

**CHAPTER 2**

**P.P.2.1**  $i = V/R = 110/15 = \underline{7.333 \text{ A}}$

**P.P.2.2** (a)  $v = iR = 3 \text{ mA}[10 \text{ kohms}] = \underline{30 \text{ V}}$

(b)  $G = 1/R = 1/10 \text{ kohms} = \underline{100 \mu\text{S}}$

(c)  $p = vi = 30 \text{ volts}[3 \text{ mA}] = \underline{90 \text{ mW}}$

**P.P.2.3**  $p = vi$  which leads to  $i = p/v = [30 \cos^2(t) \text{ mW}]/[15\cos(t) \text{ mA}]$

or  $i = \underline{2\cos(t) \text{ mA}}$

$R = v/i = 15\cos(t)\text{V}/2\cos(t)\text{mA} = \underline{7.5 \text{ k}\Omega}$

**P.P.2.4** 5 branches and 3 nodes. The 1 ohm and 2 ohm resistors are in parallel. The 4 ohm resistor and the 10 volt source are also in parallel.

**P.P.2.5** Applying KVL to the loop we get:

$$-32 + 4i - (-8) + 2i = 0 \text{ which leads to } i = 24/6 = 4\text{A}$$

$$v_1 = 4i = \underline{16 \text{ V}} \quad \text{and} \quad v_2 = -2i = \underline{-8 \text{ V}}$$

**P.P.2.6** Applying KVL to the loop we get:

$$-70 + 10i + 2v_x + 5i = 0$$

But,  $v_x = 10i$  and  $v_0 = -5i$ . Hence,

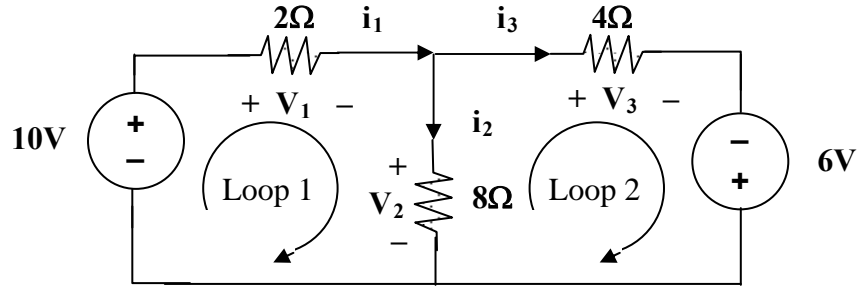
$$-70 + 10i + 20i + 5i = 0 \text{ which leads to } i = 2 \text{ A.}$$

$$\text{Thus, } v_x = \underline{20\text{V}} \quad \text{and} \quad v_0 = \underline{-10 \text{ V}}$$

**P.P.2.7** Applying KCL,  $0 = -9 + i_0 + [i_0/4] + [v_0/8]$ , but  $i_0 = v_0/2$

Which leads to:  $9 = (v_0/2) + (v_0/8) + (v_0/8)$  thus,  $v_0 = \underline{12\text{ V}}$  and  $i_0 = \underline{6\text{ A}}$

**P.P.2.8**



At the top node,  $0 = -i_1 + i_2 + i_3$  or  $i_1 = i_2 + i_3$  (1)

For loop 1  $-10 + V_1 + V_2 = 0$   
or  $V_1 = 10 - V_2$  (2)

For loop 2  $-V_2 + V_3 - 6 = 0$   
or  $V_3 = V_2 + 6$  (3)

Using (1) and Ohm's law, we get

$$(V_1/2) = (V_2/8) + (V_3/4)$$

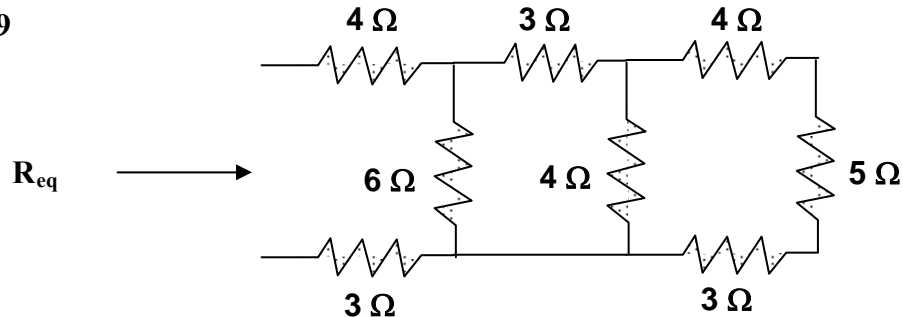
and now using (2) and (3) in the above yields

$$[(10 - V_2)/2] = (V_2/8) + (V_2 + 6)/4$$

or  $[7/8]V_2 = 14/4$  or  $V_2 = \underline{4\text{ V}}$

$V_1 = 10 - V_2 = \underline{6\text{ V}}$ ,  $V_3 = 4 + 6 = \underline{10\text{ V}}$ ,  $i_1 = (10 - 4)/2 = \underline{3\text{ A}}$ ,  
 $i_2 = 4/8 = \underline{500\text{ mA}}$ ,  $i_3 = \underline{2.5\text{ A}}$

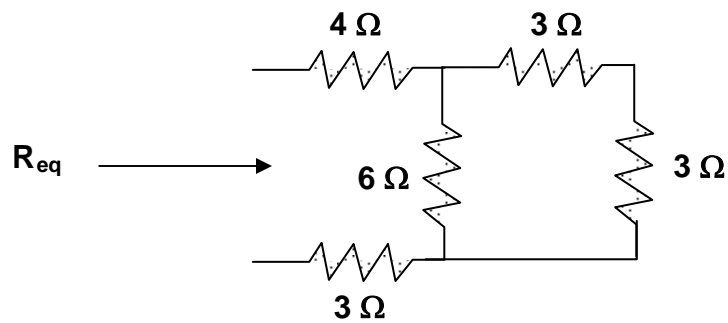
**P.P.2.9**



Combining the 4 ohm, 5 ohm, and 3ohm resistors in series gives  $4+3+5 = 12$ .

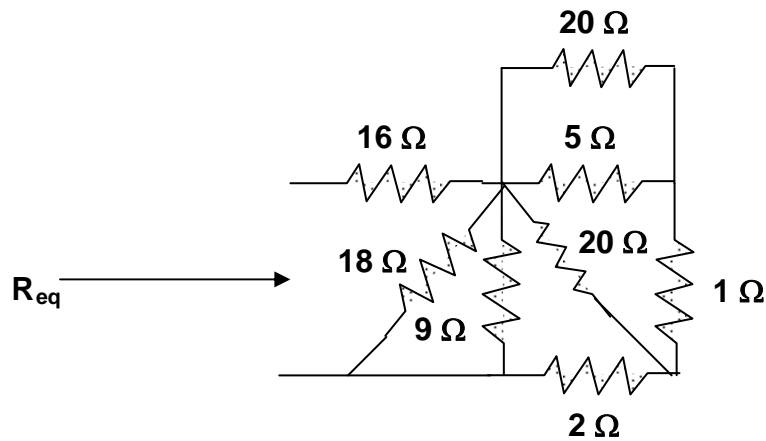
But, 4 in parallel with 12 produces  $[4 \times 12] / [4 + 12] = 48 / 16 = 3 \text{ohm}$ .

So that the equivalent circuit is shown below.



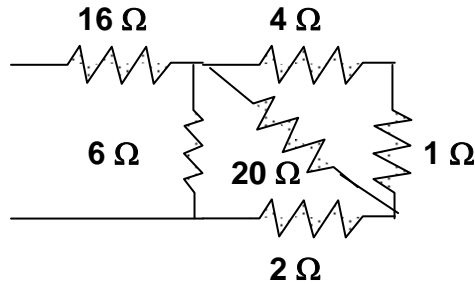
Thus,  $R_{eq} = 4 + 3 + [6 \times 6] / [6 + 6] = \underline{10 \Omega}$

**P.P.2.10**

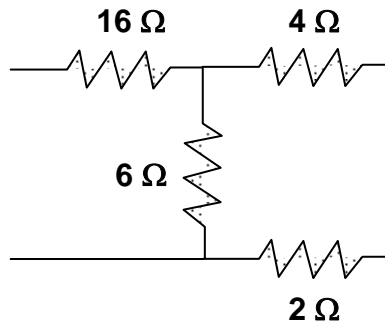


Combining the 9 ohm resistor and the 18 ohm resistor yields  $[9 \times 18] / [9 + 18] = 6 \text{ ohms}$ .

Combining the 5 ohm and the 20 ohm resistors in parallel produces  $[5 \times 20 / (5 + 20)] = 4$  ohms. We now have the following circuit:



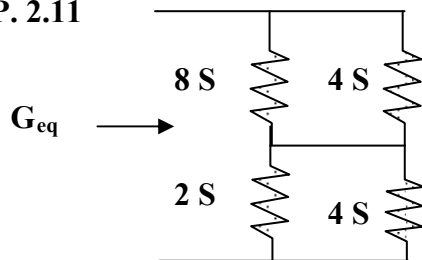
The 4 ohm and 1 ohm resistors can be combined into a 5 ohm resistor in parallel with a 20 ohm resistor. This will result in  $[5 \times 20 / (5 + 20)] = 4$  ohms and the circuit shown below:



The 4 ohm and 2 ohm resistors are in series and can be replaced by a 6 ohm resistor. This gives a 6 ohm resistor in parallel with a 6 ohm resistor,  $[6 \times 6 / (6 + 6)] = 3$  ohms. We now have a 3 ohm resistor in series with a 16 ohm resistor or  $3 + 16 = 19$  ohms. Therefore:

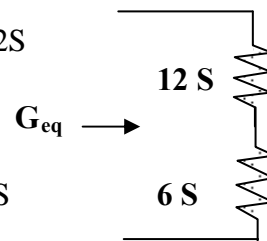
$$R_{eq} = \underline{19 \text{ ohms}}$$

**P.P. 2.11**



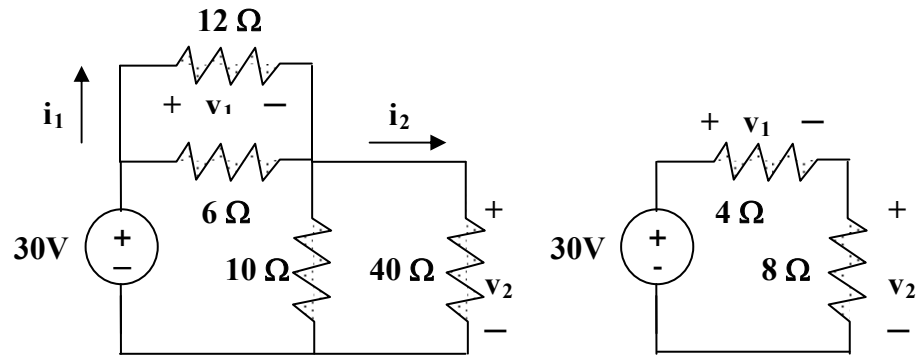
$$8 \parallel 4 = 8 + 4 = 12 \text{ S}$$

$$2 \parallel 4 = 2 + 4 = 6 \text{ S}$$



12 S in series with 6 S =  $\{12 \times 6 / (12 + 6)\} = 4$  or:  $G_{eq} = \underline{4 \text{ S}}$

**P.P.2.12**



$$6 \parallel 12 = [6 \times 12 / (6 + 12)] = 4 \text{ ohm} \quad \text{and} \quad 10 \parallel 40 = [10 \times 40 / (10 + 40)] = 8 \text{ ohm.}$$

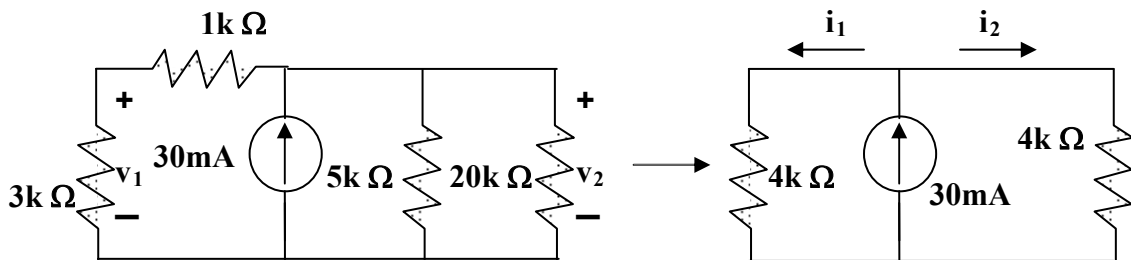
Using voltage division we get:

$$v_1 = [4 / (4 + 8)] (30) = \underline{10 \text{ volts}}, \quad v_2 = [8 / 12] (30) = \underline{20 \text{ volts}}$$

$$i_1 = v_1 / 12 = 10 / 12 = \underline{833.3 \text{ mA}}, \quad i_2 = v_2 / 40 = 20 / 40 = \underline{500 \text{ mA}}$$

$$P_1 = v_1 i_1 = 10 \times 10 / 12 = \underline{8.333 \text{ watts}}, \quad P_2 = v_2 i_2 = 20 \times 0.5 = \underline{10 \text{ watts}}$$

**P.P.2.13**



Using current division,  $i_1 = i_2 = (30 \text{ mA})(4 \text{ kohm} / (4 \text{ kohm} + 4 \text{ kohm})) = 15 \text{ mA}$

$$(a) \quad v_1 = (3 \text{ kohm})(15 \text{ mA}) = \underline{45 \text{ volts}}$$

$$v_2 = (4 \text{ kohm})(15 \text{ mA}) = \underline{60 \text{ volts}}$$

$$(b) \quad \text{For the 3k ohm resistor, } P_1 = v_1 \times i_1 = 45 \times 15 \times 10^{-3} = \underline{675 \text{ mw}}$$

$$\text{For the 20k ohm resistor, } P_2 = (v_2)^2 / 20\text{k} = \underline{180 \text{ mw}}$$

- (c) The total power supplied by the current source is equal to:  
 $P = v_2 \times 10 \text{ mA} = 60 \times 30 \times 10^{-3} = \underline{1.8 \text{ W}}$

**P.P.2.14**

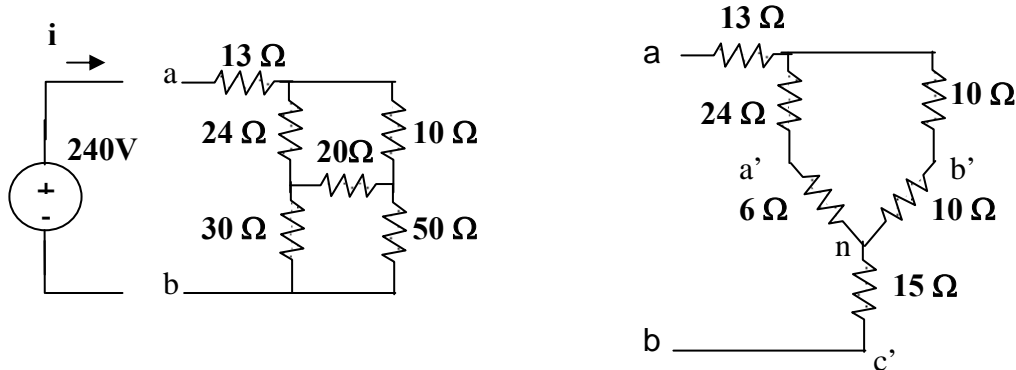
$$R_a = [R_1 R_2 + R_2 R_3 + R_3 R_1] / R_1 = [10 \times 20 + 20 \times 40 + 40 \times 10] / 10 = \underline{140 \text{ ohms}}$$

$$R_b = [R_1 R_2 + R_2 R_3 + R_3 R_1] / R_2 = 1400 / 20 = \underline{70 \text{ ohms}}$$

$$R_c = [R_1 R_2 + R_2 R_3 + R_3 R_1] / R_3 = 1400 / 40 = \underline{35 \text{ ohms}}$$

**P.P.2.15**

We first find the equivalent resistance, R. We convert the delta sub-network to a wye connected form as shown below:



$$R_{a'n} = 20 \times 30 / [20 + 30 + 50] = 6 \text{ ohms}, R_{b'n} = 20 \times 50 / 100 = 10 \text{ ohms}$$

$$R_{c'n} = 30 \times 50 / 100 = 15 \text{ ohms.}$$

$$\text{Thus, } R_{ab} = 13 + [(24 + 6) \parallel (10 + 10)] + 15 = 28 + 30 \times 20 / (30 + 20) = \underline{40 \text{ ohms.}}$$

$$i = 240 / R_{ab} = 240 / 40 = \underline{6 \text{ amps}}$$

**P.P.2.16**

For the parallel case,  $v = v_0 = 110 \text{ volts.}$

$$p = vi \implies i = p/v = 40/110 = \underline{364 \text{ mA}}$$

For the series case,  $v = v_0/N = 110/10 = 11 \text{ volts}$

$$i = p/v = 40/11 = \underline{3.64 \text{ amps}}$$

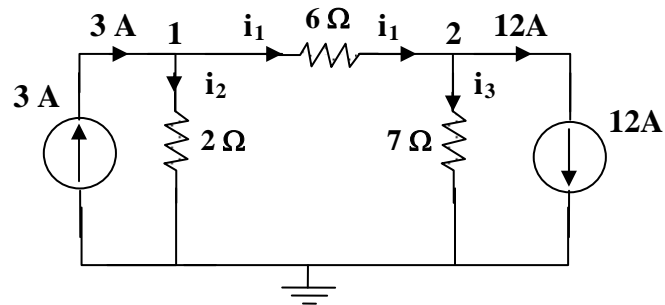
**P.P.2.17**

We use equation (2.61)

(a)  $R_1 = 50 \times 10^{-3} / (1 - 10^{-3}) = 0.05/999 = \underline{50 \text{ m}\Omega \text{ (shunt)}}$

(b)  $R_2 = 50 \times 10^{-3} / (100 \times 10^{-3} - 10^{-3}) = 50/99 = \underline{505 \text{ m}\Omega \text{ (shunt)}}$

(c)  $\mathbf{R_3 = 50 \times 10^{-3} / (10 \times 10^{-3} - 10^{-3}) = 50/9 = \underline{\underline{5.556 \Omega \text{ (shunt)}}}$

**CHAPTER 3****P.P.3.1**

At node 1,

$$-3 + i_1 + i_2 = 0 \text{ or } \frac{v_1 - v_2}{6} + \frac{v_1 - 0}{2} = 3$$

$$\text{or } 4v_1 - v_2 = 18 \quad (1)$$

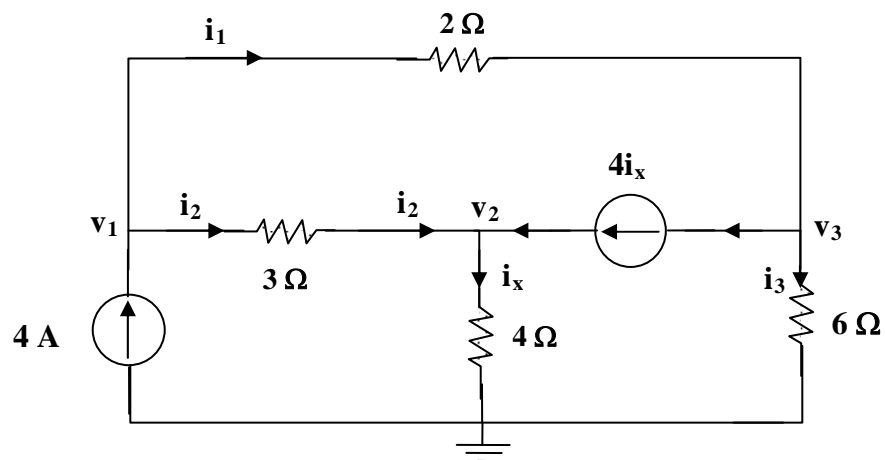
At node 2,

$$-i_1 + i_3 + 12 = 0 \text{ or } i_1 = 12 + i_3 \text{ or } \frac{v_1 - v_2}{6} = 12 + \frac{v_2 - 0}{7}$$

$$\text{or } 7v_1 - 13v_2 = 504 \quad (2)$$

Solving (1) and (2) gives

$$v_1 = -6 \text{ V}, v_2 = -42 \text{ V}$$

**P.P.3.2**

At node 1,

$$-4 + i_1 + i_2 = 0 = -4 + \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{3}$$

$$\text{or } 5v_1 - 2v_2 - 3v_3 = 24 \quad (1)$$

At node 2,

$$-i_2 + i_x - 4i_x = 0 = -i_2 - 3i_x = 0 \text{ where } i_x = [(v_2 - 0)/4] \text{ or}$$

$$\frac{v_1 - v_2}{3} + 3\frac{v_2}{4} = 0 \text{ which leads to } 4v_1 + 5v_2 = 0 \quad (2)$$

At node 3,

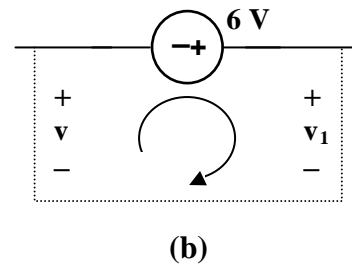
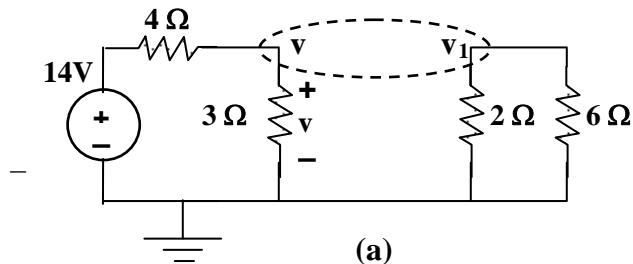
$$-i_1 + i_3 + 4i_x = 0 = \frac{v_3 - v_1}{2} + \frac{v_3 - 0}{6} + 4\frac{v_2}{4}$$

$$\text{or } -3v_1 + 6v_2 + 4v_3 = 0 \quad (3)$$

Solving (1) to (3) gives

$$v_1 = \mathbf{32 \text{ V}}, v_2 = \mathbf{-25.6 \text{ V}}, v_3 = \mathbf{62.4 \text{ V}}$$

### P.P.3.3



At the supernode in Fig. (a),

$$\frac{14 - v}{4} = \frac{v}{3} + \frac{v_1}{2} + \frac{v_1}{6}$$

$$\text{or } 42 = 7v + 8v_1 \quad (1)$$

Applying KVL to the loop in Fig. (b),

$$-v - 6 + v_1 = 0 \longrightarrow v_1 = v + 6 \quad (2)$$

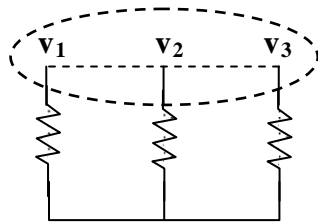
Solving (1) and (2),

$$v = -400 \text{ mV}$$

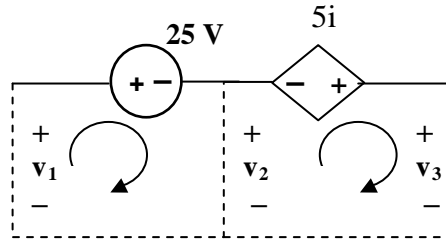
$$v_1 = v + 6 = 5.6, i_1 = \frac{v_1}{2} = 2.8$$

$$i_1 = 2.8 \text{ A}$$

**P.P.3.4**



(a)



(b)

From Fig. (a),

$$\frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0 \longrightarrow 6v_1 + 3v_2 + 4v_3 = 0 \quad (1)$$

From Fig. (b),

$$-v_1 + 25 + v_2 = 0 \longrightarrow v_1 = v_2 + 25 \quad (2)$$

$$-v_2 - 5i + v_3 = 0 \longrightarrow v_3 = v_2 + 2.5v_1 \quad (3)$$

Solving (1) to (3), we obtain

$$v_1 = 7.608 \text{ V}, v_2 = -17.39 \text{ V}, v_3 = 1.6305 \text{ V}$$

**P.P.3.5** We apply KVL to the two loops and obtain

$$-45 + 2i_1 + 12(i_1 - i_2) + 4i_1 = 0 \text{ or}$$

$$-45 + 18i_1 - 12i_2 = 0 \text{ which leads to } 3i_1 - 2i_2 = 7.5 \quad (1)$$

$$12(i_2 - i_1) + 9i_2 + 30 + 3i_2 = 0 \text{ or}$$

$$30 + 24i_2 - 12i_1 = 0 \text{ which leads to } -3i_1 + 6i_2 = -7.5 \quad (2)$$

From (1) and (2) we get

$$i_1 = 2.5 \text{ A}, i_2 = 0 \text{ A}$$

**P.P.3.6** For mesh 1,

$$-16 + 6i_1 - 2i_2 - 4i_3 = 0 \longrightarrow 3i_1 - i_2 - 2i_3 = 8 \quad (1)$$

For mesh 2,

$$10i_2 - 2i_1 - 8i_3 - 10i_0 = 0 = -i_1 + 5i_2 - 9i_3 \quad (2)$$

But  $i_0 = i_3$ ,

$$18i_3 - 4i_1 - 8i_2 = 0 \longrightarrow -2i_1 - 4i_2 + 9i_3 = 0 \quad (3)$$

From (1) to (3),

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \end{vmatrix} = 135 - 8 - 18 - 20 - 108 - 9 = -28$$

$$\Delta_1 = \begin{vmatrix} 8 & -1 & -2 \\ 0 & 5 & -9 \\ 0 & -4 & 9 \end{vmatrix} = 360 - 288 = 72$$

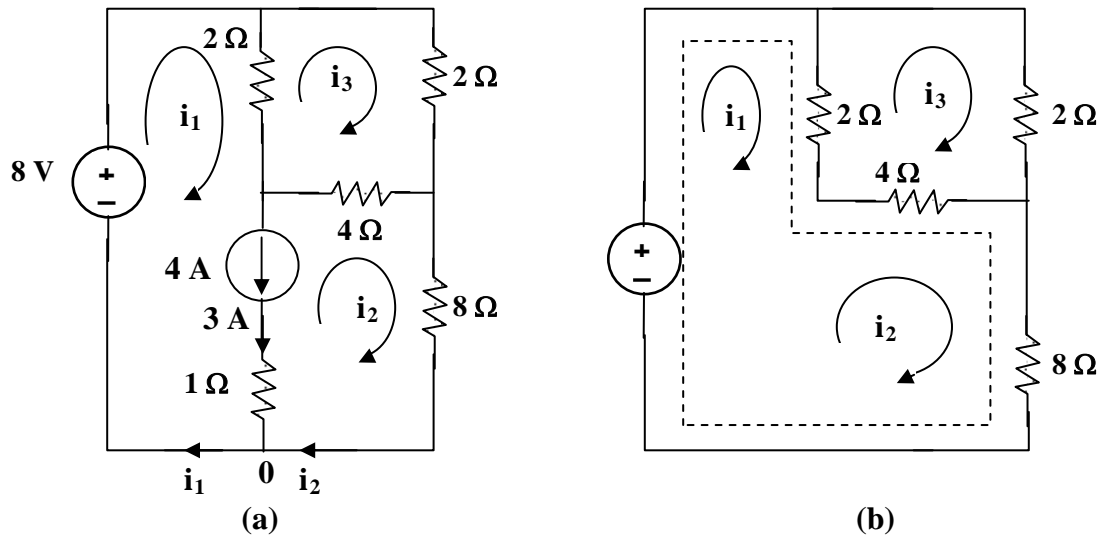
$$\Delta_2 = \begin{vmatrix} 3 & 8 & -2 \\ -1 & 0 & -9 \\ -2 & 0 & 9 \end{vmatrix} = 144 + 72 = 216$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 8 \\ -1 & 5 & 0 \\ 3 & -1 & 8 \\ -1 & 5 & 0 \end{vmatrix} = 32 + 80 = 112$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{72}{-28} = -2.571, \quad i_2 = \frac{\Delta_2}{\Delta} = \frac{216}{-28} = -7.714, \quad i_3 = \frac{\Delta_3}{\Delta} = \frac{112}{-28} = -4 \text{ A}$$

$$I_o = i_3 = -4 \text{ A}$$

**P.P.3.7**



For the supermesh,

$$-8 + 2i_1 - 2i_3 + 12i_2 - 4i_3 = 0 \text{ or } i_1 + 6i_2 - 3i_3 = 4 \quad (1)$$

For mesh 3,

$$8i_3 - 2i_1 - 4i_2 = 0 \text{ or } -i_1 - 2i_2 + 4i_3 = 0 \quad (2)$$

At node 0 in Fig. (a),

$$i_1 = 4 + i_2 \longrightarrow i_1 - i_2 = 4$$

Solving (1) to (3) yields

$$i_1 = 4.632 \text{ A}, \quad i_2 = 631.6 \text{ mA}, \quad i_3 = 1.4736 \text{ A}$$

**P.P.3.8**  $G_{11} = 1/(1) + 1/(20) + 1/(5) = 1.25$ ,  $G_{12} = -1/(5) = -0.2$ ,  
 $G_{33} = 1/(4) + 1 = 1.25$ ,  $G_{44} = 1/(1) + 1/(4) = 1.25$ ,  
 $G_{12} = -1/(5) = -0.2$ ,  $G_{13} = -1$ ,  $G_{14} = 0$ ,  
 $G_{21} = -0.2$ ,  $G_{23} = 0 = G_{26}$ ,  
 $G_{31} = -1$ ,  $G_{32} = 0$ ,  $G_{34} = -1/4 = -0.25$ ,  
 $G_{41} = 0$ ,  $G_{42} = 0$ ,  $G_{43} = 0.25$ ,  
 $i_1 = 0$ ,  $i_2 = 3+2 = 5$ ,  $i_3 = -3$ ,  $i_4 = 2$ .

