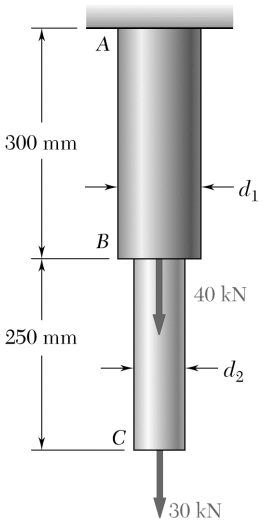
CHAPTER 1



PROBLEM 1.1

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.

SOLUTION

(*a*) Rod *AB*

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi d_1^2}$$

$$d_1 = \sqrt{\frac{4P}{\pi \sigma_{AB}}} = \sqrt{\frac{(4)(70 \times 10^3)}{\pi (175 \times 10^6)}} = 22.6 \times 10^{-3} \text{ m} \qquad d_1 = 22.6 \text{ mm} \blacktriangleleft$$

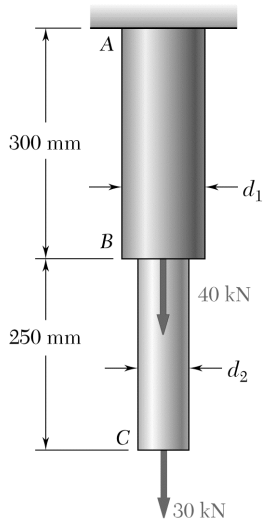
(*b*) Rod *BC*

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{P}{\frac{\pi}{4} d_2^2} = \frac{4P}{\pi d_2^2}$$

$$d_2 = \sqrt{\frac{4P}{\pi \sigma_{BC}}} = \sqrt{\frac{(4)(30 \times 10^3)}{\pi (150 \times 10^6)}} = 15.96 \times 10^{-3} \text{ m} \qquad d_2 = 15.96 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.2

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*

$$P = 40 + 30 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{ Pa} \qquad \sigma_{AB} = 35.7 \text{ MPa} \blacktriangleleft$$

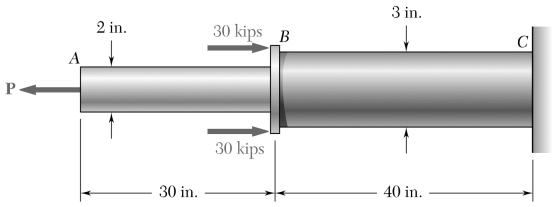
(*b*) Rod *BC*

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{ Pa} \qquad \sigma_{BC} = 42.4 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{3.1416}$$

$$= 0.31831P$$

$$A_{BC} = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{(2)(30) - P}{A_{AB}}$$

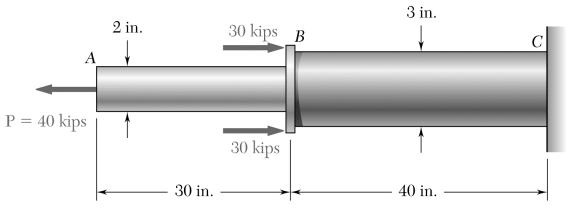
$$= \frac{60 - P}{7.0686} = 8.4883 - 0.14147P$$

Equating σ_{AB} to $2\sigma_{BC}$

$$0.31831P = 2(8.4883 - 0.14147P) \qquad P = 28.2 \text{ kips} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.4



In Prob. 1.3, knowing that $P = 40$ kips, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

PROBLEM 1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

(*a*) Rod *AB*

$P = 40$ kips (tension)

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi(2)^2}{4} = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{40}{3.1416} \qquad \sigma_{AB} = 12.73 \text{ ksi} \blacktriangleleft$$

(*b*) Rod *BC*

$F = 40 - (2)(30) = -20$ kips, i.e., 20 kips compression.

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi(3)^2}{4} = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-20}{7.0686} \qquad \sigma_{BC} = -2.83 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.5

Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

For the bolt, $\sigma_b = \dfrac{F_b}{A_b} = \dfrac{4P_b}{\pi d_b^2}$ or $P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

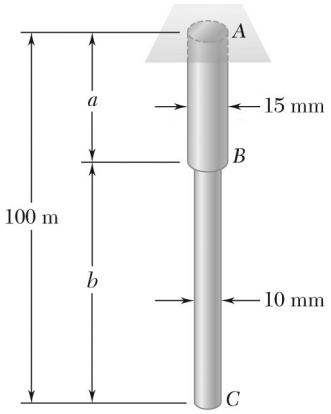
For the spacer, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)}\, d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16) \qquad d_s = 25.2 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.6

Two brass rods AB and BC, each of uniform diameter, will be brazed together at B to form a nonuniform rod of total length 100 m, which will be suspended from a support at A as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod AB for which the maximum normal stress in ABC is minimum, (*b*) the corresponding value of the maximum normal stress.

SOLUTION

Areas: $$A_{AB} = \frac{\pi}{4}(15\text{ mm})^2 = 176.71\text{ mm}^2 = 176.71\times10^{-6}\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}(10\text{ mm})^2 = 78.54\text{ mm}^2 = 78.54\times10^{-6}\text{m}^2$$

From geometry, $$b = 100 - a$$

Weights: $$W_{AB} = \rho g\, A_{AB}\ell_{AB} = (8470)(9.81)(176.71\times10^{-6})\,a = 14.683a$$

$$W_{BC} = \rho g\, A_{BC}\ell_{BC} = (8470)(9.81)(78.54\times10^{-6})(100 - a) = 652.59 - 6.526a$$

Normal stresses:

At A, $$P_A = W_{AB} + W_{BC} = 652.59 + 8.157a \qquad (1)$$

$$\sigma_A = \frac{P_A}{A_{AB}} = 3.6930\times10^6 + 46.160\times10^3 a$$

At B, $$P_B = W_{BC} = 652.59 - 6.526a \qquad (2)$$

$$\sigma_B = \frac{P_B}{A_{BC}} = 8.3090\times10^6 - 83.090\times10^3 a$$

(*a*) Length of rod AB. The maximum stress in ABC is minimum when $\sigma_A = \sigma_B$ or

$$4.6160\times10^6 - 129.25\times10^3 a = 0$$

$$a = 35.71\text{ m} \qquad \ell_{AB} = a = 35.7\text{ m} \blacktriangleleft$$

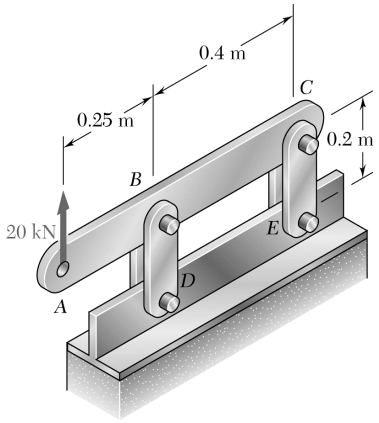
(*b*) Maximum normal stress.

$$\sigma_A = 3.6930\times10^6 + (46.160\times10^3)(35.71)$$

$$\sigma_B = 8.3090\times10^6 - (83.090\times10^3)(35.71)$$

$$\sigma_A = \sigma_B = 5.34\times10^6\text{ Pa} \qquad \sigma = 5.34\text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

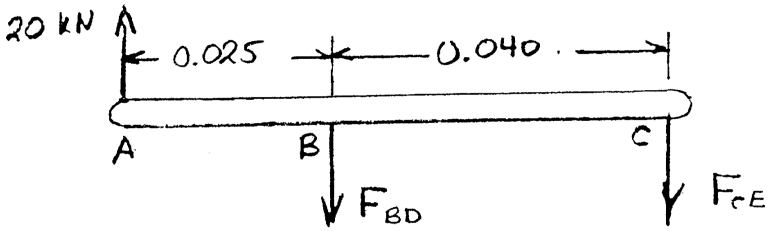


PROBLEM 1.7

Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$\Sigma M_C = 0$: $(0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$

$F_{BD} = 32.5 \times 10^3 \text{N}$ Link *BD* is in tension.

$\Sigma M_B = 0$: $-(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0$

$F_{CE} = -12.5 \times 10^3 \text{N}$ Link *CE* is in compression.

Net area of one link for tension $= (0.008)(0.036 - 0.016) = 160 \times 10^{-6}\text{m}^2$.

For two parallel links, $A_{\text{net}} = 320 \times 10^{-6}\text{m}^2$

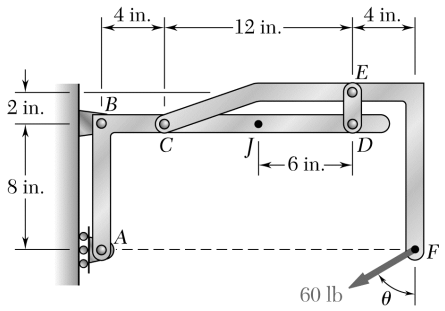
(*a*) $\sigma_{BD} = \dfrac{F_{BD}}{A_{\text{net}}} = \dfrac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6$ $\sigma_{BD} = 101.6$ MPa ◀

Area for one link in compression $= (0.008)(0.036) = 288 \times 10^{-6}\text{m}^2$.

For two parallel links, $A = 576 \times 10^{-6}\text{m}^2$

(*b*) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^{-6}$ $\sigma_{CE} = -21.7$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

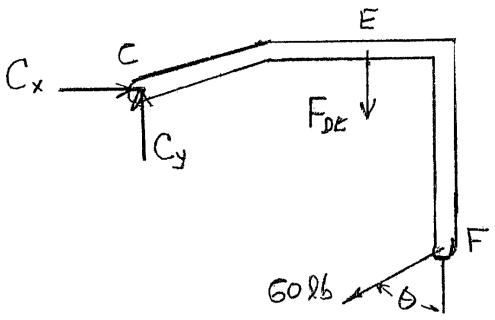


PROBLEM 1.8

Knowing that the link *DE* is $\frac{1}{8}$ in. thick and 1 in. wide, determine the normal stress in the central portion of that link when (*a*) $\theta = 0°$, (*b*) $\theta = 90°$.

SOLUTION

Use member *CEF* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad -12\,F_{DE} - (8)(60\sin\theta) - (16)(60\cos\theta) = 0$$

$$F_{DE} = -40\sin\theta - 80\cos\theta \text{ lb.}$$

$$A_{DE} = (1)\left(\frac{1}{8}\right) = 0.125 \text{ in.}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

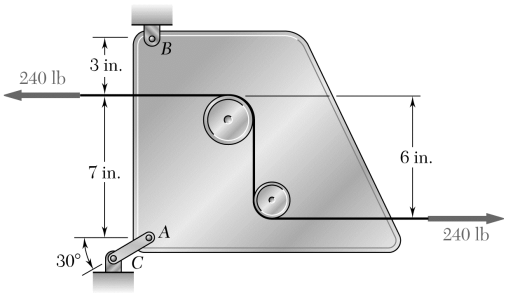
(*a*) $\underline{\theta = 0}$: $F_{DE} = -80$ lb.

$$\sigma_{DE} = \frac{-80}{0.125} \qquad \sigma_{DE} = -640 \text{ psi} \blacktriangleleft$$

(*b*) $\underline{\theta = 90°}$: $F_{DE} = -40$ lb.

$$\sigma_{DE} = \frac{-40}{0.125} \qquad \sigma_{DE} = -320 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

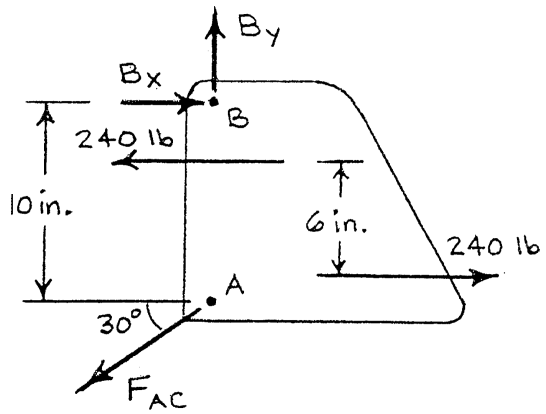


PROBLEM 1.9

Link AC has a uniform rectangular cross section $\frac{1}{16}$ in. thick and $\frac{1}{4}$ in. wide. Determine the normal stress in the central portion of the link.

SOLUTION

Free Body Diagram of Plate



Note that the two 240-lb forces form a couple of moment

$$(240 \text{ lb})(6 \text{ in.}) = 1440 \text{ lb} \cdot \text{in.}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 1440 \text{ lb} \cdot \text{in} - (F_{AC} \cos 30°)(10 \text{ in.}) = 0$$

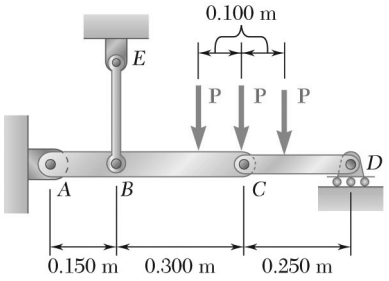
$$F_{AC} = 166.277 \text{ lb.}$$

Area of link: $$A_{AC} = \left(\frac{1}{16} \text{ in.}\right)\left(\frac{1}{4} \text{ in.}\right) = 0.015625 \text{ in.}^2$$

Stress: $$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{166.277}{0.015625} = 10640 \text{ psi}$$

$\sigma_{AC} = 10.64$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

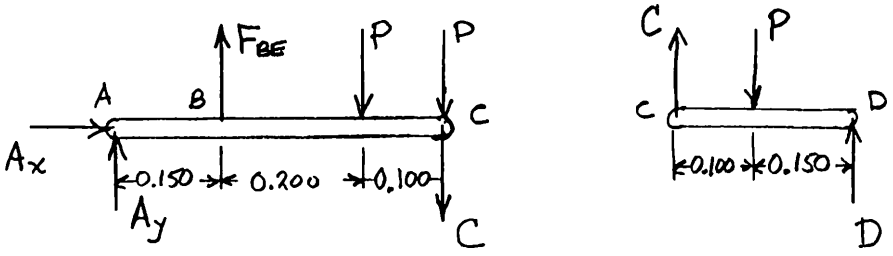


PROBLEM 1.10

Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod BE for which the normal stress in that portion is +100 MPa.

SOLUTION

Draw free body diagrams of AC and CD.



Free Body CD: $+\circlearrowright \Sigma M_D = 0$: $0.150P - 0.250C = 0$

$$C = 0.6P$$

Free Body AC: $+\circlearrowright M_A = 0$: $0.150F_{BE} - 0.350P - 0.450P - 0.450C = 0$

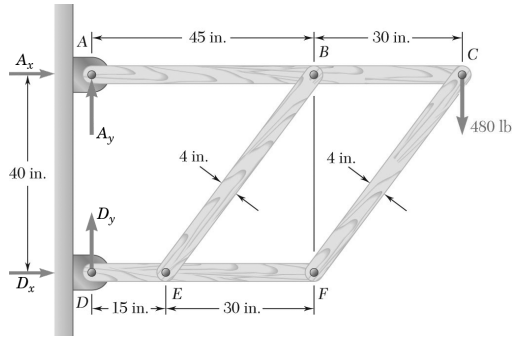
$$F_{BE} = \frac{1.07}{0.150}P = 7.1333\,P = (7.133)(4\text{ kN}) = 28.533\text{ kN}$$

Required area of BE: $\sigma_{BE} = \dfrac{F_{BE}}{A_{BE}}$

$$A_{BE} = \frac{F_{BE}}{\sigma_{BE}} = \frac{28.533 \times 10^3}{100 \times 10^6} = 285.33 \times 10^{-6}\text{m}^2$$

$$A_{BE} = 285\text{ mm}^2 \blacktriangleleft$$

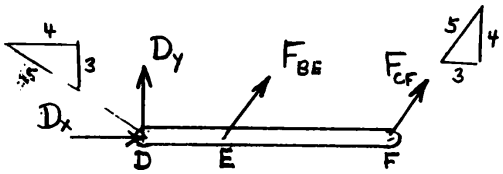
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.11

The frame shown consists of *four* wooden members, *ABC, DEF, BE,* and *CF*. Knowing that each member has a 2×4-in. rectangular cross section and that each pin has a 1/2-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

SOLUTION



Add support reactions to figure as shown.

Using entire frame as free body,

$$\Sigma M_A = 0: \quad 40\,D_x - (45 + 30)(480) = 0$$

$$D_x = 900 \text{ lb}.$$

Use member DEF as free body.

Reaction at D must be parallel to F_{BE} and F_{CF}.

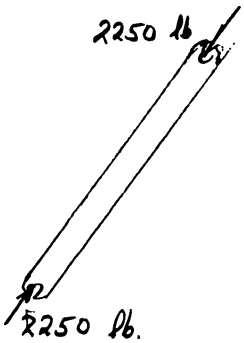
$$D_y = \frac{4}{3} D_x = 1200 \text{ lb}.$$

$$\Sigma M_F = 0: \quad -(30)\left(\frac{4}{5} F_{BE}\right) - (30 + 15) D_Y = 0$$

$$F_{BE} = -2250 \text{ lb}.$$

$$\Sigma M_E = 0: \quad (30)\left(\frac{4}{5} F_{CE}\right) - (15) D_Y = 0$$

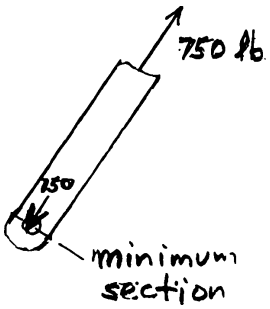
$$F_{CE} = 750 \text{ lb}.$$



Stress in compression member *BE*

Area: $A = 2 \text{ in} \times 4 \text{ in} = 8 \text{ in}^2$

(*a*) $$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{-2250}{8} \qquad \sigma_{BE} = -281 \text{ psi} \blacktriangleleft$$



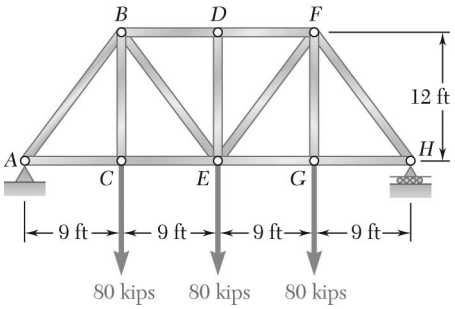
Minimum section area occurs at pin.

$$A_{\min} = (2)(4.0 - 0.5) = 7.0 \text{ in}^2$$

Stress in tension member *CF*

(*b*) $$\sigma_{CF} = \frac{F_{CF}}{A_{\min}} = \frac{750}{7.0} \qquad \sigma_{CF} = 107.1 \text{ psi} \blacktriangleleft$$

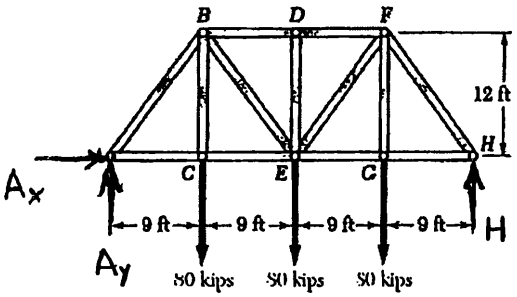
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.12

For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.

SOLUTION

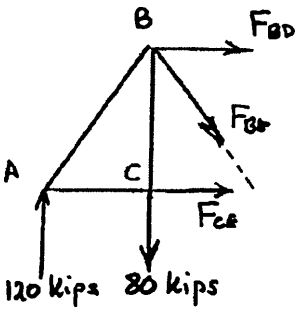


Use entire truss as free body.

$$+\circlearrowleft \Sigma M_H = 0: \quad (9)(80) + (18)(80) + (27)(80) - 36A_y = 0$$

$$A_y = 120 \text{ kips}$$

Use portion of truss to the left of a section cutting members *BD*, *BE*, and *CE*.

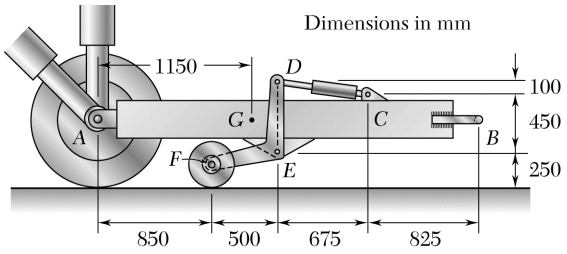


$$+\uparrow \Sigma F_y = 0: \quad 120 - 80 - \frac{12}{15}F_{BE} = 0 \qquad \therefore F_{BE} = 50 \text{ kips}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50 \text{ kips}}{5.87 \text{ in}^2}$$

$$\sigma_{BE} = 8.52 \text{ ksi} \blacktriangleleft$$

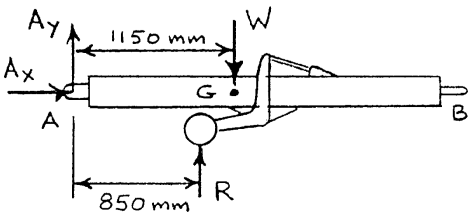
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.13

An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

SOLUTION

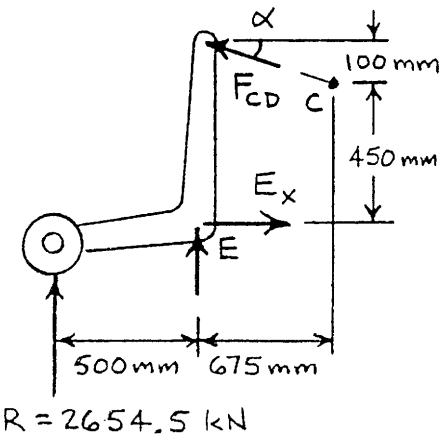


FREE BODY – ENTIRE TOW BAR:

$$W = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962.00 \text{ N}$$

$$+\circlearrowright \Sigma M_A = 0: \quad 850R - 1150(1962.00 \text{ N}) = 0$$

$$R = 2654.5 \text{ N}$$



FREE BODY – BOTH ARM & WHEEL UNITS:

$$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$$

$$+\circlearrowright \Sigma M_E = 0: \quad (F_{CD}\cos\alpha)(550) - R(500) = 0$$

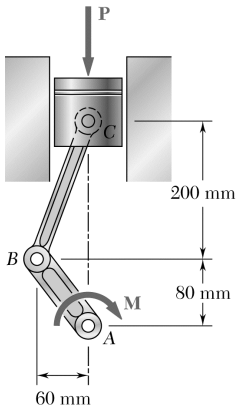
$$F_{CD} = \frac{500}{550\cos 8.4270°}(2654.5 \text{ N})$$

$$= 2439.5 \text{ N} \quad \text{(comp.)}$$

$$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5 \text{ N}}{\pi(0.0125 \text{ m})^2}$$

$$= -4.9697 \times 10^6 \text{ Pa} \qquad \sigma_{CD} = -4.97 \text{ MPa} \blacktriangleleft$$

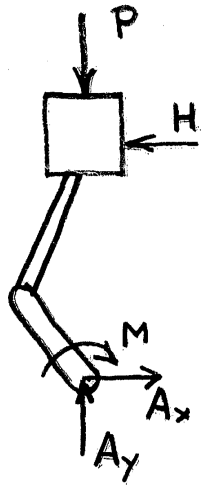
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.14

A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

SOLUTION



Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

$$+\circlearrowleft \Sigma M_A = 0: \quad (0.280\ \text{m})H - 1500\ \text{N}\cdot\text{m} = 0$$

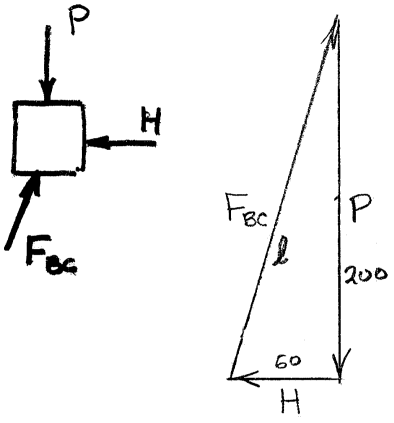
$$H = 5.3571\times10^3\ \text{N}$$

Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81\ \text{mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86\times10^3\ \text{N}$$

(*a*) $P = 17.86$ kN ◀



$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643\times10^3\ \text{N}$$

Rod *BC* is a compression member. Its area is

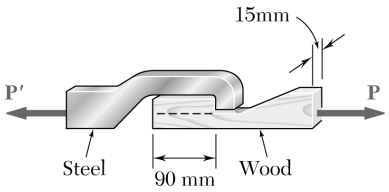
$$450\ \text{mm}^2 = 450\times10^{-6}\ \text{m}^2$$

Stress,

$$\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.643\times10^3}{450\times10^{-6}} = -41.4\times10^6\ \text{Pa}$$

(*b*) $\sigma_{BC} = -41.4$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.15

When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

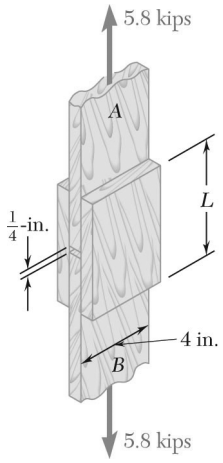
SOLUTION

Area being sheared: $A = 90\text{ mm} \times 15\text{ mm} = 1350\text{ mm}^2 = 1350 \times 10^{-6}\text{m}^2$

Force: $P = 8 \times 10^3\text{ N}$

Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^6\text{ Pa}$ $\tau = 5.93\text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



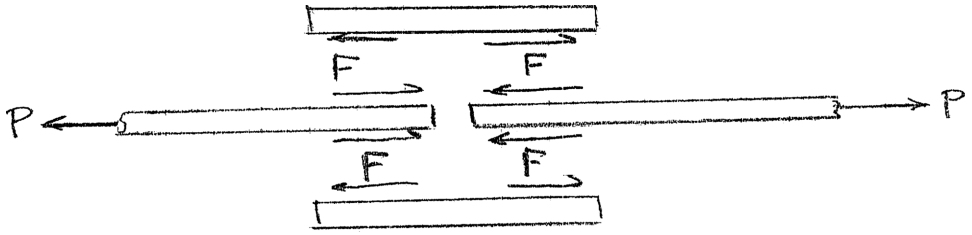
PROBLEM 1.16

The wooden members A and B are to be joined by plywood splice plates, that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ in., determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 120 psi.

SOLUTION

There are four separate areas that are glued. Each of these areas transmits one half the 5.8 kip force. Thus

$$F = \frac{1}{2}P = \frac{1}{2}(5.8) = 2.9 \text{ kips} = 2900 \text{ lb.}$$



Let l = length of one glued area and $w = 4$ in. be its width.

For each glued area, $A = lw$

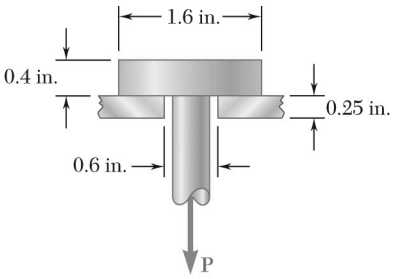
Average shearing stress: $\tau = \dfrac{F}{A} = \dfrac{F}{lw}$

The allowable shearing stress is $\tau = 120$ psi

Solving for l, $l = \dfrac{F}{\tau w} = \dfrac{2900}{(120)(4)} = 6.0417 \text{ in.}$

Total length L: $L = l + (\text{gap}) + l = 6.0417 + \dfrac{1}{4} + 6.0417$ $L = 12.33 \text{ in.}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.17

A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For steel:

$$A_1 = \pi dt = \pi(0.6)(0.4)$$
$$= 0.7540 \text{ in}^2$$

$$\tau_1 = \frac{P}{A} \therefore P = A_1\tau_1 = (0.7540)(18)$$
$$= 13.57 \text{ kips}$$

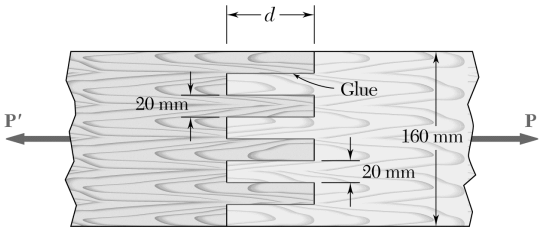
For aluminum:

$$A_2 = \pi dt = \pi(1.6)(0.25) = 1.2566 \text{ in}^2$$

$$\tau_2 = \frac{P}{A_2} \therefore P = A_2\tau_2 = (1.2566)(10) = 12.57 \text{ kips}$$

Limiting value of P is the smaller value, so $P = 12.57$ kips ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.18

Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.

SOLUTION

Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$.

Let $t = 22$ mm.

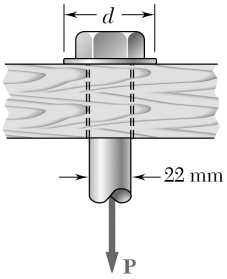
Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3} \text{m}^2$$

$$= 1.32404 \times 10^3 \text{mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2 \qquad d = 60.2 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.19

The load **P** applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

SOLUTION

Steel rod: $A = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6}\text{m}^2$

$$\sigma = 35 \times 10^6 \text{Pa}$$
$$P = \sigma A = (35 \times 10^6)(380.13 \times 10^{-6})$$
$$= 13.305 \times 10^3 \text{N}$$

Washer: $\sigma_b = 5 \times 10^6 \text{Pa}$

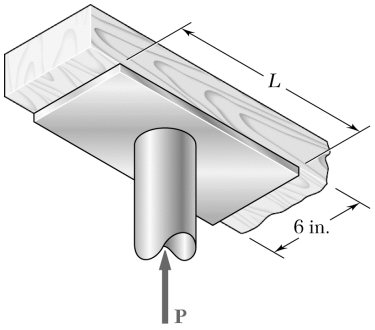
Required bearing area:

$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3}\text{m}^2$$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

$$d^2 = d_i^2 + \frac{4A_b}{\pi}$$
$$= (0.025)^2 + \frac{(4)(2.6609 \times 10^{-3})}{\pi}$$
$$= 4.013 \times 10^{-3}\text{m}^2$$
$$d = 63.3 \times 10^{-3}\text{m} \qquad d = 63.3 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.20

The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

SOLUTION

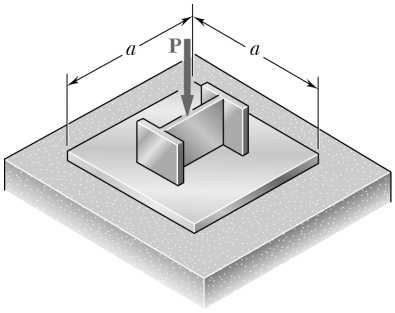
Bearing area: $A_b = Lw$

$$\sigma_b = \frac{P}{A_b} = \frac{P}{Lw}$$

$$L = \frac{P}{\sigma_b w} = \frac{20 \times 10^3}{(400)(6)} = 8.33 \text{ in.}$$

$L = 8.33$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.21

An axial load **P** is supported by a short $W8 \times 40$ column of cross-sectional area $A = 11.7\,\text{in.}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

SOLUTION

For the column $\sigma = \dfrac{P}{A}$ or

$$P = \sigma A = (30)(11.7) = 351 \text{ kips}$$

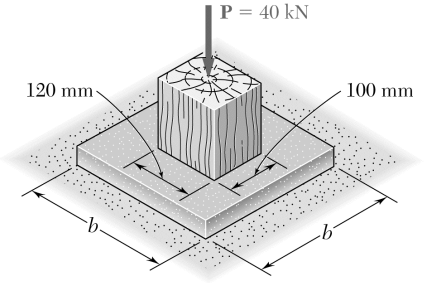
For the $a \times a$ plate, $\sigma = 3.0$ ksi

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117 \text{ in}^2$$

Since the plate is square, $A = a^2$

$$a = \sqrt{A} = \sqrt{117} \qquad a = 10.82 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.22

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40\,\text{kN} = 40 \times 10^3\,\text{N}$$

$$A = (100)(120) = 12 \times 10^3\,\text{mm}^2 = 12 \times 10^{-3}\,\text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.333 \times 10^6\,\text{Pa} \qquad 3.33\ \text{MPa} \blacktriangleleft$$

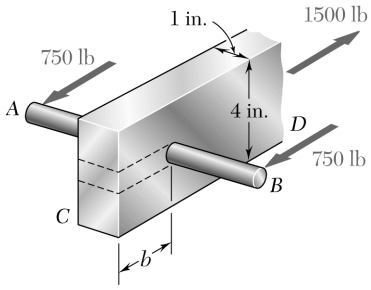
(*b*) Footing area. $P = 40 \times 10^3\,\text{N}$ $\sigma = 145\ \text{kPa} = 45 \times 10^3\,\text{Pa}$

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586\ \text{m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525\ \text{m} \qquad b = 525\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.23

A $\frac{5}{8}$-in.-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 100 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(*a*) Maximum normal stress in the wood

$$A_{\text{net}} = (1)\left(4 - \frac{5}{8}\right) = 3.375 \text{ in.}^2$$

$$\sigma = \frac{P}{A_{\text{net}}} = \frac{1500}{3.375} = 444 \text{ psi} \qquad \sigma = 444 \text{ psi} \blacktriangleleft$$

(*b*) Distance *b* for $\tau = 100$ psi

For sheared area see dotted lines.

