SOLUTIONS MANUAL

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ENGINEERING MECHANICS of SOLIDS

SECOND EDITION

EGOR P. POPOV

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PRENTICE HALL, Upper Saddle River, NJ 07458

Acquisitions Editor: Eric Svendsen
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Production Editor: Michele Wells
Supplement Cover Manager: Paul Gourhan
Supplement Cover Designer: PM Workshop Inc.
Manufacturing Buyer: Pat Brown

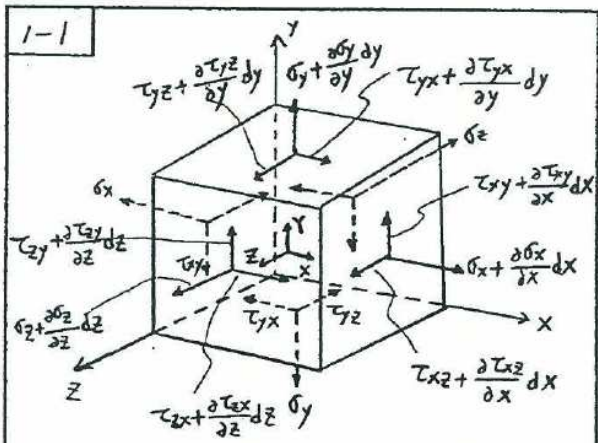
Printed in the United States of America

10 9 8 7 6 5 4

ISBN 0-13-080431-2

Prentice-Hall International (UK) Limited,London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada Inc., Toronto
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall of Japan, Inc., Tokyo
Pearson Education Asia Pte. Ltd., Singapore
Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

1-1



$$\Sigma F_x = (\sigma_x + \frac{\partial\sigma_x}{\partial x}dx)\,dy\cdot dz + (\tau_{yx} + \frac{\partial\tau_{yx}}{\partial y}dy)\,dx\,dz + (\tau_{zx} + \frac{\partial\tau_{zx}}{\partial z}dz)\,dx\cdot dy - \sigma_x\cdot dy\cdot dz - \tau_{yx}\cdot dx\cdot dz - \tau_{zx}\cdot dx\cdot dy + X = 0$$

$$\partial\sigma_x\, dx\cdot dy\cdot dz + \frac{\partial\tau_{yx}}{\partial y}dx\cdot dy\cdot dz + \frac{\partial\tau_{zx}}{\partial z}dx\cdot dy\cdot dz + X = 0$$

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} + X = 0$$

1-2

$$\Sigma F_r = (\sigma_r + \frac{\partial\sigma_r}{\partial r}dr)(r+dr)\,d\theta + (\tau_{\theta r} + \frac{\partial\tau_{\theta r}}{\partial\theta}d\theta)dr - \tau_{\theta r}dr - \sigma_r\, r\, d\theta - (\sigma_\theta + \frac{\partial\sigma_\theta}{\partial\theta}d\theta)dr\frac{d\theta}{2} - \sigma_\theta\, dr\frac{d\theta}{2} = 0$$

$$\sigma_r\, dr\, d\theta + r\frac{\partial\sigma_r}{\partial r}dr\,d\theta + \frac{\partial\sigma_r}{\partial r}(dr)^2 d\theta + \frac{\partial\tau_{\theta r}}{\partial\theta}dr\,d\theta - \sigma_\theta\, dr\,d\theta - \frac{1}{2}\frac{\partial\sigma_\theta}{\partial\theta}(d\theta)^2 dr = 0$$

$$\sigma_r + r\frac{\partial\sigma_r}{\partial r} + \frac{\partial\sigma_r}{\partial r}\cancel{dr}^{\,0} + \frac{\partial\tau_{\theta r}}{\partial\theta} - \sigma_\theta - \frac{1}{2}\frac{\partial\sigma_\theta}{\partial\theta}\cancel{d\theta}^{\,0} = 0$$

$$\rightarrow \frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta r}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\Sigma F_\theta = (\tau_{r\theta} + \frac{\partial\tau_{r\theta}}{\partial r}dr)(r+dr)\,d\theta + (\sigma_\theta + \frac{\partial\sigma_\theta}{\partial\theta}d\theta)\,dr + (\tau_{\theta r} + \frac{\partial\tau_{\theta r}}{\partial\theta}d\theta)dr\frac{d\theta}{2} + \tau_{\theta r}dr\frac{d\theta}{2} - \tau_{r\theta}\, r\, d\theta - \sigma_\theta\, dr = 0$$

$$\tau_{r\theta}\, dr\, d\theta + \frac{\partial\tau_{r\theta}}{\partial r}r\,dr\,d\theta + \frac{\partial\tau_{r\theta}}{\partial r}(dr)^2 d\theta + \frac{\partial\sigma_\theta}{\partial\theta}dr\,d\theta + \tau_{\theta r}\,dr\,d\theta + \frac{1}{2}\frac{\partial\tau_{\theta r}}{\partial\theta}(d\theta)^2 dr = 0$$

$$\tau_{r\theta} + \frac{\partial\tau_{r\theta}}{\partial r}r + \frac{\partial\tau_{r\theta}}{\partial r}\cancel{dr}^{\,0} + \frac{\partial\sigma_\theta}{\partial\theta} + \tau_{\theta r} + \frac{1}{2}\frac{\partial\tau_{\theta r}}{\partial\theta}\cancel{d\theta}^{\,0} = 0$$

$$\rightarrow \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{2\tau_{r\theta}}{r} = 0$$

1-3

$$\sigma = \frac{P}{A} = \frac{110}{7.08} = 15.54\ \text{ksi}$$

$$\sigma = \frac{P}{A} = \frac{110}{6.09} = 18.06\ \text{ksi}$$

1-4

$$P = 50\times 40 = 2000\ \text{lb}$$

$$\sigma_1 = \frac{2000}{5.5^2} = 66.1\ \text{psi}$$

$$\sigma_2 = \frac{2000}{6^2} = 55.6\ \text{psi}$$

$$\sigma_3 = \frac{2000}{16^2} = 7.8\ \text{psi}$$

1-5

magnitude of the applied force:

$$P = \sigma A = 150\times(30\times 10 + 30\times 12) = 150\times 660 = 99000\ \text{N}$$

1-5

The location of stress resultant : (at the centroid)

$$\frac{1}{660}\left[30\times10\times\frac{10}{2}+30\times12\times\left(10+\frac{30}{2}\right)\right]$$

$= 15.91\ mm$ below the top

1-6

$$A = 79.5\times9.5+102\times9.5 = 1724.25\ mm^2$$

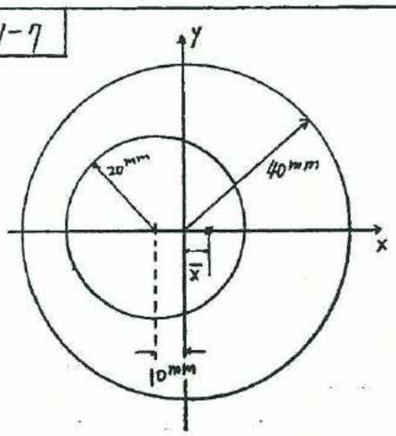
$$\bar{x} = \frac{1}{2}\times\frac{102\times9.5^2}{1724.25}+\frac{79.5\times9.5\times(\frac{1}{2}\times79.5+9.5)}{1724.25}$$

$= 24.24\ mm$ from the left side

$$\bar{y} = \frac{1}{2}\times\frac{102^2\times9.5}{1724.25}+\frac{1}{2}\times\frac{9.5^2\times79.5}{1724.25}$$

$= 30.74\ mm$ below the top

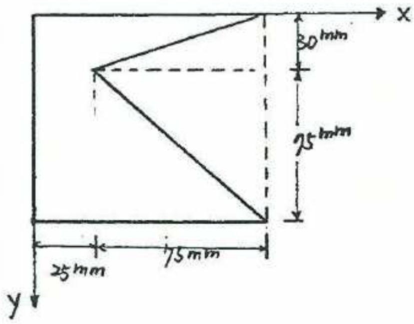
1-7



$$\bar{x} = \frac{\pi 40^2\times0+(-\pi 20^2)\cdot(-10)}{\pi 40^2+(-\pi 20^2)}$$

$= 2\ mm$

1-8



$$A = 100\times105-\frac{1}{2}\times105\times75 = 6562.5\ mm^2$$

$$\bar{x} = \frac{1}{2}\times\frac{100^2\times105}{6262.5}-\frac{1}{2}\times\frac{105\times75\times(50+25)}{6562.5}$$

$= 35\ mm$

$$\bar{y} = \frac{1}{2}\times\frac{100\times105^2}{6562.5}-\frac{1}{2}\times\frac{30\times75\times20}{6562.5}-\frac{1}{2}\times\frac{75^2\times(30+25)}{6562.5}$$

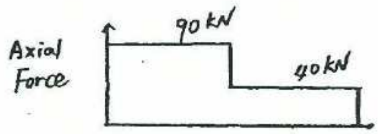
$= 57\ mm$

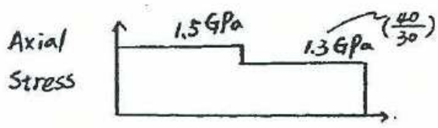
1-9

(a) $\sigma_{max} = \frac{P_1+P_2}{A_1} = \frac{50+40}{60}$

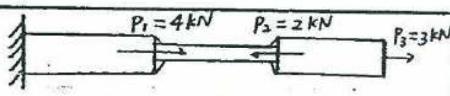
$= 1.5\ GPa$

(b)

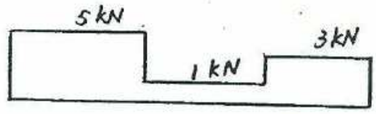




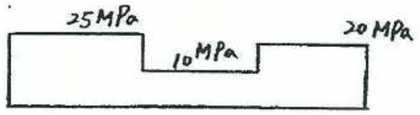
1-10



Axial Force

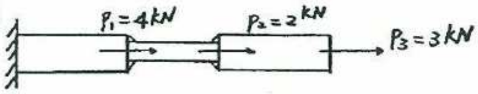


Axial Stress

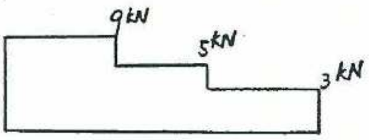


$\sigma_{max} = 25\ MPa$

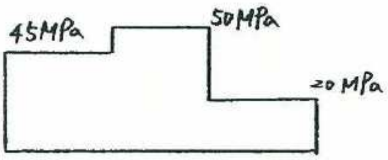
1-11



Axial Force

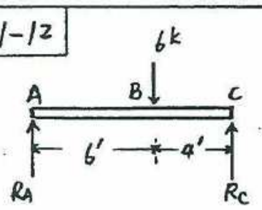


Axial Stress



$\sigma_{max} = 50\ MPa$

1-12



$$R_C = \frac{6 \times 6}{10} = 3.6k$$

$$R_A = \frac{6 \times 4}{10} = 2.4k$$

$$\sigma = \frac{P}{A}$$

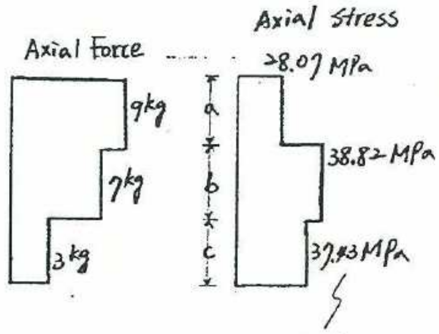
$$\sigma_A = \frac{2.40}{4 \times 5.5} = 0.109\ ksi\ (-)$$

$$\sigma_B = \frac{6.00}{30.3} = 0.198\ ksi\ (-)$$

$$\sigma_C = \frac{3.60}{30.3} = 0.119\ ksi\ (-)$$

1-13

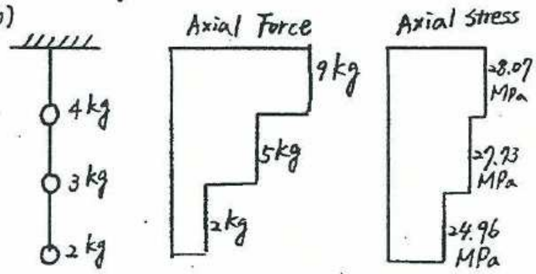
(a)



$$\frac{3 \times 9.8}{\frac{1}{4}\pi \cdot 1^2}$$

the highest wire is b

(b)



1-14

$$\sigma_\theta = \frac{P}{A}\cos^2\theta$$

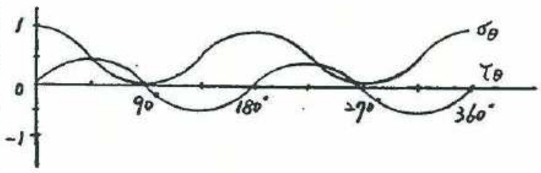
σ_θ :

θ°	0	45	90	135	180	225	270	315	360
$\cos^2\theta$	1	.5	0	.5	1	.5	0	.5	1

$$\tau_\theta = \frac{P}{A}\sin\theta\cos\theta$$

τ_θ :

θ°	0	45	90	135	180	225	270	315	360
$\sin\theta\cos\theta$	0	.5	0	-.5	0	.5	0	-.5	0



Maxima $\begin{cases} \sigma = 0^\circ, 180^\circ \\ \tau = 45^\circ, 225^\circ \end{cases}$

minima $\begin{cases} \sigma = 90^\circ, 270^\circ \\ \tau = 135^\circ, 315^\circ \end{cases}$

1-15

$$\sigma_\theta = \tau_\theta$$

$$\left|\frac{P}{A}\cos^2\theta\right| = \left|\frac{P}{A}\sin\theta\cos\theta\right|$$

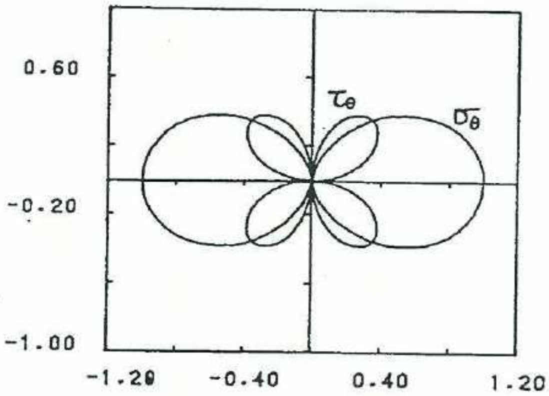
$$|\cos\theta| = |\sin\theta|$$

$$|\tan\theta| = 1$$

$$\theta = \pm\tan^{-1}(1) \rightarrow \theta = \pm 45^\circ$$

1-16

$$\sigma_\theta = \frac{P}{A}\cos^2\theta \qquad \tau_\theta = \frac{P}{A}\sin\theta\cos\theta$$

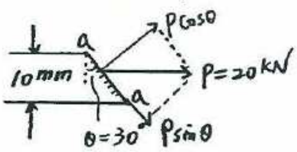


maxima $\begin{cases} \sigma = 0^\circ, 180^\circ \\ \tau = 45^\circ, 225^\circ \end{cases}$

minima $\begin{cases} \sigma = 90^\circ, 270^\circ \\ \tau = 135^\circ, 315^\circ \end{cases}$

1-17 A)

section a-a



$P\cos\theta = 17.32\ kN$

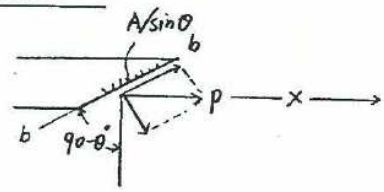
$Area = A/\cos 30^\circ = 100/\cos 30^\circ = 115.47\ mm^2$

$$\sigma = \frac{P}{A} = \frac{P\cos\theta}{115.47} = \frac{17.32\times10^3}{115.47} = 150\ MPa$$

$$\tau = \frac{P}{A} = \frac{P\sin\theta}{115.47} = \frac{20(\sin 30^\circ)\times10^3}{115.47}$$

$= 86.6\ MPa$

section b-b



$$Area = \frac{A}{\sin\theta} = \frac{100}{\sin 30^\circ} = 200\ mm^2$$

$$\sigma = \frac{P}{A} = \frac{20\sin 30^\circ\times10^3}{200} = 50\ MPa$$

$$\tau = \frac{P}{A} = \frac{20\cos 30^\circ\times10^3}{200} = 86.6\ MPa$$

B)

$$\sigma_{aa} = \frac{P}{A}\cos^2\theta = \frac{20\times10^3}{100}\cos^2(30^\circ)$$

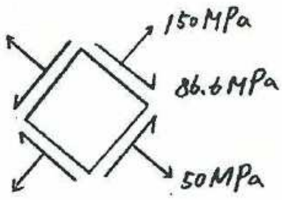
$= 150\ MPa$

$$\tau_{a\text{-}a,\,b\text{-}b} = \frac{P}{A}\cos\theta\sin\theta$$

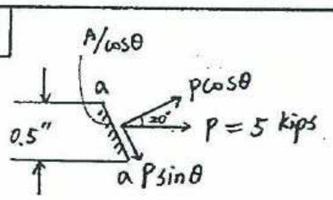
$$= \frac{20\times10^3}{100}(\cos 30^\circ)(\sin 30^\circ)$$

$= 86.6\ MPa$

$$\sigma_{b\text{-}b} = \frac{20\times10^3}{100}\cos^2(60^\circ) = 50\ MPa$$



1-18



(A) section a-a

$$A = A/\cos\theta = \frac{0.5^2}{\cos 20^\circ} = 0.266 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{P\cos\theta}{0.266} = \frac{5\times\cos 20^\circ}{0.266}$$

$$= 17.66 \text{ ksi}$$

$$\tau = \frac{P}{A} = \frac{P\sin\theta}{0.266} = \frac{5\times\sin 20^\circ}{0.266}$$

$$= 6.43 \text{ ksi}$$

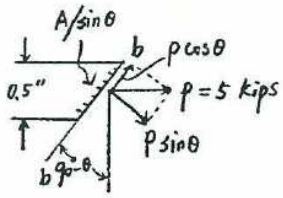
(B) $\sigma = \frac{P}{A}\cos^2\theta = \frac{5}{0.5^2}\times\cos^2 20^\circ$

$= 17.66$ ksi (eq. 1-7a)

$\tau = \frac{P}{A}\sin\theta\cos\theta = \frac{5}{0.5^2}\sin 20^\circ\cos 20^\circ$

$= 6.43$ ksi (eq. 1-7b)

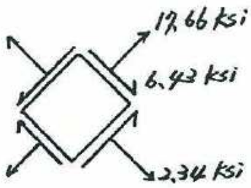
(C)



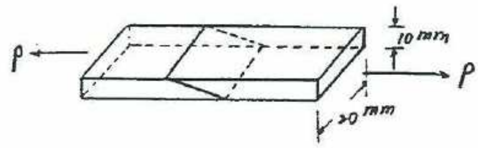
section b-b

$$\sigma = \frac{5}{0.5^2}\times\cos^2 70^\circ = 2.34 \text{ ksi}$$

$$\tau = \frac{5}{0.5^2}\times\sin 70^\circ\cos 70^\circ = 6.43 \text{ ksi}$$



1-19



$$\tau_\theta = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\cos\theta\sin\theta$$

$$\tau = 10 = \frac{P(\cos 20^\circ)(\sin 20^\circ)}{20\times 10}$$

$$P = \frac{10\times 20\times 10}{(\cos 20^\circ)(\sin 20^\circ)} = 6223 N$$

$$= 6.2 \text{ kN}$$

1-20

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\times 19^2 = 283.53 \text{ mm}^2$$

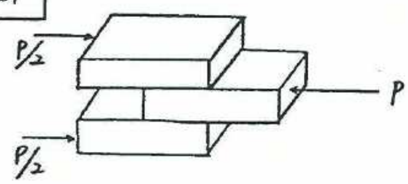
$$\sigma = \frac{P}{A}\cos^2\theta = \frac{100\times 10^3}{283.53}\times\cos^2 60^\circ$$

$$= 88.17 \text{ MPa}$$

$$\tau = \frac{P}{A}\sin\theta\cos\theta = \frac{100\times 10^3}{283.53}\times\sin 60^\circ\cos 60^\circ$$

$$= 152.72 \text{ MPa}$$

1-21

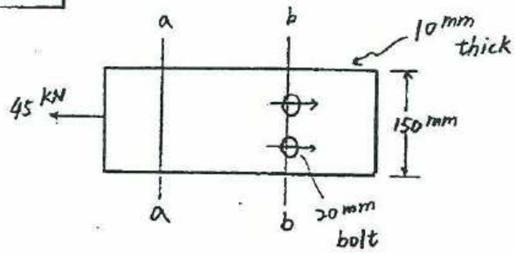


A (Shear Area) $= 40\times 80 = 3200 \text{ mm}^2$

$= 3.2\times 10^{-3} \text{ m}^2$

$$\tau = \frac{20/2 \text{ k}}{3.2\times 10^{-3}\text{ m}^2} = 3.125 \text{ MPa}$$

1-22



(a) $\sigma_{aa} = \dfrac{P}{A} = \dfrac{45\times10^3}{10\times150} = 30\ MPa$.

(b) $\sigma_{bb} = \dfrac{P}{A} = \dfrac{45\times10^3}{10\,(150-40)} = 40.91\ MPa$

(c) $\tau = \dfrac{V}{A} = \dfrac{45\times10^3}{2(\pi\times10^2)} = 71.62\ MPa$

(d) $\sigma_{br} = \dfrac{P}{A} = \dfrac{45\times10^3}{2\times(20\times10)} = 112.5\ MPa$

1-23

maximum normal stress :

$$\frac{70\times10^3}{10\times(60-20)} = 175\ MPa$$

Bearing stress :

$$\frac{P}{td} = \frac{70\times10^3}{20\times20} = 175\ MPa$$

for the middle plate

$$\frac{P/2}{t_1 d} = \frac{70\times10^3/2}{20\times20} = 87.5\ MPa$$

for the outer plate

Shear stress :

$$\frac{P/2}{\frac{1}{4}\pi d^2} = \frac{70\times10^3/2}{\frac{1}{4}\pi\cdot20^2} = 111.41\ MPa$$

1-24

$T = P\cdot R = 10\times185.75 = 1857.5\ N\text{-}m$

$P' = \dfrac{T}{r} = \dfrac{1857.5}{40} = 46.44\ kN$

$\tau = \dfrac{P'}{A} = \dfrac{46.44\times10^3}{6\times4} = 1935\ MPa$

1-25

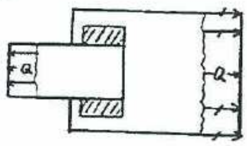
shear stress :

$$\frac{\text{shear force}}{\text{bolt area}} = \frac{16.5}{\frac{1}{4}\cdot\pi\cdot1^2} = 21\ ksi$$

tension stress in the material :

$$\frac{\text{tension force}}{\text{material area}} = \frac{64}{\frac{1}{4}\cdot\pi\cdot3^2} = 9.05\ ksi$$

1-26

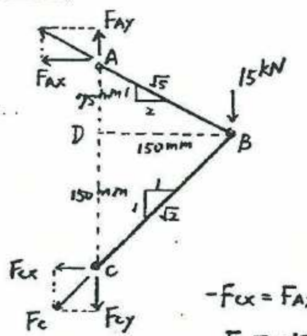


$A = \dfrac{W}{\sqrt2}\ in^2/in$

$\tau = \dfrac{Q}{2A} = \dfrac{Q}{\sqrt2\,W} = 21$

$Q = 29.7W\ \ kips/in$

1-27



$\Sigma F_x = 0 \rightarrow (+)$

$-F_{AX} - F_{cx} = 0$

$\Sigma F_y = 0 \uparrow (+)$

$-F_{cy} + F_{Ay} - 15 = 0$

$F_{cy} = -\frac{2}{3}\times15 = -10\ kN$

$F_{Ay} = \frac{1}{3}\times15 = 5\ kN$

$-F_{cx} = F_{AX} = 10\ kN$

$F_c = -10\sqrt2 = -14.14\ kN\ (comp.)$

1-27

$F_A = 5\sqrt{5} = 11.18\ kN$ (tension)

tensile stress in main bar AB:

$$\sigma_{AB} = \frac{F_A}{A} = \frac{11.18 \times 10^3}{6 \times 15} = 124.23\ MPa$$

tensile stress in clevis of bar AB:

$$(\sigma_{AB})_{clevis} = \frac{F_A}{A_{net}} = \frac{11.18 \times 10^3}{2 \times 5 \times (25-10)} = 74.54\ MPa$$

bearing between pin c & the clevis:

$$\sigma_b = \frac{F_c}{A_{bearing}} = \frac{14.14 \times 10^3}{10 \times 5 \times 2} = 141.42\ MPa$$

bearing between pin C & the main plate:

$$\sigma_b = \frac{F_c}{A} = \frac{14.14 \times 10^3}{10 \times 6} = 235.70\ MPa$$

double shear in pin C:

$$\tau = \frac{F_c}{A} = \frac{14.14 \times 10^3}{2\pi (10/2)^2} = 90.03\ MPa$$

1-28

$Area = 2\pi R l = 2\pi \times (15 \times 10^{-3}) \times 0.4$

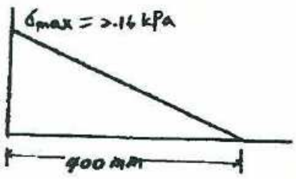
$= 0.038\ m^2$

$P = 10 \times 0.038 = 0.38\ N$

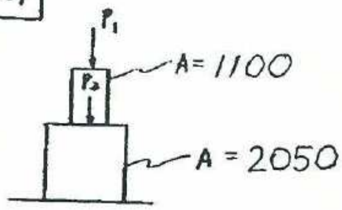
Cross section area:

$\pi R^2 - \pi r^2 = \pi (15^2 - 13^2) = 175.93\ mm^2$

$$\sigma_{max} = \frac{0.38}{175.93} = 2.16\ kPa$$



1-29



(a) $P_1 = A\sigma$

$= (1100)(100)/1000 = 110\ kN$

$P_1 + P_2 = (2050)(100)/1000 = 205$

$P_1 + 200 = 205$

$P_1 = 5\ kN$

$\Rightarrow P_1 = 5\ kN$

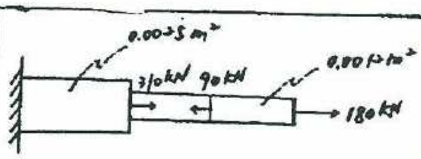
(b)

$P_1 + 65 = 205$

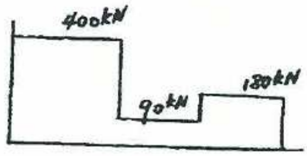
$P_1 = 140$

$\Rightarrow P_1 = 110\ KN$ governs

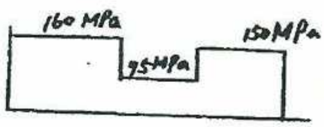
1-30



Axial Force

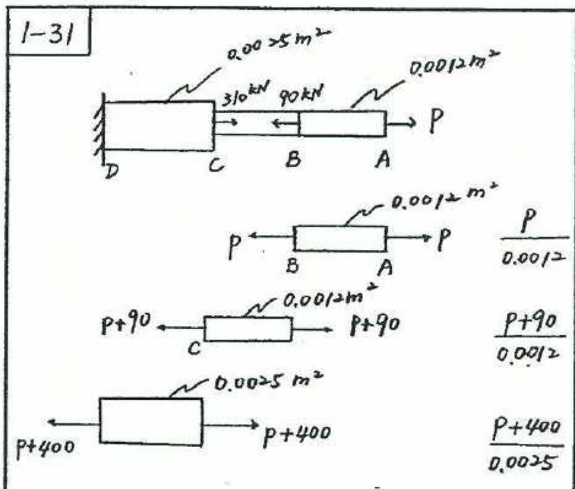


Axial stress



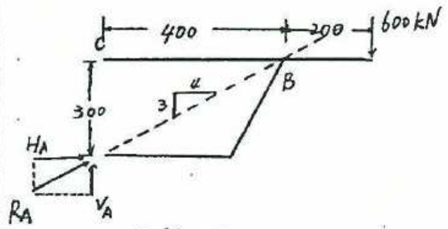
$\sigma_{max} = 160\ MPa$

1-31



$$\frac{P+90}{0.0012} = \frac{P+400}{0.0025} \qquad P = 196.15\ kN$$

1-32



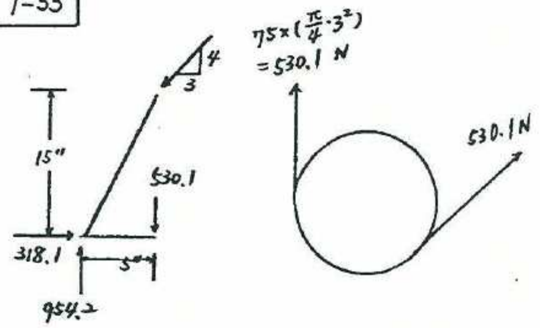
$\Sigma M_c = 0$

$$H_A = \frac{600 \times 600}{300} = 1200\ kN$$

$$R_A = 1200 \times \frac{5}{4} = 1500\ kN$$

$$\tau_A = \frac{1500}{2 \times (\frac{\pi 6^2}{4})} = 26.53\ MPa$$

1-33



$$F_{AB} = 75 \cdot \frac{\pi}{4} \cdot 3^2 = 530.1\ N$$

$$\Sigma M_A = 0\ , \quad -\frac{4}{5}P + \frac{9}{5}P = 530.1\ N$$

$$\frac{3}{5}P = 318.1\ N\ , \quad \frac{4}{5}P = 424.1\ N$$

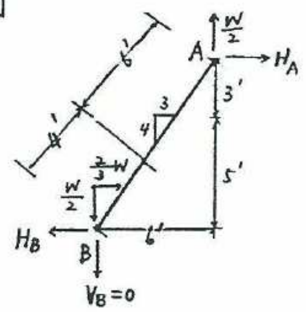
$$R_A = \sqrt{318.1^2 + 954.2^2} = 1005.8\ N$$

$$R_B = \sqrt{424.1^2 + 848.2^2} = 948.3\ N$$

$$\tau_A = \frac{V_A}{A} = \frac{1005.8}{0.022 \times 2} = 23051\ MPa$$
$$= 23.05\ GPa$$

$$\tau_B = \frac{V_B}{A} = \frac{948.3}{0.022 \times 2} = 21552\ MPa$$
$$= 21.55\ GPa$$

1-34



$+\circlearrowleft\ \Sigma M_A = 0$

$$-H_B \times 8 + \frac{W}{2} \times 6 + \frac{2}{3}W \times 3 = 0$$

(From the symmetry of the structure, $V_B = 0$)

$$H_B = \frac{5}{8}W = \frac{5}{8} \times 15 = 9.375\ k$$

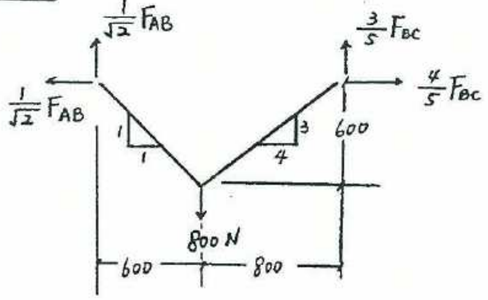
$$\tau_B = \frac{9.375}{2 \times \frac{\pi \times 1^2}{4}} = 5.97\ ksi$$

$$H_A = H_B - \frac{2}{3}W = 9.375 - \frac{2}{3} \times 15 = -0.625\ k$$

$$R_A = \sqrt{0.625^2 + 7.5^2} = 7.53\ k$$

$$\tau_A = \frac{7.53}{2 \times \pi \times 1^2/4} = 4.79\ ksi$$

1-35



$\Sigma F_x = 0$

$$F_{AB} \frac{1}{\sqrt{2}} = F_{BC} \frac{4}{5} \Rightarrow F_{BC} = 0.8839\, F_{AB}$$

$\Sigma F_y = 0$

$$F_{AB} \frac{1}{\sqrt{2}} + F_{BC} \frac{3}{5} = 800$$

$$\Rightarrow F_{AB} = 646.49 \text{ N}$$

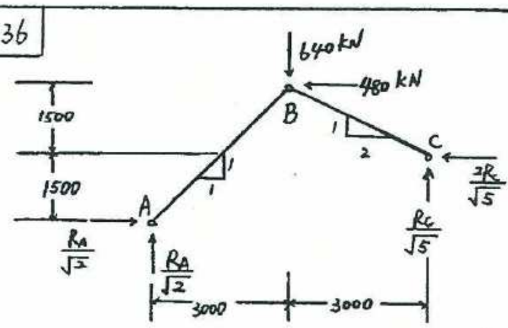
$$F_{BC} = 571.43 \text{ N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{646.49}{\pi \left(\frac{2.68}{2}\right)^2} = 114.60 \text{ MPa}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{571.43}{\pi \left(\frac{2.52}{2}\right)^2} = 114.57 \text{ MPa}$$

$114.60 \approx 114.57 \Rightarrow$ Good Design

1-36



$\Sigma M_A = 0$

$$480 \times 3000 - 640 \times 3000 + R_c \times \frac{2}{\sqrt{5}} \times 1500 + R_c \frac{1}{\sqrt{5}} \times 6000 = 0$$

$$R_c = -119.26 \text{ kN}$$

$\Sigma F_x = 0$

$$\frac{R_A}{\sqrt{2}} + 119.26 \times \frac{2}{\sqrt{5}} - 480 = 0$$

$$R_A = 373.33 \text{ kN}$$

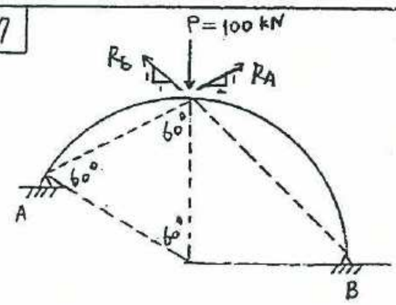
$$\sigma_A = \frac{373.33 \times 10^3}{\frac{1}{4}\pi \times 102^2} = 45.69 \text{ MPa}$$

(comp.)

$$\sigma_c = \frac{119.26 \times 10^3}{\frac{1}{4}\pi \times 102^2} = 14.6 \text{ MPa}$$

(tension)

1-37



$$R_{Ax} = R_{Bx}$$

$$R_{Ay} = \frac{1}{3} \times 100 = 33.3 \text{ kN}$$

$$R_{By} = \frac{2}{3} \times 100 = 66.7 \text{ kN}$$

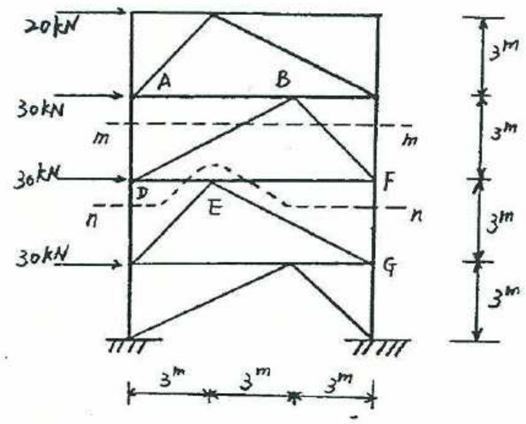
$$R_A = \sqrt{3}\, R_{Ay} = 33.3\sqrt{3} = 57.73 \text{ kN}$$

$$R_B = \sqrt{2}\, R_{By} = 66.7\sqrt{2} = 94.28 \text{ kN}$$

$$\tau_A = \frac{V_A}{A} = \frac{57.73 \times 10^3}{\frac{\pi}{4} \times 20^2} = 183.8 \text{ MPa}$$

$$\tau_B = \frac{V_B}{A} = \frac{94.28 \times 10^3}{\frac{\pi}{4} \times 20^2} = 300.0 \text{ MPa}$$

1-38



section m-m

$\Sigma F_H = 0$

$$-\frac{2}{\sqrt{5}} P_{BD} + \frac{1}{\sqrt{2}} P_{BF} + 50 = 0$$

joint B , $\Sigma F_v = 0$

$$\frac{1}{\sqrt{5}} P_{BD} + \frac{1}{\sqrt{2}} P_{BF} = 0$$

$$50 = \frac{3}{\sqrt{5}} P_{BD} \qquad P_{BD} = 37.27 \text{ kN}$$

joint D , $\Sigma F_H = 0$

$$P_{DE} + \frac{2}{\sqrt{5}} P_{BD} + 30 = 0$$

$$P_{DE} = -30 - \frac{2}{\sqrt{5}} \times 37.27 = -63.34 \text{ kN}$$

section n-n

$+\circlearrowleft \ \Sigma M_D = 0$

$$-30 \times 3 - 20 \times 6 - P_{FG} \times 9 = 0$$

$$P_{DG} = -32.89 \text{ kN}$$

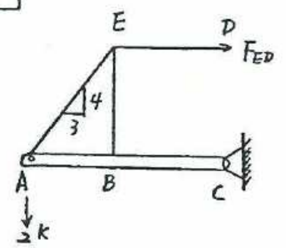
stress :

$$\sigma_{BD} = \frac{37.27 \times 10^3}{160} = 232.94 \text{ MPa (T)}$$

$$\sigma_{DE} = \frac{63.34 \times 10^3}{130} = 487.23 \text{ MPa (c)}$$

$$\sigma_{FG} = \frac{32.89 \times 10^3}{400} = 82.23 \text{ MPa (c)}$$

1-39



$$\Sigma M_c = 0 \Rightarrow F_{ED}(4) - 2(6) = 0$$

$$F_{ED} = \frac{12}{4} = 3 \text{ k}$$

$$\Sigma F_h = 0 \Rightarrow F_{AE} \frac{3}{5} - 3 = 0$$

$$F_{AE} = 5 \text{ k}$$

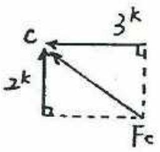
Force par bar = 2.5 k

$$\sigma = \frac{2.5 \times 10^3}{6 \times 25} = 16.67 \text{ MPa}$$

Double shear at C

$$\Sigma F_h = 0 \Rightarrow F_{hc} = 3^k$$

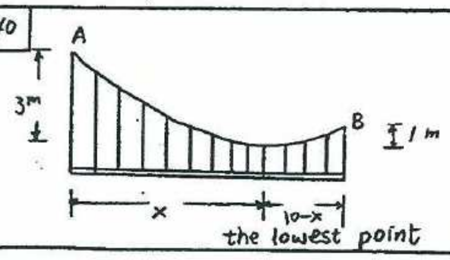
$$\Sigma F_v = 0 \Rightarrow F_{vc} = 2^k$$



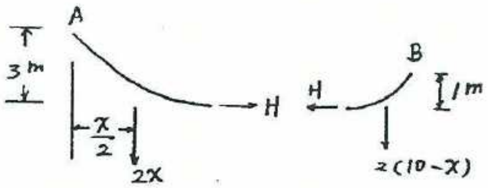
$$F_c = \sqrt{3^2 + 2^2} = 3.61 \text{ k}$$

$$\text{Pin shear} = \frac{F}{A} = \left(\frac{3.61}{\pi \frac{19^2}{4}}\right)\frac{1}{2} = 6.36 \text{ MPa}$$

1-40



1-40



$\Sigma M_A = 0$

$3 \cdot H = 2x \cdot \frac{x}{2} \qquad H = \frac{1}{3}x^2$

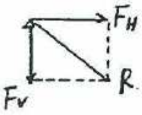
$\Sigma M_B = 0$

$\frac{2(10-x)^2}{2} = H \qquad H = (10-x)^2$

$\rightarrow \frac{1}{3}x^2 = (10-x)^2$

$x = 6.34\ m \quad , H = 13.40\ kN$

at support



$F_v = 2x = 12.68\ kN$

$R = \sqrt{13.4^2 + 12.68^2} = 18.45\ kN$

$A = \frac{R}{\sigma_y} \times 2 = \frac{18.45 \times 10^3}{1000} \times 2$

$= 36.9\ mm^2$

1-41

$F_1 = (R_1 + R_2 + R_3)\, m\omega^2$

$= (0.6 + 1.2 + 1.8)(0.5)(4 \times 2\pi)^2$

$= 1136.98\ N$

$F_2 = (R_2 + R_3)\, m\omega^2$

$= (1.2 + 1.8)(0.5)(4 \times 2\pi)^2$

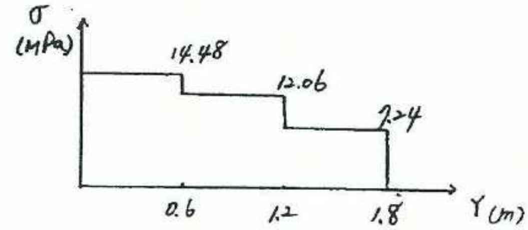
$= 947.48\ N$

$F_3 = R_3\, m\omega^2 = (1.8)(0.5)(4 \times 2\pi)^2 = 568.49\ N$

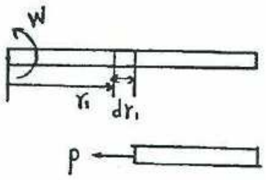
$\sigma_1 = \frac{1136.98}{\pi(5)^2} = 14.48\ MPa$

$\sigma_2 = \frac{947.48}{\pi(5)^2} = 12.06\ MPa$

$\sigma_3 = \frac{568.49}{\pi(5)^2} = 7.24\ MPa$



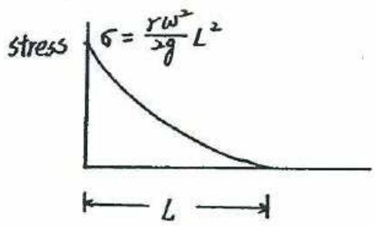
1-42



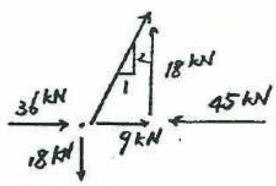
$P = \int_r^L \left(\frac{\gamma A\, dr_1}{g}\right) \omega^2 dr_1$

$= \frac{\gamma A \omega^2}{2g}(L^2 - r^2)$

$\sigma = \frac{P}{A} = \frac{\gamma\omega^2}{2g}(L^2 - r^2)$



1-43

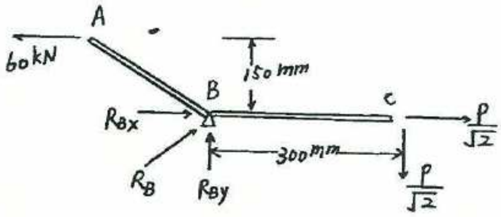


$\frac{45 \times 10^3}{2(\frac{\pi}{4}d^2)} \le 20$

$d \ge 37.85\ mm$

$d = 40\ mm$

1-44



$\Sigma M_B = 0 \qquad \frac{P}{\sqrt{2}} \times 300 = 60 \times 150$

$\frac{P}{\sqrt{2}} = 30\ kN$

$\Sigma F_H = 0 \qquad 30 + R_{Bx} - 60 = 0$

$R_{Bx} = 30\ kN$

$\Sigma F_V = 0 \qquad R_{By} = 30\ kN$

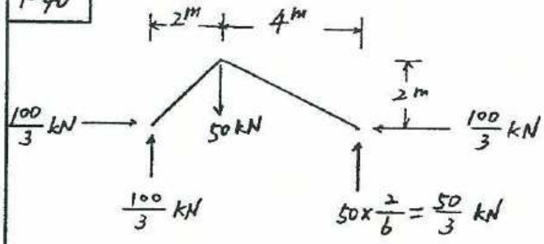
$\therefore R_B = 30\sqrt{2} = 42.43\ kN$

$$\tau = \frac{R_B}{2 \times \frac{\pi}{4} D^2}$$

$$D = \sqrt{\frac{R_B}{\frac{\pi}{2}\tau}} = \sqrt{\frac{42.43 \times 10^3}{\frac{\pi}{2} \times 100}}$$

$= 16.44\ mm$

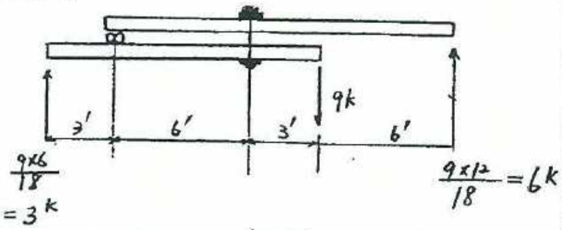
1-46



$$\frac{P}{A} = \frac{\tau}{F.S}$$

$$\frac{\frac{100}{3}}{200a} = \frac{3.5}{5} \qquad a = b = 23.81\ mm$$

1-47



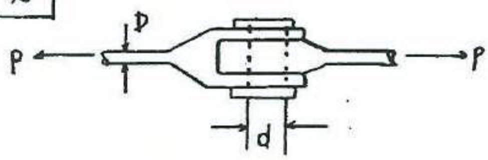
$$\text{tension} = \frac{6 \times 15}{6} = 15^k$$

$$A = \frac{P}{\sigma} = \frac{15}{18} = 0.833\ in^2$$

use $1\frac{1}{4}$" bolt

$$A_{br} = \frac{15}{0.5} = 30\ in^2$$

1-45



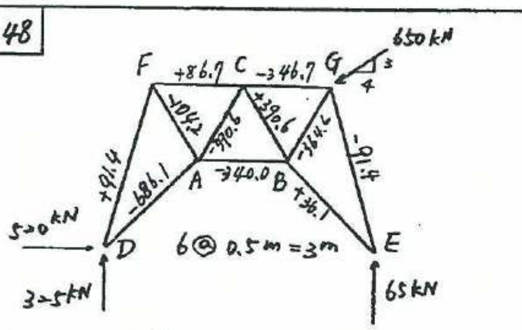
$$\sigma_a = P \cdot \frac{4}{\pi D^2}$$

$$\tau_a = \frac{P}{2} \cdot \frac{4}{\pi d^2}$$

$$\tau_a = \frac{\sigma_a}{2}$$

$$P \cdot \frac{4}{\pi D^2} = 2 \cdot \frac{P}{2} \cdot \frac{4}{\pi d^2} \qquad \therefore D = d$$

1-48



$\sigma_{All} = 140\ MPa$

Tension members : $\overline{DF}$, $\overline{FC}$, $\overline{BE}$, $\overline{CB}$

$$A_{DF} = \frac{P}{\sigma} = \frac{91.4 \times 10^3\ N}{140 \times 10^6\ N/m^2} = 652.8 \approx 653\ mm^2$$

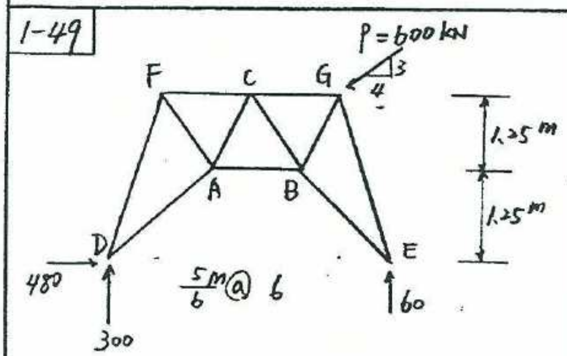
1-48

$$A_{FC} = \frac{P}{\sigma} = \frac{86.7\times10^3}{140} = 619.3 \simeq 620\ mm^2$$

$$A_{BC} = \frac{390.6\times10^3}{140} = 2790\ mm^2$$

$$A_{BE} = \frac{36.1\times10^3}{140} = 257.9 \approx 258\ mm^2$$

1-49

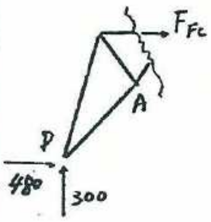


$\Sigma F_x = 0 \quad R_{Dx} = 480\ kN(\rightarrow)$

$\Sigma M_D = 0 \ \circlearrowright + \quad 360\times\frac{25}{6} - 480\times2.5 - R_E\times5 = 0$

$R_E = 60\ kN\ (\uparrow)$

$\Sigma F_y = 0 \quad R_{Dy} = 300\ kN(\uparrow)$

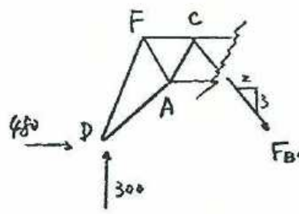


$\Sigma M_A = 0 \ \circlearrowright +$

$F_{FC}\times1.25 + 300\times\frac{10}{6} - 480\times1.25 = 0$

$F_{FC} = 80\ kN$ (tension)

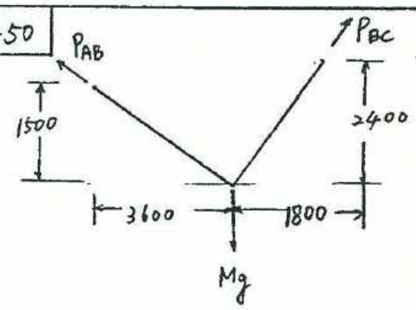
$$A_{FC} = \frac{F_{FC}}{\sigma_{allow}} = \frac{80\times10^3}{140} = 571.43 \simeq 572\ mm^2$$



$F_{BC} = 300\times\frac{\sqrt{13}}{3} = 100\sqrt{13} = 360.56\ kN$ (tension)

$$A_{BC} = \frac{F_{BC}}{\sigma_{allow}} = \frac{360.56\times10^3}{140} = 2575.39 \approx 2576\ mm^2$$

1-50



$$\begin{cases} \frac{12}{13}P_{AB} = \frac{3}{5}P_{BC} \\ \frac{5}{13}P_{AB} + \frac{4}{5}P_{BC} = Mg \end{cases}$$

$P_{AB} = \frac{13}{21}Mg \ , \quad P_{BC} = \frac{20}{21}Mg$

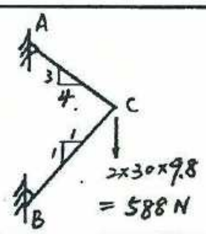
$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{13}{21}\cdot\frac{Mg}{200} = 3.1\ Mg\ kPa$ (control)

$\sigma_{BC} = \frac{20}{21}\cdot\frac{Mg}{400} = 2.38\ Mg\ kPa$

$\sigma_w = \frac{\sigma_u}{F.S} = \frac{800}{2} = 3.1\ Mg$

$M = 13.169\ kg$

1-51



1-51

$F_{AC,v} = +\frac{3}{7} \times 588 = 252\ N$

$F_{AC} = \frac{5}{3} \times 252 = 420\ N$ (tension)

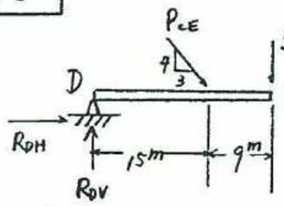
$A_{AC} = \frac{F_{AC}}{\sigma_t} = \frac{420}{140} = 3\ mm^2$

$F_{BC,v} = \frac{-4}{7} \times 588 = -336\ N$

$F_{BC} = \sqrt{2} \times (-336) = -475.18$ (Comp.)

$A_{BC} = \frac{F_{BC}}{\sigma_c} = \frac{-475.18}{-96} = 5\ mm^2$

1-52



$\Sigma M_D = 0$

$24 \times 500 = \frac{4}{5} P_{CE} \times 15$

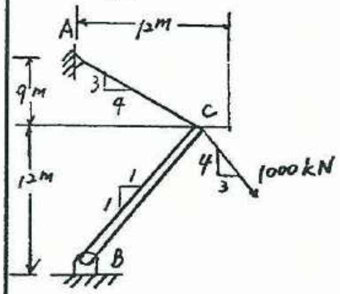
$P_{CE} = 1000\ kN$

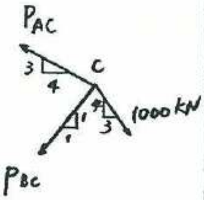
$\Sigma F_v = 0$

$R_{DV} + \frac{4}{5} P_{CE} - 500 = 0$

$R_{DV} = 300\ kN\ (\downarrow)$

$R_{DH} = \frac{3}{5} P_{CE} = 600\ kN\ (\rightarrow)$





$P_{AC} = 1000\ kN$, $P_{BC} = -282.8\ kN$

$R_{AH} = \frac{4}{5} P_{AC} = 800\ kN\ (\leftarrow)$

$R_{AV} = \frac{3}{5} P_{AC} = 600\ kN\ (\uparrow)$

$R_{BH} = \frac{1}{\sqrt{2}} P_{BC} = 200\ kN\ (\rightarrow)$

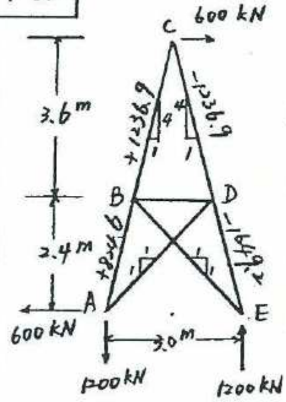
$R_{BV} = \frac{1}{\sqrt{2}} P_{BC} = 200\ kN\ (\uparrow)$

$A_{AC} = \frac{P_{AC}}{\sigma_{all}} = \frac{1000 \times 10^3}{140} = 7143\ mm^2$

$A_{BC} = \frac{P_{BC}}{\sigma_{all}} = \frac{282.8 \times 10^3}{100} = 2828\ mm^2$

$A_{CE} = \frac{P_{CE}}{\sigma_{all}} = \frac{1000 \times 10^3}{140} = 7143\ mm^2$

1-53



$A_{AB} = \frac{824.6}{14} = 58.90\ cm^2$

$A_{BC} = \frac{1236.9}{14} = 88.35\ cm^2$

$A_{BE} = \frac{565.7}{14} = 40.41\ cm^2$

$A_{AD} = \frac{565.7}{14} = 40.41\ cm^2$

$A_{BD} = \frac{500}{10} = 50.00\ cm^2$

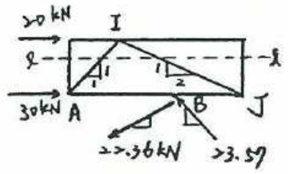
$A_{CD} = \frac{1236.9}{10} = 123.69\ cm^2$

$A_{DE} = \frac{1649.2}{10} = 164.92\ cm^2$

1-54

from problem 1-38, $P_{BF} = \frac{-\sqrt{2}}{\sqrt{5}} P_{BD}$

$= -23.57\ kN$



section l-l

$\Sigma F_h = 0 \quad 20 + \frac{2}{\sqrt{5}} P_{IJ} - \frac{1}{\sqrt{2}} P_{AI} = 0$

joint I, $\Sigma F_v = 0 \quad \frac{1}{\sqrt{2}} P_{AI} + \frac{1}{\sqrt{5}} P_{IJ} = 0$

$\Rightarrow P_{IJ} = -14.9\ kN$, $P_{AI} = -\frac{\sqrt{2}}{\sqrt{5}} P_{IJ} = 9.43\ kN$

joint A, $\Sigma F_h = 0 \quad P_{AB} + \frac{1}{\sqrt{2}} P_{AI} + 30 = 0$

$\Rightarrow P_{AB} = -30 - \frac{1}{\sqrt{2}} P_{AI} = -36.88\ kN$

1-54

$\Sigma M_J = 0$

$P_{AD} \times 9 + 22.36 \times \frac{1}{\sqrt{5}} \times 3 = 23.57 \times \frac{1}{\sqrt{2}} \times 3$

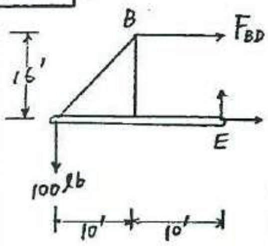
$P_{AD} = 2.22\ kN$

$A_{AB} = \frac{P_{AB}}{\sigma_{all}} = \frac{36.88 \times 10^3}{85} \simeq 434\ mm^2$

$A_{AD} = \frac{P_{AD}}{\sigma_{all}} = \frac{2.22 \times 10^3}{120} \simeq 18\ mm^2$

$A_{BF} = \frac{P_{BF}}{\sigma_{all}} = \frac{23.57 \times 10^3}{85} \simeq 277\ mm^2$

1-55



$\Sigma M_E = 0$

$F_{BD} \times 16 = 100 \times 20$

$\Rightarrow F_{BD} = 125\ lbf$

Joint B, $\Sigma F_h = 0$

$F_{AB} \times \frac{10}{\sqrt{10^2 + 16^2}} = 125$

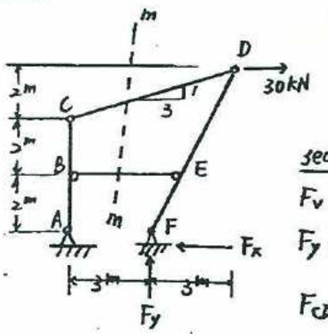
$\Rightarrow F_{AB} = 235.8\ lbf$

$A_{req} = \frac{F_{AB}}{\sigma_w} = \frac{235.8}{40 \times 10^3} = 5.90 \times 10^{-3}\ in^2$

$D = \sqrt{\frac{4 A_{req}}{\pi}} = \sqrt{\frac{4 \times 5.90 \times 10^{-3}}{\pi}}$

$= 0.087\ in$

1-56



$\Sigma M_A = 0$

$F_y \times 3 = 30 \times 6$

$F_y = 60\ kN$

section m-m

$F_v = 0$

$F_y = F_{CD} \times \frac{1}{\sqrt{10}}$

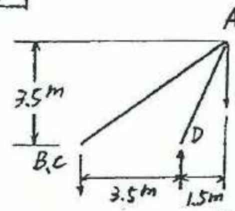
$F_{CD} = 189.74\ kN$

$\sigma_w = \frac{\sigma_u \times 0.8}{F.S} = \frac{1250 \times 0.8}{1.5} = 666.67\ MPa$

$A = \frac{F_{CD}}{\sigma_w} = \frac{189.74 \times 10^3}{666.67} = 248.61\ mm^2$

$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 248.61}{\pi}} = 17.8\ mm$

1-57



$\Sigma M_D = 0,$

$V_{AB} = \frac{200 \times 1.5}{2 \times 3.5}$

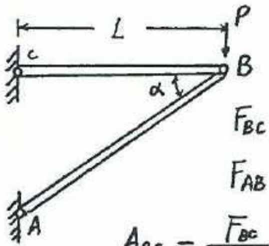
$= 42.86\ kN$

$R_{AB} = R_{AC} = V_{AB} \frac{\sqrt{(3.5)^2 + [(5)^2 + (2.5)^2]}}{3.5}$

$= 42.86 \times \frac{6.6}{3.5} = 80.82\ kN$

$A = \frac{\pi d^2}{4} = \frac{R}{\sigma_{all}} = \frac{80.82}{12},\quad d = 2.93\ cm$

1-58



$F_{BC} = P \cot \alpha$

$F_{AB} = \frac{P}{\sin \alpha}$

$A_{BC} = \frac{F_{BC}}{\sigma} = \frac{P}{\sigma} \cot \alpha$

$A_{AB} = \frac{F_{AB}}{\sigma} = \frac{P}{\sigma} \frac{1}{\sin \alpha}$

$V = L A_{BC} + \frac{L}{\cos \alpha} A_{AB}$

$= \frac{PL}{\sigma} \left(\cot \alpha + \frac{1}{\sin \alpha \cos \alpha} \right)$

$\frac{\partial V}{\partial \alpha} = 0, \quad \cos^2 \alpha = \frac{1}{3} \quad \therefore \alpha = 54.7^\circ$

1-59

(1) $z = \mu_z - \sigma_z$ $\qquad W = \frac{z-\mu_z}{\sigma_z} = \frac{(\mu_z-\sigma_z)-\mu_z}{\sigma_z}$

$= -1$

$z = \mu_z + \sigma_z$ $\qquad W = \frac{(\mu_z+\sigma_z)-\mu_z}{\sigma_z} = 1$

$\therefore P(\mu_z-\sigma_z < z < \mu_z+\sigma_z)$

$= P(-1 < W < 1)$

$= P(W<1) - P(-1<W)$

$= 0.8413 - 0.1587$ *

$= 0.6827$

(2) $z = \mu_z - 2\sigma_z$ $\qquad W = \frac{(\mu_z-2\sigma_z)-\mu_z}{\sigma_z} = -2$

$z = \mu_z + 2\sigma_z$ $\qquad W = 2$

$\therefore P(\mu_z-2\sigma_z < z < \mu_z+2\sigma_z)$

$= P(-2<W<2)$

$= P(W<2) - P(-2<W)$

$= 0.9772 - 0.0228$ *

$= 0.9545$

* See "Probability and Statistics for Engineering and Scientists" 5ed. by R.E. Walpole and R.H. Myers. <Table A.3>

1-60

from Fig. 1-26 (c)

strength (unit: 10^2 psi)	freq.	X_i	$X_i-\bar{X}$	$(X_i-\bar{X})^2$
20~22	1	2100	-1840	3385600
22-24	1	2300	-1640	2689600
24-26	2	2500	-1440	2073600
26-28	3	2700	-1240	1537600
28-30	6	2900	-1040	1081600
30~32	11	3100	-840	705600
32~34	12	3300	-640	409600
34~36	17	3500	-440	193600
36~38	21	3700	-240	57600
38~40	22	3900	-40	1600
40~42	20	4100	160	25600
42~44	17	4300	360	129600
44~46	13	4500	560	313600
46~48	9	4700	760	577600
48~50	11	4900	960	921600
50~52	7	5100	1160	1345600
52~54	1	5300	1360	1849600
54~56	1	5500	1560	2433600
56~58	0	5700	1760	0
58~60	0	5900	1960	0
60~62	1	6100	2160	4665600
Total	176			79033600

$$S = \sqrt{\frac{1}{n}(X_i-\bar{X})^2}$$

$$= \sqrt{\frac{79033600}{176}}$$

$= 670$ psi

2-1

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta/L} = \frac{(29500)/\frac{\pi}{4}(0.013)^2}{(2.2\times10^{-4})/0.2} = 202\times10^9\ \frac{N}{m^2}$$

2-2

$$A = \max \text{ of } \begin{cases} \dfrac{P}{\sigma} = \dfrac{5\times10^3}{150} = 33.33\ mm^2 \\[2mm] \dfrac{PL}{\Delta E} = \dfrac{(5\times10^3)(10^4)}{(3)(2\times10^5)} = 83.33\ mm^2 \end{cases}$$

$$\frac{\pi}{4}d^2 = 83.33 \longrightarrow d = 10.3\ mm$$

Use 11 mm rod

2-3

(a) $$\frac{PL_1}{AE_{st}} = \frac{PL_2}{AE_{Al}}$$

$$L_1 = \frac{E_{st}}{E_{Al}} L_2 = \frac{200}{70} L_2$$

$$L_2 = \frac{900}{(1+\frac{20}{7})} = 233\ mm \quad , \quad L_1 = 667\ mm$$

(b) $$\Delta = \Sigma \frac{PL}{EA} = \frac{1.2\times10^5}{\pi(25)^2}\left(\frac{667}{2\times10^5} + \frac{233}{7\times10^4}\right)$$

$$\Delta = 0.407\ mm$$

2-4

$$\varepsilon = \frac{\Delta}{L} = \text{CONSTANT}$$

$$\frac{0.035}{200} = \frac{600}{L} \quad , \quad L = 3429\ m$$

2-5

$$\bar{\sigma} = \sigma(1+\varepsilon)$$

$$A = A_0/(1+\varepsilon)$$

	Compression					tension				
strain (%)	-0.1	-0.2	-1	-2	-4	0.1	0.2	1	2	4
eng. stress	18	36	72	100	128	18	36	72	85	92
true stress	18.0	35.9	71.3	98.0	122.9	18.0	36.1	72.7	86.7	95.7
true area (A_0)	1.00	1.00	1.01	1.02	1.04	1.00	1.00	0.99	0.98	0.96

2-6

$$R_0^2 - (R_0 - 300)^2 = (25000)^2$$

$$R_0 = 1041816.667\ mm$$

$$\theta_0 = 2\sin^{-1}\frac{25000}{R_0}$$

$$L_0 \approx R\theta = (1041816.667)\left(\frac{\pi}{180}\times\theta_0\right) = 50004.8\ mm$$

$$L = L_0[1 + (16\times10^{-6})(70)] = 50060.8\ mm$$

$$2R\sin^{-1}\frac{25000}{R} = 50060.8 \longrightarrow R \approx 293163$$

$$(293163)^2 - (293163 - S)^2 = (25000)^2$$

$$S = 1068\ mm$$

Approximate by parabola, $Y = \frac{4H}{L^2}(X-L)X$

$$S \approx \int_0^L \sqrt{1+\left(\frac{\partial Y}{\partial X}\right)^2}\, dX$$

2-7

$$\varepsilon_z = \frac{P}{AE} = \frac{200\times10^3}{\frac{\pi}{4}(60)^2(85\times10^3)} = 8.322\times10^{-4}$$

$$\frac{\Delta d}{60} = \nu\varepsilon_z \longrightarrow \Delta d = 60(0.3)(8.322\times10^{-4})$$
$$= 0.015\ mm$$

2-8

$$P_{st} = \varepsilon E A/\nu = \alpha\Delta T E A/\nu$$
$$= (11.7\times10^{-6})(200)(200)\left(\frac{\pi}{4}\times20^2\right)/0.3 = 490\ kN$$

$$P_{al} = (23.4\times10^{-6})(200)(70)\left(\frac{\pi}{4}\times20^2\right)/0.35 = 294\ kN$$

2-9

(a) $$F = \frac{5(9.81)}{\cos30°} = 56.64$$

$$\frac{F}{\sigma_{all}} = \frac{\pi}{4}d^2 \longrightarrow d = \sqrt{\frac{4(56.64)}{\pi(300)}} = 0.49\ mm$$

(b) $$\Delta = \frac{\sigma}{E}L_0 = \frac{300}{(200\times10^3)}(1.5\times10^3) = 2.25\ mm$$

2-10

U_T (tool steel) ≈ 11 ksi

U_T (Low-alloy high-strength steel) ≈ 17 ksi

U_T (Low-carbon steel) ≈ 18 ksi

2-11

(a) $\dot{\sigma} + \frac{E}{\eta}\sigma = E\dot{\varepsilon}$

But $\frac{d\varepsilon}{dt} = Const = c = \dot{\varepsilon}$

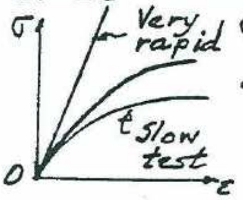
or $\varepsilon = ct$ or $t = \frac{\varepsilon}{c}$

Hence $\dot{\sigma} + \frac{E}{\eta}\sigma = Ec$

Solution: $\sigma = Ae^{-\frac{E}{\eta}t} + \eta c$

or $\sigma = Ae^{-\frac{E\varepsilon}{\eta c}} + \eta c$

Since $\sigma = 0$ for $\varepsilon = 0$, $\sigma = \eta c\left(1 - e^{-E\varepsilon/\eta c}\right)$

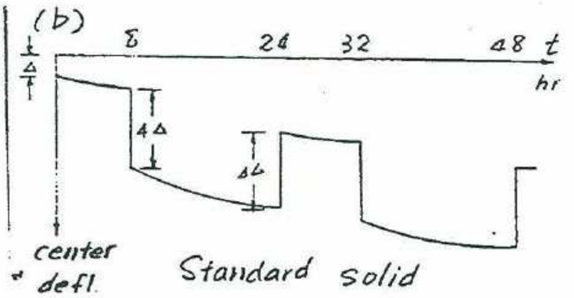


and $\frac{d\sigma}{d\varepsilon} = Ee^{-E\varepsilon/\eta c}$
$= Ee^{-Et/\eta}$

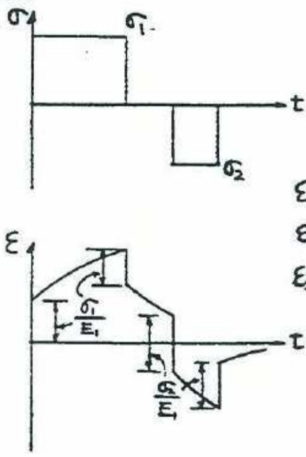
(b) At $t = 0$, $E = Const$

(c) Can be misleading

(b)



2-12



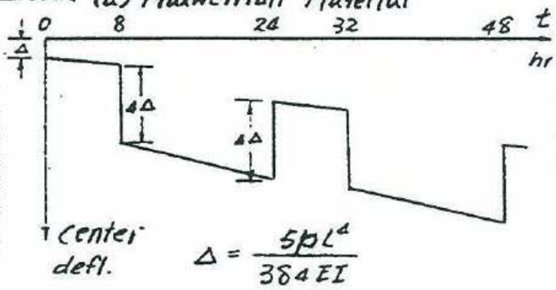
$\varepsilon = \varepsilon_1 + \varepsilon_2$

ε_1 : due to E_1 & η_1

ε_2 : due to E_2 & η_2

2-13

(a) Maxwellian Material



$\Delta = \frac{5pL^4}{384EI}$

p = empty wt. / unit length

2-14

$\dot{\varepsilon} = B\sigma^n$

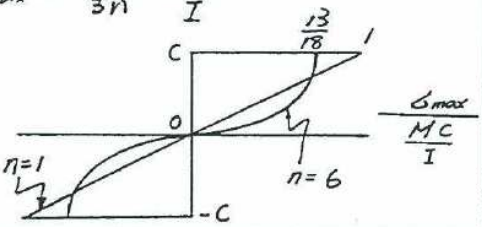
$$\frac{\sigma^n}{\sigma_{max}^n} = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{max}} = \frac{y}{c} \rightarrow \sigma = \left(\frac{y}{c}\right)^{\frac{1}{n}}\sigma_{max}$$

$$M = 2\int_0^c \sigma y\,dA = \frac{2b\sigma_{max}}{c^{\frac{1}{n}}}\int_0^c y^{\frac{n+1}{n}}dy$$

$$M = \frac{2b\sigma_{max}}{c^{\frac{1}{n}}}\ \frac{n}{2n+1}\ c^{\frac{2n+1}{n}}$$

$$M = \frac{n}{2n+1}\,2bc^2\sigma_{max} = \frac{3n}{2n+1}\ \frac{I\sigma_{max}}{c}$$

$$\sigma_{max} = \frac{2n+1}{3n}\ \frac{Mc}{I}$$



2-15

from Fig. 2-5, the ultimate stress for steel is 186 Mpa.

for aluminum is 150 MPa

2-16

(a) from Fig. 2-5

Al: $\frac{2\times10^6}{5.3\times10^6} + \frac{4\times10^5}{8.7\times10^5} = 0.84 < 1$ (O.K.)

St: $\frac{2\times10^6}{1.1\times10^6} + \frac{4\times10^5}{2\times10^5} = 3.82 > 1$ (N.G.)

(b)

Al: $\frac{5\times10^6}{1.8\times10^7} + \frac{4\times10^8}{5\times10^8} = 1.08 > 1$ (N.G.)

St: $0 + 0 < 1$ (O.K.)

3-1

$$\sigma = \frac{P}{A} = \frac{10\times10^3}{\frac{\pi}{4}d^2} \le 100$$

$d \geqslant 11.28$ mm

$$\epsilon = \frac{\sigma}{E} = \frac{\frac{10\times10^3}{\pi d^2/4}}{45\times10^3} \le 0.1\%$$

$d \geqslant 16.82$ mm (governs)

magnesium alloy elastic modulus:
45 GPa for tension or compression.

3-2

(a) $A_{req} = \frac{5\times10^3}{150} = 33.33 \ mm^2$

$$A_{req} = \frac{PL}{\Delta E} = \frac{5\times10^3\times10\times10^3}{4\times210\times10^3} = 59.52 \ mm^2$$

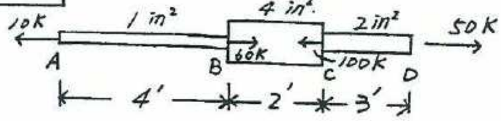
(governs)

$\longrightarrow$ stiffness control.

(b) $P = K\Delta$

$\Rightarrow K = \frac{P}{\Delta} = \frac{5}{4} = 1.25 \ KN/mm$

3-3



(a) $\Sigma F = 0$, $P_2 = 60K$

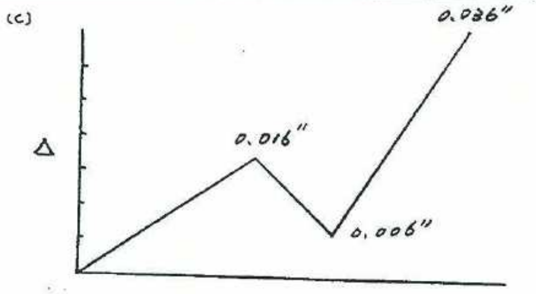
(b) $\Delta = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD}$

$$= \frac{10(4\times12)}{1(30\times10^3)} + \frac{(-50)(2\times12)}{4(30\times10^3)} + \frac{50(3\times12)}{2(30\times10^3)}$$

$= 0.016 - 0.010 + 0.03$

$= 0.036''$

(c)



(d) $f_{eff.} = \Sigma \frac{L_i}{(EA)_i}$

$$= \frac{1}{30\times10^3}\left(\frac{4\times12}{1} + \frac{2\times12}{4} + \frac{3\times12}{2}\right)$$

$= 2.4\times10^{-3} \ in/_K$

$K_{eff.} = \frac{1}{2.4\times10^{-3}} = 416.67 \ K/in$

3-4

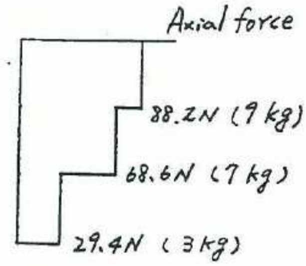
$$\Delta = \frac{PL}{AE} = \frac{mgL}{AE}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{AE}{mL}}$$

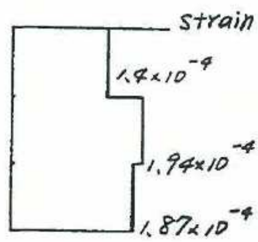
$$= \frac{1}{2\pi}\sqrt{\frac{\frac{\pi}{4}\cdot20^2\cdot180\times10^3}{2\times0.4}} = 1.3 \ KH_z$$

3-5

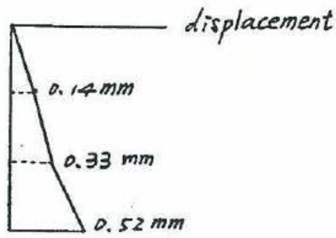
(a)



(b)

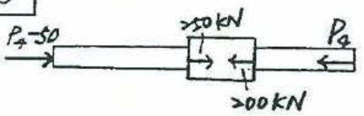


(c)



0 0

3-6

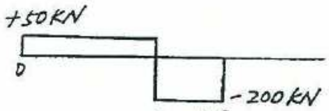


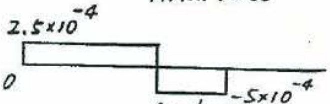
$$\Delta = \Sigma\left(\frac{PL}{EA}\right)_i$$

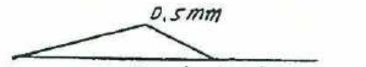
$$= -\frac{(P_4-50)\times 2000}{1000\times E} - \frac{(P_4+200)\times 1000}{2000\times E} - \frac{P_4\times 1500}{1000\times E}$$

$$= 0$$

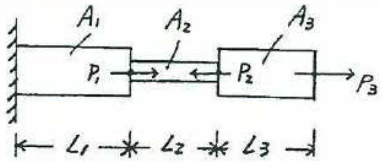
$$\rightarrow \quad P_4 = 0$$







3-7

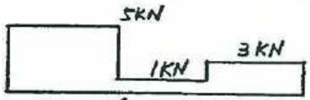


$P_1 = 4$ KN $\quad L_1 = 0.6m \quad A_1 = 2\times10^{-4}m^2$

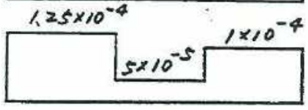
$P_2 = 2$ KN $\quad L_2 = 0.5m \quad A_2 = 1\times10^{-4}m^2$

$P_3 = 3$ KN $\quad L_3 = 0.4m \quad A_3 = 1.5\times10^{-4}m^2$

Axial force



Axial strain

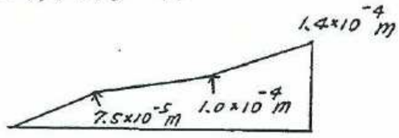


$$\Delta = \frac{PL}{AE} = \epsilon L$$

$$\Delta_1 = 1.25\times10^{-4}(0.6) = 7.5\times10^{-5}\ m$$

$$\Delta_2 = \Delta_1 + \Delta_2 = (7.5\times10^{-5}) + (5\times10^{-5})(0.5)$$
$$= 1.0\times10^{-4}\ m$$

$$\Delta_3 = \Delta_1 + \Delta_2 + \Delta_3$$
$$= 1.0\times10^{-4} + (1\times10^{-4})(0.4)$$
$$= 1.4\times10^{-4}\ m$$



Axial displacement

$$K = \frac{P}{\Delta} \quad \text{(use unit Force)}$$

$$\Delta = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

$$= \frac{1}{200\times10^6}\left(\frac{0.6}{2\times10^{-4}} + \frac{0.5}{1\times10^{-4}} + \frac{0.4}{1.5\times10^{-4}}\right)$$

$$= 5.33\times10^{-5}$$

$$\Rightarrow K = \frac{P}{\Delta} = \frac{1}{5.33\times10^{-5}} = 1.875\times10^4\ KN/m$$

3-8

(a) $\sigma_{AL} = E\epsilon = 70\times10^3\times873\times10^{-6}$

$= 61.1\ MPa = \sigma_{st}$

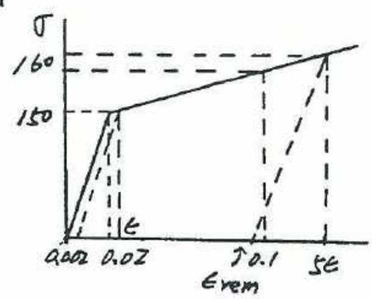
$P = \frac{\pi}{4}d^2\sigma = \frac{\pi}{4}\times50^2\times61.1 = 120\ KN$

(b) $\epsilon_{st} = \frac{\sigma_{st}}{E_{st}} = \frac{61.1}{200\times10^3} = 305.5\times10^{-6}$

$\Delta = [(305.5\times1500) + (873\times500)]\times10^{-6}$

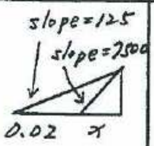
$= 0.89\ mm$

3-9



$$125(0.02 + x) = 7500\times x$$

$x = 0.00034$



$\epsilon = 0.0034 + 0.22 = 0.2234$

$5\epsilon = 0.1117$

$\sigma = 150 + (0.1117 - 0.02) \times 125 = 161.4625$ MPa

$F = 161.4625 \times 0.2 = 32.29$ kN

$\epsilon_{rem} = 0.1117 - \frac{161.4625}{2500} = 0.09$

$\Delta_{rem} = 0.09 \times 600 = 54$ mm.