Hence,

$$\begin{bmatrix} 1.25 & -0.2 & -1 & 0 \\ -0.2 & 0.2 & 0 & 0 \\ -1 & 0 & 1.25 & -0.25 \\ 0 & 0 & -0.25 & 1.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -3 \\ 2 \end{bmatrix}$$

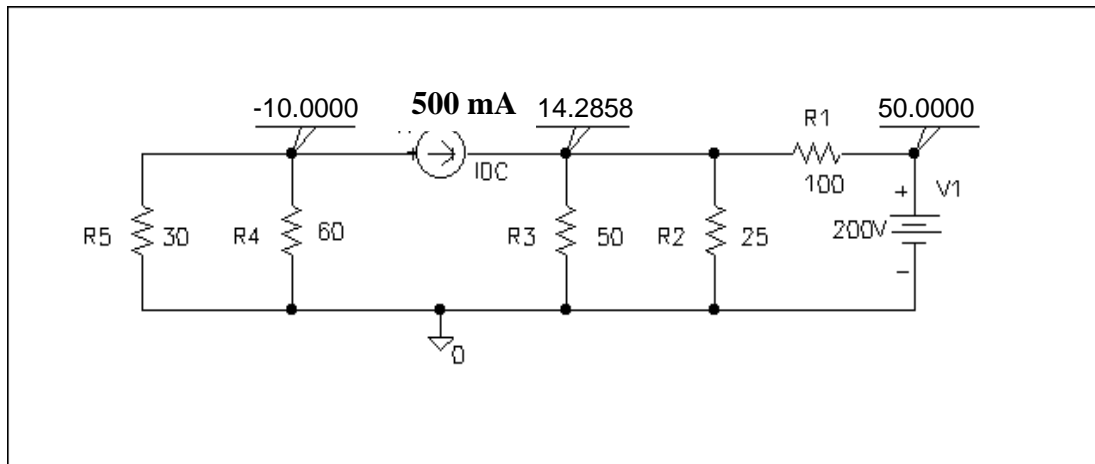
**P.P.3.9**  $R_{11} = 50 + 20 + 80 = 150$ ,  $R_{22} = 20 + 30 + 15 = 65$ ,  
 $R_{33} = 30 + 20 = 50$ ,  $R_{44} = 15 + 80 = 95$ ,  
 $R_{55} = 20 + 60 = 80$ ,  $R_{12} = -40$ ,  $R_{13} = 0$ ,  $R_{14} = -80$ ,  
 $R_{15} = 0$ ,  $R_{21} = -40$ ,  $R_{23} = -30$ ,  $R_{24} = -15$ ,  $R_{25} = 0$ ,  
 $R_{31} = 0$ ,  $R_{32} = -30$ ,  $R_{34} = 0$ ,  $R_{35} = -20$ ,  
 $R_{41} = -80$ ,  $R_{42} = -15$ ,  $R_{43} = 0$ ,  $R_{45} = 0$ ,  
 $R_{51} = 0$ ,  $R_{52} = 0$ ,  $R_{53} = -20$ ,  $R_{54} = 0$ ,  
 $v_1 = 30$ ,  $v_2 = 0$ ,  $v_3 = -12$ ,  $v_4 = 20$ ,  $v_5 = -20$

Hence the mesh-current equations are

$$\begin{bmatrix} 150 & -40 & 0 & -80 & 0 \\ -40 & 65 & -30 & -15 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -15 & 0 & 95 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -12 \\ 20 \\ -20 \end{bmatrix}$$

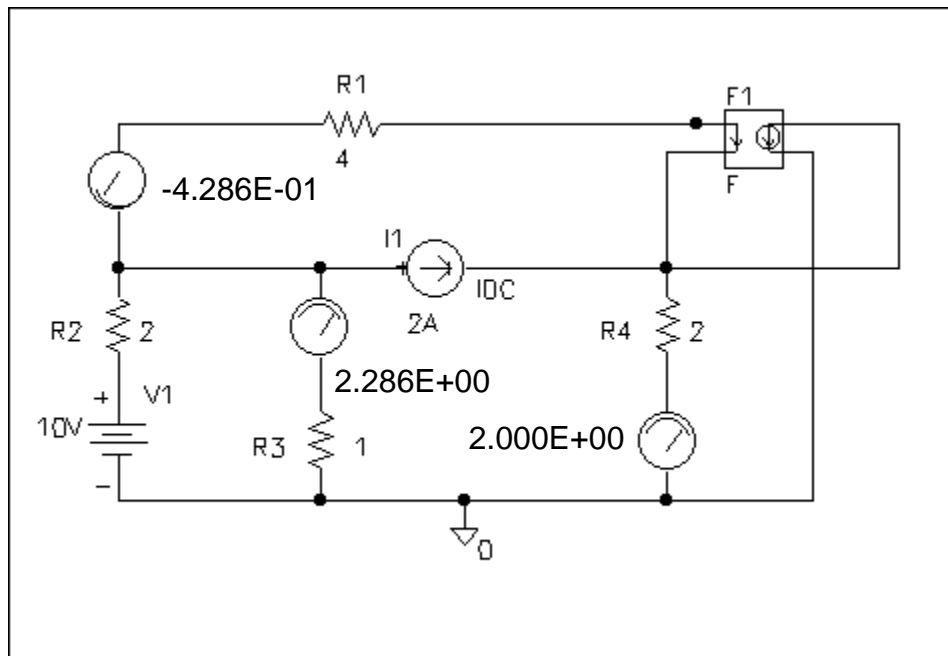
**P.P.3.10** The schematic is shown below. It is saved and simulated by selecting Analysis/Simulate. The results are shown on the viewpoints:

$$v_1 = -10 \text{ V}, v_2 = 14.286 \text{ V}, v_3 = 50 \text{ V}$$



**P.P.3.11** The schematic is shown below. After saving it, it is simulated by choosing Analysis/Simulate. The results are shown on the IPROBES.

$$i_1 = -428.6 \text{ mA}, i_2 = 2.286 \text{ A}, i_3 = 2 \text{ A}$$



**P.P.3.12** For the input loop,

$$-5 + 10 \times 10^3 I_B + V_{BE} + V_0 = 0 \quad (1)$$

For the outer loop,

$$-V_0 - V_{CE} - 500 I_0 + 12 = 0 \quad (2)$$

But  $V_0 = 200 I_E \quad (3)$

Also  $I_C = \beta I_B = 100 I_B, \alpha = \beta / (1 + \beta) = 100 / (101)$

$$I_C = \alpha I_E \longrightarrow I_E = I_C / (\alpha) = \beta I_B / (\alpha)$$

$$I_E = 100 (101 / (100)) I_B = 101 I_B \quad (4)$$

From (1), (3) and (4),

$$10,000 I_B + 200(101) I_B = 5 - V_{BE}$$

$$I_B = \frac{5 - 0.7}{10,000 + 20,000} = 142.38 \mu\text{A}$$

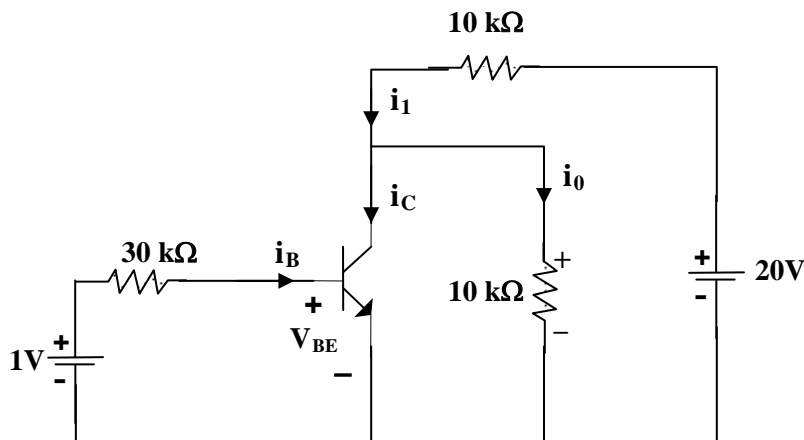
$$V_0 = 200 I_E = 20,000 I_B = \mathbf{2.876 \text{ V}}$$

From (2),

$$V_{CE} = 12 - V_0 - 500 I_C = 9.124 - 500 \times 100 \times 142.38 \times 10^{-6}$$

$$V_{CE} = \mathbf{1.984 \text{ V}} \text{ \{often, this is rounded to 2.0 volts\}}$$

**P.P.3.13**



$$i_B = \frac{1-0.7}{30k} = 10\mu\text{A}, \quad i_C = \beta i_B = 0.8 \text{ mA}$$

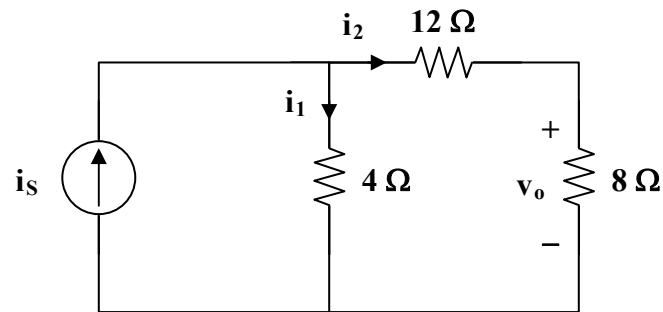
$$i_1 = i_C + i_0 \quad (1)$$

Also,  $-10ki_0 - 10ki_1 + 20 = 0 \longrightarrow i_1 = 2 \text{ mA} - i_0 \quad (2)$

Equating (1) and (2),

$$2 \text{ mA} - i_0 = 0.8 \text{ mA} + i_0 \longrightarrow i_0 = \mathbf{600 \mu\text{A}}$$

$$v_0 = 20 ki_0 = 20 \times 10^3 \times 600 \times 10^{-6} = \mathbf{12 \text{ V}}$$

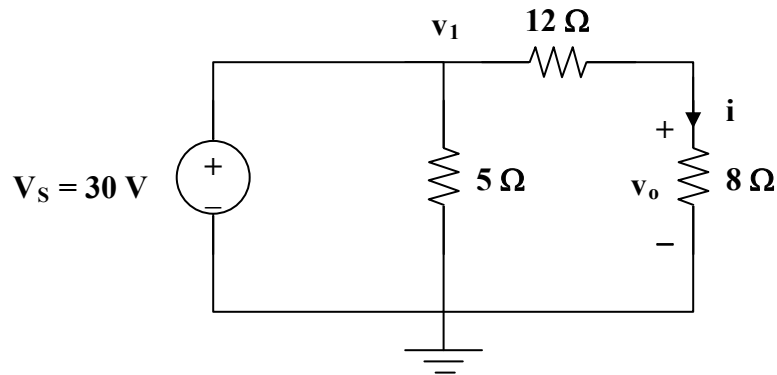
**CHAPTER 4****P.P.4.1**

By current division,  $i_2 = \frac{4}{4+12+8}i_s = \frac{1}{6}i_s$

$$v_o = 8i_2 = \frac{4}{3}i_s$$

When  $i_s = 30\ \text{A}$ ,  $v_o = \frac{4}{3}(30) = \mathbf{40\ \text{V}}$

When  $i_s = 45\ \text{A}$ ,  $v_o = \frac{4}{3}(45) = \mathbf{60\ \text{V}}$

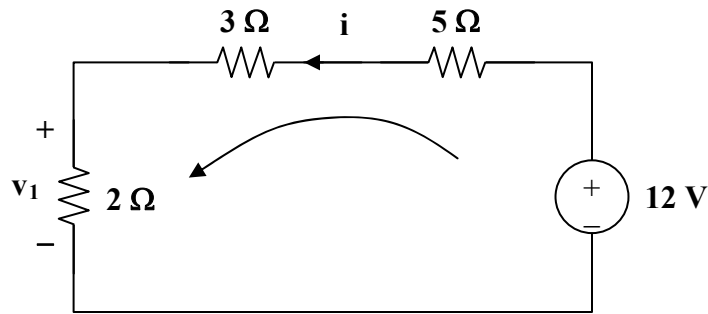
**P.P.4.2**

Let  $v_o = 1\ \text{volt}$ . Then  $i = \frac{1}{8}$  and  $v_1 = \frac{1}{8}(12 + 8) = 2.5$

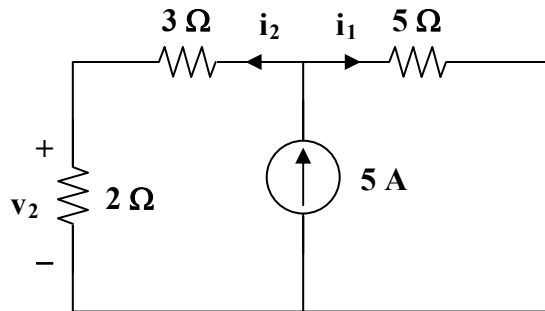
giving  $v_s = 2.5\ \text{V}$ .

If  $v_s = 40\ \text{V}$ , then  $v_o = (40/2.5)(1) = \mathbf{16\ \text{V}}$

**P.P.4.3** Let  $v_0 = v_1 + v_2$ , where  $v_1$  and  $v_2$  are contributions to the 12-V and 5-A sources respectively.



(a)



(b)

To get  $v_1$ , consider the circuit in Fig. (a).

$$(2 + 3 + 5)i - 12 = 0 \text{ or } i = 12/(10) = 1.2 \text{ A}$$

$$v_1 = 2i = 2.4 \text{ V}$$

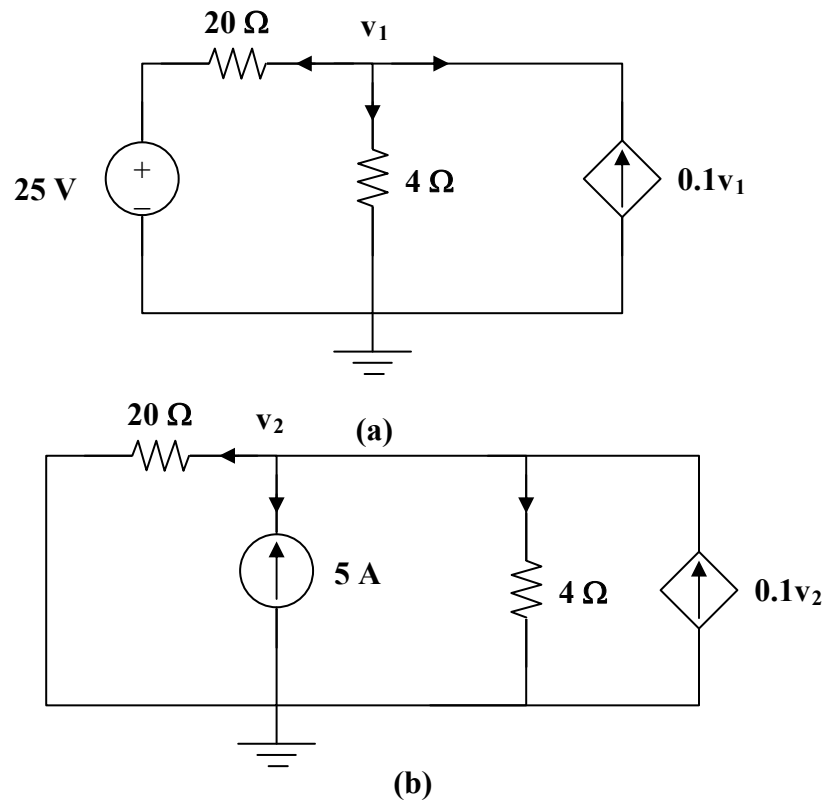
To get  $v_2$ , consider the circuit in Fig. (b).

Since the resistors are equal ( $5 = 2 + 3$ ) then the current divides equally and  $i_1 = i_2 = 5/2 = 2.5$  and  $v_2 = 2i_2 = 5 \text{ V}$

Thus,

$$v = v_1 + v_2 = 2.4 + 5 = 7.4 \text{ V}$$

**P.P.4.4** Let  $v_x = v_1 + v_2$ , where  $v_1$  and  $v_2$  are due to the 25-V and 5-A sources respectively.



To obtain  $v_1$ , consider Fig. (a).

$$-0.1v_1 + \frac{v_1 - 25}{20} + \frac{v_1 - 0}{4} = 0 \quad \text{or} \quad 0.2v_1 = 25/20 = 1.25 \quad \text{or} \quad v_1 = 6.25 \text{ V}$$

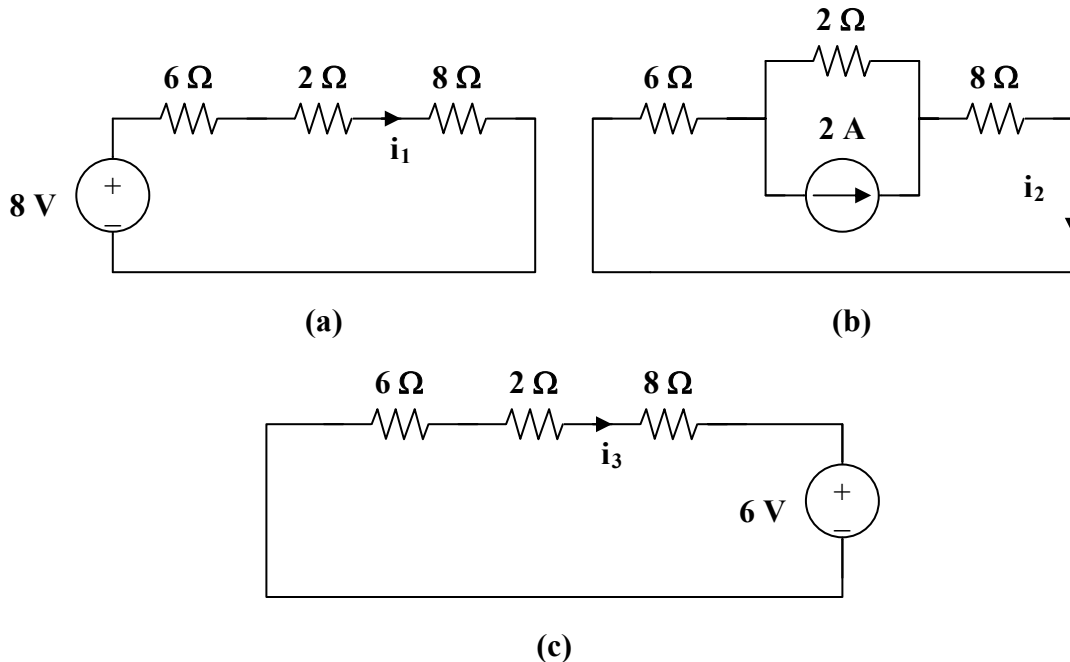
For  $v_2$ , consider Fig. (b).

$$-5 - 0.1v_2 + \frac{v_2 - 0}{20} + \frac{v_2 - 0}{4} = 0 \quad \text{or} \quad 0.2v_2 = 5 \quad \text{or} \quad v_2 = 25 \text{ V}$$

$$v_x = v_1 + v_2 = \mathbf{31.25 \text{ V}}$$

**P.P.4.5** Let  $i = i_1 + i_2 + i_3$

where  $i_1$ ,  $i_2$ , and  $i_3$  are contributions due to the 8-V, 2-A, and 6-V sources respectively.



For  $i_1$ , consider Fig. (a),  $i_1 = \frac{8}{6+2+8} = 0.5 \text{ A}$

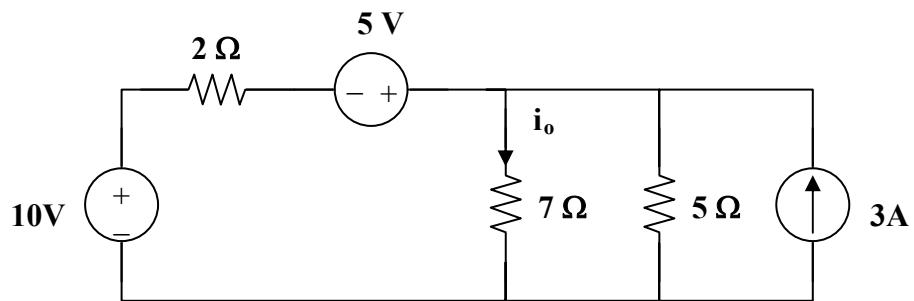
For  $i_2$ , consider Fig. (b). By current division,  $i_2 = \frac{2}{2+14}(2) = 0.25$

For  $i_3$ , consider Fig. (c),  $i_3 = \frac{-6}{16} = -0.375 \text{ A}$

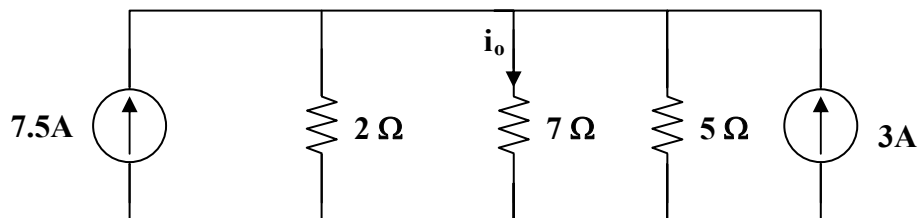
Thus,  $i = i_1 + i_2 + i_3 = 0.5 + 0.25 - 0.375 = 375 \text{ mA}$

**P.P.4.6** Combining the 6- $\Omega$  and 3- $\Omega$  resistors in parallel gives  $6 \parallel 3 = \frac{6 \times 3}{9} = 2 \Omega$ .

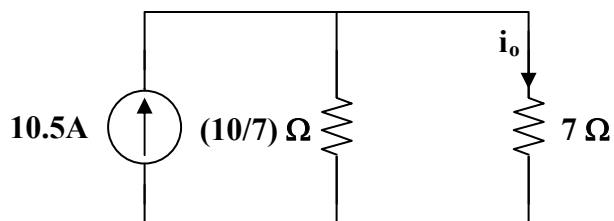
Adding the 1- $\Omega$  and 4- $\Omega$  resistors in series gives  $1 + 4 = 5 \Omega$ . Transforming the left current source in parallel with the 2- $\Omega$  resistor gives the equivalent circuit as shown in Fig. (a).



(a)



(b)



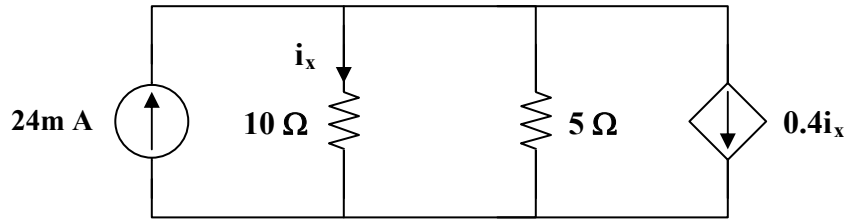
(c)

Adding the 10-V and 5-V voltage sources gives a 15-V voltage source. Transforming the 15-V voltage source in series with the 2-Ω resistor gives the equivalent circuit in Fig. (b). Combining the two current sources and the 2-Ω and 5-Ω resistors leads to the circuit in Fig. (c). Using circuit division,

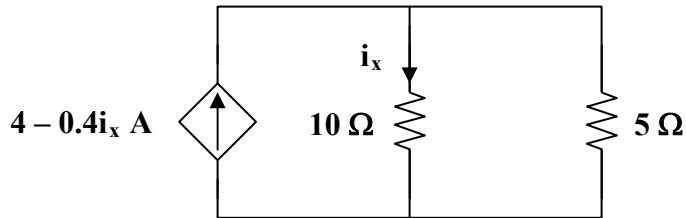
$$i_o = \frac{\frac{10}{7}}{\frac{10}{7} + 7} (10.5) = 1.78 \text{ A}$$

**P.P.4.7** We transform the dependent voltage source as shown in Fig. (a). We combine the two current sources in Fig. (a) to obtain Fig. (b). By the current division principle,

$$i_x = \frac{5}{15} (0.024 - 0.4i_x) \longrightarrow i_x = 7.059 \text{ mA}$$

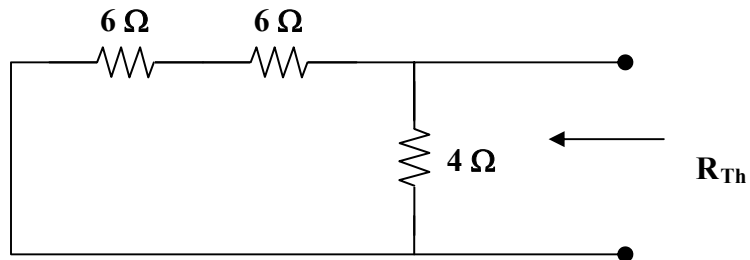


(a)

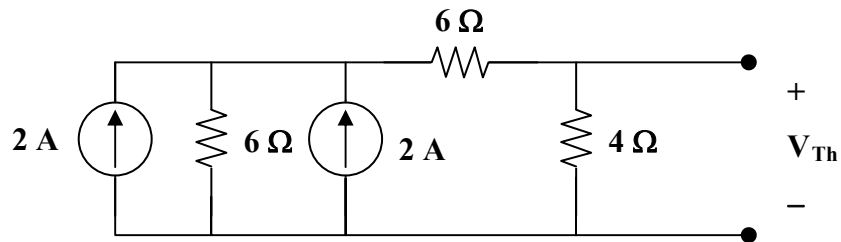


(b)

**P.P.4.8** To find  $R_{Th}$ , consider the circuit in Fig. (a).



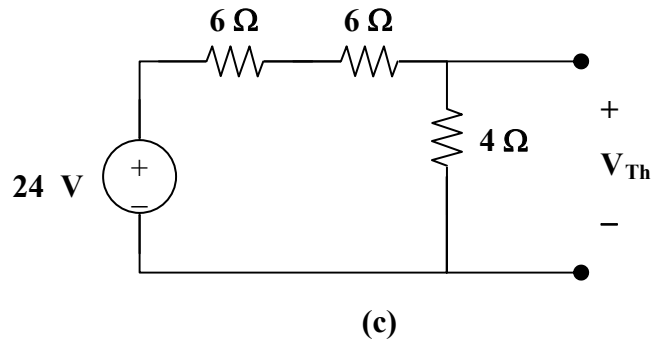
(a)



(b)

$$R_{Th} = (6 + 6) \parallel 4 = \frac{12 \times 4}{18} = 3 \Omega$$

To find  $V_{Th}$ , we use source transformations as shown in Fig. (b) and (c).

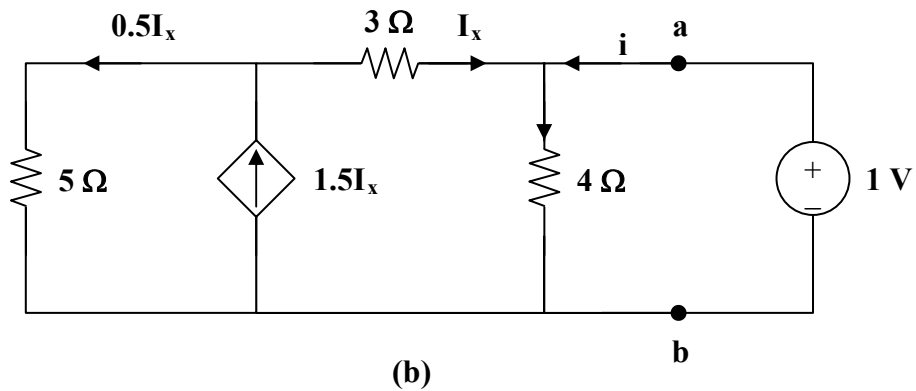
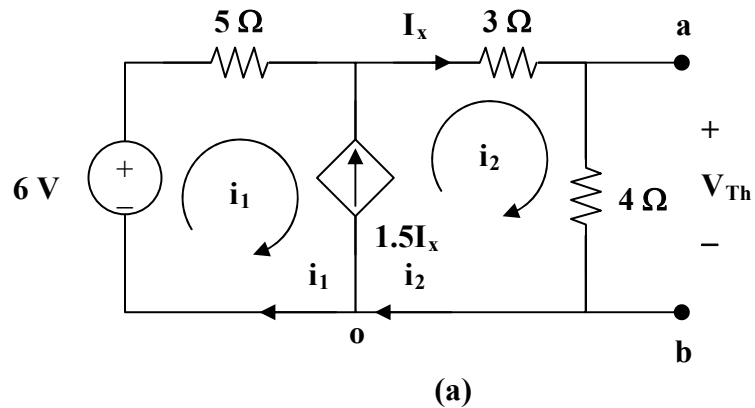


Using current division in Fig. (c),

$$V_{Th} = \frac{4}{4+12}(24) = 6 \text{ V}$$

$$i = \frac{V_{Th}}{R_{Th} + 1} = \frac{6}{3+1} = 1.5 \text{ A}$$

**P.P.4.9** To find  $V_{Th}$ , consider the circuit in Fig. (a).



$$I_x = i_2$$

$$i_2 - i_1 = 1.5I_x = 1.5i_2 \longrightarrow i_2 = -2i_1 \quad (1)$$

$$\text{For the supermesh, } -6 + 5i_1 + 7i_2 = 0 \quad (2)$$

From (1) and (2),  $i_2 = 4/3\text{A}$

$$V_{Th} = 4i_2 = \mathbf{5.333V}$$

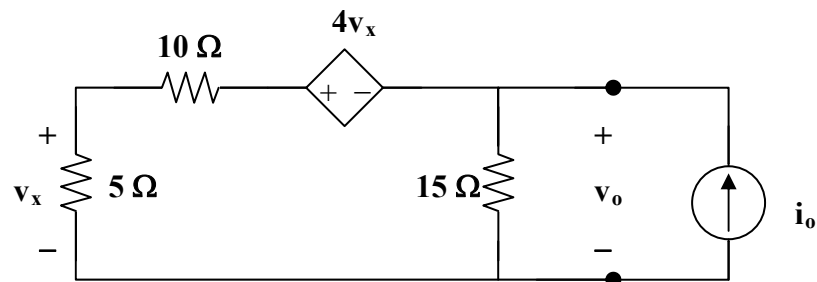
To find  $R_{Th}$ , consider the circuit in Fig. (b). Applying KVL around the outer loop,

$$5(0.5I_x) - 1 - 3I_x = 0 \longrightarrow I_x = -2$$

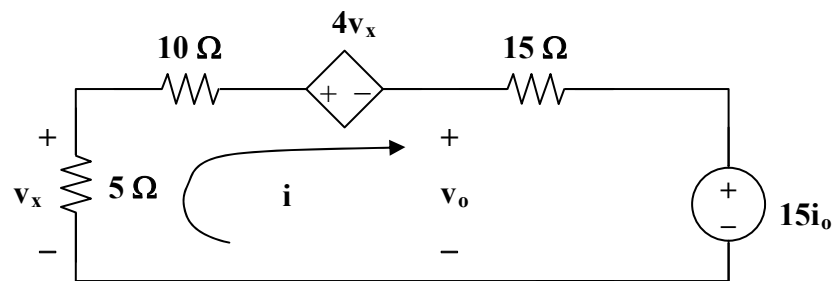
$$i = \frac{1}{4} - I_x = 2.25$$

$$R_{Th} = \frac{1}{i} = \frac{1}{2.25} = \mathbf{444.4 \text{ m}\Omega}$$

**P.P.4.10** Since there are no independent sources,  $V_{Th} = \mathbf{0}$



(a)



(b)

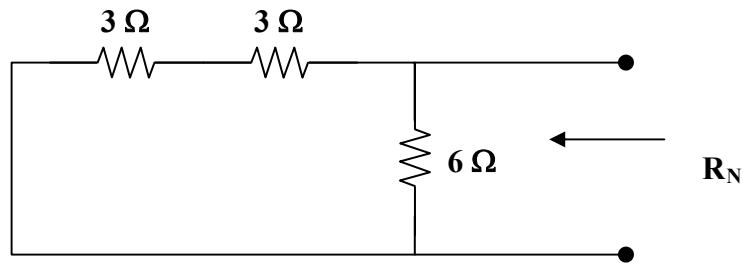
To find  $R_{Th}$ , consider Fig.(a). Using source transformation, the circuit is transformed to that in Fig. (b). Applying KVL, .

But  $v_x = -5i$ . Hence,  $30i - 20i + 15i_o = 0 \longrightarrow 10i = -15i_o$

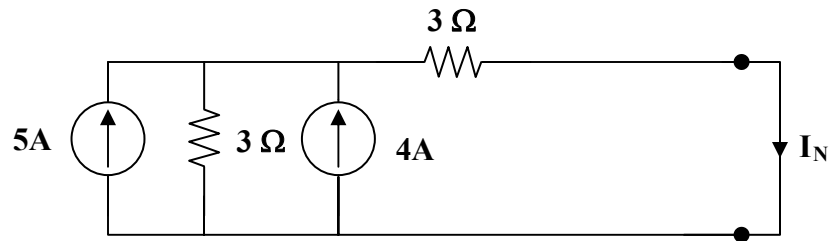
$v_o = (15i + 15i_o) = 15(-1.5i_o + i_o) = -7.5i_o$

$R_{Th} = v_o/i_o = -7.5\Omega$  It needs to be noted that this negative resistance indicates we must have an active source (a dependent source).

#### P.P.4.11



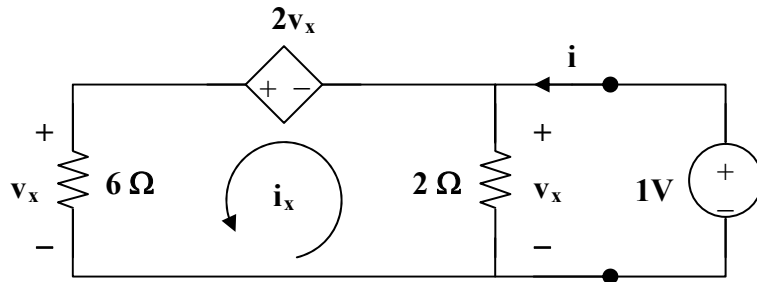
(a)



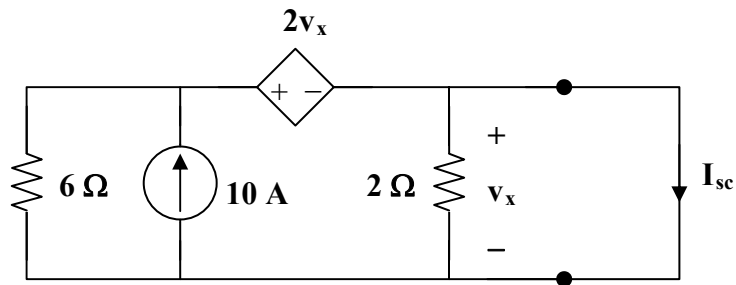
(b)

From Fig. (a),  $R_N = (3 + 3) \parallel 6 = 3\Omega$

From Fig. (b),  $I_N = \frac{1}{2}(5 + 4) = 4.5A$

**P.P.4.12**

(a)



(b)

To get  $R_N$  consider the circuit in Fig. (a). Applying KVL,  $6i_x - 2v_x - 1 = 0$

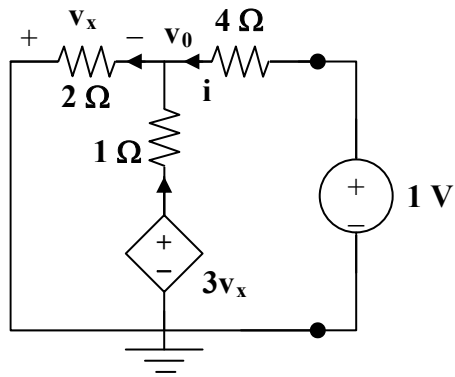
But  $v_x = 1$ ,  $6i_x = 3 \longrightarrow i_x = 0.5$

$$i = i_x + \frac{v_x}{2} = 0.5 + 0.5 = 1$$

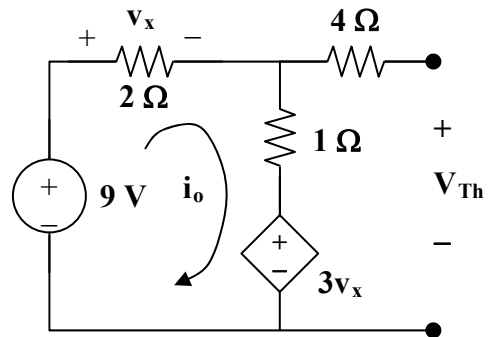
$$R_N = R_{Th} = \frac{1}{i} = 1\Omega$$

To find  $I_N$ , consider the circuit in Fig. (b). Because the  $2\Omega$  resistor is shorted,  $v_x = 0$  and the dependent source is inactive. Hence,  $I_N = i_{sc} = 10A$ .