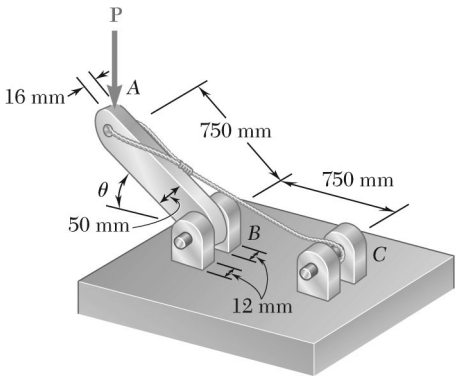
$$\tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{1500}{(2)(1)(100)} = 7.50 \text{ in.} \qquad b = 7.50 \text{ in.} \blacktriangleleft$$

(*c*) Average bearing stress on the wood

$$\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1500}{\left(\frac{5}{8}\right)(1)} = 2400 \text{ psi} \qquad \sigma_b = 2400 \text{ psi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

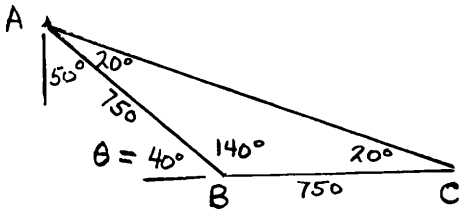


PROBLEM 1.24

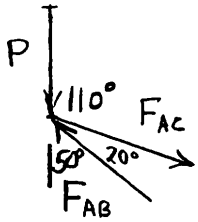
Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

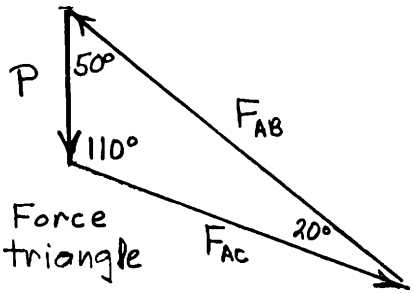
SOLUTION

Geometry: Triangle *ABC* is an isoseles triangle with angles shown here.



Use joint A as a free body.





Law of sines applied to force triangle

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°}$$

$$= \frac{(9)\sin 110°}{\sin 20°} = 24.73 \text{ kN}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.24 *(Continued)*

(*a*) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \text{ where } F_{AB} = 24.73 \times 10^3 \text{N}$$

$$d^2 = \frac{2F_{AB}}{\pi\tau} = \frac{(2)(24.73 \times 10^3)}{\pi(120 \times 10^6)} = 131.18 \times 10^{-6} \text{m}^2$$

$$d = 11.45 \times 10^{-3} \text{m} \qquad 11.45 \text{ mm} \blacktriangleleft$$

(*b*) Bearing stress in *AB* at *A*.

$$A_b = td = (0.016)(11.45 \times 10^{-3}) = 183.26 \times 10^{-6} \text{m}^2$$

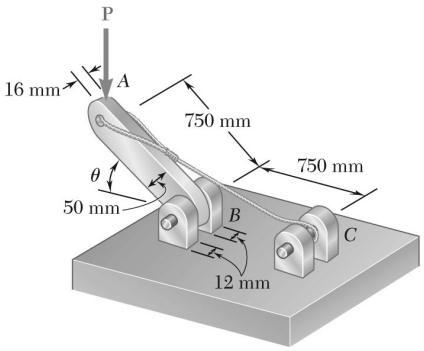
$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.73 \times 10^3}{183.26 \times 10^{-6}} = 134.9 \times 10^6 \qquad 134.9 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing stress in support brackets at *B*.

$$A = td = (0.012)(11.45 \times 10^{-3}) = 137.4 \times 10^{-6} \text{m}^2$$

$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.73 \times 10^3)}{137.4 \times 10^{-6}} = 90.0 \times 10^6 \qquad 90.0 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

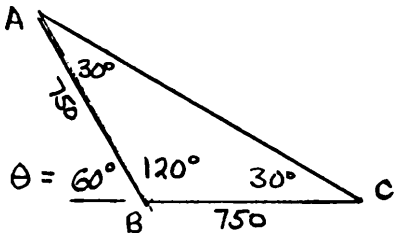


PROBLEM 1.25

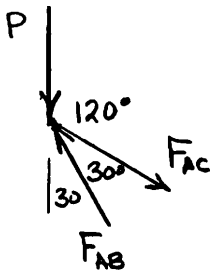
Determine the largest load **P** which may be applied at *A* when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at *B* must not exceed 120 MPa and that the average bearing stress in member *AB* and in the bracket at *B* must not exceed 90 MPa.

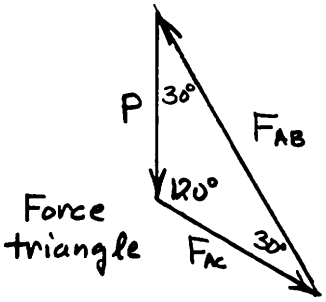
SOLUTION

Geometry: Triangle ABC is an isoseles triangle with angles shown here.



Use joint *A* as a free body.





Law of sines applied to force triangle

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB} \sin 30°}{\sin 120°} = 0.57735\, F_{AB}$$

$$P = \frac{F_{AC} \sin 30°}{\sin 30°} = F_{AC}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.25 ***(Continued)***

If shearing stress in pin at B is critical,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54 \times 10^{-6})(120 \times 10^{6}) = 18.850 \times 10^{3}\text{N}$$

If bearing stress in member AB at bracket at A is critical,

$$A_b = td = (0.016)(0.010) = 160 \times 10^{-6}\text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160 \times 10^{-6})(90 \times 10^{6}) = 14.40 \times 10^{3}\text{N}$$

If bearing stress in the bracket at B is critical,

$$A_b = 2td = (2)(0.012)(0.010) = 240 \times 10^{-6}\text{m}^2$$

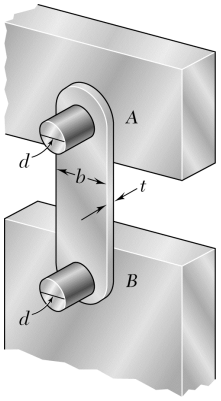
$$F_{AB} = A_b\sigma_b = (240 \times 10^{-6})(90 \times 10^{6}) = 21.6 \times 10^{3}\text{N}$$

Allowable F_{AB} is the smallest, i.e., $14.40 \times 10^{3}\text{N}$

Then from Statics $P_{\text{allow}} = (0.57735)(14.40 \times 10^{3})$

$= 8.31 \times 10^{3}\text{N}$ 8.31 kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.26

Link AB, of width $b = 50$ mm and thickness $t = 6$ mm, is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is − 140 MPa, and that the average shearing stress in each of the two pins is 80 MPa, determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.

SOLUTION

Rod AB is in compression.

$$A = bt \quad \text{where} \quad b = 50 \text{ mm} \quad \text{and} \quad t = 6 \text{ mm}$$

$$A = (0.050)(0.006) = 300 \times 10^{-6} \text{m}^2$$

$$P = -\sigma A = -(-140 \times 10^{6})(300 \times 10^{-6})$$

$$= 42 \times 10^{3} \text{N}$$

For the pin,

$$A_p = \frac{\pi}{4} d^2 \quad \text{and} \quad \tau = \frac{P}{A_p}$$

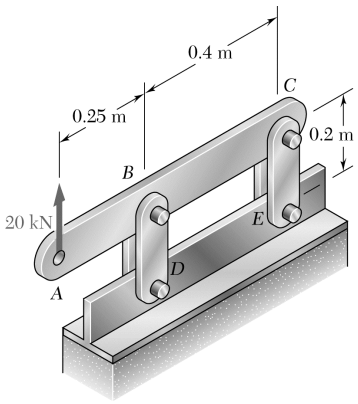
$$A_p = \frac{P}{\tau} = \frac{42 \times 10^{3}}{80 \times 10^{6}} = 525 \times 10^{-6} \text{m}^2$$

(*a*) Diameter d

$$d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{(4)(525 \times 10^{-6})}{\pi}} = 2.585 \times 10^{-3} \text{m} \qquad d = 25.9 \text{ mm} \blacktriangleleft$$

(*b*) Bearing stress $\qquad \sigma_b = \dfrac{P}{dt} = \dfrac{42 \times 10^{3}}{(25.85 \times 10^{-3})(0.006)} = 271 \times 10^{6} \text{ Pa} \qquad \sigma_b = 271 \text{ MPa} \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



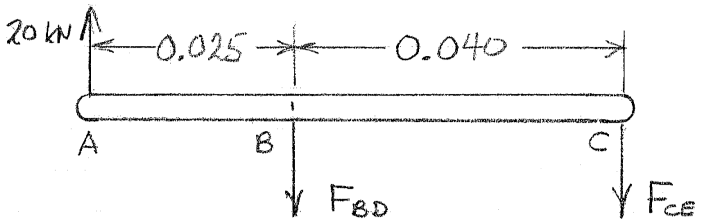
PROBLEM 1.27

For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10×50-mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(*a*) Shear pin at *B* $\quad \tau = \dfrac{F_{BD}}{2A}$ for double shear,

where $\quad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad \tau = 80.8 \text{ MPa} \blacktriangleleft$$

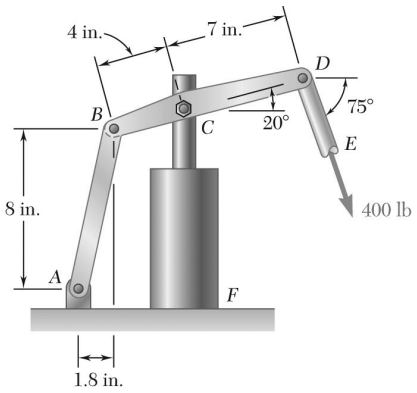
(*b*) Bearing: link *BD* $\quad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad \sigma_b = 127.0 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing in *ABC* at *B* $\quad A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad \sigma_b = 203 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



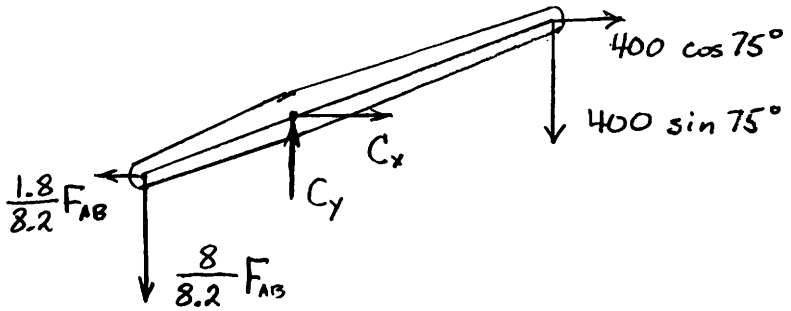
PROBLEM 1.28

The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$-in.-diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

SOLUTION

Use member *BCD* as a free body, and note that *AB* is a two force member.

$$l_{AB} = \sqrt{8^2 + 1.8^2} = 8.2 \text{ in.}$$



$$+\circlearrowleft \Sigma M_C = 0: \quad (4\cos 20°)\left(\frac{8}{8.2}F_{AB}\right) - (4\sin 20°)\left(\frac{1.8}{8.2}F_{AB}\right)$$
$$-(7\cos 20°)(400\sin 75°) - (7\sin 20°)(400\cos 75°) = 0$$
$$3.36678F_{AB} - 2789.35 = 0 \qquad \therefore F_{AB} = 828.49 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{1.8}{8.2}F_{AB} + C_x + 400\cos 75° = 0$$
$$C_x = \frac{(1.8)(828.49)}{8.2} - 400\cos 75° = 78.34 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{8}{8.2}F_{AB} + C_y - 400\sin 75° = 0$$
$$C_y = \frac{(8)(828.49)}{8.2} + 400\sin 75° = 1194.65 \text{ lb}$$
$$C = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.28 ***(Continued)***

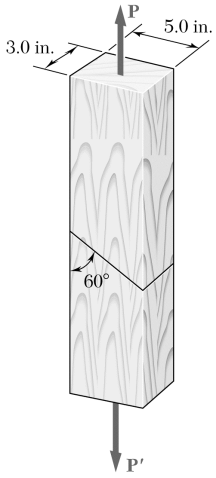
(*a*) Shearing stress in the bolt: $P = 1197.2$ lb $\quad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}\left(\dfrac{3}{8}\right)^2 = 0.11045 \text{ in}^2$

$$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{psi} = 10.84 \text{ ksi} \blacktriangleleft$$

(*b*) Bearing stress at *C* in member *BCD*: $P = 1197.2$ lb $\quad A_b = dt = \left(\dfrac{3}{8}\right)\left(\dfrac{5}{8}\right) = 0.234375 \text{ in}^2$

$$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{psi} = 5.11 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.29

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

SOLUTION

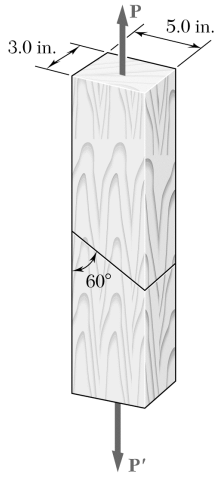
$$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$$

$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(1400)(\cos 30°)^2}{15} \qquad \sigma = 70.0 \text{ psi} \blacktriangleleft$$

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1400)\sin 60°}{(2)(15)} \qquad \tau = 40.4 \text{ psi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.30

Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

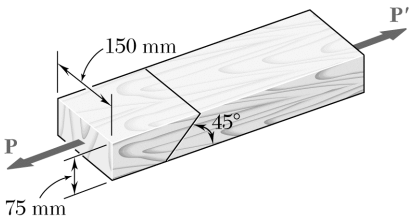
$$\theta = 90° - 60° = 30°$$

$$\sigma = \frac{P\cos^2\theta}{A_0}$$

(*a*) $$P = \frac{\sigma A_0}{\cos^2\theta} = \frac{(75)(15)}{\cos^2 30°} = 1500 \text{ lb}$$ $P = 1.500$ kips ◀

(*b*) $$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1500)\sin 60°}{(2)(15)}$$ $\tau = 43.3$ psi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.31

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

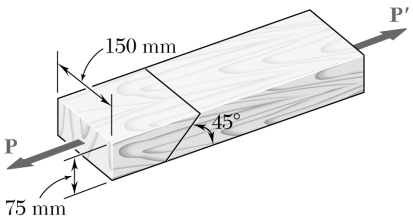
$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}} = 489 \times 10^3 \text{ Pa} \qquad \sigma = 489 \text{ kPa} \blacktriangleleft$$

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})} = 489 \times 10^3 \text{ Pa} \qquad \tau = 489 \text{ kPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.32

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{Pa}$$

$$\tau = \frac{P \sin 2\theta}{2A_0}$$

(*a*) $$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°}$$

$$= 13.95 \times 10^3 \text{N}$$

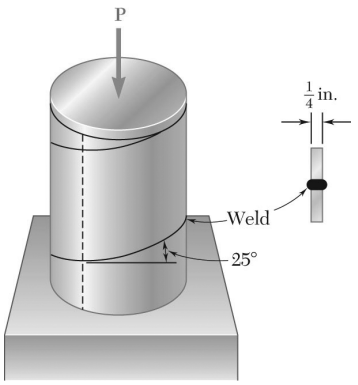
$P = 13.95$ kN ◀

(*b*) $$\sigma = \frac{P \cos^2 \theta}{A_0} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}}$$

$$= 620 \times 10^3 \text{Pa}$$

$\sigma = 620$ kPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.33

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 12\,\text{ksi}$ and $\tau = 7.2\,\text{ksi}$, determine the magnitude P of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 12\,\text{in.} \quad r_o = \frac{1}{2}d_o = 6\,\text{in.}$$

$$r_i = r_o - t = 6 - 0.25 = 5.75\,\text{in.}$$

$$A_0 = \pi(r_o^2 - r_i^2) = \pi(6^2 - 5.75^2) = 9.228\,\text{in}^2$$

$$\theta = 25°$$

Based on $|\sigma| = 12\,\text{ksi}$: $\sigma = \dfrac{P}{A_0}\cos^2\theta$

$$P = \frac{A_0\sigma}{\cos^2\theta} = \frac{(9.228)(12\times10^3)}{\cos^2 25°} = 134.8\times10^3\,\text{lb}$$

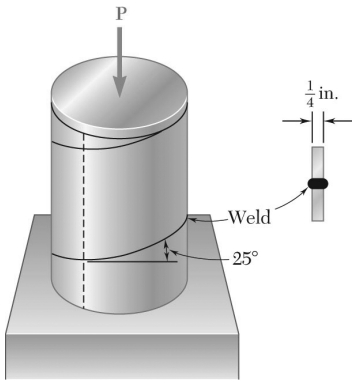
Based on $|\tau| = 7.2\,\text{ksi}$: $\tau = \dfrac{P}{2A_0}\sin 2\theta$

$$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(9.288)(7.2\times10^3)}{\sin 50°} = 174.5\times10^3\,\text{lb}$$

The smaller calculated value of P is the allowable value.

$$P = 134.8\times10^3\,\text{lb} \qquad P = 134.8\,\text{kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.34

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 66 kip axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$$d_o = 12 \text{ in.} \quad r_o = \frac{1}{2} d_o = 6 \text{ in.}$$

$$r_i = r_o - t = 6 - 0.25 = 5.75 \text{ in.}$$

$$A_0 = \pi(r_o^2 - r_i^2) = \pi(6^2 - 5.75^2) = 9.228 \text{ in}^2$$

$$\theta = 25°$$

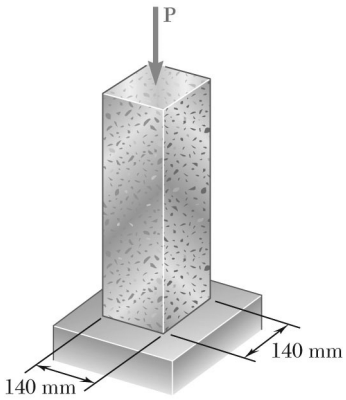
Normal stress:

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(66 \times 10^3)\cos^2 25°}{9.228} = 5875 \text{ psi} \qquad \sigma = 5.87 \text{ ksi} \blacktriangleleft$$

Shearing stress:

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(66 \times 10^3)\sin 50°}{(2)(9.228)} = 2739 \text{ psi} \qquad \tau = 2.74 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.35

A 1060-kN load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$$A_0 = (140\text{ mm})(140\text{ mm}) = 19.6\times10^{3}\text{mm}^2 = 19.6\times10^{-3}\text{m}^2$$

$$P = 1060\times10^{3}\text{N}$$

$$\sigma = \frac{P}{A_0}\cos^2\theta = \frac{1060\times10^{3}}{19.6\times10^{-3}}\cos^2\theta = 54.082\times10^{6}\cos^2\theta$$

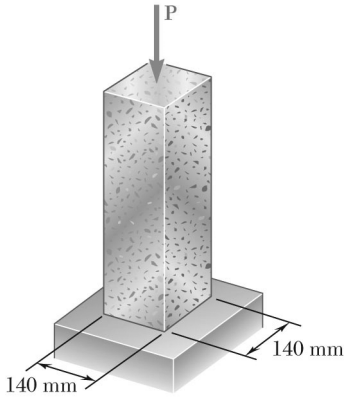
(*a*) Maximum tensile stress = 0 at $\theta = 90°$.

Maximum compressive stress = 54.1×10^{6} at $\theta = 0°$. $|\sigma|_{\max} = 54.1\text{ MPa}$ ◀

(*b*) Maximum shearing stress:

$$\tau_{\max} = \frac{P}{2A_0} = \frac{1060\times10^{3}}{(2)(19.6\times10^{-3})} = 27.0\times10^{6}\text{ Pa at }\theta = 45°. \qquad \tau_{\max} = 27.0\text{ MPa}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.36

A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 18 MPa, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

SOLUTION

$$A_0 = (140\text{ mm})(140\text{ mm}) = 19.6\times10^{3}\text{ mm}^2 = 19.6\times10^{-3}\text{ m}^2$$

$$\tau_{\max} = 18\text{ MPa} = 18\times10^{6}\text{ Pa}$$

$$\theta = 45^\circ \text{ for plane of } \tau_{\max}$$

(*a*) Magnitude of *P*. $\tau_{\max} = \dfrac{|P|}{2A_0}$ so $P = 2A_0\,\tau_{\max}$

$$P = (2)(19.6\times10^{-3})(18\times10^{6}) = 705.6\times10^{3}\text{ N} \qquad P = 706\text{ kN} \blacktriangleleft$$

(*b*) Orientation. $\sin 2\theta$ is maximum when $2\theta = 90^\circ$ $\qquad \theta = 45^\circ \blacktriangleleft$

(*c*) Normal stress at $\theta = 45^\circ$.

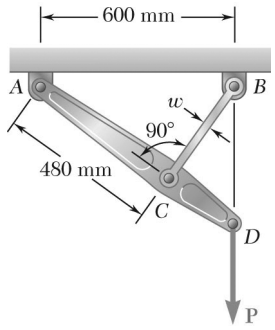
$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(705.8\times10^{3})\cos^2 45^\circ}{19.6\times10^{-3}} = 18.00\times10^{6}\text{ Pa} \qquad \sigma = 18.00\text{ MPa} \blacktriangleleft$$

(*d*) Maximum normal stress: $\sigma_{\max} = \dfrac{P}{A_0}$

$$\sigma_{\max} = \frac{705.8\times10^{3}}{19.6\times10^{-3}} = 36.0\times10^{6}\text{ Pa}$$

$$\sigma_{\max} = 36.0\text{ MPa (compression)} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

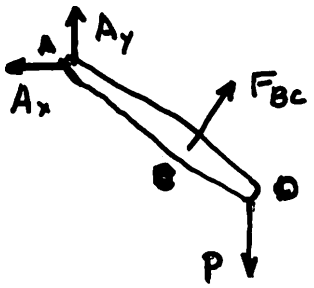


PROBLEM 1.37

Link *BC* is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P**?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - (600)P = 0$$

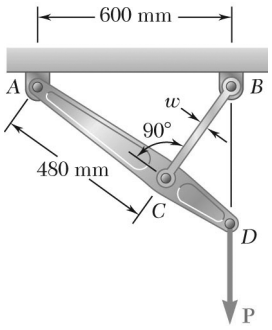
$$F_{BC} = \frac{600}{480}P = \frac{(600)(16\times10^3)}{480} = 20\times10^3\,\text{N}$$

Ultimate load for member *BC*: $F_U = \sigma_U A$

$$F_U = (480\times10^6)(0.006)(0.025) = 72\times10^3\,\text{N}$$

Factor of safety: $\text{F.S.} = \dfrac{F_U}{F_{BC}} = \dfrac{72\times10^3}{20\times10^3}$ F.S. = 3.60 ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

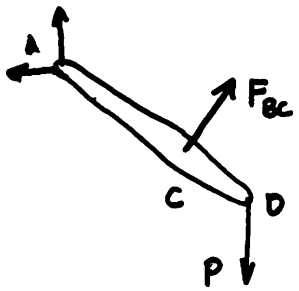


PROBLEM 1.38

Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - 600P = 0$$

$$F_{BC} = \frac{600P}{480} = \frac{(600)(20\times10^3)}{480} = 25\times10^3\text{N}$$

For a factor of safety F.S. = 3, the ultimate load of member *BC* is

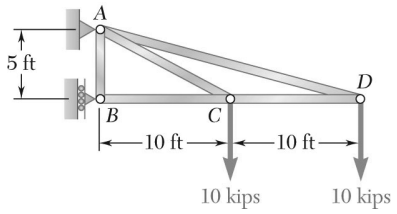
$$F_U = (\text{F.S.})(F_{BC}) = (3)(25\times10^3) = 75\times10^3\text{N}$$

But $F_U = \sigma_U A$ $\quad \therefore A = \dfrac{F_U}{\sigma_U} = \dfrac{75\times10^3}{450\times10^6} = 166.67\times10^{-6}\text{m}^2$

For a rectangular section $\quad A = wt$ or $w = \dfrac{A}{t} = \dfrac{166.67\times10^{-6}}{0.006}$

$$w = 27.8\times10^{-3}\text{m or } 27.8\text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



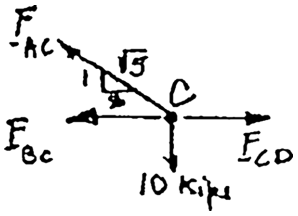
PROBLEM 1.39

A $\frac{3}{4}$-in.-diameter rod made of the same material as rods *AC* and *AD* in the truss shown was tested to failure and an ultimate load of 29 kips was recorded. Using a factor of safety of 3.0, determine the required diameter (*a*) of rod *AC*, (*b*) of rod *AD*.

SOLUTION

Forces in *AC* and *AD*.

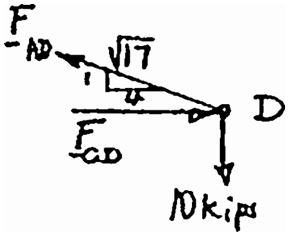
Joint *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{AC} - 10 \text{ kips} = 0$$

$$F_{AC} = 22.36 \text{ kips } T$$

Joint *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{17}} F_{AD} - 10 \text{ kips} = 0$$

$$F_{AD} = 41.23 \text{ kips } T$$

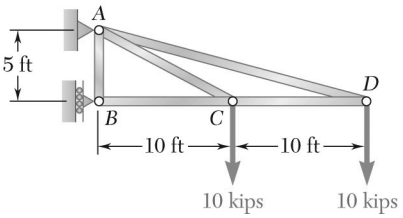
Ultimate stress. From test on $\frac{3}{4}$-in. rod: $\sigma_U = \dfrac{P_U}{A} = \dfrac{29 \text{ kips}}{\frac{1}{4}\pi(\frac{3}{4})^2} = 65.64 \text{ ksi}$

Allowable stress: $\sigma_{all} = \dfrac{\sigma_U}{F.S.} = \dfrac{65.64 \text{ ksi}}{3.0} = 21.88 \text{ ksi}$

(*a*) Diameter of rod *AC*. $\sigma_{\text{all}} = \dfrac{F_{AC}}{\frac{1}{4}\pi d^2}$ $d^2 = \dfrac{4F_{AC}}{\pi\sigma_{\text{all}}} = \dfrac{4(22.36)}{\pi(21.88)} = 1.301$ $d = 1.141$ in. ◀

(*b*) Diameter of rod *AD*. $d^2 = \dfrac{4F_{AD}}{\pi\sigma_{\text{all}}} = \dfrac{4(41.23)}{\pi(21.88)} = 2.399$ $d = 1.549$ in. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



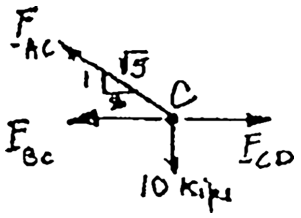
PROBLEM 1.40

In the truss shown, members *AC* and *AD* consist of rods made of the same metal alloy. Knowing that *AC* is of 1-in. diameter and that the ultimate load for that rod is 75 kips, determine (*a*) the factor of safety for *AC*, (*b*) the required diameter of *AD* if it is desired that both rods have the same factor of safety.

SOLUTION

Forces in AC and *AD*.

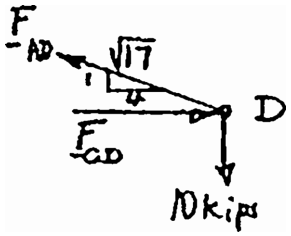
Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{AC} - 10 \text{ kips} = 0$$

$$F_{AC} = 22.36 \text{ kips } T$$

Joint *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{17}} F_{AD} - 10 \text{ kips} = 0$$

$$F_{AD} = 41.23 \text{ kips } T$$

(*a*) Factor of safety for *AC*. $\text{F.S.} = \dfrac{P_U}{F_{AC}}$ $\text{F.S.} = \dfrac{75 \text{ kips}}{22.36 \text{ kips}}$ $\text{F.S.} = 3.35$ ◀

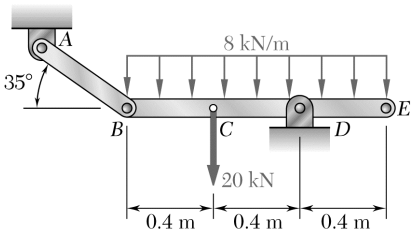
(*b*) For the same factor of safety in *AC* and *AD*, $\sigma_{AD} = \sigma_{AC}$.

$$\frac{F_{AD}}{A_{AD}} = \frac{F_{AC}}{A_{AC}}$$

$$A_{AD} = \frac{F_{AD}}{F_{AC}} A_{AC} = \frac{41.23}{22.36} \frac{\pi}{4} (1)^2 = 1.4482 \text{ in}^2$$

Required diameter: $d_{AD} = \sqrt{\dfrac{4A_{AD}}{\pi}} = \sqrt{\dfrac{(4)(1.4482)}{\pi}}$ $d_{AD} = 1.358 \text{ in.}$ ◀

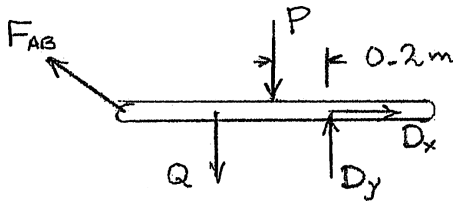
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.41

Link *AB* is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for *AB* for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at *A* and *B*.

SOLUTION



$$P = (1.2)(8) = 9.6 \text{ kN}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(0.8)(F_{AB}\sin 35°) + (0.2)(9.6) + (0.4)(20) = 0$$

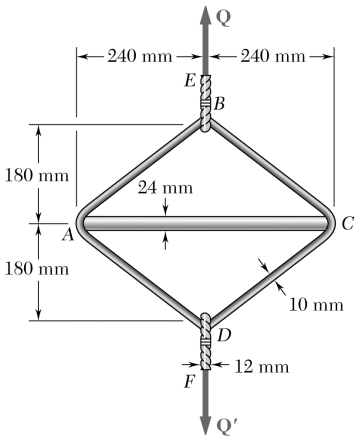
$$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{\text{ult}}}{F.S.}$$

$$A_{AB} = \frac{(F.S.)F_{AB}}{\sigma_{\text{ult}}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6}$$

$$= 168.1 \times 10^{-6} \text{m}^2 \qquad A_{AB} = 168.1 \text{ mm}^2 \blacktriangleleft$$

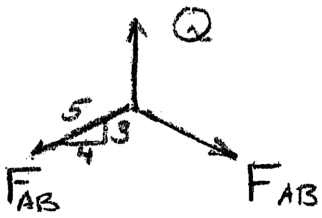
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.42

A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 480 MPa and that the ultimate strength of the aluminum used for the rod is 260 MPa, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

SOLUTION



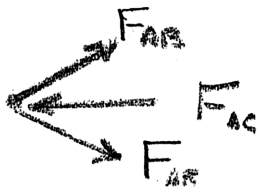
Using joint *B* as a free body and considering symmetry,

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0 \quad Q = \frac{6}{5} F_{AB}$$

Using joint *A* as a free body and considering symmetry,

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$



Based on strength of cable *BF*:

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop:

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

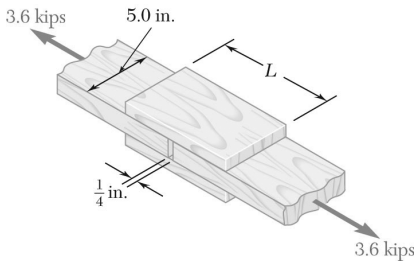
Based on strength of rod *AC*:

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2 = \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest, $\therefore \quad Q_U = 45.24 \times 10^3 \text{ N}$

Allowable load: $$Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3 \text{ N} \qquad Q = 15.08 \text{ kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.43

Two wooden members shown, which support a 3.6 kip load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 360 psi and the clearance between the members is $\frac{1}{4}$ in. Determine the required length L of each splice if a factor of safety of 2.75 is to be achieved.

SOLUTION

There are 4 separate areas of glue. Let l be the length of each area and $w = 5$ in. its width. Then the area is $A = lw$.

