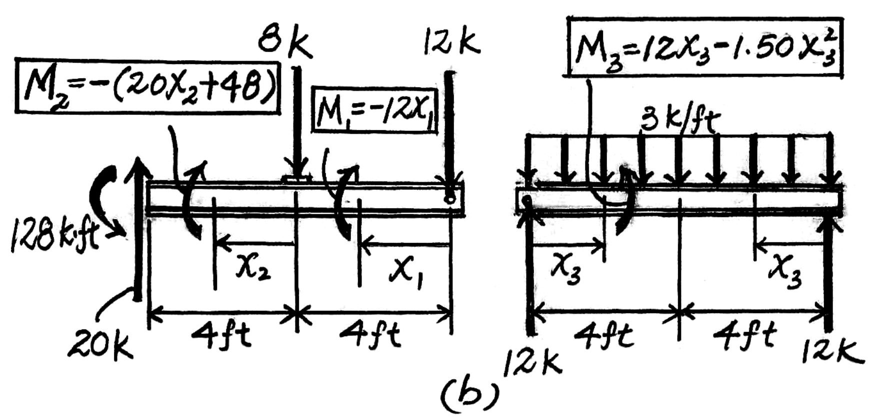
$$1\text{ k}\cdot\Delta_D = \int_0^L \frac{mM}{EI}dx = \int_0^{4\text{ ft}} \frac{(-0.5x_1)(-12x_1)}{EI}dx_1 + \int_0^{4\text{ ft}} \frac{[-0.5(x_2+4)][-(20x_2+48)]}{EI}dx_2$$

$$+\,2\int_0^{4\text{ ft}} \frac{(-0.5x_3)(12x_3 - 1.50x_3^2)}{EI}dx_3$$

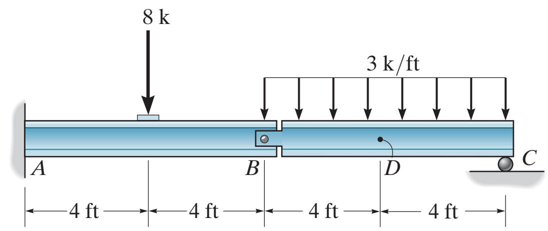
$$1\text{ k}\cdot\Delta_D = \frac{1397.33\text{ k}^2\cdot\text{ft}^3}{EI}$$

$$\Delta_D = \frac{1397\text{ k}\cdot\text{ft}^3}{EI}\downarrow$$ **Ans.**





9–43. Determine the displacement at point D. Use Castigliano's theorem. EI is constant.



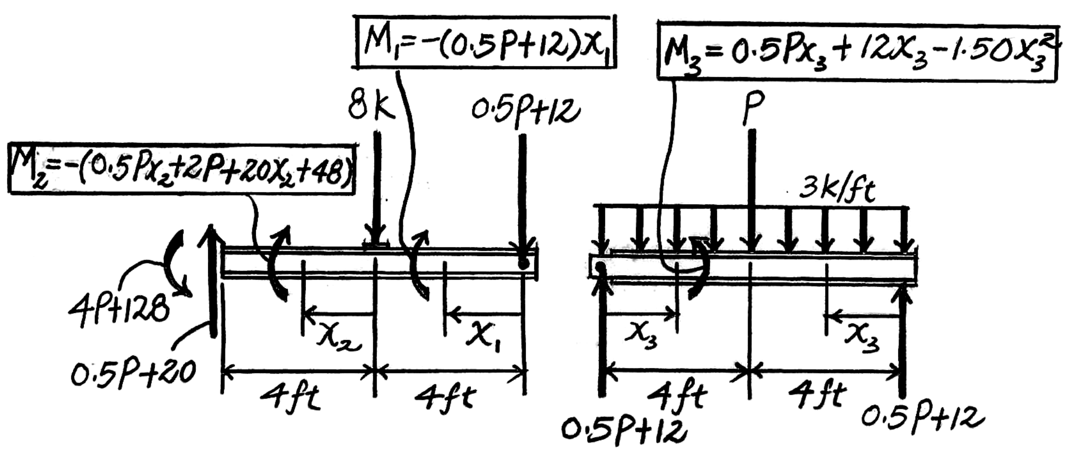
The moment functions are shown in Fig. a. Here, $\dfrac{\partial M_1}{\partial P} = -0.5x$, $\dfrac{\partial M_2}{\partial P} = -(0.5x_2 + 2)$ and $\dfrac{\partial M_3}{\partial P} = 0.5x_3$. Also set $P = 0$, $M_1 = -12x_1$, $M_2 = -(20x_2 + 48)$ and $M_3 = 12x_3 - 1.50x_3^2$. Thus,

$$\Delta_D = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{4\text{ ft}} \frac{(-12x_1)(-0.5x_1)}{EI}dx_1$$

$$+ \int_0^{4\text{ ft}} \frac{[-(20x_2 + 48)][-(0.5x_2 + 2)]}{EI}dx_2$$

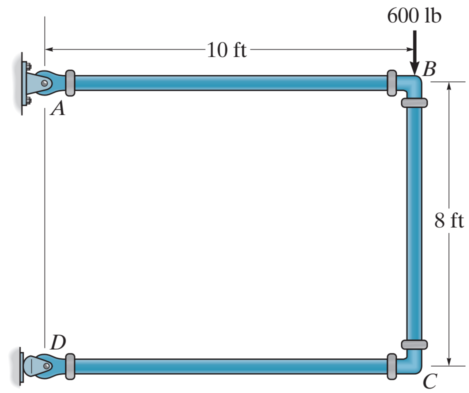
$$+ 2\int_0^{4\text{ ft}} \frac{(12x_3 - 1.50x_3^2)(0.5x_3)}{EI}dx_3$$

$$= \frac{1397.33 \text{ k}\cdot\text{ft}^3}{EI} = \frac{1397 \text{ k}\cdot\text{ft}^3}{EI} \downarrow$$ **Ans.**



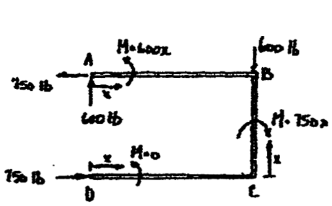
(a)

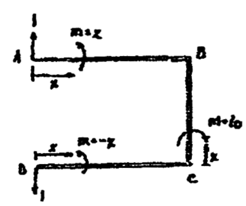
***9–44.** Use the method of virtual work and determine the vertical deflection at the rocker support *D*. *EI* is constant.



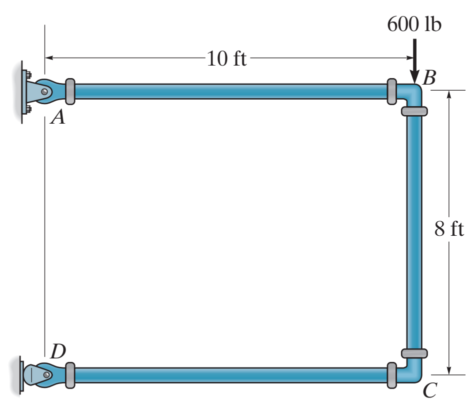
$$(\Delta_D)_x = \int_0^L \frac{mM}{EI}dx = \int_0^{Lo} \frac{(x)(600x)dx}{EI} + \int_0^L \frac{(10)(750x)dx}{EI} + 0$$

$$= \frac{440\ \text{k}\cdot\text{ft}^3}{EI} \downarrow \qquad \textbf{Ans.}$$





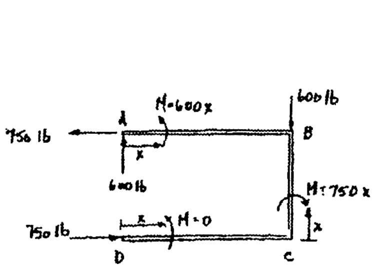
9–45. Solve Prob. 9–44 using Castigliano's theorem.

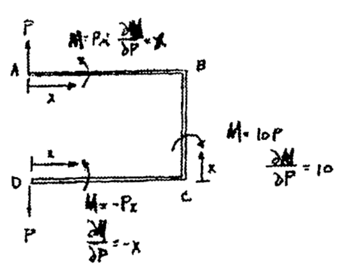


Set $P = 0$,

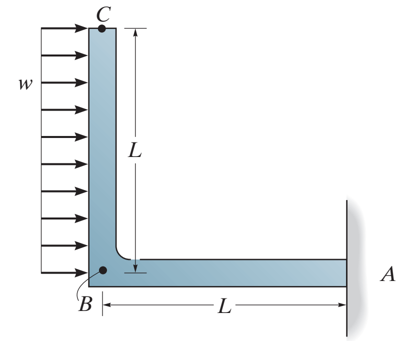
$$(\Delta_D)_v = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = \int_0^{10} \frac{(600x)(x)dx}{EI} + \int_0^{1} \frac{(750x)(10)dx}{EI} + 0$$

$$= \frac{440\ \text{k}\cdot\text{ft}^3}{EI} \downarrow \qquad \textbf{Ans.}$$





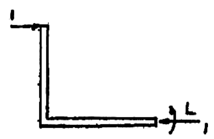
9–46. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of the end C. Use the method of virtual work.

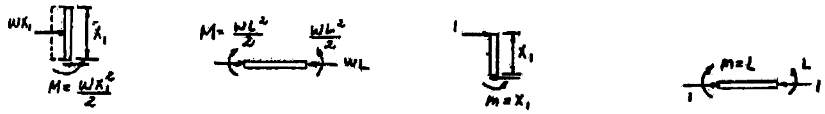


$$1 \cdot \Delta_{C_h} = \int_0^L \frac{mM}{EI} dx$$

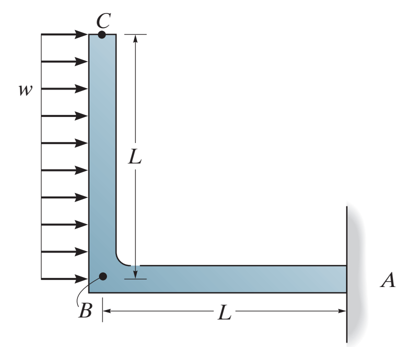
$$\Delta_{C_h} = \frac{l}{EI}\left[\int_0^L (1x_1)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (1L)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{5wL^4}{8EI}$$ **Ans.**





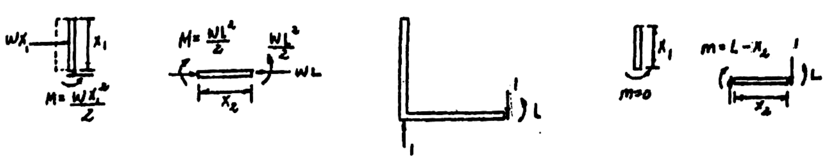
9–47. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the vertical displacement of point B. Use the method of virtual work.



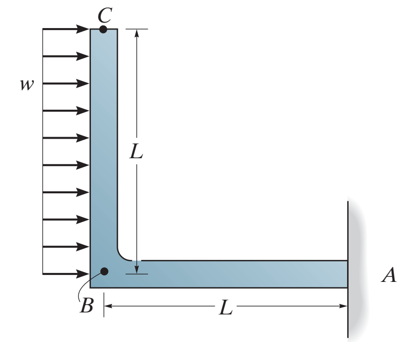
$$l \cdot \Delta_{B_v} = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_{B_v} = \frac{l}{EI}\left[\int_0^L (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (L - x_2)\left(\frac{wL^2}{2}\right)dx_2\right]$$

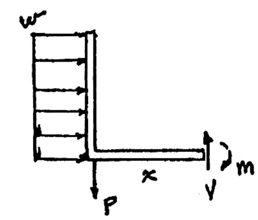
$$= \frac{wL^4}{4EI}$$ **Ans.**



***9–48.** Solve Prob. 9–47 using Castigliano's theorem.



P does not influence moment within vertical segment.



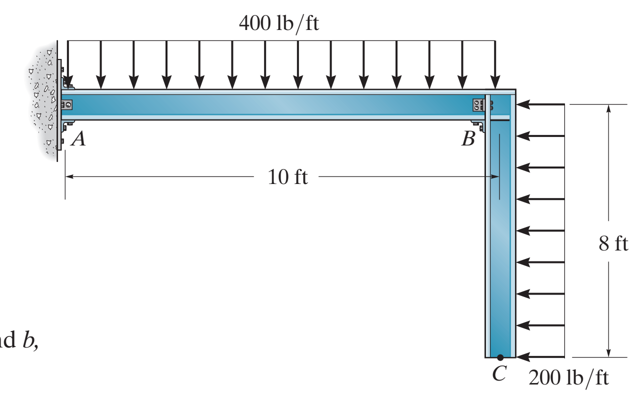
$$M = Px - \frac{wL^2}{2}$$

$$\frac{\partial M}{\partial P} = x$$

Set $P = 0$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \left(-\frac{wL^2}{2}\right)(x)\frac{dx}{EI} = \frac{wL^4}{4EI}$$ **Ans.**

9–49. Determine the horizontal displacement of point C. EI is constant. Use the method of virtual work.

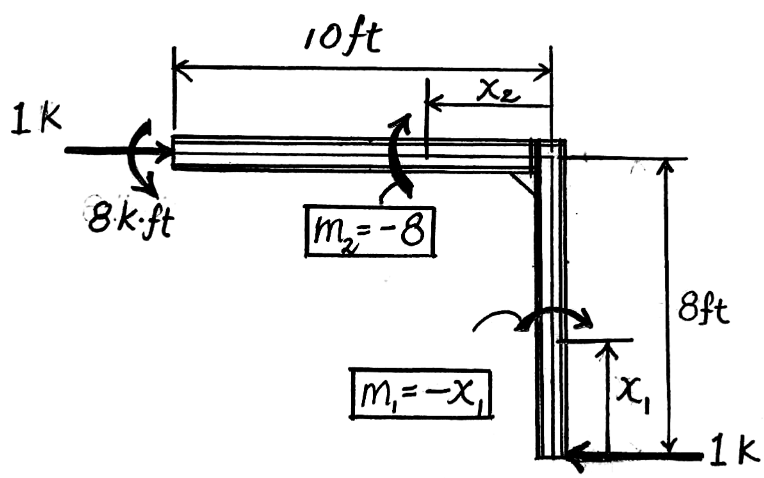


Referring to the virtual and real moment functions shown in Fig. *a* and *b*, respectively,

$$1\text{ k}\cdot\Delta_{C_h} = \int_0^L \frac{mM}{EI}dx = \int_0^{8\text{ ft}} \frac{(-x_1)(-0.1x_1^2)}{EI}dx_1 + \int_0^{10\text{ ft}} \frac{(-8)[-(0.2x_2^2 + 6.40)]}{EI}dx_2$$

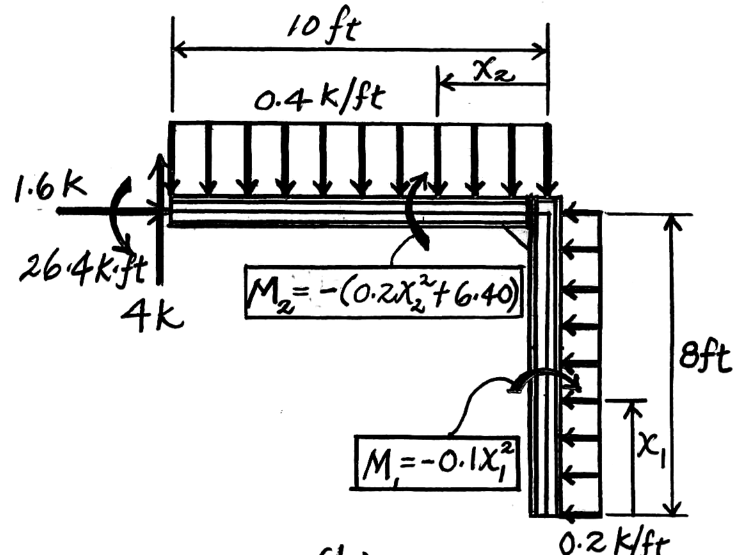
$$1\text{ k}\cdot\Delta_{C_h} = \frac{1147.73\text{ k}^2\cdot\text{ft}^3}{EI}$$

$$\Delta_{C_h} = \frac{1148\text{ k}\cdot\text{ft}^3}{EI} \leftarrow$$ **Ans.**



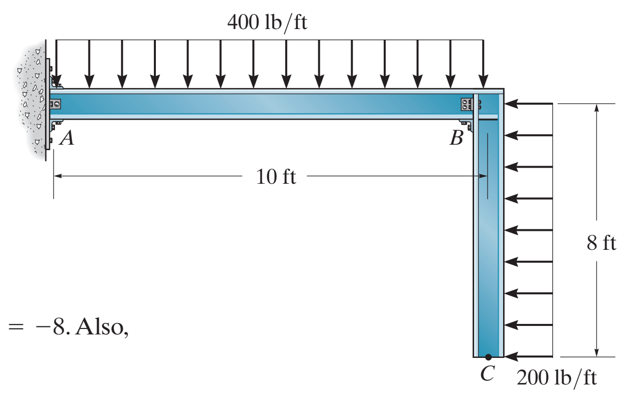
(a)

9–49. Continued



(b)

9–50. Solve Prob. 9–49 using Castigliano's theorem.

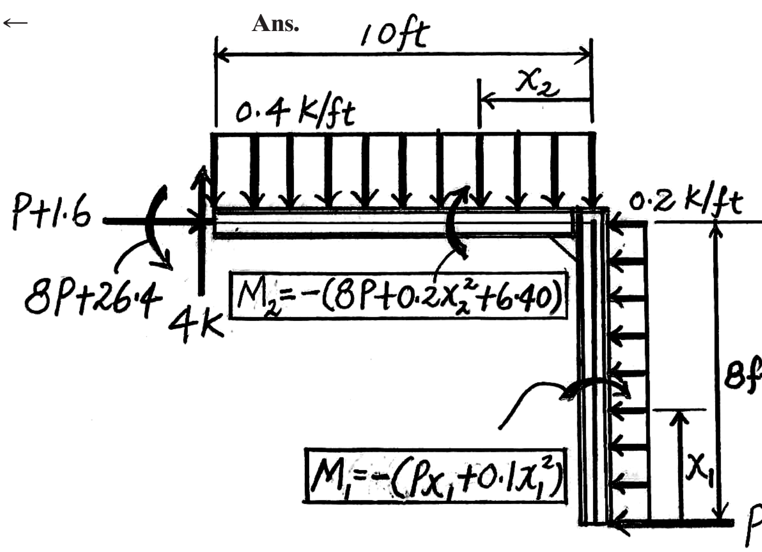


The moment functions are shown in Fig. *a*. Here, $\frac{\partial M_1}{\partial P} = -x_1$ and $\frac{\partial M_2}{\partial P} = -8$. Also, set $P = 0$, then $M_1 = -0.1x_1^2$ and $M_2 = -(0.2x_2^2 + 6.40)$.

Thus,

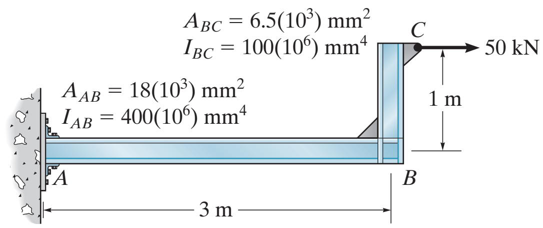
$$\Delta_{C_h} = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{8\text{ ft}} \frac{(-0.1x_1^2)(-x_1)}{EI}dx_1 + \int_0^{10\text{ ft}} \frac{[-(0.2x_2^2 + 6.40)](-8)}{EI}dx_2$$

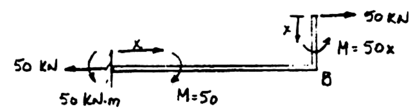
$$= \frac{1147.73\text{ k}\cdot\text{ft}^3}{EI} = \frac{1148\text{ k}\cdot\text{ft}^3}{EI} \leftarrow \qquad \textbf{Ans.}$$



(a)

9–51. Determine the vertical deflection at C. The cross-sectional area and moment of inertia of each segment is shown in the figure. Take $E = 200$ GPa. Assume A is a fixed support. Use the method of virtual work.



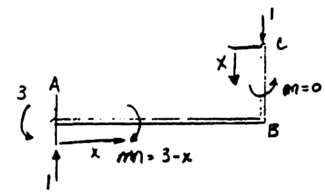


$$(\Delta_C)_v = \int_0^L \frac{mM}{EI}dx = \int_0^3 \frac{(3-x)(50)(10^3)dx}{EI_{AB}} + 0$$

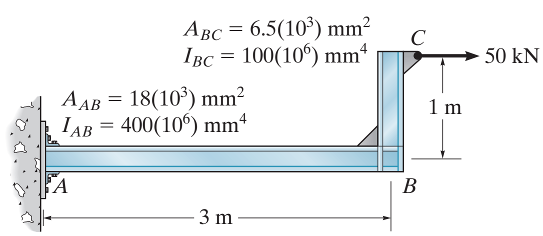
$$= \frac{[150(10^3)x - 25(10^3)x^2]_0^3}{EI_{AB}}$$

$$= \frac{225(10^3)}{200(10^9)(400)(10^6)(10^{-12})}$$

$$= 2.81\ \text{mm} \downarrow$$ **Ans.**



***9–52.** Solve Prob. 9–51, including the effect of shear and axial strain energy.

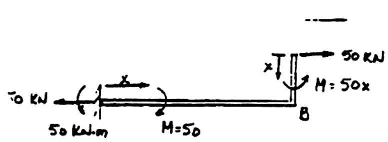


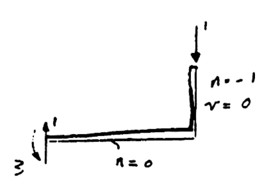
See Prob. 9–51 for the effect of bending.

$$U = \sum \frac{nNL}{AE} + \int_0^L K\left(\frac{vV}{GA}\right)dx$$

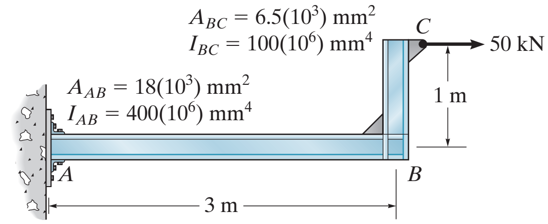
Note that each term is zero since n and N or v and V do not occur simultaneously in each member. Hence,

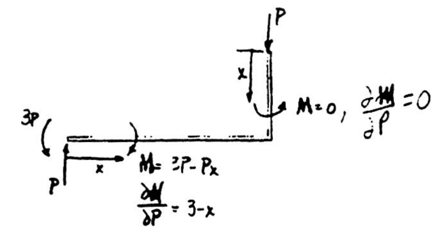
$$(\Delta_C)_v = 2.81\ mm \downarrow$$ **Ans**





9–53. Solve Prob. 9–51 using Castigliano's theorem.





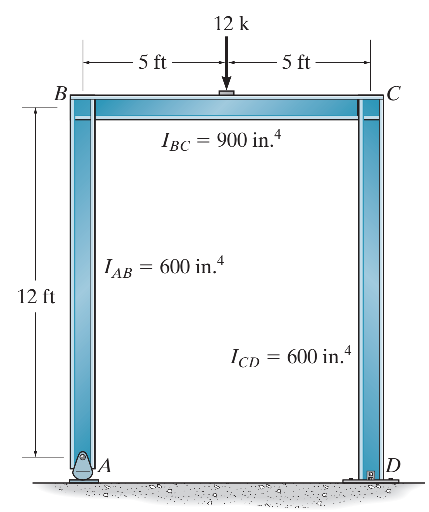
$$(\Delta_C)_v = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = \int_0^3 \frac{(50)(10^3)(3-x)dx}{EI_{AB}} + 0$$

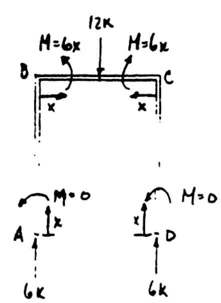
$$= \frac{[150(10^3)x - 25(10^3)x^2]_0^3}{EI_{AB}}$$

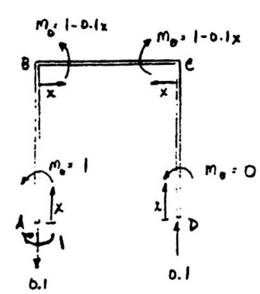
$$= \frac{225(10^3)}{200(10^9)(400)(10^6)(10^{-12})}$$

$$= 2.81\ \text{mm} \downarrow$$ **Ans.**

9–54. Determine the slope at A. Take $E = 29(10^3)$ ksi. The moment of inertia of each segment of the frame is indicated in the figure. Assume D is a pin support. Use the method of virtual work.



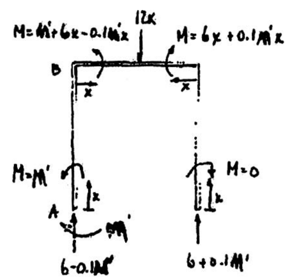


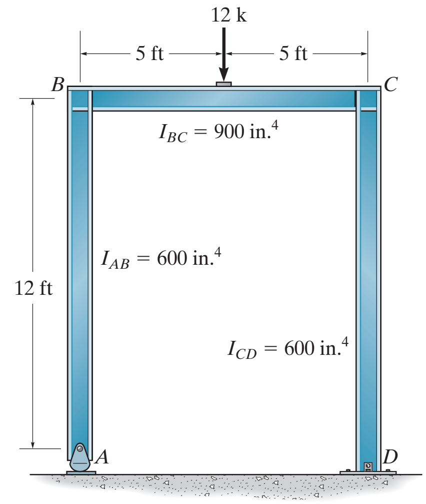


$$\theta_A = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^5 \frac{(1-0.1x)(6x)dx}{EI_{BC}} + \int_0^5 \frac{(0.1x)(6x)dx}{EI_{BC}} + 0 + 0$$

$$= \frac{(75 - 25 + 25)}{EI_{BC}} = \frac{75(144)}{29(10^3)(900)} = 0.414(10^{-3})\ \text{rad}$$ **Ans.**

9–55. Solve Prob. 9–54 using Castigliano's theorem.



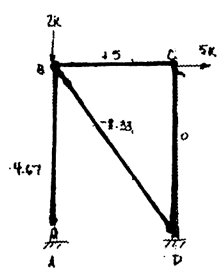


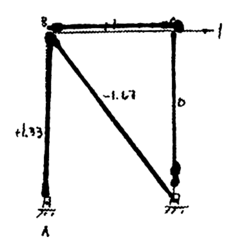
Set $M' = 0$,

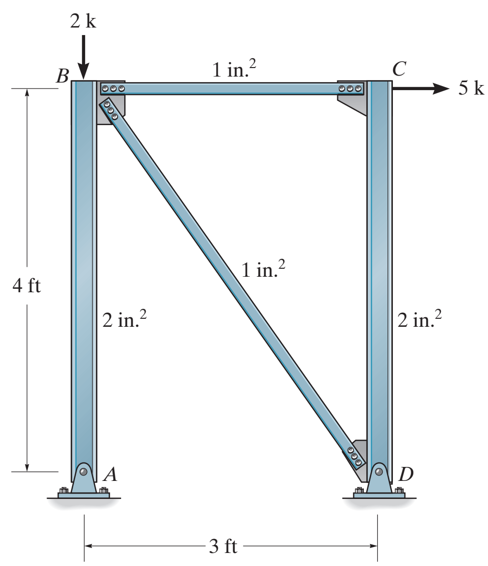
$$\theta_A = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial M'}\right)dx = \int_0^5 \frac{(6x)(1 - 0.1x)dx}{EI_{BC}} + \int_0^5 \frac{(6x)(0.1x)dx}{EI_{BC}} + 0 + 0$$

$$= \frac{(75 - 25 + 25)}{EI_{\partial C}} = \frac{75(144)}{29(10^3)(900)} = 0.414(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$

***9–56.** Use the method of virtual work and determine the horizontal deflection at C. The cross-sectional area of each member is indicated in the figure. Assume the members are pin connected at their end points. $E = 29(10^3)$ ksi.



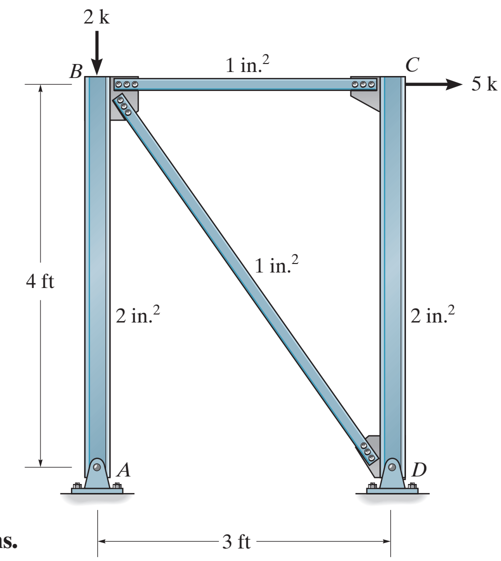




$$(\Delta_c)_h = \sum \frac{nNL}{AE} = \frac{1.33(4.667)(4)(12)}{2(29)(10^3)} + \frac{(1)(5)(3)(12)}{(1)(29)(10^3)} + 0 + \frac{(-8.33)(-1.667)(5)(12)}{(1)(29)(10^3)}$$

$$= 0.0401 \text{ in.} \rightarrow \qquad \textbf{Ans.}$$

9–57. Solve Prob. 9–56 using Castigliano's theorem.



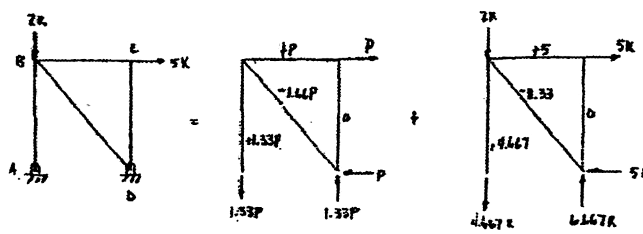
Member	N force	$\frac{\partial N}{\partial P}$
AB	$1.33P + 4.667$	1.33
BC	$P + 5$	1
BD	$-1.667P - 8.33$	-1.667
CD	0	0

Set $P = 0$,

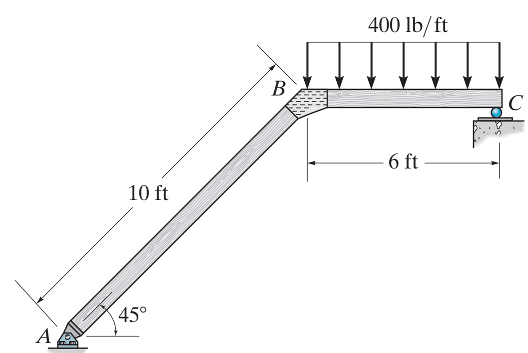
$$(\Delta_c)_h = N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{(4.667)(1.33)(4)(12)}{2(29)(10^3)} + \frac{(5)(1)(3)(12)}{(1)(29)(10^3)} + 0$$

$$+ \frac{(-8.33)(-1.667)(5)(12)}{(1)(29)(10^3)}$$

$$= 0.0401 \text{ in.} \rightarrow$$ **Ans.**



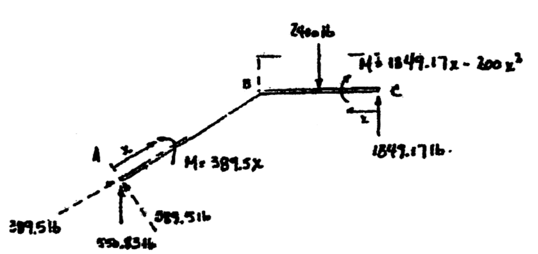
9–58. Use the method of virtual work and determine the horizontal deflection at C. E is constant. There is a pin at A, and assume C is a roller and B is a fixed joint.

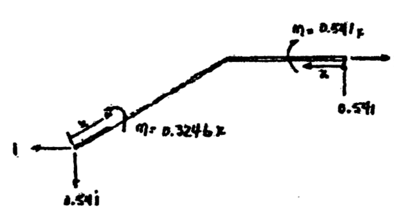


$$(\Delta_c)_h = \int_0^L \frac{mM}{EI}dx = \int_0^4 \frac{(0.541x)(1849.17x - 200x^2)dx}{EI} + \int_0^{10} \frac{(0.325x)(389.5x)dx}{EI}$$

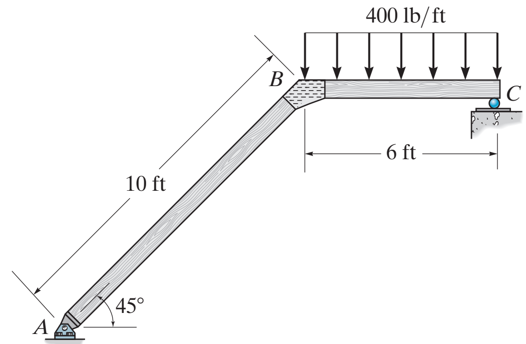
$$= \frac{1}{EI}\left[(333.47x^3 - 27.05x^4)\Big|_0^6 + (42.15x^3)\Big|_0^{10}\right]$$

$$= \frac{79.1\text{k.ft}^3}{EI} \rightarrow$$ **Ans.**





9–59. Solve Prob. 9–58 using Castigliano's theorem.

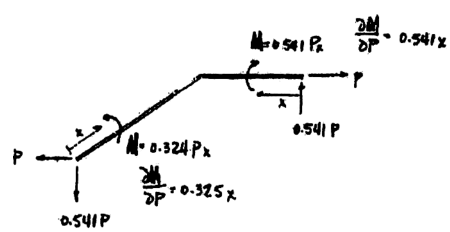


Set $P = 0$.

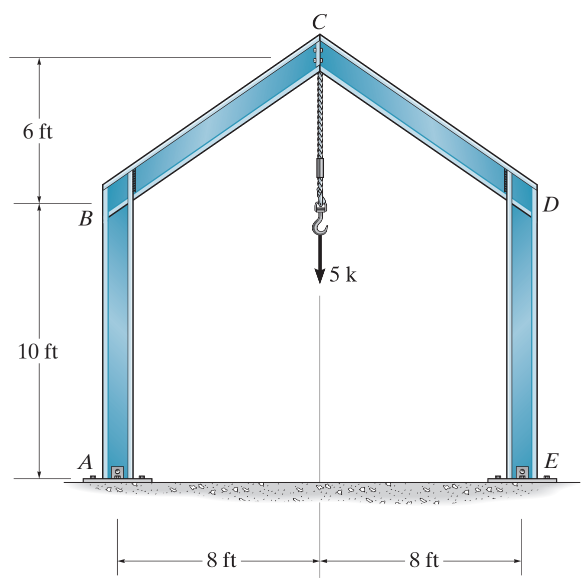
$$(\Delta_c)_h = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = \int_0^4 \frac{(1849.17x - 200x^2)(0.541x)dx}{EI} + \int_0^{10} \frac{(389.5x)(0.325x)dx}{EI}$$

$$= \frac{1}{EI}\left[(333.47x^3 - 27.5x^4)\big|_0^6 + (42.15x^3)\big|_0^{10}\right]$$

$$= \frac{79.1\text{k.ft}^3}{\text{EI}} \rightarrow$$ **Ans.**

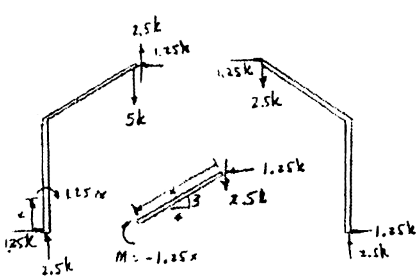


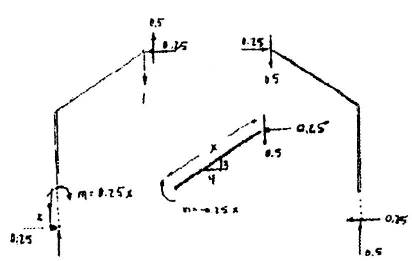
***9–60.** The frame is subjected to the load of 5 k. Determine the vertical displacement at C. Assume that the members are pin connected at A, C, and E, and fixed connected at the knee joints B and D. EI is constant. Use the method of virtual work.



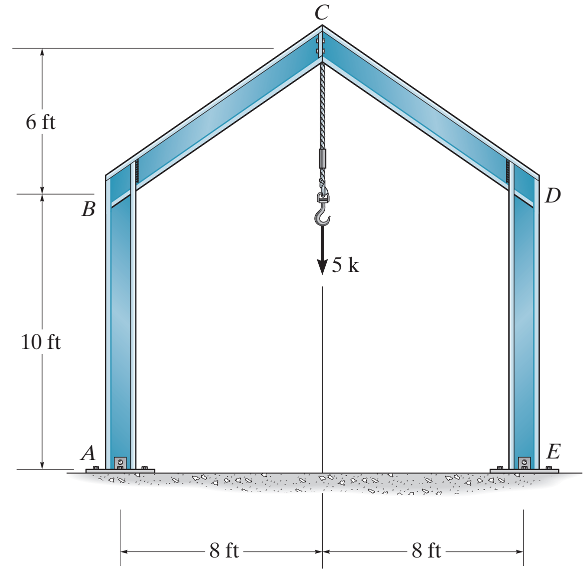
$$(\Delta_c)_v = \int_0^L \frac{mM}{EI}dx = 2\left[\int_0^{10} \frac{(0.25x)(1.25x)dx}{EI} + \int_0^{10} \frac{(-0.25x)(-1.25x)dx}{EI}\right]$$

$$= \frac{1.25(10^3)}{3EI} = \frac{4.17\ \text{k} \cdot \text{ft}^3}{EI} \downarrow$$ **Ans.**





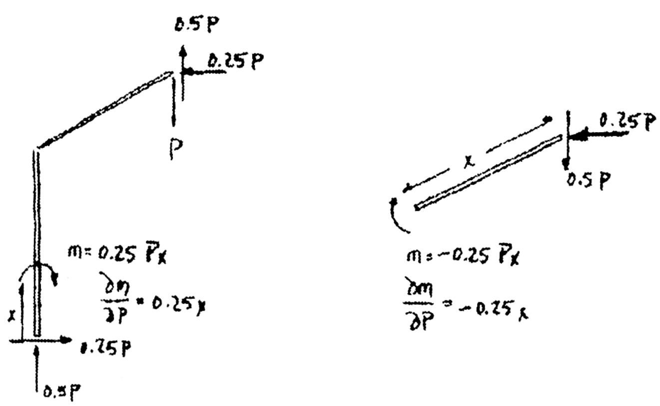
9–61. Solve Prob. 9–60 using Castigliano's theorem.



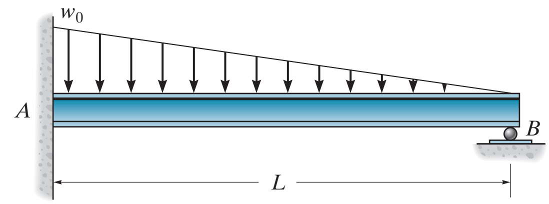
Set $P = 5$ k.

$$(\Delta_c)_v = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = 2\left[\int_0^{10}\frac{(1.25x)(0.25x)\,dx}{EI} + \int_0^{10}\frac{(-1.25x)(-0.25x)\,dx}{EI}\right]$$

$$= \frac{1.25(10^3)}{3EI} = \frac{4.17\ \text{k}\cdot\text{ft}^3}{EI}\downarrow$$ **Ans.**



10–1. Determine the reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).

$\xrightarrow{+} \sum F_x = 0; \qquad A_x = 0$ **Ans.**

$$+\uparrow \sum F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad [1]$$

$$\circlearrowleft + \sum M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad [2]$$

Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B{}' = \frac{w_0 L^4}{30EI}\downarrow \qquad v_B{}'' = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition requires

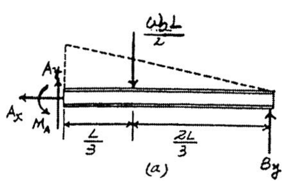
$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

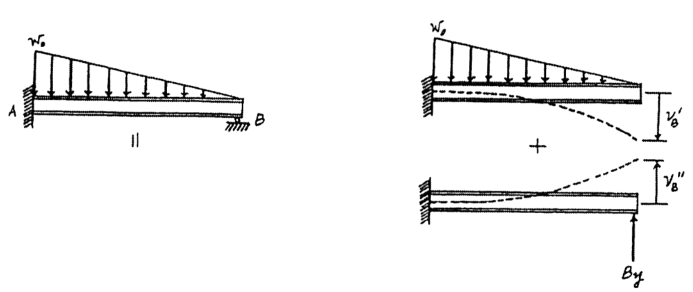
$$0 = \frac{w_0 L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$B_y = \frac{w_0 L}{10}$$ **Ans.**

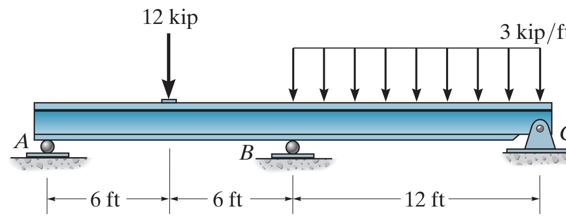
Substituting B_y into Eqs. [1] and [2] yields.

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15}$$ **Ans.**

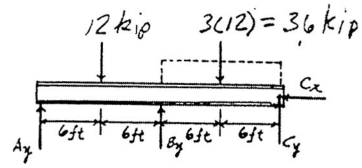




10–2. Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



Support Reactions: FBD(a).



$\overset{+}{\rightarrow} \sum F_x = 0; \qquad C_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y + B_y + C_y - 12 - 36.0 = 0$ [1]

$\circlearrowleft + \sum M_A = 0; \qquad B_y(12) + C_y(24) - 12(6) - 36.0(18) = 0$ [2]

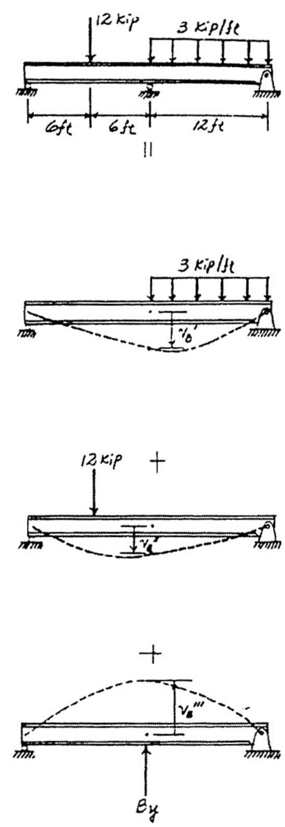
Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B{}' = \frac{5wL^4}{768EI} = \frac{5(3)(24^4)}{768EI} = \frac{6480 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$

$$v_B{}'' = \frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$$

$$= \frac{12(6)(12)}{6EI(24)}(24^2 - 6^2 - 12^2) = \frac{2376 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$

$$v_B{}''' = \frac{PL^3}{48EI} = \frac{B_y(24^3)}{48EI} = \frac{288B_y \text{ ft}^3}{EI} \uparrow$$



The compatibility condition requires

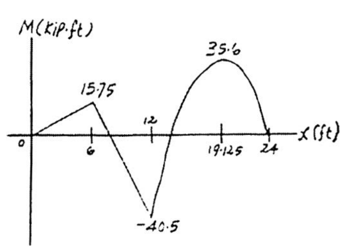
$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}'' + v_B{}'''$$

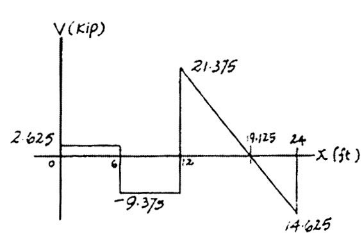
$$0 = \frac{6480}{EI} + \frac{2376}{EI} + \left(-\frac{288B_y}{EI}\right)$$

$B_y = 30.75$ kip **Ans.**

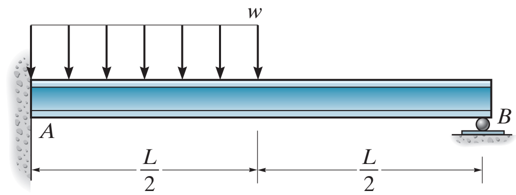
Substituting B_y into Eqs. [1] and [2] yields,

$A_y = 2.625$ kip $\quad C_y = 14.625$ kip **Ans.**

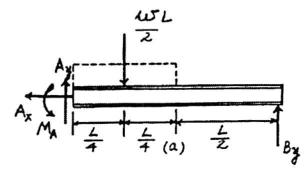




10–3. Determine the reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).



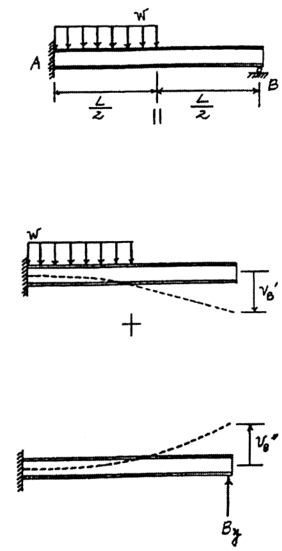
$$\overset{+}{\rightarrow} \quad \sum F_x = 0; \quad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \qquad A_y + B_y - \frac{wL}{2} = 0 \qquad [1]$$

$$\circlearrowleft + \sum M_A = 0; \qquad B_y(L) + M_A - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0 \qquad [2]$$

Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B{}' = \frac{7wL^4}{384EI}\downarrow \qquad v_B{}'' = \frac{PL^3}{3EI} = \frac{B_yL^3}{3EI}\uparrow$$



The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

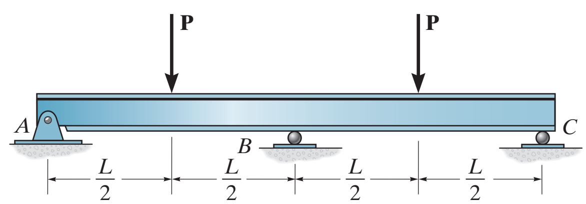
$$0 = \frac{7wL^4}{384EI} + \left(-\frac{B_yL^3}{3EI}\right)$$

$$B_y = \frac{7wL}{128} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. [1] and [2] yields,

$$A_y = \frac{57wL}{128} \qquad M_A = \frac{9wL^2}{128} \qquad \textbf{Ans.}$$

10–4. Determine the reactions at the supports A, B, and C; then draw the shear and moment diagrams. EI is constant.

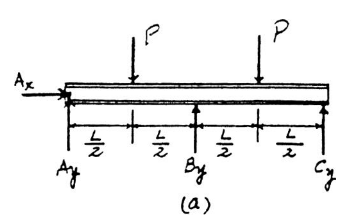


Support Reactions: FBD(a).

$$\overset{+}{\rightarrow} \quad \sum F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \qquad A_y + B_y + C_y - 2p = 0 \qquad [1]$$

$$\circlearrowleft + \sum M_A = 0; \qquad B_yL + C_y(2L) - P\left(\frac{L}{2}\right) - P\left(\frac{3L}{2}\right) = 0 \qquad [2]$$



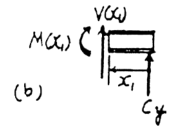
Moment Functions: FBD(b) and (c).

$$M(x_1) = C_yx_1$$

$$M(x_2) = C_yx_2 - Px_2 + \frac{PL}{2}$$

***10–4. Continued**

Slope and Elastic Curve:

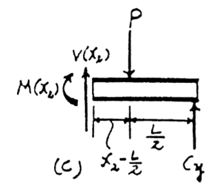


$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = C_y x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = C_y x_1$$

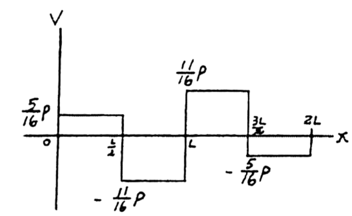
$$EI\frac{dv_1}{dx_1} = \frac{C_y}{2}x_1^2 + C_1 \quad [3]$$



$$EI\,v_1 = \frac{C_y}{6}x_1^3 + C_1x_1 + C_2 \quad [4]$$

For $M(x_2) = C_y x_2 - Px_2 + \frac{PL}{2}$,

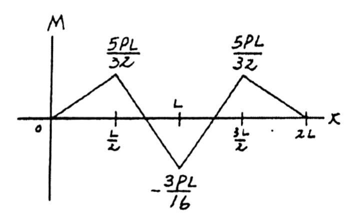
$$EI\frac{d^2v_2}{dx_2^2} = C_y x_2 - Px_2 + \frac{PL}{2}$$



$$EI\frac{dv_2}{dx_2} = \frac{C_y}{2}x_2^2 - \frac{P}{2}x_2^2 + \frac{PL}{2}x_2 + C_3 \quad [5]$$

$$EIv_2 = \frac{C_y}{6}x_2^3 - \frac{P}{6}x_2^4 + \frac{PL}{4}x_2^2 + C_3x_2 + C_4 \quad [6]$$

Boundary Conditions:



$v_1 = 0$ *at* $x_1 = 0$. From Eq. [4] $C_2 = 0$

Due to symmetry, $\frac{dv_2}{dx_2} = 0$ *at* $x_2 = L$. From Eq. [5],

$$0 = \frac{C_yL^2}{2} - \frac{PL^2}{2} + \frac{PL^2}{2} + C_3 \qquad C_3 = \frac{C_yL^2}{2}$$

$v_2 = 0$ *at* $x_2 = L$. From Eq. [6],

$$0 = \frac{C_yL^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{4} + \left(-\frac{C_yL^2}{2}\right)L + C_4$$

$$C_4 = \frac{C_yL^3}{3} - \frac{PL^3}{12}$$

Continuity Conditions:

At $x_1 = x_2 = \frac{L}{2}$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs. [3] and [5],

$$\frac{C_y}{2}\left(\frac{L}{2}\right)^2 + C_1 = \frac{C_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + \frac{PL}{2}\left(\frac{L}{2}\right) - \frac{C_yL^2}{2}$$

$$C_1 = \frac{PL^2}{8} - \frac{C_yL^2}{2}$$

At $x_1 = x_2 = \frac{L}{2}$, $v_1 = v_2$. From Eqs. [4] and [6].

$$\frac{C_y}{6}\left(\frac{L}{2}\right)^3 + \left(\frac{PL^2}{8} - \frac{C_yL^2}{2}\right)\left(\frac{L}{2}\right)$$

***10–4. Continued**

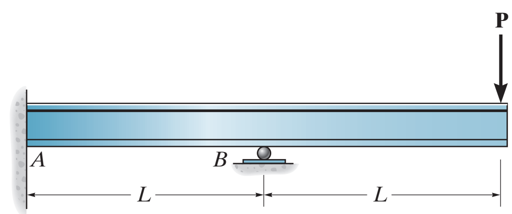
$$= \frac{C_y}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{L}{2}\right)^3 + \frac{PL}{4}\left(\frac{L}{2}\right)^2 + \left(-\frac{C_yL^2}{2}\right)\left(\frac{L}{2}\right) + \frac{C_yL^3}{3} - \frac{PL^3}{12}$$

$$C_y = \frac{5}{16}P$$ **Ans.**

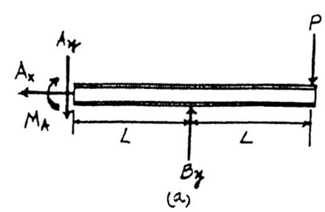
Substituting C_y into Eqs. [1] and [2],

$$B_y = \frac{11}{8}P \quad A_y = \frac{5}{16}P$$ **Ans.**

10–5. Determine the reactions at the supports, then draw the shear and moment diagram. *EI* is constant.



Support Reactions: FBD(a) .

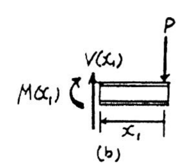


$\overset{+}{\rightarrow} \sum F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad B_y - A_y - P = 0$ [1]

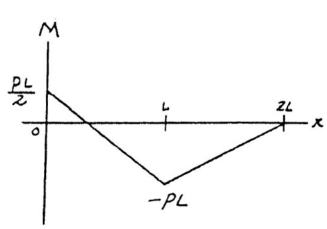
$\circlearrowleft + \sum M_B = 0; \qquad A_yL - M_A - PL = 0$ [2]

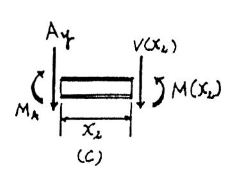
Moment Functions: FBD(b) and (c).



$$M(x_1) = -Px_1$$

$$M(x_2) = M_A - A_yx_2$$





10–5. Continued

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$.

$$EI\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad [3]$$

$$EI\,v_1 = -\frac{P}{6}x_1^3 + C_1x_1 + C_2 \qquad [4]$$

For $M(x_2) = M_A - A_yx_2$

$$EI\frac{d^2v_2}{dx_2^2} = M_A - A_yx_2$$

$$EI\frac{dv_2}{dx_2} = M_Ax_2 - \frac{A_y}{2}x_2^2 + C_3 \qquad [5]$$

$$EI\,v_2 = \frac{M_A}{2}x_2^2 - \frac{A_y}{6}x_2^3 + C_3x_2 + C_4 \qquad [6]$$

Boundary Conditions:

$v_2 = 0$ *at* $x_2 = 0$. From Eq. [6], $C_4 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = 0$. From Eq. [5], $C_3 = 0$

$v_2 = 0$ at $x_2 = L$. From Eq. [6].

$$0 = \frac{M_AL^2}{2} - \frac{A_yL^3}{6} \qquad [7]$$

Solving Eqs. [2] and [7] yields.

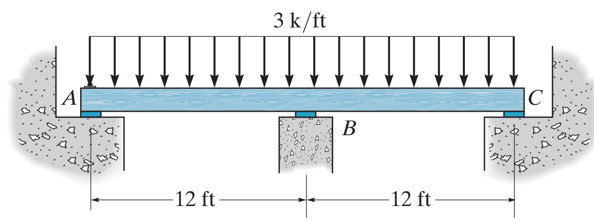
$$M_A = \frac{PL}{2} \qquad A_y = \frac{3P}{2}$$ **Ans.**

Substituting the value of A_y into Eq. [1],

$$B_y = \frac{5P}{2}$$ **Ans.**

Note: The other boundary and continuity conditions can be used to determine the constants C_1 and C_2 which are not needed here.

10–6. Determine the reactions at the supports, then draw the moment diagram. Assume B and C are rollers and A is pinned. The support at B settles downward 0.25 ft. Take $E = 29(10^3)$ ksi, $I = 500$ in^4.



Compatibility Equation. Referring to Fig. *a*,

$$\Delta'_B = \frac{5wL_{AC}^4}{384EI} = \frac{5(3)(24^4)}{384EI} = \frac{12960 \text{ k}\cdot\text{ft}^3}{EI}$$

$$= \frac{12960(12^3) \text{ k}\cdot\text{in}^3}{[29(10^3) \text{ k/in}^2](500 \text{ in}^4)}$$

$$= 1.544 \text{ in} \downarrow$$

$$f_{BB} = \frac{L_{AC}^3}{48EI} = \frac{24^3}{48EI} = \frac{288 \text{ ft}^3}{EI}$$

$$= \frac{288(12^3) \text{ in}^3}{[29(10^3) \text{ k/in}^2](500 \text{ in}^4)}$$

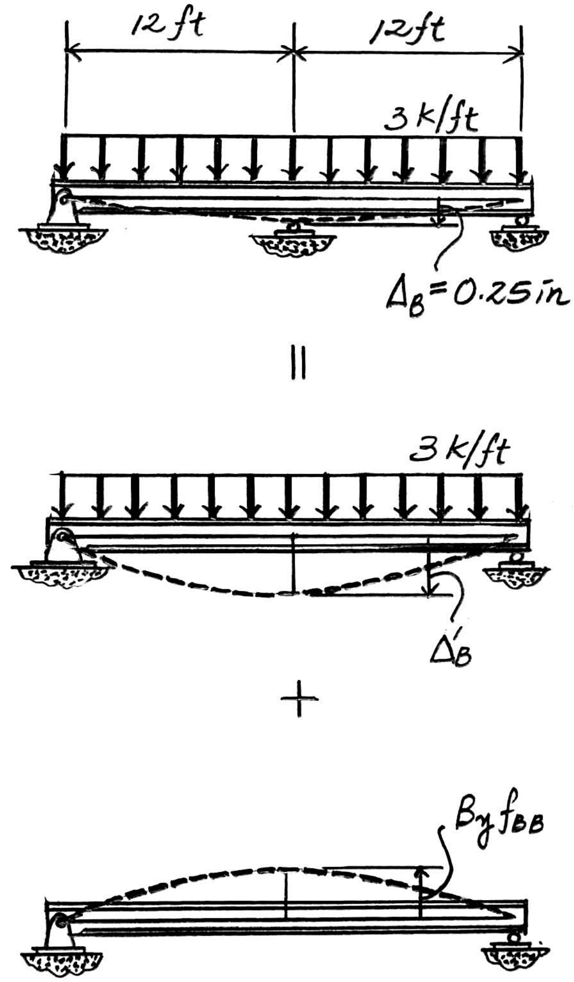
$$= 0.03432 \frac{\text{in}}{\text{k}} \uparrow$$

Using the principle of superposition,

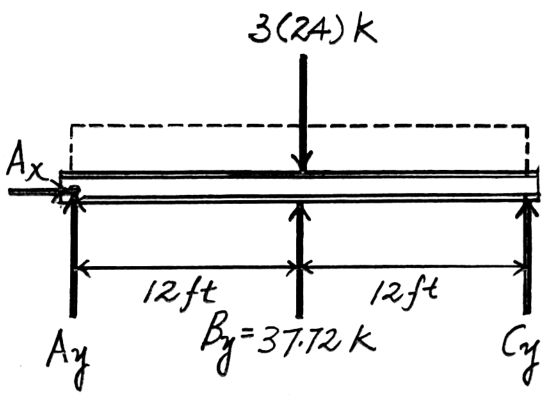
$$\Delta_B = \Delta'_B + B_y f_{BB}$$

$$(+\downarrow)\ 0.25 \text{ in} = 1.544 \text{ in} + B_y\left(-0.03432 \frac{\text{in}}{\text{k}}\right)$$

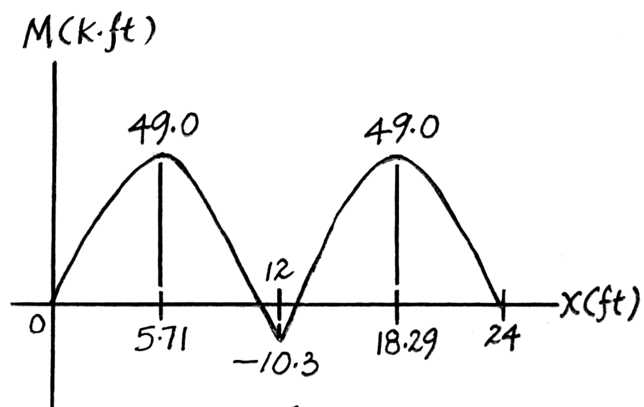
$$B_y = 37.72 \text{ k} = 37.7 \text{ k}$$ **Ans.**



(a)



(b)



(c)

10–6. Continued

Equilibrium. Referring to the FBD in Fig. *b*

$\xrightarrow{+} \sum F_x = 0; \qquad A_x = 0$ **Ans.**

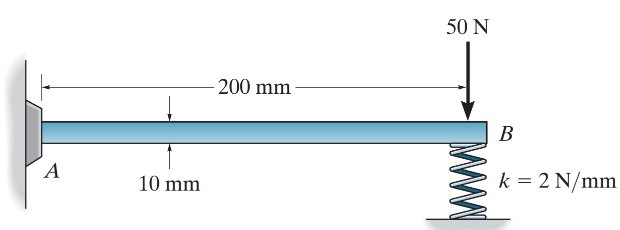
$\circlearrowleft + \sum M_A = 0; \qquad C_y(24) + 37.72(12) - 3(24)(12) = 0$

$C_y = 17.14 \text{ k} = 17.1 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y + 37.72 + 17.14 - 3(24) = 0$

$A_y = 17.14 \text{ k} = 17.1 \text{ k}$ **Ans.**

10–7. Determine the deflection at the end *B* of the clamped A-36 steel strip. The spring has a stiffness of $k = 2$ N/mm. The strip is 5 mm wide and 10 mm high. Also, draw the shear and moment diagrams for the strip.



$$I = \frac{1}{12}(0.005)(0.01)^3 = 0.4166(10^{-9}) \text{ m}^4$$

$$(\Delta_B)_1 = \frac{PL^3}{3EI} = \frac{50(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.0016 \text{ m}$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{2000\Delta_B(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.064\ \Delta_B$$

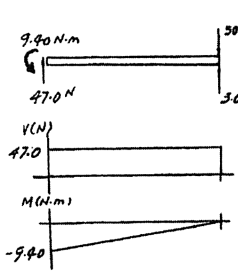
Compatibility Condition:

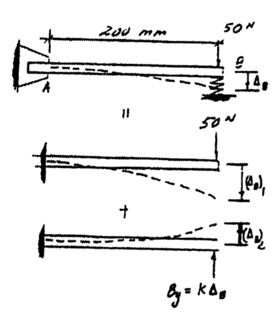
$+\downarrow \ \Delta_B = (\Delta_B)_1 - (\Delta_B)_2$

$\Delta_B = 0.0016 - 0.064\Delta_B$

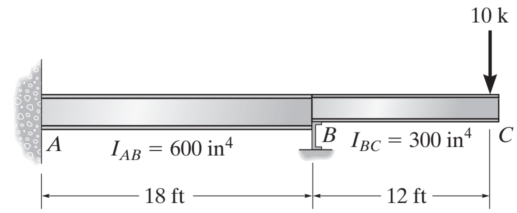
$\Delta_B = 0.001503 \text{ m} = 1.50 \text{ mm}$ **Ans.**

$B_y = k\Delta_B = 2(1.5) = 3.00 \text{ N}$



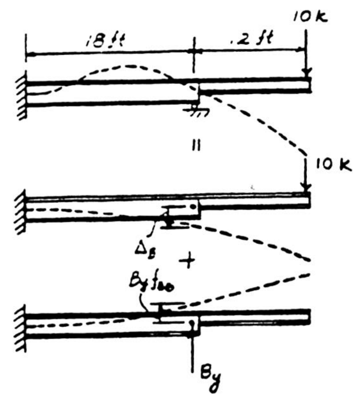


***10–8.** Determine the reactions at the supports. The moment of inertia for each segment is shown in the figure. Assume the support at B is a roller. Take $E = 29(10^3)$ ksi.



Compatibility Equation:

$(+\downarrow) \qquad \Delta_B - B_y f_{BB} = 0 \qquad (1)$



Use conjugate beam method:

$\zeta + \sum M_B' = 0; \qquad M_B' + \dfrac{2160}{EI_{AB}}(9) + \dfrac{1620}{EI_{AB}}(12) = 0$

$$\Delta_B = M_B' = -\frac{38\,880}{EI_{AB}}$$

$\zeta + \sum M_B' = 0; \qquad M_B' - \dfrac{162}{EI_{AB}}(12) = 0$

$$f_{BB} = M_B' = \frac{1944}{EI_{AB}}$$

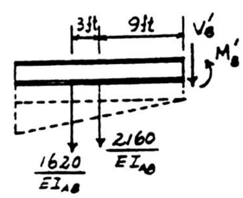
From Eq. 1 $\qquad \dfrac{38\,880}{EI_{AB}} - \dfrac{1944}{EI_{AB}} B_y = 0$

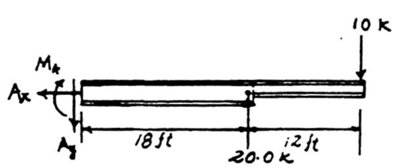
$B_y = 20\text{ k}$ **Ans.**

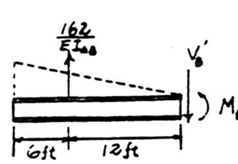
$A_y = 10\text{ k}$ **Ans.**

$M_A = 60\text{ k}\cdot\text{ft}$ **Ans.**

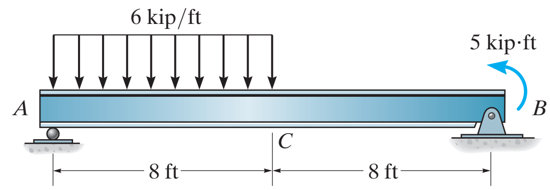
$A_x = 0$ **Ans.**







10–9. The simply supported beam is subjected to the loading shown. Determine the deflection at its center *C*. *EI* is constant.



Elastic Curves: The elastic curves for the uniform distributed load and couple moment are drawn separately as shown.

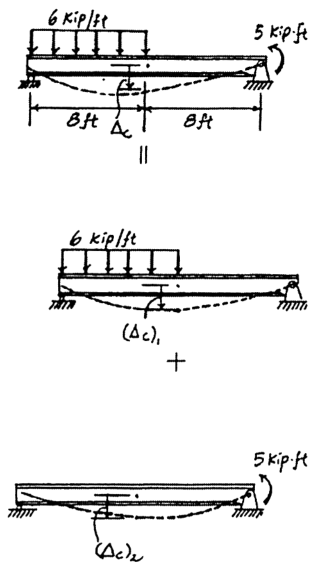
Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$(\Delta_C)_1 = \frac{-5wL^4}{768EI} = \frac{-5(6)(16^4)}{768EI} = \frac{2560\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(\Delta_C)_2 = \frac{M_o x}{6EIL}(x^2 - 3Lx + 2L^2)$$

$$= -\frac{5(8)}{6EI(16)}[8^2 - 3(16)(8) + 2(16^2)]$$

$$= \frac{80\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$



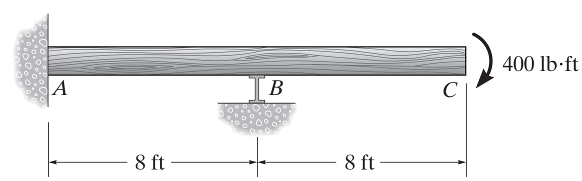
The displacement at C is

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{2560}{EI} + \frac{80}{EI}$$

$$= \frac{2640\ \text{kip}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans.}$$

10–10. Determine the reactions at the supports, then draw the moment diagram. Assume the support at *B* is a roller. *EI* is constant.



Compatibility Equation:

$$(+\downarrow) \qquad \Delta_B - 2_B - B_y f_{BB} = 0 \qquad (1)$$

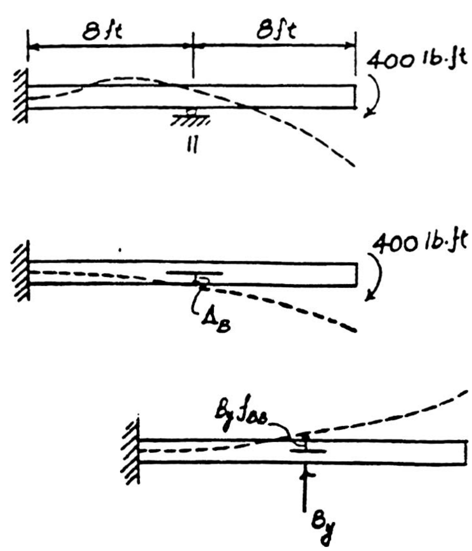
Use conjugate beam method:

$$\circlearrowleft + \sum M_{B'} = 0; \qquad M_{B'} + \frac{3200}{EI}(4) = 0$$

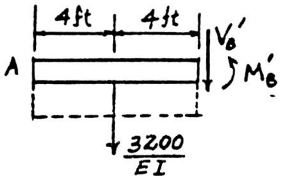
$$\Delta_B = M_{B'} = -\frac{12\ 800}{EI}$$

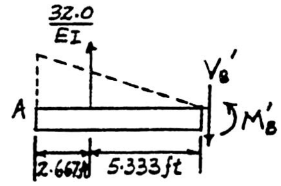
$$\circlearrowleft + \sum M_{B'} = 0; \qquad M_{B'} - \frac{32}{EI}(5.333) = 0$$

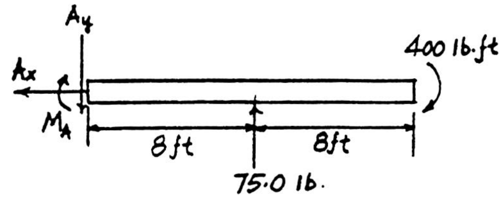
$$f_{BB} = M_{B'} = \frac{170.67}{EI}$$



10–10. Continued







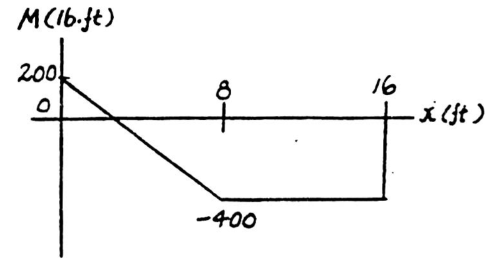
From Eq. 1 $\frac{12\,800}{EI} - B_y\left(\frac{170.67}{EI}\right) = 0$

$B_y = 75$ lb **Ans.**

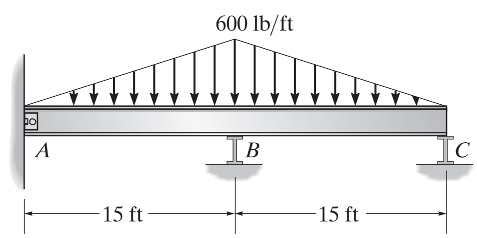
$A_x = 0$ **Ans.**

$A_y = 75$ lb **Ans.**

$M_A = 200$ lb · ft **Ans.**



10–11. Determine the reactions at the supports, then draw the moment diagram. Assume A is a pin and B and C are rollers. EI is constant.



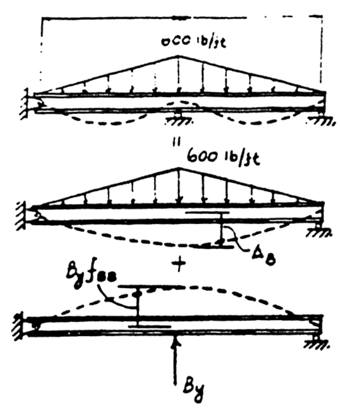
Compatibility Equation:

$(+\downarrow) \qquad \Delta_B - B_y f_{BB} = 0 \qquad (1)$

Use virtual work method:

$$\Delta_B = \int_0^L \frac{mM}{EI}dx = 2\int_0^{15} \frac{(4.5x - 0.00667x^3)(-0.5x)}{EI}dx = -\frac{4050}{EI}$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = 2\int_0^{15} \frac{(-0.5x)^2}{EI}dx = \frac{562.5}{EI}$$



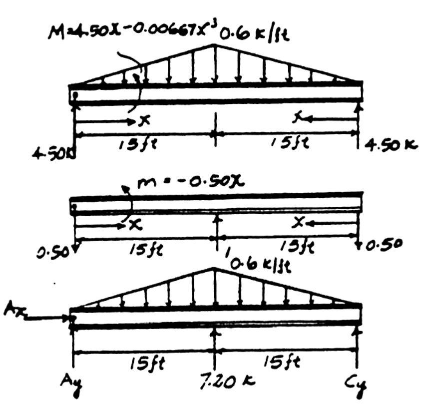
From Eq. 1 $\quad \frac{4050}{EI} - B_y \frac{562.5}{EI} = 0$

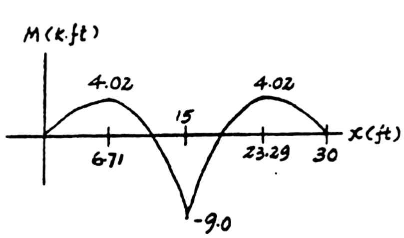
$B_y = 7.20$ k **Ans.**

$A_y = 0.900$ k **Ans.**

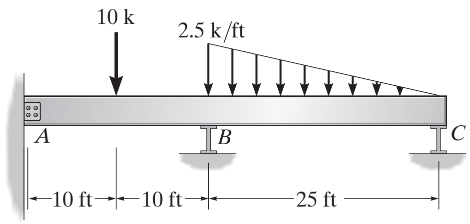
$A_x = 0$ **Ans.**

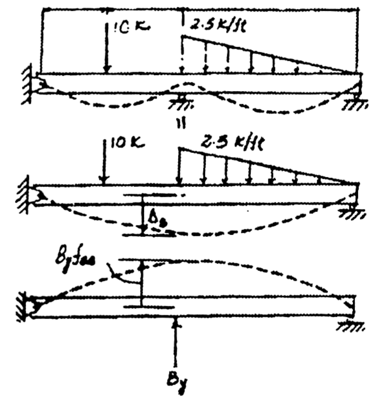
$C_y = 0.900$ k **Ans.**





***10–12.** Determine the reactions at the supports, then draw the moment diagram. Assume the support at A is a pin and B and C are rollers. EI is constant.





Compatibility Equation:

$$(+\downarrow) \qquad \Delta_B - B_y f_{BB} = 0 \qquad (1)$$

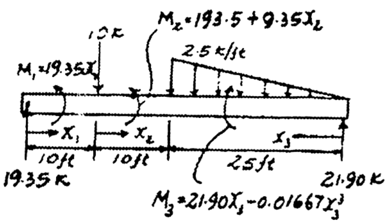
Use virtual work method:

$$\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{10} \frac{(-0.5556x_1)(19.35x_1)}{EI}dx_1$$

$$+ \int_0^{10} \frac{(-5.556 - 0.5556x_2)(193.5 + 9.35x_2)}{EI}dx_2$$

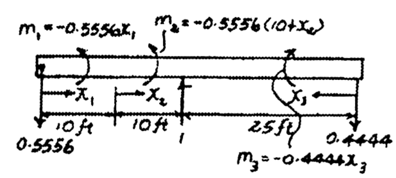
$$+ \int_0^{25} \frac{(-0.4444x_3)(21.9x_3 - 0.01667x_3^3)}{EI}dx_3$$

$$= -\frac{60\,263.53}{EI}$$



$$f_{BB} = \int_0^{10} \frac{(-0.5556x_1)^2}{EI}dx_1 + \int_0^{25} \frac{(-0.4444x_3)^2}{EI}dx_3 + \int_0^{10} \frac{(-5.556 - 0.5556x_2)^2}{EI}dx_2$$

$$= \frac{1851.85}{EI}$$



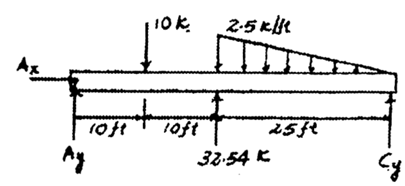
From Eq. 1 $\qquad \dfrac{60\,262.53}{EI} - B_y\dfrac{1851.85}{EI} = 0$

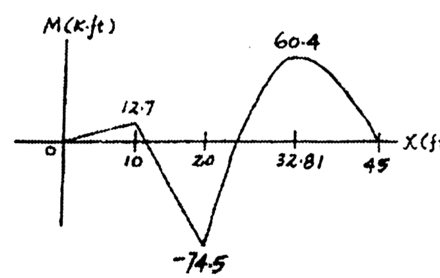
$B_y = 32.5\,\text{k}$ **Ans.**

$A_x = 0$ **Ans.**

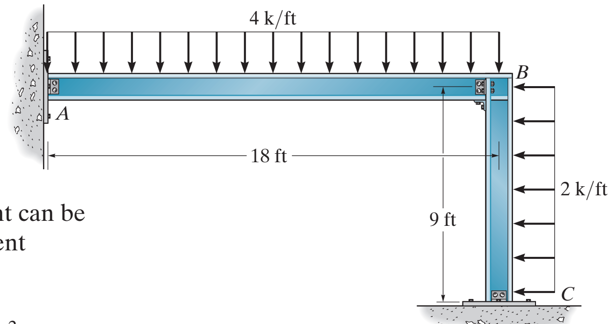
$A_y = 1.27\,\text{k}$ **Ans.**

$C_y = 7.44\,\text{k}$ **Ans.**





10–13. Determine the reactions at the supports. Assume A and C are pins and the joint at B is fixed connected. EI is constant.



Compatibility Equation: Referring to Fig a, the necessary displacement can be determined using virtual work method, using the real and virtual moment functions shown in Fig. b and c,

$$\Delta'_{C_n} = \int_0^L \frac{mM}{EI}dx = \int_0^{18\text{ ft}} \frac{(0.5x_1)(31.5x_1 - 2x_1^2)}{EI}dx_1 + \int_0^{9\text{ ft}} \frac{(x_2)(-x_2^2)}{EI}dx_2$$

$$= \frac{2733.75}{EI} \rightarrow$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_0^{18\text{ ft}} \frac{(0.5x_1)(0.5x_1)}{EI}dx_1 + \int_0^{9\text{ ft}} \frac{(x_2)(x_2)}{EI}dx_2$$

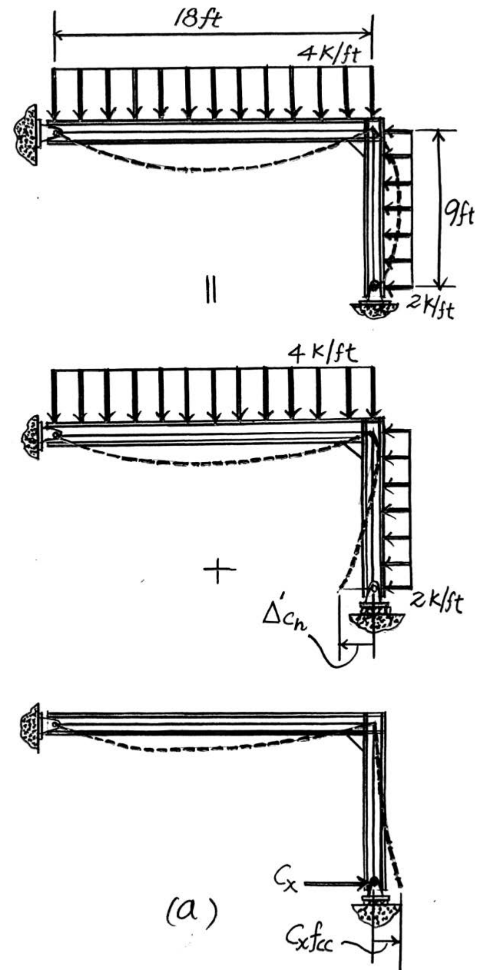
$$= \frac{729}{EI} \rightarrow$$

Using the principle of superposition,

$$\Delta_{C_n} = \Delta'_{Cn} + C_x f_{CC}$$

$$O = \frac{2733.75}{EI} + C_x\left(\frac{729}{EI}\right)$$

$$C_x = -3.75\text{k} = 3.75\text{k} \leftarrow \qquad \textbf{Ans.}$$



Equilibrium: Referring to the FBD of the frame in Fig. d,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 2(9) - 3.75 = 0$$

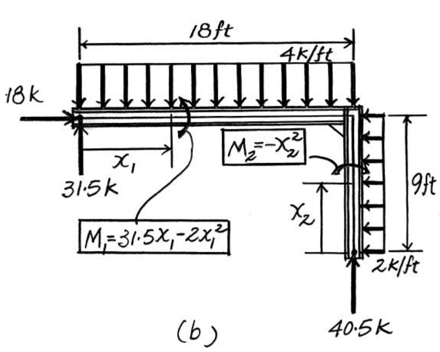
$$A_x = 21.75\text{k} \qquad \textbf{Ans.}$$

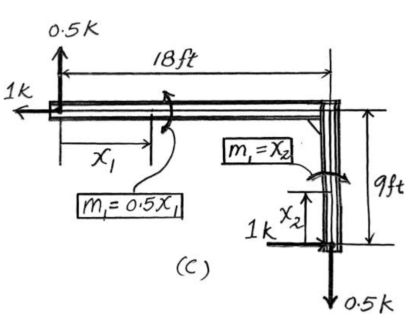
$$\circlearrowleft + \sum M_A = 0; \quad C_y(18) - 4(18)(9) - 2(9)(4.5) - 3.75(9) = 0$$

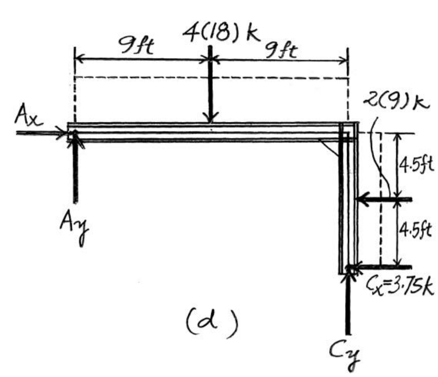
$$C_y = 42.375\text{k} = 42.4\text{k} \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad A_y + 42.375 - 4(18) = 0$$

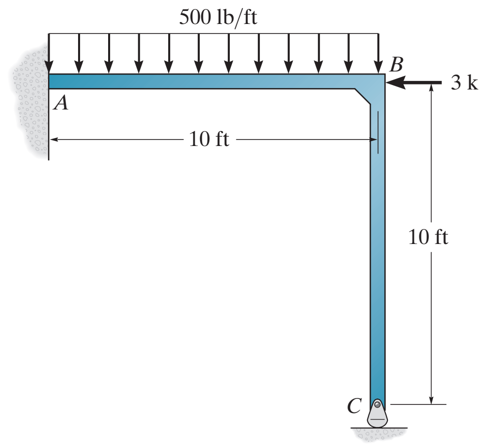
$$A_y = 29.625\text{k} = 29.6\text{k} \qquad \textbf{Ans.}$$







10–14. Determine the reactions at the supports. EI is constant.



Compatibility Equation:

$$(+\downarrow) \qquad 0 = \Delta_C - C_y f_{CC} \qquad (1)$$

Use virtual work method

$$\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^{10} \frac{(x_1)(-0.25x_1^2)}{EI}dx_1 = \frac{-625}{EI}$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_0^{10} \frac{(x_1)^2}{EI}dx_1 = \frac{333.33}{EI}$$

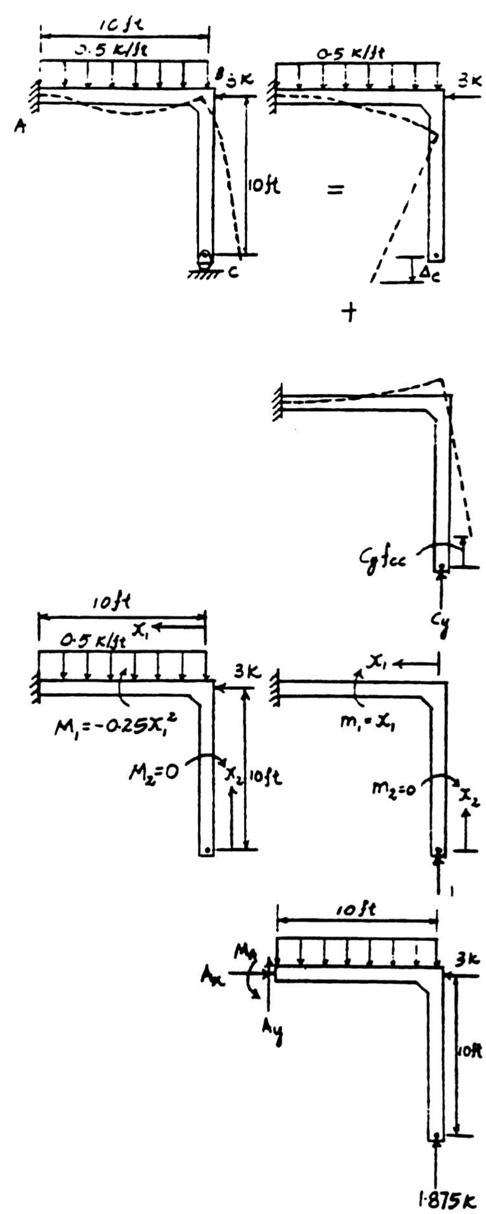
From Eq. 1 $\qquad 0 = \frac{625}{EI} - \frac{333.33}{EI}C_y$

$C_y = 1.875\,\text{k}$ **Ans.**

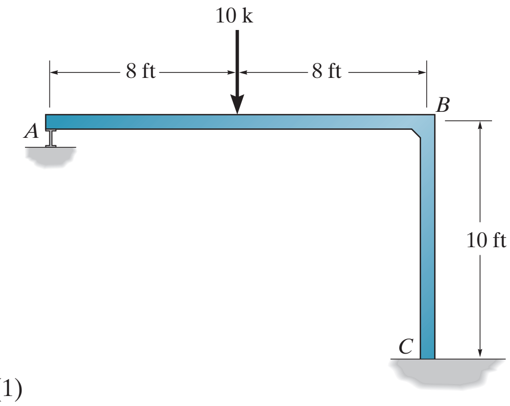
$A_x = 3.00\,\text{k}$ **Ans.**

$A_y = 3.125\,\text{k}$ **Ans.**

$M_A = 6.25\,\text{k} \cdot \text{ft}$ **Ans.**



10–15. Determine the reactions at the supports, then draw the moment diagram for each member. EI is constant.