**P.P.4.13** We first need to find  $R_{Th}$  and  $V_{Th}$ . To find  $R_{Th}$ , we consider the circuit in Fig. (a).



(a)



(b)

Applying KCL at the top node gives

$$\frac{1 - v_o}{4} + \frac{3v_x - v_o}{1} = \frac{v_o}{2}$$

But  $v_x = -v_o$ . Hence

$$\frac{1 - v_o}{4} - 4v_o = \frac{v_o}{2} \longrightarrow v_o = 1/(19)$$

$$i = \frac{1 - v_o}{4} = \frac{1 - \frac{1}{19}}{4} = \frac{9}{38}$$

$$R_{Th} = 1/i = 38/(9) = 4.222\Omega$$

To find  $V_{Th}$ , consider the circuit in Fig. (b),

$$-9 + 2i_o + i_o + 3v_x = 0$$

But  $v_x = 2i_o$ . Hence,

$$9 = 3i_o + 6i_o = 9i_o \longrightarrow i_o = 1A$$

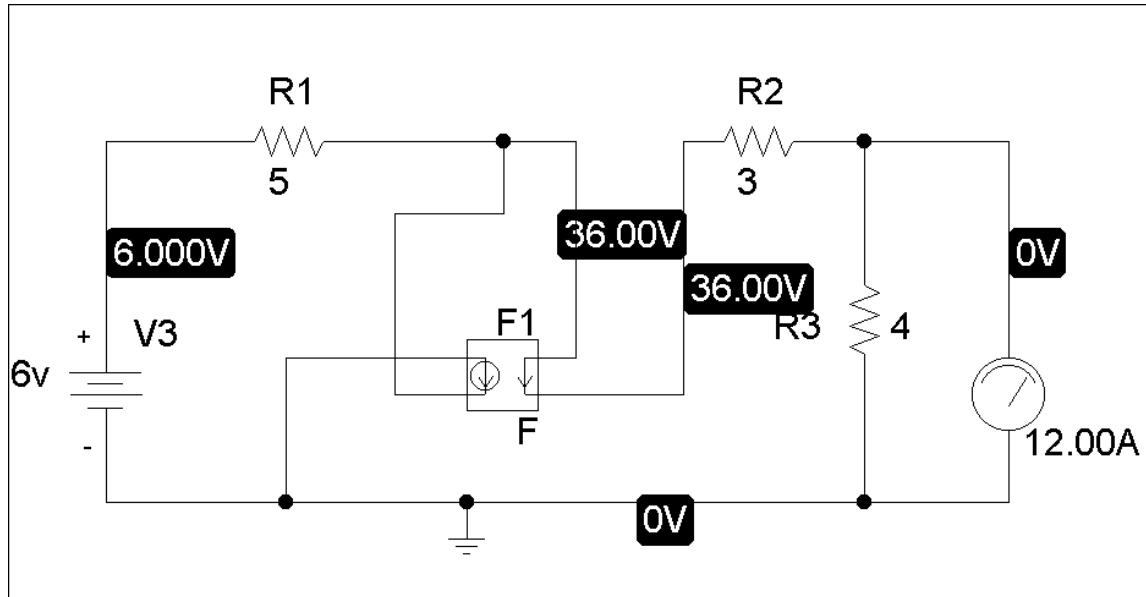
$$V_{Th} = 9 - 2i_o = 7V$$

$$R_L = R_{Th} = 4.222\Omega$$

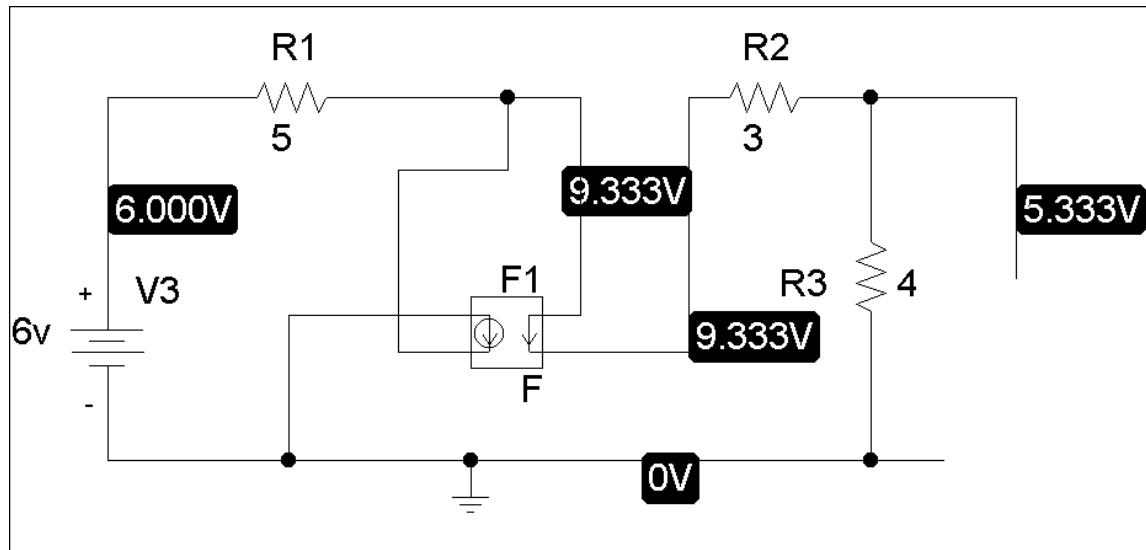
$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{49}{4(4.222)} = 2.901 W$$

**P.P.4.14**  
find  $V_{Th}$  and  $R_{th}$ .

We will use PSpice to find  $V_{oc}$  and  $I_{sc}$  which then can be used to

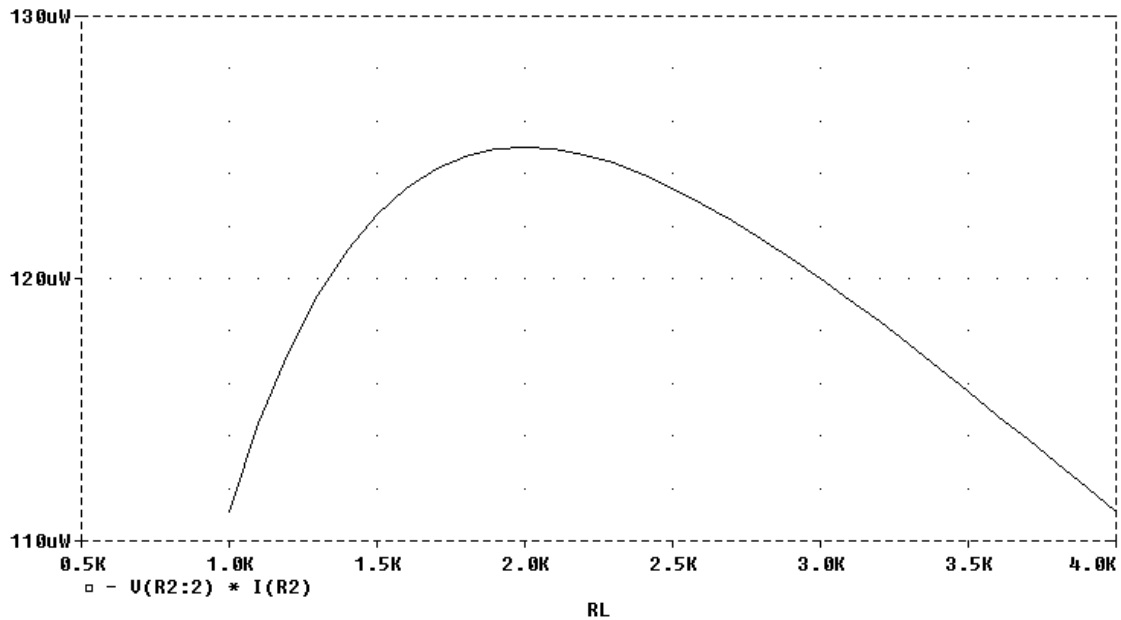


Clearly  $I_{sc} = 12\text{ A}$

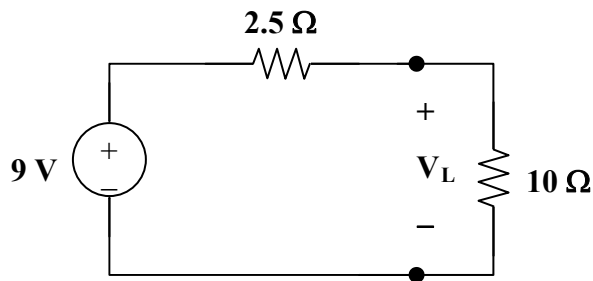


Clearly  $V_{Th} = I_{oc} = 5.333\text{ volts}$ .  $R_{Th} = V_{oc}/I_{sc} = 5.333/12 = 444.4\text{ m}\Omega$ .

**P.P.4.15** The schematic is the same as that in Fig. 4.56 except that the 1-k $\Omega$  resistor is replaced by 2-k $\Omega$  resistor. The plot of the power absorbed by  $R_L$  is shown in the figure below. From the plot, it is clear that the maximum power occurs when  $R_L = 2\text{k}\Omega$  and it is  $125\ \mu\text{W}$ .



**P.P.4.16**  $V_{Th} = 9\text{V}$ ,  $R_{Th} = (v_{oc} - V_L) \frac{R_L}{V_L} = (9 - 1) \frac{20}{8} = 2.5\Omega$



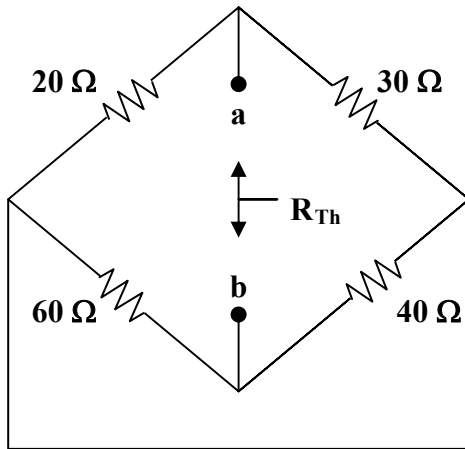
$$V_L = \frac{10}{10 + 2.5} (9) = 7.2\text{ V}$$

**P.P.4.17**  $R_1 = R_3 = 1\text{k}\Omega$ ,  $R_2 = 3.2\text{k}\Omega$

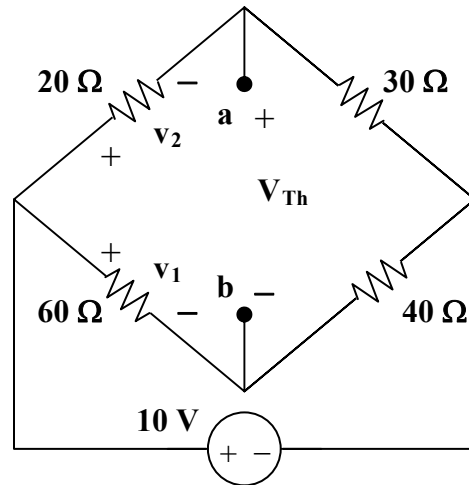
$$R_x = \frac{R_3}{R_1} R_2 = R_2 = 3.2 \text{ k}\Omega$$

**P.P.4.18** We first find  $R_{Th}$  and  $V_{Th}$ . To get  $R_{Th}$ , consider the circuit in Fig. (a).

$$\begin{aligned} R_{Th} &= 20 \parallel (30 + 60) \parallel 40 = \frac{20 \times 30}{50} + \frac{60 \times 40}{100} \\ &= 12 + 24 = 36\Omega \end{aligned}$$



(a)



(b)

To find  $V_{Th}$ , we use Fig. (b). Using voltage division,

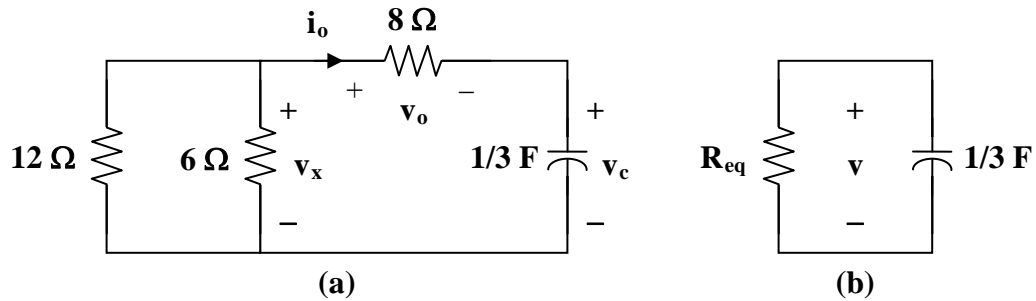
$$v_1 = \frac{60}{100}(16) = 9.6, \quad v_2 = \frac{20}{50}(16) = 6.4$$

$$\text{But } -v_1 + v_2 + v_{Th} = 0 \quad \longrightarrow \quad v_{Th} = v_1 - v_2 = 9.6 - 6.4 = 3.2\text{V}$$

$$I_G = \frac{V_{Th}}{R_{Th} + R_m} = \frac{3.2}{3.6 + 1.4} = 64\text{mA}$$

## CHAPTER 7

**P.P.7.1** The circuit in Fig. (a) is equivalent to the one shown in Fig. (b).



$$R_{eq} = 8 + 12 \parallel 6 = 12 \Omega$$

$$\tau = R_{eq}C = (12)(1/3) = 4 \text{ s}$$

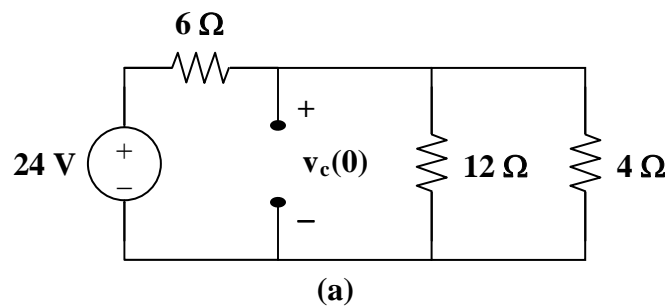
$$v_c = v_c(0)e^{-t/\tau} = 60e^{-t/4} = 60e^{-0.25t} \text{ V}$$

$$v_x = \frac{4}{4+8}v_c = 20e^{-0.25t} \text{ V}$$

$$v_x = v_o + v_c \longrightarrow v_o = v_x - v_c = -40e^{-0.25t} \text{ V}$$

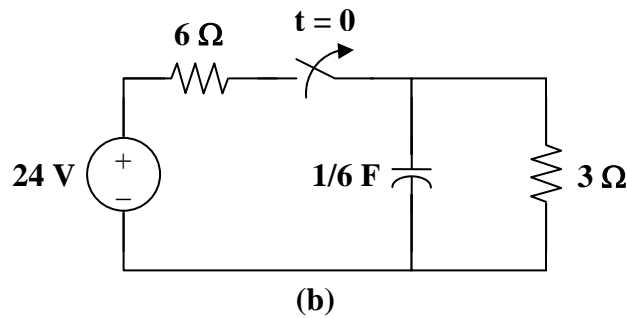
$$i_o = \frac{v_o}{8} = -5e^{-0.25t} \text{ A.}$$

**P.P.7.2** When  $t < 0$ , the switch is closed as shown in Fig. (a).



$$R_{eq} = 4 \parallel 12 = 3 \Omega \qquad v_c(0) = \frac{3}{3+6}(24) = 8 \text{ V}$$

When  $t > 0$ , the switch is open as shown in Fig. (b).



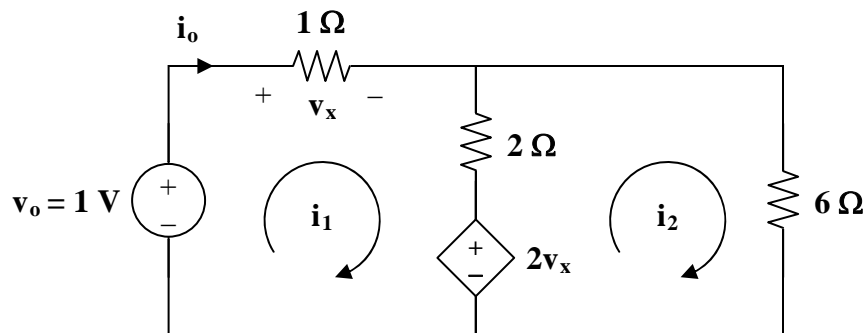
$$\tau = R_{eq}C = (3)(1/6) = 1/2 \text{ s}$$

$$v(t) = v_c(0)e^{-t/\tau} = 8e^{-2t} \text{ V}$$

$$w_c(0) = \frac{1}{2}Cv_c^2(0) = \frac{1}{2} \times \frac{1}{6} \times 64 = 5.333\text{J}$$

**P.P.7.3** This can be solved in two ways.

Method 1: Find  $R_{th}$  at the inductor terminals by inserting a voltage source.



Applying mesh analysis gives

$$\begin{aligned} \text{Loop 1:} \quad -1 + 3i_1 - 2i_2 + 2v_x &= 0, & \text{where } v_x &= li_1 \\ 5i_1 - 2i_2 &= 1 & (1) \end{aligned}$$

$$\begin{aligned} \text{Loop 2:} \quad 8i_2 - 2i_1 - 2v_x &= 0 = 8i_2 - 2i_1 - 2i_1 \\ i_2 &= \frac{1}{2}i_1 & (2) \end{aligned}$$

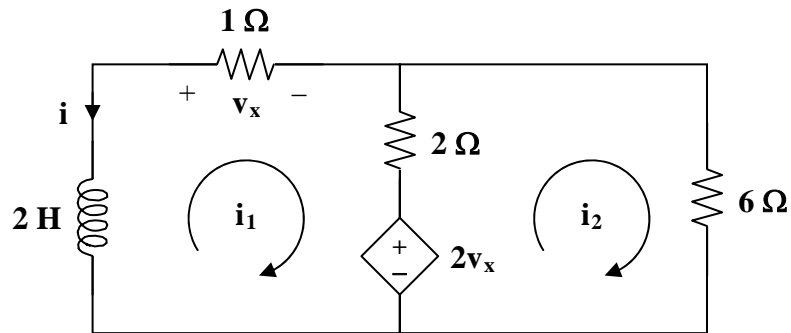
From (1) and (2),  $5i_1 - 1i_1 = 1$  or

$$i_o = i_1 = (1/4) \text{ A}$$

$$R_{th} = \frac{v_o}{i_o} = 4 \Omega, \quad \tau = \frac{L}{R} = \frac{2}{4} = \frac{1}{2} \text{ s}$$

$$i(t) = 12e^{-2t} \text{ A}$$

Method 2: We can obtain  $i$  using mesh analysis.



Applying KVL to the loops, we obtain

$$\begin{aligned} \text{Loop 1:} \quad 2 \frac{di_1}{dt} + 3i_1 - 2i_2 + 2v_x &= 0 & \text{where } v_x &= li_1 \\ 2 \frac{di_1}{dt} + 5i_1 - 2i_2 &= 0 & (3) \end{aligned}$$

$$\begin{aligned} \text{Loop 2:} \quad 8i_2 - 2i_1 - 2v_x &= 0 \\ i_2 &= \frac{1}{2}i_1 & (4) \end{aligned}$$

Substituting (4) into (3) yields

$$\begin{aligned} 2 \frac{di_1}{dt} + 5i_1 - li_1 &= 0 \\ \text{or } \frac{di_1}{dt} + 2i_1 &= 0 \\ i_1 &= Ae^{-2t} \end{aligned}$$

$$i = -i_1 = Be^{-2t}$$

$$i(0) = 12 = B$$

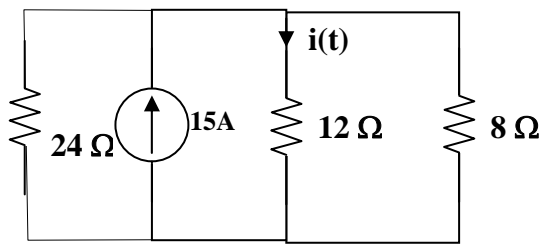
$$i(t) = 12e^{-2t} \text{ A}$$

Therefore,

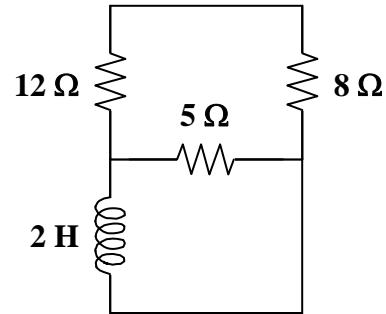
$$i(t) = 12e^{-2t} \text{ A}$$

and  $v_x(t) = -1i(t) = -12e^{-2t} \text{ V}$  for all  $t > 0$ .

**P.P.7.4** For  $t < 0$ , the equivalent circuit is shown in Fig. (a).



(a)



(b)

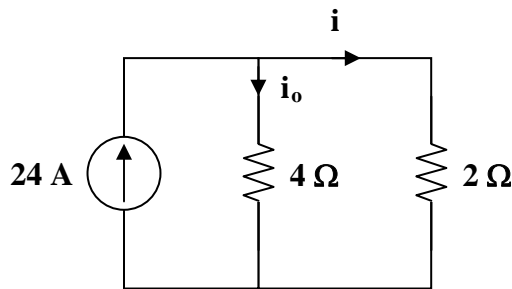
$$i(0) = 15 \left[ \frac{1}{\left\{ \frac{1}{24} + \frac{1}{12} + \frac{1}{8} \right\}} \right] / 12 = (15 \times 24 / 6) / 12 = 5 \text{ A}$$

For  $t > 0$ , the current source and 24-ohm is cut off and the RL circuit is shown in Fig. (b).

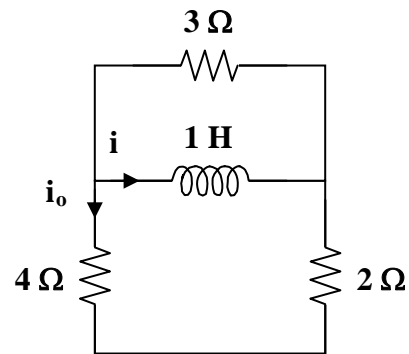
$$R_{eq} = (12 + 8) \parallel 5 = 20 \parallel 5 = 4 \Omega, \quad \tau = \frac{L}{R_{eq}} = \frac{2}{4} = 0.5$$

$$i(t) = i(0)e^{-2t} = 5e^{-2t} \text{ amps, for all } t > 0.$$

**P.P.7.5** For  $t < 0$ , the switch is closed. The inductor acts like a short so the equivalent circuit is shown in Fig. (a).



(a)



(b)

$$i = \frac{4}{4+2}(24) = 16 \text{ A}, \quad i_o = 24 - 16 = 8 \text{ A}, \quad v_o = 2i = 32 \text{ V}$$

For  $t > 0$ , the current source is cut off so that the circuit becomes that shown in Fig. (b). The Thevenin equivalent resistance at the inductor terminals is

$$R_{th} = (4 + 2) \parallel 3 = 2 \Omega, \quad \tau = \frac{L}{R_{th}} = \frac{1}{2}$$

$$i_o = \frac{3(-i)}{6+3} = \frac{-1}{3}i = -5.333e^{-2t} \text{ A} \quad \text{and} \quad v_o = -2i_o = 10.667e^{-2t} \text{ V}$$

Thus,

$$i = \begin{cases} 16 \text{ A} & t < 0 \\ 16e^{-2t} \text{ A} & t > 0 \end{cases} \quad i_o = \begin{cases} 8 \text{ A} & t < 0 \\ -5.333e^{-2t} \text{ A} & t > 0 \end{cases} \quad v_o = \begin{cases} 32 \text{ V} & t < 0 \\ 10.667e^{-2t} \text{ V} & t > 0 \end{cases}$$

**P.P.7.6**

$$i(t) = \begin{cases} 0 & t < 0 \\ 10 & 0 < t < 2 \\ -10 & 2 < t < 4 \end{cases}$$

$$i(t) = 10[u(t) - u(t-2)] - 10[u(t-2) - u(t-4)]$$

$$i(t) = \mathbf{10[u(t) - 2u(t-2) + u(t-4)]A}$$

Let  $I = \int_{-\infty}^t i \, dt$ .

For  $t < 0$ ,  $I = 0$ .

For  $0 < t < 2$ ,  $I = \int_0^t 10 \, dt = 10t$

For  $2 < t < 4$ ,  $I = \int_0^2 10 \, dt - 10 \int_2^t dt = 20 - 10t \Big|_2^t = 40 - 10t$

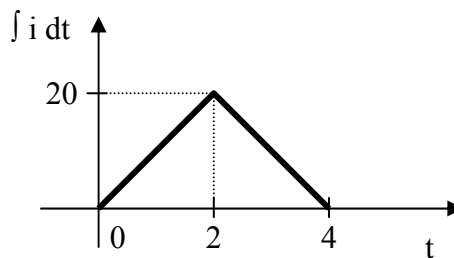
For  $t > 4$ ,  $I = 20 - 10t \Big|_2^4 = 0$

Thus,

$$I = \begin{cases} 0 & t < 0 \\ 10t & 0 < t < 2 \\ 40 - 10t & 2 < t < 4 \\ 0 & t > 4 \end{cases}$$

or  $I = \mathbf{10[r(t) - 2r(t-2) + r(t-4)]A}$

which is sketched below



**P.P.7.7**

$$i(t) = \begin{cases} 2 - 2t & 0 < t < 2 \\ -6 + 2t & 2 < t < 3 \\ 0 & \text{otherwise} \end{cases}$$

$$i(t) = (2 - 2t)[u(t) - u(t-2)] + (-6 + 2t)[u(t-2) - u(t-3)]$$

$$i(t) = 2u(t) - 2tu(t) + 4(t-2)u(t-2) - 2(t-3)u(t-3)$$

$$i(t) = [2u(t) - 2r(t) + 4r(t-2) - 2r(t-3)]A$$

Remember the singularity function,  $r(t)$ , is a ramp function equal to  $t$  for all values of  $t > 0$  and equal to zero for all values of  $t < 0$ .

**P.P.7.8**

$$h(t) = -4[u(t) - u(t-2)] + (3t-8)[u(t-2) - u(t-6)]$$

$$h(t) = -4u(t) + 4u(t-2) + 3tu(t-2) - 8u(t-2) - 3tu(t-6) + 8u(t-6)$$

$$h(t) = -4u(t) + (4-8+6)u(t-2) + 3(t-2)u(t-2) - 3(t-6)u(t-6) + (-18+8)u(t-6)$$

$$h(t) = -4u(t) + 2u(t-2) + 3(t-2)u(t-2) - 3(t-6)u(t-6) - 10u(t-6)$$

$$h(t) = -4u(t) + 2u(t-2) + 3r(t-2) - 10u(t-6) - 3r(t-6).$$

**P.P.7.9**

(a) 
$$\int_{-\infty}^{\infty} (t^3 + 5t^2 + 10)\delta(t+3) dt = t^3 + 5t^2 + 10 \Big|_{t=-3}$$

$$= -27 + 45 + 10 = \mathbf{28}$$

(b) 
$$\int_0^{10} \delta(t-\pi)\cos(3t) dt = \cos(3\pi) = \mathbf{-1}$$

**P.P.7.10** For  $t < 0$ , the capacitor acts like an open circuit.  
 $v(0^-) = v(0^+) = v(0) = 15$

For  $t > 0$ ,  $[(v(\infty)-15)/2] + [(v(\infty)-(-7.5))/6] = 0$  or  $(4/6)v(\infty) = 7.5 - 1.25 = 6.25$  or  
 $v(\infty) = 9.375 \text{ V}$

$$R_{th} = 2 \parallel 6 = \frac{3}{2} \Omega, \quad \tau = R_{th}C = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = 9.375 + (15 - 9.375)e^{-2t}$$

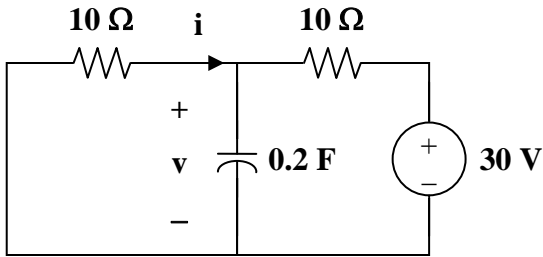
$$v(t) = \mathbf{(9.375 + 5.625e^{-2t}) \text{ V for all } t > 0}$$

At  $t = 0.5$ ,  $v(0.5) = 6.25 + 3.75e^{-1} = 6.25 + 1.3795 = \mathbf{7.63 \text{ V}}$

**P.P.7.11** For  $t < 0$ , only the left portion of the circuit is operational at steady state.  
 $v(0^-) = v(0^+) = v(0) = 20$ ,  $i(0) = 0$

For  $t > 0$ ,  $20u(-t) = 0$  so that the voltage source is replaced by a short circuit.

Transforming the current source leads to the circuit below.



$$v(\infty) = \frac{5}{15}(30) = 10$$

$$R_{th} = 5 \parallel 10 = \frac{10}{3} \Omega, \quad \tau = R_{th} C = \frac{10}{3} \times 0.2 = \frac{2}{3}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

$$v(t) = 10 + (20 - 10)e^{-3t/2}$$

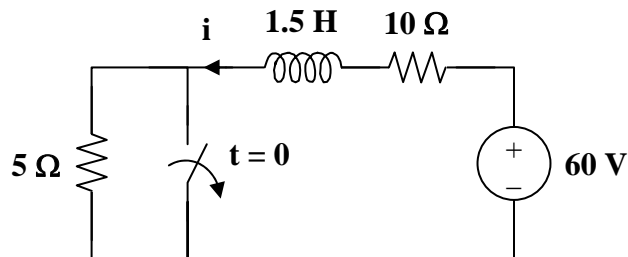
$$v(t) = 10(1 + e^{-1.5t})$$

$$i(t) = \frac{-v(t)}{5} = -2(1 + e^{-1.5t})$$

$$i(t) = \begin{cases} 0 & t < 0 \\ -2(1 + e^{-1.5t}) \text{ A} & t > 0 \end{cases}$$

$$v(t) = \begin{cases} 20 \text{ V} & t < 0 \\ 10(1 + e^{-1.5t}) \text{ V} & t > 0 \end{cases}$$

**P.P.7.12** Applying source transformation, the circuit is equivalent to the one below.



At  $t < 0$ , the switch is closed so that the 5 ohm resistor is short circuited.

$$i(0^-) = i(0) = \frac{60}{10} = 6 \text{ A}$$

For  $t > 0$ , the switch is open.

$$R_{th} = 10 + 5 = 15, \quad \tau = \frac{L}{R_{th}} = \frac{1.5}{15} = 0.1$$

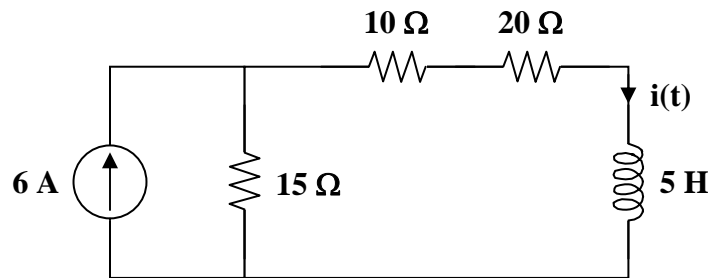
$$i(\infty) = \frac{60}{10+5} = 4\text{A}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = 4 + (6 - 4)e^{-10t}$$

$$i(t) = (4 + 2e^{-10t}) \text{ A for all } t > 0$$

**P.P.7.13** For  $0 < t < 2$ , the given circuit is equivalent to that shown below.



Since switch  $S_1$  is open at  $t = 0^-$ ,  $i(0^-) = 0$ . Also, since  $i$  cannot jump,  $i(0) = i(0^-) = 0$ .

$$i(\infty) = \frac{90}{15+10+20} = 2 \text{ A}$$

$$R_{th} = 45 \Omega, \quad \tau = \frac{L}{R_{th}} = \frac{5}{45} = \frac{1}{9}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}$$

$$i(t) = 2 + (0 - 2)e^{-9t}$$

$$i(t) = 2(1 - e^{-9t}) \text{ A}$$

When switch  $S_2$  is closed, the 20 ohm resistor is short-circuited.

$$i(2^+) = i(2^-) = 2(1 - e^{-18}) \cong 2$$

This will be the initial current

$$i(\infty) = \frac{90}{15+10} = 3.6 \text{ A}$$

$$R_{th} = 25 \Omega, \quad \tau = \frac{5}{25} = \frac{1}{5}$$

$$i(t) = i(\infty) + [i(2^+) - i(\infty)] e^{-(t-2)/\tau}$$

$$i(t) = 3.6 + (2 - 3.6)e^{-5(t-2)}$$

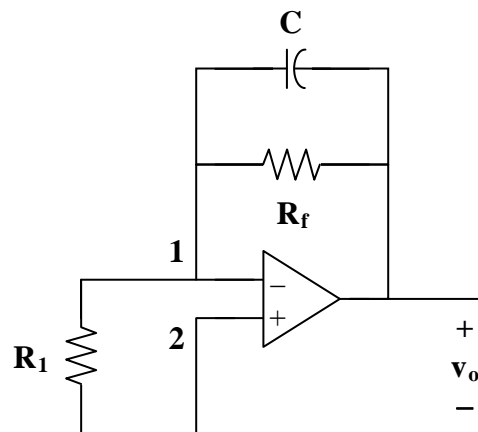
$$i(t) = 3.6 - 1.6e^{-5(t-2)}$$

$$\text{Thus, } i(t) = \begin{cases} 0 & t < 0 \\ 2(1 - e^{-9t}) \text{ A} & 0 < t < 2 \\ 3.6 - 1.6e^{-5(t-2)} \text{ A} & t > 2 \end{cases}$$

$$\text{At } t = 1, \quad i(1) = 2(1 - e^{-9}) = \mathbf{1.9997 \text{ A}}$$

$$\text{At } t = 3, \quad i(3) = 3.6 - 1.6e^{-5} = \mathbf{3.589 \text{ A}}$$

**P.P.7.14** The op amp circuit is shown below.



Since nodes 1 and 2 must be at the same potential, there is no potential difference across  $R_1$ . Hence, no current flows through  $R_1$ . Applying KCL at node 1,

$$\frac{v}{R_f} + C \frac{dv}{dt} = 0 \quad \longrightarrow \quad \frac{dv}{dt} + \frac{v}{CR_f} = 0$$

which is similar to Eq. (7.4).