Each glue area transmits one half of the total load.

$$F = \left(\frac{1}{2}\right)(3.6 \text{ kips}) = 1.8 \text{ kips}$$

Required ultimate load for each glue area:

$$F_U = (F.S.)\,F = (2.75)(1.8) = 4.95 \text{ kips}$$

Required length of each glue area:

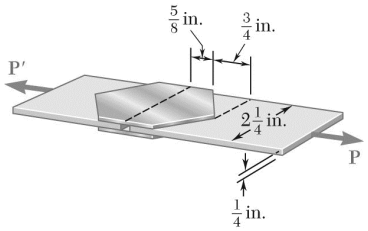
$$F_U = \tau_U A = \tau_U lw$$

$$l = \frac{F_U}{\tau_U w} = \frac{4.95 \times 10^3}{(360)(5)} = 2.75 \text{ in.}$$

Total length of splice: $L = l + \frac{1}{4}\text{ in.} + l$

$$L = 2.75 + 0.25 + 2.75 \qquad L = 5.75 \text{ in.} \blacktriangleleft$$

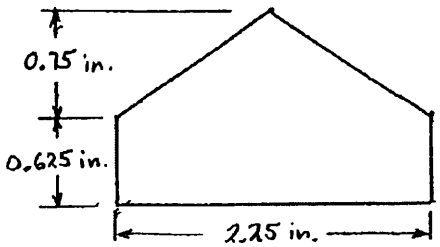
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.44

Two plates, each $\frac{1}{8}$ in. thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surface is 130 psi, determine the factor of safety with respect to shear when $P = 325$ lb.

SOLUTION



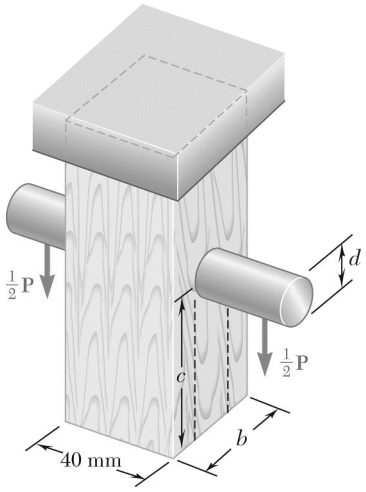
Bond area: (See figure)

$$A = \frac{1}{2}(2.25)(0.75) + (2.25)(0.625) = 2.25 \text{ in}^2$$

$$P_U = 2A\tau_U = (2)(2.25)(130) = 585 \text{ lb.}$$

$$F.S. = \frac{P_U}{P} = \frac{585}{325} = 1.800 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.45

A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin:

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\text{N}$$

Based on tension in wood:

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\text{N}$$

Based on double shear in the wood:

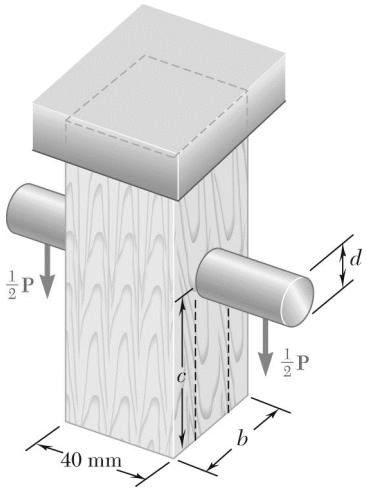
$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\text{N}$$

Use smallest $P_U = 32.8\times10^3\text{N}$

Allowable: $P = \frac{P_U}{F.S.} = \frac{32.8\times10^3}{3.2} = 10.25\times10^3\text{N}$ 10.25 kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.46

For the support of Prob. 1.45, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

PROBLEM 1.45 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

$$P = 20\,\text{kN} = 20 \times 10^3\,\text{N}$$

(*a*) Pin:
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 2.01.06 \times 10^{-6}\,\text{m}^2$$

Double shear:
$$\tau = \frac{P}{2A} \quad \tau_U = \frac{P_U}{2A}$$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6) = 58.336 \times 10^3\,\text{N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3} \qquad F.S. = 2.92 \blacktriangleleft$$

(*b*) Tension in wood:
$$P_U = 58.336 \times 10^3\,\text{N} \quad \text{for same F.S.}$$

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where} \quad w = 40\,\text{mm} = 0.040\,\text{m}$$

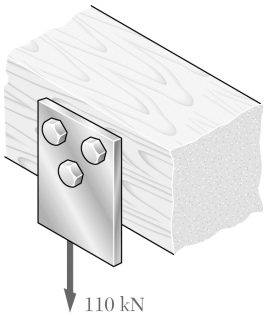
$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^{-3}\,\text{m} \qquad b = 40.3\,\text{mm} \blacktriangleleft$$

Shear in wood:
$$P_U = 58.336 \times 10^3\,\text{N} \quad \text{for same F.S.}$$

Double shear; each area is $A = wc$
$$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3}\,\text{m} \qquad c = 97.2\,\text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.47

Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

SOLUTION

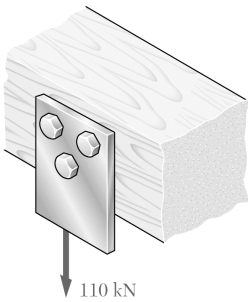
For each bolt, $P = \dfrac{110}{3} = 36.667 \text{ kN}$

Required: $P_U = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi\tau_U}} = \sqrt{\frac{(4)(122.83\times10^3)}{\pi(360\times10^6)}} = 20.8\times10^{-3}\text{m} \qquad d = 20.8 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.48

Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

SOLUTION

For each bolt, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2 = 254.47 \times 10^{-6} \text{m}^2$

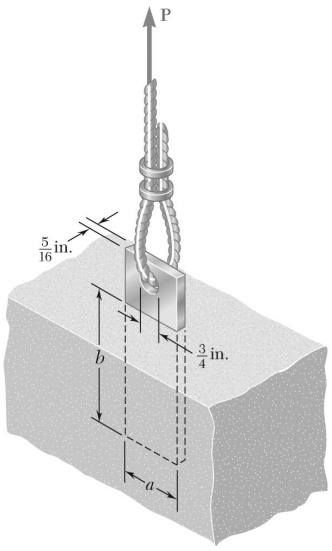
$$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$$
$$= 91.609 \times 10^3 \text{N}$$

For the three bolts, $P_U = (3)(91.609 \times 10^3) = 274.83 \times 10^3 \text{N}$

Factor of safety:

$$F.S. = \frac{P_U}{P} = \frac{274.83 \times 10^3}{110 \times 10^3} \qquad F.S. = 2.50 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

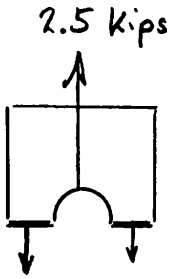


PROBLEM 1.49

A steel plate $\frac{5}{16}$ in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

SOLUTION

Based on tension in plate:



$$A = (a - d)t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a - d)t}{P}$$

Solving for *a*,

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)\left(\frac{5}{16}\right)}$$

(*a*) $a = 1.550$ in. ◀

Based on shear between plate and concrete slab,

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b \qquad \tau_U = 0.300 \text{ ksi}$$

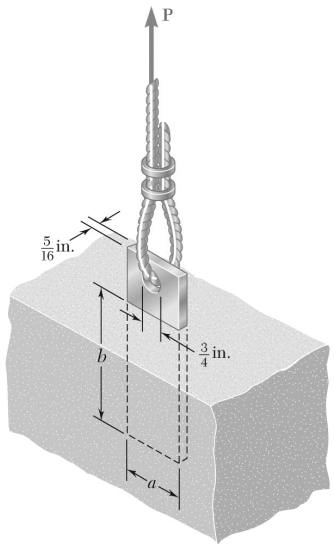
$$P_U = \tau_U A = 2\tau_U (a + t)b \qquad F.S. = \frac{P_U}{P}$$

Solving for *b*,

$$b = \frac{(F.S.)P}{2(a + t)\tau_U} = \frac{(3.6)(2.5)}{(2)\left(1.550 + \frac{5}{16}\right)(0.300)}$$

(*b*) $b = 8.05$ in. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



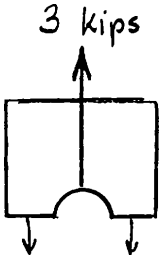
PROBLEM 1.50

Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.

PROBLEM 1.49 A steel plate $\frac{5}{16}$ in, thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

SOLUTION

Based on tension in plate:



$$A = (a - d)t$$

$$= \left(2 - \frac{3}{4}\right)\left(\frac{5}{16}\right) = 0.3906 \text{ in}^2$$

$$P_U = \sigma_U A$$

$$= (36)(0.3906) = 14.06 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{14.06}{3} = 4.69$$

Based on shear between plate and concrete slab:

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2\left(2 + \frac{5}{16}\right)(7.5)$$

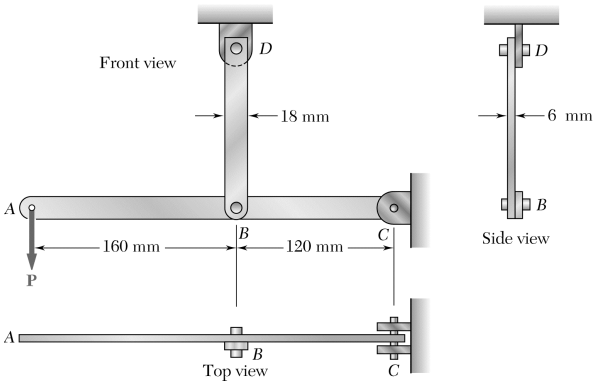
$$A = 34.69 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = (0.300)(34.69) = 10.41 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{10.41}{3} = 3.47$$

Actual factor of safety is the smaller value. $F.S. = 3.47$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

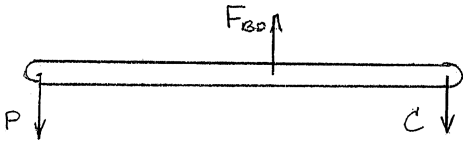


PROBLEM 1.51

In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at *A*. Note that link *BD* is not reinforced around the pin holes.

SOLUTION

Use free body *ABC*.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \qquad (2)$$

Tension on net section of link *BD*.

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-10)(10^{-3}) = 6.40\times10^3\,\text{N}$$

Shear in pins at *B* and *D*.

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(10\times10^{-3})^2 = 3.9270\times10^3\,\text{N}$$

Smaller value of F_{BD} is 3.9270×10^3 N.

From (1) $$P = \left(\frac{3}{7}\right)(3.9270\times10^3) = 1.683\times10^3\,\text{N}$$

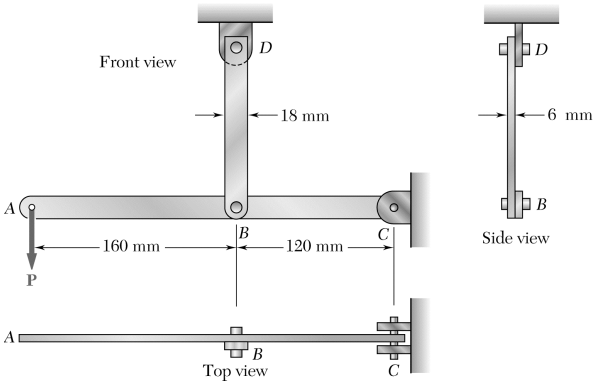
Shear in pin at *C*. $$C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\,\text{N}$$

From (2) $$P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\,\text{N}$$

Smaller value of *P* is allowable value. $P = 1.683\times10^3$ N $P = 1.683$ kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.52

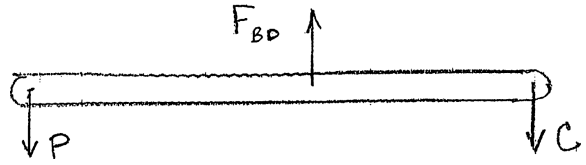


Solve Prob. 1.51, assuming that the structure has been redesigned to use 12-mm-diameter pins at *B* and *D* and no other change has been made.

PROBLEM 1.51 In the steel structure shown, a 6-mm-diameter pin is used at *C* and 10-mm-diameter pins are used at *B* and *D*. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link *BD*. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at *A*. Note that link *BD* is not reinforced around the pin holes.

SOLUTION

Use free body *ABC*.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \qquad (2)$$

Tension on net section of link *BD*.

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{400\times10^6}{3}\right)(6\times10^{-3})(18-12)(10^{-3}) = 4.80\times10^3\,\text{N}$$

Shear in pins at *B* and *D*.

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(12\times10^{-3})^2 = 5.6549\times10^3\,\text{N}$$

Smaller value of F_{BD} is 4.80×10^3 N.

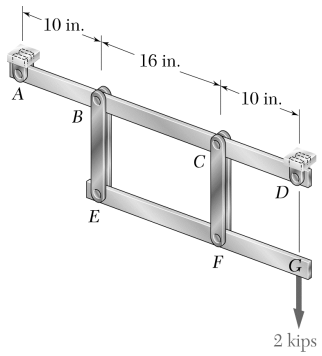
From (1), $$P = \left(\frac{3}{7}\right)(4.80\times10^3) = 2.06\times10^3\,\text{N}$$

Shear in pin at *C*. $$C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150\times10^6}{3}\right)\left(\frac{\pi}{4}\right)(6\times10^{-3})^2 = 2.8274\times10^3\,\text{N}$$

From (2), $$P = \left(\frac{3}{4}\right)(2.8274\times10^3) = 2.12\times10^3\,\text{N}$$

Smaller value of *P* is the allowable value. $P = 2.06\times10^3$ N $\qquad P = 2.06$ kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

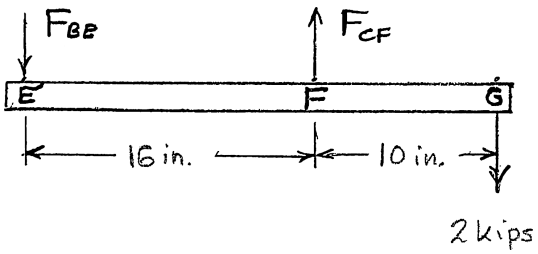


PROBLEM 1.53

Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a uniform rectangular cross section $\frac{1}{4}$ in. thick and 1 in. wide, and is made of a steel with an ultimate strength in tension of 60 ksi. The pins at *C* and *F* each have a $\frac{1}{2}$-in. diameter and are made of a steel with an ultimate strength in shear of 25 ksi. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION

Use member EFG as free body.



$$+\circlearrowleft \Sigma M_E = 0: \quad 16F_{CF} - (26)(2) = 0$$

$$F_{CF} = 3.25 \text{ kips}$$

Failure by tension in links *CF*. (2 parallel links)

Net section area for 1 link: $A = (b - d)t = (1 - \frac{1}{2})(\frac{1}{4}) = 0.125 \text{ in}^2$

$$F_U = 2A\sigma_U = (2)(0.125)(60) = 15 \text{ kips}$$

Failure by double shear in pins.

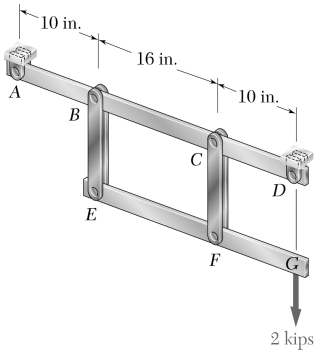
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196350 \text{ in}^2$$

$$F_U = 2A\tau_U = (2)(0.196350)(25) = 9.8175 \text{ kips}$$

Actual ultimate load is the smaller value. $F_U = 9.8175$ kips

Factor of safety: $$F.S. = \frac{F_U}{F_{CF}} = \frac{9.8175}{3.25}$$ $F.S. = 3.02$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



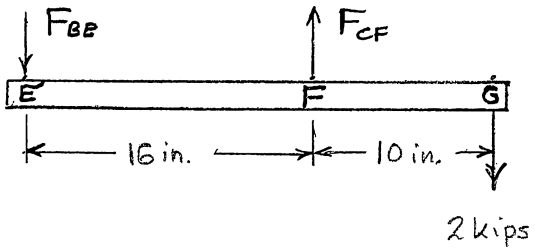
PROBLEM 1.54

Solve Prob. 1.53, assuming that the pins at *C* and *F* have been replaced by pins with a $\frac{3}{4}$-in diameter.

PROBLEM 1.53 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a uniform rectangular cross section $\frac{1}{4}$ in. thick and 1 in. wide, and is made of a steel with an ultimate strength in tension of 60 ksi. The pins at *C* and *F* each have a $\frac{1}{2}$-in. diameter and are made of a steel with an ultimate strength in shear of 25 ksi. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION

Use member *EFG* as free body.



$$+\circlearrowleft \Sigma M_E = 0: \quad 16F_{CF} - (26)(2) = 0$$

$$F_{CF} = 3.25 \text{ kips}$$

Failure by tension in links *CF*. (2 parallel links)

Net section area for 1 link: $A = (b - d)t = (1 - \frac{3}{4})(\frac{1}{4}) = 0.0625 \text{ in}^2$

$$F_U = 2A\sigma_U = (2)(0.0625)(60) = 7.5 \text{ kips}$$

Failure by double shear in pins.

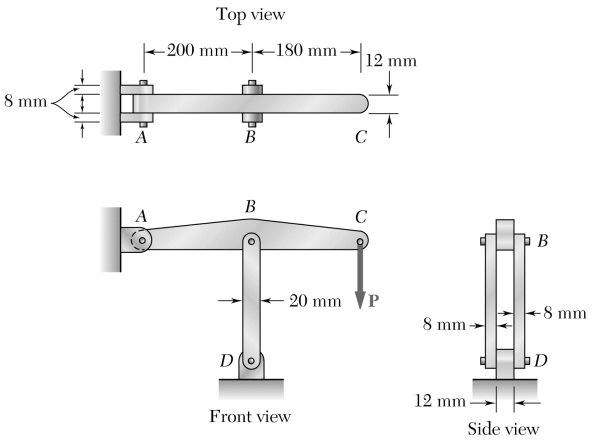
$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$F_U = 2A\tau_U = (2)(0.44179)(25) = 22.09 \text{ kips}$$

Actual ultimate load is the smaller value. $F_U = 7.5$ kips

Factor of safety: $$F.S. = \frac{F_U}{F_{CF}} = \frac{7.5}{3.25}$$ $F.S. = 2.31$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

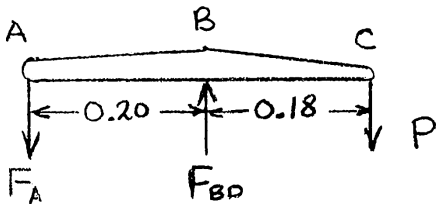


PROBLEM 1.55

In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18\,P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\,F_{BD} - 0.38\,P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.008)^2 = 50.266\times10^{-6}\text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(50.266\times10^{-6})}{3.0} = 3.351\times10^3\text{N}$$

$$P = \frac{10}{9}F_A = 3.72\times10^3\text{N}$$

Based on double shear in pins at *B* and *D*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54\times10^3\text{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97\times10^3\text{N}$$

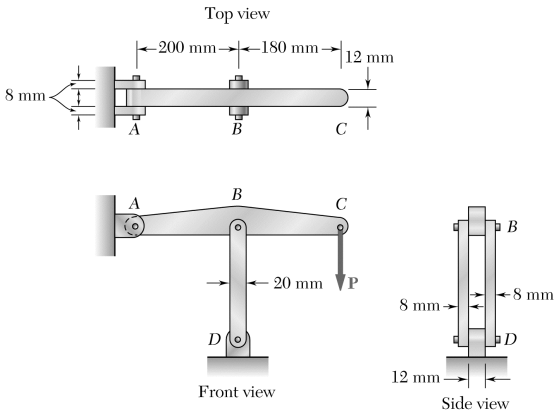
Based on compression in links *BD*: For one link, $A = (0.020)(0.008) = 160\times10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7\times10^3\text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04\times10^3\text{N}$$

Allowable value of *P* is smallest, $\therefore P = 3.72\times10^3\text{N}$ $\qquad P = 3.72\text{ kN}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



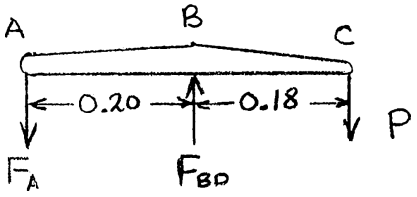
PROBLEM 1.56

In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18\,P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\,F_{BD} - 0.38\,P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54\times10^{-6}\text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(78.54\times10^{-6})}{3.0} = 5.236\times10^3\text{N}$$

$$P = \frac{10}{9}F_A = 5.82\times10^3\text{N}$$

Based on double shear in pins at *B* and *D*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54\times10^3\text{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97\times10^3\text{N}$$

Based on compression in links *BD*: For one link, $A = (0.020)(0.008) = 160\times10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7\times10^3\text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04\times10^3\text{N}$$

Allowable value of *P* is smallest, $\therefore P = 3.97\times10^3\text{N}$ $\qquad P = 3.97\text{ kN}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.57

The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

(*a*) $$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)\left(\frac{1}{2} \times 160\right) + (1.5)\left(\frac{3}{4} \times 2 \times 195\right)}{0.85}$$

$$P_U = 629 \text{ lb} \blacktriangleleft$$

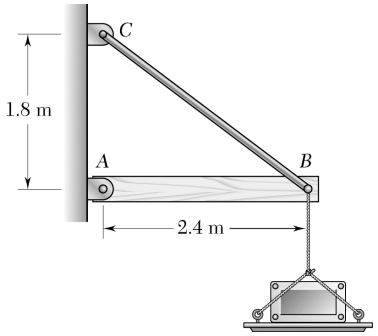
Conventional factor of safety.

$$P = P_D + P_L = \frac{1}{2} \times 160 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

(*b*) $$F.S. = \frac{P_U}{P} = \frac{629}{372.5}$$

$$F.S. = 1.689 \blacktriangleleft$$

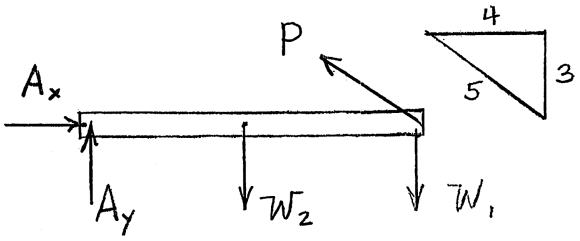
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.58

A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4W_1 - 1.2W_2$$

$$\therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$$

For dead loading, $W_1 = (40)(9.81) = 392.4$ N, $W_2 = (50)(9.81) = 490.5$ N

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{N}$$

For live loading, $W_1 = mg \quad W_2 = 0 \quad P_L = \frac{5}{3}mg$

From which $m = \frac{3}{5}\frac{P_L}{g}$

Design criterion. $\gamma_D P_D + \gamma_L P_L = \phi P_U$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^{-3})}{1.6}$$

$$= 5.920 \times 10^3 \text{N}$$

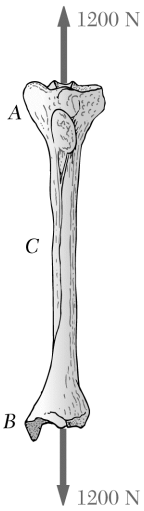
(*a*) Allowable load. $m = \frac{3}{5}\frac{5.92 \times 10^3}{9.81}$ $\quad m = 362$ kg ◀

Conventional factor of safety.

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{N}$$

(*b*) $F.S. = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3}$ $\quad F.S. = 1.718$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.59

A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \dfrac{\pi}{4}(d_1^2 - d_2^2)$

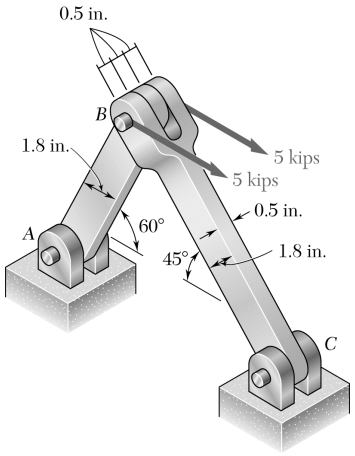
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.9 \times 10^{-6}\,\text{m}^2$$

$$d_2 = 14.93 \times 10^{-3}\,\text{m} \qquad d_2 = 14.93\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

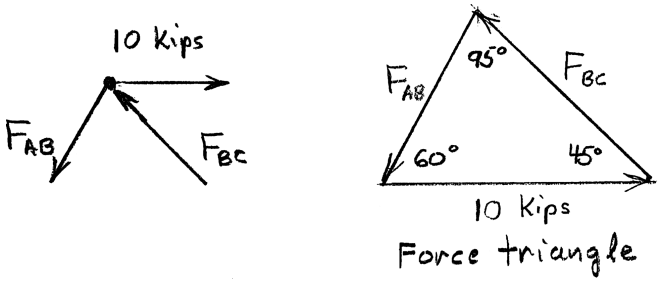


PROBLEM 1.60

Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.

SOLUTION

Use joint B as free body.



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$$F_{AB} = 7.3205 \text{ kips}$$

$$F_{BC} = 8.9658 \text{ kips}$$

Link AB is a tension member.

Minimum section at pin. $A_{\text{net}} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

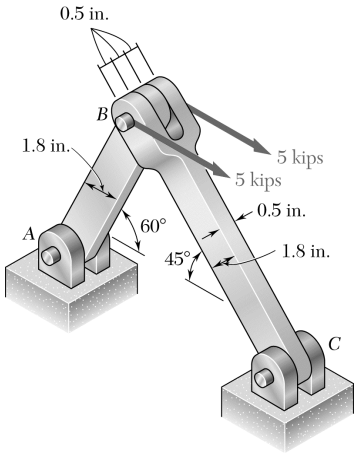
(*a*) Stress in AB : $\sigma_{AB} = \dfrac{F_{AB}}{A_{\text{net}}} = \dfrac{7.3205}{0.5}$ $\sigma_{AB} = 14.64 \text{ ksi}$ ◀

Link BC is a compression member.

Cross sectional area is $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(*b*) Stress in BC: $\sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-8.9658}{0.9}$ $\sigma_{BC} = -9.96 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



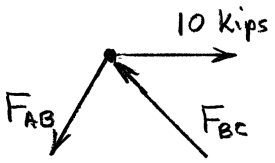
PROBLEM 1.61

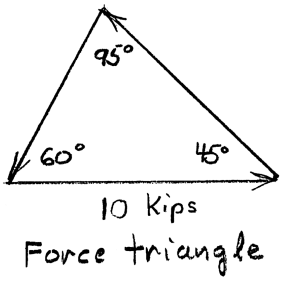
For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

PROBLEM 1.60 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.





Law of Sines

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{10}{\sin 95°} \qquad F_{BC} = 8.9658 \text{ kips}$$

(*a*) Shearing stress in pin at *C*. $\tau = \dfrac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad \tau = 8.92 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.61 *(Continued)*

(*b*) Bearing stress at *C* in member *BC*. $\sigma_b = \dfrac{F_{BC}}{A}$

$$A = td = (0.5)(0.8) = 0.4 \text{ in}^2$$

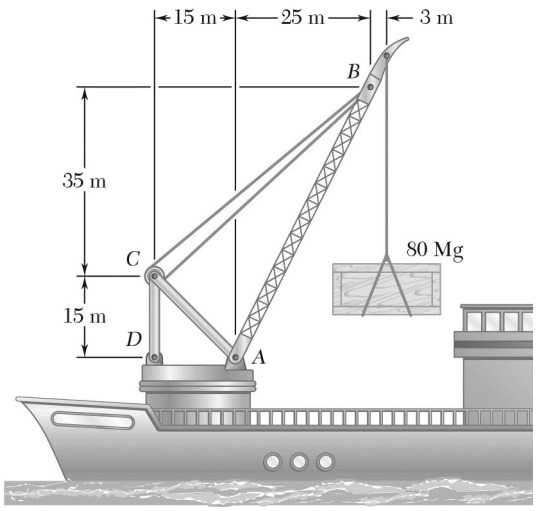
$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad \sigma_b = 22.4 \text{ ksi} \blacktriangleleft$$

(*c*) Bearing stress at *B* in member *BC*. $\sigma_b = \dfrac{F_{BC}}{A}$

$$A = 2td = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad \sigma_b = 11.21 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



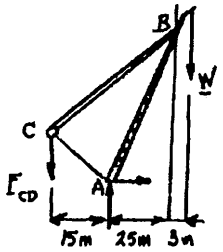
PROBLEM 1.62

In the marine crane shown, link *CD* is known to have a uniform cross section of 50 × 150 mm. For the loading shown, determine the normal stress in the central portion of that link.

SOLUTION

Weight of loading: $W = (80\text{ Mg})(9.81\text{ m/s}^2) = 784.8\text{ kN}$

Free Body: Portion *ABC*



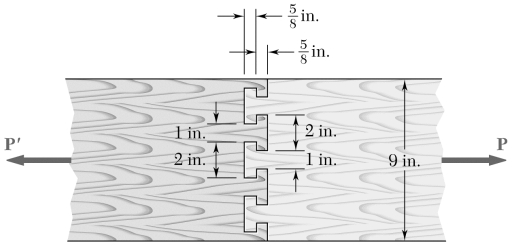
$$+\circlearrowleft \sum M_A = 0: \quad F_{CD}(15\text{ m}) - W(28\text{ m}) = 0$$

$$F_{CD} = \frac{28}{15}W = \frac{28}{15}(784.8\text{ kN})$$

$$F_{CD} = +1465\text{ kN}$$

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{+1465 \times 10^3\text{ N}}{(0.050\text{ m})(0.150\text{ m})} = +195.3 \times 10^6\text{ Pa} \qquad \sigma_{CD} = +195.3\text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.63

Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude P of the axial load that will cause the joint to fail.

SOLUTION

Six areas must be sheared off when the joint fails. Each of these areas has dimensions $\frac{5}{8}$ in. $\times \frac{1}{2}$ in., its area being

$$A = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} \text{ in}^2 = 0.3125 \text{ in}^2$$

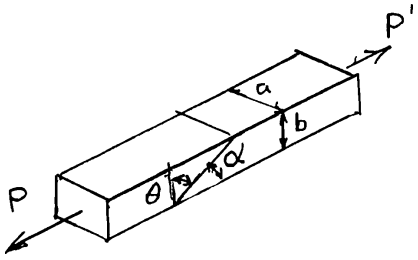
At failure, the force carried by each area is

$$F = \tau A = (1.20 \text{ ksi})(0.3125 \text{ in}^2) = 0.375 \text{ kips}$$

Since there are six failure areas,

$$P = 6F = (6)(0.375) \qquad P = 2.25 \text{ kips} \blacktriangleleft$$

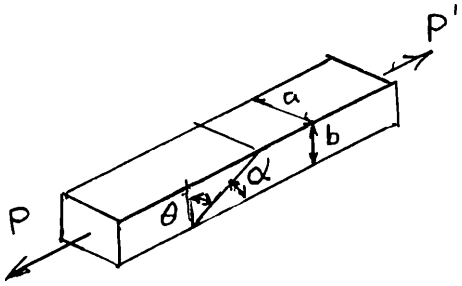
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.64

Two wooden members of uniform rectangular cross section of sides $a = 100$ mm and $b = 60$ mm are joined by a simple glued joint as shown. Knowing that the ultimate stresses for the joint are $\sigma_U = 1.26$ MPa in tension and $\tau_U = 1.50$ MPa in shear, and that $P = 6$ kN, determine the factor of safety for the joint when (*a*) $\alpha = 20°$, (*b*) $\alpha = 35°$, (*c*) $\alpha = 45°$. For each of these values of α, also determine whether the joint will fail in tension or in shear if P is increased until rupture occurs.

SOLUTION



Let $\theta = 90° - \alpha$ as shown.

From the text book:

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

or $$\sigma = \frac{P}{A_0}\sin^2\alpha \tag{1}$$

$$\tau = \frac{P}{A_0}\sin\alpha\cos\alpha \tag{2}$$

$A_0 = ab = (100 \text{ mm})(60 \text{ mm}) = 6000 \text{ mm}^2 = 6 \times 10^{-3}\text{m}^2$

$\sigma_U = 1.26 \times 10^6 \text{ Pa}$ $\qquad$ $\tau_U = 1.50 \times 10^6 \text{Pa}$

Ultimate load based on tension across the joint:

$$(P_U)_\sigma = \frac{\sigma_U A_0}{\sin^2\alpha} = \frac{(1.26 \times 10^6)(6 \times 10^{-3})}{\sin^2\alpha}$$
$$= \frac{7560}{\sin^2\alpha} = \frac{7.56}{\sin^2\alpha}\text{kN}$$

Ultimate load based on shear across the joint:

$$(P_U)_\tau = \frac{\tau_U A_0}{\sin\alpha\cos\alpha} = \frac{(1.50 \times 10^6)(6 \times 10^{-3})}{\sin\alpha\cos\alpha}$$
$$= \frac{9000}{\sin\alpha\cos\alpha} = \frac{9.00}{\sin\alpha\cos\alpha}\text{kN}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.64 ***(Continued)***

(*a*) $\alpha = 20°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 20°} = 64.63 \text{ kN}$

$$= (P_U)_\tau = \frac{9.00}{\sin 20° \cos 20°} = 28.00 \text{ kN}$$

The smaller value governs. The joint will fail in shear and $P_U = 28.00$ kN.

$$F.S. = \frac{P_U}{P} = \frac{28.00}{6} \qquad F.S. = 4.67 \blacktriangleleft$$

(*b*) $\alpha = 35°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 35°} = 22.98 \text{ kN}$

$$(P_U)_\tau = \frac{9.00}{\sin 35° \cos 35°} = 19.155 \text{ kN}$$

The joint will fail in shear and $P_U = 19.155$ kN.

$$F.S. = \frac{P_U}{P} = \frac{19.155}{6} \qquad F.S. = 3.19 \blacktriangleleft$$

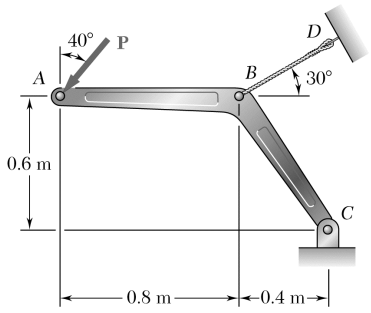
(*c*) $\alpha = 45°$: $(P_U)_\sigma = \dfrac{7.56}{\sin^2 45°} = 15.12 \text{ kN}$

$$(P_U)_\tau = \frac{9.00}{\sin 45° \cos 45°} = 18.00 \text{ kN}$$

The joint will fail in tension and $P_U = 15.12$ kN.

$$F.S. = \frac{P_U}{P} = \frac{15.12}{6} \qquad F.S. = 2.52 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

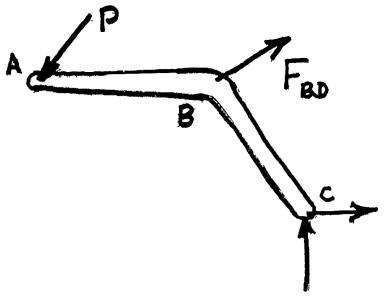


PROBLEM 1.65

Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.