Compatibility Equation:

$$(+\downarrow) \qquad 0 = \Delta_A - A_y f_{AA} \qquad (1)$$

Use virtual work method

$$\Delta_A = \int_0^L \frac{mM}{EI}dx = \int_0^8 \frac{(8 + x_2)(-10x_2)}{EI}dx_2 + \int_0^{10} \frac{(16)(-80)}{EI}dx_3 = \frac{-17066.67}{EI}$$

$$f_{AA} = \int_0^L \frac{mm}{EI}dx = \int_0^8 \frac{(x_1)^2}{EI}dx_1 + \int_0^8 \frac{(8 + x_2)^2}{EI}dx_2 + \int_0^{10} \frac{(16)^2}{EI}dx_3 = \frac{3925.33}{EI}$$

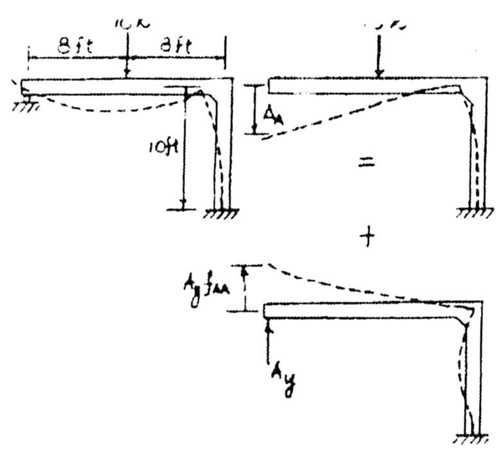
From Eq. 1 $\qquad 0 = \dfrac{17066.67}{EI} - \dfrac{3925.33}{EI}A_y$

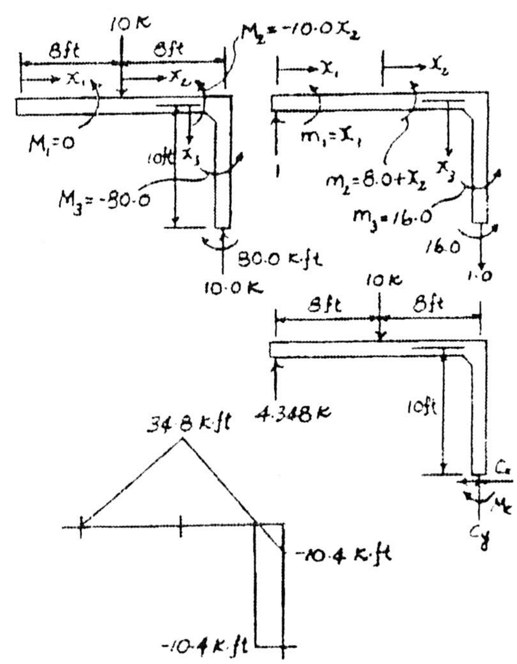
$A_y = 4.348\,\text{k} = 4.35\,\text{k}$ **Ans.**

$C_x = 0\,\text{k}$ **Ans.**

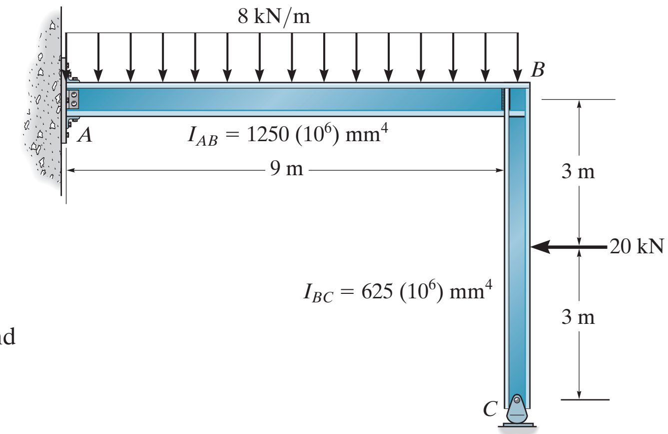
$C_y = 5.65\,\text{k}$ **Ans.**

$M_C = 10.4\,\text{k} \cdot \text{ft}$ **Ans.**





***10–16.** Determine the reactions at the supports. Assume A is fixed connected. E is constant.



Compatibility Equation. Referring to Fig. *a*, and using the real and virtual moment function shown in Fig. *b* and *c*, respectively,

$$\Delta'_{C_v} = \int_0^L \frac{mM}{EI}dx = \int_0^{9\,m} \frac{(-x_3)[-(4x_3^2 + 60)]}{EI_{AB}}dx_3 = \frac{8991}{EI_{AB}} \downarrow$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_0^{9\,m} \frac{(-x_3)(-x_3)}{EI_{AB}}dx_3 = \frac{243}{EI_{AB}} \downarrow$$

Using the principle of superposition,

$$\Delta_{C_v} = \Delta'_{C_v} + C_y f_{CC}$$

$$(+\downarrow) \quad 0 = \frac{8991}{EI_{AB}} + C_y\left(\frac{243}{EI_{AB}}\right)$$

$$C_y = -37.0\text{ kN} = 37.0\text{ kN} \uparrow$$ **Ans.**

Equilibrium. Referring to the FBD of the frame in Fig. *d*,

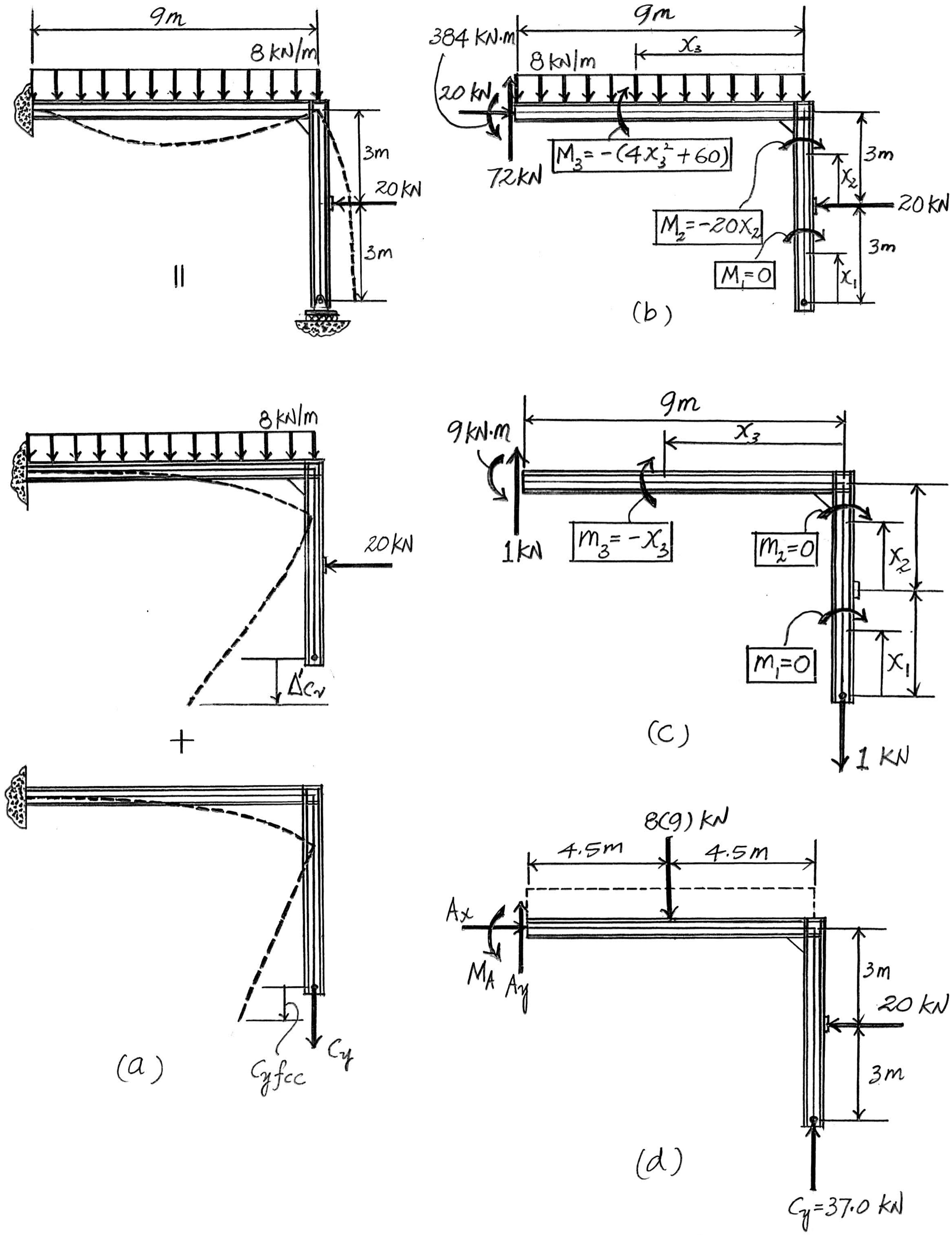
$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x - 20 = 0 \quad A_x = 20\text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $\quad M_A + 37.0(9) - 8(9)(4.5) - 20(3) = 0$

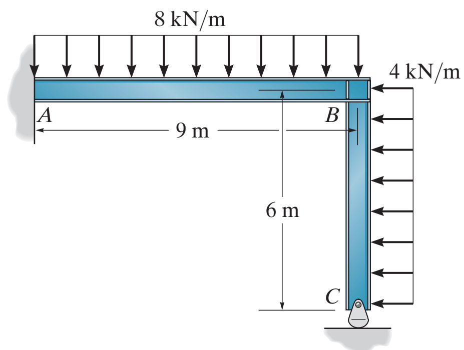
$M_A = 51.0\text{ kN}\cdot\text{m}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad A_y + 37.0 - 8(9) = 0 \quad A_y = 35.0\text{ kN}$ **Ans.**

10–16. Continued



10–17. Determine the reactions at the supports. EI is constant.



Compatibility Equation:

$$(+\downarrow) \qquad 0 = \Delta_C - C_y f_{CC} \qquad (1)$$

Use virtual work method:

$$\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^9 \frac{(-x_1 + 9)(72x_1 - 4x_1^2 - 396)}{EI}dx_1 = \frac{-9477}{EI}$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_o^9 \frac{(-x_1 + 9)^2}{EI}dx_1 = \frac{243.0}{EI}$$

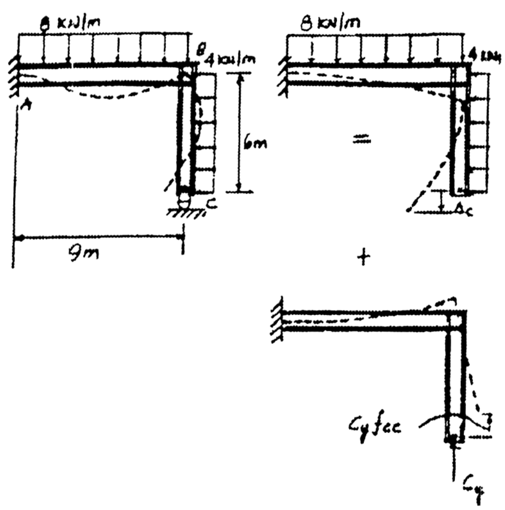
From Eq. 1 $\qquad 0 = \dfrac{9477}{EI} - \dfrac{243.0}{EI}C_y$

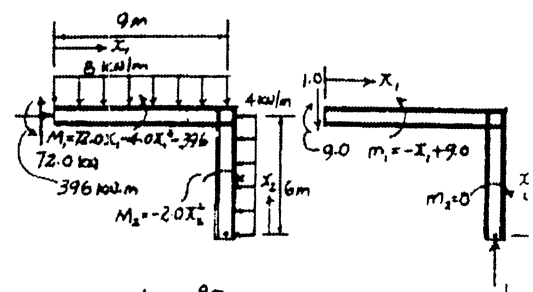
$C_y = 39.0 \text{ kN}$

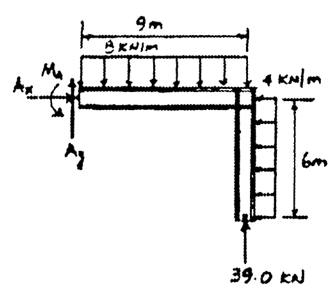
$A_y = 33.0 \text{ kN}$

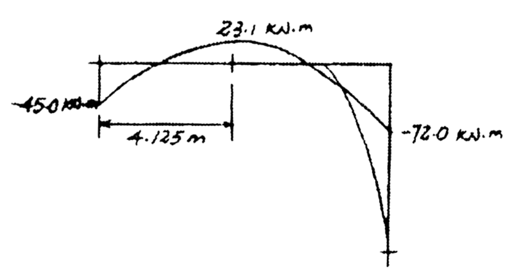
$A_x = 24.0 \text{ kN}$

$M_A = 45.0 \text{ kN} \cdot \text{m}$

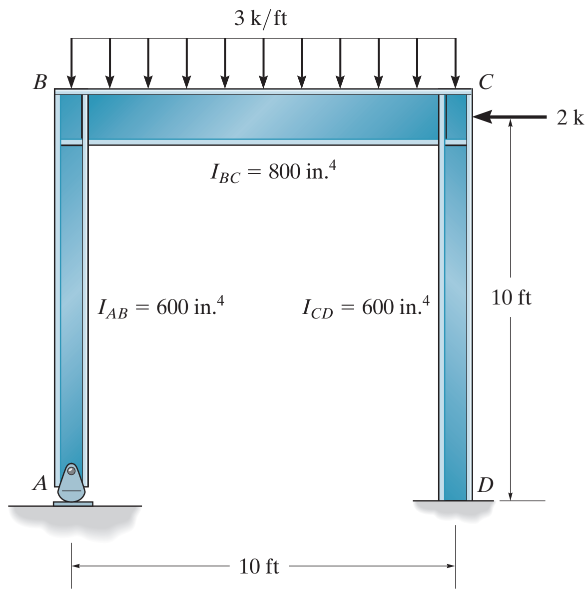








10–18. Determine the reactions at the supports A and D. The moment of inertia of each segment of the frame is listed in the figure. Take $E = 29(10^3)$ ksi.



$$\Delta_A = \int_0^L \frac{mM}{EI}dx = 0 + \int_0^{10} \frac{(lx)(\frac{3}{2}x^2)}{EI_{BC}}dx + \int_0^{10} \frac{(10)(170-2x)}{EI_{CD}}dx$$

$$= \frac{18.8125}{EI_{CD}}$$

$$f_{AA} = \int_0^L \frac{m^2}{EI}dx = 0 + \int_0^{10} \frac{x^2}{EI_{BC}}dx + \int_0^{10} \frac{10^2}{EI_{CD}}dx = \frac{1250}{EI_{CD}}$$

$$+\downarrow \Delta_A + A_y f_{AA} = 0$$

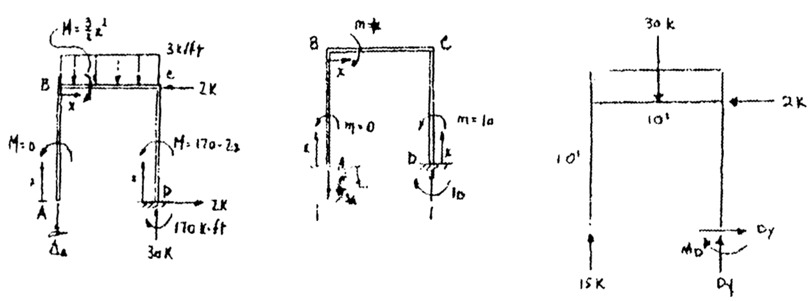
$$\frac{18{,}812.5}{EI_{CD}} + A_y\left(\frac{1250}{EI_{CD}}\right) = 0$$

$$A_y = -15.0 \text{ k} \qquad \textbf{Ans.}$$

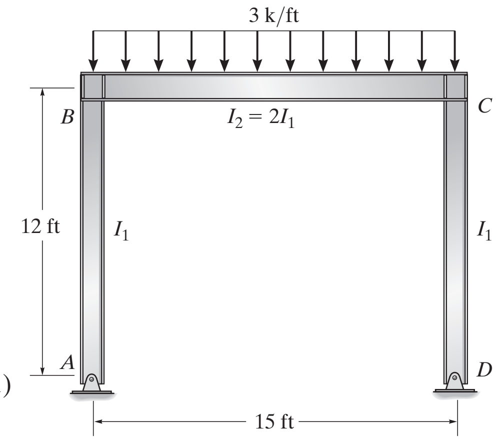
$+\uparrow \sum F_y = 0; \quad -30 + 15 + D_y = 0; \quad D_y = 15.0$ k **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad D_x = 2$ k **Ans.**

$\circlearrowleft + \sum M_D = 0; \quad 15.0(10) - 2(10) - 30(5) + M_D = 0; \quad M_D = 19.5$ k·ft **Ans.**



10–19. The steel frame supports the loading shown. Determine the horizontal and vertical components of reaction at the supports A and D. Draw the moment diagram for the frame members. E is constant.



Compatibility Equation:

$$\Delta_D + D_x f_{DD} = 0 \qquad (1)$$

Use virtual work method:

$$\Delta_D = \int_0^L \frac{mM}{EI}dx = 0 + \int_0^{15} \frac{12(22.5x - 1.5x^2)}{E(2I_1)}dx + 0 = \frac{5062.5}{EI_1}$$

$$f_{DD} = \int_0^L \frac{mm}{EI}dx = 2\int_o^{12} \frac{(1x)^2}{EI_1}dx + \int_0^{15} \frac{(12)^2}{E(2I_1)}dx = \frac{2232}{EI_1}$$

From Eq. 1

$$\frac{5062.5}{EI_1} + D_x\frac{2232}{EI_1} = 0$$

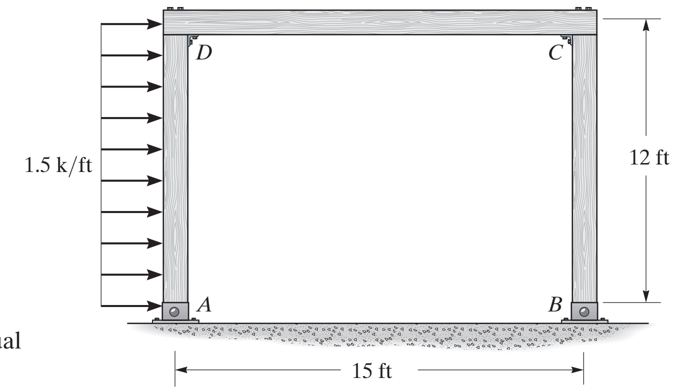
$D_x = -2.268\text{ k} = -2.27\text{ k}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad -45(7.5) + D_y(15) = 0 \quad D_y = 22.5\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 22.5 - 45 + A_y = 0; \quad A_y = 22.5\text{ k}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 2.268 = 0; \quad A_x = 2.27\text{ k}$ **Ans.**

***10–20.** Determine the reactions at the supports. Assume A and B are pins and the joints at C and D are fixed connections. EI is constant.



Compatibility Condition: Referring to Fig. a, the real and virtual moment functions shown in Fig. b and c, respectively,

$$\Delta'_{B_h} = \int_0^L \frac{mM}{EI}dx = \int_0^{12\text{ ft}} \frac{x_1(18x_1 - 0.75x_1^2)}{EI}dx_1 + \int_0^{15\text{ ft}} \frac{12(7.20x_2)}{EI}dx_2 + 0$$

$$= \frac{16200}{EI} \quad \rightarrow$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^{12\text{ ft}} \frac{x_1(x_1)}{EI}dx_1 + \int_0^{15\text{ ft}} \frac{12(12)}{EI}dx_2 + \int_0^{12\text{ ft}} \frac{x_3(x_3)}{EI}dx_3$$

Using the principle of superposition, Fig. a,

$$\Delta_{B_h} = \Delta'_{Bh} + B_x f_{BB}$$

$$(\overset{+}{\rightarrow}) \qquad 0 = \frac{16200}{EI} + B_x\left(\frac{3312}{EI}\right)$$

$B_x = -4.891\text{ k} = 4.89\text{ k} \leftarrow$ **Ans.**

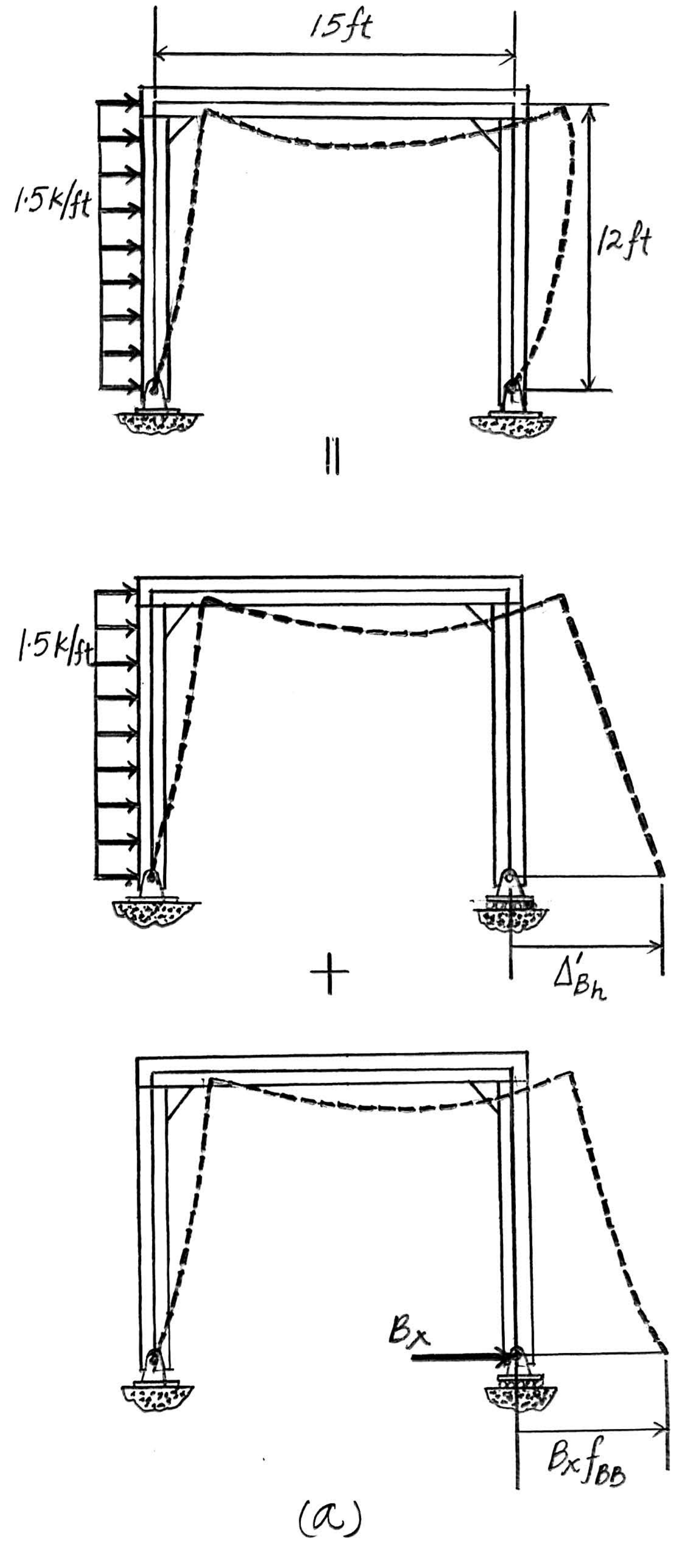
Equilibrium: Referring to the FBD of the frame in Fig. d,

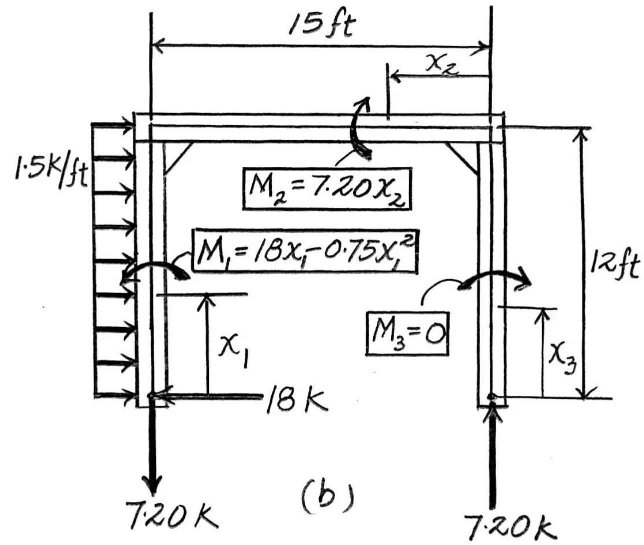
$\overset{+}{\rightarrow} \sum F_x = 0; \qquad 15(12) - 4.891 - A_x = 0 \qquad A_x = 13.11\text{ k} = 13.1\text{ k}$ **Ans.**

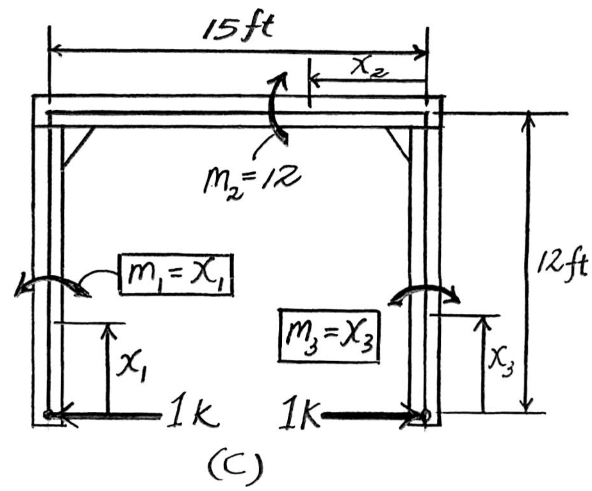
$\circlearrowleft + \sum M_A = 0; \qquad B_y(15) - 1.5(12)(6) = 0 \qquad B_y = 7.20\text{ k}$ **Ans.**

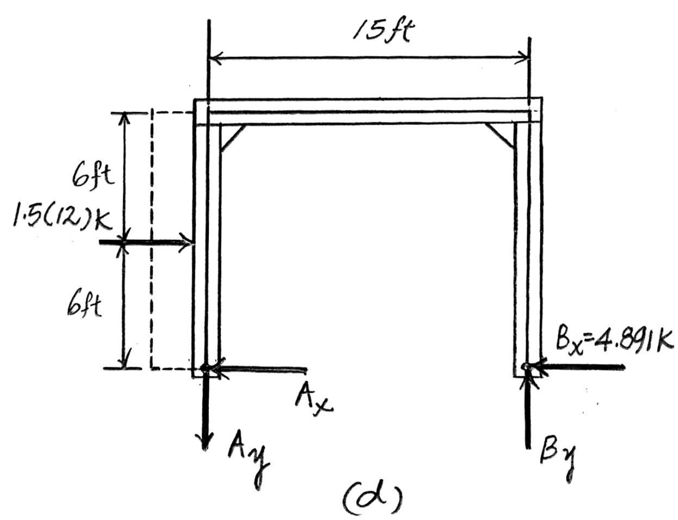
$+\uparrow \sum F_y = 0; \qquad 7.20 - A_y = 0 \qquad A_y = 7.20\text{ k}$ **Ans.**

10–20. Continued

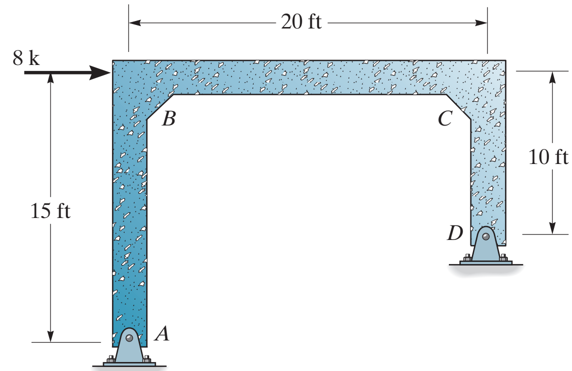








10–21. Determine the reactions at the supports. Assume A and D are pins. EI is constant.



Compatibility Equation: Referring to Fig. *a*, and the real and virtual moment functions shown in Fig. *b* and *c*, respectively.

$$\Delta' D_h = \int_0^L \frac{mM}{EI} dx = \int_0^{15\text{ ft}} \frac{(x_1)(8x_1)}{EI} dx_1 + \int_0^{20\text{ ft}} \frac{(0.25x_2 + 10)(6x_2)}{EI} dx_2 + 0$$

$$= \frac{25000}{EI} \quad \rightarrow$$

$$f_{DD} = \int_0^L \frac{mm}{EI} dx = \int_0^{15\text{ ft}} \frac{(x_1)(x_1)}{EI} dx_1 + \int_0^{20\text{ ft}} \frac{(0.25x_2 + 10)(0.25x_2 + 10)}{EI} dx_2$$

$$+ \int_0^{10\text{ ft}} \frac{(x_3)(x_3)}{EI} dx_3$$

$$= \frac{4625}{EI} \quad \rightarrow$$

Using the principle of superposition, Fig. *a*,

$$\Delta_{D_h} = \Delta'_{Dh} + D_x f_{DD}$$

$$(\overset{+}{\rightarrow}) \qquad 0 = \frac{25000}{EI} + D_x\left(\frac{4625}{EI}\right)$$

$$D_x = -5.405 \text{ k} = 5.41\ k \quad \leftarrow$$ **Ans.**

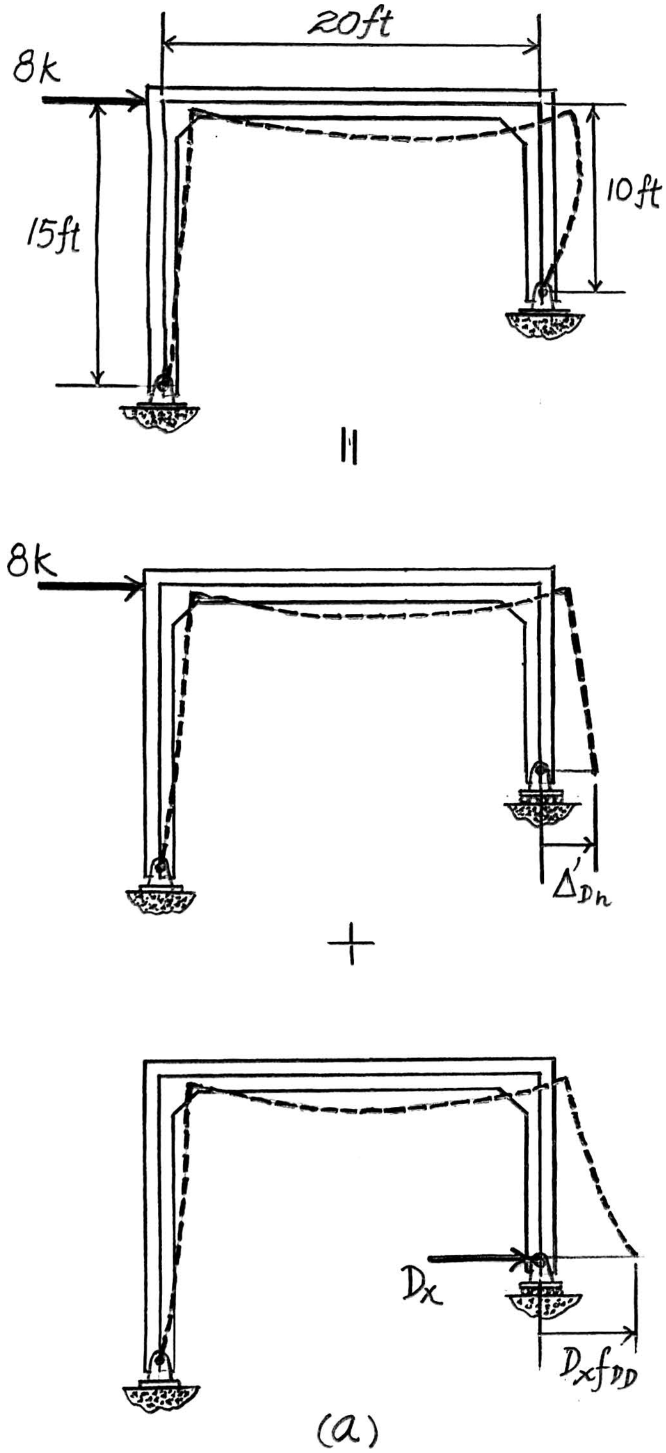
Equilibrium:

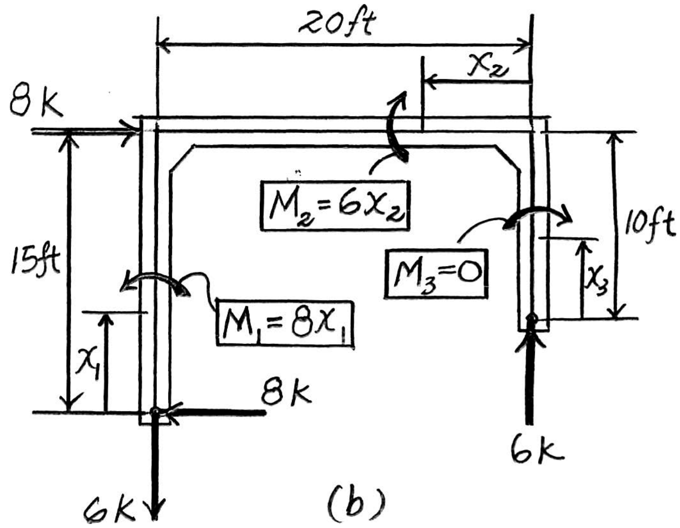
$\overset{+}{\rightarrow} \sum F_x = 0; \quad 8 - 5.405 - A_x = 0 \qquad A_x = 2.5946 \text{ k} = 2.59 \text{ k}$ **Ans.**

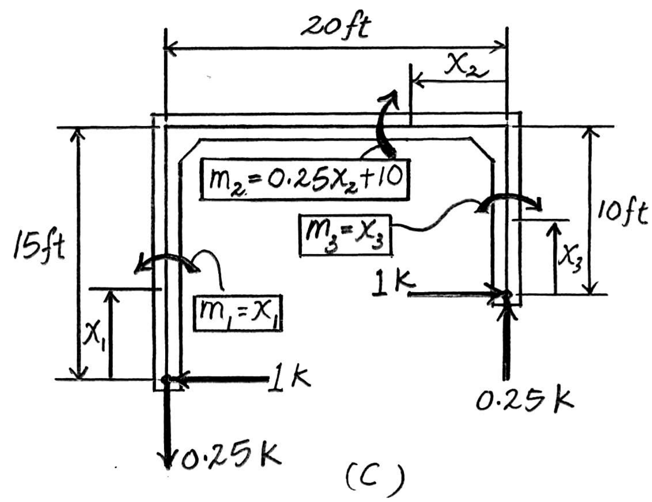
$\circlearrowleft + \sum M_A = 0; \quad D_y(20) + 5.405(5) - 8(15) = 0 \qquad D_y = 4.649 \text{ k} = 4.65 \text{ k}$ **Ans.**

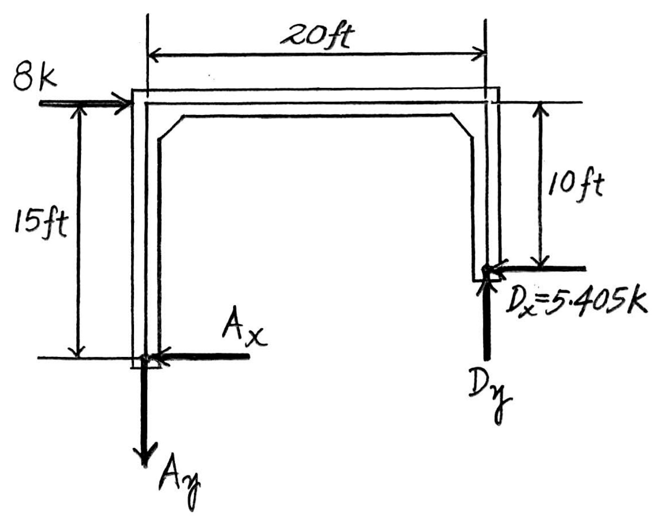
$+\uparrow \sum F_y = 0; \quad 4.649 - A_y = 0 \qquad A_y = 4.649 \text{ k} = 4.65 \text{ k}$ **Ans.**

10–21. Continued

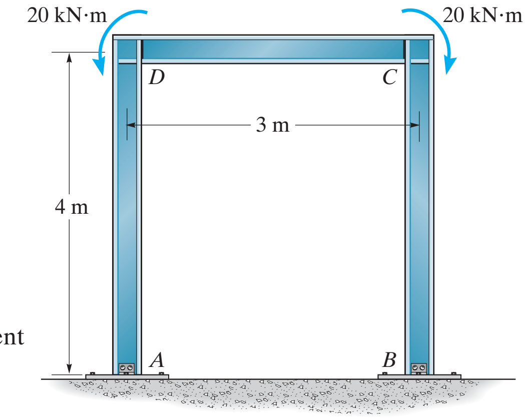








10–22. Determine the reactions at the supports. Assume A and B are pins. EI is constant.



Compatibility Condition: Referring to Fig. a, and the real and virtual moment functions shown in Fig. b and c, respectively,

$$\Delta'_{B_h} = \int_0^L \frac{mM}{EI}dx = 0 + \int_0^{3\,\text{m}} \frac{(-4)(-20)}{EI}dx_2 + 0 = \frac{240}{EI} \quad \leftarrow$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^{4\,\text{m}} \frac{(-x_1)(-x_1)}{EI}dx_1 + \int_0^{3\,\text{m}} \frac{(-4)(-4)}{EI}dx_2 + \int_0^{4\,\text{m}} \frac{(-x_3)(-x_3)}{EI}dx_3$$

$$= \frac{90.67}{EI} \quad \leftarrow$$

Applying the principle of superposition, Fig. a,

$$\Delta_{B_h} = \Delta'_{B_h} + B_x f_{BB}$$

$$(\overset{+}{\leftarrow}) \qquad 0 = \frac{240}{EI} + B_x\left(\frac{90.67}{EI}\right)$$

$$B_x = -2.647 \text{ kN} = 2.65 \text{ kN} \quad \rightarrow$$ **Ans.**

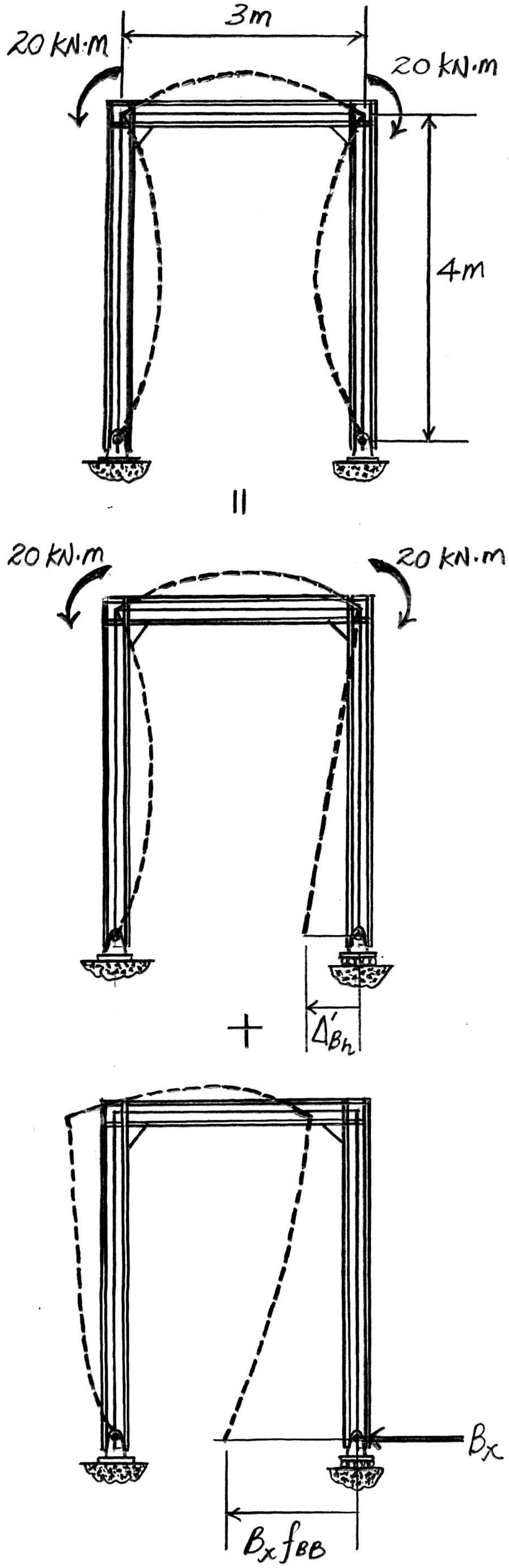
Equilibrium: Referring to the FBD of the frame shown in Fig. d,

$\overset{+}{\leftarrow} \sum F_x = 0; \qquad A_x - 2.647 = 0 \qquad A_x = 2.647 \text{ kN} = 2.65 \text{ kN}$ **Ans.**

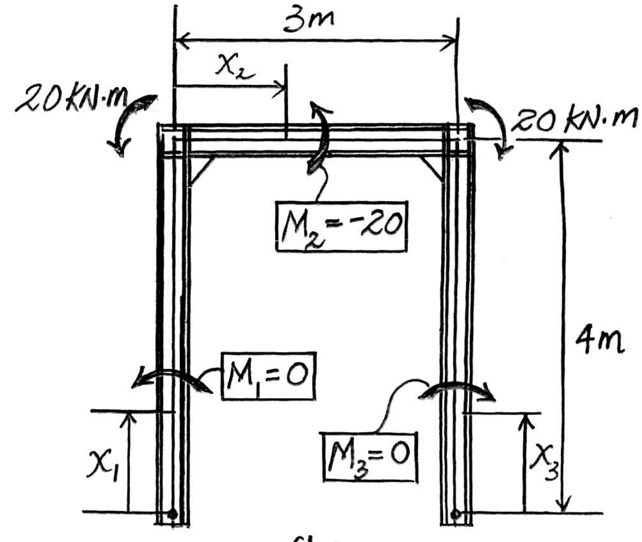
$\circlearrowleft + \sum M_A = 0; \qquad B_y(3) + 20 - 20 = 0 \qquad B_y = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y = 0$ **Ans.**

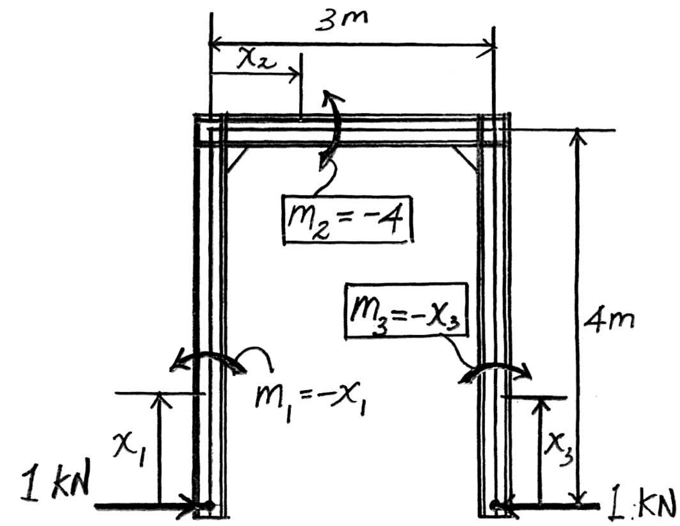
10–22. Continued



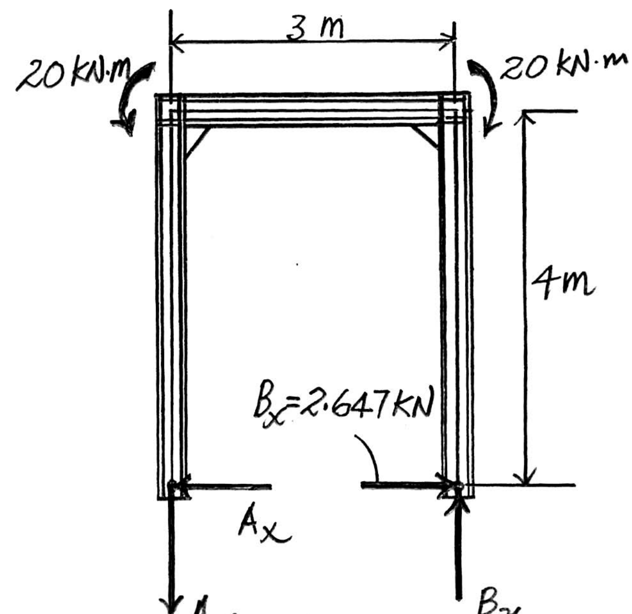
(a)



(b)

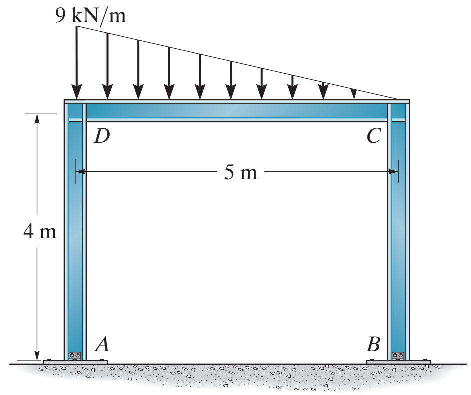


(c)



(d)

10–23. Determine the reactions at the supports. Assume A and B are pins. EI is constant.



Compatibility Equation: Referring to Fig. *a*, and the real and virtual moment functions in Fig. *b* and *c*, respectively,

$$\Delta' B_h = \int_0^L \frac{mM}{EI}dx = 0 + \int_0^{5\text{ m}} \frac{4(7.50x_2 - 0.3x_2^3)}{EI}dx_2 + 0 = \frac{187.5}{EI} \quad \rightarrow$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^{4\text{ m}} \frac{(x_1)(x_1)}{EI}dx_1 + \int_0^{5\text{ m}} \frac{4(4)}{EI}dx_2 + \int_0^{4\text{ m}} \frac{(x_3)(x_3)}{EI}dx_3$$

$$= \frac{122.07}{EI} \quad \rightarrow$$

Applying to the principle of superposition, Fig. *a*,

$$\Delta_{B_h} = \Delta'_{B_h} + B_x f_{BB}$$

$$(\xrightarrow{+}) \qquad 0 = \frac{187.5}{EI} + B_x\left(\frac{122.07}{EI}\right)$$

$$B_x = -1.529 \text{ kN} = 1.53 \text{ kN} \quad \leftarrow \qquad \textbf{Ans.}$$

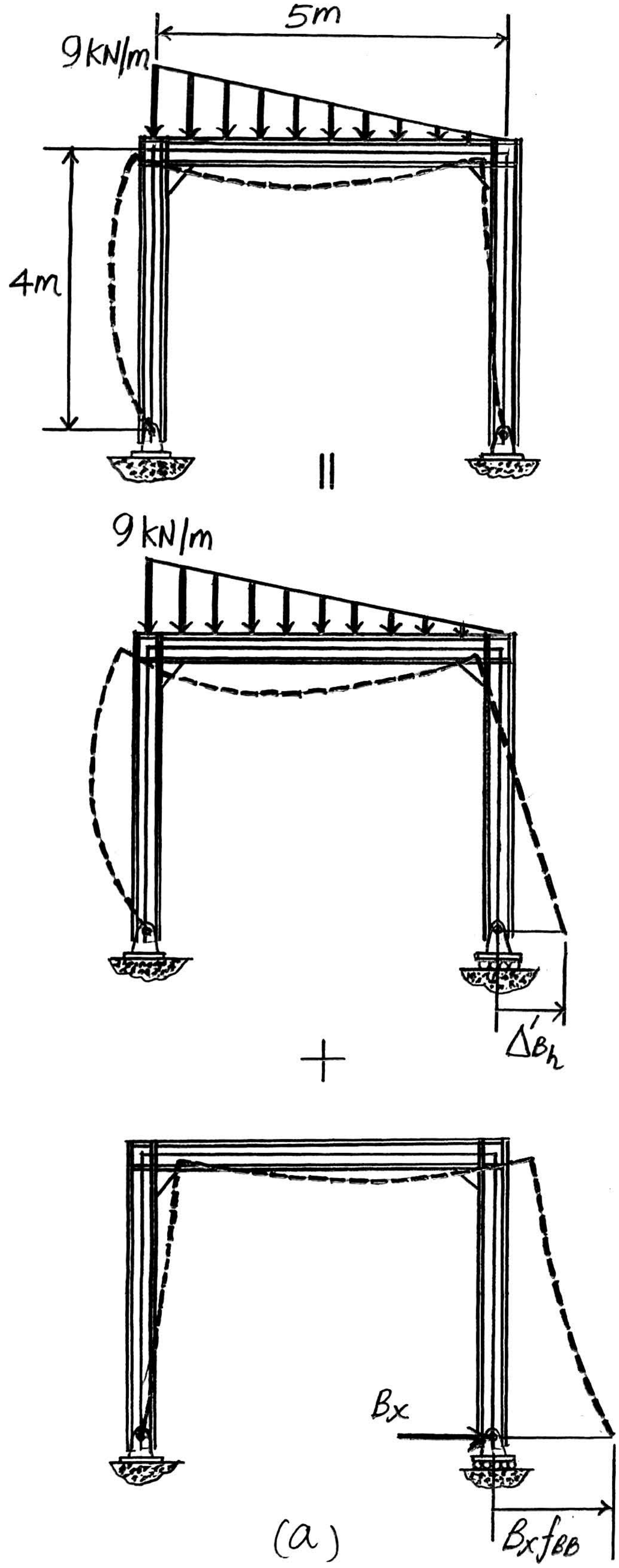
Equilibrium: Referring to the FBD of the frame in Fig. *d*,

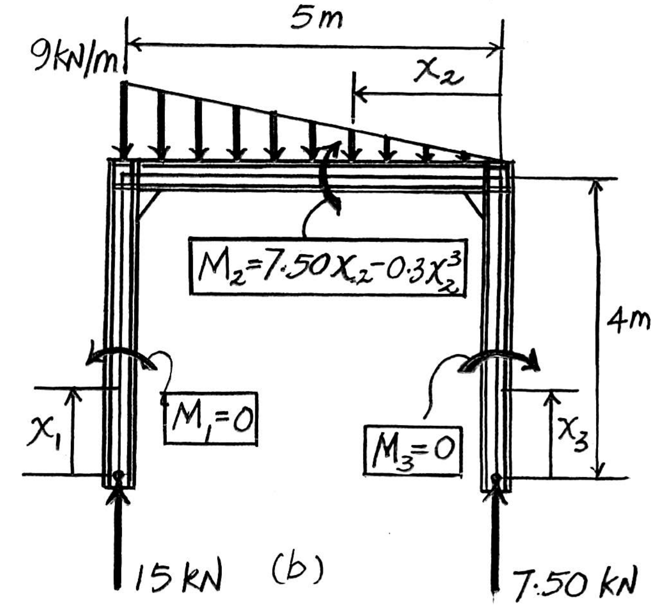
$$\xrightarrow{+} \sum F_x = 0; \qquad A_x - 1.529 = 0 \qquad A_x = 1.529 \text{ kN} = 1.53 \text{ kN} \qquad \textbf{Ans.}$$

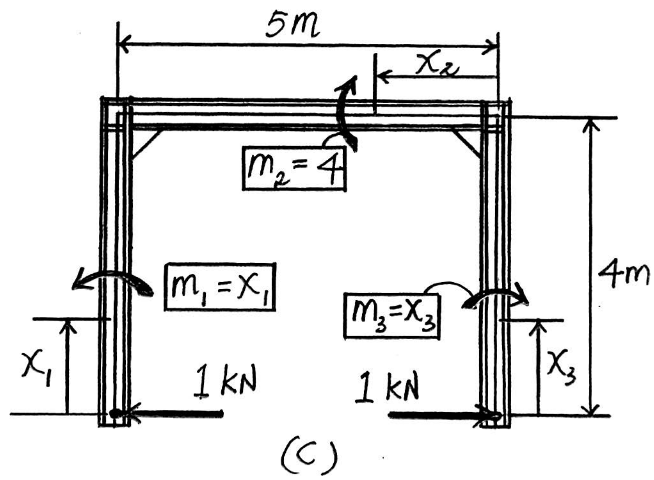
$$\circlearrowleft + \sum M_A = 0; \quad B_y(5) - \frac{1}{2}(9)(5)(1.667) = 0 \quad B_y = 7.50 \text{ kN} \qquad \textbf{Ans.}$$

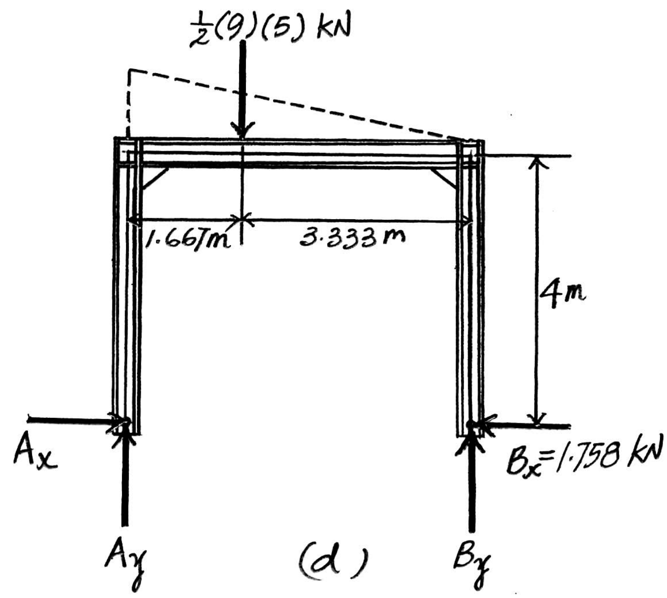
$$\circlearrowleft + \sum M_B = 0; \quad \frac{1}{2}(9)(5)(3.333) - A_y(5) = 0 \quad A_y = 15.0 \text{ kN} \qquad \textbf{Ans.}$$

10–23. Continued

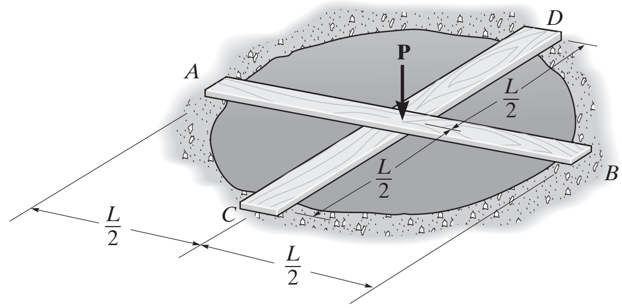








***10–24.** Two boards each having the same EI and length L are crossed perpendicular to each other as shown. Determine the vertical reactions at the supports. Assume the boards just touch each other before the load **P** is applied.



$$\Delta_E' = \Delta_E'$$

$$\Delta_E' = M_{E'} = -\frac{(P - E_y)L^2}{16EI}\left(\frac{L}{2}\right) + \frac{(P - E_y)L^2}{16EI}\left(\frac{L}{6}\right)$$

$$= -\frac{(P - E_y)L^3}{48EI}$$

$$\Delta_E'' = M_{E''} = \frac{E_yL^2}{16EI}\left(\frac{L}{6}\right) - \frac{E_yL^2}{16EI}\left(\frac{L}{2}\right)$$

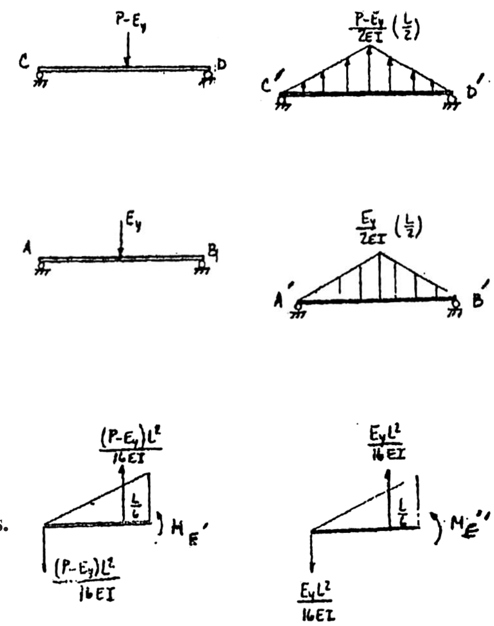
$$= -\frac{E_yL^3}{48EI}$$

$$\Delta_E' = \Delta_E''$$

$$-\frac{(P - E_y)L^3}{48EI} = -\frac{E_yL^3}{48EI}$$

$$-(P - E_y) = -E_y$$

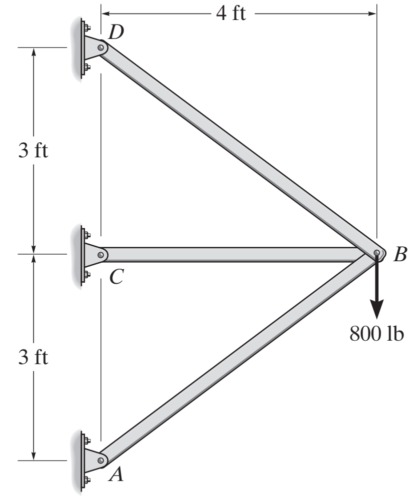
$$E_y = \frac{P}{2}$$ **Ans.**



For equilibrium:

$$A_y = B_y = C_y = D_y = \frac{P}{4}$$ **Ans.**

10–25. Determine the force in each member of the truss. *AE* is constant.



Compatibility Equation:

$$0 = \Delta_{AB} + F_{AB} f_{ABAB} \qquad (1)$$

Use virtual work method:

$$\Delta_{AB} = \sum \frac{nNL}{AE} = \frac{(1.0)(1.333)(5)}{AE} + \frac{(-1.6)(-1.067)(4)}{AE} = \frac{13.493}{AE}$$

$$f_{ABAB} = \sum \frac{nnL}{AE} = \frac{2(1)^2(5)}{AE} + \frac{(-1.6)^2(4)}{AE} = \frac{20.24}{AE}$$

From Eq. 1 $\qquad 0 = \dfrac{13.493}{AE} + \dfrac{20.24}{AE} F_{AB}$

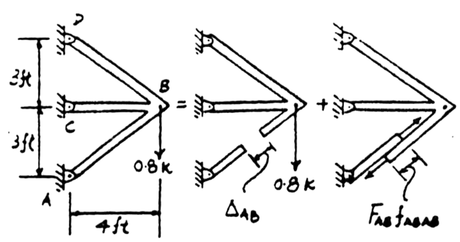
$F_{AB} = -0.667\text{ k} = 0.667\text{ k (C)}$ **Ans.**

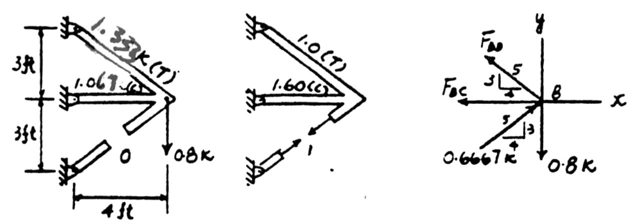
Joint B:

$+\uparrow \sum F_y = 0; \qquad \frac{3}{5} F_{BD} + \left(\frac{3}{5}\right) 0.6666 - 0.8 = 0$

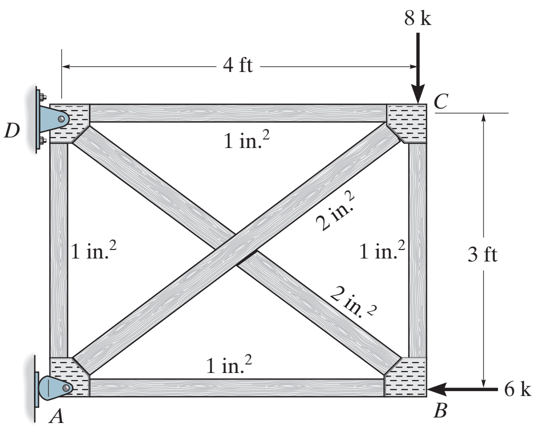
$F_{BD} = 0.667\text{ k (T)}$ **Ans.**

$\overset{+}{\leftarrow} \sum F_x = 0; \qquad F_{BC} = 0$ **Ans.**



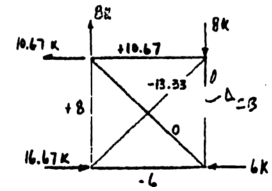


10–26. Determine the force in each member of the truss. The cross-sectional area of each member is indicated in the figure. $E = 29(10^3)$ ksi. Assume the members are pin connected at their ends.



$$\Delta_{CB} = \sum \frac{nNL}{AE} = \frac{1}{E}\left[\frac{(1.33)(10.67)(4)}{1} + \frac{(1.33)(-6)(4)}{1} + \frac{(1)(8)(3)}{1}\right] + \left[\frac{(-1.667)(-13.33)(5)}{2}\right]$$

$$= \frac{104.4}{E}$$

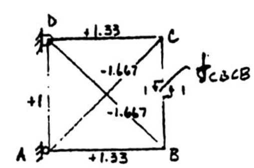


$$f_{CBCB} = \sum \frac{n^2L}{AE} = \frac{1}{E}\left[\frac{2(1.33)^2(4)}{1} + \frac{2(1)^2(3)}{1} + \frac{2(-1.667)^2(5)}{2}\right]$$

$$= \frac{34.1}{E}$$

$$\Delta_{CB} + F_{CB} f_{CBCB} = 0$$

$$\frac{104.4}{E} + F_{CB}\left(\frac{34.1}{E}\right) = 0$$

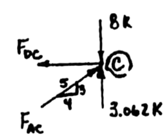


$$F_{CB} = -3.062 \text{ k} = 3.06 \text{ k (C)}$$ **Ans.**

Joint C:

$+\uparrow \sum F_y = 0;$ $\quad \frac{3}{5}F_{AC} - 8 + 3.062 = 0;$

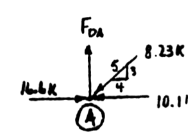
$F_{AC} = 823$ k (C) **Ans.**



$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad \frac{4}{5}(8.23) - F_{DC} = 0;$

$F_{DC} = 6.58$ k (T) **Ans.**

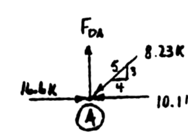
Joint B:



$+\uparrow \sum F_y = 0;$ $\quad -3.062 + \left(\frac{3}{5}\right)(F_{DB}) = 0;$

$F_{DB} = 5.103 \text{ k} = 5.10$ k (T) **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad F_{AB} - 6 - 5.103\left(\frac{4}{5}\right) = 0;$



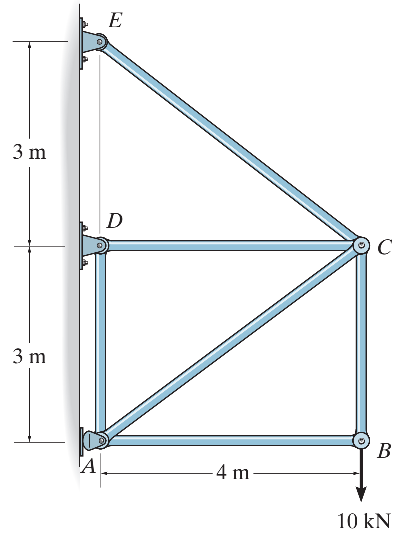
$F_{AB} = 10.1$ k (C) **Ans.**

Joint A:

$+\uparrow \sum F_y = 0;$ $\quad -8.23 + \left(\frac{3}{5}\right)F_{DA} = 0;$

$F_{DA} = 4.94$ k (T) **Ans.**

10–27. Determine the force in member AC of the truss. AE is constant.



Compatibility Equation: Referring to Fig. a, and using the real force and virtual force in each member shown in Fig. b and c, respectively,

$$\Delta'_{AC} = \sum\frac{nNL}{AE} = \frac{1(16.67)(5)}{AE} + \frac{(-1.60)(-13.33)(4)}{AE} = \frac{168.67}{AE}$$

$$f_{ACAC} = \sum\frac{n^2L}{AE} = 2\left[\frac{(1^2)(5)}{AE}\right] + \frac{[(-1.60)^2](4)}{AE} + \frac{[(-0.6)^2](3)}{AE}$$

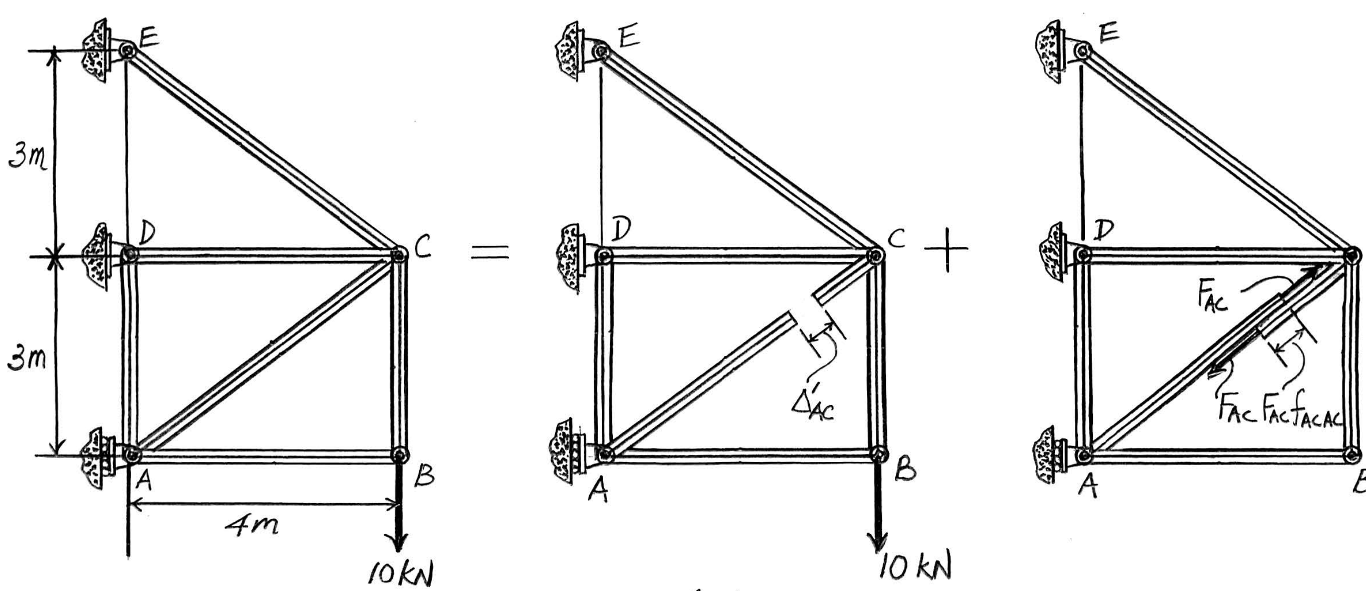
$$= \frac{21.32}{AE}$$

Applying the principle of superposition, Fig. a

$$\Delta_{AC} = \Delta'_{AC} + F_{AC}f_{ACAC}$$

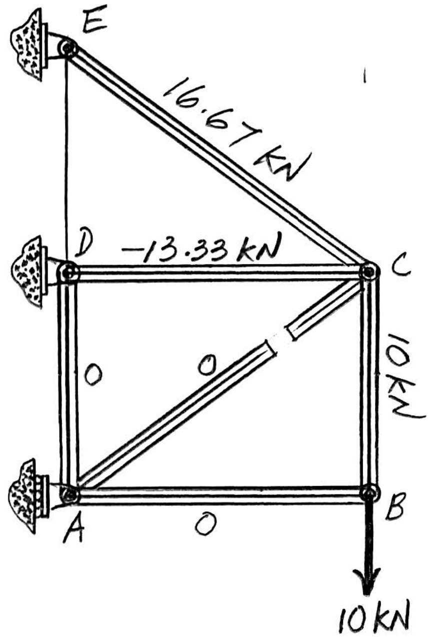
$$0 = \frac{168.67}{AE} + F_{AC}\left(\frac{21.32}{AE}\right)$$

$F_{AC} = -7.911 \text{ kN} = 7.91 \text{ kN (C)}$ **Ans.**

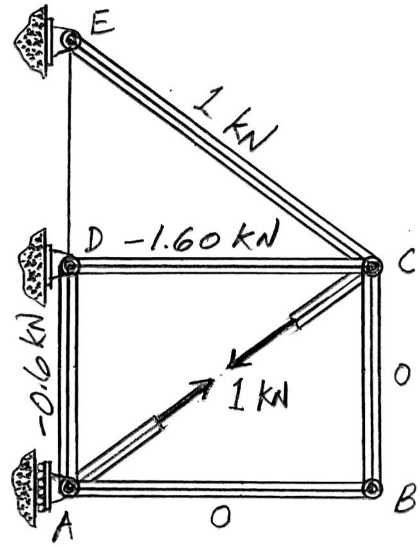


(a)

10–27. Continued

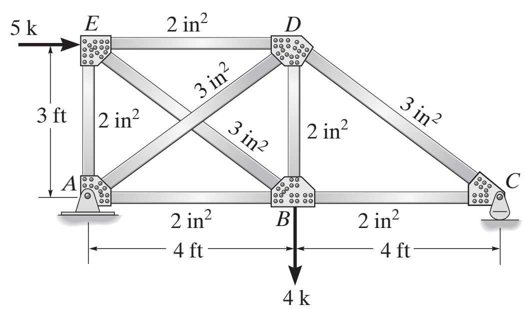


(b)



(c)

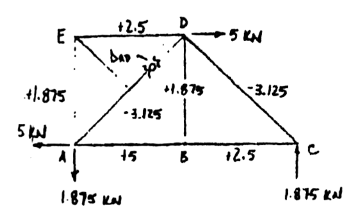
***10–28.** Determine the force in member AD of the truss. The cross-sectional area of each member is shown in the figure. Assume the members are pin connected at their ends. Take $E = 29(10^3)$ ksi.



$$\Delta_{AD} = \sum \frac{nNL}{AE} = \frac{1}{E}\left[\frac{1}{2}(-0.8)(2.5)(4) + (2)\left(\frac{1}{2}\right)(-0.6)(1.875)(3)\right.$$

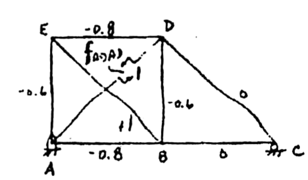
$$\left. + \frac{1}{2}(-0.8)(5)(4) + \frac{1}{3}(1)(-3.125)(5)\right]$$

$$= -\frac{20.583}{E}$$



$$f_{ADAD} = \sum \frac{n^2L}{AE} = \frac{1}{E}\left[2\left(\frac{1}{2}\right)(-0.8)^2(4) + 2\left(\frac{1}{2}\right)(-0.6)^2(3) + 2\left(\frac{1}{3}\right)(1)^2(5)\right]$$

$$= \frac{6.973}{E}$$

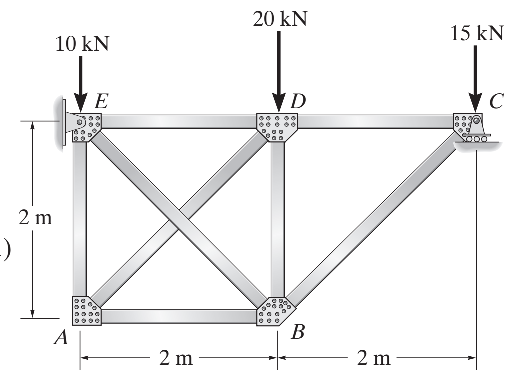


$$\Delta_{AD} + F_{AD} f_{ADAD} = 0$$

$$-\frac{20.583}{E} + F_{AD}\left(\frac{6.973}{E}\right) = 0$$

$F_{AD} = 2.95$ kN (T) **Ans.**

10–29. Determine the force in each member of the truss. Assume the members are pin connected at their ends. AE is constant.



Compatibility Equation:

$$0 = \Delta_{AD} + F_{AD} f_{ADAD} \qquad (1)$$

Use virtual work method

$$\Delta_{AD} = \sum \frac{nNL}{AE} = \frac{(-0.7071)(-10)(2)}{AE} + \frac{(-0.7071)(-20)(2)}{AE} + \frac{(1)(14.142)(2.828)}{AE}$$

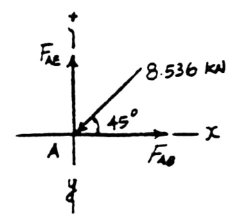
$$= \frac{82.43}{AE}$$

$$f_{ADAD} = \sum \frac{nnL}{AE} = \frac{4(-0.7071)^2(2)}{AE} + \frac{2(1)^2(2.828)}{AE} = \frac{9.657}{AE}$$

From Eq. 1

$$0 = \frac{82.43}{AE} + \frac{9.657}{AE} F_{AD}$$

$F_{AD} = -8.536 \text{ kN} = 8.54 \text{ kN (C)}$ **Ans.**



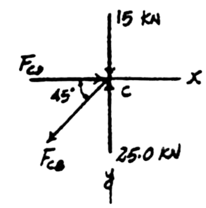
Joint *A*:

$+\uparrow \sum F_y = 0;$ $\quad F_{AE} - 8.536 \sin 45° = 0$

$F_{AE} = 6.04 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0;$ $\quad F_{AB} - 8.536 \cos 45° = 0$

$F_{AB} = 6.036 \text{ kN} = 6.04 \text{ kN (T)}$ **Ans.**



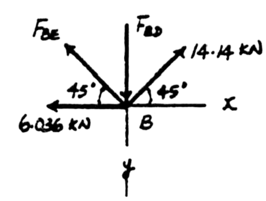
Joint *C*:

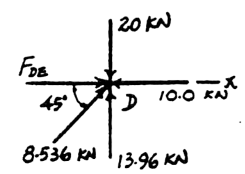
$+\uparrow \sum F_y = 0;$ $\quad -F_{CB} \sin 45° - 15 + 25 = 0$

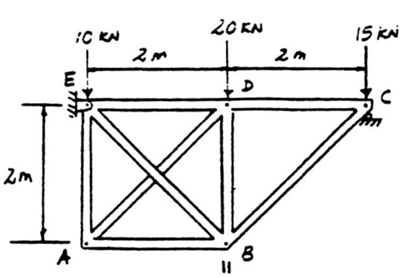
$F_{CB} = 14.14 \text{ kN} = 14.1 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0;$ $\quad F_{CD} - 14.14 \cos 45° = 0$

$F_{CD} = 10.0 \text{ kN (C)}$ **Ans.**







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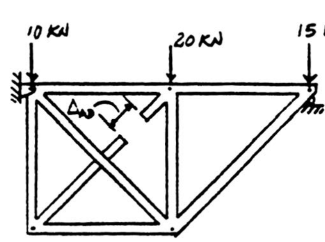

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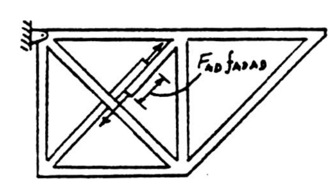
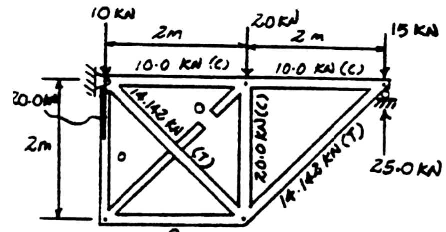


10–29. Continued

Joint *B*:



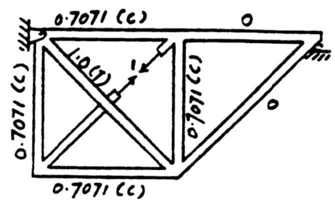
$\overset{+}{\leftarrow} \sum F_x = 0;$ $\quad F_{BE} \cos 45° + 6.036 - 14.14 \cos 45° = 0$

$F_{BE} = 5.606 \text{ kN} = 5.61 \text{ kN (T)}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad -F_{BD} + 5.606 \sin 45° + 14.14 \sin 45° = 0$

$F_{BD} = 13.96 \text{ kN} = 14.0 \text{ kN (C)}$ **Ans.**

Joint *D*:

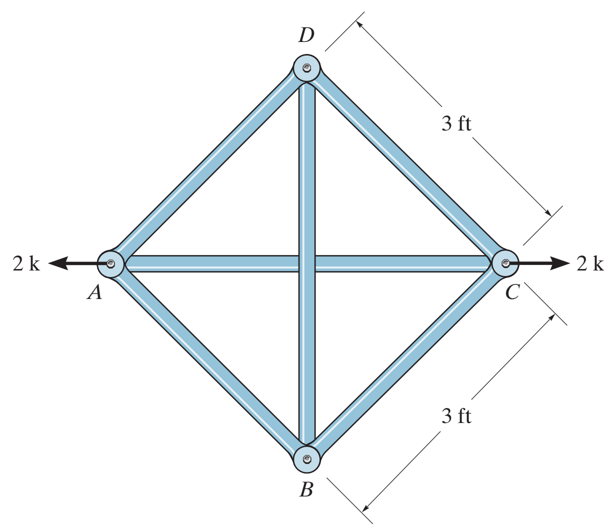


$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad F_{DE} + 8.536 \cos 45° - 10 = 0$

$F_{DE} = 3.96 \text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 8.536 \sin 45° + 13.96 - 20 = 0$ (Check) **Ans.**

10–30. Determine the force in each member of the pin-connected truss. AE is constant.



$$\Delta_{AC} = \sum \frac{nNL}{AE} = \frac{1}{AE}[(-0.707)(1.414)(3)(4) + (1)(-2)\sqrt{18}]$$

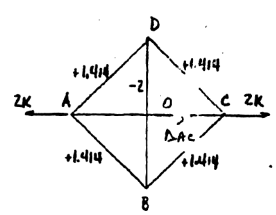
$$= -\frac{20.485}{AE}$$

$$f_{ACAC} = \sum \frac{n^2 L}{AE} = \frac{1}{AE}[4(-0.707)^2(3) + 2(1)^2\sqrt{18}]$$

$$= \frac{14.485}{AE}$$

$$\Delta_{AC} + F_{AC} f_{ACAC} = 0$$

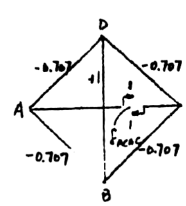
$$-\frac{20.485}{AE} + F_{AC}\left(\frac{14.485}{AE}\right) = 0$$

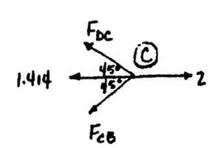


$F_{AC} = 1.414 \text{ k} = 1.41 \text{ k (T)}$ **Ans.**

Joint *C*:

$+\uparrow \sum F_y = 0;$ $\quad F_{DC} = F_{CB} = F$ **Ans.**





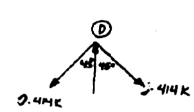
$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad 2 - 1.414 - 2F(\cos 45°) = 0;$

$F_{DC} = F_{CB} = 0.414 \text{ k (T)}$ **Ans.**

Due to symmetry:

$F_{AD} = F_{AB} = 0.414 \text{ k (T)}$ **Ans.**

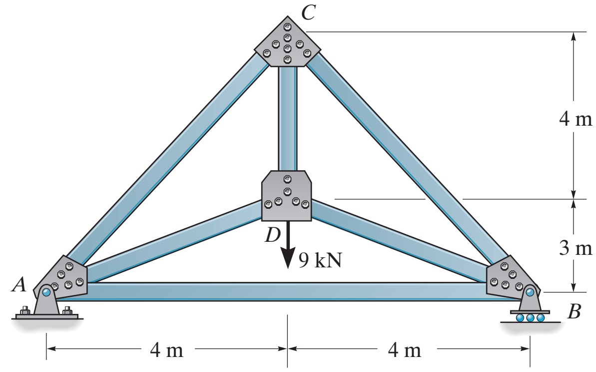
Joint *D*:



$+\uparrow \sum F_y = 0;$ $\quad F_{DB} - 2(0.414)(\cos 45°) = 0;$

$F_{DB} = 0.586 \text{ k (C)}$ **Ans.**

10–31. Determine the force in member *CD* of the truss. *AE* is constant.



Compatibility Equation: Referring to Fig. *a* and using the real and virtual force in each member shown in Fig. *b* and *c*, respectively,

$$\Delta'_{CD} = \sum \frac{nNL}{AE} = 2\left[\frac{0.8333(-7.50)(5)}{AE}\right] + \frac{(-0.3810)(6.00)(8)}{AE} = -\frac{80.786}{AE}$$

$$f_{CDCD} = \sum \frac{n^2L}{AE} = 2\left[\frac{(-0.5759)^2(\sqrt{65})}{AE}\right] + 2\left[\frac{0.8333^2(5)}{AE}\right]$$

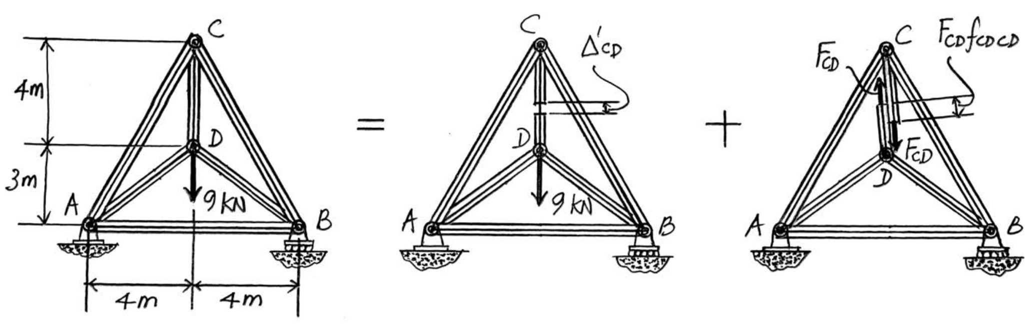
$$+ \frac{(-0.3810)^2(8)}{AE} + \frac{1^2(4)}{AE} = \frac{17.453}{AE}$$

Applying the principle of superposition, Fig. *a*,

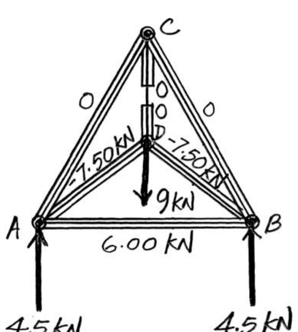
$$\Delta_{CD} = \Delta'_{CD} + F_{CD} f_{CDCD}$$

$$0 = -\frac{80.786}{AE} + F_{CD}\left(\frac{17.453}{AE}\right)$$

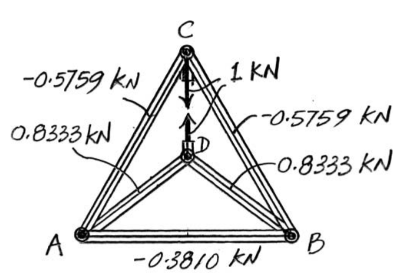
$$F_{CD} = 4.63 \text{ kN (T)}$$ **Ans.**



(a)

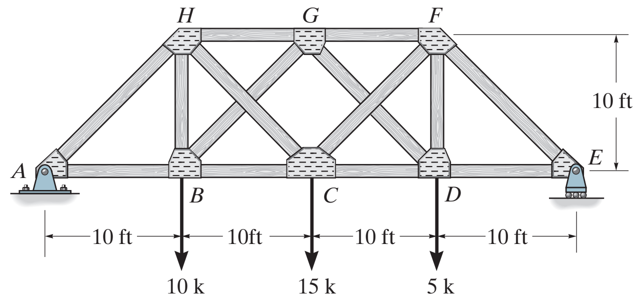


(b)



(c)

***10–32.** Determine the force in member GB of the truss. AE is constant.



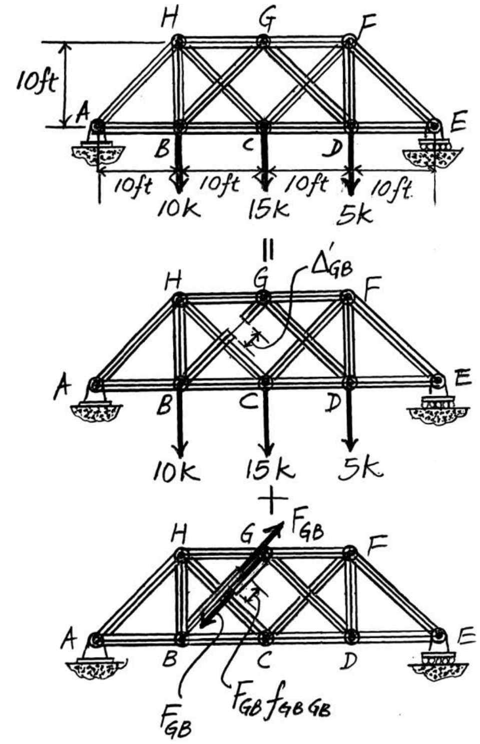
Compatibility Equation: Referring to Fig. a, and using the real and virtual force in each member shown in Fig. b and c, respectively,

$$\Delta'_{GB} = \sum \frac{nNL}{AE} = \frac{1}{AE}\Big[(-0.7071)(10)(10) + (-0.7071)(16.25)(10) + 0.7071(13.75)(10) + 0.7071(5)(10) + 0.7071(-22.5)(10) + (-0.7071)(-22.5)(10) + 1(8.839)(14.14) + (-1)(12.37)(14.14)\Big]$$

$$= -\frac{103.03}{AE}$$

$$f_{GBGB} = \sum \frac{n^2L}{AE} = 3\left[\frac{0.7071^2(10)}{AE}\right] + 3\left[\frac{(-0.7071)^2(10)}{AE}\right] + 2\left[\frac{(-1)^2(14.14)}{AE}\right] + 2\left[\frac{(1^2)(14.14)}{AE}\right]$$

$$= \frac{86.57}{AE}$$



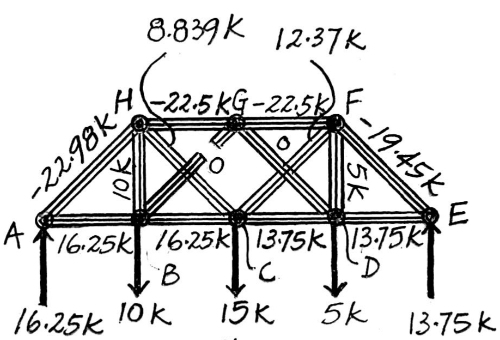
(a)

Applying the principle of superposition, Fig. a

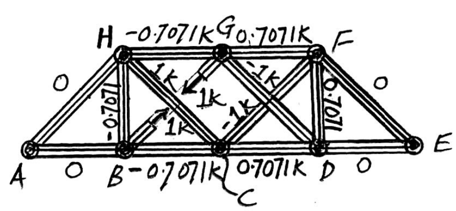
$$\Delta_{GB} = \Delta_{GB} + F_{GB}f_{GBGB}$$

$$0 = \frac{-103.03}{AE} + F_{GB}\left(\frac{86.57}{AE}\right)$$

$F_{GB} = 1.190 \text{ k} = 1.19 \text{ k}(T)$ **Ans.**

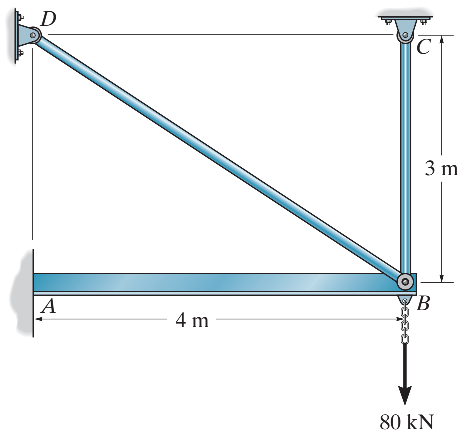


(b)



(c)

10–33. The cantilevered beam AB is additionally supported using two tie rods. Determine the force in each of these rods. Neglect axial compression and shear in the beam. For the beam, $I_b = 200(10^6)\ \text{mm}^4$, and for each tie rod, $A = 100\ \text{mm}^2$. Take $E = 200$ GPa.