Hence,

$$v(t) = v_o e^{-t/\tau}, \quad \tau = R_f C$$

$$v(0) = v_o = 4, \quad \tau = (50 \times 10^3)(10 \times 10^{-6}) = 0.5$$

$$v(t) = 4e^{-2t} \text{ V}, \quad t > 0$$

Alternatively, since no current flows through  $R_1$ , the feedback loop forms a first order RC circuit with  $v(0) = 4$  and  $\tau = R_f C = 0.5$ . Hence,

$$v(t) = 4e^{-2t} \text{ V}, \quad t > 0$$

To get to  $v_o$  from  $v$ , we notice that  $v$  is the potential difference between node 1 and the output terminal, i.e.

$$0 - v_o = v \quad \longrightarrow \quad v_o = -v \quad \text{or} \quad v_o(t) = \mathbf{-4e^{-2t} \text{ V}, \quad t > 0}$$

**P.P.7.15** Let  $v_1$  be the potential at the inverting terminal.

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$$

where  $\tau = RC = 100 \times 10^3 \times 10^{-6} = 0.1$ ,  $v(0) = 0$

$$v_1 = 0 \text{ for all } t$$

$$v_1 - v_o = v \tag{1}$$

For  $t > 0$ , the switch is closed and the op amp circuit is an inverting amplifier with

$$v_o(\infty) = \frac{-100}{10} (4 \text{ mV}) = -40 \text{ mV}$$

From (1),

$$v(\infty) = 0 - v_o(\infty) = 40 \text{ mV}$$

Thus,  $v(t) = 40(1 - e^{-10t})u(t) \text{ mV}$

$$v_o = v_1 - v = -v$$

$$v_o = 40(e^{-10t} - 1)u(t) \text{ mV}$$

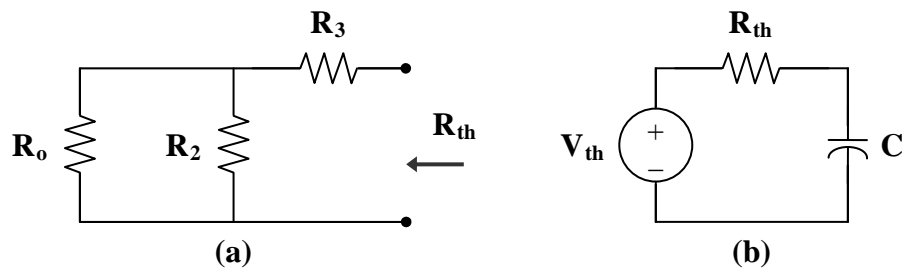
**P.P.7.16** This is a noninverting amplifier so that the output of the op amp is

$$v_a = \left(1 + \frac{R_f}{R_1}\right) v_i$$

$$v_{th} = v_a = \left(1 + \frac{R_f}{R_1}\right) v_i = \left(1 + \frac{40}{20}\right) 4.5 u(t) = 13.5 u(t)$$

To get  $R_{th}$ , consider the circuit shown in Fig. (a), where  $R_o$  is the output resistance of the op amp. For an ideal op amp,  $R_o = 0$  so that

$$R_{th} = R_3 = 10 \text{ k}\Omega$$



$$\tau = R_{th} C = 10 \times 10^3 \times 2 \times 10^{-6} = \frac{1}{50}$$

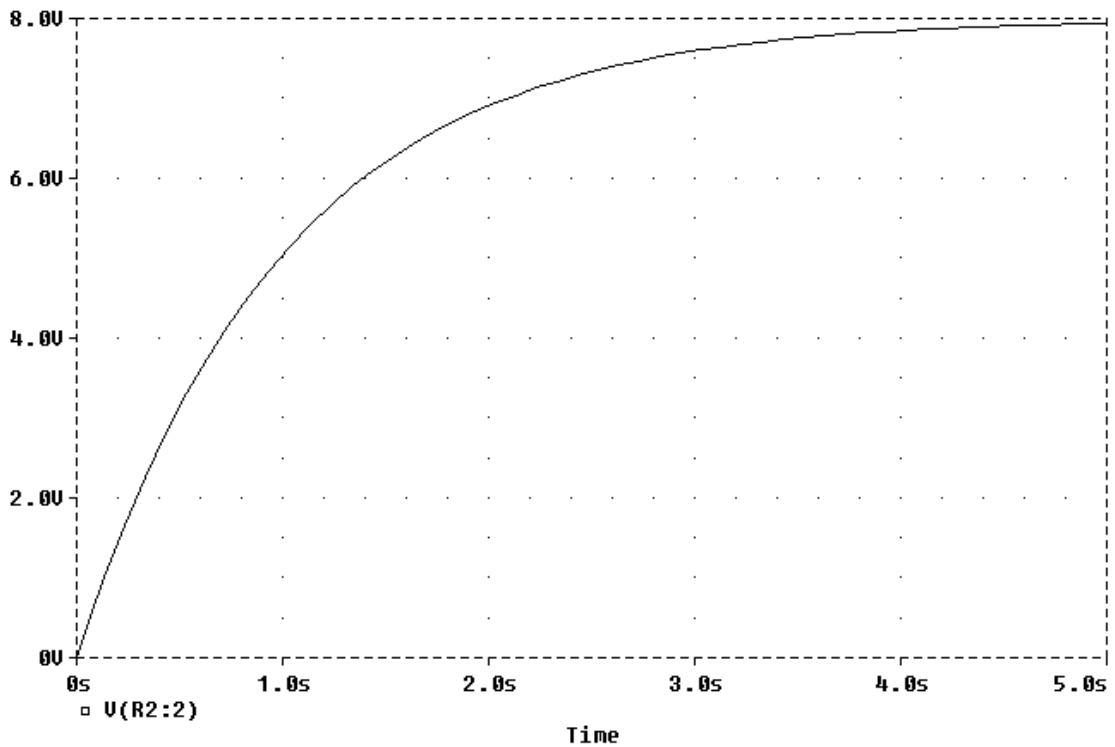
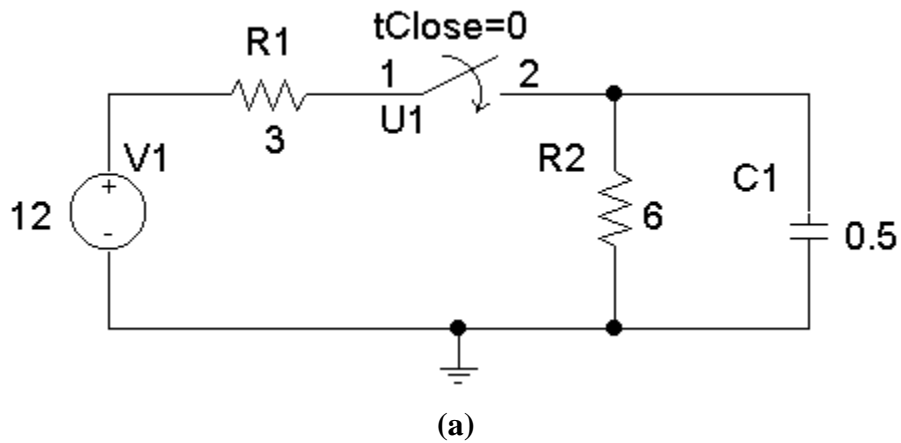
The Thevenin equivalent circuit is shown in Fig. (b), which is a first order circuit.

Hence,

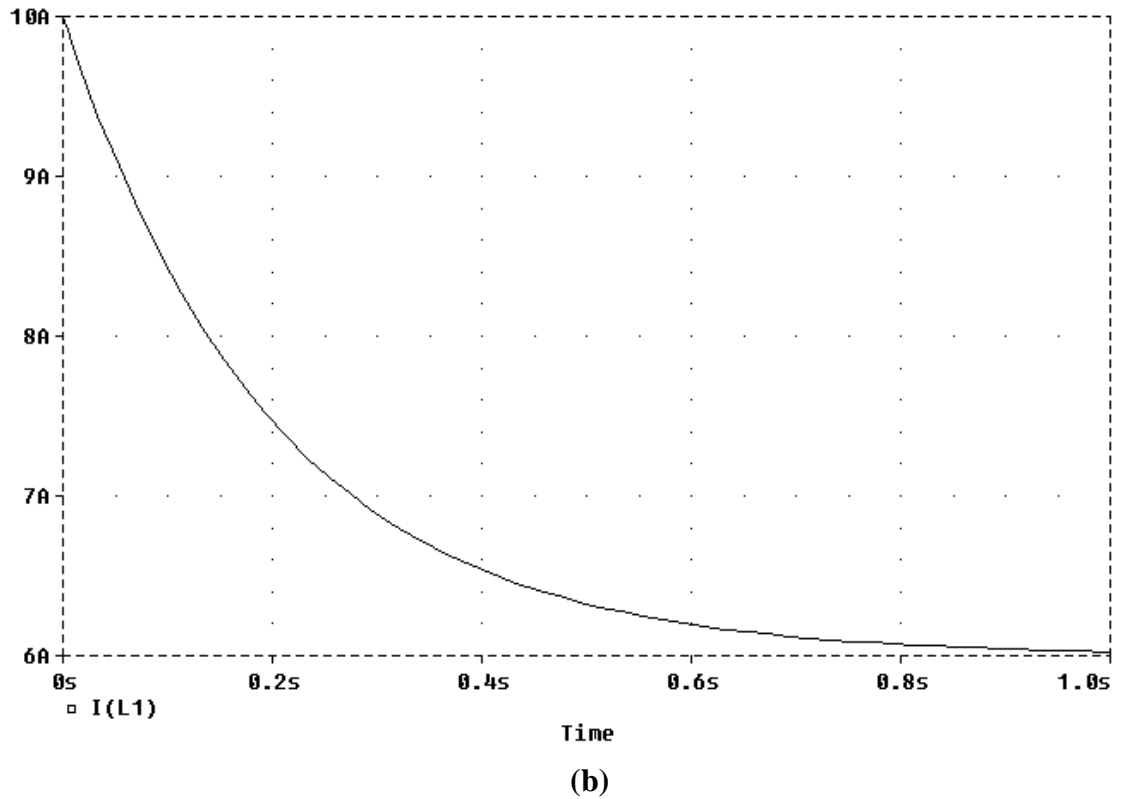
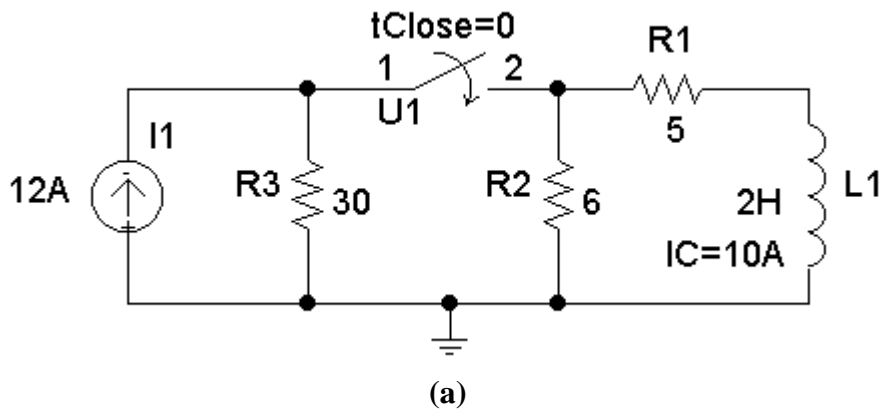
$$v_o(t) = 13.5(1 - e^{-t/\tau})u(t)$$

$$v_o(t) = 13.5(1 - e^{-50t})u(t) \text{ V}$$

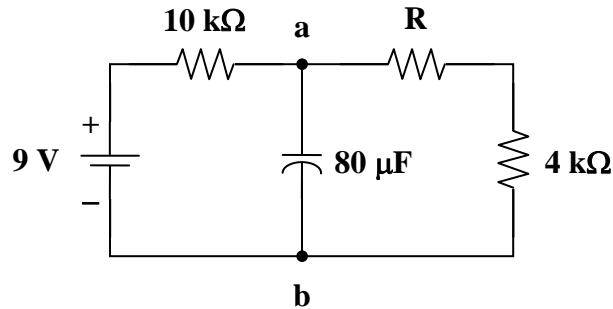
**P.P.7.17** The schematic is shown in Fig. (a). Construct and save the schematic. Select Analysis/Setup/Transient to change the Final Time to 5 s. Set the Print Step slightly greater than 0 (20 ns is default). The circuit is simulated by selecting Analysis/Simulate. In the Probe menu, select Trace/Add and display V(R2:2) as shown in Fig. (b).



**P.P.7.18** The schematic is shown in Fig. (a). While constructing the circuit, rotate L1 counterclockwise through  $270^\circ$  so that current  $i(t)$  enters pin 1 of L1 and set  $IC = 10$  for L1. After saving the schematic, select Analysis/Setup/Transient to change the Final Time to 1 s. Set the Print Step slightly greater than 0 (20 ns is default). The circuit is simulated by selecting Analysis/ Simulate. After simulating the circuit, select Trace/Add in the Probe menu and display  $I(L1)$  as shown in Fig. (b).



**P.P.7.19**  $v(0) = 0$ . When the switch is closed, we have the circuit shown below.



We find the Thevenin equivalent at terminals a-b.

$$R_{th} = (R + 4) \parallel 10 = \frac{10(R + 4)}{R + 14}$$

$$v_{th} = v(\infty) = \frac{R + 4}{R + 14}(9)$$

$$v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}, \quad \tau = R_{th}C$$

$$v(t) = v(\infty)(1 - e^{-t/\tau})$$

Since  $v(0) = 0$ ,

$$i(t) = \frac{v(t)}{R + 4} = \frac{9}{R + 4}(1 - e^{-t/\tau}) \text{ mA}$$

Assuming  $R$  is in  $k\Omega$ ,

$$120 \times 10^{-6} = \frac{9}{R + 14}(1 - e^{-t_0/\tau}) \times 10^{-3}$$

$$(0.12) \frac{R + 14}{9} = 1 - e^{-t_0/\tau}$$

or 
$$e^{-t_0/\tau} = 1 - \frac{0.12R + 1.68}{9} = \frac{7.32 - 0.12R}{9}$$

$$t_0 = \tau \ln\left(\frac{9}{7.32 - 0.12R}\right)$$

$$t_0 = \frac{10(R + 4)}{R + 14} \times 80 \times 10^{-6} \times \ln\left(\frac{9}{7.32 - 0.12R}\right)$$

When  $R = 0$ ,

$$t_0 = \frac{40 \times 80 \times 10^{-6}}{14} \times \ln\left(\frac{9}{7.32}\right) = 0.04723 \text{ s}$$

When  $R = 6 \text{ k}\Omega$ ,

$$t_0 = \frac{100}{20} \times 80 \times 10^{-6} \times \ln\left(\frac{9}{6.6}\right) = 0.124 \text{ s}$$

The time delay is **between 47.23 ms and 124 ms.**

**P.P.7.20**

(a)  $q = CV = (2 \times 10^{-3})(80) = \mathbf{160 \text{ mC}}$

(b)  $W = \frac{1}{2} CV^2 = \frac{1}{2} (2 \times 10^{-3})(6400) = \mathbf{6.4 \text{ J}}$

(c)  $\Delta I = \frac{\Delta q}{\Delta t} = \frac{0.16}{0.8 \times 10^{-3}} = \mathbf{200 \text{ A}}$

(d)  $p = \frac{\Delta w}{\Delta t} = \frac{6.4}{0.8 \times 10^{-3}} = \mathbf{8 \text{ kW}}$

(e)  $\Delta t = \frac{\Delta q}{\Delta I} = \frac{0.16}{5 \times 10^{-3}} = \mathbf{32 \text{ s}}$

**P.P.7.21**  $\tau = \frac{L}{R} = \frac{500 \times 10^{-3}}{200} = 2.5 \text{ ms}$

$$i(0) = 0, \quad i(\infty) = \frac{110}{200} = 550 \text{ mA}$$

$$i(t) = 550(1 - e^{-t/\tau}) \text{ mA}$$

$$350 \text{ mA} = i(t_0) = 550(1 - e^{-t_0/\tau}) \text{ mA}$$

$$\frac{35}{55} = 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{20}{55}$$

$$e^{t_0/\tau} = \frac{55}{20}$$

$$t_0 = \tau \ln\left(\frac{55}{20}\right) = 2.5 \ln\left(\frac{55}{20}\right) \text{ ms}$$

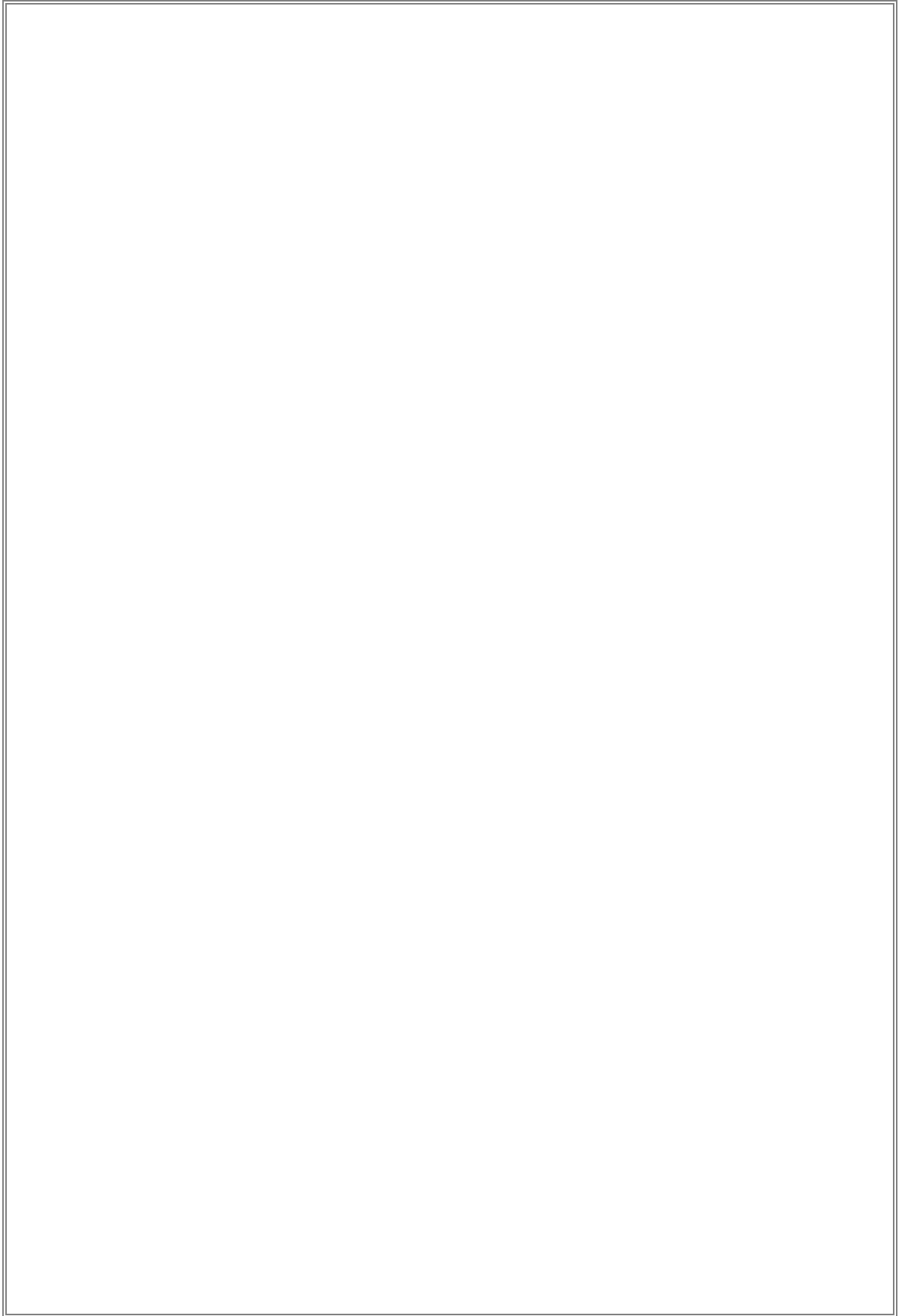
$$t_0 = \mathbf{2.529 \text{ ms}}$$

**P.P.7.22**

(a)  $t = 5\tau = \frac{5L}{R} = \frac{5 \times 20 \times 10^{-3}}{5} = \mathbf{20 \text{ ms}}$

(b)  $W = \frac{1}{2} LI^2 = \frac{1}{2} (20 \times 10^{-3}) \left(\frac{12}{5}\right)^2 = \mathbf{57.6 \text{ mJ}}$

(c)  $V = L \frac{di}{dt} = 20 \times 10^{-3} \left(\frac{12/5}{2 \times 10^{-6}}\right) = \mathbf{24 \text{ kV}}$



**CHAPTER 9**

**P.P.9.1**      amplitude = **30**  
                  phase = **-75°**  
                  angular frequency ( $\omega$ ) =  $4\pi$  = **12.57 rad/s**  
                  period (T) =  $\frac{2\pi}{\omega}$  = **0.5 s**  
                  frequency (f) =  $\frac{1}{T}$  = **2 Hz**

**P.P.9.2**       $i_1 = -4 \sin(\omega t + 55^\circ) = 4 \cos(\omega t + 55^\circ + 90^\circ)$   
                   $i_1 = 4 \cos(\omega t + 145^\circ)$ ,                       $\omega = 377 \text{ rad/s}$

Compare this with

$$i_2 = 5 \cos(\omega t - 65^\circ)$$

indicates that the phase angle between  $i_1$  and  $i_2$  is

$$145^\circ + 65^\circ = 210^\circ$$

Thus,                       **$i_1$  leads  $i_2$  by  $210^\circ$**

**P.P.9.3**      (a)       $(5 + j2)(-1 + j4) = -5 + j20 - j2 - 8 = -13 + j18$   
                   $5 \angle 60^\circ = 2.5 + j4.33$   
                   $(5 + j2)(-1 + j4) - 5 \angle 60^\circ = -15.5 + j13.67$   
                   $[(5 + j2)(-1 + j4) - 5 \angle 60^\circ]^* = -15.5 - j13.67 = 20.67 \angle 221.41^\circ$

(b)       $3 \angle 40^\circ = 2.298 + j1.928$   
              $10 + j5 + 3 \angle 40^\circ = 12.298 + j6.928 = 14.115 \angle 29.39^\circ$   
              $-3 + j4 = 5 \angle 126.87^\circ$   
              $\frac{10 + j5 + 3 \angle 40^\circ}{-3 + j4} = \frac{14.115 \angle 29.39^\circ}{5 \angle 126.87^\circ} = 2.823 \angle -97.48^\circ$   
              $2.823 \angle -97.48^\circ = -0.3675 - j2.8$   
              $10 \angle 30^\circ = 8.66 + j5$   
              $\frac{10 + j5 + 3 \angle 40^\circ}{-3 + j4} + 10 \angle 30^\circ + j5 = 8.293 + j7.2$

**P.P.9.4**      (a)       $v = 7 \cos(2t + 40^\circ)$

The phasor form is

$$\mathbf{V} = 7 \angle 40^\circ \text{ V}$$

(b)      Since  $-\sin(A) = \cos(A + 90^\circ)$ ,

$$i = -4 \sin(10t + 10^\circ) = 4 \cos(10t + 10^\circ + 90^\circ)$$

$$i = 4 \cos(10t + 100^\circ)$$

The phasor form is

$$\mathbf{I} = 4 \angle 100^\circ \text{ A}$$

**P.P.9.5**

(a) Since  $-1 = 1 \angle \pm 180^\circ$  (we can use either sign)

$$\mathbf{V} = -25 \angle 40^\circ = 25 \angle (40^\circ - 180^\circ) = 25 \angle -140^\circ$$

The sinusoid is

$$v(t) = 25 \cos(\omega t - 140^\circ) \text{ V or } 25 \cos(\omega t + 220^\circ) \text{ V}$$

(b)  $\mathbf{I} = j(12 - j5) = 5 + j12 = 13 \angle 67.38^\circ$

The sinusoid is

$$i(t) = 13 \cos(\omega t + 67.38^\circ) \text{ A}$$

**P.P.9.6**

$$\begin{aligned} \text{Let } v(t) &= -10 \sin(\omega t - 30^\circ) + 20 \cos(\omega t + 45^\circ) \\ &= 10 \cos(\omega t - 30^\circ + 90^\circ) + 20 \cos(\omega t + 45^\circ) \end{aligned}$$

Taking the phasor of each term

$$\mathbf{V} = 10 \angle 60^\circ + 20 \angle 45^\circ$$

$$\mathbf{V} = 5 + j8.66 + 14.142 + j14.142$$

$$\mathbf{V} = 19.142 + j22.8 = 29.77 \angle 49.98^\circ$$

Converting  $\mathbf{V}$  to the time domain

$$v(t) = 29.77 \cos(\omega t + 49.98^\circ) \text{ V}$$

**P.P.9.7**

Given that

$$2 \frac{dv}{dt} + 5v + 10 \int v dt = 50 \cos(5t - 30^\circ)$$

we take the phasor of each term to get

$$2j\omega \mathbf{V} + 5 \mathbf{V} + \frac{10}{j\omega} \mathbf{V} = 50 \angle -30^\circ, \quad \omega = 5$$

$$\mathbf{V} [j10 + 5 - j(10/5)] = \mathbf{V} (5 + j8) = 50 \angle -30^\circ$$

$$\mathbf{V} = \frac{50 \angle -30^\circ}{5 + j8} = \frac{50 \angle -30^\circ}{9.434 \angle 58^\circ}$$

$$\mathbf{V} = 5.3 \angle -88^\circ$$

Converting  $\mathbf{V}$  to the time domain

$$v(t) = 5.3 \cos(5t - 88^\circ) \text{ V}$$

**P.P.9.8**

For the capacitor,

$$\mathbf{V} = \mathbf{I} / (j\omega C), \quad \text{where } \mathbf{V} = 10 \angle 30^\circ, \quad \omega = 100$$

$$\mathbf{I} = j\omega C \mathbf{V} = (j100)(50 \times 10^{-6})(10 \angle 30^\circ)$$

$$\mathbf{I} = 50 \angle 120^\circ \text{ mA}$$

$$i(t) = 50 \cos(100t + 120^\circ) \text{ mA}$$

**P.P.9.9**

$$\mathbf{V}_s = 20 \angle 30^\circ, \quad \omega = 10$$

$$\mathbf{Z} = 4 + j\omega L = 4 + j2$$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{20\angle 30^\circ}{4 + j2} = \frac{20\angle 30^\circ(4 - j2)}{16 + 4} = 4.472\angle 3.43^\circ$$

$$\mathbf{V} = j\omega L \mathbf{I} = j2 \mathbf{I} = (2\angle 90^\circ)(4.472\angle 3.43^\circ) = 8.944\angle 93.43^\circ$$

Therefore,  $v(t) = 8.944 \sin(10t + 93.43^\circ) \text{ V}$   
 $i(t) = 4.472 \sin(10t + 3.43^\circ) \text{ A}$

### P.P.9.10

Let  $\mathbf{Z}_1$  = impedance of the 1-mF capacitor in series with the 100- $\Omega$  resistor

$\mathbf{Z}_2$  = impedance of the 1-mF capacitor

$\mathbf{Z}_3$  = impedance of the 8-H inductor in series with the 200- $\Omega$  resistor

$$\mathbf{Z}_1 = 100 + \frac{1}{j\omega C} = 100 + \frac{1}{j(10)(1 \times 10^{-3})} = 80 - j100$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(1 \times 10^{-3})} = -j100$$

$$\mathbf{Z}_3 = 200 + j\omega L = 200 + j(10)(8) = 200 + j80$$

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = \mathbf{Z}_1 + \mathbf{Z}_2 \mathbf{Z}_3 / (\mathbf{Z}_2 + \mathbf{Z}_3)$$

$$\mathbf{Z}_{in} = 100 - j100 + \frac{-j100 \times (200 + j80)}{-j100 + 200 + j80}$$

$$\mathbf{Z}_{in} = 100 - j100 + 49.52 - j95.04$$

$$\mathbf{Z}_{in} = [149.52 - j195] \Omega$$

**P.P.9.11** In the frequency domain,

the voltage source is  $\mathbf{V}_s = 20\angle 100^\circ$

the 0.5-H inductor is  $j\omega L = j(10)(0.5) = j5$

the  $\frac{1}{20}$ -F capacitor is  $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = -j2$

Let  $\mathbf{Z}_1$  = impedance of the 0.5-H inductor in parallel with the 10- $\Omega$  resistor

and  $\mathbf{Z}_2$  = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \parallel j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \quad \text{and} \quad \mathbf{Z}_2 = -j2$$

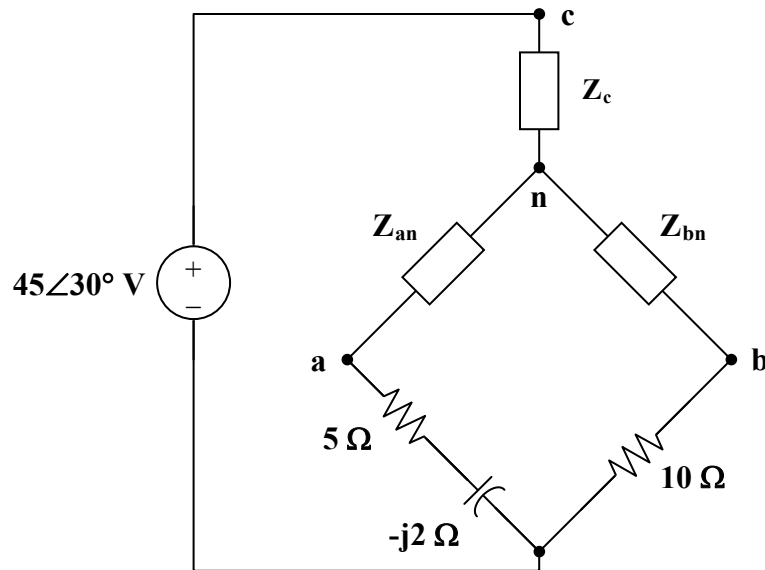
$$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2) \mathbf{V}_s$$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2} (50\angle 30^\circ) = \frac{-j(50\angle 30^\circ)}{1 + j} = \frac{50\angle (30^\circ - 90^\circ)}{\sqrt{2}\angle 45^\circ}$$

$$\mathbf{V}_o = 35.36\angle -105^\circ$$

$$v_o(t) = 35.36 \cos(10t - 105^\circ) \text{ V}$$

**P.P.9.12** We need to find the equivalent impedance via a delta-to-wye transformation as shown below.



$$\mathbf{Z}_{an} = \frac{j4(8 + j5)}{j4 + 8 + j5 - j3} = \frac{4(-5 + j8)}{8 + j6} = 0.32 + j3.76$$

$$\mathbf{Z}_{bn} = \frac{-j3(8 + j5)}{8 + j6} = \frac{3(5 - j8)(8 - j6)}{100} = -0.24 - j2.82$$

$$\mathbf{Z}_{cn} = \frac{j4(-j3)}{8 + j6} = \frac{12(8 - j6)}{100} = 0.96 - j0.72$$

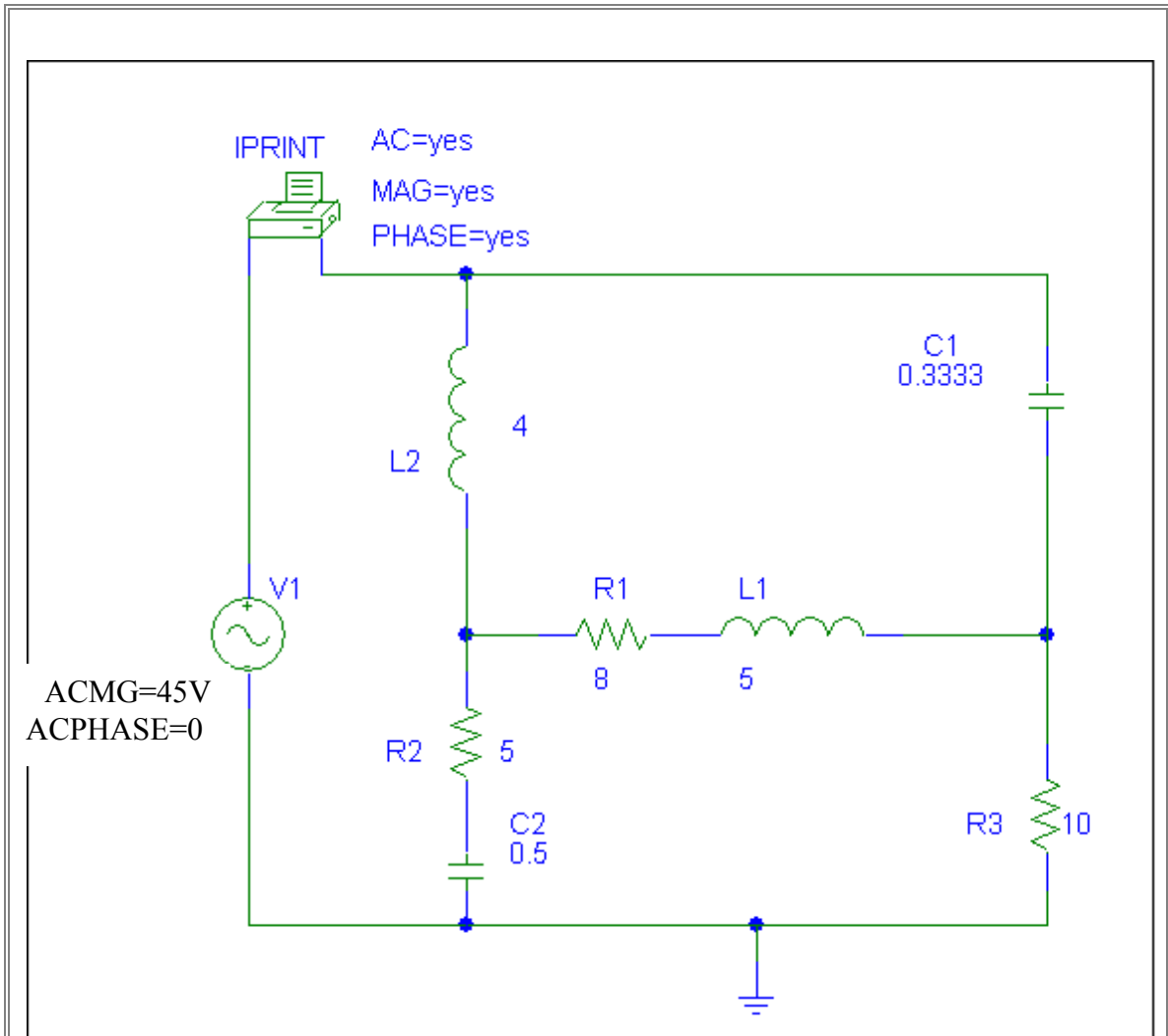
The total impedance from the source terminals is

$$\begin{aligned} \mathbf{Z} &= \mathbf{Z}_{cn} + (\mathbf{Z}_{an} + 5 - j2) \parallel (\mathbf{Z}_{bn} + 10) \\ \mathbf{Z} &= \mathbf{Z}_{cn} + (5.32 + j1.76) \parallel (9.76 - j2.82) \\ \mathbf{Z} &= \mathbf{Z}_{cn} + \frac{(5.32 + j1.76)(9.76 - j2.82)}{(5.32 + j1.76) + (9.76 - j2.82)} \\ \mathbf{Z} &= 0.96 - j0.72 + 3.744 + j0.4074 \\ \mathbf{Z} &= 4.704 - j0.3126 = 4.714 \angle -3.802^\circ \end{aligned}$$

Therefore,

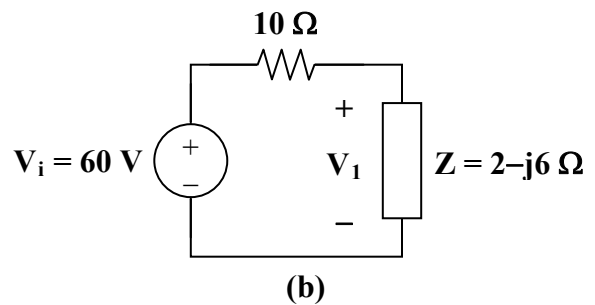
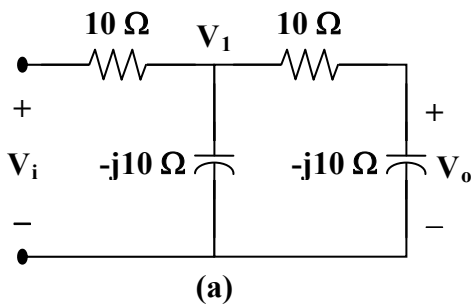
$$\begin{aligned} \mathbf{I} &= \mathbf{V} / \mathbf{Z} = \frac{45 \angle 30^\circ}{4.714 \angle -3.802^\circ} \\ \mathbf{I} &= \mathbf{9.546 \angle 33.8^\circ A} \end{aligned}$$

Let us now check this using PSpice. The solution produces the magnitude of  $\mathbf{I} = 9.946\text{E}+00$ , and the phase angle =  $33.803\text{E}+00$ , which agrees with the above answer.