SOLUTION

Use member *ABC* as a free body, and note that member *BD* is a two-force member.



$$+\circlearrowleft \Sigma M_c = 0: \quad (P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

$$1.30493P - 0.71962F_{BD} = 0$$

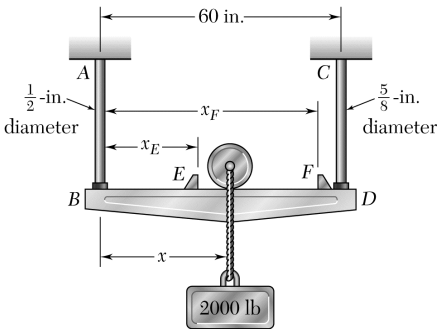
$$F_{BD} = 1.81335\ P = (1.81335)(16\times 10^3) = 29.014\times 10^3 \text{N}$$

$$F_U = 100\times 10^3 \text{N}$$

$$F.S. = \frac{F_U}{F_{BD}} = \frac{100\times 10^3}{29.014\times 10^3}$$

$$F.S. = 3.45 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.66

The 2000-lb load can be moved along the beam *BD* to any position between stops at *E* and *F*. Knowing that $\sigma_{all} = 6$ ksi for the steel used in rods *AB* and *CD*, determine where the stops should be placed if the permitted motion of the load is to be as large as possible.

SOLUTION

Permitted member forces:

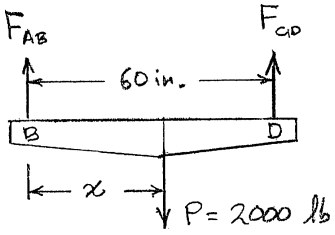
$$AB: \quad (F_{AB})_{max} = \sigma_{all} A_{AB} = (6)\left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)^2$$

$$= 1.17810 \text{ kips}$$

$$CD: \quad (F_{CD})_{max} = \sigma_{all} A_{CD} = (6)\left(\frac{\pi}{4}\right)\left(\frac{5}{8}\right)^2$$

$$= 1.84078 \text{ kips}$$

Use member *BEFD* as a free body.



$$P = 2000 \text{ lb} = 2.000 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(60)F_{AB} + (60 - x_E)P = 0$$

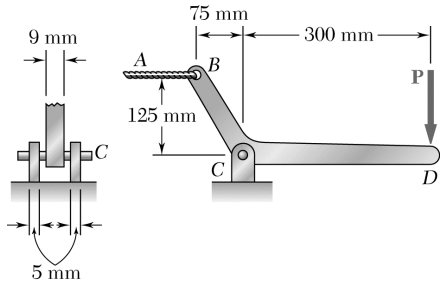
$$60 - x_E = \frac{60 F_{AB}}{P} = \frac{(60)(1.17810)}{2.000}$$

$$= 35.343 \qquad x_E = 24.7 \text{ in.} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 60F_{CD} - x_F P = 0$$

$$x_F = \frac{60F_{CD}}{P} = \frac{(60)(1.84078)}{2.000} \qquad x_F = 55.2 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

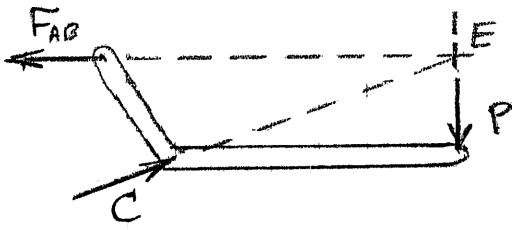


PROBLEM 1.67

Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at *C* for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at *C*, (*c*) the corresponding bearing stress in each support bracket at *C*.

SOLUTION

Draw free body diagram of *BCD*. Since *BCD* is a 3-force member, the reaction at *C* is directed toward Point *E*, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325$ mm

$$+\uparrow \Sigma F_y = 0: \quad \frac{125}{325} C - P = 0 \quad C = 2.6P = (2.6)(750) = 1950 \text{ N}$$

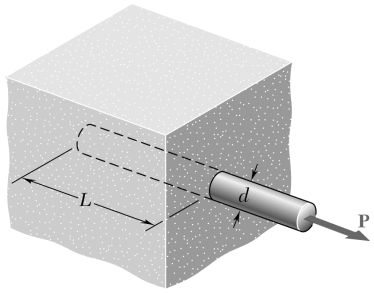
(*a*) $$\tau_{\text{pin}} = \frac{\frac{1}{2}C}{A_{\text{pin}}} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} \quad d = \sqrt{\frac{2C}{\pi\tau_{\text{pin}}}} = \sqrt{\frac{(2)(1950)}{\pi(40\times10^6)}} = 5.57\times10^{-3}\text{m}$$ $d = 5.57$ mm ◀

(*b*) $$\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{1950}{(5.57\times10^{-3})(9\times10^{-3})} = 38.9\times10^6\text{Pa}$$ $\sigma_b = 38.9$ MPa ◀

(*c*) $$\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{1950}{(2)(5.57\times10^{-3})(5\times10^{-3})} = 35.0\times10^6\text{Pa}$$ $\sigma_b = 35.0$ MPa ◀

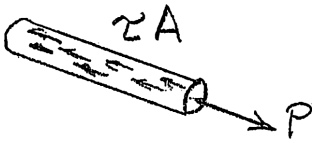
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 1.68



A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

SOLUTION



For shear, $A = \pi dL$

$$P = \tau_{all} A = \tau_{all} \pi dL$$

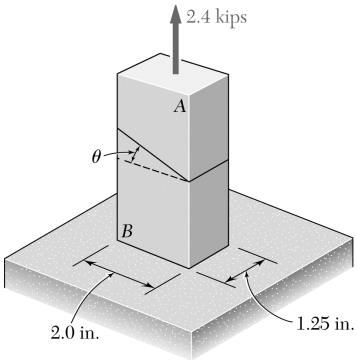
For tension, $A = \dfrac{\pi}{4} d^2$

$$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$$

Equating, $\tau_{all} \pi dL = \sigma_{all} \dfrac{\pi}{4} d^2$

Solving for L, $L_{min} = \sigma_{all} d / 4 \tau_{all}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.69

The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in.}^2$$

$$P = 2.4 \text{ kips}$$

$$P_U = (F.S.)P = 7.2 \text{ kips}$$

Based on tensile stress:

$$\sigma_U = \frac{P_U}{A_0}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_0}{P_U} = \frac{(2.5)(2.50)}{7.2} = 0.86806$$

$$\cos\theta = 0.93169 \qquad \theta = 21.3° \qquad \theta > 21.3°$$

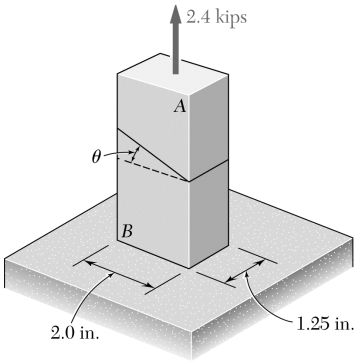
Based on shearing stress: $$\tau_U = \frac{P_U}{A_0}\sin\theta\cos\theta = \frac{P_U}{2A_0}\sin 2\theta$$

$$\sin 2\theta = \frac{2A_0\tau_U}{P_U} = \frac{(2)(2.50)(1.3)}{7.2} = 0.90278$$

$$2\theta = 64.52° \qquad \theta = 32.3° \qquad \theta < 32.3°$$

Hence, $21.3° < \theta < 32.3°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 1.70

The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in}^2$$

At the optimum angle, $(F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \dfrac{P}{A_0}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \dfrac{\sigma_U A_0}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_0}{P\cos^2\theta}$$

Shearing stress: $\tau = \dfrac{P}{A_0}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \dfrac{\tau_U A_0}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_0}{P\sin\theta\cos\theta}$$

Equating: $\dfrac{\sigma_U A_0}{P\cos^2\theta} = \dfrac{\tau_U A_0}{P\sin\theta\cos\theta}$

Solving: $\dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{\tau_U}{\sigma_U} = \dfrac{1.3}{2.5} = 0.520$

(*a*) $\theta_{\text{opt}} = 27.5°$ ◀

(*b*) $P_U = \dfrac{\sigma_U A_0}{\cos^2\theta} = \dfrac{(12.5)(2.50)}{\cos^2 27.5°} = 7.94 \text{ kips}$

$$F.S. = \frac{P_U}{P} = \frac{7.94}{2.4}$$

$F.S. = 3.31$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

CHAPTER 2

PROBLEM 2.1

An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

SOLUTION

(*a*) $\sigma_U = 400\times10^6\ \text{Pa} \qquad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(5)^2 = 19.635\ \text{mm}^2 = 19.635\times10^{-6}\ \text{m}^2$

$$P_U = \sigma_U A = (400\times10^6)(19.635\times10^{-6}) = 7854\ \text{N}$$

$$P_{\text{all}} = \frac{P_U}{F.S} = \frac{7854}{3.2} = 2454\ \text{N} \qquad P_{\text{all}} = 2.45\ \text{kN} \blacktriangleleft$$

(*b*) $$\delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635\times10^{-6})(200\times10^9)} = 50.0\times10^{-3}\ \text{m} \qquad \delta = 50.0\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.2

A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29\times10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$: $\quad 0.04 \text{ in.} = \dfrac{(2000 \text{ lb})(5.5\times12 \text{ in.})}{A(29\times10^6 \text{ psi})}$

$$A = \frac{1}{4}\pi d^2 = 0.11379 \text{ in}^2$$

$$d = 0.38063 \text{ in.} \qquad d = 0.381 \text{ in.} \blacktriangleleft$$

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{2000 \text{ lb}}{0.11379 \text{ in}^2} = 17580 \text{ psi}$ $\qquad \sigma = 17.58 \text{ ksi} \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.3

Two gage marks are placed exactly 10 in. apart on a $\frac{1}{2}$-in.-diameter aluminum rod with $E = 10.1 \times 10^6$ psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(*a*) $\delta = 10.009 - 10.000 = 0.009$ in.

$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} \qquad \sigma = \frac{E\delta}{L} = \frac{(10.1 \times 10^6)(0.009)}{10} = 9.09 \times 10^3 \text{ psi} \qquad \sigma = 9.09 \text{ ksi} \blacktriangleleft$$

(*b*) $F.S. = \dfrac{\sigma_U}{\sigma} = \dfrac{16}{9.09}$ $\qquad F.S. = 1.760 \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.4

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$, or $P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\text{m}^2$

$$P = \frac{(0.045\text{ m})(19.6350\times10^{-6}\text{ m}^2)(200\times10^{9}\text{ N/m}^2)}{18\text{ m}} = 9817.5\text{ N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\text{ N}}{19.6350\times10^{-6}\text{ m}^2} = 500\times10^{6}\text{ Pa}$ $\sigma = 500$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.5

A polystyrene rod of length 12 in. and diameter 0.5 in. is subjected to an 800-lb tensile load. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the elongation of the rod, (*b*) the normal stress in the rod.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.5)^2 = 0.19635 \text{ in}^2$$

(*a*) $$\delta = \frac{PL}{AE} = \frac{(800)(12)}{(0.19635)(0.45 \times 10^6)} = 0.1086$$ $\delta = 0.1086$ in. ◀

(*b*) $$\sigma = \frac{P}{A} = \frac{800}{0.19635} = 4074 \text{ psi}$$ $\sigma = 4.07$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.6

A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3$ GPa and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(*a*) Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{1.1}{100} = 0.011$

Stress: $\sigma = E\varepsilon = (3.3 \times 10^9)(0.011) = 36.3 \times 10^6 \text{ Pa}$

$$\sigma = \frac{P}{A}$$

Area: $A = \dfrac{P}{\sigma} = \dfrac{8.5}{36.3 \times 10^6} = 234.16 \times 10^{-9} \text{ m}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(234.16 \times 10^{-9})}{\pi}} = 546 \times 10^{-6} \text{ m}$ $d = 0.546$ mm ◀

(*b*) Stress: $\sigma = 36.3$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.7

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.

SOLUTION

$$\delta = \Delta L = L - L_0 = 250.18 - 250.00 = 0.18 \text{ mm}$$

$$\varepsilon = \frac{\delta}{L_0} = \frac{0.18 \text{ mm}}{250 \text{ mm}} = 0.00072$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{6000}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{53.052 \times 10^6}{0.00072} = 73.683 \times 10^9 \text{ Pa} \qquad E = 73.7 \text{ GPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.8

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$;

Thus, $L = \dfrac{EA\delta}{P} = \dfrac{E\delta}{\sigma} = \dfrac{(10.1 \times 10^6)(0.05)}{14 \times 10^3}$

$L = 36.1$ in. ◀

(*b*) $\sigma = \dfrac{P}{A}$;

Thus, $A = \dfrac{P}{\sigma} = \dfrac{127.5 \times 10^3}{14 \times 10^3}$ $A = 9.11 \text{ in}^2$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.9

An aluminum control rod must stretch 0.08 in. when a 500-lb tensile load is applied to it. Knowing that $\sigma_{all} = 22$ ksi and $E = 10.1 \times 10^6$ psi, determine the smallest diameter and shortest length that can be selected for the rod.

SOLUTION

$$P = 500 \text{ lb}, \quad \delta = 0.08 \text{ in.} \quad \sigma_{all} = 22 \times 10^3 \text{ psi}$$

$$\sigma = \frac{P}{A} < \sigma_{all} \quad A > \frac{P}{\sigma_{all}} = \frac{500}{22 \times 10^3} = 0.022727 \text{ in}^2$$

$$A = \frac{\pi}{4} d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.022727)}{\pi}} \qquad d_{min} = 0.1701 \text{ in.} \blacktriangleleft$$

$$\sigma = E\varepsilon = \frac{E\delta}{L} < \sigma_{all}$$

$$L > \frac{E\delta}{\sigma_{all}} = \frac{(10.1 \times 10^6)(0.08)}{22 \times 10^3} = 36.7 \text{ in.} \qquad L_{min} = 36.7 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.10

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

SOLUTION

$\sigma = 180 \times 10^6$ Pa $\quad P = 40 \times 10^3$ N

$E = 105 \times 10^9$ Pa $\quad \delta = 2.5 \times 10^{-3}$ m

(*a*) $\quad \delta = \dfrac{PL}{AE} = \dfrac{\sigma L}{E}$

$$L = \frac{E\delta}{\sigma} = \frac{(105\times10^9)(2.5\times10^{-3})}{180\times10^6} = 1.45833 \text{ m}$$

$L = 1.458$ m ◀

(*b*) $\quad \sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{40\times10^3}{180\times10^6} = 222.22\times10^{-6}\text{m}^2 = 222.22 \text{ mm}^2$$

$A = a^2 \quad a = \sqrt{A} = \sqrt{222.22}$

$a = 14.91$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.11

A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200$ GPa, determine the required diameter of the rod.

SOLUTION

$L = 4$ m

$\delta = 3 \times 10^{-3}$ m, $\sigma = 150 \times 10^{6}$ Pa

$E = 200 \times 10^{9}$ Pa, $P = 10 \times 10^{3}$ N

Stress: $\sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{10 \times 10^{3}}{150 \times 10^{6}} = 66.667 \times 10^{-6}\text{m}^2 = 66.667 \text{ mm}^2$$

Deformation: $\delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(10 \times 10^{3})(4)}{(200 \times 10^{9})(3 \times 10^{-3})} = 66.667 \times 10^{-6}\text{m}^2 = 66.667 \text{ mm}^2$$

The larger value of A governs: $A = 66.667$ mm^2

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(66.667)}{\pi}} \qquad d = 9.21 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.12

A nylon thread is to be subjected to a 10-N tension. Knowing that $E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress criterion:

$$\sigma = 40 \text{ MPa} = 40 \times 10^6 \text{ Pa} \quad P = 10 \text{ N}$$

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{10 \text{ N}}{40 \times 10^6 \text{ Pa}} = 250 \times 10^{-9} \text{m}^2$$

$$A = \frac{\pi}{4} d^2: \quad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{250 \times 10^{-9}}{\pi}} = 564.19 \times 10^{-6} \text{m}$$

$$d = 0.564 \text{ mm}$$

Elongation criterion:

$$\frac{\delta}{L} = 1\% = 0.01$$

$$\delta = \frac{PL}{AE}:$$

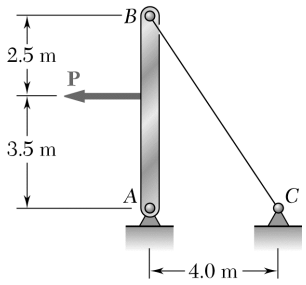
$$A = \frac{P/E}{\delta/L} = \frac{10 \text{ N}/3.2 \times 10^9 \text{ Pa}}{0.01} = 312.5 \times 10^{-9} \text{ m}^2$$

$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{312.5 \times 10^{-9}}{\pi}} = 630.78 \times 10^{-6} \text{m}^2$$

$$d = 0.631 \text{ mm}$$

The required diameter is the larger value: $d = 0.631$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



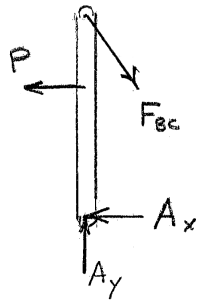
PROBLEM 2.13

The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar *AB* as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509F_{BC}$$

Considering allowable stress: $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

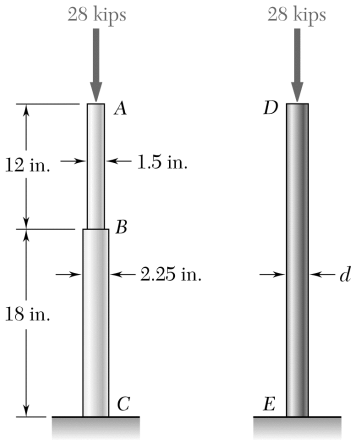
Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N} \qquad P = 1.988 \text{ kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.14

The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod.

$$\begin{aligned}\delta_A &= \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} \\ &= \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right) \\ &= \frac{28\times10^3}{10.1\times10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right) \\ &= 0.031376 \text{ in.}\end{aligned}$$

Steel rod. $\delta = 0.031376$ in.

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28\times10^3)(30)}{(29\times10^6)(0.031376)} = 0.92317 \text{ in}^2$$

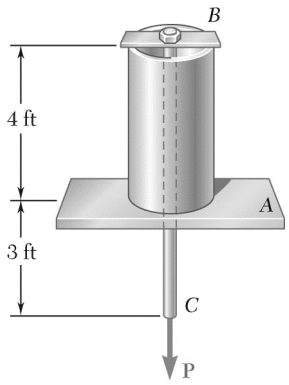
$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28\times10^3}{24\times10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value. $A = 1.1667$ in^2

Diameter:

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.6667)}{\pi}} \qquad d = 1.219 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.15

A 4-ft section of aluminum pipe of cross-sectional area 1.75 in^2 rests on a fixed support at *A*. The $\frac{5}{8}$-in.-diameter steel rod *BC* hangs from a rigid bar that rests on the top of the pipe at *B*. Knowing that the modulus of elasticity is 29×10^6 psi for steel, and 10.4×10^6 psi for aluminum, determine the deflection of point *C* when a 15-kip force is applied at *C*.

SOLUTION

Rod *BC*:

$$L_{BC} = 7 \text{ ft} = 84 \text{ in.} \quad E_{BC} = 29 \times 10^6 \text{ psi}$$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.625)^2 = 0.30680 \text{ in}^2$$

$$\delta_{C/B} = \frac{PL_{BC}}{E_{BC} A_{BC}} = \frac{(15 \times 10^3)(84)}{(29 \times 10^6)(0.30680)} = 0.141618 \text{ in.}$$

Pipe *AB*:

$$L_{AB} = 4 \text{ ft} = 48 \text{ in.} \quad E_{AB} = 10.4 \times 10^6 \text{ psi}$$

$$A_{AB} = 1.75 \text{ in}^2$$

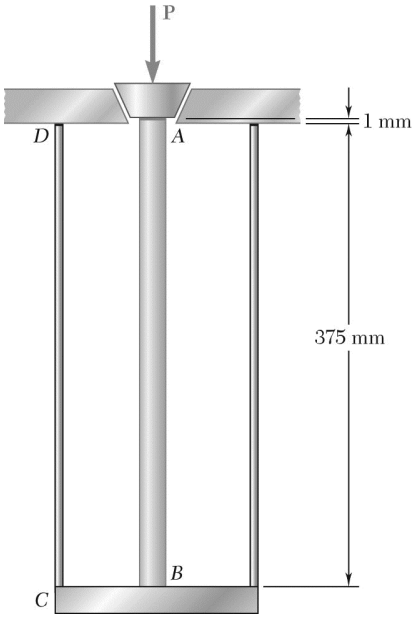
$$\delta_{B/A} = \frac{PL_{AB}}{E_{AB} A_{AB}} = \frac{(15 \times 10^3)(48)}{(10.4 \times 10^6)(1.75)} = 39.560 \times 10^{-3} \text{ in.}$$

Total:

$$\delta_C = \delta_{B/A} + \delta_{C/B} = 39.560 \times 10^{-3} + 0.141618 = 0.181178 \text{ in.}$$

$$\delta_C = 0.1812 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.16

The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{ m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

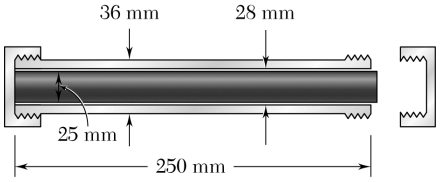
Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9}) P$$

$$P = 21.547 \times 10^3 \text{ N} \qquad P = 21.5 \text{ kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.17



A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12\times10^{-6}\text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87\times10^{-6}\text{ m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70\times10^9)(402.12\times10^{-6})} = 8.8815\times10^{-9}P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = \frac{P(0.250)}{(105\times10^6)(490.87\times10^{-6})} = -4.8505\times10^{-9}P$$

$$\delta^* = \left(\frac{1}{4}\text{turn}\right)\times1.5 \text{ mm} = 0.375 \text{ mm} = 375\times10^{-6}\text{ m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad \text{or} \quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815\times10^{-9}P + 4.8505\times10^{-9}P = 375\times10^{-6}$$

$$P = \frac{0.375\times10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308\times10^3\text{ N}$$

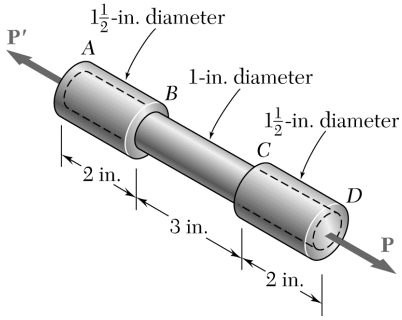
(*a*) $$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308\times10^3}{402.12\times10^{-6}} = 67.9\times10^6\text{ Pa}$$ $\sigma_{\text{tube}} = 67.9$ MPa ◀

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308\times10^3}{490.87\times10^{-6}} = -55.6\times10^6\text{ Pa}$$ $\sigma_{\text{rod}} = -55.6$ MPa ◀

(*b*) $$\delta_{\text{tube}} = (8.8815\times10^{-9})(27.308\times10^3) = 242.5\times10^{-6}\text{ m}$$ $\delta_{\text{tube}} = 0.2425$ mm ◀

$$\delta_{\text{rod}} = -(4.8505\times10^{-9})(27.308\times10^3) = -132.5\times10^{-6}\text{ m}$$ $\delta_{\text{rod}} = -0.1325$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.18

The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(*a*) $$\delta = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \Sigma \frac{L_i}{A_i}$$

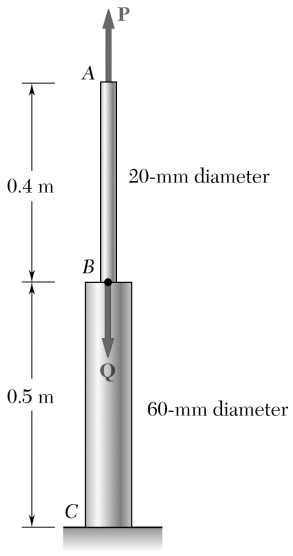
$$P = E\delta \left(\Sigma \frac{L_i}{A_i} \right)^{-1} \quad A_i = \frac{\pi}{4} d_i^2$$

	L, in.	d, in.	A, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$$P = (29 \times 10^6)(0.002)(6.083)^{-1} = 9.353 \times 10^3 \text{ lb} \qquad P = 9.53 \text{ kips} \blacktriangleleft$$

(*b*) $$\delta_{BC} = \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{9.535 \times 10^3}{29 \times 10^6}(3.8197) \qquad \delta = 1.254 \times 10^{-3} \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.19

Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$

Force in member *AB* is *P* tension.

Elongation:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})} = 72.756\times10^{-6}\,\text{m}$$

Force in member *BC* is $Q - P$ compression.

Shortening:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})} = 2.5263\times10^{-9}(Q-P)$$

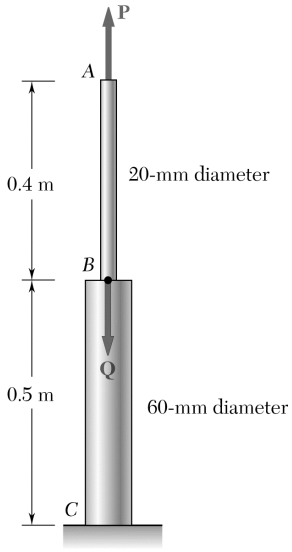
For zero deflection at *A*, $\delta_{BC} = \delta_{AB}$

$$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3\,\text{N}$$

$$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3\,\text{N} \qquad Q = 32.8\ \text{kN} \blacktriangleleft$$

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}\,\text{m}$ $\qquad \delta_{AB} = 0.0728\ \text{mm} \downarrow \blacktriangleleft$

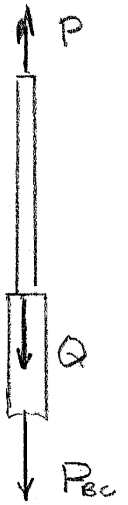
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.20

The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point A, (*b*) point B.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6\times10^3\,\text{N}$$

$$P_{BC} = P - Q = 6\times10^3 - 42\times10^3 = -36\times10^3\,\text{N}$$

$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

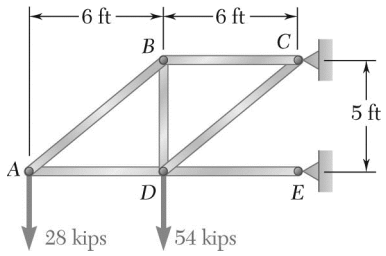
$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}E_A} = \frac{(6\times10^3)(0.4)}{(314.16\times10^{-6})(70\times10^9)} = 109.135\times10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{(-36\times10^3)(0.5)}{(2.8274\times10^{-3})(70\times10^9)} = -90.947\times10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135\times10^{-6} - 90.947\times10^{-6}\,\text{m} = 18.19\times10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm}\uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9\times10^{-6}\,\text{m} = -0.0909\text{ mm}$ $\qquad$ or $\qquad \delta_B = 0.0919\text{ mm}\downarrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



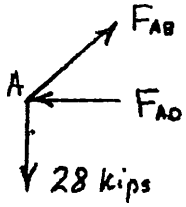
PROBLEM 2.21

Members AB and BC are made of steel ($E = 29 \times 10^6$ psi) with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member AB, (*b*) member BC.

SOLUTION

(*a*) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in.}$

Use joint A as a free body.

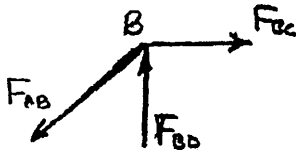


$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)} \qquad \delta_{AB} = 0.1767 \text{ in.} \blacktriangleleft$$

(*b*) Use joint B as a free body.

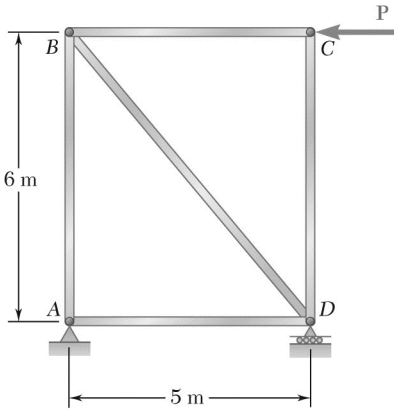


$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{ lb.}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)} \qquad \delta_{BC} = 0.1304 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.22

The steel frame (E = 200 GPa) shown has a diagonal brace BD with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3}\text{m}, \quad A_{BD} = 1920\ \text{mm}^2 = 1920 \times 10^{-6}\text{m}^2$$

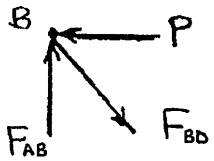
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810\ \text{m}, \quad E_{BD} = 200 \times 10^9\text{Pa}$$

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}}$$

$$F_{BD} = \frac{E_{BD}A_{BD}\delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3\text{N}$$

Use joint B as a free body. $\overset{+}{\rightarrow}\Sigma F_x = 0$:



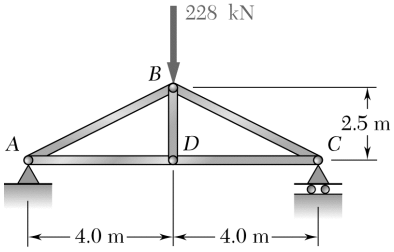
$$\frac{5}{7.810}F_{BD} - P = 0$$

$$P = \frac{5}{7.810}F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3\text{N}$$

$$P = 50.4\ \text{kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.23

For the steel truss $(E = 200\ \text{GPa})$ and loading shown, determine the deformations of the members *AB* and *AD*, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

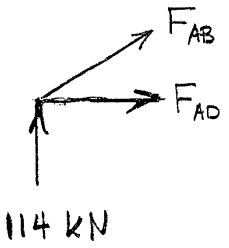
SOLUTION

Statics: Reactions are 114 kN upward at *A* and *C*.