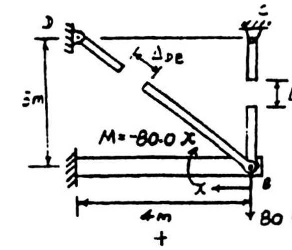


Compatibility Equations:

$$\Delta_{DB} + F_{DB}f_{DBDB} + F_{CB}f_{DBDB} = 0 \qquad (1)$$

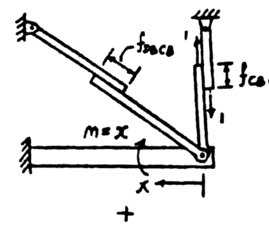
$$\Delta_{CB} + F_{DB}f_{CBDB} + F_{CB}f_{CBCB} = 0 \qquad (2)$$

Use virtual work method



$$\Delta_{DB} = \int_0^L \frac{mM}{EI}dx = \int_0^4 \frac{(0.6x)(-80x)}{EI}dx = -\frac{1024}{EI}$$

$$\Delta_{CB} = \int_0^L \frac{mM}{EI}dx = \int_0^4 \frac{(1x)(-80x)}{EI}dx = -\frac{1706.67}{EI}$$

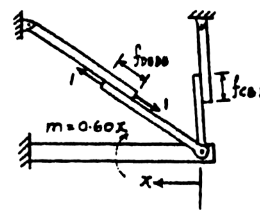


$$f_{CBCB} = \int_0^L \frac{mm}{EI}dx + \sum\frac{nnL}{AE} = \int_0^4 \frac{(1x)^2}{EI}dx + \frac{(1)^2(3)}{AE} = \frac{21.33}{EI} + \frac{3}{AE}$$

$$f_{DBDB} = \int_0^L \frac{mm}{EI}dx + \sum\frac{nnL}{AE} = \int_0^4 \frac{(0.6x)^2}{EI}dx + \frac{(1)^2(5)}{AE} = \frac{7.68}{EI} + \frac{5}{AE}$$

$$f_{DBCB} = \int_0^4 \frac{(0.6x)(1x)}{EI} = \frac{12.8}{AE}.$$

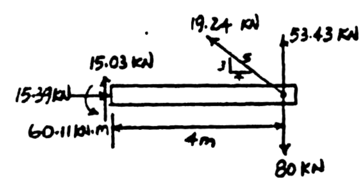
From Eq. 1



$$\frac{-1024}{E(200)(10^{-4})} + F_{DB}\left[\frac{7.68}{E(200)(10^{-4})} + \frac{5}{E(100)(10^{-4})}\right] + F_{CB}\left[\frac{12.8}{E(200)(10^{-4})}\right] = 0$$

$$0.0884F_{DB} + 0.064F_{CB} = 5.12$$

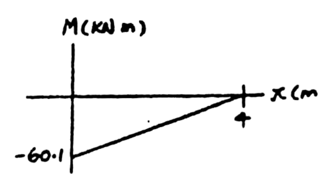
From Eq. 2



$$-\frac{1706.67}{E(200)(10^{-6})} + F_{DB}\frac{12.8}{E(200)(10^{-6})} + F_{CB}\left[\frac{21.33}{E(200)(10^{-6})} + \frac{3}{E(200)(10^{-6})}\right] = 0$$

$$0.064F_{DB} + 0.13667F_{CB} = 8.533$$

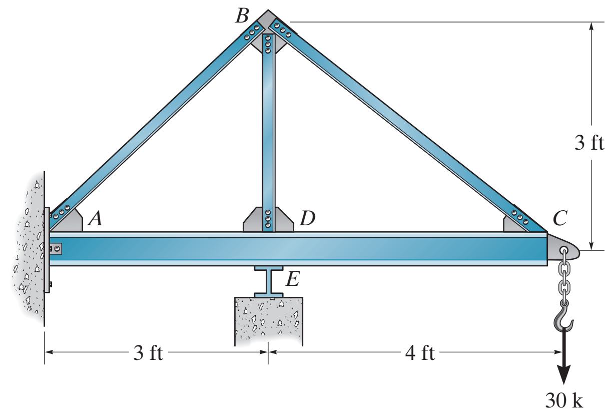
Solving



$F_{DB} = 19.24\ \text{kN} = 19.2\ \text{kN}$ **Ans.**

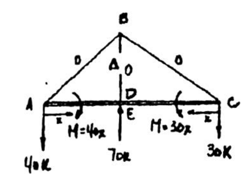
$F_{CB} = 53.43\ \text{kN} = 53.4\ \text{kN}$ **Ans.**

10–34. Determine the force in members AB, BC and BD which is used in conjunction with the beam to carry the 30-k load. The beam has a moment of inertia of $I = 600\,\text{in}^4$, the members AB and BC each have a cross-sectional area of 2 in^2, and BD has a cross-sectional area of 4 in^2. Take $E = 29(10^3)$ ksi. Neglect the thickness of the beam and its axial compression, and assume all members are pin-connected. Also assume the support at F is a pin and E is a roller.



$$\Delta = \int_0^L \frac{mM}{EI} = \sum \frac{nNL}{AE} = \int_0^3 \frac{(0.57143x)(40x)}{EI}dx + \int_0^4 \frac{(0.42857x)(30x)}{EI}dx + 0$$

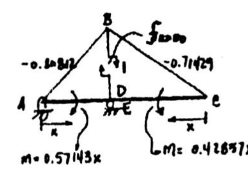
$$= \frac{480}{EI}$$



$$f_{BDBD} = \int_0^L \frac{m^2}{EI}dx + \sum \frac{n^2L}{AE} = \int_0^3 \frac{(0.57143x)^2 dx}{EI} + \int_0^4 \frac{(0.42857x)^2 dx}{EI}$$

$$+ \frac{(1)^2(3)}{4E} + \frac{(0.80812)^2\sqrt{18}}{2E} + \frac{(0.71429)^2(5)}{2E}$$

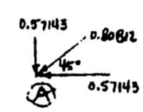
$$= \frac{6.8571}{EI} + \frac{3.4109}{E}$$



$$\Delta + F_{BD} f_{BDBD} = 0$$

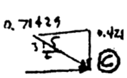
$$\frac{480(12^3)}{E(600)} + F_{BD}\left(\frac{6.8571(12^3)}{E(600)} + \frac{3.4109(12)}{E}\right) = 0$$

$$F_{BD} = -22.78\text{ k} = 22.8\text{ k }(C)$$ **Ans.**



Joint ***B:***

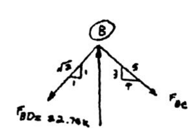
$$\overset{+}{\rightarrow} \sum F_x = 0; \quad -F_{AB}\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{4}{5}\right)F_{BC} = 0;$$



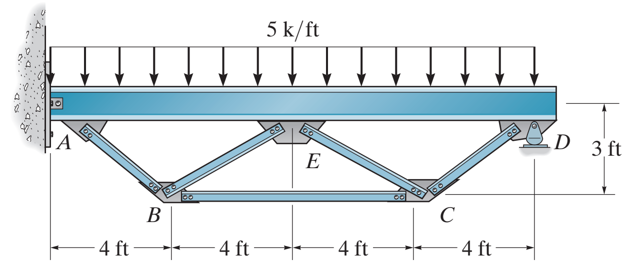
$$+\uparrow \sum F_y = 0; \quad 22.78 - \left(\frac{3}{5}\right)F_{BC} - F_{AB}\left(\frac{1}{\sqrt{2}}\right) = 0;$$

$$F_{AB} = 18.4\text{ k (T)}$$ **Ans.**

$$F_{BC} = 16.3\text{ k (T)}$$ **Ans.**



10–35. The trussed beam supports the uniform distributed loading. If all the truss members have a cross-sectional area of 1.25 in^2, determine the force in member BC. Neglect both the depth and axial compression in the beam. Take $E = 29(10^3)$ ksi for all members. Also, for the beam $I_{AD} = 750$ in^4. Assume A is a pin and D is a rocker.



Compatibility Equation: Referring to Fig. a, and using the real and virtual loadings in each member shown in Fig. b and c, respectively,

$$\Delta'_{BC} = \int_0^L \frac{mM}{EI}dx + \sum\frac{nNL}{AE} = 2\int_0^{8\text{ ft}} \frac{(-0.375x)(40x-25x^2)}{EI}dx + 0$$

$$= -\frac{3200\text{ k}\cdot\text{ft}^3}{EI} = -\frac{3200(12^2)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](750\text{ in}^2)} = -0.254$$

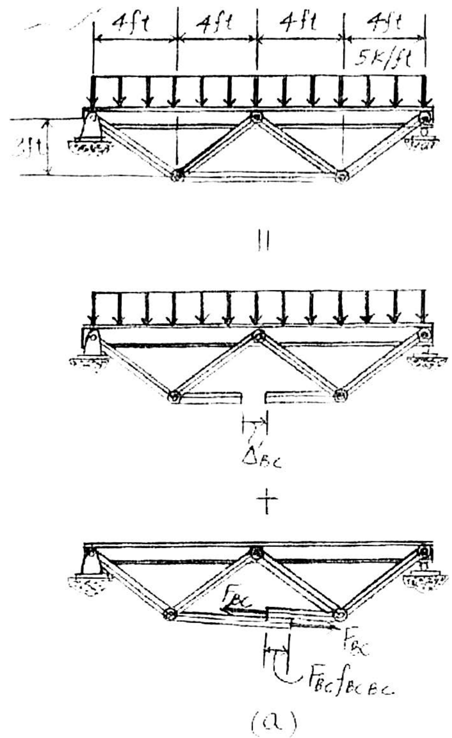
$$f_{BCBC} = \int_0^L \frac{m^2}{EI}dx + \sum\frac{n^2L}{AE} = 2\int_0^{8\text{ ft}} \frac{(-0.375x)^2}{EI}dx$$

$$+ \frac{1}{AE}[1^2(8) + 2(0.625^2)(5) + 2(-0.625)^2]$$

$$= \frac{48\text{ ft}^3}{EI} + \frac{15.8125\text{ ft}}{AE}$$

$$= \frac{48(12^2)\text{ in}^3}{[29(10)^3\text{ k/in}^3](750\text{ in}^4)} + \frac{15.8125(12)\text{ in}}{(1.25\text{ in}^2)[29(10^3)\text{ k/in}^2]}$$

$$= 0.009048\text{ in/k}$$

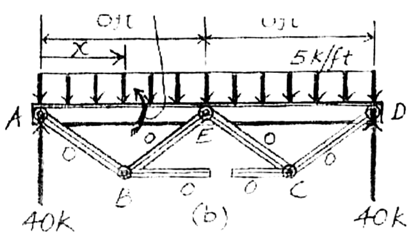


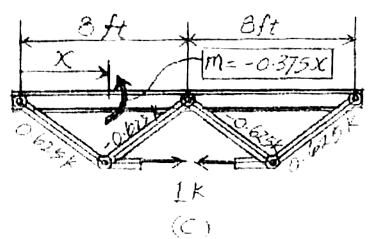
Applying principle of superposition, Fig. a

$$\Delta_{BC} = \Delta'_{BC} + F_{BC}f_{BCBC}$$

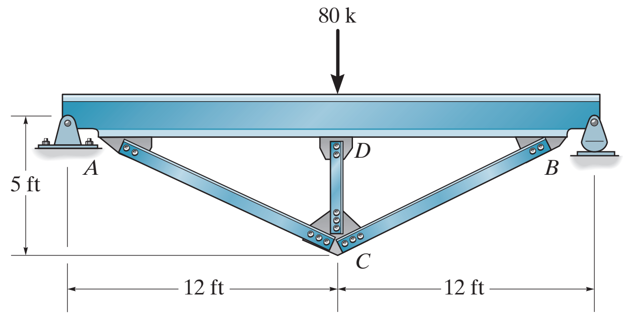
$$0 = -0.2542\text{ in} + F_{BC}(0.009048\text{ in/k})$$

$$F_{BC} = 28.098\text{ k (T)} = 28.1\text{ k (T)}$$ **Ans.**





***10–36.** The trussed beam supports a concentrated force of 80 k at its center. Determine the force in each of the three struts and draw the bending-moment diagram for the beam. The struts each have a cross-sectional area of 2 in^2. Assume they are pin connected at their end points. Neglect both the depth of the beam and the effect of axial compression in the beam. Take $E = 29(10^3)$ ksi for both the beam and struts. Also, for the beam $I = 400\,\text{in}^4$.



$$\Delta_{CD} = \int_0^L \frac{mM}{EI}dx + \sum \frac{nNL}{AE} = 2\int_0^{12} \frac{(0.5x)(40x)}{EI}dx = \frac{23040}{EI}$$

$$f_{CDCD} = \int_0^L \frac{m^2}{EI}dx + \sum \frac{n^2L}{AE} = 2\int_0^{12} \frac{(0.5x)^2}{EI}dx + \frac{(1)^2(5)}{AE} + \frac{2(1.3)^2(13)}{AE}$$

$$= \frac{288}{EI} + \frac{48.94}{AE}$$

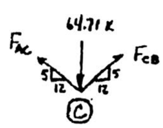
$$\Delta_{CD} + F_{CD} f_{CDCD} = 0$$

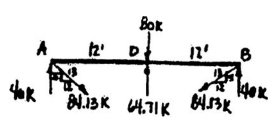
$$= \frac{23{,}040}{\frac{400}{12^4}} + F_{CD}\left(\frac{288}{\frac{400}{12^4}} + \frac{48.94}{\frac{2}{14^4}}\right) = 0$$

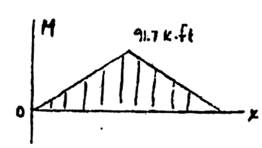
$$F_{CD} = -64.71 = 64.7\,\text{k (C)}$$ **Ans.**

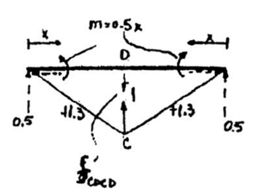
Equilibrium of joint C:

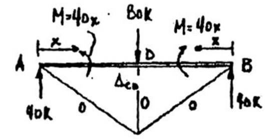
$$F_{CD} = F_{AC} = 84.1\,\text{k (T)}$$ **Ans.**



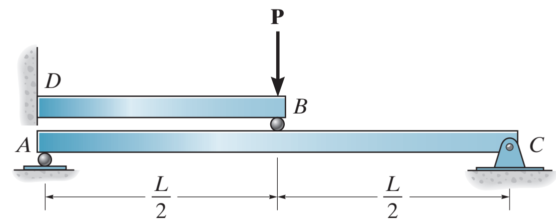








10–37. Determine the reactions at support C. EI is constant for both beams.



Support Reactions: FBD(a).

$$\xrightarrow{+} \sum F_x = 0; \qquad C_x = 0$$ **Ans.**

$$\circlearrowleft + \sum M_A = 0; \qquad C_y(L) - B_y\left(\frac{L}{2}\right) = 0$$ [1]

Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B = \frac{PL^3}{48EI} = \frac{B_yL^3}{48EI} \downarrow$$

$$v_B{}' = \frac{PL_{3D}^3}{3EI} = \frac{P(\frac{L}{2})^3}{3EI} = \frac{PL^3}{24EI} \downarrow$$

$$v_B{}'' = \frac{PL_{3\,D}^3}{3EI} = \frac{B_yL^3}{24EI} \uparrow$$

The compatibility condition requires

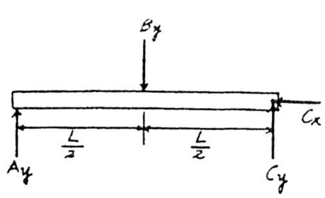
$$(+\downarrow) \qquad v_B = v_B{}' + v_B{}''$$

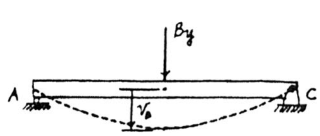
$$\frac{B_yL^3}{48EI} = \frac{PL^3}{24EI} + \left(-\frac{B_yL^3}{24EI}\right)$$

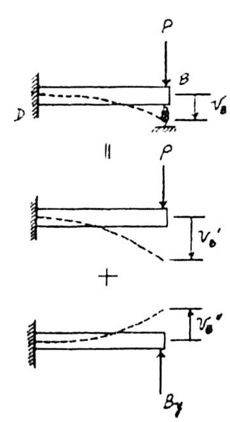
$$B_y = \frac{2P}{3}$$

Substituting B_y into Eq. [1] yields,

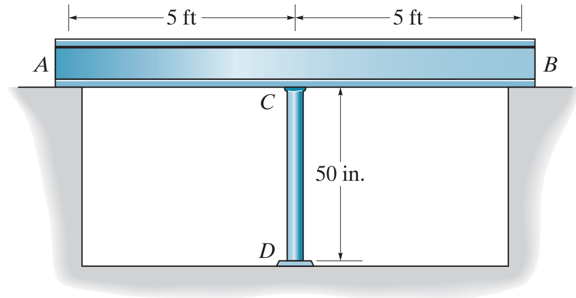
$$C_y = \frac{P}{3}$$ **Ans.**





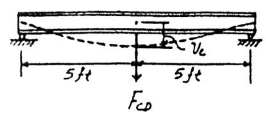


10–38. The beam AB has a moment of inertia $I = 475\ \text{in}^4$ and rests on the smooth supports at its ends. A 0.75-in.-diameter rod CD is welded to the center of the beam and to the fixed support at D. If the temperature of the rod is decreased by 150°F, determine the force developed in the rod. The beam and rod are both made of steel for which $E = 200$ GPa and $\alpha = 6.5(10^{-6})/\text{F}°$.



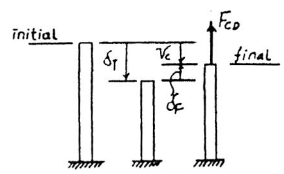
Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_C = \frac{PL^3}{48EI} = \frac{F_{CD}(120^3)}{48(29)(10^3)(475)} = 0.002613F_{CD} \downarrow$$



Using the axial force formula,

$$\delta_F = \frac{PL}{AE} = \frac{F_{CD}(50)}{\frac{6}{4}(0.75^2)(29)(10^3)} = 0.003903F_{CD} \uparrow$$



The thermal contraction is,

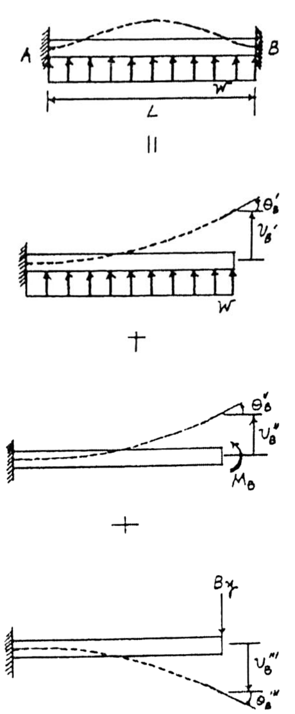
$$\delta_T = \alpha \Delta T L = 6.5(10^{-6})(150)(50) = 0.04875 \text{ in.} \downarrow$$

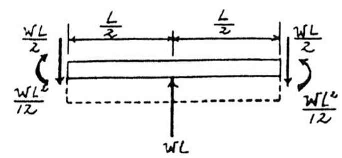
The compatibility condition requires

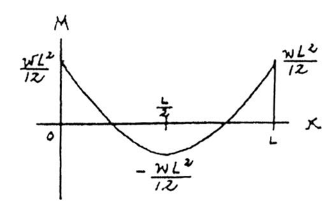
$$(+\downarrow) \qquad v_C = \delta_T + \delta_F$$

$$0.002613F_{CD} = 0.04875 + (-0.003903F_{CD})$$

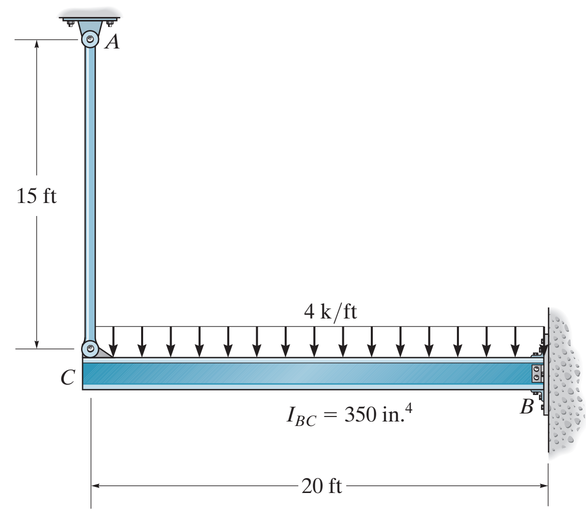
$$F_{CD} = 7.48 \text{ kip}$$ **Ans.**







10–39. The cantilevered beam is supported at one end by a $\frac{1}{2}$-in.-diameter suspender rod AC and fixed at the other end B. Determine the force in the rod due to a uniform loading of 4 k/ft. $E = 29(10^3)$ ksi for both the beam and rod.



$$\Delta_{AC} = \int_0^L \frac{mM}{EI}dx + \sum \frac{nNL}{AE} = \int_0^2 \frac{(1x)(-2x^2)}{EI}dx + 0 = -\frac{80.000}{EI}$$

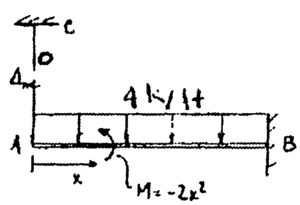
$$f_{ACAC} = \int_0^L \frac{m^2}{EI}\,dx + \sum \frac{n^2L}{AE} = \int_0^{20} \frac{x^2}{EI}\,dx + \frac{(1)^2(15)}{AE} = \frac{2666.67}{EI} + \frac{15}{AE}$$

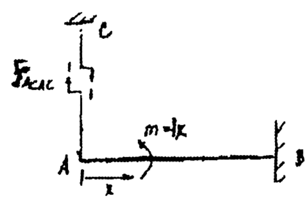
$+\downarrow \qquad \Delta_{AC} + F_{AC} f_{ACAC} = 0$

$$-\frac{80.000}{EI} + F_{AC}\left(\frac{2666.67}{EI} + \frac{15}{AE}\right) = 0$$

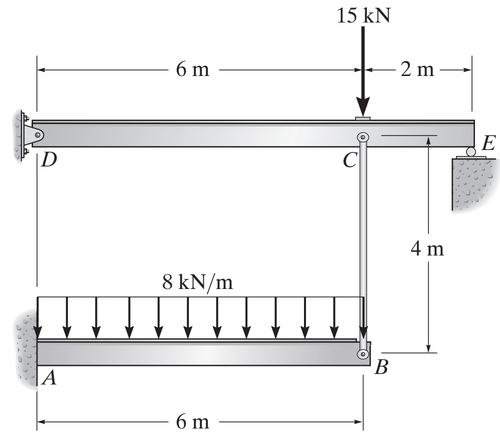
$$-\frac{80.000}{\frac{330}{12*}} + F_{AC}\left(\frac{2666.67}{\frac{350}{17*}} + \frac{15}{\pi(\frac{0.23}{12})^2}\right) = 0$$

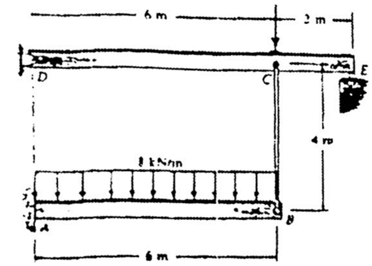
$F_{AC} = 28.0$ k **Ans.**





***10–40.** The structural assembly supports the loading shown. Draw the moment diagrams for each of the beams. Take $I = 100(10^6)\ \text{mm}^4$ for the beams and $A = 200\ \text{mm}^2$ for the tie rod. All members are made of steel for which $E = 200$ GPa.





Compatibility Equation

$$0 = \Delta_{CB} + F_{CB} f_{CBCB} \qquad (1)$$

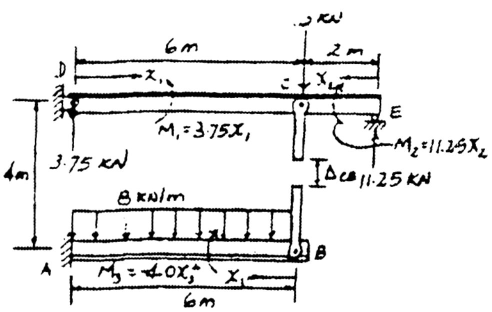
Use virtual work method

$$\Delta_{CB} = \int_0^L \frac{mM}{EI}dx = \int_0^6 \frac{(0.25x_1)(3.75x_1)}{EI}dx_1 + \int_0^2 \frac{(0.75x_2)(11.25x_2)}{EI}dx_2 + \int_0^6 \frac{(1x_3)(-4x_3^2)}{EI}dx_3$$

$$= \frac{-1206}{EI}$$

$$f_{CBCB} = \int_0^L \frac{mm}{EI}dx + \sum \frac{nnL}{AE} = \int_0^6 \frac{(0.25x_1)^2}{EI}dx_1 + \int_0^2 \frac{(0.75x_2)^2}{EI}dx_2 + \int_0^6 \frac{(1x_3)^2}{EI}dx_3 + \frac{(1)^2(4)}{AE}$$

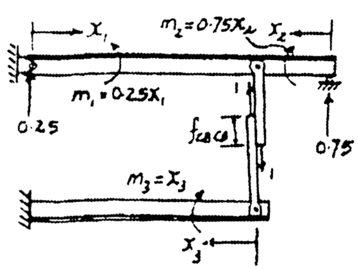
$$= \frac{78.0}{EI} + \frac{4.00}{AE}$$

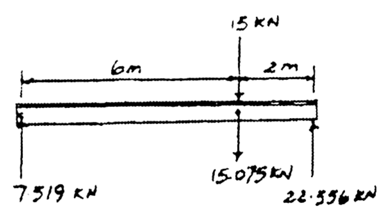


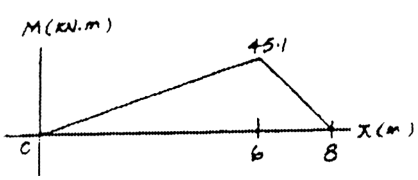
From Eq.1

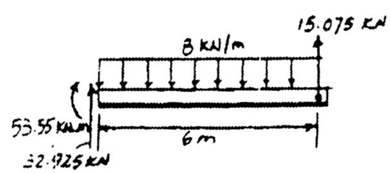
$$-\frac{1206}{E100(10^{-6})} + F_{CB}\left[\frac{78.0}{E(100)(10^{-6})} + \frac{4.00}{200(10^{-6})E}\right] = 0$$

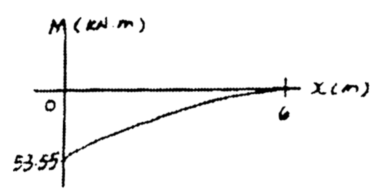
$F_{CB} = 15.075\ \text{kN (T)} = 15.1\ \text{kN (T)}$



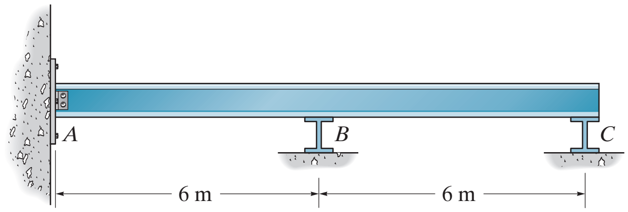








10–41. Draw the influence line for the reaction at C. Plot numerical values at the peaks. Assume A is a pin and B and C are rollers. EI is constant.



The primary real beam and qualitative influence line are shown in Fig. *a* and its conjugate beam is shown in Fig. *b*. Referring to Fig. *c*,

$$f_{AC} = M'_A = 0, \qquad f_{BC} = M'_B = 0 \qquad f_{CC} = M'_C = \frac{144}{EI}$$

The maximum displacement between A and B can be determined by referring to Fig *d*.

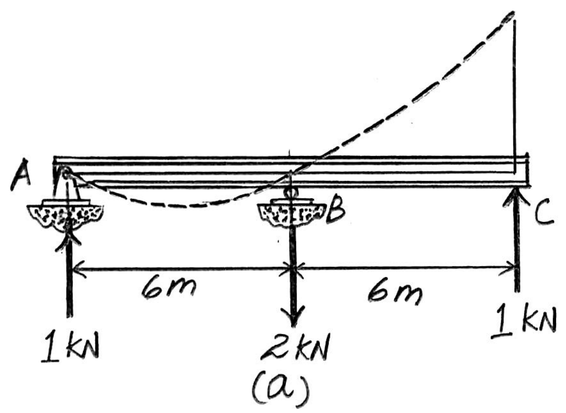
$$+\uparrow \sum F_y = 0; \qquad \frac{1}{2}\left(\frac{x}{EI}\right)x - \frac{6}{EI} = 0 \quad x = \sqrt{12}\text{ m}$$

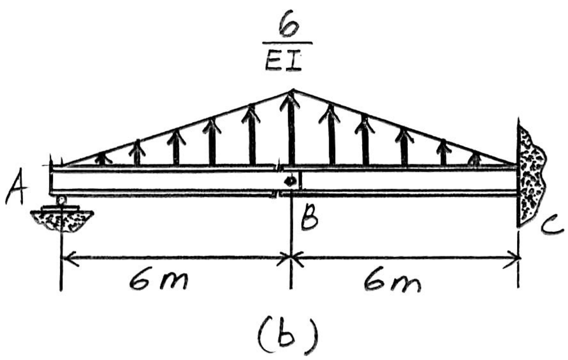
$$\circlearrowleft + \sum M = 0; \qquad M'_{\text{max}} + \frac{6}{EI}\left(\sqrt{12}\right) - \frac{1}{2}\left(\frac{\sqrt{12}}{EI}\right)\left(\sqrt{12}\right)\left(\frac{\sqrt{12}}{3}\right) = 0$$

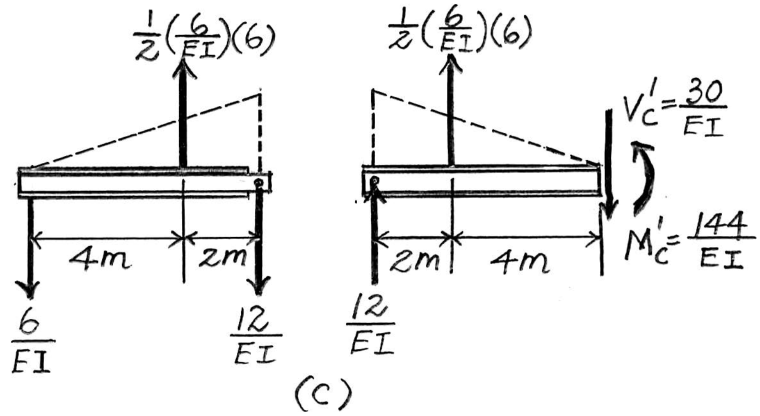
$$f_{\text{max}} = -\frac{13.86}{EI}$$

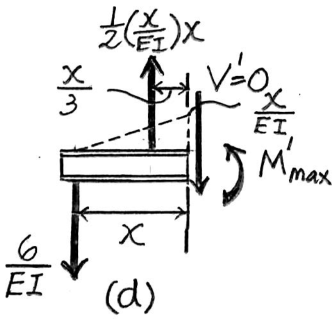
Dividing f's by f_{CC}, we obtain

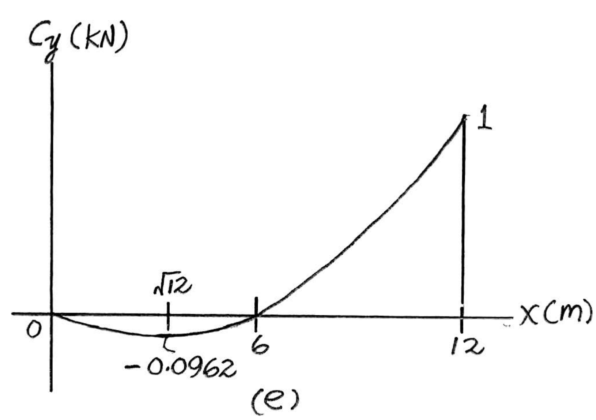
x (m)	0	$\sqrt{12}$	6	12
C_y (kN)	0	–0.0962	0	1



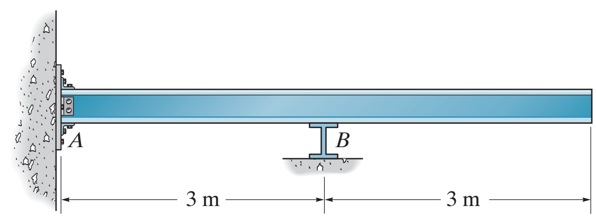








10–42. Draw the influence line for the moment at A. Plot numerical values at the peaks. Assume A is fixed and the support at B is a roller. EI is constant.



The primary real beam and qualitative influence line are shown in Fig. *a* and its conjugate beam is shown in Fig. *b*. Referring to Fig. *c*,

$$\alpha_{AA} = \frac{1}{EI}, \quad f_{AA} = M'_A = 0, \quad f_{BA} = M'_B = 0, \quad f_{CA} = M'_C = \frac{3}{2EI}$$

The maximum displacement between A and B can be determined by referring to Fig. *d*,

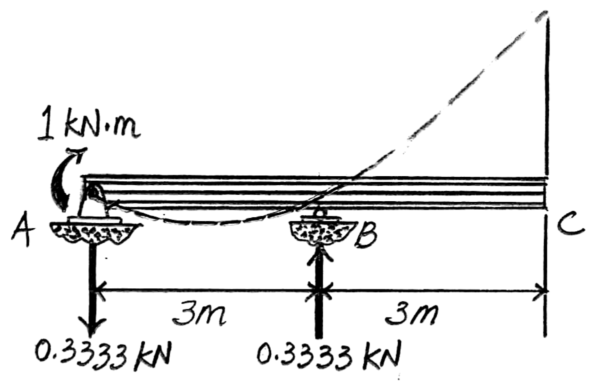
$$+\uparrow \sum F_y = 0; \qquad \frac{1}{2}\left(\frac{x}{3EI}\right)x - \frac{1}{2EI} = 0 \qquad x = \sqrt{3}\text{ m}$$

$$\zeta + \sum M = 0; \qquad \frac{1}{2}\left(\frac{\sqrt{3}}{3EI}\right)\left(\sqrt{3}\right)\left(\frac{\sqrt{3}}{3}\right) - \frac{1}{2EI}\left(\sqrt{3}\right) - M'_{max} = 0$$

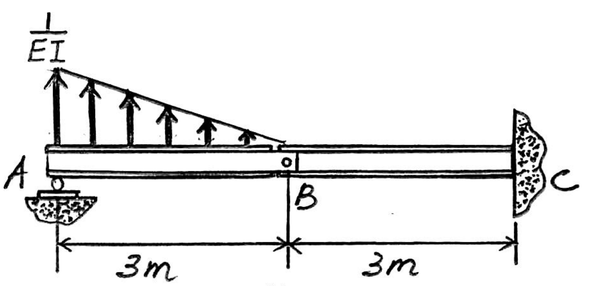
$$f_{max} = M'_{max} = -\frac{0.5774}{EI}$$

Dividing f's by α_{AA}, we obtain

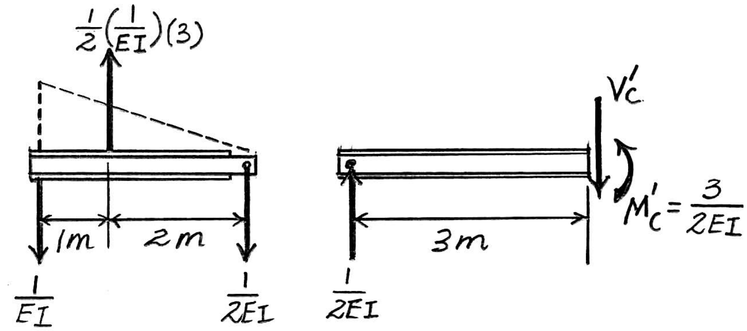
x (m)	0	1.268	3	6
M_A (kN · m)	0	–0.577	0	1.50



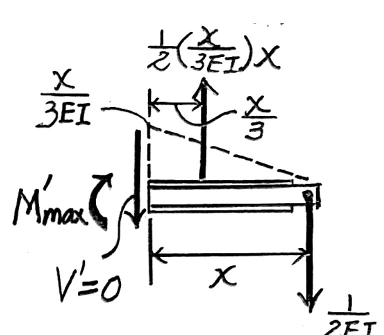
(a)



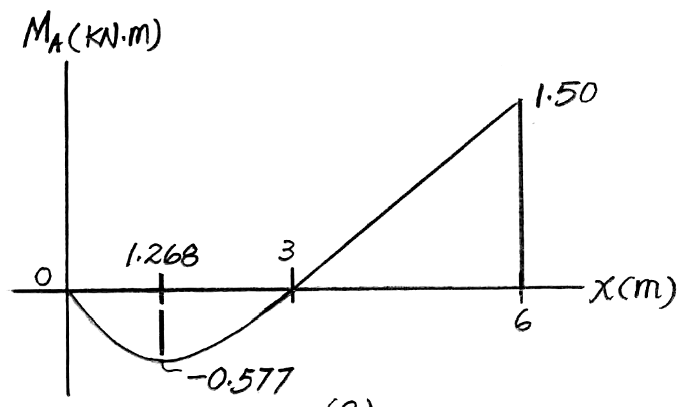
(b)



(c)

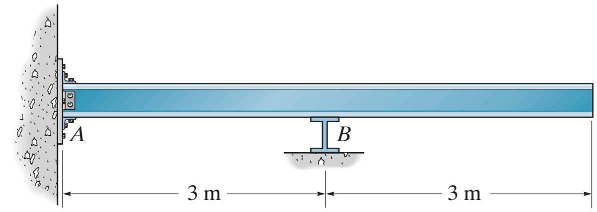


(d)



(e)

10–43. Draw the influence line for the vertical reaction at B. Plot numerical values at the peaks. Assume A is fixed and the support at B is a roller. EI is constant.



The primary real bean and qualitative influence line are shown in Fig. *a* and its conjugate beam is shown in Fig. *b*. Referring to Fig. *c*,

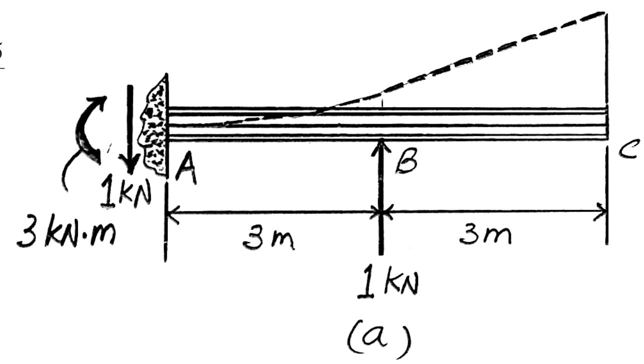
$$\zeta + \sum M_B = 0; \qquad M'_B - \frac{1}{2}\left(\frac{3}{EI}\right)(3)(2) = 0 \quad f_{BB} = M'_B = \frac{9}{EI}$$

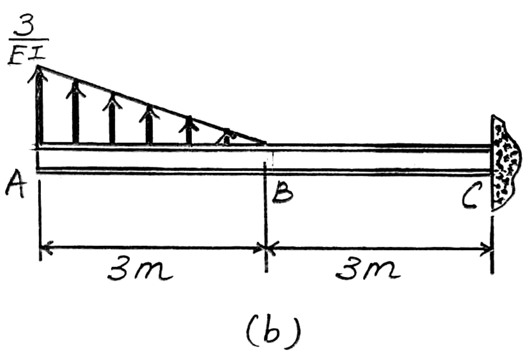
Referring to Fig. *d*,

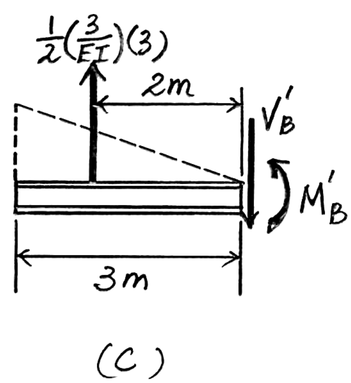
$$\zeta + \sum M_C = 0; \qquad M'_C - \frac{1}{2}\left(\frac{3}{EI}\right)(3)(5) = 0 \qquad f_{CB} = M'_C = \frac{22.5}{EI}$$

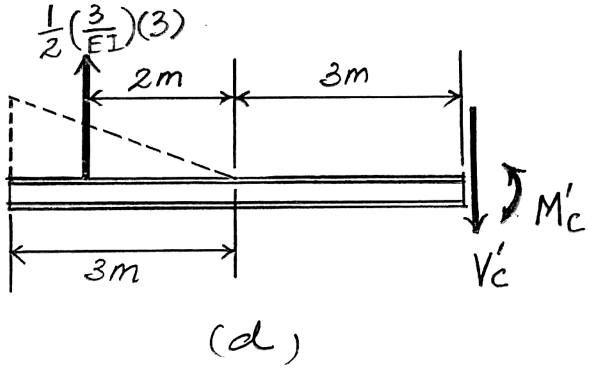
Also, $f_{AB} = 0$. Dividing f's by f_{BB}, we obtain

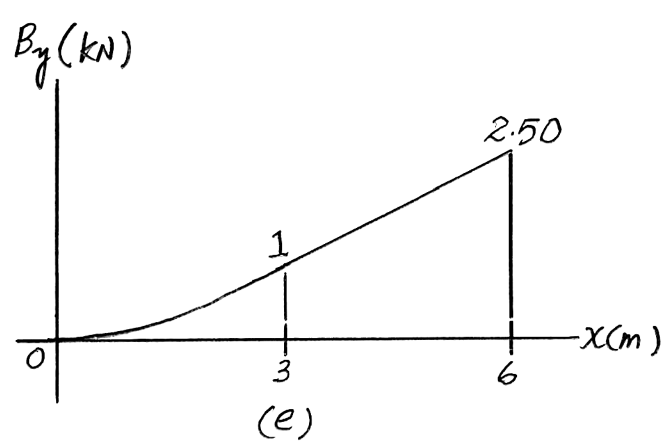
x (m)	0	3	6
B_y (kN)	0	1	2.5



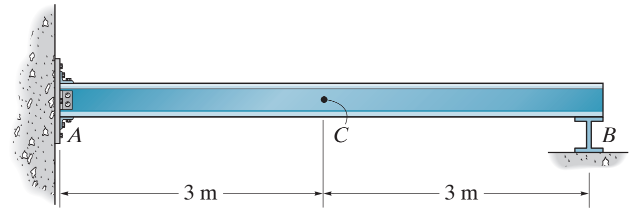








***10–44.** Draw the influence line for the shear at C. Plot numerical values every 1.5 m. Assume A is fixed and the support at B is a roller. EI is constant.



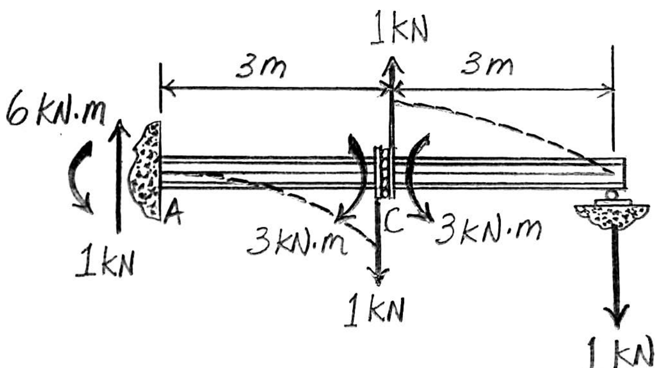
The primary real beam and qualitative influence line are shown in Fig. *a*, and its conjugate beam is shown in Fig. *b*. Referring to Figs. *c*, *d*, *e* and *f*,

$$f_{OC} = M'_0 = 0 \quad f_{1.5\,C} = M'_{1.5} = -\frac{6.1875}{EI} \quad f_{3\bar{C}} = M'_{3^-} = --\frac{22.5}{EI}$$

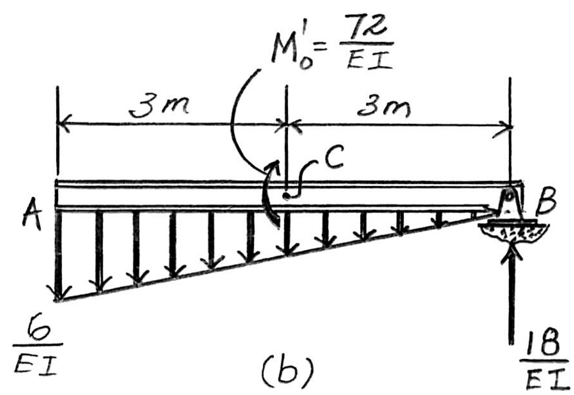
$$f_{3\overset{+}{C}} = M'_{3+} = \frac{49.5}{EI} \quad f_{4.5\,C} = M'_{4.5} = \frac{26.4375}{EI} \quad f_{6\,C} = M'_6 = 0$$

Dividing f's by $M'_0 = \dfrac{72}{EI}$, we obtain

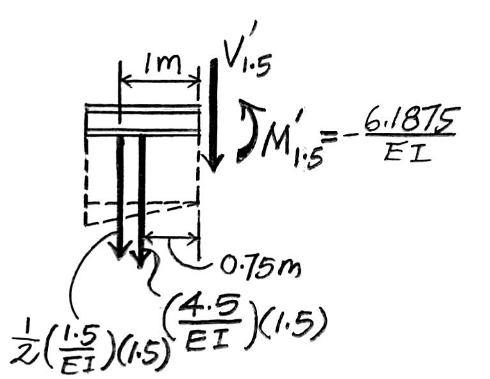
x (m)	0	1.5	3^-	3^+	4.5	6
V_C (kN)	0	–0.0859	–0.3125	0.6875	0.367	0



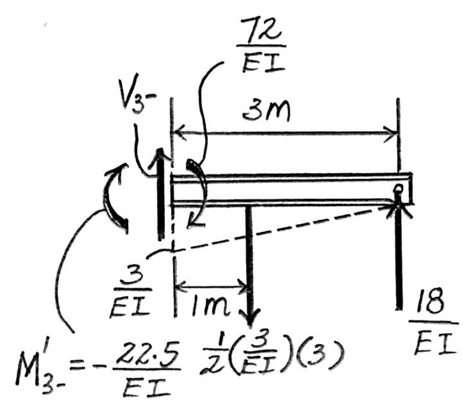
(a)



(b)

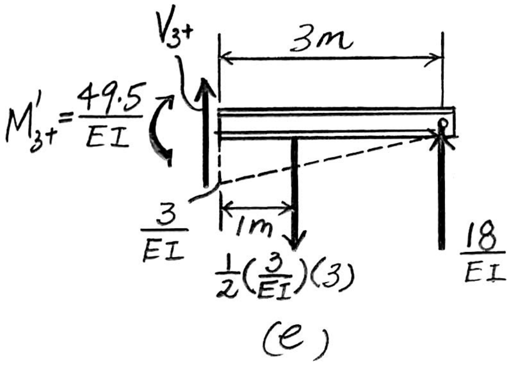


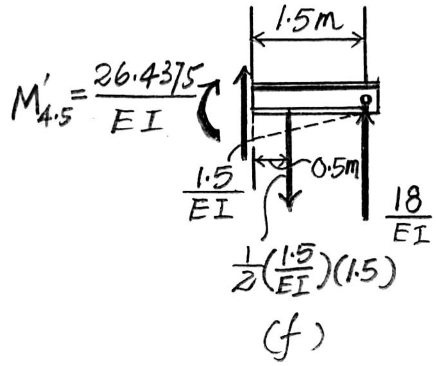
(c)

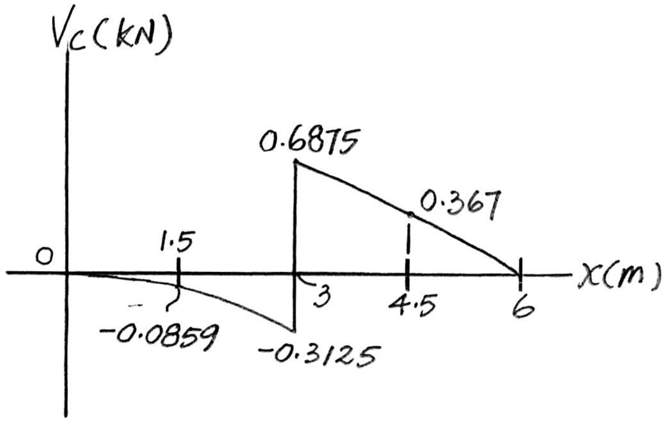


(d)

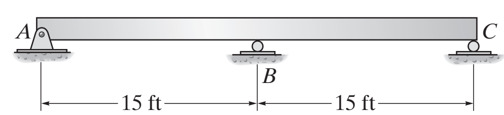
10–44. Continued







10–45. Draw the influence line for the reaction at C. Plot the numerical values every 5 ft. EI is constant.



$x = 0$ ft

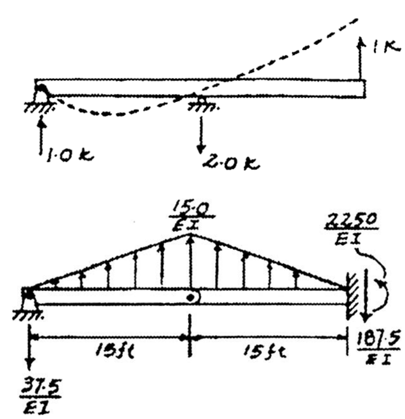
$$\Delta_0 = M_0' = 0$$

$x = 5$ ft

$$\Delta_5 = M_5' = \frac{12.5}{EI}1.667 - \frac{37.5}{EI}(5) = -\frac{166.67}{EI}$$

$x = 10$ ft

$$\Delta_{10} = M_{10}' = \frac{50}{EI}3.333 - \frac{37.5}{EI}(10) = -\frac{208.33}{EI}$$



10–45. Continued

$x = 15$ ft

$$\Delta_{15} = M_{15}' = 0$$

$x = 20$ ft

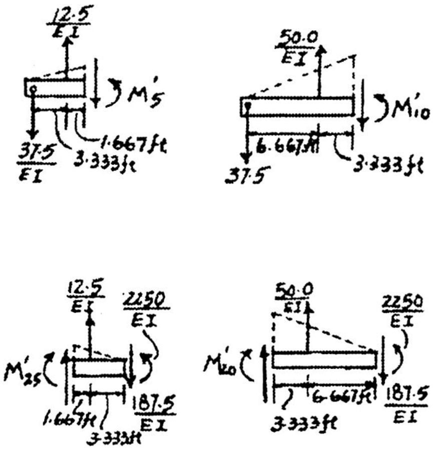
$$\Delta_{20} = M_{20}' = \frac{2250}{EI} + \frac{50}{EI}3.333 - \frac{187.5}{EI}(10) = \frac{541.67}{EI}$$

$x = 25$ ft

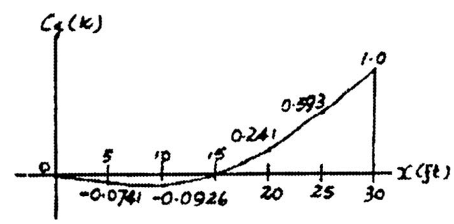
$$\Delta_{25} = M_{25}' = \frac{2250}{EI} + \frac{12.5}{EI}1.667 - \frac{187.5}{EI}(5) = \frac{1333.33}{EI}$$

$x = 30$ ft

$$\Delta_{30} = M_{30}' = \frac{2250}{EI}$$

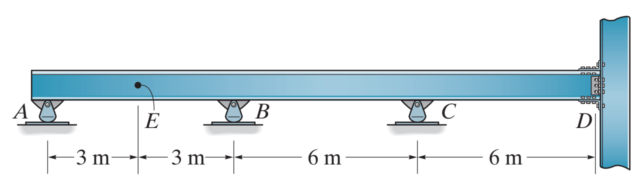


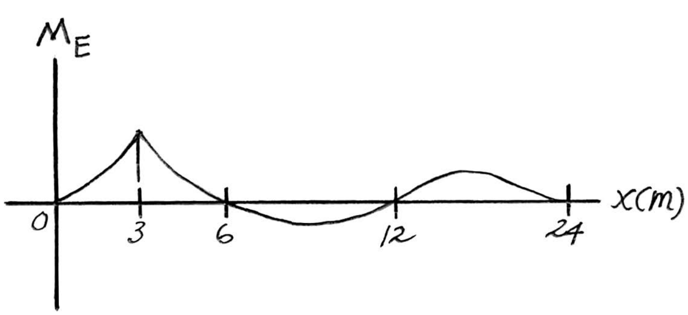
x	Δ_i/Δ_{30}
0	0
5	–0.0741
10	–0.0926
15	0
20	0.241
25	0.593
30	1.0

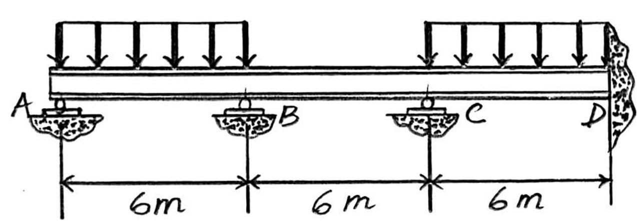


At 20 ft: $C_y = 0.241$ k **Ans.**

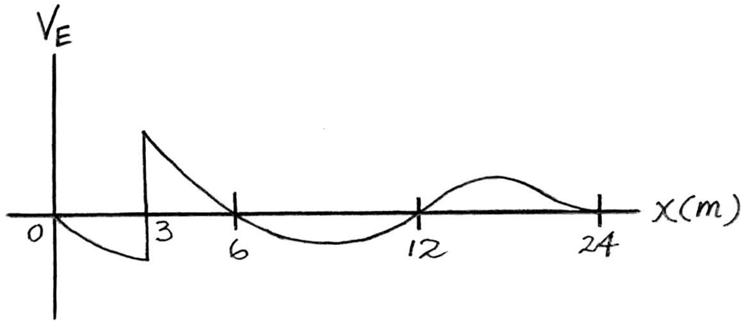
10–46. Sketch the influence line for (a) the moment at E, (b) the reaction at C, and (c) the shear at E. In each case, indicate on a sketch of the beam where a uniform distributed live load should be placed so as to cause a maximum positive value of these functions. Assume the beam is fixed at D.

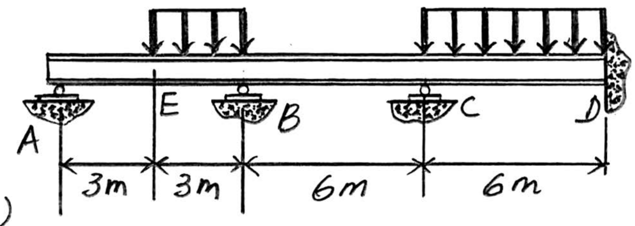


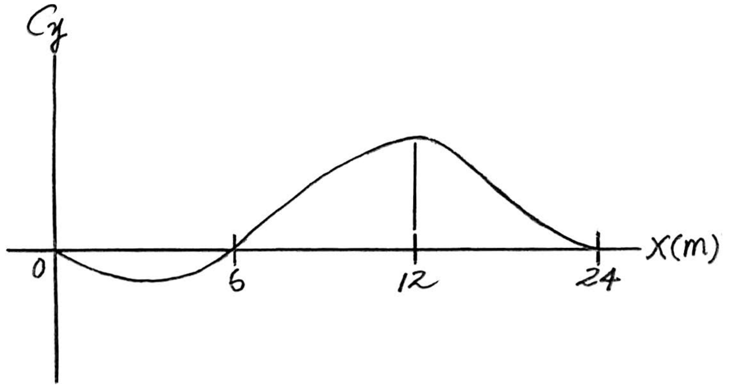


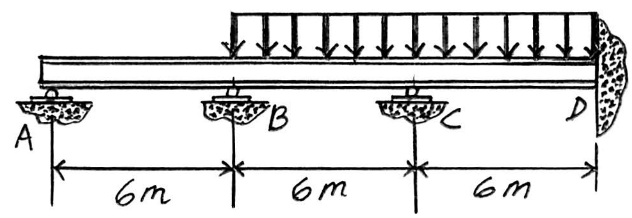


10–46. Continued

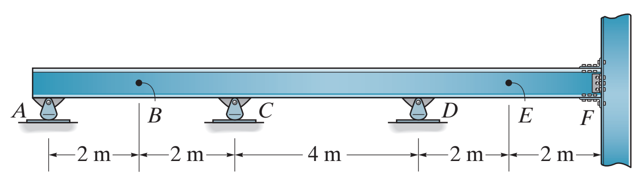


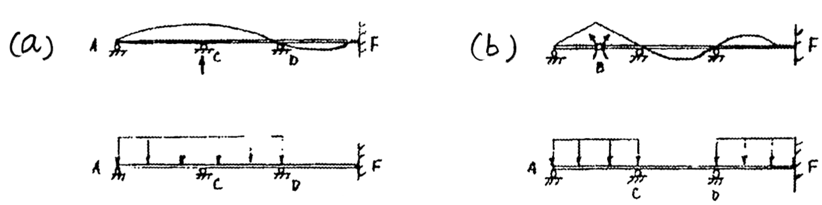


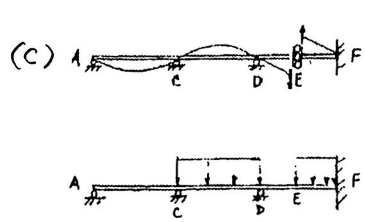




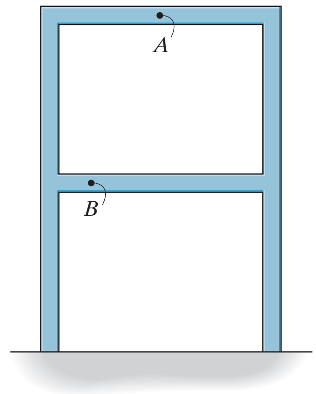
10–47. Sketch the influence line for (a) the vertical reaction at *C*, (b) the moment at *B*, and (c) the shear at *E*. In each case, indicate on a sketch of the beam where a uniform distributed live load should be placed so as to cause a maximum positive value of these functions. Assume the beam is fixed at *F*.



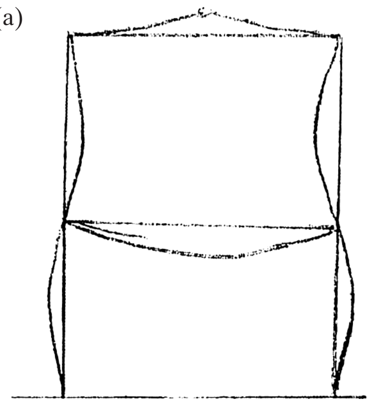




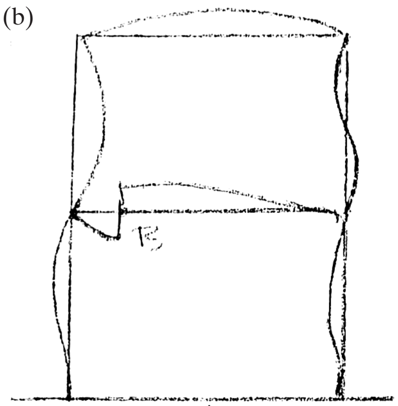
***10–48.** Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



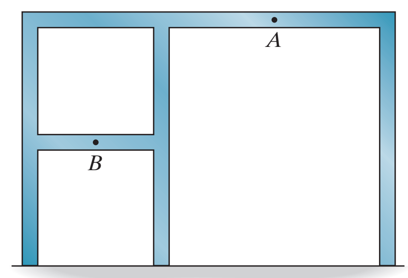
(a)



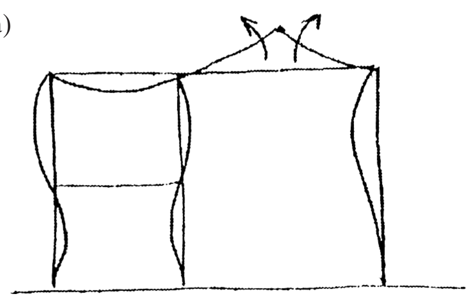
(b)



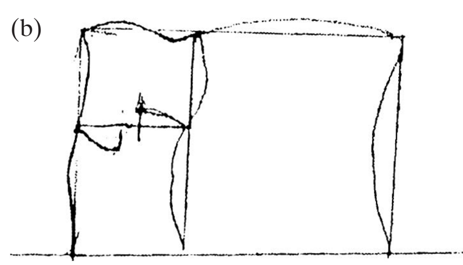
10–49. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



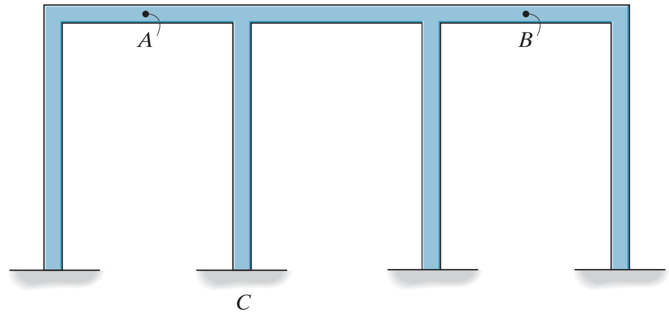
(a)

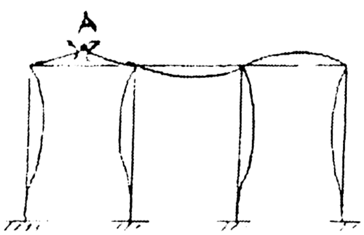


(b)

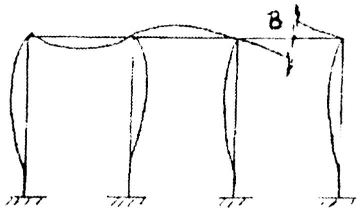


10–50. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at A and (b) the shear at B.



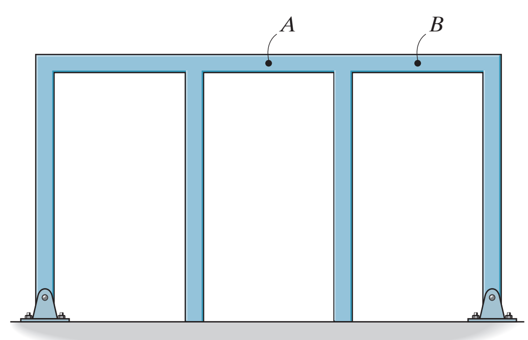


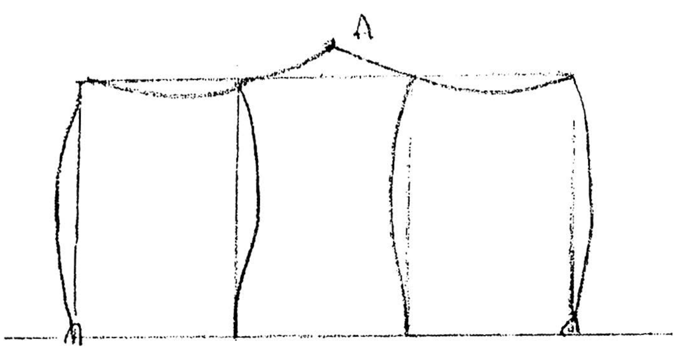
(a)



(b)

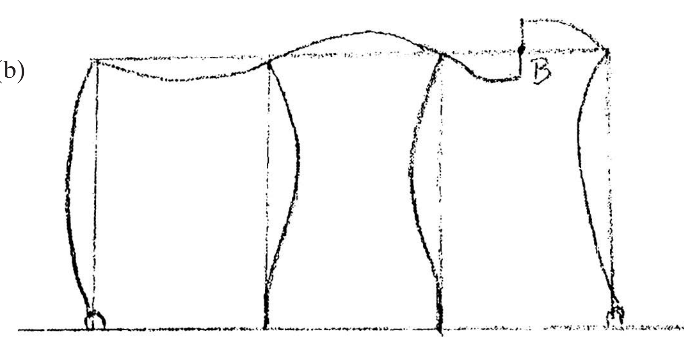
10–51. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at A and (b) the shear at B.



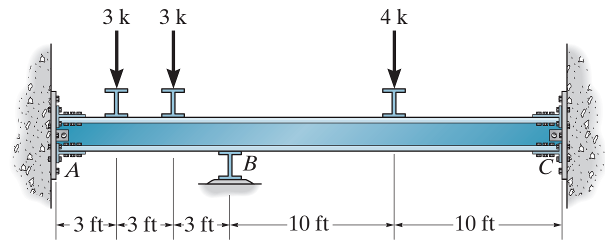


(a)

(b)



11–1. Determine the moments at A, B, and C and then draw the moment diagram. EI is constant. Assume the support at B is a roller and A and C are fixed.



Fixed End Moments. Referring to the table on the inside back cover

$$(\text{FEM})_{AB} = -\frac{2PL}{9} = -\frac{2(3)(9)}{9} = -6\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{2PL}{9} = \frac{2(3)(9)}{9} = 6\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{4(20)}{8} = -10\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = \frac{PL}{8} = \frac{4(20)}{8} = 10\text{ k}\cdot\text{ft}$$

Slope-Deflection Equations. Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left(\frac{I}{9}\right)[2(0) + \theta_B - 3(0)] + (-6) = \left(\frac{2EI}{9}\right)\theta_B - 6 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{9}\right)[2\theta_B + 0 - 3(0)] + 6 = \left(\frac{4EI}{9}\right)\theta_B + 6 \qquad (2)$$

For span BC,

$$M_{BC} = 2E\left(\frac{I}{20}\right)[2\theta_B + 0 - 3(0)] + (-10) = \left(\frac{EI}{5}\right)\theta_B - 10 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{20}\right)[2(0) + \theta_B - 3(0)] + (10) = \left(\frac{EI}{10}\right)\theta_B + 10 \qquad (4)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Substitute Eq. 2 and 3 into (5),

$$\left(\frac{4EI}{9}\right)\theta_B + 6 + \left(\frac{EI}{5}\right)\theta_B - 10 = 0 \qquad \theta_B = \frac{180}{29EI}$$

Substitute this result into Eqs. 1 to 4,

$M_{AB} = -4.621\text{ k}\cdot\text{ft} = -4.62\text{ k}\cdot\text{ft}$ **Ans.**

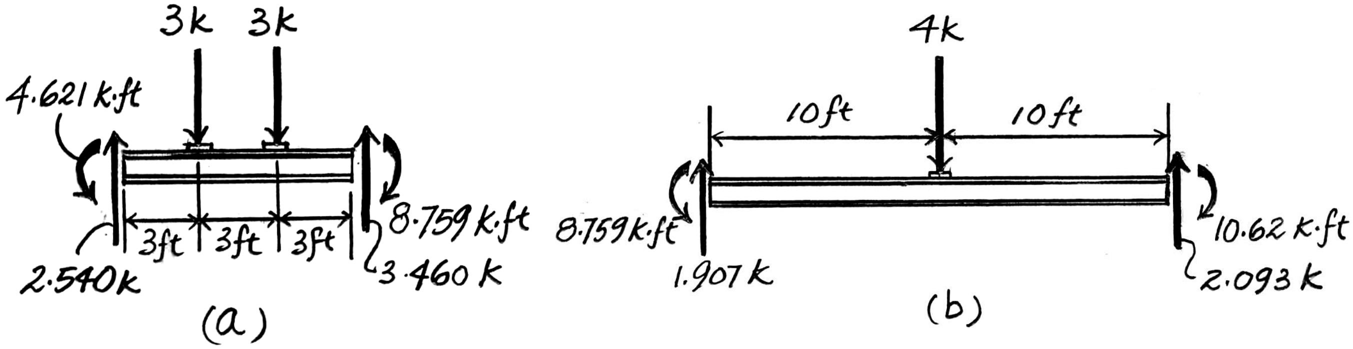
$M_{BA} = 8.759\text{ k}\cdot\text{ft} = 8.76\text{ k}\cdot\text{ft}$ **Ans.**

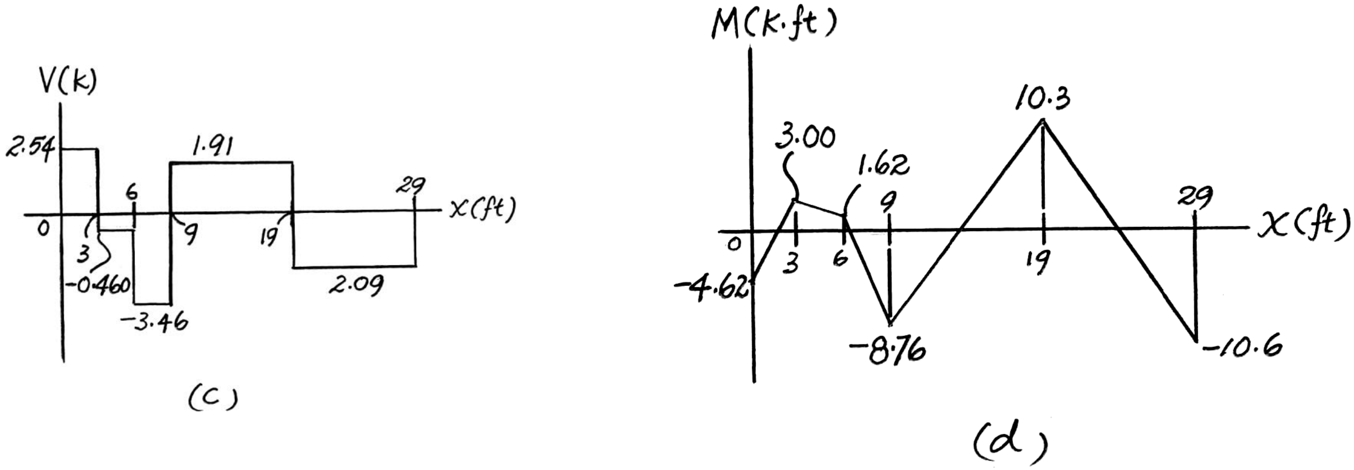
$M_{BC} = -8.759\text{ k}\cdot\text{ft} = -8.76\text{ k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 10.62\text{ k}\cdot\text{ft} = 10.6\text{ k}\cdot\text{ft}$ **Ans.**

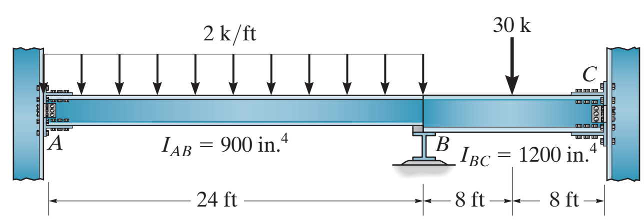
The Negative Signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have the counterclockwise rotational sense. Using these results, the shear at both ends of span AB and BC are computed and shown in Fig. a and b, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. c and d respectively.

11–1. Continued





11–2. Determine the moments at A, B, and C, then draw the moment diagram for the beam. The moment of inertia of each span is indicated in the figure. Assume the support at B is a roller and A and C are fixed. $E = 29(10^3)$ ksi.



Fixed End Moments. Referring to the table on the inside back cover,

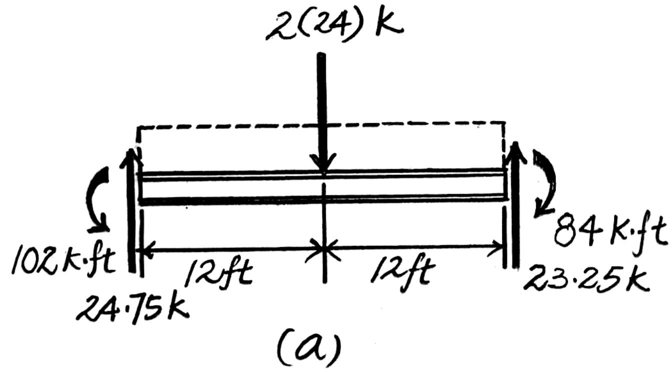
$$(FEM)_{AB} = -\frac{wL^2}{12} = -\frac{2(24^2)}{12} = -96 \text{ k} \cdot \text{ft}$$

$$(FEM)_{BA} = \frac{wL^2}{12} = \frac{2(24^2)}{12} = 96 \text{ k} \cdot \text{ft}$$

$$(FEM)_{BC} = -\frac{PL}{8} = -\frac{30(16)}{8} = -60 \text{ k} \cdot \text{ft}$$

$$(FEM)_{CB} = \frac{PL}{8} = \frac{30(16)}{8} = 60 \text{ k} \cdot \text{ft}$$

11–2. Continued



Slope-Deflection Equations. Applying Eq. 11–8,

$$M_N = 2Ek\,(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left[\frac{900\text{ in}^4}{24(12)\text{ in}}\right][2(0) + \theta_B - 3(0)] + [-96(12)\text{ k}\cdot\text{in}]$$

$$M_{AB} = 6.25E\theta_B - 1152 \qquad (1)$$

$$M_{BA} = 2E\left[\frac{900\text{ in}^4}{24(12)\text{ in}}\right][2\theta_B + 0 - 3(0)] + 96(12)\text{ k}\cdot\text{in}$$

$$M_{BA} = 12.5E\theta_B + 1152 \qquad (2)$$

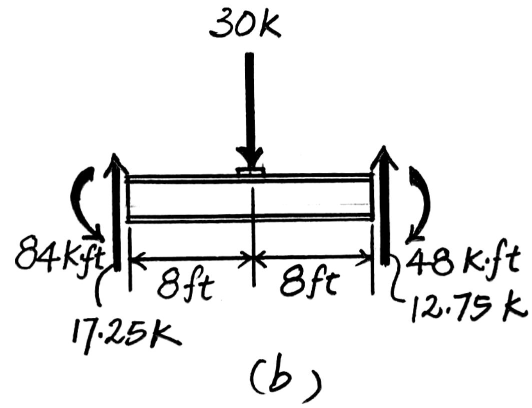
For span BC,

$$M_{BC} = 2E\left[\frac{1200\text{ in}^4}{16(12)\text{ in}}\right][2\theta_B + 0 - 3(0)] + [-60(12)\text{ k}\cdot\text{in}]$$

$$M_{BC} = 25E\theta_B - 720 \qquad (3)$$

$$M_{CB} = 2E\left[\frac{1200\text{ in}^4}{16(12)\text{ in}}\right][2(0) + \theta_B - 3(0)] + 60(12)\text{ k}\cdot\text{in}$$

$$M_{CB} = 12.5E\theta_B + 720 \qquad (4)$$



Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Substitute Eqs. 3(2) and (3) into (5),

$$12.5E\theta_B + 1152 + 25E\theta_B - 720 = 0$$

$$\theta_B = -\frac{11.52}{E}$$

Substitute this result into Eqs. (1) to (4),

$M_{AB} = -1224\text{ k}\cdot\text{in} = -102\text{ k}\cdot\text{ft}$ **Ans.**

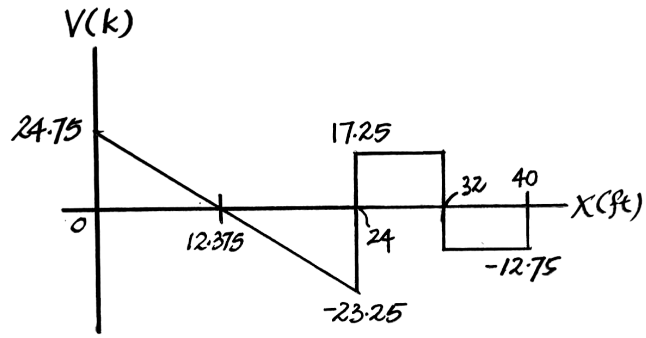
$M_{BA} = 1008\text{ k}\cdot\text{in} = 84\text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -1008\text{ k}\cdot\text{in} = -84\text{ k}\cdot\text{ft}$ **Ans.**

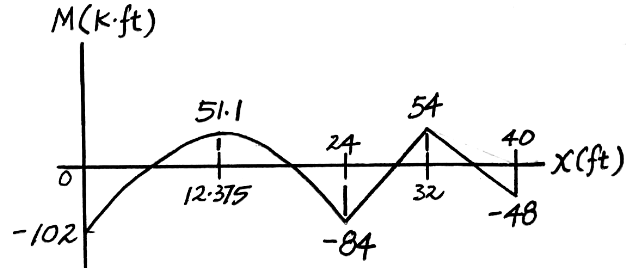
$M_{CB} = 576\text{ k}\cdot\text{in} = 48\text{ k}\cdot\text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have counterclockwise rotational senses. Using these results, the shear at both ends of spans AB and BC are computed and shown in Fig. *a* and *b*, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *c* and *d* respectively.

11–2. Continued

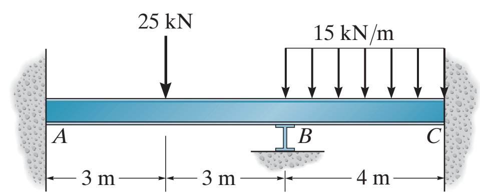


(c)

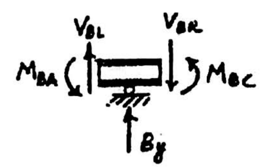


(d)

11–3. Determine the moments at the supports A and C, then draw the moment diagram. Assume joint B is a roller. EI is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

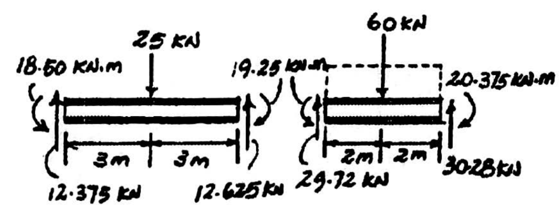


$$M_{AB} = \frac{2EI}{6}(0 + \theta_B) - \frac{(25)(6)}{8}$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) + \frac{(25)(6)}{8}$$

$$M_{BC} = \frac{2EI}{4}(2\theta_B) - \frac{(15)(4)^2}{12}$$

$$M_{CB} = \frac{2EI}{4}(\theta_B) + \frac{(15)(4)^2}{12}$$

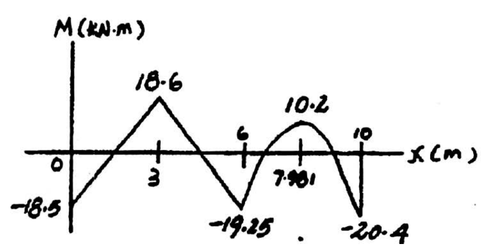


Equilibrium.

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{6}(2\theta_B) + \frac{25(6)}{8} + \frac{2EI}{4}(2\theta_B) - \frac{15(4)^2}{12} = 0$$

$$\theta_B = \frac{0.75}{EI}$$



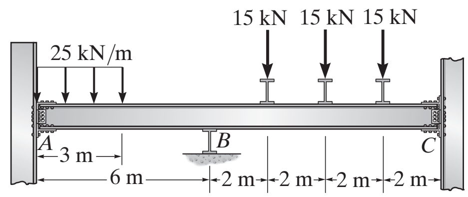
$M_{AB} = -18.5\ \text{kN}\cdot\text{m}$ **Ans.**

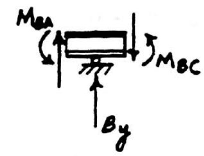
$M_{CB} = 20.375\ \text{kN}\cdot\text{m} = 20.4\ \text{kN}\cdot\text{m}$ **Ans.**

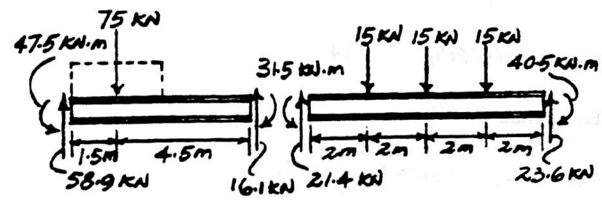
$M_{BA} = 19.25\ \text{kN}\cdot\text{m}$ **Ans.**

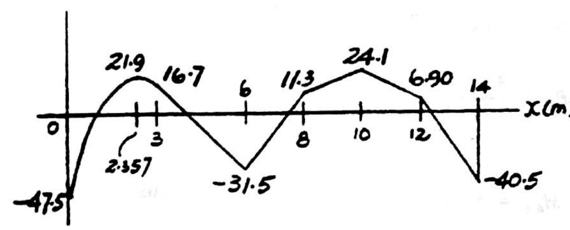
$M_{BC} = -19.25\ \text{kN}\cdot\text{m}$ **Ans.**

***11–4.** Determine the moments at the supports, then draw the moment diagram. Assume B is a roller and A and C are fixed. EI is constant.









$$(\text{FEM})_{AB} = -\frac{11(25)(6)^2}{192} = -51.5625 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BA} = \frac{5(25)(6)^2}{192} = 23.4375 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = \frac{-5(15)(8)}{16} = -37.5 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = 37.5 \text{ kN}\cdot\text{m}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{6}\right)(2(0) + \theta_B - 0) - 51.5625$$

$$M_{AB} = \frac{EI\theta_B}{3} - 51.5625 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{6}\right)(2\theta_B + 0 - 0) + 23.4375$$

$$M_{BA} = \frac{2EI\theta_B}{3} + 23.4375 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{8}\right)(2\theta_B + 0 - 0) - 37.5$$

$$M_{BC} = \frac{EI\theta_B}{2} - 37.5 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{8}\right)(2(0) + \theta_B - 0) + 37.5$$

$$M_{CB} = \frac{EI\theta_B}{4} + 37.5 \qquad (4)$$

Equilibrium.

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving:

$$\theta_B = \frac{12.054}{EI}$$

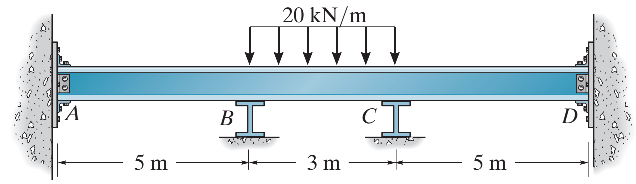
$M_{AB} = -47.5 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BA} = 31.5 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -31.5 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{CB} = 40.5 \text{ kN}\cdot\text{m}$ **Ans.**

11–5. Determine the moment at A, B, C and D, then draw the moment diagram for the beam. Assume the supports at A and D are fixed and B and C are rollers. EI is constant.