**P.P.9.13** To show that the circuit in Fig. (a) meets the requirement, consider the equivalent circuit in Fig. (b).

$$\mathbf{Z} = -j10 \parallel (10 - j10) = \frac{-j10(10 - j10)}{10 - j20} = \frac{-j(10 - j10)}{1 - j2} = 2 - j6 \Omega$$



$$\mathbf{V}_1 = \frac{2 - j6}{10 + 2 - j6} (60) = \frac{60}{3} (1 - j)$$

$$\mathbf{V}_o = \frac{-j10}{10 - j10} \mathbf{V}_1 = \left( \frac{-j}{1 - j} \right) \left( \frac{60}{3} \right) (1 - j) = -j20$$

$$\mathbf{V}_o = 20 \angle -90^\circ$$

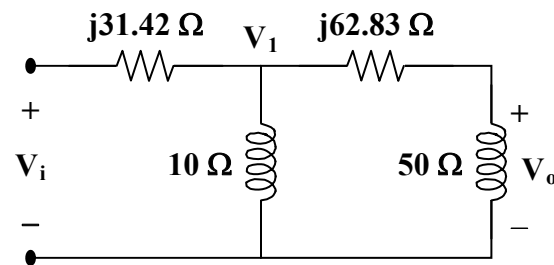
This implies that the RC circuit provides a  $90^\circ$  lagging phase shift.  
The output voltage is = **20 V**

#### P.P.9.14

the 1-mH inductor is  $j\omega L = j(2\pi)(5 \times 10^3)(1 \times 10^{-3}) = j31.42$

the 2-mH inductor is  $j\omega L = j(2\pi)(5 \times 10^3)(2 \times 10^{-3}) = j62.83$

Consider the circuit shown below.



$$\mathbf{Z} = 10 \parallel (50 + j62.83) = \frac{(10)(50 + j62.83)}{60 + j62.83}$$

$$\mathbf{Z} = 9.205 + j0.833 = 9.243 \angle 5.17^\circ$$

$$\mathbf{V}_1 = \mathbf{Z} / (\mathbf{Z} + j31.42) \mathbf{V}_i = \frac{9.243 \angle 5.17^\circ}{9.205 + j32.253} (10)$$

$$= [(9.243 \angle 5.17^\circ) / (33.54 \angle 74.07^\circ)] 10 = 2.756 \angle -68.9^\circ$$

$$\mathbf{V}_o = \frac{50}{50 + j62.83} \mathbf{V}_1 = \frac{50(2.756 \angle -68.9^\circ)}{80.297 \angle 51.49^\circ} = 1.7161 \angle -120.39^\circ$$

Therefore,

magnitude = **1.7161 V**

phase = **120.39°**

phase shift is **lagging**

**P.P.9.15**      $Z_x = (Z_3 / Z_1) Z_2$

$$Z_3 = 12 \text{ k}\Omega$$

$$Z_1 = 4.8 \text{ k}\Omega$$

$$Z_2 = 10 + j\omega L = 10 + j(2\pi)(6 \times 10^6)(0.25 \times 10^{-6}) = 10 + j9.425$$

$$Z_x = \frac{12\text{k}}{4.8\text{k}}(10 + j9.425) = 25 + j23.5625 \Omega$$

$$R_x = 25, \quad X_x = 23.5625 = \omega L_x$$

$$L_x = \frac{X_x}{2\pi f} = \frac{23.5625}{2\pi(6 \times 10^6)} = 0.625 \mu\text{H}$$

i.e. a **25- $\Omega$**  resistor in series with a **0.625- $\mu\text{H}$**  inductor.

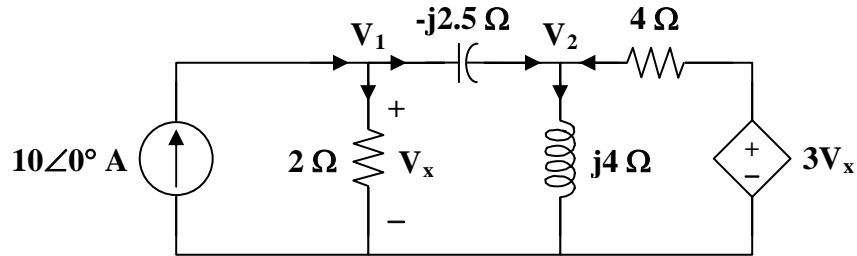
**CHAPTER 10**

**P.P.10.1**  $10\cos(2t) \longrightarrow 10\angle 0^\circ, \omega = 2$

$$2 \text{ H} \longrightarrow j\omega L = j4$$

$$0.2 \text{ F} \longrightarrow \frac{1}{j\omega C} = -j2.5$$

Hence, the circuit in the frequency domain is as shown below.



At node 1,  $10 = \frac{V_1}{2} + \frac{V_1 - V_2}{-j2.5}$

$$100 = (5 + j4)V_1 - j4V_2 \quad (1)$$

At node 2,  $\frac{V_2}{j4} = \frac{V_1 - V_2}{-j2.5} + \frac{3V_x - V_2}{4}$  where  $V_x = V_1$

$$-j2.5V_2 = j4(V_1 - V_2) + 2.5(3V_1 - V_2)$$

$$0 = -(7.5 + j4)V_1 + (2.5 + j1.5)V_2 \quad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 5 + j4 & -j4 \\ -(7.5 + j4) & 2.5 + j1.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

where  $\Delta = (5 + j4)(2.5 + j1.5) - (-j4)(-(7.5 + j4)) = 22.5 - j12.5 = 25.74\angle -29.05^\circ$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{\begin{bmatrix} 2.5 + j1.5 & j4 \\ 7.5 + j4 & 5 + j4 \end{bmatrix}}{22.5 - j12.5} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$V_1 = \frac{2.5 + j1.5}{22.5 - j12.5}(100) = \frac{2.915\angle 30.96^\circ}{25.74\angle -29.05^\circ}(100) = 11.325\angle 60.01^\circ \text{ V}$$

$$V_2 = \frac{7.5 + j4}{22.5 - j12.5}(100) = \frac{8.5\angle 28.07^\circ}{25.74\angle -29.05^\circ}(100) = 33.02\angle 57.12^\circ \text{ V}$$

In the time domain,

$$v_1(t) = 11.325\cos(2t + 60.01^\circ) \text{ V}$$

$$v_2(t) = 33.02\cos(2t + 57.12^\circ) \text{ V}$$

**P.P.10.2** The only non-reference node is a supernode.

$$\frac{75 - \mathbf{V}_1}{4} = \frac{\mathbf{V}_1}{j4} + \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{2}$$

$$75 - \mathbf{V}_1 = -j\mathbf{V}_1 + j4\mathbf{V}_2 + 2\mathbf{V}_2$$

$$75 = (1 - j)\mathbf{V}_1 + (2 + j4)\mathbf{V}_2 \quad (1)$$

The supernode gives the constraint of

$$\mathbf{V}_1 = \mathbf{V}_2 + 100\angle 60^\circ \quad (2)$$

Substituting (2) into (1) gives

$$75 = (1 - j)(40\angle 60^\circ) + (3 + j3)\mathbf{V}_2$$

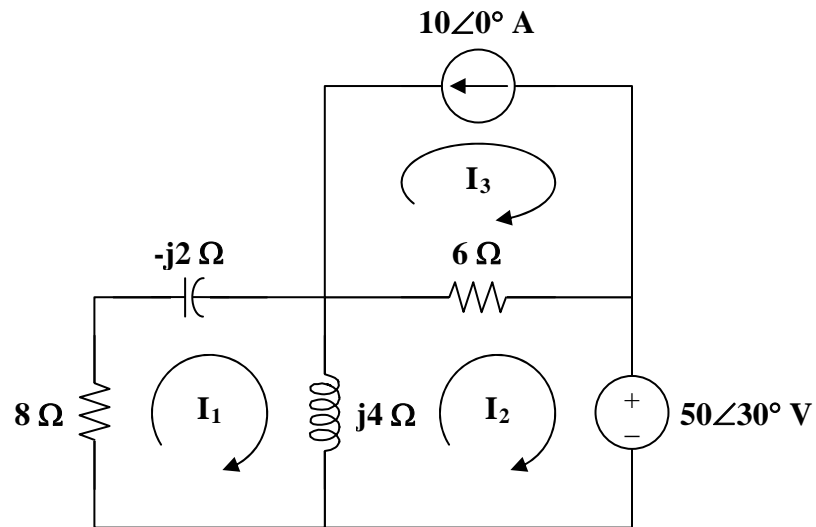
$$\mathbf{V}_2 = \frac{75 - (1 - j)(100\angle 60^\circ)}{3 + j3} = \frac{71.62\angle 210.72^\circ}{4.243\angle 45^\circ} = 16.881\angle 165.72^\circ$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 100\angle 60^\circ = (-16.358 + j4.17) + (50 + j86.6)$$

$$\mathbf{V}_1 = 33.64 + j90.77$$

Therefore,  $\mathbf{V}_1 = 96.8\angle 69.66^\circ \text{ V}$ ,  $\mathbf{V}_2 = 16.88\angle 165.72^\circ \text{ V}$

**P.P.10.3** Consider the circuit below.



For mesh 1,  $(8 - j2 + j4)\mathbf{I}_1 - j4\mathbf{I}_2 = 0$   
 $(8 + j2)\mathbf{I}_1 = j4\mathbf{I}_2 \quad (1)$

For mesh 2,  $(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 6\mathbf{I}_3 + 50\angle 30^\circ = 0$

For mesh 3,  $\mathbf{I}_3 = -10$

Thus, the equation for mesh 2 becomes

$$(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 = -60 - 50\angle 30^\circ \quad (2)$$

From (1),  $\mathbf{I}_2 = \frac{8 + j2}{j4}\mathbf{I}_1 = (0.5 - j2)\mathbf{I}_1$  (3)

Substituting (3) into (2),

$$(6 + j4)(0.5 - j2)\mathbf{I}_1 - j4\mathbf{I}_1 = -60 - 50\angle 30^\circ$$

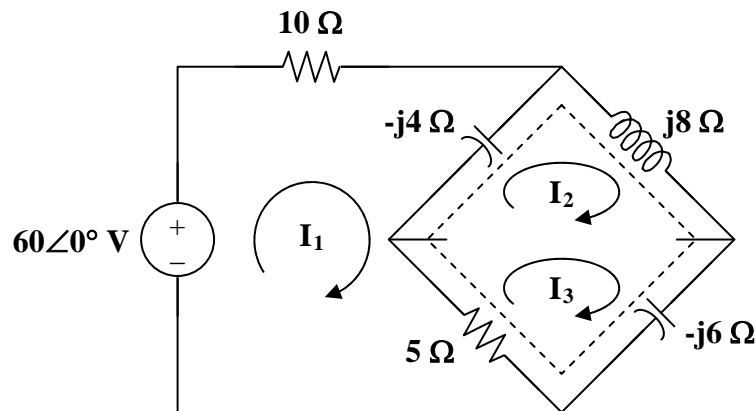
$$(11 - j14)\mathbf{I}_1 = -(103.3 + j25)$$

$$\mathbf{I}_1 = \frac{-(103.3 + j25)}{11 - j14}$$

Hence,  $\mathbf{I}_o = -\mathbf{I}_1 = \frac{103.3 + j25}{11 - j14} = \frac{106.28\angle 13.605^\circ}{17.804\angle -51.843^\circ}$

$$\mathbf{I}_o = 5.969\angle 65.45^\circ \text{ A}$$

**P.P.10.4** Meshes 2 and 3 form a supermesh as shown in the circuit below.



For mesh 1,  $-60 + (15 - j4)\mathbf{I}_1 - (-j4)\mathbf{I}_2 - 5\mathbf{I}_3 = 0$   
 $(15 - j4)\mathbf{I}_1 + j4\mathbf{I}_2 - 5\mathbf{I}_3 = 60$  (1)

For the supermesh,  $(j8 - j4)\mathbf{I}_2 + (5 - j6)\mathbf{I}_3 - (5 - j4)\mathbf{I}_1 = 0$  (2)

Also,  $\mathbf{I}_3 = \mathbf{I}_2 + 2.4$  (3)

Eliminating  $\mathbf{I}_3$  from (1) and (2)

$$(15 - j4)\mathbf{I}_1 + (-5 + j4)\mathbf{I}_2 = 72 \quad (4)$$

$$(-5 + j4)\mathbf{I}_1 + (5 - j2)\mathbf{I}_2 = -12 + j14.4 \quad (5)$$

From (4) and (5),

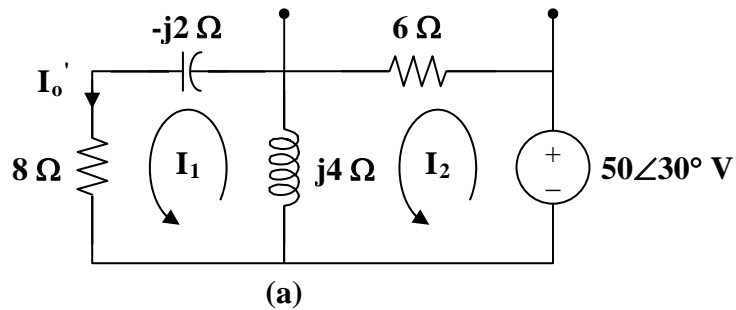
$$\begin{bmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 72 \\ -12 + j14.4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{vmatrix} = 58 - j10 = 58.86 \angle -9.78^\circ$$

$$\Delta_1 = \begin{vmatrix} 72 & -5 + j4 \\ -12 + j14.4 & 5 - j2 \end{vmatrix} = 357.6 - j24 = 358.4 \angle -3.84^\circ$$

Thus,  $\mathbf{I}_o = \mathbf{I}_1 = \frac{\Delta_1}{\Delta} = 6.089 \angle 5.94^\circ \text{ A}$

**P.P.10.5** Let  $\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o''$ , where  $\mathbf{I}_o'$  and  $\mathbf{I}_o''$  are due to the voltage source and current source respectively. For  $\mathbf{I}_o'$  consider the circuit in Fig. (a).



For mesh 1,  $(8 + j2)\mathbf{I}_1 - j4\mathbf{I}_2 = 0$   
 $\mathbf{I}_2 = (0.5 - j2)\mathbf{I}_1 \quad (1)$

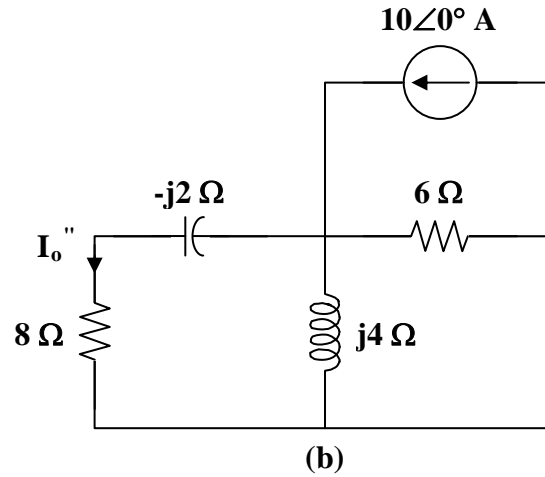
For mesh 2,  $(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 50\angle 30^\circ = 0 \quad (2)$

Substituting (1) into (2),

$$(6 + j4)(0.5 - j2)\mathbf{I}_1 - j4\mathbf{I}_1 = 50\angle 30^\circ$$

$$\mathbf{I}_o' = \mathbf{I}_1 = \frac{50\angle 30^\circ}{11 - j14} = 0.4 + j2.78$$

For  $\mathbf{I}_o''$  consider the circuit in Fig. (b).



$$\text{Let } \mathbf{Z}_1 = 8 - j2 \Omega, \quad \mathbf{Z}_2 = 6 \parallel j4 = \frac{j24}{6 + j4} = 1.846 + j2.769 \Omega$$

$$\mathbf{I}_o'' = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} (10) = \frac{(10)(1.846 + j2.769)}{9.846 + j0.77} = 2.082 + j2.65$$

$$\text{Therefore, } \mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = 2.48 + j5.43$$

$$\mathbf{I}_o = \mathbf{5.97} \angle \mathbf{65.45}^\circ \text{ A}$$

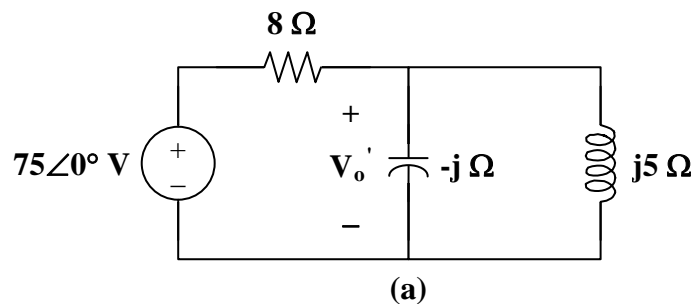
**P.P.10.6** Let  $v_o = v_o' + v_o''$ , where  $v_o'$  is due to the voltage source and  $v_o''$  is due to the current source. For  $v_o'$ , we remove the current source.

$$75 \sin(5t) \longrightarrow 75 \angle 0^\circ, \quad \omega = 5$$

$$0.2 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(5)(0.2)} = -j$$

$$1 \text{ H} \longrightarrow j\omega L = j(5)(1) = j5$$

The circuit in the frequency domain is shown in Fig. (a).



Note that  $-j \parallel j5 = -j1.25$

By voltage division,

$$\mathbf{V}_o' = \frac{-j1.25}{8 - j1.25}(75) = 11.577 \angle -81.12^\circ$$

Thus,  $v_o' = 11.577 \sin(5t - 81.12^\circ) \text{ V}$

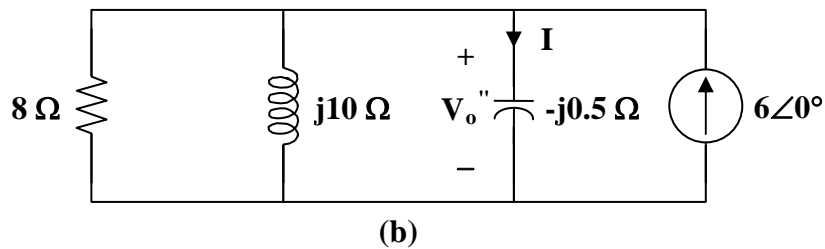
For  $v_o''$ , we remove the voltage source.

$$6 \cos(10t) \longrightarrow 6 \angle 0^\circ, \quad \omega = 10$$

$$0.2 \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.2)} = -j0.5$$

$$1 \text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

The corresponding circuit in the frequency domain is shown in Fig (b).



$$\text{Let } \mathbf{Z}_1 = -j0.5, \quad \mathbf{Z}_2 = 8 \parallel j10 = \frac{j80}{8 + j10} = 4.878 + j3.9$$

By current division,

$$\mathbf{I} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(6)$$

$$\mathbf{V}_o'' = \mathbf{I}(-j0.5) = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(6)(-j0.5) = \frac{-j(14.631 + j11.7)}{4.878 + j3.4}$$

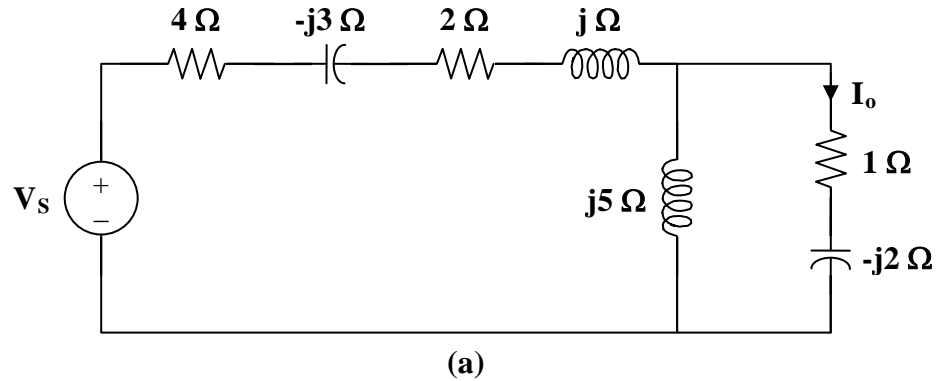
$$\mathbf{V}_o'' = \frac{18.735 \angle -51.36^\circ}{5.94 \angle 34.88^\circ} = 3.154 \angle -86.24^\circ$$

Thus,  $v_o'' = 3.154 \cos(10t - 86.24^\circ)$

Therefore,  $v_o = v_o' + v_o''$

$$v_o = [11.577 \sin(5t - 81.12^\circ) + 3.154 \cos(10t - 86.24^\circ)] \text{ V}$$

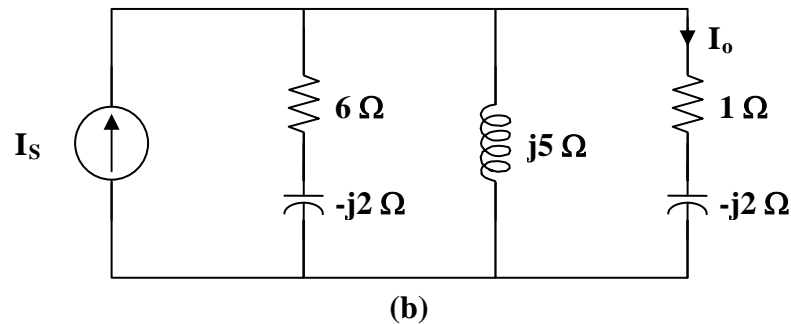
**P.P.10.7** If we transform the current source to a voltage source, we obtain the circuit shown in Fig. (a).



$$\mathbf{V}_s = \mathbf{I}_s \mathbf{Z}_s = (j12)(4 - j3) = 36 + j48$$

We transform the voltage source to a current source as shown in Fig. (b).

Let  $\mathbf{Z} = 4 - j3 + 2 + j = 6 - j2$ . Then,  $\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{36 + j48}{6 - j2} = 4.5 + j9$ .



Note that  $\mathbf{Z} \parallel j5 = \frac{(6 - j2)(j5)}{6 + j3} = \frac{10}{3}(1 + j)$ .

By current division,

$$\mathbf{I}_o = \frac{\frac{10}{3}(1 + j)}{\frac{10}{3}(1 + j) + (1 - j2)}(4.5 + j9)$$

$$\mathbf{I}_o = \frac{-60 + j120}{13 + j4} = \frac{134.16 \angle 116.56^\circ}{13.602 \angle 17.1^\circ}$$

$$\mathbf{I}_o = \mathbf{9.863 \angle 99.46^\circ \text{ A}}$$

**P.P.10.8**

When the voltage source is set equal to zero,

$$Z_{th} = 10 + (-j4) \parallel (6 + j2)$$

$$Z_{th} = 10 + \frac{(-j4)(6 + j2)}{6 - j2}$$

$$Z_{th} = 10 + 2.4 - j3.2$$

$$Z_{th} = (12.4 - j3.2) \Omega$$

By voltage division,

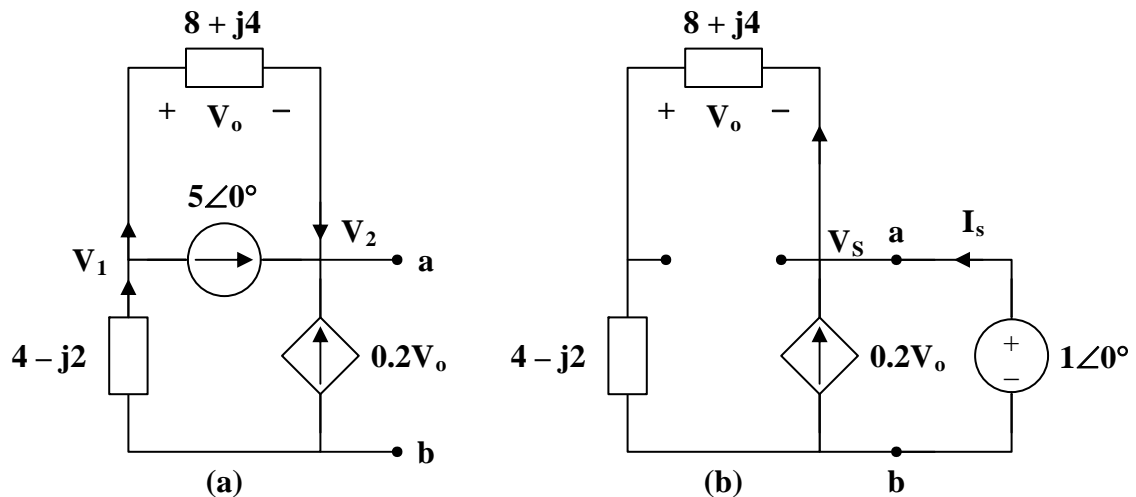
$$V_{th} = \frac{-j4}{6 + j2 - j4} (100 \angle 20^\circ) = \frac{(-j4)(100 \angle 20^\circ)}{6 - j2}$$

$$V_{th} = \frac{(4 \angle -90^\circ)(100 \angle 20^\circ)}{6.325 \angle -18.43^\circ}$$

$$V_{th} = 63.24 \angle -51.57^\circ \text{ V}$$

**P.P.10.9**

To find  $V_{th}$ , consider the circuit in Fig. (a).



At node 1,

$$\frac{0 - V_1}{4 - j2} = 5 + \frac{V_1 - V_2}{8 + j4}$$

$$-(2 + j)V_1 = 50 + (1 - j0.5)(V_1 - V_2)$$

$$50 = (1 - j0.5)V_2 - (3 + j0.5)V_1 \quad (1)$$

At node 2,  $5 + 0.2V_o + \frac{V_1 - V_2}{8 + j4} = 0$ , where  $V_o = V_1 - V_2$ .

Hence, the equation for node 2 becomes

$$5 + 0.2(V_1 - V_2) + \frac{V_1 - V_2}{8 + j4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_2 - \frac{50}{3 + j0.5} \quad (2)$$

Substituting (2) into (1),

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_2 + (50) \frac{3 + j0.5}{3 - j0.5}$$

$$0 = -50 - (2 + j)\mathbf{V}_2 + \frac{50}{37}(35 + j12)$$

$$\mathbf{V}_2 = \frac{-2.702 + j16.22}{2 + j} = 7.35 \angle 72.9^\circ$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \mathbf{7.35} \angle \mathbf{72.9^\circ} \mathbf{V}$$

To find  $\mathbf{Z}_{th}$ , we remove the independent source and insert a 1-V voltage source between terminals a-b, as shown in Fig. (b).

At node a, 
$$\mathbf{I}_s = -0.2\mathbf{V}_o + \frac{\mathbf{V}_s}{8 + j4 + 4 - j2}$$

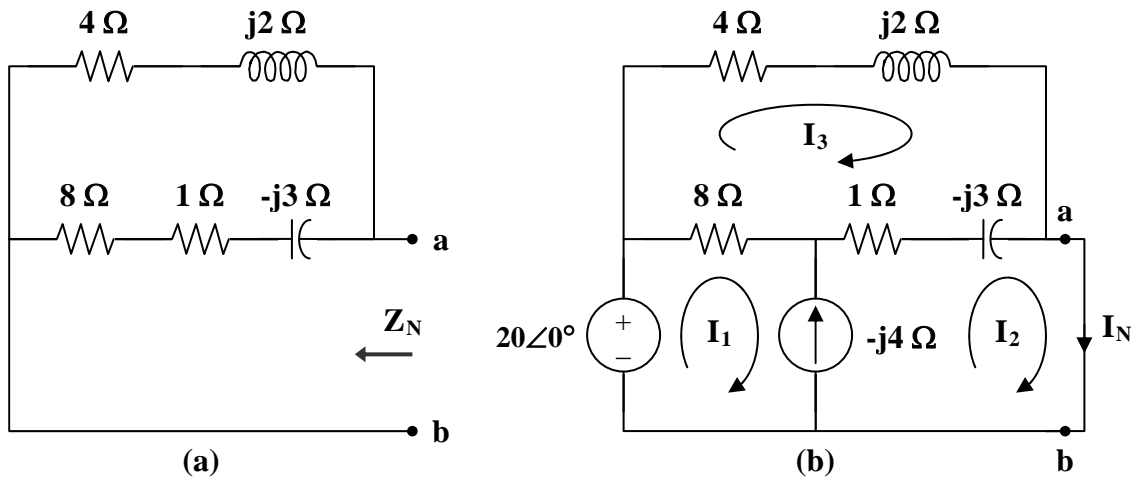
But, 
$$\mathbf{V}_s = 1 \quad \text{and} \quad -\mathbf{V}_o = \frac{8 + j4}{8 + j4 + 4 - j2} \mathbf{V}_s$$

So, 
$$\mathbf{I}_s = (0.2) \frac{8 + j4}{12 + j2} + \frac{1}{12 + j2} = \frac{2.6 + j0.8}{12 + j2}$$

and 
$$\mathbf{Z}_{th} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{1}{\mathbf{I}_s} = \frac{12 + j2}{2.6 + j0.8} = \frac{12.166 \angle 9.46^\circ}{2.72 \angle 17.10^\circ}$$

$$\mathbf{Z}_{th} = \mathbf{4.473} \angle \mathbf{-7.64^\circ} \mathbf{\Omega}$$

**P.P.10.10** To find  $Z_N$ , consider the circuit in Fig. (a).



$$Z_N = (4 + j2) \parallel (9 - j3) = \frac{(4 + j2)(9 - j3)}{13 - j}$$

$$Z_N = (3.176 + j0.706) \Omega$$

To find  $I_N$ , short-circuit terminals a-b as shown in Fig. (b). Notice that meshes 1 and 2 form a supermesh.