3-10

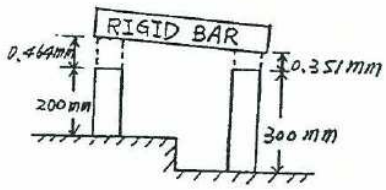
(a) $\Delta_{AL} = \alpha L \Delta T = 23.2 \times 10^{-6} \times 200 \times 100$

$= 0.464$ mm

$\Delta_{st} = \alpha_{st} L_{st} \Delta T = 11.7 \times 10^{-6} \times 300 \times 100$

$= 0.351$ mm

Inclination $\simeq \frac{0.464 - 0.351}{400} = 2.83 \times 10^{-4}$ rad



(b) $\Delta = \frac{PL}{EA} = \frac{\sigma L}{E} \Rightarrow \sigma = \frac{\Delta E}{L}$

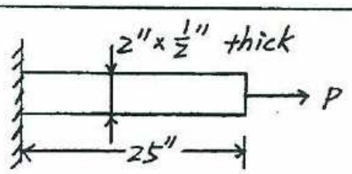
$\sigma_{AL} = \frac{\Delta_{AL} E_{AL}}{L} = \frac{0.464 \times 75 \times 10^3}{200}$

$= 174$ MPa

$\sigma_{st} = \frac{\Delta_{st} E_{st}}{L_{st}} = \frac{0.351 \times 200 \times 10^3}{300}$

$= 234$ MPa

3-11



$\Delta_x = 0.5 \times 10^{-3}$ in

$\epsilon_x = \frac{-0.3 \times 10^{-3}}{2} = -0.15 \times 10^{-3}$

$\nu = \frac{0.15 \times 10^{-3}}{\epsilon_a} = 0.25$

$\epsilon_a = 6 \times 10^{-4}$

$\sigma = E\epsilon_a = (30 \times 10^3) \times 6 \times 10^{-4}$

$= 18$ ksi

$P = 18 \times 2 \times \frac{1}{2} = 18$ k

$\Delta = 25 \times 6 \times 10^{-4} = 0.015$ in

3-12

$\epsilon_x = \frac{-16 \times 10^{-3}}{180} = -8.9 \times 10^{-5}$

$\Rightarrow 8.89 \times 10^{-5} = -\nu \frac{\sigma_y}{E}$

$\sigma_y = \frac{8.89 \times 10^{-5} \times 2 \times 10^5}{0.25} = 71.2$ N/mm^2

$P_{a\text{-}a} = \sigma_y A = 71.2 \times 180 \times 10 = 12.8 \times 10^4$ N

$2g \times 400 = P_{a\text{-}a}$

$g = \frac{12.8 \times 10^4}{800} = 160$ N/mm

$P_x = 2gx = 2 \times 160x = 320x$ N

$\Delta = \int \frac{P_x\,dx}{AE} = \frac{320}{180 \times 10 \times 2 \times 10^5} \times \frac{x^2}{2}\Big|_0^{2000}$

$= 1.8$ mm

3-13

(a) $F = \int_0^L f\,dy = \int_0^L ky^2\,dy = \frac{kL^3}{3}$

$k = \frac{3F}{L^3}$

$dF = f\,dy = ky^2\,dy$

$P = \int_0^y dP = \int_0^y ky^2\,dy = \frac{ky^3}{3} = \frac{y^3}{L^3}F$

$\Delta = \int_0^L \frac{P}{AE}\,dy = \int_0^L \frac{F}{AE}\frac{y^3}{L^3}\,dy = \frac{FL}{4AE}$

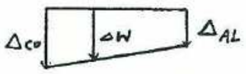
(b) $\Delta = \frac{400 \times 10^3 (10 \times 10^3)}{4(64000)(10^4)}$

$= 1.56$ mm

3-14

(a) $\Delta_{co} = \dfrac{PL}{AE} = \dfrac{\frac{2}{3}\times 2000 \times 60}{0.1\times 20\times 10^6} = 0.04''$

$\Delta_{AL} = \dfrac{PL}{AE} = \dfrac{\frac{1}{3}\times 2000\times 100}{0.2\times 10\times 10^6} = 0.033''$



$\Delta_W = \Delta_{co} - \frac{1}{3}(\Delta_{co} - \Delta_{AL})$

$= 0.04 - \frac{1}{3}(0.04 - 0.03\dot{3})$

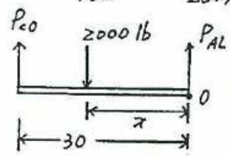
$= 0.038''$

(b) $\Delta_{co} = \Delta_{AL}$

$\Rightarrow \left(\dfrac{PL}{AE}\right)_{AL} = \left(\dfrac{PL}{AE}\right)_{co}$

$P_{AL} = \dfrac{E_{AL}A_{AL}}{L_{AL}} \times \dfrac{L_{co}}{E_{co}A_{co}} P_{co}$

$= \dfrac{10\times 10^6 \times 0.2}{100} \times \dfrac{60}{20\times 10^6 \times 0.1} P_{co} = 0.6 P_{co}$



$P_{co} + P_{AL} = 2000$

$1.6\, P_{co} = 2000$

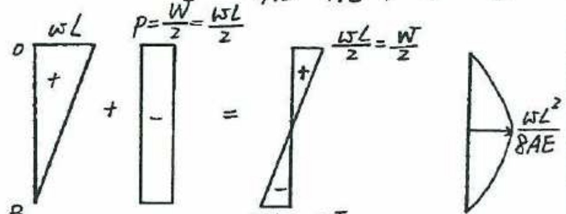
$P_{co} = 1250$ lb

$\Sigma M_0 = 0 \Rightarrow 1250\times 30 = 2000 \cdot x$

$x = 18.75$ inch from the right end

3-15

$\Delta_B = 0 \quad \dfrac{WL}{2AE} = \dfrac{PL}{AE}, \quad P = \dfrac{W}{2}$

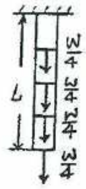


Axial force

Displacement

3-16

(a) subdivide L into 4 segments



$\Delta_4 = \sum_{i=1}^{4} \dfrac{P_iL_i}{EA}$

$= \dfrac{1}{EA}\cdot\dfrac{L}{4}\left(W + \frac{3}{4}W + \frac{2}{4}W + \frac{1}{4}W\right)$

$= \dfrac{1}{EA}\times\dfrac{L}{4}\times\dfrac{10}{4}W = \dfrac{5WL}{8EA}$

(b) subdivide L into 10 segments

$\Delta_{10} = \sum_{i=1}^{10} \dfrac{P_iL_i}{EA}$

$= \dfrac{1}{EA}\times\dfrac{L}{10}\times\dfrac{1}{10}(W + 2W + 3W + \cdots + 10W)$

$= \dfrac{L}{EA}\times\dfrac{55}{100}W = \dfrac{11WL}{20EA}$

(c) subdivide L into 20 segments

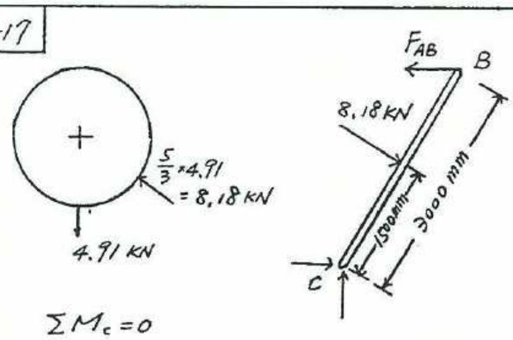
$\Delta_{20} = \sum_{i=1}^{20} \dfrac{P_iL_i}{EA}$

$= \dfrac{1}{EA}\times\dfrac{L}{20}\times\dfrac{1}{20}(W + 2W + \cdots + 20W)$

$= \dfrac{L}{EA}\times\dfrac{210W}{400} = \dfrac{21WL}{40EA}$

$\therefore \Delta_4 > \Delta_{10} > \Delta_{20} \simeq \Delta = \dfrac{WL}{2EA}$

3-17



$\Sigma M_c = 0$

$8.18 \times 1500 = F_{AB} \times 2400$

$F_{AB} = 5.11$ kN

$\Delta = \dfrac{F_{AB}L}{AE} = \dfrac{5.11\times 10^3 (1800)}{5(200\times 10^3)}$

$= 9.20$ mm

3-18

vertical bar force from C: $16\times\frac{3}{4}$

vertical bar force from D: $16\times\frac{4}{3}$

$$\Delta = \frac{PL}{AE} = \frac{1\times10^3}{100\times200}\times16\times\left(\frac{3}{4}+\frac{4}{3}\right)=1.67\ mm$$

3-19

$$\Delta = \frac{PL}{EA} = \frac{\sigma L}{E} = \frac{15\times20}{30\times10^3}$$

$= 0.01''$ (elongation of each rod)

$\Delta_B = \Delta = 0.01$ in

$\Delta_D = 0$

$\Delta_E = \Delta = 0.01$ in

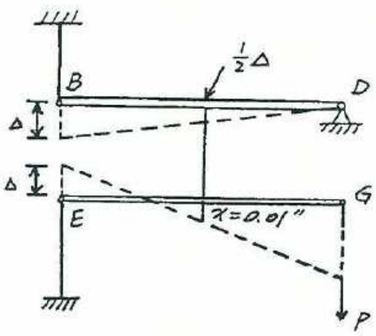
$\Delta_{CF} = \frac{15\times10}{30\times10^3} = 0.005$ in

$x - \frac{1}{2}\Delta = 0.005$

$x = 0.005 + 0.005 = 0.01$

$\frac{\Delta_G - \Delta}{2} = 0.001$

$\Delta_G = 0.02 + 0.01 = 0.03$ in



3-20

$$\Delta = \frac{wL^2}{2AE}$$

$$L^2 = \frac{2AE\Delta}{w} = \frac{2\times1\times10^6\times0.25}{1.17\times12}$$

$= 356\,000\ ft^2$

$\therefore L = 596.76\ ft$

3-21

$$\epsilon^n = E^{-1}\sigma,\quad \epsilon = \left(\frac{\sigma}{E}\right)^{\frac{1}{n}} = \left(\frac{P_x}{A_x E}\right)^{\frac{1}{n}}$$

$$\Delta = \int_0^L \epsilon\, dx = \int_0^L \left(\frac{wx}{AE}\right)^{\frac{1}{n}} dx$$

$$= \left(\frac{w}{AE}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left[x^{\frac{n+1}{n}}\right]\Big|_0^L$$

$$\Delta = \left(\frac{w}{AE}\right)^{\frac{1}{n}}(L)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)L$$

$$= \left(\frac{wL}{AE}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)L$$

3-22

(a) $\sigma_1 = 5/1 = 5$ ksi

$\sigma_2 = 5/0.5 = 10$ ksi

$$\Delta = \left[\frac{5}{16000}+\left(\frac{5}{165}\right)^3\right]\times50 + \left[\frac{10}{16000}+\left(\frac{10}{165}\right)^3\right]\times100$$

$= 0.102''$

(b) $d\epsilon = \frac{1}{16000}d\sigma + \frac{3}{165^3}\sigma^2 d\sigma$

$E = \frac{d\sigma}{d\epsilon}\Big|_{\sigma=0} = 16000$ ksi

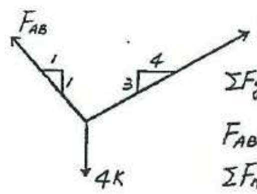
$\Delta_E = \frac{5}{E}\times50 + \frac{10}{E}\times100$

$= \frac{5}{16000}\times50 + \frac{10}{16000}\times100 = 0.078''$

$\Delta_{residual} = \Delta - \Delta_E$

$= 0.102 - 0.078 = 0.024''$

3-23



$\Sigma F_y = 0$

$F_{AB}\frac{1}{\sqrt{2}} + F_{BC}\frac{3}{5} = 20k$

$\Sigma F_x = 0$

$F_{AB}\left(\frac{1}{\sqrt{2}}\right) = F_{BC}\left(\frac{4}{5}\right)$

Solve: $\Rightarrow$ $F_{BC} = 14.3$ k

$F_{AB} = 16.15$ k

wire AB

$\underset{0\to80}{\sigma}$; $\Delta = \frac{PL}{AE} = \frac{80\times130(900\times\sqrt{2})}{130(80\times10^3)} = 1.273$ mm

σ: $80 \rightarrow$; $\Delta = \dfrac{PL}{AE} = \dfrac{(14300-10400)(900\sqrt{2})}{130(40\times10^3)}$

$= 0.955$ mm

$\Delta_{Total} = 1.27 + 0.96 = 2.23$ mm

Wire BC

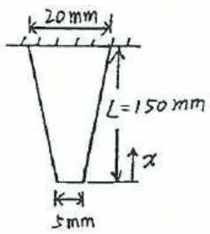
σ $0 \rightarrow 80$: $\Delta = \dfrac{PL}{AE} = \dfrac{80\times100(3.05\times10^3)}{100(80\times10^3)} = 3.05$ mm

$80 \rightarrow$: $\Delta = \dfrac{PL}{AE} = \dfrac{(16150-8000)(3.05\times10^3)}{100(40\times10^3)}$

$= 6.21$ mm

$\Delta_{Total} = 9.26$ mm

3-24



$$A_x = 4\times[5 + \frac{(20-5)}{150}x] = 4(5 + \frac{1}{10}x)$$

$$P_x = rg \times \frac{1}{2}\times 4(10 + \frac{1}{10}x)x$$

$$= rg(20 + \frac{1}{5}x)x$$

$$\Delta = \int_0^{150} \frac{P_x}{AE}dx = \int_0^{150} \frac{rg(20+\frac{1}{5}x)x}{4(5+\frac{1}{10}x)E}dx$$

$$= \int_0^{150} \frac{rg(100+x)x}{(100+2x)E}dx$$

$$= 7637.5\frac{rg}{E}$$

3-25

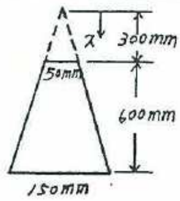
$$\Delta_A = \int_0^{L_A} \frac{P}{2E}dx = \frac{PL_A}{2E}$$

$$\Delta_B = \int_0^{L_B} \frac{P}{(1+\frac{2X}{L_B})}dx = \frac{PL_B}{2E}\ln 3$$

$$\Delta_A = \Delta_B, \quad \frac{PL_A}{2E} = \frac{PL_B}{2E}\ln 3$$

$$\frac{L_A}{L_B} = \ln 3 = 1.10$$

3-26



$d = \dfrac{x}{12}$

$A = \pi d^2 = \pi(\dfrac{x}{12})^2$

$$P = [\frac{\pi}{3}(\frac{x}{12})^2 x - \frac{\pi}{3}\times 25^2\times 300]r$$

$$= \frac{\pi r}{3}(\frac{x^3}{144} - 187500)$$

$$\Delta = \int_{300}^{900}\frac{P}{AE}dx = \int_{300}^{900}\frac{\pi r}{3}\frac{(\frac{x^3}{144}-187500)}{\pi(\frac{x}{12})^2E}dx$$

$$= \frac{r}{3E}\int_{300}^{900}\frac{x^3 - 27\times10^6}{x^2}dx$$

$$= \frac{r}{3E}[\frac{1}{2}x^2 + 27\times10^6\frac{1}{x}]\Big|_{300}^{900}$$

$$= \frac{r}{3E}(36\times10^4 - 6\times10^4) = 10^5\frac{r}{E}$$

3-27

$$P = \int_r^L (\frac{\gamma A dr}{g})\omega^2 r = \frac{\gamma A\omega^2}{2g}(L^2 - r^2)$$

$$\Delta = \int_0^L \frac{2P\,dr}{AE} = 2\int_0^L \frac{\gamma A\omega^2}{2g}(L^2-r^2)\frac{dr}{AE}$$

$$= \frac{\gamma\omega^2}{gE}[L^2 r - \frac{r^3}{3}]\Big|_0^L = \frac{2\gamma\omega^2L^3}{3gE}$$

3-28

$$F = \int_a^x -kx'dx' = -\frac{1}{2}kx'^2\Big|_a^x$$

$$= -\frac{k}{2}(x^2 - a^2)$$

$$\Delta = \int_a^{2a}\frac{F\,dx}{AE} = \int_a^{2a}\frac{-k}{2EA}(x^2-a^2)dx$$

$$= \frac{-k}{2AE}(\frac{x^3}{3} - a^2x)\Big|_a^{2a}$$

$$= -\frac{k}{2AE}[\frac{1}{3}(8a^3-a^3) - a^3] = \frac{-2ka^3}{3EA}$$

$$P = \int_a^{2a}kx\,dx = \frac{kx^2}{2}\Big|_a^{2a} = \frac{k}{2}(4a^2 - a^2)$$

$$= \frac{3ka^2}{2} \Rightarrow k = \frac{2P}{3a^2}$$

$$\therefore \Delta = -\frac{2a^3}{3EA}\times\frac{2P}{3a^2} = -\frac{4Pa}{9EA}$$

3-29

$$\begin{cases} F_{AB}\sin 26.6^\circ = F_{BC}\sin 45^\circ \\ F_{AB}\cos 26.6^\circ + F_{BC}\cos 45^\circ = 3 \end{cases} \Longrightarrow \begin{matrix} F_{AB} = 2.24 \text{ kips} \\ F_{BC} = 1.42 \text{ kips} \end{matrix}$$

$$\Delta_{AB} = \frac{2.24\times 6.71}{0.25\times 0.5\times 10.6\times 10^3} = 0.0113 \text{ in}$$

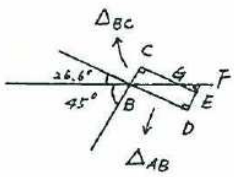
$$\Delta_{BC} = \frac{1.42\times 8.49}{0.25\times 0.875\times 10.6\times 10^3} = 5.20\times 10^{-3} \text{ in}$$

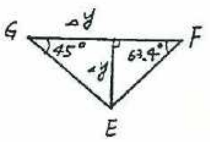
$$GF = \frac{\Delta_{AB}}{\cos 26.6^\circ} - \frac{\Delta_{BC}}{\cos 45^\circ} = 5.28\times 10^{-3} \text{ in}$$

$$GF = \Delta y + \Delta y/\tan 63.4^\circ = 5.28\times 10^{-3} \text{ in}$$

$$\therefore \Delta y = 3.52\times 10^{-3} \text{ in}$$

$$\Delta x = \frac{\Delta_{BC}}{\cos 45^\circ} + \Delta y = 0.0109 \text{ in}$$





3-30

$$\Sigma F = 0,\quad \frac{1}{\sqrt{2}}F_{AB} + \frac{\sqrt{3}}{2}F_{BC} = 10 \qquad F_{AB} = 5.177\text{ k}$$

$$\frac{1}{\sqrt{2}}F_{AB} = \frac{1}{2}F_{BC} \quad \Longrightarrow \quad F_{BC} = 7.321\text{ k}$$

$$\Delta_{AB} = \left(\frac{PL}{AE}\right)_{AB} = \frac{5.177\times 100\sqrt{2}}{10^4} = 0.07321''$$

$$\Delta_{BC} = \left(\frac{PL}{AE}\right)_{BC} = \frac{7.321\times 100\times \frac{2}{\sqrt{3}}}{10^4} = 0.08454''$$

$$EF = \sqrt{2}\times \Delta_{AB} - \frac{2}{\sqrt{3}}\times \Delta_{AB}$$

$$= \sqrt{2}\times 0.07321 - \frac{2}{\sqrt{3}}\times 0.08454$$

$$= 0.00592$$

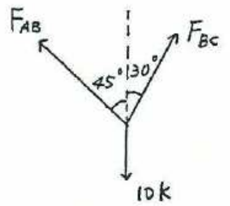
$$EF = \frac{\Delta x}{\sqrt{3}} + \Delta x = 0.00592$$

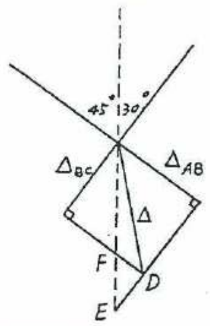
$$\Delta x = 0.00375 \text{ in}$$

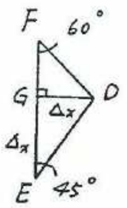
$$\Delta y = \sqrt{2}\times \Delta_{AB} - \Delta x$$

$$= \sqrt{2}\times 0.07321 - 0.00375$$

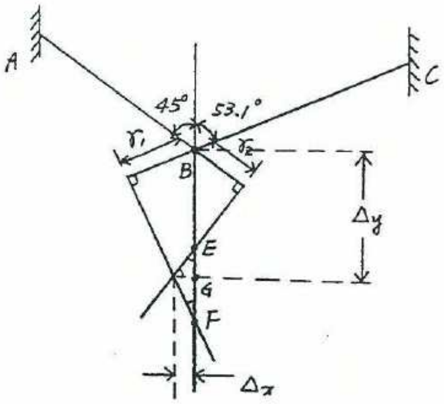
$$= 0.09978 \text{ in}$$







3-31



From Prob. 3-23

$$\gamma_1 = \frac{(14.3\times 10^3)\times(3.05\times 10^3)}{100\times 80\times 10^3} = 5.45\text{mm}$$

$$\gamma_2 = \frac{(16.15\times 10^3)\times(0.9\sqrt{2}\times 10^3)}{130\times 80\times 10^3} = 1.98\text{ mm}$$

$$BF = BE + \Delta_x \tan 45° + \frac{\Delta_x}{\tan 36.9°}$$

where $BF = \frac{\gamma_1}{\cos 53.1°}$, $BE = \frac{\gamma_2}{\cos 45°}$

$$\Delta_x = \frac{\frac{\gamma_1}{\cos 53.1°} - \frac{\gamma_2}{\cos 45°}}{\tan 45° + \frac{1}{\tan 36.9°}} = 2.69\ mm$$

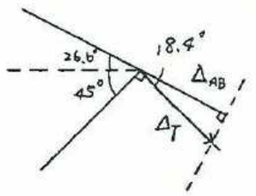
$$\Delta_y = \Delta_x + BE = 5.49\ mm$$

3-32

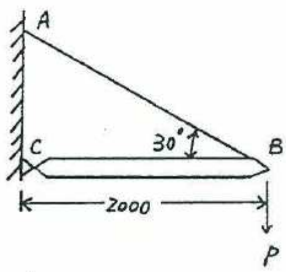
$\Delta_{AB} = 8.656 \times 10^{-3}$ in

$\Delta_{BC} = 0$

$$\Delta_T = \frac{\Delta_{AB}}{\cos 18.4°} = \frac{8.656 \times 10^{-3}}{\cos 18.4°} = 9.122 \times 10^{-3}\ in$$



3-33



(a) crane stiffness

$P = 1$ (unit force)

$P_{AB} \times \frac{1}{2} = 1$

$\Rightarrow P_{AB} = 2$ (tensile)

$P_{BC} = P_{AB} \times \frac{\sqrt{3}}{2} = 1.732$ (compressive)

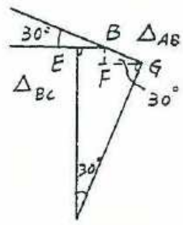
$$\Delta_{AB} = \frac{PL}{AE} = \frac{2(2000 \times \frac{2}{\sqrt{3}})}{3 \times 10^{-4}(200 \times 10^9)} = 7.70 \times 10^{-5}\ mm$$

$$\Delta_{BC} = \frac{1.732 \times 2000}{(3.2 \times 10^{-4})(200 \times 10^9)} = 5.41 \times 10^{-5}\ mm$$

$$EG = \Delta_{BC} + \frac{\sqrt{3}}{2}\Delta_{AB} = 1.21 \times 10^{-4}\ mm$$

$$x = EG \times \sqrt{3} + \frac{1}{2}\Delta_{AB} = 2.48 \times 10^{-4}\ mm$$

$$K = \frac{P}{\Delta} = \frac{1}{2.48 \times 10^{-7}} = 4.0 \times 10^6\ N/m$$



(b) Deflection from 16 kN

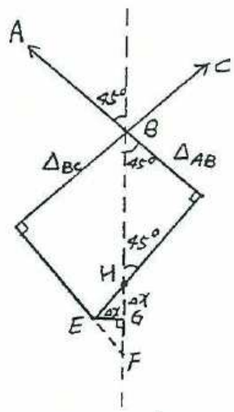
$$\Delta = \frac{P}{K} = \frac{16 \times 10^3}{4.0 \times 10^6} = 4.0 \times 10^{-3}\ m = 4.0\ mm$$

3-34

$$\Delta = \frac{mg}{K}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.0 \times 10^6}{2000}} = 7.12\ Hz$$

3-35



$\alpha = 13.0 \times 10^{-6}$ per °F

$\Delta_{AB} = 100 \times 13 \times 10^{-6} \times 900\sqrt{2}$

$= 1.655$ mm

$\Delta_{BC} = 100 \times 13 \times 10^{-6} \times 3050$

$= 3.965$ mm

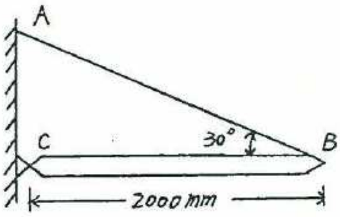
$HF = \frac{5}{3}\Delta_{BC} - \sqrt{2}\,\Delta_{AB} = 4.268$ mm

$HF = \Delta x + \Delta x \times \frac{4}{3} = 4.268$ mm

$\Delta x = 1.829$ mm

$\Delta y = \sqrt{2}\,\Delta_{AB} + \Delta x = 4.170$ mm

3-36

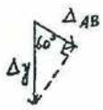


$\Delta_{BC} = 0$

$\Delta_{AB} = 11.7 \times 10^{-6} \left(200 \times \frac{2}{\sqrt{3}}\right)(80°)$

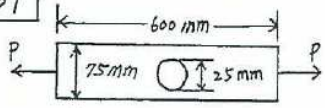
$= 2.162$ mm

$\Delta x = \Delta_{BC} = 0$



$\Delta y = \frac{\Delta_{AB}}{\cos 60°} = 4.324$ mm

3-37



from graph on fig. 3-11

$K = 2.18$

$P = \frac{\sigma A}{K} = \frac{0.22(75-25) \times 6}{2.18} = 30.28$ KN

3-38

from graph on Fig 3-11,

$K = 1.75$

$\frac{\sigma}{\sigma_{max}} = \frac{\frac{P}{A}}{K\frac{P}{A}} = \frac{1}{K} = \frac{1}{1.75} = 0.57$

3-39

$\frac{r_1}{d} = \frac{8}{40} = 0.20, \quad K_1 = 1.63$

$\Rightarrow P_1 = \frac{40 \times 10 \times 60}{1.63} = 14.7$ kN

$\frac{r_2}{d} = \frac{12}{60} = 0.20, \quad K_2 = 2.30$

$\Rightarrow P_2 = \frac{(60-24) \times 10 \times 60}{2.30} = 9.4$ kN

3-40

small bar $\frac{r}{d} = \frac{6}{20} = 0.3, \quad K_1 = 1.53$

bend $\frac{r}{d} = \frac{20}{35} = 0.57, \quad K_2 = 1.37$

pin hole $\frac{r}{d} = \frac{10}{50} = 0.2, \quad K_3 = 2.3$

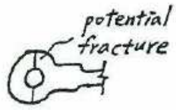
(a)

$P_1 = \frac{80}{1.53} \times 20\, t_1 = 12000$

$\Rightarrow t_1 = 11.5$ mm

$P_2 = \frac{80}{1.37} \times 35\, t_2 = 12000$

$\Rightarrow t_2 = 5.9$ mm



$P_3 = \frac{80}{2.3} \times 30\, t_3 = 12000$

$\Rightarrow t_3 = 11.5$ mm

(b) The potential fracture might occur in the pin hole section.

3-41

Number of cycles : 10×10^6

$\longrightarrow$ stress amplitude 185 MPa

$\frac{r_1}{d}=\frac{2}{10}=0.2 \Rightarrow K_1=1.63$

$\Rightarrow \sigma_{1max}=1.63\times185=301.55$ MPa

or $301.55\times10\ mm^2=3015.5$ N

$=3.0155$ kN

$\frac{r_2}{d}=\frac{4}{20}=0.2 \Rightarrow K_2=2.30$

$\Longrightarrow \sigma_{2max}=2.30\times185=425.5$ MPa

or $425.5\times(20-8)\times1=5106$ N

$=5.106$ kN

3-42

(a) from graph on Fig. 3-11,

$\frac{r}{D}=\frac{10}{60}=\frac{1}{6}$. $K=2.35$

$\sigma_{max}=K\frac{P}{A}=2.35\times\frac{300}{60\times10}=1.175$ GPa

(b) $\Delta=\sum\frac{PL}{AE}=\frac{300\times120}{200\times60\times10}+\frac{300\times120}{200\times40\times10}$

$=0.75$ mm

(c) $\Delta=0.02\times120+\frac{350\times120}{200\times60\times10}=2.75$ mm

(d) $\Delta_{res}=\Delta-\Delta_E$

$=2.75-\left(\frac{350\times120}{200\times60\times10}+\frac{350\times120}{200\times40\times10}\right)$

$=2.75-0.875=1.875$ mm

3-43 for aluminum,

$U_{r,elastic}=\frac{44^2}{2\times10.6}=91.32$ psi

$U_{r,hyper}=\frac{60^2}{2\times10.6}=169.81$ psi

for magnesium, $U_{r,elastic}=\frac{22^2}{2\times6.5}=37.23$ psi

$U_{r,hyper}=\frac{40^2}{2\times6.5}=123.08$ psi

for steel, $U_{r,elastic}=\frac{36^2}{2\times30}=21.6$ psi

$U_{r,hyper}=\frac{65^2}{2\times30}=325$ psi

3-44

for bar 12,

$u\simeq\frac{18.33\times0.5}{\frac{\pi}{4}\times0.505^2\times0.5}=91.51$ ksi

for bar 15,

$u\simeq\frac{18.33\times1}{\frac{\pi}{4}\times0.505\times3.5}=26.15$ ksi

3-45 from example 3-4

$\sigma_{AB}=17.8$ ksi, $P_{AB}=2.23$ kips

$L_{AB}=6.71''$

$\sigma_{BC}=12.9$ ksi, $P_{BC}=2.83$ kips

$L_{BC}=8.49''$

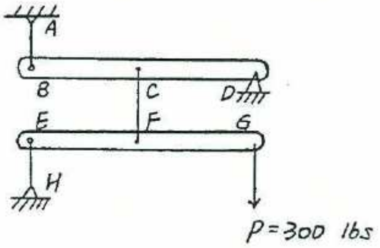
$\frac{P\Delta}{2}=\sum\frac{\sigma^2}{2E}AL=\sum\frac{\sigma PL}{2E}$

$\Delta=\frac{1}{3\times10.6\times10^3}[17.8\times2.23\times6.71+$

$12.9\times2.83\times8.49]$

$=0.018''$

3-46



Forces: $\Sigma M_F = 0 \Rightarrow F_{EH} = 300\ lbs$

$\Sigma M_E = 0 \Rightarrow F_{CF} = 600\ lbs$

$A_{CF} = \dfrac{F_{CF}}{\sigma} = \dfrac{600}{15\times10^3} = 0.04\ in^2$

$\Sigma M_D = 0 \Rightarrow F_{AB} = 300\ lbs$

$A_{AB} = \dfrac{F_{AB}}{\sigma} = \dfrac{300}{15\times10^3} = 0.02\ in^2$

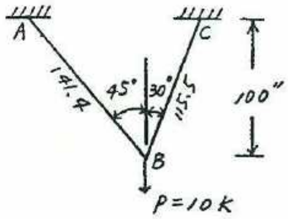
$$U = \frac{1}{2}\left[\left(\frac{P^2L}{AE}\right)_{AB} + \left(\frac{P^2L}{AE}\right)_{CF} + \left(\frac{P^2L}{AE}\right)_{EH}\right]$$

$$= \frac{1}{2}P\Delta$$

$$P\Delta = \left[\frac{0.3^2\times20}{30\times10^3\times0.02} + \frac{0.3^2\times10}{30\times10^3\times0.02} + \frac{0.6^2\times20}{30\times10^3\times0.04}\right]$$

$\therefore\ \Delta = 0.035\ in$

3-47



$\Sigma F_y = 0 \Rightarrow F_{AB}\cos45^\circ + F_{CB}\cos30^\circ = 10K$

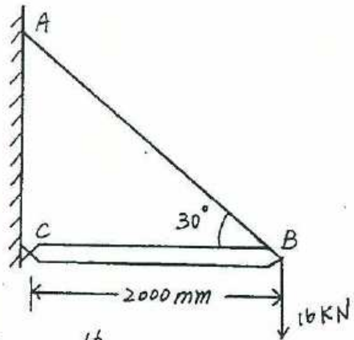
$\Sigma F_x = 0 \Rightarrow F_{AB}\sin45^\circ + F_{CB}\sin30^\circ = 0$

solve, $F_{AB} = 5.177$, $F_{CB} = 7.231$

$$u = \frac{1}{2}\frac{(5.177)^2\,141.4}{10^4} + \frac{1}{2}\frac{(7.321)^2\,115.5}{10^4} = \frac{10}{2}\Delta$$

$\Rightarrow \Delta = 0.0998\ in$

3-48



$P_{AB} = \dfrac{16}{\sin30^\circ} = 32\ kN$

$P_{BC} = 27.712\ kN$

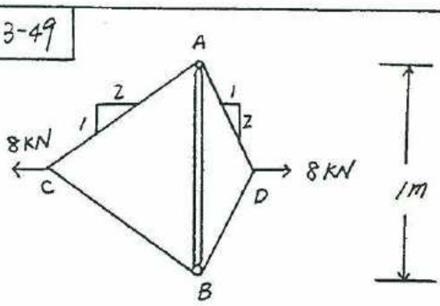
$$\frac{1}{2}P\Delta = \frac{1}{2}\frac{P_{AB}^2 L_{AB}}{A_{AB}E} + \frac{1}{2}\frac{P_{BC}^2 L_{BC}}{A_{BC}E}$$

$$\Rightarrow \frac{(32\times10^3)^2(2.31)}{(300\times10^{-6})(200\times10^9)} + \frac{(27.712\times10^3)^2(2.0)}{(320\times10^{-6})(200\times10^9)}$$

$$= 16\times10^3\,\Delta$$

$\Rightarrow \Delta = 0.004\,m = 4\,mm$

3-49



$T_{AD} = T_{DB} = 8.94\ kN$

$T_{CB} = T_{CA} = 4.47\ kN$

$T_{AB} = 8\,kN + 2\,kN = 10\ kN$

$$\frac{1}{2}\times2\times\frac{(4.47\times1000)^2\,1.118}{20\times200\times1000} + \frac{1}{2}\times2\,\frac{(8.94\times1000)^2\times0.559}{40\times200\times1000} + \frac{1}{2}\,\frac{(10\times1000)^2\times1}{10\times200\times1000} = \frac{1}{2}\times8.0\Delta$$

$\Delta = 3.42\ mm$

3-50 static force $= mg = 1.5 \times 9.81$

$= 14.715$

Bar 'A' $\Rightarrow \Delta_{ST} = \frac{PL}{AE} = \frac{14.7(2000)}{5^2 \times \pi (200 \times 10^3)}$

$= 0.0019$ mm

$\sigma_{max} = \frac{14.7}{5^2 \pi}\left(1 + \sqrt{1 + \frac{2 \times 1000}{0.0019}}\right) = 192.3$ MPa

Bar 'B' $\Rightarrow \Delta_{ST} = \frac{PL}{AE} = \frac{14.7 \times 2000}{(\frac{15}{2})^2 \pi (200 \times 10^3)}$

$= 0.0008$ mm

$\sigma_{max} = \frac{14.7}{\frac{15^2}{4}\pi}\left(1 + \sqrt{1 + \frac{2000}{0.0008}}\right) = 131.6$ MPa

Bar 'C' $\Rightarrow \Delta_{ST} = \frac{14.7(1000)}{78.5(200 \times 10^3)} + \frac{14.7(1000)}{\frac{15^2}{4}\pi (200 \times 10^3)}$

$= 1.35 \times 10^{-3}$ mm

for lower rod

$\sigma_{max} = \frac{14.7}{78.5}\left(1 + \sqrt{1 + \frac{2000}{1.35 \times 10^{-3}}}\right) = 228.1$ MPa

for upper rod

$\sigma_{max} = \frac{78.5}{176.7} \times 228.1 = 101.3$ MPa

3-51

$\Delta = 200$ mm

$u = \frac{1}{2} k \Delta^2 = \frac{1}{2} m v^2$

$\Rightarrow k = \frac{mv^2}{\Delta^2} = \frac{(1\,kg)\,3^2}{(0.02m)^2}$

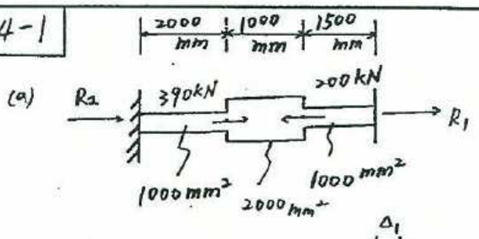
$= 45000$ N/m

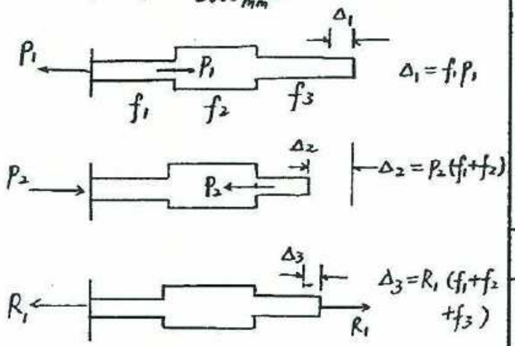
$= 45$ KN/m

0 0

4-1

(a)





$f = \dfrac{L}{AE} \Rightarrow f_1 = 1\times10^{-5}$

$f_2 = 2.5\times10^{-6}$

$f_3 = 7.5\times10^{-6}$

also $\Delta_1 + \Delta_2 + \Delta_3 = 0$

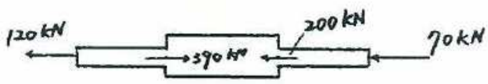
$\Rightarrow f_1P_1 + (-P_2)(f_1+f_2) + R_1(f_1+f_2+f_3) = 0$

$\Rightarrow R_1 = \dfrac{f_1P_1 + (-P_2)(f_1+f_2)}{(f_1+f_2+f_3)}$

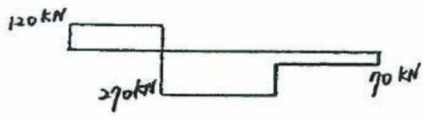
$= \dfrac{1\times10^{-5}(390) + (-200)(1\times10^{-5}+2.5\times10^{-6})}{(1\times10^{-5}) + (2.5\times10^{-6}) + (7.5\times10^{-6})}$

$= -70$ kN

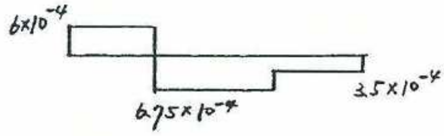
$\Sigma F = 0$, $R_2 = -120$ kN



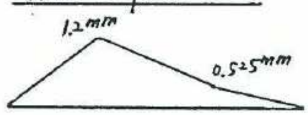
Axial force



Axial strain



Axial displacement



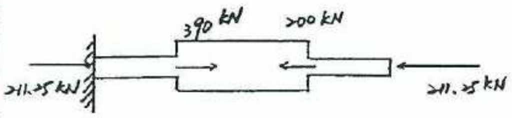
4-2

$\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 0$

$R_1 = \dfrac{f_1P_1 + (f_1+f_2)(-P_2) + \alpha L \Delta t}{f_1+f_2+f_3}$

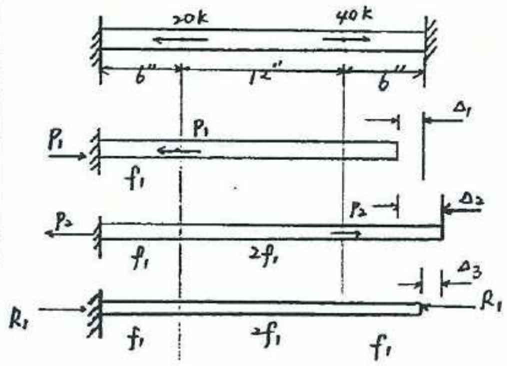
$= \dfrac{(1\times10^{-5})(390) + (1\times10^{-5}+2.5\times10^{-6})(-200) + 25\times10^{-6}\times4.5\times(-50)}{1\times10^{-5} + 2.5\times10^{-6} + 7.5\times10^{-6}}$

$= -211.25$ kN



4-3

$f = \dfrac{L}{AE} \quad \therefore f \propto L$



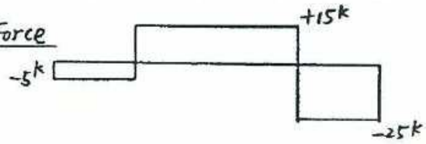
$\Delta_1 + \Delta_2 + \Delta_3 = 0$

$-P_1f_1 + P_2 3f_1 + (-R_1)(4f_1) = 0$

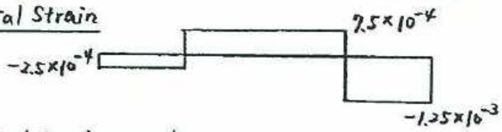
$R_1 = \dfrac{3P_2 - P_1}{4} = \dfrac{3\times40-20}{4} = 25$ kN

4-3

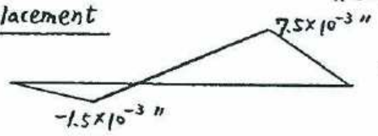
Axial Force



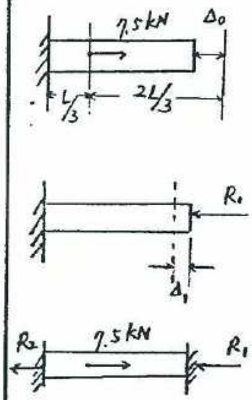
Axial Strain



Axial Displacement



4-4



From Fig. 2-25, for 5×10^8 cycles,

$S_{el} = 127$ MPa

$$\Delta_0 = \frac{7.5\times\frac{L}{3}}{EA} = \frac{2.5L}{EA}$$

$$\Delta_1 = \frac{-R_1L}{EA}$$

$$\Delta_0 + \Delta_1 = 0$$

$$\frac{2.5L}{EA} = \frac{R_1L}{EA} \qquad \therefore R_1 = 2.5\ kN$$

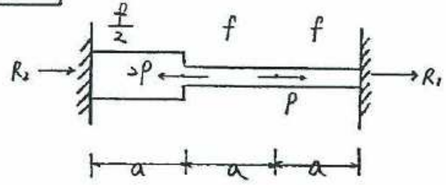
$R_2 = 7.5 - 2.5$

$= 5\ kN$ (governs)

$$\frac{R_2}{A}\times\frac{1}{F.S} = S_{el}$$

$$\frac{5\times10^3}{A}\times\frac{1}{1.8} = 127 \qquad \therefore A = 21.87\ mm^2$$

4-5

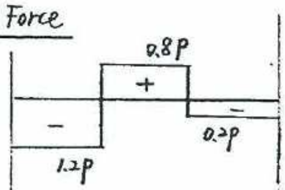


$\Delta = \Delta_0 \qquad 2P(2f) - Pf = \frac{5}{2}R_2 f$

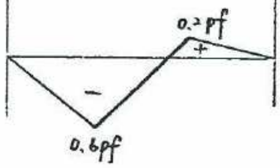
$R_2 = 1.2P$

$R_1 = -0.2P$

Axial Force

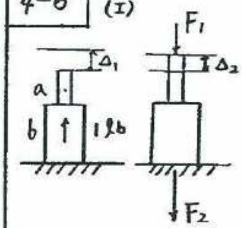


Axial Displacement



4-6

(I)

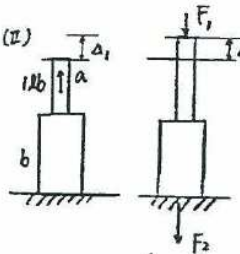


$$\Delta_1 = \frac{1\cdot\frac{L}{4}}{2A_2E} = \frac{L}{8A_2E}$$

$$\Delta_2 = -\frac{F_1\frac{L}{2}}{A_2E} - \frac{F_1\frac{L}{2}}{2A_2E} = -\frac{3F_1L}{4A_2E}$$

$$\Delta_1+\Delta_2=0 \Rightarrow F_1=\frac{1}{6},\ F_2=\frac{5}{6} \qquad \Delta_{ab} = \frac{\frac{1}{6}\times\frac{L}{4}}{A_2E} = \frac{L}{24A_2E}$$

(II)



$$\Delta_1 = \frac{1\times\frac{L}{4}}{A_2E} + \frac{1\times\frac{L}{2}}{2A_2E} = \frac{L}{2A_2E}$$

$$\Delta_2 = -\frac{3F_1L}{4A_2E}$$

$$\Delta_1+\Delta_2=0 \Rightarrow F_1=\frac{2}{3},\ F_2=\frac{1}{3}$$

$$\Delta_{ba} = \frac{\frac{1}{3}\times\frac{L}{4}}{2A_2E} = \frac{L}{24A_2E} \qquad \therefore \Delta_{ab}=\Delta_{ba}$$

4-7

statics : $2F_{AL} + F_{st} = 1 \quad \cdots (1)$

geometry : $\Delta_{AL} = \Delta_{st}$

$$\frac{F_{AL}(25\times12)}{0.2(10\times10^6)} = \frac{F_{st}(50\times12)}{0.3(30\times10^6)}$$

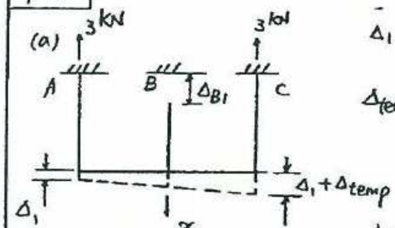
$$F_{AL} = \frac{4}{9}F_{st} \quad \cdots (2)$$

Solving (1) & (2),

$$F_{st} = \frac{9}{17}k \ , \quad F_{AL} = \frac{4}{17}k$$

4-8

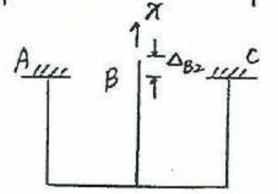
(a)



$$\Delta_1 = \frac{3\times2000}{200\times10} = 3^{mm}$$

$$\Delta_{temp} = 12.5\times10^{-6}\times2000\times(-60) = -1.5^{mm}$$

$$\Delta_{B1} = \frac{\Delta_1 + (\Delta_1+\Delta_{temp})}{2} = \Delta_1 + \frac{1}{2}\Delta_{temp} = 2.25^{mm}$$



$$\Delta_{B2} = \frac{xL}{AE} + \frac{\frac{1}{2}xL}{AE} = \frac{3xL}{2AE}$$

$\because \Delta_{B1} = \Delta_{B2}$ $\qquad 2.25 = \dfrac{3xL}{2AE}$

$x = 1.5$ kN (central wire)

$F = 3 - \frac{1.5}{2} = 2.25$ kN (side wires)

(b) For wire slack, $F_B = 0$

$\Delta_A \le \Delta_B$

$$\frac{PL}{AE} \le \alpha L \Delta T$$

$$\Rightarrow \Delta T \ge \frac{PL}{AE}\cdot\frac{1}{\alpha L} = \frac{3}{10\times200\times12.5\times10^{-6}} = 120°C$$

4-9

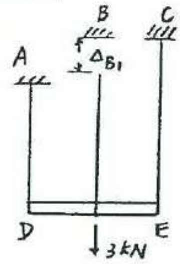
$$F_{st} = \frac{9}{17}\times500 = 264.71 \text{ lb}$$

$$\Delta = \frac{264.71\times50}{(30\times10^6)\times0.3} = 1.4706\times10^{-3} \text{ in}$$

$$f = \frac{\sqrt{g/\Delta}}{2} = \frac{\sqrt{32.2\times12/(1.4706\times10^{-3})}}{2} = 256.30 \text{ Hz}$$

4-10

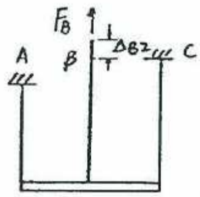
Without slipping from left support ~



$$\Delta_{AD} = \frac{PL}{AE} = \frac{1.5\times1500}{10\times200} = 1.125^{mm}$$

$$\Delta_{CE} = \frac{1.5\times2000}{10\times200} = 1.5^{mm}$$

$$\therefore \Delta_{B1} = \frac{1}{2}(\Delta_{AD}+\Delta_{CE}) = 1.375 \text{ mm}$$



$$\Delta_{AD} = \frac{\frac{1}{2}F_B\times1500}{10\times200} = 0.375F_B$$

$$\Delta_{CE} = \frac{\frac{1}{2}F_B\times2000}{10\times200} = 0.5F_B$$

$$\therefore \Delta_{B2} = \frac{1}{2}(\Delta_{AD}+\Delta_{CE}) + \frac{F_B\times2000}{10\times200} = 1.4375F_B$$

$\Delta_{B1} = \Delta_{B2} \quad \Rightarrow F_B = 0.956$ kN

$F_A = F_C = 1.022$ kN

With slipping from left support ~

$\Delta_{AD} = 1.125 + 3 = 4.125$ mm

$\Delta_{CE} = 1.5$ mm

$\Delta_{B1} = \frac{1}{2}(\Delta_{AD}+\Delta_{CE}) = 2.8125$ mm

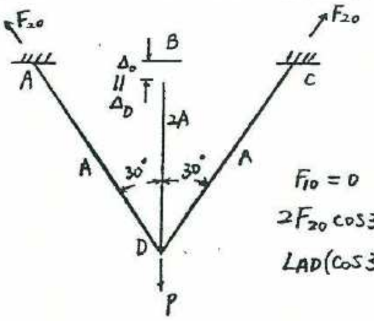
$\Delta_{B2} = 1.4375F_B$

$\Delta_{B1} = \Delta_{B2} \quad \Rightarrow F_B = 1.957$ kN

$F_A = F_C = 0.522$ kN

additional force : $\Delta F_A = \Delta F_C = -0.5$ kN , $\Delta F_B = 1$ kN

4-11



$F_{10} = 0$

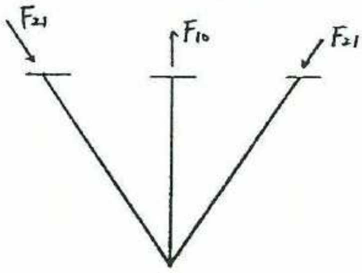
$2F_{20}\cos 30° = P$

$L_{AD}(\cos 30°) = L$

$$\Delta_{AD} = \frac{PL}{2(A)E\cos^2 30°}$$

where: $\Delta_D \cos 30° = \Delta_{AD}$

$$\Delta_D = \frac{PL}{2AE\cos^3 30°}$$



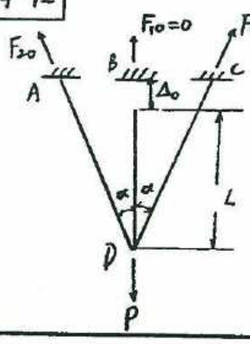
$$\Delta_1 = \frac{F_1 L}{2AE} + \frac{F_1 L}{2AE\cos^3 30°}$$

$\Delta_D - \Delta_1 = 0$

solve: $F_1 = \dfrac{P}{\cos^3 30° + 1} = 0.606P$

$F_2 = \dfrac{P - F_1}{2\cos 30°} = 0.227P$

4-12



$2F_{20}\cos\alpha = P$

$F_{20} = \dfrac{P}{2\cos\alpha} = \dfrac{2}{2\cos 30°}$

$= 1.155\ kN$

$L_{AD} = \dfrac{L}{\cos\alpha} = \dfrac{2000}{\cos 30°}$

$= 2310\ mm$

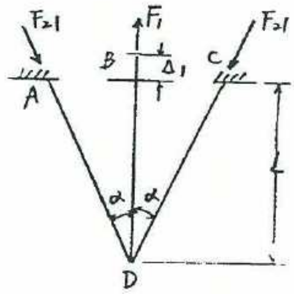
$(\Delta_{AD})_o + \Delta temp = \dfrac{F_{20}L_{AD}}{EA} + \alpha L_{AD}\Delta T$

$= \dfrac{1.155 \times 2310}{68.9 \times 40} + 23.4 \times 10^{-6} \times 2310 \times 50$

$= 3.77\ mm$

$\Delta_o \cos\alpha = (\Delta_{AD})_o + \Delta temp$

$\Delta_o = \dfrac{-3.77}{\cos 30°} = -4.35\ mm$



$\Delta_1 = \dfrac{F_1 L}{AE} + \dfrac{F_1 L}{2AE\cos^3\alpha} + \alpha L \Delta T$

$= \dfrac{F_1 \times 2000}{206.7 \times 40} + \dfrac{F_1 \times 2310}{2 \times 40 \times 68.9 \times \cos^3 30°} + 11.7 \times 10^{-6} \times 2000 \times 50$

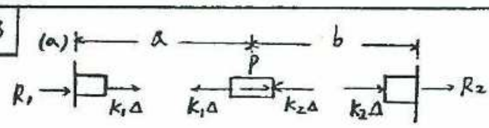
$= 0.887F_1 + 1.17$

$\Delta_1 + \Delta_0 = 0 \quad \therefore F_1 = 3.59\ kN$

$F_1 + 2F_2\cos\alpha = P \quad \therefore F_2 = -0.92\ kN$

4-13

(a)



where: $k_1 = \dfrac{A_1 E}{a}$, $k_2 = \dfrac{A_2 E}{b}$

$b = L - a$

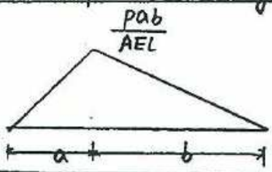
$\Rightarrow R_2 = -k_2\Delta$, $R_1 = -k_1\Delta$

$P - k_2\Delta - k_1\Delta = 0 \quad \Rightarrow \Delta = \dfrac{P}{k_1 + k_2}$

Solve: $R_1 = -\dfrac{b}{L}P$

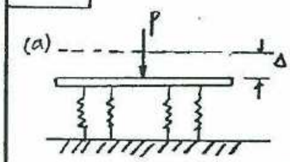
$R_2 = \dfrac{AP}{L} = -\dfrac{a}{L}P$

(b) Axial Displacement Diagram



4-14

(a)



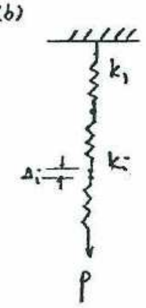
$P = \Delta k$

also, $P = \Delta k_1 + \Delta k_2 + \cdots + \Delta k_n$

$\Rightarrow k = \frac{P}{\Delta} = k_1 + k_2 + \cdots + k_n$

$\therefore k = \sum_n k_i$

(b)



$P = \Delta k$

$\Delta = \Delta_1 + \Delta_2 + \cdots + \Delta_n$

$\frac{P}{k} = \frac{P}{k_1} + \frac{P}{k_2} + \frac{P}{k_3} + \cdots + \frac{P}{k_n}$

$\therefore \frac{1}{k} = \sum_n \frac{1}{k_i}$

4-15

$\frac{1}{k} = \frac{1}{300} + \frac{1}{200} = \frac{5}{600} \Rightarrow k = 120 \, N/mm$

$k_{sys.} = 120 + 2 \times 250 = 620 \, N/mm$

$\Delta = \frac{P}{k_{sys.}} = \frac{6200}{620} = 10 \, mm$

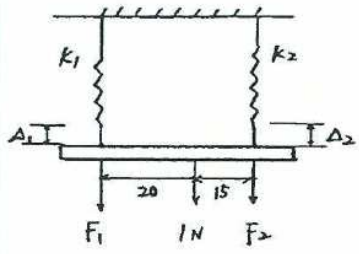
(a) Reactions: $R = k\Delta \begin{cases} = 120 \times 10 = 1200 \, N \\ \quad \text{(Top Reaction)} \\ \text{or} \\ = 250 \times 10 = 2500 \, N \\ \quad \text{(Bottom Reaction)} \end{cases}$

(b) Deflection between upper springs

$\Delta_1 = \frac{1200}{300} = 4 \, mm$

$\Delta_2 = \frac{1200}{200} = 6 \, mm$

4-16

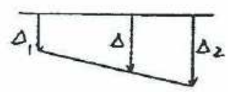


$F_1 = \frac{3}{7} N$

$F_2 = \frac{4}{7} N$

$\Delta_1 = \frac{3}{7k_1}$

$\Delta_2 = \frac{4}{7k_2}$



$$\begin{aligned} \Delta &= \Delta_1 + \frac{4}{7}(\Delta_2 - \Delta_1) \\ &= \frac{3}{7}\Delta_1 + \frac{4}{7}\Delta_2 \\ &= \frac{9}{49k_1} + \frac{16}{49k_2} \\ &= \frac{9k_2 + 16k_1}{49k_1k_2} \end{aligned}$$

$k = \frac{1}{\Delta} = \frac{49k_1k_2}{9k_2 + 16k_1}$

4-17

$P = (2k_{Al} + k_{st})\Delta$

$\Rightarrow \Delta = \frac{P}{2k_{Al} + k_{st}}$

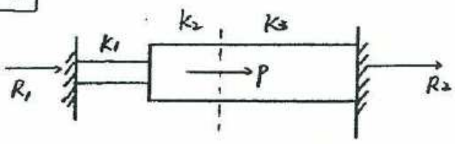
$R_{Al} = k_{Al} \cdot \Delta = \frac{P k_{Al}}{2k_{Al} + k_{st}}$

$R_{st} = k_{st} \cdot \Delta = \frac{P k_{st}}{2k_{Al} + k_{st}}$

and $k = \frac{EA}{L}$ $\quad \therefore k_{Al} : k_{st} = 4 : 9$

$P = 1^k \Rightarrow R_{Al} = \frac{4}{17} k$, $R_{st} = \frac{9}{17} k$

4-18



$k=\frac{EA}{L}$ $\quad \therefore k_1:k_2:k_3 = 1:2:1$

$\left.\begin{array}{l} R_1=\Delta K_0 \\ R_2=\Delta K_3 \end{array}\right\}$ $\quad P+\Delta K_0+\Delta K_3=0$

$\Delta=\frac{-P}{K_0+K_3}$

where $\frac{1}{K_0}=\frac{1}{K_1}+\frac{1}{K_2}$

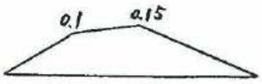
Solving:

$R_1=\frac{-PK_0}{K_0+K_3}=-\frac{2}{5}P \quad (\leftarrow)$

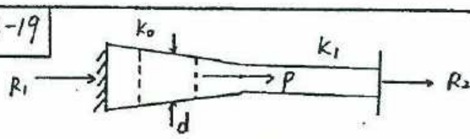
$R_2=\frac{-PK_3}{K_0+K_3}=\frac{-3}{5}P \quad (\leftarrow)$

Axial Displacement

unit: $\frac{PL}{AE}$



4-19



$R_1=+\Delta K_0$, $R_2=+\Delta K_1$

$P+\Delta K_0+\Delta K_1=0 \qquad \Delta=-\frac{P}{K_0+K_1}$

where $\frac{1}{K_0}=\int\frac{1}{K_x}dx$

$K_x=\frac{AE}{L}=\frac{(30-\frac{40}{L}x)tE}{L}$

$\frac{1}{K_0}=\frac{L}{Et}\int_0^{L/2}\frac{dx}{(30L-40x)}$

$=\frac{L}{E}\times\frac{1}{40}\times\ln 3 \qquad K_0=36.41\frac{Et}{L}$

and $K_1=\frac{E\,10t}{L/2}=20\frac{Et}{L}$

$\therefore K_0:K_1=1.82:1$

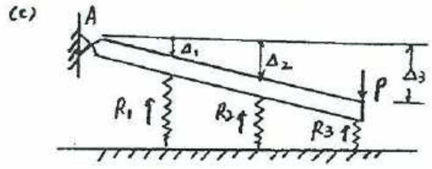
$R_1=\frac{-PK_0}{K_0+K_1}=\frac{-P\,1.82}{1.82+1}=-0.65P\ (\leftarrow)$

$R_2=-0.35P\ (\leftarrow)$

4-20

(a) 2 $\qquad$ (b) 1

(c)



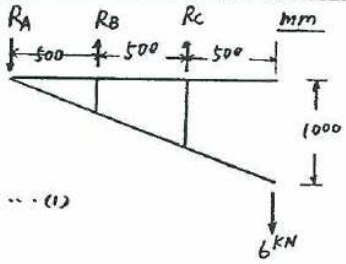
$\Sigma M_A=0$

$\Rightarrow (\Delta_1 k)a+(\Delta_2 k)2a+(\Delta_3 k)3a-P3a=0$

$\Delta_1=\frac{1}{2}\Delta_2=\frac{1}{3}\Delta_3 \qquad \Rightarrow \Delta_1=\frac{3P}{14K}$

$R_N=\Delta_N k \quad \Rightarrow R_1=\frac{3P}{14}$, $R_2=\frac{3P}{7}$, $R_3=\frac{9P}{14}$

4-21



statics:

$\Sigma M_A=0$

$R_B+2R_C=18 \quad \cdots (1)$

Geometry:

$\Delta_C=2\Delta_B \qquad \frac{R_C\times 2}{100}=2\times\frac{R_B\times 1}{80}$

$R_B=\frac{4}{5}R_C \quad \cdots (2)$

Solving (1) & (2):

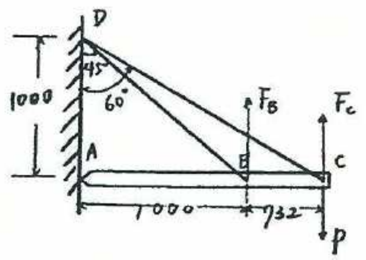
$R_B=5.14$ kN (↑)

$R_C=6.43$ kN (↑)

$\Sigma F_y=0$: $\quad R_A=6.43+5.14-6$

$=5.57$ kN (↓)

4-22



4-22

$\Sigma M_A = 0$

$1.732P = 1.732F_c + F_B \quad \cdots (1)$

$\Delta_c = 1.732\,\Delta_B$

$\Delta_c = \dfrac{F_c L}{EA\cos^3 60^\circ}$

$\Delta_B = \dfrac{F_B L}{EA\cos^3 45^\circ}$

$\rightarrow \dfrac{F_c}{\cos^3 60^\circ} = \dfrac{F_B}{\cos^3 45^\circ} = 1.732$

$\rightarrow F_c = 0.612\,F_B \quad \cdots (2)$

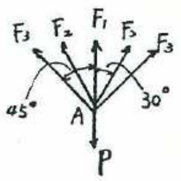
Solving: $F_B = 6.73$ kN

$F_c = 4.11$ kN

the force in wire B : $\dfrac{6.73}{\cos 45^\circ} = 9.52$ kN

in wire C : $\dfrac{4.11}{\cos 60^\circ} = 8.22$ kN

4-23



statics:

$F_1 + 2F_2\cos 30^\circ + 2F_3\cos 45^\circ = P$

Geometry: $F_2 = F_1\cos^2 30^\circ = \frac{3}{4}F_1$

$F_3 = F_1\cos^2 45^\circ = \frac{1}{2}F_1$

Solving: $F_1 = \frac{P}{3}$, $F_2 = \frac{P}{4}$, $F_3 = \frac{P}{6}$

Deflection at joint A due to load $P = 2$ MN:

$\Delta = \dfrac{PL}{AE} = \dfrac{F_1 L}{AE}$ (using center bar)

$= \dfrac{2L}{3AE}$

$= \dfrac{2\times 2000}{3\times 0.2\times 500}$

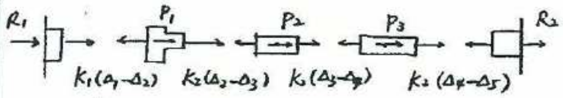
$= 13.33$ mm

4-24

(a) 5 degrees of kinematic indeterminacies

3 degrees of static indeterminacies

(b)



where $k_1 = 2K_2$

Eq.s

1) $-2k_2\Delta_2 = R_1$

2) $2K_2\Delta_2 + k_2\Delta_2 - K_2\Delta_3 = -P_1 = -3P$

3) $-K_2\Delta_2 + 2k_2\Delta_3 - K_2\Delta_4 = -P_2 = -2P$

4) $-K_2\Delta_3 + 2k_2\Delta_4 = -P_3 = -P$

5) $k_2\Delta_4 = R_2$

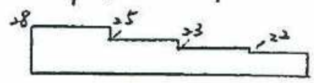
Solving: $\Delta_2 = \dfrac{-R_1}{2K_2}$, $\Delta_4 = \dfrac{+R_2}{K_2}$

$\Delta_3 = \dfrac{3P - \frac{3}{2}R_1}{k_2}$ by 2)

$\Delta_3 = \dfrac{P + 2R_2}{k_2}$ by 4)

$\Rightarrow R_1 = (P - R_2)\frac{4}{3}$

$R_1 = -28P$, $R_2 = 22P$

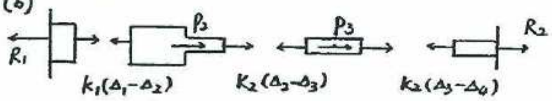


4-25

(a) 3 degrees of static indeterminacies

4 degrees of kinematic indeterminacies

(b)



$\Delta_1 = \Delta_4 = 0$

$R_1 = -K_1\Delta_2$

$K_1\Delta_2 + K_2(\Delta_2 - \Delta_3) = -P_2 = -2P$

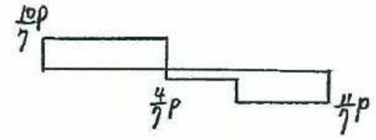
$-K_2(\Delta_2 - \Delta_3) + K_2\Delta_3 = -P_3 = -P$

$K_2\Delta_3 = R_2$

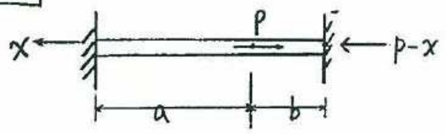
4-25 Solving where $\frac{1}{k_1} = \frac{1}{\frac{2AE}{L}} + \frac{1}{\frac{1AE}{L}}$

$$= \frac{3L}{2AE}$$

$$R_1 = \frac{10}{7}P \quad , \quad R_2 = \frac{-11}{7}P$$



4-26



$$\sigma = K\varepsilon^n$$

$$\varepsilon = \sigma^{\frac{1}{n}} K^{\frac{-1}{n}}$$

$$\varepsilon_a \cdot (a) = \varepsilon_b \cdot (b)$$

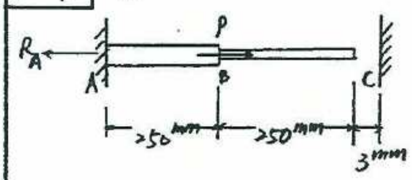
$$\sigma_a^{\frac{1}{n}} K^{-\frac{1}{n}} a = \sigma_b^{\frac{1}{n}} K^{-\frac{1}{n}} b$$

$$\sigma_a a^n = \sigma_b b^n$$

$$\left(\frac{x}{A}\right) a^n = \left(\frac{P-x}{A}\right) b^n$$

$$\therefore x = \frac{P}{\left(\frac{a}{b}\right)^n + 1}$$

4-27 (a)



Rod AB:

$$P_{yp} = \sigma_{yp} A = 200 \times 200 = 40000 N = 40 kN$$

$$u_{yp} = \frac{\sigma_{yp} L}{E} = \frac{200 \times 250}{2 \times 10^5} = 0.25 \text{ mm}$$

Rod BC:

$$P_{yp} = \sigma_{yp} A = 200 \times 100/1000 = 20 \text{ kN}$$

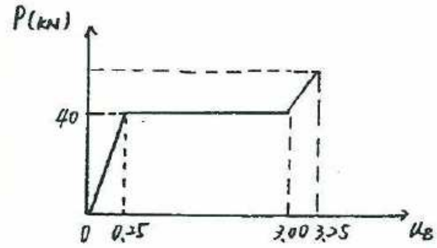
$$u_{yp} = \frac{\sigma_{yp} L}{E} = \frac{200 \times 250}{2 \times 10^5} = 0.25 \text{ mm}$$

$$P_{ult} = 40 + 20 = 60 \text{ kN}$$

$$u_B \text{ at ult} = (u_{yp})_{BC} + gap$$

$$= 0.25 + 3$$

$$= 3.25 \text{ mm}$$



(b) $3 - 0.25 = 2.75$ mm

4-28 $A_s = 4 \times (0.5)^2 \times \pi = 3.14 \text{ in}^2$

$$A_c = 14^2 - 3.14 = 192.86 \text{ in}^2$$

(a) $\varepsilon^c_{allow} = \frac{\sigma^c_{allow}}{E_c} = \frac{2000}{2 \times 10^6}$

$$= 1 \times 10^{-3}$$

$$\varepsilon^s_{allow} = \frac{\sigma^s_{allow}}{E_s} = \frac{24 \times 10^3}{30 \times 10^6}$$

$$= 0.8 \times 10^{-3} \text{ (governs)}$$

$$\therefore \varepsilon_{allow} = 0.8 \times 10^{-3}$$

$$P = P_s + P_c = \varepsilon_{allow}(A_s E_s + A_c E_c)$$

$$= 0.8 \times 10^{-3}(3.14 \times 30 \times 10^3 + 192.86 \times 2 \times 10^3)$$

$$= 384 \text{ kips}$$

4-28

(b) $P_{ult} = P_{ult}^{c} + P_{ult}^{s}$

$= 4.6 \times 192.86 + 60 \times 3.14$

$= 1096$ kips

4-29

statics: $2F_{AL} + F_{ST} = 100$ .... (1)

geometry: $\Delta_{AL} = gap + \Delta_{ST}$

$$\frac{F_{AL}(10)}{2(10\times10^3)} = 0.005 + \frac{F_{ST}(9.995)}{4(30\times10^3)}$$

$6F_{AL} - 0.9995F_{ST} = 60$ .... (2)

Solving (1) & (2):

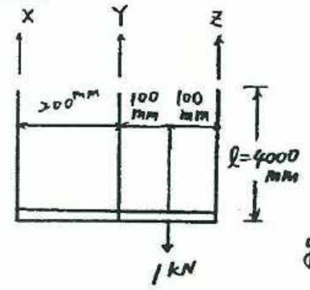
$F_{ST} = 60^k$, $F_{AL} = 20^k$

(a) $\sigma_{ST} = \dfrac{F_{ST}}{A} = \dfrac{60}{4} = 15$ ksi

(b) $\Delta_{AL} = \dfrac{F_{AL}L}{EA} = \dfrac{20\times10}{10\times10^3\times2} = 0.010$ in

(c) $P_n = 40\times4 + 60\times4 = 400$ kips

4-30



statics:

$\Sigma F_y = 0$

$X + Y + Z = 1000$ -(1)

$\Sigma M_A = 0$

$3X + Y - Z = 0$ ...(2)

geometry:

$\Delta Y = \frac{1}{2}(\Delta X + \Delta Z)$

$$\frac{Yl}{EA} = \frac{1}{2}\left(\frac{Xl}{EA} + \frac{Zl}{EA}\right) \Rightarrow X - 2Y + Z = 0 \text{ ... (3)}$$

Solving (1), (2) & (3): $X = 83.3$ N (↑)

$Y = 333.3$ N (↑)

$Z = 583.4$ N (↑)

4-31

statics: $P_{AL} = P_{ST} = P$ ... (1)

geometry: $\Delta_{AL} = \Delta_{ST}$

$$\alpha_{AL}(\Delta T)L - \sum_{AL}\frac{P_{AL}L}{AE}$$

$$= \alpha_{ST}(\Delta T)L + \frac{P_{ST}L}{AE}$$

$$12\times10^{-6}(110)(7) - \frac{P_{AL}(3)}{0.3(10^7)} - \frac{P_{AL}(4)}{0.4(10^7)}$$

$$= 6.5\times10^{-6}(110)(7) + \frac{P_{ST}(7)}{0.7\times(3\times10^7)} \text{ .... (2)}$$

Solving (1) & (2): $P = 1815$ lb

4-32

$\Delta_1 = \dfrac{-R_1L}{A_1E_1} - \alpha_1L(\Delta T)$

$\Delta_2 = \dfrac{R_2L}{A_2E_2} - \alpha_2L(\Delta T)$

$\Delta_1 = \Delta_2$

$R_1 = R_2 = \sigma A = (20\text{ MPa})(1000) = 20^{kN}$

$$\frac{-R_1L}{A_1E_1} - \alpha_1L(\Delta T) = \frac{R_2L}{A_2E_2} - \alpha_2L(\Delta T)$$

$$\Delta T = \left(\frac{R_1}{A_1E_1} + \frac{R_2}{A_2E_2}\right)\left(\frac{-1}{\alpha_1 - \alpha_2}\right) = -42.2^\circ$$

TEMP $= 60^\circ - 42.2^\circ = 17.8^\circ$C

4-33

$$\frac{F_2(L/\cos\alpha)}{AE} = \frac{F_1L}{A\frac{1}{3}E}\cos\alpha$$

$\therefore F_2 = 3F_1\cos^2\alpha$

$F_1 + 2F_2\cos\alpha = P$

$F_1 + 6F_1\cos^3\alpha = P$

$$\Rightarrow F_1 = \frac{P}{1+6\cos^3\alpha} \text{ .... (1)}$$

$$F_2 = \frac{3P\cos^2\alpha}{1+6\cos^3\alpha} \text{ .... (2)}$$

4-35

$$\Delta = \frac{(\delta P)}{k'} = \frac{212.52}{6887} = 0.0309 \text{ in}$$

$$\Delta = 2\,\frac{\gamma w^2}{gE}\left(\frac{L^3}{2} - \frac{L^3}{6}\right)$$

$$= \frac{2}{3}\,\frac{\gamma w^2 L^3}{gE}$$

4-36

From Prob. 3-28, $P_x = \frac{2P}{3a^2}x$

$$\therefore EA\frac{d^2u}{dx^2} = -\frac{2P}{3a^2}x$$

B.C. at $x = 2a$, $EA\frac{du}{dx} = -P$

$$\Rightarrow EA\frac{du}{dx} = -\frac{P}{3a^2}x^2 + c_1$$

$$-P = \frac{-P}{3a^2}\times 4a^2 + c_1$$

$$c_1 = \frac{1}{3}P$$

$$\Rightarrow EA\frac{du}{dx} = -\frac{P}{3a^2}x^2 + \frac{1}{3}P$$

$$\Delta = \frac{1}{EA}\int_a^{2a}\left(-\frac{P}{3a^2}x^2 + \frac{1}{3}P\right)dx$$

$$= \frac{1}{EA}\left(-\frac{P}{9a^2}\times 7a^3 + \frac{1}{3}P\times a\right)$$

$$= -\frac{4Pa}{9EA}$$

4-37

$$AE\frac{d^2u}{dx^2} = -\frac{\gamma A w^2}{g}r$$

$$AE\frac{du}{dx} = -\frac{\gamma A w^2}{g}\left(\frac{r^2}{2} + C_1\right)$$

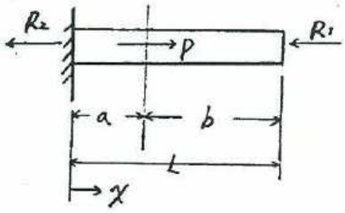
$\frac{du}{dx} = 0$, at $r = L$, $\therefore c_1 = -\frac{L^2}{2}$

$$AEu = -\frac{\gamma A w^2}{g}\left(\frac{r^3}{6} - \frac{L^2}{2}r + C_2\right)$$

$u = 0$, at $r = 0$, $\therefore c_2 = 0$

$$\Rightarrow u = -\frac{\gamma w^2}{gE}\left(\frac{r^3}{6} - \frac{L^2}{2}r\right)$$

4-38



(a) $EA\frac{d^2u}{dx^2} = -P\,\delta(x-a)$

B.C. $\begin{cases} \text{at } x = 0, & u = 0 \\ \text{at } x = L, & u = 0,\ EA\frac{du}{dx} = -R_1 \end{cases}$

$$EA\frac{du}{dx} = -P\langle x-a\rangle^0 + C_1$$

at $x = L$, $EA\frac{du}{dx} = -R_1$

$$-R_1 = -P + C_1$$

$$\therefore C_1 = P - R_1$$

$$EAu = -P\langle x-a\rangle^1 + (P-R_1)x + C_2$$

at $x = 0$, $u = 0$ $\therefore C_2 = 0$

at $x = L$, $u = 0$

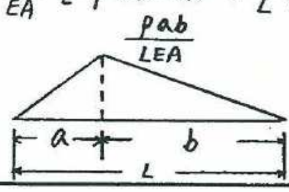
$$0 = -P(L-a) + (P-R_1)L$$

$$\Rightarrow R_1 = \frac{a}{L}P$$

$$R_2 = \frac{b}{L}P$$

(b) $u = \frac{1}{EA}\left[-P\langle x-a\rangle^1 + \left(P - \frac{a}{L}P\right)x\right]$

$$= \frac{1}{EA}\left[-P\langle x-a\rangle^1 + \frac{b}{L}Px\right]$$



5-1

$$K_s = \frac{Gab}{10} = \frac{0.64\times20\times40}{10}$$
$$= 51.2\ N/mm$$

5-3

for Aluminum,

$$U_{shear} = \frac{\tau^2}{2G} = \frac{25^2}{2\times4} = 78.125\ psi$$
$$U_r = \frac{\sigma_p^2}{2E} = \frac{60^2}{2\times10.6} = 169.81\ psi$$

for steel,

$$U_{shear} = \frac{\tau^2}{2G} = \frac{24^2}{2\times12} = 24\ psi$$
$$U_r = \frac{\sigma_p^2}{2E} = \frac{65^2}{2\times30} = 70.42\ psi$$

5-3

from eq. (5-14)

(a) state of plane stress

$\sigma_x = 20,\quad \sigma_y = 10,\quad \sigma_z = 0$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)]$$
$$= \frac{1}{30\times10^3}[0-0.25(20+10)]$$
$$= -2.5\times10^{-4}$$
$$e_x = \epsilon_x \cdot l = \frac{1}{30\times10^3}(20-0.25\times10)\times4$$
$$= 2.3\times10^{-3}\ in$$
$$e_y = \frac{1}{30\times10^3}(10-0.25\times20)\times4$$
$$= 6.67\times10^{-4}\ in$$

(b) state of plain strain

$$\sigma_z = \nu(\sigma_x+\sigma_y)$$
$$= 0.25(20+10)$$
$$= 7.5\ ksi$$

$$e_x = \frac{1}{30\times10^3}(20-0.25\times17.5)\times4$$
$$= 2.08\times10^{-3}\ in$$
$$e_y = \frac{1}{30\times10^3}(10-0.25\times27.5)\times4$$
$$= 4.17\times10^{-4}\ in$$

5-4

$$\Delta_x = a\times\frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)]$$
$$= 200\times\frac{1}{140\times10^3}[-4.8-0.35(2.40+2)]$$
$$= -9.06\times10^{-3}\ mm$$
$$\Delta_y = c\times\frac{1}{E}[\sigma_y - \nu(\sigma_x+\sigma_z)]$$
$$= 100\times\frac{1}{140\times10^3}[2.40-0.35(-4.80+2)]$$
$$= 2.41\times10^{-3}\ mm$$
$$\Delta_z = b\times\frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)]$$
$$= 120\times\frac{1}{140\times10^3}[2-0.35(-4.8+2.4)]$$
$$= 2.43\times10^{-3}\ mm$$

5-5

$\sigma_x = 2.4\,MPa$

$\sigma_y = -1.2\,MPa$

$\sigma_z = -2.0\,MPa$

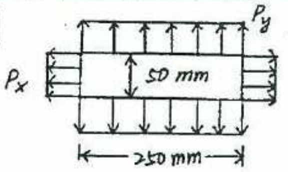
$$e_x = \frac{1}{140\times10^3}[2.4-0.35(-1.2-2.0)]\times200$$
$$= 5.029\times10^{-3}\ mm$$
$$e_y = \frac{1}{140\times10^3}[-1.2-0.35(2.4-2.0)]\times100$$
$$= -9.575\times10^{-4}\ mm$$
$$e_z = \frac{1}{140\times10^3}[-2.0-0.35(2.4-1.2)]\times120$$
$$= -2.075\times10^{-3}\ mm$$

$\sigma_x \neq 0 \qquad \sigma_y = \sigma_z = 0$

$$e'_x = \frac{1}{140\times10^3}\,\sigma_x \times 2000 = 5.029\times10^{-3}\ mm$$

$\sigma_x = 3.52\ MPa$

5-6



(a) $\sigma_x = \dfrac{100}{50\times10} = 0.2\ kN/mm^2$

$\sigma_y = \dfrac{200}{250\times10} = 0.08\ kN/mm^2$

$\sigma_z = 0$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$= \frac{1}{200}[-0.25(0.2+0.08)]$$

$$= -3.50\times10^{-4}$$

$$\Delta_{Lz} = -3.50\times10^{-4}(10) = -3.5\times10^{-3}\ mm$$

(b) $-3.50\times10^{-4} = \dfrac{1}{E}(-0.25\,\sigma_x)$

$$\sigma_x = 3.50\times10^{-4}\times\frac{200}{0.25}$$

$$= 0.28\ kN/mm^2$$

$$P_x = 0.28\times50\times10 = 140\ kN$$

5-7

$\sigma_x = \dfrac{7.5\times10^3}{15\times5} = 100\ MPa$

$\sigma_y = 0$

$\sigma_z = -\dfrac{5\times10^3}{20\times5} = -50\ MPa$

$$\epsilon_x = \frac{1}{110\times10^3}[100 - 0.33(0-50)] = 1.059\times10^{-3}$$

$$\epsilon_y = \frac{1}{110\times10^3}[0 - 0.33(100-50)] = -1.5\times10^{-4}$$

$$\epsilon_z = \frac{1}{110\times10^3}[-50 - 0.33(100+0)] = -7.545\times10^{-4}$$

5-8

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} = \frac{0.25\times200}{1.25\times0.5} = 80\ GPa$$

$$\mu = \frac{E}{1+\nu} = \frac{200}{1+0.25} = 160\ GPa$$

5-9

$$G = \frac{E}{2(1+\nu)} = \frac{180}{2(1+0.25)} = 72\ MPa$$

5-10

$$G = \frac{E}{2(1+\nu)} = \frac{16}{2(1+0.45)} = 5.52\ MPa$$

$$K = \frac{E}{3(1-2\nu)} = \frac{16}{3(1-2\times0.45)} = 53.33\ MPa$$

5-11

from eq. (5-21), $G = \dfrac{E}{2(1+\nu)}$

$$\nu = \frac{E-2G}{2G}$$

$$\nu_A = \frac{10.6 - 2\times4.0}{2\times4.0} = 0.325$$

$$\nu_s = \frac{30-2\times12}{2\times12} = 0.25$$

5-12

for alluminum,

$$\nu = \frac{E-2G}{2G} = \frac{10-2\times3.75}{2\times3.75} = 0.33$$

$$K = \frac{E}{3(1-2\nu)} = \frac{10\times10^6}{3(1-2\times0.33)} = 9.8\times10^6\ psi$$

$$= 67.55\ GPa$$

for steel,

$$\nu = \frac{30-2\times12}{2\times12} = 0.25$$

$$K = \frac{30\times10^6}{3(1-2\times0.25)} = 20\times10^6\ psi$$

$$= 137.8\ GPa$$

5-13 for glass

$$G_g = \frac{65}{2(1+0.22)} = 26.64 \ MPa$$

$$K_g = \frac{65}{3(1-2\times0.22)} = 38.69 \ MPa$$

for steel

$$G_s = \frac{200}{2(1+0.25)} = 80 \ GPa$$

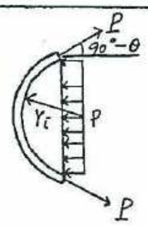
$$K_s = \frac{200}{3(1-2\times0.25)} = 133.33 \ GPa$$

for lead

$$G_\ell = \frac{15}{2(1+0.45)} = 5.17 \ GPa$$

$$K_\ell = \frac{15}{3(1-2\times0.45)} = 50 \ GPa$$

5-14



$$\Sigma F_x = 0$$

$$2P\cos(90°-\theta) = 2pLr_i\sin\theta$$

$$P = pLr_i$$

$$\sigma_1 = \frac{P}{Lt} = \frac{pr_i}{t}$$

5-15 (a) $p = \frac{0.5}{15} \times \frac{80}{5} = 0.533 \ ksi$

(b) $p = \frac{2\times0.5}{15} \times 80 = 5.33 \ ksi$

5-16

$$60\times10^6 = \frac{0.75\times80\times10^3\times9.8}{t}$$

$t = 0.0098 m$

$= 9.8 \ mm$

5-17

(a) $0.8\times160 = \frac{(9800\times0.721\times5)\times6\times10^{-3}}{(t-3)}$

$t = 4.66$ mm, use 5 mm plate

(b) $\epsilon = \frac{\Delta r}{r} = \frac{pr}{2t}\cdot\frac{1}{E}(2-\nu)$

$= \frac{0.8\times160}{2} \times \frac{1}{200\times10^3}(2-0.25)$

$= 5.6\times10^{-4}$

$\Delta d = d\epsilon = 12000\times5.6\times10^{-4}$

$= 6.72 \ mm$

5-18

(a) $\sigma_1 = \frac{pr}{t}$

$\frac{400\times10^3}{5} = \frac{20\times101\times r}{20\times10^{-3}}$

$r = 0.79 \ m, \quad d = 1.58 \ m$

(b) from table 1 of Appendix, the material may be Aluminum 2024-T4, $E = 73 \ GPa$, $\nu = 0.325$

$\epsilon = \frac{1}{73\times10^3}(80 - 0.325\times40) = 9.18\times10^{-4}$

$\Delta d = 1.58\times9.18\times10^{-4} = 1.45\times10^{-3} \ m$

5-19

$$n = \frac{F}{A\sigma} = \frac{\pi(\frac{300}{2})^2\times1.5}{195\times50} = 14.8$$

use 15 × @ 19 mm bolts

5-20 $P_{cr} = P_{bottom} = wh = 9.802\times4$

$(d\times s)P_{cr} = (2A)\sigma_{all}$

$$S = \frac{2(0.03\times0.006)\times90000}{6\times(9.802\times4)} = 0.138 \ m$$

Similary, use 0.22 m, 0.33 m & 0.66 m spacings respectively for the upper three meters.