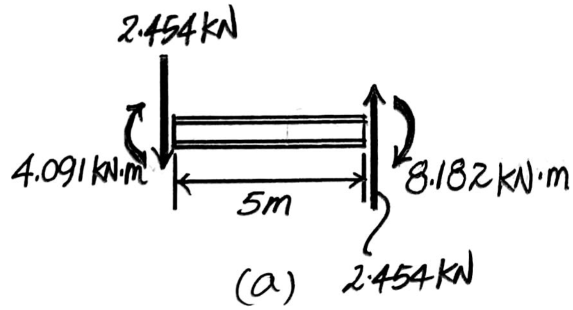


Fixed End Moments. Referring to the table on the inside back cover,

$(\text{FEM})_{AB} = 0 \qquad (\text{FEM})_{BA} = 0 \qquad (\text{FEM})_{CD} = 0 \qquad (\text{FEM})_{DC} = 0$

$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{20(3^2)}{12} = -15 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{20(3^2)}{12} = 15 \text{ kN}\cdot\text{m}$$



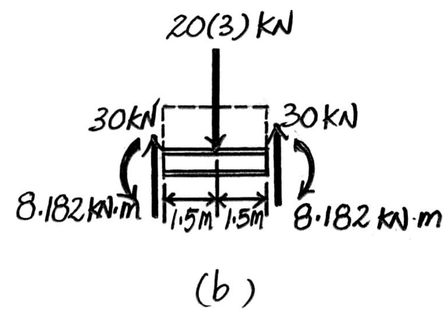
Slope-Deflection Equation. Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left(\frac{I}{5}\right)[2(0) + \theta_B - 3(0)] + 0 = \left(\frac{2EI}{5}\right)\theta_B \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{5}\right)[2\theta_B + 0 - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_B \qquad (2)$$



For span BC,

$$M_{BC} = 2E\left(\frac{I}{3}\right)[2\theta_B + \theta_C - 3(0)] + (-15) = \left(\frac{4EI}{3}\right)\theta_B + \left(\frac{2EI}{3}\right)\theta_C - 15 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{3}\right)[2\theta_C + \theta_B - 3(0)] + 15 = \left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_B + 15 \qquad (4)$$

For span CD,

$$M_{CD} = 2E\left(\frac{I}{5}\right)[2\theta_C + 0 - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_C \qquad (5)$$

$$M_{DC} = 2E\left(\frac{I}{5}\right)[2(0) + \theta_C - 3(0)] + 0 = \left(\frac{2EI}{5}\right)\theta_C \qquad (6)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{4EI}{5}\right)\theta_B + \left(\frac{4EI}{3}\right)\theta_B + \left(\frac{2EI}{3}\right)\theta_C - 15 = 0$$

$$\left(\frac{32EI}{15}\right)\theta_B + \left(\frac{2EI}{3}\right)\theta_C = 15 \qquad (7)$$

At Support C,

$$M_{CB} + M_{CD} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_B + 15 + \left(\frac{4EI}{5}\right)\theta_C = 0$$

11–5. Continued

$$\left(\frac{2EI}{3}\right)\theta_B + \left(\frac{32EI}{15}\right)\theta_C = -15 \qquad (8)$$

Solving Eqs. (7) and (8)

$$\theta_B = \frac{225}{22EI} \qquad \theta_C = -\frac{225}{22EI}$$

Substitute these results into Eqs. (1) to (6),

$M_{AB} = 4.091\ \text{kN}\cdot\text{m} = 4.09\ \text{kN}\cdot\text{m}$ **Ans.**

$M_{BA} = 8.182\ \text{kN}\cdot\text{m} = 8.18\ \text{kN}\cdot\text{m}$ **Ans.**

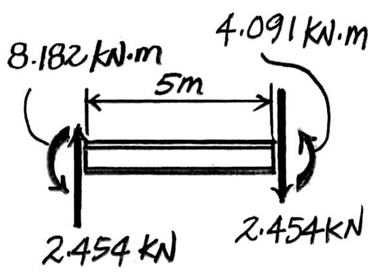
$M_{BC} = -8.182\ \text{kN}\cdot\text{m} = -8.18\ \text{kN}\cdot\text{m}$ **Ans.**

$M_{CB} = 8.182\ \text{kN}\cdot\text{m} = 8.18\ \text{kN}\cdot\text{m}$ **Ans.**

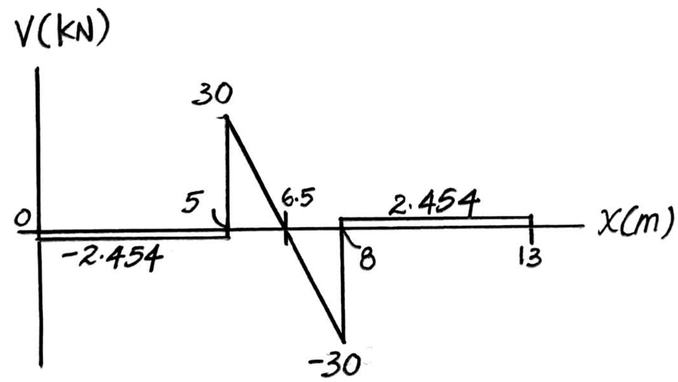
$M_{CD} = -8.182\ \text{kN}\cdot\text{m} = -8.18\ \text{kN}\cdot\text{m}$ **Ans.**

$M_{DC} = -4.091\ \text{kN}\cdot\text{m} = -4.09\ \text{kN}\cdot\text{m}$ **Ans.**

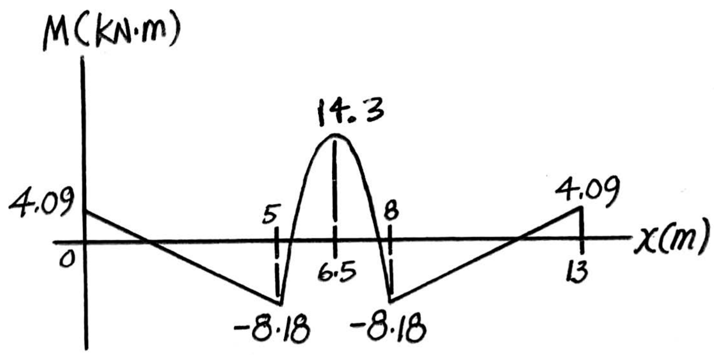
The negative sign indicates that $\mathbf{M}_{BC}$, $\mathbf{M}_{CD}$ and $\mathbf{M}_{DC}$ have counterclockwise rotational sense. Using these results, the shear at both ends of spans AB, BC, and CD are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *d*, and *e* respectively.



(c)

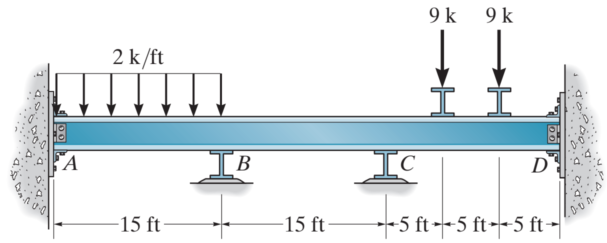


(d)



(e)

11–6. Determine the moments at A, B, C and D, then draw the moment diagram for the beam. Assume the supports at A and D are fixed and B and C are rollers. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

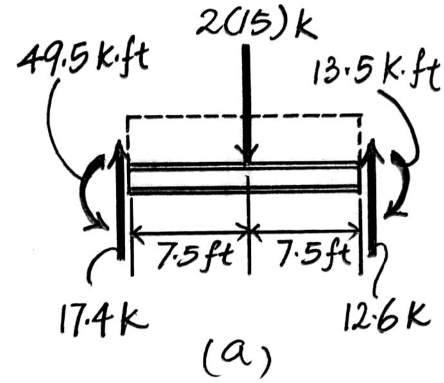
$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{2(15)^2}{12} = -37.5 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{2(15^2)}{12} = 37.5 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

$$(\text{FEM})_{CD} = \frac{-2PL}{9} = -\frac{2(9)(15)}{9} = -30 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{DC} = \frac{2PL}{9} = \frac{2(9)(15)}{9} = 30 \text{ k}\cdot\text{ft}$$



Slope-Deflection Equation. Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

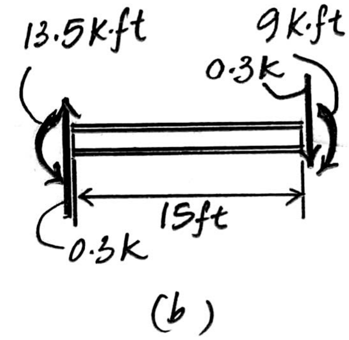
$$M_{AB} = 2E\left(\frac{I}{15}\right)[2(0) + \theta_B - 3(0)] + (-37.5) = \left(\frac{2EI}{15}\right)\theta_B - 37.5 \tag{1}$$

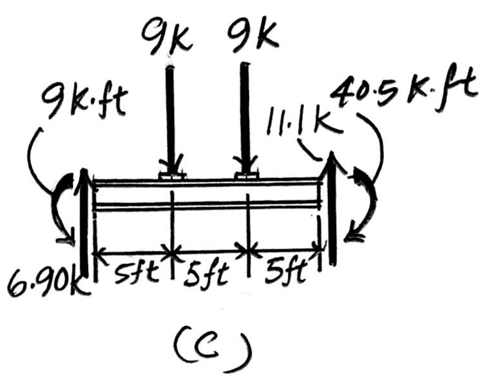
$$M_{BA} = 2E\left(\frac{I}{15}\right)[2\theta_B + 0 - 3(0)] + 37.5 = \left(\frac{4EI}{15}\right)\theta_B + 37.5 \tag{2}$$

For span BC,

$$M_{BC} = 2E\left(\frac{I}{15}\right)[2\theta_B + \theta_C - 3(0)] + 0 = \left(\frac{4EI}{15}\right)\theta_B + \left(\frac{2EI}{15}\right)\theta_C \tag{3}$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)[2\theta_C + \theta_B - 3(0)] + 0 = \left(\frac{4EI}{15}\right)\theta_C + \left(\frac{2EI}{15}\right)\theta_B \tag{4}$$





11–6. Continued

For span CD,

$$M_{CD} = 2E\left(\frac{I}{15}\right)[2\theta_C + 0 - 3(0)] + (-30) = \left(\frac{4EI}{15}\right)\theta_C - 30 \quad (5)$$

$$M_{DC} = 2E\left(\frac{I}{15}\right)[2(0) + \theta_C - 3(0)] + 30 = \left(\frac{2EI}{15}\right)\theta_C + 30 \quad (6)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{4EI}{15}\right)\theta_B + 37.5 + \left(\frac{4EI}{15}\right)\theta_B + \left(\frac{2EI}{15}\right)\theta_C = 0$$

$$\left(\frac{8EI}{15}\right)\theta_B + \left(\frac{2EI}{15}\right)\theta_C = -37.5 \quad (7)$$

At Support C,

$$M_{CB} + M_{CD} = 0$$

$$\left(\frac{4EI}{15}\right)\theta_C + \left(\frac{2EI}{15}\right)\theta_B + \left(\frac{4EI}{15}\right)\theta_C - 30 = 0$$

$$\left(\frac{8EI}{15}\right)\theta_C + \left(\frac{2EI}{15}\right)\theta_B = 30 \quad (8)$$

Solving Eqs. (7) and (8),

$$\theta_C = \frac{78.75}{EI} \qquad \theta_B = -\frac{90}{EI}$$

Substitute these results into Eqs. (1) to (6),

$M_{AB} = -49.5 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BA} = 13.5 \text{ k} \cdot \text{ft}$ **Ans.**

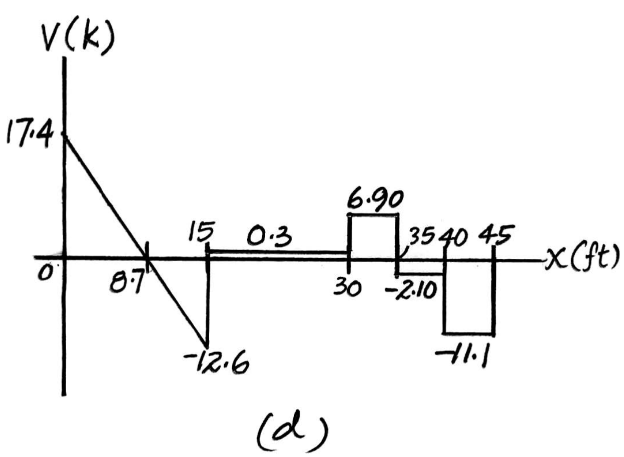
$M_{BC} = -13.5 \text{ k} \cdot \text{ft}$ **Ans.**

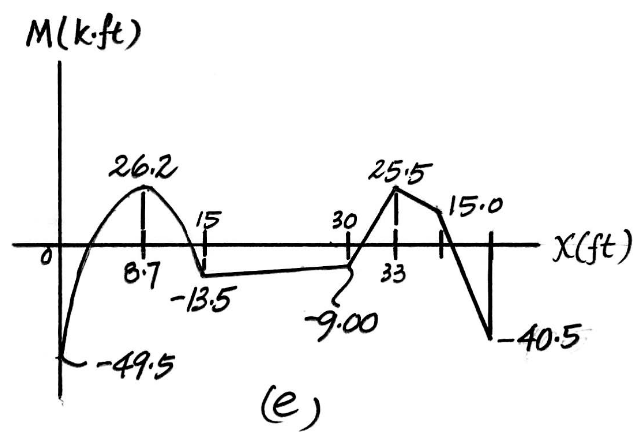
$M_{CB} = 9 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CD} = -9 \text{ k} \cdot \text{ft}$ **Ans.**

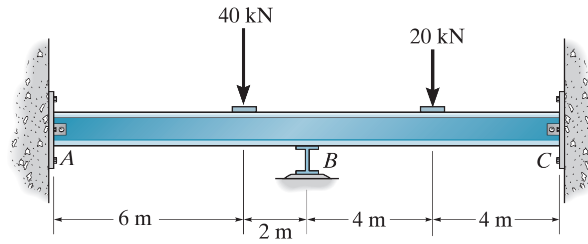
$M_{DC} = 40.5 \text{ k} \cdot \text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$, $\mathbf{M}_{BC}$ and $\mathbf{M}_{CD}$ have counterclockwise rotational sense. Using these results, the shear at both ends of spans AB, BC, and CD are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *d*, and *e* respectively.





11–7. Determine the moment at B, then draw the moment diagram for the beam. Assume the supports at A and C are pins and B is a roller. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = \left(\frac{P}{L^2}\right)\left(b^2a + \frac{a^2b}{2}\right) = \left(\frac{40}{8^2}\right)\left[6^2(2) + \frac{2^2(6)}{2}\right] = 52.5\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = -\frac{3PL}{16} = -\frac{3(20)(8)}{16} = -30\ \text{kN}\cdot\text{m}$$

Slope-Deflection Equations. Applying Eq. 11–10 Since one of the end's support for spans AB and BC is a pin.

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

For span AB,

$$M_{BA} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + 52.5 = \left(\frac{3EI}{8}\right)\theta_B + 52.5 \qquad (1)$$

For span BC,

$$M_{BC} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + (-30) = \left(\frac{3EI}{8}\right)\theta_B - 30 \qquad (2)$$

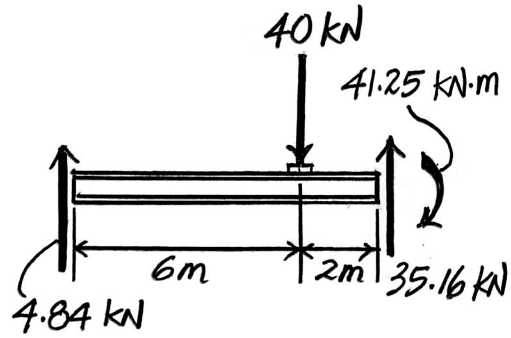
Equilibrium. At support B,

$$M_{BA} + M_{BC} = 0$$

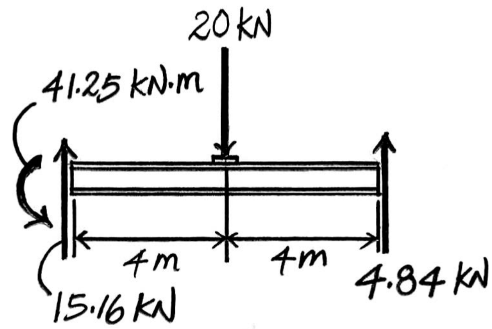
$$\left(\frac{3EI}{8}\right)\theta_B + 52.5 + \left(\frac{3EI}{8}\right)\theta_B - 30 = 0$$

$$\left(\frac{3EI}{4}\right)\theta_B = -22.5$$

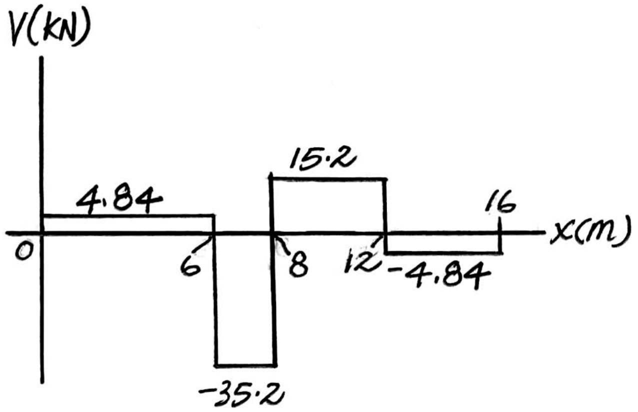
$$\theta_B = -\frac{30}{EI}$$



(a)

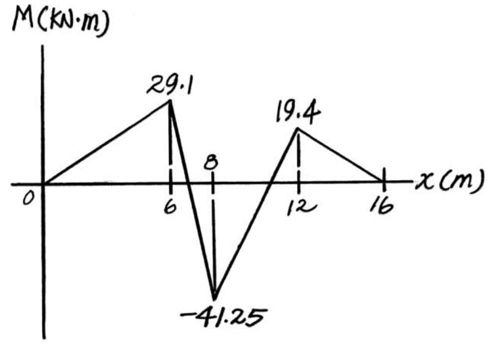


(b)



(c)

11–7. Continued



(d)

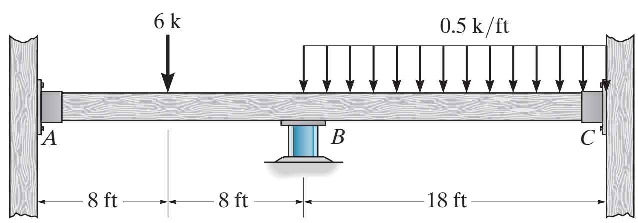
Substitute this result into Eqs. (1) and (2)

$M_{BA} = 41.25\ \text{kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -41.25\ \text{kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{M}_{BC}$ has counterclockwise rotational sense. Using this result, the shear at both ends of spans AB and BC are computed and shown in Fig. *a* and *b* respectively. Subsequently, the shear and Moment diagram can be plotted, Fig. *c* and *d* respectively.

***11–8.** Determine the moments at A, B, and C, then draw the moment diagram. EI is constant. Assume the support at B is a roller and A and C are fixed.

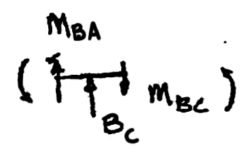


$$(\text{FEM})_{AB} = -\frac{PL}{8} = -12, \qquad (\text{FEM})_{BC} = -\frac{wL^2}{12} = -13.5$$

$$(\text{FEM})_{BA} = \frac{PL}{8} = 12, \qquad (\text{FEM})_{CB} = \frac{wL^2}{12} = 13.5$$

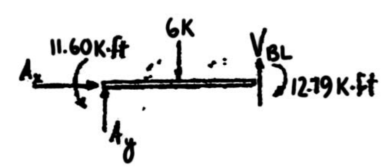
$$\theta_A = \theta_C = \psi_{AB} = \psi_{BC} = 0$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

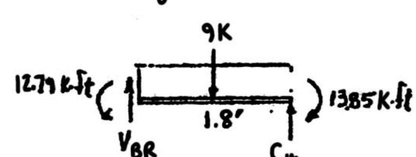


$$M_{AB} = \frac{2EI}{16}(\theta_B) - 12$$

$$M_{BA} = \frac{2EI}{16}(2\theta_B) + 12$$



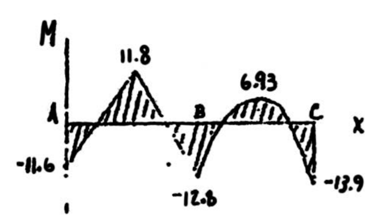
$$M_{BC} = \frac{2EI}{18}(2\theta_B) - 13.5$$



$$M_{CB} = \frac{2EI}{18}(\theta_B) + 13.5$$

Moment equilibrium at B:

$$M_{BA} + M_{BC} = 0$$



$$\frac{2EI}{16}(2\theta_B) + 12 + \frac{2EI}{18}(2\theta_B) - 13.5 = 0$$

$$\theta_B = \frac{3.1765}{EI}$$

11–8. Continued

Thus

$M_{AB} = -11.60 = -11.6 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 12.79 = 12.8 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -12.79 = -12.8 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 13.853 = 13.9 \text{ k}\cdot\text{ft}$ **Ans.**

Left Segment

$\circlearrowleft + \sum M_A = 0; \quad -11.60 + 6(8) + 12.79 - V_{BL}(16) = 0$

$V_{BL} = 3.0744 \text{ k}$

$+\uparrow \sum F_y = 0; \quad A_y = 2.9256 \text{ k}$

Right Segment

$\circlearrowleft + \sum M_B = 0; \quad -12.79 + 9(9) - C_y(18) + 13.85 = 0$

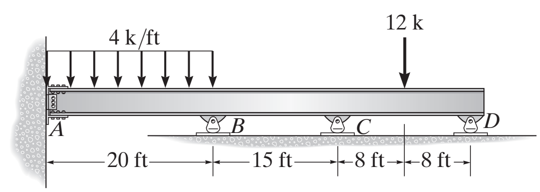
$C_y = 4.5588 \text{ k}$

$+\uparrow \sum F_y = 0; \quad V_{BK} = 4.412 \text{ k}$

At B

$B_y = 3.0744 + 4.4412 = 7.52 \text{ k}$

11–9. Determine the moments at each support, then draw the moment diagram. Assume A is fixed. EI is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{20}(2(0) + \theta_B - 0) - \frac{4(20)^2}{12}$$

$$M_{BA} = \frac{2EI}{20}(2\theta_B + 0 - 0) + \frac{4(20)^2}{12}$$

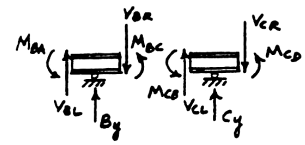
$$M_{BC} = \frac{2EI}{15}(2\theta_B + \theta_C - 0) + 0$$

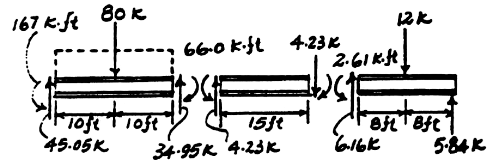
$$M_{CB} = \frac{2EI}{15}(2\theta_C + \theta_B - 0) + 0$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = \frac{3EI}{16}(\theta_C - 0) - \frac{3(12)16}{16}$$

11–9. Continued





Equilibrium.

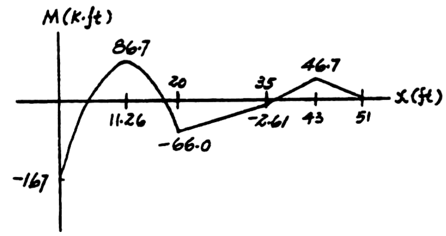
$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

Solving

$$\theta_C = \frac{178.08}{EI}$$

$$\theta_B = -\frac{336.60}{EI}$$



$M_{AB} = -167\ \text{k}\cdot\text{ft}$ **Ans.**

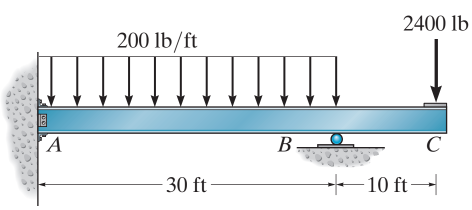
$M_{BA} = 66.0\ \text{k}\cdot\text{ft}$ **Ans.**

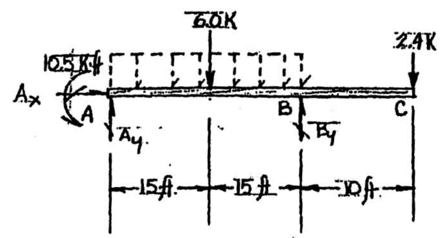
$M_{BC} = -66.0\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 2.61\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CD} = -2.61\ \text{k}\cdot\text{ft}$ **Ans.**

11–10. Determine the moments at A and B, then draw the moment diagram for the beam. EI is constant.

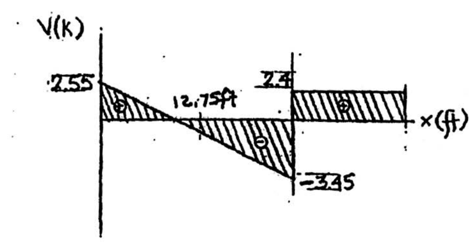




$$(\text{FEM})_{AB} = -\frac{1}{12}(w)(L^2) = -\frac{1}{12}(200)(30^2) = -15\ \text{k}\cdot\text{ft}$$

$$M_{AB} = \frac{2EI}{30}(0 + \theta_B - 0) - 15$$

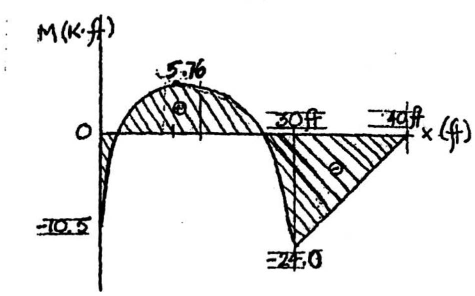
$$M_{BA} = \frac{2EI}{30}(2\theta_B + 0 - 0) + 15$$



$\sum M_B = 0; \qquad M_{BA} = 2.4(10)$

Solving,

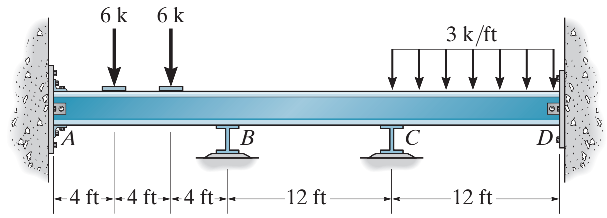
$$\theta_B = \frac{67.5}{EI}$$



$M_{AB} = -10.5\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 24\ \text{k}\cdot\text{ft}$ **Ans.**

11–11. Determine the moments at A, B, and C, then draw the moment diagram for the beam. Assume the support at A is fixed, B and C are rollers, and D is a pin. EI is constant.

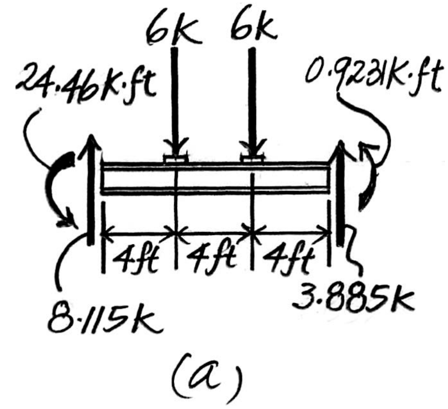


Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{2PL}{9} = -\frac{2(6)(12)}{9} = -16 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{2PL}{9} = \frac{2(6)(12)}{9} = 16 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = 0 \quad (\text{FEM})_{CD} = -\frac{wL^2}{8} = -\frac{3(12^2)}{8} = -54 \text{ k}\cdot\text{ft}$$



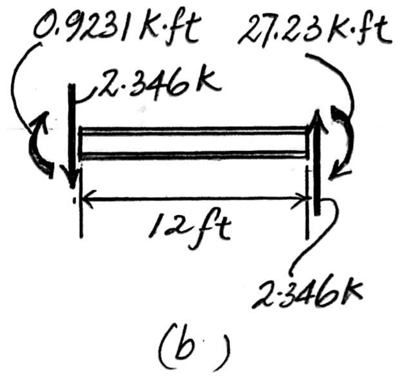
Slope-Deflection Equations. Applying Eq. 11–8, for spans AB and BC.

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_B - 3(0)] + (-16) = \left(\frac{EI}{6}\right)\theta_B - 16 \quad (1)$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 3(0)] + 16 = \left(\frac{EI}{3}\right)\theta_B + 16 \quad (2)$$



For span BC,

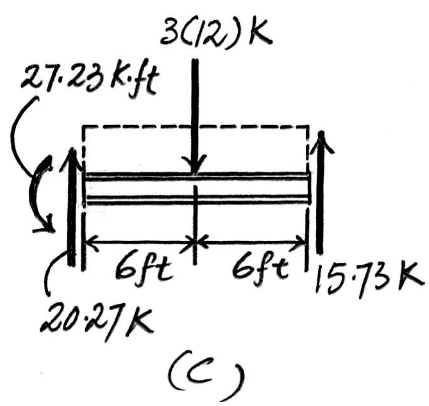
$$M_{BC} = 2E\left(\frac{I}{12}\right)[2\theta_B + \theta_C - 3(0)] + 0 = \left(\frac{EI}{3}\right)\theta_B + \left(\frac{EI}{6}\right)\theta_C \quad (3)$$

$$M_{CB} = 2E\left(\frac{I}{12}\right)[2\theta_C + \theta_B - 3(0)] + 0 = \left(\frac{EI}{3}\right)\theta_C + \left(\frac{EI}{6}\right)\theta_B \quad (4)$$

Applying Eq. 11–10 for span CD,

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = 3E\left(\frac{I}{12}\right)(\theta_C - 0) + (-54) = \left(\frac{EI}{4}\right)\theta_C - 54 \quad (5)$$



Equilibrium. At support B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{EI}{3}\right)\theta_B + 16 + \left(\frac{EI}{3}\right)\theta_B + \left(\frac{EI}{6}\right)\theta_C = 0$$

$$\left(\frac{2EI}{3}\right)\theta_B + \left(\frac{EI}{6}\right)\theta_C = -16 \quad (6)$$

At support C,

$$M_{CB} + M_{CD} = 0$$

$$\left(\frac{EI}{3}\right)\theta_C + \left(\frac{EI}{6}\right)\theta_B + \left(\frac{EI}{4}\right)\theta_C - 54 = 0$$

$$\left(\frac{7EI}{12}\right)\theta_C + \left(\frac{EI}{6}\right)\theta_B = 54 \quad (7)$$

11–11. Continued

Solving Eqs. (6) and (7)

$$\theta_C = \frac{1392}{13EI} \qquad \theta_B = -\frac{660}{13EI}$$

Substitute these results into Eq. (1) to (5)

$M_{AB} = -24.46 \text{ k}\cdot\text{ft} = -24.5 \text{ k}\cdot\text{ft}$ **Ans.**

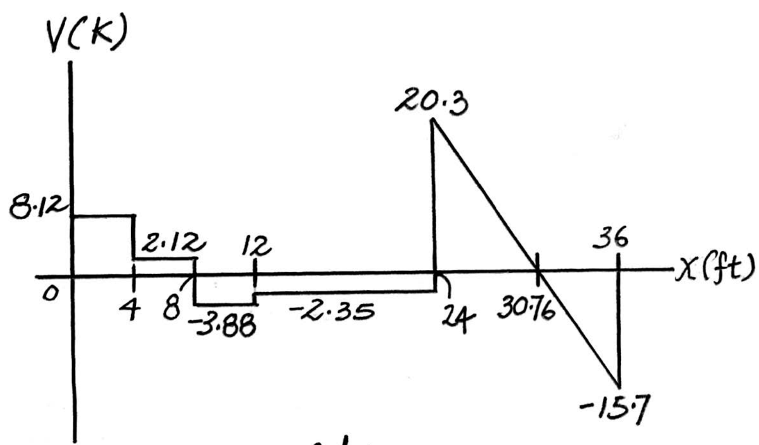
$M_{BA} = -0.9231 \text{ k}\cdot\text{ft} = -0.923 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = 0.9231 \text{ k}\cdot\text{ft} = 0.923 \text{ k}\cdot\text{ft}$ **Ans.**

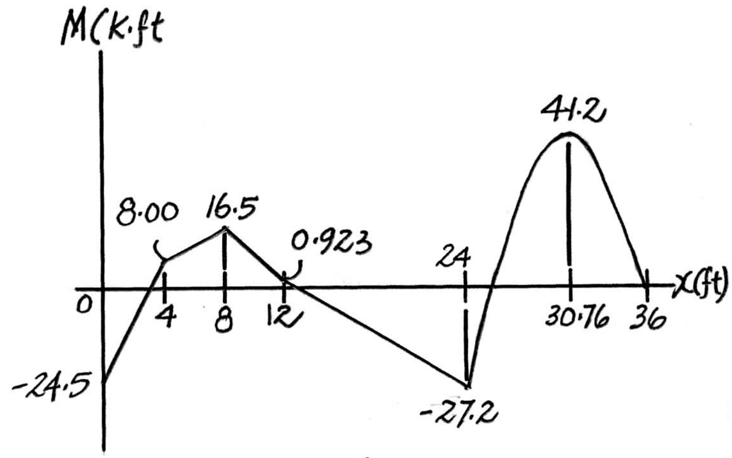
$M_{CB} = 27.23 \text{ k}\cdot\text{ft} = 27.2 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{CD} = -27.23 \text{ k}\cdot\text{ft} = -27.2 \text{ k}\cdot\text{ft}$ **Ans.**

The negative signs indicates that $\mathbf{M}_{AB}$, $\mathbf{M}_{BA}$, and $\mathbf{M}_{CD}$ have counterclockwise rotational sense. Using these results, the shear at both ends of spans AB, BC, and CD are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *d* and *e* respectively.

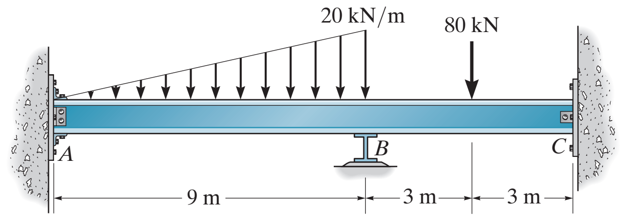


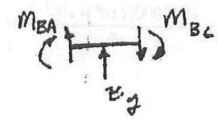
(d)



(e)

***11–12.** Determine the moments acting at A and B. Assume A is fixed supported, B is a roller, and C is a pin. EI is constant.





$$(FEM)_{AB} = \frac{wL^2}{30} = -54, \quad (FEM)_{BC} = \frac{3PL}{16} = -90$$

$$(FEM)_{BA} = \frac{wL^2}{20} = 81$$

Applying Eqs. 11–8 and 11–10,

$$M_{AB} = \frac{2EI}{9}(\theta_B) - 54$$

$$M_{BA} = \frac{2EI}{9}(2\theta_B) + 81$$

$$M_{BC} = \frac{3EI}{6}(\theta_B) - 90$$

Moment equilibrium at B:

$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI}{9}(\theta_B) + 81 + \frac{EI}{2}\theta_B - 90 = 0$$

$$\theta_B = \frac{9.529}{EI}$$

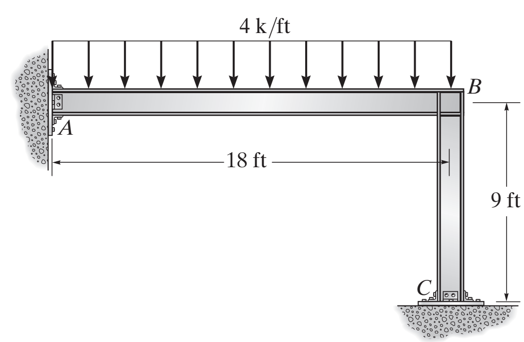
Thus,

$M_{AB} = -51.9 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BA} = 85.2 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -85.2 \text{ kN}\cdot\text{m}$ **Ans.**

11–13. Determine the moments at A, B, and C, then draw the moment diagram for each member. Assume all joints are fixed connected. EI is constant.



$$(FEM)_{AB} = \frac{-4(18)^2}{12} = -108 \text{ k}\cdot\text{ft}$$

$$(FEM)_{BA} = 108 \text{ k}\cdot\text{ft}$$

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

11–13. Continued

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{18}\right)(2(0) + \theta_B - 0) - 108$$

$$M_{AB} = 0.1111EI\theta_B - 108 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{18}\right)(2\theta_B + 0 - 0) + 108$$

$$M_{BA} = 0.2222EI\theta_B + 108 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{9}\right)(2\theta_B + 0 - 0) + 0$$

$$M_{BC} = 0.4444EI\theta_B \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{9}\right)(2(0) + \theta_B - 0) + 0$$

$$M_{CB} = 0.2222EI\theta_B \qquad (4)$$

Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving Eqs. 1–5:

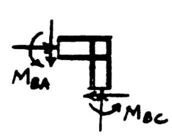
$$\theta_B = \frac{-162.0}{EI}$$

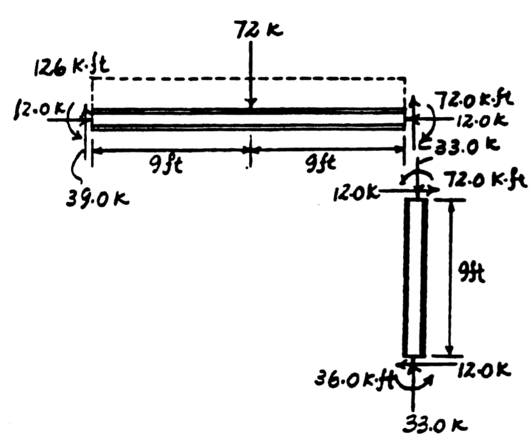
$M_{AB} = -126\ \text{k}\cdot\text{ft}$ **Ans.**

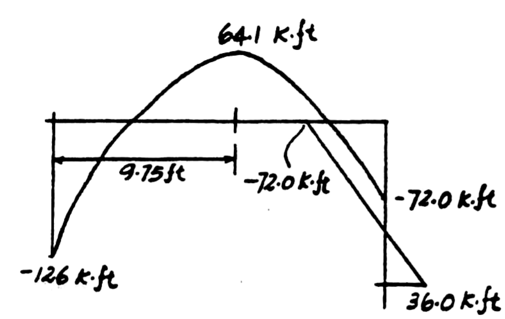
$M_{BA} = 72\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -72\ \text{k}\cdot\text{ft}$ **Ans.**

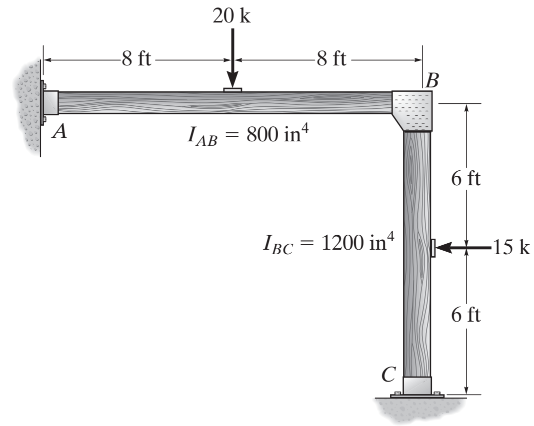
$M_{CB} = -36\ \text{k}\cdot\text{ft}$ **Ans.**







11–14. Determine the moments at the supports, then draw the moment diagram. The members are fixed connected at the supports and at joint B. The moment of inertia of each member is given in the figure. Take $E = 29(10^3)$ ksi.

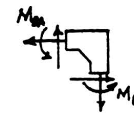


$$(\text{FEM})_{AB} = \frac{-20(16)}{8} = -40\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = 40\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = \frac{-15(12)}{8} = -22.5\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = 22.5\text{ k}\cdot\text{ft}$$



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2(29)(10^3)(800)}{16(144)}(2(0) + \theta_B - 0) - 40$$

$$M_{AB} = 20{,}138.89\theta_B - 40 \qquad (1)$$

$$M_{BA} = \frac{2(29)(10^3)(800)}{16(144)}(2\theta_B + 0 - 0) + 40$$

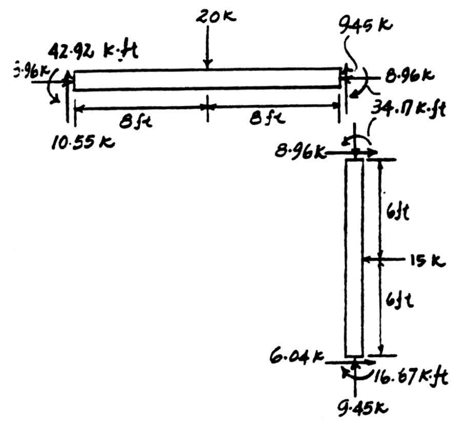
$$M_{BA} = 40{,}277.78\theta_B + 40 \qquad (2)$$

$$M_{BC} = \frac{2(29)(10^3)(1200)}{12(144)}(2\theta_B + 0 - 0) - 22.5$$

$$M_{BC} = 80{,}555.55\theta_B - 22.5 \qquad (3)$$

$$M_{CB} = \frac{2(29)(10^3)(1200)}{12(144)}(2(0) + \theta_B - 0) + 22.5$$

$$M_{CB} = 40{,}277.77\theta_B + 22.5 \qquad (4)$$

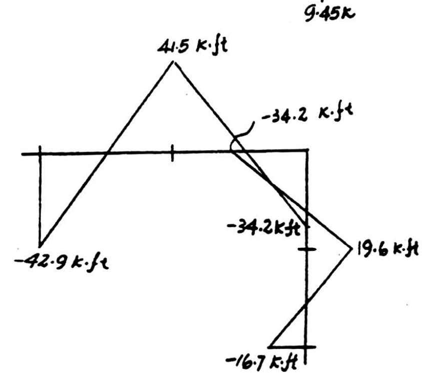


Equilibrium.

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving Eqs. 1–5:

$\theta_B = -0.00014483$



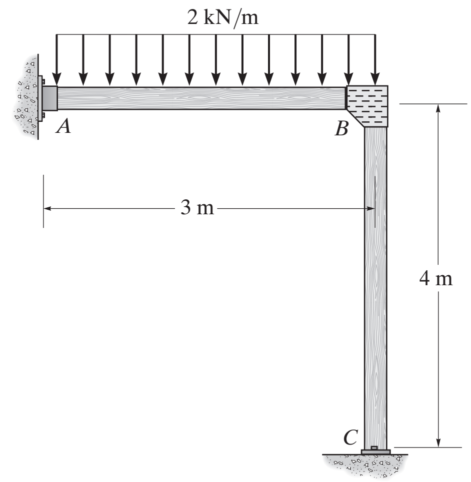
$M_{AB} = -42.9\text{ k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 34.2\text{ k}\cdot\text{ft}$

$M_{BC} = -34.2\text{ k}\cdot\text{ft}$

$M_{CB} = 16.7\text{ k}\cdot\text{ft}$ **Ans.**

11–15. Determine the moment at B, then draw the moment diagram for each member of the frame. Assume the support at A is fixed and C is pinned. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{2(3^2)}{12} = -1.50\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{2(3^2)}{12} = 1.50\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = 0$$

Slope-Deflection Equations. Applying Eq. 11–8 for member AB,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{3}\right)[2(0) + \theta_B - 3(0)] + (-1.50) = \left(\frac{2EI}{3}\right)\theta_B - 1.50 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{3}\right)[2\theta_B + 0 - 3(0)] + 1.50 = \left(\frac{4EI}{3}\right)\theta_B + 1.50 \qquad (2)$$

Applying Eq. 11–10 for member BC,

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BC} = 3E\left(\frac{I}{4}\right)(\theta_B - 0) + 0 = \left(\frac{3EI}{4}\right)\theta_B \qquad (3)$$

Equilibrium. At Joint B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_B + 1.50 + \left(\frac{3EI}{4}\right)\theta_B = 0$$

$$\theta_B = -\frac{0.72}{EI}$$

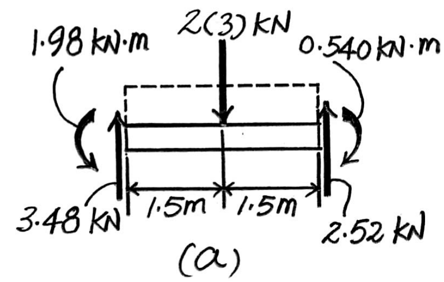
Substitute this result into Eqs. (1) to (3)

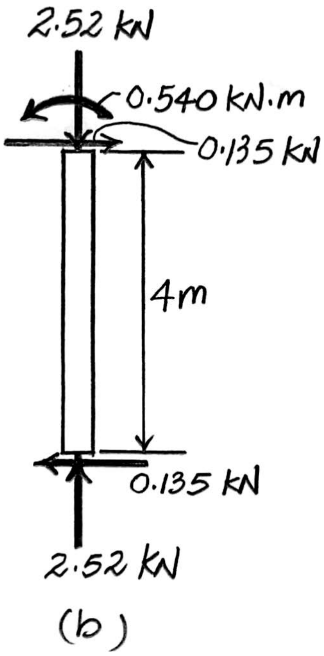
$M_{AB} = -1.98\ \text{kN}\cdot\text{m}$ **Ans.**

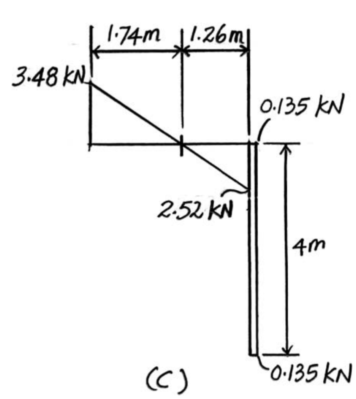
$M_{BA} = 0.540\ \text{kN}\cdot\text{m}$ **Ans.**

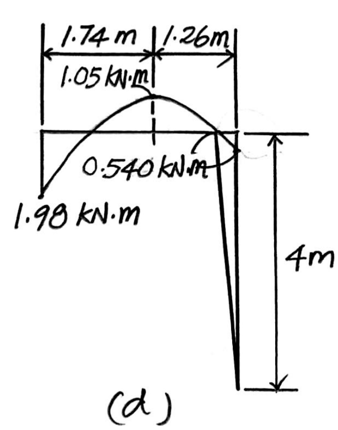
$M_{BC} = -0.540\ \text{kN}\cdot\text{m}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have counterclockwise rotational sense. Using these results, the shear at both ends of member AB and BC are computed and shown in Fig. a and b respectively. Subsequently, the shear and moment diagram can be plotted, Fig. c and d respectively.

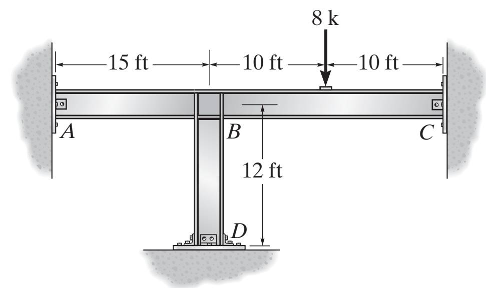


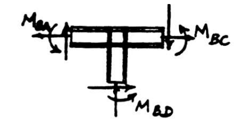






***11–16.** Determine the moments at B and D, then draw the moment diagram. Assume A and C are pinned and B and D are fixed connected. EI is constant.



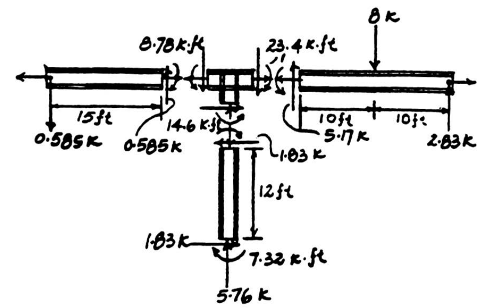


$$(\text{FEM})_{BA} = 0$$

$$(\text{FEM})_{BC} = \frac{-3(8)(20)}{16} = -30\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BD} = (\text{FEM})_{DB} = 0$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

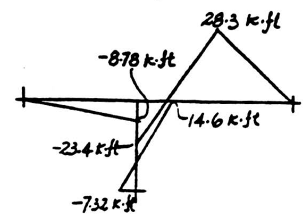


$$M_{BA} = 3E\left(\frac{I}{15}\right)(\theta_B - 0) + 0$$

$$M_{BA} = 0.2EI\theta_B \qquad (1)$$

$$M_{BC} = 3E\left(\frac{I}{20}\right)(\theta_B - 0) - 30$$

$$M_{BC} = 0.15EI\theta_B - 30 \qquad (2)$$



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BD} = 2E\left(\frac{I}{12}\right)(2\theta_B + 0 - 0) + 0$$

$$M_{BD} = 0.3333EI\theta_B \qquad (3)$$

$$M_{DB} = 2E\left(\frac{I}{12}\right)(2(0) + \theta_B - 0) + 0$$

$$M_{DB} = 0.1667EI\theta_B \qquad (4)$$

Equilibrium.

$$M_{BA} + M_{BC} + M_{BD} = 0 \qquad (5)$$

Solving Eqs. 1–5:

$$\theta_B = \frac{43.90}{EI}$$

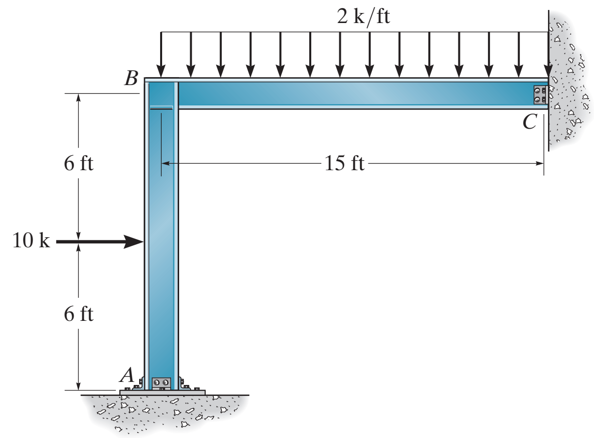
$M_{BA} = 8.78\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -23.41\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BD} = 14.63\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{DB} = 7.32\ \text{k}\cdot\text{ft}$ **Ans.**

11–17. Determine the moment that each member exerts on the joint at B, then draw the moment diagram for each member of the frame. Assume the support at A is fixed and C is a pin. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(FEM)_{AB} = -\frac{PL}{8} = -\frac{10(12)}{8} = -15\ \text{k}\cdot\text{ft} \qquad (FEM)_{BA} = \frac{PL}{8} = \frac{10(12)}{8} = 15\ \text{k}\cdot\text{ft}$$

$$(FEM)_{BC} = -\frac{wL^2}{8} = -\frac{2(15^2)}{8} = -56.25\ \text{k}\cdot\text{ft}$$

Slope Reflection Equations. Applying Eq. 11–8 for member AB,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_B - 3(0)] + (-15) = \left(\frac{EI}{6}\right)\theta_B - 15 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 3(0)] + 15 = \left(\frac{EI}{3}\right)\theta_B + 15 \qquad (2)$$

For member BC, applying Eq. 11–10

$$M_N = 3Ek(\theta_N - \psi) + (FEM)_N$$

$$M_{BC} = 3E\left(\frac{I}{15}\right)(\theta_B - 0) + (-56.25) = \left(\frac{EI}{5}\right)\theta_B - 56.25 \qquad (3)$$

Equilibrium. At joint B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{EI}{3}\right)\theta_B + 15 + \left(\frac{EI}{5}\right)\theta_B - 56.25 = 0$$

$$\theta_B = \frac{77.34375}{EI}$$

Substitute this result into Eqs. (1) to (3)

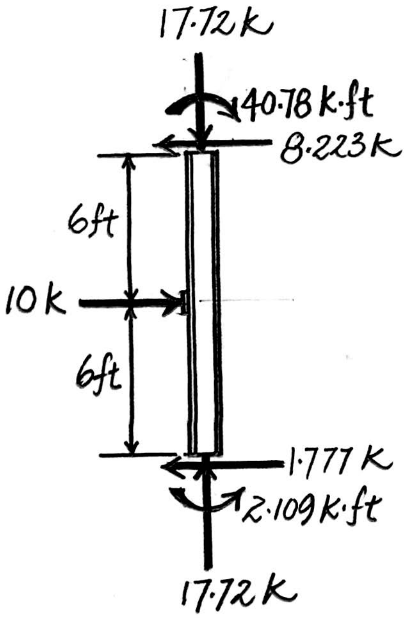
$M_{AB} = -2.109\ \text{k}\cdot\text{ft} = -2.11\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 40.78\ \text{k}\cdot\text{ft} = 40.8\ \text{k}\cdot\text{ft}$ **Ans.**

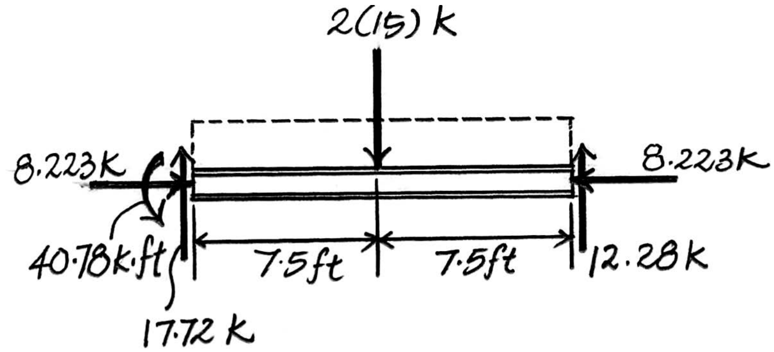
$M_{BC} = -40.78\ \text{k}\cdot\text{ft} = -40.8\ \text{k}\cdot\text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have counterclockwise rotational sense. Using these results, the shear at both ends of member AB and BC are computed and shown in Fig. *a* and *b* respectively. Subsequently, the shear and Moment diagram can be plotted, Fig. *c* and *d* respectively.

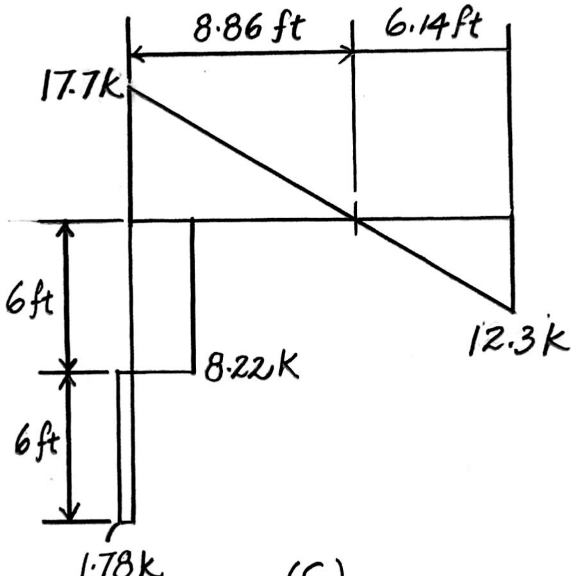
11–17. Continued



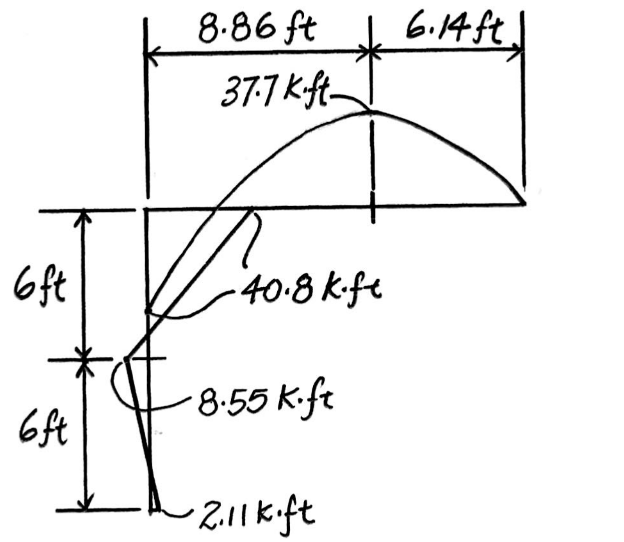
(a)



(b)

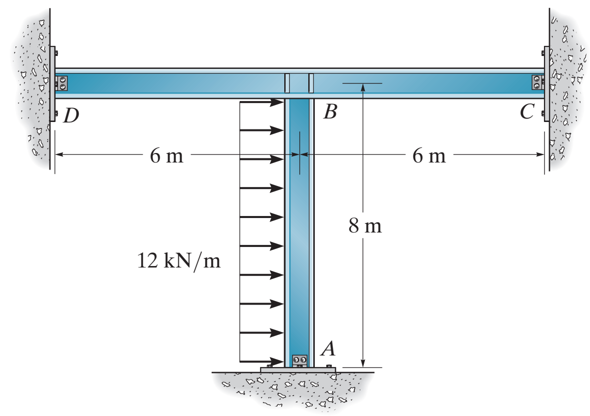


(c)



(d)

11–18. Determine the moment that each member exerts on the joint at B, then draw the moment diagram for each member of the frame. Assume the supports at A, C, and D are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = \frac{wL^2}{8} = \frac{12(8^2)}{8} = 96\ \text{kN}\cdot\text{m} \qquad (\text{FEM})_{BC} = (\text{FEM})_{BD} = 0$$

Slope-Reflection Equation. Since the far end of each members are pinned, Eq. 11–10 can be applied

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

For member AB,

$$M_{BA} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + 96 = \left(\frac{3EI}{8}\right)\theta_B + 96 \qquad (1)$$

For member BC,

$$M_{BC} = 3E\left(\frac{I}{6}\right)(\theta_B - 0) + 0 = \left(\frac{EI}{2}\right)\theta_B \qquad (2)$$

For member BD,

$$M_{BD} = 3E\left(\frac{I}{6}\right)(\theta_B - 0) + 0 = \frac{EI}{2}\theta_B \qquad (3)$$

Equilibrium. At joint B,

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$\left(\frac{3EI}{8}\right)\theta_B + 96 + \left(\frac{EI}{2}\right)\theta_B + \frac{EI}{2}\theta_B = 0$$

$$\theta_B = -\frac{768}{11EI}$$

Substitute this result into Eqs. (1) to (3)

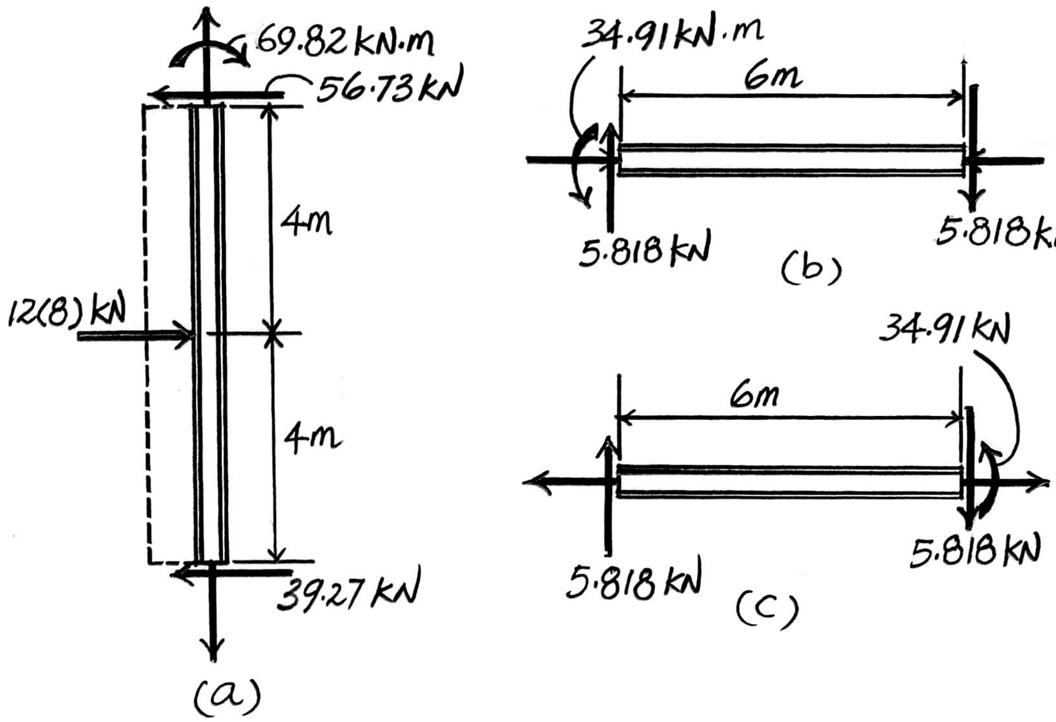
$$M_{BA} = 69.82\ \text{kN}\cdot\text{m} = 69.8\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

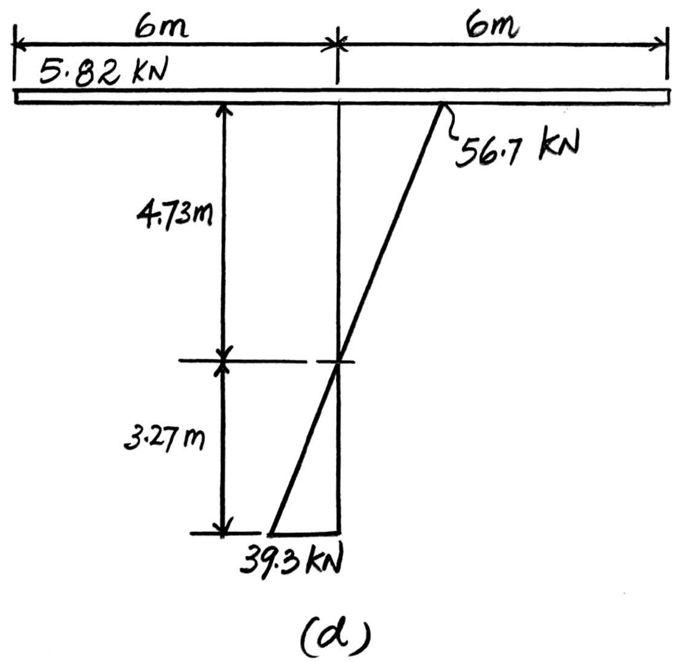
$$M_{BC} = -34.91\ \text{kN}\cdot\text{m} = -34.9\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

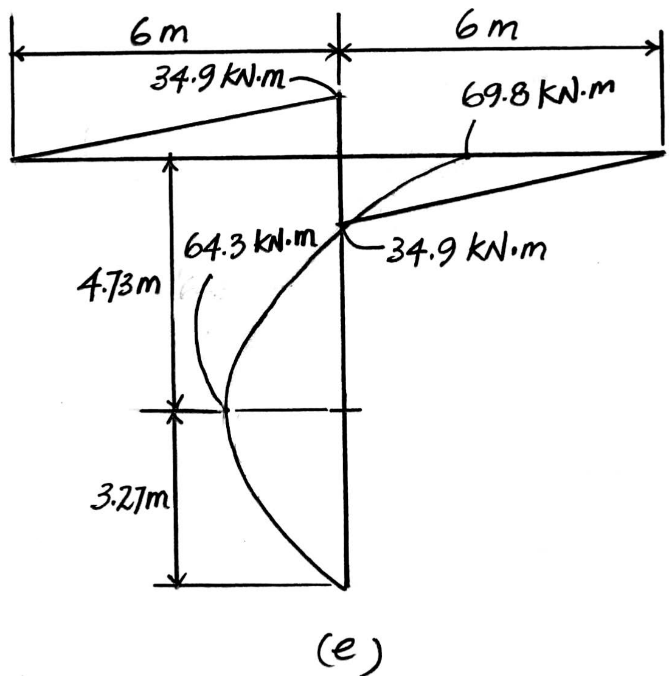
$$M_{BD} = -34.91\ \text{kN}\cdot\text{m} = -34.9\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The negative signs indicate that $\mathbf{M}_{BC}$ and $\mathbf{M}_{BD}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members AB, BC, and BD are computed and shown in Fig. a, b and c respectively. Subsequently, the shear and moment diagrams can be plotted, Fig. d and e respectively.

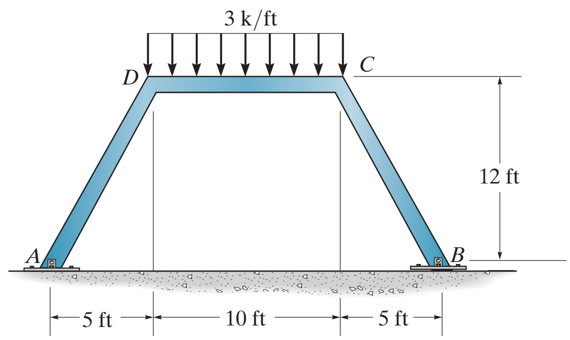
11–18. Continued







11–19. Determine the moment at joints D and C, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{DC} = -\frac{wL^2}{12} = -\frac{3(10^2)}{12} = -25\ \text{k}\cdot\text{ft} \qquad (\text{FEM})_{CD} = \frac{wL^2}{12} = \frac{3(10^2)}{12} = 25\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{DA} = (\text{FEM})_{CB} = 0$$

Slope-Deflection Equations. For member CD, applying Eq. 11–8

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{DC} = 2E\left(\frac{I}{10}\right)[2\theta_D + \theta_C - 3(0)] + (-25) = \left(\frac{2EI}{5}\right)\theta_D + \left(\frac{EI}{5}\right)\theta_C - 25 \qquad (1)$$

$$M_{CD} = 2E\left(\frac{I}{10}\right)[2\theta_C + \theta_D - 3(0)] + 25 = \left(\frac{2EI}{5}\right)\theta_C + \left(\frac{EI}{5}\right)\theta_D + 25 \qquad (2)$$

For members AD and BC, applying Eq. 11–10

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{DA} = 3E\left(\frac{I}{13}\right)(\theta_D - 0) + 0 = \left(\frac{3EI}{13}\right)\theta_D \qquad (3)$$

$$M_{CB} = 3E\left(\frac{I}{13}\right)(\theta_C - 0) + 0 = \left(\frac{3EI}{13}\right)\theta_C \qquad (4)$$

Equilibrium. At joint D,

$$M_{DC} + M_{DA} = 0$$

$$\left(\frac{2EI}{5}\right)\theta_D + \left(\frac{EI}{5}\right)\theta_C - 25 + \left(\frac{3EI}{13}\right)\theta_D = 0$$

$$\left(\frac{41EI}{65}\right)\theta_D + \left(\frac{EI}{5}\right)\theta_C = 25 \qquad (5)$$

At joint C,

$$M_{CD} + M_{CB} = 0$$

$$\left(\frac{2EI}{5}\right)\theta_C + \left(\frac{EI}{5}\right)\theta_D + 25 + \left(\frac{3EI}{13}\right)\theta_C = 0$$

$$\left(\frac{41EI}{65}\right)\theta_C + \left(\frac{EI}{5}\right)\theta_D = -25 \qquad (6)$$

Solving Eqs. (5) and (6)

$$\theta_D = \frac{1625}{28EI} \qquad \theta_C = -\frac{1625}{28EI}$$

Substitute these results into Eq. (1) to (4)

$M_{DC} = -13.39\ \text{k}\cdot\text{ft} = -13.4\ \text{k}\cdot\text{ft}$ **Ans.**

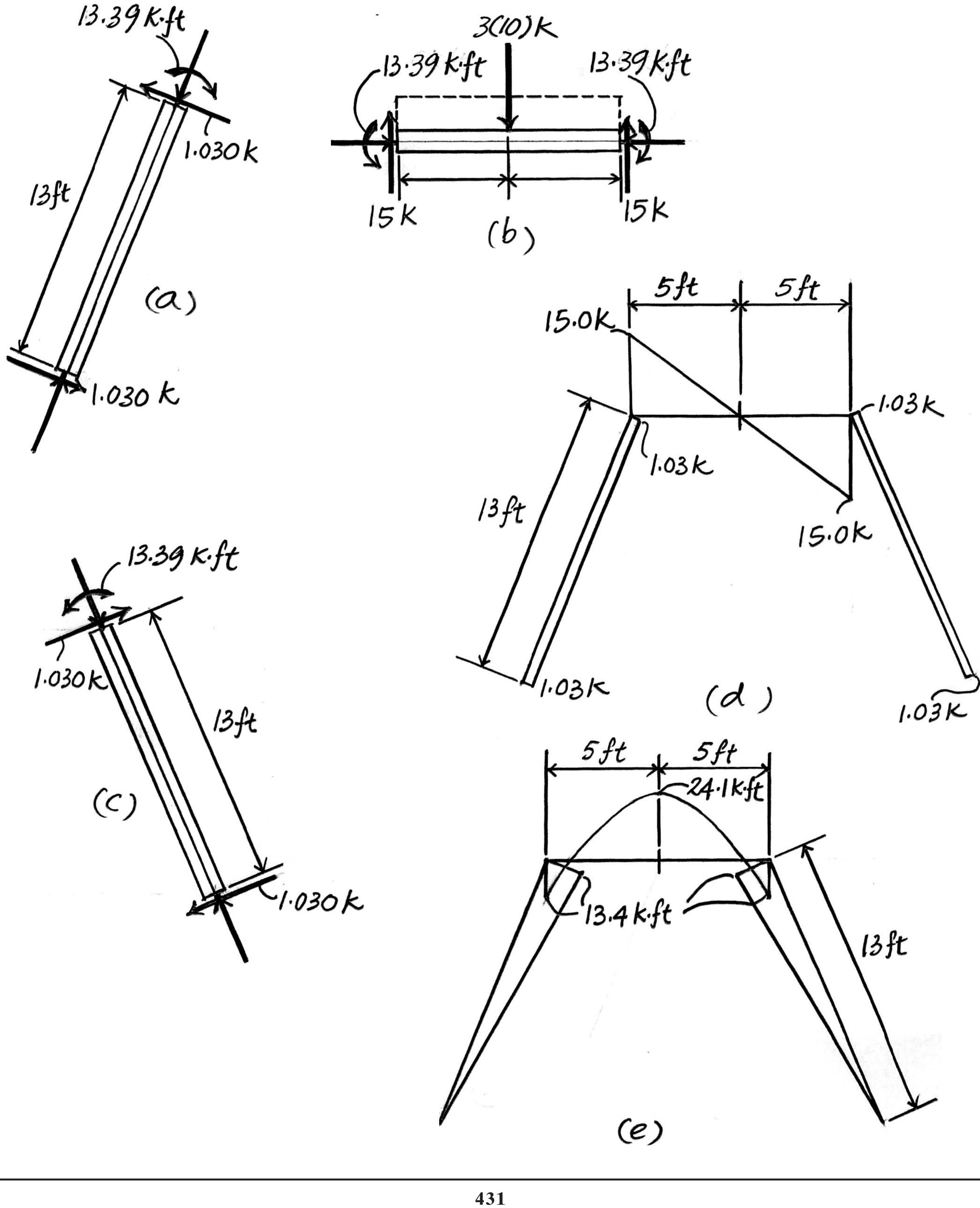
$M_{CD} = 13.39\ \text{k}\cdot\text{ft} = 13.4\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{DA} = 13.39\ \text{k}\cdot\text{ft} = 13.4\ \text{k}\cdot\text{ft}$ **Ans.**

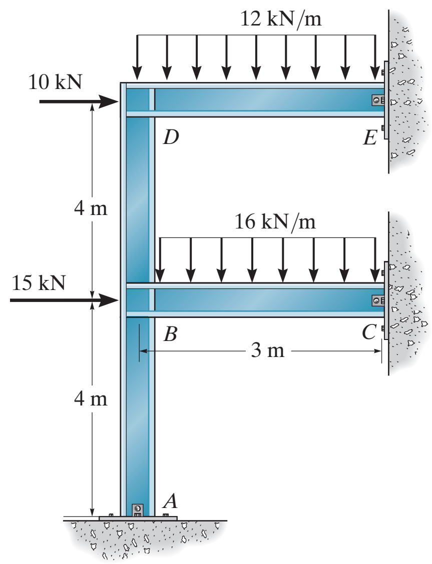
$M_{CB} = -13.39\ \text{k}\cdot\text{ft} = -13.4\ \text{k}\cdot\text{ft}$ **Ans.**

11–19. Continued

The negative signs indicate that $\mathbf{M}_{DC}$ and $\mathbf{M}_{CB}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members AD, CD, and BC are computed and shown in Fig. a, b, and c respectively. Subsequently, the shear and moment diagrams can be plotted, Fig. d and e respectively.



***11–20.** Determine the moment that each member exerts on the joints at B and D, then draw the moment diagram for each member of the frame. Assume the supports at A, C, and E are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = (\text{FEM})_{BD} = (\text{FEM})_{DB} = 0$$

$$(\text{FEM})_{BC} = -\frac{wL^2}{8} = -\frac{16(3^2)}{8} = -18 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{DE} = -\frac{wL^2}{8} = -\frac{12(3^2)}{8} = -13.5 \text{ kN}\cdot\text{m}$$

Slope-Deflection Equations. For member AB, BC, and ED, applying Eq. 11–10.

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left(\frac{I}{4}\right)(\theta_B - 0) + 0 = \left(\frac{3EI}{4}\right)\theta_B \qquad (1)$$

$$M_{BC} = 3E\left(\frac{I}{3}\right)(\theta_B - 0) + (-18) = EI\theta_B - 18 \qquad (2)$$

$$M_{DE} = 3E\left(\frac{I}{3}\right)(\theta_D - 0) + (-13.5) = EI\theta_D - 13.5 \qquad (3)$$

For member BD, applying Eq. 11–8

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BD} = 2E\left(\frac{I}{4}\right)[2\theta_B + \theta_D - 3(0)] + 0 = EI\theta_B + \left(\frac{EI}{2}\right)\theta_D \qquad (4)$$

$$M_{DB} = 2E\left(\frac{I}{4}\right)[2\theta_D + \theta_B - 3(0)] + 0 = EI\theta_D + \left(\frac{EI}{2}\right)\theta_B \qquad (5)$$

Equilibrium. At Joint B,

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$\left(\frac{3EI}{4}\right)\theta_B + EI\theta_B - 18 + EI\theta_B + \left(\frac{EI}{2}\right)\theta_D = 0$$

$$\left(\frac{11EI}{4}\right)\theta_B + \left(\frac{EI}{2}\right)\theta_D = 18 \qquad (6)$$

At joint D,

$$M_{DB} + M_{DE} = 0$$

$$EI\theta_D + \left(\frac{EI}{2}\right)\theta_B + EI\theta_D - 13.5 = 0$$

$$2EI\theta_D + \left(\frac{EI}{2}\right)\theta_B = 13.5 \qquad (7)$$

Solving Eqs. (6) and (7)

$$\theta_B = \frac{39}{7EI} \qquad \theta_D = \frac{75}{14EI}$$

11–20. Continued

Substitute these results into Eqs. (1) to (5),

$M_{BA} = 4.179\ \text{kN}\cdot\text{m} = 4.18\ \text{kN}\cdot\text{m}$ **Ans.**

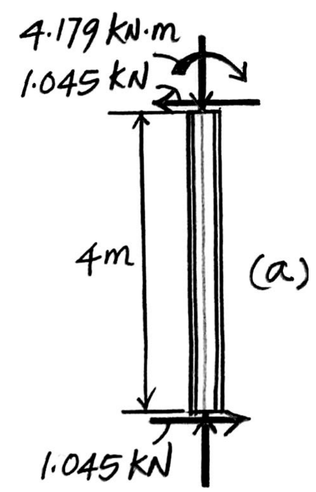
$M_{BC} = -12.43\ \text{kN}\cdot\text{m} = -12.4\ \text{kN}\cdot\text{m}$ **Ans.**

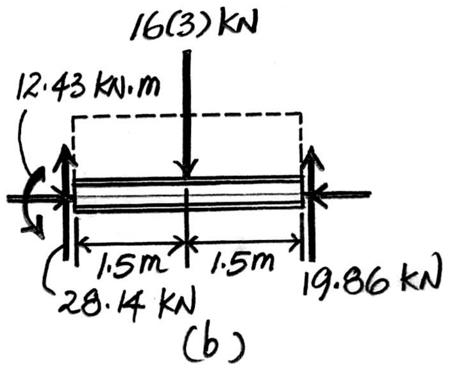
$M_{DE} = -8.143\ \text{kN}\cdot\text{m} = -8.14\ \text{kN}\cdot\text{m}$ **Ans.**

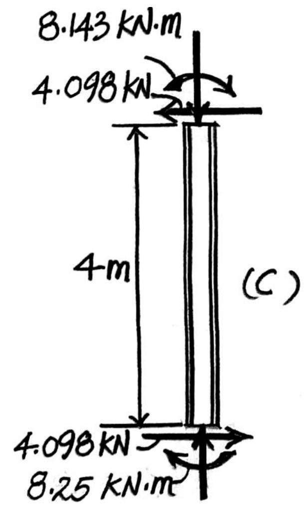
$M_{BD} = 8.25\ \text{kN}\cdot\text{m}$ **Ans.**

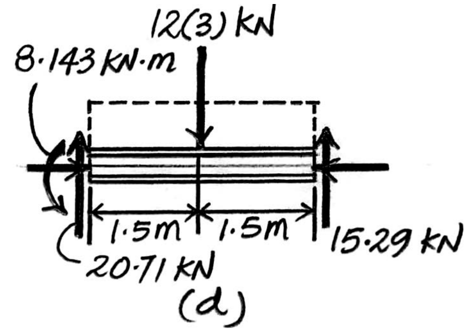
$M_{DB} = 8.143\ \text{kN}\cdot\text{m} = 8.14\ \text{kN}\cdot\text{m}$ **Ans.**

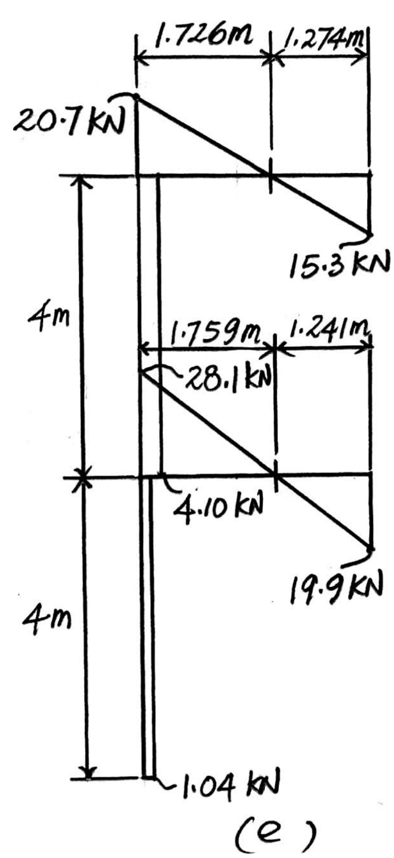
The negative signs indicate that $\mathbf{M}_{BC}$ and $\mathbf{M}_{DE}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members AB, BC, BD and DE are computed and shown on Fig. a, b, c and d respectively. Subsequently, the shear and moment diagram can be plotted, Fig. e and f.

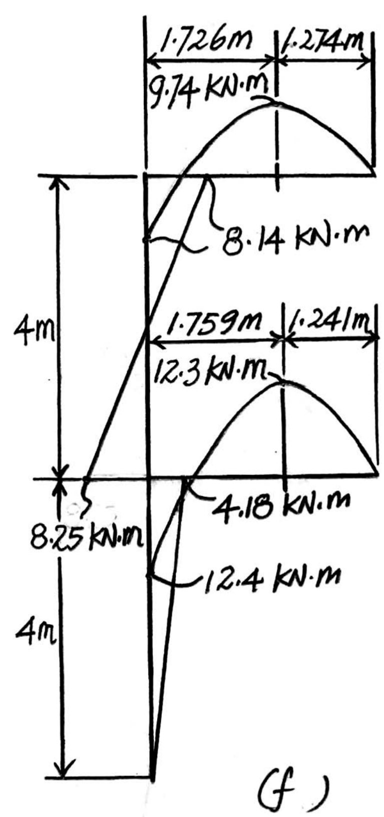




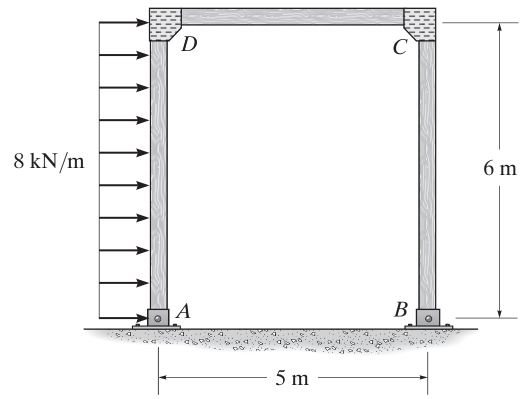








11–21. Determine the moment at joints C and D, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{DA} = \frac{wL^2}{8} = \frac{8(6^2)}{8} = 36 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{DC} = (\text{FEM})_{CD} = (\text{FEM})_{CB} = 0$$

Slope-Deflection Equations. Here, $\psi_{DA} = \psi_{CB} = \psi$ and $\psi_{DC} = \psi_{CD} = 0$

For member CD, applying Eq. 11–8,

$$M_N = 2Ek\,(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{DC} = 2E\left(\frac{I}{5}\right)[2\theta_D + \theta_C - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_D + \left(\frac{2EI}{5}\right)\theta_C \qquad (1)$$

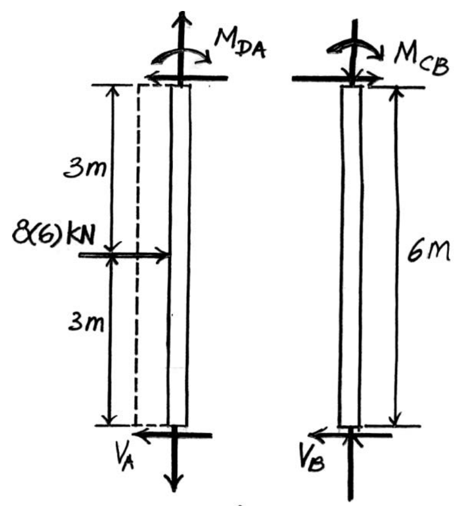
$$M_{CD} = 2E\left(\frac{I}{5}\right)[2\theta_C + \theta_D - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_C + \left(\frac{2EI}{5}\right)\theta_D \qquad (2)$$

For member AD and BC, applying Eq. 11–10

$$M_N = 3Ek\,(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{DA} = 3E\left(\frac{I}{6}\right)(\theta_D - \psi) + 36 = \left(\frac{EI}{2}\right)\theta_D - \left(\frac{EI}{2}\right)\psi + 36 \qquad (3)$$

$$M_{CB} = 3E\left(\frac{I}{6}\right)(\theta_C - \psi) + 0 = \left(\frac{EI}{2}\right)\theta_C - \left(\frac{EI}{2}\right)\psi \qquad (4)$$



(a)

Equilibrium. At joint D,

$$M_{DA} + M_{DC} = 0$$

$$\left(\frac{EI}{2}\right)\theta_D - \left(\frac{EI}{2}\right)\psi + 36 + \left(\frac{4EI}{5}\right)\theta_D + \left(\frac{2EI}{5}\right)\theta_C = 0$$

$$1.3EI\theta_D + 0.4EI\theta_C - 0.5EI\psi = -36 \qquad (5)$$

At joint C,

$$M_{CD} + M_{CB} = 0$$

$$\left(\frac{4EI}{5}\right)\theta_C + \left(\frac{2EI}{5}\right)\theta_D + \left(\frac{EI}{2}\right)\theta_C - \left(\frac{EI}{2}\right)\psi = 0$$

$$0.4EI\theta_D + 1.3EI\theta_C - 0.5EI\psi = 0 \qquad (6)$$

Consider the horizontal force equilibrium for the entire frame

$$\xrightarrow{+} \sum F_x = 0; \quad 8(6) - V_A - V_B = 0$$

Referring to the FBD of member AD and BC in Fig. *a*,

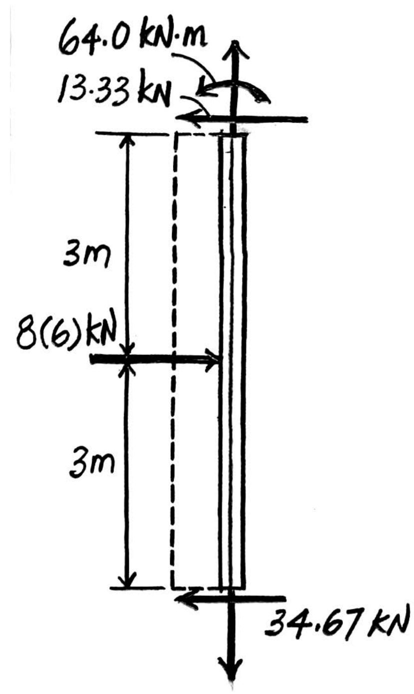
$$\circlearrowleft + \sum M_D = 0; \quad 8(6)(3) - M_{DA} - V_A(6) = 0$$

$$V_A = 24 - \frac{M_{DA}}{6}$$

and

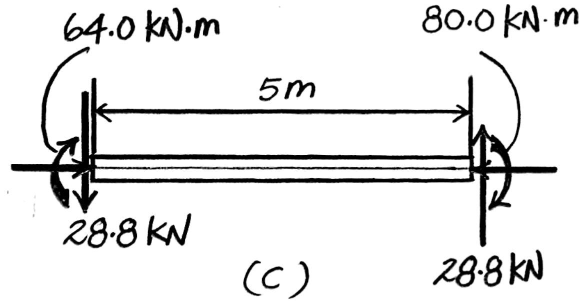
$$\circlearrowleft + \sum M_C = 0; \quad -M_{CB} - V_B(6) = 0$$

$$V_B = -\frac{M_{CB}}{6} = 0$$



(b)

11–21. Continued



Thus,

$$8(6) - \left(24 - \frac{M_{DA}}{6}\right) - \left(-\frac{M_{CB}}{6}\right) = 0$$

$$M_{DA} + M_{CB} = -144$$

$$\left(\frac{EI}{2}\right)\theta_D - \left(\frac{EI}{2}\right)\psi + 36 + \left(\frac{EI}{2}\right)\theta_C - \left(\frac{EI}{2}\right)\psi = -144$$

$$0.5EI\theta_D + 0.5EI\theta_C - EI\psi = -180 \qquad (7)$$

Solving of Eqs. (5), (6) and (7)

$$\theta_C = \frac{80}{EI} \quad \theta_D = \frac{40}{EI} \quad \psi = \frac{240}{EI}$$

Substitute these results into Eqs. (1) to (4),

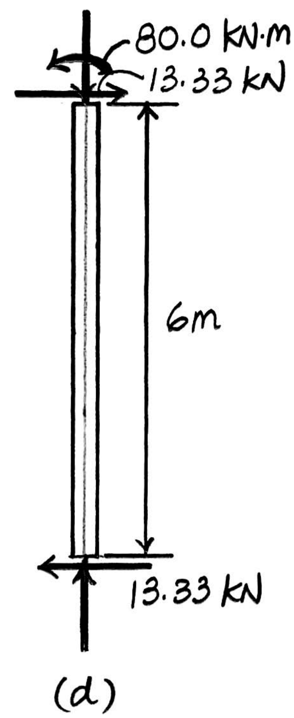
$M_{DC} = 64.0\text{ kN}\cdot\text{m}$ **Ans.**

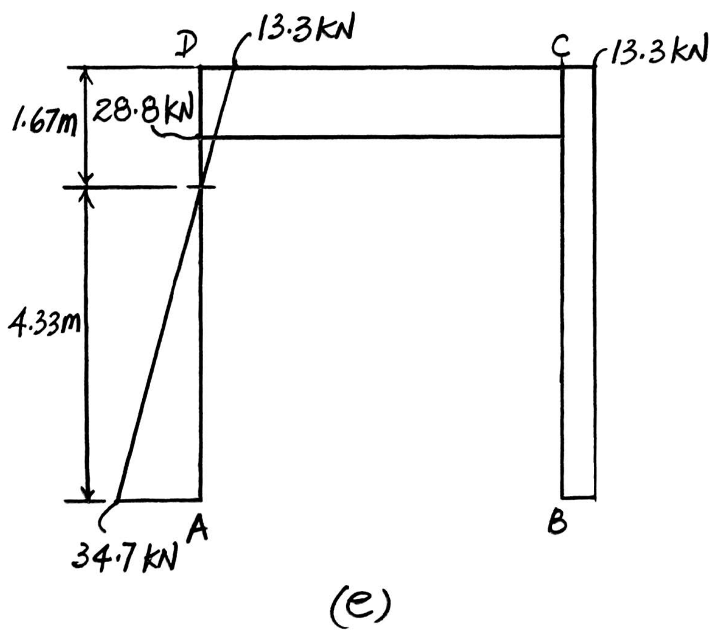
$M_{CD} = 80.0\text{ kN}\cdot\text{m}$ **Ans.**

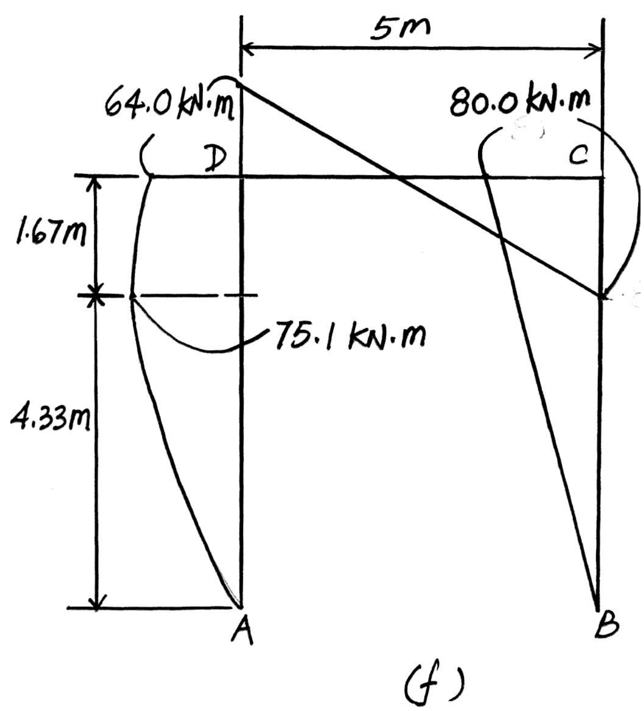
$M_{DA} = -64.0\text{ kN}\cdot\text{m}$ **Ans.**

$M_{CB} = -80.0\text{ kN}\cdot\text{m}$ **Ans.**

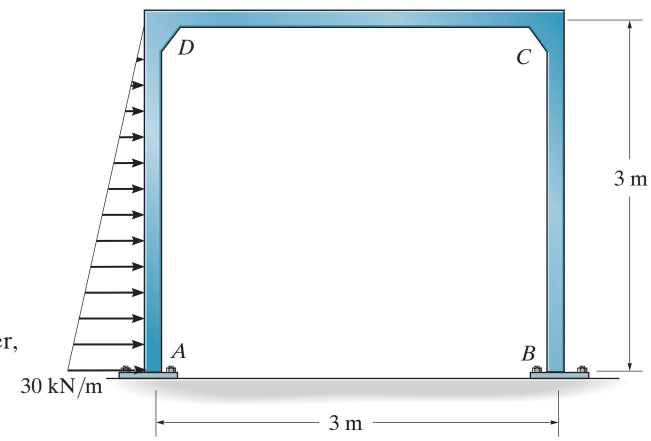
The negative signs indicate that $\mathbf{M}_{DA}$ and $\mathbf{M}_{CB}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members *AD*, *CD*, and *BC* are computed and shown in Fig. *b*, *c*, and *d*, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *e* and *f* respectively.