Member *BD* is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717\ \text{m}$$

Use joint *A* as a free body.



$$+\uparrow \Sigma F_y = 0: \ 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10\ \text{kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \ F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4\ \text{kN}$$

Member *AB*:

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})}$$

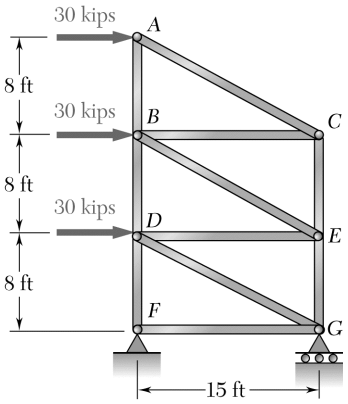
$$= -2.11 \times 10^{-3}\ \text{m} \qquad \delta_{AB} = 2.11\ \text{mm} \blacktriangleleft$$

Member *AD*:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})}$$

$$= 2.03 \times 10^{-3}\ \text{m} \qquad \delta_{AD} = 2.03\ \text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

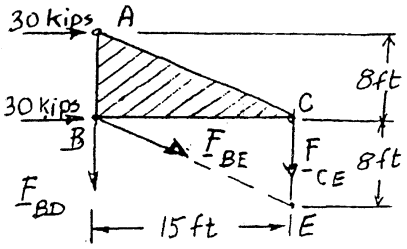


PROBLEM 2.24

For the steel truss ($E = 29\times10^6$ psi) and loading shown, determine the deformations of the members *BD* and *DE*, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

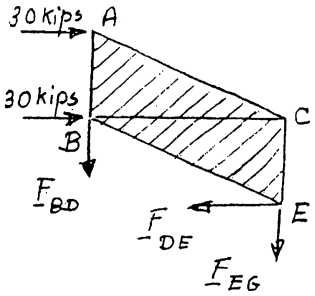
Free body: Portion *ABC* of truss



$$+\circlearrowleft \Sigma M_E = 0: \; F_{BD}(15\text{ ft}) - (30\text{ kips})(8\text{ ft}) - (30\text{ kips})(16\text{ ft}) = 0$$

$$F_{BD} = +48.0\text{ kips}$$

Free body: Portion *ABEC* of truss



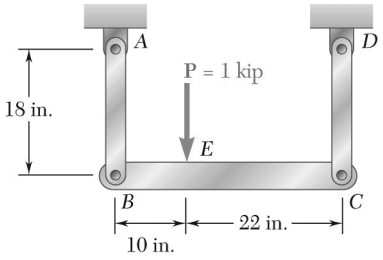
$$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \; 30\text{ kips} + 30\text{ kips} - F_{DE} = 0$$

$$F_{DE} = +60.0\text{ kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0\times10^3\text{ lb})(8\times12\text{ in.})}{(2\text{ in}^2)(29\times10^6\text{ psi})} \qquad \delta_{BD} = +79.4\times10^{-3}\text{ in.} \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0\times10^3\text{ lb})(15\times12\text{ in.})}{(3\text{ in}^2)(29\times10^6\text{ psi})} \qquad \delta_{DE} + 124.1\times10^{-3}\text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

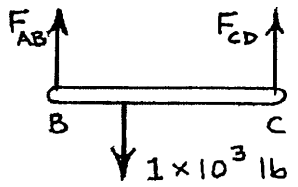


PROBLEM 2.25

Each of the links *AB* and *CD* is made of aluminum ($E = 10.9 \times 10^6$ psi) and has a cross-sectional area of 0.2 in.^{2}. Knowing that they support the rigid member *BC*, determine the deflection of point *E*.

SOLUTION

Free body *BC*:



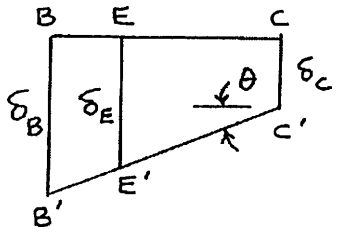
$$+\circlearrowleft \Sigma M_C = 0: \; -(32)F_{AB} + (22)(1\times10^3) = 0$$
$$F_{AB} = 687.5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \; 687.5 - 1\times10^3 + F_{CD} = 0$$
$$F_{CD} = 312.5 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB}\,L_{AB}}{EA} = \frac{(687.5)(18)}{(10.9\times10^6)(0.2)} = 5.6766\times10^{-3} \text{ in} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}\,L_{CD}}{EA} = \frac{(312.5)(18)}{(10.9\times10^6)(0.2)} = 2.5803\times10^{-3} \text{ in} = \delta_C$$

Deformation diagram:

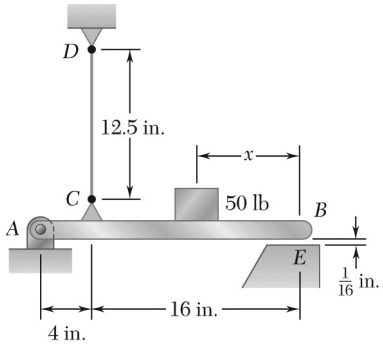


$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963\times10^{-3}}{32}$$
$$= 96.759\times10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + L_{EC}\theta$$
$$= 2.5803\times10^{-3} + (22)(96.759\times10^{-6})$$
$$= 4.7090\times10^{-3} \text{ in}$$

$$\delta_E = 4.71\times10^{-3} \text{ in} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.26

The length of the $\frac{3}{32}$-in.-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of $\frac{1}{16}$in. exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between *B* and *E*.

SOLUTION

Rigid beam *ACB* rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point *C* moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

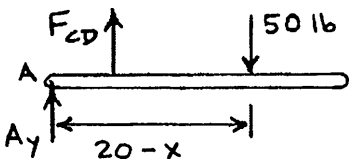
$$A_{CD} = \frac{\pi}{d} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body *ACB*:



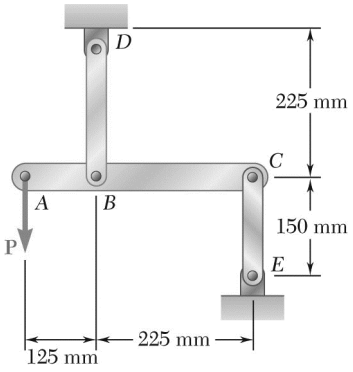
$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

For contact, $x < 3.99$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

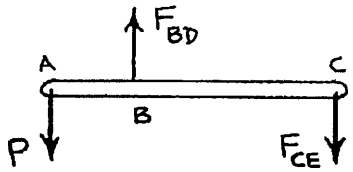


PROBLEM 2.27

Link *BD* is made of brass (E = 105 GPa) and has a cross-sectional area of 240 mm^2. Link *CE* is made of aluminum (E = 72 GPa) and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member *ABC* determine the maximum force **P** that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.35 mm.

SOLUTION

Free body member *AC*:



$$+\circlearrowleft \Sigma M_C = 0: \; 0.350P - 0.225F_{BD} = 0$$

$$F_{BD} = 1.55556P$$

$$+\circlearrowleft \Sigma M_B = 0: \; 0.125P - 0.225F_{CE} = 0$$

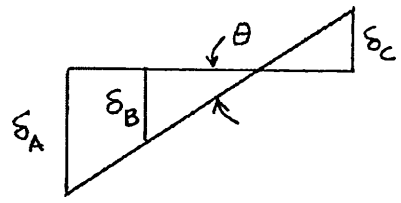
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times10^9)(240\times10^{-6})} = 13.8889\times10^{-9}\,P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times10^9)(300\times10^{-6})} = 3.8581\times10^{-9}\,P$$

Deformation Diagram:

From the deformation diagram,



Slope, $\theta = \dfrac{\delta_B + \delta_C}{L_{BC}} = \dfrac{17.7470\times10^{-9}P}{0.225} = 78.876\times10^{-9}P$

$$\delta_A = \delta_B + L_{AB}\theta$$

$$= 13.8889\times10^{-9}P + (0.125)(78.876\times10^{-9}P)$$

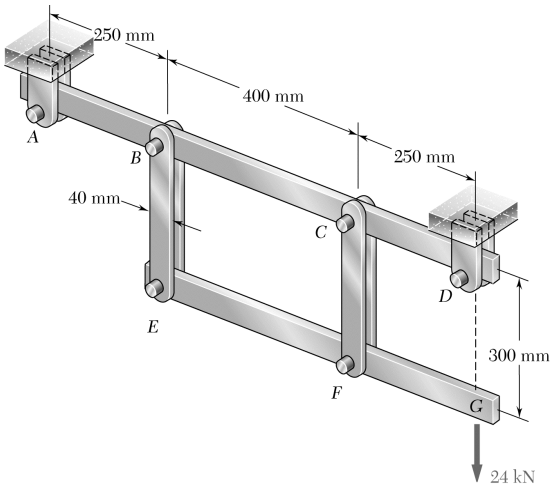
$$= 23.748\times10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times10^{-3}$ m $= 23.748\times10^{-9}P$

$$P = 14.7381\times10^3 \text{ N}$$

$$P = 14.74 \text{ kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

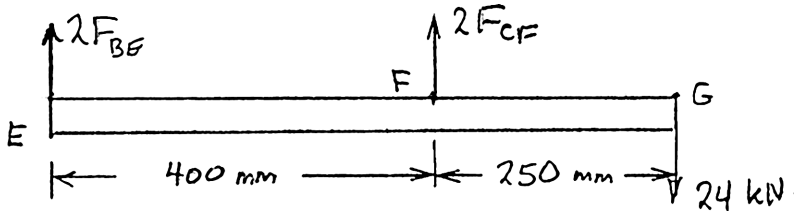


PROBLEM 2.28

Each of the four vertical links connecting the two rigid horizontal members is made of aluminum ($E = 70$ GPa) and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (c) point *G*.

SOLUTION

Statics. Free body *EFG*.



$$+\circlearrowleft \Sigma M_F = 0: \ -(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_E = 0: \ (400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2$$

$$= 400 \times 10^{-6} \text{ m}^2$$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{ m}$$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{ m}$$

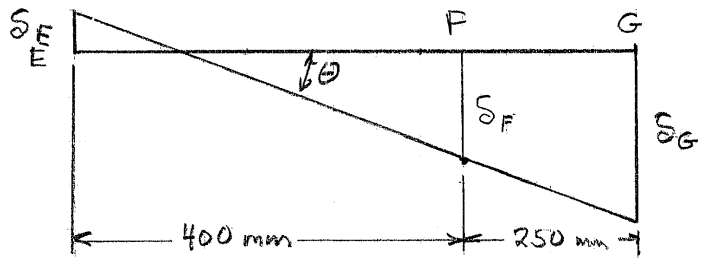
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.28 *(Continued)*

(*a*) Deflection of Point *E*. $\delta_E = |\delta_{BF}|$ $\delta_E = 80.4\ \mu\text{m} \uparrow$ ◀

(*b*) Deflection of Point *F*. $\delta_F = \delta_{CF}$ $\delta_F = 209\ \mu\text{m} \downarrow$ ◀

Geometry change.



Let θ be the small change in slope angle.

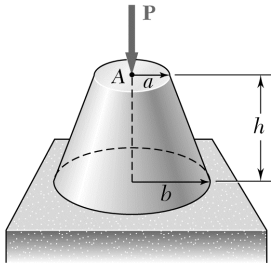
$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357\times10^{-6} + 208.93\times10^{-6}}{0.400} = 723.22\times10^{-6}\ \text{radians}$$

(*c*) Deflection of Point *G*. $\delta_G = \delta_F + L_{FG}\,\theta$

$$\delta_G = \delta_F + L_{FG}\,\theta = 208.93\times10^{-6} + (0.250)(723.22\times10^{-6})$$
$$= 389.73\times10^{-6}\,\text{m}$$

$\delta_G = 390\ \mu\text{m} \downarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

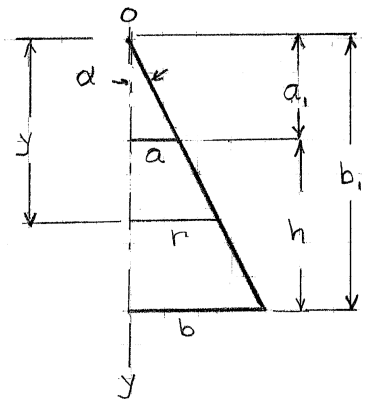


PROBLEM 2.29

A vertical load **P** is applied at the center *A* of the upper section of a homogeneous frustum of a circular cone of height *h*, minimum radius *a*, and maximum radius *b*. Denoting by *E* the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point *A*.

SOLUTION

Extend the slant sides of the cone to meet at a point *O* and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point *y*, $A = \pi r^2$

Deformation of element of height *dy*: $d\delta = \dfrac{P\,dy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation.

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1-a_1}{a_1b_1} = \frac{P(b_1-a_1)}{\pi Eab}$$

$$\delta_A = \frac{Ph}{\pi Eab}\downarrow \blacktriangleleft$$

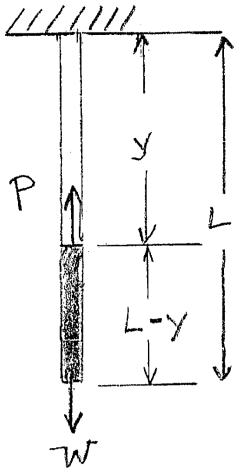
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.30

A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION

(*a*) For element at point identified by coordinate y,



P = weight of portion below the point

$$= \rho g\, A(L-y)$$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g(L-y)}{E}\,dy$$

$$\delta = \int_0^L \frac{\rho g(L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Big|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) \qquad \delta = \frac{1}{2}\frac{\rho g L^2}{E} \blacktriangleleft$$

(*b*) Total weight: $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L \qquad F = \frac{1}{2}W \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.31

The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\varepsilon_t = 2\ln(d_1/d)$.

SOLUTION

If the volume is constant, $\frac{\pi}{4}d^2L = \frac{\pi}{4}d_1^2L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2 \qquad \varepsilon_t = 2\ln\frac{d_1}{d} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.32

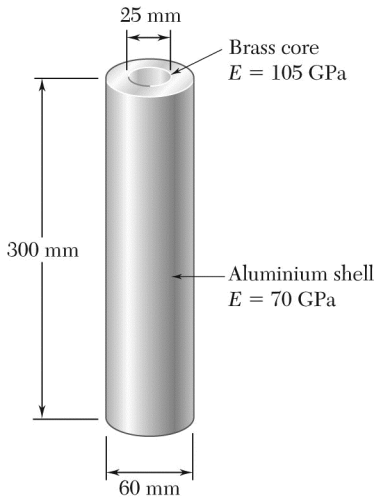
Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\varepsilon_t = \ln(1+\varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0+\delta}{L_0} = \ln\left(1+\frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus $\varepsilon_t = \ln(1+\varepsilon)$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.33

An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the aluminum shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_a = Portion of axial force carried by shell

P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus,
$$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with
$$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L}$$

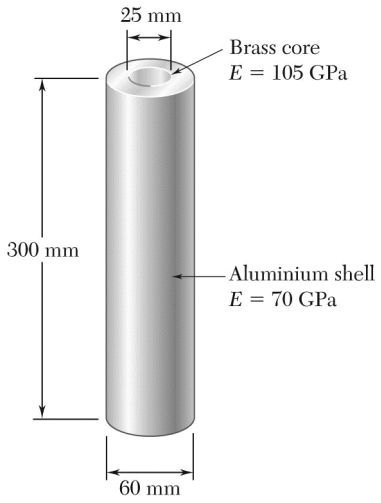
$$P = 215.10 \times 10^6 \frac{\delta}{L}$$

Strain:
$$\varepsilon = \frac{\delta}{L} = \frac{P}{215.10 \times 10^6} = \frac{200 \times 10^3}{215.10 \times 10^6} = 0.92980 \times 10^{-3}$$

(*a*) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(0.92980 \times 10^{-3}) = 65.1 \times 10^6\ \text{Pa}$ $\sigma_a = 65.1\ \text{MPa}$ ◀

(*b*) $\delta = \varepsilon L = (0.92980 \times 10^{-3})(300\ \text{mm})$ $\delta = 0.279\ \text{mm}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.34

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

SOLUTION

Let P_a = Portion of axial force carried by shell and P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus, $$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with $$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

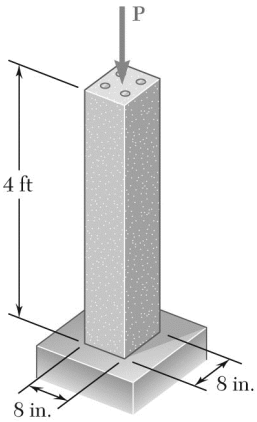
$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L} = 215.10 \times 10^6 \frac{\delta}{L}$$

with $\delta = 0.40$ mm, $L = 300$ mm

(*a*) $$P = (215.10 \times 10^6)\frac{0.40}{300} = 286.8 \times 10^3\ \text{N}$$ $P = 287$ kN ◀

(*b*) $$\sigma_b = \frac{P_b}{A_b} = \frac{E_b \delta}{L} = \frac{(105 \times 10^9)(0.40 \times 10^{-3})}{300 \times 10^{-3}} = 140 \times 10^6\ \text{Pa}$$ $\sigma_b = 140.0$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.35

A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29\times10^6$ psi and $E_c = 3.6\times10^6$ psi, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force **P** is applied to the post.

SOLUTION

$$A_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\right)^2\right] = 1.76715 \text{ in}^2$$

$$A_c = 8^2 - A_s = 62.233 \text{ in}^2$$

$$\delta_s = \frac{P_s L}{A_s E_s} = \frac{P_s(48)}{(1.76715)(29\times10^6)} = 0.93663\times10^{-6}\ P_s$$

$$\delta_c = \frac{P_c L}{A_c E_c} = \frac{P_c(48)}{(62.233)(3.6\times10^6)} = 0.21425\times10^{-6}\ P_c$$

But $\delta_s = \delta_c$: $0.93663\times10^{-6} P_s = 0.21425\times10^{-6} P_c$

$$P_s = 0.22875 P_c \tag{1}$$

Also: $$P_s + P_c = P = 150 \text{ kips} \tag{2}$$

Substituting (1) into (2): $1.22875 P_c = 150$ kips

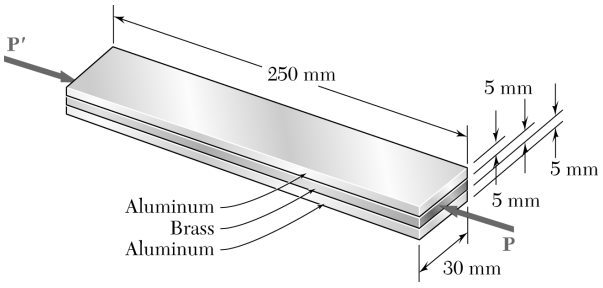
$$P_c = 122.075 \text{ kips}$$

From (1): $P_s = 0.22875(122.075) = 27.925$ kips

$$\sigma_s = -\frac{P_s}{A_s} = -\frac{27.925}{1.76715} \qquad \sigma_s = -15.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_c = -\frac{P_c}{A_c} = -\frac{122.075}{62.233} \qquad \sigma_c = -1.962 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.36

A 250-mm bar of 15×30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude P = 30 kN, and knowing that E_a = 70 GPa and E_b = 105 GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.

SOLUTION

For each layer,

$$A = (30)(5) = 150 \text{ mm}^2 = 150\times10^{-6}\text{ m}^2$$

Let P_a = load on each aluminum layer

P_b = load on brass layer

Deformation. $\delta = \dfrac{P_a L}{E_a A} = \dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a}P_a = \dfrac{105}{70}P_a = 1.5\ P_a$

Total force. $P = 2P_a + P_b = 3.5\ P_a$

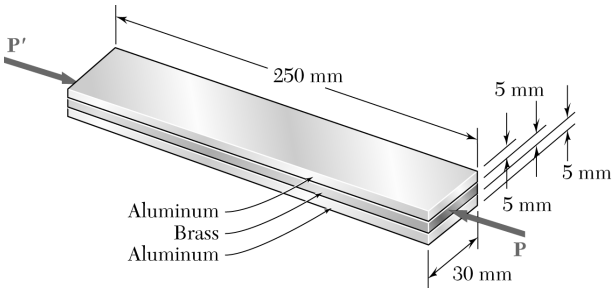
Solving for P_a and P_b, $\qquad P_a = \dfrac{2}{7}P \qquad P_b = \dfrac{3}{7}P$

(*a*) $\sigma_a = -\dfrac{P_a}{A} = -\dfrac{2}{7}\dfrac{P}{A} = -\dfrac{2}{7}\dfrac{30\times10^3}{150\times10^{-6}} = -57.1\times10^6\text{ Pa}$ $\qquad \sigma_a = -57.1\text{ MPa}$ ◀

(*b*) $\sigma_b = -\dfrac{P_b}{A} = -\dfrac{3}{7}\dfrac{P}{A} = -\dfrac{3}{7}\dfrac{30\times10^3}{150\times10^{-6}} = -85.7\times10^6\text{ Pa}$ $\qquad \sigma_b = -85.7\text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.37



Determine the deformation of the composite bar of Prob. 2.36 if it is subjected to centric forces of magnitude $P = 45$ kN.

PROBLEM 2.36 A 250-mm bar of 15×30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.

SOLUTION

For each layer,

$$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$$

Let P_a = load on each aluminum layer

P_b = load on brass layer

Deformation. $\delta = -\dfrac{P_a L}{E_a A} = -\dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5\, P_a$

Total force. $P = 2P_a + P_b = 3.5 \text{ Pa}$ $\qquad P_a = \dfrac{2}{7} P$

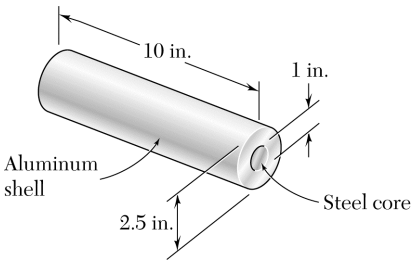
$$\delta = -\frac{P_a L}{E_a A} = -\frac{2}{7}\frac{PL}{E_a A}$$

$$= -\frac{2}{7}\frac{(45 \times 10^3)(250 \times 10^{-3})}{(70 \times 10^9)(150 \times 10^{-6})}$$

$$= -306 \times 10^{-6} \text{ m}$$

$$\delta = -0.306 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.38

Compressive centric forces of 40 kips are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 29\times10^6$ psi and $E_a = 10.1\times10^6$ psi, determine (*a*) the normal stresses in the steel core and the aluminum shell, (*b*) the deformation of the assembly.

SOLUTION

Let P_a = portion of axial force carried by shell

P_s = portion of axial force carried by core

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s}{L}\delta$$

Total force. $$P = P_a + P_s = (E_a A_a + E_s A_s)\frac{\delta}{L}$$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_a A_a + E_s A_s}$$

Data: $P = 40\times10^3$ lb

$$A_a = \frac{\pi}{4}(d_0^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0)^2 = 4.1233 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^2$$

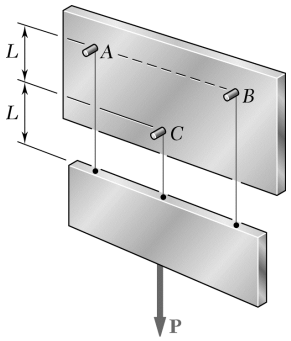
$$\varepsilon = \frac{-40\times10^3}{(10.1\times10^6)(4.1233) + (29\times10^6)(0.7854)} = -620.91\times10^{-6}$$

(*a*) $\sigma_s = E_s\varepsilon = (29\times10^6)(-620.91\times10^{-6}) = -18.01\times10^3$ psi $\quad -18.01$ ksi ◀

$\sigma_a = E_a\varepsilon = (10.1\times10^6)(620.91\times10^{-6}) = -6.27\times10^3$ psi $\quad -6.27$ ksi ◀

(*b*) $\delta = L\varepsilon = (10)(620.91\times10^{-6}) = -6.21\times10^{-3}$ $\quad \delta = -6.21\times10^{-3}$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.39

Three wires are used to suspend the plate shown. Aluminum wires of $\frac{1}{8}$-in. diameter are used at A and B while a steel wire of $\frac{1}{12}$-in. diameter is used at C. Knowing that the allowable stress for aluminum $(E_a = 10.4\times10^6 \text{ psi})$ is 14 ksi and that the allowable stress for steel $(E_s = 29\times10^6 \text{ psi})$ is 18 ksi, determine the maximum load **P** that can be applied.

SOLUTION

By symmetry, $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire:

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \quad \varepsilon_C = \frac{\delta}{L} = 2\varepsilon_A$$

Determine allowable strain.

Wires *A*&*B*: $\varepsilon_A = \dfrac{\sigma_A}{E_A} = \dfrac{14\times10^3}{10.4\times10^6} = 1.3462\times10^{-3}$

$\varepsilon_C = 2\ \varepsilon_A = 2.6924\times10^{-4}$

Wire *C*: $\varepsilon_C = \dfrac{\sigma_C}{E_C} = \dfrac{18\times10^3}{29\times10^6} = 0.6207\times10^{-3}$

$\varepsilon_A = \varepsilon_B = \dfrac{1}{2}\varepsilon_C = 0.3103\times10^{-6}$

Allowable strain for wire *C* governs, $\therefore \quad \sigma_C = 18\times10^3 \text{ psi}$

$\sigma_A = E_A\varepsilon_A \quad P_A = A_A E_A \varepsilon_A$

$$= \frac{\pi}{4}\left(\frac{1}{8}\right)^2 (10.4\times10^6)(0.3103\times10^{-6}) = 39.61 \text{ lb}$$

$P_B = 39.61 \text{ lb}$

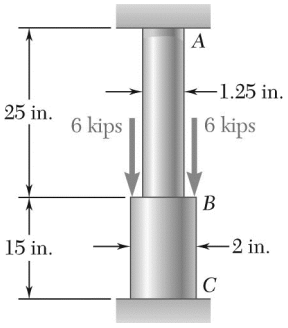
$\sigma_C = E_C\varepsilon_C \quad P_C = A_C\sigma_C = \dfrac{\pi}{4}\left(\dfrac{1}{12}\right)^2 (18\times10^3) = 98.17 \text{ lb}$

For equilibrium of the plate,

$$P = P_A + P_B + P_C = 177.4 \text{ lb}$$

$P = 177.4$ lb ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

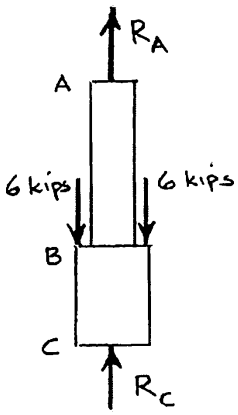


PROBLEM 2.40

A polystyrene rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the reactions at *A* and *C*, (*b*) the normal stress in each portion of the rod.

SOLUTION

(*a*) We express that the elongation of the rod is zero:



$$\delta = \frac{P_{AB}L_{AB}}{\frac{\pi}{4}d_{AB}^2 E} + \frac{P_{BC}L_{BC}}{\frac{\pi}{4}d_{BC}^2 E} = 0$$

But $P_{AB} = +R_A \qquad P_{BC} = -R_C$

Substituting and simplifying:

$$\frac{R_A L_{AB}}{d_{AB}^2} - \frac{R_C L_{BC}}{d_{BC}^2} = 0$$

$$R_C = \frac{L_{AB}}{L_{BC}}\left(\frac{d_{BC}}{d_{AB}}\right)^2 R_A = \frac{25}{15}\left(\frac{2}{1.25}\right)^2 R_A$$

$$R_C = 4.2667 R_A \qquad (1)$$

From the free body diagram: $R_A + R_C = 12$ kips (2)

Substituting (1) into (2): $5.2667\ R_A = 12$

$$R_A = 2.2785 \text{ kips} \qquad R_A = 2.28 \text{ kips} \uparrow \blacktriangleleft$$

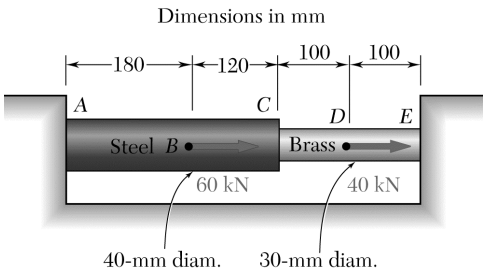
From (1): $R_C = 4.2667(2.2785) = 9.7217$ kips

$$R_C = 9.72 \text{ kips} \uparrow \blacktriangleleft$$

(*b*) $$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+R_A}{A_{AB}} = \frac{2.2785}{\frac{\pi}{4}(1.25)^2} \qquad \sigma_{AB} = +1.857 \text{ ksi} \blacktriangleleft$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{-R_C}{A_{BC}} = \frac{-9.7217}{\frac{\pi}{4}(2)^2} \qquad \sigma_{BC} = -3.09 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.41

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

$\underline{A}$ to $\underline{C}$: $E = 200\times10^{9}\,\text{Pa}$

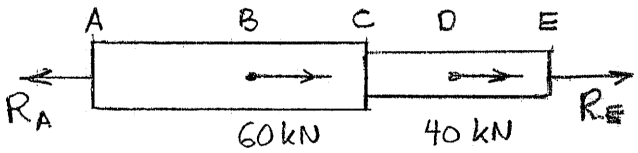
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3}\,\text{mm}^2 = 1.25664\times10^{-3}\,\text{m}^2$$

$$EA = 251.327\times10^{6}\,\text{N}$$

$\underline{C}$ to $\underline{E}$: $E = 105\times10^{9}\,\text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$EA = 74.220\times10^{6}\,\text{N}$$



$\underline{A}$ to $\underline{B}$: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327\times10^{6}}$$

$$= 716.20\times10^{-12}\,R_A$$

$\underline{B}$ to $\underline{C}$: $P = R_A - 60\times10^{3}$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0.120)}{251.327\times10^{6}}$$

$$= 447.47\times10^{-12}\,R_A - 26.848\times10^{-6}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.41 *(Continued)*

<u>*C* to *D*</u>: $P = R_A - 60\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 80.841\times10^{-6}$$

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 134.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9} R_A - 242.424\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9} R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3 \text{ N}$ $R_A = 62.8 \text{ kN} \leftarrow$ ◀

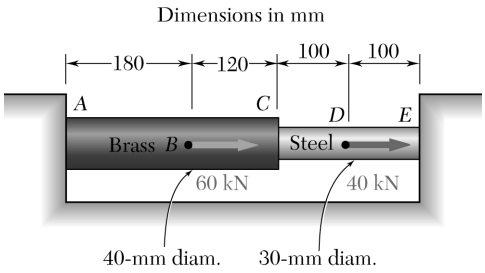
$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3 \text{ N}$ $R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9} R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6} \text{ m}$$ $\delta_C = 46.3 \ \mu\text{m} \rightarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.42

Solve Prob. 2.41, assuming that rod AC is made of brass and rod CE is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

$\underline{A}$ to $\underline{C}$:

$$E = 105\times10^{9}\ \text{Pa}$$

$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3}\ \text{mm}^2 = 1.25664\times10^{-3}\ \text{m}^2$$

$$EA = 131.947\times10^{6}\ \text{N}$$

$\underline{C}$ to $\underline{E}$:

$$E = 200\times10^{9}\ \text{Pa}$$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

$$EA = 141.372\times10^{6}\ \text{N}$$

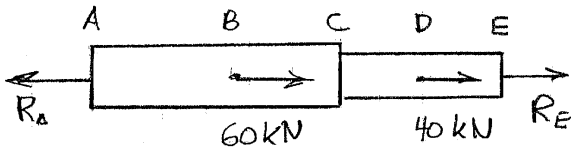
$\underline{A}$ to $\underline{B}$:

$$P = R_A$$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947\times10^{6}}$$

$$= 1.36418\times10^{-9}\,R_A$$



$\underline{B}$ to $\underline{C}$:

$$P = R_A - 60\times10^{3}$$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0.120)}{131.947\times10^{6}}$$

$$= 909.456\times10^{-12}\,R_A - 54.567\times10^{-6}$$

$\underline{C}$ to $\underline{D}$:

$$P = R_A - 60\times10^{3}$$

$$L = 100\ \text{mm} = 0.100\ \text{m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0.100)}{141.372\times10^{6}}$$

$$= 707.354\times10^{-12}\,R_A - 42.441\times10^{-6}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.42 *(Continued)*

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{141.372\times10^6}$$

$$= 707.354\times10^{-12} R_A - 70.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.68834\times10^{-9} R_A - 167.743\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.68834\times10^{-9} R_A - 167.743\times10^{-6} = 0 \quad R_A = 45.479\times10^3 \text{ N}$ $\quad R_A = 45.5 \text{ kN} \leftarrow$ ◀

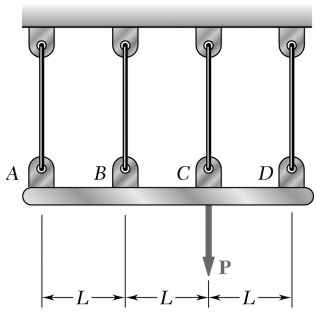
$R_E = R_A - 100\times10^3 = 45.479\times10^3 - 100\times10^3 = -54.521\times10^3$ $\quad R_E = 54.5 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times10^{-9} R_A - 54.567\times10^{-6}$

$$= (2.27364\times10^{-9})(45.479\times10^3) - 54.567\times10^{-6}$$

$= 48.8\times10^{-6} \text{ m}$ $\quad \delta_C = 48.8\,\mu\text{m} \rightarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.43

The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P** shown.

SOLUTION

Deformations Let θ be the rotation of bar *ABCD* and δ_A, δ_B, δ_C and δ_D be the deformations of wires *A*, *B*, *C*, and *D*.