5-21

$$\sigma_1 = \frac{P_i r_i}{t} = \frac{120 \times 59}{1} = 7080 \text{ psi}$$

$$\sigma_{2\,(accurate)} = \frac{P_i r_i^2}{r_o^2 - r_i^2} = \frac{120 \times 59^2}{60^2 - 59^2} = 3510 \text{ psi}$$

$$\epsilon = \frac{1}{E}(\sigma_1 - \nu \sigma_2) = \frac{1}{29 \times 10^6}[7080 - 0.25 \times 3510] = 2.13 \times 10^{-4} \text{ in/in}$$

$$\Delta_c = \pi d \epsilon = \pi \times 119 \times 2.13 \times 10^{-4} = 0.080''$$

$$\Delta_d = d\epsilon = 119 \times 2.13 \times 10^{-4} = 0.025''$$

5-22

$$\alpha(\Delta T)L = \frac{PL}{EA}$$

(a) $$20 \times 10^{-6}(150) = \frac{P}{70\,(5 \times 50)}, \quad P = 52.5 \text{ kN}$$

$$\sigma_{hoop} = \frac{52.5 \times 10^3}{5 \times 50} = 210 \text{ MPa}$$

(b) $$\sigma_{bearing} = \frac{F}{A} = \frac{52.5 \times 10^3 \times 2}{200 \times 50} = 10.5 \text{ MPa}$$

5-23

$$\sigma_1 = \frac{1 \times 1000}{10} = 100 \text{ MPa}$$

$$\Delta d_1 = \frac{2000}{200 \times 10^3}(100 - 0.3 \times 50) = 0.85 \text{ mm}$$

$$\Delta d_2 = 2000 \times 12 \times 10^{-6} \cdot (-50) = -1.2 \text{ mm}$$

$$\Delta d_3 = -2000 \times 18 \times 10^{-6}(-50) = -1.8 \text{ mm}$$

$$\sigma = E \times (\Delta d_1 + \Delta d_2 - \Delta d_3)/d = 140 \times 10^3 \times \frac{1.45}{2000} = 101.5 \text{ MPa}$$

5-24

static : $P_s = P_B = P$

geometry : $\Delta_s = \Delta_B$

$$\frac{P_s(\pi D)}{E_s A_s} = \alpha_B(\Delta T)(\pi D) - \frac{P_B(\pi D)}{E_B A_B}$$

$$\frac{P}{30 \times 10^6 \times (1 \times 0.25)} = 10.7 \times 10^{-6}(100) - \frac{P}{16 \times 10^6 \times (1 \times 0.25)}$$

$$P = 2791.30 \text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{2.79}{1 \times 0.25} = 11.17 \text{ ksi}$$

5-25

$$\sigma_{1s} = \frac{(2 - \bar{P}) \times 200}{4} = 100 - 50\bar{P}$$

$$\sigma_{1a} = \frac{\bar{P} \times 200}{4} = 50\bar{P}$$

$$\Delta_{ds} = \Delta_{da}$$

$$\frac{100 - 50\bar{P}}{200} = \frac{50\bar{P}}{70}$$

$$\bar{P} = 0.518 \text{ MPa}$$

$$\sigma_{1a} = 50 \times 0.518 = 25.93 \simeq 26 \text{ MPa}$$

5-26

statics: $P_s = P_a = P$

geometry: $\Delta_s = \Delta_a$

$$\alpha_s \Delta T(\pi D_s) + \frac{P_s(\pi D_s)}{E_s A_s} = \alpha_a \Delta T(\pi D_a) - \frac{P_a(\pi D_a)}{E_a A_a}$$

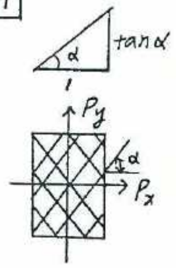
$$1.2 \times 10^{-6} \times 40 \times 280 + \frac{P_s \times 280}{200 \times 280 \times 0.8} = 23 \times 10^{-6} \times 40 \times 300 - \frac{P_a \times 300}{70 \times \frac{\pi}{4} \times 20^2}$$

$$0.01989185 2 \times P = 0.26556$$

$$P = 13.2 \text{ kN}$$

$$\sigma_{s1} = \frac{13.2 \times 10^3}{280 \times 0.8} = 58.93 \text{ MPa}$$

5-27



$P_y = F_n \sin\alpha$

$P_x = F_n \cos\alpha$

$$\frac{P_y}{1} = 2\,\frac{P_x}{\tan\alpha}$$

$$F_n \sin\alpha = 2F_n \frac{\cos^2\alpha}{\sin\alpha}$$

$\tan^2\alpha = 2$; $\alpha = 55°$

5-28

From eq. 5-40,

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0$$

Let $r = e^t$, $dr = e^t dt$ or $\frac{dt}{dr} = e^{-t}$

$$\frac{du}{dt} = \frac{du}{dt}\cdot\frac{dt}{dr} = e^{-t}\frac{du}{dt}$$

$$\frac{d^2u}{dr^2} = \frac{d(e^{-t})}{dt}\cdot\frac{dt}{dr}\frac{du}{dt} + e^{-t}\frac{d^2u}{dt^2}\frac{dt}{dr}$$

$$= e^{-2t}\left(\frac{du}{dt} - \frac{d^2u}{dt^2}\right)$$

$$e^{-2t}\left(\frac{d^2u}{dt^2} - \frac{du}{dt}\right) + \frac{1}{e^t}\left(e^{-t}\frac{du}{dt}\right) - \frac{u}{(e^t)^2} = 0$$

$$\frac{\partial^2 u}{\partial t^2} - u = 0$$

solution: $u(t) = A_1 e^t + A_2 e^{-t}$

or $u(r) = A_1 r + \frac{A_2}{r}$

5-29

Let $\beta = \frac{r_o}{r_i}$

from eq. 5-49,

$$\sigma_{t\,max} = \frac{P_i r_i^2}{r_o^2 - r_i^2}(1+\beta^2)$$

$$\sigma_{t\,ave} = \frac{F_t}{A} = \frac{1}{(r_o - r_i)\times 1}\int_{r_i}^{r_o} \sigma_t\,(1\times dr)$$

$$= \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(\frac{1}{r_o - r_i}\right)\left[\int_{r_i}^{r_o} dr + \int_{r_i}^{r_o}\frac{r_o^2}{r^2}dr\right]$$

$$= \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(\frac{1}{r_o - r_i}\right)(r_o - r_i)\left[1 + \frac{r_o}{r_i}\right]$$

$$= \frac{P_i r_i^2}{r_o^2 - r_i^2}(1+\beta)$$

$$\therefore \frac{\sigma_{t\,max}}{\sigma_{t\,ave}} = \frac{1+\beta^2}{1+\beta}$$

5-30

$$\sigma_{t\,max} = P_i r_i^2\left[\frac{1}{r_o^2 - r_i^2} + \frac{r_o^2}{(r_o^2 - r_i^2)r_i^2}\right]$$

$$\sigma_{t\,max} = P_i\left[\frac{r_i^2}{r_o^2 - r_i^2} + \frac{1}{1 - \left(\frac{r_i}{r_o}\right)^2}\right]$$

$$\lim_{r_o\to\infty}\sigma_{t\,max} = \lim_{r_o\to\infty} P_i\left(\frac{r_i}{r_o^2 - r_i^2} + \frac{1}{1-\left(\frac{r_i}{r_o}\right)^2}\right)$$

$$= P_i(0+1) = P_i$$

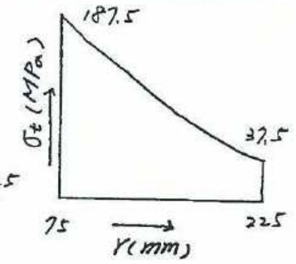
5-31

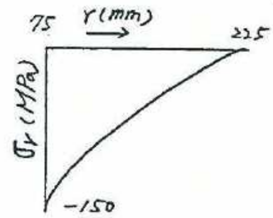
$r_i = 75\,mm$, $r_o = 225\,mm$, $P_i = 150\,MPa$

(a) $$\sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) = 18.75\left(1 - \frac{50625}{r^2}\right)\ MPa$$

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) = 18.75\left(1 + \frac{50625}{r^2}\right)\ MPa$$

r(mm)	75	100	125	150	175	200	225
$-\sigma_r$(MPa)	150	76.2	42.0	23.4	12.2	5.0	0
σ_t (MPa)	187.5	113.7	79.5	60.9	49.7	42.5	37.5





(b) $$\tau_{max} = \frac{P_i r_o^2}{r_o^2 - r_i^2} = \frac{150(225^2)}{225^2 - 75^2} = 168.8\,MPa$$

(c) $$A_1 = \frac{(1+\nu)(1-2\nu)}{E}\left(\frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}\right)$$

$$= \frac{1.25\times 0.5}{200\times 10^3}\left(\frac{150\times 75^2}{225^2 - 75^2}\right) = 5.86\times 10^{-5}$$

$$A_2 = \frac{1+\nu}{E}\left(\frac{P_i r_i^2 r_o^2}{r_o^2 - r_i^2}\right) = \frac{1.25}{200\times10^3}\left(\frac{150\times75^2\times225^2}{225^2-75^2}\right) = 5.93$$

$$u = A_1 r + A_2\left(\frac{1}{r}\right)$$

$$u_i = 5.86\times10^{-5}(75) + \frac{5.93}{75} = 8.35\times10^{-2}\ mm$$

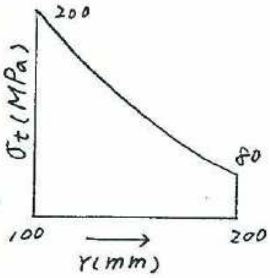
$$u_o = 5.86\times10^{-5}(225) + \frac{5.93}{225} = 3.96\times10^{-2}\ mm$$

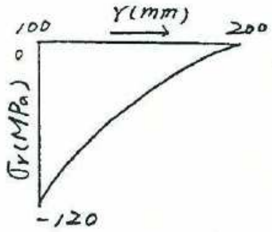
5-32 $r_i = 100\ mm,\ r_o = 200\ mm,\ P_i = 120\ MPa$

(a) $$\sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) = 40\left(1 - \frac{40000}{r^2}\right) MPa$$

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) = 40\left(1 + \frac{40000}{r^2}\right) MPa$$

r (mm)	100	125	150	175	200
$-\sigma_r$ (MPa)	120	62.4	31.1	12.2	0
σ_t (MPa)	200	142.4	111.1	92.2	80





(b) $$\tau_{max} = \frac{P_i r_o^2}{r_o^2 - r_i^2} = \frac{120\times200^2}{200^2-100^2} = 160\ MPa$$

(c) $$A_1 = \frac{(1+\nu)(1-2\nu)}{E}\left(\frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}\right) = \frac{1.25\times0.5}{200\times10^3}\left(\frac{120\times100^2}{200^2-100^2}\right) = 1.25\times10^{-4}$$

$$A_2 = \frac{1+\nu}{E}\left(\frac{P_i r_i^2 r_o^2}{r_o^2 - r_i^2}\right) = \frac{1.25}{200\times10^3}\left(\frac{120\times200^2\times100^2}{200^2-100^2}\right) = 10$$

$$u_i = 1.25\times10^{-4}(100) + \frac{10}{1000} = 11.25\times10^{-2}\ mm$$

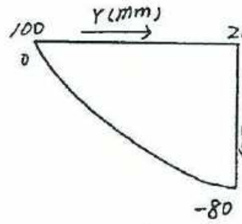
$$u_o = 1.25\times10^{-4}(200) + \frac{10}{200} = 7.5\times10^{-2}\ mm$$

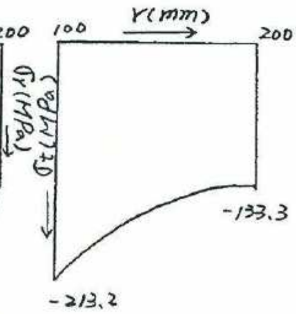
5-33 $r_i = 100\ mm,\ r_o = 200\ mm,\ P_o = 80\ MPa$

(a) $$\sigma_r = -\frac{P_o r_o^2}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right) = -106.6\left(1 - \frac{10000}{r^2}\right) MPa$$

$$\sigma_t = -\frac{P_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right) = -106.6\left(1 + \frac{10000}{r^2}\right) MPa$$

r (mm)	100	125	150	175	200
$-\sigma_r$ (MPa)	0	38.8	59.2	71.8	80.0
$-\sigma_t$ (MPa)	213.2	174.8	154.0	141.4	133.3





(b) $$\tau_{max} = \frac{(\sigma_t)_{max}}{2} = -\frac{213.2}{2} = -106.6\ MPa$$

(c) $$A_1 = \frac{(1+\nu)(1-2\nu)}{E}\left(-\frac{P_o r_o^2}{r_o^2 - r_i^2}\right) = \frac{1.25\times0.5}{200\times10^3}\times\frac{(-80)(200^2)}{200^2-100^2} = -3.33\times10^{-4}$$

$$A_2 = \frac{1+\nu}{E}\left(\frac{-P_o r_i^2 r_o^2}{r_o^2 - r_i^2}\right) = \frac{1.25}{200\times10^3}\left(\frac{-80\times100^2\times200^2}{200^2-100^2}\right) = -6.67$$

$$u = A_1 r + \frac{A_2}{r}$$

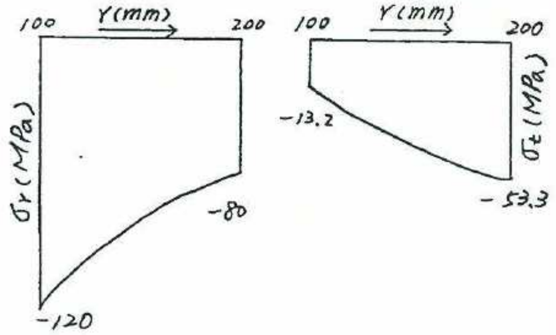
$$u_i = -3.33\times10^{-4}(100) - \frac{6.67}{100} = -0.1\ mm$$

$$u_o = -3.33\times10^{-4}(200) - \frac{6.67}{200} = -0.1\ mm$$

5-34 $P_i = 120\,MPa$, $P_o = 80\,MPa$

superposing Prob. 5-32 and 5-33

$r(mm)$	100	125	150	175	200
$-\sigma_r(MPa)$	120	101.2	90.3	84.0	80.0
$-\sigma_t(MPa)$	13.2	32.4	42.9	49.2	53.3



(b) $\tau_{max} = \dfrac{\sigma_{tmax} - \sigma_{rmax}}{2}$

$= \dfrac{-13.2 + 120}{2} = 53.4\ MPa$

(c) $u_i = 11.25\times10^{-2} - 0.1 = 1.25\times10^{-2}\,mm$

$u_o = 7.5\times10^{-2} - 0.1 = 2.5\times10^{-2}\,mm$

5-35 $F_o = P_o(A_{proj.})_o = 80(400\times1) = 32\,kN (\rightarrow)$

$F_i = P_i(A_{proj.})_i = -160(200\times1) = -32\,kN (\leftarrow)$

$C_1 = \dfrac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} = \dfrac{160\times200^2\times100^2}{200^2-100^2} = 1.066\times10^6$

$F_t = 2\int_{100}^{200} \sigma_t(1\times dr) = 2\int_{100}^{200}(C_1 + \frac{C_2}{r^2})\,dr$

$= 2(100C_1 + \frac{C_2}{200}) = 0$

$\Sigma F_x = F_o + F_i + F_t = 32 - 32 + 0 = 0$

5-36 $\tau_{max} = \dfrac{\sigma_{yp}}{2} = \dfrac{P_i r_o^2}{r_o^2 - r_i^2} = \dfrac{P_i}{1-(\frac{2}{r_o})^2}$

(a) $\dfrac{40}{2} = \dfrac{8\times2}{1-(\frac{2}{r_o})^2}$

$1-(\frac{2}{r_o})^2 = \frac{4}{5}$, $\quad r_o = 4.47''$

$d_o = 8.94''$

(b) $P_{ult} = \sigma_{yp}\ln(\frac{a}{b})$

$3(-8) = 40\ln(\frac{2}{b})$

$\frac{2}{b} = e^{-(\frac{24}{40})} = 0.549$

$b = 3.64\,in$

$d_o = 2b = 7.29\,in$

5-37 inner cylinder : $r_i = 3''$, $r_o = 5''$, $P_i = 0$

(a) $A_1 = \dfrac{(1+0.25)(1-0.5)}{30\times10^3}\times\dfrac{(-P_o)(25)}{(25-9)}$

$= -3.26\times10^{-5}P_o$

$A_2 = \dfrac{(1+0.25)}{30\times10^3}\times\dfrac{(-P_o)(9)(25)}{(25-9)}$

$= -5.86\times10^{-4}P_o$

$u_i(5'') = [3.26\times10^{-5}(5) + \frac{5.86\times10^{-4}}{5}](-P_o)$

$= -2.80\times10^{-4}P_o\ in$

outer cylinder: $r_i = 5''$, $r_o = 8''$, $P_o = 0$

$A_1 = \dfrac{(1+0.25)(1-0.5)}{30\times10^3}\times\dfrac{P_i(25)}{(64-25)} = 1.34\times10^{-5}P_i$

$A_2 = \dfrac{(1+0.25)}{30\times10^3}\times\dfrac{P_i(25)(64)}{64-25} = 1.71\times10^{-3}P_i$

$u_o(5'') = [(1.34\times10^{-5})(5) + \frac{1.71\times10^{-3}}{5}]P_i$

$= 4.09\times10^{-4}P_i\ in$

$|u_i(5'')| + |u_o(5'')| = \frac{0.01}{2}$

$(2.80 + 4.09)\times10^{-4}P = 0.005$

$P = 7.26\ ksi$

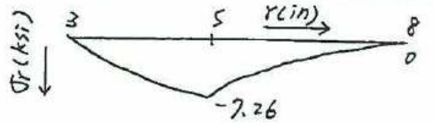
(b) inner cylinder : $r_i = 3''$, $r_o = 5''$, $P_o = 7.26\,ksi$

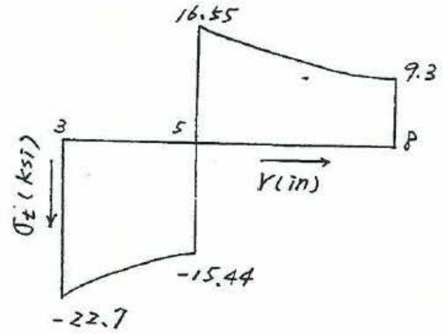
$\sigma_r = -\dfrac{7.26(25)}{25-9}(1-\frac{9}{r^2}) = -11.35(1-\frac{9}{r^2})$

$$\sigma_t = \frac{7.26(25)}{64-25}\left(1+\frac{64}{r^2}\right) = 4.65\left(1+\frac{64}{r^2}\right)$$

r (in)	3.0	3.5	4.0	4.5	5.0
$-\sigma_r$ (ksi)	0.0	3.01	4.97	6.31	7.26
$-\sigma_t$ (ksi)	22.7	19.69	17.73	16.39	15.44

r(in)	5.0	5.5	6.0	6.5	7.0	7.5	8.0
$-\sigma_r$ (ksi)	7.25	5.19	3.62	2.39	1.42	0.64	0.0
σ_t (ksi)	16.55	14.49	12.92	11.69	10.72	9.94	9.3





(c) $\sigma_t = 20 + 22.7 = 42.7$ ksi

$$\sigma_t(r_i) = \frac{P_i(r_i^2 + r_o^2)}{r_o^2 - r_i^2}$$

$$42.7 = \frac{P_i(3^2+8^2)}{8^2-3^2}, \quad P_i = 32.2 \text{ ksi}$$

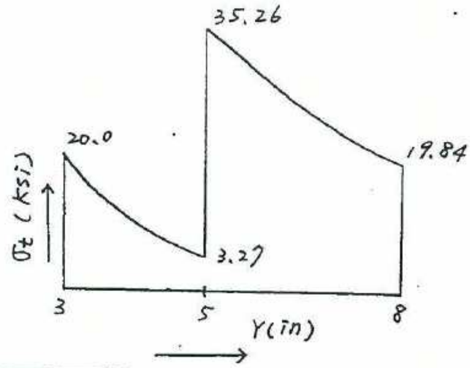
(d) due to $P_i = 32.2$ ksi

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left(1+\frac{r_o^2}{r^2}\right) = 5.27\left(1+\frac{64}{r^2}\right)$$

r (in)	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
σ_t (ksi)	42.74	32.79	26.39	21.96	18.77	16.44	14.67	13.29	12.11	11.23	10.54

superposing,

r (in)	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
σ_t (ksi)	20.0	13.10	8.66	5.57	3.27						
					35.26	30.93	27.59	24.98	22.83	21.17	19.84



5-38 For a thin disk, $\sigma_z = 0$

$$\frac{du}{dr} = \epsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_t)$$

$$\text{or } \sigma_r - \nu\sigma_t = E\frac{du}{dr} \quad \cdots\cdots (1)$$

$$\frac{u}{r} = \epsilon_t = \frac{1}{E}(-\nu\sigma_r + \sigma_t)$$

$$\text{or } -\nu\sigma_r + \sigma_t = E\frac{u}{r} \quad \cdots\cdots (2)$$

solving (1) & (2)

$$\sigma_r = \frac{E}{(1-\nu^2)}\left(\frac{u}{r} + \nu\frac{du}{dr}\right)$$

$$\sigma_t = \frac{E}{(1-\nu^2)}\left(\nu\frac{u}{r} + \frac{du}{dr}\right)$$

$$F_{inertia} = \frac{\gamma r\, d\phi\, dr}{g}\omega^2 r$$

$$\sigma_{inertia} = \frac{F_{inertia}}{d\phi\, dr} = \frac{\gamma\omega^2 r^2}{g}$$

$$\sigma_t - \sigma_r - r\frac{d\sigma_r}{dr} - \frac{\gamma\omega^2 r^2}{g} = 0$$

$$\frac{E}{(1-\nu^2)}\left[\nu\frac{u}{r} + \frac{du}{dr}\right] - \frac{E}{(1-\nu^2)}\left[\frac{u}{r} + \nu\frac{du}{dr}\right]$$

$$-\frac{rE}{(1-\nu^2)}\left[\frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + \nu\frac{d^2u}{dr^2}\right] = 0$$

Simplifying,

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + \frac{(1-\nu^2)}{\nu E}\frac{\gamma\omega^2 r}{g} = 0$$

5-39

$r_i = a, \quad r_o = 2a$

(a) $\sigma_r = \sigma_{yp} \ln\left(\frac{r}{c}\right) - \frac{\sigma_{yp}}{2}\left(\frac{b^2 - c^2}{b^2}\right)$

for $c = 1.5a$

$P_i = \sigma_r(a) = 350\left[\ln\left(\frac{a}{1.5a}\right) - \frac{1}{2}\frac{(2a)^2 - (1.5a)^2}{(2a)^2}\right]$

$= 350\,[-0.41 - 0.22]$

$= -220.5 \text{ MPa}$

(b) plastic zone: $0 < r < 1.5a$

$\sigma_r = 350\left[\ln\left(\frac{r}{1.5a}\right) - 0.22\right]$

$\sigma_t = 350 + \sigma_r$

elastic zone: $1.5a < r < 2a$

$\sigma_r = \frac{76.6}{\left(\frac{2}{1.5}\right)^2 - 1}\left(1 - \frac{4a^2}{r^2}\right) = 98.44\left(1 - \frac{4a^2}{r^2}\right)$

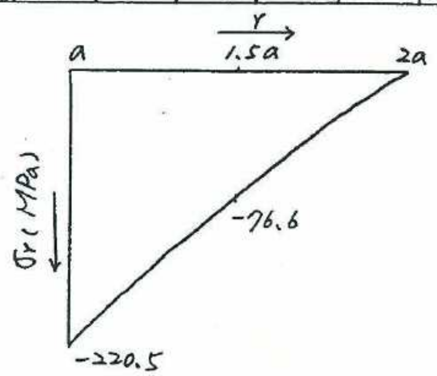
$\sigma_t = \frac{26.6}{\left(\frac{2}{1.5}\right)^2 - 1}\left(1 + \frac{4a^2}{r^2}\right) = 98.44\left(1 + \frac{4a^2}{r^2}\right)$

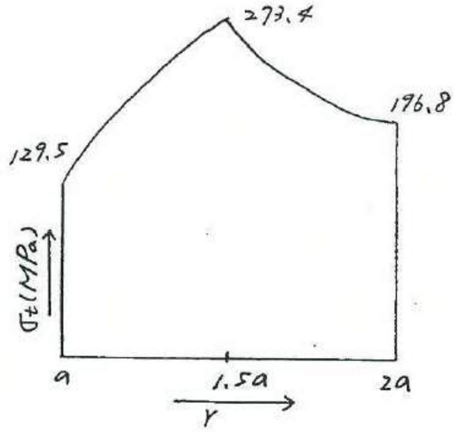
plastic zone:

r	a	$1.1a$	$1.2a$	$1.3a$	$1.4a$	$1.5a$
$-\sigma_r$ (MPa)	220.5	185.1	154.7	126.6	100.7	76.6
σ_t (MPa)	129.5	164.9	195.3	223.4	249.3	273.4

elastic zone:

r	$1.5a$	$1.6a$	$1.7a$	$1.8a$	$1.9a$	$2.0a$
$-\sigma_r$ (MPa)	76.6	55.4	37.8	23.1	10.6	0.0
σ_t (MPa)	273.4	252.3	234.6	219.9	207.5	196.8





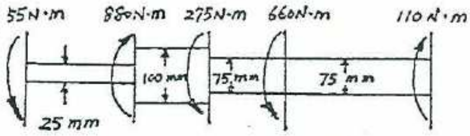
(c) $P_{ult} = \sigma_r(a) = \sigma_{yp} \ln\left(\frac{a}{b}\right)$

$= 350 \ln \frac{a}{2a}$

$= 350\,(-0.693)$

$= -242.6 \text{ MPa}$

6-1



$T_1 = 55\,N\cdot m,\ T_2 = 825\,N\cdot m,\ T_3 = 550\,N\cdot m,\ T_4 = 110\,N\cdot m$

$$\tau_1 = \frac{T_1 C_1}{J_{p1}} = \frac{55\left(\frac{25}{2}\times10^{-3}\right)}{\frac{\pi}{2}\left(\frac{25}{2}\times10^{-3}\right)^4} = 1.79\times10^7\ N/m^2$$

$$\tau_2 = \frac{T_2 C_2}{J_{p2}} = \frac{825(50\times10^{-3})}{\frac{\pi}{2}(50\times10^{-3})^4} = 4.2\times10^6\ N/m^2$$

$$\tau_3 = \frac{T_3 C_3}{J_{p3}} = \frac{550\left(\frac{75}{2}\times10^{-3}\right)}{\frac{\pi}{2}\left(\frac{75}{2}\times10^{-3}\right)^4} = 6.64\times10^6\ N/m^2$$

$$\therefore\ \tau_{max} = \tau_1 = 1.79\times10^7\ N/m^2$$

6-2

$$\frac{T_2}{T_1} = \frac{I_{p2}}{I_{p1}} = \frac{\frac{\pi}{2}(r^4-b^4)}{\frac{\pi}{2}r^4}$$

$$T_2 = T_1 \cdot \frac{(100)^4-(50)^4}{(100)^4} = 0.938\,T_1$$

$$\text{Loss} = 100\% - 93.8\% = 6.2\%$$

6-3

$$\tau_{max} = \frac{Tc}{I_p}$$

$$\tau_{max} = \frac{T\cdot 20}{\frac{\pi}{32}(40)^4} = \frac{T\cdot 30}{\frac{\pi}{32}(60^4 - D_i^4)}$$

$$T\cdot(60 - D_i^4) = T\cdot\frac{30}{20}(40)^4$$

$$D_i = 54.95\ mm$$

$$t = \frac{60-54.95}{2} = 2.525\ mm$$

$$\frac{W_s}{W_t} = \frac{(40)^2}{(60)^2-(54.95)^2} = \frac{1}{9.675}$$

6-4

$$\tau = \sigma_1 = -\sigma_2$$

$$\varepsilon = 0.004,\quad E = 180,\quad G = 70$$

$$\sigma_1 = -\sigma_2 = 180(0.004) = 0.72 = \tau$$

$$\phi = \frac{T\cdot L}{G\cdot I_p},\quad \tau = \frac{T\cdot c}{I_p} \Rightarrow T = \frac{\tau I_p}{c}$$

$$\therefore\ \phi = \frac{\tau\cdot I_p}{c}\cdot\frac{L}{G I_p} = \frac{(0.72)(500)}{(10)(70)}$$

$$\phi = 0.514\ \text{radians}$$

6-5

(a) $T = \dfrac{159\times400}{2} = 31{,}800\ N.m$

$$\tau_{max} = \frac{Tc}{I_p} = \frac{31800\,(0.050)}{\frac{\pi}{2}(0.050)^4}$$

$$\tau_{max} = 161.96\ MPa$$

(b) $(100)^3\cdot 2 = d^3\cdot 4$

$$d = 79.4\ mm$$

6-6

$$T = \frac{63000\ Hp}{N} = \frac{63000\times90}{630} = 9000\ \text{in-lbs}$$

$$\frac{I_p}{c} = \frac{T}{\tau} = \frac{9000}{5750} = 1.565 = \frac{\pi}{2}c^3$$

$$c = \sqrt[3]{\frac{2}{\pi}1.565} = 1.0\ in,\quad d = 2.0\ in$$

6-7

$$T_L = T_S,\quad f_L = \frac{28}{90}f_S$$

Same material, same allow. τ_{max}

$$d_L^3\cdot f_L = d_S^3\cdot f_S$$

$$d_L^3\,\frac{28}{90}f_S = (12)^3 f_S$$

$$d_L^3 = \frac{90}{28}(12)^3$$

$$d_L = 17.7\ mm$$

6-8 $T_i = \frac{159(500)}{30} = 2650$ N.m

$$\tau_{max} = 60\times10^6 = \frac{T_i c_i}{I_p} = \frac{2650 c_i}{\frac{\pi}{2} c_i^4}$$

$$c_i^3 = \frac{2(2650)}{\pi\ 60\ 10^6} \quad c_i = 0.030\ m = 30mm$$

$d_i = 2c_i = 60$ mm dia for the internal gear

Larger gear is of the same material

Hence, $d_L^3 f_L = d_i^3 f_i$

$$d_L^3 = d_i^3 \frac{f_i}{f_L} = (60)^3 \frac{30}{10}$$

$d_L = 86.5$ mm

6-9 From the S-N diagram in Fig 2-25, for $N = 5\times10^8$ cycles, $S = 131$ MPa

$$\tau = \frac{\sigma}{2} = \frac{131}{2} = 65.5 \text{ MPa}$$

$$\frac{T}{\tau} = \frac{I_p}{c}, \quad T = 65.5\times10^6 \frac{\pi}{2}(0.02)^3 = 823.1$$

Using a safety factor of 1.8

$$T_{allow} = \frac{823.1}{1.8} = 457.28 \text{ N.m}$$

6-10 (a) $T = \frac{63000\ Hp}{N} = \frac{63000\ (300)}{75}$

$T = 252{,}000$ in-lbs

$$\frac{I_p}{c} = \frac{T}{\tau} = \frac{252{,}000}{8{,}000} = 31.5$$

$$\frac{I_p}{c} = \frac{\pi}{2}\frac{\left[c^4 - (c/1.2)^4\right]}{c} = 0.813c^3 = 31.5$$

$c^3 = 38.732, \quad c = 3.38$ in

$D_o = 2c = 6.77$ in

$D_i = \frac{D_o}{1.2} = 5.64$ in

(b) $\frac{I_p}{c} = \frac{T}{\tau} = 31.5 = \frac{\pi}{2}c^3$

$c^3 = 20.054, \quad c = 2.72$ in

$D = 2c = 5.44$ in

dia of equiv solid shaft

6-11 $T = \frac{63000}{N} Hp$

$$T_{25} = \frac{63000\ (25)}{26.3} = 6.00\times10^4 \text{ in-lb}$$

$$T_{75} = \frac{63000\ (75)}{26.3} = 1.80\times10^5 \text{ in-lb}$$

$$\frac{I_p}{c} = \frac{T}{\tau}; \quad \left(\frac{I_p}{c}\right)_{left} = \frac{6\times10^4}{6000} = 10 \text{ in}^3$$

$$\frac{\pi}{2}c^3 = 10, \quad c = \left(\frac{20}{\pi}\right)^{1/3} = 1.85 \text{ in}$$

$$\left(\frac{J}{c}\right)_{right} = \frac{1.8\times10^5}{6000} = 30, \quad c = \left(\frac{60}{\pi}\right)^{1/3} = 2.67 \text{ in}$$

$\therefore D_{left} = 3.71$ in. $\quad D_{right} = 5.35$ in

6-12 see fig. 6-13

$$r = 12\ mm, \quad \frac{r}{\frac{1}{2}d} = \frac{12}{\frac{1}{2}\times75} = 0.32$$

$$\frac{D}{d} = \frac{150}{75} = 2.00$$

by extrapolation, $K = 1.3$

$$\tau = K\frac{Tc}{I_p} = 1.3\,\frac{2.7\times(37.5\times10^{-3})}{\frac{\pi}{2}(37.5\times10^{-3})^4}\times10^{-3}$$

$= 42.37$ MPa

$$r = 3\ mm, \quad \frac{r}{\frac{1}{2}d} = \frac{3}{\frac{1}{2}\times75} = 0.08$$

$$\frac{D}{d} = 2.00$$

$K = 1.85$

$$\tau = \frac{1.85}{1.3}(42.37) = 60.30 \text{ MPa}$$

0 0

6-13

$$T = \frac{63000 \times 110}{100} = 69300 \text{ in-lb}$$

$$K = \frac{\tau I_P}{TC} = \frac{8000\left(\frac{\pi}{2} \times 2^4\right)}{69300\,(2)} = 1.45$$

$$\frac{D}{d} = \frac{6}{4} = 1.5$$

from Fig. 4-13, $\frac{r}{\frac{1}{2}d} = 0.155$

$$r = (0.155)\frac{4}{2} = 0.310 \text{ in.}$$

6-14 (a) $D = 40$, $d = 20$, $r = 1$

$$\frac{D}{d} = 2, \quad \frac{r}{d/2} = 0.1$$

From Fig 6-13 of stress concentration factors,

$K \simeq 1.75$ for $\frac{D}{d} = 2$ and $\frac{r}{d/2} = 0.1$

$$\tau_{max} = K \frac{Tc}{I_p} \quad \left(\text{compute } \frac{c}{I_p} \text{ from } d\right)$$

$$= T.(1.75)\frac{2}{\pi\,(0.010)^3}$$

$$\tau_{max} = 1{,}114{,}085\ T \text{ Pascals}$$

or $1.114\ T$ MPa

(b) Here $\frac{D}{d} = \frac{30}{18} = 1.67$

and $\frac{r}{d/2} = \frac{2}{9} = 0.222$

$K \simeq 1.4$ (from Fig 6-13)

$$\tau_{max} = K\frac{TC}{I_p} = T\,(1.4)\frac{2}{\pi\,(0.009)^3}$$

$$\tau_{max} = 1{,}222{,}589\ T \text{ Pascals}$$

or $1.223\ T$ MPa

Shaft assembly (a) causes lower stresses than the assembly (b), a more generous 2mm fillet radius in case (a) would further reduce the stress

6-15

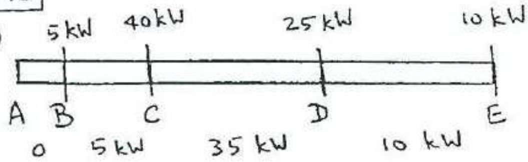
$$\phi = \frac{TL}{GI_p} = \frac{\tau I_p}{c}\frac{L}{GI_p} = \frac{\tau L}{cG}$$

$$L = \frac{\phi\, c\, G}{\tau} = \frac{2\pi\,(0.003)(27 \times 10^9)}{42 \times 10^6}$$

$$= 12.12 \text{ m}$$

6-16

(a)



$$T_{max} = \frac{159\,(kW)_{max}}{f} = \frac{159 \times 35}{3}$$

$$T_{max} = 1855 \text{ N.m}$$

$$I_p = \frac{\pi r^4}{2} = \frac{\pi}{2}(0.025)^4 = 6.14 \times 10^{-7} \text{ m}^4$$

$$I_p/c = \frac{6.14 \times 10^{-7}}{0.025} = 2.4544 \times 10^{-5}$$

$$\tau_{AB} = 0$$

$$\tau_{CD} = T\frac{C}{I_p} = \frac{1855}{2.4544 \times 10^{-5}} = 75.58 \times 10^6 \text{ N/m}^2$$

or 75.58 MPa

$$\tau_{BC} = \frac{5}{35}\tau_{CD} = \frac{5}{35}(75.58) = 10.8 \text{ MPa}$$

$$\tau_{DE} = \frac{10}{35}\tau_{CD} = \frac{10}{35}(75.58) = 21.6 \text{ MPa}$$

(b) from eqn (6-15)

$$\phi_{AE} = -\phi_{AB} - \phi_{BC} + \phi_{CD} + \phi_{DE}$$

$\phi_{AB} = 0$ for $T_{AB} = 0$

$$\phi_{BC} = \frac{1855\,(5/35)\,3}{6.14 \times 10^{-7} \times 84 \times 10^9} = 0.0154$$

$$\phi_{CD} = \frac{1855 \times 6}{6.14 \times 10^{-7} \times 84 \times 10^9} = 0.2158$$

$$\phi_{DE} = 0.2158\,(10/35) = 0.0616$$

Then, $\phi_{AE} = 0.262$ rad $= 15.01$ deg.

6-17 $L = 6, \quad D_i/D_o = 0.5$

(a)

$$I_p = \frac{TL}{\phi G} = \frac{1 \times 6}{0.1 \frac{2\pi}{360}(12\times10^6)}$$

$$I_p = 2.865 \times 10^{-4}$$

$$I_p = \frac{\pi}{2}\left[c^4 - \left(\frac{c}{2}\right)^4\right] = 2.865\times10^{-4}$$

$$c^4 = 1.945 \times 10^{-4}$$

$$c = 0.12 \text{ in}, \quad D_o = 2c = 0.24 \text{ in}$$

(b) From eqn. (6-19)

$$k_t = \frac{\pi \times 6 \times 12\times10^6}{\frac{1}{0.12^2} - \frac{1}{0.24^2}} = 4.34\times10^6 \text{ in-lb/rad}$$

6-18 (a) $L = 1000$ mm, $c = 30$ mm

Criterion: strength

$$\tau_{max, steel} = 2\,\tau_{max, Al}$$

$$\frac{Tc}{I_{p\,st}} = 2\frac{Tc}{I_{p\,Al}}$$

$$\frac{\pi c^4}{2} = 2\,\frac{\pi}{2}(c^4 - r_i^4)$$

$$r_i = 25.2 \text{ mm}$$

Criterion: stiffness

$$\phi_{Al} = \phi_{steel}$$

$$\frac{TL}{G_{Al} I_{p\,Al}} = \frac{TL}{G_{st} I_{p\,st}}$$

$$28\,\frac{\pi c^4}{2} = 84\,\frac{\pi}{2}(c^4 - r_i^4)$$

$$r_i = 27.1 \text{ mm}$$

(b) Strength controls — use $r_i = 25$ mm

6-19 $\phi = \dfrac{TL}{I_p G}$

$\phi_{A1} = \phi_A$ due to 600 N.m torque

$\phi_{A2} = \phi_A$ due to rotation of gear C

$$\phi_{A1} = \frac{600 \times 6}{\frac{\pi}{2}(0.03)^4(84\times10^9)} = 0.0337 \text{ rad.}$$

$$\phi_C = \frac{(400/200)\,600\times3}{\frac{\pi}{2}(0.03)^4(84\times10^9)} = 0.0337 \text{ rad}$$

$$\phi_{A2} = 2\phi_C = 0.0674 \text{ rad}$$

$$\phi_A = \phi_{A1} + \phi_{A2} = 0.1011 \text{ rad} = 5.79^\circ$$

6-20 (a)

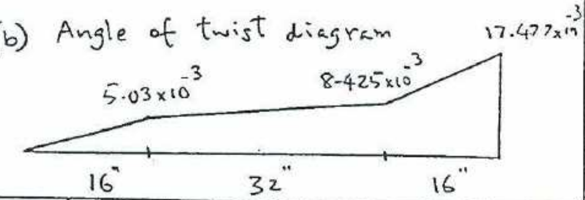
$I_{p3''} = 7.952 \text{ in}^4, \quad \phi_1 = 5.030\times10^{-3}$

$I_{p2''} = 1.571 \text{ in}^4, \quad \phi_2 = 3.395\times10^{-3}$

$I_{p_o} = 1.473 \text{ in}^4, \quad \phi_3 = 9.052\times10^{-3}$

$$\phi = \phi_1 + \phi_2 + \phi_3 = 17.477\times10^{-3} \text{ rad} = 1.0^\circ$$

(b) Angle of twist diagram



6-21 $T = \dfrac{119 \times hp}{f}$ and $\phi = \dfrac{TL}{I_p G}$

$$\phi = 6^\circ = 6\times\frac{\pi}{180} \text{ rad} = \frac{119\,hp\,(0.3)}{(20)\frac{\pi}{2}(0.006)^4(84\times10^9)}$$

$$hp = 10$$

Power input required: 10 hp

6-22 From eqn. (6-19)

$$K_t = \frac{\pi \times 1 \times 70\times10^6}{\frac{1}{0.020^2} - \frac{1}{0.030^2}}$$

$$K_t = 1.583\times10^5 \text{ N.m/rad}$$

Torsional flexibility, $f_t = 1/K_t$

$$f_t = 6.32\times10^{-6} \frac{\text{rad}}{\text{N.m}}$$

6-23 $L = 0.5\,m$, $d = 0.024\,m$, $G = 83\,MPa$

$$k_t = \frac{I_p G}{L} = \frac{\pi}{2}\frac{(0.012)^4 (83\times10^6)}{0.5}$$

$$k_t = 5.407$$

$$I_{mz} = mR^2/2 = 2R^2/2 = R^2$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_t}{I_{mz}}} = \frac{1}{2\pi}\sqrt{\frac{5.407}{R^2}}$$

$$f_n = 0.37/R$$

(b) Using Al alloy:

$$k_t = \frac{\pi}{32} d^4 \frac{(26\times10^9)}{0.5} = 5.105\times10^9 d^4$$

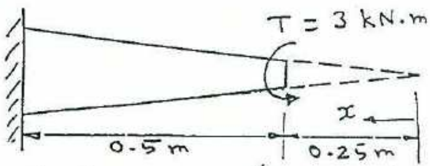
$$f_n = \frac{0.37}{R} = \frac{1}{2\pi}\sqrt{\frac{5.105\times10^9 d^4}{R^2}}$$

$$\frac{0.37}{R} = \frac{1}{2\pi R}(7.145\times10^4\, d^2)$$

$$d = 0.0057\,m \text{ or } 5.7\,mm$$

6-24

(a)



$$I_p = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{x}{10}\right)^4 = \frac{\pi x^4}{2\times10^4}$$

$$\phi = \int\frac{T\,dx}{I_p G} = \frac{3000}{200\times10^9}\frac{2\times10^4}{\pi}\int_{0.25}^{0.75}\frac{dx}{x^4}$$

$$= \frac{3}{\pi}10^{-4}\left|-\frac{x^{-3}}{3}\right|_{0.25}^{0.75}$$

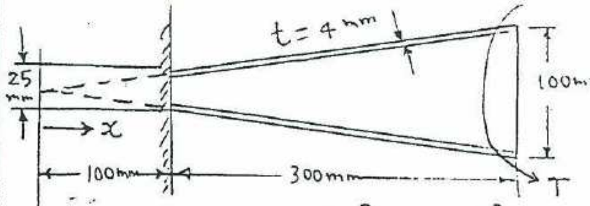
$$= \frac{1}{\pi}10^{-4}\left(\frac{1}{0.25^3} - \frac{1}{0.75^3}\right)$$

$$\phi = 1.962\times10^{-3}\ rad$$

(b) $$f_t = \frac{\phi}{T} = \frac{1.962\times10^{-3}}{3000}$$

$$f_t = 6.539\times10^{-7}\ (rad/N.m)$$

6-25



$$c = \frac{x}{8},\quad I_p = 2\pi c^3 t = 2\pi\left(\frac{x}{8}\right)^3(0.004)$$
$$= 4.9\times10^{-5}x^3$$

$$\phi = \int_L\frac{T\,dx}{I_p G} = \frac{T}{G}\int_{0.1}^{0.4}\frac{dx}{4.9\times10^{-5}x^3}$$

$$= \frac{T}{4.9\times10^{-5}G}\left|-\frac{x^{-2}}{2}\right|_{0.1}^{0.4}$$

$$\phi = \frac{T}{4.9\times10^{-5}G}\,\frac{1}{2}\left(\frac{1}{0.1^2} - \frac{1}{0.4^2}\right)$$

$$k_t = \frac{T}{\phi} = 1.045\times10^{-6}\,G\ (N.m/rad)$$

(b) $f_t = 1/k_t = 9.569\times10^5\,G$ (rad/N.m)

6-26

$$I_p G\frac{d^2\phi}{dx^2} = -t_x = -kx$$

$$T = I_p G\phi' = -\frac{1}{2}kx^2 + c_1$$

Since $T(0) = 0$, $c_1 = 0$

$$I_p G\phi = -\frac{1}{6}kx^3 + c_2$$

Since $\phi(L) = 0$, $c_2 = kL^3/6$

$$\phi(0) = \frac{c_2}{I_p G} = \frac{kL^3}{6 I_p G}$$

6-27

$$T_A = \int_0^L\frac{t_o}{L}x\,dx = \frac{t_o L}{2}$$

$$T(x) = T_A - \int_0^x\frac{t_o}{L}x\,dx = \frac{t_o L}{2} - \frac{t_o x^2}{2L}$$

$$\phi_B = \int\frac{T(x)}{G I_p}dx = \frac{t_o L^2}{3 G I_p}$$

6-28

$$T_A = \int_0^2\left(200 + 240\frac{x}{2}\right)dx = 640$$

$$T(x) = 640 - \int_0^x(200 + 120x)\,dx = 640 - 200x - 60x^2$$

$$\phi(B) = \int_0^2\frac{T(x)\,dx}{G I_p} = \frac{720}{G I_p}$$

0 0

6-29 $E_{st} = 210\ GPa,\ E_{AL} = \frac{1}{3} E_{st} = 70\ GPa$

(a) Using Table 1 of Appendix,

$G_{st} = 83\ GPa,\quad G_{AL} = 26\ GPa$

$I_{P\,st} = \frac{\pi}{2}(0.025)^4 = 6.14 \times 10^{-7}\ m^4$

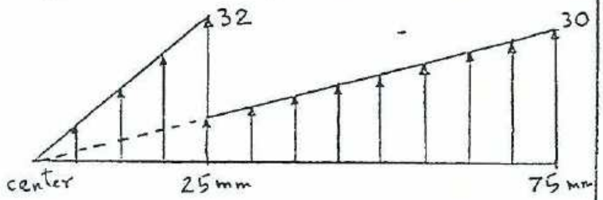
$I_{P\,AL} = \frac{\pi}{2}\left[(0.075)^4 - (0.025)^4\right] = 4.91 \times 10^{-5}\ m^4$

$\tau_{st} = G_{st}\frac{1}{3}\gamma_{max},\quad \tau_{AL} = G_{AL}\gamma_{max}$

$T = 20 = \frac{1}{0.075}\left[\left(83\times10^9 \times 6.14\times10^{-7} \times \frac{1}{3}\right) + (26\times10^9 \times 4.91\times10^{-5})\right]\gamma_{max}$

$\gamma_{max} = 1.16 \times 10^{-6}\ rad$

$\tau_{st} = 32\ kPa,\quad \tau_{AL} = 30\ kPa$



(b) $L = 1\ m$

$\phi = \frac{32 \times 1}{83\times10^6 \times 25\times10^{-3}} + \frac{30 \times 1}{26\times10^6 \times 75\times10^{-3}}$

$= 3.08 \times 10^{-5}$

$k_t = \frac{T}{\phi} = \frac{20}{3.08\times10^{-5}} = 6.49\times10^5\ (kN.m/rad)$

$f_t = \frac{1}{k_t} = 1.54\times10^{-6}\ (rad/kN.m)$

6-30 $\phi_s = \phi_t \qquad \frac{T_s L}{G I_{Ps}} = \frac{T_t L}{G I_{Pt}}$

$\frac{T_s}{T_t} = \frac{I_{Ps}}{I_{Pt}} = \frac{\pi(0.025)^4/32}{2\pi(0.025)^3(0.002)} = 0.195$

$T_s + T_t = T$

$0.195\,T_t + T_t = T,\quad T_t = 0.837\,T$

6-31

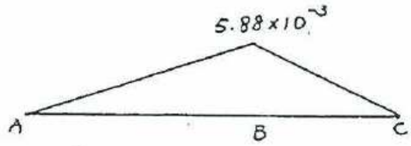
$T_s = (1 - 0.837) \times 200 = 33\ N.m$

6-32

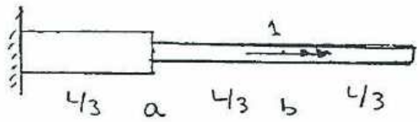
$T_1 = \frac{\phi \times 12\times10^3 \times 2\pi}{15} = 5\times10^3\,\phi\ (K\text{-}in)$

$T_2 = \frac{\phi\times12\times10^3\times\pi}{10\times2} = 1.8\times10^3\,\phi\ (K\text{-}in)$

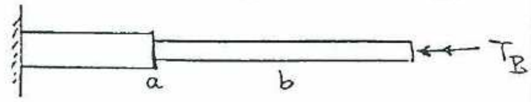
$T_1 + T_2 + T = 0 \Rightarrow \phi = 5.88\times10^{-3}\ (rad)$



6-33 (a) unit torque applied at b



$\phi_{end} = \frac{L}{3 I_{P2} G} + \frac{L}{3 I_{P1} G} = \frac{4}{3}\frac{L}{3 I_{P2} G}$



$\phi_{end} = \frac{2}{3}\frac{L\,T_B}{G I_{P2}} + \frac{L}{3}\frac{T_B}{G I_{P1}} = \frac{7}{3}\frac{L\,T_B}{3 I_{P2} G}$

$\therefore\ T_B = 4/7 \text{ and } T_A = 3/7$

$\phi_{ab} = \frac{3}{7}\cdot\frac{L}{3 G I_{P1}} = \frac{L}{7 G I_{P1}}$

(b) unit torque applied at a

$\phi_{end} = \frac{L}{3 G I_{P1}} = \frac{7}{3}\frac{T_B' L}{3 G I_{P2}}$

$T_B' = 1/7$

$\phi_{ba} = \frac{L}{7\times3 G I_{P2}} = \frac{L}{7 G I_{P1}}$

$\therefore\ \phi_{ab} = \phi_{ba} = \frac{L}{7 G I_{P1}}$

6-34 Flexibility, $f_t = \phi_{aa}$ due to unit torque at the location where the disk is attached

$$\phi_{aa} = \frac{T_A\,(0.2)}{G I_p} = \frac{T_B\,(0.4)}{G I_p}$$

$$\therefore\ T_A = 2T_B, \quad T_B = 1/3 \text{ and } T_A = 2/3$$

$$\phi_{aa} = \frac{2}{3}\,\frac{(0.2)}{G I_p} = f_t$$

$$k_t = 1/f_t = \frac{3 G I_p}{0.4} = 7.5\,G I_p$$

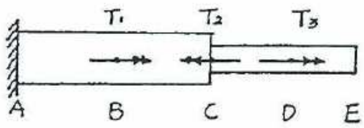
Frequency: $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k_t}{I_m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_t}{mR^2/2}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{7.5\,G\,\pi\,(0.020)^4}{2 \times 50\,\frac{R^2}{2}}}$$

$$\therefore f = 4.37 \times 10^{-5}\,\frac{\sqrt{G}}{R}$$

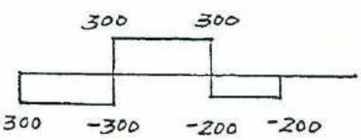
6-35

(a)

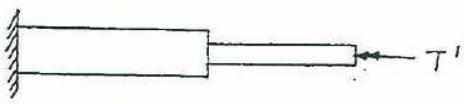


$$J_1 = \frac{\pi}{2}\left(\frac{2.83}{2}\right)^4, \quad J_2 = \frac{\pi}{2}\left(\frac{2.38}{2}\right)^4$$

from Table 1 of Appendix $G = 3.75$ Ksi



$$\phi_{end} = \frac{(-300+300)}{J_1 G}\times 30 + \frac{200 \cdot 25}{J_2 G} = 0.423$$



$$\frac{-T'L_1}{J_1 G} + \frac{-T'L_2}{J_2 G} + 0.423 = 0$$

$$T' = 62.4 \ (lb-in)$$

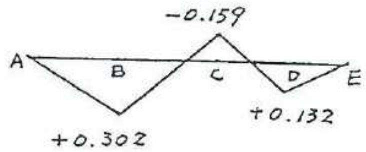
(b) $\phi_{AB} = -\left(\dfrac{-300\times 30}{J_1 G} + \dfrac{62.4 \times 30}{J_1 G}\right) = 0.302\,(rad)$

$$\phi_{BC} = -\left(\frac{300\times 30}{J_1 G} + \frac{62.4\times 30}{J_1 G}\right) = -0.461\,(rad)$$

$$\phi_{CD} = -\left(\frac{-200\times 25}{J_2 G} + \frac{62.4\times 25}{J_2 G}\right) = 0.291\,(rad)$$

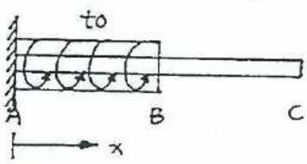
$$\phi_{DE} = -\left(\frac{62.4\times 25}{J_2 G}\right) = -0.132\ (rad)$$

check $\phi_{AE} = \phi_{AB} + \phi_{BC} + \phi_{CD} + \phi_{DE} = 0$ O.K.



6-36

(a) solved by forced method



$$T_A = \frac{t_o}{2}L$$

$$T(x) = -\frac{t_o L}{2} + t_o x, \quad x \le \frac{L}{2}$$

$$\phi_{AC} = -\int_0^{\frac{L}{2}} \frac{T(x)}{JG}\,dx = +\frac{t_o L^2}{8JG}$$

$$\frac{T_c L}{JG} + \phi_{AC} = 0 \Rightarrow T_c = \frac{t_o L}{8}$$

(b) $\phi_x = -\displaystyle\int_0^x \frac{T(x)}{JG}\,dx - \frac{T_c X}{JG} \qquad x \le \frac{L}{2}$

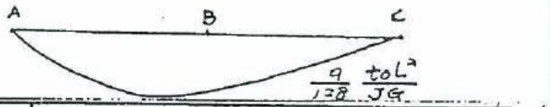
$$= \frac{1}{JG}\left(\frac{3 t_o L}{8}x - \frac{t_o}{2}x^2\right)$$

Let $\dfrac{d\phi}{dx} = 0 \Rightarrow x = \dfrac{3}{8}L$

$$\phi_x = \frac{t_o L^2}{8JG} - \frac{T_c X}{JG} \qquad x > \frac{L}{2}$$

6-36 (contd.)

x	0	$\frac{L}{8}$	$\frac{L}{4}$	$\frac{3L}{8}$	$\frac{L}{2}$	$\frac{3L}{4}$	L
$\phi \cdot \frac{JG}{t_o L^2}$	0	$\frac{5}{128}$	$\frac{1}{16}$	$\frac{9}{128}$	$\frac{1}{16}$	$\frac{1}{32}$	0



6-37

(a) $\frac{T_B L}{JG} = \frac{t_o L^2}{3JG}$. $\quad T_B = \frac{t_o L}{3} \longleftarrow$

(b) $\phi_x = \frac{-1}{JG}\int_0^x T(x)\,dx + \frac{t_o L}{3JG}x$

$= \frac{-1}{JG}\left(\frac{t_o L}{6}x + \frac{t_o x^3}{6L}\right)$

$\frac{d\phi}{dx} = 0 \Rightarrow x = L/\sqrt{3}$

check $x = L$, $\phi_L = 0$ O.K.

6-38

$JG\phi'' = -\frac{t_o}{L}x$

$JG\phi = -\frac{t_o}{6L}x^3 + C_1 x + C_2$

$\phi(0) = C_2 = 0$

$T = JG\phi' = -\frac{t_o}{2L}x^2 + C_1$

$T(L) = 0 \Rightarrow C_1 = \frac{t_o L}{2}$

$\phi(L) = -\frac{t_o L^2}{6JG} + \frac{t_o L^2}{2} = \frac{t_o L^2}{3}$

6-39

$JG\phi'' = -440 + \frac{240}{2}x$

$JG\phi' = T = -440x + 60x^2 + C_1$

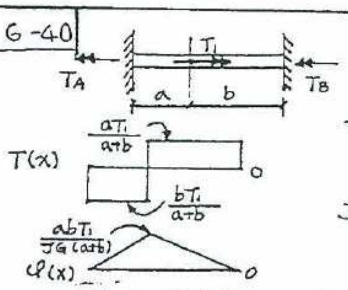
$T_{end} = 0 \Rightarrow C_1 = 0$

$JG\phi = -220x^2 + 20x^3 + C_2$

$\phi(2) = 0 \Rightarrow C_2 = 720$

$\phi_{end} = \phi(0) = \frac{720}{JG}$

6-40



$JG\frac{d^2\varphi}{dx^2} = -t_x$

$= -T_1\langle x-a\rangle^{-1}$

$T = JG\frac{d\varphi}{dx}$

$= -T_1\langle x-a\rangle^0 + C_1$

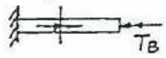
$JG\varphi = -T_1\langle x-a\rangle^1 + C_1 x + C_2$

Since $\varphi(0) = 0$, $C_2 = 0$; also since

$\varphi(L) = \varphi(a+b) = 0$, $C_1 = (b/L)T_1$

$T(0) = C_1 = T_A = \frac{b}{a+b}T_1$

$T(L) = T_B = -\frac{a}{a+b}T_1$



6-41

(a) $JG\phi'' = -t_o \qquad x \le \frac{L}{2}$

$JG\phi' = -t_o x + C_1$

$JG\phi = -\frac{t_o}{2}x^2 + C_1 x + C_2$

$\phi(0) = 0 \Rightarrow C_2 = 0$

$JG\phi'(\frac{L}{2}) = -\frac{t_o L}{2} + C_1$

$JG\phi(\frac{L}{2}) = -\frac{t_o L^2}{8} + C_1\frac{L}{2}$

(b) $JG\phi'' = 0 \qquad x \ge \frac{L}{2}$

$JG\phi = C_1' x + C_2'$

$\phi(L) = 0 \Rightarrow C_2' = -C_1' L$

$JG\phi'(\frac{L}{2}) = C_1'$

$JG\phi(\frac{L}{2}) = -\frac{C_1' L}{2}$

$C_1' = -\frac{t_o L}{2} + C_1$

$\frac{t_o L^2}{4} - \frac{C_1}{2}L = -\frac{t_o L^2}{8} + C_1\frac{L}{2}$

$C_1 = \frac{3}{8}t_o L$, $\quad C_1' = \frac{-1}{8}t_o L$

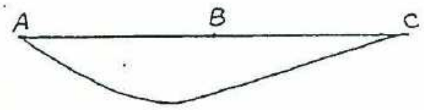
$T_A = JG\phi'(0) = \frac{3}{8}t_o L \longleftarrow$

$T_B = JG\phi'(L) = \frac{1}{8}t_o L \longleftarrow$

(angle of twist diagram on the next page)

6-41 (cont'd)

x	0	$\frac{L}{8}$	$\frac{L}{4}$	$\frac{3L}{8}$	$\frac{L}{2}$	$\frac{3L}{4}$	L
$\frac{JG}{t_0 L^2}\phi$	0	$\frac{5}{128}$	$\frac{1}{16}$	$\frac{9}{128}$	$\frac{1}{16}$	$\frac{1}{32}$	0



$\frac{9}{128}\frac{t_0 L^2}{JG}$

6-42 (a) $J_1 = 98.2\times10^{-3}d^4$, $J_2 = 31.06\times10^{-3}d^4$

$\tau_{1max} = 5.09\frac{T_1}{d^3}$, $\tau_{2max} = 12.07\frac{T_1}{d^3}$

$V_1 = \pi(\frac{d}{2})^2\times a = 785.4\times10^{-3}d^2a$

$V_2 = \pi(\frac{3}{4}\times\frac{d}{2})^2\times a = 442\times10^{-3}d^2a$

$\frac{T_1\phi}{2} = \frac{\tau_{1max}^2}{2G}\times\frac{1}{2}V_1 + \frac{\tau_{2max}^2}{2G}\times\frac{1}{2}V_2$

$\phi = 42.37\ T_1/d^4G$

(b) check by eq. 6-16

$\phi = \frac{T_1 a}{J_1 G} + \frac{T_1 a}{J_2 G}$

$= \frac{T_1 a}{d^4 G}\left(\frac{32}{\pi} + \frac{32\times4^4}{\pi\times3^4}\right)$

$= 42.4\ \frac{T_1 a}{d^4 G}$ O.K.

6-43 $T = \Sigma F.r = \Sigma(\tau A)r$

(a) $T = 8\left[(10.5)\pi(3/8)^2\right]5$

$= 185.55$ k-in

(b) $T = \frac{63000}{N}H_p$

$H_p = \frac{300\ (185.55\times10^3)}{63000} = 883.6$

6-44 $F = A\,\tau_{All}$

$F_o = 0.2(16) = 3.2$ k.