11–22. Determine the moment at joints A, B, C, and D, then draw the moment diagram for each member of the frame. Assume the supports at A and B are fixed. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = -\frac{wL^2}{20} = -\frac{30(3^2)}{20} = 13.5\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{DA} = \frac{wL^2}{30} = \frac{30(3^2)}{30} = 9\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{DC} = (\text{FEM})_{CD} = (\text{FEM})_{CB} = (\text{FEM})_{BC} = 0$$

Slope-Deflection Equations. Here, $\psi_{AD} = \psi_{DA} = \psi_{BC} = \psi_{CB} = \psi$ and $\psi_{CD} = \psi_{DC} = 0$

Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For member AD,

$$M_{AD} = 2E\left(\frac{I}{3}\right)[2(0) + \theta_D - 3\psi] + (-13.5) = \left(\frac{2EI}{3}\right)\theta_D - 2EI\psi - 13.5 \quad (1)$$

$$M_{DA} = 2E\left(\frac{I}{3}\right)(2\theta_D + 0 - 3\psi) + 9 = \left(\frac{4EI}{3}\right)\theta_D - 2EI\psi + 9 \quad (2)$$

For member CD,

$$M_{DC} = 2E\left(\frac{I}{3}\right)[2\theta_D + \theta_C - 3(0)] + 0 = \left(\frac{4EI}{3}\right)\theta_D + \left(\frac{2EI}{3}\right)\theta_C \quad (3)$$

$$M_{CD} = 2E\left(\frac{I}{3}\right)[2\theta_C + \theta_D - 3(0)] + 0 = \left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_D \quad (4)$$

For member BC,

$$M_{BC} = 2E\left(\frac{I}{3}\right)[2(0) + \theta_C - 3\psi] + 0 = \left(\frac{2EI}{3}\right)\theta_C - 2EI\psi \quad (5)$$

$$M_{CB} = 2E\left(\frac{I}{3}\right)[2\theta_C + 0 - 3\psi] + 0 = \left(\frac{4EI}{3}\right)\theta_C - 2EI\psi \quad (6)$$

Equilibrium. At Joint D,

$$M_{DA} + M_{DC} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_D - 2EI\psi + 9 + \left(\frac{4EI}{3}\right)\theta_D + \left(\frac{2EI}{3}\right)\theta_C = 0$$

$$\left(\frac{8EI}{3}\right)\theta_D + \left(\frac{2EI}{3}\right)\theta_C - 2EI\psi = -9 \quad (7)$$

At joint C,

$$M_{CD} + M_{CB} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_D + \left(\frac{4EI}{3}\right)\theta_C - 2EI\psi = 0$$

11–22. Continued

$$\left(\frac{2EI}{3}\right)\theta_D + \left(\frac{8EI}{3}\right)\theta_C - 2EI\psi = 0 \qquad (8)$$

Consider the horizontal force equilibrium for the entire frame,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad \frac{1}{2}(30)(3) - V_A - V_B = 0$$

Referring to the FBD of members AD and BC in Fig. a

$$\circlearrowleft + \sum M_D = 0; \quad \frac{1}{2}(30)(3)(2) - M_{DA} - M_{AD} - V_A(3) = 0$$

$$V_A = 30 - \frac{M_{DA}}{3} - \frac{M_{AD}}{3}$$

and

$$\circlearrowleft + \sum M_C = 0; \quad -M_{CB} - M_{BC} - V_B(3) = 0$$

$$V_B = -\frac{M_{CB}}{3} - \frac{M_{BC}}{3}$$

Thus,

$$\frac{1}{2}(30)(3) - \left(30 - \frac{M_{DA}}{3} - \frac{M_{AD}}{3}\right) - \left(-\frac{M_{CB}}{3} - \frac{M_{BC}}{3}\right) = 0$$

$$M_{DA} + M_{AD} + M_{CB} + M_{BC} = -45$$

$$\left(\frac{4EI}{3}\right)\theta_D - 2EI\psi + 9 + \left(\frac{2EI}{3}\right)\theta_D - 2EI\psi - 13.5 + \left(\frac{4EI}{3}\right)\theta_C - 2EI\psi$$

$$+ \left(\frac{2EI}{3}\right)\theta_C - 2EI\psi = -45$$

$$2EI\theta_D + 2EI\theta_C - 8EI\psi = -40.5 \qquad (9)$$

Solving of Eqs. (7), (8) and (9)

$$\theta_C = \frac{261}{56EI} \quad \theta_D = \frac{9}{56EI} \quad \psi = \frac{351}{56EI}$$

Substitute these results into Eq. (1) to (6),

$M_{AD} = -25.93 \text{ kN}\cdot\text{m} = -25.9 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{DA} = -3.321 \text{ kN}\cdot\text{m} = -3.32 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{DC} = 3.321 \text{ kN}\cdot\text{m} = 3.32 \text{ kN}\cdot\text{m}$ **Ans.**

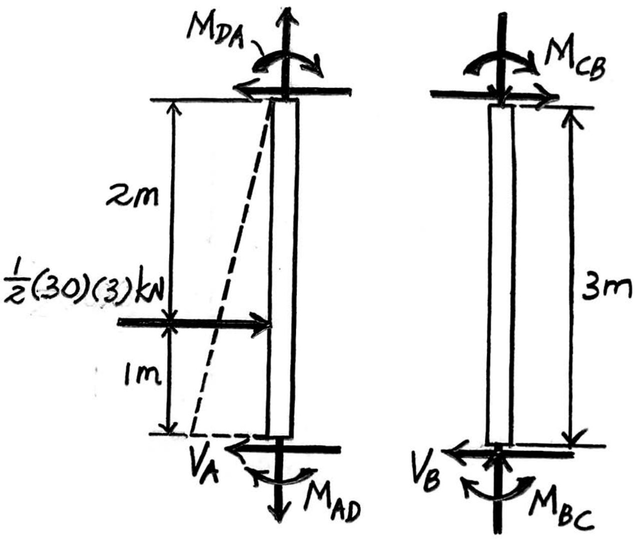
$M_{CD} = 6.321 \text{ kN}\cdot\text{m} = 6.32 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -9.429 \text{ kN}\cdot\text{m} = -9.43 \text{ kN}\cdot\text{m}$ **Ans.**

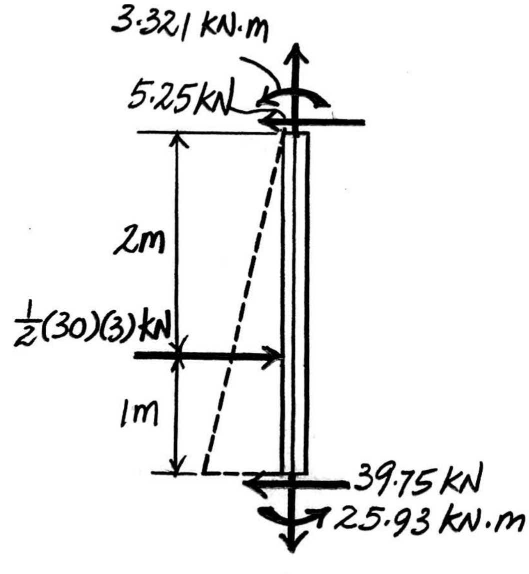
$M_{CB} = -6.321 \text{ kN}\cdot\text{m} = -6.32 \text{ kN}\cdot\text{m}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AD}$, $\mathbf{M}_{DA}$, $\mathbf{M}_{BC}$ and $\mathbf{M}_{CB}$ have counterclockwise rotational sense.Using these results, the shear at both ends of members AD, CD and BC are computed and shown on Fig. b, c and d, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. e and d respectively.

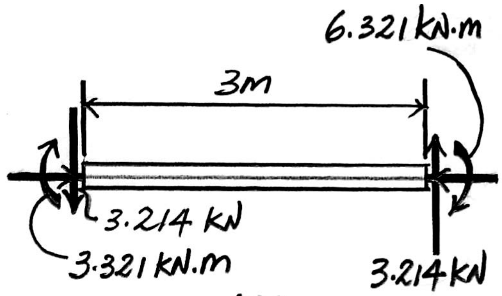
11–22. Continued



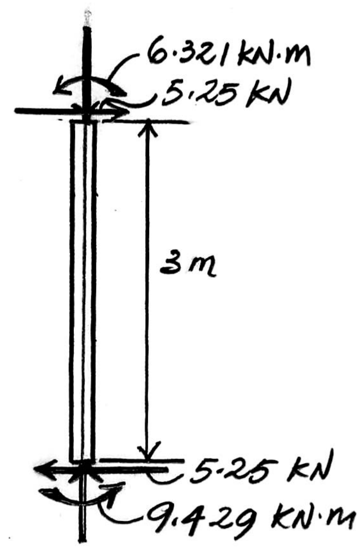
(a)



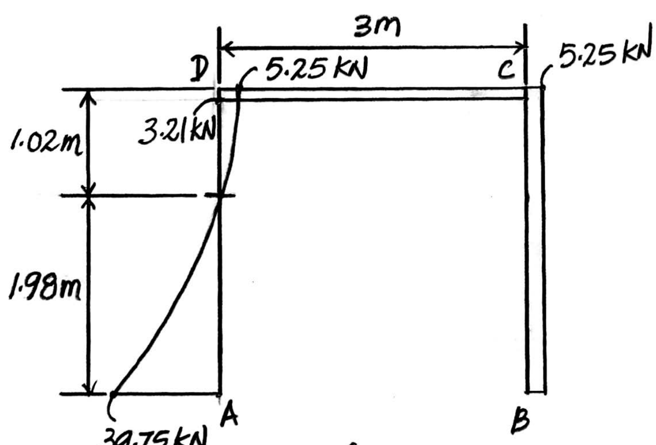
(b)



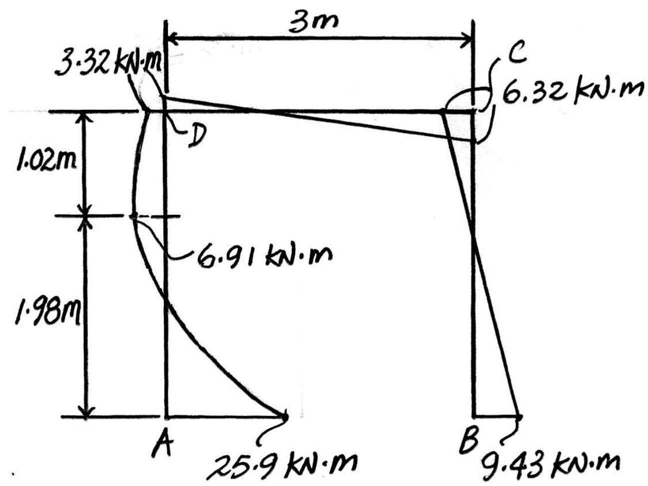
(c)



(d)

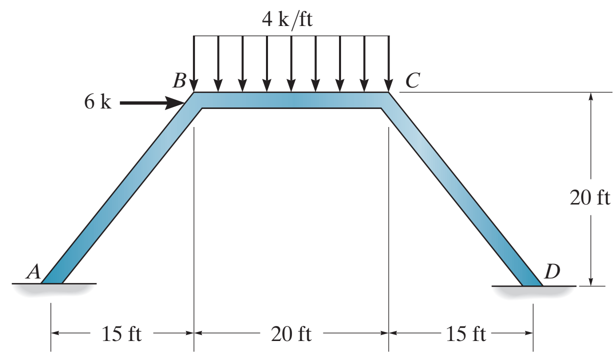


(e)



(f)

11–23. Determine the moments acting at the supports A and D of the battered-column frame. Take $E = 29(10^3)$ ksi, $I = 600$ in^4.



$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -1600 \text{ k}\cdot\text{in.} \quad (\text{FEM})_{CB} = \frac{wL^2}{12} = 1600 \text{ k}\cdot\text{in.}$$

$$\theta_A = \theta_D = 0$$

$$\psi_{AB} = \psi_{CD} = \frac{\Delta}{25}$$

$$\psi_{BC} = -\frac{1.2\Delta}{20}$$

$$\psi_{BC} = -1.5\psi_{CD} = -1.5\psi_{AB}$$

$$\psi = -1.5\psi \qquad (\text{where } \psi = \psi_{BC}, \psi = \psi_{AB} = \psi_{CD})$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{600}{25(12)}\right)(0 + \theta_B - 3\psi) + 0 = 116{,}000\theta_B - 348{,}000\psi$$

$$M_{BA} = 2E\left(\frac{600}{25(12)}\right)(2\theta_B + 0 - 3\psi) + 0 = 232{,}000\theta_B - 348{,}000\psi$$

$$M_{BC} = 2E\left(\frac{600}{20(12)}\right)(2\theta_B + \theta_C - 3(-1.5\psi)) - 1600$$

$$= 290{,}000\theta_B + 145{,}000\theta_C + 652{,}500\psi - 1600$$

$$M_{CB} = 2E\left(\frac{600}{20(12)}\right)(2\theta_C + \theta_B - 3(-1.5\psi)) + 1600$$

$$= 290{,}000\theta_C + 145{,}000\theta_B + 652{,}500\psi - 1600$$

$$M_{CD} = 2E\left(\frac{600}{20(12)}\right)(2\theta_C + 0 - 3\psi) + 0$$

$$= 232{,}000\theta_C - 348{,}000\psi$$

$$M_{DC} = 2E\left(\frac{600}{25(12)}\right)(0 + \theta_C - 3\psi) + 0$$

$$= 116{,}000\theta_C - 348{,}000\psi$$

Moment equilibrium at B and C:

$$M_{BA} + M_{BC} = 0$$

$$522{,}000\theta_B + 145{,}000\theta_C + 304{,}500\psi = 1600 \qquad (1)$$

$$M_{CB} + M_{CD} = 0$$

$$145{,}000\theta_B + 522{,}000\theta_C + 304{,}500\psi = -1600 \qquad (2)$$

11–23. Continued

using the FBD of the frame,

$\circlearrowleft + \sum M_0 = 0;$

$$M_{AB} + M_{DC} - \left(\frac{M_{BA} + M_{AB}}{25(12)}\right)(41.667)(12)$$

$$-\left(\frac{M_{DC} + M_{CD}}{25(12)}\right)(41.667)(12) - 6(13.333)(12) = 0$$

$$-0.667M_{AB} - 0.667M_{DC} - 1.667M_{BA} - 1.667M_{CD} - 960 = 0$$

$$464{,}000\theta_B + 464{,}000\theta_C - 1{,}624{,}000\psi = -960$$

Solving Eqs. (1), (2) and (3),

$\theta_B = 0.004030$ rad

$\theta_C = -0.004458$ rad

$\psi = 0.0004687$ in.

$M_{AB} = 25.4\ \text{k} \cdot \text{ft}$ **Ans.**

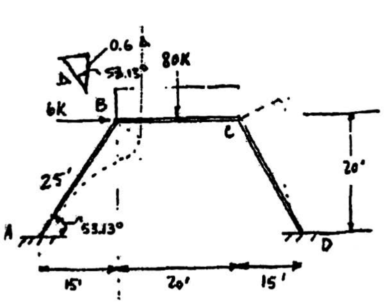
$M_{BA} = 64.3\ \text{k} \cdot \text{ft}$

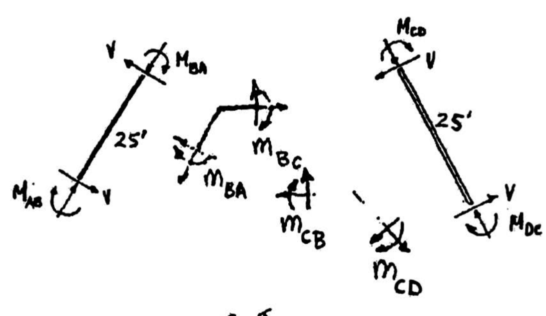
$M_{BC} = -64.3\ \text{k} \cdot \text{ft}$

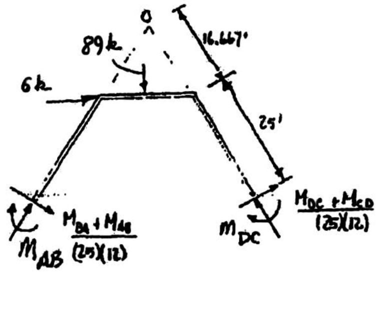
$M_{CB} = 99.8\ \text{k} \cdot \text{ft}$

$M_{CD} = -99.8\ \text{k} \cdot \text{ft}$

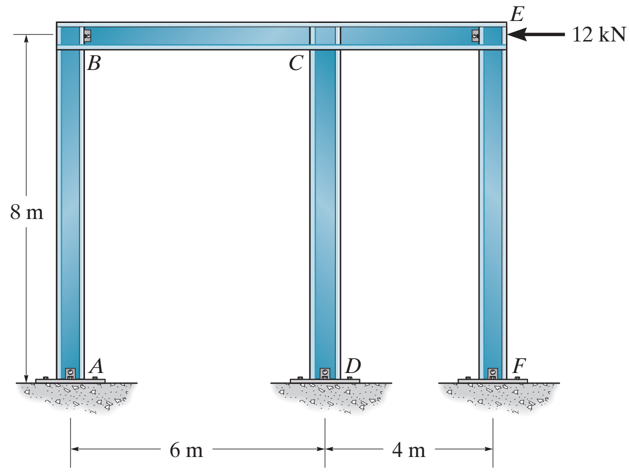
$M_{DC} = -56.7\ \text{k} \cdot \text{ft}$ **Ans.**







***11–24.** Wind loads are transmitted to the frame at joint E. If A, B, E, D, and F are all pin connected and C is fixed connected, determine the moments at joint C and draw the bending moment diagrams for the girder BCE. EI is constant.



$\psi_{BC} = \psi_{CE} = 0$

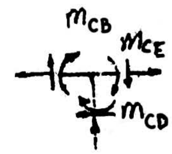
$\psi_{AB} = \psi_{CD} = \psi_{CF} = \psi$

Applying Eq. 11–10,

$$M_{CB} = \frac{3EI}{6}(\theta_C - 0) + 0$$

$$M_{CE} = \frac{3EI}{4}(\theta_C - 0) + 0$$

$$M_{CD} = \frac{3EI}{8}(\theta_C - \psi) + 0 \quad (1)$$

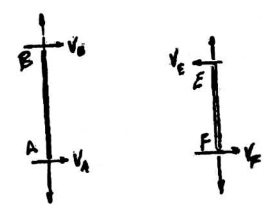


Moment equilibrium at C:

$$M_{CB} + M_{CE} + M_{CD} = 0$$

$$\frac{3EI}{6}(\theta_C) + \frac{3EI}{4}(\theta_C) + \frac{3EI}{8}(\theta_C - \psi) = 0$$

$$\psi = 4.333\theta_C \quad (2)$$

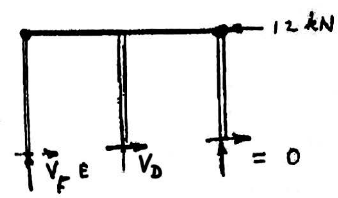


From FBDs of members AB and EF:

$\circlearrowleft + \sum M_B = 0; \quad V_A = 0$

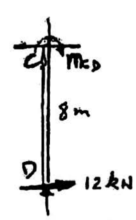
$\circlearrowleft + \sum M_E = 0; \quad V_F = 0$

Since AB and FE are two-force members, then for the entire frame:



$\overset{+}{\rightarrow} \sum F_E = 0; \quad V_D - 12 = 0; \quad V_D = 12 \text{ kN}$

From FBD of member CD:



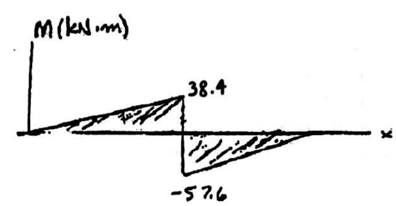
$\circlearrowleft + \sum M_C = 0; \quad M_{CD} - 12(8) = 0$

$M_{CD} = 96 \text{ kN}\cdot\text{m}$ **Ans.**

From Eq. (1),

$$96 = \frac{3}{8}EI(\theta_C - 4.333\theta_C)$$

$$\theta_C = \frac{-76.8}{EI}$$



From Eq. (2),

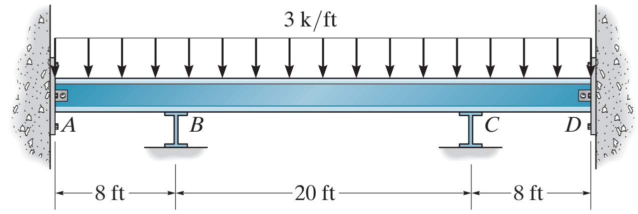
$$\psi = \frac{-332.8}{EI}$$

Thus,

$M_{CB} = -38.4 \text{ kN}\cdot\text{in}$ **Ans.**

$M_{CE} = -57.6 \text{ kN}\cdot\text{m}$ **Ans.**

12–1. Determine the moments at B and C. EI is constant. Assume B and C are rollers and A and D are pinned.



$$\text{FEM}_{AB} = \text{FEM}_{CD} = -\frac{wL^2}{12} = -16, \qquad \text{FEM}_{BA} = \text{FEM}_{DC} = \frac{wL^2}{12} = 16$$

$$\text{FEM}_{BC} = -\frac{wL^2}{12} = -100 \qquad \text{FEM}_{CB} = \frac{wL^2}{12} = 100$$

$$K_{AB} = \frac{3EI}{8}, \qquad K_{BC} = \frac{4EI}{20}, \qquad K_{CD} = \frac{3EI}{8}$$

$$\text{DF}_{AB} = 1 = DF_{DC}$$

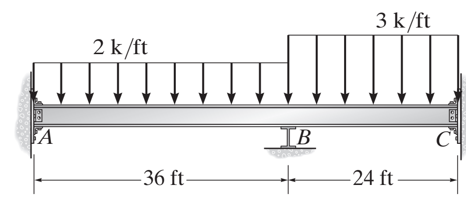
$$\text{DF}_{BA} = \text{DF}_{CD} = \frac{\frac{3EI}{8}}{\frac{3EI}{8} + \frac{4EI}{20}} = 0.652$$

$$\text{DF}_{BA} = \text{DF}_{CB} = 1 - 0.652 = 0.348$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	1	0.652	0.348	0.348	0.652	1
FEM	–16	16	–100	100	–16	16
	16	54.782	29.218	–29.218	–54.782	–16
		8	–14.609	14.609	–8	
		4.310	2.299	–2.299	–4.310	
			–1.149	1.149		
		0.750	0.400	–0.400	–0.750	
			–0.200	0.200		
		0.130	0.070	–0.070	–0.130	
			–0.035	0.035		
		0.023	0.012	–0.012	–0.023	
$\sum M$	0	84.0	–84.0	84.0	–84.0	0 k · ft

Ans.

12–2. Determine the moments at A, B, and C. Assume the support at B is a roller and A and C are fixed. EI is constant.



$$(\text{DF})_{AB} = 0 \qquad (\text{DF})_{BA} = \frac{I/36}{I/36 + I/24} = 0.4$$

$$(\text{DF})_{BC} = 0.6 \qquad (\text{DF})_{CB} = 0$$

$$(\text{FEM})_{AB} = \frac{-2(36)^2}{12} = -216 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BA} = 216 \text{ k} \cdot \text{ft}$$

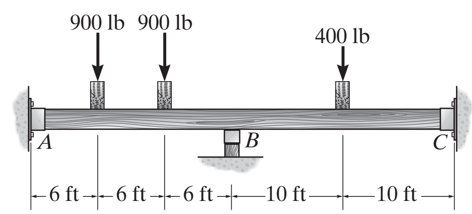
$$(\text{FEM})_{BC} = \frac{-3(24)^2}{12} = -144 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = 144 \text{ k} \cdot \text{ft}$$

Joint	A	B		C
Mem.	AB	BA	BC	CB
DF	0	0.4	0.6	0
FEM	–216	216	–144	144
		–28.8	–43.2	
	–14.4			–21.6
$\sum M$	–230	187	–187	–122 k · ft

Ans.

12–3. Determine the moments at A, B, and C, then draw the moment diagram. Assume the support at B is a roller and A and C are fixed. EI is constant.



$$(\text{DF})_{AB} = 0 \qquad (\text{DF})_{BA} = \frac{I/18}{I/18 + I/20} = 0.5263$$

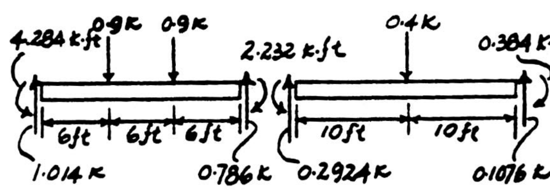
$$(DF)_{CB} = 0 \qquad (DF)_{BC} = 0.4737$$

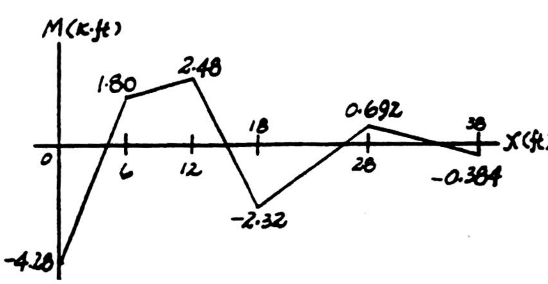
$$(\text{FEM})_{AB} = \frac{-2(0.9)(18)}{9} = -3.60 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BA} = 3.60 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BC} = \frac{-0.4(20)}{8} = -1.00 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = 1.00 \text{ k} \cdot \text{ft}$$

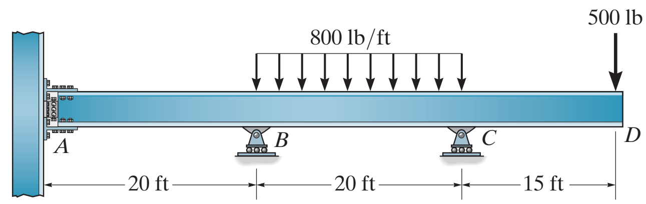




Joint	A	B		C
Mem.	AB	BA	BC	CB
DF	0	0.5263	0.4737	0
FEM	–3.60	3.60	–1.00	1.00
		–1.368	–1.232	
	–0.684			–0.616
$\sum M$	–4.28	2.23	–2.23	0.384 k · ft

Ans.

***12–4.** Determine the reactions at the supports and then draw the moment diagram. Assume *A* is fixed. *EI* is constant.



$$\text{FEM}_{BC} = -\frac{wL^2}{12} = -26.67, \quad \text{FEM}_{CB} = \frac{wL^2}{12} = 26.67$$

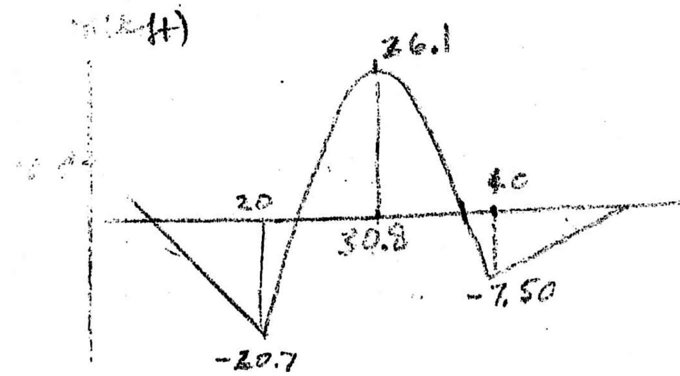
$$M_{CD} = 0.5(15) = 7.5 \text{ k} \cdot \text{ft}$$

$$K_{AB} = \frac{4EI}{20}, \quad K_{BC} = \frac{4EI}{20}$$

$$\text{DF}_{AB} = 0$$

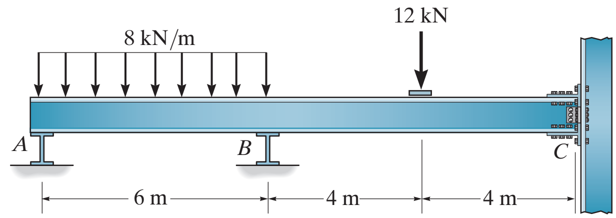
$$\text{DF}_{BA} = \text{DF}_{BC} = \frac{\frac{4EI}{20}}{\frac{4EI}{20} + \frac{4EI}{20}} = 0.5$$

$$\text{DF}_{CB} = 1$$



Joint	*A*	*B*		*C*	
Member	*AB*	*BA*	*BC*	*CB*	*CD*
DF	0	0.5	0.5	1	0
FEM			–26.67	26.67	–7.5
		13.33	13.33	–19.167	
	6.667		–9.583	6.667	
		4.7917	4.7917	–6.667	
	2.396		–3.333	2.396	
		1.667	1.667	–2.396	
	0.8333		–1.1979	0.8333	
		0.5990	0.5990	–0.8333	
	0.2994		–0.4167	0.2994	
		0.2083	0.2083	–0.2994	
	0.1042		–0.1497	0.1042	
		0.07485	0.07485	–0.1042	
	10.4	20.7	–20.7	7.5	–7.5 k · ft

12–5. Determine the moments at B and C, then draw the moment diagram for the beam. Assume C is a fixed support. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{BA} = \frac{3EI}{L_{BA}} = \frac{3EI}{6} = \frac{EI}{2} \qquad K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{8} = \frac{EI}{2}$$

$$(\text{DF})_{AB} = 1 \qquad (\text{DF})_{BA} = \frac{EI/2}{EI/2 + EI/2} = 0.5$$

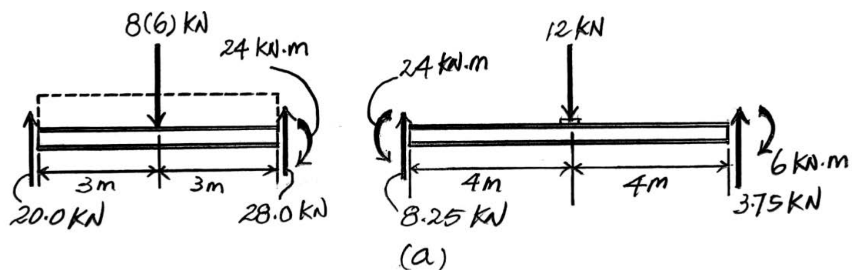
$$(\text{DF})_{BC} = \frac{EI/2}{EI/2 + EI/2} = 0.5 \qquad (\text{DF})_{CB} = 0$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = \frac{wL^2}{8} = \frac{8(6^2)}{8} = 36 \text{ kN} \cdot \text{m}$$

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{12(8)}{8} = -12 \text{ kN} \cdot \text{m}$$

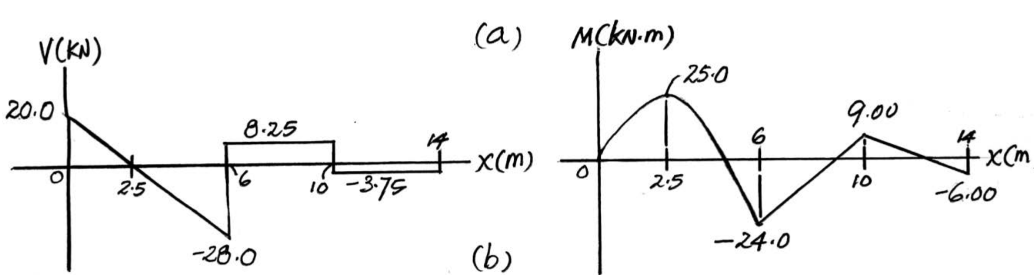
$$(\text{FEM})_{CB} = \frac{PL}{8} = \frac{12(8)}{8} = 12 \text{ kN} \cdot \text{m}$$



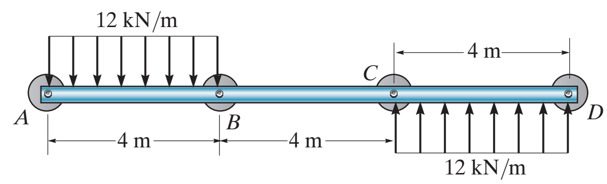
Moment Distribution. Tabulating the above data,

Joint	A	B		C
Member	AB	BA	BC	CB
DF	1	0.5	0.5	0
FEM	0	36	–12	12
Dist.		–12	–12	
				–6
$\sum M$	0	24	–24	6

Using these results, the shear and both ends of members AB and BC are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b.



12–6. Determine the moments at B and C, then draw the moment diagram for the beam. All connections are pins. Assume the horizontal reactions are zero. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{3EI}{L_{AB}} = \frac{3EI}{4} \qquad K_{BC} = \frac{6EI}{L_{BC}} = \frac{6EI}{4} = \frac{3EI}{2}$$

$$(\text{DF})_{AB} = 1 \quad (\text{DF})_{BA} = \frac{3EI/4}{3EI/4 + 3EI/2} = \frac{1}{3} \quad (\text{DF})_{BC} = \frac{3EI/2}{3EI/4 + 3EI/2} = \frac{2}{3}$$

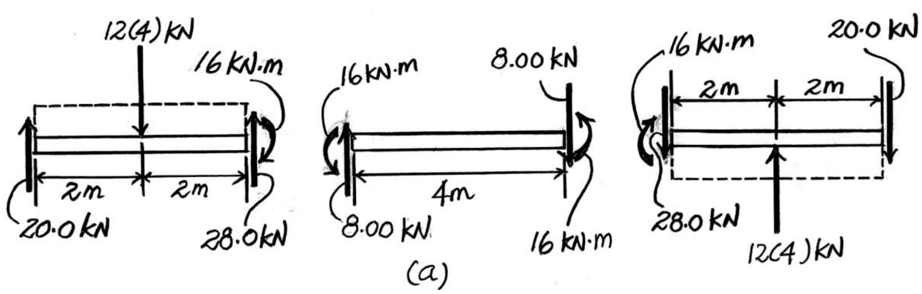
Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = \frac{wL^2}{8} = \frac{12(4^2)}{8} = 24 \text{ kN}\cdot\text{m} \qquad (\text{FEM})_{BC} = 0$$

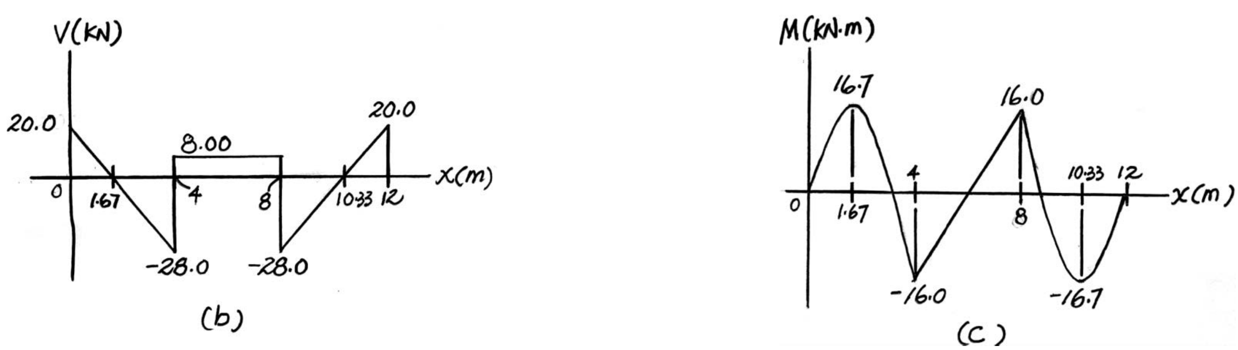
Moment Distribution. Tabulating the above data,

Joint	A	B	
Member	AB	BA	BC
DF	1	1/3	2/3
FEM	0	24	0
Dist.		–8	–16
$\sum M$	0	16	–16

Using these results, the shear at both ends of members AB, BC, and CD are computed and shown in Fig. a. Subsequently the shear and moment diagram can be plotted, Fig. b and c, respectively.

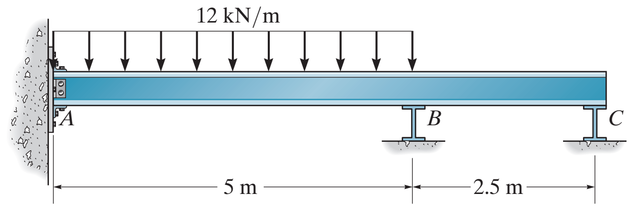


(a)



(b) (c)

12–7. Determine the reactions at the supports. Assume A is fixed and B and C are rollers that can either push or pull on the beam. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{4EI}{L_{AB}} = \frac{4EI}{5} = 0.8EI \qquad K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{2.5} = 1.2EI$$

$$(\text{DF})_{AB} = 0 \qquad (\text{DF})_{BA} = \frac{0.8EI}{0.8EI + 1.2EI} = 0.4$$

$$(\text{DF})_{BC} = \frac{1.2.EI}{0.8EI + 1.2EI} = 0.6$$

$$(\text{DF})_{CB} = 1$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{12(5^2)}{12} = -25 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{12(5^2)}{12} = 25 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

Moment Distribution. Tabulating the above data,

Joint	A	B		C
Member	AB	BA	BC	CB
DF	0	0.4	0.6	1
FEM	–25	25	0	0
Dist.		–10	–15	
CO	–5			
$\sum M$	–30	15	–15	

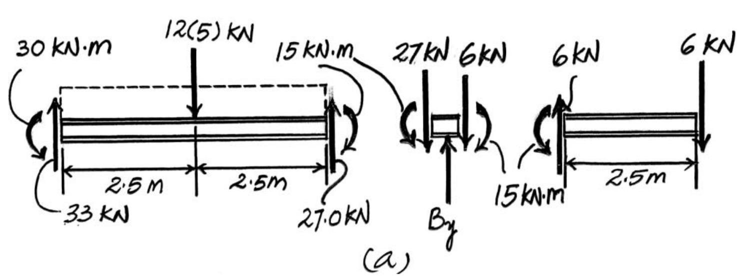
Ans.

Using these results, the shear at both ends of members AB and BC are computed and shown in Fig. *a*.

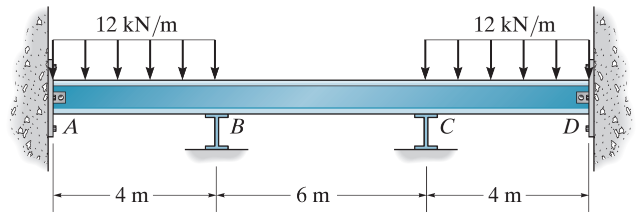
From this figure,

$A_x = 0 \quad A_y = 33 \text{ kN} \uparrow \quad B_y = 27 + 6 = 33 \text{ kN} \uparrow$ **Ans.**

$M_A = 30 \text{ kN}\cdot\text{m}$ ↺ $\quad C_y = 6 \text{ kN} \downarrow$ **Ans.**



***12–8.** Determine the moments at B and C, then draw the moment diagram for the beam. Assume the supports at B and C are rollers and A and D are pins. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{3EI}{L_{AB}} = \frac{3EI}{4} \qquad K_{BC} = \frac{2EI}{L_{BC}} = \frac{2EI}{6} = \frac{EI}{3}$$

$$(\text{DF})_{AB} = 1 \quad (\text{DF})_{BA} = \frac{3EI/4}{3EI/4 + 3EI/3} = \frac{9}{13} \quad (\text{DF})_{BC} = \frac{EI/3}{3EI/4 + EI/3} = \frac{4}{13}$$

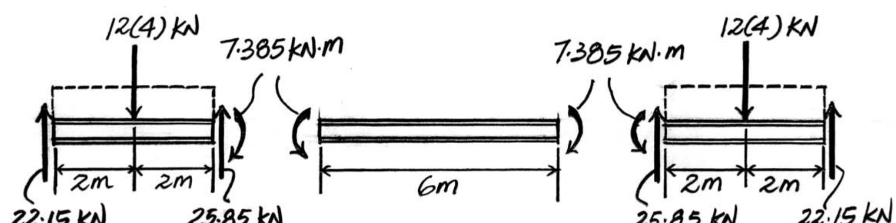
Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = (\text{FEM})_{BC} = 0 \qquad (\text{FEM})_{BA} = \frac{wL^2}{8} = \frac{12(4^2)}{8} = 24 \text{ kN}\cdot\text{m}$$

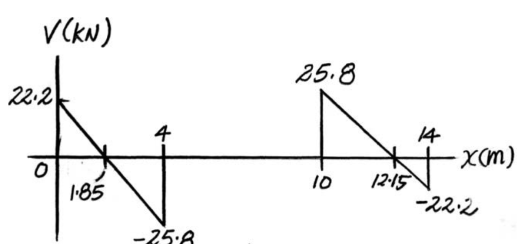
Moment Distribution. Tabulating the above data,

Joint	A	B	
Member	AB	BA	BC
DF	1	$\frac{9}{13}$	$\frac{4}{13}$
FEM	0	24	0
Dist.		–16.62	–7.385
$\sum M$	0	7.385	–7.385

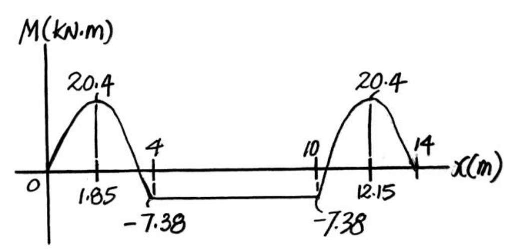
Using these results, the shear at both ends of members AB, BC, and CD are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.



(a)

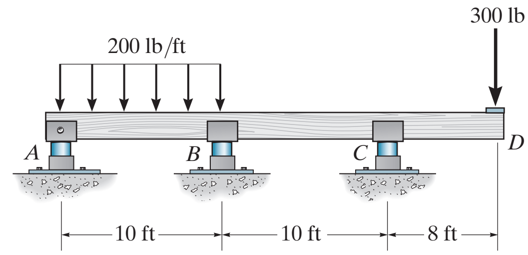


(b)



(c)

12–9. Determine the moments at B and C, then draw the moment diagram for the beam. Assume the supports at B and C are rollers and A is a pin. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{3EI}{L_{AB}} = \frac{3EI}{10} = 0.3EI \qquad K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{10} = 0.4EI.$$

$$(\text{DF})_{BA} = \frac{0.3EI}{0.3EI + 0.4EI} = \frac{3}{7} \qquad (\text{DF})_{BC} = \frac{0.4EI}{0.3EI + 0.4EI} = \frac{4}{7}$$

$$(\text{DF})_{CB} = 1 \qquad (\text{DF})_{CD} = 0$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{CD} = -300(8) = 2400 \text{ lb} \cdot \text{ft} \qquad (\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

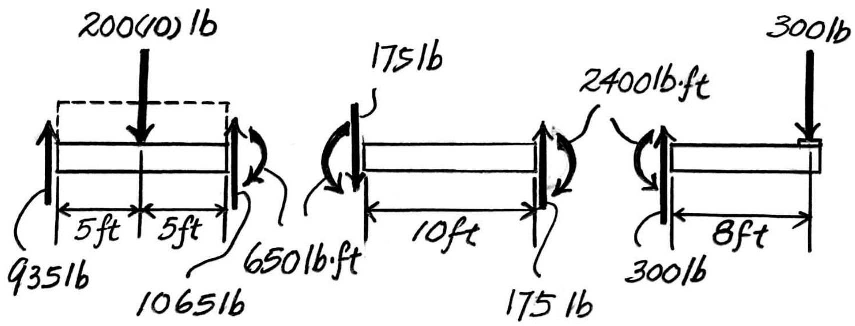
$$(\text{FEM})_{BA} = \frac{wL_{AB}^2}{8} = \frac{200(10^2)}{8} = 2500 \text{ lb} \cdot \text{ft}$$

Moment Distribution. Tabulating the above data,

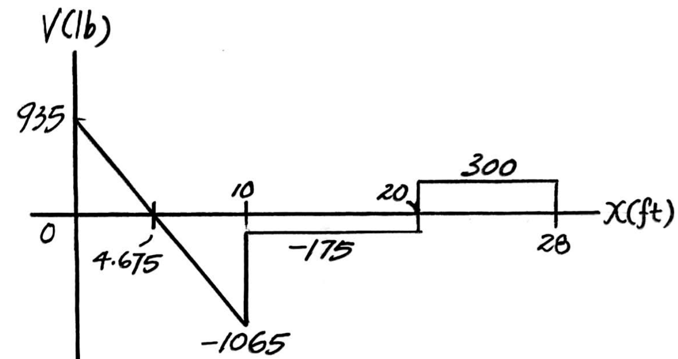
Joint	A	B		C	
Member	AB	BA	BC	CB	CD
DF	1	3/7	4/7	1	0
FEM	0	2500	0	0	–2400
Dist.		–1071.43	–1428.57	2400	
CO			1200	–714.29	
Dist.		–514.29	–685.71	714.29	
CO			357.15	–342.86	
Dist.		–153.06	–204.09	342.86	
CO			171.43	–102.05	
Dist.		–73.47	–97.96	102.05	
CO			51.03	–48.98	
Dist.		–21.87	–29.16	48.98	
CO			24.99	–14.58	
Dist.		–10.50	–13.99	14.58	
CO			7.29	–7.00	
Dist.		–3.12	–4.17	7.00	
CO			3.50	–2.08	
Dist.		–1.50	–2.00	2.08	
CO			1.04	–1.00	
Dist.		–0.45	–0.59	1.00	
CO			0.500	–0.30	
Dist.		–0.21	–0.29	0.30	
CO			0.15	–0.15	
Dist.		–0.06	–0.09	0.15	
CO			0.07	–0.04	
Dist.		–0.03	–0.04	0.04	
$\sum M$	0	650.01	–650.01	2400	–2400

Using these results, the shear at both ends of members AB, BC, and CD are computed and shown in Fig. *a*. Subsequently, the shear and moment diagrams can be plotted, Fig. *b* and *c*, respectively.

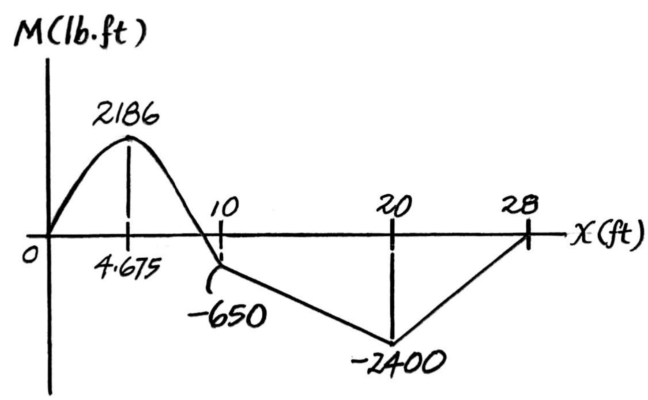
12–9. Continued



(a)

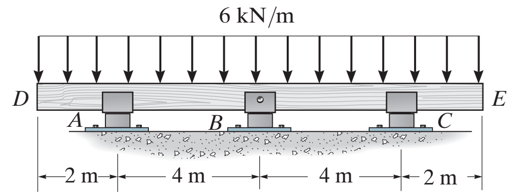


(b)



(c)

12–10. Determine the moment at B, then draw the moment diagram for the beam. Assume the supports at A and C are rollers and B is a pin. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{4EI}{L_{AB}} = \frac{4EI}{4} = EI \qquad K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{4} = EI$$

$$(\text{DF})_{AB} = 1 \quad (\text{DF})_{AD} = 0 \quad (\text{DF})_{BA} = (\text{DF})_{BC} = \frac{EI}{EI + EI} = 0.5$$

$$(\text{DF})_{CB} = 1 \qquad (\text{DF})_{CE} = 0$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = 6(2)(1) = 12\ \text{kN}\cdot\text{m} \qquad (\text{FEM})_{CE} = -6(2)(1) = -12\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{AB} = \frac{-wL_{AB}^2}{12} = -\frac{6(4^2)}{12} = -8\ \text{kN}\cdot\text{m}$$

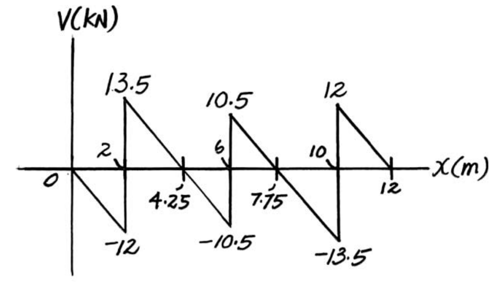
$$(\text{FEM})_{BA} = \frac{wL_{AB}^2}{12} = \frac{6(4^2)}{12} = 8\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = \frac{-wL_{BC}^2}{12} = -\frac{6(4^2)}{12} = -8\ \text{kN}\cdot\text{m}$$

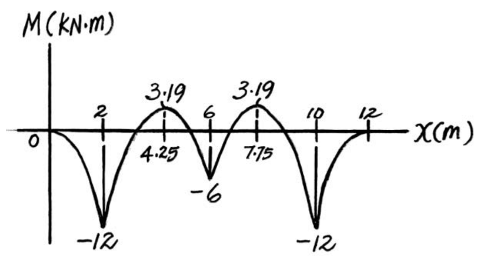
$$(\text{FEM})_{CB} = \frac{wL_{BC}^2}{12} = \frac{6(4^2)}{12} = 8\ \text{kN}\cdot\text{m}$$

Moment Distribution. Tabulating the above data,

Joint	A		B		C	
Member	AD	AB	BA	BC	CB	CE
DF	0	1	0.5	0.5	1	0
FEM	12	–8	8	–8	8	–12
Dist.		–4	0	0	4	
CO			–2	2		
$\sum M$	12	–12	6	–6	12	–12

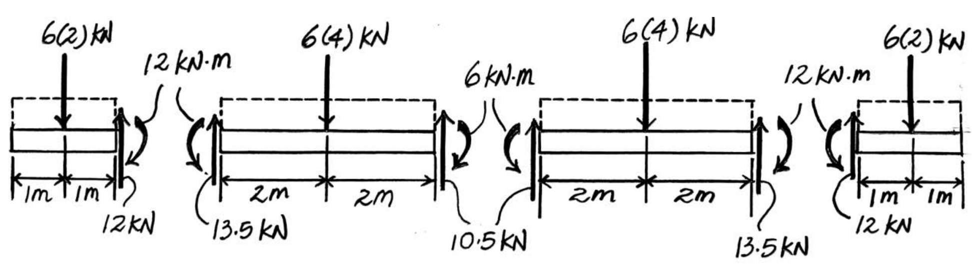


(b)



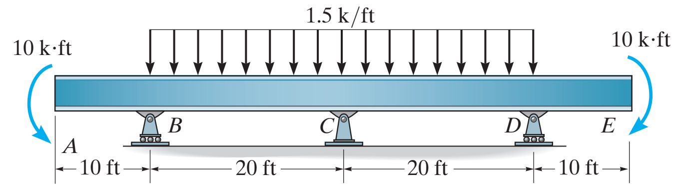
(c)

Using these results, the shear at both ends of members AD, AB, BC, and CE are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.



(a)

12–11. Determine the moments at B, C, and D, then draw the moment diagram for the beam. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{20} = 0.2\ EI \qquad K_{CD} = \frac{4EI}{L_{CD}} = \frac{4EI}{20} = 0.2\ EI$$

$$(\text{DF})_{BA} = (\text{DF})_{DE} = 0 \qquad (\text{DF})_{BC} = (\text{DF})_{DC} = 1$$

$$(\text{DF})_{CB} = (\text{DF})_{CD} = \frac{0.2EI}{0.2EI + 0.2EI} = 0.5$$

Fixed End Moments. Referring to the table on the inside back cover,

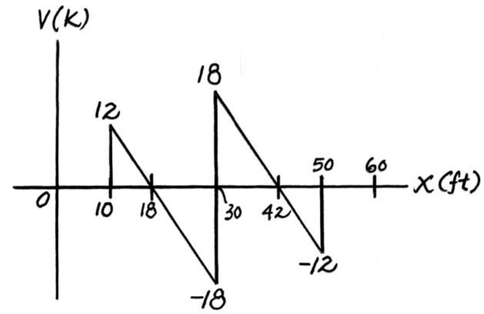
$$(\text{FEM})_{BA} = 10\ \text{k}\cdot\text{ft} \qquad (\text{FEM})_{DE} = -10\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CD} = -\frac{wL^2}{12} = -\frac{1.5(20^2)}{12} = -50\ \text{k}\cdot\text{ft}$$

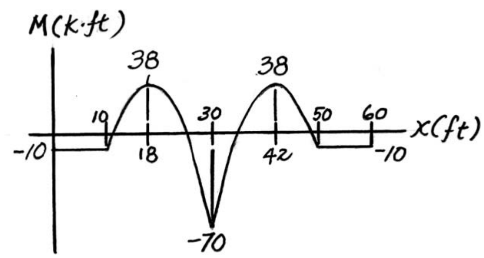
$$(\text{FEM})_{CB} = (\text{FEM})_{DC} = \frac{wL^2}{12} = -\frac{1.5(20^2)}{12} = 50\ \text{k}\cdot\text{ft}$$

Moment Distribution. Tabulating the above data,

Joint	B		C		D	
Member	BA	BC	CB	CD	DC	DE
DF	0	1	0.5	0.5	1	0
FEM	10	–50	50	–50	50	–10
Dist.		40	0	0	–40	
CO			20	–20		
$\sum M$	10	–10	70	–70	10	–10

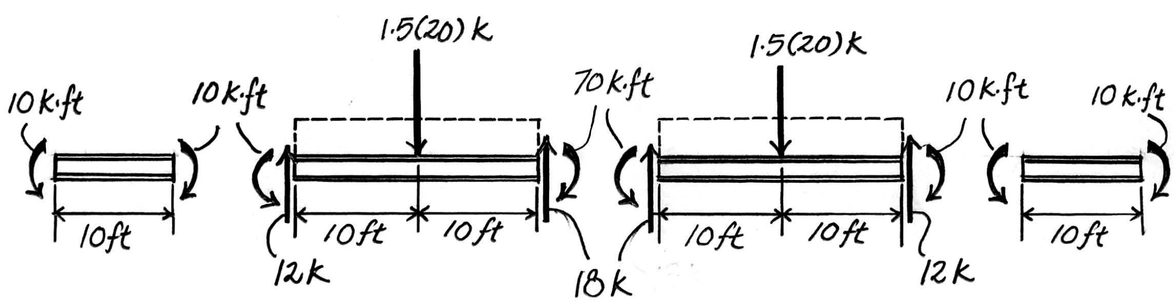


(b)



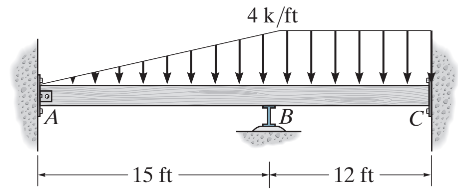
(c)

Using these results, the shear at both ends of members AB, BC, CD, and DE are computed and shown in Fig. a . Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.



(a)

***12–12.** Determine the moment at *B*, then draw the moment diagram for the beam. Assume the support at *A* is pinned, *B* is a roller and *C* is fixed. *EI* is constant.



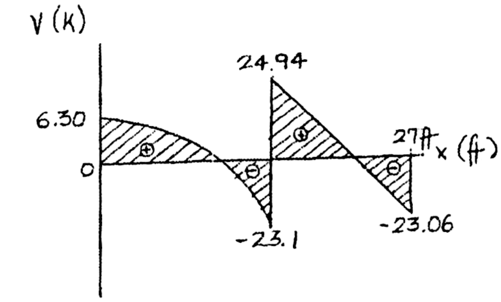
$$\text{FEM}_{AB} = \frac{wL^2}{30} = \frac{4(15^2)}{30} = 30\ \text{k}\cdot\text{ft}$$

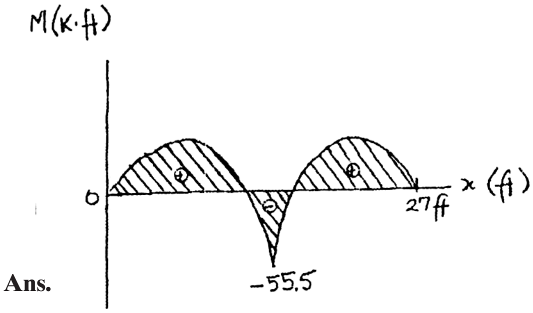
$$\text{FEM}_{BA} = \frac{wL^2}{20} = \frac{4(15^2)}{20} = 45\ \text{k}\cdot\text{ft}$$

$$\text{FEM}_{BC} = \frac{wL^2}{12} = \frac{(4)(12^2)}{12} = 48\ \text{k}\cdot\text{ft}$$

$$\text{FEM}_{CB} = 48\ \text{k}\cdot\text{ft}$$

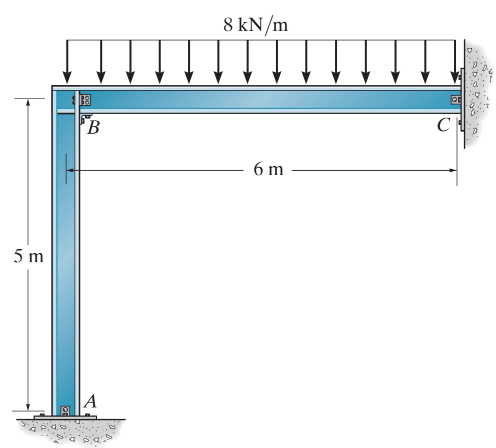
Joint	*A*	*B*		*C*
Member	*AB*	*BA*	*BC*	*CB*
DF	1	0.375	0.625	0
FEM	–30	45	–48	48
	30	1.125	1.875	
		15		0.9375
		–5.625	–9.375	
				–4.688
$\sum M$	0	55.5	–55.5	44.25





$M_B = -55.5\ \text{k}\cdot\text{ft}$ **Ans.**

12–13. Determine the moment at *B*, then draw the moment diagram for each member of the frame. Assume the supports at *A* and *C* are pins. *EI* is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{6} = 0.5\ EI$$

$$K_{BA} = \frac{3EI}{L_{AB}} = \frac{3EI}{5} = 0.6\ EI$$

$$(\text{DF})_{AB} = (\text{DF})_{CB} = 1 \qquad (\text{DF})_{BC} = \frac{0.5EI}{0.5EI + 0.6EI} = \frac{5}{11}$$

$$(\text{DF})_{BA} = \frac{0.6EI}{0.5EI + 0.6EI} = \frac{6}{11}$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{CB} = (\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

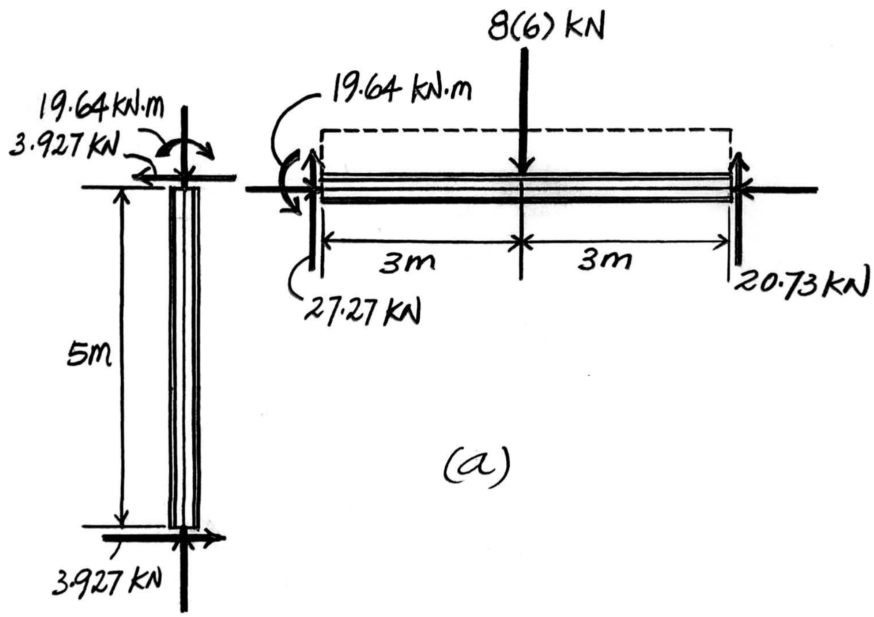
$$(\text{FEM})_{BC} = -\frac{wL_{BC}^2}{8} = -\frac{8(6^2)}{8} = -36\ \text{kN}\cdot\text{m}$$

12–13. Continued

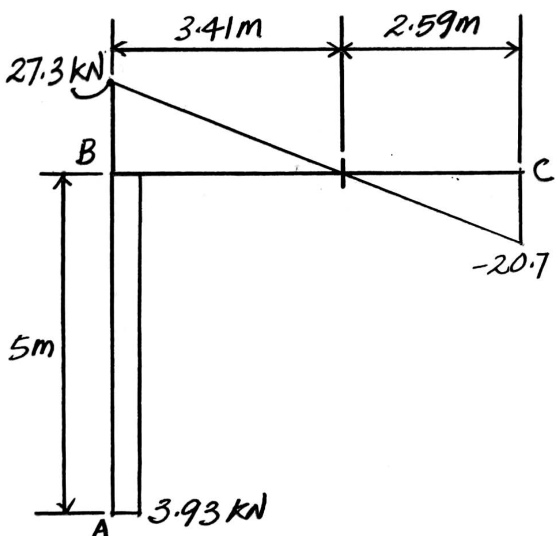
Moment Distribution. Tabulating the above data,

Joint	A	B		C
Member	AB	BA	BC	CB
DF	1	$\frac{6}{11}$	$\frac{5}{11}$	1
FEM	0	0	–36	0
Dist.		19.64	16.36	
$\sum M$	0	19.64	–19.64	0

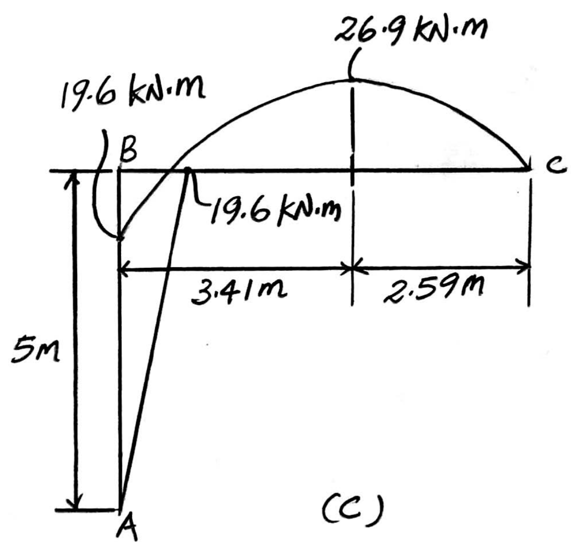
Using these results, the shear at both ends of member *AB* and *BC* are computed and shown in Fig. *a*. Subsequently, the shear and moment diagram can be plotted, Fig. *b* and *c*, respectively.



(a)

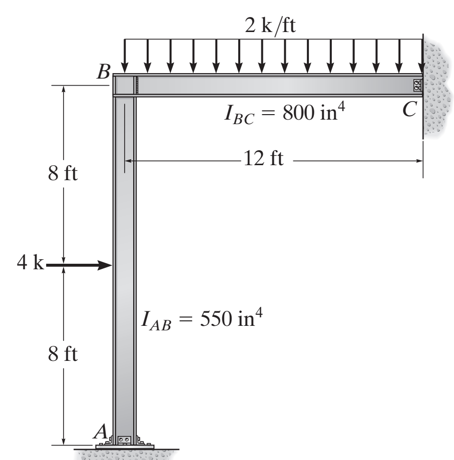


(b)



(c)

12–14. Determine the moments at the ends of each member of the frame. Assume the joint at B is fixed, C is pinned, and A is fixed. The moment of inertia of each member is listed in the figure. $E = 29(10^3)$ ksi.



$$(\text{DF})_{AB} = 0$$

$$(\text{DF})_{BA} = \frac{4(0.6875I_{BC})/16}{4(0.6875I_{BC})/16 + 3I_{BC}/12} = 0.4074$$

$$(\text{DF})_{BC} = 0.5926 \qquad (\text{DF})_{CB} = 1$$

$$(\text{FEM})_{AB} = \frac{-4(16)}{8} = -8\ \text{k}\cdot\text{ft}$$

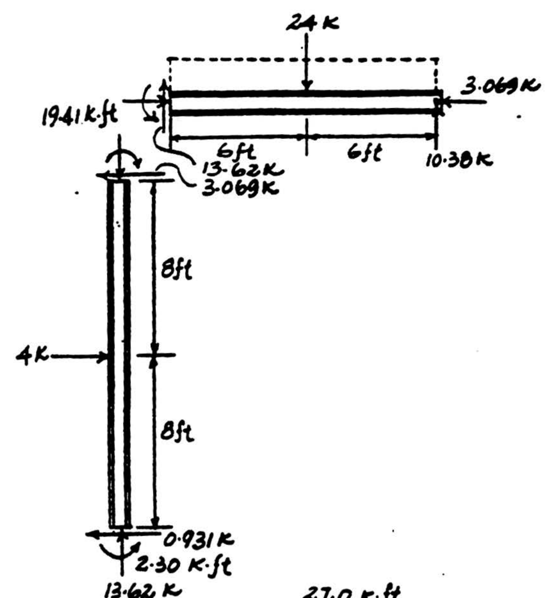
$$(\text{FEM})_{BA} = 8\ \text{k}\cdot\text{ft}$$

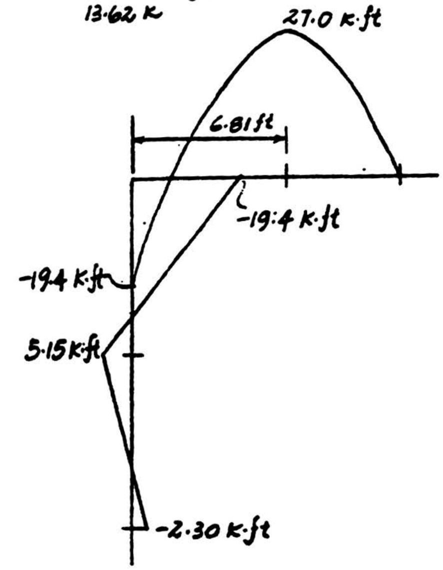
$$(\text{FEM})_{BC} = \frac{-2(12^2)}{12} = -24\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = 24\ \text{k}\cdot\text{ft}$$

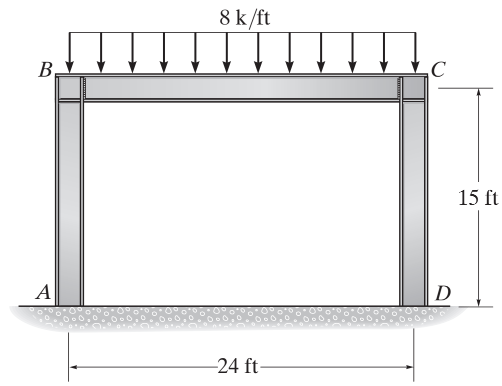
Joint	A	B		C
Mem.	AB	BA	BC	CB
DF	0	0.4047	0.5926	1
FEM	–8.0	8.0	–24.0	24.0
		6.518	9.482	–24.0
	3.259 ↙		–12.0	
		4.889	7.111	
	2.444 ↙			
$\sum M$	–2.30	19.4	–19.4	0

Ans.





12–15. Determine the reactions at A and D. Assume the supports at A and D are fixed and B and C are fixed connected. EI is constant.



$$(\text{DF})_{AB} = (\text{DF})_{DC} = 0$$

$$(\text{DF})_{BA} = (\text{DF})_{CD} = \frac{I/15}{I/15 + I/24} = 0.6154$$

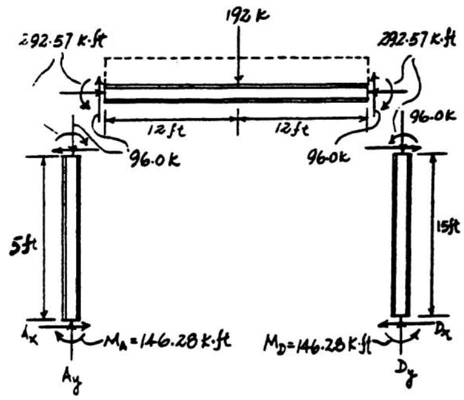
$$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.3846$$

$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

$$(\text{FEM})_{BC} = \frac{-8(24)^2}{12} = -384 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = 384 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$



Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.6154	0.3846	0.3846	0.6154	0
FEM			−384	384		
		236.31	147.69	−147.69	−236.31	
	118.16		−73.84	73.84		−118.16
		45.44	28.40	−28.40	−45.44	
	22.72		−14.20	14.20		−22.72
		8.74	5.46	−5.46	−8.74	
	4.37		−2.73	2.73		−4.37
		1.68	1.05	−1.05	−1.68	
	0.84		−0.53	0.53		−0.84
		0.32	0.20	−0.20	−0.33	
	0.16		−0.10	0.10		−0.17
		0.06	0.04	−0.04	−0.06	
	0.03		−0.02	0.02		−0.03
		0.01	0.01	−0.01	−0.01	
$\sum M$	146.28	292.57	−292.57	292.57	−292.57	−146.28

Thus from the free-body diagrams:

$A_x = 29.3$ k **Ans.**

$A_y = 96.0$ k **Ans.**

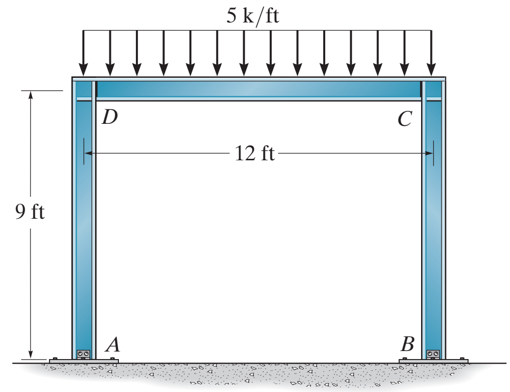
$M_A = 146$ k · ft **Ans.**

$D_x = 29.3$ k **Ans.**

$D_y = 96.0$ k **Ans.**

$M_D = 146$ k · ft **Ans.**

***12–16.** Determine the moments at D and C, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins and D and C are fixed joints. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AD} = K_{BC} = \frac{3EI}{L} = \frac{3EI}{9} = \frac{EI}{3} \qquad K_{CD} = \frac{4EI}{L} = \frac{4EI}{12} = \frac{EI}{3}$$

$$(\text{DF})_{AD} = (\text{DF})_{BC} = 1 \qquad (\text{DF})_{DA} = (\text{DF})_{DC} = (\text{DF})_{CD} = \text{DF}_{CB} = \frac{EI/3}{EI/3 + EI/3} = \frac{1}{2}$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = (\text{FEM})_{DA} = (\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

$$(\text{FEM})_{DC} = -\frac{wL_{CD}^2}{12} = -\frac{5(12^2)}{12} = -60 \text{ k}\cdot\text{ft}$$

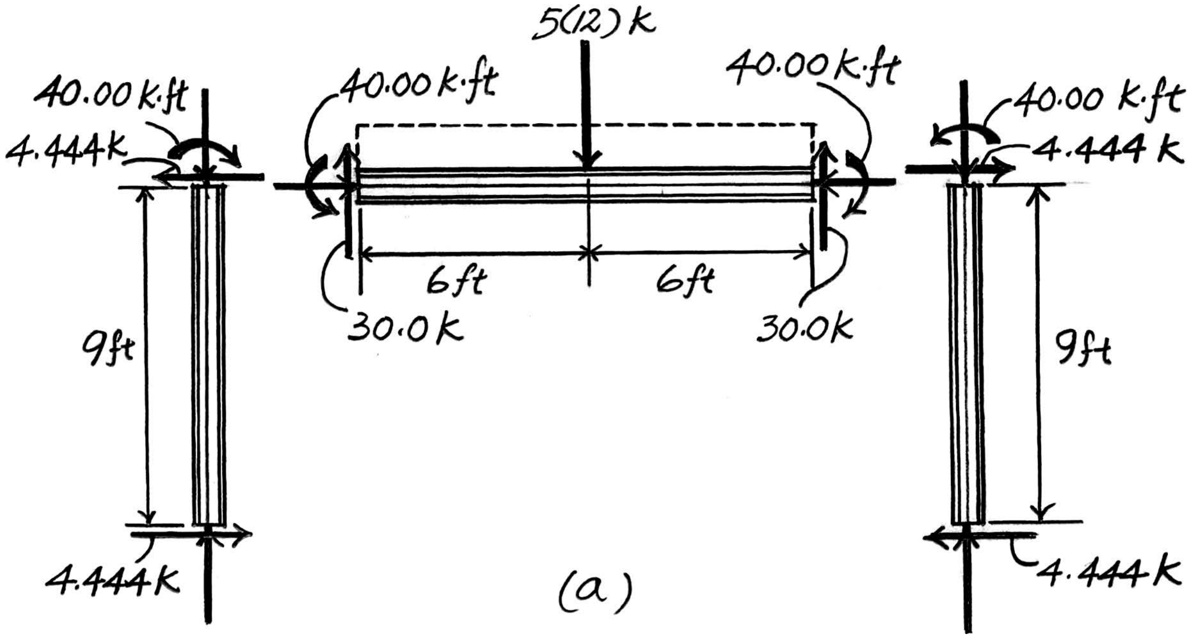
$$(\text{FEM})_{CD} = \frac{wL_{CD}^2}{12} = \frac{5(12^2)}{12} = 60 \text{ k}\cdot\text{ft}$$

Moments Distribution. Tabulating the above data,

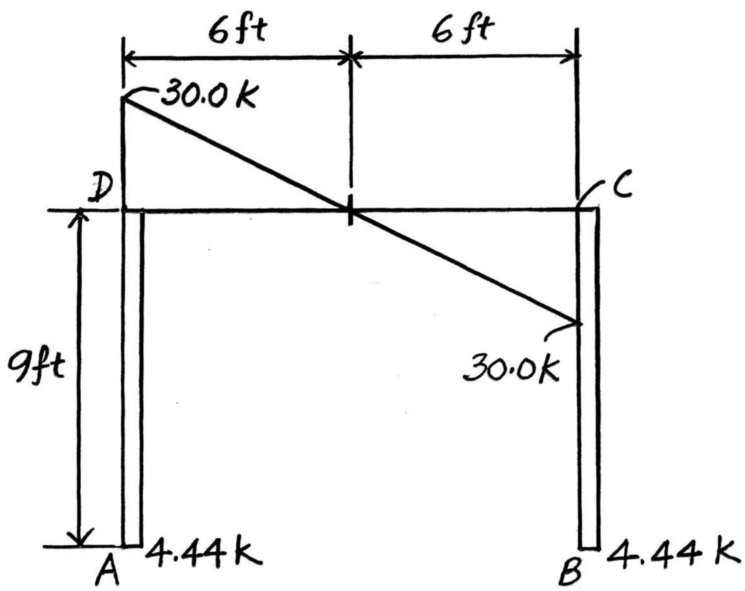
Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	0.5	0.5	0.5	0.5	1
FEM	0	0	−60	60	0	0
Dist.		30	30	−30	−30	
CO			−15	15		
Dist.		7.50	7.50	−7.50	−7.50	
C0			−3.75	3.75		
Dist.		1.875	1.875	−1.875	−1.875	
C0			−0.9375	0.9375		
Dist.		0.4688	0.4688	−0.4688	−0.4688	
C0			−0.2344	0.2344		
Dist.		0.1172	0.1172	−0.1172	−0.1172	
C0			−0.0586	0.0586		
Dist.		0.0293	0.0293	−0.0293	−0.0293	
C0			−0.0146	0.0146		
Dist.		0.0073	0.0073	−0.0073	−0.0073	
$\sum M$	0	40.00	−40.00	40.00	−40.00	

Using these results, the shear at both ends of members AD, CD, and BC are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted.

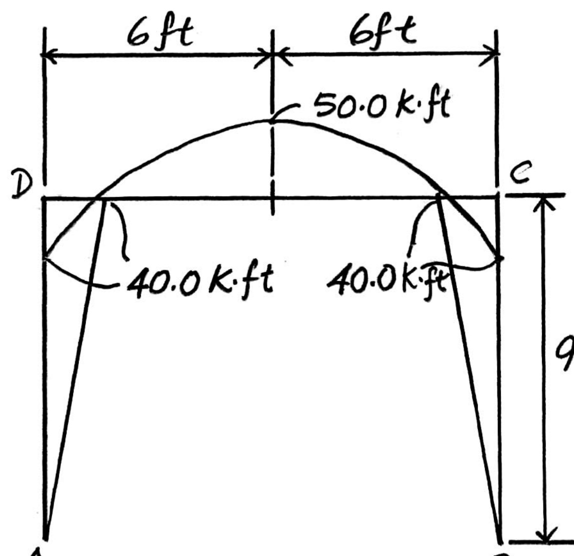
12–16. Continued



(a)

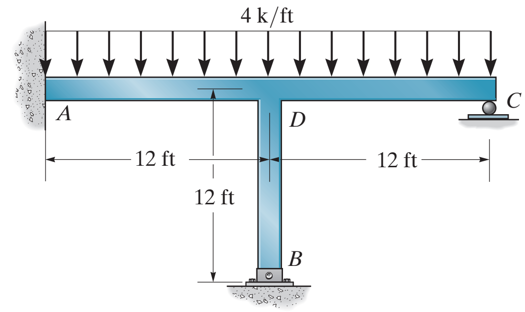


(b)



(c)

12–17. Determine the moments at the fixed support A and joint D and then draw the moment diagram for the frame. Assume B is pinned.



Member Stiffness Factor and Distribution Factor.

$$K_{AD} = \frac{4EI}{L_{AD}} = \frac{4EI}{12} = \frac{EI}{3} \qquad K_{DC} = K_{DB} = \frac{3EI}{L} = \frac{3EI}{12} = \frac{EI}{4}$$

$$(\text{DF})_{AD} = O \qquad (\text{DF})_{DA} = \frac{EI/3}{EI/3 + EI/4 + EI/4} = 0.4$$

$$(\text{DF})_{DC} = (\text{DF})_{DB} = \frac{EI/4}{EI/3 + EI/4 + EI/4} = 0.3$$

$$(\text{DF})_{CD} = (\text{DF})_{BD} = 1$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = -\frac{wL_{AD}^2}{12} = -\frac{4(12^2)}{12} = -48 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{DA} = \frac{wL_{AD}^2}{12} = \frac{4(12^2)}{12} = 48 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{DC} = -\frac{wL_{CD}^2}{8} = -\frac{4(12^2)}{8} = -72 \text{ k}\cdot\text{ft}$$

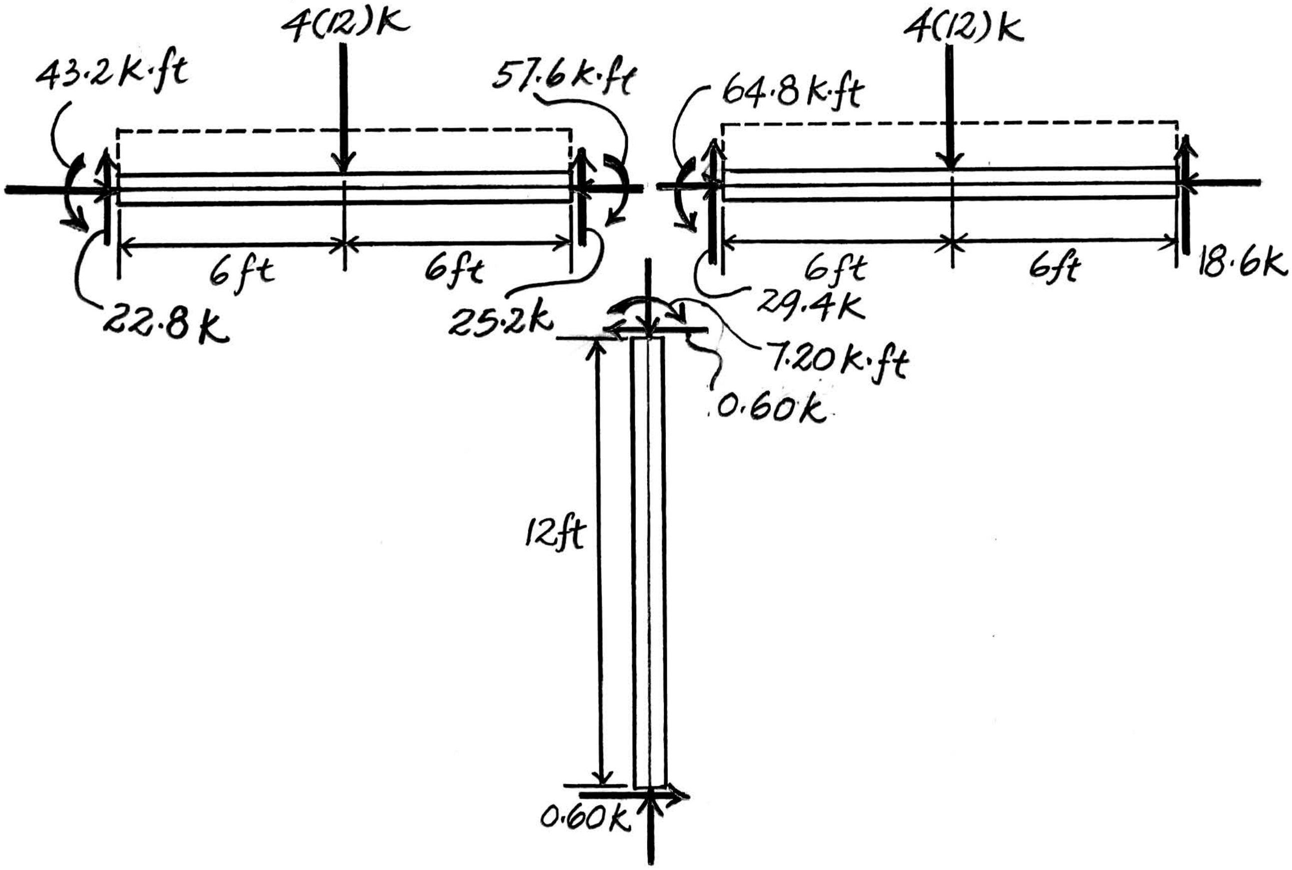
$$(\text{FEM})_{CD} = (\text{FEM})_{BD} = (\text{FEM})_{DB} = 0$$

Moments Distribution. Tabulating the above data,

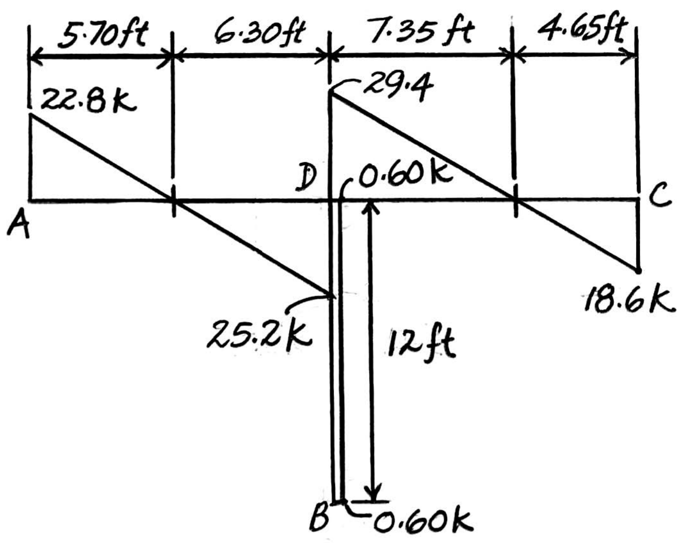
Joint	A	D			C	B
Member	AD	DA	DB	DC	CD	BD
DF	0	0.4	0.3	0.3	1	1
FEM	−48	48	0	−72	0	0
Dist.		9.60	7.20	7.20		
CO	4.80					
$\sum M$	−43.2	57.6	7.20	−64.8	0	0

Using these results, the shears at both ends of members AD, CD, and BD are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.