For the supermesh,  $-20 + 8I_1 + (1 - j3)I_2 - (9 - j3)I_3 = 0$  (1)

Also,  $I_1 = I_2 + j4$  (2)

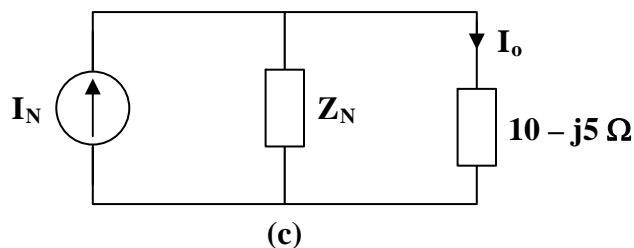
For mesh 3,  $(13 - j)I_3 - 8I_1 - (1 - j3)I_2 = 0$  (3)

Solving for  $I_2$ , we obtain

$$I_N = I_2 = \frac{50 - j62}{9 - j3} = \frac{79.65 \angle -51.11^\circ}{9.487 \angle -18.43^\circ}$$

$$I_N = 8.396 \angle -32.68^\circ \text{ A}$$

Using the Norton equivalent, we can find  $I_o$  as in Fig. (c).



By current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + 10 - j5} \mathbf{I}_N = \frac{3.176 + j0.706}{13.176 - j4.294} (8.396 \angle -32.68^\circ)$$

$$\mathbf{I}_o = \frac{(3.254 \angle 12.53^\circ)(8.396 \angle -32.68^\circ)}{13.858 \angle -18.05^\circ}$$

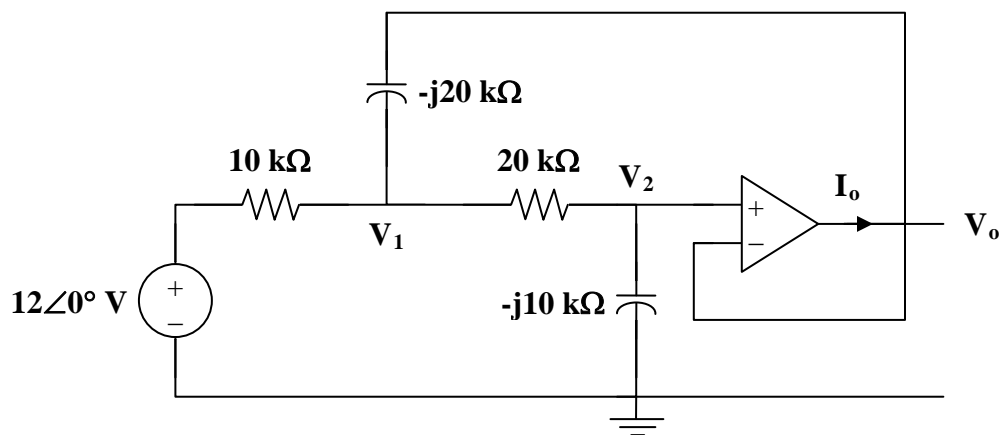
$$\mathbf{I}_o = 1.9714 \angle -2.10^\circ \text{ A}$$

**P.P.10.11**

$$10 \text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5 \times 10^3)(10 \times 10^{-9})} = -j20 \text{ k}\Omega$$

$$20 \text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5 \times 10^3)(20 \times 10^{-9})} = -j10 \text{ k}\Omega$$

Consider the circuit in the frequency domain as shown below.



As a voltage follower,  $\mathbf{V}_2 = \mathbf{V}_o$

At node 1,

$$\frac{12 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$24 = (3 + j)\mathbf{V}_1 - (1 + j)\mathbf{V}_o \quad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-j10}$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o \quad (2)$$

Substituting (2) into (1) gives

$$24 = j6\mathbf{V}_o \quad \text{or} \quad \mathbf{V}_o = 4 \angle -90^\circ$$

Hence,  $v_o(t) = 4\cos(5000t - 90^\circ) V$   
 $v_o(t) = \mathbf{4\sin(5,000t) V}$

Now,  $I_o = \frac{V_o - V_1}{-j20k}$

But from (2)  $V_o - V_1 = -j2V_o = -8$

$$I_o = \frac{-8}{-j20k} = -j400 \mu A$$

Hence,  $i_o(t) = 400\cos(5000t - 90^\circ) \mu A$

$$i_o(t) = \mathbf{400\sin(5,000t) \mu A}$$

**P.P.10.12** Let  $Z = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$

$$\frac{V_s}{V_o} = \frac{R}{R + Z}$$

The loop gain is

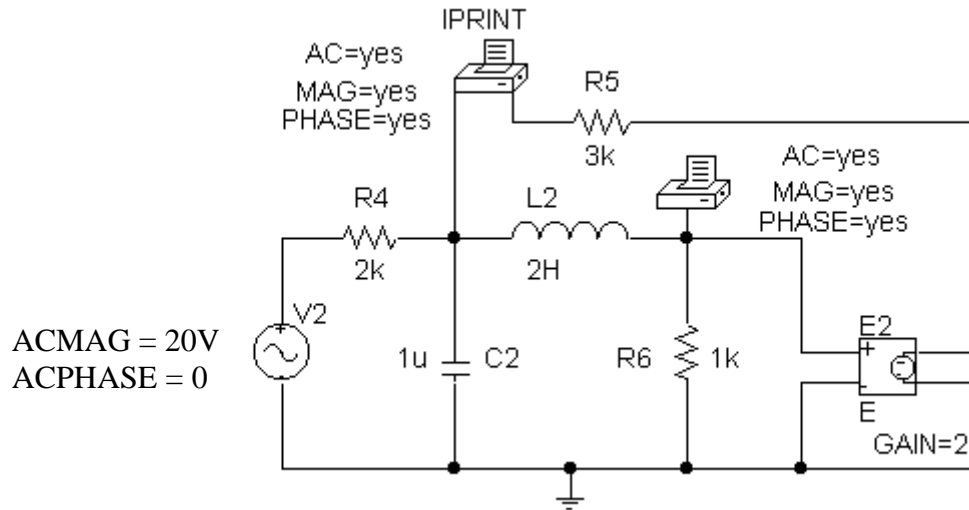
$$1/G = \frac{V_s}{V_o} = \frac{R}{R + Z} = \frac{R}{R + \frac{R}{1 + j\omega RC}} = \frac{1 + j\omega RC}{2 + j\omega RC}$$

where  $\omega RC = (1000)(10 \times 10^3)(1 \times 10^{-6}) = 10$

$$1/G = \frac{1 + j10}{2 + j10} = \frac{10.05 \angle 84.29^\circ}{10.2 \angle 78.69^\circ}$$

$$G = \mathbf{1.0147 \angle -5.6^\circ}$$

**P.P.10.13** The schematic is shown below.



Since  $\omega = 2\pi f = 3000 \text{ rad/s} \longrightarrow f = 477.465 \text{ Hz}$ . Setup/Analysis/AC Sweep as Linear for 1 point starting and ending at a frequency of 447.465 Hz. When the schematic is saved and run, the output file includes

Frequency	IM(V_PRINT1)	IP(V_PRINT1)
4.775E+02	1.088E-03	-5.512E+01
Frequency	VM(\$N_0005)	VP(\$N_0005)
4.775E+02	5.364E-01	-1.546E+02

From the output file, we obtain

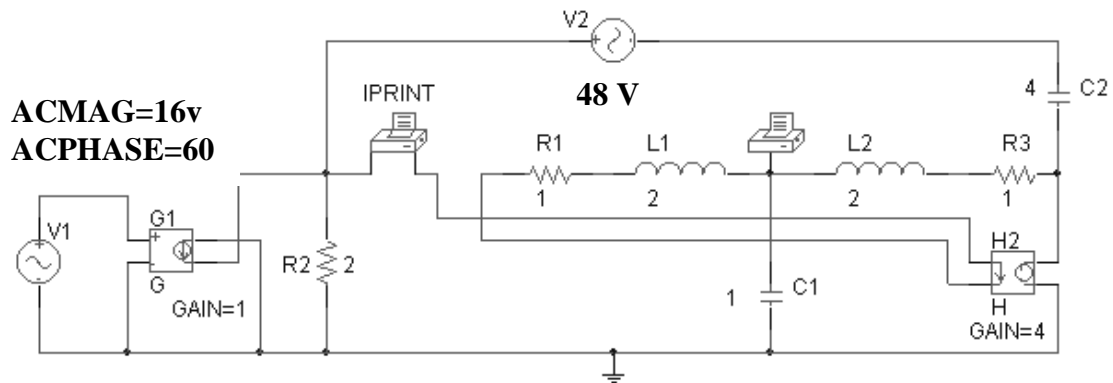
$$\mathbf{V}_o = 0.2682 \angle -154.6^\circ \text{ V} \quad \text{and} \quad \mathbf{I}_o = 0.544 \angle -55.12^\circ \text{ mA}$$

Therefore,

$$v_o(t) = 536.4 \cos(3,000t - 154.6^\circ) \text{ mV}$$

$$i_o(t) = 1.088 \cos(3,000t - 55.12^\circ) \text{ mA}$$

**P.P.10.14** The schematic is shown below.



Since PSpice does not allow the use of complex impedances, we need to convert the complex impedances into values of capacitance and inductance. We select  $\omega = 1$  rad/s which generates  $f = 0.15915$  Hz. We use this to obtain the values of capacitances, where  $C = 1/\omega X_c$ , and inductances, where  $L = X_L/\omega$ . Since AC current sources in PSpice does not allow the use of phase angles but AC voltages do, we can replace the current source with a voltage controlled current source. Thus we not have created an AC current source with a magnitude and a phase.

To obtain the desired output use Setup/Analysis/AC Sweep as Linear for 1 point starting and ending at a frequency of 0.15915 Hz. When the schematic is saved and run, the output file includes

Frequency	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	10.336E+00	1.580E+02
Frequency	VM(\$N_0004)	VP(\$N_0004)
1.592E-01	39.368E+00	4.478E+01

From the output file, we obtain

$$\mathbf{V_x = 39.37 \angle 44.78^\circ \text{ V} \quad \text{and} \quad \mathbf{I_x = 10.336 \angle 158^\circ \text{ A}}$$

**P.P.10.15** 
$$C_{eq} = \left(1 + \frac{R_2}{R_1}\right) C = \left(1 + \frac{10 \times 10^6}{10 \times 10^3}\right) (10 \times 10^{-9}) = \mathbf{10 \mu F}$$

**P.P.10.16** If  $R = R_1 = R_2 = 2.5 \text{ k}\Omega$  and  $C = C_1 = C_2 = 1 \text{ nF}$

$$f_o = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(2.5 \times 10^3)(1 \times 10^{-9})} = \mathbf{63.66 \text{ kHz}}$$

**CHAPTER 11**

**P.P.11.1**  $i(t) = 33\sin(10t + 60^\circ) = 33\cos(10t - 30^\circ)$   
 $v(t) = 330\cos(10t + 20^\circ)$

$$p(t) = v(t)i(t) = (330)(33)\cos(10t + 20^\circ)\cos(10t - 30^\circ)$$

$$p(t) = \frac{1}{2} \cdot 10890[\cos(20t + 20^\circ - 30^\circ) + \cos(20 - (-30^\circ))]$$

$$p(t) = (3.5 + 5.445\cos(20t - 10^\circ)) \text{ kW}$$

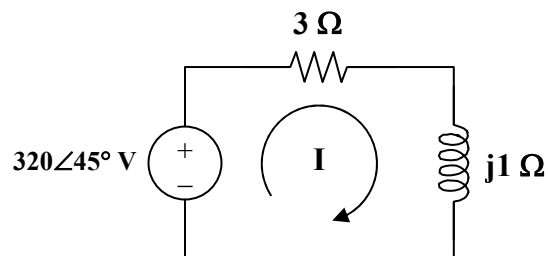
$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = 3.5 \text{ kW}$$

**P.P.11.2**  $V = IZ = 1320\angle 8^\circ$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2} (1320)(33) \cos(8^\circ - 30^\circ) = 20.19 \text{ kW}$$

**P.P.11.3**



$$I = \frac{320\angle 45^\circ}{3 + j} = 101.19\angle 26.57^\circ$$

For the resistor,

$$I_R = I = 101.19\angle 26.57^\circ$$

$$V_R = 3I = 303.6\angle 26.57^\circ$$

$$P_R = \frac{1}{2} V_m I_m = \frac{1}{2} (303.6)(101.19) = 15.361 \text{ kW}$$

For the inductor,

$$\mathbf{I}_L = 101.19 \angle 26.57^\circ$$

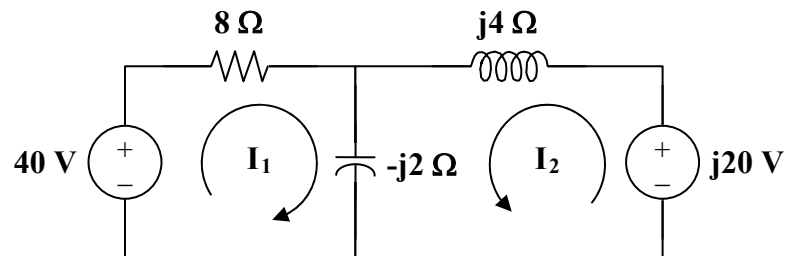
$$\mathbf{V}_L = j\mathbf{I}_L = 101.19 \angle (26.57^\circ + 90^\circ) = 101.19 \angle 116.57^\circ$$

$$P_L = \frac{1}{2}(101.19)^2 \cos(90^\circ) = \mathbf{0 \text{ W}}$$

The average power supplied is

$$P = \frac{1}{2}(320)(101.19) \cos(45^\circ - 26.57^\circ) = \mathbf{15.361 \text{ kW}}$$

**P.P.11.4** Consider the circuit below.



For mesh 1,

$$\begin{aligned} -40 + (8 - j2)\mathbf{I}_1 + (-j2)\mathbf{I}_2 &= 0 \\ (4 - j)\mathbf{I}_1 - j\mathbf{I}_2 &= 20 \end{aligned} \quad (1)$$

For mesh 2,

$$\begin{aligned} -j20 + (j4 - j2)\mathbf{I}_2 + (-j2)\mathbf{I}_1 &= 0 \\ -j\mathbf{I}_1 + j\mathbf{I}_2 &= j10 \end{aligned} \quad (2)$$

In matrix form,

$$\begin{bmatrix} 4 - j & -j \\ -j & j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

$$\Delta = 2 + j4, \quad \Delta_1 = -10 + j20, \quad \Delta_2 = 10 + j60$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = 5 \angle 53.14^\circ \quad \text{and} \quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = 13.6 \angle 17.11^\circ$$

For the 40-V voltage source,

$$\mathbf{V}_s = 40 \angle 0^\circ$$

$$\mathbf{I}_1 = 5 \angle 53.14^\circ$$

$$P_s = \frac{-1}{2}(40)(5) \cos(-53.14^\circ) = \mathbf{-60 \text{ W}}$$

For the j20-V voltage source,

$$\mathbf{V}_s = 20\angle 90^\circ$$

$$\mathbf{I}_2 = 13.6\angle 17.11^\circ$$

$$P_s = \frac{-1}{2}(20)(13.6)\cos(90^\circ - 17.11^\circ) = -40 \text{ W}$$

For the resistor,

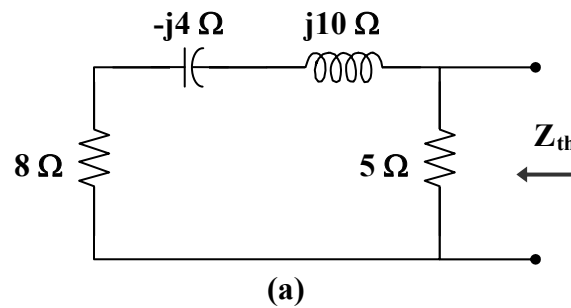
$$I = |\mathbf{I}_1| = 5$$

$$V = 8|\mathbf{I}_1| = 40$$

$$P = \frac{1}{2}(40)(5) = 100 \text{ W}$$

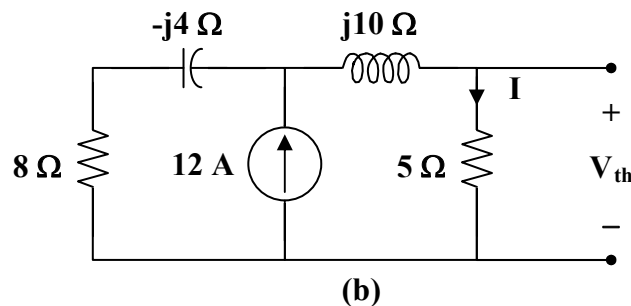
The average power absorbed by the inductor and capacitor is **zero watts**.

**P.P.11.5** We first obtain the Thevenin equivalent circuit across  $\mathbf{Z}_L$ .  $\mathbf{Z}_{Th}$  is obtained from the circuit in Fig. (a).



$$\mathbf{Z}_{Th} = 5 \parallel (8 - j4 + j10) = \frac{(5)(8 + j6)}{13 + j6} = 3.415 + j0.7317$$

$\mathbf{V}_{Th}$  is obtained from the circuit in Fig. (b).



By current division,

$$\mathbf{I} = \frac{8 - j4}{8 - j4 + j10 + 5}(12)$$

$$\mathbf{V}_{Th} = 5\mathbf{I} = \frac{(60)(8 - j4)}{13 + j6} = 37.5 \angle -51.34^\circ$$

$$\mathbf{Z}_L = (\mathbf{Z}_{Th})^* = [3.415 - j0.7317] \Omega$$

$$P_{\max} = \frac{|\mathbf{V}_{Th}|^2}{8R_L} = \frac{(37.5)^2}{(8)(3.415)} = 51.47 \text{ W}$$

**P.P.11.6** We first find  $\mathbf{Z}_{Th}$  and  $\mathbf{V}_{Th}$  across  $R_L$ .

Let  $\mathbf{Z}_1 = 80 + j60$

$$\mathbf{Z}_2 = 90 \parallel (-j30) = \frac{(90)(-j30)}{90 - j30} = 9(1 - j3)$$

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 \parallel \mathbf{Z}_2 = \frac{(80 + j60)(9 - j27)}{80 + j60 + 9 - j27} = 17.181 - j24.57 \Omega$$

$$\mathbf{V}_{Th} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} (120 \angle 60^\circ) = \frac{(9)(1 - j3)}{89 + j33} (120 \angle 60^\circ)$$

$$\mathbf{V}_{Th} = 35.98 \angle -31.91^\circ$$

$$R_L = |\mathbf{Z}_{Th}| = 30 \Omega$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{35.98 \angle -31.91^\circ}{47.181 - j24.57} = 0.6764 \angle -4.4^\circ$$

The maximum average power absorbed by  $R_L$  is

$$P_{\max} = \frac{1}{2} |\mathbf{I}|^2 R_L = \frac{1}{2} (0.6764)^2 (30) = 6.863 \text{ W}$$

**P.P.11.7** 
$$i(t) = \begin{cases} 16t & 0 < t < 1 \\ 32 - 16t & 1 < t < 2 \end{cases} \quad T = 2$$

$$I_{rms}^2 = \frac{1}{T} \int_0^T i^2 dt = \frac{1}{2} \left[ \int_0^1 (16t)^2 dt + \int_1^2 (32 - 16t)^2 dt \right]$$

$$I_{rms}^2 = \frac{256}{2} \left[ \int_0^1 t^2 dt + \int_1^2 (4 - 4t + t^2) dt \right]$$

$$I_{rms}^2 = 128 \left[ \frac{1}{3} + \left( 4t - 2t^2 + \frac{t^3}{3} \right) \Big|_1^2 \right] = \frac{256}{3}$$

$$I_{rms} = \sqrt{\frac{256}{3}} = 9.238 \text{ A}$$

$$P = I_{rms}^2 R = (9.238^2)(9) = 768 \text{ W}$$

**P.P.11.8**  $T = \pi$ ,  $v(t) = 100\sin(t)$ ,  $0 < t < \pi$

$$V_{rms}^2 = \frac{1}{T} \int_0^T v^2 dt = \frac{1}{\pi} \int_0^\pi (100\sin(t))^2 dt$$

$$V_{rms}^2 = \frac{10^4}{\pi} \int_0^\pi \frac{1}{2} [1 - \cos(2t)] dt = 5000$$

$$V_{rms} = 70.71 \text{ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{5000}{6} = 833.3 \text{ W}$$

**P.P.11.9** The load impedance is  
 $Z = 60 + j40 = 72.11 \angle 33.7^\circ \Omega$

The power factor is

$$\text{pf} = \cos(33.7^\circ) = 0.8321 \text{ lagging}$$

Since the load is inductive

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{320 \angle 10^\circ}{72.11 \angle 33.7^\circ} = 4.438 \angle -23.69^\circ \text{ A}$$

The apparent power is

$$\mathbf{S} = \mathbf{V}_{rms}(\mathbf{I}_{rms})^* = 0.5(320)(4.438) \angle (10^\circ - (-23.69^\circ)) = 710 \angle 33.69^\circ \text{ VA}$$

**P.P.11.10** The total impedance as seen by the source is

$$\mathbf{Z} = 10 + j4 \parallel (8 - j6) = 10 + \frac{(j4)(8 - j6)}{8 - j2}$$

$$\mathbf{Z} = 12.69 \angle 20.62^\circ$$

The power factor is

$$\text{pf} = \cos(20.62^\circ) = 0.936 \text{ (lagging)}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{165 \angle 0^\circ}{12.69 \angle 20.62^\circ} = 13.002 \angle -20.62^\circ$$

The average power supplied by the source is equal to the power absorbed by the load.

$$P = I_{rms}^2 R = (13.002)^2 (11.88) = 1,062 \text{ W} = 2.008 \text{ kW}$$

$$\text{or } P = V_{rms} I_{rms} \text{pf} = (165)(13.002)(0.936) = 2.008 \text{ kW}$$

**P.P.11.11**

$$(a) \quad \mathbf{S} = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^* = (110 \angle 85^\circ)(0.4 \angle -15^\circ)$$

$$\mathbf{S} = 44 \angle 70^\circ \text{ VA}$$

$$S = |\mathbf{S}| = 44 \text{ VA}$$

$$(b) \quad \mathbf{S} = 44 \angle 70^\circ = 15.05 + j41.35$$

$$P = 15.05 \text{ W}, \quad Q = 41.35 \text{ VAR}$$

$$(c) \quad \text{pf} = \cos(70^\circ) = 0.342 \quad (\text{lagging})$$

$$\mathbf{Z} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{110 \angle 85^\circ}{0.4 \angle -15^\circ} = 275 \angle 70^\circ$$

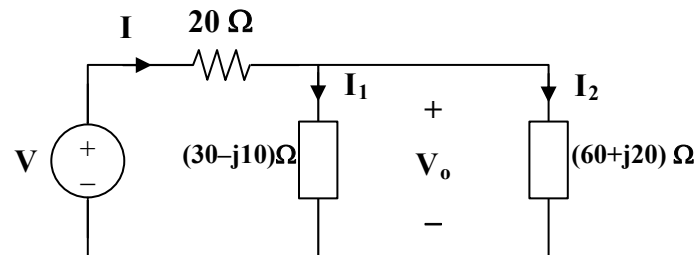
$$\mathbf{Z} = 94.06 + j258.4 \Omega$$

**P.P.11.12**

$$(a) \quad \text{If } \mathbf{Z} = 250 \angle -75^\circ, \quad \text{pf} = \cos(-75^\circ) = 0.2588 \quad (\text{leading})$$

$$(b) \quad Q = S \sin \theta \longrightarrow S = \frac{Q}{\sin \theta} = \frac{-100 \text{ kVAR}}{\sin(-75^\circ)} = 103.53 \text{ kVA}$$

$$(c) \quad S = \frac{V_{\text{rms}}^2}{|\mathbf{Z}|} \longrightarrow V_{\text{rms}} = \sqrt{S \cdot |\mathbf{Z}|} = \sqrt{(103530)(250)} = 5.087 \text{ kV}$$

**P.P.11.13** Consider the circuit below.

Let  $\mathbf{I}_2$  be the current through the  $60\text{-}\Omega$  resistor.

$$P = I_2^2 R \longrightarrow I_2^2 = \frac{P}{R} = \frac{240}{60} = 4$$

$$\mathbf{I}_2 = 2 \text{ (rms)}$$

$$\mathbf{V}_o = \mathbf{I}_2 (60 + j20) = 120 + j40$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{30 - j10} = 3.2 + j2.4$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 5.2 + j2.4$$

$$\mathbf{V} = 20\mathbf{I} + \mathbf{V}_o = (104 + j48) + (120 + j40)$$

$$\mathbf{V} = 224 + j88 = \mathbf{240.7} \angle \mathbf{21.45^\circ} \mathbf{V}_{\text{rms}}$$

For the 20- $\Omega$  resistor,

$$\mathbf{V} = 20\mathbf{I} = 204 + j48 = 114.54 \angle 24.8^\circ$$

$$\mathbf{I} = 5.2 + j2.4 = 5.727 \angle 24.8^\circ$$

$$\mathbf{S} = \mathbf{VI}^* = (114.54 \angle 24.8^\circ)(5.727 \angle -24.8^\circ)$$

$$\mathbf{S} = \mathbf{656 VA}$$

For the (30 - j10)- $\Omega$  impedance,

$$\mathbf{V}_o = 120 + j40 = 126.5 \angle 18.43^\circ$$

$$\mathbf{I}_1 = 3.2 + j2.4 = 4 \angle 36.87^\circ$$

$$\mathbf{S}_1 = \mathbf{V}_o \mathbf{I}_1^* = (126.5 \angle 18.43^\circ)(4 \angle -36.87^\circ)$$

$$\mathbf{S}_1 = 506 \angle -18.44^\circ = \mathbf{[480 - j160] VA}$$

For the (60 + j20)- $\Omega$  impedance,

$$\mathbf{I}_2 = 2 \angle 0^\circ$$

$$\mathbf{S}_2 = \mathbf{V}_o \mathbf{I}_2^* = (126.5 \angle 18.43^\circ)(2 \angle -0^\circ)$$

$$\mathbf{S}_2 = 253 \angle 18.43^\circ = \mathbf{[240 + j80] VA}$$

The overall complex power supplied by the source is

$$\mathbf{S}_T = \mathbf{VI}^* = (240.67 \angle 21.45^\circ)(5.727 \angle -24.8^\circ)$$

$$\mathbf{S_T} = 1378.3\angle-3.35^\circ = \mathbf{[1376-j80] VA}$$

**P.P.11.14**

For load 1,

$$P_1 = 2000, \quad \text{pf} = 0.75 = \cos \theta_1 \longrightarrow \theta_1 = -41.41^\circ$$

$$P_1 = S_1 \cos \theta_1 \longrightarrow S_1 = \frac{P_1}{\cos \theta_1} = 2666.67$$

$$Q_1 = S_1 \sin \theta_1 = -176.85$$

$$S_1 = P_1 + jQ_1 = 2000 - j1763.85 \quad (\text{leading})$$

For load 2,

$$P_2 = 4000, \quad \text{pf} = 0.95 = \cos \theta_2 \longrightarrow \theta_2 = 18.19^\circ$$

$$S_2 = \frac{P_2}{\cos \theta_2} = 4210.53$$

$$Q_2 = S_2 \sin \theta_2 = 1314.4$$

$$S_2 = P_2 + jQ_2 = 4000 + j1314.4 \quad (\text{lagging})$$

The total complex power is

$$S = S_1 + S_2 = [6 - j0.4495] \text{ kVA}$$

$$\text{pf} = \frac{P}{|S|} = \frac{6000}{6016.18} = \mathbf{0.9972} \quad (\text{leading})$$

**P.P.11.15**  $\text{pf} = 0.85 = \cos \theta \longrightarrow \theta = 31.79^\circ$

$$Q = S \sin \theta \longrightarrow S = \frac{Q}{\sin \theta} = \frac{140}{\sin(31.79^\circ)} = 265.8 \text{ kVA}$$

$$P = S \cos \theta = 225.93 \text{ kW}$$

For  $\text{pf} = 1 = \cos \theta_1 \longrightarrow \theta_1 = 0^\circ$

Since P remains the same,

$$P = P_1 = S_1 \cos \theta_1 \longrightarrow S_1 = \frac{P_1}{\cos \theta_1} = 225.93$$

$$Q_1 = S_1 \sin \theta_1 = 0$$

The difference between the new  $Q_1$  and the old Q is  $Q_c$ .

$$Q_c = 140 \text{ kVAR} = \omega C V_{\text{rms}}^2$$

$$C = \frac{140 \times 10^3}{(2\pi)(60)(110)^2} = \mathbf{30.69 \text{ mF}}$$

**P.P.11.16** The wattmeter measures the average power from the source.

Let  $\mathbf{Z}_1 = 4 - j2$

$$\mathbf{Z}_2 = 12 \parallel j9 = \frac{(12)(j9)}{12 + j9} = 4.32 + j5.76$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = 8.32 + j3.76 = 9.13 \angle 24.32^\circ$$

$$\mathbf{S} = \mathbf{VI}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(120)^2}{9.13 \angle -24.32^\circ} = 1577.2 \angle 24.32^\circ \text{ VA}$$

$$P = |\mathbf{S}| \cos \theta = \mathbf{1.437 \text{ kW}}$$

**P.P.11.17** Demand charge =  $\$5 \times 32,000 = \$160,000$

Energy charge for the first 50,000 kWh =  $\$0.08 \times 50,000 = \$4,000$

The remaining energy =  $500,000 - 50,000 = 450,000 \text{ kWh}$

Charge for this bill =  $\$0.05 \times 450,000 = \$22,500$

Total bill =  $\$160,000 + \$4,000 + \$22,500 = \mathbf{\$186,500}$

**P.P.11.18** Energy consumed =  $800 \text{ kW} \times 20 \times 26 = 416,000 \text{ kWh}$

The power factor of 0.88 exceeds 0.85 by  $3 \times 0.01$ . Hence, there is a power factor credit which amounts to an energy credit of

$$416,000 \times \frac{0.1}{100} \times 3 = 1248 \text{ kWh}$$

Total energy billed =  $416,000 - 1,248 = 414,752 \text{ kWh}$

Energy cost =  $\$0.06 \times 414,752 = \mathbf{\$24,885.12}$

**CHAPTER 12**

**P.P.12.1** For the abc sequence,  $\mathbf{V}_{an}$  leads  $\mathbf{V}_{bn}$  by  $120^\circ$  and  $\mathbf{V}_{bn}$  leads  $\mathbf{V}_{cn}$  by  $120^\circ$ .