$F_i = \frac{5}{8}[0.5(16)] = 5$ k

$T = \Sigma Fr$

$T = 6(3.2\times4 + 5\times2.5) = 151.80$ k-in

6-45 $\tau = \frac{Tc}{I_P} = \frac{800\ (0.010)}{\frac{\pi}{2}(0.010)^4} = 509$ MPa

6-46

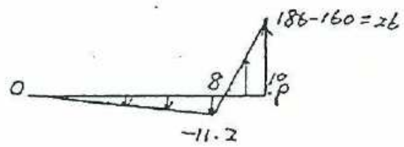
(a) $T = \int_0^8 2\pi(\frac{160}{8})\rho^3 d\rho + \int_8^{10} 160\ 2\pi\rho^2 d\rho$

$= 40\pi\frac{\rho^4}{4}\Big|_0^8 + 320\pi\frac{\rho^3}{3}\Big|_8^{12}$

$= 292210$ (N·mm)

$= 292.9$ (N·m)

(b) $\tau_{max} = \frac{292210\times10}{\frac{\pi}{2}(10)^4} = 186$ (MPa)



6-47 $\frac{\gamma_{max}}{10} = \frac{0.25}{1000} \Rightarrow \gamma_{max} = 0.0025$

$\rho_e = \frac{0.002}{0.0025}\times10 = 8$ (cm)

from prob. 4-38 we get

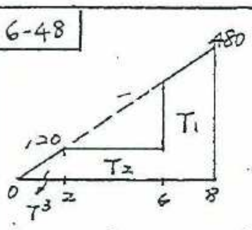
$T = 292210$ N-mm

$\phi_R = \frac{292210\times1000}{\frac{\pi}{2}(10^4)\times80\times10^3} = 0.232$

$\Delta\phi = 0.25 - 0.232 = 0.018$ (rad)

$= 1.0$ (degree)

6-48



$$\rho_e = \frac{120}{480} \times 8 = 2$$

$$T_1 = 480 \times \frac{\pi}{2} \times \frac{(8^4 - 6^4)}{8} = 263894$$

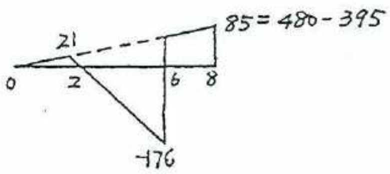
$$T_3 = 120 \times \frac{\pi}{2} \times \frac{2^4}{2} = 1508$$

$$T_2 = \int_2^6 120 \times 2\pi \rho^2 d\rho = 52276$$

$$T = T_1 + T_2 + T_3 = 317678 \text{ N-mm} = 318 \text{ N-m}$$

6-49

(a) $$\tau_{max} = \frac{317678 \times 8}{\frac{\pi}{2} \times 8^4} = 395 \text{ (MPa)}$$



(b) initially $$\theta_i = \frac{T_1}{JG} = 5.0 \times 10^{-4} \text{ rad}$$

release $$\theta_f = \frac{T}{JG} = 4.11 \times 10^{-4} \text{ rad}$$

$$\Delta\theta = (5 - 4.11) \times 10^{-4} = 0.89 \times 10^{-4} \text{ rad}$$

6-50

Area $= \pi(5)^2 = 78.54 \text{ cm}^2$

Rectangle : $a = 2.5$ cm, $b = 31.42$ cm

Square : $s = \sqrt{78.54} = 8.86$ cm

$$\frac{\tau_c}{\tau_s} = \frac{Tc/J}{T/\alpha_s s^3} = \frac{5(8.86)^3(.208)}{(\pi/2)(5)^4} = 0.74$$

$$\frac{\tau_R}{\tau_s} = \frac{T/\alpha_R ba^2}{T/\alpha_s s^3} = \frac{(8.86)^3(.208)}{(0.333)31.42(2.5)^2} = 2.21$$

$$\tau_s : \tau_c : \tau_R = 1 : 0.74 : 2.21$$

$$\frac{\varphi_c}{\varphi_s} = \frac{TL/JG}{TL/\beta_s s^4 G} = \frac{0.141(8.86)^4}{(\pi/2)(5)^4} = 0.886$$

6-50 (contd.)

$$\frac{\varphi_R}{\varphi_s} = \frac{\beta_s s^4}{\beta_R a^3 b} = \frac{0.141(8.86)^4}{(0.333)(2.5)^3(31.42)} = 5.32$$

$$\varphi_s : \varphi_c : \varphi_R = 1 : 0.886 : 5.32$$

6-51

$$\frac{\tau_{slot}}{\tau} = \frac{\dfrac{T}{0.33(2\pi R t^2)}}{\dfrac{TR}{2\pi R^3 t}} = \frac{3R}{t}$$

$$\frac{T_{slot}}{T} = \frac{\varphi G(0.33 \times 2\pi R t^3)}{\varphi G(2\pi R^3 t)} = \frac{t^2}{3R^2}$$

6-52

$$4 \cdot T_{bar} + T_{tube} = T$$

Bar: $b = 2$, $t = 1$ $b/t = 2 \Rightarrow \alpha = 0.246$

$\beta = 0.229$

$$\frac{T_{tube}}{T_{bar}} = \frac{\pi/2\,(2^4 - 1.5^4)}{0.229 \times 2 \times 1^3} = 37.5$$

$$4\, T_{bar} + 37.5\, T_{bar} = T$$

$$T_{bar} = 0.0241\, T$$

$$T_{tube} = 0.9036\, T$$

$$T_{tube} = \frac{\tau I_p}{c} = 10\, \frac{\pi}{2}(2^4 - 1.5^4) \cdot \frac{1}{2}$$

$$= 85.9 \text{ k-in}$$

$$T = T_{tube}/0.9036 = 95.1 \text{ k-in}$$

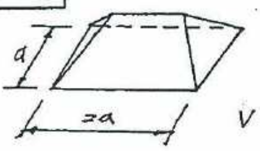
$$T_{bar} = \tau \cdot \alpha b t^2 = 10\,(0.246) \times 2 \times 1^2$$

$$= 4.92 \text{ k-in}$$

$$T = T_{bar}/0.0241 = 204.1 \text{ k-in}$$

$\therefore$ Max $T = 95.1$ k-in

6-53



$$\tau_{yp} = \frac{h}{a/2} \; ; \; h = \frac{a}{2}\tau_{yp}$$

$$V = \frac{1}{2} h a^2 + \frac{1}{3} h a^2$$

$$= \frac{1}{2}\frac{a}{2}\tau_{yp} \cdot a^2 + \frac{1}{3}\frac{a}{2}\tau_{yp} a^2$$

6-53 (contd.)

$$V = \frac{a^3}{4}\tau_{yp} + \frac{a^3}{6}\tau_{yp} = \frac{5a^3\tau_{yp}}{12}$$

$$T_{ult.} = 2V = 2\times\frac{5a^3\tau_{yp}}{12} = \frac{5}{6}a^3\tau_{yp}$$

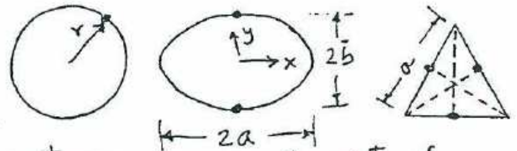
6-54

$$\frac{b}{t} = \frac{120}{9} = 13.3 \Rightarrow \beta \approx 0.312$$

$$J_o = 2\times\frac{\pi}{2}(20^4 - 12^4) + 0.312\times120\times9^3$$

$$= 465\times10^3 \quad mm^4$$

6-55 (a) Membrane analogy



Max stress occurs at point of max slope of membrane over these shapes

Circle: max stress around the circumference

Ellipse: max stress at $x=0$, $y=\pm b$

Equilateral triangle: max stress at mid-side locations

(b) $T = 1000$ N.m

$r = b = 0.015$ m, $a = 0.020$ m

Let $G = 84$ GPa for steel

Circle: $\tau_{max} = \frac{Tc}{I_p} = \frac{2T}{\pi r^3}$

$$\tau_{max} = \frac{2\times1000}{\pi(0.015)^3} = 188.6\times10^6\ Pa$$

$$\phi = \frac{TL}{I_pG} = \frac{2T}{\pi r^4}\frac{L}{G} = \frac{2\times1000\times L}{\pi(0.015)^4\,84\times10^9}$$

$$\phi = 0.15\,L$$

Ellipse: $\tau_{max} = \frac{2T}{\pi ab^2} = \frac{2\times1000}{\pi(0.020)(0.015)^2}$

$$\tau_{max} = 141.5\times10^6\ Pa$$

$$\phi = \frac{a^2+b^2}{\pi a^3b^3}\frac{TL}{G} = \frac{(0.02^2+0.015^2)\,1000\,L}{\pi(0.02)^3(0.015)^3\,84\times10^9}$$

$$\phi = 0.088\,L$$

Equilateral triangle:

$$\tau_{max} = \frac{20T}{a^3} = \frac{20\times1000}{0.02^3} = 2.5\times10^9\ Pa$$

$$\phi = \frac{80TL}{a^4\sqrt{3}\,G} = \frac{80\times1000\,L}{(0.02)^4\sqrt{3}\,84\times10^9}$$

$$\phi = 3.44\,L$$

6-56 $A = 1200\ mm^2 = 0.0012\ m^2$

Circle: $A = \pi r^2 = 0.0012$, $r = 0.01954$ mm

$$\tau_{max} = \frac{2T}{\pi r^3} = \frac{2T}{Ar} = \frac{2\times1000}{(0.0012)(0.01954)}$$

$$\tau_{max} = 85.3\times10^6\ Pa \text{ or } 85.3\ MPa$$

Ellipse: $A = \pi ab$

Let $b = 10$ mm $= 0.010$ m

$$\tau_{max} = \frac{2T}{\pi ab^2} = \frac{2T}{A.b} = \frac{2\times1000}{(0.0012)(0.010)}$$

$$\tau_{max} = 166.7\times10^6\ Pa \text{ or } 166.7\ MPa$$

Equilateral triangle:

$$A = \frac{1}{2}.a.h = \frac{1}{2}a.a\sin60^\circ = \frac{\sqrt{3}}{4}a^2$$

$$A = \frac{\sqrt{3}}{4}a^2 = 0.0012,\quad a = 0.0526\,m$$

$$\tau_{max} = \frac{20T}{a^3} = \frac{20\times1000}{(0.0526)^3}$$

$$\tau_{max} = 137\times10^6\ Pa \text{ or } 137\ MPa$$

6-57 For the circle, ellipse and triangle to have the same torsional rigidity:

$$I_{p\,circle} = K_{ellipse} = K_{triangle}$$

$$\frac{\pi r^4}{2} = \frac{\pi a^3b^3}{(a^2+b^2)} = \frac{a^4\sqrt{3}}{80}$$

6-57 (contd.)

Triangle & circle: $\frac{a^4\sqrt{3}}{80} = \frac{\pi r^4}{2}$

$$a^4 = \frac{40\pi}{\sqrt{3}} r^4$$

$$a = 2.919\ r$$

Ellipse & circle: $\frac{\pi a^3 b^3}{a^2+b^2} = \frac{\pi r^4}{2}$

Say, e.g., that $b = a/2$

Then,

$$\frac{\pi a^3 . a^3/8}{a^2 + a^2/4} = \frac{\pi r^4}{2}$$

$$a = 1.495\ r$$

and $b = a/2 = 0.748\ r$

6-58 $A = \pi(12)^2 = 452.4\ in^2$

$$q = \frac{T}{2A} = \frac{1000}{2\times 452.4} = 1.11\ lb/in$$

$$\tau_{max} = \frac{q}{t_{min}} = \frac{1.11}{0.1} = 11.1\ psi$$

$$\theta = \frac{T}{4A^2G}\oint\frac{ds}{t} = \frac{1000}{4(452.4)^2G}\left(\frac{12\pi}{0.1}+\frac{12\pi}{0.2}\right)$$

$$= \frac{1000\times 180\pi}{4\times 452^2 G}$$

$$= \frac{0.691}{G}\ rad/in$$

6-59 $T = 2Ⓐq = 2Ⓐ\tau_{max}t$

$$= 2\times\left(\frac{\pi}{2}20^2 + 20\times 40 + 20\times 100\right)\times 10^{-6} \times 20\times 10^6 \times 0.004$$

$$T = 548.53\ N.m$$

Since shear flow is constant and $\tau = q/t$, increasing the thickness of the inclined plates would reduce τ and help reduce stress concentration effect at the sharp corner.

6-60 (a) $K_{t.bar} = \beta b t^3 \frac{G}{L}$

$$= 0.229\times 60\times 30^3 \frac{G}{L}$$

$$= 370980 \times \frac{G}{L}$$

$$K_{t.box} = 4A^2/\oint ds/t \times \frac{G}{L}$$

$$= \frac{4\times(40\times 100)^2}{150/3 + 100/4}\times\frac{G}{L}$$

$$= 853333\frac{G}{L}$$

$$K_{total} = 1224313\ \frac{G}{L}$$

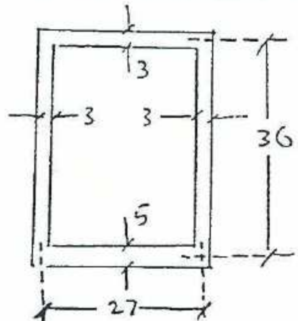
$T_{bar} = 0.30T$, $T_{box} = 0.70T$

$$\tau_{bar} = \frac{0.30T}{0.246\times 60\times 30^2} = 3.39\ MPa$$

$$\tau_{box} = \frac{0.70T}{2\times 40\times 100\times 3} = 4.38\ MPa$$

(b) $\theta = \frac{\phi}{L} = \frac{T}{K_t L} = \frac{150}{1224313\times 25} = 4.9\times 10^{-6}\ (rad)$

6-61



$$Ⓐ = (0.027)(0.036) = 9.72\times 10^{-4}$$

$$\tau = \frac{T}{2Ⓐt_{min}} = \frac{T}{2\times 9.72\times 10^{-4}(0.003)}$$

Since $\tau = 120\times 10^6\ Pa$,

$$T = 120\times 10^6 \times 2\times 9.72\times 10^{-4}\times 0.003$$

$$T = 699.8\ N.m$$

$$k_t = \frac{4Ⓐ^2\ G/L}{\oint ds/t} = \frac{4(9.72\times 10^{-4})^2\ G/L}{\frac{0.099}{0.003} + \frac{0.027}{0.005}}$$

6-61 (cont'd)

$$k_t = 9.842\times10^{-8}\ G/L$$

Flexibility in torsion:

$$f_t = \frac{1}{k_t} = 1.016\times10^{7}\ L/G$$

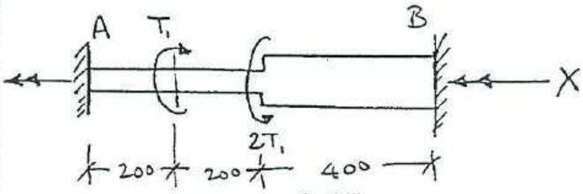
6-62 Solid bar: $\phi = \dfrac{TL}{\beta b t^3 G}$

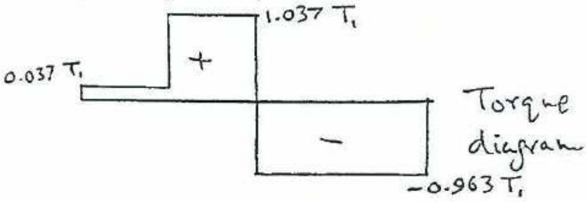
Tube: $\phi = \dfrac{TL}{G}\,\dfrac{\oint ds/t}{4Ⓐ^2}$

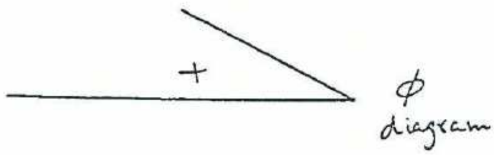
Solid bar: $\beta b t^3 = 0.141\times20\times20^3$

$= 2.256\times10^4$

Tube: $\dfrac{4Ⓐ^2}{\oint ds/t} = \dfrac{4\times(30\times30)^2}{120/1.5} = 4.05\times10^4$







Cut at B and reapply torque X:

$$\phi = \frac{2T_1(200)}{G(2.256\times10^4)} + \frac{T_1(200)}{G(2.256\times10^4)} = \frac{X(400)}{G(2.256\times10^4)} + \frac{X(400)}{G(4.05\times10^4)}$$

Solving,

$$X = T_B = \underline{0.963\,T_1}$$

From $\Sigma M_x = 0$, $T_A = \underline{0.037\,T_1}$

Tube: $T_{max} = \tau_{max}\,2Ⓐ\,t$

$T_{max} = 40\times10^6\times2\times0.030^2\times0.0015$

$= 108\ \text{N.m} = 0.963\,T_1$

$\therefore\ T_1 = \underline{112.15}\ \text{N.m}$

Square bar: $T_{max} = \tau_{max}\,\alpha\,b t^2$

$T_{max} = 40\times10^6\times0.208\times0.02\times0.02^2$

$= 1.331\ \text{N.m} = 0.037\,T_1$

$\therefore\ T_1 = \underline{35.98}\ \text{N.m}$

Governs

$$\Delta_{tip} = 0.250\,\phi$$

$$\phi = \frac{X}{G}\,\frac{400}{4.05\times10^4} = \frac{0.963\,T_1}{G}\,(9.875\times10^{-3})$$

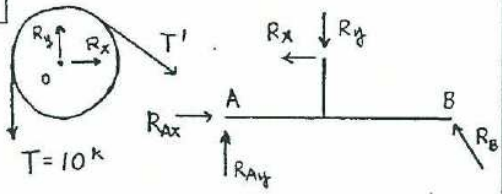
$$\phi = 9.51\times10^{-3}\,\frac{T_1}{G}$$ [note that 9.51×10^{-3} was derived using mm units]

$$\phi = 9.51\times10^{6}\,\frac{T_1}{G}$$ [using consistent meter units]

$$\Delta_{tip} = 0.25\times9.51\times10^{6}\times\frac{35.98}{200\times10^9}$$

$$\Delta_{tip} = 0.00043\ \text{m} \quad\text{or}\quad 0.43\ \text{mm}$$

7-1



$\Sigma M_o = 0 \quad \rightarrow \quad T' = T = 10^k$

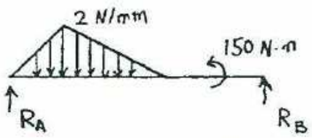
$R_x = -\frac{2}{\sqrt{5}} T' = -4\sqrt{5}$

$R_y = T + \frac{1}{\sqrt{5}} T' = 10 + 2\sqrt{5}$

$$\begin{cases} R_{Ax} - \frac{1}{\sqrt{2}} R_B = -4\sqrt{5} \\ R_{Ay} + \frac{1}{\sqrt{2}} R_B = 10 + 2\sqrt{5} \\ 10 R_{Ay} - 4 R_x - 6 R_y = 0 \end{cases}$$

$$\Rightarrow \begin{cases} R_{Ax} = 0.422^K \\ R_{Ay} = 5.106^K \\ R_B = 13.246^K \end{cases}$$

7-2



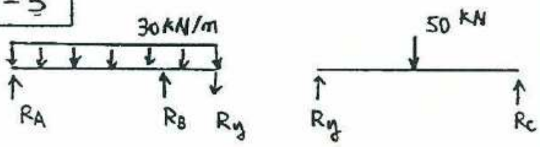
$\Sigma M_A = 0$

$R_B = \frac{1}{1500}\left(\frac{1}{2} \times 300 \times 2 \times 200 + \frac{1}{2} \times 600 \times 2 \times 500 - 150 \times 10^3\right)$

$= 140\ N$

$\Sigma F_y = 0\ , \quad R_A = \frac{1}{2} \times 900 \times 2 - 140 = 760 N$

7-3



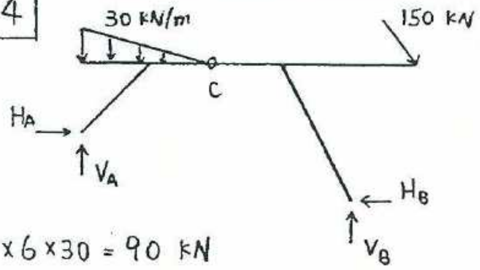
$R_y = R_c = 25\ KN$

$\Sigma M_A = 0$

$R_B = \frac{1}{3}(4 \times 30 \times 2 + 25 \times 4) = 113.33\ KN$

$R_A = 4 \times 30 + R_y - R_B = 31.67\ KN$

7-4



$\frac{1}{2} \times 6 \times 30 = 90\ kN$

$\Sigma M_B = 0 \ \rightarrow\ 3H_A + 12 V_A = 10 \times 90 - 3 \times 120 - 6 \times 90$

$\Sigma M_C = 0 \ \rightarrow\ -3H_A + 6 V_A = 4 \times 90$

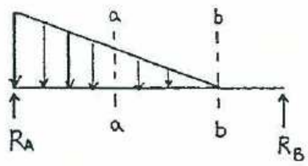
$\therefore\ H_A = -80\ kN\ , \quad V_A = 20\ kN$

$\Sigma M_A = 0 \ \rightarrow\ 3 H_B - 12 V_B = -2 \times 90 - 15 \times 120 - 3 \times 90$

$\Sigma M_c = 0 \ \rightarrow\ 6 H_B - 6 V_B = -9 \times 120$

$\therefore\ H_B = 10\ kN\ , \quad V_B = 190\ kN$

7-5



$\Sigma M_B = 0 \quad \rightarrow \quad R_A = \frac{1}{8}\left(\frac{1}{2} \times 6 \times 8 \times 6\right) = 18^k$

$\Sigma M_A = 0 \quad \rightarrow \quad R_B = \frac{1}{8}\left(\frac{1}{2} \times 6 \times 8 \times 2\right) = 6^k$

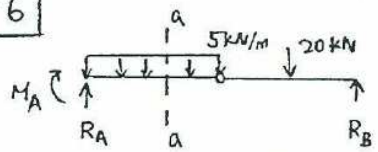
$V_a = -6 + \frac{1}{2} \times 3 \times 4 = 0$

$M_a = 6 \times 5 - \frac{1}{2} \times 3 \times 4 \times 1 = 24\ \text{k-ft}$ (↻)

$V_b = -6\ K$

$M_b = 6 \times 2 = 12\ \text{k-ft}$ (↻)

7-6



$R_B = 10\ kN$

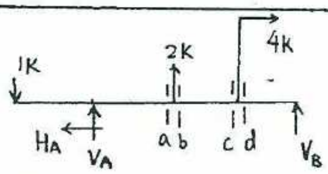
$R_A = 8\times5 + 20 - 10 = 50\ kN$

$M_A = 16\times10 - 12\times20 - 8\times5\times4 = -240\ kN\text{-}m$

$V_a = 50 - 5\times5 = 25\ kN$

$M_a = -240 + 5\times50 - 5\times5\times2.5 = -52.5\ kN\text{-}m$

7-7



$H_A = 4\,K$, $V_A = \frac{1}{10}(14 - 2\times6 - 4\times5) = -1.8$

$V_B = 1 - 2 + 1.8 = 0.8\,K$

$P_a = 4\,K$, $V_a = -1 - 1.8 = -2.8\,K$

$M_a = -8\times1 - 4\times1.8 = -15.2\ k\text{-}ft$

$P_b = 4K$, $V_b = -2.8 + 2 = -0.8\,K$

$M_b = -15.2\ K\text{-}ft$

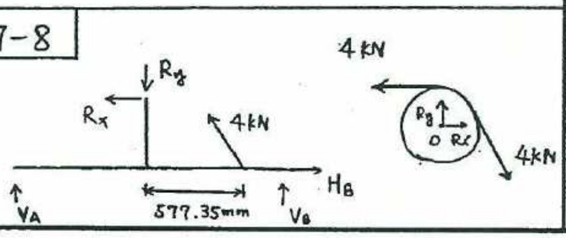
$P_c = 4\,K$. $V_c = -0.8\,K$

$M_c = -11\times1 - 7\times1.8 + 2\times3 = -17.6\ k\text{-}ft$

$P_d = 0\ k$. $V_d = -0.8\,K$

$M_d = -17.6 + 5\times4 = 2.4\ k\text{-}ft$

7-8



$R_x = 4 - 2 = 2\,KN$, $R_y = 2\sqrt{3}\ KN$

$V_A = \frac{1}{2000}(4\times750) = 1.5\ KN$

$V_B = -1.5\ KN$, $H_B = 4\,KN$

$P_a = 0\ KN$, $V_a = 1.5\ KN$; $M_a = 1.5\ kN\text{-}m$

$P_b = 2\,kN$, $V_b = 1.5 - 2\sqrt{3} = -1.96\ kN$

$M_b = 1.5\times1 - 2\times0.5 = 0.5\ KN\cdot m$

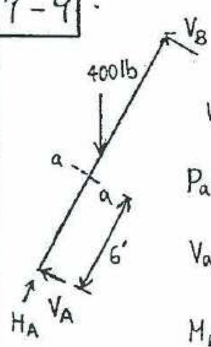
$P_c = 2\,kN$, $V_c = -1.96\ kN$

$M_c = 1.5\times1.57735 - 2\times0.5 - 2\sqrt{3}\times0.57735 = -0.63\,kN\text{-}m$

$P_d = 4\,KN$, $V_d = 1.5\ kN$

$M_d = -1.5\times0.42265 = -0.63\ kN\text{-}m$

7-9



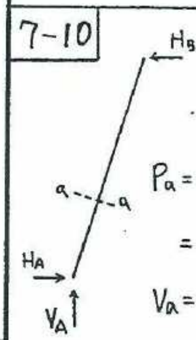
$V_A = V_B = 400\times4/4\sqrt{13} = 110.94\ lb$

$P_a = -\frac{3}{\sqrt{13}}400 = -332.82\ lb$

$V_a = \frac{2}{\sqrt{13}}400 - 110.94 = 110.94\ lb$

$M_a = 110.94\times6 = 665.64\ lb\text{-}ft$

7-10



$V_A = 10\times\sqrt{5}\times9.81 = 219.36N$

$H_A = H_B = 219.36\times0.5/2 = 54.84N$

$P_a = -\left[\frac{2}{\sqrt{5}}\times(219.36 - 10\times0.8\times9.81) + \frac{1}{\sqrt{5}}\times54.84\right]$

$= -150.53N$

$V_a = \frac{1}{\sqrt{5}}(219.36 - 10\times0.8\times9.81) - \frac{2}{\sqrt{5}}\times54.84$

$= 13.95N$

$M_a = \frac{0.8}{\sqrt{5}}(219.36 - \frac{1}{2}\times10\times0.8\times9.81 - 2\times54.84)$

$= 25.2\ N\text{-}m$

7-11

$$P_a = \frac{P}{2}(\cos 90^\circ - \sin 90^\circ) = -\frac{P}{2}$$

$$V_a = -\frac{P}{2}(\cos 90^\circ + \sin 90^\circ) = -\frac{P}{2}$$

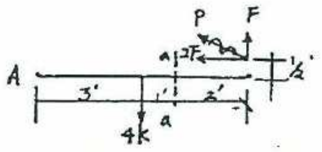
$$M_a = \frac{PR}{2}(0-1-1) = -PR$$

$$P_b = \frac{P}{2}(\cos 45^\circ - \sin 45^\circ) = 0$$

$$V_b = -\frac{P}{2}(\cos 45^\circ + \sin 45^\circ) = -\frac{\sqrt{2}}{2}P$$

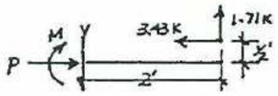
$$M_b = \frac{-PR}{2}(\sin 45^\circ + 1 - \cos 45^\circ) = \frac{-PR}{2}$$

7-12



$\Sigma M_A = 0, \quad 6F + 2F(\tfrac{1}{2}) - 4(3) = 0$

$F = 1.71\ k \uparrow, \quad 2F = 3.43\ k \leftarrow$



$P = 3.43\ k\ (-)$

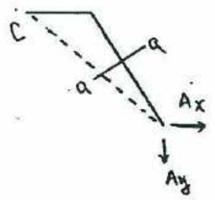
$V = 1.71\ k\ (-)$

$M = 3.43(\tfrac{1}{2}) + 1.71(2)$

$= 5.14\ \text{k-ft}\ (+)$

7-13

$\Sigma M_B = 0 \longrightarrow A_y = \frac{1}{3}(4 \times 40) = 53.33\ \text{kN}$



$\Sigma M_c = 0$

$A_x = \frac{2}{1.4} A_y = 76.19\ \text{kN}$

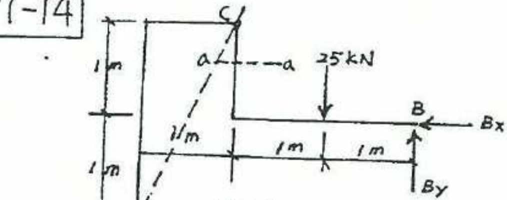
$P_a = \frac{1}{\sqrt{2.96}}(A_x + 1.4A_y)$

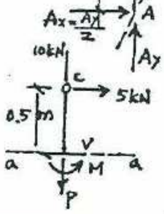
$= 87.68\ \text{kN}$

$V_a = \frac{1}{\sqrt{2.96}}(1.4A_x - A_y) = 31\ \text{kN}$

$M_a = 0.7A_x - 0.5A_y = 26.67\ \text{kN}$

7-14





$\Sigma M_B = 0$

$25(1) + (\tfrac{A_y}{2}) \times 1 - 3A_y = 0$

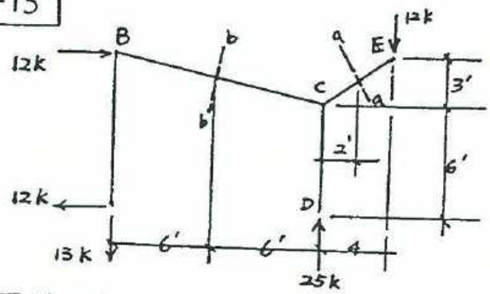
$A_y = 10\ \text{kN} \uparrow \quad A_x = 5\ \text{kN} \rightarrow$

$P = 10\ \text{kN}\ (+)$

$V = 5\ \text{kN}\ (+)$

$M = 5(0.5) = 2.5\ \text{kN}\cdot\text{m}\ (+)$

7-15



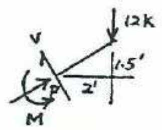
$\Sigma M_A = 0, \quad 12(9) + 16(12) - 12(R_D) = 0$

$R_D = 25\ k \uparrow$

$\Sigma F_x = 0, \quad A_x = 12\ k \leftarrow$

$\Sigma F_y = 0, \quad A_y = 25 - 12 = 13\ k \downarrow$

section a-a

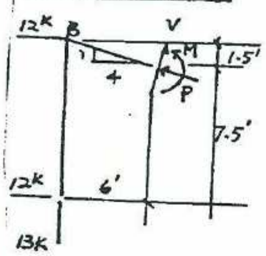


$P = 12(\tfrac{3}{5}) = 7.2\ k\ (-)$

$V = 12(\tfrac{4}{5}) = 9.6\ k\ (+)$

$M = 12(2) = 24\ k\ (-)$

section b-b



$P = 13(\tfrac{1}{\sqrt{17}})$

$= 3.15\ k\ (-)$

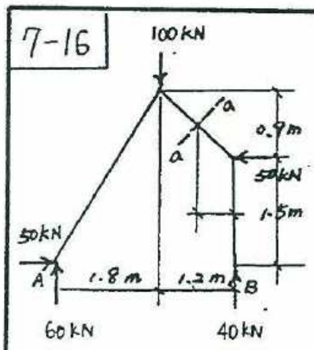
$V = 13(\tfrac{4}{\sqrt{17}})$

$= 12.61\ k\ (-)$

$M = 12(9) - 13(6)$

$= 30\ \text{k-ft}\ (+)$

7-16

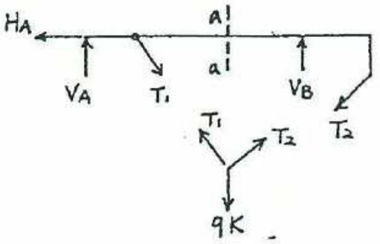


$P_a = -\frac{3}{5}\times 40 - \frac{4}{5}\times 50$
$= -64$ kN

$V_a = \frac{4}{5}\times 40 - \frac{3}{5}\times 50$
$= 2$ kN

$M_a = 0.6\times 40 - 0.45\times 50$
$= 1.5$ kN-m

7-17



$\frac{T_1}{\sqrt{5}} = \frac{T_2}{\sqrt{2}}$, $\frac{2T_1}{\sqrt{5}} + \frac{T_2}{\sqrt{2}} = 9$

$T_2 = 3\sqrt{2}$, $T_1 = 3\sqrt{5}$

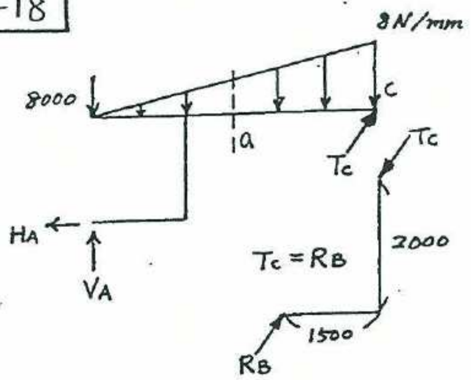
$\Sigma M_A = 0$ $V_B = 6$ (K) ↑

$\Sigma P = 0$ $P_a = -3$ (K)

$\Sigma V = 0$ $V_a = -3$ (K)

$\Sigma M_a = 0$ $M_a = 6$ k-ft

7-18



$\Sigma M_A = 0$ $T_c = 40/3 = 13.3$ kN

$\Sigma H = 0$ $H_A = 3/5\ T_c = 8$ kN

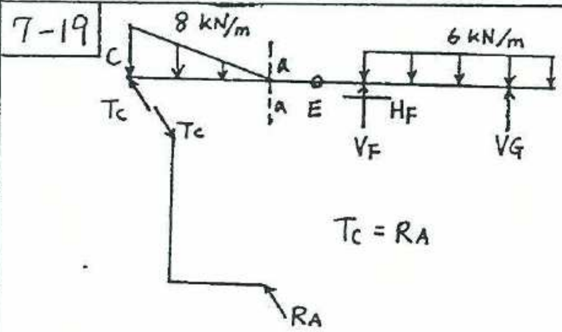
$\Sigma V = 0$ $V_A = 12 + 8 - 4/5\ T_c = 9.3$ kN

$P_a = \frac{3}{5} T_c = 8$ kN

$V_a = \frac{(4+8)\times 1.5}{2} - \frac{4}{5} T_c = \cancel{-1.67}$ kN

$M_a = \frac{4}{5} T_c \times 1.5 - 9\times\frac{5}{9}\times 1.5 = 8.5$ kN-m

7-19



$T_c = R_A$

$\Sigma M_E = 0$ $T_c = 11.25$ kN

$P_a = \frac{3}{5} T_c = 6.75$ kN (tension)

$V_a = 12 - \frac{4}{5} T_c = -3.0$ kN

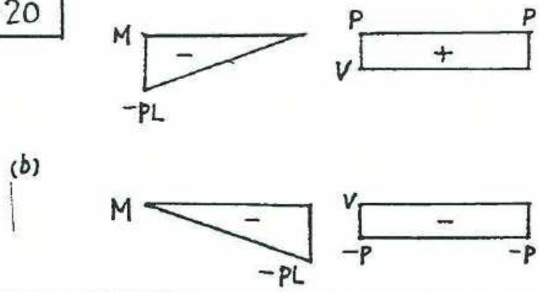
$M_a = \frac{-1}{2}\times 8\times 3\times 2 + \frac{4}{5} T_c \times 3 = +3$ kN-m

$H_F = P_a = 6.75$ kN →

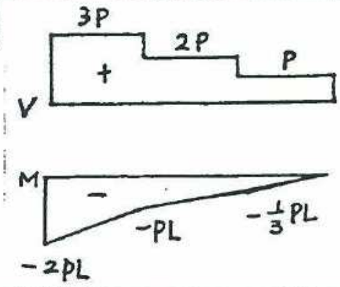
$\Sigma M_F = 0$ $V_G = 15.75$ kN ↑

$\Sigma V = 0$ $V_F = 9$ kN ↑

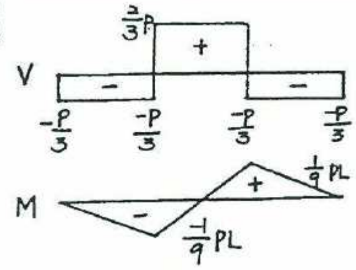
7-20



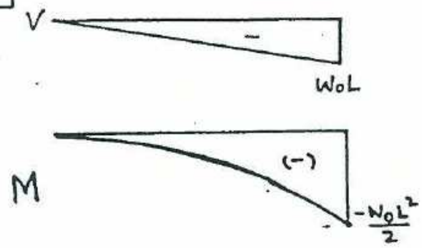
7-21



7-22



7-23



7-24

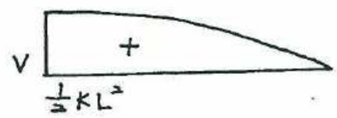
$V' = -Kx \qquad V = \frac{-K}{2}x^2 + C$

$x = L,\ V = 0 \Rightarrow C = \frac{K}{2}L^2$

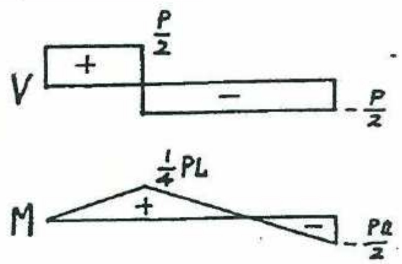
$M' = \frac{-K}{2}x^2 + \frac{K}{2}L^2$

$M = -\frac{K}{6}x^3 + \frac{K}{2}L^2x + C_1$

$x = L,\ M = 0 \Rightarrow C_1 = \frac{-KL^3}{3}$



7-25



7-26

$M''(x) = -4x, \qquad x \le 3$

$M'(x) = -2x^2 + C_1, \quad x \le 3$

$M'(x) = C_2 \qquad x > 3$

$C_1 = R_A = \frac{1}{6}\left(\frac{1}{2}\times3\times12\times4\right) = 12$

$C_2 = 12 - \frac{1}{2}\times3\times12 = -6$

when $x = 0$, $M(0) = 0$

$$\Rightarrow \begin{cases} M(x) = -\frac{2}{3}x^3 + 12x & x \le 3 \\ M(x) = -6x + 36 & x > 3 \end{cases}$$

7-27

$R_A = \frac{1}{9}\left(\frac{1}{2}\times6\times6\times2\right) = 4\ kN$

$M(x) = 4x, \qquad x < 3$

$M(x) = 4x - \frac{1}{2}(x-3)\times(x-3)\times\frac{1}{3}(x-3)$

$= 4x - \frac{1}{6}(x-3)^3, \quad x \ge 3$

7-28

$R_A = \frac{1}{6}\left(\frac{1}{2}\times3\times6\times5\right) = \frac{15}{2}\ kN$

$M(x) = \frac{15}{2}x - \frac{6}{2}x^2 + \frac{1}{2}x\cdot2x\cdot\frac{x}{3}$

$= \frac{15}{2}x - 3x^2 + \frac{1}{3}x^3 \qquad x \le 3$

$M(x) = \frac{15}{2}x - \frac{1}{2}\times3\times6(x-1)$

$= 9 - \frac{3}{2}x \qquad x > 3$

7-29

$R_A = R_B = \frac{W_oL}{2},$

$M = \frac{W_oL}{2}x - \frac{W_o}{2}x^2 + M_A$

7-30

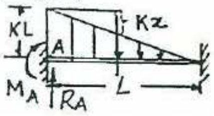
$M = R_Ax - \frac{K}{2}x^2\cdot\frac{2}{3}x$

$M' = R_A - Kx^2 \qquad R_B = R_A - KL^2$

$R_A + R_B = \frac{1}{2}KL^2, \quad R_A = \frac{3}{4}KL^2$

$R_B = -\frac{1}{4}KL^2$

7-31



$$M(x) = M_A + R_A x - KLx\frac{x}{2} + \frac{1}{2}xKx\frac{2}{3}x = M_A + R_A x - \frac{KLx^2}{2} + \frac{Kx^3}{3}$$

7-32

$M(\theta) = 1000 \times 0.2 \sin\theta = 200 \sin\theta$

$P(\theta) = 1000 \times \sin\theta$ (tension)

$V(\theta) = 1000 \cos\theta$

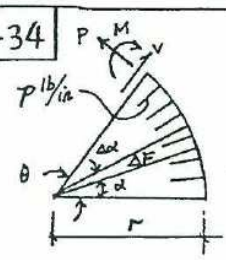
7-33

$M(\theta) = \frac{-PR}{2}(\sin\theta + 1 - \cos\theta)$

$V(\theta) = \frac{-P}{2}(\cos\theta + \sin\theta)$

$P(\theta) = \frac{P}{2}(\cos\theta - \sin\theta)$

7-34

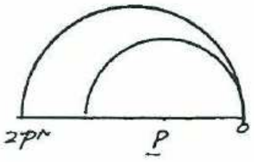


$\Delta F = pr\Delta\alpha$

$\Delta P = pr\Delta\alpha \sin(\theta-\alpha)$

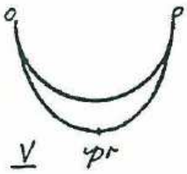
$\Delta V = -pr\Delta\alpha \cos(\theta-\alpha)$

$\Delta M = pr^2\Delta\alpha \sin(\theta-\alpha)$



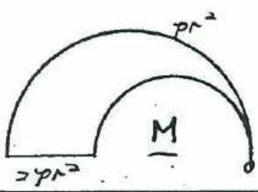
$P = \int_0^\theta pr\sin(\theta-\alpha)d\alpha$

$P = pr(1-\cos\theta)$



$V = \int_0^\theta -pr\cos(\theta-\alpha)d\alpha$

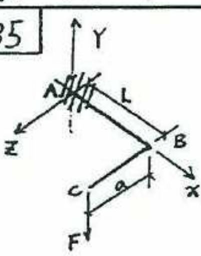
$V = -pr\sin\theta = pr\sin(-\theta)$



$M = \int_0^\theta pr^2\sin(\theta-\alpha)d\alpha$

$M = pr^2(1-\cos\theta)$

7-35



bar AB: $V = +F$, $M = -F(L-x)$, $T = Fa$

bar BC: $V = +F$, $M = -F(a-z)$, $T = 0$

7-36

see solution of Prob. 7-24

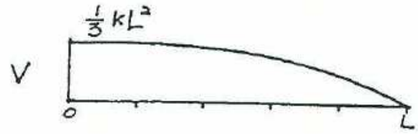
7-37

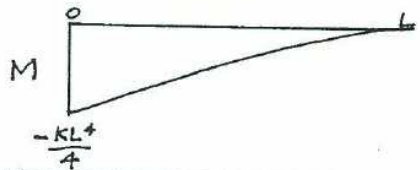
$V' = -kx^2$, $V = -\frac{k}{3}x^3 + C_1$

since $x = L$, $V = 0$ so $C_1 = \frac{1}{3}kL^3$

$M = -\frac{k}{12}x^4 + \frac{1}{3}kL^3x + C_2$

since $x = L$, $M = 0$ so $C_2 = -\frac{K}{4}L^4$





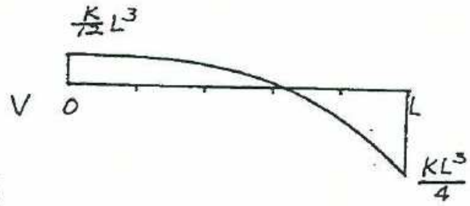
7-38

$V' = -kx^2$, $V = -\frac{k}{3}x^3 + C_1$

$M = -\frac{k}{12}x^4 + C_1x + C_2$

since $x = 0$, L. $M = 0$

so $C_1 = \frac{k}{12}L^3$, $C_2 = 0$



7-39

$V' = K\sin\frac{2\pi}{L}x$

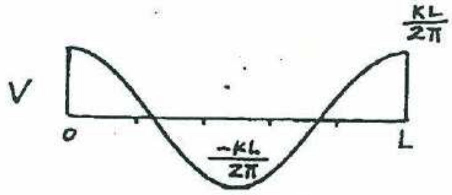
$V = -\frac{KL}{2\pi}\cos\frac{2\pi}{L}x + C_1$

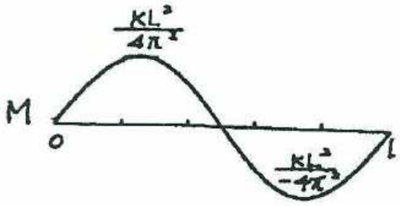
$M = -\frac{KL^2}{4\pi^2}\sin\frac{2\pi}{L}x + C_1x + C_2$

0 0

since $x=0, L$ $M=0$.

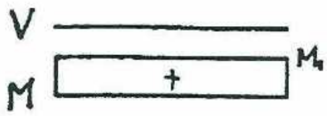
so $C_1=0$, $C_2=0$





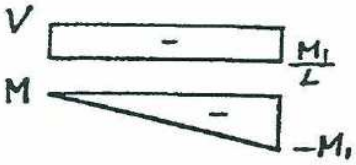
7-40

$V=0$, $M=M_1$



7-41

$V=\frac{-M_1}{L}$ $\quad M=\frac{-M_1}{L}x$

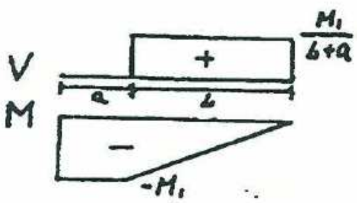


7-42

$M=C_1x+C_2$, $V=C_1$

$x=0$, $M=-M_1$, $C_2=-M_1$

$x=L+a$, $M=0$, $C_1=\frac{M_1}{L+a}$



7-43

$V=2P \quad 0\le x\le a$

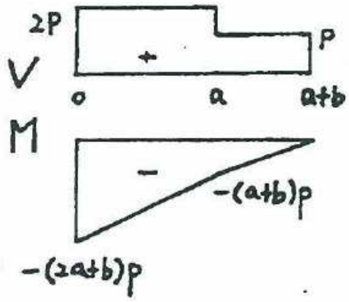
$V=P \quad a<x<a+b$

$M_1=2Px+C_1 \quad 0\le x\le a$

$M_2=Px+C_2 \quad a\le x\le a+b$

$x=a+b \quad M=0 \Rightarrow C_2=-(a+b)P$

$x=a,\ M_1=M_2 \Rightarrow C_1=-(2a+b)P$



7-44

$V=0 \quad 0\le x\le a$

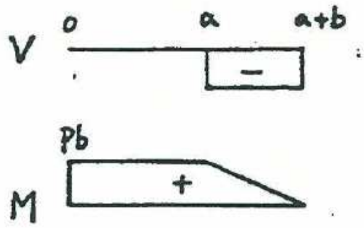
$V=-P \quad a<x\le b+a$

$M=C_1 \quad 0\le x\le a$

$M=-Px+C_2 \quad a<x\le b+a$

$x=0 \quad M=Pb=C_1$

$x=a+b$, $M=0$, $C_2=P(a+b)$



7-45

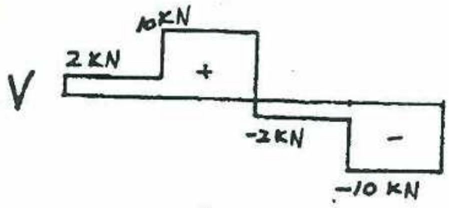
$V=2$, $M=2x+C_1$, $0\le x\le 3$

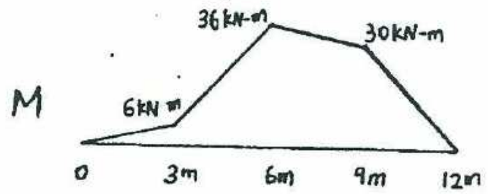
$V=10$, $M=10x+C_2$, $3\le x\le 6$

$V=-2$, $M=-2x+C_3$, $6\le x\le 9$

$V=-10$, $M=-10x+C_4$, $9\le x\le 12$

$x=0$, $M=0$, $C_1=0$
$x=3$, $M=6$, $C_2=24$
$x=6$, $M=36$, $C_3=48$
$x=9$, $M=30$, $C_4=120$
$x=12$, $M=0$, O.K





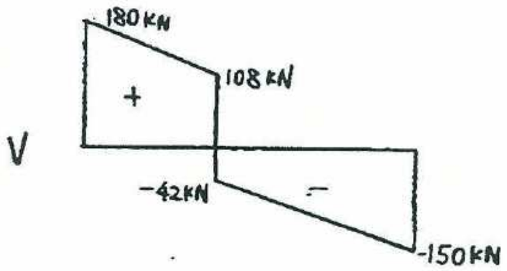
7-46

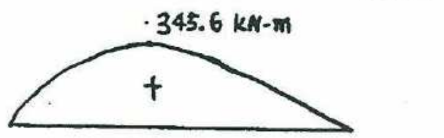
$V(x) = -30x + 180 \qquad 0 \le x \le 2.4$

$V(x) = -30x + 30 \qquad 2.4 < x \le 6$

$M(x) = -15x^2 + 180x \qquad 0 \le x \le 2.4$

$M(x) = -15x^2 + 30x + 360 \qquad 2.4 < x \le 6$





7-47

$V = \frac{-1}{3}P \qquad 0 \le x \le a$

$V = \frac{2}{3}P \qquad a < x \le 2a$

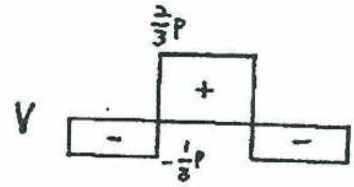
$V = -\frac{1}{3}P \qquad 2a < x \le 3a$

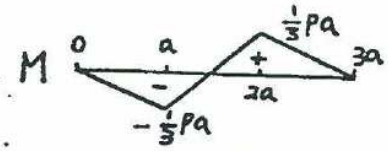
$M = -\frac{1}{3}Px + C_1 \qquad 0 \le x \le a$

$M = \frac{2}{3}Px + C_2 \qquad a \le x \le 2a$

$M = -\frac{1}{3}Px + C_3 \qquad 2a \le x \le 3a$

$x=0$, $M=0$. $C_1=0$
$x=a$, $M=-\frac{Pa}{3}$. $C_2=-Pa$
$x=2a$, $M=\frac{Pa}{3}$. $C_3=Pa$
$x=3a$, $M=0$ O.K.

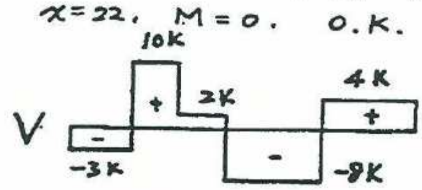


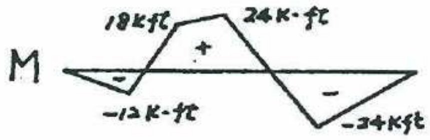


7-48

$V=-3$, $M=-3x+C_1 \qquad 0 \le x \le 4$
$V=10$, $M=10x+C_2 \qquad 4 \le x \le 7$
$V=2$, $M=2x+C_3 \qquad 7 \le x \le 10$
$V=-8$, $M=-8x+C_4 \qquad 10 \le x \le 16$
$V=4$, $M=4x+C_5 \qquad 16 \le x \le 22$

$x=0$, $M=0$, $C_1=0$
$x=4$, $M=-12$, $C_2=-52$
$x=7$, $M=18$, $C_3=4$
$x=10$, $M=24$, $C_4=104$
$x=16$, $M=-24$, $C_5=-88$
$x=22$, $M=0$. O.K.





0 0

7-49 $V_0 = \frac{1}{2} \times 9 \times 120 = 540$ kN

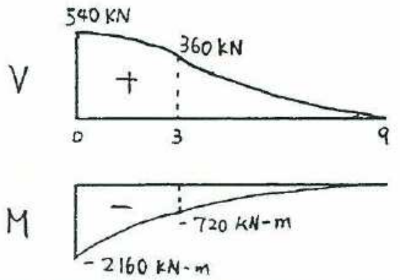
$$M_0 = -\frac{120}{2}(3\times2+6\times5) = -2160 \text{ kN-m}$$

$V(x) = -20x^2+540 \qquad 0 \le x \le 3$

$V(x) = 10(9-x)^2 = 10x^2-180x+810 \qquad 3 \le x \le 9$

$M(x) = -\frac{20}{3}x^3+540x-2160 \qquad 0 \le x \le 3$

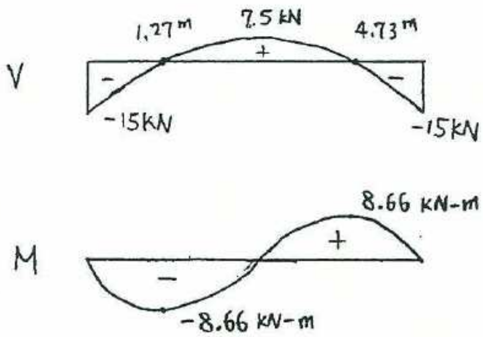
$M(x) = \frac{10}{3}x^3-90x^2+810x-2430 \qquad 3 \le x \le 9$



7-50 $V_0 = -\frac{1}{6}\left(\frac{1}{2}\times3\times15\times4\right) = -15$ kN

$V(x) = -15+15x-\frac{5}{2}x^2$

$M(x) = -15x+\frac{15}{2}x^2-\frac{5}{6}x^3$



7-51 $4q_0\times20 = 500\times12\times12-300\times6\times3$

$$q_0 = 832.5 \text{ kN/m}$$

$V(x) = 832.5x \qquad 0 \le x \le 4$

$V(x) = (832.5)(4)-500(x-4)$

$= -500x+5330 \qquad 4 \le x \le 16$

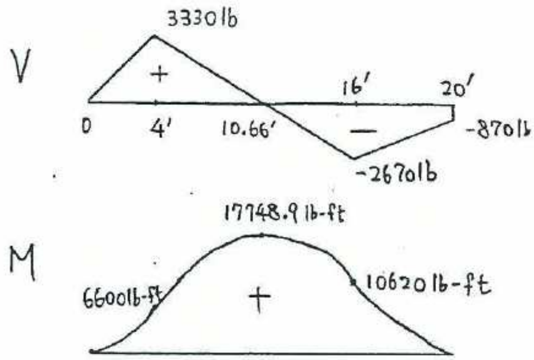
$V(x) = (-500)(16)+5330+200(x-16)$

$= 200x-5870 \qquad 16 \le x \le 22$

$M(x) = 416.25x^2 \qquad 0 \le x \le 4$

$M(x) = -250x^2+5330x-10660 \qquad 4 \le x \le 16$

$M(x) = 100x^2-5870x+78940 \qquad 16 \le x \le 22$



7-52 $V(x) = -5x+100 \qquad 0 \le x \le 20$

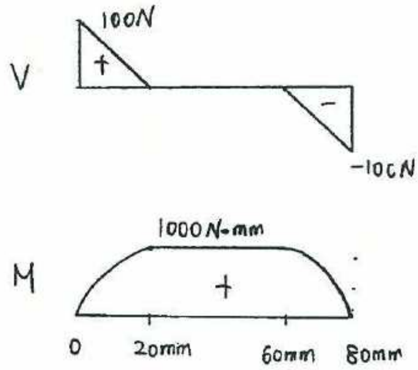
$V(x) = 0 \qquad 20 \le x \le 60$

$V(x) = -5x+300 \qquad 60 \le x \le 80$

$M(x) = -2.5x^2+100x \qquad 0 \le x \le 20$

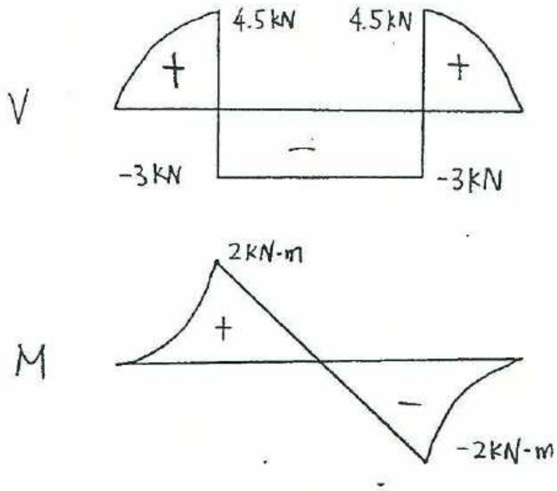
$M(x) = 1000 \qquad 20 \le x \le 60$

$M(x) = -2.5x^2+300x-8000 \qquad 60 \le x \le 80$



7-53

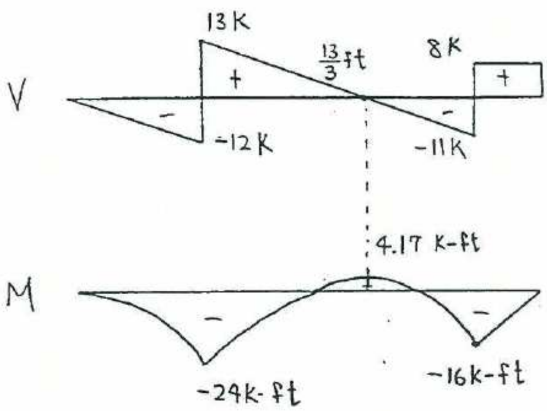
$R = \frac{1}{3}\left(\frac{1}{2} \times 1.5 \times 6 \times 5\right) = 7.5 \text{ kN}$



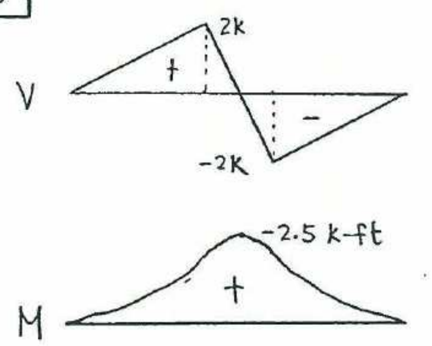
7-54

$R_\ell = \frac{1}{8}(3 \times 12 \times 6 - 8 \times 2) = 25$

$R_r = \frac{1}{8}(3 \times 12 \times 2 + 8 \times 10) = 19$

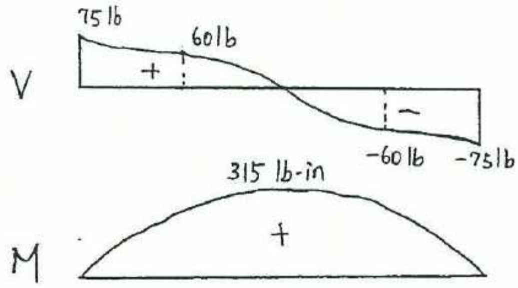


7-55



7-56

$R = \frac{1}{2}(3 \times 10 + 3 \times 40) = 75 \text{ lb}$



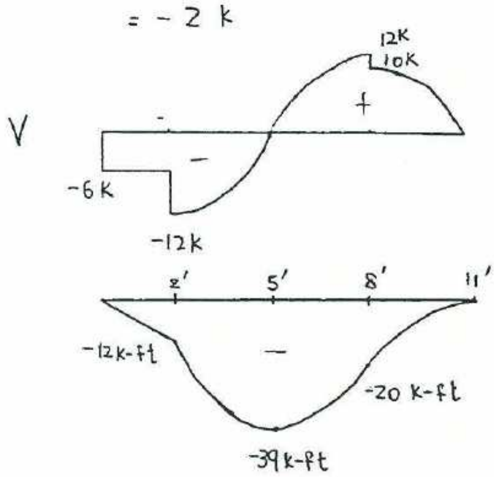
7-57

$R_\ell = \frac{1}{6}\left(6 \times 8 - \frac{1}{2} \times 6 \times 8 \times 3 - \frac{1}{2} \times 3 \times 6 \times 2\right)$

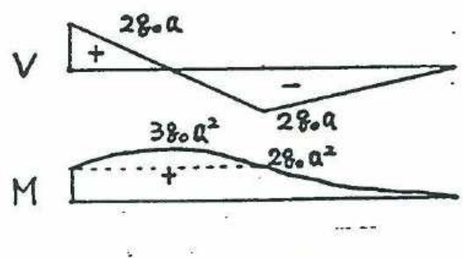
$= -7 \text{ k}$

$R_r = \frac{1}{6}\left(\frac{1}{2} \times 3 \times 6 \times 8 - \frac{1}{2} \times 6 \times 8 \times 3 - 6 \times 2\right)$

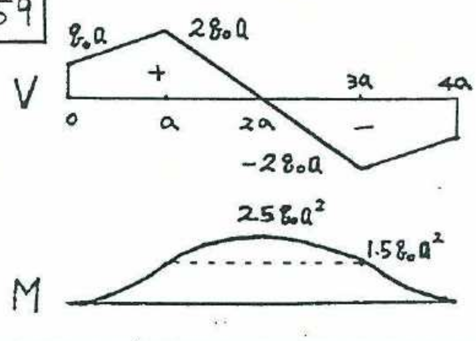
$= -2 \text{ k}$



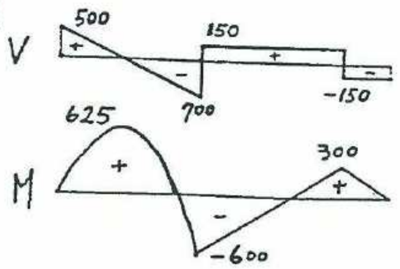
7-58



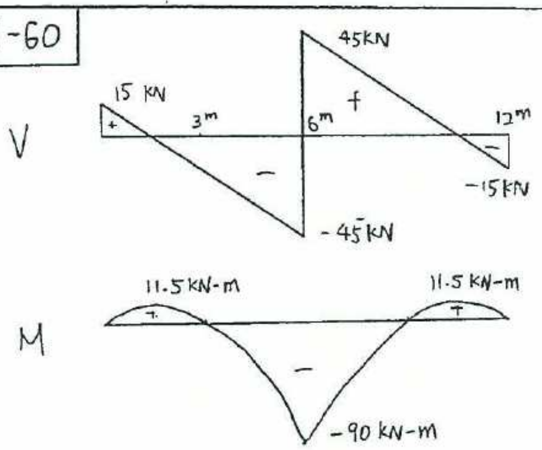
7-59



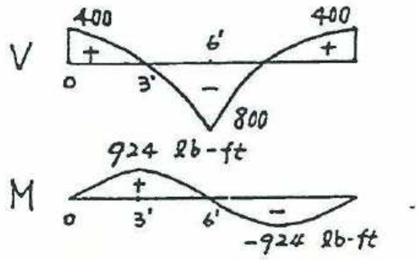
7-63



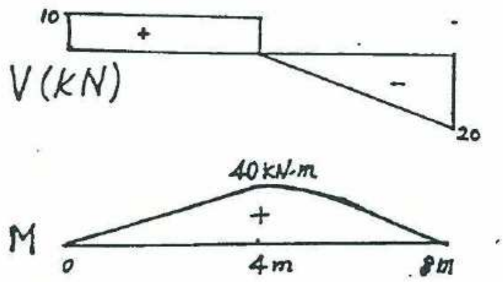
7-60



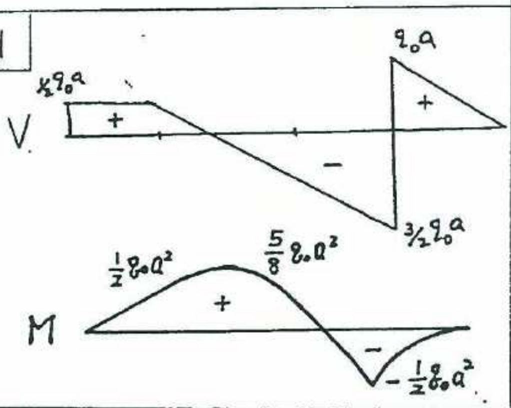
7-64



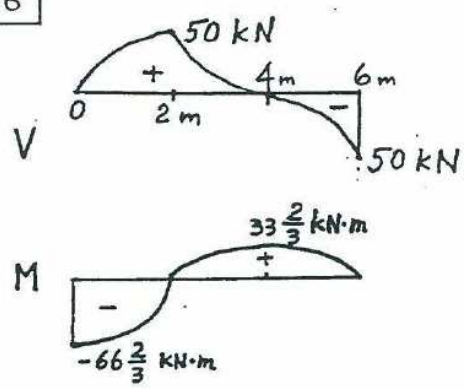
7-65



7-61

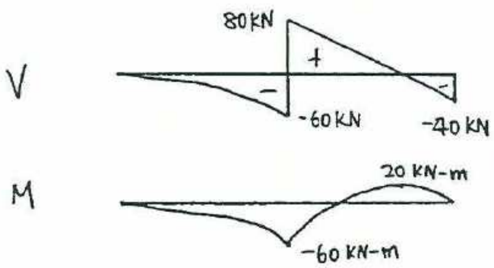


7-66

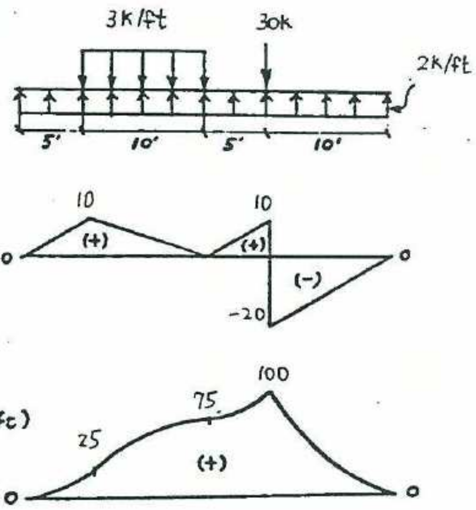


7-62

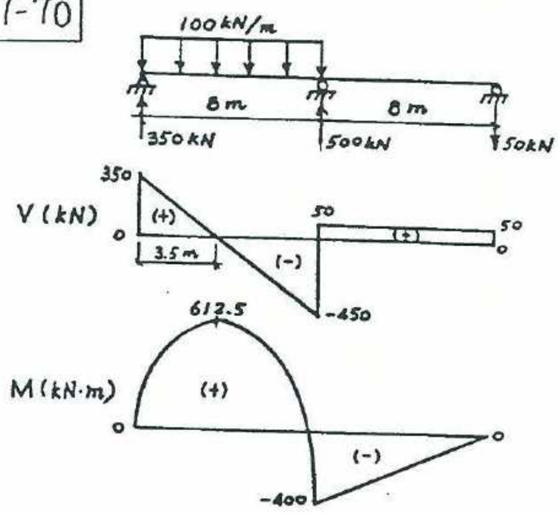
$R_l = \frac{1}{3}\left(\frac{1}{2}\times 3\times 40\times 4 + 3\times 40\times 1.5\right) = 140$ KN



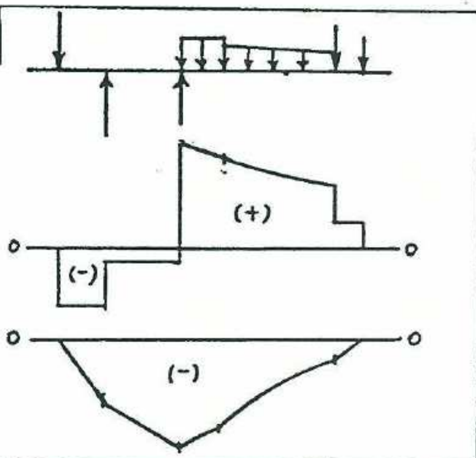
7-67



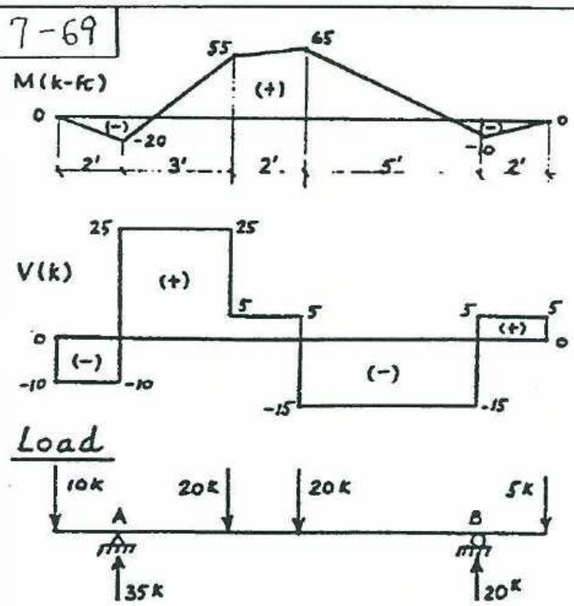
7-70



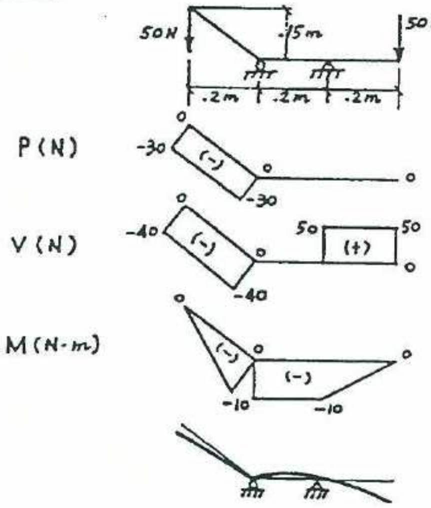
7-68



7-69



7-71



$$S = \frac{M}{\sigma_{all}} = \frac{10000}{90} = 111.11\ mm^3$$

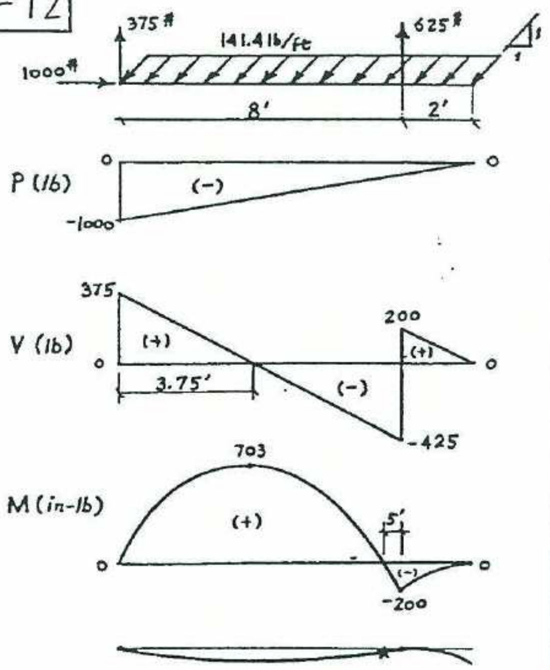
$$S = \frac{\pi R^3}{4} = 111.11\ ,\quad R = 5.21\ mm$$

use 11 mm diameter solid shaft

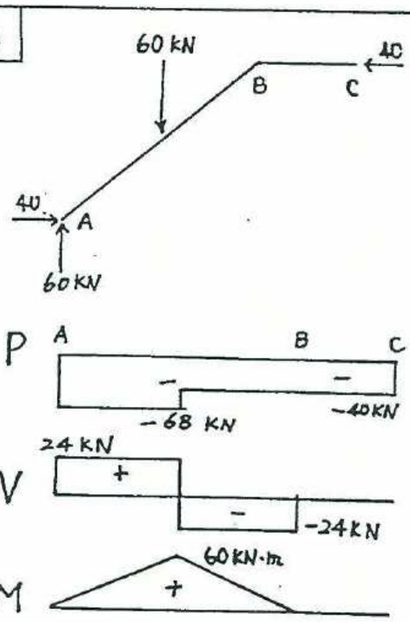
$$\tau_{max} = \frac{4V}{3A} = \frac{4\times 50}{3\times(\pi\times 5.5^2)} = 0.70\ MPa$$

$< \tau_{all}$ ∴ o.k.