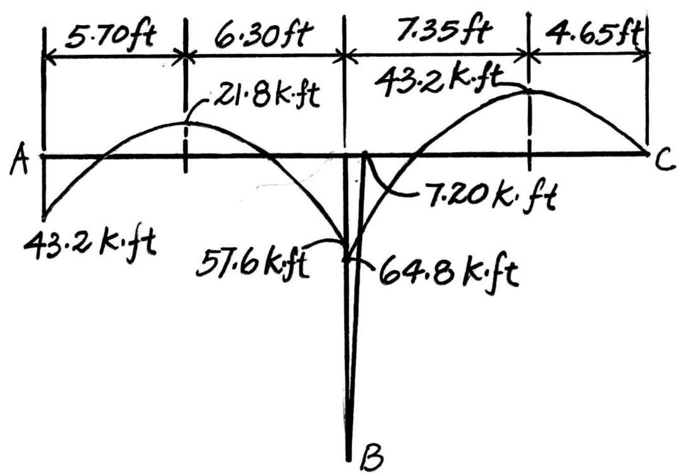
12–17. Continued



(a)

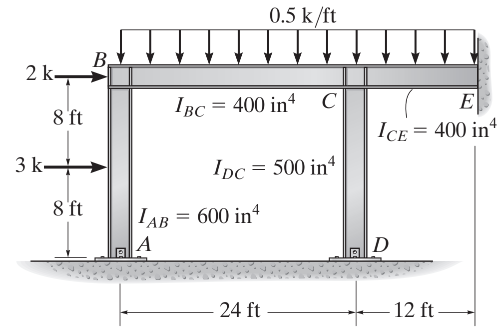


(b)



(c)

12–18. Determine the moments at each joint of the frame, then draw the moment diagram for member *BCE*. Assume *B*, *C*, and *E* are fixed connected and *A* and *D* are pins. $E = 29(10^3)$ ksi.



$(DF)_{AB} = (DF)_{DC} = 1 \quad (DF)_{DC} = 0$

$$(DF)_{BA} = \frac{3(A1.5I_{BC})/16}{3(1.5I_{BC})/16 + 4I_{BC}/24} = 0.6279$$

$(DF)_{BC} = 0.3721$

$$(DF)_{CB} = \frac{4I_{BC}/24}{4I_{BC}/24 + 3(1.25I_{BC})/16 + 4I_{BC}/12} = 0.2270$$

$(DF)_{CD} = 0.3191$

$(DF)_{CE} = 0.4539$

$$(FEM)_{AB} = \frac{-3(16)}{8} = -6\text{ k}\cdot\text{ft}$$

$(FEM)_{BA} = 6\text{ k}\cdot\text{ft}$

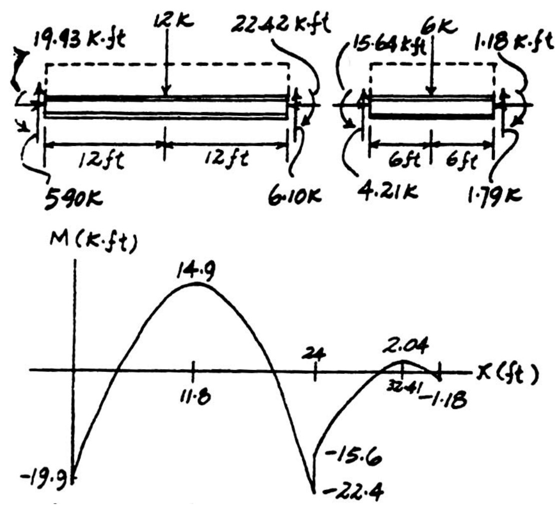
$$(FEM)_{BC} = \frac{-(0.5)(24)^2}{12} = -24\text{ k}\cdot\text{ft}$$

$(FEM)_{CB} = 24\text{ k}\cdot\text{ft}$

$$(FEM)_{CE} = \frac{-(0.5)(12)^2}{12} = -6\text{ k}\cdot\text{ft}$$

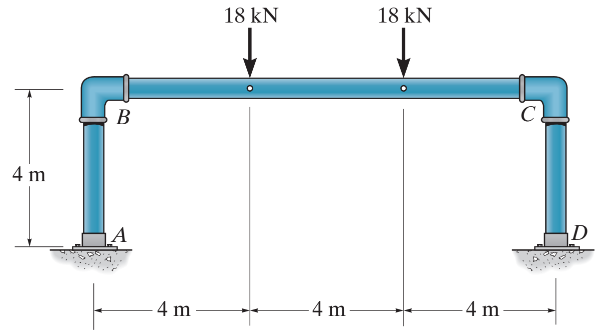
$(FEM)_{EC} = 6\text{ k}\cdot\text{ft}$

$(FEM)_{CD} = (FEM)_{DC} = 0$



Joint	*A*	*B*		*C*		*E*		*D*
Mem.	*AB*	*BA*	*BC*	*CB*	*CD*	*CE*	*EC*	*DC*
DF	1	0.6279	0.3721	0.2270	0.3191	0.4539	0	1
FEM	−6.0	6.0	−24.0	24.0		−6.0	6.0	
	6.0	11.30	6.70	−4.09	−5.74	−8.17		
		3.0	−2.04	3.35			−4.09	
		−0.60	−0.36	−0.76	−1.07	−1.52		
			−0.38	−0.18			−0.76	
		0.24	0.14	0.04	0.06	0.08		
			0.02	0.07			0.04	
		−0.01	−0.01	−0.02	−0.02	−0.03		
							−0.02	
$\sum M$	0	19.9	−19.9	22.4	−6.77	−15.6	1.18	0

12–19. The frame is made from pipe that is fixed connected. If it supports the loading shown, determine the moments developed at each of the joints. *EI* is constant.



$$\text{FEM}_{BC} = -\frac{2PL}{9} = -48, \quad \text{FEM}_{CB} = \frac{2PL}{9} = 48$$

$$K_{AB} = K_{CD} = \frac{4EI}{4}, \quad K_{BC} = \frac{4EI}{12}$$

$$\text{DF}_{AB} = \text{DF}_{DC} = 0$$

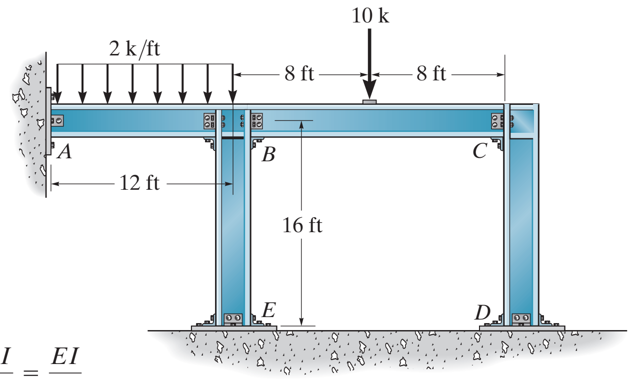
$$\text{DF}_{BA} = \text{DF}_{CD} = \frac{\frac{4EI}{5}}{\frac{4EI}{4} + \frac{4EI}{12}} = 0.75$$

$$\text{DF}_{BC} = \text{DF}_{CB} = 1 - 0.75 = 0.25$$

Joint	*A*	*B*		*C*		*D*
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	0	0.75	0.25	0.25	0.75	0
FEM			–48	48		
		36	12	–12	–36	
	18		–6	6		–18
		4.5	1.5	–1.5	–4.5	
	2.25		–0.75	0.75		–2.25
		0.5625	0.1875	–0.1875	–0.5625	
	0.281		–0.0938	0.0938		–0.281
		0.0704	0.0234	–0.0234	–0.0704	
	20.6	41.1	–41.1	41.1	–41.1	–20.6

Ans.

***12–20.** Determine the moments at B and C, then draw the moment diagram for each member of the frame. Assume the supports at A, E, and D are fixed. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{4EI}{L_{AB}} = \frac{4EI}{12} = \frac{EI}{3} \qquad K_{BC} = K_{BE} = K_{CD} = \frac{4EI}{L} = \frac{4EI}{16} = \frac{EI}{4}$$

$$(\text{DF})_{AB} = (\text{DF})_{EB} = (\text{DF})_{DC} = 0 \quad (\text{DF})_{BA} = \frac{EI/3}{EI/3 + EI/4 + EI/4} = 0.4$$

$$(\text{DF})_{BC} = (\text{DF})_{BE} = \frac{EI/4}{EI/3 + EI/4 + EI/4} = 0.3$$

$$(\text{DF})_{CB} = (\text{DF})_{CD} = \frac{EI/4}{EI/4 + EI/4} = 0.5$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{wL_{AB}^2}{12} = -\frac{2(12^2)}{12} = -24 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL_{AB}^2}{12} = \frac{2(12^2)}{12} = 24 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{PL_{BC}}{8} = -\frac{10(16)}{8} = -20 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{PL_{BC}}{8} = \frac{10(16)}{8} = 20 \text{ k}\cdot\text{ft}$$

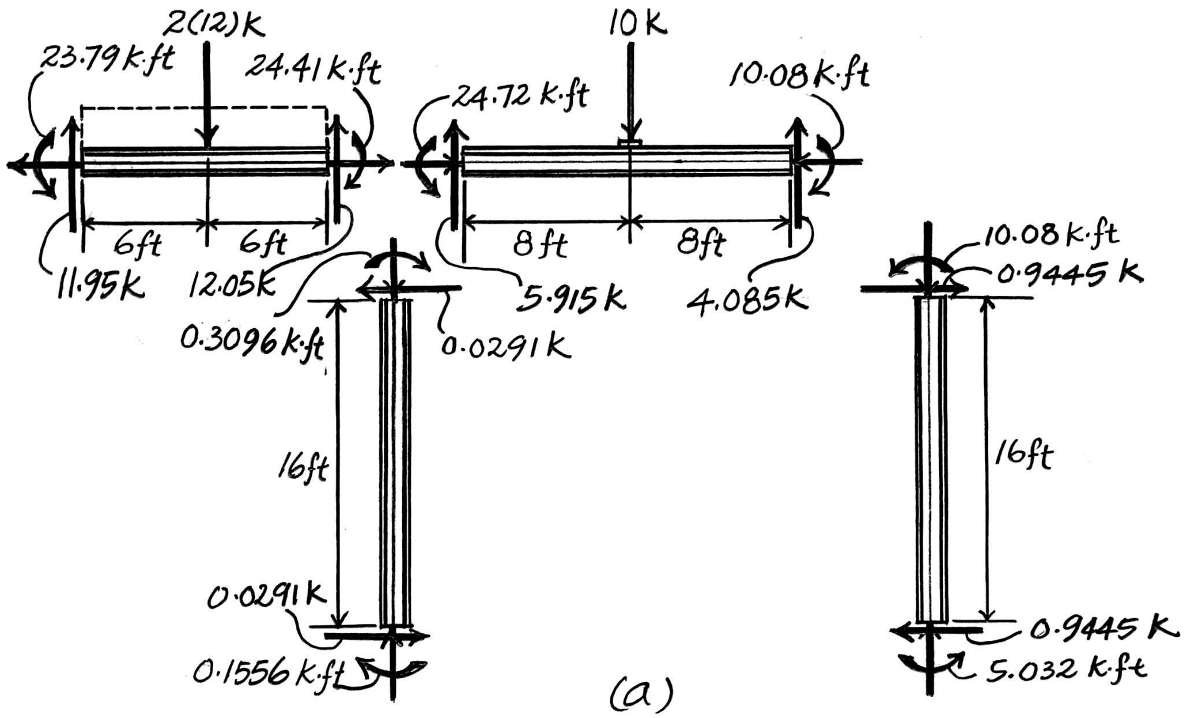
$$(\text{FEM})_{BE} = (\text{FEM})_{EB} = (\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$

Moment Distribution. Tabulating the above data,

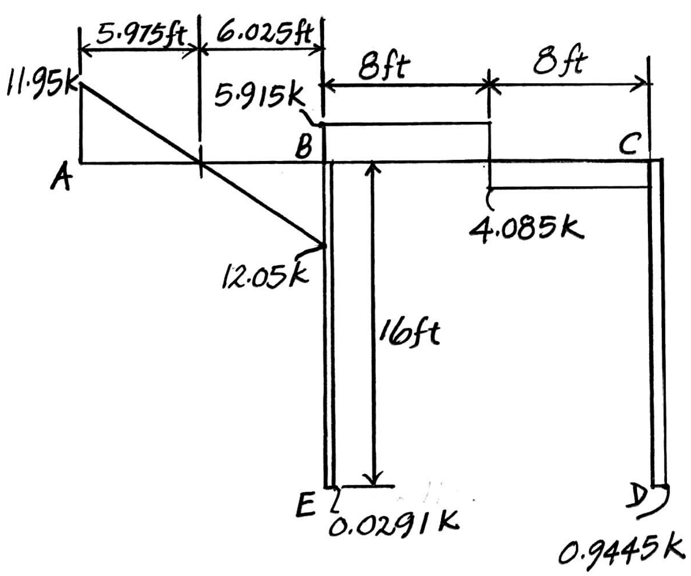
Joint	A	B			C		D	E
Member	AB	BA	BE	BC	CB	CD	DC	EB
DF	0	0.4	0.3	0.3	0.5	0.5	0	0
FEM	–24	24	0	–20	20	0	0	0
Dist.		–1.60	–1.20	–1.20	–10	–10		
CO	–0.80			–5	–0.60		–5	–0.6
Dist.		2.00	1.50	1.50	0.30	0.30		
CO	1.00			0.15	0.75		0.15	0.75
Dist.		–0.06	–0.045	–0.045	–0.375	–0.375		
CO	–0.03			–0.1875	–0.0225		–0.1875	–0.0225
Dist.		0.075	0.05625	0.05625	0.01125	0.01125		
CO	0.0375			0.005625	0.028125		0.005625	0.028125
Dist.		–0.00225	–0.0016875	–0.0016875	–0.01406	–0.01406		
$\sum M$	–23.79	24.41	0.3096	–24.72	10.08	–10.08	–5.031	0.1556

Using these results, the shear at both ends of members AB, BC, BE, and CD are computed and shown in Fig. *a*. Subsequently, the shear and moment diagram can be plotted.

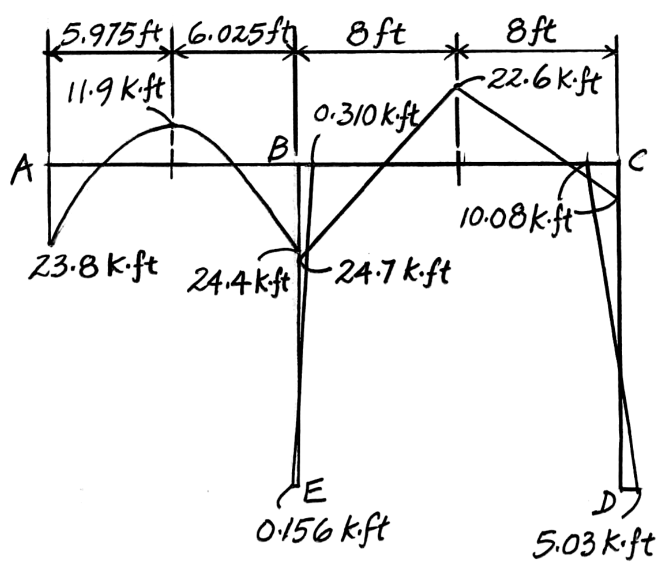
12–20. Continued



(a)

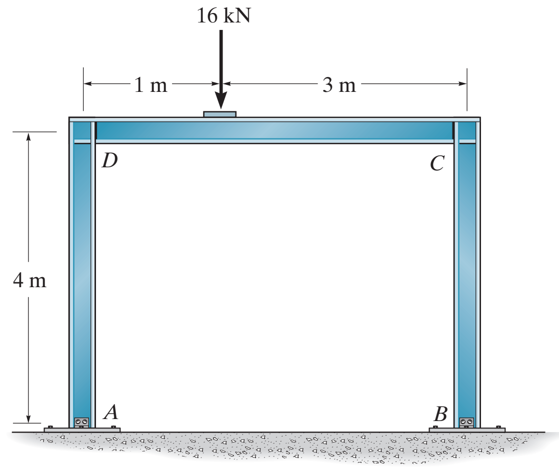


(b)



(c)

12–21. Determine the moments at D and C, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Moment Distribution. No sidesway, Fig. b.

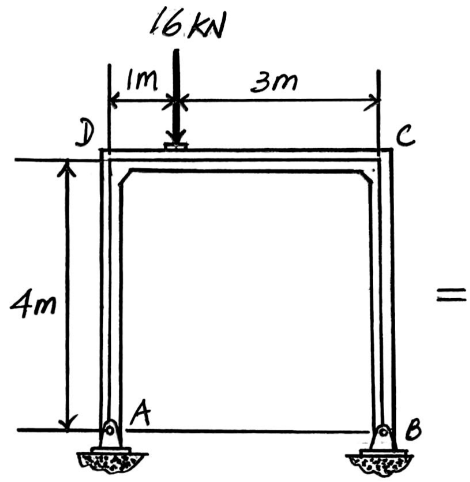
$$K_{DA} = K_{CB} = \frac{3EI}{L} = \frac{3EI}{4} \qquad K_{CD} = \frac{4EI}{L} = \frac{4EI}{4} = EI$$

$$(\text{DF})_{AD} = (\text{DF})_{BC} = 1 \quad (\text{DF})_{DA} = (\text{DF})_{CB} = \frac{3EI/4}{3EI/4 + EI} = \frac{3}{7}$$

$$(\text{DF})_{DC} = (\text{DF})_{CD} = \frac{EI}{3EI/4 + EI} = \frac{4}{7}$$

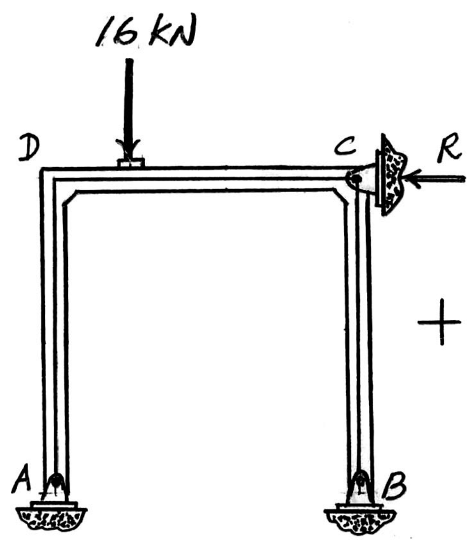
$$(\text{FEM})_{DC} = -\frac{Pb^2a}{L^2} = -\frac{16(3^2)(1)}{4^2} = -9 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CD} = -\frac{Pa^2b}{L^2} = -\frac{16(1^2)(3)}{4^2} = 3 \text{ kN}\cdot\text{m}$$



(a)

Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	1
FEM	0	0	–9	3	0	0
Dist.		3.857	5.143	–1.714	–1.286	
CO			–0.857	2.572		
Dist.		0.367	0.490	–1.470	–1.102	
CO			–0.735	0.245		
Dist.		0.315	0.420	–0.140	–0.105	
CO			–0.070	0.210		
Dist.		0.030	0.040	–0.120	–0.090	
CO			–0.060	0.020		
Dist.		0.026	0.034	–0.011	–0.009	
CO			–0.006	0.017		
Dist.		0.003	0.003	–0.010	–0.007	
$\sum M$	0	4.598	–4.598	2.599	–2.599	0



(b)

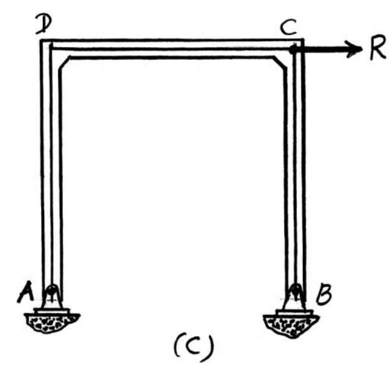
Using these results, the shears at A and B are computed and shown in Fig. d. Thus, for the entire frame

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 1.1495 - 0.6498 - R = 0 \quad R = 0.4997 \text{ kN}$

12–21. Continued

For the frame in Fig. *e*,

Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	1
FEM	0	–10	0	0	–10	0
Dist.		4.286	5.714	5.714	4.286	
CO			2.857	2.857		
Dist.		–1.224	–1.633	–1.633	–1.224	
CO			–0.817	–0.817		
Dist.		0.350	0.467	0.467	0.350	
CO			0.234	0.234		
Dist.		–0.100	–0.134	–0.134	–0.100	
CO			–0.067	–0.067		
Dist.		0.029	0.038	0.038	0.029	
CO			0.019	0.019		
Dist.		–0.008	–0.011	–0.011	–0.008	
$\sum M$	0	–6.667	6.667	6.667	–6.667	0



Using these results, the shears at A and B caused by the application of R' are computed and shown in Fig. *f*. For the entire frame,

$\xrightarrow{+} \Sigma F_x = 0; \quad R'1.667 - 1.667 = 0 \quad R' = 3.334 \text{ kN}$

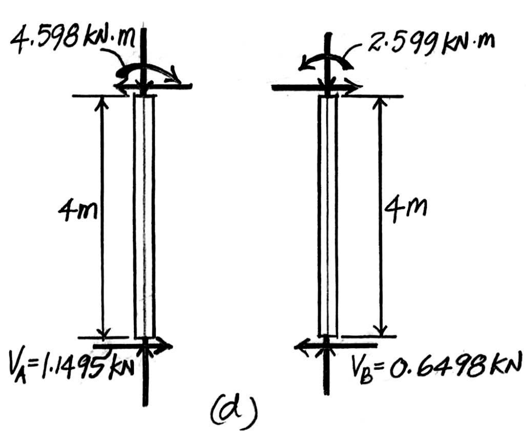
Thus,

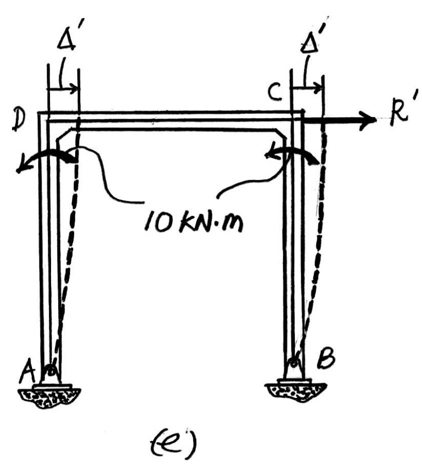
$$M_{DA} = 4.598 + (-6.667)\left(\frac{0.4997}{3.334}\right) = 3.60 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

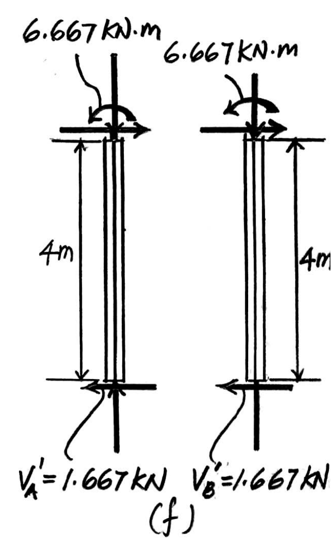
$$M_{DC} = -4.598 + (6.667)\left(\frac{0.4997}{3.334}\right) = -3.60 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

$$M_{CD} = 2.599 + (6.667)\left(\frac{0.4997}{3.334}\right) = -3.60 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

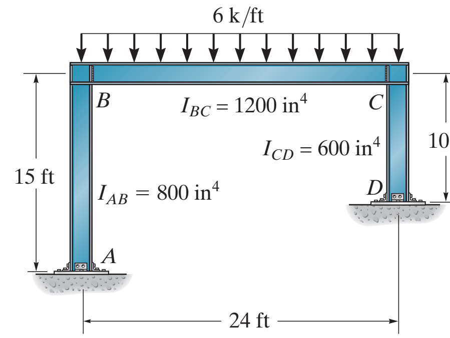
$$M_{CB} = 2.599 + (-6.667)\left(\frac{0.4997}{3.334}\right) = -3.60 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$







12–22. Determine the moments acting at the ends of each member. Assume the supports at A and D are fixed. The moment of inertia of each member is indicated in the figure. $E = 29(10^3)$ ksi.



Consider no sideway

$$(\text{DF})_{AB} = (\text{DF})_{DC} = 0$$

$$(\text{DF})_{BA} = \frac{(\frac{1}{12}I_{BC})/15}{(\frac{1}{12}I_{BC})/15 + I_{BC}/24} = 0.5161$$

$$(\text{DF})_{BC} = 0.4839$$

$$(\text{DF})_{CB} = \frac{I_{BC}/24}{0.5I_{BC}/10 + I_{BC}/24} = 0.4545$$

$$(\text{DF})_{CD} = 0.5455$$

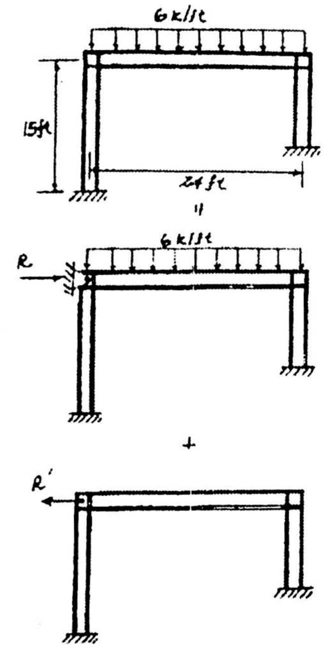
$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

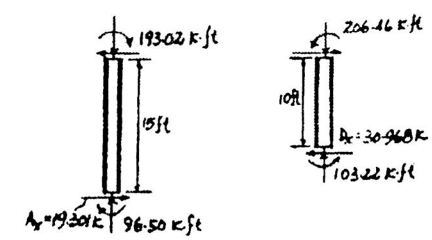
$$(\text{FEM})_{BC} = \frac{-6(24)^2}{12} = -288 \text{ k}\cdot\text{ft}$$

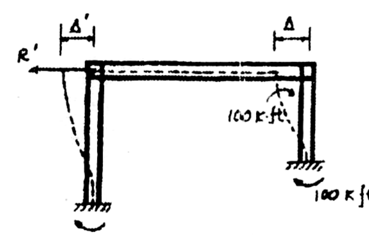
$$(\text{FEM})_{CB} = 288 \text{ k}\cdot\text{ft}$$

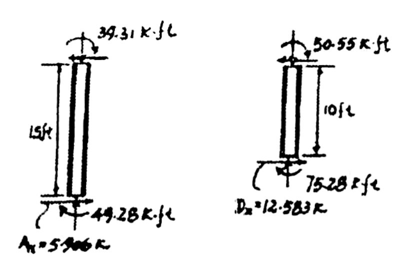
$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$

Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.5161	0.4839	0.4545	0.5455	0
FEM			−288	288		
		148.64	139.36	−130.90	−157.10	
	74.32		−65.45	69.68		−78.55
		33.78	31.67	−31.67	38.01	
	16.89		−15.84	15.84		−19.01
		8.18	7.66	−7.20	−8.64	
	4.09		−3.60	3.83		−4.32
		1.86	1.74	−1.74	−2.09	
	0.93		−0.87	0.87		−1.04
		0.45	0.42	−0.40	−0.47	
	0.22		0.20	0.21		−0.24
		0.10	0.10	−0.10	−0.11	
	0.05		−0.05	0.05		−0.06
		0.02	0.02	−0.02	−0.03	
$\sum M$	96.50	193.02	−193.02	206.46	−206.46	−103.22









12–22. Continued

$\overset{+}{\rightarrow} \sum F_x = 0$ (for the frame without sideway)

$R + 19.301 - 30.968 = 0$

$R = 11.666 \text{ k}$

$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 100 = \frac{6E(0.75I_{AB})\Delta'}{10^2}$$

$$\Delta' = \frac{100(10^2)}{6E(0.75I_{AB})}$$

$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = \frac{6EI_{AB}\Delta'}{15^2} = \left(\frac{6EI_{AB}}{15^2}\right)\left(\frac{100(10^2)}{6E(0.75I_{AB})}\right) = 59.26 \text{ k} \cdot \text{ft}$$

Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.5161	0.4839	0.4545	0.5455	0
FEM	59.26	59.26			100	100
		−30.58	−28.68	−45.45	−54.55	
	−15.29		−22.73	−14.34		−27.28
		11.73	11.00	6.52	7.82	
	5.87		3.26	5.50		3.91
		−1.68	−1.58	−2.50	−3.00	
	−0.84		−125	−0.79		−1.50
		0.65	0.60	0.36	0.43	
	0.32		0.18	0.30		0.22
		−0.09	−0.09	−0.14	−0.16	
	−0.05		−0.07	−0.04		−0.08
		0.04	0.03	0.02	0.02	
	0.02		0.01	0.02		0.01
$\sum M$	49.28	39.31	−39.31	−50.55	50.55	75.28

$R' = 5.906 + 12.585 = 18.489 \text{ k}$

$$M_{AB} = 96.50 + \left(\frac{11.666}{18.489}\right)(49.28) = 128 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{BA} = 193.02 - \left(\frac{11.666}{18.489}\right)(39.31) = 218 \text{ k} \cdot \text{ft}$$ **Ans.**

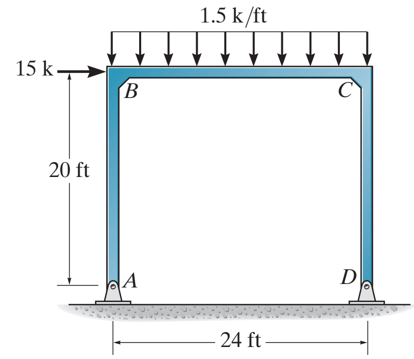
$$M_{BC} = -193.02 + \left(\frac{11.666}{18.489}\right)(-39.31) = 218 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{CB} = 206.46 - \left(\frac{11.666}{18.489}\right)(-50.55) = 175 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{CD} = -206.46 + \left(\frac{11.666}{18.489}\right)(50.55) = 175 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{DC} = -103.21 + \left(\frac{11.666}{18.489}\right)(75.28) = -55.7 \text{ k} \cdot \text{ft}$$ **Ans.**

12–23. Determine the moments acting at the ends of each member of the frame. *EI* is the constant.



Consider no sideway

$(\text{DF})_{AB} = (\text{DF})_{DC} = 1$

$(\text{DF})_{BA} = (\text{DF})_{CD} = \dfrac{3I/20}{3I/20 + 4I/24} = 0.4737$

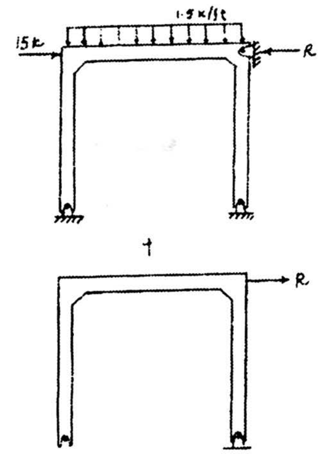
$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.5263$

$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$

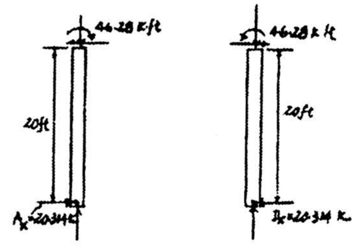
$(\text{FEM})_{BC} = \dfrac{-1.5(24)^2}{12} = -72 \text{ k} \cdot \text{ft}$

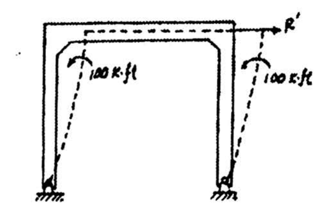
$(\text{FEM})_{CB} = 72 \text{ k} \cdot \text{ft}$

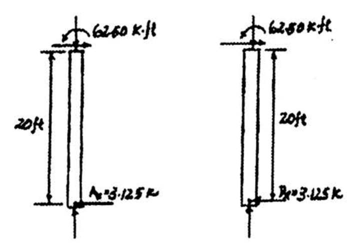
$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$



Joint	*A*	*B*		*C*		*D*
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	1	0.4737	0.5263	0.5263	0.4737	1
FEM			−72.0	72.0		
		34.41	37.89	−37.89	−34.11	
			−18.95	18.95		
		8.98	9.97	−9.97	−8.98	
			−4.98	4.98		
		2.36	2.62	−2.62	−2.36	
			−1.31	1.31		
		0.62	0.69	−0.69	−0.62	
			−0.35	0.35		
		0.16	0.18	−0.18	−0.16	
			−0.09	0.09		
		0.04	0.05	−0.05	−0.04	
			−0.02	0.02		
		0.01	0.01	−0.01	−0.01	
$\sum M$		46.28	−46.28	46.28	−46.28	







12–23. Continued

$\overset{+}{\leftarrow} \Sigma F_x = 0$ (for the frame without sidesway)

$R + 2.314 - 2.314 - 15 = 0$

$R = 15.0 \text{ k}$

Joint	*A*	*B*		*C*		*D*
Mem.	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	1	0.4737	0.5263	0.5263	0.4737	1
FEM		–100			–100	
		47.37	52.63	52.63	47.37	
			26.32	26.32		
		–12.47	–13.85	–13.85	–12.47	
			–6.93	–6.93		
		3.28	3.64	3.64	3.28	
			1.82	1.82		
		–0.86	–0.96	–0.96	–0.86	
			–0.48	–0.48		
		0.23	0.25	0.25	0.23	
			0.13	0.13		
		–0.06	–0.07	–0.07	–0.06	
			–0.03	–0.03		
		0.02	0.02	0.02	0.02	
		–62.50	62.50	62.50	–62.50	

$R' = 3.125 + 3.125 = 6.25 \text{ k}$

$M_{BA} = 46.28 + \left(\frac{15}{6.25}\right)(-62.5) = -104 \text{ k} \cdot \text{ft}$ **Ans.**

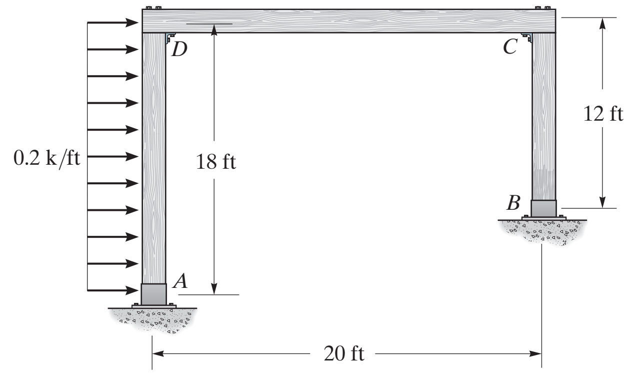
$M_{BC} = -46.28 + \left(\frac{15}{6.25}\right)(62.5) = 104 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = 46.28 + \left(\frac{15}{6.25}\right)(62.5) = 196 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CD} = -46.28 + \left(\frac{15}{6.25}\right)(-62.5) = -196 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{AB} = M_{DC} = 0$ **Ans.**

***12–24.** Determine the moments acting at the ends of each member. Assume the joints are fixed connected and A and B are fixed supports. EI is constant.



Moment Distribution. No sidesway, Fig. b,

$$K_{AD} = \frac{4EI}{L_{AD}} = \frac{4EI}{18} = \frac{2EI}{9} \qquad K_{CD} = \frac{4EI}{L_{CD}} = \frac{4EI}{20} = \frac{EI}{5}$$

$$K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{12} = \frac{EI}{3}$$

$$(\text{DF})_{AD} = (\text{DF})_{BC} = 0 \qquad (\text{DF})_{DA} = \frac{2EI/59}{2EI/9 + EI/5} = \frac{10}{9}$$

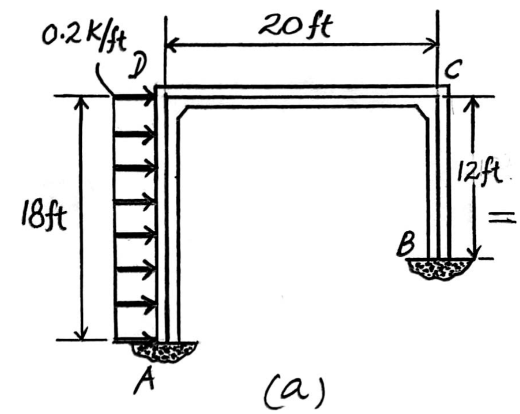
$$(\text{DF})_{DC} = \frac{EI/5}{2EI/9 + EI/5} = \frac{9}{19}$$

$$(\text{DF})_{CD} = \frac{EI/5}{EI/5 + EI/3} = \frac{3}{8} \qquad (\text{DF})_{CB} = \frac{EI/3}{EI/5 + EI/3} = \frac{5}{8}$$

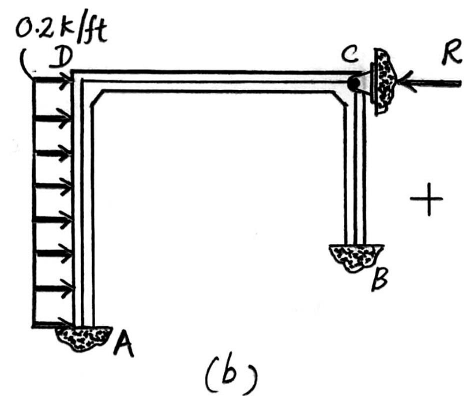
$$(\text{FEM})_{AD} = -\frac{wL_{AD}^2}{12} = -\frac{0.2(18^2)}{12} = -5.40 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{DA} = \frac{wL_{AD}^2}{12} = \frac{0.2(18^2)}{12} = 5.40 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{DC} = (\text{FEM})_{CD} = (\text{FEM})_{CB} = (\text{FEM})_{BC} = 0$$



Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	0	$\frac{10}{19}$	$\frac{9}{19}$	$\frac{3}{8}$	$\frac{5}{8}$	0
FEM	–5.40	5.40	0	0	0	0
Dist.		–2.842	–2.558			
CO	–1.421			–1.279		
Dist.				0.480	0.799	
CO			0.240			0.400
Dist.		–0.126	–0.114			
CO	–0.063			–0.057		
Dist.				0.021	0.036	
CO			0.010			0.018
Dist.		–0.005	–0.005			
$\sum M$	–6.884	2.427	–2.427	–0.835	0.835	0.418



12–24. Continued

Using these results, the shears at A and B are computed and shown in Fig. d. Thus, for the entire frame,

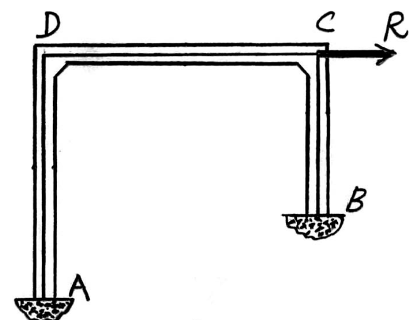
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 0.2(18) + 0.104 - 2.048 - R = 0 \quad R = 1.656 \text{ k}$

For the frame in Fig. e,

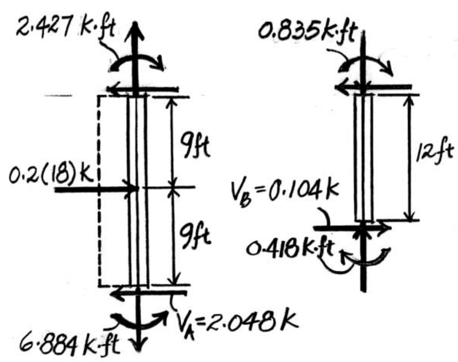
$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = -10 \text{ k} \cdot \text{ft}; \quad -\frac{6EI\Delta'}{L^2} = -10 \quad \Delta' = \frac{240}{EI}$$

$$(\text{FEM})_{AD} = (\text{FEM})_{DA} = -\frac{6EI\Delta'}{L^2} = -\frac{6EI(240/EI)}{18^2} = -4.444 \text{ k} \cdot \text{ft}$$

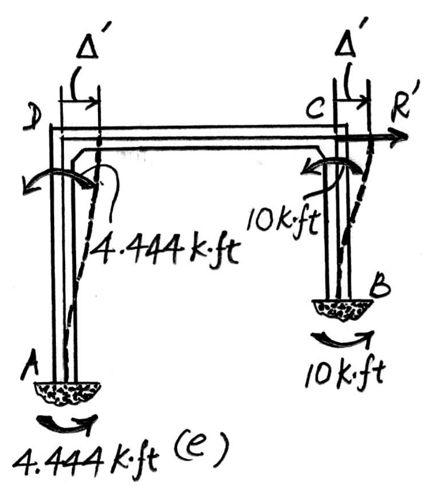
Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	0	$\frac{10}{19}$	$\frac{9}{19}$	$\frac{3}{8}$	$\frac{5}{8}$	0
FEM	–4.444	–4.444			–10	–10
Dist.		2.339	2.105	3.75	6.25	
CO	1.170		1.875	1.053		3.125
Dist.		–0.987	–0.888	–0.395	–0.658	
CO	–0.494		–0.198	–0.444		–0.329
Dist.		0.104	0.094	0.767	0.277	
CO	0.052		0.084	0.047		0.139
Dist.		0.044	–0.040	–0.018	–0.029	
CO	–0.022		–0.009	–0.020		–0.015
Dist.		0.005	0.004	0.008	0.012	
CO	0.003		0.004	0.002		0.006
Dist.		–0.002	–0.002	–0.001	–0.001	
$\sum M$	–3.735	–3.029	3.029	4.149	–4.149	–7.074



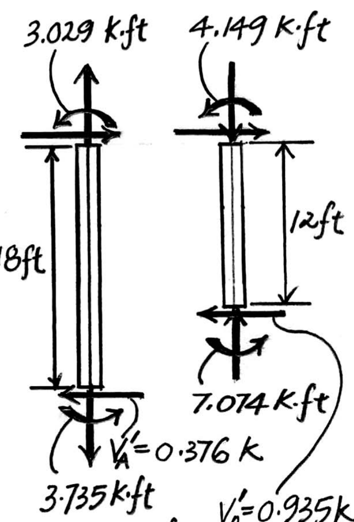
(c)



(d)



(e)



(f)

12–24. Continued

Using these results, the shears at both ends of members AD and BC are computed and shown in Fig. *f*. For the entire frame,

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad R' - 0.376 - 0.935 = 0 \quad R' = 1.311 \text{ k}$$

Thus,

$$M_{AD} = -6.884 + \left(\frac{1.656}{1.311}\right)(-3.735) = 11.6 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{DA} = 2.427 + \left(\frac{1.656}{1.311}\right)(-3.029) = -1.40 \text{ k}\cdot\text{ft}$$ **Ans.**

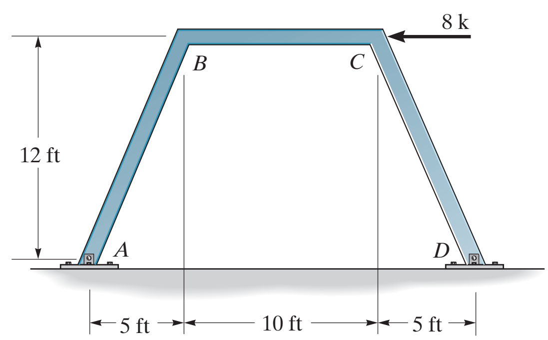
$$M_{DC} = -2.427 + \left(\frac{1.656}{1.311}\right)(3.029) = 1.40 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{CD} = -0.835 + \left(\frac{1.656}{1.311}\right)(4.149) = 4.41 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{CB} = 0.835 + \left(\frac{1.656}{1.311}\right)(-4.149) = -4.41 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{CD} = 0.418 + \left(\frac{1.656}{1.311}\right)(-7.074) = -8.52 \text{ k}\cdot\text{ft}$$ **Ans.**

12–25. Determine the moments at joints B and C, then draw the moment diagram for each member of the frame. The supports at A and D are pinned. EI is constant.



Moment Distribution. For the frame with **P** acting at C, Fig. *a*,

$$K_{AB} = K_{CD} = \frac{3EI}{L} = \frac{3EI}{13} \quad K_{BC} = \frac{4EI}{10} = \frac{2EI}{5}$$

$$(\text{DF})_{AB} = (\text{DF})_{DC} = 1 \quad (\text{DF})_{BA} = (\text{DF})_{CD} = \frac{3EI/13}{3EI/13 + 2EI/5} = \frac{15}{41}$$

$$(\text{DF})_{BC} = (\text{DF})_{CB} = \frac{2EI/5}{3EI/13 + 2EI/5} = \frac{26}{41}$$

$$(\text{FEM})_{BA} = (\text{FEM})_{CD} = 100 \text{ k}\cdot\text{ft}; \quad \frac{3EI\Delta'}{L^2} = 100 \quad \Delta' = \frac{16900}{3EI}$$

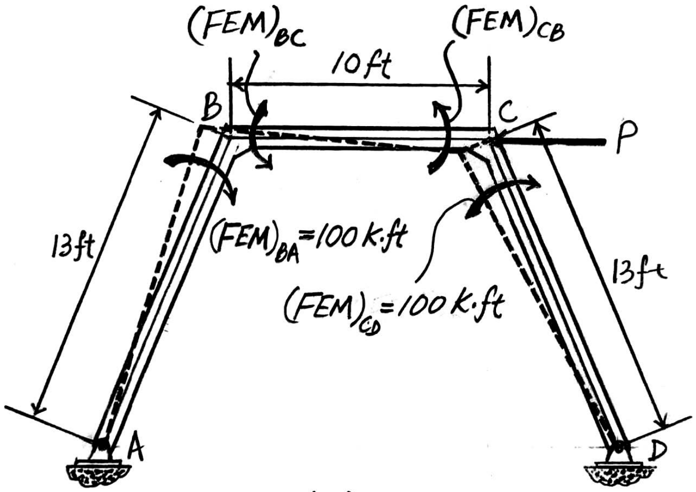
From the geometry shown in Fig. *b*,

$$\Delta'_{BC} = \frac{5}{13}\Delta' + \frac{5}{13}\Delta' = \frac{10}{13}\Delta'$$

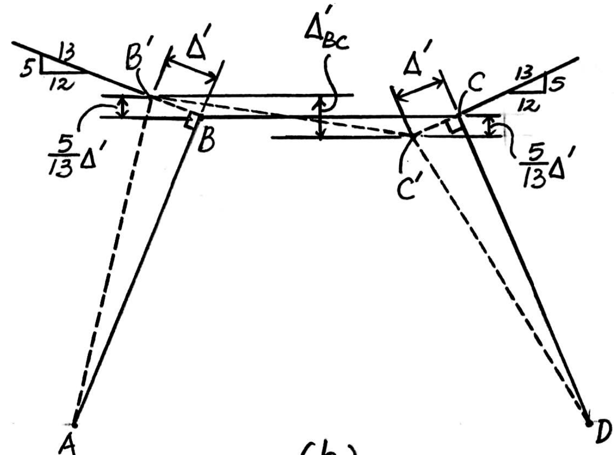
Thus

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = -\frac{6EI\Delta'_{BC}}{L_{BC}^2} = -\frac{6EI\left(\frac{10}{13}\right)\left(\frac{16900}{3EI}\right)}{10^2} = -260 \text{ k}\cdot\text{ft}$$

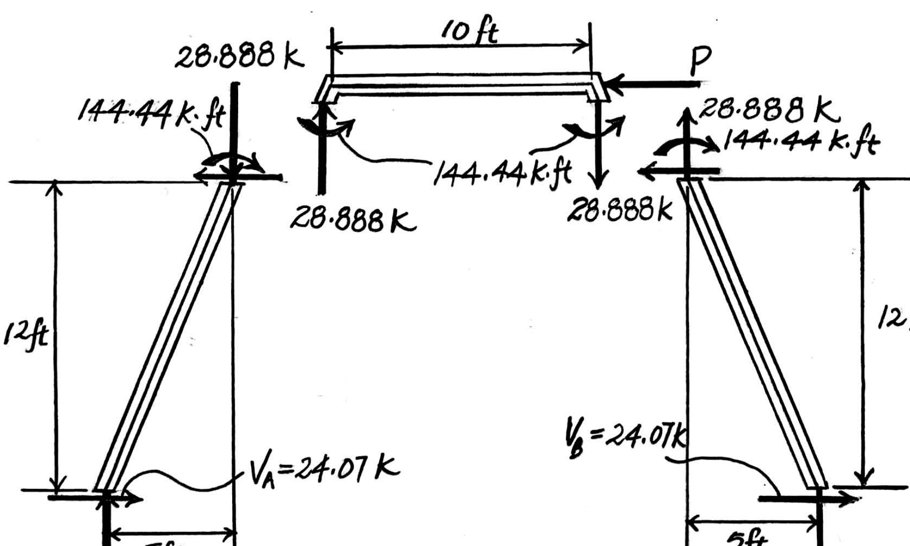
12–25. Continued



(a)



(b)



(c)

12–25. Continued

Joint	*A*	*B*		*C*		*D*
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	1	15/41	26/41	26/41	15/41	1
FEM	0	100	−260	−260	100	0
Dist.		58.54	101.46	101.46	58.54	
CO			50.73	50.73		
Dist.		18.56	−32.17	−32.17	−18.56	
CO			−16.09	−16.09		
Dist.		5.89	10.20	10.20	5.89	
CO			5.10	5.10		
Dist.		−1.87	−3.23	−3.23	−1.87	
CO			−1.62	−1.62		
Dist.		0.59	1.03	1.03	0.59	
CO			0.51	0.51		
Dist.		−0.19	−0.32	−0.32	−0.19	
CO			−0.16	−0.16		
Dist.		0.06	0.10	0.10	0.06	
CO			0.05	0.05		
Dist.		−0.02	−0.03	−0.03	−0.02	
$\sum M$	0	144.44	−144.44	−144.44	−144.44	0

Using these results, the shears at *A* and *D* are computed and shown in Fig. *c*. Thus for the entire frame,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 24.07 + 24.07 - P = 0 \quad P = 48.14 \text{ k}$$

Thus, for **P** = 8 k,

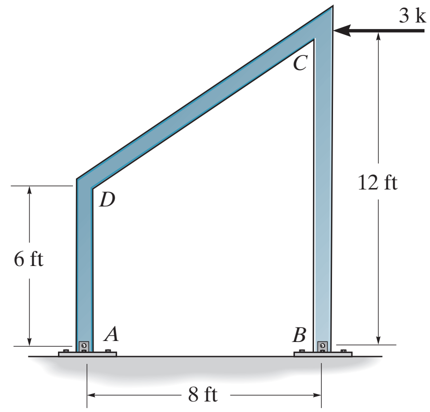
$$M_{BA} = \left(\frac{8}{48.14}\right)(144.44) = 24.0 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{BC} = \left(\frac{8}{48.14}\right)(-144.44) = -24.0 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{CB} = \left(\frac{8}{48.14}\right)(-144.44) = -24.0 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{CD} = \left(\frac{8}{48.14}\right)(144.44) = 24.0 \text{ k} \cdot \text{ft}$$ **Ans.**

12–26. Determine the moments at C and D, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Moment Distribution. For the frame with **P** acting at C, Fig. a,

$$K_{AD} = \frac{3EI}{L_{AD}} = \frac{3EI}{6} = \frac{EI}{2} \quad K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{12} = \frac{EI}{4}$$

$$K_{CD} = \frac{4EI}{L_{CD}} = \frac{4EI}{10} = \frac{2EI}{5}$$

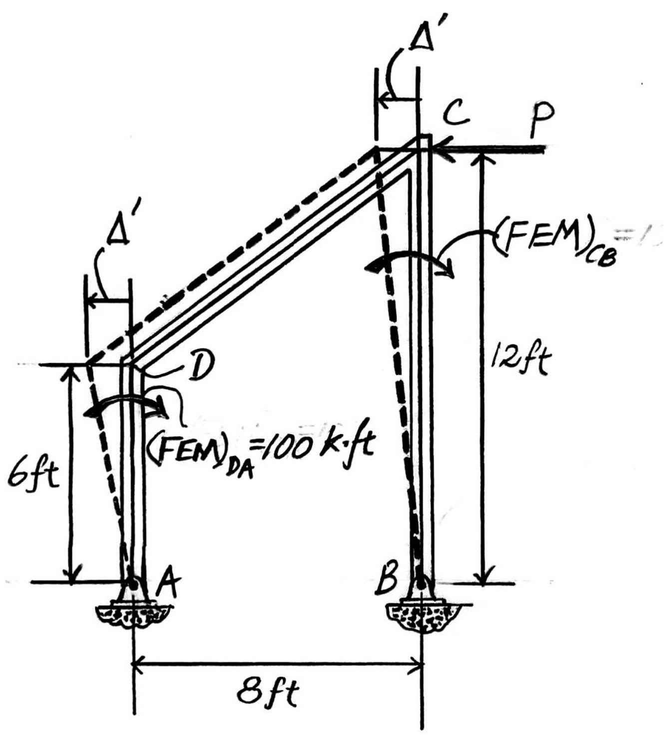
$$(\text{DF})_{AD} = (\text{DF})_{BC} = 1 \quad (\text{DF})_{DA} = \frac{EI/2}{EI/2 + 2EI/5} = \frac{5}{9}$$

$$(\text{DF})_{DC} = \frac{2EI/5}{EI/2 + 2EI/5} = \frac{4}{9}$$

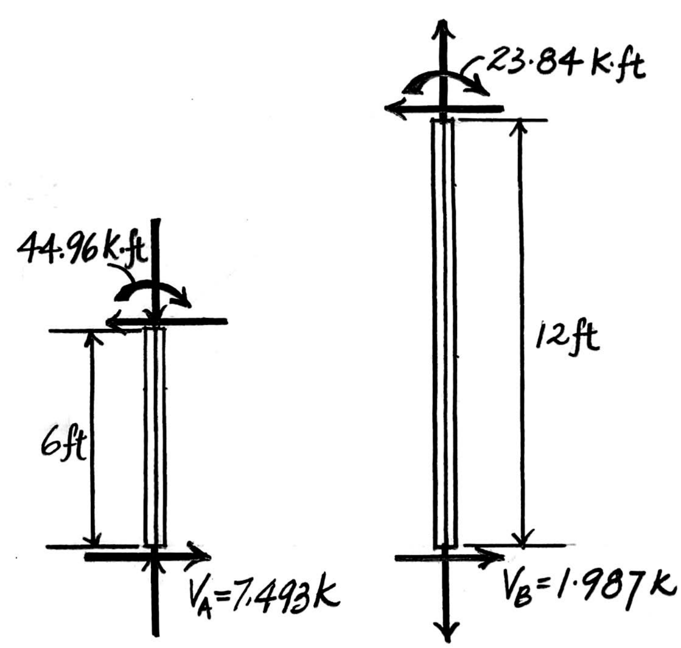
$$(\text{DF})_{CD} = \frac{2EI/5}{2EI/5 + EI/4} = \frac{8}{13} \quad (\text{DF})_{CB} = \frac{EI/4}{2EI/5 + EI/4} = \frac{5}{13}$$

$$(\text{FEM})_{DA} = 100\ \text{k}\cdot\text{ft}; \quad \frac{3EI\Delta'}{L_{DA}^2} = 100 \quad \Delta' = \frac{1200}{EI}$$

$$(\text{FEM})_{CB} = \frac{3EI\Delta'}{L_{CB}^2} = \frac{3EI(1200/EI)}{12^2} = 25\ \text{k}\cdot\text{ft}$$



(a)



(b)

12–26. Continued

Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{8}{13}$	$\frac{5}{13}$	1
FEM	0	100	0	0	25	0
Dist.		−55.56	−44.44	−15.38	−9.62	
CO			−7.69	−22.22		
Dist.		4.27	3.42	13.67	8.55	
CO			6.84	1.71		
Dist.		−3.80	−3.04	−1.05	−0.66	
CO			−0.53	−1.52		
Dist.		0.29	0.24	0.94	0.58	
CO			0.47	0.12		
Dist.		−0.26	−0.21	−0.07	−0.05	
CO			−0.04	−0.11		
Dist.		−0.02	−0.02	0.07	0.04	
$\sum M$	0	44.96	−44.96	−23.84	23.84	0

Using the results, the shears at A and B are computed and shown in Fig. c. Thus, for the entire frame,

$$\xrightarrow{+} \sum F_X = 0; \quad 7.493 + 1.987 - P = 0 \quad P = 9.480\ k$$

Thus, for $P = 3$ k,

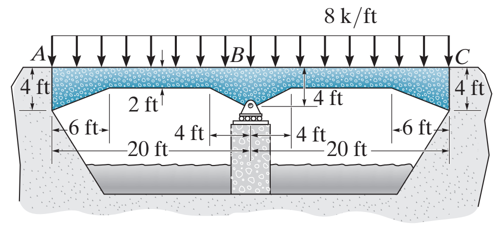
$$M_{DA} = \left(\frac{3}{9.480}\right)(44.96) = 14.2\ \text{k}\cdot\text{ft}$$ **Ans.**

$$M_{DC} = \left(\frac{3}{9.480}\right)(-44.96) = -14.2\ \text{k}\cdot\text{ft}$$ **Ans.**

$$M_{CD} = \left(\frac{3}{9.480}\right)(-23.84) = -7.54\ \text{k}\cdot\text{ft}$$ **Ans.**

$$M_{CB} = \left(\frac{3}{9.480}\right)(23.84) = 7.54\ \text{k}\cdot\text{ft}$$ **Ans.**

13–1. Determine the moments at A, B, and C by the moment-distribution method. Assume the supports at A and C are fixed and a roller support at B is on a rigid base. The girder has a thickness of 4 ft. Use Table 13–1. E is constant. The haunches are straight.



$$a_A = \frac{6}{20} = 0.3 \qquad a_B = \frac{4}{20} = 0.2$$

$$r_A = r_B = \frac{4-2}{2} = 1$$

From Table 13–1,

For span AB,

$C_{AB} = 0.622 \qquad C_{BA} = 0.748$

$K_{AB} = 10.06 \qquad K_{BA} = 8.37$

$$K_{BA} = \frac{K_{BA}EI_C}{L} = \frac{8.37EI_C}{20} = 0.4185EI_C$$

$(\text{FEM})_{AB} = -0.1089(8)(20)^2 = -348.48 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{BA} = 0.0942(8)(20)^2 = 301.44 \text{ k}\cdot\text{ft}$

For span BC,

$C_{BC} = 0.748 \qquad C_{CB} = 0.622$

$K_{BC} = 8.37 \qquad K_{CB} = 10.06$

$K_{BC} = 0.4185EI_C$

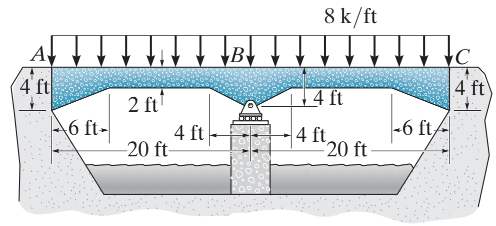
$(\text{FEM})_{BC} = -301.44 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{CB} = 348.48 \text{ k}\cdot\text{ft}$

Joint	A	B		C
Mem.	AB	BA	BC	CB
K		$0.4185EI_C$	$0.4185EI_C$	
DF	0	0.5	0.5	0
COF	0.622	0.748	0.748	0.622
FEM	–348.48	301.44	–301.44	348.48
		0	0	
$\sum M$	–348.48	301.44	–301.44	348.48 k · ft

Ans.

13–2. Solve Prob. 13–1 using the slope-deflection equations.



$a_A = \dfrac{6}{20} = 0.3 \qquad a_B = \dfrac{4}{20} = 0.2$

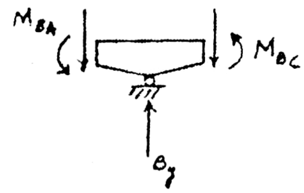
$r_A = r_B = \dfrac{4-2}{2} = 1$

For span AB,

$C_{AB} = 0.622 \qquad C_{BA} = 0.748$

$K_{AB} = 10.06 \qquad K_{BA} = 8.37$

$K_{BA} = \dfrac{K_{BA}EI_C}{L} = \dfrac{8.37EI_C}{20} = 0.4185EI_C$



$(\text{FEM})_{AB} = -0.1089(8)(20)^2 = -348.48 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{BA} = 0.0942(8)(20)^2 = 301.44 \text{ k}\cdot\text{ft}$

For span BC,

$C_{BC} = 0.748 \qquad C_{CB} = 0.622$

$K_{BC} = 8.37 \qquad K_{CB} = 10.06$

$K_{BC} = 0.4185EI_C$

$(\text{FEM})_{BC} = -301.44 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{CB} = 348.48 \text{ k}\cdot\text{ft}$

$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$

$M_{AB} = 0.503EI(0 + 0.622\theta_B - \;) - 348.48$

$M_{AB} = 0.312866EI\theta_B - 348.8 \qquad (1)$

$M_{BA} = 0.4185EI(\theta_B + 0 - 0) + 301.44$

$M_{BA} = 0.4185EI\theta_B + 301.44 \qquad (2)$

$M_{BC} = 0.4185EI(\theta_B + 0 - 0) - 301.44$

$M_{BC} = 0.4185EI\theta_B - 301.44 \qquad (3)$

$M_{CB} = 0.503EI(0 + 0.622\theta_B - 0) + 348.48$

$M_{CB} = 0.312866EI\theta_B - 348.48 \qquad (4)$

Equilibrium.

$M_{BA} + M_{BC} = 0 \qquad (5)$

Solving Eqs. 1–5:

$\theta_B = 0$

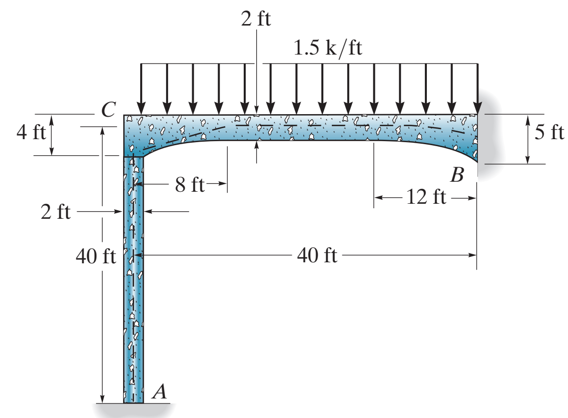
$M_{AB} = -348 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 301 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -301 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 348 \text{ k}\cdot\text{ft}$ **Ans.**

13–3. Apply the moment-distribution method to determine the moment at each joint of the parabolic haunched frame. Supports A and B are fixed. Use Table 13–2. The members are each 1 ft thick. E is constant.



The necessary data for member BC can be found from Table 13–2.

Here,

$$a_C = \frac{8}{40} = 0.2 \qquad a_B = \frac{12}{40} = 0.3 \qquad r_C = \frac{4-2}{2} = 1.0 \qquad r_B = \frac{5-2}{2} = 1.5$$

Thus,

$$C_{CB} = 0.735 \qquad C_{BC} = 0.589 \qquad K_{CB} = 7.02 \qquad K_{BC} = 8.76$$

Then,

$$K_{CB} = \frac{K_{CB}EI_C}{L_{BC}} = \frac{7.02E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.117E$$

The fixed end moment are given by

$$(\text{FEM})_{CB} = -0.0862(1.5)(40^2) = -206.88 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = 0.1133(1.5)(40^2) = 271.92 \text{ k}\cdot\text{ft}$$

Since member AC is prismatic

$$K_{CA} = \frac{4EI}{L_{AC}} = \frac{4E\left[\frac{1}{12}(1)(2)^3\right]}{40} = 0.0667E$$

Tabulating these data;

Joint	A	C		B
Mem	AC	CA	CB	BC
K		$0.0667E$	$0.117E$	
DF	0	0.3630	0.6370	0
COF	0	0.5	0.735	0
FEM			–206.88	
Dist.		75.10	131.78	271.92
CO	37.546			96.86
$\sum M$	37.546	75.10	–75.10	368.78

Thus,

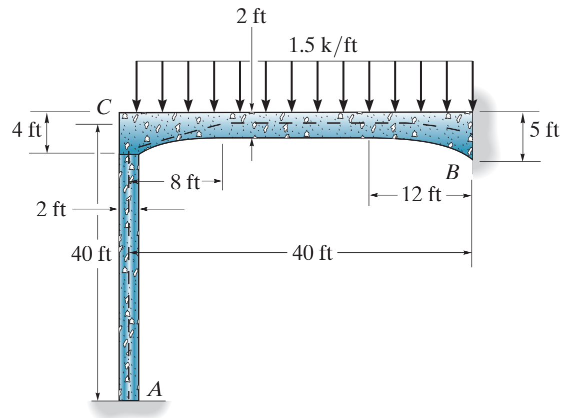
$M_{AC} = 37.55 \text{ k}\cdot\text{ft} = 37.6 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{CA} = 75.10 \text{ k}\cdot\text{ft} = 75.1 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{CB} = -75.10 \text{ k}\cdot\text{ft} = -75.1 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = 368.78 \text{ k}\cdot\text{ft} = 369 \text{ k}\cdot\text{ft}$ **Ans.**

***13–4.** Solve Prob. 13–3 using the slope-deflection equations.



The necessary data for member BC can be found from Table 13.2.

Here,

$$a_C = \frac{8}{40} = 0.2 \qquad a_B = \frac{12}{40} = 0.3 \qquad r_C = \frac{4-2}{2} = 1.0 \quad r_B = \frac{5-2}{2} = 1.5$$

Thus,

$$C_{CB} = 0.735 \qquad C_{BC} = 0.589 \qquad K_{CB} = 7.02 \qquad K_{BC} = 8.76$$

Then,

$$K_{CB} = \frac{K_{CB}EI_C}{L_{BC}} = \frac{7.02E\left[\frac{1}{12}(1)(2)^3\right]}{40} = 0.117E$$

$$K_{BC} = \frac{K_{BC}EI_C}{L_{BC}} = \frac{8.76E\left[\frac{1}{12}(1)(2)^3\right]}{40} = 0.146E$$

The fixed end moment are given by

$$(\text{FEM})_{CB} = -0.0862(1.5)(40)^2 = -206.88\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = 0.1133(1.5)(40)^2 = 271.92\ \text{k}\cdot\text{ft}$$

For member BC, applying Eq. 13–8,

$$M_N = K_N[\theta_N + C_N\theta_F - \psi(HC_N)] + (\text{FEM})_N$$

$$M_{CB} = 0.117E[\theta_c + 0.735(0) - 0(1 + 0.735)] + (-206.88) = 0.117E\theta_c - 206.88 \qquad (1)$$

$$M_{BC} = 0.146E[0 + 0.589\theta_C - 0(1 + 0.589)] + 271.92 = 0.085994E\theta_C + 271.92 \qquad (2)$$

Since member AC is prismatic, Eq. 11–8 is applicable

$$M_N = 2EK(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AC} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right][2(0) + \theta_C - 3(0)] + 0 = 0.03333E\theta_C \qquad (3)$$

$$M_{CA} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right][2\theta_C + 0 - 3(0)] + 0 = 0.06667E\theta_C \qquad (4)$$

Moment equilibrium of joint C gives

$$M_{CA} + M_{CB} = 0$$

$$0.06667E\theta_C + 0.117E\theta_C - 206.88 = 0$$

$$\theta_C = \frac{1126.39}{E}$$

13–4. Continued

Substitute this result into Eqs. (1) to (4),

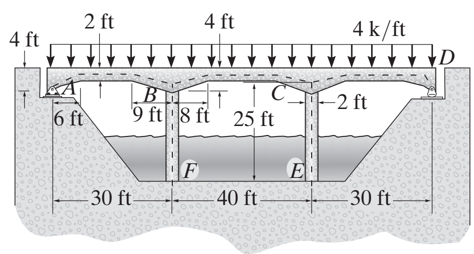
$M_{CB} = -75.09 \text{ k} \cdot \text{ft} = -75.1 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BC} = 368.78 \text{ k} \cdot \text{ft} = 369 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{AC} = 37.546 \text{ k} \cdot \text{ft} = 37.5 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CA} = 75.09 \text{ k} \cdot \text{ft} = 75.1 \text{ k} \cdot \text{ft}$ **Ans.**

13–5. Use the moment-distribution method to determine the moment at each joint of the symmetric bridge frame. Supports at F and E are fixed and B and C are fixed connected. Use Table 13–2. Assume E is constant and the members are each 1 ft thick.



For span AB,

$$a_A = \frac{6}{30} = 0.2 \qquad a_B = \frac{9}{30} = 0.3$$

$$r_A = r_B = \frac{4 - 2}{2} = 1$$

From Table 13–2,

$C_{AB} = 0.683 \qquad C_{BA} = 0.598$

$k_{AB} = 6.73 \qquad k_{BA} = 7.68$

$$K_{AB} = \frac{6.73EI}{30} = 0.2243EI$$

$$K_{BA} = \frac{7.68EI}{30} = 0.256EI$$

$$K_{BA} = 0.256EI[1 - (0.683)(0.598)]$$
$$= 0.15144EI$$

$$(\text{FEM})_{AB} = -0.0911(4)(30)^2 = -327.96 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BA} = 0.1042(4)(30)^2 = 375.12 \text{ k} \cdot \text{ft}$$

13–5. Continued

For span CD,

$C_{DC} = 0.683 \qquad C_{CD} = 0.598$

$K_{DC} = 6.73 \qquad K_{CD} = 7.68$

$K_{DC} = 0.2243EI$

$K_{CD} = 0.256EI$

$K_{CD} = 0.15144EI$

$(\text{FEM})_{CD} = -375.12 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{DC} = 327.96 \text{ k} \cdot \text{ft}$

For span BC,

$$a_B = a_C = \frac{8}{40} = 0.2$$

$$r_A = r_{CB} = \frac{4 - 2}{2} = 1$$

From Table 13–2,

$C_{BC} = C_{CB} = 0.619$

$k_{BC} = k_{CB} = 6.41$

$$K_{BC} = K_{CB} = \frac{6.41EI}{40} = 0.16025EI$$

$(\text{FEM})_{BC} = -0.0956(4)(40)^2 = -611.84 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 611.84 \text{ k} \cdot \text{ft}$

For span BF,

$C_{BF} = 0.5$

$$K_{BF} = \frac{4EI}{25} = 0.16EI$$

$(\text{FEM})_{BF} = (\text{FEM})_{FB} = 0$

For span CE,

$C_{CE} = 0.5$

$K_{CE} = 0.16EI$

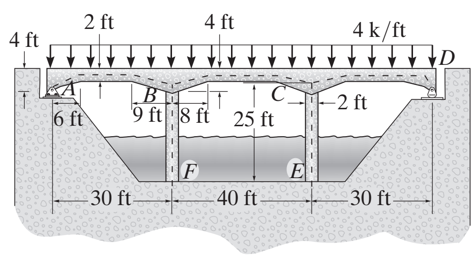
$(\text{FEM})_{CE} = (\text{FEM})_{EC} = 0$

13–5. Continued

Joint	A	F	B			C			E	D
Member	AB	FB	BF	BA	BC	CB	CD	CE	EC	DC
DF	1	0	0.3392	0.3211	0.3397	0.3397	0.3211	0.3392	0	1
COF	0.683		0.5	0.598	0.619	0.619	0.598	0.5		0.683
FEM	−327.96			375.12	−611.84	611.84	−375.12			332.96
	327.96		80.30	76.01	80.41	−80.41	−76.01	−80.30		−327.96
		40.15		224.00	−49.77	49.77	−224.00		−40.15	
			−59.09	−55.95	−59.19	59.19	55.95	59.19		
		−29.55			36.64	−36.64			29.55	
			−12.42	−11.77	−12.45	12.45	11.77	12.42		
		−6.21			7.71	−7.71			6.21	
			−2.61	−2.48	−2.62	2.62	2.48	2.61		
		−1.31			1.62	−1.62			1.31	
			−0.55	−0.52	−0.55	0.55	0.52	0.55		
		−0.27			0.34	−0.34			−0.27	
			−0.11	−0.11	−0.12	0.12	0.11	0.11		
		−0.5			0.07	−0.07			0.05	
			−0.03	−0.02	−0.02	0.02	0.02	0.03		
$\sum$	0	2.76	5.49	604	−609	609	−604	5.49	−2.76	0

k · ft **Ans.**

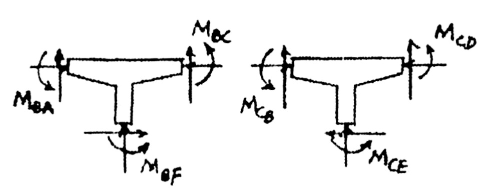
13–6. Solve Prob. 13–5 using the slope-deflection equations.



See Prob. 13–19 for the tabulated data

$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 - C_N)] + (\text{FEM})_N$

For span AB,



$M_{AB} = 0.2243EI(\theta_A + 0.683\theta_B - 0) - 327.96$

$M_{AB} = 0.2243EI\theta_A + 0.15320EI\theta_B - 327.96$ (1)

$M_{BA} = 0.256EI(\theta_B + 0.598\theta_A - 0) + 375.12$

$M_{BA} = 0.256EI\theta_B + 0.15309EI\theta_A + 375.12$ (2)

For span BC,

$M_{BC} = 0.16025EI(\theta_B + 0.619\theta_C - 0) - 611.84$

$M_{BC} = 0.16025EI\theta_B + 0.099194EI\theta_C - 611.84$ (3)

$M_{CB} = 0.16025EI(\theta_C + 0.619\theta_B - 0) + 611.84$

$M_{CB} = 0.16025EI\theta_C + 0.099194EI\theta_B + 611.84$ (4)

13–6. Continued

For span CD,

$M_{CD} = 0.256EI(\theta_C + 0.598\theta_D - 0) - 375.12$

$M_{CD} = 0.256EI\theta_C + 0.15309EI\theta_D - 375.12$ (5)

$M_{DC} = 0.2243EI(\theta_D + 0.683\theta_C - 0) + 327.96$

$M_{DC} = 0.2243EI\theta_D + 0.15320EI\theta_C + 327.96$ (6)

For span BF,

$M_{BF} = 2E\left(\frac{1}{25}\right)(2\theta_B + 0 - 0) + 0$

$M_{BF} = 0.16EI\theta_B$ (7)

$M_{FB} = 2E\left(\frac{1}{25}\right)(2(0) + \theta_B - 0) + 0$

$M_{FB} = 0.08EI\theta_B$ (8)

For span CE,

$M_{CE} = 2E\left(\frac{1}{25}\right)(2\theta_C + 0 - 0) + 0$

$M_{CE} = 0.16EI\theta_C$ (9)

$M_{EC} = 2E\left(\frac{1}{25}\right)(2(0) + \theta_C - 0) + 0$

$M_{EC} = 0.08EI\theta_C$ (10)

Equilibrium equations:

$M_{AB} = 0$ (11)

$M_{DC} = 0$ (12)

$M_{BA} + M_{BC} + M_{BF} = 0$ (13)

$M_{CB} + M_{CE} + M_{CD} = 0$ (14)

Solving Eq. 1–14,

$\theta_A = \frac{1438.53}{EI} \qquad \theta_B = \frac{34.58}{EI} \qquad \theta_C = \frac{-34.58}{EI} \qquad \theta_D = \frac{-1438.53}{EI}$

$M_{AB} = 0$ **Ans.**

$M_{BA} = 604 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BC} = -610 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BF} = 5.53 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{FB} = 2.77 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = 610 \text{ k} \cdot \text{ft}$ **Ans.**

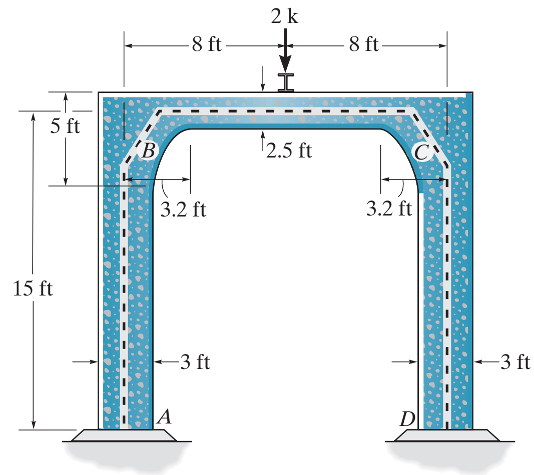
$M_{CD} = -604 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CE} = -5.53 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{EC} = -2.77 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{DC} = 0$ **Ans.**

13–7. Apply the moment-distribution method to determine the moment at each joint of the symmetric parabolic haunched frame. Supports A and D are fixed. Use Table 13–2. The members are each 1 ft thick. E is constant.



$$a_B = a_C = \frac{3.2}{16} = 0.2$$

$$r_B = r_C = \frac{5 - 2.5}{2.5} = 1$$

$$C_{BC} = C_{CB} = 0.619$$

$$k_{BC} = k_{CB} = 6.41$$

$$(\text{FEM})_{BC} = -0.1459(2)(16) = -4.6688 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = 4.6688 \text{ k} \cdot \text{ft}$$

$$K_{BC} = K_{CB} = \frac{k_{BC}EI_C}{L} = \frac{6.41(E)\left(\frac{1}{12}\right)(1)(2.5)^3}{16} = 0.5216E$$

$$K_{BA} = K_{CO} = \frac{4EI}{L} = \frac{4E\left[\frac{1}{12}(1)(3)^3\right]}{15} = 0.6E$$

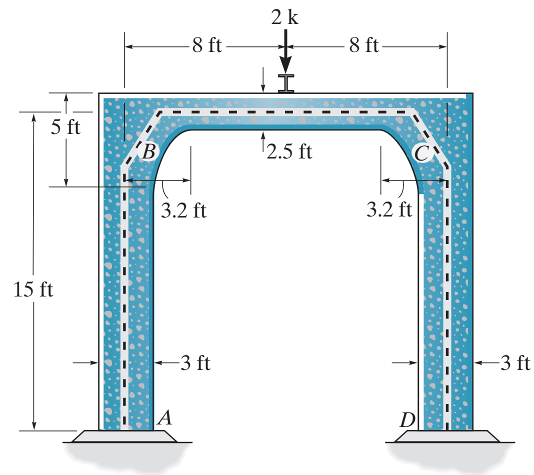
$$(\text{DF})_{BA} = (\text{DF})_{CD} = \frac{0.6E}{0.5216E + 0.6E} = 0.535$$

$$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.465$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.535	0.465	0.465	0.535	0
COF	0.5	0.5	0.619	0.619	0.5	0.5
FEM			−4.6688	4.6688		
		2.498	2.171	−2.171	−2498	
	1.249		−1.344	1.344		−1.249
		0.7191	0.6249	−0.6249	−0.7191	
	0.359		−0.387	0.387		−0.359
		0.207	0.180	−0.180	−0.207	
	0.103		−0.111	0.111		−0.103
		0.059	0.052	−0.052	−0.059	
	0.029		−0.032	0.032		−0.029
		0.017	0.015	−0.015	−0.017	
	0.008		−0.009	0.009		−0.008
		0.005	0.004	−0.004	−0.005	
	0.002		−0.002	0.002		0.002
		0.001	0.001	−0.001	−0.001	
Σ	1.750	3.51	−3.51	3.51	−3.51	−1.75 k · ft

Ans.

***13–8.** Solve Prob. 13–7 using the slope-deflection equations.



$$a_B = a_C = \frac{3.2}{16} = 0.2$$

$$r_B = r_C = \frac{5 - 2.5}{2.5} = 1$$

$$C_{BC} = C_{CB} = 0.619$$

$$k_{BC} = k_{CB} = 6.41$$

$$(\text{FEM})_{BC} = 0.1459(2)(16) = -4.6688 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = -4.6688 \text{ k} \cdot \text{ft}$$

$$K_{BC} = K_{CB} = \frac{k_{BC} E I_c}{L} = \frac{6.41(E)\left(\frac{1}{12}\right)(1)(2.5)^2}{16} = 0.5216E$$

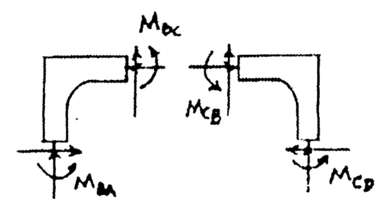
$$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{15}(0 + \theta_B - 0) + 0$$

$$M_{BA} = \frac{2EI}{15}(2\theta_B + 0 - 0) + 0$$

$$M_{CD} = \frac{2EI}{15}(2\theta_C + 0 - 0) + 0$$

$$M_{DC} = \frac{2EI}{15}(0 + \theta_C - 0) + 0$$



$$M_{BC} = 0.5216E(\theta_B + 0.619(\theta_C) - 0) - 4.6688$$

$$M_{CB} = 0.5216E(\theta_C + 0.619(\theta_B) - 0) + 4.6688$$

Equilibrium.

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

Or,

$$\frac{2E\left(\frac{1}{12}\right)(1)(3)^3}{15}(2\theta_B) + 0.5216E[\theta_B + 0.619\theta_C] - 4.6688 = 0$$

$$1.1216\theta_B + 0.32287\theta_C = \frac{4.6688}{E} \qquad (1)$$

$$\frac{2E\left(\frac{1}{12}\right)(1)(3)^3}{15}(2\theta_C) + 0.5216E[\theta_C + 0.619\theta_B] + 4.6688 = 0$$

$$1.1216\theta_C + 0.32287\theta_B = -\frac{4.6688}{E} \qquad (2)$$

13–8. Continued

Solving Eqs. 1 and 2:

$$\theta_B = -\theta_C = \frac{5.84528}{E}$$

$M_{AB} = 1.75 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BA} = 3.51 \text{ k} \cdot \text{ft}$ **Ans.**

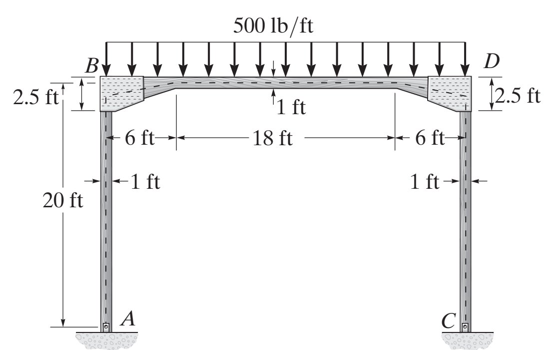
$M_{BC} = -3.51 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = 3.51 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CD} = -3.51 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{DC} = -1.75 \text{ k} \cdot \text{ft}$ **Ans.**

13–9. Use the moment-distribution method to determine the moment at each joint of the frame. The supports at A and C are pinned and the joints at B and D are fixed connected. Assume that E is constant and the members have a thickness of 1 ft. The haunches are straight so use Table 13–1.



For span BD,

$$a_B = a_D = \frac{6}{30} = 0.2$$

$$r_A = r_B = \frac{2.5 - 1}{1} = 1.5$$

From Table 13–1,

$$C_{BD} = C_{DB} = 0.691$$

$$k_{BD} = k_{DB} = 9.08$$

$$K_{BD} = K_{DB} = \frac{kEI_C}{L} = \frac{9.08EI}{30} = 0.30267EI$$

$$(\text{FEM})_{BD} = -0.1021(0.5)(30^2) = -45.945 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{DB} = 45.945 \text{ k} \cdot \text{ft}$$

For span AB and CD,

$$K_{BA} = K_{DC} = \frac{3EI}{20} = 0.15EI$$

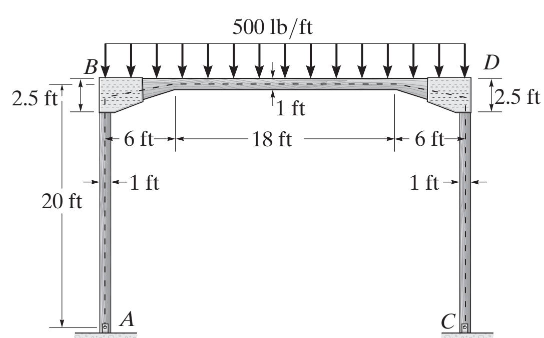
$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = (\text{FEM})_{DC} = (\text{FEM})_{CD} = 0$$

13–9. Continued

Joint	A	B		D		C
Mem.	AB	BA	BD	DB	DC	CD
K		$0.15EI$	$0.3026EI$	$0.3026EI$	$0.15EI$	
DF	1	0.3314	0.6686	0.6686	0.3314	1
COF		0	0.691	0.691	0	
FEM			−45.95	45.95		
		15.23	30.72	−30.72	−15.23	
			−21.22	21.22		
		7.03	14.19	−14.19	−7.03	
			−9.81	9.81		
		3.25	6.56	−6.56	−3.25	
			−4.53	4.53		
		1.50	3.03	−3.03	−1.50	
			−2.09	2.09		
		0.69	1.40	−1.40	−0.69	
			−0.97	0.97		
		0.32	0.65	−0.65	−0.32	
			−0.45	0.45		
		0.15	0.30	−0.30	−0.15	
			−0.21	0.21		
		0.07	0.14	−0.14	−0.07	
			−0.10	0.10		
		0.03	0.06	−0.06	−0.03	
			−0.04	0.04		
		0.01	0.03	−0.03	−0.01	
$\sum M$	0	28.3	−28.3	28.3	−28.3	0 k · ft

Ans.

13–10. Solve Prob. 13–9 using the slope-deflection equations.



See Prob. 13–17 for the tabular data.