Hence,

$$\mathbf{V}_{an} = 110\angle(30^\circ + 120^\circ) = \mathbf{110\angle150^\circ V}$$

$$\mathbf{V}_{cn} = 110\angle(30^\circ - 120^\circ) = \mathbf{110\angle-90^\circ V}$$

**P.P.12.2**

(a)  $\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle30^\circ - 120\angle-90^\circ$   
 $\mathbf{V}_{ab} = (103.92 + j60) + j120$   
 $\mathbf{V}_{ab} = \mathbf{207.8\angle60^\circ V}$

Alternatively, using the fact that  $\mathbf{V}_{ab}$  leads  $\mathbf{V}_{an}$  by  $30^\circ$  and has a magnitude of  $\sqrt{3}$  times that of  $\mathbf{V}_{an}$ ,

$$\mathbf{V}_{ab} = \sqrt{3}(120)\angle(30^\circ + 30^\circ) = \mathbf{207.8\angle60^\circ V}$$

Following the abc sequence,

$$\mathbf{V}_{bc} = \mathbf{207.8\angle-60^\circ V}$$

$$\mathbf{V}_{ca} = \mathbf{207.8\angle\pm180^\circ V}$$

(b)  $\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}}$

$$\mathbf{Z} = (0.4 + j0.3) + (24 + j19) + (0.6 + j0.7)$$

$$\mathbf{Z} = 25 + j20 = 32\angle38.66^\circ$$

$$\mathbf{I}_a = \frac{120\angle30^\circ}{32\angle38.66^\circ} = \mathbf{3.75\angle-8.66^\circ A}$$

Following the abc sequence,

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \mathbf{3.75\angle-128.66^\circ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle -240^\circ = \mathbf{3.75\angle111.34^\circ A}$$

**P.P.12.3**

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{120\angle -20^{\circ}}{20\angle 40^{\circ}} = \mathbf{6\angle -60^{\circ} A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^{\circ} = \mathbf{6\angle 180^{\circ} A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120^{\circ} = \mathbf{6\angle 60^{\circ} A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^{\circ} = 6\sqrt{3}\angle -90^{\circ} = \mathbf{10.392\angle -90^{\circ} A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^{\circ} = \mathbf{10.392\angle 150^{\circ} A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^{\circ} = \mathbf{10.392\angle 30^{\circ} A}$$

**P.P.12.4** In a delta load, the phase current leads the line current by  $30^{\circ}$  and has a magnitude  $\frac{1}{\sqrt{3}}$  times that of the line current. Hence,

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_a}{\sqrt{3}}\angle 30^{\circ} = \frac{9.609}{\sqrt{3}}\angle 65^{\circ} = \mathbf{5.548\angle 65^{\circ} A}$$

$$\mathbf{Z}_{\Delta} = 18 + j12 = 21.63\angle 33.69^{\circ} \Omega$$

$$\mathbf{V}_{AB} = \mathbf{I}_{AB}\mathbf{Z}_{\Delta} = (5.548\angle 65^{\circ})(21.63\angle 33.69^{\circ})$$

$$\mathbf{V}_{AB} = \mathbf{120\angle 98.69^{\circ} V}$$

**P.P.12.5**  $\mathbf{Z}_Y = 12 + j15 = 19.21\angle 51.34^{\circ}$

After converting the  $\Delta$ -connected source to a Y-connected source,

$$\mathbf{V}_{an} = \frac{240}{\sqrt{3}}\angle (150^{\circ} - 30^{\circ}) = 138.56\angle -15^{\circ}$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{138.56\angle -15^{\circ}}{19.21\angle 51.34^{\circ}} = \mathbf{7.21\angle -66.34^{\circ} A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^{\circ} = \mathbf{7.21\angle 173.66^{\circ} A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^{\circ} = \mathbf{7.21\angle 53.66^{\circ} A}$$

**P.P.12.6**

For the source,

$$\mathbf{S} = 3 \mathbf{V}_p \mathbf{I}_p^* = (3)(120 \angle 30^\circ)(3.75 \angle 8.66^\circ)$$

$$\mathbf{S} = -1350 \angle 38.66^\circ = [-1.054.2 - j0.8433] \text{ kVA}$$

For the load,

$$\mathbf{S} = 3 |\mathbf{I}_p|^2 \mathbf{Z}$$

where

$$\mathbf{Z} = 24 + j19 = 30.61 \angle 38.37^\circ$$

$$\mathbf{I}_p = 3.75 \angle -8.66^\circ$$

$$\mathbf{S} = (3)(3.75)^2 (30.61 \angle 38.37^\circ)$$

$$\mathbf{S} = 1291.36 \angle 38.37^\circ = [1.012 + j0.8016] \text{ kVA}$$

**P.P.12.7**

$$P = S \cos \theta \longrightarrow S = \frac{P}{\cos \theta} = \frac{30 \times 10^3}{0.85} = 35.29 \text{ kVA}$$

$$S = \sqrt{3} V_L I_L \longrightarrow I_L = \frac{S}{\sqrt{3} V_L} = \frac{35.29 \times 10^3}{\sqrt{3} (440)} = 46.31 \text{ A}$$

Alternatively,

$$P_p = \frac{30 \times 10^3}{3} = 10 \text{ kW}, \quad V_p = \frac{440}{\sqrt{3}} \text{ V}$$

$$P_p = V_p I_p \cos \theta$$

$$I_p = \frac{P_p}{V_p \cos \theta} = \frac{(10 \times 10^3) \sqrt{3}}{(440)(0.85)} = 46.31 \text{ A}$$

**P.P.12.8**

(a) For load 1,

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{840}{\sqrt{3}}$$

$$\mathbf{I}_{a1} = \frac{\mathbf{V}_a}{\mathbf{Z}_p} = \frac{840 \angle 0^\circ}{\sqrt{3}} \cdot \frac{1}{30 + j40} = 9.7 \angle -53.13^\circ$$

$$\mathbf{S}_1 = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \frac{(840)^2}{50 \angle -53.15^\circ} = 14.112 \angle 53.13^\circ \text{ kVA}$$

For load 2,

$$S_2 = \frac{P_2}{\cos \theta_2} = \frac{48}{0.8} = 60 \text{ kVA}$$

$$Q_2 = S_2 \sin \theta_2 = (60)(0.6) = 36 \text{ kVAR}$$

$$S_2 = 48 + j36 \text{ kVA}$$

$$S = S_1 + S_2 = [56.47 + j47.29] \text{ kVA}$$

$$S = 73.65 \angle 39.94^\circ \text{ kVA}$$

$$\text{with pf} = \cos(39.94^\circ) = 0.7667$$

$$(b) \quad Q_c = P(\tan \theta_{\text{old}} - \tan \theta_{\text{new}})$$
$$Q_c = (56.47)(\tan 39.94^\circ - \tan 0^\circ) = 47.29 \text{ kVAR}$$

For each capacitor, the rating is **15.76 kVAR**

$$(c) \quad \text{At unity pf, } S = P = 56.47 \text{ kVA}$$

$$I_L = \frac{S}{\sqrt{3} V_L} = \frac{56470}{\sqrt{3}(840)} = \mathbf{38.81 \text{ A}}$$

### P.P.12.9

The phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{440 \angle 0^\circ}{10 - j5} = 39.35 \angle 26.56^\circ = 35.2 + j17.595$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{440 \angle -120^\circ}{16} = 27.5 \angle -120^\circ = -13.75 - j23.82$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{440 \angle 120^\circ}{8 + j6} = 44 \angle 83.13^\circ = 5.263 + j43.68$$

The line currents are

$$I_a = I_{AB} - I_{CA} = (35.2 + j17.595) - (5.263 + j43.68)$$
$$= 29.94 - j26.08 = \mathbf{39.71 \angle -41.06^\circ \text{ A}}$$

$$I_b = I_{BC} - I_{AB} = -48.95 - j41.42 = \mathbf{64.12 \angle -139.8^\circ \text{ A}}$$

$$I_c = I_{CA} - I_{BC} = 19.013 + j67.5 = \mathbf{70.13 \angle 74.27^\circ \text{ A}}$$

**P.P.12.10**

The phase currents are

$$\mathbf{I}_{AB} = \frac{220\angle 0^\circ}{-j5} = j44$$

$$\mathbf{I}_{BC} = \frac{220\angle 0^\circ}{j10} = 22\angle 30^\circ$$

$$\mathbf{I}_{CA} = \frac{220\angle 120^\circ}{10} = 22\angle -120^\circ$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = (j44) - (-11 - j19.05)$$

$$\mathbf{I}_a = 11 + j63.05 = \mathbf{64\angle 80.1^\circ A}$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = (19.05 + j11) - (j44)$$

$$\mathbf{I}_b = 19.05 - j33 = \mathbf{38.1\angle -60^\circ A}$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = (-11 - j19.05) - (19.05 + j11)$$

$$\mathbf{I}_c = -30.05 - j30.05 = \mathbf{42.5\angle 225^\circ A}$$

The real power is absorbed by the resistive load

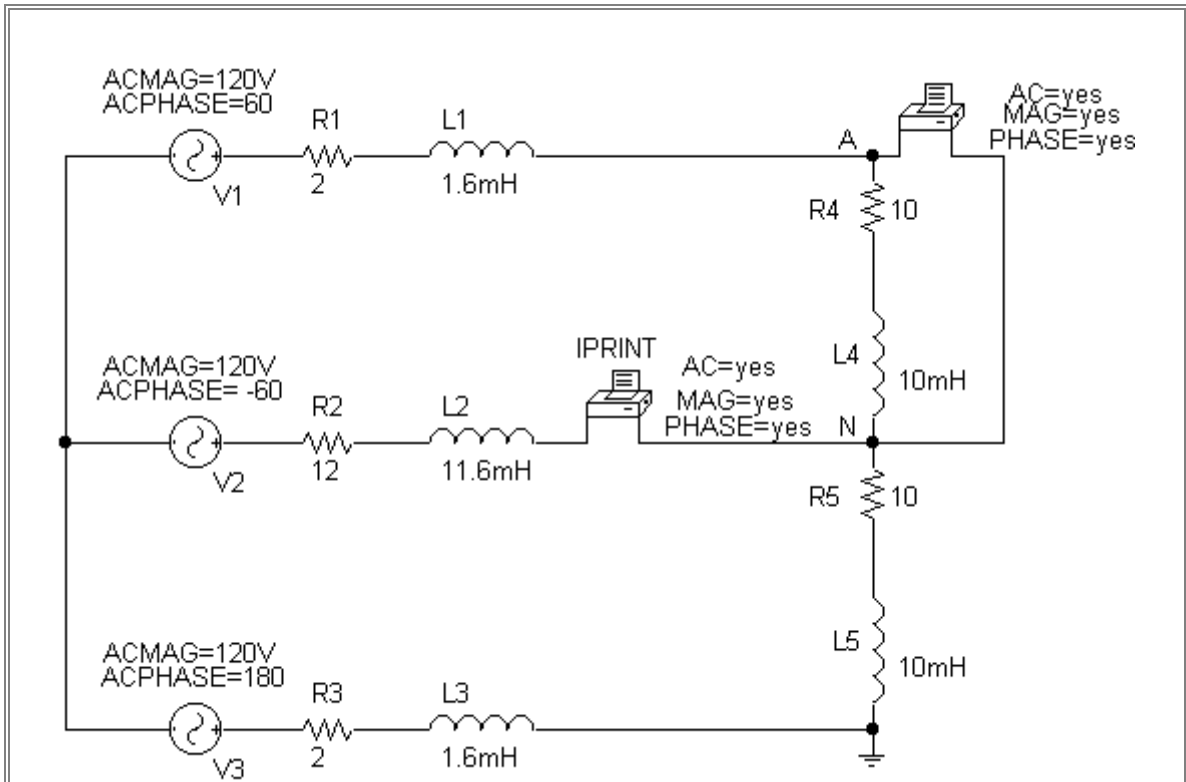
$$P = |\mathbf{I}_{CA}|^2 (10) = (22)^2 (10) = \mathbf{4.84 kW}$$

**P.P.12.11** The schematic is shown below. First, use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 1, *Start Freq* = 100, and *End Freq* = 100. Once the circuit is saved and simulated, we obtain an output file whose contents include the following results.

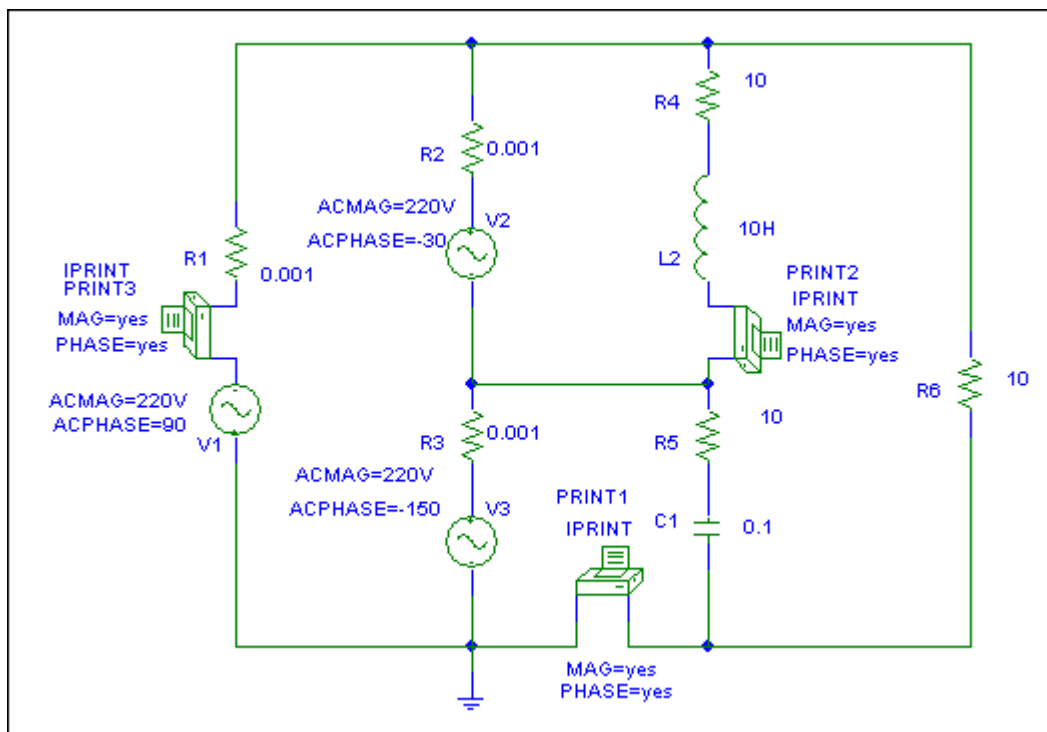
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000E+02	8.547E+00	-9.127E+01
FREQ	VM(A,N)	VP(A,N)
1.000E+02	1.009E+02	6.087E+01

From this we obtain,

$$\mathbf{V}_{an} = \mathbf{100.9\angle 60.87^\circ V}, \quad \mathbf{I}_{bB} = \mathbf{8.547\angle -91.27^\circ A}$$



**P.P.12.12** The schematic is shown below.



In this case, we may assume that  $\omega = 1 \text{ rad/s}$ , so that  $f = 1/2\pi = 0.1592 \text{ Hz}$ . Hence,  $L = X_L/\omega = 10$  and  $C = 1/\omega X_c = 0.1$ .

Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 1, *Start Freq* = 0.1592, and *End Freq* = 0.1592. Once the circuit is saved and simulated, we obtain an output file whose contents include the following results.

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	3.724E+01	8.379E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.555E+01	-7.501E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	2.468E+01	-9.000E+01

From this we obtain,

$$\mathbf{I_{ca} = 24.68 \angle -90^\circ \text{ A} \quad I_{cC} = 37.25 \angle 83.79^\circ \text{ A} \quad I_{AB} = 15.55 \angle -75.01^\circ \text{ A}}$$

### P.P.12.13

(a) If point o is connected to point B,  $P_2 = \mathbf{0 \text{ W}}$

$$P_1 = \text{Re}(\mathbf{V_{AB} I_a^*})$$

$$P_1 = (440)(39.71) \cos(0^\circ + 41.06^\circ) = \mathbf{13.175 \text{ kW}}$$

$$P_3 = \text{Re}(\mathbf{V_{CB} I_c^*})$$

$$\text{where } \mathbf{V_{CB} = -V_{BC} = 240 \angle (-120^\circ + 180^\circ) = 240 \angle 60^\circ}$$

$$P_3 = (440)(70.13) \cos(60^\circ - 74.27^\circ) = \mathbf{29.91 \text{ kW}}$$

(b) Total power is =  $(13.175 + 29.91) \text{ kW} = \mathbf{43.08 \text{ kW}}$ .

**P.P.12.14**  $V_L = 208 \text{ V}$  ,  $P_1 = -560 \text{ W}$  ,  $P_2 = 800 \text{ W}$

(a)  $P_T = P_1 + P_2 = -560 + 800 = \mathbf{240 \text{ W}}$

(b)  $Q_T = \sqrt{3} (P_2 - P_1) = \sqrt{3} (800 + 560) = \mathbf{2.356 \text{ kVAR}}$

(c)  $\tan \theta = \frac{Q_T}{P_T} = \frac{2355.6}{240} = 9.815 \longrightarrow \theta = 84.18^\circ$

pf =  $\cos \theta = \mathbf{0.1014}$  (**lagging / inductive**)

It is inductive because  $P_2 > P_1$

(d) For a Y-connected load,

$$I_p = I_L, \quad V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

$$P_p = V_p I_p \cos \theta \longrightarrow I_p = \frac{80}{(120)(0.1014)} = 6.575 \text{ A}$$

$$Z_p = \frac{V_p}{I_p} = \frac{120}{6.575} = 18.25$$

$$\mathbf{Z_p = Z_p \angle \theta = 18.25 \angle 84.18^\circ \Omega}$$

The impedance is **inductive**.

**P.P.12.15**      $\mathbf{Z_\Delta = 30 - j40 = 50 \angle -53.13^\circ}$

The equivalent Y-connected load is

$$\mathbf{Z_Y = \frac{Z_\Delta}{3} = 16.67 \angle -53.13^\circ}$$

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_L = \frac{V_p}{|Z_Y|} = \frac{254}{16.67} = 15.24$$

$$P_1 = V_L I_L \cos(\theta + 30^\circ)$$

$$P_1 = (440)(15.24) \cos(-53.13^\circ + 30^\circ) = \mathbf{6.167 \text{ kW}}$$

$$P_2 = V_L I_L \cos(\theta - 30^\circ)$$

$$P_2 = (440)(15.24) \cos(-53.13^\circ - 30^\circ) = \mathbf{802.1 \text{ W}}$$

$$P_T = P_1 + P_2 = \mathbf{6.969 \text{ kW}}$$

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(802.1 - 6167)$$

$$Q_T = \mathbf{-9.292 \text{ kVAR}}$$

**CHAPTER 13****P.P. 13.1** For mesh 1,

$$141.42 + j141.42 = 4(1 + j2)\mathbf{I}_1 + j\mathbf{I}_2 \quad (1)$$

For mesh 2,

$$0 = j\mathbf{I}_1 + (10 + j5)\mathbf{I}_2 \quad (2)$$

For the matrix form 
$$\begin{bmatrix} 141.42 + j141.42 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j8 & j \\ j & 10 + j5 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = j100, \Delta_2 = 141.42 - j141.42$$

$$\mathbf{I}_2 = \Delta_2 / \Delta = (141.42 - j141.42) / j100$$

$$\mathbf{V}_o = 10\mathbf{I}_2 = 10(-1.4142 - j1.4142) = \mathbf{20\angle-135^\circ V}$$

**P.P. 13.2** Since  $\mathbf{I}_1$  enters the coil with reactance  $2\Omega$  and  $\mathbf{I}_2$  enters the coil with reactance  $6\Omega$ , the mutual voltage is positive. Hence, for mesh 1,

$$100\angle 60^\circ = (5 + j2 + j6 - j3 \times 2)\mathbf{I}_1 - j6\mathbf{I}_2 + j3\mathbf{I}_2$$

or 
$$100\angle 60^\circ = (5 + j2)\mathbf{I}_1 - j3\mathbf{I}_2 \quad (1)$$

For mesh 2, 
$$0 = (j6 - j4)\mathbf{I}_2 - j6\mathbf{I}_1 + j3\mathbf{I}_1$$

or 
$$\mathbf{I}_2 = 1.5\mathbf{I}_1 \quad (2)$$

Substituting this into (1), 
$$100\angle 60^\circ = (5 - j2.5)\mathbf{I}_1$$

$$\mathbf{I}_1 = (100\angle 60^\circ) / (5.59\angle -26.57^\circ) = \mathbf{17.889\angle 86.57^\circ A}$$

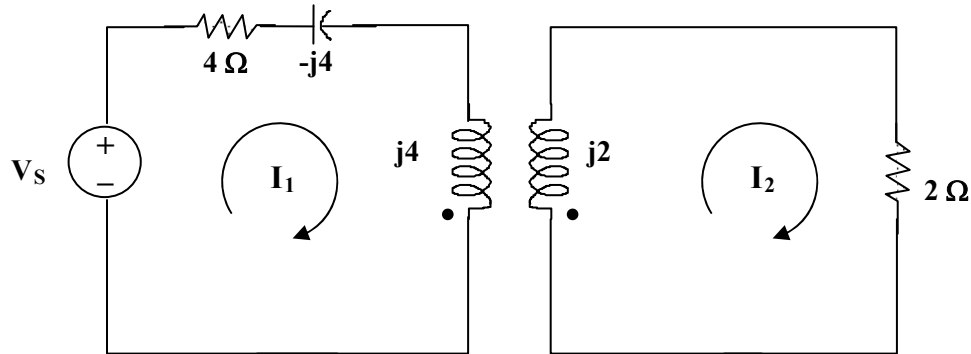
$$\mathbf{I}_2 = 1.5\mathbf{I}_1 = \mathbf{26.83\angle 86.57^\circ A}$$

**P.P. 13.3** The coupling coefficient is,  $k = m / \sqrt{L_1 L_2} = 1 / \sqrt{2 \times 1} = \mathbf{0.7071}$ 

To obtain the energy stored, we first obtain the frequency-domain circuit shown below.

$$100\cos(\omega t) \text{ becomes } 100\angle 0^\circ, \omega = 2$$

$1\text{H}$  becomes  $j\omega 1 = j2$   
 $2\text{H}$  becomes  $j\omega 2 = j4$   
 $(1/8)\text{F}$  becomes  $1/j\omega C = -j4$



For mesh 1,  $100 = (4 - j4 + j4)\mathbf{I}_1 - j2\mathbf{I}_2$   
 or  $50 = 2\mathbf{I}_1 - j\mathbf{I}_2$  (1)

For mesh 2,  $-j2\mathbf{I}_1 + (2 + j2)\mathbf{I}_2 = 0$   
 or  $\mathbf{I}_1 = (1 - j)\mathbf{I}_2$  (2)

Substituting (2) into (1),  $(2 - j3)\mathbf{I}_2 = 50$

$$\mathbf{I}_2 = 50/(2 - j3) = 13.87\angle 56.31^\circ$$

$$\mathbf{I}_1 = 19.658\angle 11.31^\circ$$

In the time domain,  
 $i_1 = 19.658\cos(2t + 11.31^\circ)$   
 $i_2 = 13.87\cos(2t + 56.31^\circ)$

At  $t = 1.5$ ,  $2t = 3 \text{ rad} = 171.9^\circ$

$$i_1 = 19.658\cos(171.9^\circ + 11.31^\circ) = -19.62 \text{ A}$$

$$i_2 = 13.87\cos(171.9^\circ + 56.31^\circ) = -9.25 \text{ A}$$

The total energy stored in the coupled inductors is given by,

$$\begin{aligned}
 W &= 0.5L_1(i_1)^2 + 0.5L_2(i_2)^2 - 0.5M(i_1i_2) \\
 &= 0.5(2)(-19.62)^2 + 0.5(1)(-9.25)^2 - (1)(-19.62)(-9.25) \\
 &= \mathbf{246.2 \text{ J}}
 \end{aligned}$$

**P.P. 13.4**

$$\begin{aligned} \mathbf{Z}_{in} &= 4 + j8 + [3^2/(j10 - j6 + 6 + j4)] \\ &= 4 + j8 + 9/(6 + j8) \\ &= \mathbf{8.58\angle 58.05^\circ \Omega} \end{aligned}$$

The current from the voltage is,

$$\mathbf{I} = \mathbf{V/Z} = 40\angle 0^\circ / 8.58\angle 58.05^\circ = \mathbf{4.662\angle -58.05^\circ \text{ A}}$$

**P.P. 13.5**

$$L_1 = 10, L_2 = 4, M = 2$$

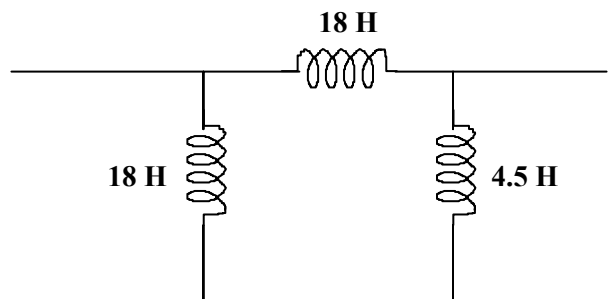
$$L_1 L_2 - M^2 = 40 - 4 = 36$$

$$L_A = (L_1 L_2 - M^2)/(L_2 - M) = 36/(4 - 2) = \mathbf{18 \text{ H}}$$

$$L_B = (L_1 L_2 - M^2)/(L_1 - M) = 36/(10 - 2) = \mathbf{4.5 \text{ H}}$$

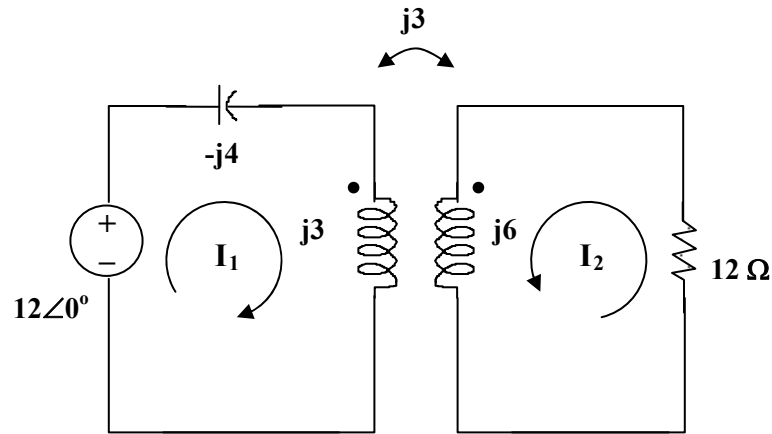
$$L_C = (L_1 L_2 - M^2)/M = 36/2 = \mathbf{18 \text{ H}}$$

Hence, we get the  $\pi$  equivalent circuit as shown below.



**P.P. 13.6**

If we reverse the direction of  $I_2$  so that we replace  $I_2$  by  $-I_2$ , we have the circuit shown in Figure (a).



(a)

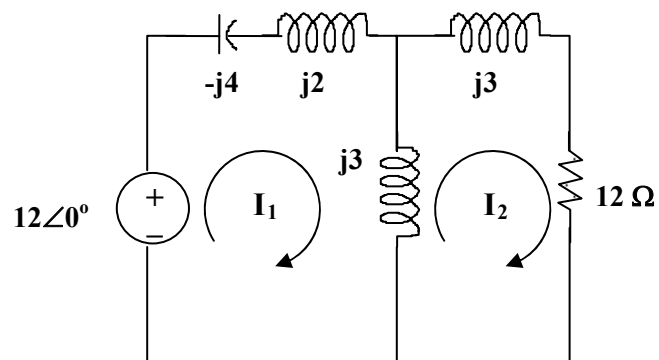
We now replace the coupled coil by the T-equivalent circuit and assume  $\omega = 1$ .

$$L_a = 5 - 3 = 2 \text{ H}$$

$$L_b = 6 - 3 = 3 \text{ H}$$

$$L_c = 3 \text{ H}$$

Hence the equivalent circuit is shown in Figure (b). We apply mesh analysis.



(b)

$$12 = i_1(-j4 + j2 + j3) + j3i_2$$

$$\text{or } 12 = ji_1 + j3i_2 \quad (1)$$

Loop 2 produces,

$$0 = j3i_1 + (j3 + j3 + 12)i_2$$

$$\text{or } i_1 = (-2 + j4)i_2 \quad (2)$$

Substituting (2) into (1),  $12 = (-4 + j)i_2$ , which leads to  $i_2 = 12/(-4 + j)$

$$I_2 = -i_2 = 12/(4 - j) = \mathbf{2.91\angle 14.04^\circ \text{ A}}$$

$$I_1 = i_1 = (-2 + j4)i_2 = 12(2 - j4)/(4 - j) = \mathbf{13\angle -49.4^\circ \text{ A}}$$

### P.P. 13.7

(a)  $n = V_2/V_1 = 110/2200 = \mathbf{1/20}$  (a step-down transformer)

(b)  $S = V_1 I_1 = 2200 \times 5 = \mathbf{11 \text{ kVA}}$

(c)  $I_2 = I_1/n = 5/(1/20) = \mathbf{100 \text{ A}}$

### P.P. 13.8

resulting in

The  $16 - j24$ -ohm impedance can be reflected to the primary

$$Z_{in} = 2 + (16 - j24)/16 = 3 - j1.5$$

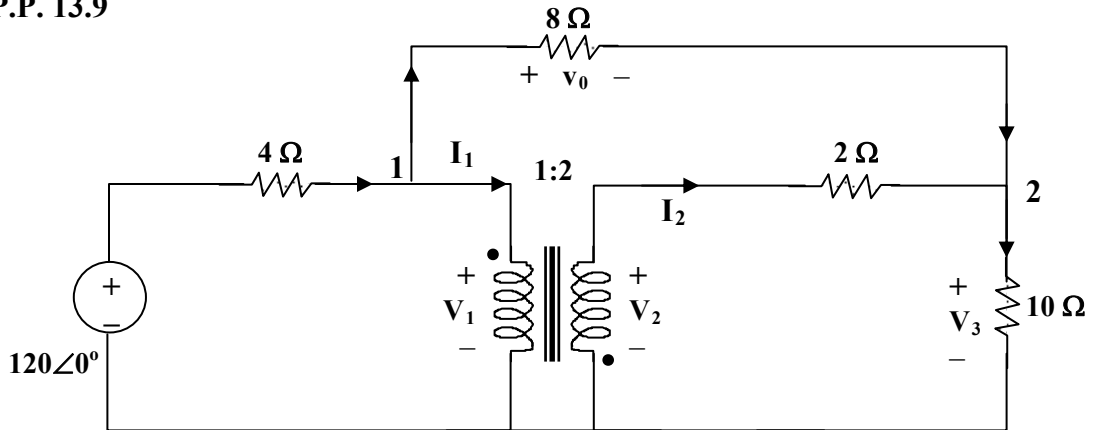
$$I_1 = 240/(3 - j1.5) = 240/(3.354\angle -26.57^\circ) = 71.56\angle 26.57^\circ$$

$$I_2 = -I_1/n = -17.89\angle 26.57^\circ$$

$$V_o = -j24i_2 = (24\angle -90^\circ)(-17.89\angle 26.57^\circ) = \mathbf{429.4\angle 116.57^\circ \text{ V}}$$

$$S_1 = V_1 I_1 = (240)(71.56\angle 26.57^\circ) = \mathbf{17.174\angle -26.57^\circ \text{ kVA.}}$$

**P.P. 13.9**



Consider the circuit shown above.

$$\text{At node 1,} \quad (120 - V_1)/4 = I_1 + (V_1 - V_3)/8 \quad (1)$$

$$\text{At node 2,} \quad [(V_1 - V_3)/8] + [(V_2 - V_3)/2] = (V_3)/8 \quad (2)$$

$$\text{At the transformer terminals,} \quad V_2 = -2V_1 \text{ and } I_2 = -I_1/2 \quad (3)$$

$$\text{But } I_2 = (V_2 - V_3)/2 = -I_1/2 \text{ which leads to } I_1 = (V_3 - V_2)/1 = V_3 + 2V_1.$$

Substituting all of this into (1) and (2) leads to,

$$(120 - V_1)/4 = V_3 + 2V_1 + (V_1 - V_3)/8 \text{ which leads } 240 = 19V_1 + 7V_3 \quad (4)$$

$$[(V_1 - V_3)/8] + [(-2V_1 - V_3)/2] = V_3/8 \text{ which leads to} \\ V_3 = -7V_1/6 \quad (5)$$

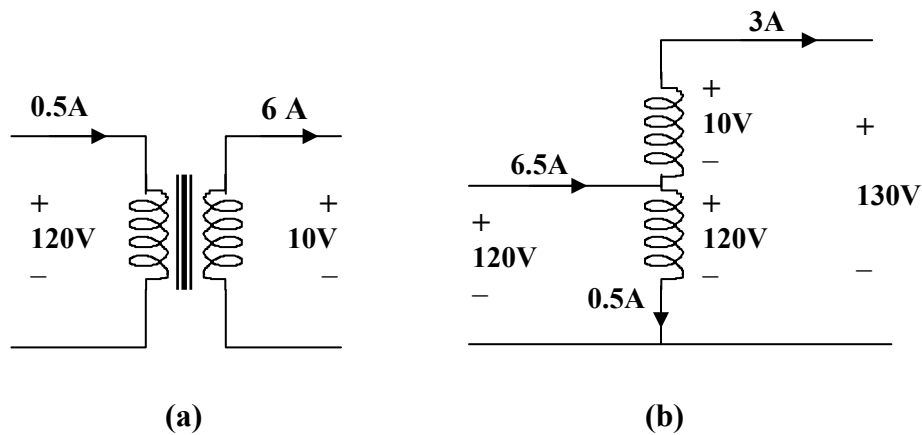
From (4) and (5),

$$240 = 10.833V_1 \text{ or } V_1 = 22.155 \text{ volts}$$

$$V_3 = -7V_1/6 = -25.85 \text{ volts}$$

$$V_o = V_1 - V_3 = \mathbf{48 \text{ volts}}$$

**P.P. 13.10** We should note that the current and voltage of each winding of the autotransformer in Figure (b) are the same for the two-winding transformer in Figure (a).



For the two-winding transformer,

$$S_1 = 120 \times 0.5 = 60 \text{ VA}$$

$$S_2 = 6(10) = 60 \text{ VA}$$

For the autotransformer,

$$S_1 = 120(6.5) = 780 \text{ VA}$$

$$S_2 = 130(6) = 780 \text{ VA}$$

**P.P. 13.11**  $(I_2)^* = S_2/V_2 = 16,000/1000 = 16 \text{ A}$

Since  $S_1 = V_1(I_1)^* = V_2(I_2)^* = S_2$ ,  $V_2/V_1 = I_1/I_2$ ,  $1000/2500 = I_1/32$ ,

or  $I_1 = 1000 \times 16 / 2500 = 6.4 \text{ A}$ .

At the top, KCL produces  $I_1 + I_o = I_2$ , or  $I_o = I_2 - I_1 = 16 - 6.4 = 9.6 \text{ A}$ .