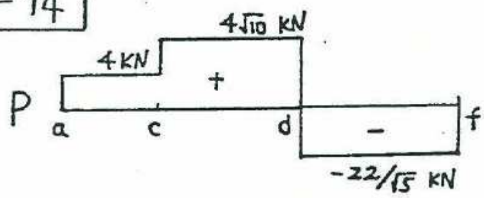
7-72

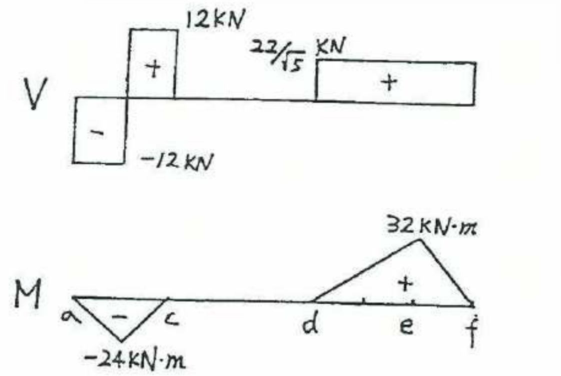


7-73



7-74

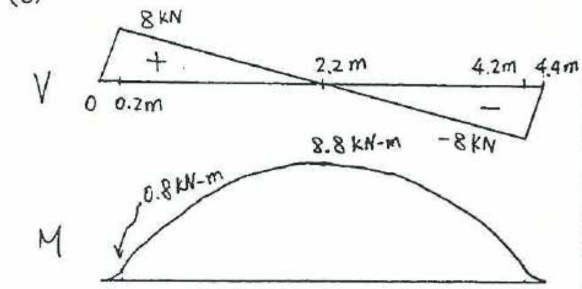




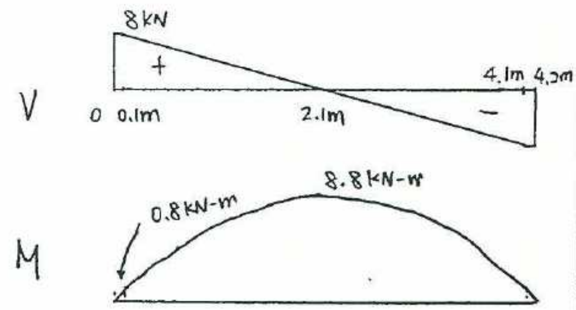
7-75

(a) $R = 4 \times 4 / 0.4 = 40$ KN/m

(b)



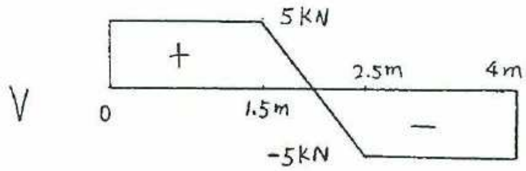
(c) $R' = 4 \times 4 / 2 = 8$ KN

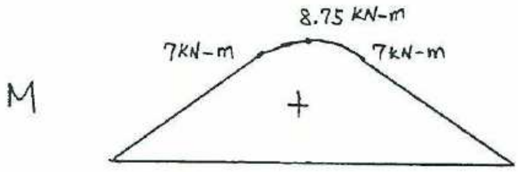


(d) center to center of reaction forces gives a more accurate result

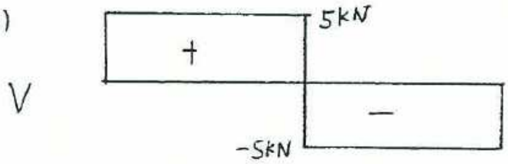
7-76 (a) $R = 10 \times 1/2 = 5\ kN$

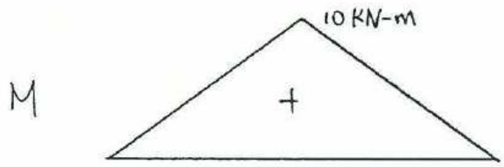
(b)



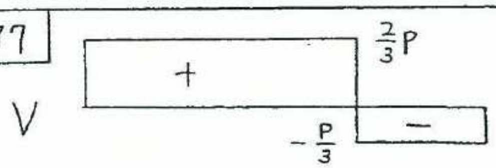


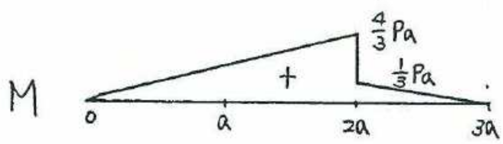
(c)



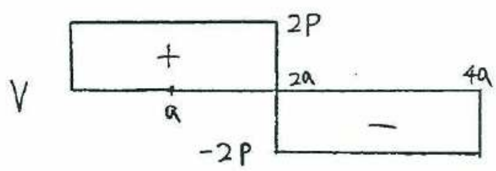


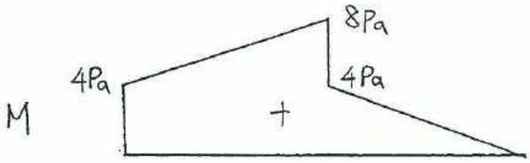
7-77



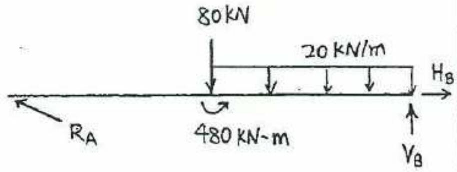


7-78 $R_1 = \frac{1}{4a}(-4Pa + 4P \times 3a) = 2P$



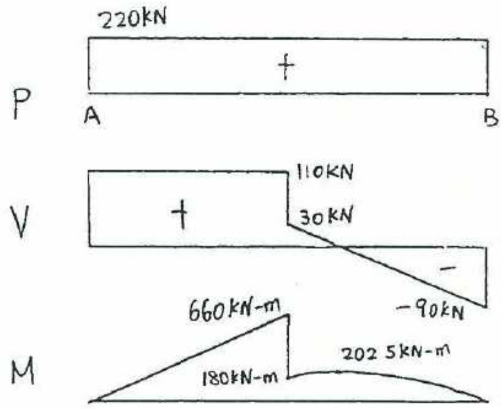


7-79

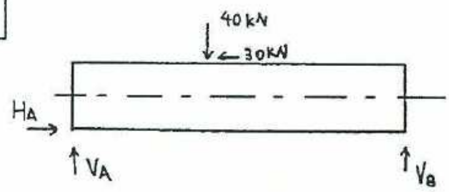


$R_A = \frac{\sqrt{5}}{12}(480 + 80 \times 6 + 20 \times 6 \times 3) = 110\sqrt{5}\ kN$

$H_B = 220\ kN$, $V_B = 90\ kN$

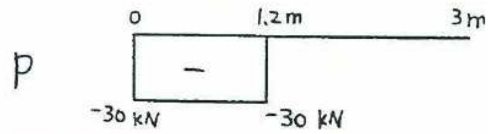


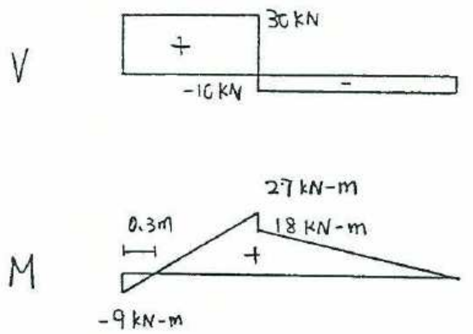
7-80



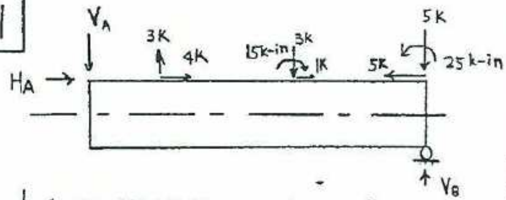
$V_A = \frac{1}{3}(40 \times 1.8 + 30 \times 0.6) = 30\ kN$

$H_A = 30\ kN$, $V_B = 40 - V_A = 10\ kN$



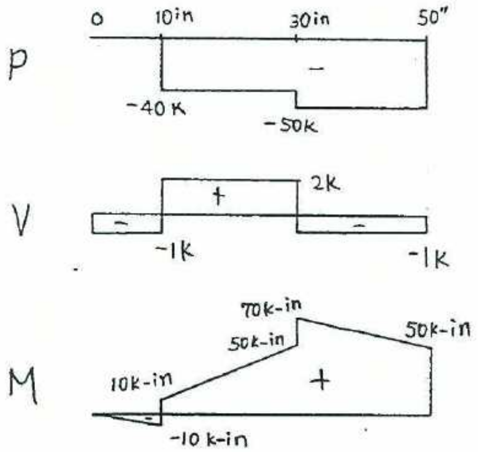


7-81

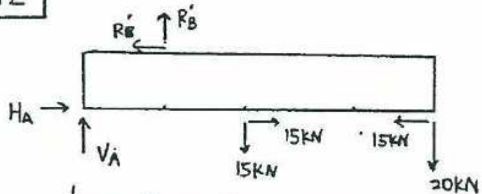


$$V_B = \frac{1}{50}(-25+15+5\times50+3\times30-3\times10) = 6\text{ k}$$

$$V_A = 1\text{K} \quad , \quad H_A = 0\text{ K}$$

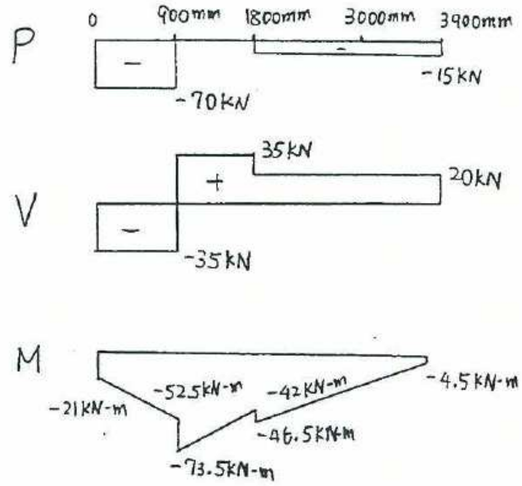


7-82

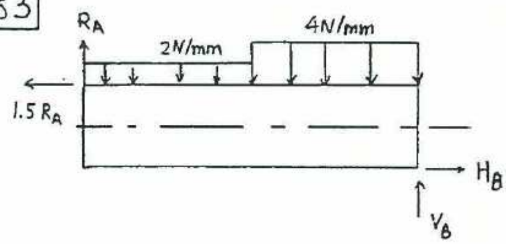


$$R_B' = \frac{1}{(900+600)}(20\times3900+15\times1800) = 70\text{KN}$$

$$V_A = -35\text{ kN} \quad , \quad H_A = 70\text{ kN}$$

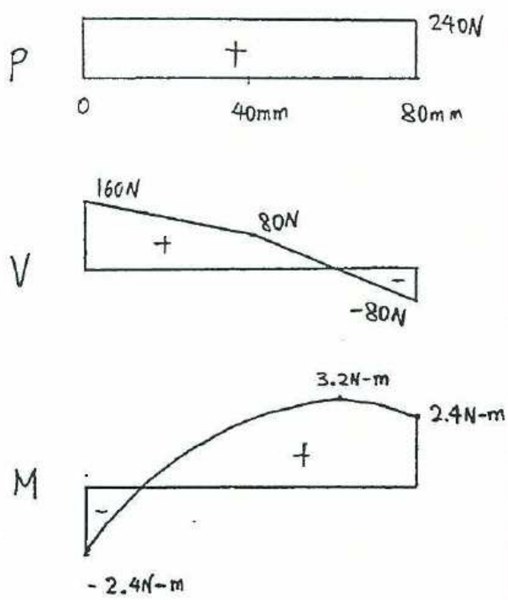


7-83

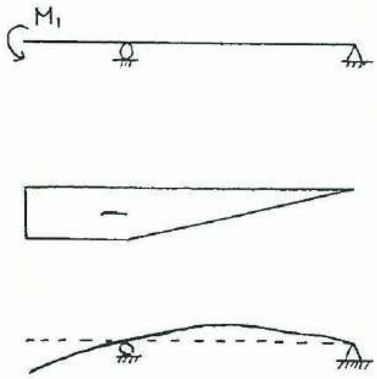


$$R_A = \frac{1}{80-30}(2\times40\times60+4\times40\times20) = 160\text{ N}$$

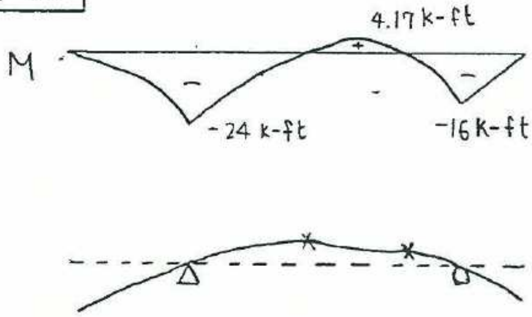
$$V_B = 2\times40+4\times40-160 = 80\text{ N} \quad , \quad H_B = 240\text{N}$$



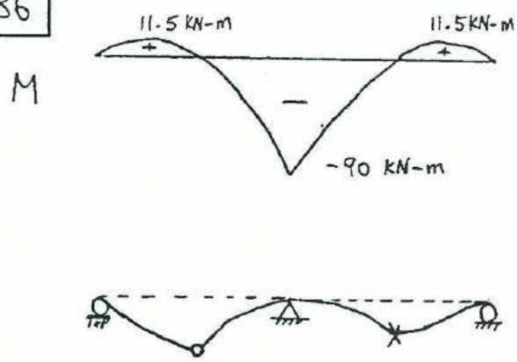
7-84



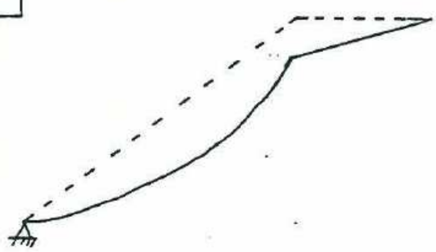
7-85



7-86



7-87



7-88

$$\frac{d^2M}{dx^2} = -q_0\langle x-\tfrac{L}{2}\rangle^0$$

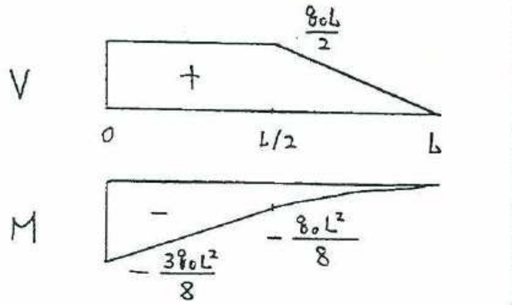
$$M(x) = -\tfrac{1}{2}q_0\langle x-\tfrac{L}{2}\rangle^2 + C_1x + C_2$$

$$V(L)=0 \rightarrow -q_0(\tfrac{L}{2}) + C_1 = 0 \rightarrow C_1 = \frac{q_0L}{2}$$

$$M(L)=0 \rightarrow -\frac{q_0L^2}{8} + \frac{q_0L^2}{2} + C_2 = 0 \rightarrow C_2 = -\frac{3q_0L^2}{8}$$

$$M(x) = \frac{q_0L}{2}\langle x-0\rangle^1 - \frac{3q_0L^2}{8}\langle x-0\rangle^0 - q_0\langle x-\tfrac{L}{2}\rangle^2$$

$$V(x) = \frac{q_0L}{2}\langle x-0\rangle^0 - \frac{3q_0L^2}{8}\langle x-0\rangle^{-1}_{*} - \frac{q_0}{2}\langle x-\tfrac{L}{2}\rangle^1$$



7-89

$$\frac{d^2M}{dx} = -\frac{40}{3}\langle x-1\rangle^1$$

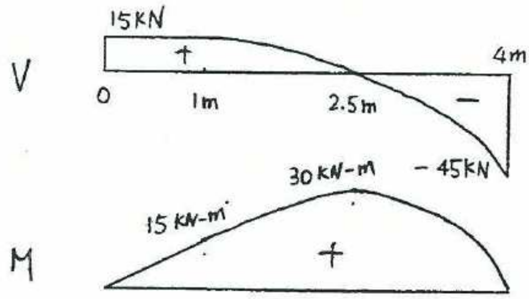
$$M(x) = -\frac{20}{9}\langle x-1\rangle^3 + C_1x + C_2$$

$$M(0)=0 \rightarrow C_2 = 0$$

$$M(4)=0 \rightarrow C_1 = 15$$

$$M(x) = 15\langle x-0\rangle^1 - \frac{20}{9}\langle x-1\rangle^3$$

$$V(x) = 15\langle x-0\rangle^0 - \frac{20}{3}\langle x-1\rangle^2$$



7-90

$$\frac{d^2M}{dx^2} = -\frac{100}{3.6}\langle x-0\rangle^1 + \frac{225}{4.5}\langle x-3.6\rangle^1$$

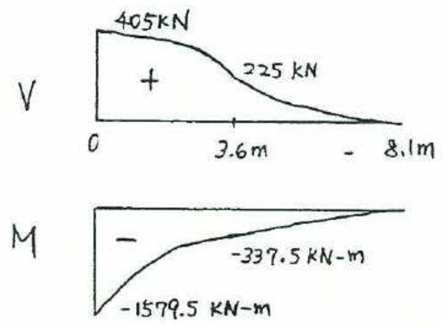
$$M(x) = -\frac{125}{27}\langle x-0\rangle^3 + \frac{25}{3}\langle x-3.6\rangle^3 + C_1x + C_2$$

$$V(8.1) = 0 \rightarrow C_1 = 405$$

$$M(8.1) = 0 \rightarrow C_2 = -1579.5$$

$$M(x) = 405\langle x-0\rangle^1 - 1579.5\langle x-0\rangle^0 - \frac{125}{27}\langle x-0\rangle^3 + \frac{25}{3}\langle x-3.6\rangle^3$$

$$V(x) = 405\langle x-0\rangle^0 - 1579.5\langle x-0\rangle_*^{-1} - \frac{125}{9}\langle x-0\rangle^2 + 25\langle x-3.6\rangle^2$$



7-91

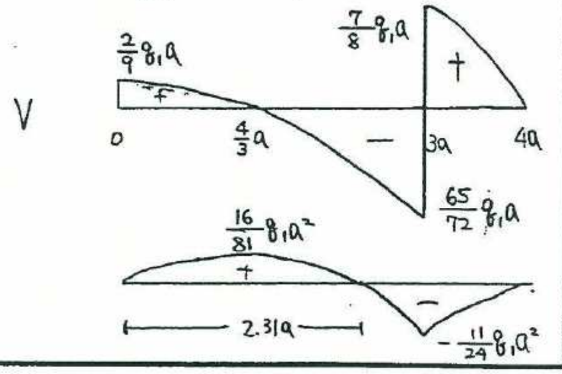
$$\frac{d^2M}{dx^2} = -\frac{q_1}{4a}\langle x-0\rangle^1 + C_1\langle x-0\rangle_*^{-1} + C_2\langle x-3a\rangle_*^{-1}$$

$$M(x) = -\frac{q_1}{24a}\langle x-0\rangle^3 + C_1\langle x-0\rangle^1 + C_2\langle x-3a\rangle^1$$

$$\left.\begin{array}{l} V(4a) = 0 \rightarrow -2q_1a + C_1 + C_2 = 0 \\ M(4a) = 0 \rightarrow -\frac{8}{3}q_1a + 4C_1 + C_2 = 0 \end{array}\right\} \begin{array}{l} C_1 = \frac{2}{9}q_1a \\ C_2 = \frac{16}{9}q_1a \end{array}$$

$$M(x) = -\frac{q_1}{24a}\langle x-0\rangle^3 + \frac{2}{9}q_1a\langle x-0\rangle^1 + \frac{16}{9}q_1a\langle x-3a\rangle^1$$

$$V(x) = -\frac{q_1}{8a}\langle x-0\rangle^2 + \frac{2}{9}q_1a\langle x-0\rangle^0 + \frac{16}{9}q_1a\langle x-3a\rangle$$



7-92

$$\frac{d^2M}{dx^2} = -150\langle x-0\rangle^0 + 600\langle x-0\rangle^1 - 1200\langle x-0.25\rangle^1$$

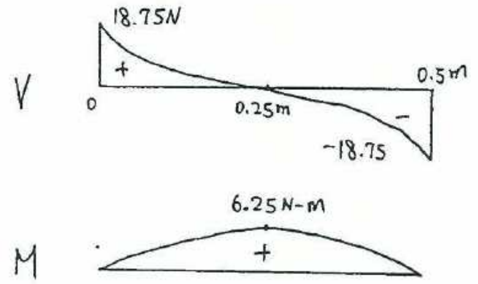
$$M(x) = -75\langle x-0\rangle^2 + 100\langle x-0\rangle^3 - 200\langle x-0.25\rangle^3 + C_1x + C_2$$

$$M(0) = 0 \longrightarrow C_2 = 0$$

$$M(0.5) = 0 \longrightarrow C_1 = 18.75$$

$$M(x) = 18.75\langle x-0\rangle^1 - 75\langle x-0\rangle^2 + 100\langle x-0\rangle^3 - 200\langle x-0.25\rangle^3$$

$$V(x) = 18.75\langle x-0\rangle^0 - 150\langle x-0\rangle^1 + 300\langle x-0\rangle^2 - 600\langle x-0.25\rangle^2$$



7-93

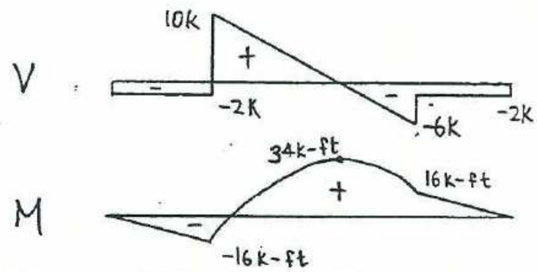
$$\frac{d^2M}{dx^2} = -2\langle x-0\rangle_*^{-1} + C_1\langle x-8\rangle_*^{-1} - \langle x-8\rangle^0 + \langle x-24\rangle^0 + C_2\langle x-24\rangle_*^{-1} + 2\langle x-32\rangle_*^{-1}$$

$$M(x) = -2\langle x-0\rangle^1 + C_1\langle x-8\rangle^1 - \frac{1}{2}\langle x-8\rangle^2 + \frac{1}{2}\langle x-24\rangle^2 + C_2\langle x-24\rangle^1 + 2\langle x-32\rangle^1$$

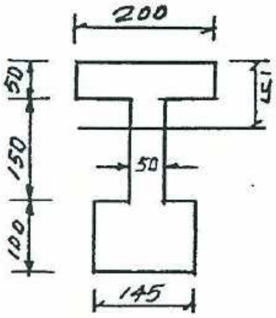
$$\left.\begin{array}{l} V(32) = 0 \rightarrow -2 + C_1 - 24 + 8 + C_2 + 2 = 0 \\ M(32) = 0 \rightarrow -64 + 24C_1 - 288 + 32 + 8C_2 = 0 \end{array}\right\} \begin{array}{l} C_1 = 12 \\ C_2 = 4 \end{array}$$

$$M(x) = -2\langle x-0\rangle^1 + 12\langle x-8\rangle^1 - \frac{1}{2}\langle x-8\rangle^2 + \frac{1}{2}\langle x-24\rangle^2 + 4\langle x-24\rangle^1 + 2\langle x-32\rangle^1$$

$$V(x) = -2\langle x-0\rangle^0 + 2\langle x-8\rangle^0 - \langle x-8\rangle^1 + \langle x-24\rangle^1 + 4\langle x-24\rangle^0 + 2\langle x-32\rangle^0$$



8-1



$A = 145\times 100 + 150\times 50 + 200\times 50 = 32\,000\ mm^2$

$\bar{y} = \frac{1}{32000}(145\times 100\times 250 + 50\times 150\times 125 + 200\times 50\times 25)$

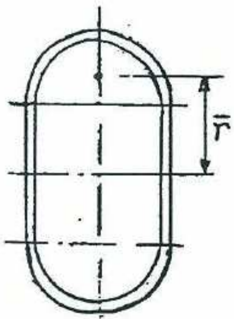
$\bar{y} = 150.4\ mm = 0.15\ m$

$I = \frac{1}{12}\times 0.145\times 0.1^3 + 0.145\times 0.1\times 0.1^2 + \frac{1}{12}\times 0.05\times 0.15^3 + 0.15\times 0.05\times 0.025^2 + \frac{1}{12}\times 0.2\times 0.05^2 + 0.05\times 0.2\times 0.125^2$

$= 3.34\times 10^{-4}\ m^4$

$M = \frac{\sigma_N I}{C} = \frac{165\times 10^3\times 3.34\times 10^{-4}}{0.15} = 367\ KN\cdot m$

8-2



From Table 2 of Appendix:

$\bar{r} = \frac{2}{\pi}R_{avg} + \frac{0.125}{2} = 0.094\ m$

$I_o \approx 0.095\pi R_{avg}^3 t = 4.66\times 10^{-8}\ m^4$

$A = \pi R_{avg} t = 1.96\times 10^{-4}\ m^2$

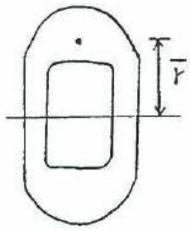
$I = 2\times \frac{1}{12}\times 0.00125\times 0.125^3 + 2(4.66\times 10^{-8} + 1.96\times 10^{-4}\times 0.094) = 3.73\times 10^{-5}\ m^4$

$M = \frac{\sigma_N I}{C} = \frac{165\times 10^3\times 3.75\times 10^{-5}}{0.1125} = 55\ KN\cdot m$

8-3

$\bar{r} = \frac{4(0.008)}{3\pi} + \frac{0.016}{2} = 1.14\times 10^{-2}\ m$

$I_o = 0.11R^4 = 0.11(0.008)^4 = 4.51\times 10^{-10}\ m$



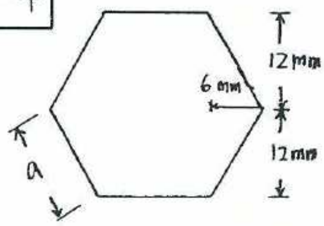
$A = \frac{\pi R^2}{2} = 1.005\times 10^{-4}\ in^2$

$I = \frac{2}{12}(0.003)(0.016)^3 + 2\times 4.51\times 10^{-10} + 2(1.005\times 10^{-4})(1.14\times 10^{-2})^2$

$= 2.91\times 10^{-8}\ m^4$

$M = \frac{\sigma_N I}{C} = \frac{(165\times 10^6)(2.91\times 10^{-8})}{(0.016)} = 300\ N\cdot m$

8-4



$a = \frac{12}{\sin 60^\circ} = 13.86\ mm$

$I = \frac{1}{12}(13.86)(24)^3 + \frac{4}{36}(6)(12)^3 + 4(\frac{1}{2}\times 6\times 12)(4)^2$

$= 19422\ mm^4$

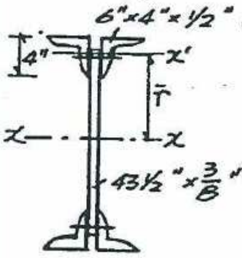
$M = \frac{\sigma_N I}{C} = \frac{(165\times 10^6)(19422\times 10^{-12})}{(12\times 10^{-3})} = 267\ N\cdot m$

8-5

$I_{x'} = 6.27\ in^4,\ A = 4.75\ in^2$

$\bar{r} = 22 - 0.987 = 21.01\ in$

6"×4"×1/2" angles



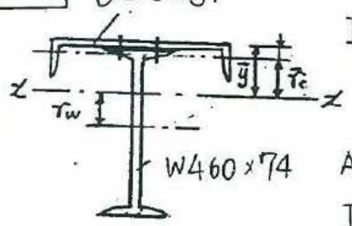
$I_x = 4(6.27 + 4.75\times 21.01^2) + \frac{1}{12}\times\frac{3}{8}\times 43.5^3$

$= 10\,984\ in^4$

$M = \frac{\sigma_N I}{C} = \frac{24\times 10984}{22}\times\frac{1}{12}$

$= 1000\ K\text{-}ft$

8-6



$A_c = 3930\ mm^2$

$I_c = 1.61 \times 10^6\ mm^4$

$\bar{x} = 17.7\ mm$

$A_w = 9460\ mm^2$

$I_w = 333 \times 10^6\ mm^4$

$$\bar{y} = \frac{(3930)(17.7) + (9460)(7.16 + 457/2)}{(3930 + 9460)}$$

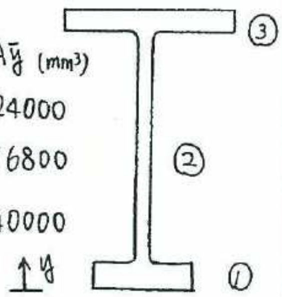
$= 171.7\ mm$ (from top)

$$I = 333 \times 10^6 + (9460)(171.7 - 235.66)^2 + 1.61 \times 10^6 + (3930)(171.7 - 17.7)^2 = 467 \times 10^6\ mm^4$$

$$M = \frac{\sigma_N I}{c} = \frac{(165 \times 10^3)(467 \times 10^{-6})}{(292.5 \times 10^{-3})} = 263\ kN \cdot m$$

8-7

	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$
①	2400	10	24000
②	4480	160	716800
③	4000	310	1240000



$$\bar{y} = \frac{24000 + 716800 + 1240000}{2400 + 4480 + 4000} = 182\ mm$$

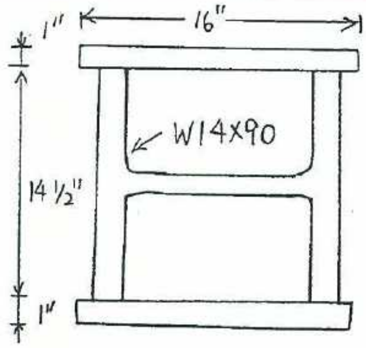
$$I = \left[\tfrac{1}{12}(120)(20)^3 + (2400)(172)^2\right] + \left[\tfrac{1}{12}(16)(280)^3 + (4480)(22)^2\right] + \left[\tfrac{1}{12}(200)(20)^3 + (4000)(128)^2\right] = 168188587\ mm^4$$

$$M = \frac{\sigma_N I}{c} = \frac{(165 \times 10^3)(168188587 \times 10^{-12})}{(182 \times 10^{-3})} = 152\ kN \cdot m$$

8-8

W14×90

$I_{yy} = 362$

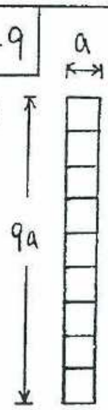


$$I = 362 + 2\left[\tfrac{1}{12}(16)(1)^3 + (16)(7.75)^2\right] = 2286.7$$

$$M = \frac{\sigma_N I}{c} = \frac{(24)(2286.7)}{(8.25)} = 6652\ k\text{-}in$$

8-9

(a)



horizontal axis:

$$I = \tfrac{1}{12}(a)(9a)^3 = 60.75a^4$$

$$I/c = \frac{60.75a^4}{4.5a} = 13.5a^3$$

vertical axis:

$$I = \tfrac{1}{12}(9a)(a^3) = 0.75a^3$$

$$I/c = \frac{0.75a^4}{0.5a} = 1.5a^3$$

(b)

horizontal axis:

$$I = \tfrac{1}{12}(a)(3a)^3 + 2\left[\tfrac{1}{12}(3a)(a)^3 + (3a^2)(2a)^2\right] = 16.75a^4$$

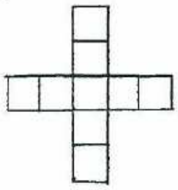
$$I/c = \frac{16.75a^4}{2.5a} = 6.7a^3$$

vertical axis:

$$I = \tfrac{1}{12}(3a)(a)^3 + 2\left[\tfrac{1}{12}a(3a)^3\right] = 4.75a^4$$

$$I/c = \frac{4.75a^4}{1.5} = 3.17a^3$$

(c) both axes:



$$I = \frac{1}{12}(a)(5a)^3 + \frac{1}{12}(4a)(a)^3$$

$$= 10.75a^4$$

$$I/c = \frac{10.75a^4}{2.5a} = 4.3a$$

8-10 $S_x = \frac{I_x}{C_x}$, $S_y = \frac{I_y}{C_y}$

S310×74 : $S_x = \frac{127\times10^6}{305/2} = 833\times10^3\ mm^3$

$S_y = \frac{6.53\times10^6}{139/2} = 94\times10^3\ mm^3$

W360×134 : $S_x = \frac{415\times10^6}{356/2} = 2330\times10^3\ mm^3$

$S_y = \frac{151\times10^6}{369/2} = 818\times10^3\ mm^3$

C 380×50 : $S_x = \frac{131\times10^6}{381/2} = 688\times10^3\ mm^3$

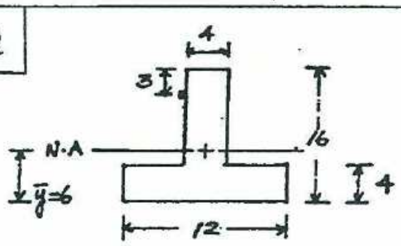
$S_y = \frac{3.38\times10^6}{(86.4-20)} = 50.9\times10^3\ mm^3$

8-11 $\sigma = E\epsilon = (200\times10^3)(0.0002) = 40\,MPa$

$M = \sigma S = (40\times10^3)(1460\times10^{-6}) = 58.4\ kN\cdot m$

$w = \frac{2M}{L^2} = \frac{2(58.4)}{(1)^2} = 116.8\ kN$

8-12



$\sigma_A = E\epsilon_A = (200\times10^9)(50\times10^{-3}) = 10^{10}\ Pa$

$M_A = \frac{\sigma_A I}{c} = \frac{(10)^{10}(2176\times10^{-12})}{(7\times10^{-3})} = 3108.6\ N\cdot m$

and $M_A = 0.3P$ from the simple supported beam

$P = \frac{M_A}{0.3} = 10.36\ KN$

8-13 $\bar{y} = \frac{4\times6\times3 - 3\times2\times3.5}{4\times6 - 3\times2} = 2.83''$

$M_C = \frac{\sigma I}{c} = \frac{15\times65.5}{3.17} = 310\ K\text{-}in$

$M_T = \frac{\sigma I}{c} = \frac{10\times65.5}{2.83} = 231\ K\text{-}in.$

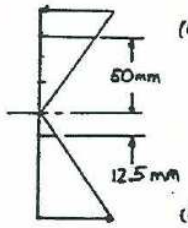
8-14 $\bar{y} = \frac{(80\cdot120\cdot60) - (\pi\cdot30^2\times70)}{(80\cdot120) - (\pi\cdot30^2)} = 55.83\ mm$

$M_C = \frac{\sigma_C I}{c} = \frac{(150\times10^6)(10.483\times10^{-6})}{0.06417}\times10^{-3} = 24.5\ kN\cdot m$

$M_T = \frac{\sigma_T I}{c} = \frac{(100\times10^6)(10.483\times10^{-6})}{0.05583}\times10^{-3} = 18.8\ kN\cdot m$

8-15 $I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3(10^{-12})$

$= 2.81\times10^{-5}\ m^4$



(a) $C = \sigma_{ave}A = \frac{My_{ave}}{I}A$

$= \frac{(1.6\times10^{-3})(5\times10^{-3})(0.1)(.05)}{2.81\times10^{-5}}$

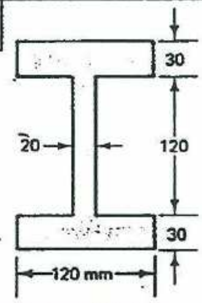
$= .142\ MN = 142\ KN$

(b) $T = \frac{(1.6\times10^{-3})(1.25\times10^{-2})(.05)(.025)}{2.81\times10^{-5}}$

$= 8.90\times10^{-3}\ MN = 8.90\ KN$

8-16 $F = \int_{A_1}\sigma\,dA = \int_{A_1}\frac{My}{I}dA = \frac{M}{I}\int_{A_1}y\,dA = \frac{MQ}{I}$

8-17



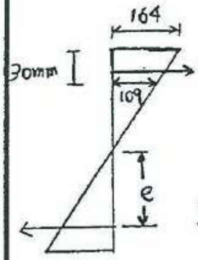
$I = \frac{1}{12}\times20\times120^3$

$+ \frac{1}{12}\times120\times30^3$

$+ (120\times30)(75)^2$

$= 43.92\times10^6\ mm^4$

$$\sigma = \frac{Mc}{I} = \frac{(80\times10^6)(90)}{43.92\times10^6} = 164\ N/mm^2$$



$$T = (30\times120)\left(\frac{164+109}{2}\right) + \left(60\times20\times\frac{109}{2}\right) = 556800N$$

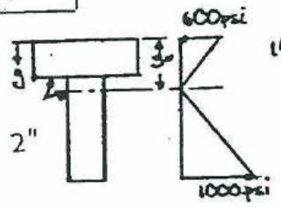
$$e = \frac{1}{(556800)}\left[\frac{1}{3}(60\times20\times109\times60) + \frac{1}{2}(30\times120)(164-109)(80) + (30\times120\times137\times75)\right]$$

$$= 71.8\ mm$$

$$T\cdot 2e = (556.8)(71.8\times10^{-3}) \doteq 80\ kN\cdot m$$

8-18

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{2(6)(1+5)}{2(2)(6)} = 3''$$



(a) $\sigma_t = \frac{Mc_t}{I} = \frac{(2270)(12)(3)}{(136)}$

≈ 600 psi

$\sigma_b = \frac{5}{3}\sigma_t = 1000$ psi

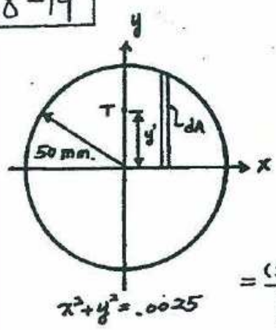
(b) $= \Sigma \sigma_{AVE} A$

$= \frac{1}{2}(600+200)(12) + \frac{1}{2}(200)(2)(1)$

$= 5000$ lb.

(c) $T = \frac{1}{2}(1000)(5)(2) = 5000$ lb. same as b).

8-19



$$dT = \frac{\sigma}{2}dA$$

$$T = \int \frac{\sigma}{2}dA = 2\int_0^{.05}\frac{My}{2I}y\,dx$$

$$= \frac{M}{I}\int_0^{.05}(.0025 - x^2)dx$$

$$= \frac{M}{I}\left(.0025x - \frac{x^3}{3}\right)\Big]_0^{.05}$$

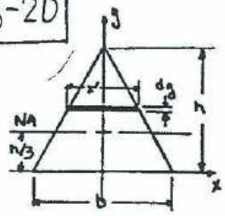
$$= \frac{(3500\times10^{-3})}{\frac{\pi(.05)^4}{4}}\left[.0025(.05) - \frac{.05^3}{3}\right]$$

$$= 59.4\ KN$$

$$M = T(2y')$$

$$y' = \frac{3.5}{2(59.4)} = .029m = 29\ mm$$

8-20



$$I_{xx} = I_o + Ad^2$$

$$I_{xx} = \int y^2 dA$$

$$dA = x'dy = b\left(\frac{h-y}{h}\right)dy$$

$$= b\left(1 - \frac{y}{h}\right)dy$$

$$I_{xx} = b\int_0^h y^2\left(1-\frac{y}{h}\right)dy$$

$$= b\left(\frac{1}{3}y^3 - \frac{1}{4h}y^4\right)\Big]_0^h$$

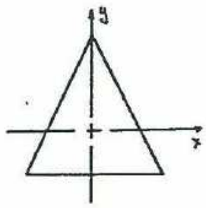
$$= b\left(\frac{1}{3}h^3 - \frac{1}{4}h^3\right) = \frac{1}{12}bh^3$$

$$I_{xx} = I_o + Ad^2$$

$$I_o = I_{xx} - Ad^2$$

$$= \frac{1}{12}bh^3 - \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$$

$$= \frac{1}{12}bh^3 - \frac{1}{18}bh^3 = \frac{1}{36}bh^3$$



$$T = \int \sigma dA$$

$$dA = x'dy = b\left(\frac{2}{3} - \frac{y}{h}\right)dy$$

$$T = \int\left(\frac{My}{I}\right)b\left(\frac{2}{3} - \frac{y}{h}\right)dy$$

$$= \frac{36M}{h^3}\int_0^{\frac{2}{3}h} y\left(\frac{2}{3} - \frac{y}{h}\right)dy$$

$$= \frac{36M}{h^3}\left(\frac{1}{3}y^2 - \frac{1}{3}\frac{y^3}{h}\right)\Big]_0^{\frac{2}{3}h}$$

$$= \frac{16M}{9h} = \frac{16(4000)(10^{-6})}{9(.15)}$$

$$= 47.4\ KN$$

$$y_T = \frac{\int \sigma y dA}{\int \sigma dA}$$

$$\int \sigma y dA = \frac{Mb}{I}\int_0^{\frac{2}{3}h} y^2\left(\frac{2}{3} - \frac{y}{h}\right)dy$$

$$= \frac{36M}{h^3}\left(\frac{2}{9}y^3 - \frac{1}{4}\frac{y^4}{h}\right)\Big]_0^{\frac{2}{3}h}$$

$$= \frac{16}{27}M$$

$$y_T = \frac{16(4000)}{27(47.4)} = 50mm\ (\text{from NA})$$

$C = T = 47.4$ KN (from equilibrium)

$M = T$ (Lever arm)

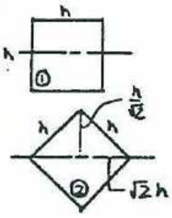
$$\text{lever arm} = y_T + y_C = \frac{M}{T}$$

$$y_C = \frac{M}{T} - y_T = \frac{4000}{47.4\times10^3} - .05$$

$= 34.4$ mm from NA

8-21

$$M = \frac{\sigma I}{c}$$



$$M_1 = \frac{\frac{1}{12}h\,h^3}{\frac{h}{2}}\sigma = \frac{1}{6}h^3\sigma$$

$$I_2 = 2\Big[\tfrac{1}{36}(\sqrt{2}h)\big(\tfrac{h}{\sqrt{2}}\big)^3 + \tfrac{1}{2}(\sqrt{2}h)\big(\tfrac{h}{\sqrt{2}}\big)\big(\tfrac{h}{3\sqrt{2}}\big)^2\Big] = \frac{h^4}{12}$$

$$M_2 = \frac{\frac{1}{12}h^4\sigma}{\frac{h}{\sqrt{2}}} = \frac{\sqrt{2}}{12}h^3\sigma$$

$$\frac{M_1}{M_2} = \frac{\frac{1}{6}h^3\sigma}{\frac{\sqrt{2}}{12}h^3\sigma} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

8-22

$$I = \frac{2}{12}\big[(1-k)a\sqrt{2}\big]\Big[\frac{\sqrt{2}}{2}(1-k)a\Big]^3 + 2\int_0^{\frac{\sqrt{2}}{2}(1-k)a} x^2 \times \sqrt{2}\,ka \cdot dx = \frac{1}{12}(1+3k)(1-k)^3 a^4$$

$$S = \frac{I}{c} = \frac{\sqrt{2}}{6}(1+3k)(1-k)^2a^3$$

$$\frac{dS}{dk} = \frac{\sqrt{2}}{6}a^3\,[9k^2 - 10k + 1] = 0 \quad k = \tfrac{1}{9} \text{ or } 1$$

$k = \frac{1}{9} \quad \therefore \; S' = \frac{\sqrt{2}}{6}a^3\left(\frac{256}{243}\right)$

$k = 0 \qquad S = \frac{\sqrt{2}}{6}a^3$

$$\frac{\sigma'}{\sigma} = \frac{S}{S'} = \frac{243}{256} = 95\%$$

$$\therefore\; 1 - \sigma'/\sigma = 5\%$$

8-23

$$\epsilon_{AB} = \frac{0.025}{200} = 0.000125$$

$$\epsilon_{CD} = \frac{-0.23}{200} = -0.00115$$

Tension at top and compression at bottom $\longrightarrow$ Negative moment

$$\frac{\epsilon_{top} + 0.00115}{0.000125 + 0.00115} = \frac{150}{125}$$

$\epsilon_{top} = 0.00038$

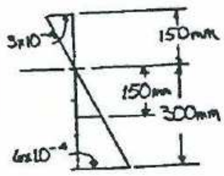
$\sigma_{top} = E\,\epsilon_{top} = (200\times10^3)(0.00038) = 76$ kPa (tension)

$$\frac{\epsilon_{bottom} + 0.000125}{0.000125 + 0.00115} = \frac{160}{125}$$

$\epsilon_{bottom} = 0.001507$

$\sigma_{bottom} = E\,\epsilon_{bottom} = (200\times10^3)(0.001507) = 301.4$ kPa (compression)

8-24



$Cb = 450\left(\frac{6}{9}\right) = 300$ mm

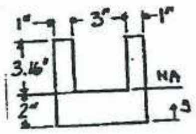
$P = \epsilon_{ave.}\,EA$

$= \left(\frac{1}{2}\right)(6\times10^{-4})(2\times10^5)(.1)(.15)$

$= .9$ MN

$= 900$ KN

8-25



$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$= \frac{2(3)(1) + 2(1)(5.16)\left(\frac{5.16}{2}\right)}{2(3) + 2(1)(5.16)}$$

$= 2''$

$$I = \tfrac{1}{12}(5)(2)^3 + 2(5)(1)^2 + 2\left(\tfrac{1}{12}\right)(1)\,3.16^3 + 2(1)(3.16)\left(\tfrac{3.16}{2}\right)^2$$

$= 34.4$ in^4

$$\sigma = \epsilon E = \frac{My}{I}$$

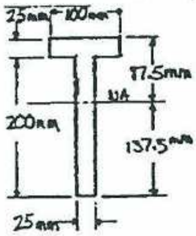
$$= \frac{16Py}{I}$$

$$P = \frac{\epsilon EI}{16y} = \frac{(900\times10^{-6})(30\times10^6)(10^{-3})(34.4)}{16(2.16)}$$

$= 26.9$ K.

8-26

$$\bar{y} = \frac{100(25)(12.5)+200(25)(125)}{(100)(25)+(200)(25)} = 87.5\text{mm from top}$$



$$I = \frac{1}{12}(.1)(.025)^3+(.1)(.025)(.075)^2 + \frac{1}{12}(.025)(.2)^3+(.025)(.2)(.0375)^2$$

$$= 3.79\times10^{-5}\,m^4$$

$$M = (\tfrac{2}{3}F)(1) = \tfrac{2}{3}F \text{ KN}\cdot m$$

for F acting up !

$$\sigma_t = \frac{Mc}{I} = \frac{(.667F)(.0875)}{(3.79\times10^{-5})} = \frac{20.000}{1.5}$$

$$\sigma_b = \frac{(.667F)(.1375)}{3.79\times10^{-5}} = \frac{40000}{1.5} \rightarrow F = \frac{F = 8.66\text{ KN}^*}{11.0\text{KN} > 8.66}$$

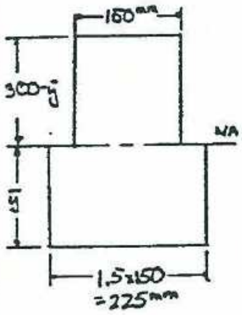
for F acting down :

$$\sigma_t = \frac{(.667F)(.0875)}{3.79\times10^{-5}} = \frac{40000}{1.5} \rightarrow F = 17.3^{KN}$$

$$\sigma_b = \frac{(.667F)(.1375)}{3.79\times10^{-5}} = \frac{20000}{1.5} \rightarrow F = 5.51$$

8-27

$$n = \frac{E_T}{E_C} = 1.5 \qquad \Sigma M_{NA} = 0$$



$$225\bar{y}(\tfrac{\bar{y}}{2}) = 150(300-\bar{y})(\tfrac{300-\bar{y}}{2})$$

$$\bar{y} = 135\text{ mm}, (300-\bar{y}) = 165\text{mm}$$

$$I = \tfrac{1}{3}(.150)(.165)^3 + \tfrac{1}{3}(.225)(.135)^3$$

$$= 4.09\times10^{-4}\,m^4$$

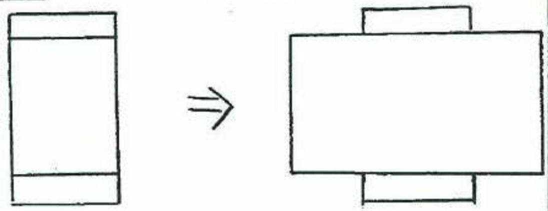
$$\sigma_t = \frac{1.5(240000)(10^{-6})(.135)}{(4.09\times10^{-4})}$$

$$= 119\text{ M}\cdot\text{N/m}^2$$

$$\sigma_c = \frac{(240000)(10^{-6})(.165)}{(4.09\times10^{-4})} = 96.8\text{ MN/m}^2$$

8-28

Alternative Solution



$$n = \frac{E_s}{E_b} = \frac{200}{86} = 2.3256$$

$$I = \frac{1}{12}(\tfrac{200}{86}\times40)(40)^3 + 2\left[\tfrac{1}{12}(40)(10)^3+(400)(25)^2\right]$$

$$= 1002791\text{ mm}^4$$

$$M_1 = \frac{\frac{\sigma_{max}}{n}I}{C_1} = \frac{\frac{40\times10^6}{2.3256}(1002791\times10^{-12})}{(20\times10^{-3})} = 862$$

$$M_2 = \frac{\sigma_{max}I}{C_2} = \frac{(40\times10^6)(1002791\times10^{-12})}{(30\times10^{-3})} = 1337$$

$$\therefore M_{max} = M_1 = 862\text{ N}\cdot\text{m}$$

8-29

$$n = \frac{E_s}{E_b} = \frac{200}{86} = 2.3256$$

$$I = \frac{1}{12}(20+30\times\tfrac{200}{86})(60)^3 = 1615814\text{ mm}^4$$

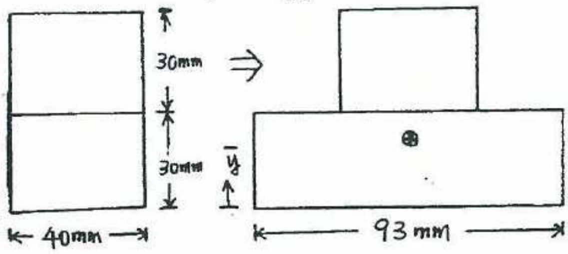
$$M_{max} = \frac{\frac{\sigma_{max}}{n}I}{C} = \frac{\frac{40\times10^6}{2.3256}(1615814\times10^{-12})}{(30\times10^{-3})}$$

$$= 926.4\text{ N}\cdot\text{m}$$

Alternative Solution

8-30

$$n = \frac{E_s}{E_b} = \frac{200}{86} = 2.3256$$



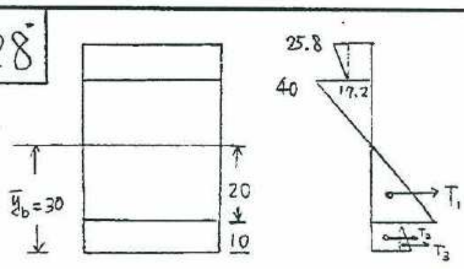
$$\bar{y} = \frac{(93\times30\times15)+(40\times30\times45)}{(93\times30)+(40\times30)} = 24\text{ mm}$$

$$I = \tfrac{1}{12}(93)(30)^3+(93\times30)(9)^2+\tfrac{1}{12}(40)(30)^3+(40\times30)(21)^2$$

$$= 1054440\text{ mm}^4$$

Alternative solution

8-28



Since the cross section is symmetry about the horizontal axis, $\bar{y}_b = 30mm$

The maximum stress in steel is 40MPa

The maximum stress in brass

$$\sigma_b' = 40\left(\frac{30}{20}\right)\left(\frac{86}{200}\right) = 25.8\ MPa < 40MPa$$

The stress in brass at contact surface

$$\sigma_b^2 = 25.8\left(\frac{20}{30}\right) = 17.2\ MPa$$

The flexural strength

$$M = 2\left[\left(\frac{1}{2}\times 40\times 20\times 40\right)\times\frac{0.04}{3} + (40\times 10\times 17.2)\times 0.025 + \frac{1}{2}\times 40\times 10\times(25.8-17.2)\times\left(0.02+\frac{0.02}{3}\right)\right]$$

$$= 862.4\ N\cdot m$$

8-29

Since the cross section is symmetry about the horizontal axis, the neutral axis located at middle.

The maximum stress in steel is 40MPa

The maximum stress in brass

$$\sigma_b = 40\left(\frac{86}{200}\right) = 17.2\ MPa < 40\ MPa$$

The flexural strength

$$M = 2\left[\left(\frac{1}{2}\times 30\times 30\times 40\right) + 2\times\left(\frac{1}{2}\times 10\times 30\times 17.2\right)\right]\times 0.02$$

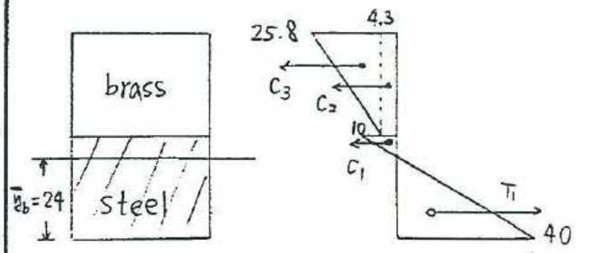
$$= 926.4\ N\cdot m$$

8-30

$$\bar{y}_b = \frac{\int_0^{30}(200)y_b\,dy_b + \int_{30}^{60}(86)y_b\,dy_b}{\int_0^{30}200\,dy_b + \int_{30}^{60}86\,dy_b}$$

$$= \frac{200\times\frac{30^2}{2} + 86\times\frac{60^2-30^2}{2}}{200\times 30 + 86\times 30}$$

$$= 24\ mm$$



The maximum stress in steel is 40MPa

The stress in steel at contact surface

$$\sigma_s' = 40\left(\frac{6}{24}\right) = 10\ MPa$$

The maximum stress in brass

$$\sigma_b' = 40\left(\frac{36}{24}\right)\left(\frac{86}{200}\right) = 25.8\ MPa < 40MPa$$

The stress in bruss at contact surface

$$\sigma_b^2 = 25.8\left(\frac{6}{36}\right) = 4.3MPa$$

Based on these stress quantities

$$T_1 = \frac{1}{2}\times 40\times 24\times 40 = 19200\ N$$

$$C_1 = \frac{1}{2}\times 40\times 6\times 10 = 1200N$$

$$C_2 = 40\times 30\times 4.3 = 5160N$$

$$C_3 = \frac{1}{2}\times 40\times 30\times(25.8-4.3) = 12900\ N$$

The flexural strength

$$M = T_1\times 0.016 + C_1\times 0.004 + C_2\times 0.021 + C_3\times 0.026$$

$$= 755.76\ N\cdot m$$

$$M_1 = \frac{\frac{\sigma_{max}}{n} I}{C_1} = \frac{\frac{40\times10^6}{2.3256}(1054440\times10^{-12})}{(24\times10^{-3})} = 756$$

$$M_2 = \frac{\sigma_{max} I}{C_2} = \frac{(40\times10^6)(1054440\times10^{-12})}{(36\times10^{-3})} = 1172$$

$$\therefore M_{max} = M_1 = 756 \text{ N·m}$$

8-31

$$\bar{y}_b = \frac{\int_A E_i y_b dA}{\int_A E_i dA} = \frac{15\times50\times80\times(40+20)+40\times50\times20\times10}{15\times50\times80+40\times50\times20} = 40 \text{ mm}$$

$$(EI)^* = \int_A E_i Y^2 dA = 15\times10^3\left(\tfrac{1}{12}\cdot50\cdot80^3+50\cdot80\cdot20^2\right) + 40\times10^3\left(\tfrac{1}{12}\cdot50\cdot20^3+50\cdot20\cdot30^2\right) = 9.2\times10^{10}$$

$$\sigma_1 = E_1\frac{M_2}{(EI)^*}y_1 = 15\times10^3\frac{12\times10^6\times60}{9.2\times10^{10}} = 117.4 \text{ N/mm}^2 \text{ (MPa)}$$

$$\sigma_2 = E_2\frac{M_2}{(EI)^*}y_2 = \frac{12\times10^6}{9.2\times10^{10}}\,40\times40\times10^3 = 208.7 \text{ MPa}$$

8-32

$$\frac{r}{d} = \frac{4}{20} = 0.2 \quad \& \quad \frac{h}{r} = \frac{8}{4} = 2$$

we can find $K = 1.5$

at middle spans: $\sigma_{max} = \frac{M_{max}}{S} = \frac{256\times10^3}{1568} = 163\times10^6 \text{ N/mm}^2 = 163 \text{ MPa}$

at transition points: $M_{max} = \frac{80\times160\times40}{2} - \frac{80\times40^2}{2} = 192\times10^3 \text{ N·mm}$

$$\sigma_{max} = K\cdot\frac{M}{S} = (1.5)\frac{192\times10^3}{800} = 360 \text{ MPa}$$

8-33 from preceding problem

$\sigma_{max} = 163$ MPa at middle span

at transition points $K = 1.5$, $S = 800 \text{ mm}^3$

$$\sigma_{max} = K\cdot\frac{M}{S} = 163 \text{ MPa} \Rightarrow M = 86.9 \text{ N·m}$$

$$M = \frac{80\cdot160\cdot x}{2} - \frac{80\cdot x^2}{2} = 86.9\times10^3$$

$$\Rightarrow x = 15 \text{ mm}$$

8-34

$$\frac{r}{d} = \frac{2}{10} = 0.2, \quad \frac{h}{r} = \frac{5}{2} = 2.5$$

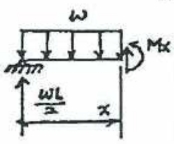
we can find $K \approx 1.52$

$$M = \frac{\sigma S}{K} = \frac{(500\times10^6)(0.01)^2/6}{1.52} = 5482.456$$

$$M = \frac{P}{2}(0.015) \rightarrow P = 730994 \text{ N/m}$$

8-35

$$\sigma_{max} = \frac{M_{max}C}{I} = \frac{\frac{wL^2}{8}\left(\frac{h}{2}\right)}{I} = \frac{wL^2h}{16I}$$



$$M_x = \frac{wx}{2}(L-x)$$

$$U = \int_0^L \frac{M_x^2}{2EI}dx = \frac{1}{2EI}\int_0^L M_x^2 dx$$

$$U = \frac{1}{2EI}\int_0^L \frac{w^2x^2}{4}(L^2 - 2Lx + x^2)\,dx$$

$$= \frac{w^2}{8EI}\int_0^L (L^2x^2 - 2Lx^3 + x^4)\,dx$$

$$= \frac{w^2L^5}{8EI}\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right)$$

$$= \frac{w^2L^5}{8\times30EI}$$

$$= \left(\frac{wL^2h}{16I}\right)^2 \frac{16^2\left(\frac{bh^3}{12}\right)}{h^2}\frac{L}{8\times30E} = \frac{\sigma_{max}^2}{2E}\left(\frac{8}{45}AL\right)$$

8-36

$$M = -Px$$

$$U = \int_0^L \frac{M^2}{2EI}dx = \frac{P^2}{2EI}\int_0^L x^2 dx = \frac{P^2L^3}{6EI}$$

$$U = \left(\frac{PL\frac{h}{2}}{I}\right)^2 \frac{LI}{(\frac{h}{2})^2 6E} = \sigma_{max}^2 \frac{\frac{Ah^2}{12}L}{(\frac{h^2}{4})6E}$$

$$= \frac{\sigma_{max}^2}{2E}\left(\frac{Vol}{9}\right)$$

8-37

$$M_y = \frac{\sigma_y I}{c} = \frac{\sigma_y \pi r^4}{r4}$$

$$M_u = \sigma_y \left(\frac{1}{2}\pi r^2\right)\left(\frac{2\times 4r}{3\pi}\right)$$

$$\frac{M_u}{M_y} = \frac{(8)(4)}{6\pi} = 1.7$$

8-38

$$M_y = \frac{\sigma_y I}{c} = \left(\frac{\sigma_y}{.15}\right)(2)\left[\frac{(.2)(.15)^3}{36} + \frac{(.2)(.15)}{2}(.05)^2\right]$$

$$= 7.5\times10^{-4}\,\sigma_y$$

$$M_u = \sigma_y\left(\tfrac{1}{2}\right)(.2)(.15)(.1) = 1.5\times10^{-3}\sigma_y$$

$$\frac{M_u}{M_y} = \frac{1.5\times10^{-3}\sigma_y}{7.5\times10^{-4}\sigma_y} = 2.00$$

8-39

W200 x 36

$Z_x = 380\times10^3\ mm^3$, $S = 342\times10^3\ mm^3$

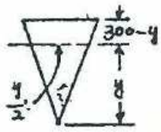
$$\frac{M_p}{M_{yp}} = \frac{380\times10^3}{342\times10^3} = 1.11$$

8-40

$$M_y = \frac{\sigma_y}{.2}\left(\frac{1}{36}\right)(.15)(.3)^3 = 5.63\times10^{-4}\sigma_y$$

at M_u, stress is constant and $C = T \rightarrow$

Area above N.A = Area below NA



$$\frac{1}{2}(y)\left(\frac{y}{2}\right) = \frac{1}{2}\left(\frac{y}{2}+150\right)(300-y)$$

$$\frac{y^2}{4} = -\frac{y^2}{4} + 22500$$

$$y = 212\ mm$$

$$M_u = \sigma_y\left[\tfrac{1}{2}(.212)\left(\tfrac{.212}{2}\right)\left(\tfrac{.212}{3}\right) + \left(\tfrac{.212}{2}\right)(.088)\left(\tfrac{.088}{2}\right)\right.$$

$$\left. + (.022)(.088)\left(\tfrac{.088}{2}\right)\right] = 1.29\times10^{-3}\sigma_y$$

$$\frac{M_u}{M_y} = \frac{1.29\times10^{-3}\sigma_y}{5.63\times10^{-4}\sigma_y} = 2.29$$

8-41

$$\bar{y} = \frac{2\left(\frac{1}{4}\right) + \left(\frac{7}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}+\frac{7}{4}\right)}{\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)+2} = 1.185''$$ from top

$$I = \tfrac{1}{12}(4)(.5)^3 + 2(.935)^2 + \tfrac{1}{12}(.5)(3.5)^3$$

$$+ \left(\tfrac{3.5}{2}\right)(1.065)^2 = 5.56\ in^4$$

$$M_y = \frac{\sigma_y I}{c} = \left(\frac{5.56}{2.815}\right)\sigma_y = 1.98\,\sigma_y$$

for equal areas, $\rightarrow$ NA in flange

$4x = \frac{1}{2}\left(\frac{7}{2}\right) + \left(\frac{1}{2}-x\right)4 \rightarrow x = .469''$ from top

$$M_u = \sigma_y\left[.469(4)\left(\tfrac{.469}{2}\right) + \left(\tfrac{3.5}{2}\right)(1.75+.031)\right.$$

$$\left. + (.031)(4)\left(\tfrac{.031}{2}\right)\right] = 3.56\,\sigma_y$$

$$\frac{M_u}{M_y} = \frac{3.56\,\sigma_y}{1.98\,\sigma_y} = 1.80$$

8-42

when ultimate moment occurs, the neutral axis is 130mm from bottom

$$M_u = \int \sigma_y \cdot y\,dA = \sigma_y \int y\,dA$$

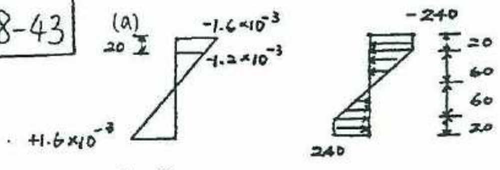
$$= 200\times10^3\,[0.2\times0.05\times0.145 + 0.05\times0.12\times0.06$$

$$+ 0.05\times0.03\times0.015 + 0.145\times0.1\times0.08]$$

$$= 599\ KN$$

8-43

(a)



$$M = 2\left[240\times20\times100\times(60+10)+\tfrac{1}{2}\times240\times10\times60\times\tfrac{2}{3}\times60\right]$$

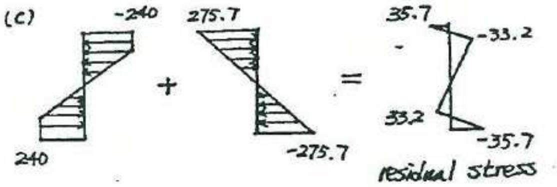
$$= 72.96\times10^{6}\ \mathrm{N\cdot mm} = 72.96\ \mathrm{kN\cdot m}$$

(b) $\sigma_{max} = \dfrac{MC}{I} = \dfrac{72.96\times10^{6}\times80}{21.17\times10^{6}} = 275.7\ \mathrm{N/mm^2}$

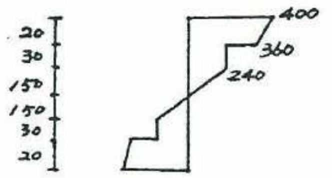
$$\epsilon = \sigma_{max}/E = \frac{275.7\times10^{6}}{200\times10^{9}} = 1.38\times10^{-3}$$

$$\epsilon_{res} = -1.6\times10^{-3}+1.38\times10^{-3} = -0.22\times10^{-3}$$

(c)



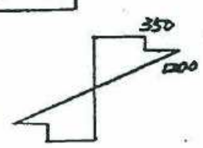
8-44



$$M = 2\left[360\times20\times240\times190+\tfrac{40}{2}\times20\times240\times(180+\tfrac{2}{3}\times20)+240\times60\times20\times(120+\tfrac{60}{2})+\tfrac{240}{2}\times120\times20\times\tfrac{2}{3}\times120\right]$$

$$= 826\times10^{6}\ \mathrm{N\cdot mm} = 826\ \mathrm{kN\cdot m}$$

8-45

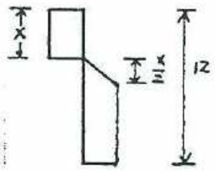


$$M = \left[350\times4\times40\times(40-4)+\tfrac{1}{2}\times1200\times16\times40\times\tfrac{2}{3}\times2\times16\right]$$

$$= 102\times10^{5}\ \mathrm{N\cdot mm}$$

$$M = \frac{P\times400}{4} \Rightarrow P = 102\ \mathrm{kN}$$

8-46



$$10\times2\times16+10\times(x-2)\times4$$

$$= \frac{10}{2}\times\frac{x}{2}\times4+10\times(12-\tfrac{3}{2}x)\times4$$

$$x = 2.66\ \mathrm{in}$$

$$M = 10\times2\times16\times(2.66-1)+10\times(2.66-2)\cdot4\cdot\frac{(2.66-2)}{2}+\frac{10}{2}\cdot\frac{2.66}{2}\cdot4\cdot\frac{2.66}{3}+10\times8.01\times4\times5.34$$

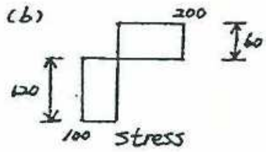
$$= 2321\ \mathrm{k\text{-}in}$$

8-47

(a) $M_{yp} = \dfrac{\sigma I}{c} = \dfrac{100\times48.6\times10^{6}}{90}$

$$= 54\times10^{6}\ \mathrm{N\cdot mm} = 54\ \mathrm{kN\cdot m}$$

(b)



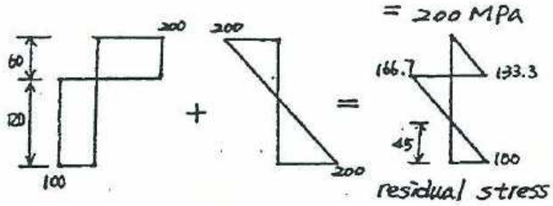
$$M_u = 100\times60\times200\times30+100\times120\times100\times60$$

$$= 108\times10^{6}\ \mathrm{N\cdot mm}$$

$$= 108\ \mathrm{kN\cdot m}$$

(c) $\sigma_{max} = \dfrac{MC}{I} = \dfrac{108\times10^{3}\times90}{48.6\times10^{6}} = 200\ \mathrm{N/mm^2}$

$$= 200\ \mathrm{MPa}$$



$$C = 100\times60\times\tfrac{1}{2}\times133.3+100\times45\times\tfrac{1}{2}\times100$$

$$= 400+225 = 625\ \mathrm{kN}$$

$$T = 100\times75\times\tfrac{1}{2}\times166.7 = 625\ \mathrm{kN}$$

$$C = T$$

$$\Sigma M_{bottom} = 225\times45/3+400(\tfrac{60}{3}+120)-625\times(\tfrac{2}{3}\times75+45) = 0$$

$\therefore$ equilibrium

0 0

8-48

$$\bar{y}_b = \frac{\int_A E_i y_b dA}{\int_A E_i dA}$$

$$= \frac{70\times15\times40\times20+10\times40\times60\times40+20\times20\times30\times10}{15\times20\times40+10\times40\times60+20\times20\times30}$$

$$= 40 \text{ mm}$$

$$(EI)^* = \int_A E_i Y^2 dA = 10\times10^3(\tfrac{1}{12}\cdot60\cdot40^3) + 15\times10^3(\tfrac{1}{12}\cdot40\cdot20^3+40\cdot20\cdot30^2) + 30\times10^3(\tfrac{1}{12}\cdot20\cdot20^3+20\cdot20\cdot30^2) = 2.56\times10^{10}$$

$$\sigma_1 = E_1\frac{M_z}{(EI)^*}y_1 = 15\times10^3\times\frac{10\times10^6}{2.56\times10^{10}}\times40 = 234.375 \text{ MPa}$$

$$\sigma_2 = E_2\frac{M_z}{(EI)^*}y_2 = 10\times10^3\times\frac{10\times10^6}{2.56\times10^{10}}\times20 = 78.125 \text{ MPa}$$

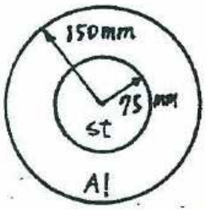
$$\sigma_3 = E_3\frac{M_z}{(EI)^*}y_3 = 30\times10^3\times\frac{10\times10^6}{2.56\times10^{10}}\times40 = 468.75 \text{ MPa}$$

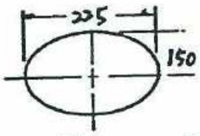
8-49

$$\sigma_{AL} = \frac{(80)(10^{-3})(.179)}{(3.01\times10^{-4})} = -47.6 \text{ MN/m}^2$$

$$\sigma_{St} = n\frac{MC}{I} = 3\times\frac{(80)(10^{-3})(.121)}{(3.01\times10^{-4})} = 96.5 \text{ MN/m}^2$$

8-50





$$n = \frac{E_{st}}{E_{AL}} = 3$$

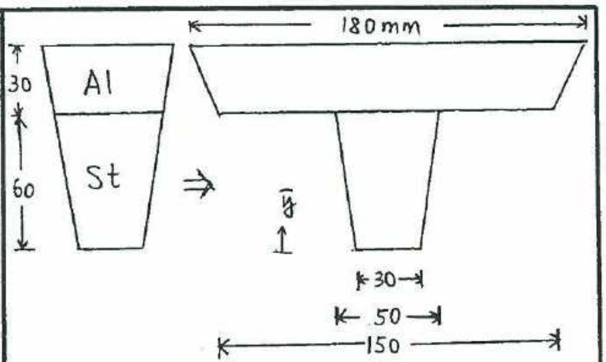
$$I_t = \frac{\pi}{4}(.225)(.075)^3 + \frac{\pi}{4}(.15^4 - .075^4) = 4.47\times10^{-4} \text{ m}^4$$

$$\sigma_{AL} = \frac{MC}{I_t} = \frac{(80)(10^{-3})(.15)}{(4.47\times10^{-4})} = \pm26.8 \text{ MN/m}^2$$

$$\sigma_{St} = n\frac{MC}{I} = \frac{3(80)(10^{-3})(.075)}{(4.47\times10^{-4})} = \pm40.3 \text{ MN/m}^2$$

8-51

$$n = \frac{E_{st}}{E_{AL}} = \frac{210}{70} = 3$$



$$\bar{y} = \frac{(30\times60\times30)+(10\times60\times40)+(150\times30\times75)+(15\times30\times80)}{(30\times60)+(10\times60)+(150\times30)+(15\times30)}$$

$$= 61.43 \text{ mm}$$

$$I = \tfrac{1}{12}(30)(60)^3 + (30\times60)(31.43)^2 + 2\left[\tfrac{1}{36}(10)(60)^3 + (\tfrac{1}{2}\times10\times60)(21.43)^2\right] + \tfrac{1}{12}(150)(30)^3 + (150\times30)(13.57)^2 + 2\left[\tfrac{1}{36}(15)(30)^3 + (\tfrac{1}{2}\times15\times30)(18.57)^2\right]$$

$$= 4.06\times10^6 \text{ mm}^4$$

$$\sigma_{Al} = \frac{(80\times10^6)(28.57)}{(4.06\times10^6)} = 563 \text{ MPa}$$

$$\sigma_{St} = \frac{3(80\times10^6)(61.43)}{(4.06\times10^6)} = 3631 \text{ MPa}$$

8-52

$$n = \frac{E_{st}}{E_w} = \frac{30\times10^6}{1.2\times10^6} = 25$$

transformed steel areas are $\frac{1}{2}$" × 50" (top) and $\frac{1}{2}$" by 150" (bottom)

$$\bar{y} = \frac{(\tfrac{1}{2})(50)(\tfrac{1}{4}) + 8(12)(6.5) + \tfrac{1}{2}(150)(12.75)}{(\tfrac{1}{2})(50) + 8(12) + \tfrac{1}{2}(150)}$$

$$= 8.09\text{" from top}$$

$$I_t = \frac{(50)(\tfrac{1}{2})^3}{12} + (50)(\tfrac{1}{2})(7.84)^2 + \frac{(8)(12)^3}{12} + (8)(12)(1.6)^2 + \frac{(150)(\tfrac{1}{2})^3}{12} + (150)(\tfrac{1}{2})(4.66)^2$$

$$= 4565 \text{ in}^4$$

$$M_w = \frac{\sigma_w I}{C_w} = \frac{(1.2)(4565)}{7.6} = 721 \text{ k·in}$$

$$M_{st} = \frac{\sigma_{st} I}{n C_{st}} = \frac{(20)(4565)}{(25)(8.09)} = 451 \text{ k·in (control)}$$

8-53

$$n = \frac{E_{st}}{E_w} = \frac{30}{1.2} = 25$$

transformed steel – 12.5" × 12"

transformed area – [8 + 2(12.5)] by 12"

= 33" × 12"

$$I_t = \frac{(33)(12)^3}{12} = 4750 \text{ in}^4$$

$$M_{st} = \frac{20(4750)}{6(25)} = 633^{\text{k-in}} \text{ (controls)}$$

$$M_w = \frac{1.2(4750)}{6} = 950^{\text{k-in}}$$

8-54

steel – 10 mm ϕ @ 80 mm = 9.82×10^{-4} m²/m

$$\frac{(1)x^2}{2} = 12(9.82\times10^{-4})(.125 - x) \qquad x = kd, \quad d = .125 m$$

$x = 0.44$ m

$$M_t = \sigma_{st} A_{st}(d - x) = (150)(10^3)(9.82\times10^{-4}) \times (.125 - .044) = 11.9 \text{ KN·m/m of slab}$$

8-55

assume NA is below cutout areas

$x = kd$, $d = 28"$

$$\frac{12x^2}{2} + \frac{6(x-6)^2}{2} = 36(28 - x)$$

$x = 10.76"$ → assumption ok

$$I = \frac{12(10)^3}{3} + \frac{2(3)(4)^3}{3} + 36(18)^2 = 15{,}800 \text{ in}^4$$

$$M = \frac{\sigma I}{nc} = \frac{20(15{,}800)}{12(18)} = 1460^{\text{k-in}} = 122^{\text{k-ft}}$$

8-56

(a) $$S = \frac{bh^2}{6} = \frac{(.05)(.1)^2}{6} = 8.33\times10^{-5} \text{ m}^3$$

$$\sigma = \frac{M}{S} = \frac{(2083)(10^{-6})}{8.33\times10^{-5}} = 25.0 \text{ MN/m}^2$$

(b) $\bar{r} = .25$ m, $r_o = .25 + .05 = .3$ m, $r_i = .2$ m

$$R = \frac{h}{\ln(r_o/r_i)} = \frac{.1}{\ln(.3/.2)} = .24663 \text{ m}$$

$$\sigma_i = \frac{M(R - r_i)}{r_i A(\bar{r} - R)} = \frac{(2083)(10^{-6})(.24663 - .2)}{(.2)(.005)(.25 - .24663)} = 28.8 \text{ MN/m}^2$$

$$\sigma_o = \frac{M(R - r_o)}{r_o A(\bar{r} - R)} = \frac{(2083)(10^{-6})(.24663 - .3)}{(.3)(.05)(.25 - .24663)} = -22.0 \text{ MN/m}^2$$

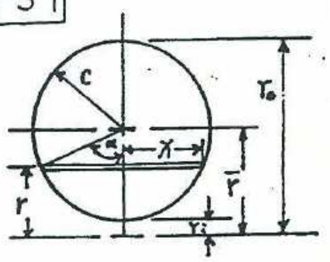
(c) $\bar{r} = .075$ m, $r_o = .125$ m, $r_i = .025$ m

$$R = \frac{.1}{\ln(.125/.025)} = .06213 \text{ m}$$

$$\sigma_i = \frac{(2083)(10^{-6})(.06213 - .025)}{(.025)(.005)(.01287)} = 48.1 \text{ MN/m}^2$$

$$\sigma_o = \frac{(2083)(10^{-6})(.06213 - .125)}{(.125)(.005)(.01287)} = -16.2 \text{ MN/m}^2$$

8-57



$$R = \frac{A}{\int_A \frac{dA}{r}} \qquad A = \pi c^2$$

$$\int_A \frac{dA}{r} = \int \frac{2x\,dr}{r}$$

$$= \int_0^{\pi} \frac{2c\sin\alpha}{\bar{r} + c\cos\alpha} c\sin\alpha\, d\alpha$$

$$= \int_0^{\pi} \frac{2c^2(1 - \cos^2\alpha)\,d\alpha}{\bar{r} + c\cos\alpha}$$

$$= 2\int_0^{\pi} \frac{c^2 - c^2\cos^2\alpha + \bar{r}^2 - \bar{r}^2}{\bar{r} + c\cos\alpha}\, d\alpha$$

$$= 2\int_0^{\pi}\left(\frac{c^2 - \bar{r}^2}{\bar{r} + c\cos\alpha} + \bar{r} - c\cos\alpha\right) d\alpha$$

$$= 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$R = \frac{\pi c^2}{2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)} = \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{2}$$

0 0

8-58

$$R = \frac{\bar{r}^2 + \sqrt{\bar{r}^2 - c^2}}{2} = \frac{3 + \sqrt{3^2 - 12}}{2} = 2.915''$$

$$\sigma = \frac{M(R-r)}{rA(\bar{r}-R)} \rightarrow M = \frac{\sigma r A(\bar{r}-R)}{(R-\bar{r})}$$

$$M_o = \frac{12(4)\pi(1)^2(3-2.915)}{2.915-4} = 11.8^{k\text{-}in}$$

$$M_i = \frac{12(2)(\pi)(1)^2(3-2.915)}{2.915-2} = 7.00^{k\text{-}in}$$

0 0

9-1

$$M = \frac{WL}{8} = \frac{4 \times 6}{8} = 3 \, kN \cdot m$$

$M_{zz} = M\cos\alpha = 2.819$.

$M_{yy} = M\sin\alpha = 1.026$.

$$I_{zz} = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \, mm^4$$

$$I_{yy} = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \, mm^4$$

$$\sigma_{max} = \frac{M_{zz} C_1}{I_{zz}} + \frac{M_{yy} C_2}{I_{yy}}$$

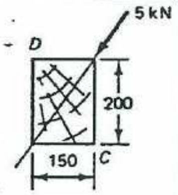
$$= \frac{2.819 \times 100}{100} + \frac{1.026 \times 75}{56.25}$$

$$= 4.187 \, N/mm^2$$

$$= 4.187 \, MPa.$$

9-2

$$M = \frac{PL}{4} = \frac{5 \times 6}{4} = 7.5 \, kN\text{-}m$$



$$M_y = 7.5 \times \frac{3}{5} = 4.5 \, kN\text{-}m$$

$$M_z = 7.5 \times \frac{4}{5} = 6 \, kN\text{-}m.$$

$$I_z = \frac{1}{12}(0.15)(0.2)^3 = 1 \times 10^{-4} \, m^4$$

$$I_y = \frac{1}{12}(0.2)(0.15)^3 = 5.625 \times 10^{-5} \, m^4.$$

$$\sigma_{max} = \pm \left(\frac{M_z C_1}{I_z} + \frac{M_y C_2}{I_y} \right)$$

$$= \pm \left(\frac{6 \times 0.1}{1 \times 10^{-4}} + \frac{4.5 \times 0.075}{5.625 \times 10^{-5}} \right)$$

$$= 12 \, MPa.$$

$$\tan\beta = \frac{I_z}{I_y} \tan\alpha$$

$$= \frac{1 \times 10^{-4}}{5.625 \times 10^{-5}} \left(\frac{3}{4} \right)$$

$$\therefore \beta = 53.1^\circ$$

9-3

$$M = PL = 100 \times 0.2 = 20 \, N\text{-}m$$

$$M_y = M\cos 30^\circ = 17.3 \, Nm$$

$$M_z = M\sin 30^\circ = 10.0 \, Nm$$

$$I_z = \frac{1}{12}(0.35)(0.45)^3 - \frac{1}{12}(0.25)(0.35)^3$$

$$= 1.76 \times 10^{-3} \, m^4$$

$$I_y = \frac{1}{12}(0.45)(0.35)^3 - \frac{1}{12}(0.35)(0.25)^3$$

$$= 1.15 \times 10^{-3} \, m^4.$$

$$\sigma_{max} = \left(\frac{M_z C_1}{I_z} + \frac{M_y C_2}{I_y} \right)$$

$$= \frac{10.0 \times 0.225}{1.76 \times 10^{-3}} + \frac{17.3 \times 0.0175}{1.15 \times 10^{-3}}$$

$$= 391 \, Pa$$

9-4

From appendix 8:

$$I_z = I_y = 279.3 \, in^4$$

$$= 116 \times 10^6 \, mm^4$$

$$= 1.16 \times 10^{-4} \, m^4$$

$$M_1 = P_1 L = 50 \times 0.4 = 20 \, N\text{-}m$$

$$M_2 = P_2 L = 60 \times 0.4 = 24 \, N\text{-}m.$$

$$M_z = M_1 + M_2 \cos 20^\circ = 42.6 \, N\text{-}m$$

$$M_x = M_2 \sin 20^\circ = 8.21 \, N\text{-}m.$$

$$\sigma_{max} = \frac{M_z C_1}{I} + \frac{M_x C_2}{I}$$

$$= \frac{42.6 \times 0.162}{1.16 \times 10^{-4}} + \frac{8.21 \times 0.162}{1.16 \times 10^{-4}}$$

$$= \pm 70.7 \, kPa.$$

$$\sigma_{min} = \frac{M_z C_1}{I} - \frac{M_x C_2}{I}$$

$$= \frac{42.6 \times 0.162}{1.16 \times 10^{-4}} - \frac{42.6 \times 0.162}{1.16 \times 10^{-4}}$$

$$= \pm 47.8 \, kPa.$$

9-5

From appendix 8.

$$I = 72.99 \, in^4$$

$$= 3.02 \times 10^{-5} \, m^4$$

$$M_z = P_1 L = 20 \times 0.55 = 11 \, N\text{-}m$$

$$M_y = P_2 L = 30 \times 0.3 = 9 \, N\text{-}m$$

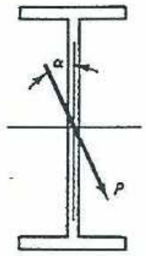
$$\sigma_{max} = \frac{M_z C_1}{I} + \frac{M_x C_2}{I}$$

$$= \frac{11 \times 0.1095}{3.02 \times 10^{-5}} + \frac{9 \times 0.1095}{3.02 \times 10^{-5}}$$

$$= 72.6 \, kPa$$

9-6

$M = (3-1)P = 2P$



$M_x = M\cos\alpha = 2P\cos\alpha$

$M_y = M\sin\alpha = 2P\sin\alpha$

$I_x = 127 \times 10^6\ mm^4$

$I_y = 6.53 \times 10^6\ mm^4$

$$\sigma_{max} = \frac{M_x c_x}{I_x} + \frac{M_y c_y}{I_y}$$

$$= \frac{2P\cos\alpha\left(\frac{305}{2}\right)(1000)}{127\times10^6} + \frac{2P\sin\alpha\left(\frac{129}{2}\right)(1000)}{6.53\times10^6}$$

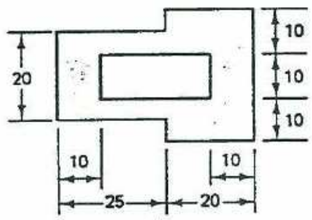
$$= P\left[2.40\times10^{-3}\cos\alpha + 2.13\times10^{-2}\sin\alpha\right]$$

$\alpha = 0°$, $\sigma_{max} = 2.40P$

$\alpha = 1°$, $\sigma_{max} = 2.77P$

$\alpha = 5°$, $\sigma_{max} = 4.25P$.

9-7



$$I = \frac{1}{12}(0.045)(0.03)^3$$

$$-\frac{1}{12}(0.025)(0.01)^3$$

$$-2\left[\frac{1}{12}(0.025)(0.005)^3 + (0.025)(0.005)(0.125)^2\right]$$

$$= 5.96\times10^{-8}\ m^4$$

$$\sigma_{max} = \frac{MC}{I} = \frac{500\times0.015}{5.96\times10^{-8}}$$

$$= 125.9\ MPa$$

9-8

$P_x = 64.3\ N$, $P_y = 76.6\ N$.

a. $I_{xx} = 114624\ mm^4$

$I_{yy} = 69184\ mm^4$

$$\sigma_c = \frac{M_x C_1}{I_x} + \frac{M_y C_2}{I_y}$$

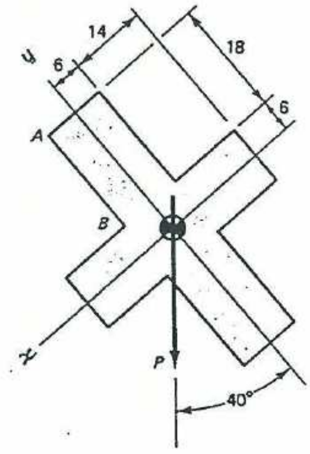
$$= \frac{76.6\times150\times24}{114624} + \frac{64.3\times150\times6}{69184} = 3.24\ N/mm^2$$

$$\sigma_D = \frac{M_x C_2}{I_x} + \frac{M_y C_1}{I_y}$$

$$= \frac{76.6\times150\times6}{114624}$$

$$+ \frac{64.3\times150\times24}{69184}$$

$$= 3.39\ N/mm^2$$

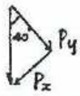


b. $$\sigma_G = \frac{76.6\times150\times y}{114624}$$

$$-\frac{64.3\times150\times6}{69184}$$

$= 0$

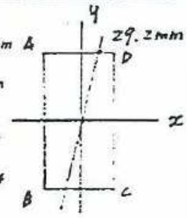
$y = 8.34\ mm$



9-9

$M_y = 15\times0.48 = 7.2\ kN\text{-}m$

$M_x = 10\times0.6 = 6\ kN\text{-}m$



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 6.67\times10^{-5}\ m^4$$

$$I_y = \frac{1}{12}(0.2)(0.1)^3 = 1.67\times10^{-5}\ m^4$$

$$\sigma_A = \frac{6\times0.1}{6.67\times10^{-5}} + \frac{7.2\times0.05}{1.67\times10^{-5}}$$

$$= 9 + 21.6 = 30.6\ MPa$$

$\sigma_B = -9 + 21.6 = 12.6\ MPa$

$\sigma_C = -9 - 21.6 = -30.6\ MPa$

$\sigma_D = 9 - 21.6 = -12.6\ MPa$

$$x = \frac{12.6\times100}{12.6+30.6} = 29.2\ mm$$

9-10

$P = 400\ kN$.

$$M = Pe = 400\left(200 - \frac{247}{2}\right)$$

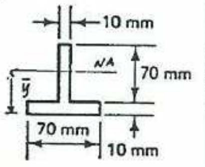
$= 30.6\ kN\text{-}m$.