For span AB,

$$M_N = 3E\frac{I}{L}[\theta_N - \psi] + (\text{FEM})_N$$

$$M_{BA} = 3E\left(\frac{I}{20}\right)(\theta_B - 0) + 0$$

$$M_{BA} = \frac{3EI}{20}\theta_B \qquad (1)$$

13–10. Continued

For span BD,

$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$

$M_{BD} = 0.30267EI(\theta_B + 0.691\theta_D - 0) - 45.945$

$M_{BD} = 0.30267EI\theta_B + 0.20914EI\theta_D - 45.945$ (2)

$M_{DB} = 0.30267EI(\theta_D + 0.691\theta_B - 0) + 45.945$

$M_{DB} = 0.30267EI\theta_D + 0.20914EI\theta_B - 45.945$ (3)

For span DC,

$M_N = 3E\frac{I}{L}[\theta_N - \psi] + (\text{FEM})_N$

$M_{DC} = 3E\left(\frac{I}{20}\right)(\theta_D - 0) + 0$

$M_{DC} = \frac{3EI}{20}\theta_D$ (4)

Equilibrium equations,

$M_{BA} + M_{BD} = 0$ (5)

$M_{DB} + M_{DC} = 0$ (6)

Solving Eqs. 1–6:

$\theta_B = \frac{188.67}{EI} \qquad \theta_D = -\frac{188.67}{EI}$

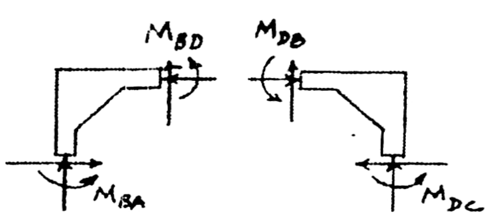
$M_{BA} = 28.3 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BD} = -28.3 \text{ k} \cdot \text{ft}$ **Ans.**

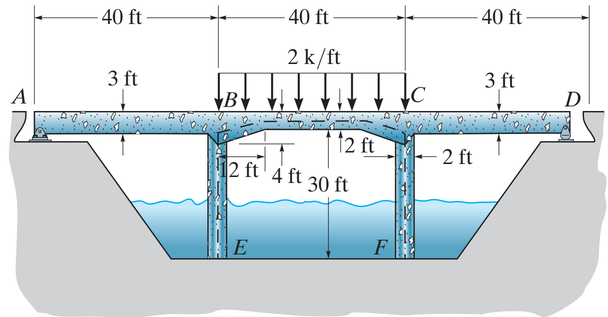
$M_{DB} = 28.3 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{DC} = -28.3 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{AB} = M_{CD} = 0$ **Ans.**



13–11. Use the moment-distribution method to determine the moment at each joint of the symmetric bridge frame. Supports F and E are fixed and B and C are fixed connected. The haunches are straight so use Table 13–2. Assume E is constant and the members are each 1 ft thick.



The necessary data for member BC can be found from Table 13–1.

Here,

$$a_B = a_C = \frac{12}{40} = 0.3$$

$$r_B = r_C = \frac{4-2}{2} = 1.0$$

Thus,

$C_{BC} = C_{CB} = 0.705 \qquad K_{BC} = K_{CB} = 10.85$

Since the stimulate and loading are symmetry, Eq. 13–14 applicable.

Here,

$$K_{BC}\frac{K_{BC}EI_C}{L_{BC}} = \frac{10.85E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.18083E$$

$$K'_{BC} = K_{BC}(1 - C_{BC}) = 0.18083E(1 - 0.705) = 0.05335E$$

The fixed end moment are given by

$$(\text{FEM})_{BC} = -0.1034(2)(40^2) = -330.88 \text{ k}\cdot\text{ft}$$

Since member AB and BE are prismatic

$$K_{BE} = \frac{4EI}{L_{BA}} = \frac{4E\left[\frac{1}{12}(1)(2^3)\right]}{30} = 0.08889E$$

$$K_{BA} = \frac{3EI}{L_{BA}} = \frac{3E\left[\frac{1}{12}(1)(3^3)\right]}{40} = 0.16875E$$

Tabulating these data,

Joint	A	B			E
Member	AB	BA	BC	BE	EB
K		$0.16875E$	$0.05335E$	$0.08889E$	
DF	1	0.5426	0.1715	0.2859	0
COF		0	0.705	0.5	
FEM			−330.88		
Dist		179.53	56.75	94.60	
CO					47.30
$\sum M$		179.53	−274.13	94.60	47.30

13–11. Continued

Thus,

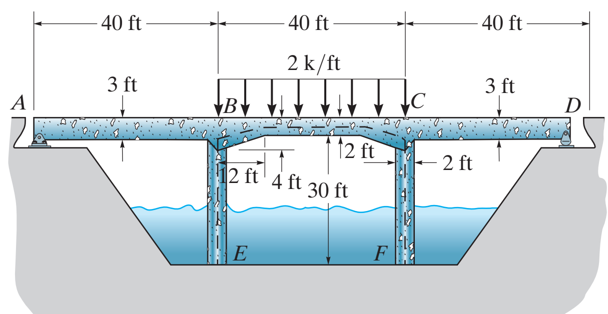
$M_{CD} = M_{BA} = 179.53 \text{ k} \cdot \text{ft} = 180 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CF} = M_{BE} = 94.60 \text{ k} \cdot \text{ft} = 94.6 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = M_{BC} = -274.13 \text{ k} \cdot \text{ft} = 274 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{FC} = M_{EB} = 47.30 \text{ k} \cdot \text{ft} = 47.3 \text{ k} \cdot \text{ft}$ **Ans.**

***13–12.** Solve Prob. 13–11 using the slope-deflection equations.



The necessary data for member BC can be found from Table 13–1

Here,

$$a_B = a_C = \frac{12}{40} = 0.3 \qquad r_B = r_C = \frac{4-2}{2} = 1.0$$

Thus,

$$C_{BC} = C_{CB} = 0.705 \qquad K_{BC} = K_{CB} = 10.85$$

Then,

$$K_{BC} = K_{CB} = \frac{K_{BC} E I_C}{L_{BC}} = \frac{10.85E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.1808E$$

The fixed end moment's are given by

$$(\text{FEM})_{BC} = -0.1034(2)(40^2) = -330.88 \text{ k} \cdot \text{ft}.$$

For member BC, applying Eq. 13–8. Here, due to symatry,

$\theta_C = -\theta_B$

$$M_N = K_N[\theta_N + C_N\theta_F - \psi(HC_N)] + (\text{FEM})_N$$

$$M_{BC} = 0.1808E[\theta_B + 0.705(-\theta_B) - 0\,(1 + 0.705)] + (-330.88)$$
$$= 0.053346E\theta_B - 330.88 \qquad (1)$$

13–12. Continued

For prismatic member BE, applying Eq. 11–8.

$$M_N = 2EK(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BE} = 2E\left[\frac{\frac{1}{12}(1)(3)^3}{30}\right][2\theta_B + 0 - 3(0)] + 0 = 0.08889E\theta_B \qquad (2)$$

$$M_{EB} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{30}\right][2(0) + \theta_B - 3(0) + 0 = 0.04444E\theta_B \qquad (3)$$

For prismatic member AB, applying Eq. 11–10

$$M_N = 3EK(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right](\theta_B - 0) + 0 = 0.16875E\theta_B \qquad (4)$$

Moment equilibrium of joint B gives

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$0.16875E\theta_B + 0.053346E\theta_B - 330.88 + 0.08889E\theta_B = 0$$

$$\theta_B = \frac{1063.97}{E}$$

Substitute this result into Eq. (1) to (4)

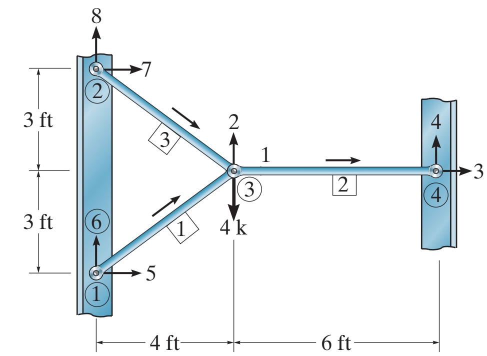
$M_{CB} = M_{BC} = -274.12\ \text{k}\cdot\text{ft} = -274\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CF} = M_{BE} = 94.58\ \text{k}\cdot\text{ft} = 94.6\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{FC} = M_{EB} = 47.28\ \text{k}\cdot\text{ft} = 47.3\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CD} = M_{BA} = 179.55\ \text{k}\cdot\text{ft} = 180\ \text{k}\cdot\text{ft}$ **Ans.**

14–1. Determine the stiffness matrix **K** for the assembly. Take $A = 0.5\text{ in}^2$ and $E = 29(10^3)$ ksi for each member.



Member 1: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-0}{5} = 0.6$

$$\mathbf{k}_1 = \frac{AE}{60}\begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix}$$

Member 2: $\lambda_x = \dfrac{10-4}{6} = 1; \qquad \lambda_y = \dfrac{3-3}{6} = 0$

$$\mathbf{k}_2 = \frac{AE}{72}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

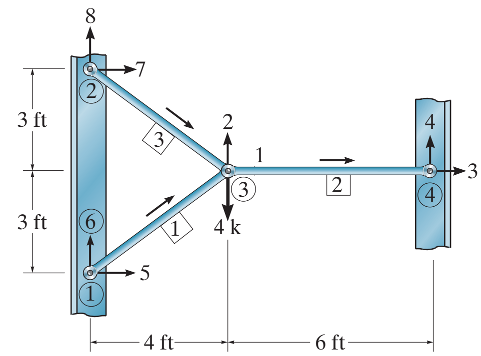
Member 3: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-6}{5} = -0.6$

$$\mathbf{k}_3 = \frac{AE}{60}\begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix}$$

Assembly stiffness matrix: $K = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$

$$\mathbf{K} = \begin{bmatrix} 510.72 & 0 & -201.39 & 0 & -154.67 & -116 & -154.67 & 116 \\ 0 & 174 & 0 & 0 & -116 & -87.0 & 116 & -87.0 \\ -201.39 & 0 & 201.39 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -154.67 & -116 & 0 & 0 & 154.67 & 116 & 0 & 0 \\ -116 & -87.0 & 0 & 0 & 116 & 87.0 & 0 & 0 \\ -154.67 & 116 & 0 & 0 & 0 & 0 & 154.67 & -116 \\ 116 & -87.0 & 0 & 0 & 0 & 0 & -116 & 87.0 \end{bmatrix}$$ **Ans.**

14–2. Determine the horizontal and vertical displacements at joint ③ of the assembly in Prob. 14–1.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad \mathbf{Q}_k = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Use the assembly stiffness matrix of Prob. 14–1 and applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 0 \\ -4 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 510.72 & 0 & -201.39 & 0 & -154.67 & -116 & -154.67 & 116 \\ 0 & 174 & 0 & 0 & -116 & -87.0 & 116 & -87.0 \\ -201.39 & 0 & 201.39 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -154.67 & -116 & 0 & 0 & 154.67 & 116 & 0 & 0 \\ -116 & -87.0 & 0 & 0 & 116 & 87.0 & 0 & 0 \\ -154.67 & 116 & 0 & 0 & 0 & 0 & 154.67 & -116 \\ 116 & -87.0 & 0 & 0 & 0 & -0 & -116 & 87.0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Partition matrix

$$0 = 510.72(D_1) + 0(D_2)$$

$$-4 = 0(D_1) + 174(D_2)$$

Solving

$$D_1 = 0$$

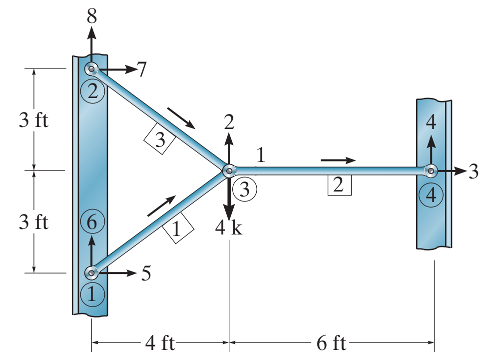
$$D_2 = -0.022990 \text{ in.}$$

Thus,

$$D_1 = 0 \qquad \textbf{Ans.}$$

$$D_2 = -0.0230 \text{ in.} \qquad \textbf{Ans.}$$

14–3. Determine the force in each member of the assembly in Prob. 14–1.



From Prob. 14–2.

$$D_1 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = 0 \quad D_2 = -0.02299$$

To calculate force in each member, use Eq. 14–23.

$$q_F = \frac{AE}{L}[-\lambda_x \quad -\lambda_y \quad \lambda_x \quad \lambda_y]\begin{bmatrix} D_{N_x} \\ D_{N_y} \\ D_{F_x} \\ D_{F_y} \end{bmatrix}$$

Member 1: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-0}{5} = 0.6$

$$q_1 = \frac{AE}{L}[-0.8 \quad -0.6 \quad 0.8 \quad 0.6]\begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.02299 \end{bmatrix}$$

$$q_1 = \frac{0.5(29(10^3))}{60}(0.6)(-0.02299) = -3.33\text{ k} = 3.33\text{ k (C)}$$ **Ans.**

Member 2: $\lambda_x = \dfrac{10-4}{6} = 1; \qquad \lambda_y = \dfrac{3-3}{6} = 0$

$$q_2 = \frac{AE}{L}[-1 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0 \\ -0.02299 \\ 0 \\ 0 \end{bmatrix}$$

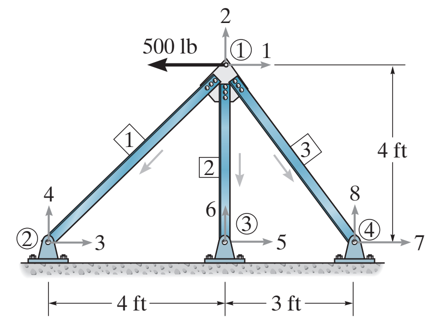
$$q_2 = 0$$ **Ans.**

Member 3: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-6}{5} = -0.6$

$$q_3 = \frac{AE}{L}[-0.8 \quad 0.6 \quad 0.8 \quad -0.6]\begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.02299 \end{bmatrix}$$

$$q_3 = \frac{0.5(29(10^3))}{60}(-0.6)(-0.02299) = 3.33\text{ k (T)}$$ **Ans.**

***14–4.** Determine the stiffness matrix **K** for the truss. Take $A = 0.75 \text{ in}^2$, $E = 29(10^3)$ ksi.



Member 1: $\lambda_x = \dfrac{0 - 4}{\sqrt{32}} = -0.7071 \qquad \lambda_y = \dfrac{0 - 4}{\sqrt{32}} = -0.7071$

$$\mathbf{k}_1 = AE\begin{bmatrix} 0.08839 & 0.08839 & -0.08839 & -0.08839 \\ 0.08839 & 0.08839 & -0.08839 & -0.08839 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 \end{bmatrix}$$

Member 2: $\lambda_x = \dfrac{4 - 4}{4} = 0 \qquad \lambda_y = \dfrac{0 - 4}{4} = -1$

$$\mathbf{k}_2 = AE\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & -0.25 \\ 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0.25 \end{bmatrix}$$

Member 3: $\lambda_x = \dfrac{7 - 4}{5} = 0.6 \qquad \lambda_y = \dfrac{0 - 4}{5} = -0.8$

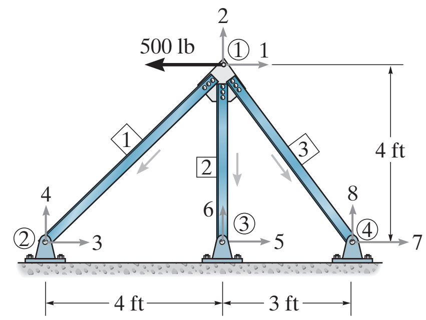
$$\mathbf{k}_3 = AE\begin{bmatrix} 0.072 & -0.096 & -0.072 & 0.096 \\ -0.096 & 0.128 & 0.096 & -0.128 \\ -0.072 & 0.096 & 0.072 & -0.096 \\ 0.096 & -0.128 & -0.096 & 0.128 \end{bmatrix}$$

Structure stiffness matrix

$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$

$$\mathbf{K} = AE\begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.128 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.128 & 0 & 0 & 0 & 0 & -0.096 & 0.128 \end{bmatrix} \quad \textbf{Ans.}$$

14–5. Determine the horizontal displacement of joint ① and the force in member [2]. Take $A = 0.75 \text{ in}^2$, $E = 29(10^3)$ ksi.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_k = \begin{bmatrix} -500 \\ 0 \end{bmatrix}$$

Use the structure stiffness matrix of Prob. 14–4 and applying $\mathbf{Q} = \mathbf{KD}$. We have

$$\begin{bmatrix} -500 \\ 0 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = AE \begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.128 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.128 & 0 & 0 & 0 & 0 & -0.096 & 0.128 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Partition matrix

$$\begin{bmatrix} -500 \\ 0 \end{bmatrix} = AE \begin{bmatrix} 0.16039 & -0.00761 \\ -0.00761 & 0.46639 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-500 = AE(0.16039D_1 - 0.00761D_2) \qquad (1)$$

$$0 = AE(-0.00761D_1 + 0.46639D_2) \qquad (2)$$

Solving Eq. (1) and (2) yields:

$$D_1 = \frac{-3119.82}{AE} = \frac{-3119.85(12 \text{ in./ft})}{0.75 \text{ in}^2(26)(10^6) \text{ lb/in}^2} = -0.00172 \text{ in.}$$ **Ans.**

$$D_2 = \frac{-50.917}{AE}$$

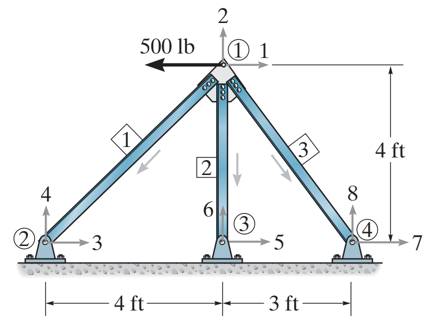
For member 2

$\lambda_x = 0, \quad \lambda_y = -1, \quad L = 4$ ft

$$q_2 = \frac{AE}{4}[0 \quad 1 \quad 0 \quad -1]\frac{1}{AE}\begin{bmatrix} -3119.85 \\ -50.917 \\ 0 \\ 0 \end{bmatrix}$$

$$= -12.73 \text{ lb} = 12.7 \text{ lb (C)}$$ **Ans.**

14–6. Determine the force in member $\boxed{2}$ if its temperature is increased by 100°F. Take $A = 0.75\text{ in}^2$, $E = 29(10^3)$ ksi, $\alpha = 6.5(10^{-6})/°\text{F}$.



$$\begin{bmatrix} (Q_1)_0 \\ (Q_2)_0 \\ (Q_3)_0 \\ (Q_4)_0 \end{bmatrix} = AE(6.5)(10^{-6})(+100)\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} = AE\begin{bmatrix} 0 \\ -650 \\ 0 \\ 650 \end{bmatrix}(10^{-4})$$

Use the structure stiffness matrix of Prob. 14–4.

$$\begin{bmatrix} -500 \\ 0 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = AE\begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.1280 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.1280 & 0 & 0 & 0 & 0 & -0.096 & 0.1280 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$+ AE\begin{bmatrix} 0 \\ -650 \\ 0 \\ 650 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}(10^{-6})$$

$$\frac{-500}{(0.75)(29)(10^6)} = 0.16039D_1 - 0.00761D_2 + 0$$

$$0 = -0.00761D_1 + 0.46639D_2 - 650(10^{-6})$$

Solving yields

$$D_1 = -77.837(10^{-6})\text{ ft}$$

$$D_2 = 1392.427(106^{-6})\text{ ft}$$

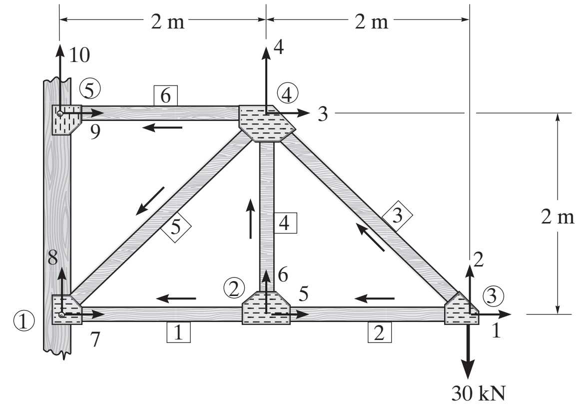
For member 2

$$\lambda_x = 0, \qquad \lambda_y = -1, \qquad L = 4\text{ ft}$$

$$q_2 = \frac{0.75(29)(10^6)}{4}[0 \quad 1 \quad 0 \quad -1]\begin{bmatrix} -77.837 \\ 1392.427 \\ 0 \\ 0 \end{bmatrix}(10^{-6}) - 0.75(29)(10^6)(6.5)(10^{-6})(100)$$

$$= 7571.32 - 14\,137.5 = -6566.18\text{ lb} = 6.57\text{ k(C)}$$ **Ans.**

14–7. Determine the stiffness matrix **K** for the truss. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



The origin of the global coordinate system will be set at joint ①.

For member $\boxed{1}$, $L = 2$ m. $\quad \lambda_x = \dfrac{0-2}{2} = -1 \qquad \lambda_y = \dfrac{0-0}{2} = 0$

$$\mathbf{k}_1 = \frac{0.0015[200(10^9)]}{2}\begin{array}{c} \begin{matrix} 5 & 6 & 7 & 8 \end{matrix} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

$$= \begin{array}{c} \begin{matrix} 5 & 6 & 7 & 8 \end{matrix} \\ \left[\begin{array}{cccc} 150(10^6) & 0 & -150(10^6) & 0 \\ 0 & 0 & 0 & 0 \\ -150(10^6) & 0 & 150(10^6) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

For member $\boxed{2}$, $L = 2$ m. $\quad \lambda_x = \dfrac{2-4}{2} = -1 \qquad \lambda_y = \dfrac{0-0}{2} = 0$

$$\mathbf{k}_2 = \frac{0.0015[200(10^9)]}{2}\begin{array}{c} \begin{matrix} 1 & 2 & 5 & 6 \end{matrix} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

$$= \begin{array}{c} \begin{matrix} 1 & 2 & 5 & 6 \end{matrix} \\ \left[\begin{array}{cccc} 150(10^6) & 0 & -150(10^6) & 0 \\ 0 & 0 & 0 & 0 \\ -150(10^6) & 0 & 150(10^6) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

For member $\boxed{3}$, $L = 2\sqrt{2}$m. $\quad \lambda_x = \dfrac{2-4}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{2} \qquad \lambda_y = \dfrac{2-0}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$$\mathbf{k}_3 = \frac{0.0015[200(10^9)]}{2\sqrt{2}}\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \left[\begin{array}{cccc} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$= \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \left[\begin{array}{cccc} 53.033(10^6) & -53.033(10^6) & -53.033(10^6) & 53.033(10^6) \\ -53.033(10^6) & 53.033(10^6) & 53.033(10^6) & -53.033(10^6) \\ -53.033(10^6) & 53.033(10^6) & 53.033(10^6) & -53.033(10^6) \\ 53.033(10^6) & -53.033(10^6) & -53.033(10^6) & 53.033(10^6) \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

14–7. Continued

For member $\boxed{4}$, $L = 2$ m.

$$\lambda_x = \frac{2-2}{2} = 0 \qquad \lambda_y = \frac{2-0}{2} = 1$$

$$\mathbf{k}_4 = \frac{0.0015[200(10^9)]}{2}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 150(10^6) & 0 & -150(10^6) \\ 0 & 0 & 0 & 0 \\ 0 & -150(10^6) & 0 & 150(10^6) \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$

For member $\boxed{5}$, $L = 2\sqrt{2}$ m.

$$\lambda_x = \frac{0-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \lambda_y = \frac{2-0}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\mathbf{k}_5 = \frac{0.0015[200(10^9)]}{2\sqrt{2}}\begin{array}{c} \begin{array}{cccc} 3 & 4 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{array}\right] \end{array}\begin{array}{c} \\ 3 \\ 4 \\ 7 \\ 8 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 3 & 4 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} 53.033(10^6) & 53.033(10^6) & -53.033(10^6) & -53.033(10^6) \\ -53.033(10^6) & 53.033(10^6) & -53.033(10^6) & -53.033(10^6) \\ -53.033(10^6) & -53.033(10^6) & 53.033(10^6) & 53.033(10^6) \\ 53.033(10^6) & -53.033(10^6) & 53.033(10^6) & 53.033(10^6) \end{array}\right] \end{array}\begin{array}{c} \\ 3 \\ 4 \\ 7 \\ 8 \end{array}$$

For member $\boxed{6}$, $L = 2$ m.

$$\lambda_x = \frac{0-2}{2} = -1 \qquad \lambda_y = \frac{2-2}{2} = 0$$

$$\mathbf{k}_6 = \frac{0.0015[200(10^9)]}{2}\begin{array}{c} \begin{array}{cccc} 3 & 4 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 3 \\ 4 \\ 9 \\ 10 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 3 & 4 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} 150(10^6) & 0 & -150(10^6) & 0 \\ 0 & 0 & 0 & 0 \\ -150(10^6) & 0 & 150(10^6) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 3 \\ 4 \\ 9 \\ 10 \end{array}$$

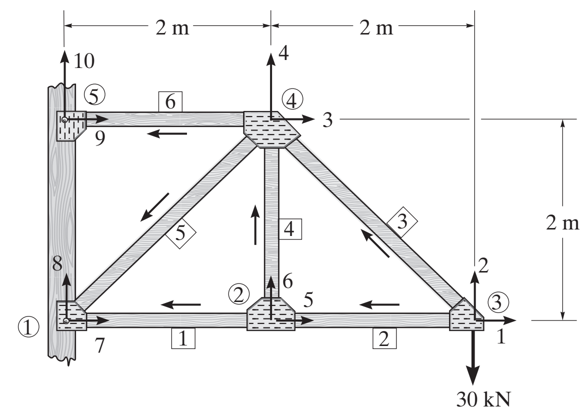
14–7. Continued

Structure stiffness matrix is a 10×10 matrix since the highest code number is 10. Thus,

$$
\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix}
$$

$$
\mathbf{K} = \begin{bmatrix}
203.033 & -53.033 & -53.033 & 53.033 & -150 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 53.033 & -53.033 & 0 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 256.066 & 0 & 0 & 0 & -53.033 & -53.033 & -150 & 0 \\
53.033 & -53.033 & 0 & 256.066 & 0 & -150 & -53.033 & -53.033 & 0 & 0 \\
-150 & 0 & 0 & 0 & 300 & 0 & -150 & 0 & 0 & 0 \\
0 & 0 & 0 & -150 & 0 & 150 & 0 & 0 & 0 & 0 \\
0 & 0 & -53.033 & -53.033 & -150 & 0 & 203.033 & 53.033 & 0 & 0 \\
0 & 0 & -53.033 & -53.033 & 0 & 0 & 53.033 & 53.033 & 0 & 0 \\
0 & 0 & -150 & 0 & 0 & 0 & 0 & 0 & 150 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix}
(10^6) \qquad \textbf{Ans.}
$$

***14–8.** Determine the vertical displacement at joint ② and the force in member [5]. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa.



Here,

$$
Q_k = \begin{bmatrix} 0 \\ -30(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}
\qquad
D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\begin{matrix} 7 \\ 8 \\ 9 \\ 10 \end{matrix}
$$

Then, applying $\mathbf{Q} = \mathbf{KD}$

$$
\begin{bmatrix} 0 \\ -30(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \end{bmatrix}
=
\begin{bmatrix}
203.033 & -53.033 & -53.033 & 53.033 & -150 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 53.033 & -53.033 & 0 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 256.066 & 0 & 0 & 0 & -53.033 & -53.033 & -150 & 0 \\
53.033 & -53.033 & 0 & 256.066 & 0 & -150 & -53.033 & -53.033 & 0 & 0 \\
-150 & 0 & 0 & 0 & 300 & 0 & -150 & 0 & 0 & 0 \\
0 & 0 & 0 & -150 & 0 & 150 & 0 & 0 & 0 & 0 \\ \hline
0 & 0 & -53.033 & -53.033 & -150 & 0 & 203.033 & 53.033 & 0 & 0 \\
0 & 0 & -53.033 & -53.033 & 0 & 0 & 53.033 & 53.033 & 0 & 0 \\
0 & 0 & -150 & 0 & 0 & 0 & 0 & 0 & 150 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
(10^6)
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

14–8. Continued

From the matrix partition, $Q_k = K_{11}D_u + K_{12}D_k$ is given by

$$\begin{bmatrix} 0 \\ -30(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 203.033 & -53.033 & -53.033 & 53.033 & -150 & 0 \\ -53.033 & 53.033 & 53.033 & -53.033 & 0 & 0 \\ -53.033 & 53.033 & 256.066 & 0 & 0 & 0 \\ -53.033 & -53.033 & 0 & 256.066 & 0 & -150 \\ -150 & 0 & 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & -150 & 0 & 150 \end{bmatrix}(10^6)\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = [203.033D_1 - 53.033D_2 - 53.033D_3 + 53.033D_4 - 150D_5](10^6) \quad (1)$$

$$-30(10^3) = [-53.033D_1 + 53.033D_2 + 53.033D_3 - 53.033D_4](10^6) \quad (2)$$

$$0 = [-53.033D_1 + 53.033D_2 + 256.066D_3](10^6) \quad (3)$$

$$0 = [53.033D_1 - 53.033D_2 + 256.066D_4 - 150D_6](10^6) \quad (4)$$

$$0 = [-150D_4 + 300D_5](10^6) \quad (5)$$

$$0 = [-150D_4 + 150D_6](10^6) \quad (6)$$

Solving Eqs (1) to (6),

$D_1 = -0.0004$ m $\quad D_2 = -0.0023314$ m $\quad D_3 = 0.0004$ m $\quad D_4 = -0.00096569$ m

$D_5 = -0.0002$ m $\quad D_6 = 0.00096569$ m $\quad = 0.000966$ m **Ans.**

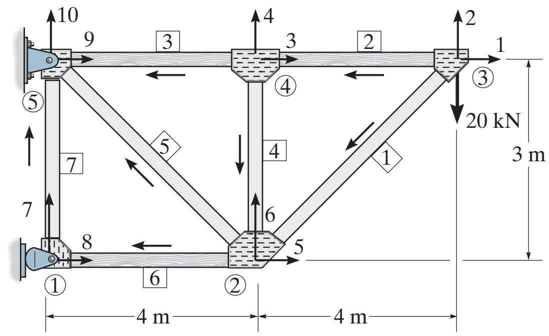
Force in member $\boxed{5}$. Here $\lambda_x = -\frac{\sqrt{2}}{2}$, $\lambda_y = -\frac{\sqrt{2}}{2}$ and $L = 2\sqrt{2}$ m

Applying Eqs 14–23,

$$(q_5)_F = \frac{0.0015[200(10^9)]}{2\sqrt{2}}\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}\begin{bmatrix} \\ \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 7 \\ 8 \end{matrix}$$

$$= -42.4 \text{ kN}$$ **Ans.**

14–9. Determine the stiffness matrix **K** for the truss. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



The origin of the global coordinate system will be set at joint ①.

For member [1], $L = 5$ m, $\lambda_x = \dfrac{4-8}{5} = -0.8$ and $\lambda_y = \dfrac{0-3}{5} = -0.6$

$$\mathbf{k}_1 = \frac{0.0015[200(10^9)]}{5}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 38.4 & 28.8 & -38.4 & -28.8 \\ 28.8 & 21.6 & -28.8 & -21.6 \\ -38.4 & -28.8 & 38.4 & 28.8 \\ -28.8 & -21.6 & 28.8 & 21.6 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}(10^6)$$

For member [2], $L = 4$ m, $\lambda_x = \dfrac{4-8}{4} = -1$ and $\lambda_y = \dfrac{3-3}{0} = 0$

$$\mathbf{k}_2 = \frac{0.0015[200(10^9)]}{4}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 75 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 \\ -75 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}(10^6)$$

14–9. Continued

For member $\boxed{3}$, $L = 4$ m, $\lambda_x = \dfrac{0 - 4}{4} = -1$ and $\lambda_y = \dfrac{3 - 3}{4} = 0$

$$\mathbf{k}_3 = \frac{0.0015[200(10^9)]}{4}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 9 \\ 10 \end{matrix} \quad (\text{columns } 3,\ 4,\ 9,\ 10)$$

$$= \begin{bmatrix} 75 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 \\ -75 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 9 \\ 10 \end{matrix}(10^6) \quad (\text{columns } 3,\ 4,\ 9,\ 10)$$

For member $\boxed{4}$, $L = 3$ m, $\lambda_x = \dfrac{4 - 4}{3} = 0$ and $\lambda_y = \dfrac{0 - 3}{3} = -1$

$$\mathbf{k}_4 = \frac{0.0015[200(10^9)]}{3}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix} \quad (\text{columns } 3,\ 4,\ 5,\ 6)$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & -100 \\ 0 & 0 & 0 & 0 \\ 0 & -100 & 0 & 100 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix}(10^6) \quad (\text{columns } 3,\ 4,\ 5,\ 6)$$

For member $\boxed{5}$, $L = 5$ m, $\lambda_x = \dfrac{0 - 4}{5} = -0.8$ and $\lambda_y = \dfrac{3 - 0}{5} = 0.6$

$$\mathbf{k}_5 = \frac{0.0015[200(10^9)]}{5}\begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix}\begin{matrix} 5 \\ 6 \\ 9 \\ 10 \end{matrix} \quad (\text{columns } 5,\ 6,\ 9,\ 10)$$

$$= \begin{bmatrix} 38.4 & -28.8 & -38.4 & 28.8 \\ -28.8 & 21.6 & 28.8 & -21.6 \\ -38.4 & 28.8 & 38.4 & -28.8 \\ 28.8 & -21.6 & -28.8 & 21.6 \end{bmatrix}\begin{matrix} 5 \\ 6 \\ 9 \\ 10 \end{matrix}(10^6) \quad (\text{columns } 5,\ 6,\ 9,\ 10)$$

14–9. Continued

For member $\boxed{6}$, $L = 4$ m, $\lambda_x = \dfrac{0-4}{4} = -1$ and $\lambda_y = \dfrac{0-0}{4} = 0$

$$\mathbf{k}_6 = \frac{0.0015[200(10^9)]}{4}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 8 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}\begin{array}{c} 5 \\ 6 \\ 8 \\ 7 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 5 & 6 & 8 & 7 \end{array} \\ \begin{bmatrix} 75 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 \\ -75 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}\begin{array}{c} 5 \\ 6 \\ 8 \\ 7 \end{array}(10^6)$$

For member $\boxed{7}$, $L = 3$ m, $\lambda_x = \dfrac{0-0}{3} = 0$ and $\lambda_y = \dfrac{3-0}{3} = 1$

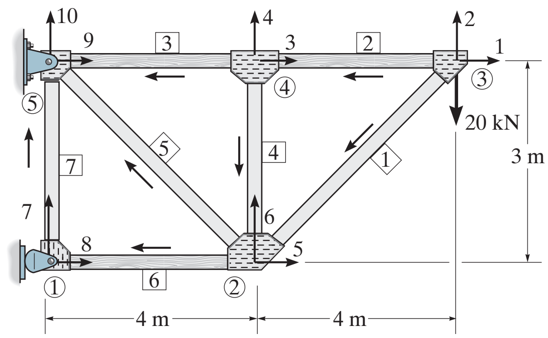
$$\mathbf{k}_7 = \frac{0.0015[200(10^9)]}{3}\begin{array}{c} \begin{array}{cccc} 8 & 7 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array}\begin{array}{c} 8 \\ 7 \\ 9 \\ 10 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 8 & 7 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & -100 \\ 0 & 0 & 0 & 0 \\ 0 & -100 & 0 & 100 \end{bmatrix} \end{array}\begin{array}{c} 8 \\ 7 \\ 9 \\ 10 \end{array}(10^6)$$

Structure stiffness matrix is a 10×10 matrix since the highest code number is 10.
Thus,

$$\begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 & 0 & 0 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 & 0 & 0 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 & 0 & 0 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 & -75 & -38.4 & 28.8 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 & 0 & 28.8 & -21.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -100 \\ 0 & 0 & 0 & 0 & -75 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & -75 & 0 & -38.4 & 28.8 & 0 & 0 & 113.4 & -28.8 \\ 0 & 0 & 0 & 0 & 28.8 & -21.6 & -100 & 0 & -28.8 & 121.6 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array}(10^6) \qquad \textbf{Ans.}$$

14–10. Determine the force in member [5]. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



Here,

$$Q_k = \begin{bmatrix} 0 \\ -20(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} \qquad D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 8 \\ 9 \\ 10 \end{matrix}$$

Then applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 0 \\ -20(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline Q \\ Q_9 \\ Q_{10} \end{bmatrix} = \left[\begin{array}{ccccccc|ccc} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 & 0 & 0 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 & 0 & 0 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 & 0 & 0 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 & -75 & -38.4 & 28.8 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 & 0 & 28.8 & -21.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -100 \\ \hline 0 & 0 & 0 & 0 & -75 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & -75 & 0 & -38.4 & 28.8 & 0 & 0 & 113.4 & -28.8 \\ 0 & 0 & 0 & 0 & 28.8 & -21.6 & -100 & 0 & -28.8 & 121.6 \end{array}\right](10^6) \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$ is given by

$$\begin{bmatrix} 0 \\ -20(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & 28.8 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}(10^6) \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = (113.4D_1 + 28.8D_2 - 75D_3 - 384D_5 - 28.8D_6)(10^6) \qquad (1)$$

$$-20(10^3) = (28.8D_1 + 21.6D_2 - 28.8D_5 - 21.6D_6(10^6)) \qquad (2)$$

$$0 = (-75D_1 + 150D_3)(10^6) \qquad (3)$$

$$0 = (100D_4 - 100D_6)(10^6) \qquad (4)$$

$$0 = (-38.4D_1 - 28.8D_2 + 151.8D_5)(10^6) \qquad (5)$$

$$0 = (-28.8D_1 - 21.6D_2 + 100D_4 + 143.2D_6)(10^6) \qquad (6)$$

$$0 = (100D_7)(10^6) \qquad (7)$$

14–10. Continued

Solving Eqs (1) to (7)

$D_1 = 0.000711 \quad D_2 = -0.00470 \quad D_3 = 0.000356 \quad D_4 = -0.00187$

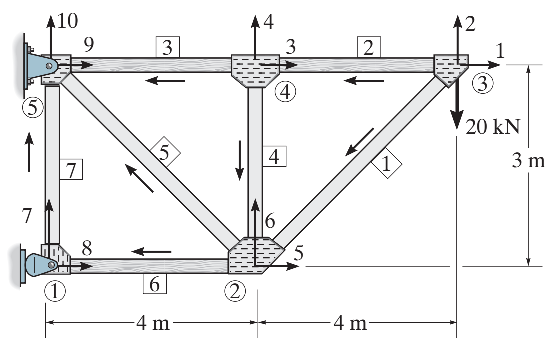
$D_5 = -0.000711 \quad D_6 = -0.00187 \quad D_7 = 0$

Force in member $\boxed{5}$. Here $L = 5\ m$, $\lambda_x = -0.8$ and $\lambda_y = 0.6$.

$$(q_5)_F = \frac{0.0015[200(10^9)]}{5}\begin{bmatrix} 0.8 & -0.6 & -0.8 & 0.6 \end{bmatrix}\begin{bmatrix} -0.000711 \\ -0.00187 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 5 \\ 6 \\ 9 \\ 10 \end{matrix}$$

$= 33.3\ \text{kN}$ **Ans.**

14–11. Determine the vertical displacement of node ② if member $\boxed{6}$ was 10 mm too long before it was fitted into the truss. For the solution, remove the 20-k load. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



For member $\boxed{6}$, $L = 4$ m, $\lambda_x = -1, \lambda_y = 0$ and $\Delta_L = 0.01$ m. Thus,

$$\begin{bmatrix} (Q_5)_0 \\ (Q_6)_0 \\ (Q_7)_0 \\ (Q_8)_0 \end{bmatrix} = \frac{0.00015[200(10^9)](0.001)}{4}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.75 \\ 0 \\ 0.75 \\ 0 \end{bmatrix}\begin{matrix} 5 \\ 6 \\ 8 \\ 7 \end{matrix}(10^6)$$

Also

$$Q_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} \quad \text{and } D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 8 \\ 9 \\ 10 \end{matrix}$$

Applying $\mathbf{Q} = \mathbf{KD} + \mathbf{Q}_0$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_8 \\ Q_9 \\ Q_{10} \end{bmatrix} = \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 & 0 & 0 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 & 0 & 0 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 & 0 & 0 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 & -75 & -38.4 & 28.8 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 & 0 & 28.8 & -21.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -100 \\ 0 & 0 & 0 & 0 & -75 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & -75 & 0 & -38.4 & 28.8 & 0 & 0 & 113.4 & -28.8 \\ 0 & 0 & 0 & 0 & 28.8 & -21.6 & -100 & 0 & -28.8 & 121.6 \end{bmatrix}(10^6)\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.75 \\ 0 \\ 0 \\ 0.75 \\ 0 \\ 0 \end{bmatrix}(10^6)$$

14–11. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k + (\mathbf{Q}_k)_0$

$$\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0\\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0\\ -75 & 0 & 150 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 100 & 0 & -100 & 0\\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0\\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 100\end{bmatrix}(10^6)\begin{bmatrix}D_1\\D_2\\D_3\\D_4\\D_5\\D_6\\D_7\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\\-0.75\\0\\0\end{bmatrix}(10^6)$$

Expanding this matrix equality,

$0 = (113.4D_1 + 28.8D_2 - 75D_3 - 38.4D_5 - 28.8D_6)(10^6)$ (1)

$0 = (28.8D_1 + 21.6D_2 - 28.8D_5 - 21.6D_6)(10^6)$ (2)

$0 = (-75D_1 + 150D_3)(10^6)$ (3)

$0 = (100D_4 - 100D_6)(10^6)$ (4)

$0 = (-38.4D_1 - 28.8D_2 + 151.8D_5)(10^6) + [-0.75(10^6)]$ (5)

$0 = (-28.8D_1 - 21.6D_2 - 100D_4 + 143.2D_6)(10^6)$ (6)

$0 = (100D_7)(10^6)$ (7)

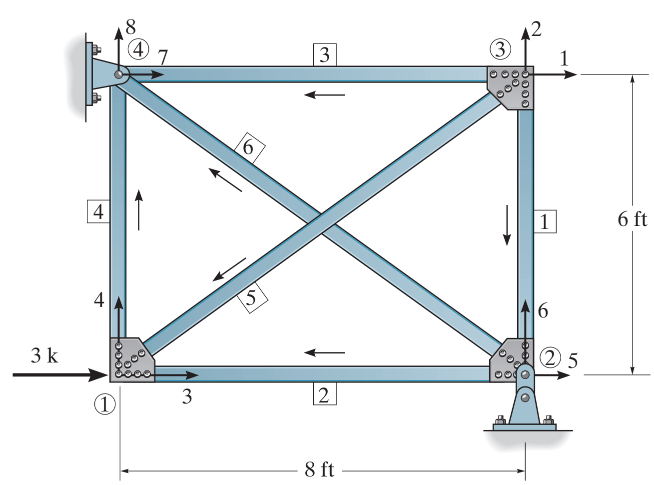
Solving Eqs. (1) to (7)

$D_1 = 0 \quad D_2 = 0.02667 \quad D_3 = 0 \quad D_4 = 0.01333$

$D_5 = 0.01 \quad D_6 = 0.01333 \quad D_7 = 0$

$D_6 = 0.0133$ m **Ans.**

***14–12.** Determine the stiffness matrix **K** for the truss. Take $A = 2$ in^2, $E = 29(10^3)$ ksi.



The origin of the global coordinate system is set at joint ①.

For member [1], $L = 6(12) = 72$ in.,

$$\lambda_x = \frac{8-8}{6} = 0 \text{ and } \lambda_y = \frac{0-6}{6} = -1$$

$$\mathbf{k}_1 = \frac{2[29(10^3)]}{72}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1\\ 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 1\end{bmatrix}\begin{matrix}1\\2\\5\\6\end{matrix}$$

(columns: 1 2 5 6)

14–12. Continued

$$= \begin{array}{c} \begin{matrix} 1 & 2 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 805.56 & 0 & -805.56 \\ 0 & 0 & 0 & 0 \\ 0 & -805.56 & 0 & 805.56 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \end{matrix} \end{array}$$

For member $\boxed{2}$, $L = 8(12) = 96$ in., $\lambda_x = \dfrac{0-8}{8} = -1$ and $\lambda_y = \dfrac{0-0}{8} = 0.$

$$\mathbf{k}_2 = \frac{2[29(10^3)]}{96} \begin{array}{c} \begin{matrix} 5 & 6 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 3 \\ 4 \end{matrix} \end{array}$$

$$= \begin{array}{c} \begin{matrix} 5 & 6 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 604.17 & 0 & -604.17 & 0 \\ 0 & 0 & 0 & 0 \\ -604.17 & 0 & 604.17 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 3 \\ 4 \end{matrix} \end{array}$$

For member $\boxed{3}$, $L = 8(12) = 96$ in., $\lambda_x = \dfrac{0-8}{8} = -1$ and $\lambda_y = \dfrac{6-6}{8} = 0.$

$$\mathbf{k}_3 = \frac{2[29(10^3)]}{96} \begin{array}{c} \begin{matrix} 1 & 2 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 7 \\ 8 \end{matrix} \end{array}$$

$$= \begin{array}{c} \begin{matrix} 1 & 2 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 604.17 & 0 & -604.17 & 0 \\ 0 & 0 & 0 & 0 \\ -604.17 & 0 & 604.17 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 7 \\ 8 \end{matrix} \end{array}$$

For member $\boxed{4}$, $L = 6(12) = 72$ in., $\lambda_x = \dfrac{0-0}{6} = 0$, and $\lambda_y = \dfrac{6-0}{6} = 1$

$$\mathbf{k}_4 = \frac{2[29(10^3)]}{72} \begin{array}{c} \begin{matrix} 3 & 4 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 7 \\ 8 \end{matrix} \end{array}$$

14–12. Continued

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 805.56 & 0 & -805.56 \\ 0 & 0 & 0 & 0 \\ 0 & -805.56 & 0 & 805.56 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 7 \\ 8 \end{matrix} \quad \text{(columns 3, 4, 7, 8)}$$

For member $\boxed{5}$, $L = 10(12) = 120$ in., $\lambda_x = \dfrac{0 - 8}{10} = -0.8$ and $\lambda_y = \dfrac{0 - 6}{10} = -0.6$.

$$\mathbf{k}_5 = \frac{2[29(10^3)]}{120} \begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \text{(columns 1, 2, 3, 4)}$$

$$= \begin{bmatrix} 309.33 & 232 & -309.33 & -232 \\ 232 & 174 & -232 & -174 \\ -309.33 & -232 & 309.33 & 232 \\ -232 & -174 & 232 & 174 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \text{(columns 1, 2, 3, 4)}$$

For member $\boxed{6}$, $L = 10(12) = 120$ in., $\lambda_x = \dfrac{0 - 8}{10} = -0.8$ and $\lambda_y = \dfrac{6 - 0}{10} = 0.6$.

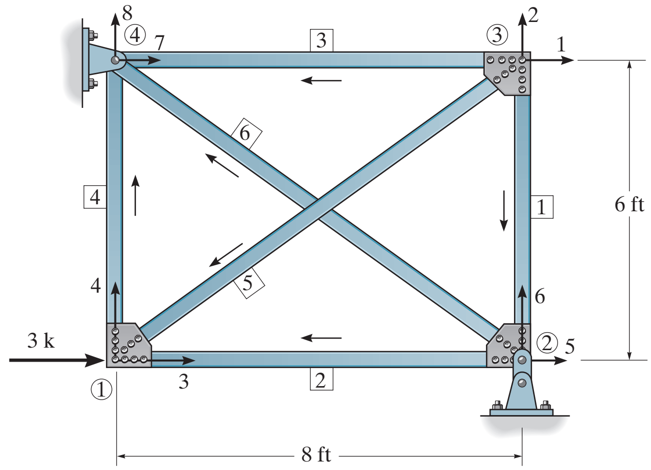
$$\mathbf{k}_6 = \frac{2[29(10^3)]}{120} \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix} \quad \text{(columns 5, 6, 7, 8)}$$

$$= \begin{bmatrix} 309.33 & -232 & -309.33 & 232 \\ -232 & 174 & 232 & -174 \\ -309.33 & 232 & 309.33 & -232 \\ 232 & -174 & -232 & 174 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix} \quad \text{(columns 5, 6, 7, 8)}$$

The structure stiffness matrix is a 8×8 matrix since the highest code number is 8. Thus,

$$\begin{bmatrix} 913.5 & 232 & -309.33 & -232 & 0 & 0 & -604.17 & 0 \\ 232 & 979.56 & -232 & -174 & 0 & -805.56 & 0 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 & 0 & 0 & 0 \\ -232 & -174 & 232 & 979.56 & 0 & 0 & 0 & -805.56 \\ 0 & 0 & -604.17 & 0 & 913.5 & -232 & -309.33 & 232 \\ 0 & -805.66 & 0 & 0 & -232 & 979.56 & 232 & -174 \\ -604.17 & 0 & 0 & 0 & -309.33 & 232 & 913.5 & -232 \\ 0 & 0 & 0 & -805.56 & 232 & -174 & -232 & 979.56 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} \quad \text{(columns 1–8)}$$

14–13. Determine the horizontal displacement of joint ② and the force in member [5]. Take $A = 2\text{ in}^2$, $E = 29(10^3)$ ksi. Neglect the short link at ②.



Here,

$$Q_k = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \qquad D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 6 \\ 7 \\ 8 \end{matrix}$$

Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ \hline Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \left[\begin{array}{ccccc|ccc} 913.5 & 232 & -309.33 & -232 & 0 & 0 & -604.17 & 0 \\ 232 & 979.56 & -232 & -174 & 0 & -805.56 & 0 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 & 0 & 0 & 0 \\ -232 & -174 & 232 & 979.56 & 0 & 0 & 0 & -805.56 \\ 0 & 0 & -604.17 & 0 & 913.5 & -232 & -309.33 & 232 \\ \hline 0 & -805.56 & 0 & 0 & -232 & 979.56 & 232 & -174 \\ -604.17 & 0 & 0 & 0 & -309.33 & 232 & 913.5 & -232 \\ 0 & 0 & 0 & -805.56 & 232 & -174 & -232 & 979.56 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition; $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 913.5 & 232 & -309.33 & -232 & 0 \\ 232 & 979.56 & -232 & -174 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 \\ -232 & -174 & 232 & 979.56 & 0 \\ 0 & 0 & -604.17 & 0 & 913.5 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = 913.5D_1 + 232D_2 - 309.33D_3 - 232D_4 \qquad (1)$$

$$0 = 232D_1 + 979.59D_2 - 232D_3 - 174D_4 \qquad (2)$$

$$3 = -309.33D_1 - 232D_2 + 913.5D_3 + 232D_4 - 604.17D_5 \qquad (3)$$

$$0 = -232D_1 - 174D_2 + 232D_3 + 979.56D_4 \qquad (4)$$

$$0 = -604.17D_3 + 913.5D_5 \qquad (5)$$

Solving Eqs. (1) to (5),

$D_1 = 0.002172 \qquad D_2 = 0.001222 \qquad D_3 = 0.008248 \qquad D_4 = -0.001222$

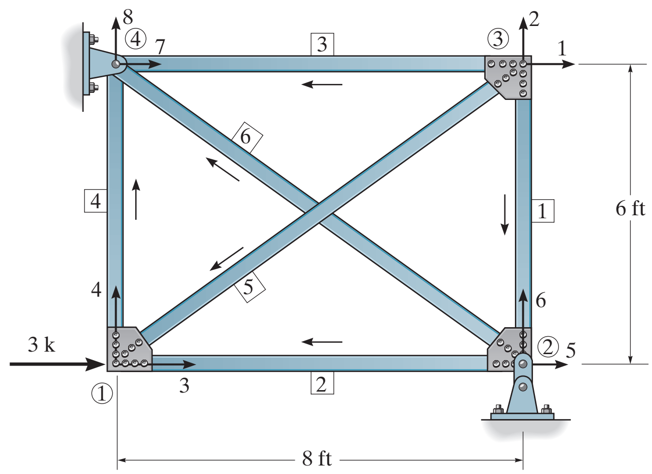
$D_5 = 0.005455 = 0.00546$ m **Ans.**

Force in Member [5]. Here, $L = 10(12) = 120$ in., $\lambda_x = -0.8$ and $\lambda_y = -0.6$

$$(q_5)_F = \frac{2[29(10^3)]}{120} \begin{bmatrix} 0.8 & 0.6 & -0.8 & -0.6 \end{bmatrix} \begin{bmatrix} 0.002172 \\ 0.001222 \\ 0.008248 \\ -0.001222 \end{bmatrix} \begin{matrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{matrix}$$

$= 1.64$ k (C) **Ans.**

14–14. Determine the force in member [3] if this member was 0.025 in. too short before it was fitted onto the truss. Take $A = 2\ \text{in}^2$. $E = 29(10^3)$ ksi. Neglect the short link at ②.



For member [3], $L = 8(12) = 96$ in $\lambda_x = -1$, $\lambda_y = 0$ and $\Delta L = -0.025$. Thus,

$$\begin{bmatrix}(Q_1)_0\\(Q_2)_0\\(Q_7)_0\\(Q_8)_0\end{bmatrix} = \frac{2[29(10^3)](-0.025)}{96}\begin{bmatrix}-1\\0\\1\\0\end{bmatrix} = \begin{bmatrix}15.10\\0\\-15.10\\0\end{bmatrix}\begin{matrix}1\\2\\7\\8\end{matrix}$$

Also,

$$\mathrm{Q}_k = \begin{bmatrix}0\\0\\3\\0\\0\end{bmatrix}\begin{matrix}1\\2\\3\\4\\5\end{matrix} \quad \text{and} \quad \mathrm{D}_k = \begin{bmatrix}0\\0\\0\end{bmatrix}\begin{matrix}6\\7\\8\end{matrix}$$

$$\begin{bmatrix}0\\0\\3\\0\\0\\Q_6\\Q_7\\Q_8\end{bmatrix} = \begin{bmatrix}913.5 & 232 & -309.33 & -232 & 0 & 0 & -604.17 & 0\\ 232 & 979.56 & -232 & -174 & 0 & -805.56 & 0 & 0\\ -309.33 & -232 & 913.5 & 232 & -604.17 & 0 & 0 & 0\\ -232 & -174 & 232 & 979.56 & 0 & 0 & 0 & -805.56\\ 0 & 0 & -604.17 & 0 & 913.5 & -232 & -309.33 & 232\\ 0 & -805.56 & 0 & 0 & -232 & 979.56 & 232 & -174\\ -604.17 & 0 & 0 & 0 & -309.33 & 232 & 913.5 & -232\\ 0 & 0 & 0 & -805.56 & 232 & -174 & -232 & -979.56\end{bmatrix}\begin{bmatrix}D_1\\D_2\\D_3\\D_4\\D_5\\0\\0\\0\end{bmatrix} + \begin{bmatrix}15.10\\0\\0\\0\\0\\0\\-15.10\\0\end{bmatrix}$$

Applying $\mathbf{Q} = \mathbf{KD} + \mathbf{Q}_0$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k + (\mathbf{Q}_k)_0$,

$$\begin{bmatrix}0\\0\\3\\0\\0\end{bmatrix} = \begin{bmatrix}913.5 & 232 & -309.33 & -232 & 0\\ 232 & 979.56 & -232 & -174 & 0\\ -309.33 & -232 & 913.5 & 232 & -604.17\\ -232 & -174 & 232 & 979.56 & 0\\ 0 & 0 & -604.17 & 0 & 913.5\end{bmatrix}\begin{bmatrix}D_1\\D_2\\D_3\\D_4\\D_5\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}15.10\\0\\0\\0\\0\end{bmatrix}$$

Expanding this matrix equality,

$$0 = 913.5D_1 + 232D_2 - 309.33D_3 - 232D_4 + 15.10 \qquad (1)$$

$$0 = 232D_1 + 979.56D_2 - 232D_3 - 174D_4 \qquad (2)$$

$$3 = -309.33D_1 - 232D_2 + 913.5D_3 + 232D_4 - 604.17D_5 \qquad (3)$$

$$0 = -232D_1 - 174D_2 + 232D_3 + 979.56D_4 \qquad (4)$$

$$0 = -604.17D_3 + 913.5D_5 \qquad (5)$$

Solving Eqs. (1) to (5),

$D_1 = -0.01912 \quad D_2 = 0.003305 \quad D_3 = -0.002687 \quad D_4 = -0.003305$

$D_5 = -0.001779$

14–14. Continued

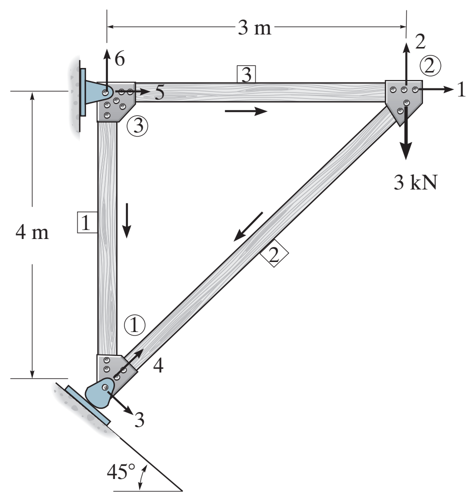
Force in member [3]. Here, $L = 8(12) = 96$ in., $\lambda_x = -1$, $\lambda_y = 0$ and

$$(q_F)_0 = \frac{-2[29(10^3)](-0.025)}{96} = 15.10 \text{ k}$$

$$(q_3)_F = \frac{2[29(10^3)]}{96} \begin{bmatrix}1 & 0 & -1 & 0\end{bmatrix} \begin{bmatrix}-0.01912 \\ 0.003305 \\ 0 \\ 0\end{bmatrix} + 15.10$$

$$= 3.55 \text{ k (T)}$$ **Ans.**

14–15. Determine the stiffness matrix **K** for the truss. AE is constant.

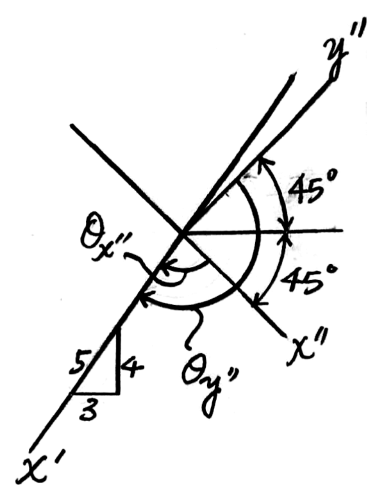


The origin of the global coordinate system is set at joint ①.

For member [2], $L = 5$ m. Referring to Fig. a, $\theta''_x = 180° - 45° - \sin^{-1}\left(\frac{4}{5}\right) = 81.87°$
$\theta''_y = 171.87°$. Thus, $\chi''_x = \cos\theta_{x''} = \cos 81.87° = 0.14142$ and
$\lambda_{y''} = \cos\theta_{y''} = \cos 171.87° = -0.98995$

Also, $\lambda_x = \frac{0-3}{5} = -0.6$ and $\lambda_y = \frac{0-4}{5} = -0.8$

$$\mathbf{k}_1 = AE \begin{bmatrix} 0.072 & 0.096 & 0.01697 & -0.11879 \\ 0.096 & 0.128 & 0.02263 & -0.15839 \\ 0.01697 & 0.02263 & 0.004 & -0.028 \\ -0.11879 & -0.15839 & -0.028 & 0.196 \end{bmatrix} \begin{matrix}1\\2\\3\\4\end{matrix}$$
(columns 1, 2, 3, 4)



(a)

For member [1], $L = 4$ m. Referring to Fig. b, $\theta_{x''} = 45°$ and $\theta_{y''} = 135°$.
Thus, $\lambda_{x''} = \cos 45° = \frac{\sqrt{2}}{2}$ and $\lambda_{y''} = \cos 135° = -\frac{\sqrt{2}}{2}$.

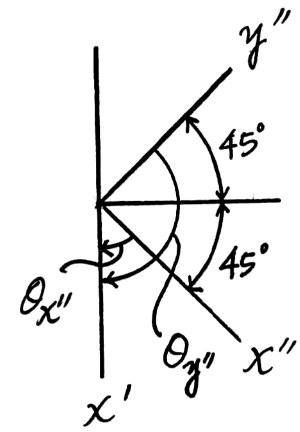
Also, $\lambda_x = 0$ and $\lambda_y = -1$.

$$\mathbf{k}_2 = AE \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.17678 & -0.17678 \\ 0 & 0.17678 & 0.125 & -0.125 \\ 0 & -0.17678 & -0.125 & 0.125 \end{bmatrix} \begin{matrix}5\\6\\3\\4\end{matrix}$$
(columns 5, 6, 3, 4)

For member [3], $L = 3$ m, $\lambda_x = 1$ and $\lambda_y = 0$.

$$\mathbf{k}_3 = AE \begin{bmatrix} 0.33333 & 0 & -0.33333 & 0 \\ 0 & 0 & 0 & 0 \\ -0.33333 & 0 & 0.33333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix}5\\6\\1\\2\end{matrix}$$
(columns 5, 6, 1, 2)

14–15. Continued



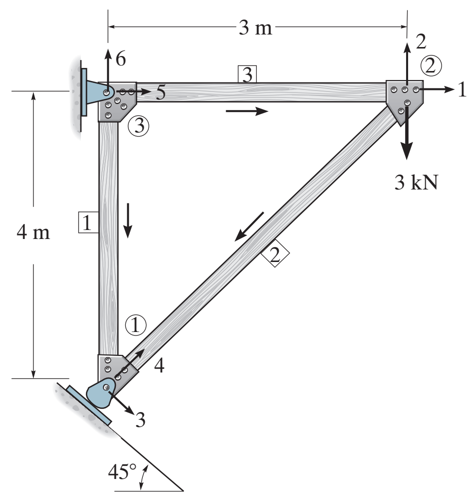
(b)

The structure stiffness matrix is a 6 × 6 matrix since the highest code number is 6. Thus,

$$\mathbf{k} = AE \begin{bmatrix} 0.40533 & 0.096 & 0.01697 & -0.11879 & -0.33333 & 0 \\ 0.096 & 0.128 & 0.02263 & -0.15839 & 0 & 0 \\ 0.01697 & 0.02263 & 0.129 & -0.153 & 0 & 0.17678 \\ -0.11879 & -0.15839 & -0.153 & 0.321 & 0 & -0.17678 \\ -0.33333 & 0 & 0 & 0 & 0.33333 & 0 \\ 0 & 0 & 0.17678 & -0.17678 & 0 & 0.25 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

(columns 1, 2, 3, 4, 5, 6)

***14–16.** Determine the vertical displacement of joint ② and the support reactions. AE is constant.



Here,

$$Q_k = \begin{bmatrix} 0 \\ -3(10^3) \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \text{ and } D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \end{matrix}$$

Applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 0 \\ -3(10^3) \\ 0 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = AE \begin{bmatrix} 0.40533 & 0.096 & 0.01697 & -0.11879 & -0.33333 & 0 \\ 0.096 & 0.128 & 0.02263 & -0.15839 & 0 & 0 \\ 0.01697 & 0.02263 & 0.129 & -0.153 & 0 & 0.17678 \\ -0.11879 & -0.15839 & -0.153 & 0.321 & 0 & -0.17678 \\ -0.33333 & 0 & 0 & 0 & 0.33333 & 0 \\ 0 & 0 & 0.17678 & -0.17678 & 0 & 0.25 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition; $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$\begin{bmatrix} 0 \\ -3(10^3) \\ 0 \end{bmatrix} = AE \begin{bmatrix} 0.40533 & 0.096 & 0.01697 \\ 0.096 & 0.128 & 0.02263 \\ 0.011697 & 0.02263 & 0.129 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = AE(0.40533\, D_1 + 0.096\, D_2 + 0.01697\, D_3) \qquad (1)$$

$$-3(10^3) = AE(0.096\, D_1 + 0.128\, D_2 + 0.02263\, D_3) \qquad (2)$$

$$0 = AE(0.01697\, D_1 + 0.02263\, D_2 + 0.0129\, D_3) \qquad (3)$$

14–16. Continued

Solving Eqs. (1) to (3),

$$D_1 = \frac{6.750(10^3)}{AE} \quad D_3 = \frac{4.2466(10^3)}{AE}$$

$$D_2 = \frac{-29.250(10^3)}{AE} = \frac{29.3(10^3)}{AE} \quad \downarrow$$ **Ans.**

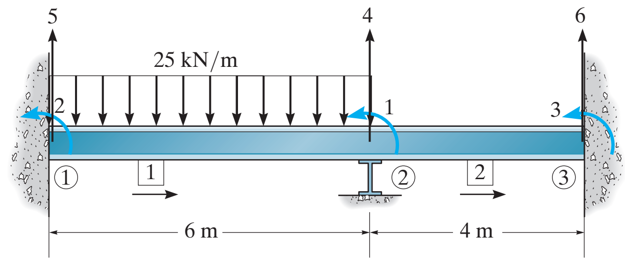
Again, the matrix partition $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$ gives

$$\begin{bmatrix} Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = AE \begin{bmatrix} -0.11879 & -0.15839 & -0.153 \\ -0.33333 & 0 & 0 \\ 0 & 0 & 0.17678 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 6.750(10^3) \\ -29.250(10^3) \\ 4.2466(10^3) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$Q_4 = 3.182(10^3)\text{ N} = 3.18\text{ kN}$ $\quad Q_5 = -2.250(10^3)\text{ N} = -2.25\text{ kN}$ **Ans.**

$Q_6 = 750\text{ N}$ **Ans.**

15–1. Determine the moments at ① and ③. Assume ② is a roller and ① and ③ are fixed. *EI* is constant.



Member Stiffness Matrices. For member [1],

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.66667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333EI$$

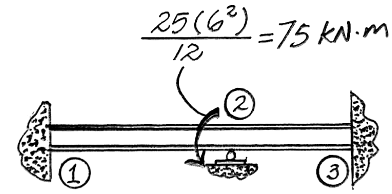
$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 5 & 2 & 4 & 1 \end{array} \\ \left[\begin{array}{cccc} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{array}\right] \end{array} \begin{array}{c} 5 \\ 2 \\ 4 \\ 1 \end{array}$$

For member [2],

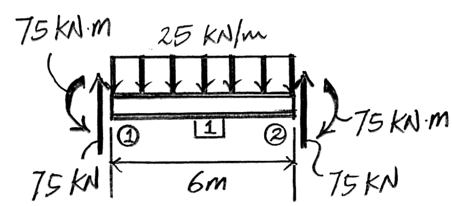
$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 4 & 1 & 6 & 3 \end{array} \\ \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{array}\right] \end{array} \begin{array}{c} 4 \\ 1 \\ 6 \\ 3 \end{array}$$



(a)



(b)

Known Nodal Loads and Deflection. The nodal load acting on the unconstrained degree of freedom (Code number 1) is shown in Fig. *a*. Thus;

$$\mathbf{Q}_k = [75]\,1 \qquad \text{and} \qquad \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6 × 6 matrix since the highest Code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$

$$\left[\begin{array}{c} 75 \\ \hline Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{array}\right] = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{c|ccccc} 1.6667 & 0.33333 & 0.5 & 0.20833 & 0.16667 & -0.375 \\ \hline 0.33333 & 0.66667 & 0 & -0.16667 & 0.16667 & 0 \\ 0.5 & 0 & 1.00 & 0.375 & 0 & -0.375 \\ 0.20833 & -0.16667 & 0.375 & 0.24306 & -0.05556 & -0.1875 \\ 0.16667 & 0.16667 & 0 & -0.05556 & 0.05556 & 0 \\ -0.375 & 0 & -0.375 & -0.1875 & 0 & 0.1875 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \left[\begin{array}{c} D_1 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$75 = 1.66667EID_1 + 0 \qquad D_1 = \frac{45}{EI}$$

15–1. Continued

Also, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_2 = 0.33333EI\left(\frac{45}{EI}\right) + 0 = 15 \text{ kN}\cdot\text{m}$$

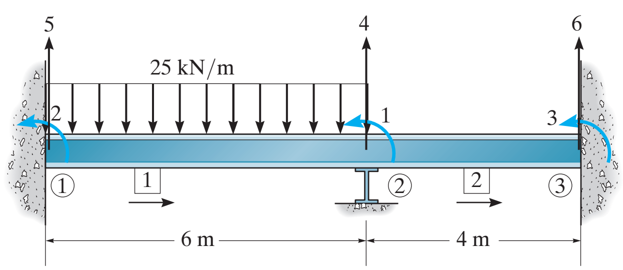
$$Q_3 = 0.5EI\left(\frac{45}{EI}\right) + 0 = 22.5 \text{ kN}\cdot\text{m}$$

Superposition of these results and the (FEM) in Fig. *b*,

$M_1 = 15 + 75 = 90 \text{ kN}\cdot\text{m}$ ↺ **Ans.**

$M_3 = 22.5 + 0 = 22.5 \text{ kN}\cdot\text{m}$ ↺ **Ans.**

15–2. Determine the moments at ① and ③ if the support ② moves upward 5 mm. Assume ② is a roller and ① and ③ are fixed. $EI = 60(10^6)\ \text{N}\cdot\text{m}^2$.



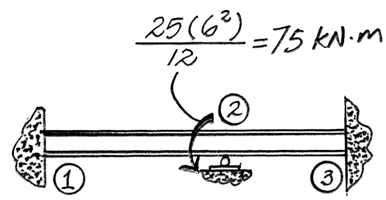
Member Stiffness Matrices. For member [1],

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556\,EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667\,EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.66667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333\,EI$$

$$\mathbf{k}_1 = EI\begin{bmatrix} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{bmatrix}$$

(columns 5, 2, 4, 1; rows 5, 2, 4, 1)

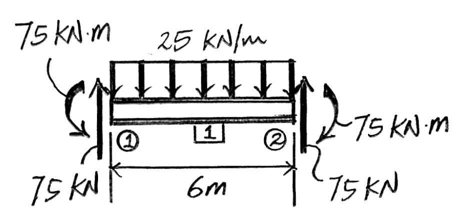


(a)

For member [2],

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$



(b)

15–2. Continued

$$\mathbf{k}_2 = EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \begin{matrix} 4 \\ 1 \\ 6 \\ 3 \end{matrix} \quad \text{(columns: 4, 1, 6, 3)}$$

Known Nodal Loads and Deflection. The nodal load acting on the unconstrained degree of freedoom (code number 1) is shown in Fig. *a*. Thus,

$$Q_k = [75(10^3)]\,1 \qquad \text{and} \qquad D_k = \begin{bmatrix} 0 \\ 0 \\ 0.005 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Load-Displacement Relation. The structure stiffness matrix is a 6 × 6 matrix since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{kD}$

$$\begin{bmatrix} 75(10^3) \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{bmatrix} 1.66667 & 0.33333 & 0.5 & 0.20833 & 0.16667 & -0.375 \\ 0.33333 & 0.66667 & 0 & -0.16667 & 0.16667 & 0 \\ 0.5 & 0 & 1.00 & 0.375 & 0 & -0.375 \\ 0.20833 & -0.16667 & 0.375 & 0.24306 & -0.05556 & -0.1875 \\ 0.16667 & 0.16667 & 0 & -0.05556 & 0.05556 & 0 \\ -0.375 & 0 & -0.375 & -0.1875 & 0 & 0.1875 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{bmatrix} D_1 \\ 0 \\ 0 \\ 0.005 \\ 0 \\ 0 \end{bmatrix} \quad \text{(columns: 1, 2, 3, 4, 5, 6)}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$75(10^3) = [1.6667D_1 + 0.20833(0.005)][60(10^6)]$$

$$D_1 = 0.125(10^{-3}) \text{ rad}$$

Using this result and apply, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_2 = \{0.33333[0.125(10^{-3})] + (-0.16667)(0.005)\}[60(10^6)] = -47.5 \text{ kN} \cdot \text{m}$$

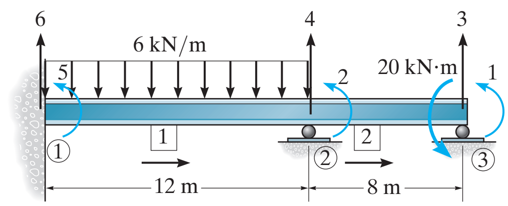
$$Q_3 = \{0.5[0.125(10^{-3})] + 0.375(0.005)\}[60(10^6)] = 116.25 \text{ kN} \cdot \text{m}$$

Superposition these results to the (FEM) in Fig. *b*,

$M_1 = -47.5 + 75 = 27.5 \text{ kN} \cdot \text{m}$ **Ans.**

$M_3 = 116.25 + 0 = 116.25 \text{ kN.m} = 116 \text{ kN} \cdot \text{m}$ **Ans.**

15–3. Determine the reactions at the supports. Assume the rollers can either push or pull on the beam. EI is constant.



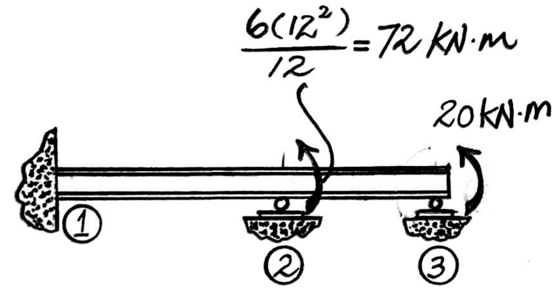
Member Stiffness Matrices. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{12^3} = 0.006944EI \qquad \frac{6EI}{L^2} = \frac{6EI}{12^2} = 0.041667EI$$

$$\frac{4EI}{L} = \frac{4EI}{12} = 0.333333EI \qquad \frac{2EI}{L} = \frac{2EI}{12} = 0.166667EI$$

$$\mathbf{k}_1 = EI \begin{bmatrix} 0.006944 & 0.041667 & -0.006944 & 0.041667 \\ 0.041667 & 0.333333 & -0.041667 & 0.166667 \\ -0.006944 & -0.041667 & 0.006944 & -0.041667 \\ 0.041667 & 0.166667 & -0.041667 & 0.333333 \end{bmatrix} \begin{matrix} 6 \\ 5 \\ 4 \\ 2 \end{matrix}$$

(column code numbers: 6, 5, 4, 2)



(a)

For member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{8^3} = 0.0234375EI \qquad \frac{6EI}{L^2} = \frac{6EI}{8^2} = 0.09375EI$$

$$\frac{4EI}{L} = \frac{4EI}{8} = 0.5EI \qquad \frac{2EI}{L} = \frac{2EI}{8} = 0.25EI$$

$$\mathbf{k}_2 = EI \begin{bmatrix} 0.0234375 & 0.09375 & -0.0234375 & 0.09375 \\ 0.09375 & 0.5 & -0.09375 & 0.25 \\ -0.0234375 & -0.09375 & 0.0234375 & -0.09375 \\ 0.09375 & 0.25 & -0.09375 & 0.5 \end{bmatrix} \begin{matrix} 4 \\ 2 \\ 3 \\ 1 \end{matrix} \begin{bmatrix} 0 \\ 87 \\ 0 \\ -3.76 \end{bmatrix}$$

(column code numbers: 4, 2, 3, 1)

Known Nodal Loads And Deflection. The nodal loads acting on the unconstrained degree of freedoom (code number 1 and 2) are shown in Fig. *a*. Thus,

$$\mathbf{Q}_k = \begin{bmatrix} 20 \\ 72 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Load-Displacement Relation. The structure stiffness matrix is a 6×6 matrix since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 20 \\ 72 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{bmatrix} 0.5 & 0.25 & -0.09375 & 0.09375 & 0 & 0 \\ 0.25 & 0.833333 & -0.09375 & 0.052083 & 0.166667 & 0.041667 \\ -0.09375 & -0.09375 & 0.0234375 & -0.0234375 & 0 & 0 \\ 0.09375 & 0.052083 & -0.0234375 & 0.0303815 & -0.041667 & -0.006944 \\ 0 & 0.166667 & 0 & -0.041667 & 0.333333 & 0.041667 \\ 0 & 0.041667 & 0 & -0.006944 & 0.041667 & 0.006944 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(column code numbers: 1, 2, 3, 4, 5, 6)

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

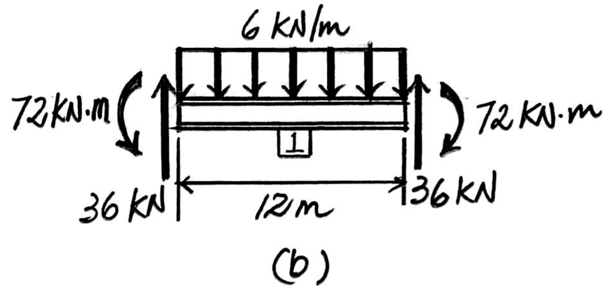
$$20 = EI[0.5D_1 + 0.25D_2] \qquad (1)$$

$$72 = EI[0.25D_1 + 0.833333D_2] \qquad (2)$$

15–3. Continued

Solving Eqs. (1) and (2),

$$D_1 = -\frac{3.7647}{EI} \qquad D_2 = \frac{87.5294}{EI}$$



Also, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$\begin{bmatrix} Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{bmatrix} -0.09375 & -0.09375 \\ 0.09375 & 0.052083 \\ 0 & 0.166667 \\ 0 & 0.041667 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} -3.7647 \\ 87.5294 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_3 = -0.09375(-3.7647) + (-0.09375)(87.5294) = -7.853 \text{ kN}$$

$$Q_4 = 0.09375(-3.7647) + 0.052083(87.5294) = 4.206 \text{ kN}$$

$$Q_5 = 0 + 0.166667(87.5294) = 14.59 \text{ kN}\cdot\text{m}$$

$$Q_6 = 0 + 0.041667(87.5294) = 3.647 \text{ kN}$$

Superposition these results with the (FEM) in Fig. *b*,

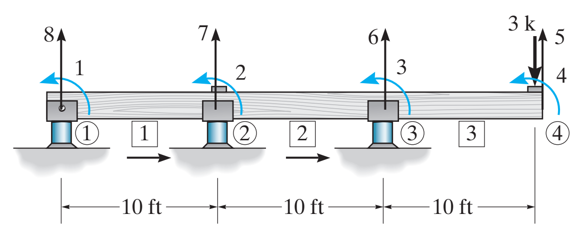
$R_3 = -7.853 + 0 = -7.853 \text{ kN} = 7.85 \text{ kN} \downarrow$ **Ans.**

$R_4 = 4.206 + 36 = 40.21 \text{ kN} = 40.2 \text{ kN} \uparrow$ **Ans.**

$M_5 = 14.59 + 72 = 86.59 \text{ kN}\cdot\text{m} = 86.6 \text{ kN}\cdot\text{m} \uparrow$ **Ans.**

$R_6 = 3.647 + 36 = 39.64 \text{ kN} = 39.6 \text{ kN} \uparrow$ **Ans.**

***15–4.** Determine the reactions at the supports. Assume ① is a pin and ② and ③ are rollers that can either push or pull on the beam. *EI* is constant.



Member Stiffness Matrices. For member [1], [2] and [3],

$$\frac{12EI}{L^3} = \frac{12EI}{10^3} = 0.012 \qquad \frac{6EI}{L^2} = \frac{6EI}{10^2} = 0.06$$

$$\frac{4EI}{L} = \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{L} = \frac{2EI}{10} = 0.2$$

$$\mathbf{k}_1 = EI \begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix}$$

(Columns labeled 8, 1, 7, 2; rows labeled 8, 1, 7, 2.)

15–4. Continued

$$\mathbf{k}_2 = EI\begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix}\begin{matrix} 7 \\ 2 \\ 6 \\ 3 \end{matrix} \quad (\text{columns } 7,\ 2,\ 6,\ 3)$$

$$\mathbf{k}_3 = EI\begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix}\begin{matrix} 6 \\ 3 \\ 5 \\ 4 \end{matrix} \quad (\text{columns } 6,\ 3,\ 5,\ 4)$$

Known Nodal Load and Deflection. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, 4 and 5) is

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \text{ and } \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 6 \\ 7 \\ 8 \end{matrix}$$

Load-Displacement Relation. The structure stiffness matrix is a 8 × 8 matrix since the highest code number is 8. Applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3 \\ \hline Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = EI\left[\begin{array}{ccccc|ccc} 0.4 & 0.2 & 0 & 0 & 0 & 0 & -0.06 & 0.06 \\ 0.2 & 0.8 & 0.2 & 0 & 0 & -0.06 & 0 & 0.06 \\ 0 & 0.2 & 0.8 & 0.2 & -0.06 & 0 & 0.06 & 0 \\ 0 & 0 & 0.2 & 0.4 & -0.06 & 0.06 & 0 & 0 \\ 0 & 0 & -0.06 & -0.06 & 0.012 & -0.012 & 0 & 0 \\ \hline 0 & -0.06 & 0 & 0.06 & -0.012 & 0.024 & -0.012 & 0 \\ -0.06 & 0 & 0.06 & 0 & 0 & -0.012 & 0.024 & -0.012 \\ 0.06 & 0.06 & 0 & 0 & 0 & 0 & -0.012 & 0.012 \end{array}\right]\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

(columns 1, 2, 3, 4, 5, 6, 7, 8)

From the matrix partition, $\mathbf{Q}_k = \mathbf{k}_{11}\mathbf{D}_u + \mathbf{k}_{12}\mathbf{D}_k$,

$$0 = 04D_1 + 0.2D_2 \quad (1)$$

$$0 = 0.2D_1 + 0.8D_2 + 0.2D_3 \quad (2)$$

$$0 = 0.2D_2 + 0.8D_3 + 0.2D_4 - 0.06D_5 \quad (3)$$

$$0 = 0.2D_3 + 0.4D_4 - 0.06D_5 \quad (4)$$

$$-3 = -0.06D_3 - 0.06D_4 + 0.012D_5 \quad (5)$$

Solving Eq. (1) to (5)

$$D_1 = -12.5 \quad D_2 = 25 \quad D_3 = -87.5 \quad D_4 = -237.5 \quad D_5 = -1875$$

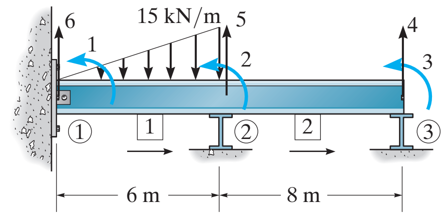
Using these results, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{k}_{22}\mathbf{D}_k$

$Q_6 = 6.75$ kN **Ans.**

$Q_7 = -4.5$ kN **Ans.**

$Q_8 = -0.75$ kN **Ans.**

15–5. Determine the support reactions. Assume ② and ③ are rollers and ① is a pin. EI is constant.



Member Stiffness Matrices. For member [1]

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.066667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.033333EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 6 & 1 & 5 & 2 \end{array} \\ \left[\begin{array}{cccc} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{array}\right] \end{array} \begin{array}{c} 6 \\ 1 \\ 5 \\ 2 \end{array}$$

For Member [2],

$$\frac{12EI}{L^3} = \frac{12EI}{8^3} = 0.0234375EI \qquad \frac{6EI}{L^2} = \frac{6EI}{8^2} = 0.09375EI$$

$$\frac{4EI}{L} = \frac{4EI}{8} = 0.5EI \qquad \frac{2EI}{L} = \frac{2EI}{8} = 0.025EI$$

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 5 & 2 & 4 & 3 \end{array} \\ \left[\begin{array}{cccc} 0.0234375 & 0.09375 & -0.0234375 & 0.09375 \\ 0.09375 & 0.5 & -0.09375 & 0.25 \\ -0.0234375 & -0.09375 & 0.0234375 & -0.09375 \\ 0.09375 & 0.25 & -0.09375 & 0.5 \end{array}\right] \end{array} \begin{array}{c} 5 \\ 2 \\ 4 \\ 3 \end{array}$$

Known Nodal Load and Deflection. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, and 3) are shown in Fig. *a*

$$\mathbf{Q}_k = \left[\begin{array}{c} 0 \\ 36 \\ 0 \end{array}\right] \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \quad \text{and} \quad \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6×6 matirx since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\left[\begin{array}{c} 0 \\ 36 \\ 0 \\ \hline Q_4 \\ Q_5 \\ Q_6 \end{array}\right] = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 0.66667 & 0.33333 & 0 & 0 & 0.16667 & 0.16667 \\ 0.33333 & 1.16667 & 0.25 & -0.09375 & -0.07292 & 0.16667 \\ 0 & 0.25 & 0.5 & -0.09375 & 0.09375 & 0 \\ \hline 0 & -0.09375 & -0.09375 & 0.0234375 & -0.0234375 & 0 \\ -0.16667 & -0.07292 & 0.09375 & -0.0234375 & 0.0789931 & -0.05556 \\ 0.16667 & 0.16667 & 0 & 0 & -0.05556 & 0.05556 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \left[\begin{array}{c} D_1 \\ D_2 \\ D_3 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{k}_{11}\mathbf{D}_u + \mathbf{k}_{12}\mathbf{D}_k$

$$0 = 0.66667D_1 + 0.33333D_2 \qquad (1)$$

$$36 = 0.33333D_1 + 1.16667D_2 + 0.25D_3 \qquad (2)$$

$$0 = 0.25D_2 + 0.5D_3 \qquad (3)$$

15–5. Continued

Solving Eqs. (1) to (3),

$$D_1 = \frac{-20.5714}{EI}$$

$$D_2 = \frac{41.1429}{EI}$$

$$D_3 = \frac{-20.5714}{EI}$$

Using these results and apply $\mathbf{Q}_u = \mathbf{k}_{21}\,\mathbf{D}_u + {}_+\mathbf{k}_{22}\,\mathbf{D}_k$

$$Q_4 = 0 + (-0.09375EI)\left(\frac{41.1429}{EI}\right) + (-0.09375EI)\left(-\frac{20.5714}{EI}\right) = -1.929 \text{ kN}$$

$$Q_5 = -0.16667EI\left(-\frac{20.5714}{EI}\right) + (-0.07292EI)\left(\frac{41.1429}{EI}\right)$$

$$+ 0.09375EI\left(-\frac{20.5714}{EI}\right)$$

$$= -1.500 \text{ kN}$$

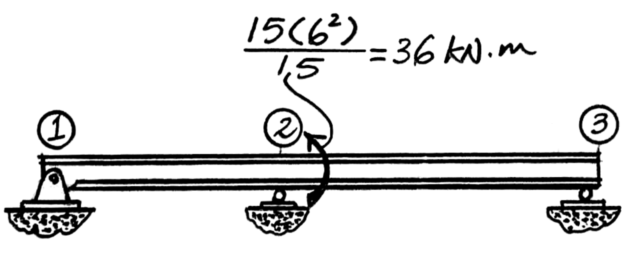
$$Q_6 = 0.16667EI\left(-\frac{20.5714}{EI}\right) + 0.16667EI\left(\frac{41.1429}{EI}\right) = 3.429 \text{ kN}$$

Superposition these results with the FEM show in Fig. *b*

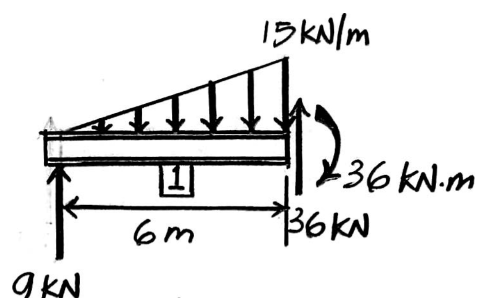
$R_4 = -1.929 + 0 = -1.929 \text{ kN} = 1.93 \text{ kN} \downarrow$ **Ans.**

$R_5 = -1.500 + 36 = 34.5 \text{ kN} \uparrow$

$R_6 = 3.429 + 9 = 12.43 \text{ kN} = 12.4 \text{ kN} \uparrow$ **Ans.**

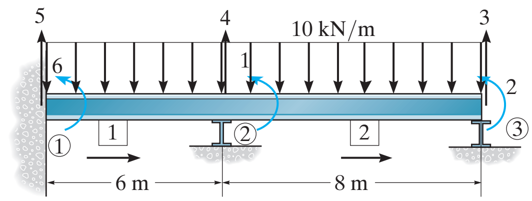


(a)



(b)

15–6. Determine the reactions at the supports. Assume ① is fixed ② and ③ are rollers. EI is constant.