**P.P. 13.12**

(a)  $S_T = (\sqrt{3})V_L I_L$ , but  $S_T = P_T/\cos\theta = 40 \times 10^6 / 0.85 = 47.0588 \text{ MVA}$

$$I_{LS} = S_T / (\sqrt{3})V_{LS} = 47.0588 \times 10^6 / [(\sqrt{3})12.5 \times 10^3] = \mathbf{2.174 \text{ kA}}$$

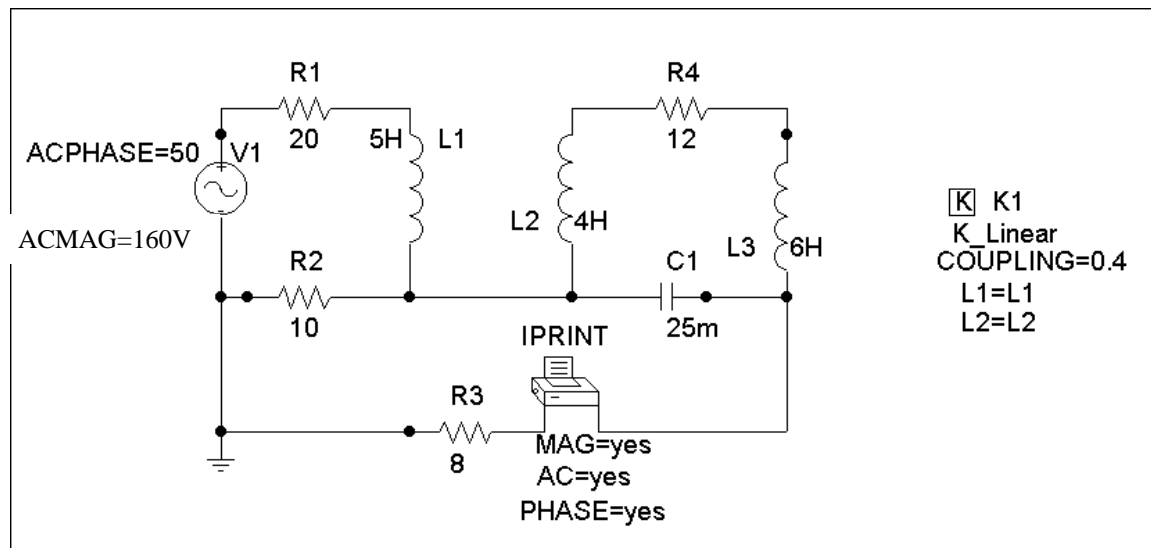
(b)  $V_{LS} = 12.5 \text{ kV}$ ,  $V_{LP} = 625 \text{ kV}$ ,  $n = V_{LS}/V_{LP} = 12.5/625 = \mathbf{0.02}$

(c)  $I_{LP} = nI_{LS} = 0.02 \times 2173.6 = \mathbf{43.47 \text{ A}}$

or  $I_{LP} = S_T / [(\sqrt{3})V_{LP}] = 47.0588 \times 10^6 / [(\sqrt{3})625 \times 10^3] = \mathbf{43.47 \text{ A}}$

(d) The load carried by each transformer is  $(1/3)S_T = \mathbf{15.69 \text{ MVA}}$

**P.P. 13.13** The process is essentially the same as in Example 13.13. We are given the coupling coefficient,  $k = 0.4$ , and can determine the operating frequency from the value of  $\omega = 4$  which implies that  $f = 4/(2\pi) = 0.6366 \text{ Hz}$ .



Saving and then simulating produces,

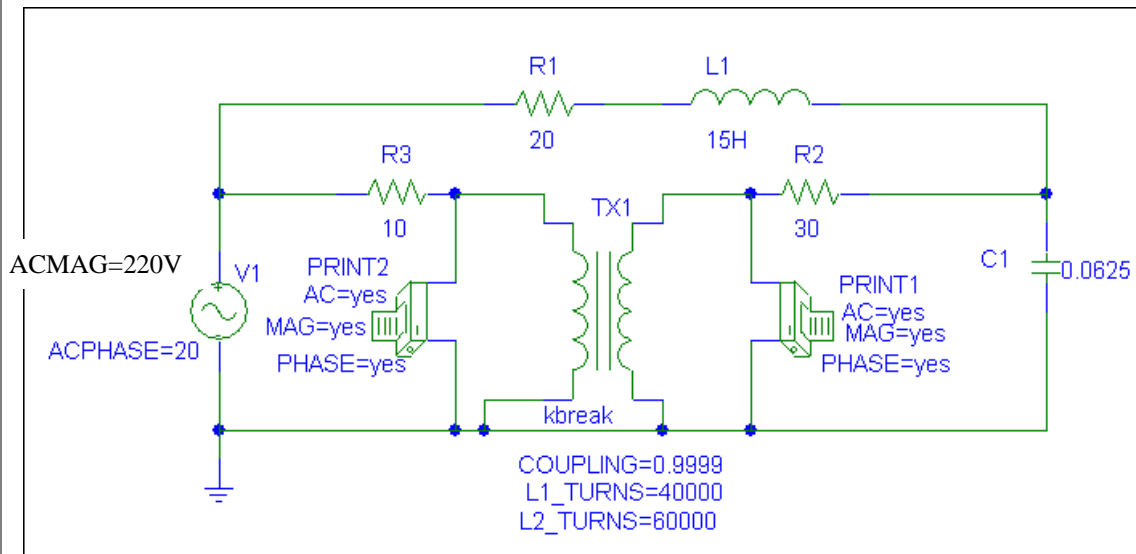
$$i_o = 2.012\cos(4t + 68.52^\circ) \text{ A}$$

**P.P. 13.14** Following the same basic steps in Example 13.14, we first assume  $\omega = 1$ . This then leads to following determination of values for the inductor and the capacitor.

$$j15 = j\omega L \text{ leads to } L = 15 \text{ H}$$

$$-j16 = 1/(\omega C) \text{ leads to } C = 62.5 \text{ mF}$$

The schematic is shown below.



FREQ	VM(\$N_0005,0)	VP(\$N_0005,0)
1.592E-01	1.530E+02	2.185E+00
FREQ	VM(\$N_0001,0)	VP(\$N_0001,0)
1.592E-01	2.302E+02	2.091E+00

Thus,

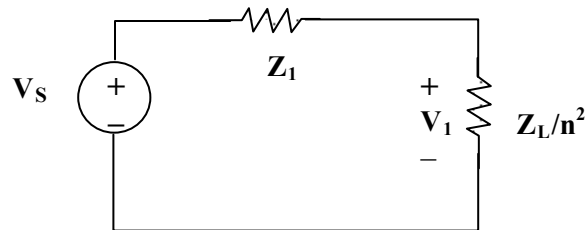
$$\mathbf{V}_1 = 153\angle 2.18^\circ \text{ V}$$

$$\mathbf{V}_2 = 230.2\angle 2.09^\circ \text{ V}$$

Note, if we divide  $V_2$  by  $V_1$  we get  $1.5046\angle-0.09^\circ$  which is in good agreement that the transformer is ideal with a voltage ratio of 1:1.5 (or 2:3)!

**P.P. 13.15**  $V_2/V_1 = 120/13,200 = 1/110 = 1/n$

**P.P. 13.16**



As in Example 13.16,  $n^2 = Z_L/Z_1 = 400/(2.5 \times 10^3) = 4/25$ ,  $n = 0.4$

By voltage division,  $V_1 = V_s/2$  (since  $Z_1 = Z_L/n^2$ ), therefore  $V_1 = 60/2 = 30$  volts, and

$$V_2 = nV_1 = (0.4)(30) = 12 \text{ volts}$$

**P.P. 13.17**

(a)  $S = 12 \times 60 + 350 + 4,500 = 5.57 \text{ kW}$

(b)  $I_P = S/V_P = 5570/2400 = 2.321 \text{ A}$

**CHAPTER 14**

**P.P.14.1**      $H(\omega) = \frac{V_o}{V_s} = \frac{j\omega L}{R + j\omega L}$

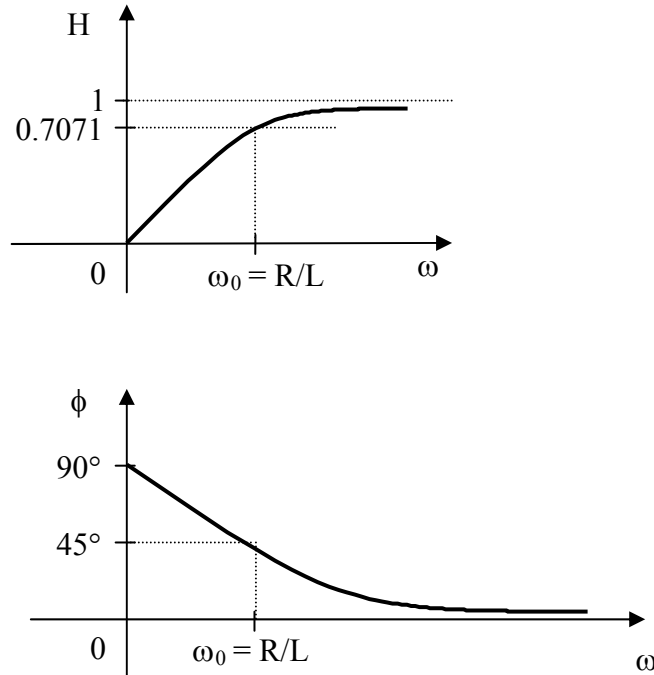
$$H(\omega) = \frac{j\omega L/R}{1 + j\omega L/R} = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}$$

where  $\omega_0 = \frac{R}{L}$ .

$$H = |H(\omega)| = \frac{\omega/\omega_0}{\sqrt{1 + (\omega/\omega_0)^2}} \qquad \phi = \angle H(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

- At  $\omega = 0$ ,      $H = 0$ ,      $\phi = 90^\circ$
- As  $\omega \rightarrow \infty$ ,    $H = 1$ ,      $\phi = 0^\circ$
- At  $\omega = \omega_0$ ,    $H = \frac{1}{\sqrt{2}}$ ,      $\phi = 90^\circ - 45^\circ = 45^\circ$

Thus, the sketches of H and  $\phi$  are shown below.



**P.P.14.2** The desired transfer function is the input impedance.

$$\mathbf{Z}_i(s) = \frac{\mathbf{V}_o(s)}{\mathbf{I}_o(s)} = \left( 10 + \frac{1}{s/20} \right) \parallel (6 + 2s)$$

$$\mathbf{Z}_i(s) = \frac{(10 + 20/s)(6 + 2s)}{10 + 20/s + 6 + 2s} = \frac{10(s + 2)(s + 3)}{s^2 + 8s + 10}$$

The poles are at

$$p_{1,2} = \frac{-8 \pm \sqrt{64 - 40}}{2} = \mathbf{-1.5505, -6.449}$$

The zeros are at

$$z_1 = \mathbf{-2}, \quad z_2 = \mathbf{-3}.$$

**P.P.14.3** 
$$\mathbf{H}(\omega) = \frac{1 + j\omega/2}{(j\omega)(1 + j\omega/10)}$$

$$H_{\text{db}} = 20 \log_{10} |1 + j\omega/2| - 20 \log_{10} |j\omega| - 20 \log_{10} |1 + j\omega/10|$$

$$\phi = -90^\circ + \tan^{-1}(\omega/2) - \tan^{-1}(\omega/10)$$

**The magnitude and the phase plots are as shown in Fig. 14.14.**

**P.P.14.4** 
$$\mathbf{H}(\omega) = \frac{(50/400)j\omega}{(1 + j\omega/4)(1 + j\omega/10)^2}$$

$$H_{\text{db}} = -20 \log_{10} |8| + 20 \log_{10} |j\omega| - 20 \log_{10} |1 + j\omega/4| - 40 \log_{10} |1 + j\omega/10|$$

$$\phi = 90^\circ - \tan^{-1}(\omega/4) - 2 \tan^{-1}(\omega/10)$$

**The magnitude and the phase plots are as shown in Fig. 14.16.**

**P.P.14.5** 
$$\mathbf{H}(\omega) = \frac{10/400}{(j\omega) \left( 1 + \frac{j\omega 8}{40} + \left( \frac{j\omega}{20} \right)^2 \right)}$$

$$H_{\text{db}} = -20 \log_{10} |40| - 20 \log_{10} |j\omega| - 20 \log_{10} |1 + j\omega/5 - \omega^2/400|$$

$$\phi = -90^\circ - \tan^{-1} \left( \frac{0.2\omega}{1 - \omega^2/400} \right)$$

**The magnitude and the phase plots are as shown in Fig. 14.18.**

**P.P.14.6**

The gain is = 40 db =  $20\log_{10}(\text{gain})$  or the gain = 100.

A zero at  $\omega = 5$ ,  $1 + j\omega/5$

A pole at  $\omega = 10$ ,  $\frac{1}{1 + j\omega/10}$

Two poles at  $\omega = 100$ ,  $\frac{1}{(1 + j\omega/100)^2}$

Hence,

$$\mathbf{H(\omega)} = \frac{100(1 + j\omega/5)}{(1 + j\omega/10)(1 + j\omega/100)^2} = \frac{100(1/5)(5 + j\omega)}{(1/100,000)(10 + j\omega)(100 + j\omega)^2}$$

$$\mathbf{H(\omega)} = \frac{2,000,000(s + 5)}{(s + 10)(s + 100)^2}$$

**P.P.14.7**

$$(a) \quad Q = \frac{\omega_0 L}{R} \longrightarrow \omega_0 = \frac{QR}{L} = \frac{(50)(4)}{25 \times 10^{-3}} = 8 \times 10^3 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(64 \times 10^6)(25 \times 10^{-3})}$$

$$C = \mathbf{0.625 \mu F}$$

$$(b) \quad B = \frac{\omega_0}{Q} = \frac{8 \times 10^3}{50} = \mathbf{160 \text{ rad/s}}$$

Since  $Q > 10$ ,

$$\omega_1 = \omega_0 - \frac{B}{2} = 8000 - 80 = \mathbf{7920 \text{ rad/s}}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 8000 + 80 = \mathbf{8080 \text{ rad/s}}$$

$$(c) \quad \text{At } \omega = \omega_0, \quad P = \frac{V_{in}^2}{2R} = \frac{100^2}{8} = \mathbf{1.25 \text{ kW}}$$

$$\text{At } \omega = \omega_1, \quad P = 0.5 \cdot \frac{V_{in}^2}{2R} = \mathbf{0.625 \text{ kW}}$$

$$\text{At } \omega = \omega_2, \quad P = 0.5 \cdot \frac{V_{in}^2}{2R} = \mathbf{0.625 \text{ kW}}$$

**P.P.14.8**  $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20 \times 10^{-3})(5 \times 10^{-9})}} = 10^5 = \mathbf{100 \text{ krad/s}}$

$$Q = \frac{R}{\omega_0 L} = \frac{100 \times 10^3}{(10^5)(20 \times 10^{-3})} = \mathbf{50}$$

$$B = \frac{\omega_0}{Q} = \frac{10^5}{50} = \mathbf{2 \text{ krad/s}}$$

Since  $Q > 10$ ,

$$\omega_1 = \omega_0 - \frac{B}{2} = 100,000 - 1,000 = \mathbf{99 \text{ krad/s}}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 100,000 + 1,000 = \mathbf{101 \text{ krad/s}}$$

**P.P.14.9**  $Z = j\omega 0.01 + 20 \parallel \frac{2000}{j\omega} = j\omega 0.01 + \frac{20}{1 + j\omega/100}$

$$Z = j\omega 0.01 + \frac{20(1 - j\omega/100)}{1 + \omega^2/10^4}$$

$$\text{Im}(Z) = 0 \longrightarrow \omega 0.01 - \frac{0.2\omega}{1 + \omega^2/10^4} = 0$$

$$\omega = \frac{20\omega}{1 + \omega^2/10^4} \longrightarrow 1 + \omega^2/10^4 = 20$$

Clearly,  $\omega = \mathbf{435.9 \text{ rad/s}}$

**P.P.14.10**  $H(s) = \frac{V_o}{V_i} = \frac{R_2 \parallel sL}{R_1 + R_2 \parallel sL}, \quad s = j\omega$

$$H(s) = \frac{sR_2L}{R_1R_2 + sR_1L + sR_2L}$$

$$H(\omega) = \frac{j\omega R_2L}{R_1R_2 + j\omega L(R_1 + R_2)}$$

$$H(0) = 0$$

$$H(\omega) = \lim_{\omega \rightarrow \infty} \frac{jR_2L}{R_1R_2/\omega + jL(R_1 + R_2)} = \frac{R_2}{R_1 + R_2}$$

i.e. a **highpass filter**.

The corner frequency occurs when  $H(\omega_c) = \frac{1}{\sqrt{2}} \cdot H(\infty)$ .

$$\mathbf{H}(\omega) = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{j\omega L}{j\omega L + R_1 R_2 / (R_1 + R_2)} \right)$$

$$\mathbf{H}(\omega) = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{j\omega}{j\omega + k} \right), \quad \text{where } k = \frac{R_1 R_2}{(R_1 + R_2)L}$$

At the corner frequency,

$$\frac{1}{\sqrt{2}} \cdot \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} \cdot \left| \frac{j\omega_c}{j\omega_c + k} \right|$$

$$\frac{1}{\sqrt{2}} = \frac{\omega_c}{\sqrt{\omega_c^2 + k^2}} \longrightarrow \omega_c = k = \frac{R_1 R_2}{(R_1 + R_2)L}$$

Hence, 
$$\mathbf{H}(\omega) = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{j\omega}{j\omega + \omega_c} \right)$$

and the corner frequency is

$$\omega_c = \frac{(100)(100)}{(100 + 100)(2 \times 10^{-3})} = \mathbf{25 \text{ krad/s}}$$

**P.P.14.11**  $B = 2\pi(20.3 - 20.1) \times 10^3 = 400\pi$

Assuming high Q,

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} = \frac{(2\pi)(40.4 \times 10^3)}{2} = 40.4\pi \times 10^3 \text{ rad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{40.4\pi \times 10^3}{400\pi} = \mathbf{101}$$

$$B = \frac{R}{L} \longrightarrow L = \frac{R}{B} = \frac{20 \times 10^3}{400\pi} = \mathbf{15.915 \text{ H}}$$

$$Q = \frac{1}{\omega_0 CR} \longrightarrow C = \frac{1}{\omega_0 QR}$$

$$C = \frac{1}{(40.4\pi \times 10^3)(101)(20 \times 10^3)} = \mathbf{3.9 \text{ pF}}$$

**P.P.14.12** Given  $H(\infty) = 5$  and  $f_c = 2 \text{ kHz}$

$$\omega_c = 2\pi f_c = \frac{1}{R_i C_i}$$

$$R_i = \frac{1}{2\pi f_c C_i} = \frac{1}{(2\pi)(2 \times 10^3)(0.1 \times 10^{-3})}$$

$$R_i = 795.8 \cong \mathbf{800 \Omega}$$

$$H(\infty) = \frac{-R_f}{R_i} = -5 \longrightarrow R_f = 5R_i = 3,978 \cong \mathbf{4 k\Omega}$$

**P.P.14.13**  $Q = 10, \quad \omega_0 = 20 \text{ krad/s}$

$$B = \frac{\omega_0}{Q} = 2 \text{ krad/s}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 19 \text{ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 21 \text{ krad/s}$$

Since  $\omega_1 = \frac{1}{C_2 R}$ ,

$$C_2 = \frac{1}{\omega_1 R} = \frac{1}{(19 \times 10^3)(10 \times 10^3)} = \mathbf{5.263 \text{ nF}}$$

$$C_1 = \frac{1}{\omega_2 R} = \frac{1}{(21 \times 10^3)(10 \times 10^3)} = \mathbf{4.762 \text{ nF}}$$

$$K = \frac{R_f}{R_i} = 5 \longrightarrow R_f = 5R_i = \mathbf{50 k\Omega}$$

**P.P.14.14**  $K_f = \frac{\omega'_c}{\omega_c} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4$

$$C' = \frac{C}{K_m K_f} \longrightarrow K_m = \frac{C}{C' K_f} = \frac{1}{(15 \times 10^{-9})(2\pi \times 10^4)} = \frac{10^4}{3\pi}$$

$$R' = K_m R = \frac{10^4}{3\pi} (1) = 1.061 \text{ k}\Omega$$

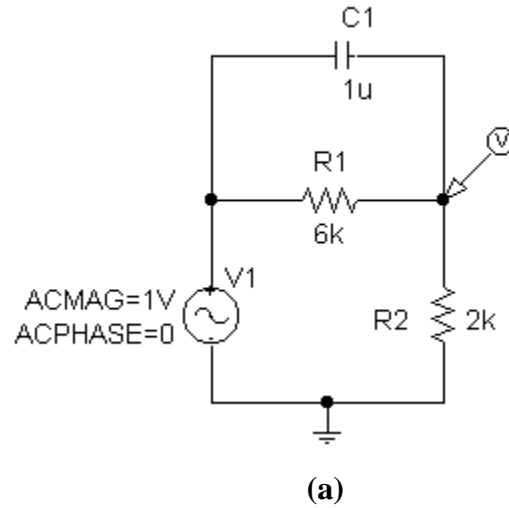
$$L' = \frac{K_m}{K_f} L = \frac{10^4}{3\pi} \cdot \frac{2}{2\pi \times 10^4} = 33.77 \text{ mH}$$

Therefore,

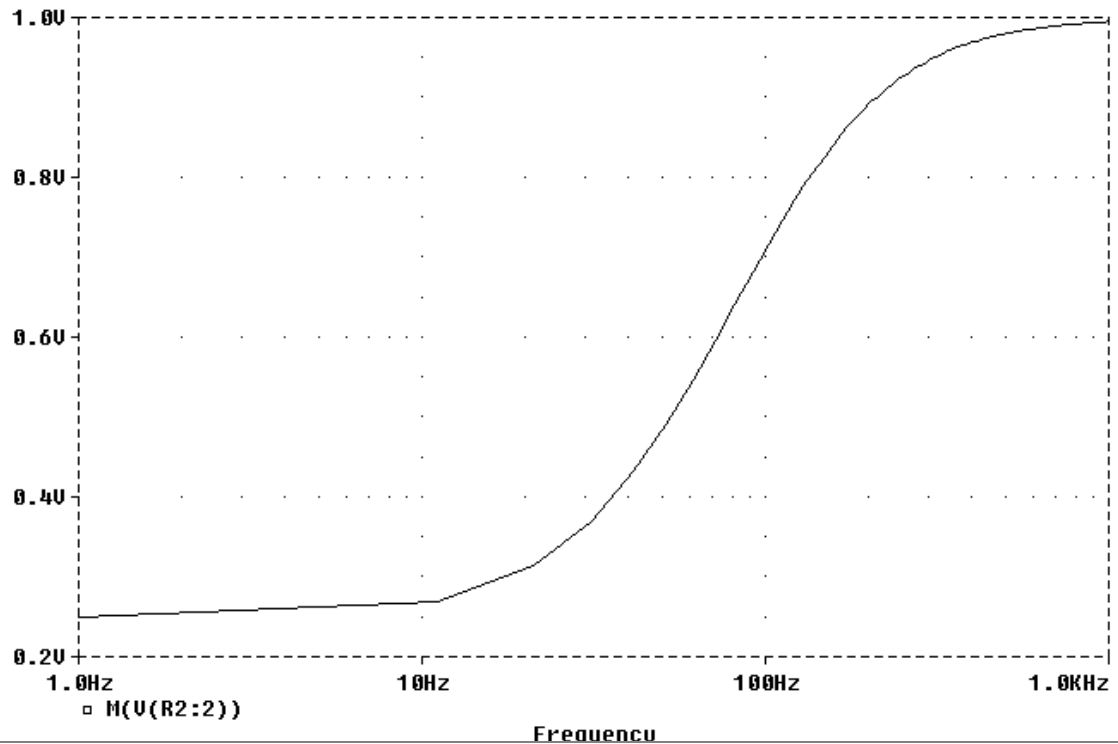
$$R'_1 = R'_2 = \mathbf{1.061 k\Omega}$$

$C_1 = C_2 = 15 \text{ nF}$   
 $L' = 33.77 \text{ mH}$

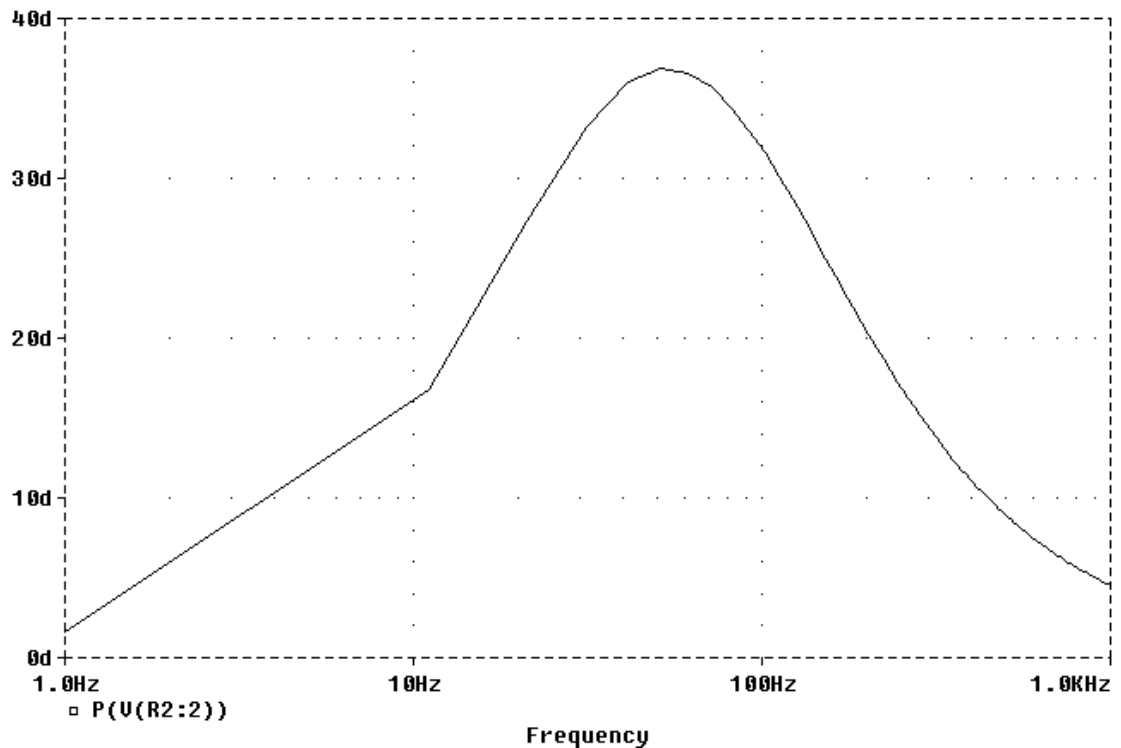
**P.P.14.15** The schematic is shown in Fig. (a).



Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 100, *Start Freq* = 1, and *End Freq* = 1K. After saving and simulating the circuit, we obtain **the magnitude and phase plots are shown in Figs. (b) and (c)**.



(b)



(c)

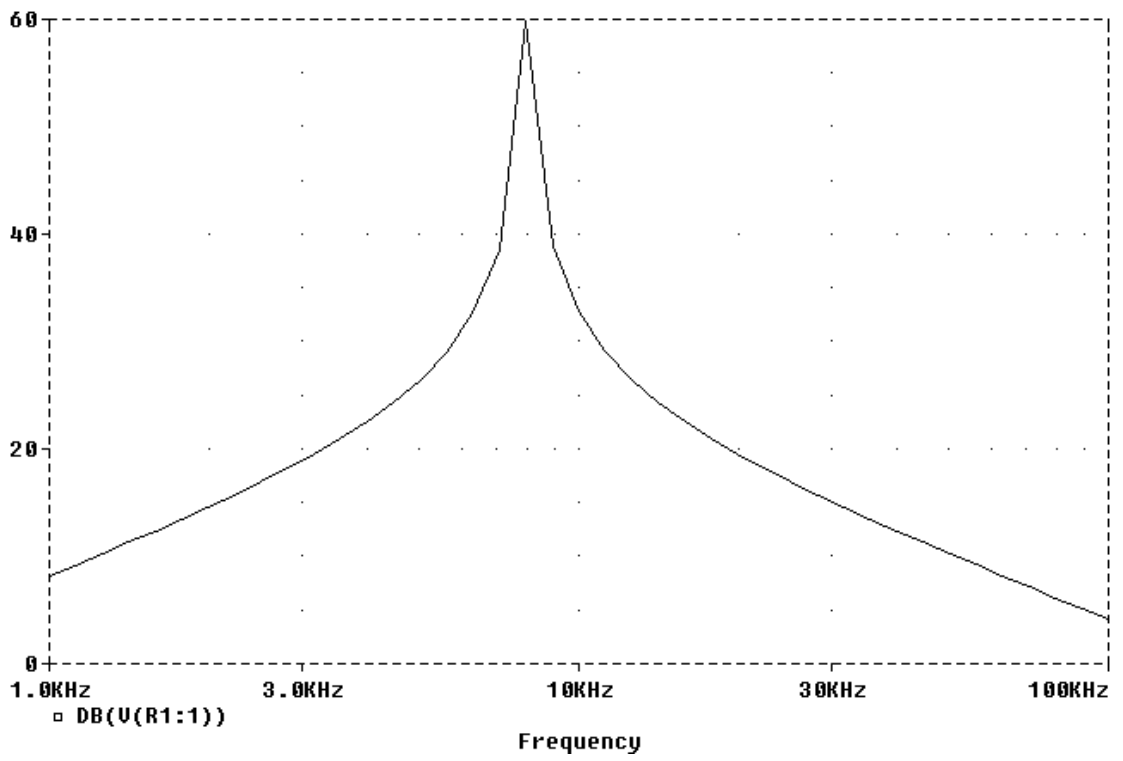
**P.P.14.16** The schematic is shown in Fig. (a).



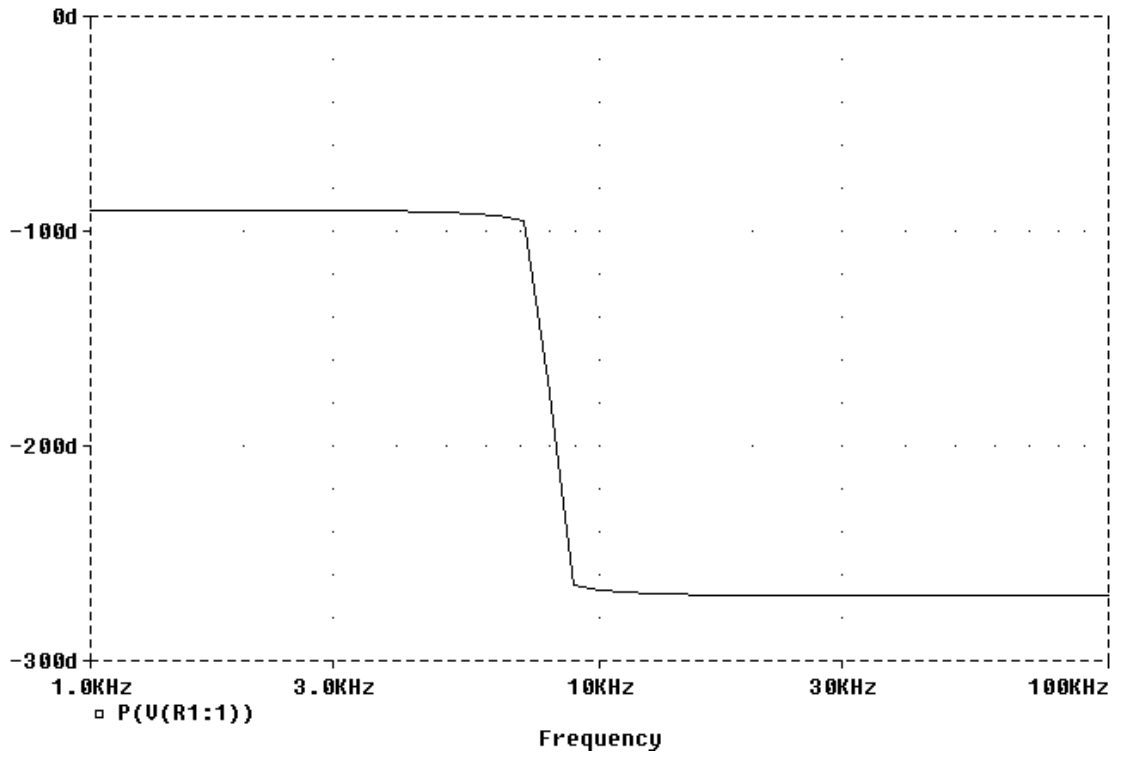
(a)

Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Decade* sweep type with these *Sweep Parameters* : *Pts/Decade* = 20, *Start Freq* = 1K, and *End Freq* = 100K. Save and simulate the circuit.

For the magnitude plot, choose **DB( )** from the **Analog Operators and Functions** list. Then, select the voltage **V(R1:1)** and OK. Another option would be to type **DB(V(R1:1))** as the **Trace Expression**. For the phase plot, choose **P( )** from the **Analog Operators and Functions** list. Then, select the voltage **V(R1:1)** and OK. Another option would be to type **VP(R1:1)** as the **Trace Expression**. **The resulting magnitude and phase plots are shown in Figs. (b) and (c).**



(b)



(c)

**P.P.14.17**  $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

or  $C = \frac{1}{4\pi^2 f_0^2 L}$

For the high end of the band,  $f_0 = 108 \text{ MHz}$

$$C_1 = \frac{1}{4\pi^2 (108^2 \times 10^{12})(4 \times 10^{-6})} = 0.543 \text{ pF}$$

For the low end of the band,  $f_0 = 88 \text{ MHz}$

$$C_2 = \frac{1}{4\pi^2 (88^2 \times 10^{12})(4 \times 10^{-6})} = 0.818 \text{ pF}$$

Therefore, C must be adjustable and be in the range **0.543 pF to 0.818 pF** .

**P.P.14.18**

For  $BP_6$ ,  $f_0 = 1336 \text{ Hz}$  and it passes frequencies in the range  $1209 \text{ Hz} < f < 1477 \text{ Hz}$  .

$$B = 2\pi(1477 - 1209) = 1683.9$$

$$L = \frac{R}{B} = \frac{600}{1683.9} = \mathbf{356 \text{ mH}}$$

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 (1336)^2 (0.356)} = \mathbf{39.83 \text{ nF}}$$

**P.P.14.19**  $C = 10 \text{ } \mu\text{F}$  and  $R_1 = R_2 = 8 \text{ } \Omega$

$$2\pi f_c = \frac{1}{R_1 C} \longrightarrow f_c = \frac{1}{2\pi R_1 C} = \frac{1}{(2\pi)(8)(10 \times 10^{-6})} = \mathbf{1.989 \text{ kHz}}$$

$$L = \frac{R_2}{2\pi f_c} = \frac{8}{(2\pi)(1.989 \times 10^3)} = \mathbf{0.64 \text{ mH}}$$