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S} = \frac{400}{0.00626} + \frac{30.6}{0.000572}$$

$= 117\ MPa$

Load is transferred by bearing at the pin. Maximum stress will probably occur where the flange ends due to a reduced S.

9-11



Link section

$$\bar{y} = \frac{0.07 \times 0.01 \times 0.005 + 0.07 \times 0.01 \times 0.045}{0.07 \times 0.01 + 0.07 \times 0.01}$$

$$= 0.025$$

$$I = \frac{1}{12} \times 0.07 \times 0.01^3 + 0.07 \times 0.01 \times 0.02^2$$

$$+ \frac{1}{12}\, 0.01 \times 0.07^3 + 0.07 \times 0.01 \times 0.02^2$$

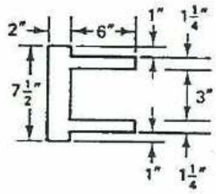
$$= 8.517 \times 10^{-7}\ m^4$$

$$A = 2 \times 0.07 \times 0.01 = 0.0014\ m^2$$

$$\frac{P}{0.0014} + \frac{P(e+0.0025)(0.025)}{8.517 \times 10^{-7}} + \frac{P}{0.0014} - \frac{P(e+0.0025)(0.055)}{8.517 \times 10^{-7}} = 0$$

$$e = 15.6\ mm$$

9-12



Section a-a

$$\bar{y} = \frac{7.5 \times 2 \times 1 + 6 \times 1.25 \times 5 \times 2}{7.5 \times 2 + 6 \times 1.25 \times 2}$$

$= 3$ in from the left.

$$I = \frac{1}{12} \cdot 7.5 \times 2^3 + 7.5 \times 2 \times 2^2$$

$$+ \frac{1}{12} \times 2.5 \times 6^3 + 2.5 \times 6 \times 2^2$$

$$= 170\ in^4$$

$$M = 23P$$

$$\sigma_t = 4000 = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{30} + \frac{23P \times 3}{170}$$

$P = 9107$ lbs ⟵ controls.

$$\sigma_c = -12000 = \frac{P}{A} - \frac{Mc}{I} = \frac{P}{30} - \frac{23P \times 5}{170}$$

$P = 18659$ lbs.

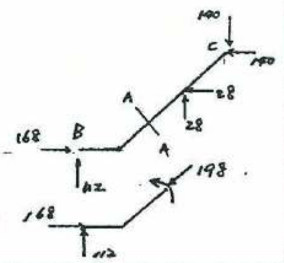
9-13

$\Sigma M_B = 0$.

$$D_x = D_y = \frac{140 \times (40-30)}{30+20}$$

$$= 28\ k.$$

$$P = \frac{168+112}{\sqrt{2}} = 198\ k$$

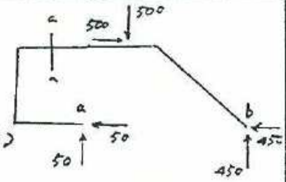


$$M = 112 \times 20 - 168 \times 10 = 560\ k \cdot in$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S} = \frac{-198}{6^2} + \frac{-560}{\frac{1}{6}(6)^3}$$

$$= -21.0\ ksi$$

9-14



$\Sigma M_a = 0$.

$$= 20 V_b - 500(12+6)$$

$V_b = N_b = 450$ lb.

$N_a = 500 - 450 = 50$ lb

$V_a = 50$ lb

$P_a = 50$ lb

$M_a = 50 \times 11 = 550$ lb-in

$$\sigma_{max} = \frac{P}{A} - \frac{Mc}{I} = \frac{50}{\frac{\pi \cdot 2^2}{4}} - \frac{550 \times 1}{\frac{\pi}{4} \times 1^4} = -684\ Psi$$

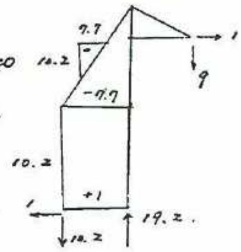
9-15

$\Sigma M_c = 0$

$$= 9V_a - 9 \times 17 - 1 \times 20$$

$\therefore V_a = 19.2\,k$, $P = -19.2\,k$.

$V_c = 10.2\,k$



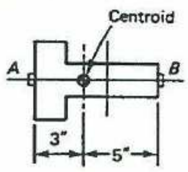
$M = 7.7 \times 2 - 1 \times 14 = 1.3\ k \cdot in$

$$\sigma_{comp} = \frac{P}{A} - \frac{Mc}{I}$$

$$= \frac{-17.2}{12^2} - \frac{1.3}{\frac{1}{6} \times 12^3} = -138\ Psi$$

9-16

$P = -F$, $M = Fe$ ↻ +



$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{-F}{A} + \frac{3Fe}{I} = \epsilon E$$

$$= -100 \times 10^{-6} \times 30 \times 10^6$$

$$= -3000\ psi$$

$$\sigma_B = \frac{P}{I} - \frac{Mc}{I} = \frac{-F}{A} - \frac{5Fe}{I} = \epsilon E = -800 \times 10^{-6} \times 30 \times 10^6$$

$$= -29000\ psi$$

$$\sigma_A - \sigma_B = \frac{8Fe}{I} = 21000, \quad \frac{Fe}{I} = \frac{21000}{8}.$$

from σ_A.

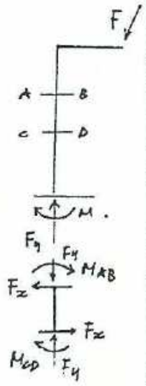
$$F = [3000 + 3(\frac{21000}{8})] \times 24 = 261\ k.$$

0 0

9-17

$$\sigma_A = -\frac{F_y}{A} + \frac{M_{AB}}{S} = 0$$

$$\sigma_B = -\frac{F_y}{A} - \frac{M_{AB}}{S} = -30$$



$$\sigma_A - \sigma_B = \frac{2M_{AB}}{S} = 30$$

$$M_{AB} = \frac{1}{2} \times 30 \times \frac{1}{6} \times 0.08 \times 1^2$$

$$= 2\ kN\text{-}m$$

$$\sigma_C = -\frac{F_y}{A} + \frac{M_{CD}}{S} = -24$$

$$\sigma_D = -\frac{F_y}{A} - \frac{M_{CD}}{S} = -6$$

$$\therefore \frac{2M_{CD}}{S} = -18 \longrightarrow M_{CD} = \frac{1}{2}(-18) \times \frac{1}{6} \times 0.08 \times 0.1^2$$

$$= -1.2\ kN\text{-}m$$

$$\sigma_A = 0.2F_x - M_{CD}$$

$$\therefore F_x = \frac{M_{AB} + M_{CD}}{0.2} = \frac{2 - 1.2}{0.2} = 4\ kN$$

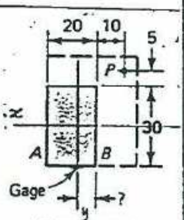
9-18

$$I_x = \frac{1}{12} \times 0.2 \times 0.3^3$$

$$= 45 \times 10^{-9}\ m^4$$

$$I_y = \frac{1}{12} \times 0.3 \times 0.2^3 = 20 \times 10^{-9}\ m^4$$

$$A = 0.03 \times 0.02 = 0.0006\ m^2$$



Top view and section

$$\sigma_G = \frac{-P}{0.0006} + \frac{P \times 0.02 \times 0.015}{45 \times 10^{-9}}$$

$$- \frac{P \times 0.02 \times x}{20 \times 10^{-9}} = 0$$

$\therefore$ $x = 5\ mm$ does not depend on P.

9-19

$$\sigma_A = E\varepsilon_A = 70 \times 10^9 \times 20 \times 10^{-6}$$

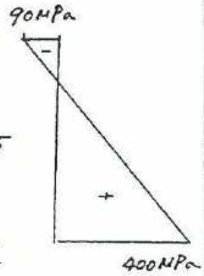
$$= 1.4\ N/mm^2$$

$$\sigma_A = \frac{P}{N} + \frac{M_x C_x}{I_x} + \frac{M_y C_y}{I_y} = 1.4$$

$$= \frac{\frac{2}{3}F}{1200} + \frac{\frac{F}{3} \times 500 \times 15}{\frac{1}{12} \times 40 \times 30^3} + \frac{\frac{2F}{3} \times 500 \times (-12)}{\frac{1}{12} \times 30 \times 40^3}$$

$$\therefore F = 420\ N.$$

9-20



$$a.\ P = \frac{1}{2} \times 400 \times 10^6 \times 0.02 \times 0.015$$

$$- \frac{1}{2} \times 90 \times 10^6 \times 0.02 \times 0.015$$

$$= 46.5\ kN$$

$$b.\ \varepsilon_{mg} = \varepsilon_{st} \longrightarrow \sigma_{mg} = \frac{E_{mg}\,\sigma_{st}}{E_{st}}$$

$$= 0.225\ \sigma_{st}$$

$$Pa = A \times 0.225\sigma_{st} \times 30 + A\sigma_{st} \times 10$$

$$= 16.75\ A\sigma_{st}$$

$$P = A \times 0.225\sigma_{st} + A\sigma_{st}$$

$$= 1.225\ A\sigma_{st}$$

$$a = 13.7\ mm$$

9-21

$$R = \frac{\bar{r}}{\ln\frac{r_o}{r_i}} = \frac{6.5}{\ln\frac{9.5}{3.5}} = 6.51$$

$$\sigma_i = \frac{P}{A} + \frac{M(R - r_i)}{r_i A(\bar{r} - R)} = \frac{19}{2 \times 6} + \frac{19 \times 6.5 \times (6.51 - 3.5)}{3.5 \times 2 \times 6 \times (6.5 - 6.51)}$$

$$= -922\ ksi$$

$$\sigma_o = \frac{P}{A} + \frac{M(R - r_o)}{r_o A(\bar{r} - R)} = \frac{19}{12} + \frac{19 \times 6.5 \times (6.51 - 9.5)}{9.5 \times 12 \times (6.5 - 6.51)}$$

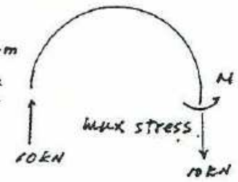
$$= 340\ ksi < 922\ ksi$$

9-22

$$M = 10 \times (0.3 - 0.04) = 2.6\ kN\text{-}m$$

$$R = \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{2} = \frac{130 + \sqrt{130^2 - 20^2}}{2}$$

$$= 129\ mm$$



$$a.\ \sigma_i = \frac{P}{A} + \frac{M(R - r_i)}{r_i A(\bar{r} - R)} = \frac{10}{\pi(0.02)^2} + \frac{2.6 \times (0.129 - 0.11)}{0.11\pi \times 0.2^2 \times (0.13 - 0.129)}$$

$$= 475\ MN/m^2$$

$$\sigma_o = \frac{P}{A} + \frac{M(R - r_o)}{r_o A(\bar{r} - R)} = \frac{10}{\pi(0.02)^2} + \frac{2.6 \times (0.129 - 0.15)}{0.15\pi \times 0.2^2 \times (0.13 - 0.129)}$$

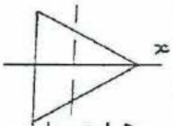
$$= -362\ MN/m^2 \qquad \therefore \sigma_{max} = 475\ MN/m^2$$

$$b.\ \frac{\sigma_{max}}{\sigma_{comp}} = \frac{475}{362} = 1.31$$

0 0

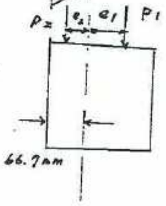
9-23

$$0 = \frac{-P_1}{\frac{1}{2}0.2^2} + \frac{P_1 e_1 \times 0.0667}{\frac{1}{36}\times 0.2^4}$$



$\therefore e_1 = 33.3\,mm$

$$0 = \frac{-P_2}{\frac{1}{2}\times 0.2^2} + \frac{P_2 e_2 (0.2 - 0.0667)}{\frac{1}{36}\times 0.2^4}$$



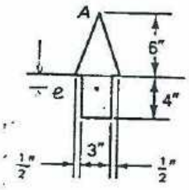
$\therefore e_2 = 16.7\,mm$

range, 50 to 100 mm.

from base of the triangular cross-section

9-24

$$\bar{y} = \frac{4\times 3\times 2 + \frac{1}{2}\times 6\times 4\times 6}{4\times 3 + \frac{1}{2}\times 6\times 4} = 4\ in$$



$$I = \frac{1}{12}\times 3\times 4^3 + 3\times 4\times 2^2 + \frac{1}{36}\times 4\times 6^3 + \frac{1}{2}\times 6\times 4\times 2^2 = 136\ in^4$$

$$\sigma_A = 0 = -\frac{P}{A} + \frac{Mc}{I} = -\frac{P}{24} + \frac{Pe\times 6}{136}$$

$e = 0.944\ in$

or 6.944 in from A.

9-25

$$I = \frac{\pi}{4}R^4;\ A = \pi R^2$$

$$\sigma = -\frac{P}{A} + \frac{Mc}{I} = -\frac{P}{\pi R^2} + \frac{PrR}{\frac{\pi R^4}{4}} = 0.$$

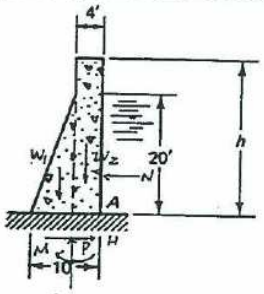
$\therefore r = \frac{R}{4}.$

9-26

$W_1 = 150\times\frac{1}{2}\times 6\times 20 = 9000\ lb/ft$

$W_2 = 150\times 4\times h = 600h\ lb/ft$

$H = 62.5\times\frac{1}{2}\times 20^2 = 12500\ lb/ft$



$P = -(W_1 + W_2) = -9000 - 600h$

$M = 9000\times(5-4) + 12500\times\frac{20}{3} - 600h\times(5-2) = 92333 - 1800h.$

$$\sigma_A = \frac{P}{A} + \frac{M}{S} = \frac{-9000-600h}{10\times 1} + \frac{92333-1800h}{\frac{1}{6}\times 1\times 10^2} = 0$$

$\sigma_A = 4640 - 168h = 0$

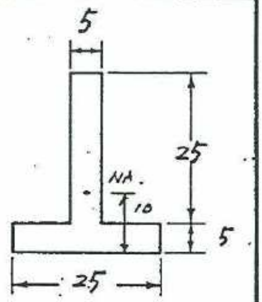
$\therefore h = 27.6\ ft.$

9-27

a. $P = -\frac{1}{2}\times 25\times 5\times 200 + \frac{1}{2}\times 5\times 5\times 200 + 20\times 5\times 200$

$= -12500 + 2500 + 2000$

$= 10\,kN.$

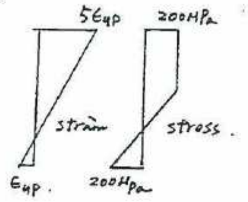


$M = 12500\times(0.005 + \frac{2}{3}(0.005))$

$- 2500\times\frac{1}{3}\times 0.005$

$+ 20000\times 0.01$

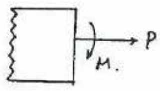
$= 300\ N\text{-}m$



b. $\sigma_{bottom} = \frac{-P}{A} + \frac{Mc}{I}$

$$= \frac{10\times 10^3}{-0.00025} + \frac{300\times 0.01}{28\times 10^{-8}}$$

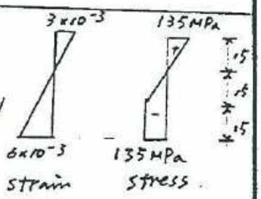
$= 104\ MPa.$



$$\varepsilon_{bottom} = -\varepsilon_{yp} + \frac{\sigma_{bottom}}{E}$$

$$= \frac{-200+104}{200\times 10^3} = -0.00048\ m/m$$

9-28



a. $P = 135\times 15\times 20 = 40{,}500\,N$

$P(22.5 + e)$

$= -\frac{1}{2}P\times 5 + \frac{1}{2}P\times 25 + P\times 37.5$

$\therefore e = 25\,mm.$

b. $$\sigma_T = -\frac{Pec}{I} + \frac{P}{A} = \frac{-40500\times 0.025\times 0.0225}{45{,}875\times 10^{-7}} + \frac{40500}{0.0009}$$

$= -105\ MPa$

$\varepsilon_T = 3\times 10^{-3} - 105\times 3\times 10^{-3}/135 = 6.67\times 10^{-4}$

$$\sigma_B = \frac{Pec}{I} + \frac{P}{A} = 195\ MPa$$

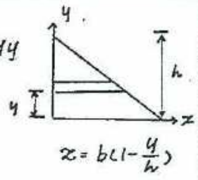
$\varepsilon_B = 195\times 3\times 10^{-3}/135 - 6\times 10^{-3}$

$= -1.67\times 10^{-3}$

0 0

9-29

a. $I_{zy} = \int_0^h \frac{b^2}{2}(1-\frac{y}{h})^2 y\,dy$



$x = b(1-\frac{y}{h})$

$= \frac{b^2}{2}(\frac{h^2}{2} - \frac{2}{3}h^2 + \frac{1}{4}h^2)$

$= \frac{b^2h^2}{24}$

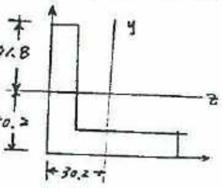
b. $I_{x_0 y_0} = I_{xy} - A d_x d_y$.

$= \frac{b^2h^2}{24} - \frac{bh}{2} \times \frac{b}{3} \times \frac{h}{3}$

$= -\frac{b^2h^2}{72}$

9-30

a. $\bar{y} = \frac{[102 \times 12.7 \times 51 + (102-12.7) \times 12.7 \times \frac{12.7}{2}]}{102 \times 12.7 + (102-12.7) \times 12.7}$



= 30.2 mm from the bottom

and left.

b. $I_y = \frac{1}{12} \times 12.7 \times 89.3^3 + 12.7 \times 89.3 \times (71.8 - 44.65)^2$

$+ \frac{1}{12} \times 102 \times 12.7^3 + 102 \times 12.7 \times (-30.2 - 6.35)^2$

$= 2.34 \times 10^6 mm^4$

$I_z = 2.34 \times 10^6 mm^4$ ($\because$ symmetry).

$I_{yz} = 0 + 12.7 \times 89.3 \times (71.8 - 44.65) \times (30.2 - 6.35)$

$+ 0 + 12.7 \times 102 \times (-30.2 + 6.35)(-51 + 30.2)$

$= 1.38 \times 10^6 mm^4$

c. $\theta_1 = \frac{1}{2} \cdot \tan^{-1}[\frac{2 \times 1.38 \times 10^6}{0}] \rightarrow \theta_1 = 90°$

$I_{max/min} = 2.34 \times 10^6 \pm \sqrt{0 + (1.38 \times 10^6)^2}$

$\therefore I_{max} = 3.72 \times 10^6 mm^4$

$I_{min} = 960 \times 10^3 mm^4$

d. $r_{min} = 0.782$ in $= 19.8628$ mm

$I_{min} = 2429.51 \times 19.8628$

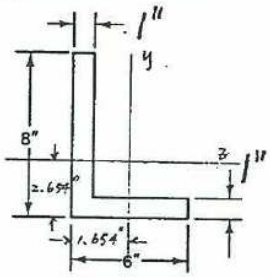
$= 959 \times 10^3 mm^2$

$I_{max} = I_x + I_y - I_{min}$

$= 3.73 \times 10^6 mm^4$

9-31

a. $\bar{z} = \frac{7 \times 1 \times \frac{1}{2} + 6 \times 1 \times 3}{7+6}$



$= 1.654$ in

$\bar{y} = \frac{8 \times 1 \times 4 + 5 \times 1 \times \frac{1}{2}}{8+5}$

$= 2.654$ in.

$I_z = \frac{1}{12} \times 1 \times 7^3 + 7 \times 1.846^2 + \frac{1}{12} \times 6 \times 1^3 + 6 \times 2.154^2$

$= 80.8 in^4$

$I_y = \frac{1}{12} \times 1 \times 7^3 + 7 \times 1.154^2 + \frac{1}{12} \times 1 \times 6^3 + 6 \times 1.346^2$

$= 38.8 in^4$

$I_{zy} = 7 \times 1 \times 1.846 \times (-1.154) + 6 \times 1 \times (-2.154) \times 1.346$

$= -32.3 in^4$

$\theta_1 = \frac{1}{2} \tan[\frac{2 \times (-32.3)}{80.8 - 38.8}] = -28.5°$.

$I_{max/min} = \frac{80.8 + 38.8}{2} \pm \sqrt{21^2 + (-32.3)^2}$

$I_{max} = 98.3 in^4$

$I_{min} = 21.3 in^4$

b. $r_{min} = 1.28$, $I_{min} = 13 \times 1.28^2 = 21.3 in^4$

$I_{max} = I_x + I_y - I_{min} = 80.8 + 38.8 - 21.3$

$= 98.3 in^4$

9-32

$I_z = \frac{2}{12} \times 30 \times 10^3 + 2 \times 10 \times 30 \times 25^2 + \frac{1}{12} \times 10 \times 60^3$

$= 560 \times 10^3 mm^4$

$I_y = \frac{1}{12} \times 10 \times 40^3 + 10 \times 40 \times 15^2 \times 2 + \frac{1}{12} \times 40 \times 10^3$

$= 290 \times 10^3 mm^4$

$I_{yz} = 2 \times 10 \times 30 \times 25 \times 20 = 300 \times 10^3 mm^4$

$\theta_1 = \frac{1}{2} \tan^{-1}[\frac{2 \times 300 \times 10^3}{560 \times 10^3 - 290 \times 10^3}] = 32.886°$.

$I_{max/min} = \frac{560 + 290}{2} \pm \sqrt{135^2 + 300^2}$

$I_{max} = 753.976 in^4$

$I_{min} = 96.0243 in^4$

9-33

$$\sigma_B = \frac{-4\times10^6 \times 5.14\times10^6}{3.89\times22.64\times10^{12} - 5.14^2\times10^{12}}\times125.7 + \frac{4\times10^6\times22.64\times10^6}{3.89\times22.64\times10^{12}-5.14^2\times10^{12}}\times4.3$$

$$= -0.3397\times125.7 + 1.4969\times24.3$$

$$= -36.3\ \text{MPa}.$$

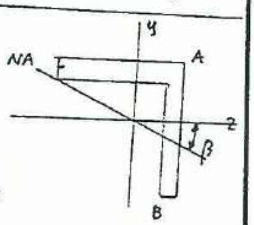
$$\sigma_F = -0.3397\times(-74.3) + 1.4969\times24.3$$

$$= 61.6\ \text{MPa}$$

$$\tan\beta = \frac{4\times10^6\times22.64\times10^6}{4\times10^6\times5.14\times10^6} = 4.40$$

$\therefore\ \beta = 77.2°$.

9-34



$$\tan\beta = \frac{M_z I_{yz}}{M_z I_y}.$$

$$= \frac{160\times10^3\times1.38\times10^6}{160\times10^3\times2.34\times10^6}$$

$\beta = 30°$, by inspection, A+B are the farthest perpendicular points from NA.

$$\sigma_A = \frac{160\times10^3\times2.34\times10^6}{2.34^2\times10^{12}-1.38^2\times10^{12}}\times30.2 + \frac{160\times10^3\times2.34\times10^6}{2.34^2\times10^{12}-1.38^2\times10^{12}}\times(-30.2)$$

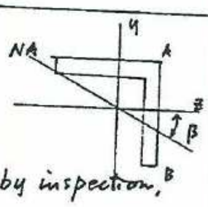
$$= 1.048\times30.2 - 0.6183(-30.2)$$

$$= 5.03\ \text{MPa}$$

$$\sigma_B = 1.048\times(-7.8) - 0.6183(-30.2+12.7)$$

$$= -6.45\ \text{MPa}.$$

9-35



$$M_z = 200\times0.8$$

$$= 160\ \text{N-m}$$

$$= 1416\ \text{lb-in}$$

$\tan\beta = \dfrac{-32.3}{38.8} \Rightarrow \beta = -39.8°$, by inspection, A+B are the farthest perpendicular points from NA.

$$\sigma_A = \frac{1416\times38.3}{80.8\times38.8-32.3^2}\times2.654 + \frac{-1416\times(-32.3)}{80.8\times38.8-32.3^2}\times1.654$$

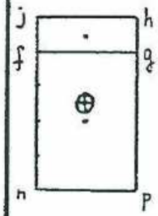
$$= 25.929\times2.654 + 21.88\times1.654 = 105.0\ \text{psi}$$

$$\sigma_B = 25.929\times(-5.346) + 21.88\times1.154$$

$$= -113.4\ \text{psi}$$

10-1

$A_{fghi} = 50 \times 150 = 7500\ mm^2$



$\bar{y}_1 = 25 + 50 + 25 = 100\ mm$

$A_{fgpn} = 4 \times 50 \times 150 = 30000\ mm^2$

$\bar{y}_2 = 25\ mm$

$A_{fghi}\,\bar{y}_1 = 750000 = A_{fgpn}\,\bar{y}_2$

10-2

$V = 250\ lb$

$I = \frac{1}{12}(6)(8)^3 = 256\ in^4$

$Q_1 = (6 \times 2)(3) = 36\ in^3$

$Q_2 = (6 \times 6)(1) = 36\ in^3$

$q = \frac{VQ}{I} = \frac{(250)(36)}{256} = 35.16\ lb/in$

$F_{nail} = \frac{35.16/2}{6} = 2.93\ lb$

10-3

$I = \frac{1}{12}(200)(250)^3 - \frac{1}{12}(100)(150)^3$

$= 232291667\ mm^4$

$Q = (100 \times 50)(100) = 500000\ mm^3$

① $V = 3\ kN$

$q = \frac{VQ}{I} = \frac{(3000)(500000)}{(232291667)} = 6.46\ N/mm$

$S = \frac{2F}{q} = \frac{2(300)}{6.46} = 93\ mm$

② $V = 2\ kN$

$q = \frac{VQ}{I} = \frac{(2000)(500000)}{(232291667)} = 4.3\ N/mm$

$S = \frac{2F}{q} = \frac{2(300)}{4.3} = 139.5\ mm$

10-4

(a) Take the design that has the smaller Q → take design (a)

(b) $Q = (2 \times 6)(4) = 48\ in^3$

$I = \frac{1}{12}(10)^4 - \frac{1}{12}(6)^4 = 725.33\ in^4$

$q = \frac{(800)(48)}{(725.33)} = 53\ lb/in$

$S = \frac{2(140)}{53} = 5.3\ in$

10-5

$I = \frac{1}{12}(200)(240)^3 - \frac{1}{12}(188)(200)^3$

$+ \frac{2}{12}(240)(20)^3 + 2(240 \times 20)(130)^2$

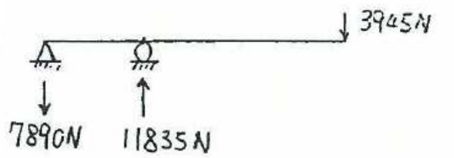
$= 251977045\ mm^4$

$Q = (240 \times 20)(130) = 624000\ mm^3$

$q = \frac{VQ}{I} = \frac{(1.5 \times 10^6)(624000)}{(251977045)} = 3715\ N/mm$

$S = \frac{2(120 \times 10^3)}{3715} = 64.6\ mm$

10-6



(a) $Q = (250 \times 50)(150) = 1875000\ mm^3$

$q = \frac{VQ}{I} = \frac{(7890)(1875000)}{(740 \times 10^6)} = 20\ N/mm$

$S = \frac{2(400)}{20} = 40\ mm$

(b) $Q = (150 \times 50)(50) = 375000\ mm^3$

$q = \frac{VQ}{I} = \frac{(7890)(375000)}{(740 \times 10^6)} = 4\ N/mm$

$S = \frac{2(400)}{4} = 200\ mm$

10-7 $I = 2(34.4\times10^6 + 4570\times100.5^2)$

$= 161\times10^6 \text{ mm}^4$

$$M = \frac{\sigma I}{y} = \frac{(160\times10^3)(161\times10^{-6})}{(0.201)} = 128 \text{ kN-m}$$

$$q = \frac{2F}{S} = \frac{2(90)}{(0.15)} = 1200$$

$$V = \frac{qI}{Q} = \frac{(1200)(161\times10^{-6})}{(4570\times100.5\times10^{-9})} = 420 \text{ kN}$$

10-8 $Q_A = (14\times\frac{1}{2})(20.25) + 2(4.75)(19)$

$= 322.25 \text{ in}^3$

$$q_A = \frac{(150)(322.25)}{(14560)} = 3.32 \text{ k/in}$$

$$S_A = \frac{11.3}{3.32} = 3.40 \text{ in}$$

$$Q_B = (14\times\tfrac{1}{2})(20.25) = 141.75 \text{ in}^3$$

$$q_B = \frac{(150)(141.75)}{(14560)} = 1.46 \text{ k/in}$$

$$S = \frac{2(6.63)}{1.46} = 9.08 \text{ in}$$

10-9 $\bar{y} = \dfrac{(3930)(228.5+7.16-17.1)}{(9460+3930)} = 64 \text{ mm}$

$I = (333\times10^6 + 9460\times(64^2) + (1.61\times10^6 + 3930\times232^2)$

$= 934\times10^6 \text{ mm}^4$

$$Q = (3930)(232) = 911760 \text{ mm}^3$$

$$q = \frac{(250\times10^3)(911760)}{(934\times10^6)} = 244 \text{ N/mm}$$

$$F = \frac{244/2}{150} = 0.8133 \text{ N}$$

$$\tau = \frac{F}{A} = \frac{0.8133}{\frac{\pi}{4}(19\times10^{-3})^2} = 2870 \text{ Pa}$$

10-10 $Q = (0.05\times0.3)(0.35) + (0.025\times0.25)(0.2)$

$= 0.0065 \text{ m}^3$

$$q = \frac{(450)(0.0065)}{(4300\times10^{-6})} = 680.23 \text{ kN/m}$$

$$\tau = \frac{qS}{2A} = \frac{(680.23\times10^{-3})(0.125)}{2(\pi\times0.011^2)} = 112 \text{ MN/m}^2$$

10-11 $q = \dfrac{(25\times10^3)(270\times184.8\times2)}{(47.6\times10^6)} = 52.4$

$(52.4)(30) = 1572$

$$F.S. = \frac{3560}{1572} = 2.26$$

10-12 $A_{rivets} = \frac{\pi}{4}(4.76)^2 = 17.13 \text{ mm}^2$

$I = 47.6\times10^6 + 2(100\times1.3)(200.65)^2 = 58\times10^6$

$Q_A = (270\times184.8\times2) + (100\times1.3\times200.65) = 125876.5$

$$q_A = \frac{(25)(125876.5\times10^{-9})}{(58\times10^{-6})} = 54.3$$

$$\tau_A = \frac{(54.3\times10^{-3})(30\times10^{-3})}{2(17.13\times10^{-6})} = 47.5 \text{ MPa}$$

$$Q_B = 100\times1.3\times200.65 = 26084.5$$

$$q_B = \frac{(25)(26084.5\times10^{-9})}{(58\times10^{-6})} = 11.2$$

$$\tau_B = \frac{(11.2\times10^{-3})(30\times10^{-3})}{2(17.13\times10^{-6})} = 9.8 \text{ MPa}$$

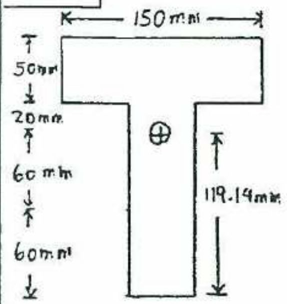
10-13

$$\tau_{max} = \frac{VQ}{It} = \frac{V(\frac{\pi}{2}r^2\times\frac{4r}{3\pi})}{(\frac{\pi}{4}r^4)(2r)} = \frac{4}{3}\frac{V}{A}$$

10-14

$$\tau_{max} = \frac{VQ}{It} = \frac{V(\pi r t\times\frac{2r}{\pi})}{(\pi r^3 t)(2t)} = \frac{2V}{2\pi r t} = \frac{2V}{A}$$

10-15



$$\bar{y}_b = \frac{(50\times150\times165+50\times140\times70)}{(50\times150+50\times140)} = 119.14 \text{ mm}$$

$$I = \frac{1}{12}\times150\times50^3 + 150\times50\times45.86^2 + \frac{1}{12}\times50\times140^3 + 50\times140\times49.14^2 = 45.67\times10^6$$

$$\tau_1 = 0$$

$$\tau_2 = \frac{(0.24)(0.15\times0.025\times0.05836)}{(45.67\times10^{-6})(0.15)} = 7.67 \text{ MPa}$$

$$\tau_3 = \frac{(0.24)(0.15\times0.05\times0.04586)}{(45.67\times10^{-6})(0.15 \text{ or } 0.05)} = 12.05 \text{ MPa or } 36.15 \text{ MPa}$$

$$\tau_4 = \frac{(0.24)(0.05\times0.12\times0.05914)}{(45.67\times10^{-6})(0.05)} = 37.29 \text{ MPa}$$

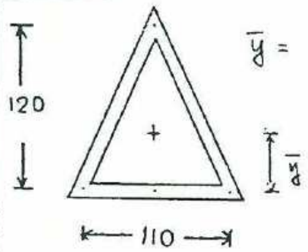
$$\tau_5 = \frac{(0.24)(0.05\times0.06\times0.08914)}{(45.67\times10^{-6})(0.05)} = 28.11 \text{ MPa}$$

$$\tau_6 = 0$$

10-17

$$L = (55^2+120^2)^{1/2} = 132 \text{ mm}$$

$$\bar{y} = \frac{2\times132\times4\times60}{(110+264)\times4} = 42.35$$



$$I = 2\left(\frac{1}{12}\times4\times132\times120^2 + 4\times132\times17.65^2\right) + \frac{1}{12}\times100\times4^3 + 100\times4\times42.35^2$$

$$= 2314110 \text{ mm}^4$$

midheight :

$$Q_1 = \left(\frac{132}{2}\times4\right)(90-42.35) = 12579.6 \text{ mm}^3$$

$$\tau_1 = \frac{(100\times10^{-3})(12579.6\times10^{-9})}{(2314110\times10^{-12})(4\times10^{-3})} = 136 \text{ MPa}$$

centroidal level :

$$Q_2 = \left[132\times\frac{(120-42.35)}{120}\times4\right]\frac{(120-42.35)}{2}$$

$$= 13264.95 \text{ mm}^3$$

$$\tau_2 = \frac{(100\times10^{-3})(13264.95\times10^{-9})}{(2314110\times10^{-12})(4\times10^{-3})} = 143 \text{ MPa}$$

10-16

$$I = \frac{1}{12}(200\times300^3 - 150\times250^3)$$

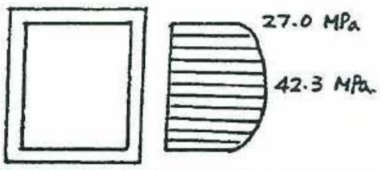
$$= 254.7\times10^6$$

$$\tau_{max} = \frac{VQ}{It} = \frac{500\times10^3\times(25\times200\times137.5+2\times25\times125\times62.5)}{254.7\times10^6\times50}$$

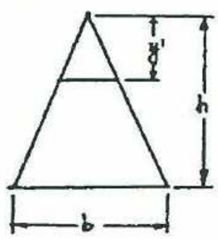
$$= 42.3 \text{ MPa}$$

$$\tau_{min} = \frac{VQ}{It} = \frac{500\times10^3\times(25\times200\times137.5)}{254.7\times10^6\times50}$$

$$= 27.0 \text{ MPa}$$



10-18



(a) $\tau = \frac{VQ}{It}$. find $\left(\frac{Q}{t}\right)_{max}$

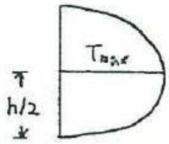
$$\frac{Q}{t} = \frac{\frac{1}{2}(\frac{1}{2}y')y'(\frac{2}{3}h-\frac{2}{3}y')}{\frac{1}{2}y'}$$

$$= \frac{1}{3}(hy'-y'^2)$$

$$\frac{d(\frac{Q}{t})}{dy'} = 0 = \frac{1}{3}(h-2y')$$

$$y' = \frac{1}{2}h$$

(b) $V = \frac{\tau I t}{Q} = \frac{(100)(\frac{1}{36} \times 25 \times 50^3 \times 12.5)}{\frac{1}{2} \times 12.5 \times 25 \times \frac{50}{3}} = 41667 N$



parabolic distribution

10-19 $I = 2(\frac{1}{12})(0.16)^4 = 1.092 \times 10^{-4}\ m^4$

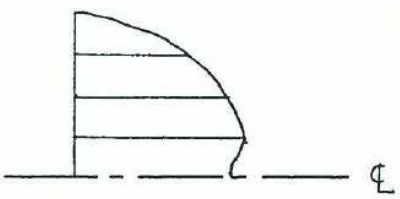
$\tau_1 = 0$

$\tau_2 = \frac{(5)(\frac{1}{2} \times 0.04^2 \times \frac{0.4}{3})}{(1.092 \times 10^{-4})(0.04)} = 120\ kPa$

$\tau_3 = \frac{(5)(\frac{1}{2} \times 0.08^2 \times \frac{0.32}{3})}{(1.092 \times 10^{-4})(0.08)} = 195\ kPa$

$\tau_4 = \frac{(5)(\frac{1}{2} \times 0.12^2 \times 0.08)}{(1.092 \times 10^{-4})(0.12)} = 220\ kPa$

$\tau_5 = \frac{(5)(\frac{1}{2} \times 0.16^2 \times \frac{0.16}{3})}{(1.092 \times 10^{-4})(0.16)} = 195\ kPa$



10-20 $V = \frac{dM}{dx} = \frac{3+2}{10} = 0.5\ K$

$\bar{y}_b = \frac{4 \times 6 \times 5 + 2 \times 8 \times 1}{4 \times 6 + 2 \times 8} = 3.4\ in$

$I = \frac{1}{12} \times 4 \times 6^3 + 4 \times 6 \times 1.6^2$

$+ \frac{1}{12} 8 \times 2^3 + 8 \times 2 \times 2.4^2$

$= 230.9\ in^4$

(a) $Q = 2 \times 6 \times 1.6 = 19.2\ in^3$

$q = \frac{VQ}{I} = \frac{(500)(19.2)}{(230.9)} = 41.58\ lb/in$

$F = (41.58)(12) = 499\ lb$

(b) $Q = 8 \times 2 \times 2.4 = 38.4\ in^3$

$\tau = \frac{VQ}{It} = \frac{(500)(38.4)}{(230.9)(4)} = 20.79\ lb/in^2$

10-21 $I = \frac{1}{12}(75)(200)^3 - 2[\frac{\pi}{4}(25)^4 + \pi(25)^2(40)^2]$

$= 43.1 \times 10^6\ mm^4$

$Q = (75 \times 60 \times 70) - \frac{\pi}{2}(25)^2(40 + \frac{4 \times 25}{3\pi}) = 265313$

$V = \frac{\tau I t}{Q} = \frac{(50)(43.1 \times 10^6)(25)}{265313} = 2.03 \times 10^5\ N$

$M = \frac{\sigma I}{c} = \frac{(100)(43.1 \times 10^6)}{(100)(10^3)} = 4.31 \times 10^4\ N\text{-}m$

10-22 $\bar{y} = \frac{14 \times 2 \times (17+9) + 2 \times 4 \times (2+2+1)}{14 \times 2 \times 2 + 2 \times 4 \times 3}$

$= 9.6\ in$

$\tau = \frac{VQ}{It} = \frac{(400)(2 \times 4 \times 7.6)}{(2640)(2)} = 4.6\ psi$

$F = \frac{VQs}{I} = \frac{(400)(2 \times 2 \times 4 \times 7.6 + 2 \times 4 \times 8.6)(6)}{2640}$

$= 173\ lb$

10-23

$\bar{y}_b = \frac{25 \times 150 \times 12.5 + 25 \times 200 \times 125 + 25 \times 200 \times 237.5}{25 \times 150 + 25 \times 200 + 25 \times 200}$

$= 135.23\ mm$

$$I = \frac{1}{12}(150)(25)^3 + (150)(25)(122.73)^2 + \frac{1}{12}(25)(200)^3 + (25)(200)(15.23)^2 + \frac{1}{12}(200)(25)^3 + (200)(25)(102.27)^2 = 127\times10^6 \text{ mm}^4$$

$$P_1 = \frac{\sigma I}{cL} = \frac{(150\times10^{-3})(127\times10^6)}{(135.23)(1000)} = 140.87 \text{ kN}$$

$$P_2 = \frac{\tau I t}{Q} = \frac{(100\times10^{-3})(127\times10^6)(25)}{(150\times25\times122.73)+(25\times115.23\times57.615)} = 507.02 \text{ KN}$$

$$P_3 = \frac{q I}{Q} = \frac{(2)(127\times10^6)}{(150\times25\times122.73)+(25\times50\times85.23)} = 448.15 \text{ KN}$$

flexure controls $\longrightarrow P_{max} = 140.87$ kN

10-24

$$\bar{y}_b = \frac{2\times20\times400\times200 + 100\times80\times40 + 100\times40\times380}{2\times20\times400 + (40+80)\times100} = 180$$

$$I = \frac{1}{12}(140\times400^3 - 100\times280^3) + 140\times400\times20^2 - 100\times280\times40^2 = 541\times10^6 \text{ mm}^4$$

(a) $V_{PLY} = \frac{\tau I t}{Q} = \frac{1.5\times541\times10^6\times40}{220\times140\times110 - 100\times180\times90} = 18360 \text{ N} = 18.36 \text{ KN}$

(b) $V_{u-N} = \frac{I f}{Q} = \frac{541\times10^6}{100\times40\times200} f = 676.25 f$

$$f = \frac{2F}{S} = \frac{2\times500}{50} = 20 \text{ N/mm}$$

$$V_{u-N} = 676.25\times20 = 13525 \text{ N} = 13.53 \text{ KN}$$

(c) $f = \frac{2F}{S} = \frac{2\times500}{25} = 40$ N/mm

$$V_{b-N} = \frac{I f}{Q} = \frac{541\times10^6\times40}{100\times80\times140} = 19321 \text{ N} = 19.32 \text{ KN}$$

upper nailing governs $V_{max} = 13.53$ KN

10-25

$$I = \frac{1}{12}(80\times220^3 - 65\times140^3) = 56.12\times10^6$$

$$Q = 40\times80\times90 + 70\times15\times35 = 324750$$

$$V = \frac{\tau I t}{Q} = \frac{(2\times10^{-3})(56.12\times10^6)(15)}{(324750)} = 5.2 \text{ kN}$$

$$\tau = \frac{VQ}{It} = \frac{(5.2\times10^3)(40\times80\times100 - 15\times12\times76)}{(56.12\times10^6)(2\times12+15)} = 0.73 \text{ MPa}$$

10-26

$$I = \frac{1}{12}(125\times500^3 - 100\times300^3) = 1077\times10^6$$

$$M = \frac{\sigma I}{c} = \frac{(10^4)(1077\times10^{-6})}{(250\times10^{-3})} = 43.1 \text{ kN}\cdot\text{m}$$

$$V_{glue} = \frac{\tau I b}{Q} = \frac{(300)(1077\times10^{-6})(0.1)}{(0.05\times0.1\times0.2)} = 32.3 \text{ kN}$$

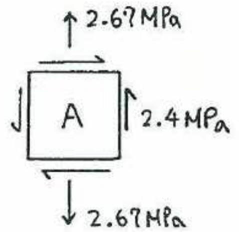
$$V_{wood} = \frac{(600)(1077\times10^{-6})(0.025)}{(0.125\times0.1\times0.2 + 0.025\times0.15\times0.075)} = 5.8 \text{ kN}$$

$\Rightarrow V_{allowable} = 5.8$ kN

10-27

$$\sigma = \frac{My}{I} = \frac{(10^4\times200)(15)}{\frac{1}{12}\times40\times150^3} = 2.67 \text{ MPa}$$

$$\tau = \frac{VQ}{It} = \frac{(10^4)(60\times40\times45)}{\frac{1}{12}\times40\times150^3\times40} = 2.4 \text{ MPa}$$



10-28 W 14×90

$M_A = M_B = \frac{1}{2}4\times20\times5 - \frac{1}{2}\times4\times5^2 = 150\ \text{k-ft}$.

$V_A = V_B = \frac{1}{2}\times4\times20 - 5\times4 = 20\ \text{K}$

$\sigma_A = \frac{My_1}{I} = \frac{(150\times12)\times0.01}{999} = 0.018\ \text{ksi}$ (tensile)

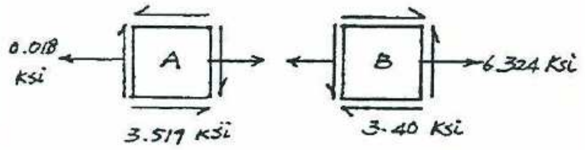
$\sigma_B = \frac{My_2}{I} = \frac{(150\times12)\times3.51}{999} = 6.324\ \text{ksi}$ (tensile)

$\tau_B = \frac{VQ}{It} = \frac{20}{999\times0.44}\left[14.52\times0.71\times(7.01-0.355) + 2.79\times0.44\times(3.51+\frac{2.79}{2})\right]$

$= 3.40\ \text{ksi}$

$\tau_A = \frac{VQ}{It} = \frac{20}{999\times0.44}\left[14.52\times0.71\times(7.01-0.355) + 6.29\times0.44\times3.155\right]$

$= 3.519\ \text{ksi}$



10-29 $M_L = \frac{2}{5}\times100\times1.2 = 48\ \text{kN}\cdot\text{m}$

$M_R = \frac{2}{5}\times100\times1.3 = 52\ \text{kN}\cdot\text{m}$

$I = \frac{1}{12}\times0.2\times0.3^3 = 4.5\times10^{-4}\ \text{m}^4$

$(\sigma_{upper})_L = (\sigma_{upper})_R = 0$

$(\sigma_{bottom})_L = \frac{My}{I} = \frac{(48\times10^{-3})(0.15)}{(4.5\times10^{-4})} = 16\ \text{MPa}$

$(\sigma_{bottom})_R = \frac{(52\times10^{-3})(0.15)}{(4.5\times10^{-4})} = 17.33\ \text{MPa}$

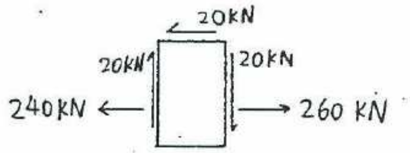
$\tau_{upper} = \frac{VQ}{It} = \frac{(\frac{2}{5}\times0.1)(0.2\times0.15\times0.075)}{(4.5\times10^{-4})(0.2)} = 1\ \text{MPa}$

$V_1 = \tau_{upper}\times100\times200 = 20\ \text{kN}$

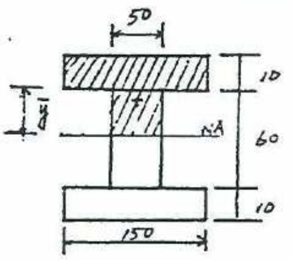
$F_L = \frac{1}{2}\times(\sigma_{bottom})_L\times150\times200 = 240\ \text{kN}$

$F_R = \frac{1}{2}\times(\sigma_{bottom})_R\times150\times200 = 260\ \text{kN}$

$V_2 = V_3 = \frac{(F_R - F_L)\times100}{100} = 20\ \text{kN}$



10-30



transformed section

$I = \frac{1}{12}(150\times80^3 - 100\times60^3) = 4.6\times10^6\ \text{mm}^4$

$\tau = \frac{VQ}{It} = \frac{5\times10^3\times(150\times10\times35 + 50\times30\times15)}{4.6\times10^6\times50}$

$= 1.63\ \text{MPa}$

10-31 $n = E_w/E_f = 2$

$t_w = 12\times2 = 24$

$\bar{y}_b = \frac{(50\times324\times475 + 48\times450\times225 + 50\times100\times25)}{50\times324 + 48\times450 + 50\times100}$

$= 296.26\ \text{mm}$

$I = \frac{1}{12}(324\times500^3 - 100\times400^3 - 176\times450^3)$

$+ (324\times500\times46.26^2 - 100\times400\times46.26^2 - 176\times450\times71.26^2)$

$= 1364.07\times10^6,\ \text{m}^4$

(a) $(\tau_w)_{max} = \frac{(20\times10^6)(50\times324\times178.74 + 48\times153.74\times76.87)}{(1364.07\times10^6)(48)}$

$= 1.06\ \text{MPa}$

0 0

(b) $\tau_{max} = \dfrac{(20\times10^3)(176\times50\times178.74)}{(1364.07\times10^6)(50+50)}$

$= 0.23$ MPa

10-32 $I = \frac{1}{12}\times56\times40^3 - 2(\frac{1}{12}\times10\times36^3+10\times36\times2^2)$

$-(\frac{1}{12}\times20\times36^3+20\times36\times2^2)$

$= 137387 \text{ mm}^4$

$\tau_1 = \dfrac{(10^4)(4\times20\times10)}{(137387)(4)} = 14.56$ MPa

$\tau_2 = \dfrac{(10^4)(4\times20\times10+4\times10\times18)}{(137387)(4)} = 27.66$ MPa

10-33 (a) $I = 2\times\frac{1}{12}(3\times100\times80^2)+2\times\frac{1}{12}(3\times80^3)+2\times\frac{1}{12}\times130\times3^3+2\times130\times3\times40^2 = 1.82\times10^6 \text{ mm}^4$

(b) $\tau_{aa} = \dfrac{VQ}{It} = \dfrac{10\times10^3\times(50\times3\times20)}{1.82\times10^6\times3}$

$= 5.49$ MPa

$\tau_{bb} = \dfrac{10\times10^3\times(35\times3\times40)}{1.82\times10^6\times3} = 7.69$ MPa

$\tau_{cc} = \dfrac{10\times10^3\times(65\times3\times40+40\times3\times20)}{1.82\times10^6\times3}$

$= 18.68$ MPa

10-34

$\tau_{aa} = \dfrac{VQ}{It} = 0.006\times\frac{1}{20}\times(200\times20\times62.1)$

$= 74.52$ MPa

$\tau_{bb} = \dfrac{VQ}{It} = 0.006\times\frac{1}{25}\times(400\times20\times62.1+62.1\sqrt{2}\times25\times\frac{62.1}{2})$

$= 135.6$ MPa

10-35 angles $\tau_{aa} = \dfrac{VQ}{It} = 0.01\times\dfrac{10\times4\times35}{4}$

$= 2.5$ MPa

plate $\tau_{aa} = 0.01\times\dfrac{40\times4\times48+2\times30\times4\times35+20\times4\times40}{4}$

$= 48.2$ MPa

10-36 Pipe: $\frac{A}{2} = 1025 \text{ mm}^2$, $t = 6.02$ mm

$\bar{R} = \frac{1}{4}(114+102) = 54$, $\bar{y} = 100+\dfrac{2\bar{R}}{\pi} = 134.38$ mm

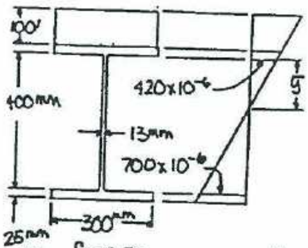
$I = \frac{1}{12}\times10\times260^3+2(2.98\times10^6+2050\times100^2)$

$= 61.6\times10^6$

$\tau_{pipe} = \dfrac{(200\times10^3)(1025\times134.38)}{(61.6\times10^6)(2\times6.02)} = 37$ MPa

$\tau_{web} = \dfrac{(200\times10^3)(10\times30\times115)}{(61.6\times10^6)(10)} = 11$ MPa

10-37



a) $\dfrac{\bar{y}}{420} = \dfrac{400}{420+700}$

$\bar{y} = 150$ mm

$P_c = \sigma_{AVE} A_t$

$\sigma_{AVE} = \epsilon_{AVE} E_t$

$P_t = \left[\left(\frac{225}{150}\right)(420\times10^{-6})\right]10^4(.1)(.3) = .189$ MN

$= \underline{1890 \text{ KN}}$

b) Assume each bolt carries equal force

# of rows $= \frac{.5}{.6} = 8.33 = 8$ # of bolts $= 16$

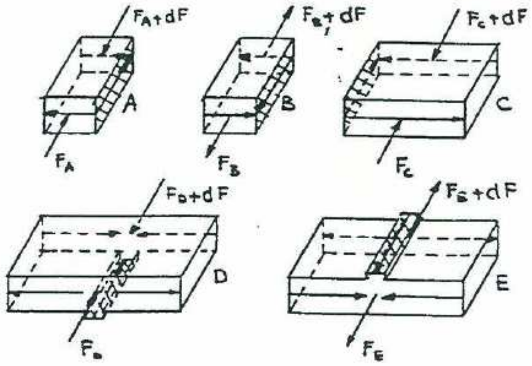
Force/bolt $= \dfrac{1890}{16} = \underline{118 \text{ KN/bolt}}$

10-38 $I = \frac{1}{12}[.15(.24)^3+.14(.2)^3] = 7.95\times10^{-5} \text{ m}^4$

$q_A = q_B = \dfrac{VQ}{I} = \dfrac{100(.03)(.02)(.11)}{7.95\times10^{-5}} = 83.0$ KN/m

$$q_C = \frac{100(.07)(.02)(.11)}{7.95\times10^{-5}} = 194\ \mathrm{kN/m}$$

$$q_D = q_E = \frac{100(.15)(.02)(.11)}{7.95\times10^{-5}} = 415\ \mathrm{kN/m}$$



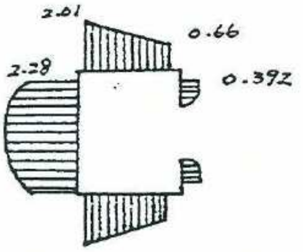
10-39 (a) $$\tau_{a\text{-}a} = \frac{VQ}{It} = \frac{7(1)(\frac{1}{2})(2)}{35.7(\frac{1}{2})} = 0.392\ \mathrm{ksi}$$

$$\tau_{bb} = \frac{7\left[1(\frac{1}{2})(2) + 3(\frac{1}{2})(2.75)\right]}{35.7(\frac{1}{2})} = 2.01\ \mathrm{ksi}$$

$$\tau_{c\text{-}c} = \frac{7\left[1(\frac{1}{2})(2) + 3(\frac{1}{2})(2.75) + 3(\frac{1}{2})(1.5)\right]}{35.7(\frac{1}{2})}$$

$$= 2.89\ \mathrm{ksi}$$

(b)



10-40 $$I = \frac{1}{12}(.002)(.05)^3 + 2(.002)(.026)(.025)^2$$

$$= 8.58\times10^{-8}\ \mathrm{m}^4$$

$Ve = F_r h_r - F_L h_L$, $h_r = h_L$, $\bar{y}_1 = \bar{y}_2 = .025\ \mathrm{m}$

$$F_r = \frac{VA_r\bar{y}}{2It}A_r = \frac{V(.002)(.02)^2(.025)}{2(8.58\times10^{-8})}$$

$$F_L = \frac{V(.002)(.006)^2(.025)}{2(8.58\times10^{-8})}$$

$$e = \frac{.002(.025)(.05)}{2(8.58\times10^{-8})}(.02^2 - .006^2)$$

$= .00530\ \mathrm{m} = 5.30\ \mathrm{mm}$ from ℄ vert. member

10-41 $$I = \frac{1}{12}(.002)(.1)^3 + 2(.002)(.04)(.04^2 + .05^2)$$

$$= 8.23\times10^{-7}\ \mathrm{m}^4$$

$Ve = h_1F_1 + h_2F_2$, $A_1 = tb = A_2$

$$F_1 = \frac{VA_1\bar{y}_1}{2It}A_1 \qquad F_2 = \frac{VA_2\bar{y}_2}{2It}A_2$$

$$e = \frac{tb^2}{4I}(h_1^2 + h_2^2) = \frac{.002(.04)^2}{4(8.23\times10^{-7})}(.08^2 + .1^2)$$

$= .0159\ \mathrm{m} = 15.9\ \mathrm{mm}$

(from ℄ of vert. member)

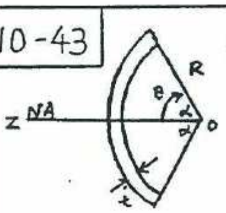
10-42 from problem 10-41, $I = 8.23\times10^{-7}$

$$e = \frac{(0.002)(0.04)^2}{4(2.23\times10^{-7})}(0.1^2 - 0.08^2)$$

$= 0.0035\ \mathrm{m} = 3.5\ \mathrm{mm}$

(from ℄ of vertical member)

10-43



$$Q_z = \int_\alpha^\theta R\sin\theta\, dA$$

$$= \int_\alpha^\theta R\sin\theta\, Rt\, d\theta$$

$$= R^2t(\cos\theta - \cos\alpha)$$

$$I = \int_{-\alpha}^{\alpha} R^2\sin^2\theta\, Rt\, d\theta = R^3t\left[\alpha - \frac{\sin 2\alpha}{2}\right]$$

$$\tau = \frac{VQ}{It} = \frac{VR^2t(\cos\theta - \cos\alpha)}{R^3t^2[\alpha - \frac{\sin 2\alpha}{2}]}$$

$$= \frac{V(\cos\theta - \cos\alpha)}{Rt[\alpha - \frac{\sin 2\alpha}{2}]}$$

Torque about o $= T = \int_{-\alpha}^{\alpha} \tau R dA$

$$= \int_{-\alpha}^{\alpha} RV \frac{\cos\theta - \cos\alpha}{[\alpha - \frac{\sin 2\alpha}{2}]} d\theta$$

$$= \frac{2R(\sin\alpha - \alpha\cos\alpha)}{\alpha - \sin\alpha\cos\alpha}$$

$$e = \frac{T}{V} = \frac{2R(\sin\alpha - \alpha\cos\alpha)}{\alpha - \sin\alpha\cos\alpha}$$

10-44

$$I = [\tfrac{1}{12}\times 4\times 40^3 + 4\times 40\times 80^2 + \tfrac{1}{12}\times 4^3\times 100 + 4\times 100\times 100^2 + \tfrac{1}{12}\times 4\times 125\times 100^2 + 125\times 4\times 50^2]\times 2$$

$$= 13.43\times 10^6 \text{ mm}^4$$

$$\tau_1 = \frac{V}{4I}Q = \frac{V(4\times a\times(60+\frac{a}{2}))}{4I}$$

$$= (60a + \frac{a^2}{2})\frac{V}{I}$$

$$F_1 = \int \tau_1 dA = \int_0^{40}(60a + \frac{a^2}{2})\frac{V}{I}\times 4\times da$$

$$= 0.0103\ V$$

$$\tau_2 = \frac{V}{4I}(40\times 4\times 80 + 4\times a\times 100)$$

$$= (3200 + 100a)\frac{V}{I}$$

$$F_2 = \int \tau_2 dA = \int_0^{100}(3200 + 100a)\frac{V}{I}\times 4\times dA$$

$$= 0.2442\ V$$

$$T = 2F_1\times 175 + 2F_2\times 100$$

$$= 2\times 0.0103V\times 175 + 2\times 0.2442V\times 100$$

$$= 52.45\ V$$

$e = \frac{T}{V} = 52.45$ mm from left corner.

10-45

$$\sigma_x = \frac{M_z(I_y z - I_{yz} y)}{I_y I_z - I_{yz}^2}$$

$$\tau = \frac{dF}{t dx} = \frac{d(\int \sigma_x dA)}{t dx}$$

$$= \frac{dM_z}{dx}\frac{1}{t(I_yI_z - I_{yz}^2)}(I_{yz}\int z dA - I_y\int y dA)$$

$$= \frac{V_y}{t(I_yI_z - I_{yz}^2)}[I_{yz}\int z dA - I_y\int y dA]$$

take the absolute value

$$\tau = \frac{V_y(I_y Q_z - I_{yz} Q_y)}{t(I_y I_z - I_{yz}^2)} = \frac{10(0.533Q_z - 0.8Q_y)}{0.1(0.533\times 2.133 - 0.8^2)}$$

$$= 107.3\,Q_z - 161\,Q_y \text{ psi}$$

Flanges: $Q_y = st(b - \frac{s}{2}) = 0.2s - 0.05s^2$

$$Q_z = st\frac{h}{2} = 0.2s$$

$$\tau_f = 107.3(0.2s) - 161(0.2s - 0.05s^2)$$

$$= 8.05s^2 - 10.74s$$

$$F_f = \int \tau_f dA$$

$$= \int_0^2 (8.05s^2 - 10.74s)\times 0.1\times ds$$

$$= 0$$

Web: $Q_y = t\times b\times\frac{b}{2} = 0.2$

$$Q_z = t\times b\times\frac{h}{2} + t\times s\times(\frac{h}{2} - \frac{s}{2})$$

$$= -0.05s^2 + 0.2s + 0.4$$

$$\tau_b = 107.3(-0.05s^2 + 0.2s + 0.4) - 161(0.2)$$

$$= -5.365s^2 + 21.46s + 10.72$$

$$\frac{F_b}{2} = \int \tau_b dA$$

$$= \int_0^2(-5.365s^2 + 21.46s + 10.72)\times 0.1\times ds$$

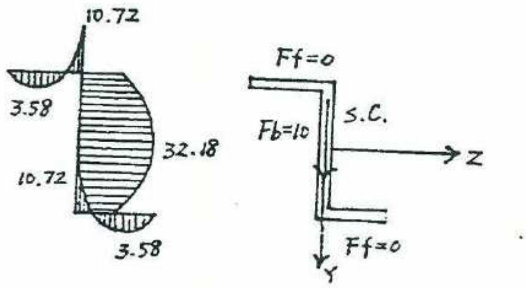
$$= 5.0 \text{ lb}$$

$$F_b = 10 \text{ lb}$$

Torque about centroid $T = 0 \times 10 = 0$

$\therefore e = 0$

Shear center passes the centroid.



10-46 $\sigma_Y = \dfrac{M_y(I_z \cdot z - I_{Yz} y)}{I_y I_z - I_{yz}^2}$

the same as Prob. 7-41

$$\tau = \frac{V_z(I_z Q_y - I_{Yz} Q_z)}{(I_y I_z - I_{yz}^2)t}$$

$$= \frac{10(2.133 Q_y - 0.8 Q_z)}{0.1(0.533 \times 2.133 - 0.8)^2}$$

$$= 429.3 Q_y - 161 Q_z \text{ psi}$$

Flanges: from Prob. 7-41

$Q_y = 0.2S - 0.05S^2$

$Q_z = 0.2S$

$\therefore \tau_f = 429.3(0.05S^2 - 0.2S) - 161(-0.2)S$

$= 53.66S - 21.465S^2$

$F_f = \int \tau_f dA$

$= \int_0^2 (53.66S - 21.465S^2) \times 0.1 ds$

$= 5.0$ lb.

Web: $Q_y = 0.2$

$Q_z = -0.05S^2 + 0.2S + 0.4$

$\tau_b = 429.3 \times 0.2 - 161(-0.05S^2 + 0.2S + 0.4)$

$= 8.05S^2 - 32.2S + 21.46$

$F_b = \int \tau_b dA$

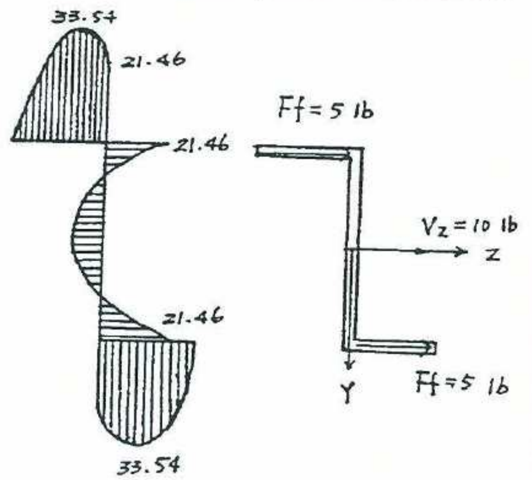
$= \int_0^2 (8.05S^2 - 32.2S + 21.46) \times 0.1 ds$

$= 0$

Torque about centroid

$eV = F_f \cdot \frac{h}{2} - F_f \cdot \frac{h}{2} = 0 \quad e = 0$

Shear center passes the centroid



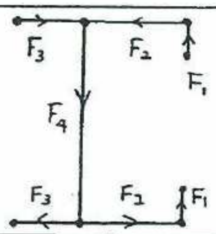
10-47 $I = 2A(a\sin\alpha)^2$

$$q = \frac{VQ}{I} = \frac{VA(a\sin\alpha)}{2A(a\sin\alpha)^2} = \frac{V}{2a\sin\alpha}$$

$$Pe = Ve = \frac{2\pi R(2\alpha)}{2\pi} aq = \frac{V\alpha R}{\sin\alpha}$$

$e = \dfrac{\alpha}{\sin\alpha} R$ from O

10-48



$I = (3A \times 75^2 + A \times 50^2) \times 2$

$= 38750A$

$$F_1 = \frac{VQ}{I} \times S = \frac{V(50A)}{(38750A)}(25) = \frac{2V}{62}$$

$$F_2 = \frac{V(50A+75A)}{(38750A)}(75) = \frac{15V}{62}$$

$$F_3 = \frac{V(75A)}{(38750A)}(50) = \frac{6V}{62}$$

$$eV = 2F_1 \times 75 + F_2 \times 150 - F_3 \times 150$$

$$e = \frac{1}{62}(300 + 2250 - 900) = 26.61 \text{ mm}$$

(left from ℄ of vertical member)

$\tau_B = 0$

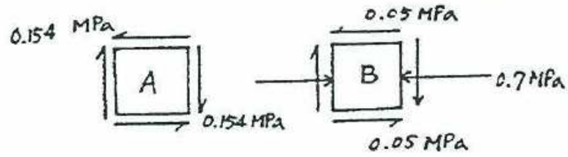
due to torsion

$A_m = (20-2)(30-2) = 504 \text{ mm}^2$

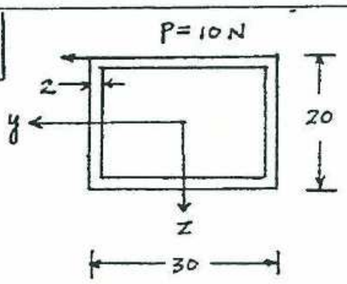
$\tau_A = \tau_B = \frac{T}{2A_m t} = \frac{100}{2 \times 504 \times 2} = 0.05 \text{ MPa}$

total $\tau_A = 0.05 + 0.104 = 0.154$ MPa

$\tau_B = 0.05$



10-49



$V = 10$ N, $M = 10 \times 100 = 1000$ N·mm

$T = 10 \times 10 = 100$ N·mm

$I_z = \frac{1}{12}(20 \times 30^3 - 16 \times 26^3) = 21{,}565 \text{ in}^4$

$I_y = \frac{1}{12}(30 \times 20^3 - 26 \times 16^3) = 11{,}125 \text{ in}^4$

$J = I_z + I_y = 32{,}690 \text{ in}^4$

due to bending

$\sigma_A = \frac{My}{I_z} = 0$

$\sigma_B = \frac{My}{I_z} = \frac{1000 \times 15}{21565} = 0.70$ MPa

due to shear

$\tau_A = \frac{VQ}{I_z t} = \frac{10(20 \times 15 \times 7.5 - 16 \times 13 \times 6.5)}{21565 \times 4}$

$= 0.104$

10-50

$M = 20 \times 100 = 2000$ N·mm

$T = 20 \times 13 = 260$ N·mm

$V = 20$ N

$I = \frac{\pi}{4}(13^4 - 9^4) = 16500 \text{ mm}^4$

$J = 2I = 33000 \text{ mm}^4$

due to bending

$\sigma_A = \frac{My}{I} = 0$

$\sigma_B = \frac{2000 \times 13}{16500} = 1.58$ MPa

due to torsion

$\tau_A = \tau_B = \frac{T}{2A_m t} = \frac{260}{2 \times \pi \times 11^2 \times 4}$

$= 0.09$ MPa

due to shear

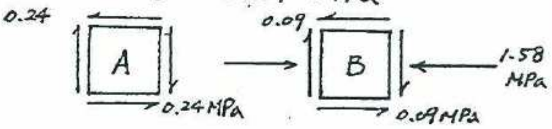
$\tau_B = 0,$

$\tau_A = \frac{VQ}{It} = \frac{20(\pi \times 13^2 \times \frac{2 \times 13}{3\pi} - \pi \times 9^2 \times \frac{2 \times 9}{3\pi})}{16500 \times 8}$

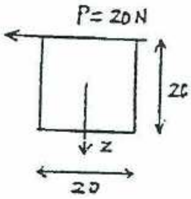
$= 0.15$ MPa

total $\tau_A = 0.09 + 0.15 = 0.24$ MPa

$\tau_B = 0.09$ MPa



10-51



$M = 20 \times 100 = 2000$ N·mm

$V = 20$ N

$T = 20 \times 10 = 200$ N·mm

$I_z = \frac{1}{12} 20^4 = 13333$ mm^4

$\alpha = 0.208$ from Eq. 4-30 & Table

due to bending

$\sigma_A = 0$

$$\sigma_B = \frac{2000 \times 10}{13333} = 1.5 \text{ MPa}$$

due to torsion

$$\tau_A = \tau_B = \frac{T}{\alpha a^3} = \frac{200}{0.208 \times 20^3} = 0.12 \text{ MPa}$$

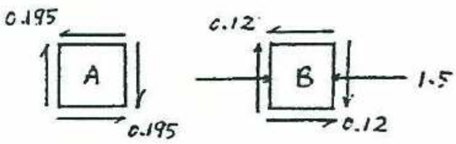
due to shear

$$\tau_A = \frac{VQ}{It} = \frac{20 \times 10 \times 20 \times 5}{13333 \times 20} = 0.075 \text{ MPa}$$

$\tau_B = 0$

total $\tau_A = 0.12 + 0.075 = 0.195$ MPa

$\tau_B = 0.12$ MPa



10-52

$$m = \frac{2\bar{r}}{d} = \frac{2\left[\frac{1}{2}(2) - \frac{1}{2}(\frac{1}{4})\right]}{\frac{1}{4}} = 7 \rightarrow K = 1.20$$

$$\Delta = \frac{64 F \bar{r}^3 N}{G d^4} = \frac{64(50)(.875)^3(8)}{11.6 \times 10^6 (.25)^4}$$

$= .378$ in.

$$\tau_{max} = K \frac{1.6 F \bar{r}}{\pi d^3} = \frac{1.2(16)(70)(.875)}{\pi (.25)^3}$$

$= 24{,}000$ psi

10-53

$$K_1 = \frac{G d^4}{64 \bar{r}^3 N} = \frac{(82 \times 10^6)(0.006)^4}{64(0.012)^3(16)} = 60$$

$$K_2 = \frac{(82 \times 10^6)(0.008)^4}{64(0.016)^3(18)} = 71$$

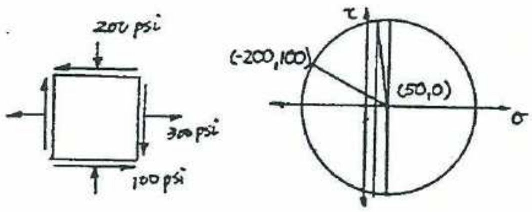
$$K = \frac{K_1 K_2}{K_1 + K_2} = \frac{60 \times 71}{60 + 71} = 32.52 \text{ kN/m}$$

$m_1 = 2\bar{r}_1 / d_1 = 2(12)/6 = 4 \rightarrow K = 1.37$

$m_2 = 2\bar{r}_2 / d_2 = 2(16)/8 = 4 \rightarrow K = 1.37$

$$\therefore F_{max} = \frac{\tau_{max} \pi d^3}{16 K \bar{r}} = \frac{(480 \times 10^3)\pi(0.006)^3}{16(1.37)(0.012)} = 1.24 \text{ kN}$$

10-54



$\text{center} = \frac{1}{2}(300 - 200) = 50$

$\text{radius} = \sqrt{(300-50)^2 + 100^2} = 269$

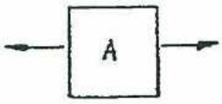
$\theta = \tan^{-1}\frac{100}{200} + 2(30) = 86.6°$

$\tau' = 269 \sin 86.6° = 269$ psi

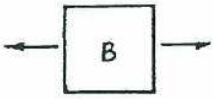
$\tau_{all} = 150$ psi < 269 psi

∴ not permissible

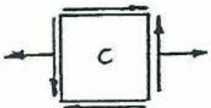
11-1.



$\sigma_A = \frac{P_1}{A}$

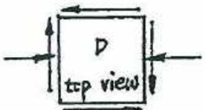


$\sigma_B = \frac{P_1}{A} + \frac{My}{I}$



$\sigma_C = \frac{P_1}{A}$

$\tau_C = \frac{VQ}{It}$

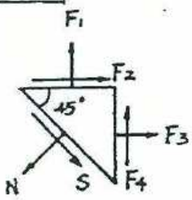


$\sigma_D = \frac{My}{I}$

$\tau_D = \frac{Tr}{J}$

$\tau_E = \frac{Tr}{J} - \frac{P_1 Q}{It}$

11-2



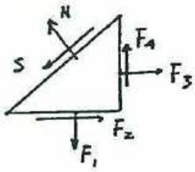
$F_1 = 30 \times \frac{1}{\sqrt{2}},\ F_2 = \frac{20}{\sqrt{2}}$

$F_3 = \frac{20}{\sqrt{2}},\ F_4 = \frac{20}{\sqrt{2}}$

$N = \frac{1}{\sqrt{2}}\left(\frac{30}{\sqrt{2}} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}}\right) = 45$

$S = \frac{1}{\sqrt{2}}\left(\frac{30}{\sqrt{2}} + \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}}\right) = 5$

$\sigma = 45$ MPa , $\tau = 5$ MPa



$F_1 = \frac{30}{\sqrt{2}}$

$F_2 = \frac{20}{\sqrt{2}}$

$F_3 = \frac{20}{\sqrt{2}}$

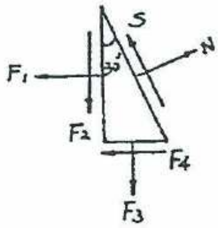
$F_4 = \frac{20}{\sqrt{2}}$

$N = \frac{1}{\sqrt{2}}\left(\frac{30}{\sqrt{2}} - \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}}\right) = 5$

$S = \frac{1}{\sqrt{2}}\left(-\frac{30}{\sqrt{2}} - \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}}\right) = -5$

$\sigma = 5$ MPa , $\tau = -5$ MPa

11-3



$F_1 = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$

$F_2 = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$

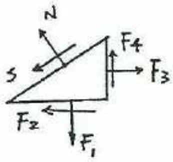
$F_3 = \frac{1}{2} \times 30 = 15$

$F_4 = \frac{1}{2} \times 20 = 10$

$N = \frac{\sqrt{3}}{2}(10\sqrt{3}+10) + \frac{1}{2}(10\sqrt{3}+15) = 39.8$

$S = \frac{\sqrt{3}}{2}(10\sqrt{3}+15) - \frac{1}{2}(10\sqrt{3}+10) = 14.3$

$\sigma = 39.8$ MPa , $\tau = 14.3$ MPa



$F_1 = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3}$

$F_2 = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

$F_3 = 20 \times \frac{1}{2} = 10$

$F_4 = 20 \times \frac{1}{2} = 10$

$N = \frac{\sqrt{3}}{2}(15\sqrt{3}-10) + \frac{1}{2}(-10\sqrt{3}+10)$

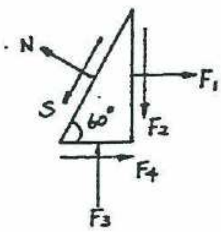
$= 10.18$

$S = \frac{1}{2}(-15\sqrt{3}+10) + \frac{\sqrt{3}}{2}(-10\sqrt{3}+10)$

$= -14.33$

$\sigma = 10.18$ MPa . $\tau = -14.33$ MPa

11-4



$F_1 = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$

$F_2 = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$

$F_3 = \frac{1}{2} \times 10 = 5$

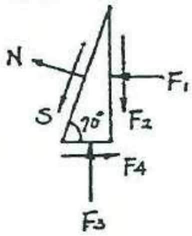
$F_4 = \frac{1}{2} \times 12 = 6$

$N = \frac{\sqrt{3}}{2}(10\sqrt{3}+6) + \frac{1}{2}(6\sqrt{3}-5) = 22.9$

$S = -\frac{1}{2}(10\sqrt{3}+6) + \frac{\sqrt{3}}{2}(5-6\sqrt{3}) = 6.99$

$\sigma = 22.9$ ksi , $\tau = 6.99$ ksi

11-5



$F_1 = 10 \times 0.940 = 9.40$

$F_2 = 6 \times 0.940 = 5.64$

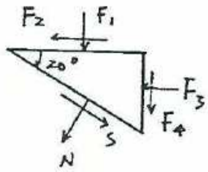
$F_3 = 8 \times 0.342 = 2.74$

$F_4 = 6 \times 0.342 = 2.05$

$N = 0.94(-9.4+2.05)+0.342(5.64-2.74)$
$= -5.92$

$S = 0.342(-9.4+2.05)+0.94(-5.64+2.74)$
$= -5.24$

$\sigma_y = -5.92$ ksi , $\tau = -5.24$ ksi



$F_1 = 8 \times 0.940 = 7.52$

$F_2 = 6 \times 0.940 = 5.64$

$F_3 = 10 \times 0.342 = 3.42$

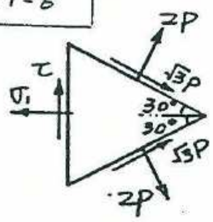
$F_4 = 6 \times 0.342 = 2.052$

$N = 0.94(-7.52-2.052)+0.342(-5.64-3.42)$
$= -12.09$

$S = 0.342(-7.52-2.052)+0.94(5.64+3.42)$
$= 5.24$

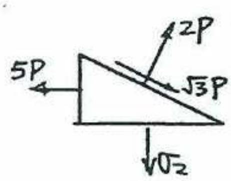
$\sigma = -12.09$ ksi , $\tau = 5.24$ ksi

11-6



$\sigma_1 = 2 \times 2P \times 0.5 + 2 \times \sqrt{3}P \times \frac{1}{\sqrt{3}}$
$= 5P$

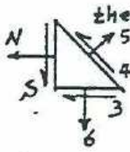
$\tau = 0$



$\sigma_2 = 2P - \sqrt{3}P \cdot \frac{1}{\sqrt{3}}$
$= P$

11-7

assume area of vertical plane $A_v = 1$
then area of inclined plane $A_i = \sqrt{2}$



$N = (5/\sqrt{2} - 4/\sqrt{2}) \times \sqrt{2} - 3 = -2$

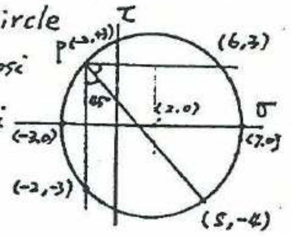
$S = (4/\sqrt{2} + 5/\sqrt{2}) \times \sqrt{2} - 6 = 3$

$\sigma_V = -2/1 = -2$ psi, $\tau_V = -3$ psi

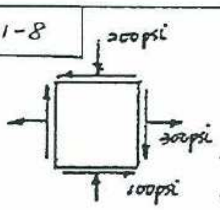
check by Mohr's circle

$\sigma_V = -2$ psi , $\sigma_H = -3$ psi

$\sigma_1 = 7$ psi , $\sigma_2 = -3$ psi



11-8



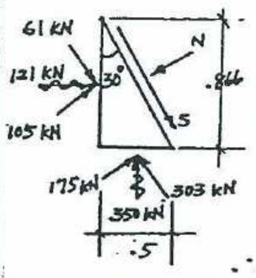
$\theta = -60°$

$\tau' = -\frac{300-(-200)}{2} \sin(-120°)$
$+(-100)\cos(-120°)$
$= 266.5$ psi

$\tau_{allowable} = 150$ psi < 266.5 psi

not permissible.