Member Stiffness Matrices. For member [1],

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.066667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333EI$$

$$\mathbf{k}_1 = EI \begin{bmatrix} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 4 \\ 1 \end{matrix} \quad (\text{columns: } 5,\ 6,\ 4,\ 1)$$

For Member [2],

$$\frac{12EI}{L^3} = \frac{12EI}{8^3} = 0.0234375EI \qquad \frac{6EI}{L^2} = \frac{8EI}{8^2} = 0.09375EI$$

$$\frac{4EI}{L} = \frac{4EI}{8} = 0.5EI \qquad \frac{2EI}{L} = \frac{2EI}{8} = 0.025EI$$

$$\mathbf{k}_2 = EI \begin{bmatrix} 0.0234375 & 0.09375 & -0.0234375 & 0.09375 \\ 0.09375 & 0.5 & -0.09375 & 0.25 \\ -0.0234375 & -0.09375 & 0.0234375 & -0.09375 \\ 0.09375 & 0.25 & -0.09375 & 0.5 \end{bmatrix} \begin{matrix} 4 \\ 1 \\ 3 \\ 2 \end{matrix} \quad (\text{columns: } 4,\ 1,\ 3,\ 2)$$

Known Nodal Load and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1 and 2) are shown in Fig. *a*

$$\mathbf{Q}_k = \begin{bmatrix} -50 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Load-Displacement Relation. The structure stiffness matrix is a 6 × 6 matirx since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} -50 \\ 0 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{bmatrix} 1.16667 & 0.25 & -0.09375 & -0.07292 & 0.16667 & 0.33333 \\ 0.25 & 0.5 & -0.09375 & 0.09375 & 0 & 0 \\ -0.09375 & -0.09375 & 0.0234375 & -0.0234375 & 0 & 0 \\ -0.07292 & 0.09375 & -0.0234375 & 0.0789931 & -0.05556 & -0.16667 \\ 0.16667 & 0 & 0 & -0.05556 & 0.05556 & 0.16667 \\ 0.33333 & 0 & 0 & -0.16667 & 0.16667 & 0.66667 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(columns: 1, 2, 3, 4, 5, 6)

15–6. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\,\mathbf{D}_u + \mathbf{K}_{12}\,\mathbf{D}_k$

$$-50 = EI(1.16667D_1 + 0.25D_2)$$

$$0 = EI\,(0.25D_1 + 0.5D_2)$$

Solving Eqs. (1) and (2),

$$D_1 = \frac{48}{EI} \qquad D_2 = \frac{24}{EI}$$

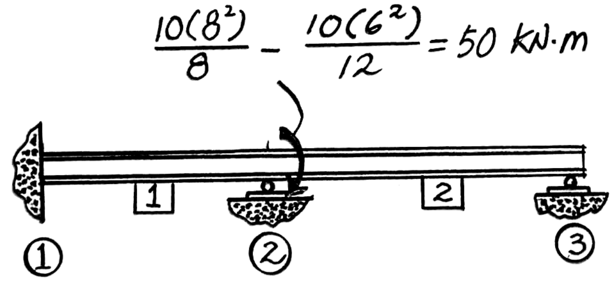
Using these results and apply in $\mathbf{Q}_u = \mathbf{K}_{21}\,\mathbf{D}_u + \mathbf{K}_{22}\,\mathbf{D}_k$

$$Q_3 = -0.09375EI\left(-\frac{48}{EI}\right) + (-0.09375EI)\left(\frac{24}{EI}\right) + 0 = 2.25\text{ kN}$$

$$Q_4 = -0.07292EI\left(-\frac{48}{EI}\right) + 0.09375EI\left(\frac{24}{EI}\right) + 0 = 5.75\text{ kN}$$

$$Q_5 = 0.16667EI\left(-\frac{48}{EI}\right) + 0 + 0 = -8.00\text{ kN}$$

$$Q_6 = (0.33333EI)\left(-\frac{48}{EI}\right) + 0 + 0 = -16.0\text{ kN}$$



(a)

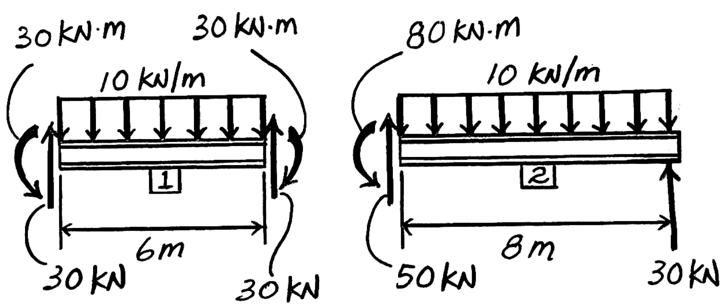
Superposition these results with the FEM show in Fig. *b*

$R_3 = 2.25 + 30 = 32.25\text{ kN}\uparrow$ **Ans.**

$R_4 = 5.75 + 30 + 50 = 85.75\text{ kN}\uparrow$ **Ans.**

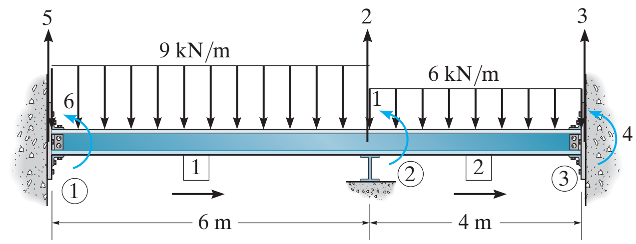
$R_5 = -8.00 + 30 = 22.0\text{ kN}\uparrow$ **Ans.**

$R_6 = -16.0 + 30 = 14.0\text{ kN}\cdot\text{m}$ ↺ **Ans.**



(b)

15–7. Determine the reactions at the supports. Assume ① and ③ are fixed and ② is a roller. EI is constant.



Member Stiffness Matrices. For member 1,

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.66667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333EI$$

$$\mathbf{k}_1 = EI \begin{bmatrix} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 2 \\ 1 \end{matrix}$$

(columns 5, 6, 2, 1)

For member 2,

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_2 = EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \begin{matrix} 2 \\ 1 \\ 3 \\ 4 \end{matrix}$$

(columns 2, 1, 3, 4)

Known Nodal Loads and Deflections. The nodal load acting on the unconstrained degree of freedom (code number 1) are shown in Fig. *a*

$$\mathbf{Q}_k = [19]\,1 \qquad \text{and} \qquad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

15–7. Continued

Load-Displacement Relation. The structure stiffness matrix is a 6 × 6 matirx since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 19 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{c:cccccc} 1.66667 & 0.20833 & -0.375 & 0.5 & 0.16667 & 0.33333 \\ \hdashline 0.20833 & 0.24306 & -0.1875 & 0.375 & -0.05556 & -0.16667 \\ -0.375 & -0.1875 & 0.1875 & -0.375 & 0 & 0 \\ 0.5 & 0.375 & -0.375 & 1.00 & 0 & 0 \\ 0.16667 & -0.05556 & 0 & 0 & 0.05556 & 0.16667 \\ 0.33333 & -0.16667 & 0 & 0 & 0.16667 & 0.66667 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} D_1 \\ \hdashline 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$

$$19 = 1.66667EID_1 \qquad D_1 = \frac{11.4}{EI}$$

Using this result and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$Q_2 = 0.20833EI\left(\frac{11.4}{EI}\right) = 2.375 \text{ kN}$$

$$Q_3 = -0.375EI\left(\frac{11.4}{EI}\right) = -4.275 \text{ kN}$$

$$Q_4 = 0.5EI\left(\frac{11.4}{EI}\right) = 5.70 \text{ kN}\cdot\text{m}$$

$$Q_5 = 0.16667\left(\frac{11.4}{EI}\right) = 1.90 \text{ kN}$$

$$Q_6 = 0.33333\left(\frac{11.4}{EI}\right) = 3.80 \text{ kN}\cdot\text{m}$$

Superposition these results with the FEM shown in Fig. *b*,

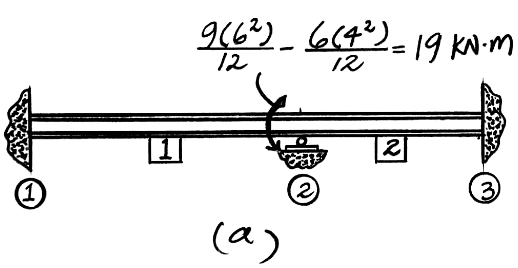
$R_2 = 2.375 + 27 + 12 = 41.375 \text{ kN} = 41.4 \text{ kN} \uparrow$ **Ans.**

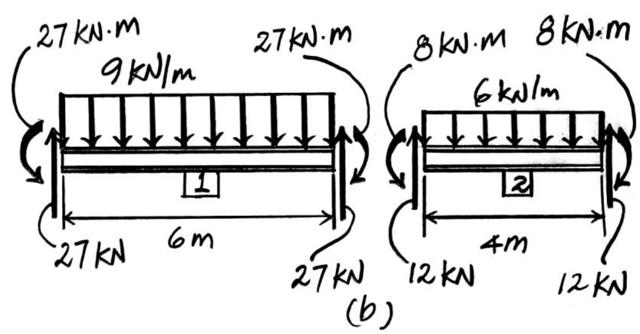
$R_3 = -4.275 + 12 = 7.725 \text{ kN} \uparrow$ **Ans.**

$R_4 = 5.70 - 8 = -2.30 \text{ kN}\cdot\text{m} = 2.30 \text{ kN.m}$ ↻ **Ans.**

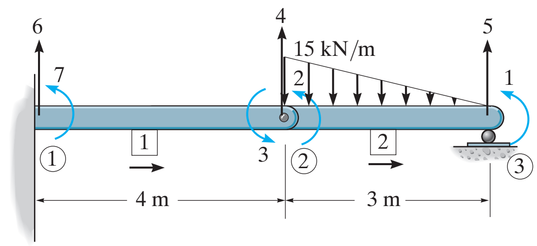
$R_5 = 1.90 + 27 = 28.9 \text{ kN} \uparrow$ **Ans.**

$R_6 = 3.80 + 27 = 30.8 \text{ kN.m} \uparrow$ **Ans.**





***15–8.** Determine the reactions at the supports. EI is constant.



Member Stiffness Matrices. For member $\boxed{1}$

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_1 = EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \begin{matrix} 6 \\ 7 \\ 4 \\ 3 \end{matrix}$$

(columns: 6, 7, 4, 3)

For member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{3^3} = 0.44444EI \qquad \frac{6EI}{L^2} = \frac{6EI}{3^2} = 0.66667EI$$

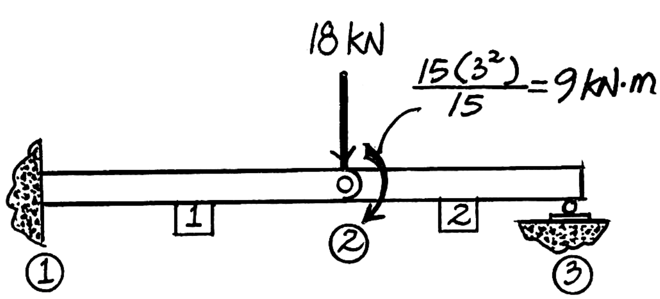
$$\frac{4EI}{L} = \frac{4EI}{3} = 1.33333EI \qquad \frac{2EI}{L} = \frac{2EI}{3} = 0.66667EI$$

$$\mathbf{k}_2 = EI \begin{bmatrix} 0.44444 & 0.66667 & -0.44444 & 0.66667 \\ 0.66667 & 1.33333 & -0.66667 & 0.66667 \\ -0.44444 & -0.66667 & 0.44444 & -0.66667 \\ 0.66667 & 0.66667 & -0.66667 & 1.33333 \end{bmatrix} \begin{matrix} 4 \\ 2 \\ 5 \\ 1 \end{matrix}$$

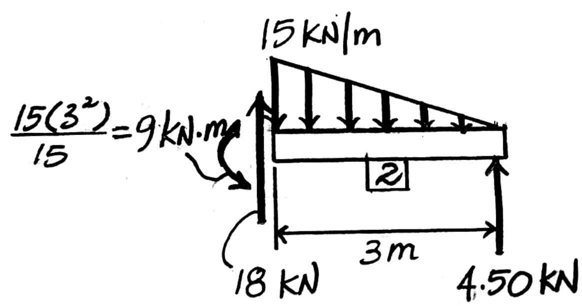
(columns: 4, 2, 5, 1)

Known Nodal Loads and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, and 4) are shown in Fig. *a* and *b*.

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ -9 \\ 0 \\ -18 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 7 \end{matrix}$$



(a)



(b)

15–8. Continued

Load-Displacement Relation. The structure stiffness matrix is a 7 × 7 matirx since the highest code number is 7. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ -9 \\ 0 \\ -18 \\ \hline Q_5 \\ Q_6 \\ Q_7 \end{bmatrix} = EI \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{cccc|ccc} 1.33333 & 0.66667 & 0 & 0.66667 & -0.66667 & 0 & 0 \\ 0.66667 & 1.33333 & 0 & 0.66667 & -0.66667 & 0 & 0 \\ 0 & 0 & 1.00 & -0.375 & 0 & 0.375 & 0.5 \\ 0.66667 & 0.66667 & -0.375 & 0.63194 & -0.44444 & -0.1875 & -0.375 \\ \hline -0.66667 & -0.66667 & 0 & -0.44444 & 0.44444 & 0 & 0 \\ 0 & 0 & 0.375 & -0.1875 & 0 & 0.1875 & 0.375 \\ 0 & 0 & 0.5 & -0.375 & 0 & 0.375 & 1.00 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$0 = EI(1.33333D_1 + 0.66667D_2 + 0.66667D_4) \quad (1)$$

$$-9 = EI(0.66667D_1 + 1.33333D_2 + 0.66667D_4) \quad (2)$$

$$0 = EI(D_3 - 0.375D_4) \quad (3)$$

$$-18 = EI(0.66667D_1 + 0.66667D_2 - 0.375D_3 + 0.63194D_4) \quad (4)$$

Solving Eqs. (1) to (4),

$$D_1 = \frac{111.167}{EI} \qquad D_2 = \frac{97.667}{EI} \qquad D_3 = -\frac{120}{EI} \qquad D_4 = -\frac{320}{EI}$$

Using these result and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$Q_5 = -0.66667EI\left(\frac{111.167}{EI}\right) + \left(-0.66667EI\right)\left(\frac{97.667}{EI}\right) + (-0.44444EI)\left(\frac{-320}{EI}\right) + 0 = 3.00 \text{ kN}$$

$$Q_6 = 0.375EI\left(-\frac{120}{EI}\right) + (-0.1875EI)\left(-\frac{320}{EI}\right) + 0 = 15.00 \text{ kN}$$

$$Q_7 = 0.5EI\left(-\frac{120}{EI}\right) + (-0.375EI)\left(-\frac{320}{EI}\right) + 0 = 60.00 \text{ kN} \cdot \text{m}$$

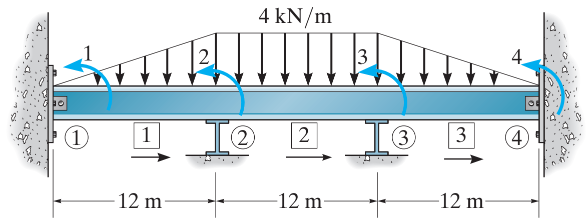
Superposition of these results with the (FEM),

$R_5 = 3.00 + 4.50 = 7.50 \text{ kN} \uparrow$ **Ans.**

$R_6 = 15.00 + 0 = 15.0 \text{ kN} \uparrow$ **Ans.**

$R_7 = 60.00 + 0 = 60.0 \text{ kN} \cdot \text{m} \circlearrowleft$ **Ans.**

15–9. Determine the moments at ② and ③. EI is constant. Assume ①, ②, and ③ are rollers and ④ is pinned.



The FEMs are shown on the figure.

$$\mathbf{Q}_k = \begin{bmatrix} -19.2 \\ -19.2 \\ 19.2 \\ 19.2 \end{bmatrix} \qquad \mathbf{D}_k = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$\mathbf{k}_1 = EI\begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{k}_2 = EI\begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{k}_3 = EI\begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$$

$$\mathbf{K} = EI\begin{bmatrix} 0.3333 & 0.16667 & 0 & 0 \\ 0.16667 & 0.6667 & 0.16667 & 0 \\ 0 & 0.16667 & 0.6667 & 0.16667 \\ 0 & 0 & 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{KD}$$

$$\begin{bmatrix} -19.2 \\ -19.2 \\ 19.2 \\ 19.2 \end{bmatrix} = EI\begin{bmatrix} 0.3333 & 0.16667 & 0 & 0 \\ 0.16667 & 0.6667 & 0.16667 & 0 \\ 0 & 0.16667 & 0.6667 & 0.16667 \\ 0 & 0 & 0.16667 & 0.3333 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$-19.2 = EI[0.3333D_1 + 0.16667D_2]$$

$$-19.2 = EI[0.16667D_1 + 0.6667D_2 + 0.16667D_3]$$

$$19.2 = EI[0.16667D_2 + 0.6667D_3 + 0.16667D_4]$$

$$19.2 = EI[0.16667D_3 + 0.16667D_4]$$

Solving,

$$D_1 = -46.08/EI$$

$$D_2 = -23.04/EI$$

$$D_3 = 23.04/EI$$

$$D_4 = 46.08/EI$$

$$\mathbf{q} = \mathbf{k}_1\mathbf{D}$$

15–9. Continued

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = EI \begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix} \begin{bmatrix} -46.08/EI \\ -23.04/EI \end{bmatrix}$$

$q_1 = EI[0.3333(-46.08/EI) + 0.16667(-23.04/EI)]$

$q_1 = -19.2 \text{ kN} \cdot \text{m}$

$q_2 = EI[0.16667(-46.08/EI) + 0.3333(-23.04/EI)]$

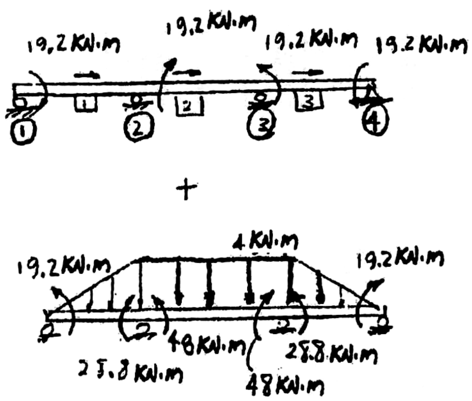
$q_2 = -15.36 \text{ kN} \cdot \text{m}$

Since the opposite FEM $= 19.2 \text{ kN} \cdot \text{m}$ is at node 1, then

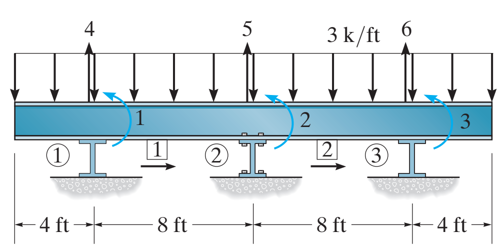
$M_1 = M_4 = 19.2 - 19.2 = 0$

Since the FEM $= -28.8 \text{ kN} \cdot \text{m}$ is at node 2, then

$M_2 = M_3 = -28.8 - 15.36 = 44.2 \text{ kN} \cdot \text{m}$ **Ans.**



15–10. Determine the reactions at the supports. Assume ② is pinned and ① and ③ are rollers. EI is constant.



Member 1

$$\mathbf{k}_1 = \frac{EI}{8} \begin{bmatrix} 0.1875 & 0.75 & -0.1875 & 0.75 \\ 0.75 & 4 & -0.75 & 2 \\ -0.1875 & -0.75 & 0.1875 & -0.75 \\ 0.75 & 2 & -0.75 & 4 \end{bmatrix}$$

15–10. Continued

Member 2

$$\mathbf{k}_2 = \frac{EI}{8}\begin{bmatrix} 0.1875 & 0.75 & -0.1875 & 0.75 \\ 0.75 & 4 & -0.75 & 2 \\ -0.1875 & -0.75 & 0.1875 & -0.75 \\ 0.75 & 2 & -0.75 & 4 \end{bmatrix}$$

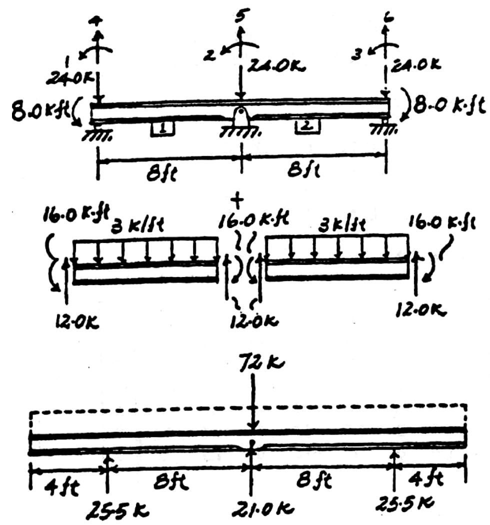
Q = KD

$$\begin{bmatrix} 8.0 \\ 0 \\ -8.0 \\ Q_4 - 24.0 \\ Q_5 - 24.0 \\ Q_6 - 24.0 \end{bmatrix} = \frac{EI}{8}\begin{bmatrix} 4 & 2 & 0 & 0.75 & -0.75 & 0 \\ 2 & 8 & 2 & 0.75 & 0 & -0.75 \\ 0 & 2 & 4 & 0 & 0.75 & -0.75 \\ 0.75 & 0.75 & 0 & 0.1875 & -0.1875 & 0 \\ -0.75 & 0 & 0.75 & -0.1875 & 0.375 & -0.1875 \\ 0 & -0.75 & -0.75 & 0 & -0.1875 & 0.1875 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8.0 = \frac{EI}{8}[4D_1 + 2D_2]$$

$$0 = \frac{EI}{8}[2D_1 + 8D_2 + 2D_3]$$

$$-8.0 = \frac{EI}{8}[2D_2 + 4D_3]$$



Solving:

$$D_1 = \frac{16.0}{EI}, \qquad D_2 = 0, \qquad D_3 = -\frac{16.0}{EI}$$

$$Q_4 - 24.0 = \frac{EI}{8}(0.75)\left(\frac{16.0}{EI}\right) + 0 + 0$$

$Q_4 = 25.5$ k **Ans.**

$$Q_5 - 24.0 = \frac{EI}{8}(-0.75)\left(\frac{16.0}{EI}\right) + 0 + \frac{EI}{8}(0.75)\left(-\frac{16.0}{EI}\right)$$

$Q_5 = 21.0$ k **Ans.**

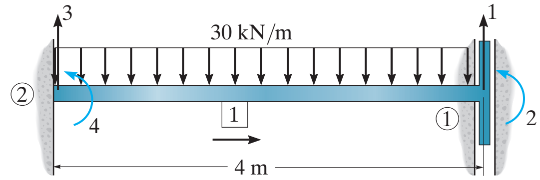
$$Q_6 - 24.0 = 0 + 0 + \frac{EI}{8}(-0.75)\left(\frac{-16.0}{EI}\right)$$

$Q_6 = 25.5$ k **Ans.**

$\circlearrowleft + \sum M_2 = 0;\quad 25.5(8) - 25.5(8) = 0$ (Check)

$+\uparrow \sum F = 0;\quad 25.5 + 21.0 + 25.5 - 72 = 0$ (Check)

15–11. Determine the reactions at the supports. There is a smooth slider at ①. EI is constant.



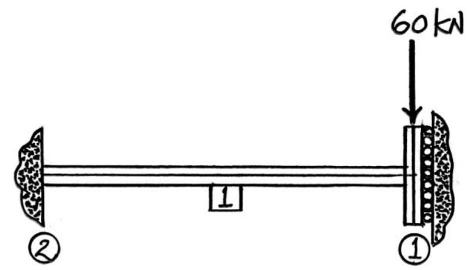
Member Stiffness Matrix. For member [1],

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_1 = EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 1 \\ 2 \end{matrix}$$

(column code numbers: 3, 4, 1, 2)



(a)

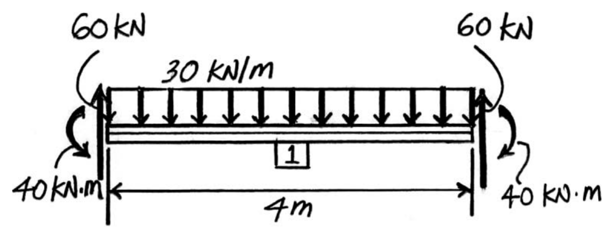
Known Nodal Loads And Deflections. The nodal load acting on the unconstrained degree of freedom (code number 1) is shown in Fig. a. Thus,

$$\mathbf{Q}_k = [-60]\, 1 \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 4 \end{matrix}$$

Load-Displacement Relation. The structure stiffness matrix is a 4 × 4 matirx since the highest code number is 4. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} -60 \\ \hline Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = EI \left[\begin{array}{c|ccc} 0.1875 & -0.375 & -0.1875 & -0.375 \\ \hline -0.375 & 1.00 & 0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & 0.375 \\ -0.375 & 0.5 & 0.375 & 1.00 \end{array}\right] \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} D_1 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

(column code numbers: 1, 2, 3, 4)



(b)

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

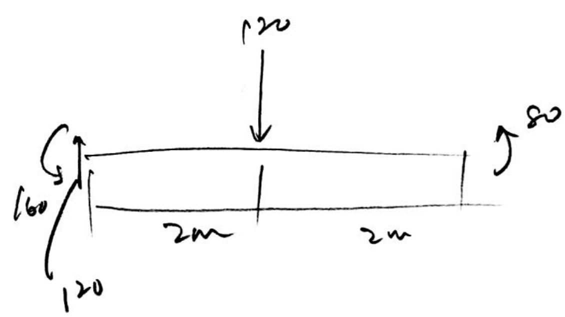
$$-60 = 0.1875EID_1 \qquad D_1 = -\frac{320}{EI}$$

Using this result, and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_2 = -0.375EI\left(-\frac{320}{EI}\right) + 0 = 120 \text{ kN}\cdot\text{m}$$

$$Q_3 = -0.1875EI\left(-\frac{320}{EI}\right) + 0 = 60 \text{ kN}$$

$$Q_4 = -0.375EI\left(-\frac{320}{EI}\right) + 0 = 120 \text{ kN}\cdot\text{m}$$



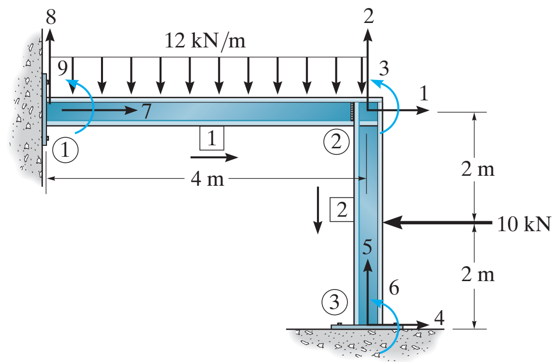
Superposition these results with the FEM shown in Fig. b,

$R_2 = 120 - 40 = 80$ kN·m ↺ **Ans.**

$R_3 = 60 + 60 = 120$ kN ↑ **Ans.**

$R_4 = 120 + 40 = 160$ kN·m ↺ **Ans.**

16–1. Determine the structure stiffness matrix **K** for the frame. Assume ① and ③ are fixed. Take $E = 200$ GPa, $I = 300(10^6)$ mm^4, $A = 10(10^3)$ mm^2 for each member.



Member Stiffness Matrices.

The orgin of the global coordinate system will be set at joint ①.

For member [1] and [2], $L = 4$m

$$\frac{AE}{L} = \frac{0.01[200(10^9)]}{4} = 500(10^6)\text{ N/m}$$

$$\frac{12EI}{L^3} = \frac{12[200(10^9)][300(10^{-6})]}{4^3} = 11.25(10^6)\text{ N/m}$$

$$\frac{6EI}{L^2} = \frac{6[200(10^9)][300(10^{-6})]}{4^2} = 22.5(10^6)\text{ N}$$

$$\frac{4EI}{L} = \frac{4[200(10^9)][300(10^{-6})]}{4} = 60(10^6)\text{ N}\cdot\text{m}$$

$$\frac{2EI}{L} = \frac{2[200(10^9)][300(10^{-6})]}{4} = 30(10^6)\text{ N}\cdot\text{m}$$

For member [1], $\lambda_x = \frac{4-0}{4} = 1$ and $\lambda_y = \frac{0-0}{4} = 0$. Thus,

$$\mathbf{k}_1 = \begin{array}{c} \begin{matrix} 7 & 8 & 9 & 1 & 2 & 3 \end{matrix} \\ \left[\begin{array}{cccccc} 500 & 0 & 0 & -500 & 0 & 0 \\ 0 & 11.25 & 22.5 & 0 & -11.25 & 22.5 \\ 0 & 22.5 & 60 & 0 & -22.5 & 30 \\ -500 & 0 & 0 & 500 & 0 & 0 \\ 0 & -11.25 & -22.5 & 0 & 11.25 & -22.5 \\ 0 & 22.5 & 30 & 0 & -22.5 & 60 \end{array}\right] \end{array} \begin{array}{c} 7 \\ 8 \\ 9 \\ 1 \\ 2 \\ 3 \end{array} (10^6)$$

For member [2], $\lambda_x = \frac{4-4}{4} = 0$ and $\lambda_y = \frac{-4-0}{4} = -1$. Thus,

$$\mathbf{k}_2 = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \left[\begin{array}{cccccc} 11.25 & 0 & 22.5 & -11.25 & 0 & 22.5 \\ 0 & 500 & 0 & 0 & -500 & 0 \\ 22.5 & 0 & 60 & -22.5 & 0 & 30 \\ -11.25 & 0 & -22.5 & 11.25 & 0 & -22.5 \\ 0 & -500 & 0 & 0 & 500 & 0 \\ 22.5 & 0 & 30 & -22.5 & 0 & 60 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} (10^6)$$

16–1. Continued

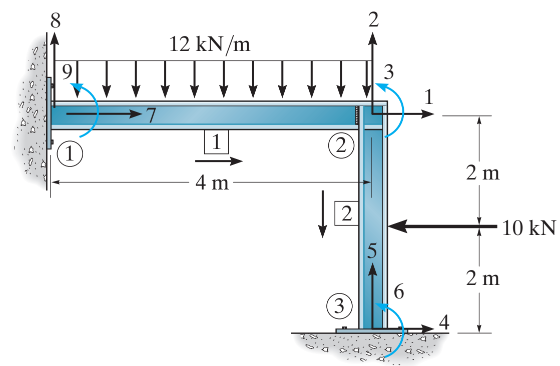
Structure Stiffness Matrix. It is a 9 × 9 matrix since the highest code number is 9. Thus,

$$
\mathbf{K} = \begin{array}{c} \\ \end{array}
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
$$

$$
\mathbf{K} = \begin{bmatrix}
511.25 & 0 & 22.5 & -11.25 & 0 & 22.5 & -500 & 0 & 0 \\
0 & 511.25 & -22.5 & 0 & -500 & 0 & 0 & -11.25 & -22.25 \\
22.5 & -22.5 & 120 & -22.5 & 0 & 30 & 0 & 22.5 & 30 \\
-11.25 & 0 & -22.5 & 11.25 & 0 & -22.5 & 0 & 0 & 0 \\
0 & -500 & 0 & 0 & 500 & 0 & 0 & 0 & 0 \\
22.5 & 0 & 30 & -22.5 & 0 & 60 & 0 & 0 & 0 \\
-500 & 0 & 0 & 0 & 0 & 0 & 500 & 0 & 0 \\
0 & -11.25 & 22.5 & 0 & 0 & 0 & 0 & 11.25 & 22.5 \\
0 & -22.5 & 30 & 0 & 0 & 0 & 0 & 25.5 & 60
\end{bmatrix}
\begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5\ (10^6) \\ 6 \\ 7 \\ 8 \\ 9 \end{array}
$$

Ans.

16–2. Determine the support reactions at the fixed supports ① and ③. Take $E = 200$ GPa, $I = 300(10^6)$ mm^4, $A = 10(10^3)$ mm^2 for each member.



Known Nodal Loads and Deflections. The nodal load acting on the unconstrained degree of freedom (code number 1, 2 and 3) are shown in Fig. *a* and *b*.

$$
\mathbf{Q}_k = \begin{bmatrix} -5(10^3) \\ -24(10^3) \\ 11(10^3) \end{bmatrix} \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}
$$

Loads-Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$,

$$
\begin{bmatrix} -5(10^3) \\ -24(10^3) \\ 11(10^3) \\ \hline Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix}
=
\left[\begin{array}{ccc|cccccc}
511.25 & 0 & 22.5 & -11.25 & 0 & 22.5 & -500 & 0 & 0 \\
0 & 511.25 & -22.5 & 0 & -500 & 0 & 0 & -11.25 & -22.5 \\
22.5 & -22.5 & 120 & -22.5 & 0 & 30 & 0 & 22.5 & 30 \\
\hline
-11.25 & 0 & -22.5 & 11.25 & 0 & -22.5 & 0 & 0 & 0 \\
0 & -500 & 0 & 0 & 500 & 0 & 0 & 0 & 0 \\
22.5 & 0 & 30 & -22.5 & 0 & 60 & 0 & 0 & 0 \\
-500 & 0 & 0 & 0 & 0 & 0 & 500 & 0 & 0 \\
0 & -11.25 & 22.5 & 0 & 0 & 0 & 0 & 11.25 & 22.5 \\
0 & -22.5 & 30 & 0 & 0 & 0 & 0 & 22.5 & 60
\end{array}\right]
(10^6)
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

16–2. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$-5(10^3) = (511.25D_1 + 22.5D_3)(10^6) \qquad (1)$$

$$-24(10^3) = (511.25D_2 - 22.5D_3)(10^6) \qquad (2)$$

$$11(10^3) = (22.5D_1 - 22.5D_2 + 120D_3)(10^6) \qquad (3)$$

Solving Eqs. (1) to (3),

$$D_1 = -13.57(10^{-6})\text{ m} \qquad D_2 = -43.15(10^{-6})\text{m} \qquad D_3 = 86.12(10^{-6})\text{ rad}$$

Using these results and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_4 = -11.25(10^6)(-13.57)(10^{-6}) + (-22.5)(10^6)(86.12)(10^{-6}) = -1.785\text{ kN}$$

$$Q_5 = -500(10^6)(-43.15)(10^{-6}) = 21.58\text{ kN}$$

$$Q_6 = 22.5(10^6)(-13.57)(10^{-6}) + 30(10^6)(86.12)(10^{-6}) = 2.278\text{ kN}\cdot\text{m}$$

$$Q_7 = -500(10^6)(-13.57)(10^{-6}) = 6.785\text{ kN}$$

$$Q_8 = -11.25(10^6)(-43.15)(10^{-6}) + 22.5(10^6)(86.12)(10^{-6}) = 2.423\text{ kN}$$

$$Q_9 = -22.5(10^6)(-43.15)(10^{-6}) + 30(10^6)(86.12)(10^{-6}) = 3.555\text{ kN}\cdot\text{m}$$

Superposition these results to those of FEM shown in Fig. *a*,

$R_4 = -1.785 + 5 = 3.214\text{ kN} = 3.21\text{ kN} \rightarrow$ **Ans.**

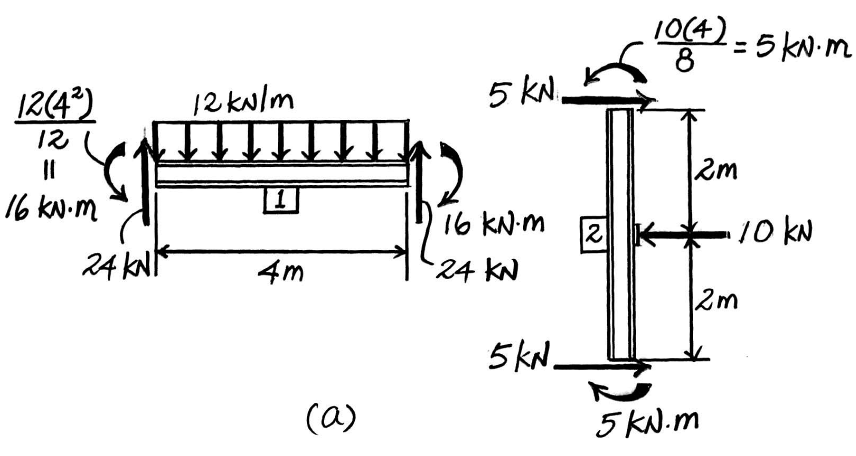
$R_5 = 21.58 + 0 = 21.58\text{ kN} = 21.6\text{ kN} \uparrow$ **Ans.**

$R_6 = 2.278 - 5 = -2.722\text{ kN}\cdot\text{m} = 2.72\text{ kN}\cdot\text{m}$ ↻ **Ans.**

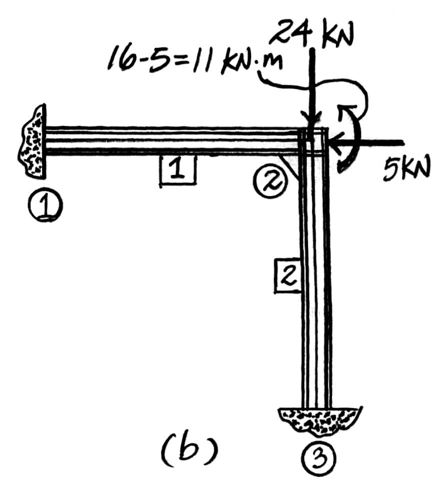
$R_7 = 6.785 + 0 = 6.785\text{ kN} = 6.79\text{ kN} \rightarrow$ **Ans.**

$R_8 = 2.423 + 24 = 26.42\text{ kN} = 26.4\text{ kN} \uparrow$ **Ans.**

$R_9 = 3.555 + 16 = 19.55\text{ kN}\cdot\text{m} = 19.6\text{ kN}\cdot\text{m}$ ↺ **Ans.**

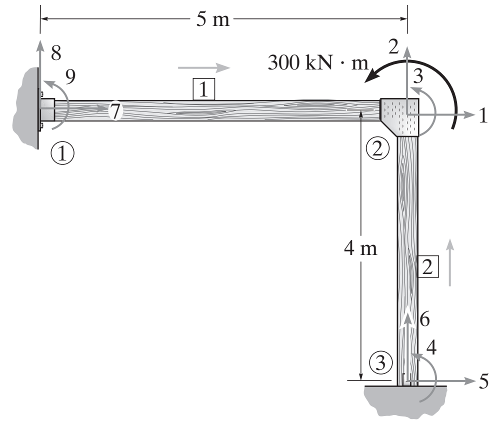


(a)



(b)

16–3. Determine the structure stiffness matrix **K** for the frame. Assume ③. is pinned and ①. is fixed. Take $E = 200$ MPa, $I = 300(10^6)$ mm^4, $A = 21(10^3)$ mm^2 for each member.



For member 1

$$\lambda_x = \frac{5-0}{5} = 1 \quad \lambda_y = 0$$

$$\frac{AE}{L} = \frac{(0.021)(200)(10^6)}{5} = 840000 \qquad \frac{12EI}{L^3} = \frac{(12)(200)(10^6)(300)(10^{-6})}{5^3} = 5760$$

$$\frac{6EI}{L^2} = \frac{6(200)(10^6)(300)(10^{-6})}{5^2} = 14400 \qquad \frac{2EI}{L} = \frac{2(200)(10^6)(300)(10^{-6})}{5} = 24000$$

$$\frac{4EI}{L} = \frac{4(200)(10^6)(300)(10^{-6})}{5} = 48000$$

$$\mathbf{k}_1 = \begin{bmatrix} 840000 & 0 & 0 & -840000 & 0 & 0 \\ 0 & 5760 & 14400 & 0 & -5760 & 14400 \\ 0 & 14400 & 48000 & 0 & -14400 & 24000 \\ -840000 & 0 & 0 & 840000 & 0 & 0 \\ 0 & -5760 & -14400 & 0 & 5760 & -14400 \\ 0 & 14400 & 24000 & 0 & -14400 & 48000 \end{bmatrix}$$

For member 2

$$\lambda_x = 0 \quad \lambda_y = \frac{0-(-4)}{4} = 1$$

$$\frac{AE}{L} = \frac{(0.021)(200)(10^6)}{5} = 1050000 \qquad \frac{12EI}{L^3} = \frac{(12)(200)(10^6)(300)(10^{-6})}{4^3} = 11250$$

$$\frac{6EI}{L^2} = \frac{6(200)(10^6)(300)(10^{-6})}{4^2} = 22500 \qquad \frac{2EI}{L} = \frac{2(200)(10^6)(300)(10^{-6})}{4} = 30000$$

$$\frac{4EI}{L} = \frac{4(200)(10^6)(300)(10^{-6})}{4} = 60000$$

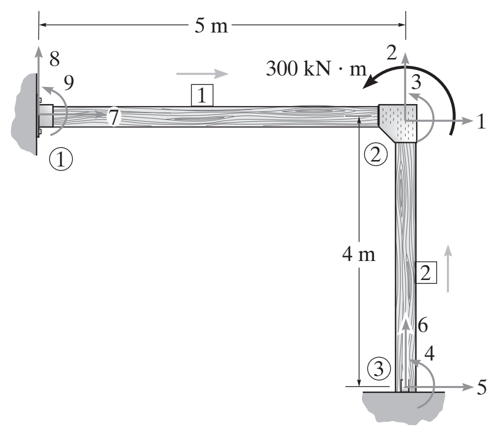
$$\mathbf{k}_2 = \begin{bmatrix} 11250 & 0 & -22500 & -11250 & 0 & -22500 \\ 0 & 1050000 & 0 & 0 & -1050000 & 0 \\ -22500 & 0 & 60000 & 22500 & 0 & 30000 \\ -11250 & 0 & 22500 & 11250 & 0 & 22500 \\ 0 & -1050000 & 0 & 0 & 1050000 & 0 \\ -22500 & 0 & 30000 & 22500 & 0 & 60000 \end{bmatrix}$$

16–3. Continued

Structure Stiffness Matrix.

$$\mathbf{K} = \begin{bmatrix} 851250 & 0 & 22500 & 22500 & -11250 & 0 & -840000 & 0 & 0 \\ 0 & 1055760 & -14400 & 0 & 0 & -1050000 & 0 & -5760 & -14400 \\ 22500 & -14400 & 108000 & 30000 & -22500 & 0 & 0 & 144000 & 24000 \\ 22500 & 0 & 30000 & 60000 & -22500 & 0 & 0 & 0 & 0 \\ -11250 & 0 & -22500 & -22500 & 11250 & 0 & 0 & 0 & 0 \\ 0 & -1050000 & 0 & 0 & 0 & 1050000 & 0 & 0 & 0 \\ -840000 & 0 & 0 & 0 & 0 & 0 & 840000 & 0 & 0 \\ 0 & -5760 & 14400 & 0 & 0 & 0 & 0 & 5760 & 14400 \\ 0 & -14400 & 24000 & 0 & 0 & 0 & 0 & 14400 & 48000 \end{bmatrix}$$ **Ans.**

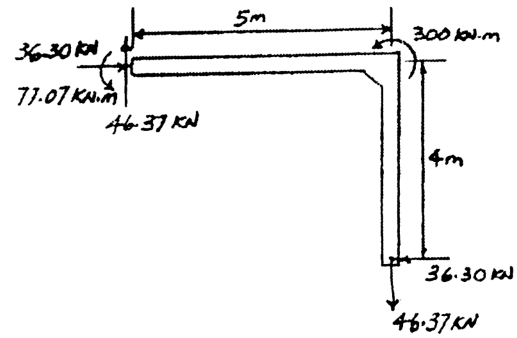
***16–4.** Determine the support reactions at ① and ③. Take $E = 200$ MPa, $I = 300(10^6)$ mm^4, $A = 21(10^3)$ mm^2 for each member.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_k = \begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} 851250 & 0 & 22500 & 22500 & -11250 & 0 & -840000 & 0 & 0 \\ 0 & 1055760 & -14400 & 0 & 0 & -1050000 & 0 & -5760 & -14400 \\ 22500 & -14400 & 108000 & 30000 & -22500 & 0 & 0 & 14400 & 24000 \\ 22500 & 0 & 30000 & 60000 & -22500 & 0 & 0 & 0 & 0 \\ -11250 & 0 & -22500 & -22500 & 11250 & 0 & 0 & 0 & 0 \\ 0 & -1050000 & 0 & 0 & 0 & 1050000 & 0 & 0 & 0 \\ -840000 & 0 & 0 & 0 & 0 & 0 & 840000 & 0 & 0 \\ 0 & -5760 & 14400 & 0 & 0 & 0 & 0 & 5760 & 14400 \\ 0 & -14400 & 24000 & 0 & 0 & 0 & 0 & 1440 & 48000 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

16–4. Continued



Partition matrix

$$\begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \end{bmatrix} = \begin{bmatrix} 851250 & 0 & 22500 & 22500 \\ 0 & 1055760 & -14400 & 0 \\ 22500 & -14400 & 108000 & 30000 \\ 22500 & 0 & 30000 & 60000 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$0 = 851250D_1 + 22500D_3 + 22500D_4$

$0 = 1055760D_2 - 14400D_3$

$300 = 22500D_1 - 14400D_2 + 108000D_3 + 30000D_4$

$0 = 22500D_1 + 30000D_3 + 60000D_4$

Solving.

$D_1 = -0.00004322$ m $\qquad D_2 = 0.00004417$ m $\qquad D_3 = 0.00323787$ rad
$D_4 = -0.00160273$ rad

$$\begin{bmatrix} Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} -11250 & 0 & -22500 & -22500 \\ 0 & -1050000 & 0 & 0 \\ -840000 & 0 & 0 & 0 \\ 0 & -5760 & 14400 & 0 \\ 0 & -14400 & 24000 & 0 \end{bmatrix} \begin{bmatrix} -0.00004322 \\ 0.00004417 \\ 0.00323787 \\ -0.00160273 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$Q_5 = -36.3$ kN **Ans.**

$Q_6 = -46.4$ kN **Ans.**

$Q_7 = 36.3$ kN **Ans.**

$Q_8 = 46.4$ kN **Ans.**

$Q_9 = 77.1$ kN·m **Ans.**

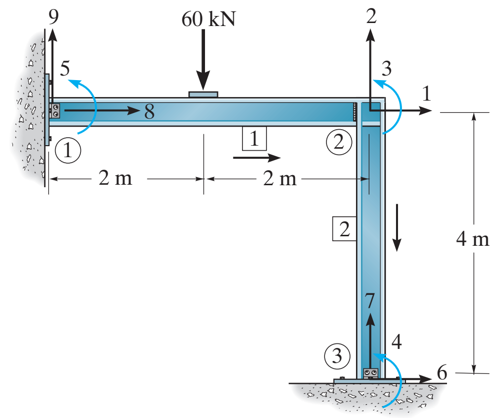
Check equilibrium

$\circlearrowleft \sum F_x = 0; \qquad 36.30 - 36.30 = 0$ (Check)

$+\uparrow \sum F_y = 0; \qquad 46.37 - 46.37 = 0$ (Check)

$\circlearrowleft + \sum M_1 = 0; \qquad 300 + 77.07 - 36.30(4) - 46.37(5) = 0$ (Check)

16–5. Determine the structure stiffness matrix **K** for the frame. Take $E = 200$ GPa, $I = 350(10^6)$ mm^4, $A = 15(10^3)$ mm^2 for each member. Joints at ① and ③ are pins.



Member Stiffness Matrices. The origin of the global coordinate system will be set at joint ①. For member $\boxed{1}$ and $\boxed{2}$, $L = 4$m.

$$\frac{AE}{L} = \frac{0.015[200(10^9)]}{4} = 750(10^6)\ \text{N/m}$$

$$\frac{12EI}{L^3} = \frac{12[200(10^9)][350(10^{-6})]}{4^3} = 13.125(10^6)\ \text{N/m}$$

$$\frac{6EI}{L^2} = \frac{4[200(10^9)][350(10^{-6})]}{4^2} = 26.25(10^6)\ \text{N}$$

$$\frac{4EI}{L} = \frac{4[200(10^9)][350(10^{-6})]}{4} = 70(10^6)\ \text{N}\cdot\text{m}$$

$$\frac{2EI}{L} = \frac{2[200(10^9)][350(10^{-6})]}{4} = 35(10^6)\ \text{N}\cdot\text{m}$$

For member $\boxed{1}$, $\lambda_x = \dfrac{4-0}{4} = 1$ and $\lambda_y = \dfrac{0-0}{4} = 0$. Thus,

$$\mathbf{k}_1 = \begin{array}{c} \begin{matrix} 8 & 9 & 5 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 750 & 0 & 0 & -750 & 0 & 0 \\ 0 & 13.125 & 26.25 & 0 & -13.125 & 26.25 \\ 0 & 26.25 & 70 & 0 & -26.25 & 35 \\ -750 & 0 & 0 & 750 & 0 & 0 \\ 0 & -13.125 & -26.25 & 0 & 13.125 & -26.25 \\ 0 & 26.25 & 35 & 0 & -26.25 & 70 \end{bmatrix} \end{array} \begin{matrix} 8 \\ 9 \\ 5 \\ 1 \\ 2 \\ 3 \end{matrix} (10^6)$$

For member $\boxed{2}$, $\lambda_x = \dfrac{4-4}{4} = 0$, and $\lambda_y = \dfrac{-4-0}{4} = -1$. Thus,

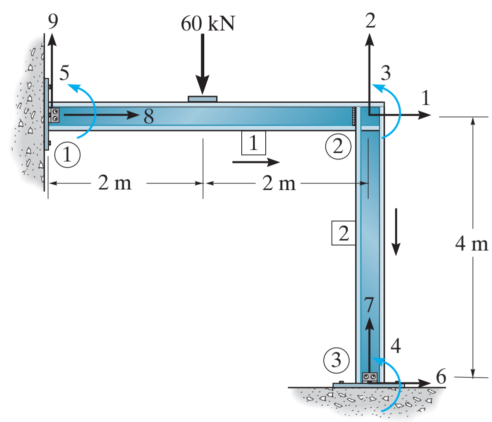
$$\mathbf{k}_2 = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 6 & 7 & 4 \end{matrix} \\ \begin{bmatrix} 13.125 & 0 & 26.25 & -13.125 & 0 & 26.25 \\ 0 & 750 & 0 & 0 & -750 & 0 \\ 26.25 & 0 & 70 & -26.25 & 0 & 35 \\ -13.125 & 0 & -26.25 & 13.125 & 0 & -26.25 \\ 0 & -750 & 0 & 0 & 750 & 0 \\ 26.25 & 0 & 35 & -26.25 & 0 & 70 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 6 \\ 7 \\ 4 \end{matrix} (10^6)$$

16–5. Continued

Structure Stiffness Matrix. It is a 9 × 9 matrix since the highest code number is 9. Thus

$$\mathbf{K} = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix} \\ \left[\begin{matrix} 763.125 & 0 & 26.25 & 26.25 & 0 & -13.125 & 0 & -750 & 0 \\ 0 & 763.125 & -26.25 & 0 & -26.25 & 0 & -750 & 0 & -13.125 \\ 26.25 & -26.25 & 140 & 35 & 35 & -26.25 & 0 & 0 & 26.25 \\ 26.25 & 0 & 35 & 70 & 0 & -26.25 & 0 & 0 & 0 \\ 0 & -26.25 & 35 & 0 & 70 & 0 & 0 & 0 & 26.25 \\ -13.125 & 0 & -26.25 & -26.25 & 0 & 13.125 & 0 & 0 & 0 \\ 0 & -750 & 0 & 0 & 0 & 0 & 750 & 0 & 0 \\ -750 & 0 & 0 & 0 & 0 & 0 & 0 & 750 & 0 \\ 0 & -13.125 & 26.25 & 0 & 26.25 & 0 & 0 & 0 & 13.125 \end{matrix}\right] \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix} (10^6) \qquad \textbf{Ans.}$$

16–6. Determine the support reactions at pins ① and ③. Take $E = 200$ GPa, $I = 350(10^6)$ mm^4, $A = 15(10^3)$ mm^2 for each member.



Known Nodal Loads and Deflections. The nodal load acting on the unconstrained degree of freedom (code numbers 1, 2, 3, 4, and 5) are shown in Fig. *a* and Fig. *b*.

$$\mathbf{Q}_k = \left[\begin{matrix} 0 \\ -41.25(10^3) \\ 45(10^3) \\ 0 \\ 0 \end{matrix}\right] \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \left[\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right] \begin{matrix} 6 \\ 7 \\ 8 \\ 9 \end{matrix}$$

Loads-Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\left[\begin{array}{c} 0 \\ -41.25(10^3) \\ 45(10^3) \\ 0 \\ 0 \\ \hline Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{array}\right] = \left[\begin{array}{ccccc|cccc} 763.125 & 0 & 26.25 & 26.25 & 0 & -13.125 & 0 & -750 & 0 \\ 0 & 763.125 & -26.25 & 0 & -26.25 & 0 & -750 & 0 & -13.125 \\ 26.25 & -26.25 & 140 & 35 & 35 & -26.25 & 0 & 0 & 26.25 \\ 26.25 & 0 & 35 & 70 & 0 & -26.25 & 0 & 0 & 0 \\ 0 & -26.25 & 35 & 0 & 70 & 0 & 0 & 0 & 26.25 \\ \hline -13.125 & 0 & -26.25 & -26.25 & 0 & 13.125 & 0 & 0 & 0 \\ 0 & -750 & 0 & 0 & 0 & 0 & 750 & 0 & 0 \\ -750 & 0 & 0 & 0 & 0 & 0 & 0 & 750 & 0 \\ 0 & -13.125 & 26.25 & 0 & 26.25 & 0 & 0 & 0 & 13.125 \end{array}\right] (10^6) \left[\begin{array}{c} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

16–6. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$0 = (763.125D_1 + 26.25D_3 + 26.25D_4)(10^6) \quad (1)$$

$$-41.25(10^3) = (763.125D_2 - 26.25D_3 - 26.25D_5)(10^6) \quad (2)$$

$$45(10^3) = (26.25D_1 - 26.25D_2 + 140D_3 + 35D_4 + 35D_5)(10^6) \quad (3)$$

$$0 = (26.25D_1 + 35D_3 + 70D_4)(10^6) \quad (4)$$

$$0 = (-26.25D_2 + 35D_3 + 70D_5)(10^6) \quad (5)$$

Solving Eqs. (1) to (5)

$D_1 = -7.3802(10^{-6}) \qquad D_2 = -47.3802(10^{-6}) \qquad D_3 = 423.5714(10^{-6})$

$D_4 = -209.0181(10^{-6}) \qquad D_5 = -229.5533(10^{-6})$

Using these results and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$Q_6 = (-13.125)(10^6) - 7.3802(10^{-6}) - 26.25(10^6)423.5714(10^{-6}) - 26.25(10^6) - 209.0181(10^{-6}) + 0 = -5.535 \text{ kN}$

$Q_7 = -750(10^6) - 47.3802(10^{-6}) + 0 = 35.535 \text{ kN}$

$Q_8 = -750(10^6) - 7.3802(10^{-6}) + 0 = 5.535 \text{ kN}$

$Q_9 = -13.125(10^6) - 47.3802(10^{-6}) + 26.25(10^6) + 423.5714(10^{-6}) + 26.25(10^6) - 229.5533(10^{-6}) + 0 = 5.715 \text{ kN}$

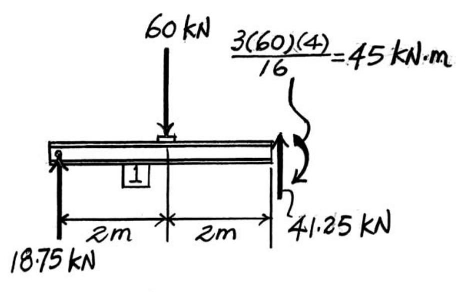
Superposition these results to those of FEM shown in Fig. *a*,

$R_6 = -5.535 \text{ kN} + 0 = 5.54 \text{ kN}$ **Ans.**

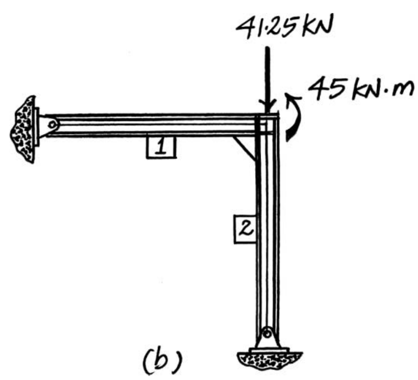
$R_7 = 35.535 + 0 = 35.5 \text{ kN}$ **Ans.**

$R_8 = 5.535 + 0 = 5.54 \text{ kN}$ **Ans.**

$R_9 = 5.715 + 18.75 = 24.5 \text{ kN}$ **Ans.**

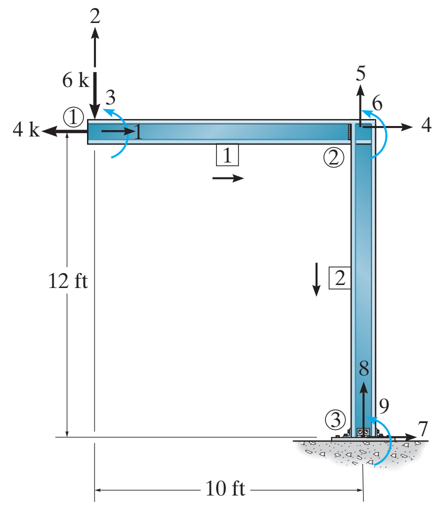


(a)



(b)

16–7. Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 650\ \text{in}^4$, $A = 20\ \text{in}^2$ for each member.



Member 1.

$$\lambda_x = \frac{10 - 0}{10} = 1 \qquad \lambda_y = 0$$

$$\frac{AE}{L} = \frac{20(29)(10^3)}{10(12)} = 4833.33 \qquad \frac{12EI}{L^3} = \frac{12(29)(10^3)(650)}{(10)^3(12)^3} = 130.90$$

$$\frac{6EI}{L^2} = \frac{6(29)(10^3)(650)}{(10)^2(12)^2} = 7854.17 \qquad \frac{4EI}{L} = \frac{4(29)(10^3)(650)}{(10)(12)} = 628333.33$$

$$\frac{2EI}{L} = \frac{2(29)(10^3)(650)}{(10)(12)} = 314166.67$$

$$\mathbf{k}_1 = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 \\ -4833.33 & 0 & 0 & 4833.33 & 0 & 0 \\ 0 & -130.90 & -7854.17 & 0 & 130.90 & -7854.17 \\ 0 & 7854.17 & 314166.67 & 0 & -7854.17 & 628333.33 \end{bmatrix}$$

Member 2.

$$\lambda_x = 0 \qquad \lambda_y = \frac{-12 - 0}{12} = -1$$

$$\frac{AE}{L} = \frac{(20)(29)(10^3)}{(12)(12)} = 4027.78 \qquad \frac{12EI}{L^3} = \frac{12(29)(10^3)(650)}{(12)^3(12)^3} = 75.75$$

$$\frac{6EI}{L} = \frac{6(29)(10^3)(650)}{(12)^2(12)^2} = 5454.28 \qquad \frac{4EI}{L} = \frac{4(29)(10^3)(650)}{(12)(12)} = 523611.11$$

$$\frac{2EI}{L^2} = \frac{2(29)(10^3)(650)}{(12)(12)} = 261805.55$$

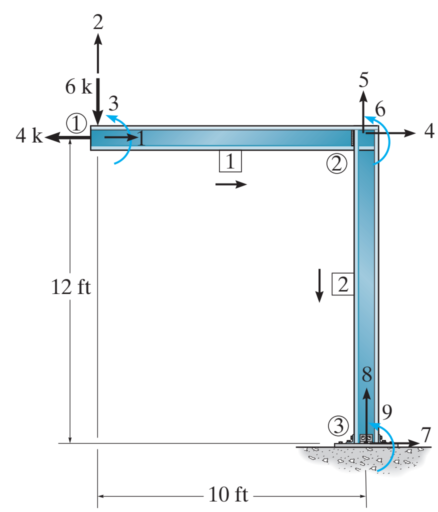
$$\mathbf{k}_2 = \begin{bmatrix} 75.75 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & 4027.78 & 0 & 0 & -4027.78 & 0 \\ 5454.28 & 0 & 523611.11 & -5454.28 & 0 & 261805.55 \\ -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 523611.11 \end{bmatrix}$$

16–7. Continued

Structure Stiffness Matrix.

$$\mathbf{K} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 & 0 & 0 & 0 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 & 0 & 0 & 0 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.37 & 0 & -4027.78 & 0 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 & -5454.28 & 0 & 261805.55 \\ 0 & 0 & 0 & -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & 0 & 0 & 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 0 & 0 & 0 & 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 523611.11 \end{bmatrix} \quad \textbf{Ans.}$$

***16–8.** Determine the components of displacement at ①. Take $E = 29(10^3)$ ksi, $I = 650\text{ in}^4$, $A = 20\text{ in}^2$ for each member.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_k = \begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 & 0 & 0 & 0 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 & 0 & 0 & 0 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.17 & 0 & -4027.78 & 0 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 & -5454.28 & 0 & 261805.55 \\ 0 & 0 & 0 & -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & 0 & 0 & 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 0 & 0 & 0 & 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 523611.11 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

16–8. Continued

Partition Matrix.

$$\begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.17 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4 = 4833.33D_1 - 4833.33D_4$

$-6 = 130.90D_2 + 7854.17D_3 - 130.90D_5 + 7854.17D_6$

$0 = 7854.17D_2 + 628333.33D_3 - 7854.17D_5 + 314166.67D_6$

$0 = -4833.33D_1 + 4909.08D_4 + 5454.28D_6$

$0 = -130.90D_2 - 7854.17D_3 + 4158.68D_5 - 7854.17D_6$

$0 = 7854.17D_2 + 314166.67D_3 + 5454.28D_4 - 7854.17D_5 + 1151944.44D_6$

Solving the above equations yields

$D_1 = -0.608$ in. **Ans.**

$D_2 = -1.12$ in. **Ans.**

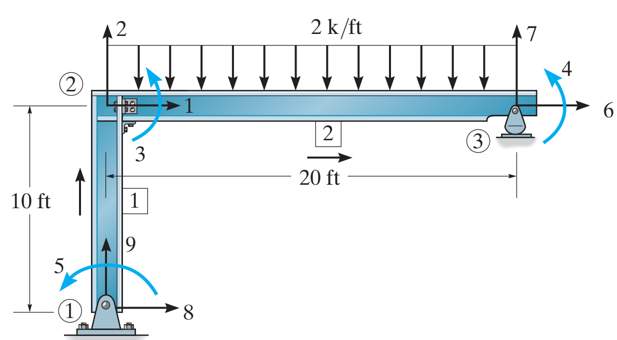
$D_3 = 0.0100$ rad **Ans.**

$D_4 = -0.6076$ in.

$D_5 = -0.001490$ in.

$D_6 = 0.007705$ rad

16–9. Determine the stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 300$ in^4, $A = 10$ in^2 for each member.



Member Stiffness Matrices. The origin of the global coordinate system will be set at joint ①. For member $\boxed{1}$, $L = 10$ ft, $\lambda_x = \dfrac{0 - 0}{10} = 0$ and $\lambda_y = \dfrac{10 - 0}{10} = 1$

$$\frac{AE}{L} = \frac{10[29(10^3)]}{10(12)} = 2416.67 \text{ k/in} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](300)}{[10(12)]^3} = 60.4167 \text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](300)}{[10(12)]^2} = 3625 \text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](300)}{10(12)} = 290000 \text{ k} \cdot \text{in}$$

16–9. Continued

$$\frac{2EI}{L} = \frac{2[29(10^3)](300)}{10(12)} = 145000 \text{ k} \cdot \text{in}$$

$$\mathbf{k}_1 = \begin{array}{c} \\ \left[\begin{array}{cccccc} \scriptstyle 8 & \scriptstyle 9 & \scriptstyle 5 & \scriptstyle 1 & \scriptstyle 2 & \scriptstyle 3 \\ 60.4167 & 0 & -3625 & -60.4167 & 0 & -3625 \\ 0 & 2416.67 & 0 & 0 & -2416.67 & 0 \\ -3625 & 0 & 290000 & 3625 & 0 & 145000 \\ -60.4167 & 0 & 3625 & 60.4167 & 0 & 3625 \\ 0 & -2416.67 & 0 & 0 & 2416.67 & 0 \\ -3625 & 0 & 145000 & 3625 & 0 & 290000 \end{array}\right] \end{array} \begin{array}{c} \\ 8 \\ 9 \\ 5 \\ 1 \\ 2 \\ 3 \end{array}$$

For member $\boxed{2}$, $L = 20$ ft, $\lambda_x = \dfrac{20 - 0}{20} = 1$ and $\lambda_y = \dfrac{10 - 10}{20} = 0$.

$$\frac{AE}{L} = \frac{10[29(10^3)]}{20(12)} = 1208.33 \text{ k/in} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](300)}{[20(12)]^3} = 7.5521 \text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](300)}{[20(12)]^2} = 906.25 \text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](300)}{20(12)} = 145000 \text{ k} \cdot \text{in}$$

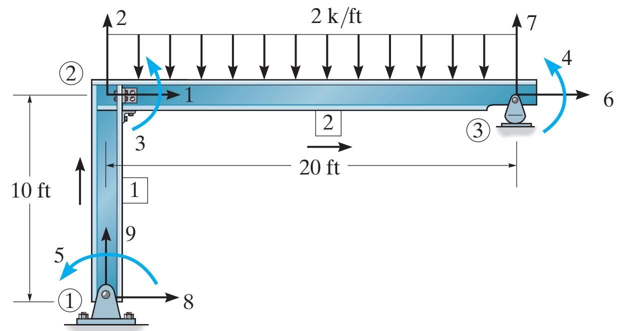
$$\frac{2EI}{L} = \frac{2[29(10^3)](300)}{20(12)} = 72500 \text{ k} \cdot \text{in}$$

$$\mathbf{k}_2 = \left[\begin{array}{cccccc} \scriptstyle 1 & \scriptstyle 2 & \scriptstyle 3 & \scriptstyle 6 & \scriptstyle 7 & \scriptstyle 4 \\ 1208.33 & 0 & 0 & -1208.33 & 0 & 0 \\ 0 & 7.5521 & 906.25 & 0 & -7.5521 & 906.25 \\ 0 & 906.25 & 145000 & 0 & -906.25 & 72500 \\ -1208.33 & 0 & 0 & 1208.33 & 0 & 0 \\ 0 & -7.5521 & -906.25 & 0 & 7.5521 & -906.25 \\ 0 & 906.25 & 72500 & 0 & -906.25 & 145000 \end{array}\right] \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 6 \\ 7 \\ 4 \end{array}$$

Structure Stiffness Matrix. It is a 9×9 matrix since the highest code number is 9. Thus,

$$\mathbf{K} = \left[\begin{array}{ccccccccc} 1268.75 & 0 & 3625 & 0 & 3625 & -1208.33 & 0 & -60.4167 & 0 \\ 0 & 2424.22 & 906.25 & 906.25 & 0 & 0 & -7.5521 & 0 & -2416.67 \\ 3625 & 906.25 & 435000 & 72500 & 145000 & 0 & -906.25 & -3625 & 0 \\ 0 & 906.25 & 72500 & 145000 & 0 & 0 & -906.25 & 0 & 0 \\ 3625 & 0 & 145000 & 0 & 290000 & 0 & 0 & -3625 & 0 \\ -1208.33 & 0 & 0 & 0 & 0 & 1208.33 & 0 & 0 & 0 \\ 0 & -7.5521 & -906.25 & -906.25 & 0 & 0 & 7.5521 & 0 & 0 \\ -60.4167 & 0 & -3625 & 0 & -3625 & 0 & 0 & 60.4167 & 0 \\ 0 & -2416.67 & 0 & 0 & 0 & 0 & 0 & 0 & 2416.67 \end{array}\right]$$

16–10. Determine the support reactions at ① and ③. Take $E = 29(10^3)$ ksi, $I = 300\text{ in}^4$, $A = 10\text{ in}^2$ for each member.



Known Nodal Loads and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, 4, 5, and 6) are shown in Fig. *a* and *b*.

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ -25 \\ -1200 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 9 \end{matrix}$$

Loads–Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$.

$$\begin{bmatrix} 0 \\ -25 \\ -1200 \\ 0 \\ 0 \\ 0 \\ \hline Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \left[\begin{array}{cccccc|ccc} 1268.75 & 0 & 3625 & 0 & 3625 & -1208.33 & 0 & -60.4167 & 0 \\ 0 & 2424.22 & 906.25 & 906.25 & 0 & 0 & -7.5521 & 0 & -2416.67 \\ 3625 & 906.25 & 435000 & 72500 & 145000 & 0 & -906.25 & -3625 & 0 \\ 0 & 906.25 & 72500 & 145000 & 0 & 0 & -906.25 & 0 & 0 \\ 3625 & 0 & 145000 & 0 & 290000 & 0 & 0 & -3625 & 0 \\ -1208.33 & 0 & 0 & 0 & 0 & 1208.33 & 0 & 0 & 0 \\ \hline 0 & -7.5521 & -906.25 & -906.25 & 0 & 0 & 7.5521 & 0 & 0 \\ -60.4167 & 0 & -3625 & 0 & -3625 & 0 & 0 & 60.4167 & 0 \\ 0 & -2416.67 & 0 & 0 & 0 & 0 & 0 & 0 & 2416.37 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

$$20 = -2416.67D_2$$

$$D_2 = -8.275862071(10^{-3})$$

$$5 = -7.5521(-8.2758)(10^{-3}) - 906.25D_3 - 906.25D_4$$

$$0 = 906.25(-8.2758)(10^{-3}) + 72500D_3 + 145000D_4$$

$$4.937497862 = -906.25D_3 - 906.25D_4$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$0 = 1268.75D_1 + 3625D_3 + 3625D_5 - 1208.33D_6 \qquad (1)$$

$$-25 = 2424.22D_2 + 906.25D_3 + 906.25D_4 \qquad (2)$$

$$-1200 = 3625D_1 + 906.25D_2 + 435000D_3 + 72500D_4 + 145000D_5 \qquad (3)$$

$$0 = 906.25D_2 + 72500D_3 + 145000D_4 \qquad (4)$$

$$0 = 3625D_1 + 145000D_3 + 290000D_5 \qquad (5)$$

$$0 = -1208.33D_1 + 1208.33D_6 \qquad (6)$$

16–10. Continued

Solving Eqs. (1) to (6)

$D_1 = 1.32$ $D_2 = -0.008276$ $D_3 = -0.011$ $D_4 = 0.005552$

$D_5 = -0.011$ $D_6 = 1.32$

Using these results and applying $\mathbf{Q}_k = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$Q_7 = -7.5521(-0.008276) - 906.25(-0.011) - 906.25(0.005552) = 5$

$Q_8 = 60.4167(1.32) - 3625(-0.011) - 3625(-0.011) = 0$

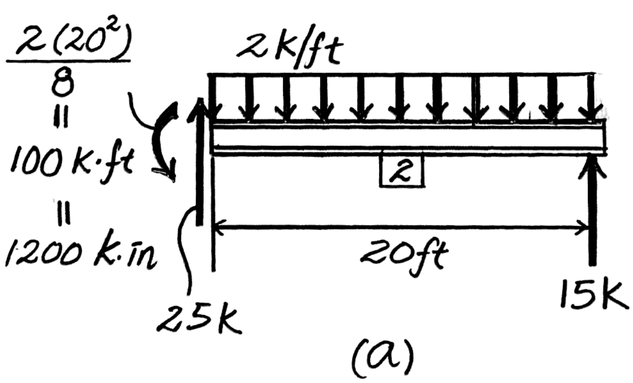
$Q_9 = -2416.67(-0.008276) = 20$

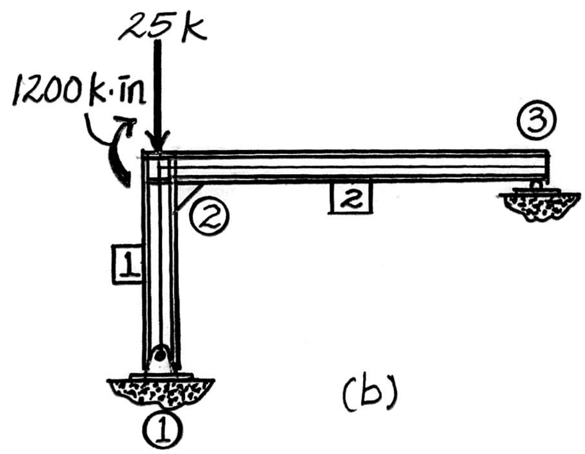
Superposition these results to those of FEM shown in Fig. *a*.

$R_7 = 5 + 15 = 20$ k **Ans.**

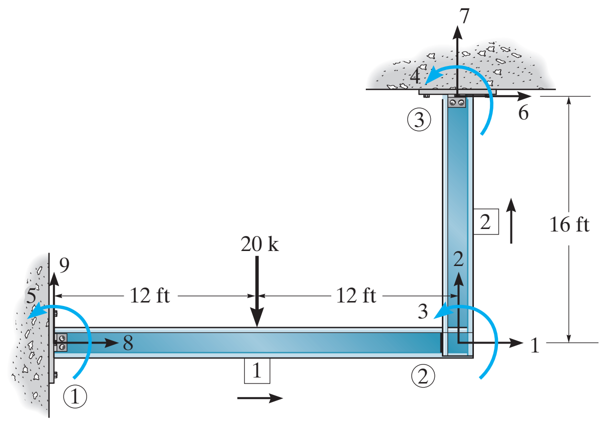
$R_8 = 0 + 0 = 0$ **Ans.**

$R_9 = 20 + 0 = 20$ k **Ans.**





16–11. Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 700\text{ in}^4$, $A = 20\text{ in}^2$ for each member.



Member Stiffness Matrices. The origin of the global coordinate system will be set at joint ①. For member 1, $L = 24$ ft, $\lambda_x = \frac{24-0}{24} = 1$ and $\lambda_y = \frac{0-0}{24} = 0$

$$\frac{AE}{L} = \frac{20[29(10^3)]}{24(12)} = 2013.89\text{ k/in} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](700)}{[24(12)]^3} = 10.1976\text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](700)}{[24(12)]^2} = 1468.46\text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](700)}{[24(12)]} = 281944\text{ k}\cdot\text{in}$$

$$\frac{2EI}{L} = \frac{2[29(10^3)](700)}{[24(12)]} = 140972\text{ k}\cdot\text{in}$$

$$\mathbf{k}_1 = \begin{bmatrix} 2013.89 & 0 & 0 & -2013.89 & 0 & 0 \\ 0 & 10.1976 & 1468.46 & 0 & -10.1976 & 1468.46 \\ 0 & 1468.46 & 281944 & 0 & -1468.46 & 140972 \\ -2013.89 & 0 & 0 & 2013.89 & 0 & 0 \\ 0 & -10.1976 & -1468.46 & 0 & 10.1976 & -1468.46 \\ 0 & 1468.46 & 140972 & 0 & -1468.46 & 281944 \end{bmatrix} \begin{matrix} 8 \\ 9 \\ 5 \\ 1 \\ 2 \\ 3 \end{matrix}$$

(columns: 8, 9, 5, 1, 2, 3)

For member 2, $L = 16$ ft, $\lambda_x = \frac{24-24}{16} = 0$ and $\lambda_y = \frac{16-0}{16} = 1.$

$$\frac{AE}{L} = \frac{20[29(10^3)]}{16(12)} = 3020.83\text{ k/in.} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](700)}{[16(12)]^3} = 34.4170\text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](700)}{[16(12)]^2} = 3304.04\text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](700)}{[16(12)]} = 422917\text{ k}\cdot\text{in}$$

$$\frac{2EI}{L} = \frac{2[29(10^3)](700)}{[16(12)]} = 211458\text{ k}\cdot\text{in}$$

16–11. Continued

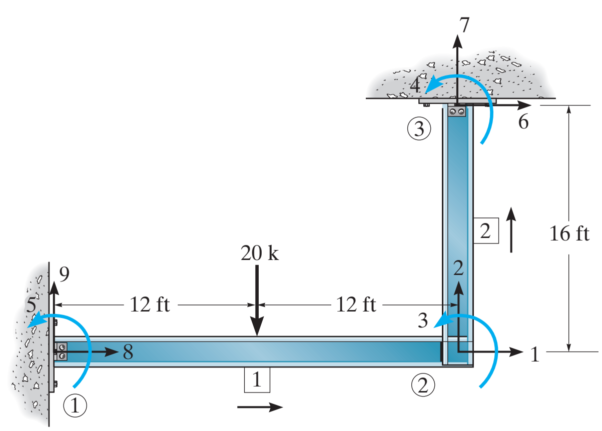
$$
\mathbf{k}_2 = \begin{array}{c} \\ \end{array}
\begin{array}{cccccc}
1 & 2 & 3 & 6 & 7 & 4 \\
\end{array}
$$

$$
\mathbf{k}_2 = \begin{bmatrix}
34.4170 & 0 & -3304.04 & -34.4170 & 0 & -3304.04 \\
0 & 3020.83 & 0 & 0 & -3020.83 & 0 \\
-3304.04 & 0 & 422917 & 3304.04 & 0 & 211458 \\
-34.4170 & 0 & 3304.04 & 34.4170 & 0 & 3304.04 \\
0 & -3020.83 & 0 & 0 & 3020.83 & 0 \\
-3304.04 & 0 & 211458 & 3304.04 & 0 & 422917
\end{bmatrix}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 6 \\ 7 \\ 4 \end{array}
$$

Structure Stiffness Matrix. It is a 9×9 matrix since the highest code number is 9. Thus,

$$
\mathbf{K} = \begin{bmatrix}
2048.31 & 0 & -3304.04 & -3304.04 & 0 & -34.4170 & 0 & -2013.89 & 0 \\
0 & 3031.03 & -1468.46 & 0 & -1468.46 & 0 & -3020.83 & 0 & -10.1976 \\
-3304.04 & -1468.46 & 704861 & 211458 & 140972 & 3304.04 & 0 & 0 & 1468.46 \\
-3304.04 & 0 & 211458 & 422917 & 0 & 3304.04 & 0 & 0 & 0 \\
0 & -1468.46 & 140972 & 0 & 281944 & 0 & 0 & 0 & 1468.46 \\
-34.4170 & 0 & 3304.04 & 3304.04 & 0 & 34.4170 & 0 & 0 & 0 \\
0 & -3020.83 & 0 & 0 & 0 & 0 & 3020.83 & 0 & 0 \\
-2013.89 & 0 & 0 & 0 & 0 & 0 & 0 & 2013.89 & 0 \\
0 & -10.1976 & 1468.46 & 0 & 1468.46 & 0 & 0 & 0 & 10.1976
\end{bmatrix}
$$

***16–12.** Determine the support reactions at the pins ① and ③. Take $E = 29(10^3)$ ksi, $I = 700\ \text{in}^4$, $A = 20\ \text{in}^2$ for each member.



Known Nodal Loads and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, 4, and 5) are shown in Fig. *a* and *b*.

$$
\mathbf{Q}_k = \begin{bmatrix} 0 \\ -13.75 \\ 1080 \\ 0 \\ 0 \end{bmatrix}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\quad \text{and} \quad
\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\begin{array}{c} 6 \\ 7 \\ 8 \\ 9 \end{array}
$$

16–12. Continued

Loads-Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ -13.75 \\ 90 \\ 0 \\ 0 \\ \hline Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \left[\begin{array}{ccccc|cccc} 2048.31 & 0 & -3304.04 & -3304.04 & 0 & -34.4170 & 0 & -2013.89 & 0 \\ 0 & 3031.03 & -1468.46 & 0 & -1468.46 & 0 & -3020.83 & 0 & -10.1976 \\ -3304.04 & -1468.46 & 704861 & 211458 & 140972 & 3304.04 & 0 & 0 & 1468.46 \\ -3304.04 & 0 & 211458 & 422917 & 0 & 3304.04 & 0 & 0 & 0 \\ 0 & -1468.46 & 140972 & 0 & 281944 & 0 & 0 & 0 & 1468.46 \\ \hline -34.4170 & 0 & 3304.04 & 3304.04 & 0 & 34.4170 & 0 & 0 & 0 \\ 0 & -3020.83 & 0 & 0 & 0 & 0 & 3020.83 & 0 & 0 \\ -2013.89 & 0 & 0 & 0 & 0 & 0 & 0 & 2013.89 & 0 \\ 0 & -10.1976 & 1468.46 & 0 & 1468.46 & 0 & 0 & 0 & 10.1976 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$0 = 2048.31D_1 - 3304.04D_3 - 3304.04D_4$ (1)

$-13.75 = 3031.03D_2 - 1468.46D_3 - 1468.46D_5$ (2)

$90 = -3304.04D_1 - 1468.46D_2 + 704861D_3 + 211458D_4 + 140972D_5$ (3)

$0 = -3304.04D_1 + 211458D_3 + 422917D_4$ (4)

$0 = -1468.46D_2 + 140972D_3 + 281944D_5$ (5)

Solving Eqs. (1) to (5),

$D_1 = 0.001668 \quad D_2 = -0.004052 \quad D_3 = 0.002043 \quad D_4 = -0.001008 \quad D_5 = -0.001042$

Using these results and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$Q_6 = -34.4170(0.001668) + 3304.04(0.002043) + 3304.04(-0.001008) = 3.360$

$Q_7 = -3020.83(-0.004052) = 12.24$

$Q_8 = -2013.89(0.001668) = -3.360$

$Q_9 = -10.1976(-0.004052) + 1468.46(0.002043) + 1468.46(-0.001008) = 1.510$

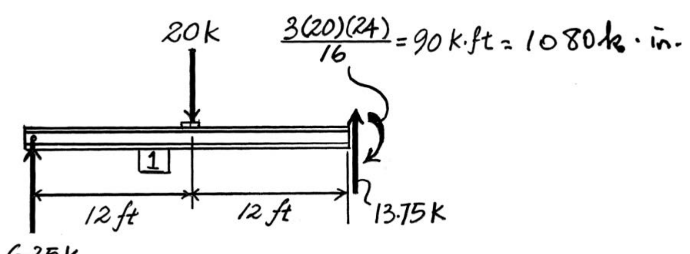
Superposition these results to those of FEM shown in Fig. *a*.

$R_6 = 3.360 + 0 = 3.36$ k **Ans.**

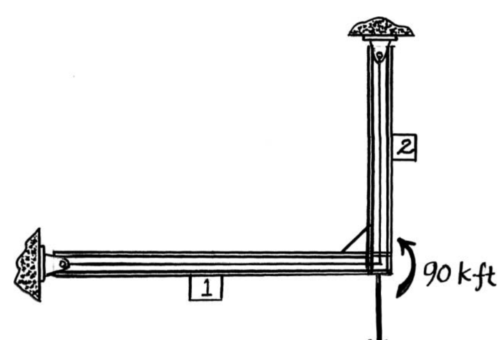
$R_7 = 12.24 + 0 = 12.2$ k **Ans.**

$R_8 = -3.360 + 0 = -3.36$ k **Ans.**

$R_9 = 1.510 + 6.25 = 7.76$ k **Ans.**



(a)



(b)