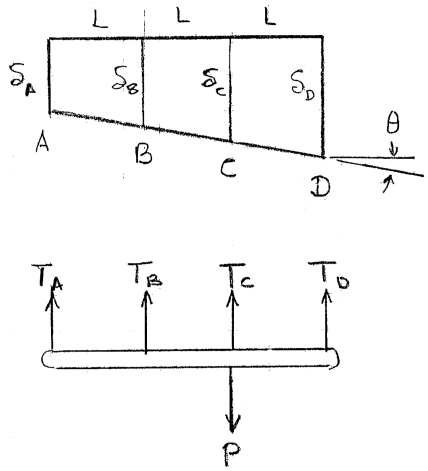
From geometry,

$$\theta = \frac{\delta_B - \delta_A}{L}$$

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta = 2\delta_B - \delta_A \tag{1}$$

$$\delta_D = \delta_A + 3L\theta = 3\delta_B - 2\delta_A \tag{2}$$



Since all wires are identical, the forces in the wires are proportional to the deformations.

$$T_C = 2T_B - T_A \tag{1′}$$

$$T_D = 3T_B - 2T_A \tag{2′}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.43 *(Continued)*

Use bar *ABCD* as a free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -2LT_A - LT_B + LT_D = 0 \tag{3}$$

$$+\uparrow \Sigma F_y = 0: \quad T_A + T_B + T_C + T_D - P = 0 \tag{4}$$

Substituting (2′) into (3) and dividing by L,

$$-4T_A + 2T_B = 0 \qquad T_B = 2T_A \tag{3′}$$

Substituting (1′), (2′), and (3′) into (4),

$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \qquad 10T_A = P$$

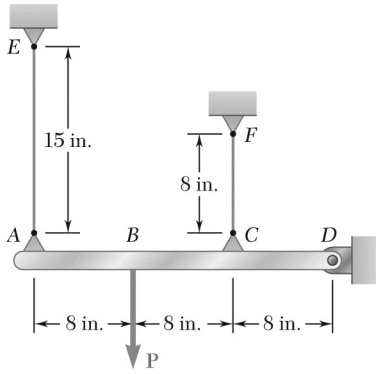
$$T_A = \frac{1}{10}P \blacktriangleleft$$

$$T_B = 2T_A = (2)\left(\frac{1}{10}\right)P \qquad T_B = \frac{1}{5}P \blacktriangleleft$$

$$T_C = (2)\left(\frac{1}{5}P\right) - \left(\frac{1}{10}P\right) \qquad T_C = \frac{3}{10}P \blacktriangleleft$$

$$T_D = (3)\left(\frac{1}{5}P\right) - (2)\left(\frac{1}{10}P\right) \qquad T_D = \frac{2}{5}P \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



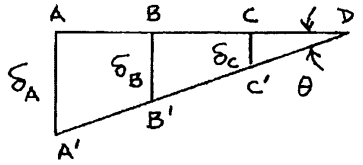
PROBLEM 2.44

The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at D. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 120-lb load **P** is applied at B, (*b*) the corresponding deflection of point B.

SOLUTION

Let θ be the rotation of bar $ABCD$.

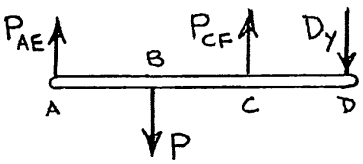
Then $\delta_A = 24\theta \qquad \delta_C = 8\theta$



$$\delta_A = \frac{P_{AE} L_{AE}}{AE}$$

$$P_{AE} = \frac{EA\delta_A}{L_{AE}} = \frac{(29\times10^6)\frac{\pi}{4}(\frac{1}{16})^2(24\theta)}{15}$$

$$= 142.353\times10^3\,\theta$$



$$\delta_C = \frac{P_{CF} L_{CF}}{AE}$$

$$P_{CF} = \frac{EA\delta_C}{L_{CF}} = \frac{(29\times10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(8\theta)}{8}$$

$$= 88.971\times10^3\,\theta$$

Using free body $ABCD$,

$+\circlearrowleft \Sigma M_D = 0$: $\quad -24P_{AE} + 16P - 8P_{CF} = 0$

$$-24(142.353\times10^3\theta) + 16(120) - 8(88.971\times10^3\theta) = 0$$

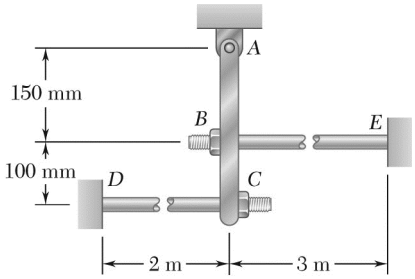
$$\theta = 0.46510\times10^{-3}\ \text{rad}\circlearrowleft$$

(*a*) $P_{AE} = (142.353\times10^3)\,(0.46510\times10^{-3})$ $\qquad P_{AE} = 66.2$ lb ◀

$P_{CF} = (88.971\times10^3)\,(0.46510\times10^{-3})$ $\qquad P_{CF} = 41.4$ lb ◀

(*b*) $\delta_B = 16\theta = 16(0.46510\times10^{-3})$ $\qquad \delta_B = 7.44\times10^{-3}$ in.↓ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

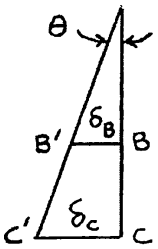


PROBLEM 2.45

The steel rods *BE* and *CD* each have a 16-mm diameter ($E = 200$ GPa); the ends of the rods are single-threaded with a pitch of 2.5 mm. Knowing that after being snugly fitted, the nut at *C* is tightened one full turn, determine (*a*) the tension in rod *CD*, (*b*) the deflection of point *C* of the rigid member *ABC*.

SOLUTION

Let θ be the rotation of bar *ABC* as shown.



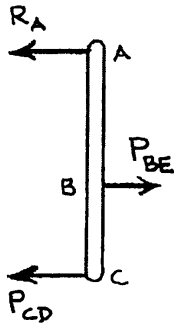
Then $\delta_B = 0.15\theta \qquad \delta_C = 0.25\theta$

But $$\delta_C = \delta_{\text{turn}} - \frac{P_{CD}L_{CD}}{E_{CD}A_{CD}}$$

$$P_{CD} = \frac{E_{CD}A_{CD}}{L_{CD}}(\delta_{\text{turn}} - \delta_C)$$

$$= \frac{(200\times10^9\ \text{Pa})\frac{\pi}{4}(0.016\ \text{m})^2}{2\ \text{m}}(0.0025\,\text{m} - 0.25\theta)$$

$$= 50.265\times10^3 - 5.0265\times10^6\,\theta$$



$$\delta_B = \frac{P_{BE}L_{BE}}{E_{BE}A_{BE}} \qquad \text{or} \qquad P_{BE} = \frac{E_{BE}A_{BE}}{L_{BE}}\delta_B$$

$$P_{BE} = \frac{(200\times10^9\ \text{Pa})\frac{\pi}{4}(0.016\ \text{m})^2}{3\ \text{m}}(0.15\theta)$$

$$= 2.0106\times10^6\,\theta$$

From free body of member *ABC*:

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.15P_{BE} - 0.25P_{CD} = 0$$

$$0.15(2.0106\times10^6\,\theta) - 0.25(50.265\times10^3 - 5.0265\times10^6\,\theta) = 0$$

$$\theta = 8.0645\times10^{-3}\ \text{rad}$$

(*a*) $P_{CD} = 50.265\times10^3 - 5.0265\times10^6(8.0645\times10^{-3})$

$$= 9.7288\times10^3\ \text{N}$$

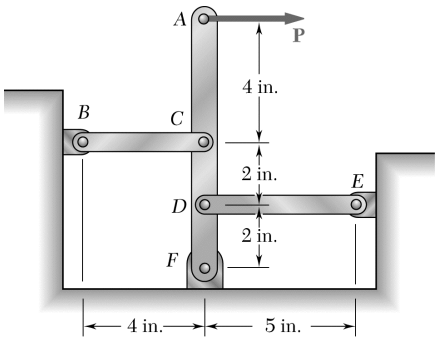
$P_{CD} = 9.73\ \text{kN}$ ◀

(*b*) $\delta_C = 0.25\theta = 0.25(8.0645\times10^{-3})$

$$= 2.0161\times10^{-3}\ \text{m}$$

$\delta_C = 2.02\ \text{mm} \leftarrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.46

Links *BC* and *DE* are both made of steel ($E = 29\times10^6$ psi) and are $\frac{1}{2}$ in. wide and $\frac{1}{4}$ in. thick. Determine (*a*) the force in each link when a 600-lb force **P** is applied to the rigid member *AF* shown, (*b*) the corresponding deflection of point *A*.

SOLUTION

Let the rigid member *ACDF* rotate through small angle θ clockwise about point *F*.

Then $\delta_C = \delta_{BC} = 4\theta$ in. $\rightarrow$

$\delta_D = -\delta_{DE} = 2\theta$ in. $\rightarrow$

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EA\delta}{L}$$

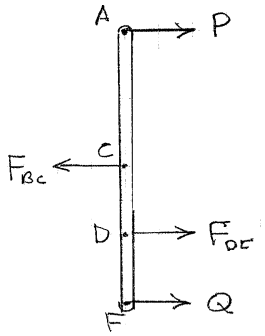
For links: $A = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in}^2$

$L_{BC} = 4$ in.

$L_{DE} = 5$ in.

$$F_{BC} = \frac{EA\,\delta_{BC}}{L_{BC}} = \frac{(29\times10^6)(0.125)(4\theta)}{4} = 3.625\times10^6\,\theta$$

$$F_{DE} = \frac{EA\delta_{DE}}{L_{DE}} = \frac{(29\times10^6)(0.125)(-2\theta)}{5} = -1.45\times10^6\,\theta$$



Use member *ACDF* as a free body.

$$+\circlearrowleft \Sigma M_F = 0: \quad 8P - 4F_{BC} + 2F_{DE} = 0$$

$$P = \frac{1}{2}F_{BC} - \frac{1}{4}F_{DE}$$

$$600 = \frac{1}{2}(3.625\times10^6)\theta - \frac{1}{4}(-1.45\times10^6)\theta = 2.175\times10^6\,\theta$$

$$\theta = 0.27586\times10^{-3} \text{ rad} \circlearrowleft$$

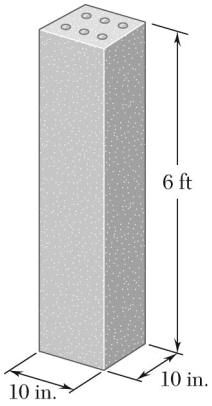
(*a*) $F_{BC} = (3.625\times10^6)(0.27586\times10^{-3})$ $\qquad F_{BC} = 1000$ lb ◀

$F_{DE} = -(1.45\times10^6)(0.27586\times10^{-3})$ $\qquad F_{DE} = -400$ lb ◀

(*b*) Deflection at Point *A*.

$\delta_A = 8\theta = (8)(0.27586\times10^{-3})$ $\qquad \delta_A = 2.21\times10^{-3}$ in. $\rightarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.47

The concrete post ($E_c = 3.6\times10^6$ psi and $\alpha_c = 5.5\times10^{-6}/°\text{F}$) is reinforced with six steel bars, each of $\frac{7}{8}$-in. diameter ($E_s = 29\times10^6$ psi and $\alpha_s = 6.5\times10^{-6}/°\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F.

SOLUTION

$$A_s = 6\frac{\pi}{4}d^2 = 6\frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let $P_c =$ tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\varepsilon_s = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T) \qquad \varepsilon_c = \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \qquad \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

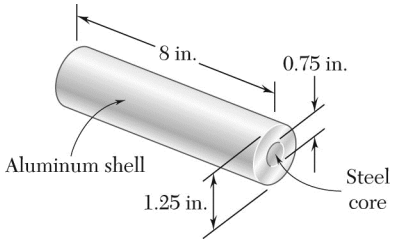
$$\left[\frac{1}{(3.6\times10^6)(96.392)} + \frac{1}{(29\times10^6)(3.6079)}\right]P_c = (1.0\times10^{-6})(65)$$

$$P_c = 5.2254\times10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254\times10^3}{96.392} = 54.210 \text{ psi} \qquad \sigma_c = 54.2 \text{ psi}$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254\times10^3}{3.6079} = -1448.32 \text{ psi} \qquad \sigma_s = -1.448 \text{ ksi}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.48

The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, $\alpha_a = 12.9 \times 10^{-6}$/°F) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.

SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise.

Let $\sigma_a = -6 \text{ ksi} = -6 \times 10^3 \text{ psi}$

$$A_a = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$

where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

(*a*) $\Delta T = 145.91°\text{F}$ $\Delta T = 145.9°\text{F}$ ◀

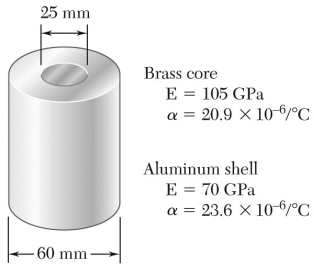
(*b*) $$\varepsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

or

$$\varepsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

$\delta = L\varepsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053$ in. $\delta = 0.01053$ in. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.49

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Brass core:

$$E = 105 \text{ GPa}$$

$$\alpha = 20.9 \times 10^{-6}/^{\circ}\text{C}$$

Aluminum shell:

$$E = 70 \text{ GPa}$$

$$\alpha = 23.6 \times 10^{-6}/^{\circ}\text{C}$$

Let L be the length of the assembly.

Free thermal expansion:

$$\Delta T = 195 - 15 = 180^{\circ}\text{C}$$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

Aluminum shell: $(\delta_T) = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

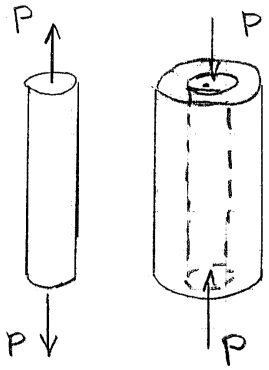
Brass core:

$$E_b = 105 \times 10^9 \text{ Pa}$$

$$A_b = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$

$$= 490.87 \times 10^{-6} \text{ m}^2$$

$$(\delta_P)_b = \frac{PL}{E_b A_b}$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.49 *(Continued)*

Aluminum shell:

$$E_a = 70\times10^9\ \text{Pa}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2)$$

$$= 2.3366\times10^3\ \text{mm}^2$$

$$= 2.3366\times10^{-3}\ \text{m}^2$$

$$\delta = (\delta_P)_b + (\delta_P)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(105\times10^9)(490.87\times10^{-6})} + \frac{1}{(70\times10^9)(2.3366\times10^{-3})}$$

$$= 25.516\times10^{-9}\ \text{N}^{-1}$$

Then

$$P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K}$$

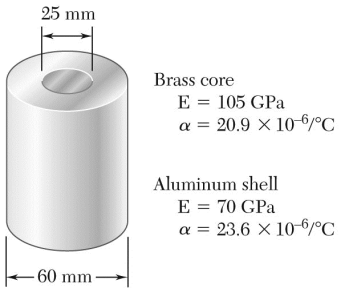
$$= \frac{(23.6\times10^{-6} - 20.9\times10^{-6})(180)}{25.516\times10^{-9}}$$

$$= 19.047\times10^3\ \text{N}$$

Stress in aluminum:

$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047\times10^3}{2.3366\times10^{-3}} = -8.15\times10^6\ \text{Pa} \qquad \sigma_a = -8.15\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.50

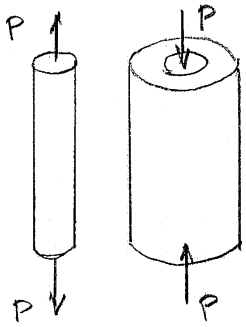
Solve Prob. 2.49, assuming that the core is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°\text{C}$) instead of brass.

PROBLEM 2.49 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Aluminum shell: $E = 70$ GPa $\alpha = 23.6\times10^{-6}/°\text{C}$

Let L be the length of the assembly.



Free thermal expansion: $\Delta T = 195 - 15 = 180°\text{C}$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_s)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200\times10^9\,\text{Pa}, \quad A_s = \dfrac{\pi}{4}(25)^2 = 490.87\ \text{mm}^2 = 490.87\times10^{-6}\,\text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70\times10^9\,\text{Pa}$

$$(\delta_P)_a = \frac{PL}{E_a A_a}$$

$$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366\times10^3\,\text{mm}^2 = 2.3366\times10^{-3}\,\text{m}^2$$

$$\delta = (\delta_P)_s + (\delta_P)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(200\times10^9)(490.87\times10^{-6})} + \frac{1}{(70\times10^9)(2.3366\times10^{-3})}$$

$$= 16.2999\times10^{-9}\,\text{N}^{-1}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

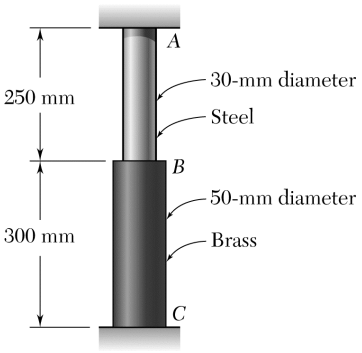
PROBLEM 2.50 ***(Continued)***

Then

$$P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6\times10^{-6} - 11.7\times10^{-6})(180)}{16.2999\times10^{-9}} = 131.41\times10^{3}\,\text{N}$$

Stress in aluminum: $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{131.19\times10^{3}}{2.3366\times10^{-3}} = -56.2\times10^{6}\,\text{Pa}$ $\sigma_a = -56.2\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.51

A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°\text{C}$) and portion *BC* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in *ABC* when there is a temperature rise of 50 °C.

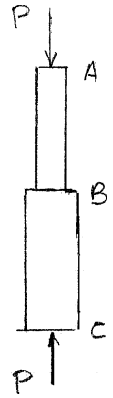
SOLUTION

$$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^{3}\ \text{mm}^2 = 1.9635\times10^{-3}\ \text{m}^2$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)\\ &= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)\\ &= 459.75\times10^{-6}\ \text{m}\end{aligned}$$



Shortening due to induced compressive force *P*:

$$\begin{aligned}\delta_P &= \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}\\ &= \frac{0.250P}{(200\times10^{9})(706.86\times10^{-6})} + \frac{0.300P}{(105\times10^{9})(1.9635\times10^{-3})}\\ &= 3.2235\times10^{-9}\,P\end{aligned}$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}\,P = 459.75\times10^{-6}$$

$$P = 142.62\times10^{3}\ \text{N}$$

$$P = 142.6\ \text{kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.52

A steel railroad track ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}$/°C) was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7\times10^{-6})(48-6)(10) = 4.914\times10^{-3}$ m

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200\times10^{9}} = 50\times10^{-12}\sigma$$

$$\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 0$$

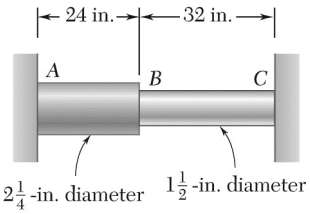
$$\sigma = -98.3\times10^{6} \text{ Pa} \qquad \sigma = -98.3 \text{ MPa}$$

(*b*) $\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 3\times10^{-3}$

$$\sigma = \frac{3\times10^{-3} - 4.914\times10^{-3}}{50\times10^{-12}}$$

$$= -38.3\times10^{6} \text{ Pa} \qquad \sigma = -38.3 \text{ MPa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



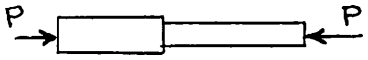
PROBLEM 2.53

A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion *BC* is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \quad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

Free thermal expansion. $\Delta T = 70°\text{F}$



$$(\delta_T)_{AB} = L_{AB}\alpha_s(\Delta T) + (24)(6.5\times10^{-6})(70) = 10.92\times10^{-3} \text{ in}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_a(\Delta T) = (32)(13.3\times10^{-6}(70) = 29.792\times10^{-3} \text{ in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 40.712\times10^{-3} \text{ in.}$

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_s A_{AB}} = \frac{24P}{(29\times10^6)(3.9761)} = 208.14\times10^{-9}\,P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_a A_{BC}} = \frac{32P}{(10.4\times10^6)(1.76715)} = 1741.18\times10^{-9}\,P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1949.32\times10^{-9}\,P$

For zero net deflection, $\delta_P = \delta_T$ $\quad 1949.32\times10^{-9}\,P = 40.712\times10^{-3}$ $\quad P = 20.885\times10^3$ lb

(*a*) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{20.885\times10^3}{3.9761} = -5.25\times10^3$ psi $\quad \sigma_{AB} = -5.25$ ksi ◀

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{20.885\times10^3}{1.76715} = -11.82\times10^3$ psi $\quad \sigma_{BC} = -11.82$ ksi ◀

(*b*) $(\delta_P)_{AB} = (208.14\times10^{-9})(20.885\times10^3) = 4.3470\times10^{-3} \text{ in.}$

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 10.92\times10^{-3} \rightarrow + 4.3470\times10^{-3} \leftarrow$ $\quad \delta_B = 6.57\times10^{-3}$ in. $\rightarrow$ ◀

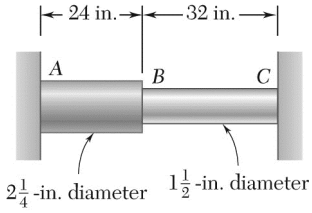
or

$(\delta_P)_{BC} = (1741.18\times10^{-9})(20.885\times10^3) = 36.365\times10^{-3} \text{ in.}$

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 29.792\times10^{-3} \leftarrow + 36.365\times10^{-3} \rightarrow = 6.57\times10^{-3} \text{ in.} \rightarrow$ (checks)

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

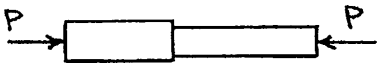
PROBLEM 2.54



Solve Prob. 2.53, assuming that portion *AB* of the composite rod is made of aluminum and portion *BC* is made of steel.

PROBLEM 2.53 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion *BC* is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \qquad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

Free thermal expansion. $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_a(\Delta T) = (24)(13.3\times10^{-6})(70) = 22.344\times10^{-3} \text{ in.}$$
$$(\delta_T)_{BC} = L_{BC}\alpha_s(\Delta T) = (32)(6.5\times10^{-6})(70) = 14.56\times10^{-3} \text{ in.}$$

Total: $$\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 36.904\times10^{-3} \text{ in.}$$

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_a A_{AB}} = \frac{24\,P}{(10.4\times10^6)(3.9761)} = 580.39\times10^{-9}\,P$$
$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_s A_{BC}} = \frac{32\,P}{(29\times10^6)(1.76715)} = 624.42\times10^{-9}\,P$$

Total: $$\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1204.81\times10^{-9}\,P$$

For zero net deflection, $\delta_P = \delta_T$ $\qquad 1204.81\times10^{-9}\,P = 36.904\times10^{-3} \qquad P = 30.631\times10^3$ lb

(*a*) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{30.631\times10^3}{3.9761} = -7.70\times10^3 \text{ psi} \qquad \sigma_{AB} = -7.70 \text{ ksi} \blacktriangleleft$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{30.631\times10^3}{1.76715} = -17.33\times10^3 \text{ psi} \qquad \sigma_{BC} = -17.33 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.54 ***(Continued)***

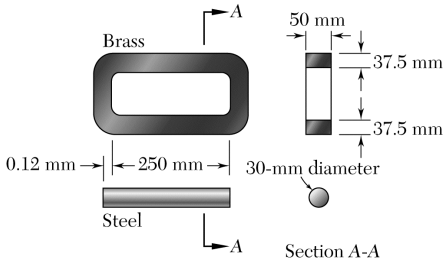
(*b*) $(\delta_P)_{AB} = (580.39\times10^{-9})(30.631\times10^{3}) = 17.7779\times10^{-3}\,\text{in.}$

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 22.344\times10^{-3} \rightarrow + 17.7779\times10^{-3} \leftarrow \qquad \delta_B = 4.57\times10^{-3}\,\text{in.} \rightarrow$ ◀

or $(\delta_P)_{BC} = (624.42\times10^{-9})(30.631\times10^{3}) = 19.1266\times10^{-3}\,\text{in.}$

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 14.56\times10^{-3} \leftarrow + 19.1266\times10^{-3} \rightarrow = 4.57\times10^{-3}\ \text{in.} \rightarrow$ (checks)

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.55

A brass link ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}$/°C) and a steel rod ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}$/°C) have the dimensions shown at a temperature of 20°C. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45°C. Determine (*a*) the final stress in the steel rod, (*b*) the final length of the steel rod.

SOLUTION

Initial dimensions at $T = 20$ °C.

Final dimensions at $T = 45$ °C.

$$\Delta T = 45 - 20 = 25\ ^\circ\text{C}$$

Free thermal expansion of each part:

Brass link: $(\delta_T)_b = \alpha_b(\Delta T)L = (20.9\times10^{-6})(25)(0.250) = 130.625\times10^{-6}\ \text{m}$

Steel rod: $(\delta_T)_s = \alpha_s(\Delta T)L = (11.7\times10^{-6})(25)(0.250) = 73.125\times10^{-6}\ \text{m}$

At the final temperature, the difference between the free length of the steel rod and the brass link is

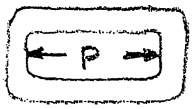
$$\delta = 120\times10^{-6} + 73.125\times10^{-6} - 130.625\times10^{-6} = 62.5\times10^{-6}\ \text{m}$$

Add equal but opposite forces P to elongate the brass link and contract the steel rod.

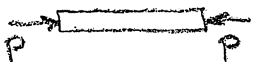
Brass link: $E = 105\times10^{9}$ Pa

$$A_b = (2)(50)(37.5) = 3750\ \text{mm}^2 = 3.750\times10^{-3}\ \text{m}^2$$

$$(\delta_P) = \frac{PL}{EA} = \frac{P(0.250)}{(105\times10^{9})(3.750\times10^{-3})} = 634.92\times10^{-12}\,P$$



Steel rod: $E = 200\times10^{9}$ Pa $\quad A_s = \dfrac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$



$$(\delta_P)_s = \frac{PL}{E_s A_s} = \frac{P(0.250)}{(200\times10^{9})(706.86\times10^{-6})} = 1.76838\times10^{-9}\,P$$

$(\delta_P)_b + (\delta_P)_s = \delta$: $\quad 2.4033\times10^{-9}\,P = 62.5\times10^{-6} \quad P = 26.006\times10^{3}\ \text{N}$

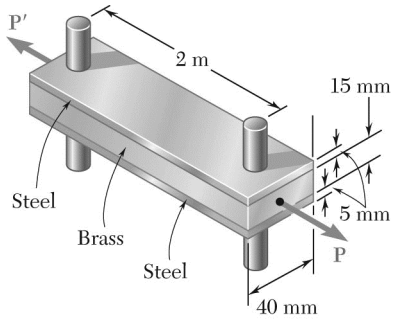
(*a*) Stress in steel rod: $\sigma_s = -\dfrac{P}{A_s} = -\dfrac{(26.006\times10^{3})}{706.86\times10^{-6}} = -36.8\times10^{6}\ \text{Pa}$ $\qquad \sigma_s = -36.8\ \text{MPa}$ ◀

(*b*) Final length of steel rod: $L_f = L_0 + (\delta_T)_s - (\delta_P)_s$

$$L_f = 0.250 + 120\times10^{-6} + 73.125\times10^{-6} - (1.76838\times10^{-9})(26.003\times10^{3})$$

$$= 0.250147\ \text{m} \qquad L_f = 250.147\ \text{mm}\ \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.56

Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7\times10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°C$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

(*a*) Required temperature change for fabrication:

$$\delta_T = 0.5 \text{ mm} = 0.5\times10^{-3} \text{ m}$$

Temperature change required to expand steel bar by this amount:

$$\delta_T = L\alpha_s\Delta T, \quad 0.5\times10^{-3} = (2.00)(11.7\times10^{-6})(\Delta T),$$
$$\Delta T = 0.5\times10^{-3} = (2)(11.7\times10^{-6})(\Delta T)$$
$$\Delta T = 21.368\,°C \qquad 21.4\,°C \blacktriangleleft$$

(*b*) Once assembled, a tensile force P^* develops in the steel, and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6}\text{ m}^2$

$$(\delta_P)_s = \frac{F^*L}{A_sE_s} = \frac{P^*(2.00)}{(400\times10^{-6})(200\times10^9)} = 25\times10^{-9}P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600\times10^{-6}\text{ m}^2$

$$(\delta_P)_b = \frac{P^*L}{A_bE_b} = \frac{P^*(2.00)}{(600\times10^{-6})(105\times10^9)} = 31.746\times10^{-9}P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit:

$$(\delta_P)_s + (\delta_P)_b = 0.5\times10^{-3}, \quad 56.746\times10^{-9}P^* = 0.5\times10^{-3}$$
$$P^* = 8.811\times10^3 \text{ N}$$

Stresses due to fabrication:

Steel: $$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.811\times10^3}{400\times10^{-6}} = 22.03\times10^6 \text{ Pa} = 22.03 \text{ MPa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.56 *(Continued)*

Brass: $$\sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811\times10^3}{600\times10^{-6}} = -14.68\times10^6 \text{ Pa} = -14.68 \text{ MPa}$$

To these stresses must be added the stresses due to the 25 kN load.

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^9)}{2.00}\delta' = 40\times10^6\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^9)}{2.00}\delta' = 31.5\times10^6\,\delta'$$

Total $$P = P_s + P_b = 25\times10^3 \text{ N}$$

$$40\times10^6\,\delta' + 31.5\times10^6\,\delta' = 25\times10^3 \qquad \delta' = 349.65\times10^{-6} \text{ m}$$

$$P_s = (40\times10^6)(349.65\times10^{-6}) = 13.986\times10^3 \text{ N}$$

$$P_b = (31.5\times10^6)(349.65\times10^{-6}) = 11.140\times10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^3}{400\times10^{-6}} = 34.97\times10^6 \text{ Pa}$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^3}{600\times10^{-6}} = 18.36\times10^6 \text{ Pa}$$

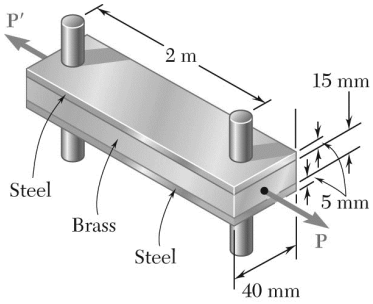
Add stress due to fabrication.

Total stresses:

$$\sigma_s = 34.97\times10^6 + 22.03\times10^6 = 57.0\times10^6 \text{ Pa} \qquad \sigma_s = 57.0 \text{ MPa}$$

$$\sigma_b = 18.36\times10^6 - 14.68\times10^6 = 3.68\times10^6 \text{ Pa} \qquad \sigma_b = 3.68 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.57

Determine the maximum load P that may be applied to the brass bar of Prob. 2.56 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

PROBLEM 2.56 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

See solution to Problem 2.56 to obtain the fabrication stresses.

$$\sigma_s^* = 22.03 \text{ MPa}$$

$$\sigma_b^* = 14.68 \text{ MPa}$$

Allowable stresses: $\sigma_{s,\text{all}} = 30$ MPa, $\sigma_{b,\text{all}} = 25$ MPa

Available stress increase from load.

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$

$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.85 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

Forces $P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$

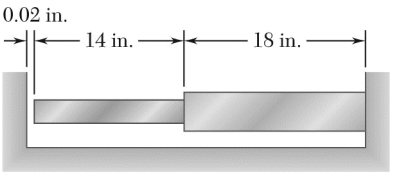
$$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^{-3} \text{ N}$$

Total allowable additional force:

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N}$$

$P = 5.70$ kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Bronze
$A = 2.4$ in.^{2}
$E = 15 \times 10^6$ psi
$\alpha = 12 \times 10^{-6}$/°F

Aluminum
$A = 2.8$ in.^{2}
$E = 10.6 \times 10^6$ psi
$\alpha = 12.9 \times 10^{-6}$/°F

PROBLEM 2.58

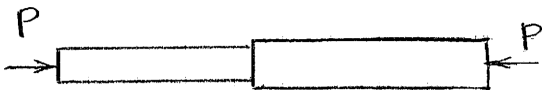
Knowing that a 0.02-in. gap exists when the temperature is 75°F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −11 ksi, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$$\sigma_a = -11 \text{ ksi} = -11 \times 10^3 \text{ psi}$$

$$P = -\sigma_a A_a = (11 \times 10^3)(2.8) = 30.8 \times 10^3 \text{ lb}$$

Shortening due to *P*:



$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(30.8 \times 10^3)(14)}{(15 \times 10^6)(2.4)} + \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)}$$

$$= 30.657 \times 10^{-3} \text{ in.}$$

Available elongation for thermal expansion:

$$\delta_T = 0.02 + 30.657 \times 10^{-3} = 50.657 \times 10^{-3} \text{ in.}$$

But $\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$

$$= (14)(12 \times 10^{-6})(\Delta T) + (18)(12.9 \times 10^{-6})(\Delta T) = 400.2 \times 10^{-6})\Delta T$$

Equating, $(400.2 \times 10^{-6})\Delta T = 50.657 \times 10^{-3}$ $\quad \Delta T = 126.6$ °F

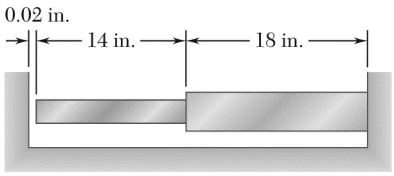
(*a*) $T_{\text{hot}} = T_{\text{cold}} + \Delta T = 75 + 126.6 = 201.6$ °F $\qquad T_{\text{hot}} = 201.6$°F ◀

(*b*) $\delta_a = L_a \alpha_a (\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (18)(12.9 \times 10^{-6})(26.6) - \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)} = 10.712 \times 10^{-3} \text{ in.}$$

$$L_{\text{exact}} = 18 + 10.712 \times 10^{-3} = 18.0107 \text{ in.} \qquad L = 18.0107 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Bronze
$A = 2.4\ \text{in.}^2$
$E = 15 \times 10^6\ \text{psi}$
$\alpha = 12 \times 10^{-6}/°\text{F}$

Aluminum
$A = 2.8\ \text{in.}^2$
$E = 10.6 \times 10^6\ \text{psi}$
$\alpha = 12.9 \times 10^{-6}/°\text{F}$

PROBLEM 2.59

Determine (*a*) the compressive force in the bars shown after a temperature rise of 180°F, (*b*) the corresponding change in length of the bronze bar.