11-9



$N = 105 + 175 = 280$ kN

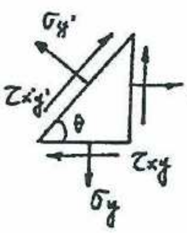
$S = 303 - 61 = 242$ kN

$\therefore \tau = 242$ kN/m²

$\tau_{all} = 280(0.5)+85$
$= 225$ kN/m²
$< \tau$

$\therefore$ state of stress NOT permissible.

11-10



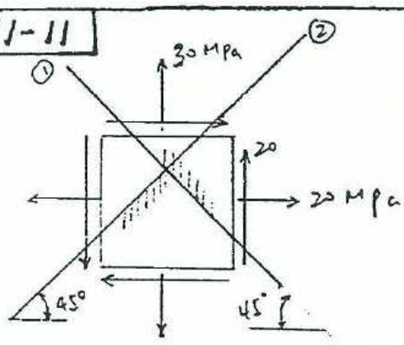
$\Sigma F_{x'} = 0$

$\tau_{x'y'} = -(\sigma_x \cos\theta + \tau_{xy}\sin\theta)\sin\theta + (\sigma_y \sin\theta + \tau_{xy}\cos\theta)\cos\theta$

$= -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

0 0

11-11



For plane ① $\theta = -135°$

$$\sigma_{x'} = \frac{30+20}{2} + \frac{20-30}{2}\cos(-270°) + 20\sin(-270°)$$
$$= 45\ \text{MPa}$$

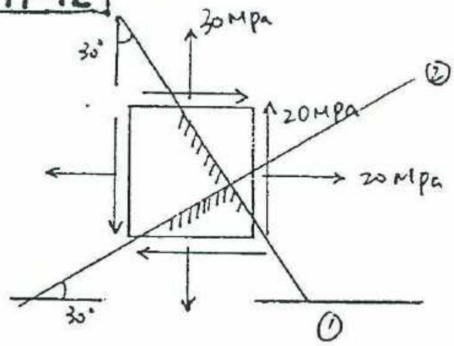
$$\tau_{x'y'} = -\frac{20-30}{2}\sin(-270°) + 20\cos(-270°)$$
$$= 5\ \text{MPa}$$

For plane ②, $\theta = 135°$

$$\sigma_{x'} = \frac{30+20}{2} + \frac{20-30}{2}\cos(270°) + 20\sin(270°)$$
$$= 5\ \text{MPa}$$

$$\tau_{x'y'} = -\frac{20-30}{2}\sin(270°) + 20\cos(270°)$$
$$= -5\ \text{MPa}$$

11-12



For plane ①, $\theta = 30°$

$$\sigma_{x'} = \frac{30+20}{2} + \frac{20-30}{2}\cos 60° + 20\sin 60°$$
$$= 39.8\ \text{MPa}$$

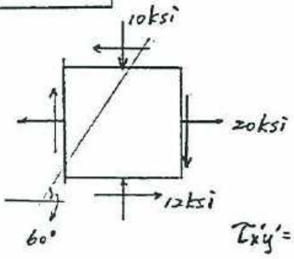
$$\tau_{x'y'} = -\frac{20-30}{2}\sin 60° + 20\cos 60°$$
$$= 14.3\ \text{MPa}$$

For plane ②, $\theta = 120°$

$$\sigma_{x'} = \frac{20+30}{2} + \frac{20-30}{2}\cos 240° + 20\sin 240°$$
$$= 10.18\ \text{MPa}$$

$$\tau_{x'y'} = -\frac{20-30}{2}\sin 240° + 20\cos 240°$$
$$= -14.33\ \text{MPa}$$

11-13



$\theta = 150°$.

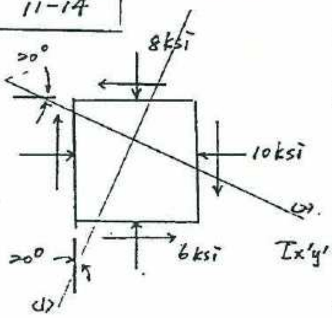
$$\sigma_{x'} = \frac{20-10}{2} + \frac{20-(-10)}{2}\cos 300° - 12\sin 300°$$

$$= 22.89 \text{ ksi}$$

$$\tau_{x'y'} = -\frac{20-(-10)}{2}\sin 300° - 12\cos 300°$$

$$= 6.99 \text{ ksi}$$

11-14



For plane (1), $\theta = 160°$

$$\sigma_{x'} = \frac{-10-8}{2} + \frac{(-10)-(-8)}{2}\cos 320° - 6\sin 320°$$

$$= -5.91 \text{ ksi}$$

$$\tau_{x'y'} = -\frac{(-10)-(-8)}{2}\sin 320° - 6\cos 320°$$

$$= -5.24 \text{ ksi}$$

For plane (2), $\theta = -110°$.

$$\sigma_{x'} = \frac{-10-8}{2} + \frac{(-10)-(-8)}{2}\cos(-220°) - 6\sin(-220°)$$

$$= -12.09 \text{ ksi}$$

$$\tau_{x'y'} = -\frac{(-10)-(-8)}{2}\sin(-220°) - 6\cos(-220°)$$

$$= 5.24 \text{ ksi}$$

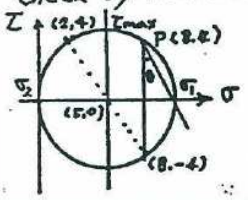
11-15

Using Eq. 11-7 and 11-8.

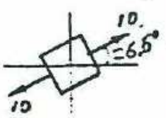
$$\sigma_1 = \frac{8+2}{2} + \sqrt{\left(\frac{8-2}{2}\right)^2 + 4^2} = 10 \text{ ksi}$$

$$\sigma_2 = 5-5 = 0, \quad \theta = \frac{1}{2}\tan^{-1}\frac{4}{(8-2)/2} = 26.6°$$

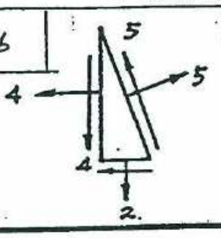
check by Mohr's circle



$\sigma_1 = 10$ ksi
$\sigma_2 = 0$
$\theta = 26.6°$
$\tau_{max} = 5$ ksi
$\sigma = 5$ ksi



11-16



$$\tau_{max} = \left[\left(\frac{8-2}{2}\right)^2 + 4^2\right]^{1/2}$$

$$= 5 \text{ ksi}$$

$$\sigma = (8+2)/2$$

$$= 5 \text{ ksi}$$

11-17

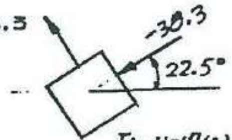


Fig. 11-17(a)

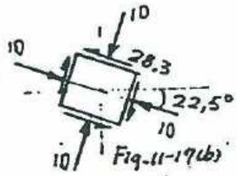


Fig. 11-17(b)

(a) $\sigma_1 = \frac{-20}{2} + \left[\left(\frac{40}{2}\right)^2 + 20^2\right]^{\frac{1}{2}} = 18.3$ ksi

$\sigma_2 = -10 - 28.3 = -38.3$ ksi

(b) $\tau_{max} = 28.3$ ksi, $\sigma = -10$ ksi

(c) $\sigma_1 + \sigma_2 = -20$ ksi

$= \sigma_x + \sigma_y$ o.k.

11-18

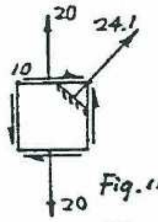


Fig. 11-18(a)

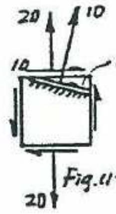


Fig. 11-18(b)

(a) $\sigma_1 = \frac{20}{2} + (10^2 + 10^2)^{\frac{1}{2}} = 24.1$ ksi

$\sigma_2 = 10 - 14.1 = -4.1$ ksi

(b) $\tau_{max} = 14.1$ ksi, $\sigma = 10$ ksi

(c) $\sigma_1 + \sigma_2 = 20$ ksi $= \sigma_x + \sigma_y$

11-19

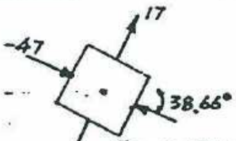


Fig. 11-19(a)

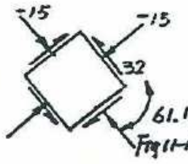


Fig. 11-19(b)

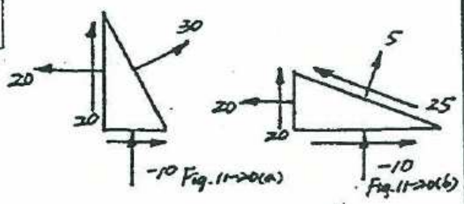
(a) $\sigma_1 = -15 + (25^2 + 20^2)^{\frac{1}{2}} = 17$ MPa

$\sigma_2 = -15 - 32.0 = -47.0$ MPa

(b) $\tau_{max} = 32.0$ MPa, $\sigma = -15$ MPa

(c) $\sigma_1 + \sigma_2 = -30 = \sigma_x + \sigma_y$

11-20



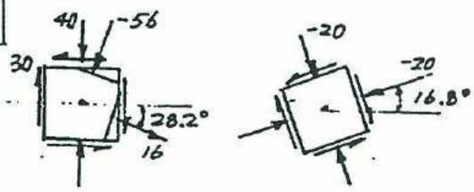
(a) $\sigma_1 = 5 + (15^2 + 20^2)^{\frac{1}{2}} = 30$ MPa

$\sigma_2 = 5 - 25 = -20$ MPa

(b) $\tau_{max} = 25$ MPa , $\sigma = 5$ MPa

(c) $\sigma_1 + \sigma_2 = 10 = \sigma_x + \sigma_y$

11-21



(a) $\sigma_1 = -20 + (20^2 + 30^2)^{\frac{1}{2}}$

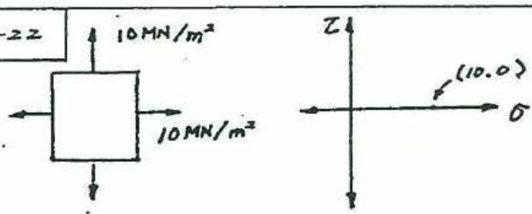
$= 16.1$ ksi

$\sigma_2 = -20 - 36.1 = -56.1$ ksi

(b) $\tau_{max} = 36.1$ ksi , $\sigma = -20$ ksi

(c) $\sigma_1 + \sigma_2 = -40$ ksi $= \sigma_x + \sigma_y$

11-22

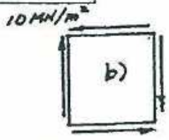


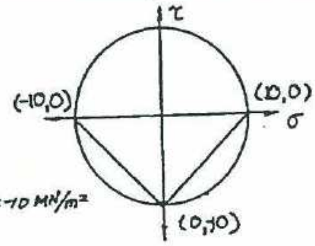
(a) Already in principal state

Mohr's circle is a single point.

(b) No shear.

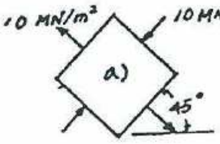
11-23





a) $\sigma_1 = 10$ MN/m², $\sigma_2 = -10$ MN/m²

$\theta = 45°$



check

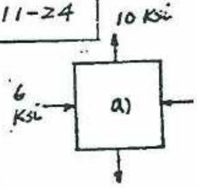
$\sigma_1 + \sigma_2 = 0 = \sigma_x + \sigma_y$

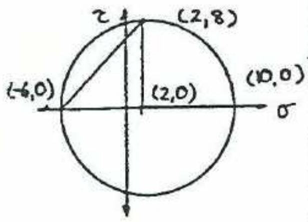
o.k.

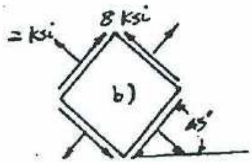
b) already in state of principal shear.

$\tau = 10$ MN/m²

11-24







a) already in principal state

$\sigma_1 = 10$, $\sigma_2 = -6$

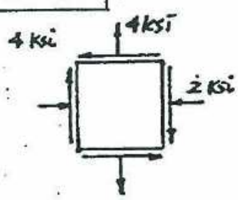
b) center $= \frac{10-6}{2} = 2$

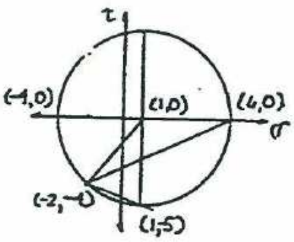
radius $= 10 - 2 = 8$

$\theta_2 = \tan^{-1}\frac{8}{8} = 45°$

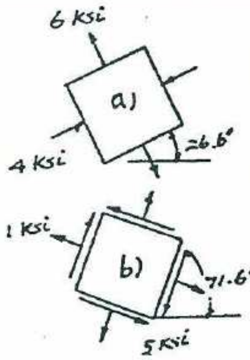
$\tau = 8$ ksi

11-25





0 0



a) center $= \frac{4-2}{2} = 1$

radius $= \sqrt{(1+2)^2+(-4)^2}$

$= 5$

$\theta_1 = \tan^{-1}\frac{4}{8} = 26.6°$

$\sigma_1 = 1+5 = 6$ ksi .

$\sigma_2 = 1-5 = -4$ ksi

b) $\tau = 5$ ksi

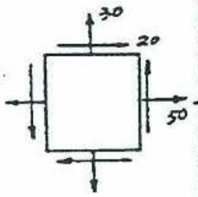
$\theta_2 = \tan^{-1}\frac{3}{1} = 71.6°$

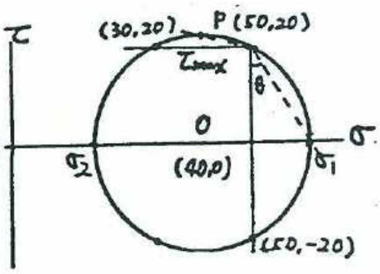
$\sigma = 1$ ksi

check $\sigma_1 + \sigma_2 = 2 = \sigma_x + \sigma_y$

o.K.

11-26





(a) radius $= (10^2+20^2)^{\frac{1}{2}} = 10\sqrt{5}$

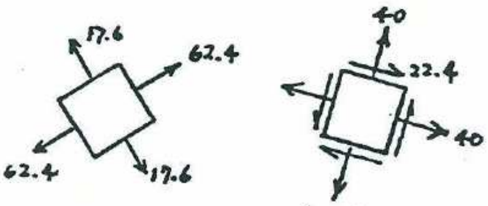
$\sigma_1 = 40 + 10\sqrt{5} = 62.4$ MPa

$\sigma_2 = 40 - 10\sqrt{5} = 17.6$ MPa

$\sigma_1 + \sigma_2 = 80 = \sigma_x + \sigma_y$ o.k.

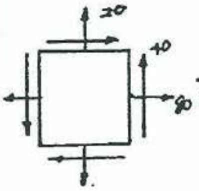
$\theta = \tan^{-1}\left(\frac{10\sqrt{5}-10}{20}\right) = 31.7°$

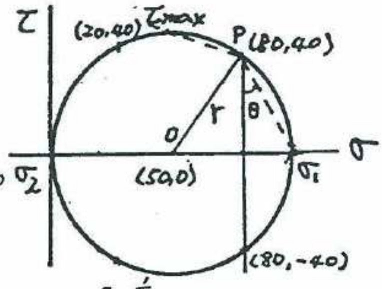
(b) $\tau_{max} = 10\sqrt{5} = 22.4$, $\sigma = 40$ MPa



(a) (b)

11-27





(a) $r = (30^2 + 40^2)^{\frac{1}{2}} = 50$

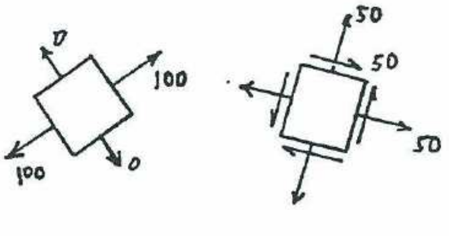
$\sigma_1 = 50+50 = 100$ psi

$\sigma_2 = 50-50 = 0$ psi

$\sigma_1 + \sigma_2 = 100 = \sigma_x + \sigma_y$ O.K.

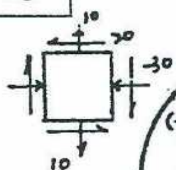
$\theta = \tan^{-1}\left(\frac{20}{40}\right) = 26.6°$

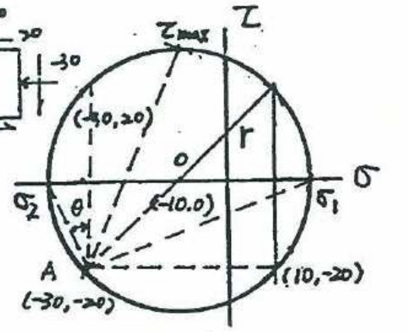
(b) $\tau_{max} = 50$ psi $\sigma = 50$ psi



(a) (b)

11-28





(a) $r = (20^2+20^2)^{\frac{1}{2}} = 20\sqrt{2}$

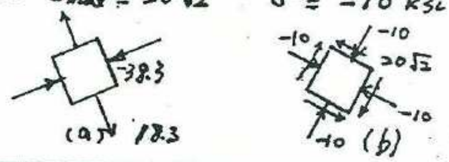
$\sigma_1 = -10 + 20\sqrt{2} = 18.3$ ksi

$\sigma_2 = -10 - 20\sqrt{2} = -38.3$ ksi

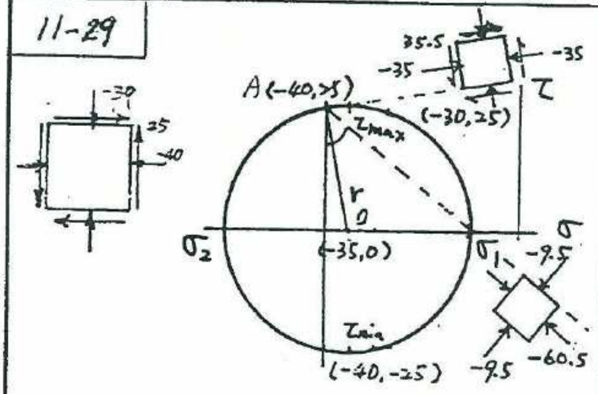
$\sigma_1 + \sigma_2 = -20 = \sigma_x + \sigma_y$ o.K.

$\theta = \tan^{-1}\left(\frac{20\sqrt{2}-20}{20}\right) = 22.5°$

(b) $\tau_{max} = 20\sqrt{2}$ $\sigma = -10$ ksi



11-29



(a) $r = (5^2 + 25^2)^{\frac{1}{2}} = 25.5$ MPa

$\sigma_1 = -35 + 25.5 = -9.5$ MPa

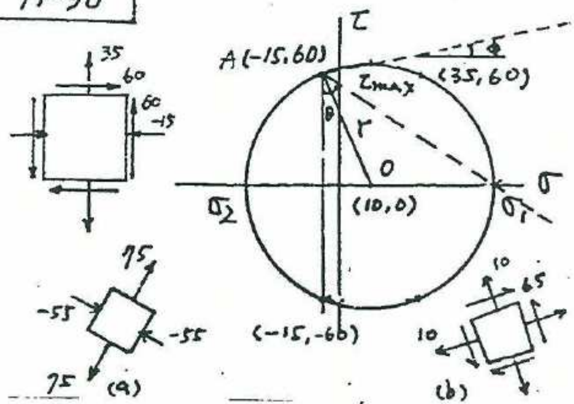
$\sigma_2 = -35 - 25.5 = -60.5$ MPa

$\sigma_1 + \sigma_2 = -70 = \sigma_x + \sigma_y$ o.K.

$\theta = \tan^{-1}\left(\frac{30.5}{25}\right) = 50.7°$

(b) $\tau_{max} = 25.5$ MPa, $\sigma = -35$ MPa

11-30



(a) $r = (25^2 + 60^2)^{\frac{1}{2}} = 65$

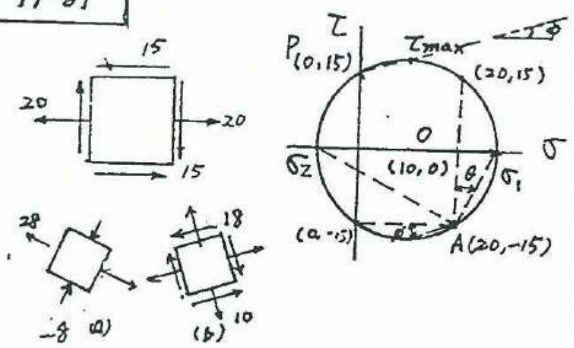
$\sigma_1 = 75$ MPa, $\sigma_2 = -55$ MPa

$\sigma_1 + \sigma_2 = 20 = \sigma_x + \sigma_y$ o.K.

(b) $\phi = \tan^{-1}\left(\frac{5}{25}\right) = 11.3°$

$\tau_{max} = 65$ MPa, $\sigma = 10$ MPa

11-31



(a) $r = (10^2 + 15^2)^{\frac{1}{2}} = 18.0$

$\sigma_1 = 10 + 18 = 28.0$ Ksi

$\sigma_2 = 10 - 18 = -8.0$ Ksi

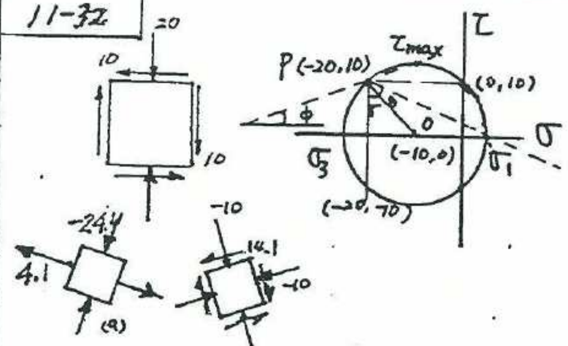
$\sigma_1 + \sigma_2 = 20 = \sigma_x + \sigma_y$ o.K.

$\theta = \tan^{-1}\left(\frac{8}{15}\right) = 28.07°$

(b) $\phi = \tan^{-1}\left(\frac{3}{10}\right) = 16.7°$

$\tau_{max} = 18.0$ Ksi $\sigma = 10$ Ksi

11-32



(a) $r = (10^2 + 10^2)^{\frac{1}{2}} = 14.1$

$\sigma_1 = -10 + 14.1 = 4.1$ Ksi

$\sigma_2 = -10 - 14.1 = -24.1$ Ksi

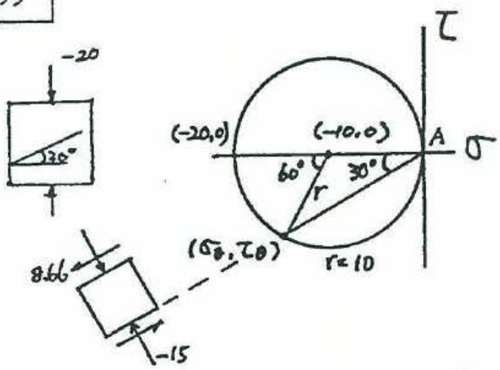
$\sigma_1 + \sigma_2 = -20$ Ksi $= \sigma_x + \sigma_y$ o.K.

$\theta = \tan^{-1}\left(\frac{4.1}{10}\right) = 22.3°$

(b) $\phi = \tan^{-1}\left(\frac{4.1}{10}\right) = 22.3°$

$\tau_{max} = 14.1$ Ksi, $\sigma = -10$ Ksi

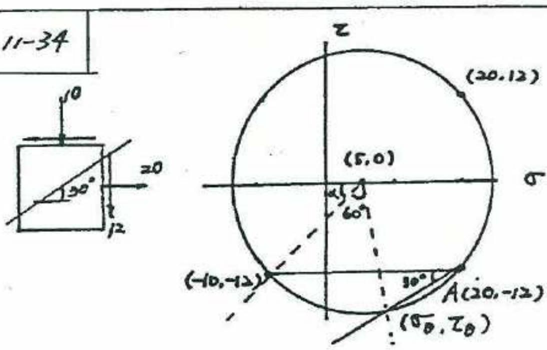
11-33



$\sigma_\theta = -10 - 10 \cdot \cos 60^\circ = -15$ Ksi

$\tau_\theta = -10 \sin 60^\circ = 8.66$ Ksi

11-34



$r = (15^2 + 12^2)^{\frac{1}{2}} = 19.2$

$\alpha = \tan^{-1}\left(\frac{12}{15}\right) = 38.7^\circ$

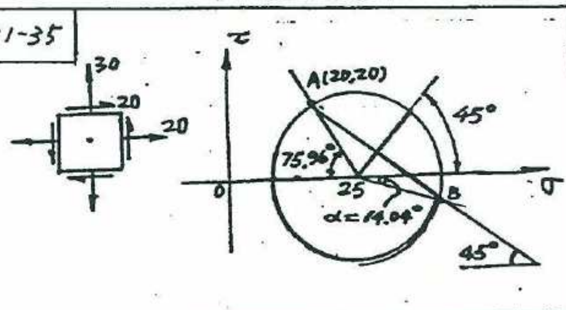
$\sigma_\theta = 5 + 19.2 \cos(180^\circ - 38.7^\circ - 60^\circ)$

$= 7.9$ Ksi

$\tau_\theta = -19.2 \sin(180^\circ - 38.7^\circ - 60^\circ)$

$= -19.0$ Ksi

11-35



$R = \sqrt{20^2 + 5^2} = 20.62$

$\sigma_x = 25 + 20.62 \cos\alpha = 45$ MPa

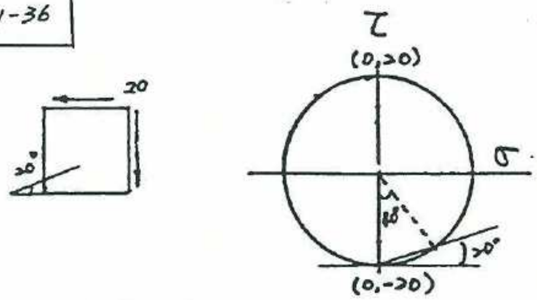
$\tau = -20.62 \sin\alpha = -5.00$ MPa

here, $\tan\frac{20}{5} = 75.96^\circ$

$\alpha = [180 - (75.96 + 45)] - 45$

$= 14.04^\circ$

11-36

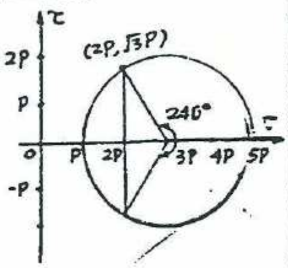


$r = 20$

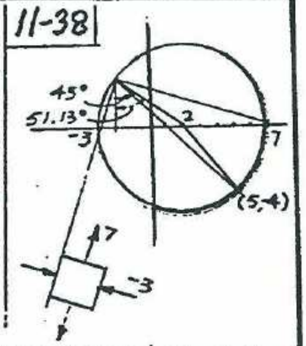
$\sigma_\theta = +20 \sin 40^\circ = +12.9$ Ksi

$\tau_\theta = -20 \cos 40^\circ = -15.3$ Ksi

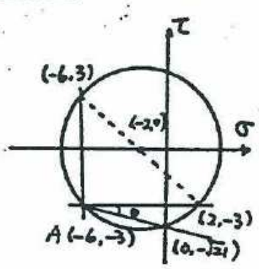
11-37



11-38



11-39



$r = (4^2 + 3^2)^{1/2} = 5$

$\tau = -\sqrt{25 - 4}$

$= -\sqrt{21} = -4.58$

$\theta = \tan^{-1}\frac{\sqrt{21} - 3}{6}$

$= 14.78^\circ$

check by wedge method

assume $A_i = 1$, $A_V = \sin\theta$, $A_H = \cos\theta$

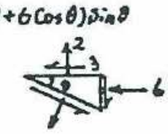
$N = (2\cos\theta - 3\sin\theta)\cos\theta$

$S = (2\sin\theta + 3\cos\theta)\cos\theta + (-3\sin\theta + 6\cos\theta)\sin\theta$

$= 3.29 + 1.29 = 4.58$

$\sigma_V = N/1.0 = 0$

$\tau_V = S/1.0 = 4.58$ ksi O.K.



b) $\theta' = 180 - 2(53.1) = 73.8°$

$\sigma_x = 3500 + 3500\cos 73.8°$

$= 4480$ KN/m²

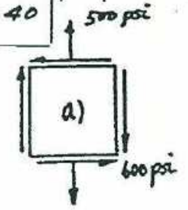
$\sigma_y = 3500 - 3500\cos 73.8°$

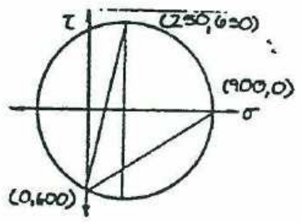
$= 2520$ KN/m²

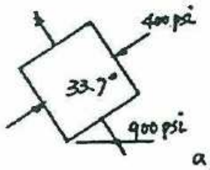
$\tau = -3500\sin 73.8°$

$= -3360$ KN/m²

11-40







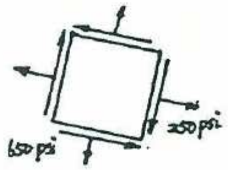
center $= \frac{1}{2}(500) = 250$ psi

radius $= \sqrt{250^2 + 600^2} = 650$

a) $\sigma_1 = 250 + 650 = 900$ psi

$\sigma_2 = 250 - 650 = -400$ psi

$\theta_1 = \tan^{-1}\frac{600}{900} = 33.7°$



b) $\tau = 650$ psi

$\sigma' = 250$ psi

$\theta_2 = \tan^{-1}\frac{600+650}{250}$

$= 78.70$

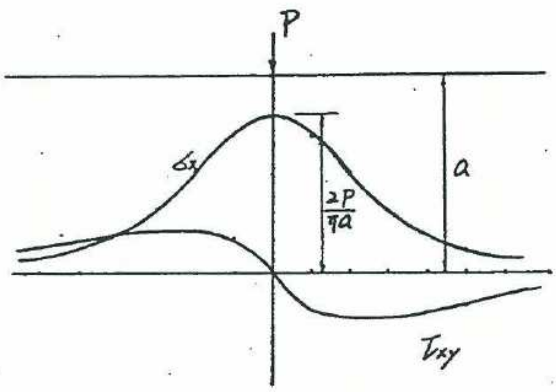
11-42

$\sigma_x = \sigma_r\cos^2\theta = -\frac{2P\cos^4\theta}{\pi a}$

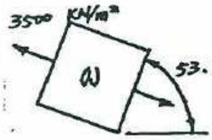
$\sigma_y = \sigma_r\sin^2\theta = -\frac{2P}{\pi a}\sin^2\theta\cos^2\theta$

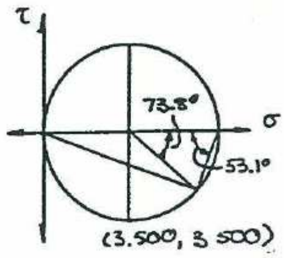
$\tau_{xy} = \sigma_r\sin\theta\cos\theta = -\frac{2P\sin\theta\cos^2\theta}{\pi r}$

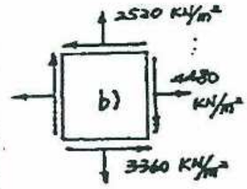
$\tau_{xy} = -\frac{2P}{\pi a}\sin\theta\cos^3\theta$



11-41







a) radius $= \tau_{max} = 3500$

$\sigma_1 = 7000$ KN/m²

$\sigma_2 = 0$

11-43

$\sigma_x = \sigma_r\cos^2\theta = \frac{-P\cos\theta}{r(\alpha + \frac{1}{2}\sin 2\alpha)}\cos^2\theta$

$\sigma_x = \frac{-P\cos^4\theta}{a(\alpha + \frac{1}{2}\sin 2\alpha)}$

(a) $\alpha = 10°$

$\sigma_{x\,max} = -\frac{P}{a(0.175 + \frac{1}{2}0.341)} = -\frac{P}{0.346a}$

$\sigma_{x\ (Eq\ 3\text{-}5)} = \frac{-P}{2a\tan 10°} = \frac{-P}{0.352a}$

(b) $\alpha = 45°$

$\sigma_{x\,max} = \frac{-P}{a(.787 + .5)} = \frac{-P}{1.287a}$

$\sigma_{x\ (Eq\ 3\text{-}5)} = \frac{-P}{2a\tan 45°} = \frac{-P}{2a}$

11-44

$$\sigma_x = \sigma_r \sin^2\theta = \frac{-P\cos\theta}{r(\alpha-\frac{1}{2}\sin 2\alpha)}\sin^2\theta$$

$$\sigma_x = \frac{-Py\sin^4\theta}{x^2(\alpha-\frac{1}{2}\sin 2\alpha)}$$

$$\tau_{xy} = \sigma_r \sin\theta\cos\theta = \frac{-P\cos^2\theta\sin\theta}{r(\alpha-\frac{1}{2}\sin 2\alpha)}$$

$$\tau_{xy} = \frac{-Py^2\sin^4\theta}{x^3(\alpha-\frac{1}{2}\sin 2\alpha)}$$

Elementary solution

$$\sigma_x = \frac{My}{I} = -\frac{Pxy}{I}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{-PQ}{I}$$

If $\alpha = 30°$ $\quad \alpha - \frac{1}{2}\sin 2\alpha = 0.09$

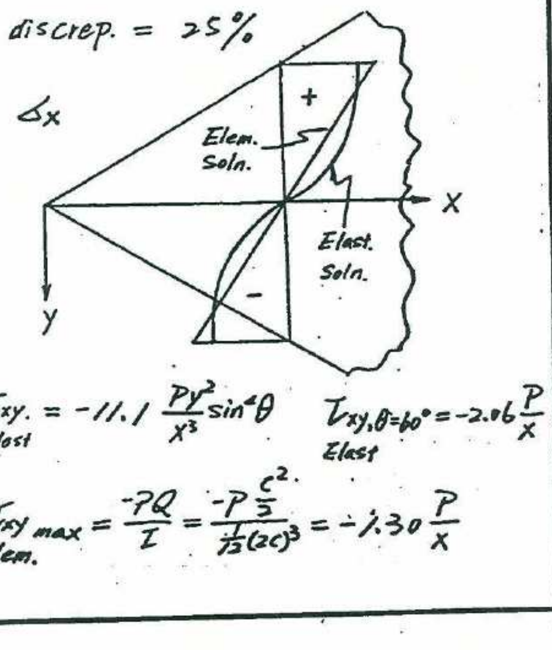
Elast. Soln. $\sigma_x = -11.1 \frac{Py}{x^2}\sin^4\theta$

$\sigma_{x,\theta=60°} = \sigma_{x\,max} = -3.61 \frac{P}{x}$

Elem Soln $\sigma_{x,max} = -\frac{Pxy}{\frac{1}{12}(2c)^3}$

$\sigma_{x\,max} = -\frac{3Pxc}{2c^3} = -4.5\frac{P}{x}$ $(c = x\tan 30°)$

discrep. = 25%



$\tau_{xy\,Elast} = -11.1\frac{Py^2}{x^3}\sin^4\theta$ $\quad \tau_{xy,\theta=60°\,Elast} = -2.06\frac{P}{x}$

$$\tau_{xy\,max\,Elem.} = \frac{-PQ}{I} = \frac{-P\frac{c^2}{2}}{\frac{1}{12}(2c)^3} = -1.30\frac{P}{x}$$

11-45 from eq. (11-23)

$(2-\sigma_n)(5-\sigma_n) - 4 = 0$

$\sigma_1 = 6, \quad \sigma_3 = 1, \quad \sigma_2 = 3$

for $n_i^{(1)}$, $n_1^{(1)} = 0$, let, $n_2^{(1)} = 1$

$(2-6)n_2^{(1)} + 2n_3^{(1)} = 0, \quad n_3^{(1)} = 2$

normalize $\Rightarrow n^{(1)} = (0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$

for $n_i^{(2)} \Rightarrow n^{(2)} = (1, 0, 0)$

for $n_i^{(3)}$. $n_1^{(3)} = 0$, Let $n_2^{(3)} = 1$

$(2-1)n_2^{(3)} + 2n_3^{(3)} = 0. \quad n_3^{(3)} = \frac{-1}{2}$

normalize $\Rightarrow n^{(3)} = (0, \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}})$

11-46 from eq. (11-23)

$\sigma_n^3 - 18\sigma_n^2 - 120\sigma_n + 1872 = 0$

(a) $I_\sigma = 18, \quad II_\sigma = -120, \quad III_\sigma = -1872$

(b) $\sigma_1 = 19.1646, \quad \sigma_2 = 9.3181$

$\sigma_3 = -10.4827$

(c) $(10 - 19.1646)n_1 + 4n_2 - 6n_3 = 0$

$4n_1 - (6 + 19.1646)n_2 + 8n_3 = 0$

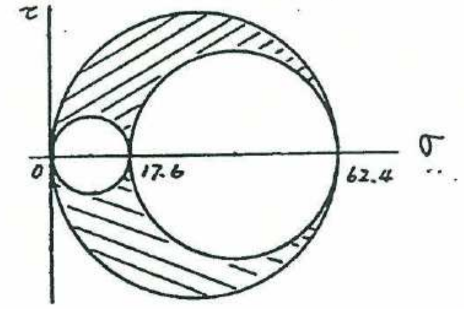
$n_1 = -0.55n_3, \quad n_2 = 0.23n_3$

$n_1^2 + n_2^2 + n_3^2 = 1, \quad n_3 = 0.86$

$n_1 = -0.47, \quad n_2 = 0.20$

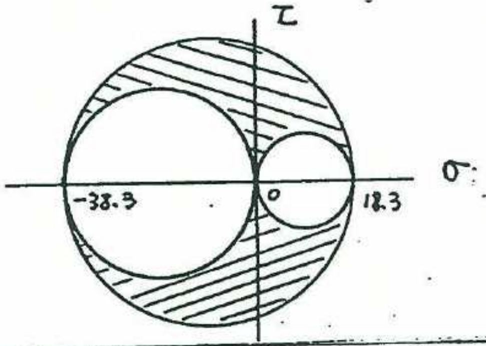
11-47 from solution of Prob. 11-26

$\sigma_1 = 62.4$, $\sigma_2 = 17.6$, $\sigma_3 = 0$

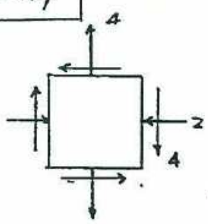


11-48 from solution of Prob. 11-28

$\sigma_1 = 18.3$, $\sigma_2 = 0$, $\sigma_3 = -38.3$



11-49



$\sigma_x = -2$, $\sigma_y = 4$, $\tau_{xy} = -4$

$\sigma_0 = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = 2/3$

$S_x = -\frac{8}{3}$, $S_y = \frac{10}{3}$, $S_z = -\frac{2}{3}$

$J_2 = -\frac{80}{9} - \frac{20}{9} + \frac{16}{9} - 16 = -\frac{228}{9}$

$J_3 = -\left(\frac{160}{27} + \frac{2}{3} \times 16\right) = -\frac{448}{27}$

$\theta = \frac{1}{3} \arccos\left[\frac{448/27}{2\left(\frac{228}{27}\right)^{3/2}}\right] = 24.31°$

$\sigma_1 = \frac{2}{3} + 2\left[\frac{228}{27}\right]^{1/2}(0.9176) = 6.0$

$\sigma_2 = \frac{2}{3} - 2\left[\frac{228}{27}\right]^{1/2}(0.11476) = 0$

$\sigma_3 = \frac{2}{3} - 2\left[\frac{228}{27}\right]^{1/2}(0.80292) = -4$

using eq. 11-33, 11-34. for σ_1

$(\sigma_x - \sigma_1)l_1 + \tau_{xy} m_1 = 0$

$0_x l_1 + 0_x m_1 + (0 - \sigma_1) n_1 = 0$

$l_1^2 + m_1^2 + n_1^2 = 1$

$\therefore l_1 = 1/\sqrt{5}$, $m_1 = -2/\sqrt{5}$, $n_1 = 0$

for σ_2 . $(\sigma_x - 0)l_2 + \tau_{xy} m_2 = 0$

$\tau_{xy} l_2 + (\sigma_y - 0) m_2 = 0$

$l_2^2 + m_2^2 + n_2^2 = 1$

$\therefore l_2 = m_2 = 0$. $n_2 = \pm 1$.

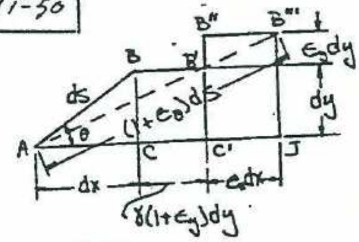
for σ_3 . $(\sigma_x - \sigma_3)l_3 + \tau_{xy} m_3 = 0$

$0_x l_3 + 0_x m_3 + (0 - \sigma_3) n_3 = 0$

$l_3^2 + m_3^2 + n_3^2 = 1$

$\therefore l_3 = 2/\sqrt{5}$, $m_3 = 1/\sqrt{5}$, $n_3 = 0$

11-50



BB' = shearing strain

B'B'' = strain in y-direction

B''B''' = strain in x-direction

$AB''' = (1 + \epsilon_0) ds$

$(AB''')^2 = [(1 + \epsilon_0) ds]^2$

$= (AJ)^2 + (JB)^2$

$= [\,(1+\epsilon_y) dy + (1 + \epsilon_x) dx]^2$

$+ [(1 + \epsilon_y) dy]^2$

$= [(1 + \epsilon_\theta) ds]^2 / ds^2$

$(1+\epsilon_\theta)^2 = [(1+\epsilon_x)\frac{dx}{ds} + (1+\epsilon_y)\frac{dy}{ds}]^2$

$+ [(1+\epsilon_y)\frac{dy}{ds}]^2$

but $\frac{dx}{ds} = \cos\theta$ and $\frac{dy}{ds} = \sin\theta$

$\therefore \epsilon_\theta = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + ..\sin\theta\cos\theta$

11-51 see Fig. 11-20

$$\beta \approx \tan\beta = \frac{-BB'\cos\theta + B''B'\sin\theta + B'''B''\cos\theta}{dy'}$$

$$= \frac{-\epsilon_x dx\cos\theta + \epsilon_y dy\sin\theta + \gamma_{xy} dy\cos\theta}{dy'}$$

but $\frac{dx}{dy'} = \sin\theta$ and $\frac{dy}{dy'} = \cos\theta$

$$\therefore \beta = -\epsilon_x \sin\theta\cos\theta + \epsilon_y \sin\theta\cos\theta + \gamma_{xy}\cos^2\theta$$

$$= -(\epsilon_x - \epsilon_y)\sin\theta\cos\theta + \gamma_{xy}\cos^2\theta$$

11-52 $\varepsilon_1 = \frac{1000}{2} + (620^2+100^2)^{\frac{1}{2}} = 1128\ \text{Mm/m}$

$\varepsilon_2 = \frac{1000}{2} - (620^2+100^2)^{\frac{1}{2}} = -128\ \text{Mm/m}$

$\tan 2\theta_1 = \frac{-200}{-1240}$, $\theta_1 = 4.58°$

11-53 $\varepsilon_1 = \frac{-1000}{2} + (300^2+400^2)^{\frac{1}{2}} = 0\ \text{Mm/m}$

$\varepsilon_2 = \frac{-1000}{2} - (300^2+400^2)^{\frac{1}{2}} = -1000\ \text{Mm/m}$

$\tan 2\theta_1 = \frac{800}{-600}$, $\theta_1 = -26.6°$

11-54 $\varepsilon_3 = -20\ \text{Mm/m}$

$(70-\lambda)(50-\lambda) - 30 = 0$

$\lambda = 60 \pm 20$

$\varepsilon_1 = 80\ \text{Mm/m}$, $\varepsilon_2 = 40\ \text{Mm/m}$

for ε_1, Let $n_1^{(1)} = 1$

$$\begin{bmatrix} -10 & -\sqrt{3} \\ -\sqrt{3} & -30 \end{bmatrix} \begin{Bmatrix} n_1^{(1)} \\ n_2^{(1)} \end{Bmatrix} = 0$$

$n_2^{(1)} = -\frac{1}{\sqrt{3}}$

normalize $n_1^{(1)} = \frac{\sqrt{3}}{2}$, $n_2^{(1)} = -\frac{1}{2}$

$n_3^{(1)} = 0$,

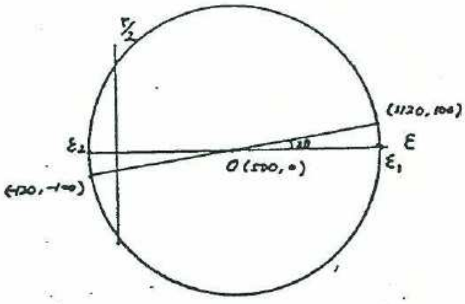
for ε_2, $n_1^{(2)} = -\sqrt{3}\, n_2^{(2)}$

$(n_1^{(2)})^2 + (n_2^{(2)})^2 = 1$, $n_3^{(2)} = 0$

$n_2^{(2)} = \frac{1}{2}$, $n_1^{(2)} = -\frac{\sqrt{3}}{2}$

for ε_3, $n^{(3)} = (0, 0, 1)$

11-55



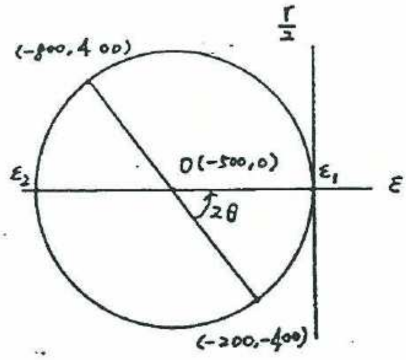
$r = [(1120+120)^2 + 200^2]^{\frac{1}{2}} \div 2 = 6$

$\varepsilon_1 = 500 + 628 = 1128\ \text{Mm/m}$

$\varepsilon_2 = 500 - 628 = -128\ \text{Mm/m}$

$\tan 2\theta = \frac{100}{1120-500}$, $\theta = 4.58°$

11-56



$r = [(-500+200)^2 + 400^2]^{\frac{1}{2}} = 500$

$\varepsilon_1 = -500 + 500 = 0\ \text{Mm/m}$

$\varepsilon_2 = -500 - 500 = -1000\ \text{Mm/m}$

$-\tan 2\theta = \frac{400}{300}$, $\theta = -26.6°$

11-57

$\gamma = 2(.00012) - (-.00022 + .00022)$

$= .00024$

$\epsilon_{1,2} = \dfrac{-.00022 + .00022}{2} \pm \sqrt{\left(\dfrac{-.00022 - .00022}{2}\right)^2 + \left(\dfrac{.00024}{2}\right)^2}$

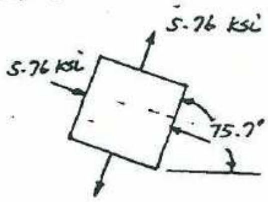
$= \pm .000251$

$\sigma_1 = \dfrac{30\times10^6}{1-.3^2}[.000251 - .3(.000251)]$

$= 5.76 \text{ ksi}$

$\sigma_2 = -5.76 \text{ ksi}$

$\theta_1 = \frac{1}{2}\tan^{-1}\dfrac{24}{-2(22)} = 75.7°$



11-58

$\epsilon_x = \epsilon_0 = .00040$

$\epsilon_y = \frac{1}{3}(2\epsilon_{60°} + 2\epsilon_{120°} - \epsilon_{0°})$

$= \frac{1}{3}(.00080 - .00120 - .00040) = -.000267$

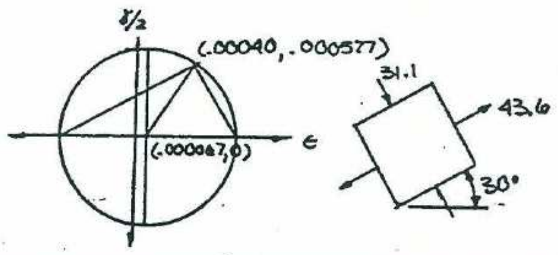
$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_{60°} - \epsilon_{120°}) = \frac{2}{\sqrt{3}}(.00040 + .00060)$

$= .001155$

$\text{center} = \dfrac{\epsilon_x + \epsilon_y}{2} = \dfrac{.00040 - .000267}{2}$

$= .000067$

$\dfrac{\gamma}{2} = .000577$



radius $= \sqrt{(.00040 - .000067)^2 + .000577^2}$

$= .000677$

$\epsilon_1 = .000067 + .000677 = .000734$

$\epsilon_2 = .000067 - .000677 = -.000600$

$\sigma_1 = \dfrac{70000}{1-.25^2}[.000734 - .25(.000600)]$

$= 43.6 \text{ MPa}$

$\sigma_2 = \dfrac{70000}{1-.25^2}[-.000600 + .25(.000734)]$

$= -31.1 \text{ MPa}$

11-59 using Eq. 11-50 and plug in

$\theta = 135°$

$\varepsilon_{135°} = \dfrac{\varepsilon_x}{2} + \dfrac{\varepsilon_y}{2} - \dfrac{\gamma_{xy}}{2}$

$\therefore \gamma_{xy} = \varepsilon_x + \varepsilon_y - 2\varepsilon_{135°}$

using $\varepsilon_{0°}, \varepsilon_{90°}, \varepsilon_{45°}$ to compute

$\gamma_{xy} = 2\varepsilon_{45°} - (\varepsilon_{0°} + \varepsilon_{90°})$

$= 2\times400 - (-120 + 1120)$

$= -200 \ \mu m/m$

using $\varepsilon_{0°}, \varepsilon_{90°}, \varepsilon_{135°}$ to compute

$\gamma_{xy} = \varepsilon_0 + \varepsilon_{90°} - 2\varepsilon_{135°}$

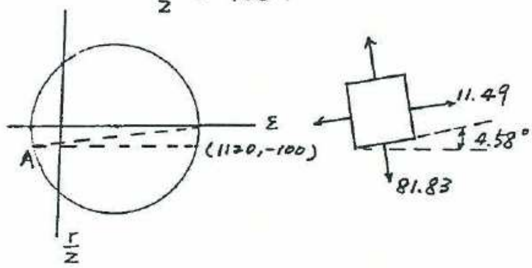
$= (-120 + 1120) - 2\times600$

$= -200 \ \mu m/m$

$\therefore$ consistent.

$\text{center} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 500 \ \mu m/m$

$\dfrac{\gamma_{xy}}{2} = -100$



radius $= \sqrt{(-120-500)^2 + (-100)^2} = 628$

$\varepsilon_1 = 500 + 628 = 1128\ \mu m/m$

$\varepsilon_2 = 500 - 628 = -128\ \mu m/m$

$\therefore\ \sigma_1 = \dfrac{70{,}000}{1-0.75^2}[1128 - 0.75\times 128]/10^6$

$= 81.83$ MPa.

$\sigma_2 = \dfrac{70{,}000}{1-0.75^2}[-128 + 0.75\times 1128]/10^6$

$= 11.49$ MPa

$\theta = \arctan\left(\dfrac{100}{2\times 628 - 8}\right) = 4.58°$

11-60

$\sigma_x + \sigma_y = 27.5$ MPa

$\therefore\ \varepsilon_x + \varepsilon_y = \dfrac{(\sigma_x+\sigma_y)(1-\nu)}{E} \times 10^6$

$= 103.125\ \mu m/m$

radius $= \dfrac{500}{2} = 250$

$\therefore\ \varepsilon_1 = \dfrac{\varepsilon_x + \varepsilon_y}{2} + 250 = 301.56$

$\varepsilon_2 = \dfrac{\varepsilon_x + \varepsilon_y}{2} - 250 = -198.43$

$\therefore\ \sigma_1 = \dfrac{200{,}000}{1-0.75^2}[301.56 - 0.75\times 198.43]/10^6$

$= 53.75$ MPa

$\sigma_2 = \dfrac{200{,}000}{1-0.75^2}[-198.43 + 0.75\times 301.56]/10^6$

$= -26.25$ MPa

$\sigma_1 + \sigma_2 = 27.5$ MPa. o.k.

12-1

$$\bar{\sigma} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{(10)+(-6)+(14)}{3} = 6$$

$$\begin{pmatrix} 10 & 4 & -6 \\ 4 & -6 & 8 \\ -6 & 8 & 14 \end{pmatrix} = \underbrace{\begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}}_{\text{spherical}} + \underbrace{\begin{pmatrix} 4 & 4 & -6 \\ 4 & -12 & 8 \\ -6 & 8 & 8 \end{pmatrix}}_{\text{deviatoric}}$$

12-2

$$\bar{\sigma} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{40+50+60}{3} = 50$$

$$\begin{pmatrix} 40 & -30 & 20 \\ -30 & 50 & 10 \\ 20 & 10 & 60 \end{pmatrix} = \underbrace{\begin{pmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{pmatrix}}_{\text{spherical}} + \underbrace{\begin{pmatrix} -10 & -30 & 20 \\ -30 & 0 & 10 \\ 20 & 10 & 10 \end{pmatrix}}_{\text{deviatoric}}$$

12-3

$$\bar{\sigma} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \frac{50+70+60}{3} = 60$$

$$\begin{pmatrix} 50 & -30 & 0 \\ -30 & 70 & -20 \\ 0 & -20 & 60 \end{pmatrix} = \underbrace{\begin{pmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{pmatrix}}_{\text{spherical}} + \underbrace{\begin{pmatrix} -10 & -30 & 0 \\ -30 & 10 & -20 \\ 0 & -20 & 0 \end{pmatrix}}_{\text{deviatoric}}$$

12-4

(a) $\sigma_y = 300$ MPa , $\sigma_x = \tau_{xy} = 0$

The three criteria have the same σ_{YP}

(b) $\sigma_x = \sigma_y = 0$, $\tau_{xy} = -150$ MPa

$\Rightarrow \sigma_1 = 150$ MPa $\quad \sigma_2 = -150$ MPa

maximum normal stress $\quad \sigma_{YP} = 150$ MPa

distortion - energy $\quad \sigma_{YP}/\sqrt{3} = 150$ MPa

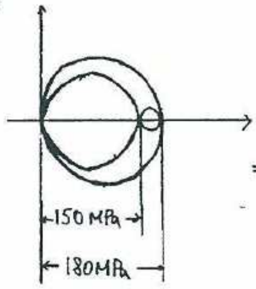
$\sigma_{YP} = 259.8$ MPa

maximum shear stress $\quad \sigma_{YP}/2 = 150$ MPa

$\sigma_{YP} = 300$ MPa

$\Rightarrow$ maximum shear stress theory control!

(c)



$\sigma_x = 150$, $\sigma_y = 180$

$\tau_{xy} = 0$

$\Rightarrow \sigma_1 = 180$, $\sigma_2 = 150$

$\sigma_3 = 0$

maximum normal stress $\quad \sigma_{YP} = 180$ MPa

distortion - energy $\quad (30)^2+(180)^2+(150)^2 = 2\sigma_{YP}^2$

$\sigma_{YP} = 167$ MPa

maximum shear stress $\quad \sigma_{YP} = 90$ MPa

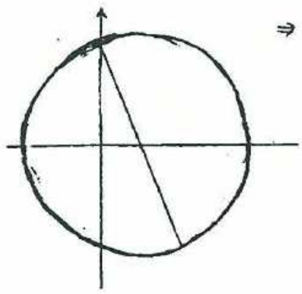
$\Rightarrow$ maximum normal stress theory control!

12-5

(a) $\sigma_x = -150$ MPa , $\sigma_y = \tau_{xy} = 0$

The three criteria have the same σ_{YP}

(b) $\sigma_x = 0$, $\sigma_y = 60$ MPa , $\tau_{xy} = 75$ MPa



$\Rightarrow \sigma_1 = 110.78$ MPa

$\sigma_2 = 0$ MPa

$\sigma_3 = -50.78$ MPa

maximum normal stress $\sigma_{YP} = 110.78$ MPa

distortion - energy

$$(161.56)^2 + (110.78)^2 + (50.78)^2 = 2\sigma_{YP}^2$$

$$\sigma_{YP} = 143.1 \text{ MPa}$$

maximum shear stress $\sigma_{YP} = \frac{\sigma_1 - \sigma_3}{2}$

$$= 80.78 \text{ MPa}$$

$\Rightarrow$ distortion-energy theory control!

(c) $\sigma_x = 75$ MPa , $\sigma_y = -100$ MPa , $\tau_{xy} = 0$

$\Rightarrow \sigma_1 = 75$ MPa , $\sigma_2 = 0$, $\sigma_3 = -100$ MPa

maximum normal stress = 100 MPa

distortion - energy $(175)^2 + (75)^2 + (100)^2 = 2\sigma_{YP}^2$

$$\sigma_{YP} = 152.07 \text{ MPa}$$

maximum shear stress $= \frac{\sigma_1 - \sigma_3}{2} = 87.5$ MPa

$\Rightarrow$ distortion-energy theory control!

12-6 from von Mises yield criterion

$$(60\alpha - 50\alpha)^2 + (50\alpha - 70\alpha)^2 + (70\alpha - 60\alpha)^2 = 2(250)^2$$

$$600\alpha^2 = 125000$$

$$\alpha = 14.43$$

12-7 Tresca :

$$\tau_{max} = \frac{\sigma_{YP}}{2} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2 \right]^{\frac{1}{2}}$$

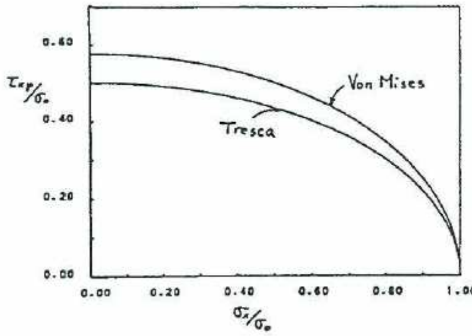
$$\left(\frac{\sigma_x}{\sigma_{YP}}\right)^2 + 4 + \left(\frac{\tau_{xy}}{\sigma_{YP}}\right)^2 = 1$$

Von Mises :

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{YP}^2$$

$$\sigma_1 \text{ or } \sigma_2 = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2 \right]^{\frac{1}{2}}$$

$$\left(\frac{\sigma_x}{\sigma_{YP}}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_{YP}}\right)^2 = 1$$



12-8 In pure shear $\sigma_1 = \tau_0$, $\sigma_2 = 0$,

$\sigma_3 = -\tau_0$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = \tau_0^2$$

for simple tension $\sigma_2 = \sigma_3 = 0$

$$\frac{2\sigma_1^2}{6} = \tau_0 \Rightarrow \sigma_1 = \sqrt{3}\,\tau_0 \text{ in Von Mises}$$

$$\tau_{max} = \frac{\sigma_1}{2} = \tau_0 \Rightarrow \sigma_1 = 2\tau_0 \text{ in Tresca}$$

for $\sigma_1 = \sigma_2$

$$\frac{2\sigma_1^2}{6} = \tau_0 \Rightarrow \sigma_1 = \sqrt{3}\,\tau_0 \text{ in Von Mises}$$

$$\tau_{max} = \frac{\sigma_1}{2} = \tau_0 \Rightarrow \sigma_1 = 2\tau_0 \text{ in Tresca}$$

12-9 (a) $\frac{5P+P}{2} = \frac{60}{2} \Rightarrow P = 10$

$\sigma_1 = 50$ ksi , $\sigma_2 = 20$ ksi , $\sigma_3 = -10$ ksi

(b) $[(5-2)^2 + (2+1)^2 + (5+1)^2]P^2 = 2 \times 60^2$

$P = 11.547$

$\sigma_1 = 57.7$ ksi , $\sigma_2 = 23.1$ ksi , $\sigma_3 = -11.5$ ksi

12-10 $\varepsilon_3 = \frac{1}{E}(\sigma_3 - \nu\sigma_1 - \nu\sigma_2) = 0$

$\because \sigma_2 = 0 \quad \therefore \sigma_3 = \nu\sigma_1$

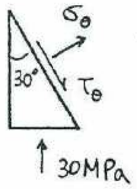
$J_2 = \frac{1}{3}\sigma_{YP}^2 \Rightarrow \sigma_1 = \frac{\sigma_{YP}}{(1-\nu+\nu^2)^{\frac{1}{2}}}$

Use Tresca :

$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{YP}}{2}$, $\sigma_3 = \nu\sigma_1$

$\sigma_1 = \frac{\sigma_{YP}}{1-\nu}$

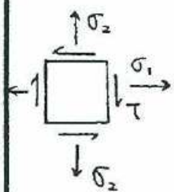
13-1



$$\sigma_\theta = \frac{-30}{2} + \frac{30}{2}\cos 60^\circ = -\frac{15}{2}\text{ MPa}$$

$$\tau_\theta = -\frac{30}{2}\sin 60^\circ = -\frac{15\sqrt{3}}{2}\text{ MPa}$$

13-2



$$\tau = \frac{T\rho}{I_p} = \frac{(200\times10^4)(50)}{\frac{\pi}{2}(50^4-46^4)} = 35.92\text{ MPa}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(1.5)(50)}{(4)} = 18.75\text{ MPa}$$

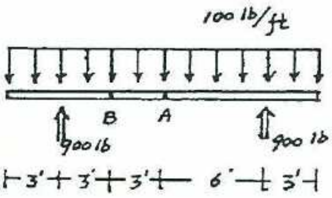
$$\sigma_2 = \frac{Pr}{2t} = \frac{(1.5)(50)}{2(4)} = 9.38\text{ MPa}$$

$$R = \sqrt{[(18.75-9.38)/2]^2 + (35.92)^2} = 36.22\text{ MPa}$$

$$\sigma_{max} = \frac{18.75+9.38}{2} + 36.22 = 50.29\text{ MPa}$$

$$\sigma_{min} = \frac{18.75+9.38}{2} - 36.22 = -22.16\text{ MPa}$$

13-3



$V_A = 0$

$M_A = (900\times6 - \frac{1}{2}\times100\times9^2)\times12$

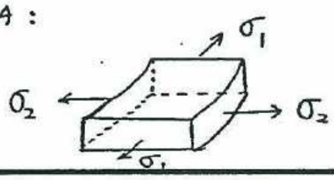
$= 16200$ lb-in

$V_B = 300$ lb

$M_B = (900\times3 - \frac{1}{2}\times100\times6^2)\times12$

$= 10800$ lb-in

element A :



$$\sigma_1 = \frac{pr}{t} = \frac{100\times20}{0.2} = 10000\text{ psi}$$

$$\sigma_2 = \frac{pr}{2t} + \frac{M_A r}{I}$$

$$= \frac{100\times20}{2\times0.2} + \frac{16200\times20}{\pi\times20^3\times0.2}$$

$$= 5000 + 64.46$$

$$= 5064.46\text{ psi}$$

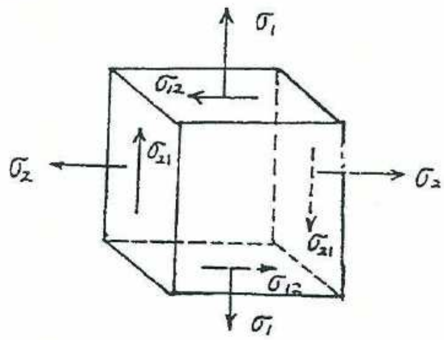
$$\frac{M_A r}{I} = 64.46\text{ psi} \ll \frac{pr}{2t} = 5000\text{ psi}$$

(due to dead weight)

$\sigma_3 \approx 0$

$\sigma_{13} = 0$ ($\because V_A = 0$)

element B :



$$\sigma_1 = \frac{pr}{t} = \frac{100\times20}{0.2} = 10000\text{ psi}$$

$$\sigma_2 = \frac{pr}{2t} + \frac{M_B\cdot0}{I} = 5000\text{ psi}$$

$\sigma_3 \approx 0$

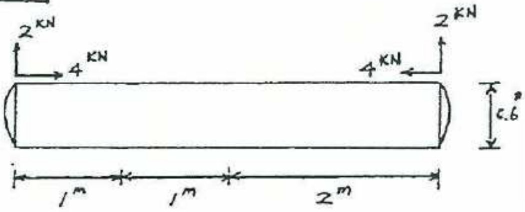
$$\sigma_{12} = \frac{V_B\cdot Q_B}{I\cdot2t} = \frac{V_B\cdot2tr^2}{\pi r^3 t\cdot2t} = \frac{V_B}{\pi rt}$$

$$= \frac{300}{\pi\times10\times0.2} = 47.74\text{ psi}$$

$$\frac{V_B Q_B}{I\cdot2t} = 47.74\text{ psi} \ll \frac{pr}{2t} = 5000\text{ psi}$$

∴ the dead weight has little influence on the total stress.

13-4



$W = 102 \times 9.81 = 1000\ N/m = 1\ ^{KN}/m$

$V_A = 0\ .\ \ M_A = \frac{1}{8} W\ell^2 = \frac{1}{8} \times 1 \times 4^2$

$= 2\ ^{KN-m}$

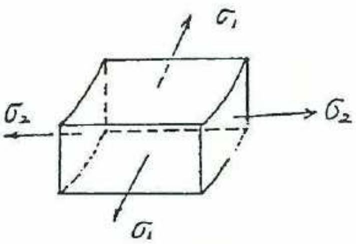
$V_B = 2 - 1 \times 1 = 1\ ^{KN}$

$M_B = 2 \times 1 - \frac{1}{2} \times 1 \times 1^2 = 1.5\ KN-m$

$P_A = P_B = -4\ ^{KN}\ ,\ \ P_i = 500\ kpa$

$M = 4 \times 0.3 = 1.2\ kN-m$

element A:



$\sigma_1 = \frac{P_i r}{t} = \frac{500 \times 300}{6} = 25\ MPa$

$\sigma_2 = \frac{P_i r}{2t} + \frac{P_A}{A} + \frac{(M_A + M) r}{I}$

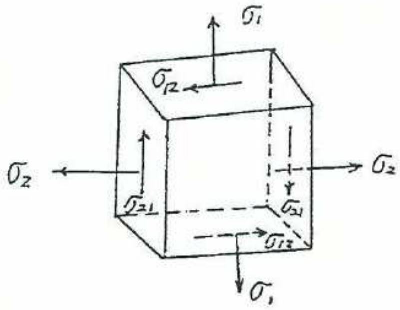
$= 12.5 - \frac{4 \times 10^{-3}}{\pi \times 0.6 \times 6 \times 10^{-3}} + \frac{3.2 \times 10^{-3} \times 0.3}{\pi \times (0.3)^3 \times 6 \times 10^{-3}}$

$= 12.5 - 0.35 + 1.89$

$= 14.04\ MPa$

$\sigma_3 \approx 0.5\ MPa \approx 0$

element B:



$\sigma_1 = \frac{P_i r}{t} = 25\ MPa$

$\sigma_2 = \frac{Pr}{2t} - \frac{P_B}{A} = 12.5 - 0.35 = 12.15\ MPa$

$\sigma_3 \approx 0$

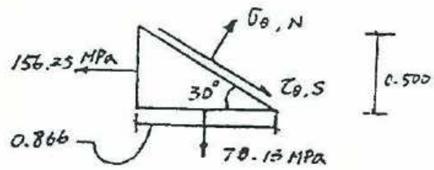
$\sigma_{12} = \frac{V_B Q_B}{I \cdot 2t} = \frac{V_B}{\pi \cdot rt} = \frac{1 \times 10^{-3}}{\pi \times 0.3 \times 6 \times 10^{-3}}$

$= 0.178\ MPa \approx 0$

13-5

$\sigma_1 = \frac{P_i r_i}{t} = \frac{1.5 \times (2.5/2)}{0.012} = 156.25\ MPa$

$\sigma_2 = \frac{1}{2} \sigma_1 = 78.13\ MPa$



$N = 156.25\ (0.5)(0.5) + 78.13(0.866)(0.866)$

$= 97.66\ MN$

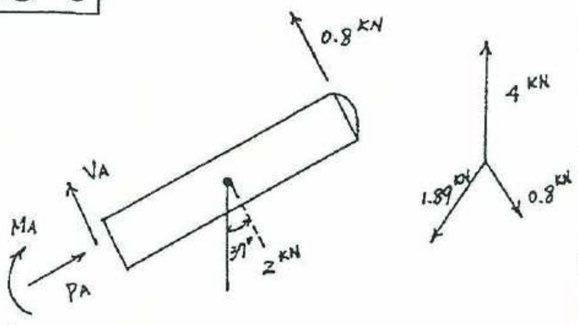
$\sigma_\theta = 97.66\ MPa$

$S = 156.25\ (0.5)(0.866) - 78.13(0.866)(0.5)$

$= 33.83\ MN$

$\tau = 33.83\ MPa$

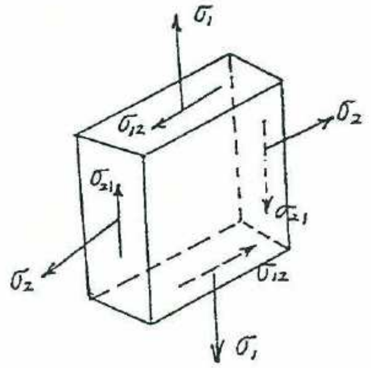
13-6



$$V_A = 2 \times \frac{4}{5} - 0.8 = 0.8 \text{ KN}$$

$$P_A = -2 \times \frac{3}{5} = -1.2 \text{ KN}$$

$$P_i = 0.5 \text{ MPa}$$



$$\sigma_1 = \frac{P_i r}{t} = \frac{500 \times 300}{6} = 25 \text{ MPa}$$

$$\sigma_2 = \frac{P_i r}{2t} + \frac{P_A}{A}$$

$$= 12.5 - \frac{1.2 \times 10^{-3}}{\pi \times 0.6 \times 6 \times 10^{-3}}$$

$$= 12.4 \text{ MPa}$$

$$\sigma_3 \approx 0$$

$$\sigma_{12} = \frac{V_A Q_A}{I \cdot 2t} = \frac{V_A}{\pi r t} = \frac{0.8 \times 10^{-3}}{\pi \times 0.3 \times 6 \times 10^{-3}}$$

$$= 0.14 \text{ MPa} \approx 0$$

13-7

$$\sigma_1 = \frac{P r_{mean}}{t} = \frac{600 \times 6.19}{0.375 \times (10)^3}$$

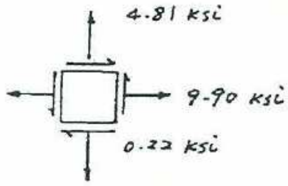
$$= 9.90 \text{ ksi}$$

$$\sigma_2 = \sigma_1 / 2 = 4.95 \text{ ksi}$$

$$\sigma_w = \frac{W}{A} = -\frac{49.56 \times 40}{14.58 \times (10)^3} = -0.14 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(40 \times 40) \times \frac{14.58}{2} \times \frac{2}{\pi} \times 6.19}{279.3 \times 0.75 \times (10)^3}$$

$$= 0.22 \text{ ksi}$$



13-8

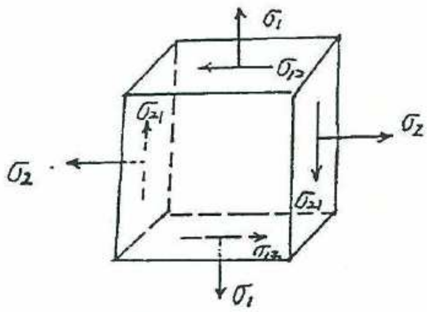
$$V_A = P = 31.4^{K}$$

$$T = 31.4 \times 10 = 314^{K\text{-}in}$$

$$M_A = 31.4 \times 20 = 628^{K\text{-}in}$$

$$P_i = 250 \text{ psi}$$

$$r = 10^{in}, \quad t = 0.25^{in}$$



(a) $$\sigma_1 = \frac{P_i r}{t} = 250 \times \frac{10}{0.25}$$

$$= 10000 \text{ psi} = 10 \text{ ksi}$$

$$\sigma_2 = \frac{P_i r}{2t} = 5 \text{ ksi}$$

$$\sigma_3 \approx 0 = \sigma_3^{P}$$

$$\sigma_{12} = \frac{V_A Q_A}{I \cdot 2t} + \frac{Tr}{J}$$

$$= \frac{V_A}{\pi r t} + \frac{T}{2\pi r^2 t}$$

$$= \frac{31.4}{\pi \times 10 \times 0.25} + \frac{314}{2\pi \times 10^2 \times 0.25}$$

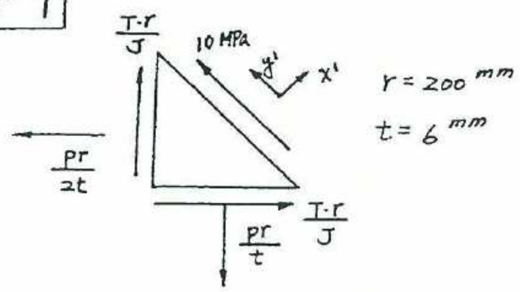
$$= 4 + 2 = 6 \text{ ksi}$$

(b) $$\sigma_1^P = \frac{10+5}{2} + \sqrt{\left(\frac{10-5}{2}\right)^2 + 6^2} = 14 \text{ ksi}$$

$$\sigma_2^P = \frac{10+5}{2} - \sqrt{\left(\frac{10-5}{2}\right)^2 + 6^2} = 1 \text{ ksi}$$

$$\tau_{max} = \frac{\sigma_1^P - \sigma_3^P}{2} = \frac{14}{2} = 7 \text{ ksi}$$

13-9



$r = 200$ mm

$t = 6$ mm

$$\Sigma F_{x'} = 0$$

$$2 \times \frac{1}{\sqrt{2}} \frac{T \cdot r}{J} - \frac{pr}{2t} \cdot \frac{1}{\sqrt{2}} - \frac{pr}{t} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\Sigma F_{y'} = 0$$

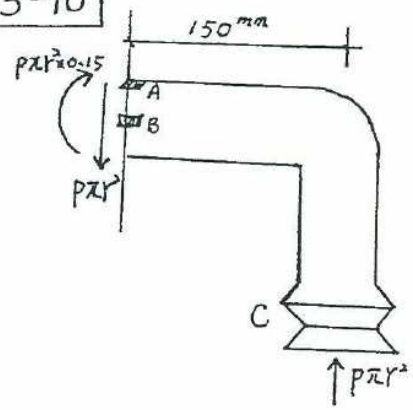
$$\frac{pr}{2t} \cdot \frac{1}{\sqrt{2}} + 10\sqrt{2} - \frac{pr}{t} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \frac{p \cdot r}{t} = 40 \text{ MPa}, \quad \frac{Tr}{J} = 10 \text{ MPa}$$

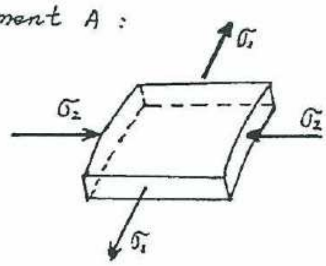
$$\Rightarrow p = 40 \times \frac{6}{200} = 1.2 \text{ MPa}$$

$$T = 2\pi r^2 t \times 10 = 15.08 \text{ kN-m}$$

13-10



element A :

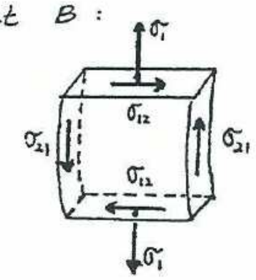


$$\sigma_1 = \frac{p \cdot r}{t} = \frac{2 \times 30}{2} = 30 \text{ MPa}$$

$$\sigma_2 = \frac{Mr}{I} = \frac{p\pi r^2 \times 0.15 \times r}{\pi r^3 t} = \frac{2 \times 0.15}{2 \times 10^{-3}}$$

$$= -150 \text{ MPa}$$

element B :

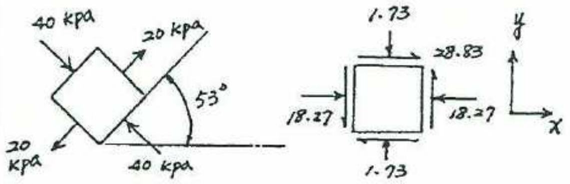


$$\sigma_1 = \frac{p \cdot r}{t} = \frac{2 \times 30}{2} = 30 \text{ MPa}$$

$$\sigma_{12} = \frac{VQ}{I \cdot 2t} = \frac{V}{\pi \cdot r \cdot t} = \frac{p\pi \cdot r^2}{\pi \cdot r \cdot t}$$

$$= \frac{pr}{t} = 30 \text{ MPa}$$

13-11



$$\sigma_x = \frac{20+(-40)}{2} + \frac{20-(-40)}{2}\cos(2\times -53^\circ)$$

$$= -18.27 \text{ kpa}$$

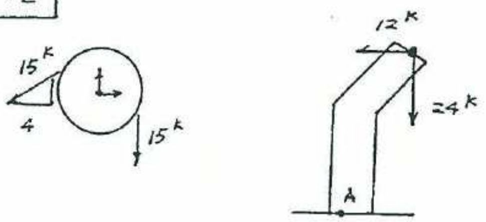
$$\sigma_y = \frac{20+(-40)}{2} + \frac{20-(-40)}{2}\cos(2\times 37^\circ)$$

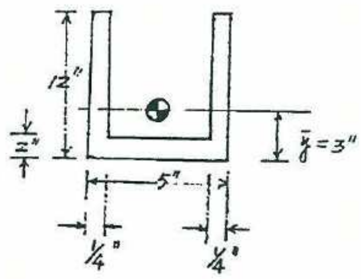
$$= -1.73 \text{ kpa.}$$

$$\tau_{xy} = -\frac{20-(-40)}{2}\sin(2\times -53^\circ)$$

$$= 28.83 \text{ kpa}$$

13-12





$$\bar{y} = \frac{12\times5\times6 - 10\times4.5\times7}{12\times5 - 10\times4.5} = \frac{45}{15} = 3''$$

$$A = 15 \text{ in}^2.$$

$$I = 2\times\left(\frac{1}{3}\times\frac{1}{4}\times9^3 + \frac{1}{3}\times\frac{1}{4}\times3^3\right) + \frac{1}{12}\times4.5\times2^3 + 4.5\times2\times2^2 = 165 \text{ in}^4$$

$$Q_A = 2\times2\times\frac{1}{4}\times8 = 8 \text{ in}^3$$

$$V_A = 12\text{ k}, \quad P_A = 24.$$

$$M_A = 12\times36 - 24(2\times12-9) = 72 \text{ k-in}$$

$$\sigma = \frac{M_A\cdot Y_A}{I} - \frac{P_A}{A} = \frac{-72\times7}{165} - \frac{24}{15}$$

$$= -4.65 \text{ ksi}$$

$$\tau = \frac{V_A\cdot Q_A}{It} = \frac{12\times8}{165\times\frac{2}{4}} = 1.16 \text{ ksi}$$

$$\sigma_{max} = \frac{-4.65}{2} + \sqrt{\left(\frac{4.65}{2}\right)^2 + (1.16)^2}$$

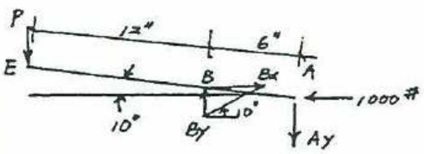
$$= 0.272 \text{ ksi}$$

$$\sigma_{min} = \frac{-4.65}{2} - \sqrt{\left(\frac{4.65}{2}\right)^2 + (1.16)^2}$$

$$= -4.93 \text{ ksi}$$

13-13

(a)



$$\Sigma F_x = 0,$$

$$B_x = 1000 \text{ lb} \rightarrow$$

$$B_y = B_x \tan 10^\circ$$

$$= 1000(0.176)$$

$$= 176.3 \text{ lb}$$

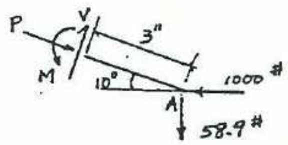
$$\Sigma M_A = 0 \quad +\circlearrowright$$

$$18\cos10^\circ P - 176(6\cos10^\circ) - 1000(6\sin10^\circ) = 0$$

$$P = 117.4 \text{ lb.}$$

$$\Sigma F_y = 0, \quad A_y = 176.3 - 117.4 = 58.9 \text{ lb}$$

(b)



$$p = 1000\cos 10^\circ - 58.9\sin 10^\circ$$
$$= 974.6 \text{ lb } (-)$$

$$V = 1000\sin 10^\circ + 58.9\cos 10^\circ$$
$$= 231.6 \text{ lb}$$

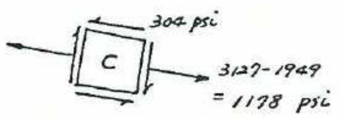
$$M = 1000(3\sin 10^\circ) + 58.9(3\cos 10^\circ)$$
$$= 694.9 \text{ lb}$$

$$\sigma_P = \frac{P}{A} = \frac{974.6}{1/4 \times 2} = 1949.2 \text{ psi } (-)$$

$$\sigma_M = \frac{My}{I} = \frac{694.9 \times (3/4)}{\frac{1}{4}(2)^3 \times 1/12} = 3127.1 \text{ psi } (+)$$

$$\tau = \frac{VQ}{It} = \frac{231.6(1/4 \times 1/4 \times 7/8)}{\frac{1}{4}(2)^3 \times \frac{1}{12}(1/4)} = 304.0 \text{ psi}$$

(c)



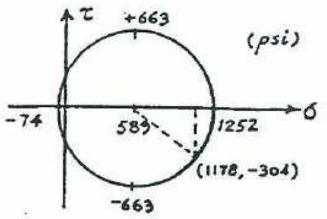
(d) center : $\frac{1178}{2} = 589$ psi

$$r = \sqrt{(589)^2 + (304)^2} = 663 \text{ psi}$$

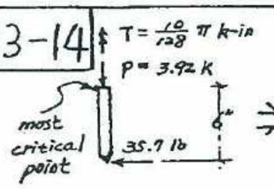
$\sigma_1 = 1252$ psi

$\sigma_2 = -74$ psi

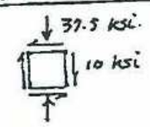
$\tau_{max} = 663$ psi



13-14



At the critical point

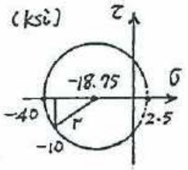


$$\sigma = \frac{P}{A} + \frac{MC}{I} = \frac{3.92}{0.196} + \frac{(0.0357\times 6)0.25}{0.00306}$$
$$= 37.5 \text{ ksi } (-)$$

$$\tau = \frac{TC}{J} = \frac{10\pi(0.25)}{128(0.00612)} = 10.0 \text{ ksi}$$

$$\text{center} = \frac{-37.5}{2} = -18.75,$$

$$r = \sqrt{(18.75)^2 + (10)^2} = 21.25$$



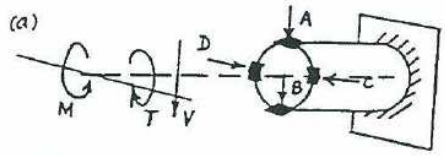
$\sigma_1 = -18.75 + 21.25 = 2.5$ ksi

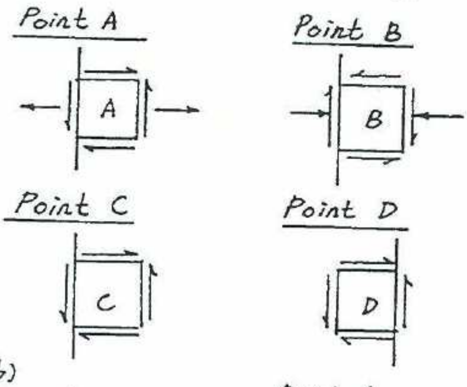
$\sigma_2 = -18.75 - 21.25 = -40.0$ ksi

$\therefore\ \sigma_{max} = -40.0$ ksi

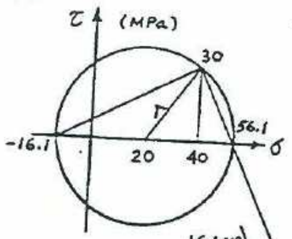
13-15

(a)





(b)

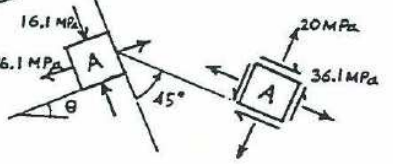


Point A

$$\text{center} = \frac{40}{2} = 20$$

$$r = \sqrt{(20)^2 + (30)^2}$$
$$= 36.1$$

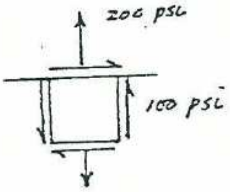
$$\theta = \tan^{-1}\left(\frac{30}{56.1}\right)$$
$$= 28.15^\circ$$



13-16

Point A

(a)

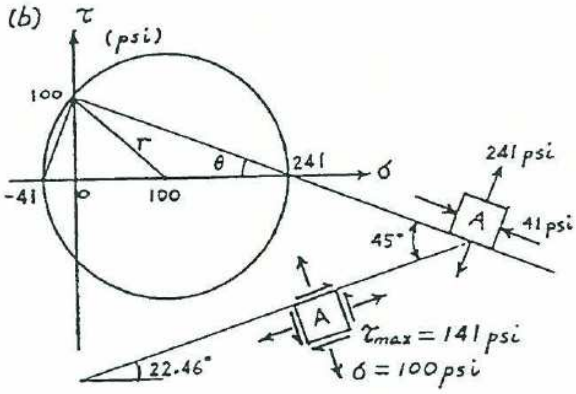


$$\text{center} = \frac{200}{2} = 100$$

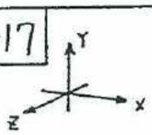
$$r = \sqrt{(100)^2 + (100)^2} = 141 \text{ psi}$$

$$\theta = \tan^{-1}\left(\frac{100}{241}\right) = 22.54^\circ$$

(b)



13-17



(a)

$$F_x = 2000 \text{ N}$$
$$F_y = -2000 \text{ N}$$
$$F_z = -1000 \text{ N}$$

$$A = 12 \times 12 = 144 \text{ mm}^2$$

$$I = \frac{12(12)^3}{12} = 1728 \text{ mm}^4$$

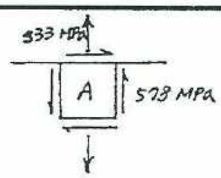
Point A

$$\sigma_A = \frac{F_y}{A} + \frac{M_{xx} \cdot C}{I} = -\frac{2000}{144} - \frac{(1000 \times 100) \times 6}{1728}$$

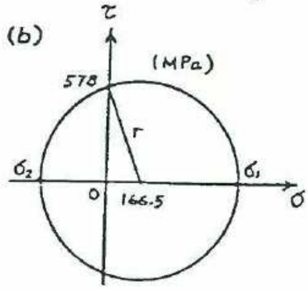
$$= 333 \text{ MPa}$$

$$\tau_A = \frac{3V}{2A} + \frac{T}{\alpha b c^2} = \frac{3}{2}\left(\frac{2000}{144}\right) + \frac{1000 \times 200}{0.208 \times 12 \times 12^2}$$

$$= 578 \text{ MPa}$$



(b)



$$\text{center} = \frac{333}{2} = 166.5$$

$$r = \sqrt{(166.5)^2 + (578)^2}$$

$$= 601.5 \text{ MPa}$$

$$\sigma_2 = 166.5 - 601.5$$
$$= -435 \text{ MPa}$$

$$\sigma_1 = 166.5 + 601.5$$
$$= 768 \text{ MPa}$$

13-18

$F_x = 135\pi N \rightarrow$, $F_y = 180\pi N \downarrow$

$$I = \frac{\pi (0.025)^4}{4} = 9.77 \times 10^{-8} \pi \text{ m}^4$$

$$J = \frac{\pi (0.025)^4}{2} = 1.95 \times 10^{-7} \pi \text{ m}^4$$

$$A = \pi (0.025)^2 = 6.25 \times 10^{-4} \pi \text{ m}^2$$

Point A

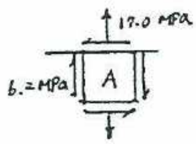
$$\sigma_A = \frac{F_y}{A} + \frac{M_{xx} C}{I} = -\frac{180\pi}{6.25 \times 10^{-4}\pi} + \frac{(180\pi \times .375)(.025)}{9.77 \times 10^{-8}\pi}$$

$$= 17.0 \text{ MPa}$$

$$\tau_A = \frac{TC}{J} + \frac{VQ}{It} = -\frac{135\pi \times 0.375(0.025)}{1.95 \times 10^{-7}\pi}$$

$$+ \frac{135\pi (6.25 \times 10^{-4}\pi/2)\left(\frac{4 \times .025}{3\pi}\right)}{9.77 \times 10^{-8}\pi (0.05)}$$

$$= -6.2 \text{ MPa}$$

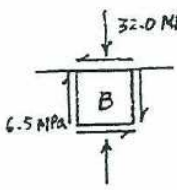


Point B

$$\sigma_B = \frac{F_y}{A} + \frac{M_{zz} \cdot x}{I} = -\frac{180\pi}{6.25 \times 10^{-4}\pi} - \frac{(135\pi \times 0.250)(.025)}{9.77 \times 10^{-8}\pi}$$

$$- \frac{(180\pi \times 0.50)(.025)}{9.77 \times 10^{-8}\pi}$$

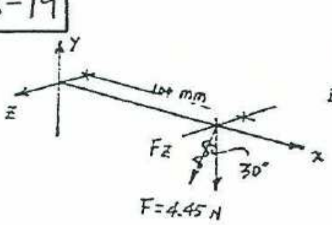
$$= -32.0 \text{ MPa}$$

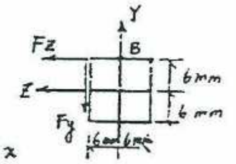


$$\tau_B = \frac{TC}{J} = \frac{-135\pi \times .375(0.025)}{1.95 \times 10^{-7}\pi}$$

$$= -6.5 \text{ MPa}$$

13-19





$A = (12)(12)$
$= 144\ mm^2$

$I = \frac{(12)(12)^3}{12}$
$= 1728\ mm^4$

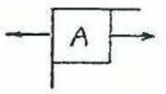
$F_z = 4.45 \sin 45° = 2.23\ N$

$F_y = 4.45 \cos 45° = 3.85\ N$

Point A

$$\sigma_A = \frac{M_{yy}C}{I} + \frac{M_{zz}C}{I} = \frac{(F_z+F_y)100\times 6}{1728}$$
$$= 2.11\ MPa$$

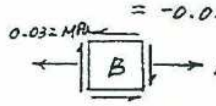
$\tau_A = 0$



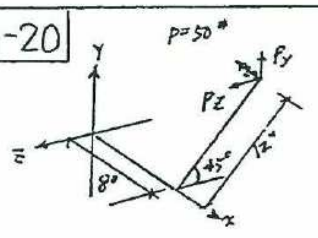
Point B

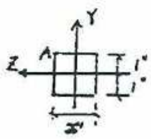
$$\sigma_B = \frac{M_{zz}C}{I} = \frac{(F_y\times 100)\times 6}{1728} = 1.34\ MPa$$

$$\tau_B = \frac{3V}{2A} + \frac{T}{\alpha bc^3} = -\frac{3F_z}{2(144)} - \frac{(F_y+F_z)\times 6}{0.208(12)(12)^3}$$
$$= -0.023 - 0.009 = -0.032\ MPa$$



13-20





$A = (2)(2) = 4\ in^2$

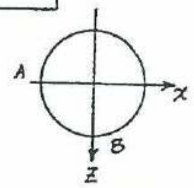
$I = \frac{(2)(2)^3}{12} = \frac{4}{3}\ in^4$

$$\sigma_A = \frac{M_{yy}C}{I} = -\frac{(P_z\times 8)(1)}{4/3} = -\frac{(50\cos 45°\times 8)\times 3}{4}$$
$$= -212.1\ psi$$

$$\tau_A = \frac{3V}{2A} + \frac{T}{\alpha bc^3} = \frac{3P_y}{2A} - \frac{P(12)}{0.208\times 2\times (2)^3}$$
$$= \frac{3(50\sin 45°)}{2(4)} - \frac{50(12)}{(0.208)(16)} = -167.0\ psi$$

13-21

$A = 1.704\ in^2,\ I = 1.530\ in^4$

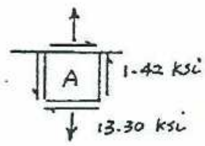


$P = 400\ lb\ (-)$
$V = 90\ lb$
$M_{zz} = 400\times 3 = 1200\ in\text{-}lb$
$M_{xx} = 90\times 10 = 900\ in\text{-}lb$
$T = 90\times 3 = 270\ in\text{-}lb$

Point A

$$\sigma_A = \frac{P}{A} + \frac{M_{zz}C}{I} = -\frac{400}{1.704} + \frac{1200\times 12\left(\frac{2.875}{2}\right)}{1.530}$$
$$= +13,295\ psi$$

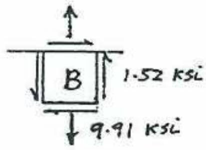
$$\tau_A = \frac{VQ}{It} + \frac{TC}{J}$$
$$= -\frac{90\left(\frac{1.704}{2}\times\frac{2}{\pi}\times 1.33\right)}{1.530\times 0.406} + \frac{270\times 12\times(1.438)}{2\times 1.530}$$
$$= 1,418\ psi$$



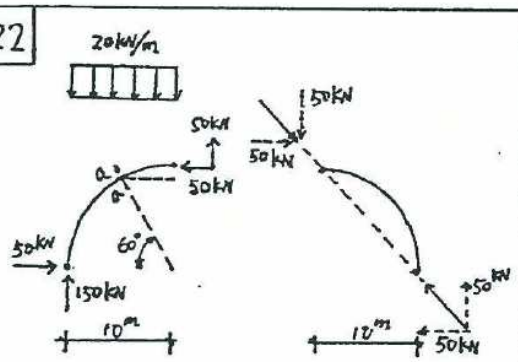
Point B

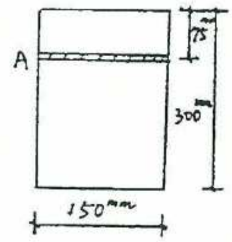
$$\sigma_B = \frac{P}{A} + \frac{M_{xx}C}{I} = -\frac{400}{1.704} + \frac{900\times 12\left(\frac{2.875}{2}\right)}{1.530}$$
$$= +9,912\ psi$$

$$\tau_B = \frac{TC}{J} = \frac{270\times 12\times(1.438)}{2\times 1.530} = 1,523\ psi$$



13-22





section a-a

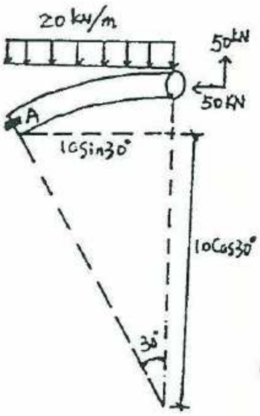
$A = 0.3 \times 0.15 = 0.045\ m^2$

$I = \frac{1}{12} \times 0.15 \times 0.3^3$

$= 3.375 \times 10^{-4}\ m^4$

$Q = 0.075 \times 0.15 \times 0.1125$

$= 1.266 \times 10^{-3}\ m^3$



$M = \frac{1}{2} \times 20 \times (10\sin 30°)^2 - 50 \times 10 \sin 30° - 50\,(10 - 10\cos 30°)$

$= -67\ ^{kN\text{-}m}$

$P = -(50 \sin 30° + 50 \sin 60°)$

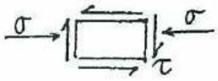
$= -68.3\ ^{kN} \leftarrow$

$V = 18.3\ ^{kN} \downarrow$

$\sigma = \frac{-68.3}{0.045} - \frac{67 \times 0.075}{3.375 \times 10^{-4}}$

$= -16.42\ MPa$

element A



$\tau = \frac{VQ}{It} = \frac{18.3 \times 1.266 \times 10^{-3}}{3.375 \times 10^{-4} \times 0.15}$

$= 0.44\ MPa$

$\sigma_2 = -\frac{16.42}{2} - \sqrt{\left(\frac{-16.42}{2}\right)^2 + 0.44^2}$

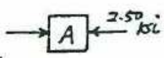
$= -16.43\ MPa$

$\sigma_1 = -\frac{16.42}{2} + \sqrt{\left(\frac{-16.42}{2}\right)^2 + 0.44^2} = 0.01\ MPa$

13-23

$V = 9.22\ k \downarrow,\ M = 9.22 \times 10 = 92.2\ k\text{-in}$

Point A

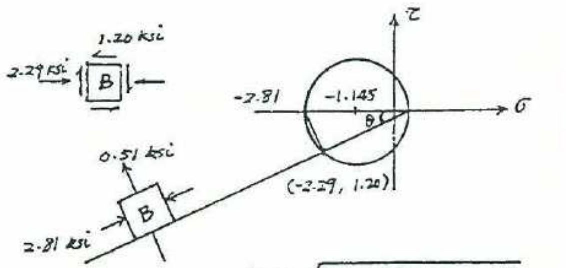


$\sigma_A = \frac{MC}{I} = \frac{92.2 \times 6}{221} = 2.50\ ksi$

Point B

$\sigma_B = \frac{5.50 \times 2.50}{6} = 2.29\ ksi$

$\tau_B = \frac{VQ}{It} = \frac{9.22\,(\frac{1}{2} \times 5)\,5.75}{221\,(0.5)} = 1.20\ ksi$



$r = \sqrt{(1.145)^2 + (1.20)^2}$

$= 1.66$

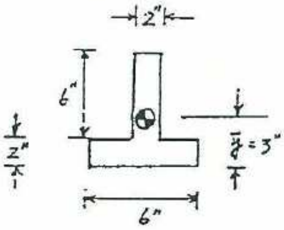
$\theta = \tan^{-1}\left(\frac{1.20}{2.80}\right) = 23.20°$

Point C $\quad \sigma_c = 0$

$\tau_c = \frac{9.22\,(\frac{1}{2} \times 5 \times 5.75 + \frac{1}{2} \times 5.5 \times 2.75)}{221 \times 0.5}$

$= 1.83\ ksi$

13-24



$\bar{y} = \frac{6 \times 2 \times 5 + 2 \times 6 \times 1}{6 \times 2 + 2 \times 6} = 3''$

$A = 24\ in^2$

$I = \frac{1}{3}\, 2 \times 5^3 + \frac{1}{3}\, 2 \times 1^3 + \frac{1}{12} \times 6 \times 2^3 + 6 \times 2 \times 2^2$

$= 100\ in^4$

$Q_A = 2 \times 2 \times 4 = 16\ in^3$

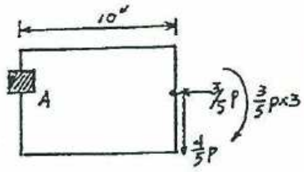
$\frac{(\frac{4}{5}P \times 10 - \frac{3}{5}P \times 3) \cdot 3}{I} - \frac{\frac{3}{5}P}{A}$

$= E \times 20 \times 10^{-6} = 600$

(a) $\Rightarrow \quad P = 3.73\ ^{K} = 3730\ ^{lb}$

(b) $V_A = \frac{4}{5} P$

$Q_A = 16$

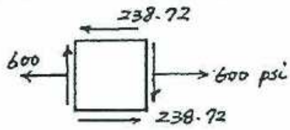


$\tau = \frac{V_A Q_A}{It} = \frac{\frac{4}{5}P \times 16}{100 \times 2}$

$= 238.72\ psi$

ref Appendix.

$G = 12\times10^6$ psi



$\therefore \gamma/2 = \dfrac{\tau}{2G}$

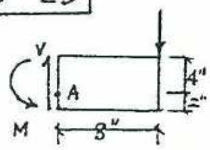
$= 9.95\times10^{-6}$

Max. principal strain ε_1

$= \frac{1}{2}(20\times10^{-6}+0)+\sqrt{\left(\frac{20\times10^{-6}}{2}\right)^2+(9.95\times10^{-6})^2}$

$\approx 24.1\times10^{-6}$

13-25



$I = \dfrac{bh^3}{12} = \dfrac{2(6)^3}{12} = 36\ in^4$

$V = P$

$M = 8P$

$\sigma_x = \dfrac{My}{I} = \dfrac{8P(1)}{36} = \dfrac{2P}{9}$

$\tau = \dfrac{VQ}{Ib} = \dfrac{P(2\times2\times2)}{36\times2} = \dfrac{P}{9}$

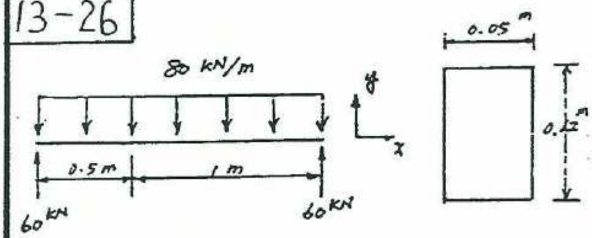
$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2+(\tau)^2}$

$120 = \dfrac{P}{9}\sqrt{(1)^2+(1)^2} = \dfrac{1.414P}{9}$

$P = \dfrac{120\times9}{1.414}$

$P = 764$ lb.

13-26



$I = \frac{1}{12}\times0.05\times0.12^3 = 7.2\times10^{-6}\ m^4$

$V = 60-80\times0.5 = 20^{kN}$

$M = 60\times0.5-\frac{1}{2}80\times0.5^2 = 20^{kN\cdot m}$

element A :

$\sigma_1 = \sigma_y = \dfrac{-80}{0.05} = -1.6\times10^3$ Kpa

$= -1.6$ MPa

$\sigma_2 = \sigma_x = -\dfrac{M\cdot\frac{h}{2}}{I}$

$= \dfrac{-20\times0.06}{7.2\times10^{-6}} = -166.7$ MPa

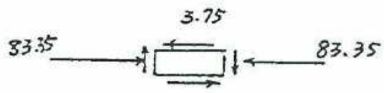
x-y direction is the principal stress and direction.

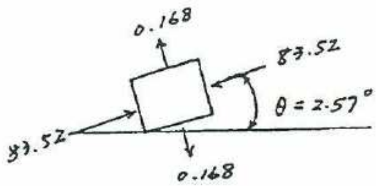
element B :

$\sigma_x = -\dfrac{M\cdot y}{I} = -166.7\times0.5 = -83.35$ MPa

$\tau_{xy} = \dfrac{VQ}{I\cdot t} = \dfrac{20\times0.03\times0.05\times0.045}{7.2\times10^{-6}\times0.05}$

$= 3.75$ MPa





$\sigma_1 = \dfrac{0+(-83.35)}{2}+\sqrt{\left(\frac{83.35}{2}\right)^2+(3.75)^2}$

$= 0.168$ MPa

$\sigma_2 = \dfrac{-83.35}{2}-\sqrt{\left(\frac{83.35}{2}\right)^2+(3.75)^2}$

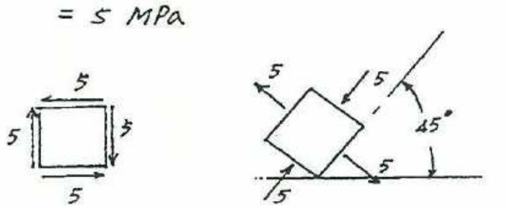
$= -83.52$ MPa

$\theta = \frac{1}{2}\tan^{-1}\dfrac{3.75\times2}{0-(-83.35)} = 2.57°$

element C :

$$\tau_{xy} = \frac{VQ}{It} = \frac{20\times 0.05\times 0.06\times 0.03}{7.2\times 10^{-6}\times 0.05}$$

$$= 5 \text{ MPa}$$

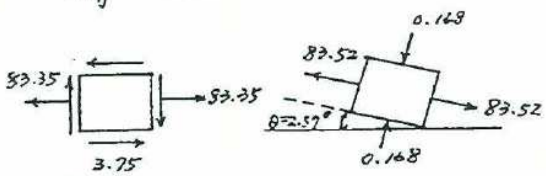


$\tau_{xy} = 5$ MPa $\sigma_1 = 5$ MPa , $\sigma_2 = -5$ MPa

element D :

$\sigma_x = 83.35$ MPa

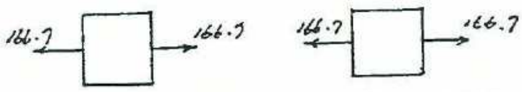
$\tau_{xy} = 3.75$ MPa



$\sigma_1 = 83.52$ MPa . $\sigma_2 = -0.168$ MPa

element E :

$$\sigma_x = \frac{M\cdot \frac{h}{2}}{I} = 166.7 \text{ MPa}$$



$\sigma_1 = 166.7$ MPa

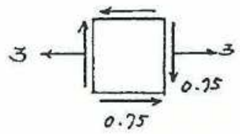
element A :

$V_A = \frac{80}{3}$ kN , $M_A = 16$ kN-m

$$\sigma_x = \frac{M_A\cdot y_A}{I} = \frac{16\times 0.1}{5.33\times 10^{-4}} = 3 \text{ MPa}$$

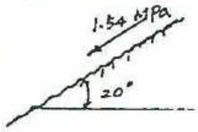
$$\tau_{xy} = \frac{V_A Q_A}{It} = \frac{80/3\times 0.1\times 0.1\times 0.15}{5.33\times 10^{-4}\times 0.1}$$

$$= 0.75 \text{ MPa}$$



$$\tau_{x'y'} = -\left(\frac{3-0}{2}\right)\sin(2\times 70^\circ) + 0.75\cos(2\times 70^\circ)$$

$$= -1.54 \text{ MPa}$$



element B :

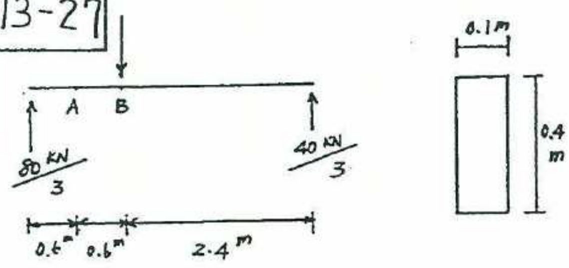
$M_B = \frac{80}{3}\times 1.2 = 32$ kN-m , $Q_B = 0$

$$\sigma_x = \frac{M_B\, y_B}{I} = \frac{32\times 0.2}{5.33\times 10^{-4}} = 12 \text{ MPa}$$

$$\tau_{x'y'} = -\left(\frac{12-0}{2}\right)\sin(2\times 70^\circ)$$

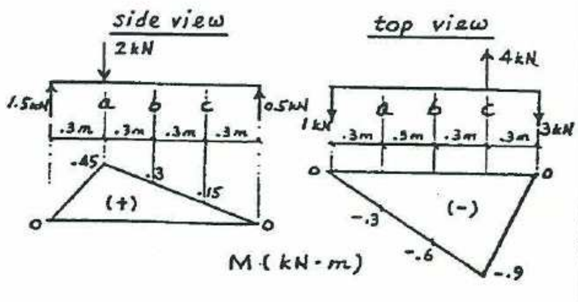
$$= -3.857 \text{ MPa}$$

13-27



$$I = \frac{1}{12}\times 0.1\times 0.4^3 = 5.33\times 10^{-4}\ m^4$$

13-28



$$\sigma_a = \pm \frac{0.45}{\frac{(.1)(.15)^2}{6}} \pm \frac{0.3}{\frac{(.15)(.1)^2}{6}} = \pm 1200 \pm 1200$$

$$= \pm 2400 \text{ kpa}$$

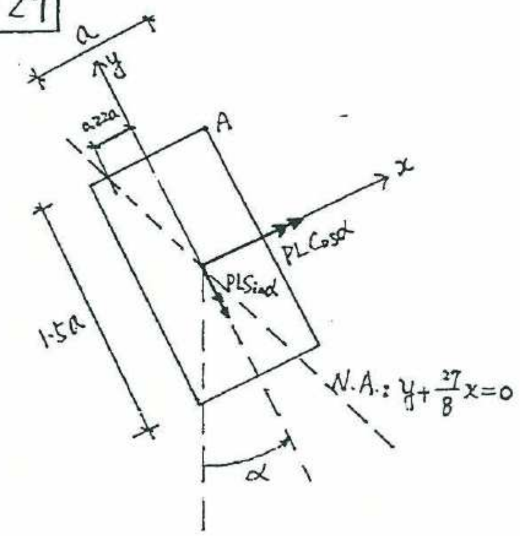
$\sigma_b < \sigma_c$ (by inspection)

$$\sigma_c = \pm \frac{0.15}{\frac{(.1)(.1)}{6}} \pm \frac{0.9}{\frac{(.1)(.1)}{6}} = \pm 900 \pm 5400$$

$$= \pm 6300 \text{ kpa}$$

$$\therefore \sigma_{max} = \sigma_c = \pm 6300 \text{ kPa}$$

13-29



$$\sigma_A(\alpha) = \frac{PL\cos\alpha \cdot (\frac{1.5}{2}a)}{\frac{1}{12}a \cdot (1.5a)^3} + \frac{PL\sin\alpha(\frac{a}{2})}{\frac{1}{12}1.5a(a)^3}$$

$$\frac{d_A(\alpha)}{d\alpha} = 0 \Rightarrow \tan\alpha = \frac{3}{2} \Rightarrow \alpha = \tan^{-1}\frac{3}{2}$$

$$= 56.3°$$

Neutral Axis :

$$\frac{PL\cos\alpha \cdot y}{\frac{1}{12}a(1.5a)^3} + \frac{PL\sin\alpha \cdot x}{\frac{1}{12}(1.5a)\cdot a^3} = 0$$

$$\frac{4}{9}y + \tan\alpha \cdot x = 0$$

$$\frac{4}{9}y + \frac{3}{2}x = 0$$

$$y + \frac{27}{8}x = 0$$

13-30

$$(\sigma_x)_{max} = \frac{Mr}{I} = \frac{Mr}{\pi(r^4/4)}$$

$$M = \frac{\pi r^3}{4}(\sigma_x)_{max} = \frac{\pi(25)^3}{4} \times \frac{120}{10^6} = 1.47 \text{ kN-m}$$

$$(\sigma_1)_{max} = \frac{(\sigma_x)_{max}}{2} + \sqrt{\left[\frac{(\sigma_x)_{max}}{2}\right]^2 + \left(\frac{Tr}{J}\right)^2}$$

$$T = \frac{\pi r^3}{2}\sqrt{\left[(\sigma_1)_{max} - \frac{(\sigma_x)_{max}}{2}\right]^2 - \left[\frac{(\sigma_x)_{max}}{2}\right]^2}$$

$$= \frac{\pi(25)^3}{2\times10^6}\sqrt{(160-60)^2 - 60^2} = 1.96 \text{ kN-m}$$

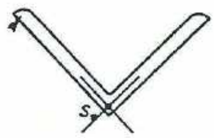
13-31

from Table $S_{xx} = 3.26\times10^4 \text{ mm}^3$

$A = 2.42\times10^3 \text{ mm}^2$

$r_{zz} = 19.9 \text{ mm}$

$$M_1 = \sigma_y S_1 = 3.26\times10^4\,\sigma_y$$



$$M_2 = \sigma_y S_2$$

$$= \sigma_y \frac{I_2}{c_2}$$

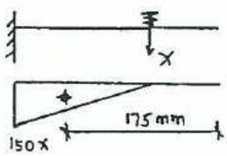
$$= \sigma_y \frac{(2.42\times10^3)(19.9)^2}{(30.2\times\sqrt{2})}$$

$$= 2.24\times10^4\,\sigma_y$$

$$\frac{M_1}{M_2} = \frac{3.26\times10^4\,\sigma_y}{2.24\times10^4\,\sigma_y} = 1.46$$

13-32

$$\Delta_T = (20\times10^{-6})(40)(4.5\times10^3) = 3.6 \text{ mm}$$



$$\frac{\frac{1}{2}(150)(150x)(175)}{20\times10^6} = 2$$

$$x = 20.317 \text{ N}$$

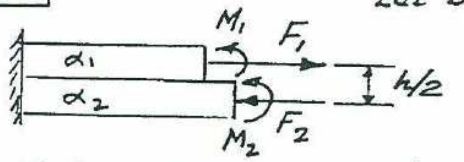
$$\Delta = \Delta_T + \Delta_x = 3.6 + \frac{x(4500)}{(6)(10^5)} + \frac{x}{20} + \frac{\frac{1}{2}(150)(150x)(100)}{20\times10^6}$$

$$= 5.91 \text{ mm}$$

number of turns = 5.91

13-33

Let $b = 1''$.



If free to expand ends do not match. Need F_1 & F_2, which cannot exist, however, without M_1 & M_2 as $\Sigma M_z = 0$.

$\Sigma F_x = 0$: $F_1 = F_2 = F$

Both bars curve alike:

$$\frac{1}{\rho} = \frac{M_1}{EI} = \frac{M_2}{EI}$$

hence

$$M_1 = M_2 = M = \frac{EI}{\rho}$$ where I for one bar,

$\Sigma M_z = 0$: $2M = Fh/2$, and

$$F = \frac{4M}{h} = \frac{4EI}{h\rho}$$ whence on

EQUATING THE STRAIN AT THE BOTTOM OF THE TOP BAR TO THE STRAIN AT THE TOP OF THE BOTTOM BAR, ONE HAS

$$\alpha_1 \delta T + \frac{F}{\frac{1}{2}hE} + \frac{h/4}{\rho}^{*} =$$

$$= \alpha_2 \delta T - \frac{F}{\frac{1}{2}hE} - \frac{h/4}{\rho}^{*}$$

OR

$$\frac{4F}{hE} + \frac{h}{2\rho} = (\alpha_2 - \alpha_1)\delta T$$

$$\frac{16\cancel{E}I}{h^2\cancel{E}\rho} + \frac{h}{2\rho} = (\alpha_2 - \alpha_1)\delta T$$

But $I = \frac{1}{12} \times 1'' \times \left(\frac{h}{2}\right)^3$, hence

$$\frac{1}{\rho} = \frac{3}{2h}(\alpha_2 - \alpha_1)\delta T$$

$$\Delta = \frac{L^2}{2\rho} = \frac{3(\alpha_2 - \alpha_1)\delta T L^2}{4h}$$

13-36

$$\tau_{max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_T^2}$$

$$80 \times 10^3 = \sqrt{\left(\frac{M \cdot r}{2I}\right)^2 + \left(\frac{T r}{J}\right)^2}$$

$$= \sqrt{\left(\frac{4 \cdot r}{2 \cdot \frac{\pi r^4}{4}}\right)^2 + \left(\frac{6 \cdot r}{\frac{\pi r^4}{2}}\right)^2}$$

$$80 \times 10^3 = \frac{1}{\pi r^3}\sqrt{8^2 + 12^2}$$

$$r = 3.86 \text{ cm}$$

$$d = 2 \times 3.86 = 7.72 \text{ cm}$$

13-35

$$\frac{r-t}{r+t} = 0.8 \Rightarrow r = 4.5t \quad — ①$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + (\tau_T)^2}$$

$$80 \times 10^3 = \sqrt{\left(\frac{Mr}{2I}\right)^2 + \left(\frac{T \cdot r}{J}\right)^2}$$

$$80 \times 10^3 = \sqrt{\left(\frac{4 \cdot r}{2 \cdot \pi r^3 \cdot t}\right)^2 + \left(\frac{6 \cdot r}{2\pi r^3 t}\right)^2}$$

$$80 \times 10^3 = \frac{1}{\pi r^2 t}\sqrt{2^2 + 3^2}$$

$$r^2 t = 14.346 \text{ cm}^3 \quad — ②$$

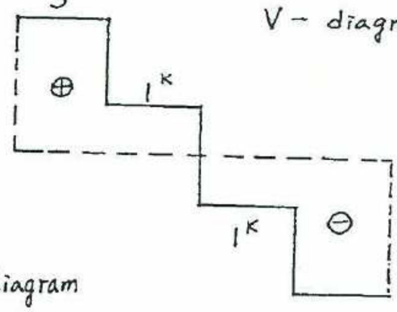
from ①, ② $\Rightarrow$ $r = 4$ cm, $t = 0.89$ cm

inside diameter $= 2(r - \frac{t}{2})$
$= 7.11$ cm

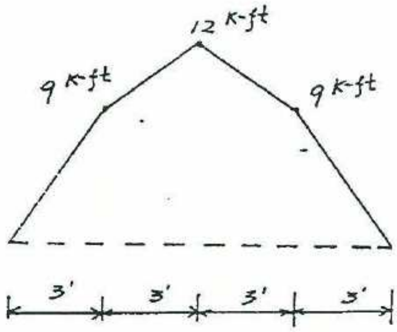
outside diameter $= 2(r + \frac{t}{2})$
$= 8.89$ cm

13-36

V- diagram



M- diagram



$$S_1 = \frac{M_{max}}{\sigma_{awB}} = \frac{12\times10^3\times12}{1250} = 115.2 \ in^3$$

$$A_1 = \frac{3}{2}\frac{V_{max}}{\tau_{aw}} = 1.5\times\frac{3\times10^3}{95} = 47.37 \ in^2$$

choose standard dressed size

$5\frac{1}{2}'' \times 11\frac{1}{2}''$ (ref table 10)

$W = 17.5 \ lb/ft$

$S = 121 \ in^3, \ A = 63.3 \ in^2$

$$S = 121 \ in^3 > S_1 + \frac{\frac{1}{8}wl^2}{\sigma_{awB}}$$

$$= 115.2 + \frac{\frac{1}{8}\times17.5\times12^2\times12}{1250}$$

$= 118.22 \ in^3$ O.K.

$$A = 63.3 \ in^2 > A_1 + \frac{3}{2}\frac{\frac{1}{2}wl}{\tau_{aw}}$$

$$= 47.37 + 1.5\,\frac{0.5\times17.5\times12}{95}$$

$= 49.03 \ in^2$ O.K.

$$l_{min} = \frac{2\times10^3}{625\times5.5} = 0.582 \ in$$

$\therefore$ choose $5.5''\times0.6''$ bearing plates under the concentrated forces.

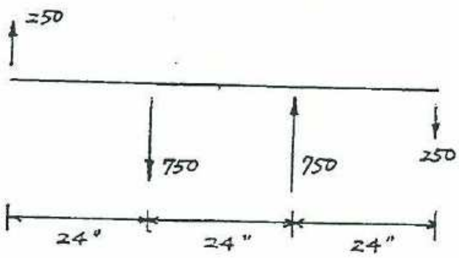
the minimum support length

$$= \frac{3\times1000 + 17.5\times12\times0.5}{625\times5.5}$$

$= 0.903 \ in$

13-37

$P = 250 \ lb$



$M_{max} = 250\times24 = 6000 \ lb-in$

$V_{max} = 500 \ lb$ can use Table 10

choose Nominal Size $3''\times4''$

$S = 5.10 \ in^3, \ A = 8.75 \ in^2$

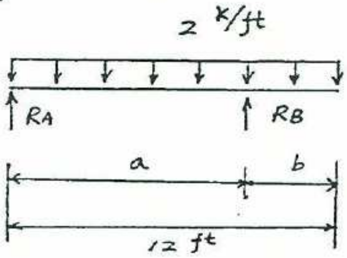
$$\sigma_{max} = \frac{M_{max}}{S} = 1176.47 \ psi < \sigma_{awB} = 1250 \ psi$$

$$\tau_{max} = \frac{3V_{max}}{2A} = 1.5\times\frac{500}{8.75}$$

$= 85.7 \ psi < \tau_{aw} = 95 psi$

O.K.

13-38



$(M_{max})_{left} = (M_{max})_{right}$

$(wl - \frac{1}{2}wl^2/a)(l - \frac{l^2}{2a}) - \frac{1}{2}w(l - \frac{l^2}{2a})$

$= \frac{1}{2}w(l-a)^2 \; \frac{1}{2}(l - \frac{l^2}{2a})^2$

$= \frac{1}{2}(l-a)^2$

$4a^4 - 8a^3 l + 4al^3 - l^4 = 0 \quad (l = 12)$

$4a^4 - 96a^3 + 6912a - 20736 = 0$

(a) solve $a = 8.485$ ft, $b = 3.515$ ft

$M_{max} = \frac{1}{2} \times 2 \times (3.515)^2 = 12.355$ k-ft

$V_{max} = R_B - 2 \times 3.515 = 16.97 - 2 \times 3.515$
$= 9.94^k$

(b) $S_{req} = \frac{M_{max}}{\sigma_{aw}} = \frac{12.355 \times 10^3 \times 12}{1250}$

$= 118.6 \text{ in}^3$

$A_{req} = \frac{3}{2} \frac{V_{max}}{\tau_{aw}} = 1.5 \times \frac{9.94 \times 10^3}{95}$

$= 157 \text{ in}^2$

choose Nominal size $12^{in} \times 14^{in}$

$A = 155 \text{ in}^2$, $S = 349 \text{ in}^3$

approximately O.K. (ref table 10)

$R_A = 7.03^k$, $R_B = 16.97^k$

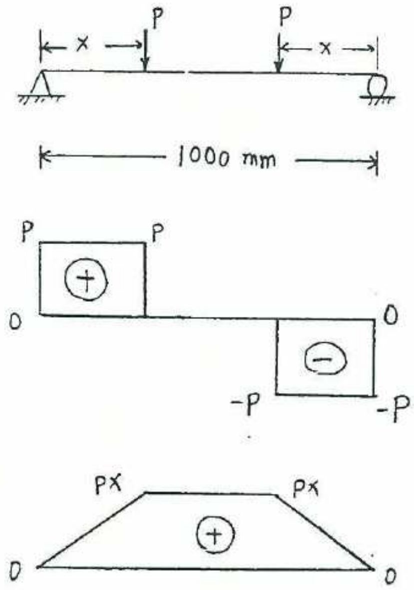
min support length at A

$l_A = \frac{R_A}{\sigma_{aw} \cdot 12} = \frac{7.03 \times 10^3}{12 \times 625} = 0.94$ in

min support length at B

$l_B = \frac{R_B}{\sigma_{aw} \times 12} = \frac{16.97 \times 10^3}{12 \times 625} = 2.26^{in}$

13-39



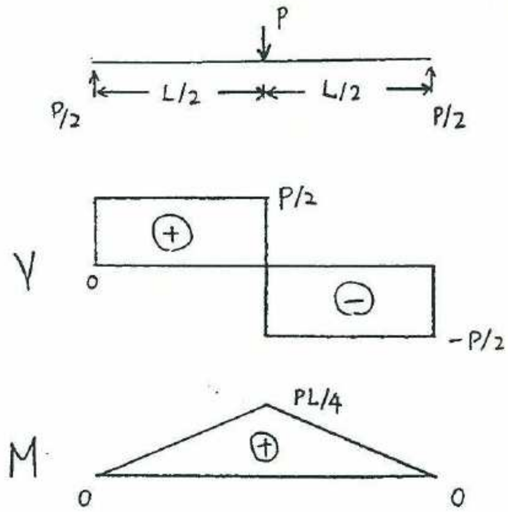
$\tau_{max} = \frac{3V}{2A} = \frac{3P}{2(15000)} = 2.5$

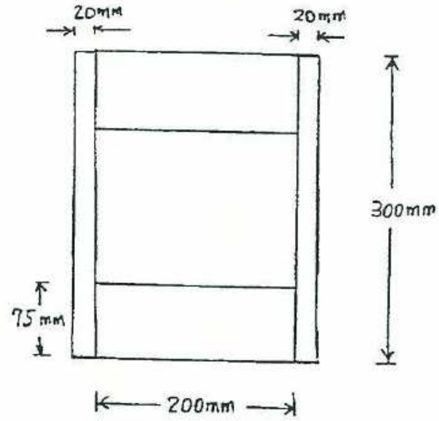
$P = 25000$ N

$\sigma_{max} = \frac{Mc}{I} = \frac{Px(75)}{\frac{1}{12}(100)(150)^3} = 10$

$X = 150$ mm

13-40





$I = \frac{1}{12}[(240)(300)^3 - (200)(150)^3]$

$= 4.8375 \times 10^8 \ mm^4$

(a) shear in glue

$V = \frac{\tau_{aw} I t}{Q} = \frac{(0.4)(4.3875 \times 10^8)(2 \times 75)}{(200 \times 75)(150 - 37.5)}$

$= 15600 \ N$

$P = 2V = 31.2 \ kN$

shear in plywood

$V = \frac{\tau_{aw} I t}{Q} = \frac{(0.8)(4.3875 \times 10^8)(2 \times 20)}{(200 \times 75 \times 112.5) + (2 \times 20 \times 150 \times 75)}$

$= 6568.4 N$

$P = 2V = 13.14 \ kN$ (control)

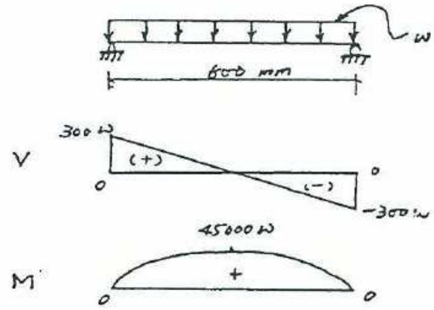
(b) $\sigma_{aw} = \frac{M_{max} C}{I} = \frac{(PL/4) C}{I} = 8$

$L = \frac{32 I}{PC} = \frac{32(4.8375 \times 10^8)}{(13140)(150)} = 7856 \ mm$

(c) $A = \frac{P}{\sigma_{aw}} = \frac{13140}{4.3} = 3055 \ mm^2$

Use 3055 mm^2 bearing plate

13-41



T section :

$\bar{y} = \frac{(1200 \times 10) + (1200 \times 50)}{(1200 \times 2)} = 30 \ mm$ from top

$I = \frac{60(20)^3}{12} + (1200)(20)^2 + \frac{(20)(60)^3}{12} + (1200)(20)^2$

$= 1.36 \times 10^6 \ mm^4$

$\sigma_{all} = \frac{MC}{I} = \frac{45000 \omega \times 50}{1.36 \times 10^6} = 4 \times 10^{-3}$

$\omega = 2.42 \times 10^{-3} \ KN/mm$

$\tau_{glue} = \frac{VQ}{It} = \frac{300 \omega \times 1200(20)}{(1.36 \times 10^6) \times 20}$

$= 400 \times 10^{-6}$

$\omega = 1.51 \times 10^{-3} \ KN/mm$

$\tau_{pl.} = \frac{VQ}{It} = \frac{300 \omega \times (50 \times 20) \times 25}{(1.36 \times 10^6) \times 20}$

$= 600 \times 10^{-6}$

$\omega = 2.18 \times 10^{-3} \ KN/mm$

Rectangular section :

$I = \frac{bh^3}{12} = \frac{(40)(60)^3}{12} = 7.2 \times 10^5 \ mm^4$

$A = (60 \times 20) \times 2 = 2400 \ mm^2$

$\sigma_{all} = \frac{MC}{I} = \frac{45000 \omega (30)}{7.2 \times 10^5} = 4 \times 10^{-3}$

$$\omega = 2.13 \times 10^{-3}\ \text{KN/mm}$$

$$\tau_{pl.} = \frac{3V}{2A} = \frac{3(300\,\omega)}{2 \times 2400} = 600 \times 10^{-3}$$

$$\omega = 3.2 \times 10^{-3}\ \text{KN/mm}$$

$\therefore$ use rectangular section with $\omega = 2.13$ KN/m

13-42

$W_L = 1\ ^{k}/_{ft} \qquad l = 24\ ft$

$$M_L = \frac{1}{8} W_L l^2 = \frac{1}{8} \times 1 \times 24 \times 1000 \times 12$$

$$= 864000\ \text{lb-in}$$

$$V_L = \frac{1}{2} W_L l = \frac{1}{2} \times 1 \times 24 = 12 \times 1000$$

$$= 12000\ \text{lb}$$

Steel

$$S_1 = \frac{M_L}{\sigma_{awB}} = \frac{864000}{24000} = 36\ in^3$$

$$A_1 = \frac{3V_L}{2\tau_{aw}} = 1.5 \times \frac{12000}{14400} = 1.25\ in^2$$

(a) Choose W 10×39

$W_D = 39\ lb/ft \qquad A = 11.5\ in^2$

$S = 42.1\ in^3$

$$S = 42.1\ in^3 > S_1 + \frac{M_D}{\sigma_{awB}}$$

$$= 36 + \frac{\frac{1}{2} \times 39 \times 24^2 \times 12}{24000}$$

$$= 41.62\ in^3 \qquad O.K.$$

$$A = 11.5\ in^2 > A_1 + \frac{3}{2}\frac{V_D}{\tau_{aw}}$$

$$= 1.25 + \frac{1.5 \times \frac{1}{2} \times 39 \times 24}{14400}$$

$$= 1.3\ in^2 \qquad O.K.$$

(b) $$\frac{W_D}{W_T} = \frac{W_D}{W_D + W_L} = \frac{0.039}{1} = 3.9\%$$

Wood

$$S_2 = \frac{M_L}{\sigma_{awB}} = \frac{864000}{1250} = 691.2\ in^3$$

$$A_2 = \frac{3}{2}\frac{V_L}{\tau_{aw}} = 1.5 \times \frac{12000}{95} = 189.47\ in^2$$

(a) Choose Nominal Size $12'' \times 22''$

$W_D = 68.7\ ^{lb}/_{ft}, \qquad A = 247\ in^2$

$S = 886\ in^3$

$$S = 886\ in^3 > S_2 + \frac{M_D}{\sigma_{awB}}$$

$$= 691.2 + \frac{\frac{1}{2} \times 68.7 \times 24^2 \times 12}{1250}$$

$$= 881.14\ in^3 \qquad O.K.$$

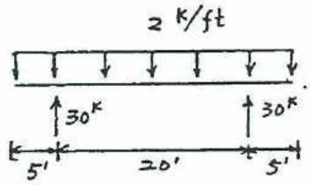
$$A = 247\ in^2 > A_2 + \frac{3}{2}\frac{V_D}{\tau_{aw}}$$

$$= 189.47 + \frac{3}{2} \times \frac{\frac{1}{2} \times 68.7 \times 24}{95}$$

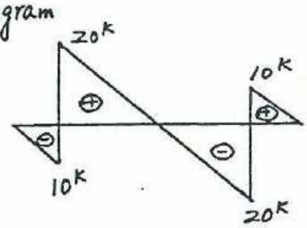
$$= 202.49\ in^2$$

(b) $$\frac{W_D}{W_T} = \frac{W_D}{W_D + W_L} = \frac{0.0687}{1} = 6.87\%$$

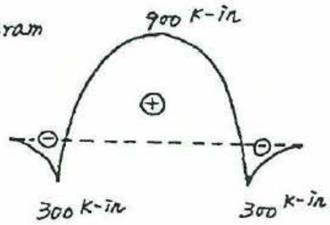
13-43



V- diagram



M- diagram



$M_{max} = 900^{\text{k-in}}$

$V_{max} = 20^{k}$

$S_{req} = \dfrac{M_{max}}{\sigma_{awB}} = \dfrac{900}{24} = 37.5 \ in^3$

$A_{req} = \dfrac{3}{2}\dfrac{V_{max}}{\tau_{aw}} = 1.5 \times \dfrac{20}{14.4} = 2.08 \ in^2$

choose W 12 × 30

$A = 8.79 \ in^2 > A_{req} = 2.08 \ in^2$

$S = 38.6 \ in^3 > S_{req} = 37.5 \ in^3$

13-44

$g_L = 75 \ lb/ft^2, \ g_D = 25 \ lb/ft^2$

(a) design for wooden joists

$W_L = 75 \times \dfrac{16}{12} = 100 \ lb/ft$

$W_D = 25 \times \dfrac{16}{12} = 33.33 \ lb/ft$

$W_T = W_L + W_D = 133.33 \ lb/ft$

$M_{max} = \dfrac{1}{8} W_T \ell^2 = \dfrac{1}{2} \times 133.33 \times (12)^2 \times 12$

$= 28800 \ lb\text{-}in$

$V_{max} = \dfrac{1}{2} W_T \ell = \dfrac{1}{2} \times 133.33 \times 12$

$= 800 \ lb$

choose Nominal Size 2" × 12"

$S = 31.6 \ in^3 > S_{req} = \dfrac{M_{max}}{\sigma_{awB}} = \dfrac{28800}{1200}$

$= 24 \ in^3$

$A = 16.9 \ in^2 > A_{req} = \dfrac{3}{2}\dfrac{V_{max}}{\tau_{aw}}$

$= 1.5 \times \dfrac{800}{100}$

$= 12 \ in^2$ o.k.

(b) design for steel beam

$W_T = (g_D + g_L) \times 12$

$= (100) \times 12 = 1200 \ lb/ft$

$M_{max} = \dfrac{1}{8} W_T \ell^2 = \dfrac{1}{8} \times 1200 \times (20)^2 \times 12$

$= 720^{\text{k-in}}$

$V_{max} = \dfrac{1}{2} W_T \cdot \ell = \dfrac{1}{2} \times 1200 \times 20$

$= 12^{k}$

choose W 12 × 26

$V_D = \dfrac{1}{2} \times 26 \times 20 = 0.26^{k}$

$M_D = \dfrac{1}{8} \times 26 \times (20)^2 \times 12$

$= 15.6^{\text{k-in}}$

$S = 33.4 \ in^3 > S_{req} = \dfrac{M_{max} + M_D}{\sigma_{awB}}$

$= \dfrac{720 + 15.6}{24}$

$= 30.65 \ in^3$

$A = 7.65 \ in^2 > A_{req} = \dfrac{3}{2}\dfrac{(V_{max} + V_D)}{\tau_{aw}}$

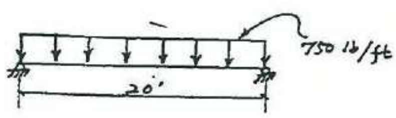
$= 1.5 \times \dfrac{12 + 0.26}{14.4}$

$= 1.277 \ in^3$ o.k.

13-45

Design Load $= 75 + 75 = 150 \ lb/ft^2$

$w = 150 \times 5 = 750 \ lb/ft$



$$M_{max} = \frac{Wl^2}{8} = \frac{750(20)^2}{8} = 37500 \text{ ft-lb}$$

$$V_{max} = \frac{Wl}{2} = \frac{750(20)}{2} = 7500 \text{ lb}$$

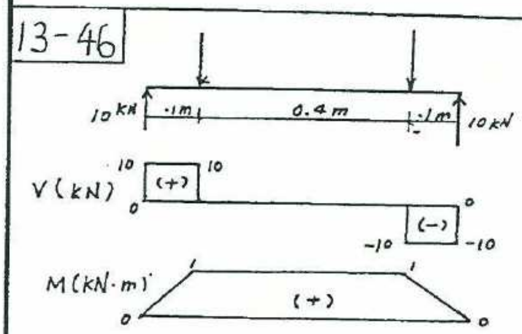
$$S = \frac{M}{\sigma_{all}} = \frac{37500 \times 12}{20{,}000} = 22.50 \text{ in}^3$$

use S 10 × 25.4 beams

$$\tau_{max} = \frac{V}{A_{web}} = \frac{7500}{10 \times 0.31} = 2420 \text{ psi} < \tau_{all}$$

∴ O.K.

13-46



$$\sigma_{all} = \frac{M}{S} = \frac{10^6}{\frac{\pi R^3}{4}} = 80, \quad R = 25.15 \text{ mm}$$

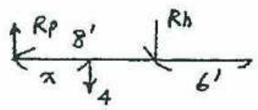
$$D = 50.31 \text{ mm}$$

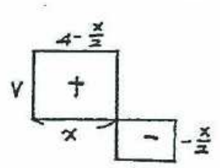
use 51 mm diameter shaft

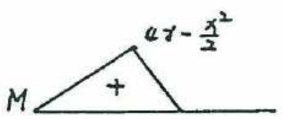
$$\tau_{max} = \frac{4V}{3A} = \frac{4 \times 10^4}{3\left(\frac{\pi \times 51^2}{4}\right)} = 6.53 \text{ MPa} < \tau_{all}$$

∴ O.K.

13-47







$$R \times 8 = 4 \times x$$

$$R = \frac{1}{2}x$$

$$x = 14 \quad R_{max} = 7 \text{ ton}$$

$$M = -42 \text{ ton-ft}$$

$$16 = \frac{42 \times 2.24 \times 12 \times C}{I} \Rightarrow \frac{I}{C} = 70.56 \text{ in}^3$$

choose S 18 × 54.7

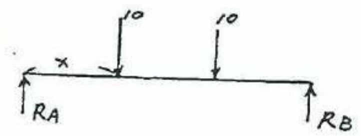
check shear strength

$$V_{max} = 7 \text{ ton} = 7 \times 2.24 = 15.68 \text{ K}$$

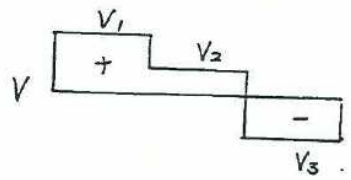
$$\tau_{max} = \frac{15.68 \times A \times C}{It} \Rightarrow \tau_{max} = 6.1 \text{ ksi} < 9.6 \text{ ksi}$$

O.K.

13-48

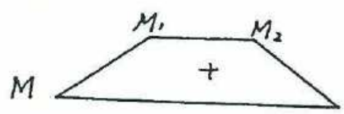


$$R_A = \frac{50}{3} - \frac{2}{900}x, \quad R_B = \frac{2x}{900} + \frac{10}{3}$$



$$V_1 = \frac{50}{3} - \frac{2x}{900}, \quad V_2 = \frac{20}{3} - \frac{2x}{900}$$

$$V_3 = -\frac{10}{3} - \frac{2x}{900}$$



$$M_1 = \frac{50}{3}x - \frac{2}{900}x^2 \qquad M_2 = \frac{70}{3}x - \frac{4x^2}{900}$$

0 0

(a) $\frac{\partial M_1}{\partial x} = \frac{50}{3} - \frac{4x}{900} = 0$, $x = 2625$

$M_{1max} = 30625$

$\frac{\partial M_2}{\partial x} = \frac{140}{3} - \frac{16x}{900} = 0$, $x = 3750$

$M_{2max} = 31250$

so, $x = 3750$ mm , $M_{max} = 31.25$ kN·m

(b) $x=0$, $V_1 = \frac{50}{3}$, $V_2 = \frac{20}{3}$, $V_3 = -\frac{10}{3}$

$x = 6000$, $V_1 = \frac{10}{3}$, $V_2 = -\frac{20}{3}$, $V_3 = -\frac{50}{3}$

so, $x=0$ or 6000 mm.

$|V_{max}| = \frac{50}{3}$

(c) $14\times10^6 = \frac{31.25\times10^3\times6}{0.1h^2}$, $h = 0.366$ m

$$\tau = \frac{\frac{50}{3}\times10^3\times0.366^2\times0.1\times0.5}{\frac{0.1}{12}\times0.366^3\times0.1}$$

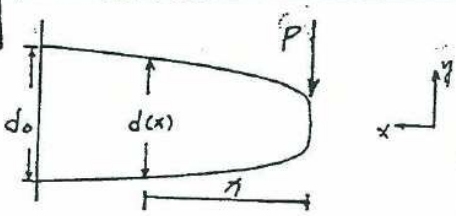
$$= \frac{1\times10^6}{0.366}$$

$\tau = 2.73\times10^6$ Pa $> 1.0\times10^6$ Pa

choose $h = 1.0$, $\tau = 1\times10^6$ Pa o.k.

number of boards $= \frac{1}{0.04} = 25$

13-49



for any $y = c$

$\sigma_x = \frac{PxC}{I(x)} = \frac{PxC}{\frac{\pi}{32}d(x)^4} = $ constant

$\therefore d(x) \propto x^{\frac{1}{4}} \Rightarrow d(x) = Ax^{\frac{1}{4}}$

$d(\ell) = A\ell^{\frac{1}{4}} = d_o$

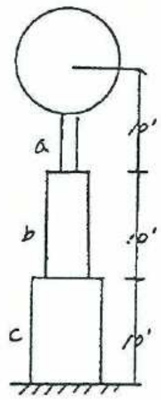
$\Rightarrow A = d_o\cdot\ell^{-\frac{1}{4}}$

$\therefore d(x) = \left(\frac{x}{\ell}\right)^{\frac{1}{4}}\cdot d_o$

plan view is the same as elevation view

13-50 existing 4" standard pipe :

$S = \frac{I}{c} = \frac{7.233}{2.25} = 3.21$ in^3



pipe a: $S_a = 1\times(1.55)$
$= 4.82$ in^3

use 5" pipe ($S=5.45$ in^3)

pipe b: $S_b = 2\times(1.55)$
$= 9.64$ in^3

use 8" pipe ($S=16.81$ in^3)

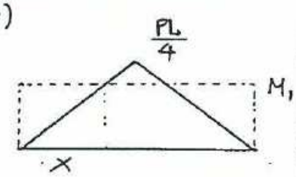
pipe c: $S_c = 3\times(1.55)$
$= 14.47$ in^3

use 10" pipe ($S=29.9$ in^3)

13-51 (a) $M_1 = \sigma_y S_1 = (160\times10^3)(1440\times10^{-6})$
$= 230.4$ kN-m

$$\frac{P(5000)}{4} = \frac{(160\times10^{-3})[212\times10^6+2\times200\times10\times(159)^2]}{(154+10)}$$

$P = 252.2$ kN



$x = \frac{2M_1}{P} = 1.827$ m

length of coverplate

$= 5 - 2\times1.827 = 1.346$ m $\longrightarrow$ use $(1.346+0.016) = 1.362$ m

$$\frac{W_0(5^2)}{8} = \frac{(252.2)(5)}{4} \rightarrow W_0 = 100.88\ kN/m$$

$$\frac{(100.88)(5)}{2}x - \frac{(100.88)}{2}x^2 = 230.4$$

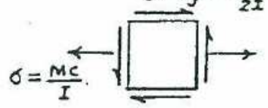
$$x = 1.203$$

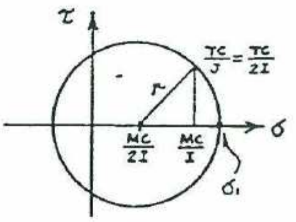
length of coverplate

$= 5 - 2\times 1.203 = 2.594\ m \rightarrow$ use

$(2.594 + 0.016) = 2.61\ m$

13-52





center $= \frac{1}{2}\left(\frac{Mc}{I}\right)$

$$r = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{2I}\right)^2}$$

(a) $$\sigma_1 = \frac{Mc}{2I} + \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{2I}\right)^2}$$

$$= \frac{c}{2I}\left(M + \sqrt{M^2 + T^2}\right) = \frac{c}{J}\left(M + \sqrt{M^2+T^2}\right)$$

(b) $J = \pi d^4/32$, $J/c = \pi d^3/16$

$$\sigma_{all} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2+T^2}\right)$$

$$d = \left(\frac{16}{\pi \sigma_{all}}\left(M + \sqrt{M^2+T^2}\right)\right)^{\frac{1}{3}}$$

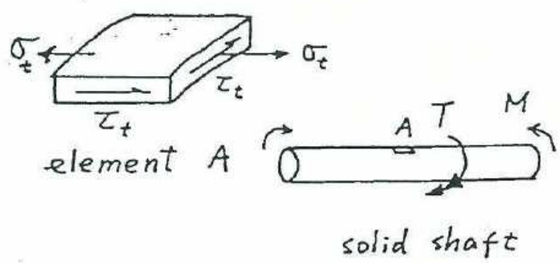
13-53

use equation (13-3)

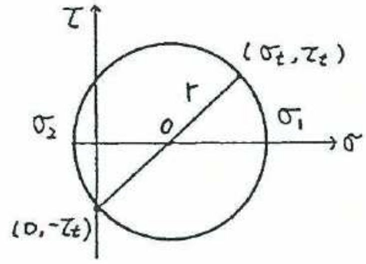
$$d = \sqrt[3]{\frac{16}{\pi \tau_{all}}(M^2+T^2)^{\frac{1}{2}}}$$

$$= \sqrt[3]{\frac{16}{\pi(50000)}(10^2+40^2)^{\frac{1}{2}}} = 0.161\ m$$

13-54



use Mohr's circle to estimate σ_1, σ_2



$r = \frac{1}{2}(\sigma_t^2 + 4\tau_t^2)^{\frac{1}{2}}$

$\sigma_1 = \frac{\sigma_t}{2} + r$

$= \frac{32.6}{c^3}$ kpa

$\sigma_2 = \frac{\sigma_t}{2} - r$

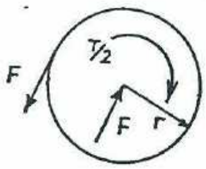
$= \frac{-19.8}{c^3}$ kpa

$$\left(\frac{32.6}{50\times10^3\times c^3}\right)^2 - \left(\frac{32.6}{50\times10^3\times c^3}\right)\left(\frac{-19.8}{50\times10^3\times c^3}\right) + \left(\frac{-19.8}{50\times10^3\times c^3}\right)^2 = 1$$

$\therefore c = 0.097\ m$, $d = 0.194\ m$

13-55

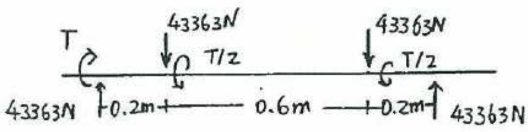
$$T = \frac{159\times 45}{11/60} = 39027\ N\cdot m$$



$F = T/2r$

$= 39027/(2\times0.45)$

$= 43363\ N$



$V_{max} = 43363N$ $\quad M_{max} = 8672.6\ N\cdot m$

use equation (13-3)

$$d = \sqrt[3]{\frac{16}{\pi(40\times10^6)}(43363^2 + 8672.6^2)^{\frac{1}{2}}}$$

$$= 0.178 \text{ m}$$

use standard procedure

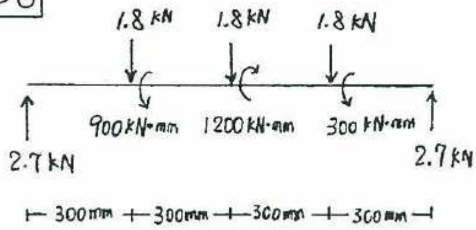
$$T_{all} = \frac{Tc}{J} + \frac{4V}{3A} = \frac{39027\times16}{\pi d^3} + \frac{4\times43363\times4}{3\pi d^3}$$

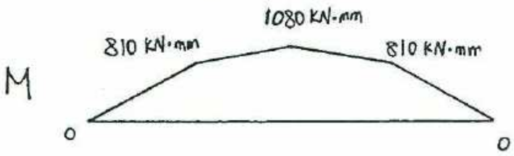
$$40\times10^6 = \frac{198763 + 73615d}{d^3}$$

$$d = 0.174 \text{ m}$$

use 18 cm diameter shaft

13-56





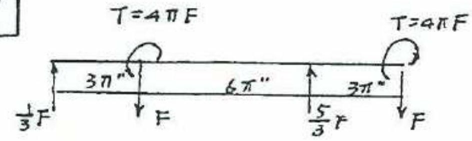
$$M_{max} = 1080 \text{ kN·mm}$$

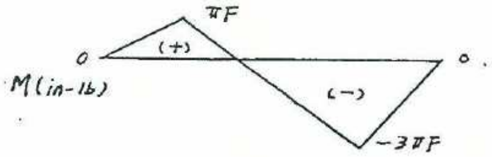
$$T_{max} = 900 \text{ kN·mm}$$

Use equation (13-3)

$$d = \sqrt[3]{\frac{16}{\pi(0.4)}(1080^2 + 900^2)^{\frac{1}{2}}} = 26.16 \text{ mm}$$

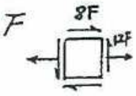
13-57





$$T_{max} = 4\pi F \qquad M_{max} = -3\pi F$$

$$\tau = \frac{TC}{J} = \frac{4\pi F\times1}{\frac{\pi(1)^4}{2}} = 8F$$



$$\sigma = \frac{MC}{I} = \frac{3\pi F\times1}{\frac{\pi(1)^4}{4}} = 12F$$

$$\tau_{max} = \sqrt{\left(\frac{12}{2}\right)^2 + (8)^2} \qquad F = 5000$$

$$F = 500 \text{ lb}$$

13-58

$$\sigma = K_1\frac{Mc}{I} = 1.6\frac{(0.3P)(0.0375)}{\frac{\pi(0.0375)^4}{4}}$$

$$= 11589P$$

$$\tau = k_2\frac{Tc}{J} = 1.2\frac{(0.6P)(0.0375)}{\frac{\pi(0.0375)^4}{2}}$$

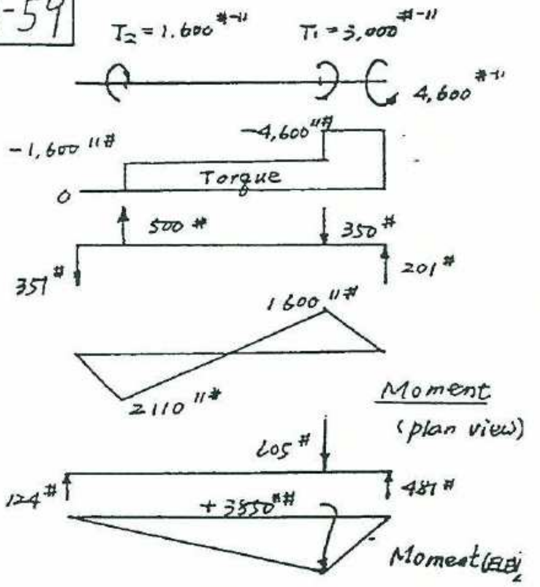
$$= 8692P$$

$$\tau_{max} = \sqrt{\left(\frac{11589P}{2}\right)^2 + (8692P)^2} = 45\times10^6$$

$$P = 4308 \text{ N}$$

0 0

13-59



$$d = \sqrt[3]{\frac{16\times1.5}{\pi\times6000}\times1000\sqrt{4.6^2+1.6^2+3.85^2}} = 2''$$

13-60

(a) from Fig. 2-25 for 10^7 cycles

$S = 190$ MPa

$$\frac{\sigma}{F.S.} = \frac{Mc}{I}$$

$$\frac{190}{2.5} = \frac{\frac{p(800)^2}{8}(100)}{\frac{\pi(100)^4}{4}}$$

$\rightarrow p = 746$ N/mm

(b) $$\tau_{all} = \frac{VQ}{It}$$

where $\tau_{all} = \frac{\sigma_{all}}{2} = \frac{(190/2.5)}{2} = 38$ MPa

$$Q = \frac{\pi d^2}{2}\,\frac{4d}{3\pi} = \frac{2d^3}{3}$$

$I = \frac{\pi d^4}{4}$, $t = 2d$

$$\frac{VQ}{It} = \frac{V\left(\frac{2d^3}{3}\right)}{\left(\frac{\pi d^4}{4}\right)(2d)} = \frac{4V}{3\pi d^2} = 38\times10^6$$

$$\frac{4(746\times400)}{3\pi d^2} = 38\times10^6$$

$d = 5.8\times10^{-2}$ m

$= 58$ mm