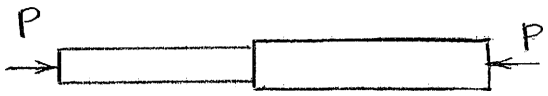
SOLUTION

Thermal expansion if free of constraint:

$$\begin{aligned}\delta_T &= L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)\\ &= (14)(12\times10^{-6})(180) + (18)(12.9\times10^{-6})(180)\\ &= 72.036\times10^{-3}\ \text{in.}\end{aligned}$$

Constrained expansion: $\delta = 0.02\ \text{in.}$

Shortening due to induced compressive force *P*:



$$\delta_P = 72.036\times10^{-3} - 0.02 = 52.036\times10^{-3}\ \text{in.}$$

But
$$\delta_P = \frac{PL_b}{E_bA_b} + \frac{PL_a}{E_aA_a} = \left(\frac{L_b}{E_bA_b} + \frac{L_a}{E_aA_a}\right)P$$

$$= \left(\frac{14}{(15\times10^6)(2.4)} + \frac{18}{(10.6\times10^6)(2.8)}\right)P = 995.36\times10^{-9}P$$

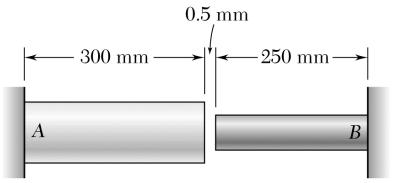
Equating,
$$995.36\times10^{-9}P = 52.036\times10^{-3}$$
$$P = 52.279\times10^3\ \text{lb}$$

(*a*) $P = 52.3\ \text{kips}$ ◀

(*b*) $\delta_b = L_b\alpha_b(\Delta T) - \dfrac{PL_b}{E_bA_b}$

$$= (14)(12\times10^{-6})(180) - \frac{(52.279\times10^3)(14)}{(15\times10^6)(2.4)} = 9.91\times10^{-3}\ \text{in.}$$ $\delta_b = 9.91\times10^{-3}\ \text{in.}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Aluminum
$A = 2000\ \text{mm}^2$
$E = 75\ \text{GPa}$
$\alpha = 23 \times 10^{-6}/°\text{C}$

Stainless steel
$A = 800\ \text{mm}^2$
$E = 190\ \text{GPa}$
$\alpha = 17.3 \times 10^{-6}/°\text{C}$

PROBLEM 2.60

At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 140 - 20 = 120\ °\text{C}$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)\\ &= 1.347\times10^{-3}\ \text{m}\end{aligned}$$

Shortening due to P to meet constraint:

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\ \text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P\\ &= 3.6447\times10^{-9}\,P\end{aligned}$$

Equating,

$$3.6447\times10^{-9}\ P = 0.847\times10^{-3}$$

$$P = 232.39\times10^3\ \text{N}$$

(*a*) $$\sigma_a = -\frac{P}{A_a} = -\frac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\ \text{Pa}$$

$$\sigma_a = -116.2\ \text{MPa} \blacktriangleleft$$

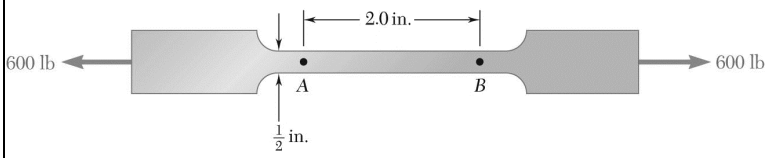
(*b*) $$\delta_a = L_a\alpha_a(\Delta T) - \frac{PL_a}{E_aA_a}$$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\ \text{m}$$

$$\delta_a = 0.363\ \text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.61



A 600-lb tensile load is applied to a test coupon made from $\frac{1}{16}$-in. flat steel plate ($E = 29\times10^6$ psi and $\nu = 0.30$). Determine the resulting change (*a*) in the 2-in. gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

SOLUTION

$$A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2\times10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{19.2\times10^3}{29\times10^6} = 662.07\times10^{-6}$$

(*a*) $\delta_x = L_0\varepsilon_x = (2.0)(662.07\times10^{-6})$ $\qquad \delta_l = 1.324\times10^{-3} \text{ in.}$ ◀

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07\times10^{-6}) = -198.62\times10^{-6}$$

(*b*) $\delta_{\text{width}} = w_0\varepsilon_y = \left(\frac{1}{2}\right)(-198.62\times10^{-6})$ $\qquad \delta_w = -99.3\times10^{-6} \text{ in.}$ ◀

(*c*) $\delta_{\text{thickness}} = t_0\varepsilon_z = \left(\frac{1}{16}\right)(-198.62\times10^{-6})$ $\qquad \delta_t = -12.41\times10^{-6} \text{ in.}$ ◀

(*d*) $A = wt = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z)$

$$= w_0t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

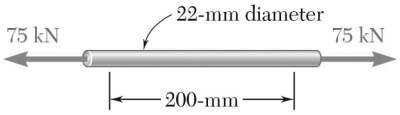
$$\Delta A = A - A_0 = w_0t_0(\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{16}\right)(-198.62\times10^{-6} - 198.62\times10^{-6} + \text{negligible term})$$

$$= -12.41\times10^{-6} \text{ in}^2 \qquad \Delta A = -12.41\times10^{-6} \text{ in}^2$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.62



In a standard tensile test, a steel rod of 22-mm diameter is subjected to a tension force of 75 kN. Knowing that $\nu = 0.3$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

SOLUTION

$P = 75 \text{ kN} = 75 \times 10^3 \text{ N}$ $\quad A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{380.13 \times 10^{-6}} = 197.301 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{197.301 \times 10^6}{200 \times 10^9} = 986.51 \times 10^{-6}$$

$$\delta_x = L\varepsilon_x = (200 \text{ mm})(986.51 \times 10^{-6})$$

(*a*) $\quad \delta_x = 0.1973 \text{ mm}$ ◀

$$\varepsilon_y = -\nu\varepsilon_x = -(0.3)(986.51 \times 10^{-6}) = -295.95 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = (22 \text{ mm})(-295.95 \times 10^{-6})$$

(*b*) $\quad \delta_y = -0.00651 \text{ mm}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.63

A 20-mm-diameter rod made of an experimental plastic is subjected to a tensile force of magnitude $P = 6$ kN. Knowing that an elongation of 14 mm and a decrease in diameter of 0.85 mm are observed in a 150-mm length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material.

SOLUTION

Let the y-axis be along the length of the rod and the x-axis be transverse.

$$A = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16\times10^{-6}\text{ m}^2 \qquad P = 6\times10^3 \text{ N}$$

$$\sigma_y = \frac{P}{A} = \frac{6\times10^3}{314.16\times10^{-6}} = 19.0985\times10^6 \text{ Pa}$$

$$\varepsilon_y = \frac{\delta_y}{L} = \frac{14 \text{ mm}}{150 \text{ mm}} = 0.093333$$

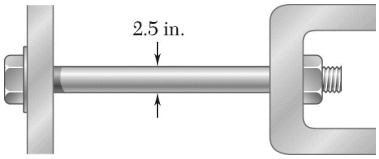
Modulus of elasticity: $E = \dfrac{\sigma_y}{\varepsilon_y} = \dfrac{19.0985\times10^6}{0.093333} = 204.63\times10^6 \text{ Pa}$ $\qquad E = 205 \text{ MPa}$ ◀

$$\varepsilon_x = \frac{\delta_x}{d} = -\frac{0.85}{20} = -0.0425$$

Poisson's ratio: $\nu = -\dfrac{\varepsilon_x}{\varepsilon_y} = -\dfrac{-0.0425}{0.093333}$ $\qquad \nu = 0.455$ ◀

Modulus of rigidity: $G = \dfrac{E}{2(1+\nu)} = \dfrac{204.63\times10^6}{(2)(1.455)} = 70.31\times10^6 \text{ Pa}$ $\qquad G = 70.3 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.64

The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the internal force in the bolt, if the diameter is observed to decrease by 0.5×10^{-3} in.

SOLUTION

$$\delta_y = -0.5 \times 10^{-3} \text{ in.} \qquad d = 2.5 \text{ in.}$$

$$\varepsilon_y = \frac{\varepsilon_y}{d} = -\frac{0.5 \times 10^{-3}}{2.5} = -0.2 \times 10^{-3}$$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}: \qquad \varepsilon_x = \frac{-\varepsilon_y}{\nu} = \frac{0.2 \times 10^{-3}}{0.3} = 0.66667 \times 10^{-3}$$

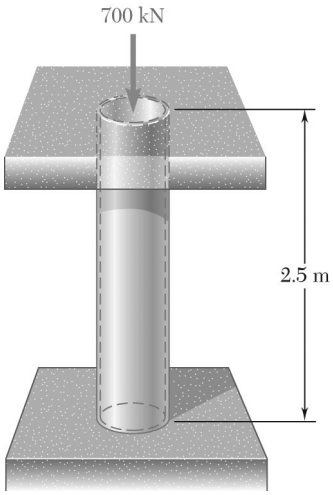
$$\sigma_x = E\varepsilon_x = (29 \times 10^6)(0.66667 \times 10^{-3}) = 19.3334 \times 10^3 \text{ psi}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2.5)^2 = 4.9087 \text{ in}^2$$

$$F = \sigma_x A = (19.3334 \times 10^3)(4.9087) = 94.902 \times 10^3 \text{ lb}$$

$$F = 94.9 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.65

A 2.5-m length of a steel pipe of 300-mm outer diameter and 15-mm wall thickness is used as a column to carry a 700-kN centric axial load. Knowing that $E = 200$ GPa and $\nu = 0.30$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$$d_o = 0.3 \text{ m} \qquad t = 0.015 \text{ m} \qquad L = 2.5 \text{ m}$$
$$d_i = d_o - 2t = 0.3 - 2(0.015) = 0.27 \text{ m} \qquad P = 700\times10^3 \text{ N}$$
$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(0.3^2 - 0.27^2) = 13.4303\times10^{-3} \text{ m}^2$$

(*a*) $$\delta = -\frac{PL}{EA} = -\frac{(700\times10^3)(2.5)}{(200\times10^9)(13.4303\times10^{-3})}$$

$$= -651.51\times10^{-6} \text{ m}$$

$\delta = -0.652$ mm ◀

$$\varepsilon = \frac{\delta}{L} = \frac{-651.51\times10^{-6}}{2.5} = -260.60\times10^{-6}$$

$$\varepsilon_{\text{LAT}} = -\nu\varepsilon = -(0.30)(-260.60\times10^{-6})$$
$$= 78.180\times10^{-6}$$

(*b*) $$\Delta d_o = d_o\varepsilon_{\text{LAT}} = (300 \text{ mm})(78.180\times10^{-6})$$

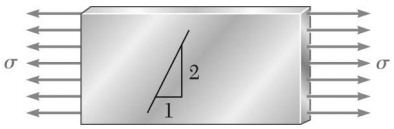
$\Delta d_o = 0.0235$ mm ◀

(*c*) $$\Delta t = t\varepsilon_{\text{LAT}} = (15 \text{ mm})(78.180\times10^{-6})$$

$\Delta t = 0.001173$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

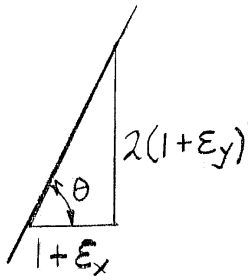
PROBLEM 2.66



An aluminum plate ($E = 74$ GPa and $v = 0.33$) is subjected to a centric axial load that causes a normal stress σ. Knowing that, before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when $\sigma = 125$ MPa.

SOLUTION

The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$



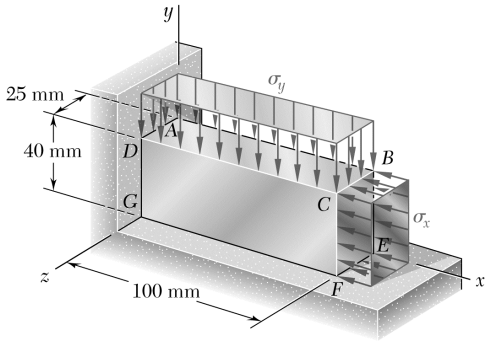
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125\times10^6}{74\times10^9} = 1.6892\times10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(1.6892\times10^{-3}) = -0.5574\times10^{-3}$$

$$\tan\theta = \frac{2(1-0.0005574)}{1+0.0016892} = 1.99551$$

$\tan\theta = 1.99551$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.67

The block shown is made of a magnesium alloy, for which $E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

(*a*) $\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\sigma_y = \nu\sigma_x = (0.35)(-180\times10^6)$$

$$= -63\times10^6 \text{ Pa} \qquad \sigma_y = -63 \text{ MPa} \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}(\sigma_x + \sigma_y) = \frac{(0.35)(-243\times10^6)}{45\times10^9} = -1.89\times10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_Z) = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{157.95\times10^6}{45\times10^9} = -3.51\times10^{-3}$$

(*b*) $A_0 = L_x L_z$

$$A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_x L_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$$

$$\Delta A = A - A_0 = L_x L_z(\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x+\varepsilon_z)$$

$$\Delta A = (100 \text{ mm})(25 \text{ mm})(-3.51\times10^{-3} - 1.89\times10^{-3}) \qquad \Delta A = -13.50 \text{ mm}^2 \blacktriangleleft$$

(*c*) $V_0 = L_x L_y L_z$

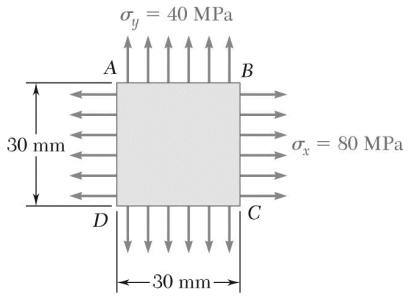
$$V = L_x(1+\varepsilon_x)L_y(1+\varepsilon_y)L_z(1+\varepsilon_z)$$

$$= L_x L_y L_z(1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x+\varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\Delta V = V - V_0 = L_x L_y L_z(\varepsilon_x+\varepsilon_y+\varepsilon_z+\text{small terms})$$

$$\Delta V = (100)(40)(25)(-3.51\times10^{-3} + 0 - 1.89\times10^{-3}) \qquad \Delta V = -540 \text{ mm}^3 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.68

A 30-mm square was scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition at the square is as shown. For $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagnonal *AC*.

SOLUTION

Given: $\sigma_x = 80$ MPa $\quad \sigma_y = 40$ MPa

Using Eq's (2.28):

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{80 - 0.3(40)}{200\times10^3} = 340\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{40 - 0.3(80)}{200\times10^3} = 80\times10^{-6}$$

(*a*) Change in length of *AB*.

$$\delta_{AB} = (AB)\varepsilon_x = (30\text{ mm})(340\times10^{-6}) = 10.20\times10^{-3}\text{ mm}$$

$$\delta_{AB} = 10.20\ \mu\text{m} \blacktriangleleft$$

(*b*) Change in length of *BC*.

$$\delta_{BC} = (BC)\varepsilon_y = (30\text{ mm})(80\times10^{-6}) = 2.40\times10^{-3}\text{ mm}$$

$$\delta_{BC} = 2.40\mu\text{m} \blacktriangleleft$$

(*c*) Change in length of diagonal *AC*.

From geometry, $(AC)^2 = (AB)^2 + (BC)^2$

Differentiate: $2(AC)\,\Delta(AC) = 2(AB)\Delta(AB) + 2(BC)\Delta(BC)$

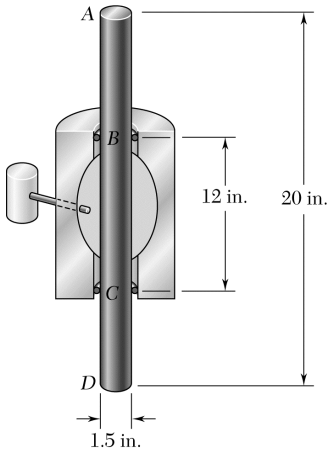
But $\Delta(AC) = \delta_{AC} \quad \Delta(AB) = \delta_{AB} \quad \Delta(BC) = \delta_{BC}$

Thus, $2(AC)\delta_{AC} = 2(AB)\delta_{AB} + 2(BC)\delta_{BC}$

$$\delta_{AC} = \frac{AB}{AC}\delta_{AB} + \frac{BC}{AC}\delta_{BC} = \frac{1}{\sqrt{2}}(10.20\ \mu\text{m}) + \frac{1}{\sqrt{2}}(2.40\mu\text{m})$$

$$\delta_{AC} = 8.91\ \mu\text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.69

The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion *BC* of the rod. Knowing that $E = 10.1\times10^{6}$ psi and $\nu = 0.36$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -P = -6000 \text{ psi} \qquad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{10.1\times10^{6}}[-6000 - (0.36)(0) - (0.36)(-6000)]$$

$$= -380.198\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{10.1\times10^{6}}[-(0.36)(-6000) + 0 - (0.36)(-6000)]$$

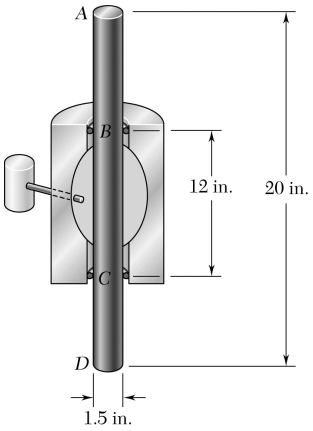
$$= 427.72\times10^{-6}$$

Length subjected to strain ε_x: $L = 12$ in.

(*a*) $\delta_y = L\varepsilon_y = (12)(427.72\times10^{-6})$ $\qquad \delta_l = 5.13\times10^{-3}$ in. ◀

(*b*) $\delta_x = d\varepsilon_x = (1.5)(-380.198\times10^{-6})$ $\qquad \delta_d = -0.570\times10^{-3}$ in. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.70

For the rod of Prob. 2.69, determine the forces that should be applied to the ends *A* and *D* of the rod (*a*) if the axial strain in portion *BC* of the rod is to remain zero as the hydrostatic pressure is applied, (*b*) if the total length *AD* of the rod is to remain unchanged.

PROBLEM 2.69 The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion *BC* of the rod. Knowing that $E = 10.1\times10^{6}$ psi and $\nu = 0.36$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

SOLUTION

Over the pressurized portion *BC*,

$$\sigma_x = \sigma_z = -p \qquad \sigma_y = \sigma_y$$

$$(\varepsilon_y)_{BC} = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}(2\nu p + \sigma_y)$$

(*a*) $(\varepsilon_y)_{BC} = 0 \qquad 2\nu p + \sigma_y = 0$

$$\sigma_y = -2\nu p = -(2)(0.36)(6000)$$

$$= -4320 \text{ psi}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

$$F = A\sigma_y = (1.76715)(-4320) = -7630 \text{ lb}$$

i.e., 7630 lb compression ◀

(*b*) Over unpressurized portions *AB* and *CD*, $\sigma_x = \sigma_z = 0$

$$(\varepsilon_y)_{AB} = (\varepsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length,

$$\delta = L_{AB}(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} + L_{CD}(\varepsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} = 0$$

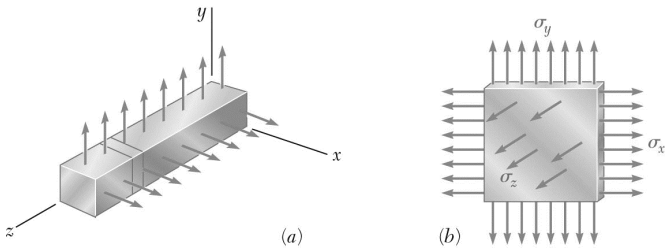
$$(20-12)\frac{\sigma_y}{E} + \frac{12}{E}(2\nu p + \sigma_y) = 0$$

$$\sigma_y = -\frac{24\nu p}{20} = -\frac{(24)(0.36)(6000)}{20} = -2592 \text{ psi}$$

$$P = A\sigma_y = (1.76715)(-2592) = -4580 \text{ lb} \qquad P = 4580 \text{ lb compression} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.71



In many situations, physical constraints prevent strain from occurring in a given direction. For example, $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain,* we can express σ_z, ε_x, and ε_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y]$$

$$\varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

SOLUTION

$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y) \qquad \blacktriangleleft$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

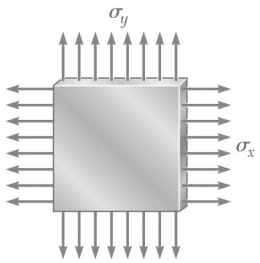
$$= \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \blacktriangleleft$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x] \qquad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.72

In many situations, it is known that the normal stress in a given direction is zero, for example, $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress,* show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y, and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + v\varepsilon_y}{1-v^2} \qquad \sigma_y = E\frac{\varepsilon_y + v\varepsilon_x}{1-v^2} \qquad \varepsilon_z = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) \tag{1}$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y) \tag{2}$$

Multiplying (2) by v and adding to (1),

$$\varepsilon_x + v\varepsilon_y = \frac{1-v^2}{E}\sigma_x \qquad \text{or} \qquad \sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) \quad \blacktriangleleft$$

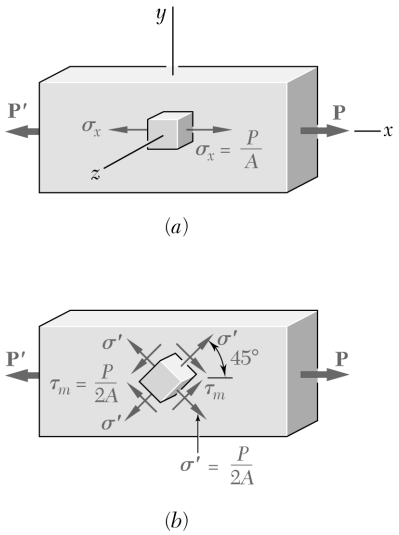
Multiplying (1) by v and adding to (2),

$$\varepsilon_y + v\varepsilon_x = \frac{1-v^2}{E}\sigma_y \qquad \text{or} \qquad \sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) \quad \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y) = -\frac{v}{\cancel{E}}\cdot\frac{\cancel{E}}{1-v^2}(\varepsilon_x + v\varepsilon_y + \varepsilon_y + v\varepsilon_x)$$

$$= -\frac{v(1+v)}{1-v^2}(\varepsilon_x + \varepsilon_y) = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y) \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.73

For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypotenuses of the triangles shown in Fig. 2.49, which represent, respectively, an element before and after deformation, (*b*) using the values of the corresponding stresses of σ' and σ_x shown in Fig. 1.38, and the generalized Hooke's law.

SOLUTION

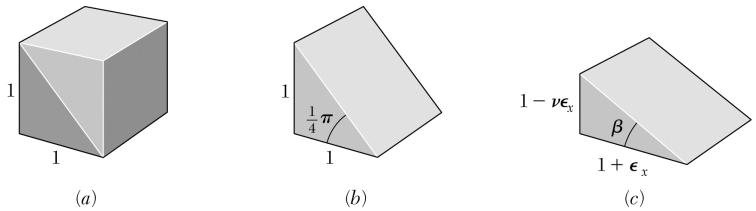


Figure 2.49

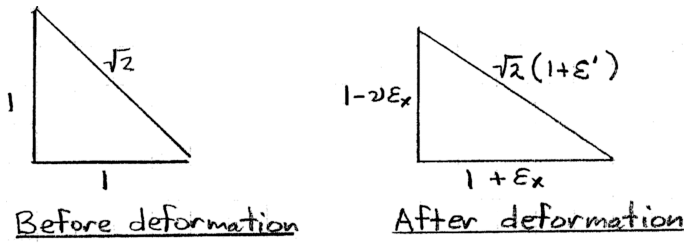
(*a*) $$[\sqrt{2}(1+\varepsilon')]^2 = (1+\varepsilon_x)^2 + (1-v\varepsilon_x)^2$$

$$2(1+2\varepsilon'+\varepsilon'^2) = 1+2\varepsilon_x+\varepsilon_x^2+1-2v\varepsilon_x+v^2\varepsilon_x^2$$

$$4\varepsilon'+2\varepsilon'^2 = 2\varepsilon_x+\varepsilon_x^2-2v\varepsilon_x+v^2\varepsilon_x^2$$

Neglect squares as small $$4\varepsilon' = 2\varepsilon_x - 2v\varepsilon_x$$

$$\varepsilon' = \frac{1-v}{2}\varepsilon_x \blacktriangleleft$$

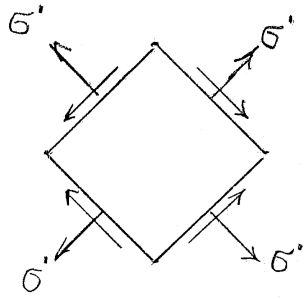


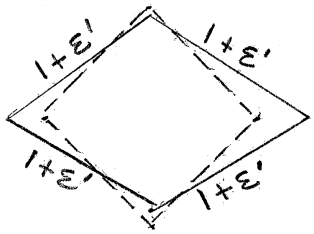
(A) **(B)**

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.73 *(Continued)*

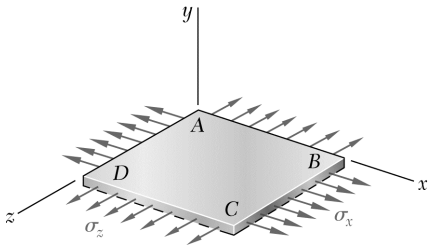
(*b*)
$$\varepsilon' = \frac{\sigma'}{E} - \frac{v\sigma'}{E}$$
$$= \frac{1-v}{E} \cdot \frac{P}{2A}$$
$$= \frac{1-v}{2E}\sigma_x$$
$$= \frac{1-v}{2}\varepsilon_x$$





◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.74

The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the *x* direction must be zero, that is, $\varepsilon_x = 0$. Denoting by *E* the modulus of elasticity and by *v* Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

SOLUTION

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

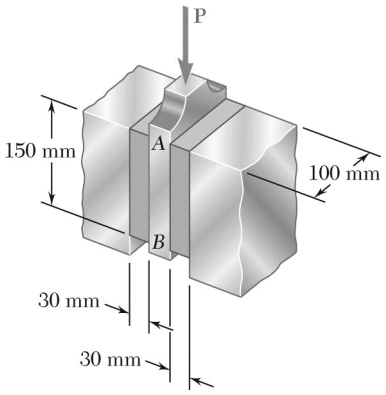
$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{1}{E}(\sigma_x - v\sigma_0)$$

(*a*) $\sigma_x = v\sigma_0$ ◀

(*b*) $\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) = \frac{1}{E}(-v^2\sigma_0 - 0 + \sigma_0) = \frac{1-v^2}{E}\sigma_0$

$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-v^2}$ ◀

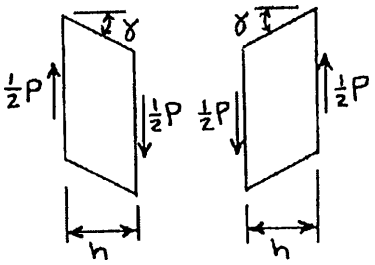
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.75

A vibration isolation unit consists of two blocks of hard rubber bonded to a plate AB and to rigid supports as shown. Knowing that a force of magnitude $P = 25$ kN causes a deflection $\delta = 1.5$ mm of plate AB, determine the modulus of rigidity of the rubber used.

SOLUTION



$$F = \frac{1}{2}P = \frac{1}{2}(25\times10^3\ \text{N}) = 12.5\times10^3\ \text{N}$$

$$\tau = \frac{F}{A} = \frac{12.5\times10^3\ \text{N})}{(0.15\ \text{m})(0.1\ \text{m})} = 833.33\times10^3\ \text{Pa}$$

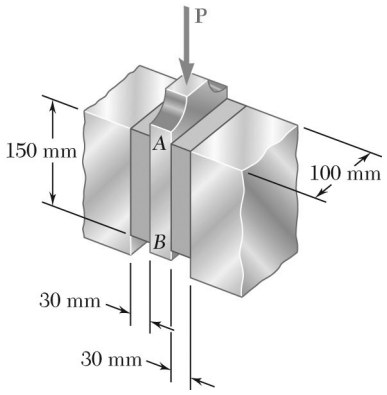
$$\delta = 1.5\times10^{-3}\ \text{m} \qquad h = 0.03\ \text{m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5\times10^{-3}}{0.03} = 0.05$$

$$G = \frac{\tau}{\gamma} = \frac{833.33\times10^3}{0.05} = 16.67\times10^6\ \text{Pa}$$

$$G = 16.67\ \text{MPa} \blacktriangleleft$$

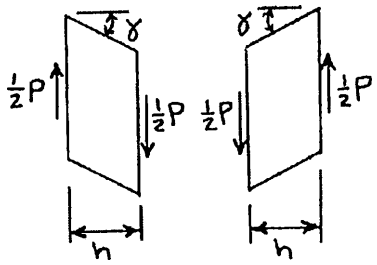
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.76

A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{h}$

Shearing stress: $\tau = G\gamma = \dfrac{G\delta}{h}$

Force: $\dfrac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$ or $P = \dfrac{2GA\delta}{h}$

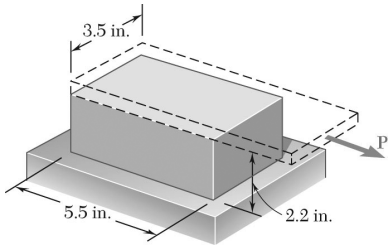
Effective spring constant: $k = \dfrac{P}{\delta} = \dfrac{2GA}{h}$

with $A = (0.15)(0.1) = 0.015\ \text{m}^2 \quad h = 0.03\ \text{m}$

$$k = \frac{2(19\times10^6\ \text{Pa})(0.015\ \text{m}^2)}{0.03\ \text{m}} = 19.00\times10^6\ \text{N/m}$$

$$k = 19.00\times10^3\ \text{kN/m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



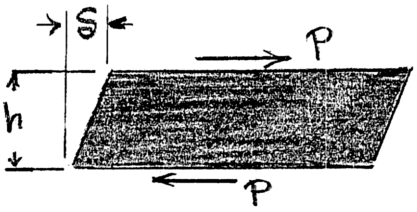
PROBLEM 2.77

The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.

SOLUTION

Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb

The area is $A = (3.5)(5.5) = 19.25 \text{ in}^2$

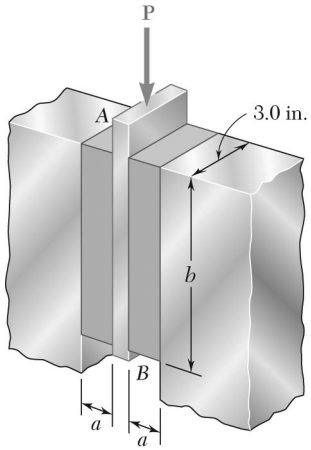


Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52 \text{ psi}$

Shearing strain: $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h} \quad \therefore \quad \delta = h\gamma = (2.2)(0.0085006)$ $\qquad \delta = 0.0187 \text{ in.}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.78

A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 220$ psi and $G = 1800$ psi. Knowing that a centric vertical force of magnitude $P = 3.2$ kips must cause a 0.1-in. vertical deflection of the plate AB, determine the smallest allowable dimensions a and b of the block.

SOLUTION

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$.

The shearing stress is $\tau = \dfrac{\frac{1}{2}P}{A}$

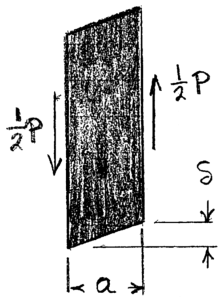
or required area $A = \dfrac{P}{2\tau} = \dfrac{3.2\times10^3}{(2)(220)} = 7.2727 \text{ in}^2$

But $A = (3.0)b$

Hence, $b = \dfrac{A}{3.0} = 2.42 \text{ in.}$ $b_{\min} = 2.42 \text{ in.}$ ◀

Use $b = 2.42$ in and $\tau = 220$ psi

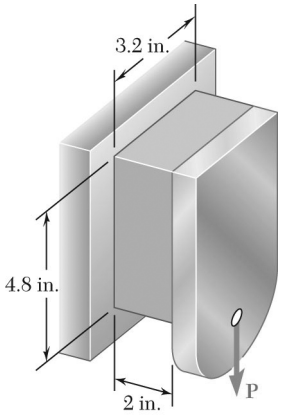
Shearing strain. $\gamma = \dfrac{\tau}{G} = \dfrac{220}{1800} = 0.12222$



But $\gamma = \dfrac{\delta}{a}$

Hence, $a = \dfrac{\delta}{\gamma} = \dfrac{0.1}{0.12222} = 0.818 \text{ in.}$ $a_{\min} = 0.818 \text{ in.}$ ◀

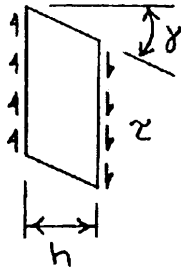
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.79

The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = 55 \times 10^3 \text{ lb}$$

$$\tau = \frac{P}{A} = \frac{55 \times 10^3}{15.36} = 3580.7 \text{ psi}$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\gamma = \frac{\tau}{G} = \frac{3580.7}{150 \times 10^3} = 23.871 \times 10^{-3}$$

$$h = 2 \text{ in.}$$

$$\delta = h\gamma = (2)(23.871 \times 10^{-3})$$

$$= 47.7 \times 10^{-3} \text{ in.}$$

$$\delta = 0.0477 \text{ in.} \downarrow \blacktriangleleft$$

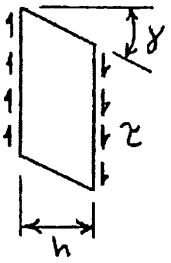
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.80

What load **P** should be applied to the plate of Prob. 2.79 to produce a $\frac{1}{16}$-in. deflection?

PROBLEM 2.79 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$\delta = \frac{1}{16}\text{in.} = 0.0625 \text{ in.}$$

$$h = 2 \text{ in.}$$

$$\gamma = \frac{\delta}{h} = \frac{0.0625}{2} = 0.03125$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\tau = G\gamma = (150 \times 10^3)(0.03125)$$

$$= 4687.5 \text{ psi}$$

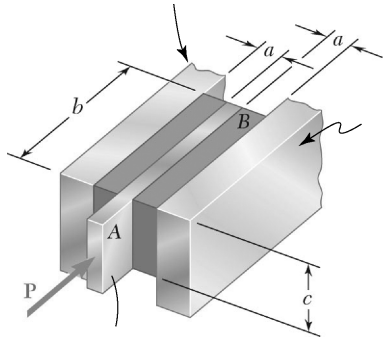
$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = \tau A = (4687.5)(15.36)$$

$$= 72 \times 10^3 \text{ lb}$$

72 kips ◀

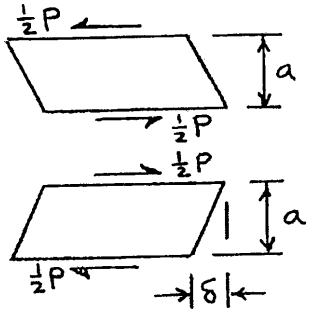
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.81

Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 45$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

SOLUTION



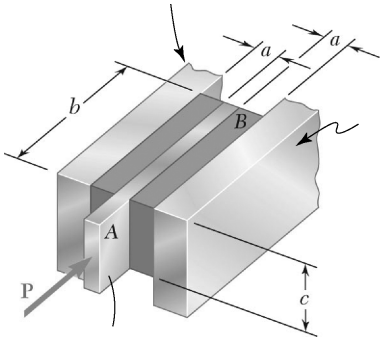
Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(12\times10^6 \text{ Pa})(0.005 \text{ m})}{1.4\times10^6 \text{ Pa}} = 0.0429 \text{ m} \qquad a = 42.9 \text{ mm} \blacktriangleleft$$

Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{45\times10^3 \text{ N}}{2(0.1 \text{ m})\,(1.4\times10^6 \text{ Pa})} = 0.1607 \text{ m} \qquad b = 160.7 \text{ mm} \blacktriangleleft$$

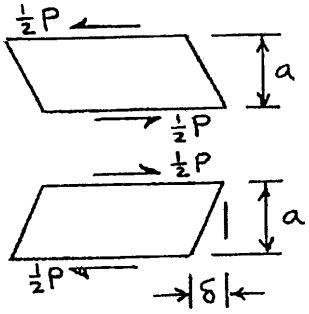
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.82

Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.

SOLUTION



Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

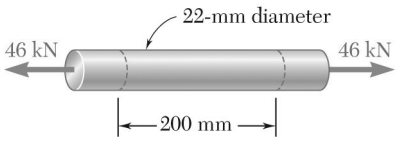
$$P = 2bc\tau = 2(0.2\text{ m})(0.125\text{ m})(1.5\times10^{3}\text{ kPa})$$ P = 75.0 kN ◀

Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$$a = \frac{G\delta}{\tau} = \frac{(10\times10^{6}\text{ Pa})(0.006\text{ m})}{1.5\times10^{6}\text{ Pa}} = 0.04\text{ m}$$ a = 40.0 mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.83*



Determine the dilatation e and the change in volume of the 200-mm length of the rod shown if (*a*) the rod is made of steel with $E = 200$ GPa and $\nu = 0.30$, (*b*) the rod is made of aluminum with $E = 70$ GPa and $\nu = 0.35$.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$P = 46 \times 10^3 \text{ N}$$

$$\sigma_x = \frac{P}{A} = 121.01 \times 10^6 \text{ Pa}$$

$$\sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\frac{\sigma_x}{E}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x - \nu\sigma_x - \nu\sigma_x) = \frac{(1-2\nu)\sigma_x}{E}$$

Volume: $V = AL = (380.13 \text{ mm}^2)(200 \text{ mm}) = 76.026 \times 10^3 \text{ mm}^3$

$$\Delta V = Ve$$

(*a*) Steel:

$$e = \frac{(1-0.60)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6} \qquad e = 242 \times 10^{-6} \blacktriangleleft$$

$$\Delta V = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40 \text{ mm}^3 \qquad \Delta V = 18.40 \text{ mm}^3 \blacktriangleleft$$

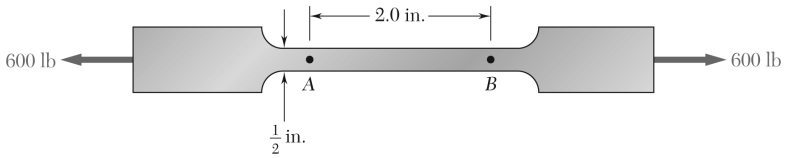
(*b*) Aluminum:

$$e = \frac{(1-0.70)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6} \qquad e = 519 \times 10^{-6} \blacktriangleleft$$

$$\Delta V = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4 \text{ mm}^3 \qquad \Delta V = 39.4 \text{ mm}^3 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.84*



Determine the change in volume of the 2-in. gage length segment *AB* in Prob. 2.61 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion *AB* from its final volume.

SOLUTION

From Problem 2.61, thickness $= \frac{1}{16}$ in., $E = 29 \times 10^6$ psi, $\nu = 0.30$.

(*a*) $A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$

Volume: $V_0 = AL_0 = (0.03125)(2.00) = 0.0625 \text{ in}^3$

$$\sigma_x = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 264.83 \times 10^{-6}$$

$$\Delta V = V_0 e = (0.0625)(264.83 \times 10^{-6}) = 16.55 \times 10^{-6} \text{ in}^3 \quad \blacktriangleleft$$

(*b*) From the solution to Problem 2.61,

$\delta_x = 1.324 \times 10^{-3}$ in., $\delta_y = -99.3 \times 10^{-6}$ in., $\delta_z = -12.41 \times 10^{-6}$ in.

The dimensions when under a 600-lb tensile load are:

Length: $L = L_0 + \delta_x = 2 + 1.324 \times 10^{-3} = 2.001324$ in.

Width: $w = w_0 + \delta_y = \frac{1}{2} - 99.3 \times 10^{-6} = 0.4999007$ in.

Thickness: $t = t_0 + \delta_z = \frac{1}{16} - 12.41 \times 10^{-6} = 0.06248759$ in.

Volume: $V = Lwt = 0.062516539 \text{ in}^3$

$$\Delta V = V - V_0 = 0.062516539 - 0.0625 = 16.54 \times 10^{-6} \text{ in}^3 \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.85*

A 6-in.-diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29\times10^6$ psi and $v = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere, $V_0 = \dfrac{\pi}{6} d_0^3$

$$= \frac{\pi}{6}(6.00)^3$$

$$= 113.097 \text{ in.}^3$$

$$\sigma_x = \sigma_y = \sigma_z = -\text{p}$$

$$= -7.1\times10^3 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)$$

$$= -\frac{(1-2v)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6}$$

$$= -97.93\times10^{-6}$$

Likewise, $\varepsilon_y = \varepsilon_z = -97.93\times10^{-6}$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79\times10^{-6}$$

(*a*) $-\Delta d = -d_0\varepsilon_x = -(6.00)(-97.93\times10^{-6}) = 588\times10^{-6}$ in. $\quad -\Delta d = 588\times10^{-6}$ in. ◀

(*b*) $-\Delta V = -V_0 e = -(113.097)(-293.79\times10^{-6}) = 33.2\times10^{-3}\text{ in}^3$ $\quad -\Delta V = 33.2\times10^{-3}\text{ in}^3$ ◀

(*c*) Let m = mass of sphere. $\quad m$ = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0(1+e)$$

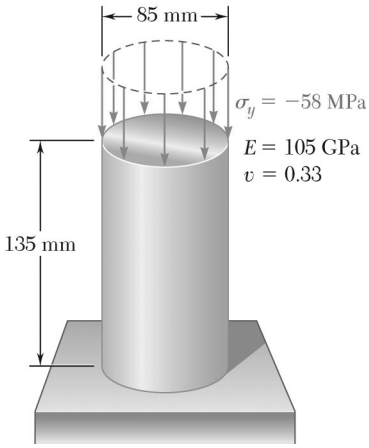
$$\frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)} \times \frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$

$$= (1 - e + e^2 - e^3 + \cdots) - 1 = -e + e^2 - e^3 + \cdots$$

$$\approx -e = 293.79\times10^{-6}$$

$$\frac{\rho-\rho_0}{\rho_0}\times100\% = (293.79\times10^{-6})(100\%) \qquad 0.0294\% \text{ ◀}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.86*

(*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_0 = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_0 = A_0 h_0 = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(*a*) $\sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E} = -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) \qquad \Delta h = -0.0746 \text{ mm} \blacktriangleleft$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} = -187.81 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) \qquad \Delta V = -143.9 \text{ mm}^3 \blacktriangleleft$$

(*b*) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$

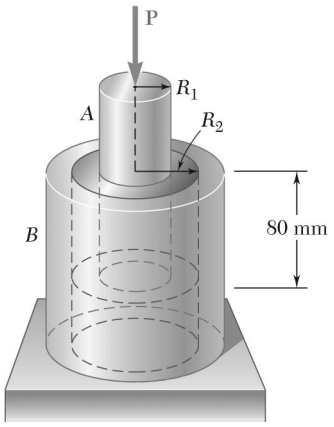
$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y = \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) \qquad \Delta h = -0.0306 \text{ mm} \blacktriangleleft$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) \qquad \Delta V = -521 \text{ mm}^3 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

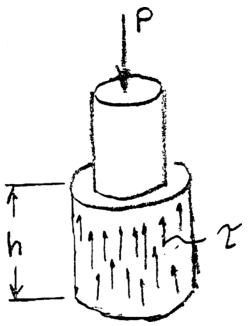


PROBLEM 2.87*

A vibration isolation support consists of a rod *A* of radius $R_1 = 10$ mm and a tube *B* of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity *G* = 12 MPa. Determine the largest allowable force **P** that can be applied to rod *A* if its deflection is not to exceed 2.50 mm.

SOLUTION

Let *r* be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



Shearing stress τ acting on a cylindrical surface of radius *r* is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

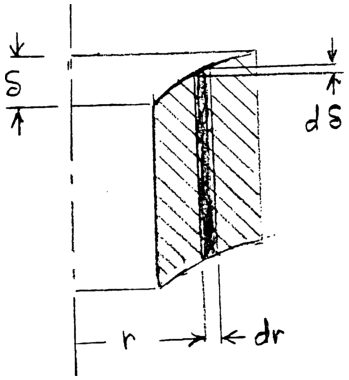
The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length *dr*,

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi Gh}\frac{dr}{r}$$



Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\,\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi Gh\delta}{\ln(R_2/R_1)}$$

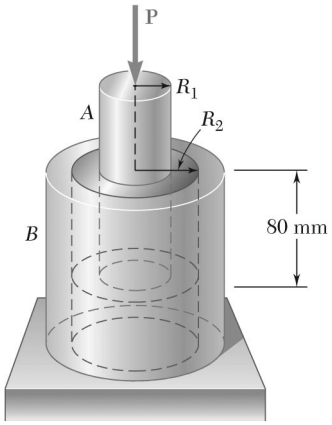
Data: $R_1 = 10$ mm $= 0.010$ m, $R_2 = 25$ mm $= 0.025$ m, $h = 80$ mm $= 0.080$ m

$$G = 12\times10^6 \text{ Pa} \qquad \delta = 2.50\times10^{-3} \text{ m}$$

$$P = \frac{(2\pi)(12\times10^6)(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46\times10^3 \text{ N}$$

16.46 kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

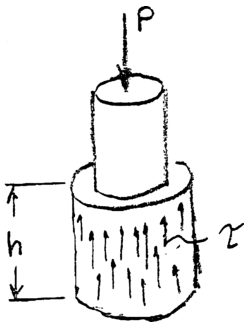


PROBLEM 2.88

A vibration isolation support consists of a rod *A* of radius R_1 and a tube *B* of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod *A*.

SOLUTION

Let *r* be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



Shearing stress τ acting on a cylindrical surface of radius *r* is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

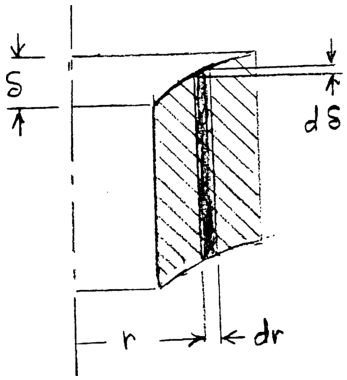
$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length *dr*,

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr$$

$$dr\delta = \frac{P}{2\pi Gh}\frac{dr}{r}$$



Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\,\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi Gh\delta}{P} = \frac{(2\pi)(10.93\times10^6)(0.080)(0.002)}{10.10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

$R_2/R_1 = 3.00$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.89*

The material constants E, G, k, and v are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (*a*) $k = GE/(9G - 3E)$ and (*b*) $v = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)}$$

(*a*) $$1 + v = \frac{E}{2G} \quad \text{or} \quad v = \frac{E}{2G} - 1$$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$k = \frac{EG}{9G - 6E} \blacktriangleleft$$

(*b*) $$\frac{k}{G} = \frac{2(1+v)}{3(1-2v)}$$

$$3k - 6kv = 2G + 2Gv$$

$$3k - 2G = 2G + 6k$$

$$v = \frac{3k - 2G}{6k + 2G} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.90*

Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint:* Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

$$G = \frac{E}{2(1+\nu)} \qquad \text{or} \qquad \frac{E}{G} = 2(1+\nu)$$

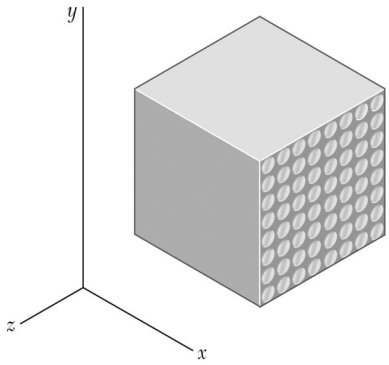
Assume $\nu > 0$ for almost all materials, and $\nu < \frac{1}{2}$ for a positive bulk modulus.

Applying the bounds, $$2 \le \frac{E}{G} < 2\left(1+\frac{1}{2}\right) = 3$$

Taking the reciprocals, $$\frac{1}{2} > \frac{G}{E} > \frac{1}{3}$$

or $$\frac{1}{3} < \frac{G}{E} < \frac{1}{2} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.91*

A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the *x* direction. The cube is constrained against deformations in the *y* and *z* directions and is subjected to a tensile load of 65 kN in the *x* direction. Determine (*a*) the change in the length of the cube in the *x* direction, (*b*) the stresses σ_x, σ_y, and σ_z.

$$E_x = 50 \text{ GPa} \quad \nu_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad \nu_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad \nu_{zy} = 0.428$$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \tag{5}$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \tag{6}$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \tag{7}$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{V_{xz}}{E_x}\sigma_x \tag{8}$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\,\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.91* *(Continued)*

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\ \sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \dfrac{1}{E_x}\sigma_x - \dfrac{\nu_{xy}}{E_x}\sigma_y - \dfrac{\nu_{xz}}{E}\sigma_z$

$$E_x = \frac{1}{E_x}[1-(0.254)(0.134993)-(0.254)(0.134993)]\sigma_x$$

$$= \frac{0.93142\sigma_x}{E_x}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600\times10^{-6}\ \text{m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6\ \text{Pa}$$

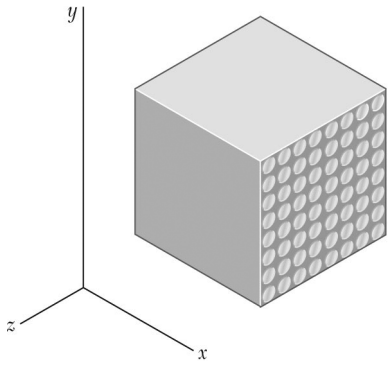
$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(*a*) $\delta_x = L_x\varepsilon_x = (40\ \text{mm})(756.78\times10^{-6})$ $\delta_x = 0.0303\ \text{mm}$ ◀

(*b*) $\sigma_x = 40.625\times10^6\ \text{Pa}$ $\sigma_x = 40.6\ \text{MPa}$ ◀

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6\ \text{Pa}$ $\sigma_y = \sigma_z = 5.48\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.92*

The composite cube of Prob. 2.91 is constrained against deformation in the z direction and elongated in the *x* direction by 0.035 mm due to a tensile load in the *x* direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the *y* direction.

$$E_x = 50 \text{ GPa} \quad \nu_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad \nu_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad \nu_{zy} = 0.428$$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \tag{5}$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \tag{6}$$

Constraint condition: $\varepsilon_z = 0$

Load condition: $\sigma_y = 0$

From Equation (3),
$$0 = -\frac{\nu_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.92* *(Continued)*

From Equation (1) with $\sigma_y = 0$,

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}[\sigma_x - 0.254\sigma_z] = \frac{1}{E_x}[1-(0.254)(0.077216)]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x\varepsilon_x}{0.98039}$$

But, $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.035 \text{ mm}}{40 \text{ mm}} = 875\times10^{-6}$

(*a*) $\sigma_x = \dfrac{(50\times10^9)(875\times10^{-6})}{0.98039} = 44.625\times10^3 \text{ Pa}$ $\sigma_x = 44.6 \text{ MPa}$ ◀

$\sigma_y = 0$ ◀

$\sigma_z = (0.077216)(44.625\times10^6) = 3.446\times10^6 \text{ Pa}$ $\sigma_z = 3.45 \text{ MPa}$ ◀

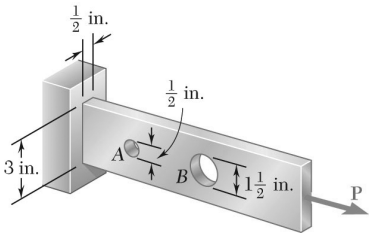
From (2), $\varepsilon_y = \dfrac{\nu_{xy}}{E_x}\sigma_x + \dfrac{1}{E_y}\sigma_y - \dfrac{\nu_{zy}}{E_z}\sigma_z$

$$= -\frac{(0.254)(44.625\times10^6)}{50\times10^9} + 0 - \frac{(0.428)(3.446\times10^6)}{15.2\times10^9}$$

$$= -323.73\times10^{-6}$$

(*b*) $\delta_y = L_y\varepsilon_y = (40 \text{ mm})(-323.73\times10^{-6})$ $\delta_y = -0.0129 \text{ mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.93

Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 6.5$ kips, determine the maximum value of the stress (*a*) at *A*, (*b*) at *B*.

SOLUTION

(*a*) At hole *A*: $r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ in.

$$d = 3 - \frac{1}{2} = 2.50 \text{ in.}$$

$$A_{\text{net}} = dt = (2.50)\left(\frac{1}{2}\right) = 1.25 \text{ in}^2$$

$$\sigma_{\text{non}} = \frac{P}{A_{\text{net}}} = \frac{6.5}{1.25} = 5.2 \text{ ksi}$$

$$\frac{2r}{D} = \frac{2\left(\frac{1}{4}\right)}{3} = 0.1667$$

From Fig. 2.60*a*, $K = 2.56$

$$\sigma_{\max} = K\sigma_{\text{non}} = (2.56)(5.2) \qquad \sigma_{\max} = 13.31 \text{ ksi} \blacktriangleleft$$

(*b*) At hole B: $r = \frac{1}{2}(1.5) = 0.75, \qquad d = 3 - 1.5 = 1.5$ in.

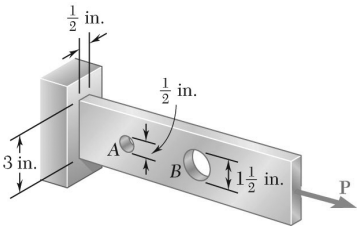
$$A_{\text{net}} = dt = (1.5)\left(\frac{1}{2}\right) = 0.75 \text{ in}^2, \qquad \sigma_{\text{non}} = \frac{P}{A_{\text{net}}} = \frac{6.5}{0.75} = 8.667 \text{ ksi}$$

$$\frac{2r}{D} = \frac{2(0.75)}{3} = 0.5$$

From Fig. 2.60*a*, $K = 2.16$

$$\sigma_{\max} = K\sigma_{\text{non}} = (2.16)(8.667) \qquad \sigma_{\max} = 18.72 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.94

Knowing that $\sigma_{all} = 16$ ksi, determine the maximum allowable value of the centric axial load **P**.

SOLUTION

At hole A:

$$r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \text{ in.}$$

$$d = 3 - \frac{1}{2} = 2.50 \text{ in.}$$

$$A_{net} = dt = (2.50)\left(\frac{1}{2}\right) = 1.25 \text{ in}^2$$

$$\frac{2r}{D} = \frac{2\left(\frac{1}{4}\right)}{3} = 0.1667$$

From Fig. 2.60*a*,

$$K = 2.56$$

$$\sigma_{max} = \frac{KP}{A_{net}} \quad \therefore \quad P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.25)(16)}{2.56} = 7.81 \text{ kips}$$

At hole B:

$$r = \frac{1}{2}(1.5) = 0.75 \text{ in}, \qquad d = 3 - 1.5 = 1.5 \text{ in.}$$

$$A_{net} = dt = (1.5)\left(\frac{1}{2}\right) = 0.75 \text{ in}^2,$$

$$\frac{2r}{D} = \frac{2(0.75)}{3} = 0.5$$

From Fig. 2.60*a*,

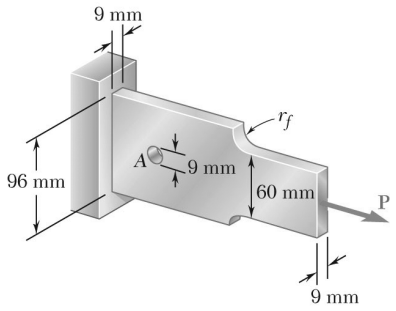
$$K = 2.16$$

$$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(0.75)(16)}{2.16} = 5.56 \text{ kips}$$

Smaller value for P controls.

$P = 5.56$ kips ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.95

Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

SOLUTION

For the circular hole, $r = \left(\frac{1}{2}\right)(9) = 4.5 \text{ mm}$

$$d = 96 - 9 = 87 \text{ mm} \qquad \frac{2r}{D} = \frac{2(4.5)}{96} = 0.09375$$

$$A_{\text{net}} = dt = (0.087 \text{ m})(0.009 \text{ m}) = 783 \times 10^{-6} \text{ m}^2$$

From Fig. 2.60*a*, $K_{\text{hole}} = 2.72$

$$\sigma_{\max} = \frac{K_{\text{hole}} P}{A_{\text{net}}}$$

$$P = \frac{A_{\text{net}} \sigma_{\max}}{K_{\text{hole}}} = \frac{(783 \times 10^{-6})(100 \times 10^{6})}{2.72} = 28.787 \times 10^{3} \text{ N}$$

(*a*) For fillet, $D = 96 \text{ mm}, d = 60 \text{ mm}$

$$\frac{D}{d} = \frac{96}{60} = 1.60$$

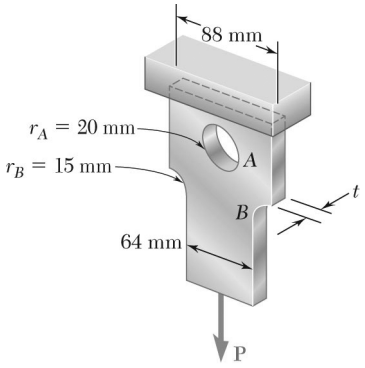
$$A_{\min} = dt = (0.060 \text{ m})(0.009 \text{ m}) = 540 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = \frac{K_{\text{fillet}} P}{A_{\min}} \quad \therefore \quad K_{\text{fillet}} = \frac{A_{\min} \sigma_{\max}}{P} = \frac{(5.40 \times 10^{-6})(100 \times 10^{6})}{28.787 \times 10^{3}} = 1.876$$

From Fig. 2.60*b*, $\frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.19d = 0.19(60)$ $\qquad r_f = 11.4 \text{ mm}$ ◀

(*b*) $P = 28.8 \text{ kN}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.96

For $P = 100$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.

SOLUTION

At the hole: $r_A = 20$ mm $\quad d_A = 88 - 40 = 48$ mm

$$\frac{2r_A}{D_A} = \frac{2(20)}{88} = 0.455$$

From Fig. 2.60*a*, $K = 2.20$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{\max}}$$

$$t = \frac{(2.20)(100 \times 10^3 \text{ N})}{(0.048 \text{ m})(125 \times 10^6 \text{ Pa})} = 36.7 \times 10^{-3} \text{ m} = 36.7 \text{ mm}$$

At the fillet: $D = 88$ mm, $\quad d_B = 64$ mm $\quad \dfrac{D}{d_B} = \dfrac{88}{64} = 1.375$

$$r_B = 15 \text{ mm} \qquad \frac{r_B}{d_B} = \frac{15}{64} = 0.2344$$

From Fig. 2.60*b*, $K = 1.70$

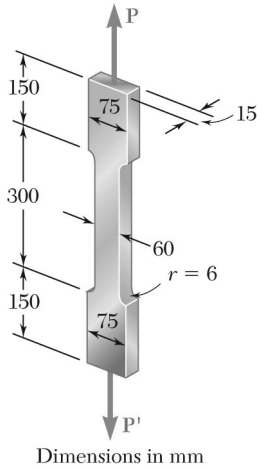
$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{KP}{d_B t}$$

$$t = \frac{KP}{d_B \sigma_{\max}} = \frac{(1.70)(100 \times 10^3 \text{ N})}{(0.064 \text{ m})(125 \times 10^6 \text{ Pa})} = 21.25 \times 10^{-3} \text{ m} = 21.25 \text{ mm}$$

The larger value is the required minimum plate thickness.

$t = 36.7$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.97

The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that E = 70 GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

SOLUTION

$$\sigma_{all} = 200\times10^{6}\ \text{Pa} \quad E = 70\times10^{9}\ \text{Pa}$$

$$A_{min} = (60\ \text{mm})(15\ \text{mm}) = 900\ \text{mm}^2 = 900\times10^{-6}\ \text{m}^2$$

(*a*) Test specimen. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \quad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60*b* $K = 1.95$ $\sigma_{max} = K\dfrac{P}{A}$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900\times10^{-6})(200\times10^{6})}{1.95} = 92.308\times10^{3}\ \text{N} \qquad P = 92.3\ \text{kN} \blacktriangleleft$$

Wide area $A^{*} = (75\ \text{mm})(15\ \text{mm}) = 1125\ \text{mm}^2 = 1.125\times10^{-3}\ \text{m}^2$

$$\delta = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} = \frac{92.308\times10^{3}}{70\times10^{9}}\left[\frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}}\right]$$

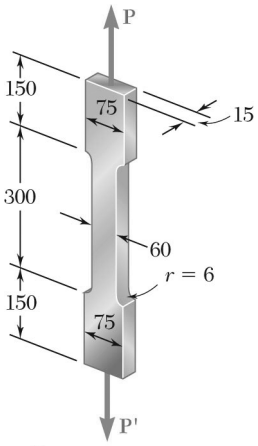
$$= 7.91\times10^{-6}\ \text{m} \qquad \delta = 0.791\ \text{mm} \blacktriangleleft$$

(*b*) Uniform bar.

$$P = A\sigma_{all} = (900\times10^{-6})(200\times10^{6}) = 180\times10^{3}\ \text{N} \qquad P = 180.0\ \text{kN} \blacktriangleleft$$

$$\delta = \frac{PL}{AE} = \frac{(180\times10^{3})(0.600)}{(900\times10^{-6})(70\times10^{9})} = 1.714\times10^{-3}\ \text{m} \qquad \delta = 1.714\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Dimensions in mm

PROBLEM 2.98

For the test specimen of Prob. 2.97, determine the maximum value of the normal stress corresponding to a total elongation of 0.75 mm.

PROBLEM 2.97 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60×15-mm rectangular cross section.

SOLUTION

$$\delta = \Sigma \frac{P_i L_i}{E_i A_i} = \frac{P}{E} \Sigma \frac{L_i}{A_i} \qquad \delta = 0.75 \times 10^{-3} \text{ m}$$

$$L_1 = L_3 = 150 \text{ mm} = 0.150 \text{ m}, \quad L_2 = 300 \text{ mm} = 0.300 \text{ m}$$

$$A_1 = A_3 = (75 \text{ mm})(15 \text{ mm}) = 1125 \text{ mm}^2 = 1.125 \times 10^{-3} \text{ m}^2$$

$$A_2 = (60 \text{ mm})(15 \text{ mm}) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\Sigma \frac{L_i}{A_i} = \frac{0.150}{1.125 \times 10^{-3}} + \frac{0.300}{900 \times 10^{-6}} + \frac{0.150}{1.125 \times 10^{-3}} = 600 \text{ m}^{-1}$$

$$P = \frac{E\delta}{\Sigma \frac{L_i}{A_i}} = \frac{(70 \times 10^9)(0.75 \times 10^{-3})}{600} = 87.5 \times 10^3 \text{ N}$$

Stress concentration. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

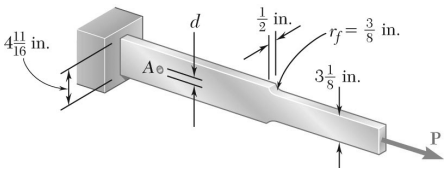
$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60*b* $K = 1.95$

$$\sigma_{max} = K \frac{P}{A_{min}} = \frac{(1.95)(87.5 \times 10^3)}{900 \times 10^{-6}} = 189.6 \times 10^6 \text{ Pa} \qquad \sigma_{max} = 189.6 \text{ MPa} \blacktriangleleft$$

Note that $\sigma_{max} < \sigma_{all}$.

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.99

A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from $\frac{1}{2}$ to $1\frac{1}{2}$ in. in $\frac{1}{4}$-in. increments. If the allowable stress in the plate is 21 ksi, determine (*a*) the diameter d of the largest bit that can be used if the allowable load **P** at the hole is to exceed that at the fillets, (*b*) the corresponding allowable load **P**.

SOLUTION

At the fillets: $\dfrac{D}{d} = \dfrac{4.6875}{3.125} = 1.5 \qquad \dfrac{r}{d} = \dfrac{0.375}{3.125} = 0.12$

From Fig. 2.60*b*, $K = 2.10$

$$A_{\min} = (3.125)(0.5) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P_{\text{all}}}{A_{\min}} = \sigma_{\text{all}}$$

$$P_{\text{all}} = \frac{A_{\min}\sigma_{\text{all}}}{K} = \frac{(1.5625)(21)}{2.10} = 15.625 \text{ kips}$$

At the hole: $A_{\text{net}} = (D - 2r)t$, K from Fig. 2.60*a*

$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \sigma_{\text{all}} \qquad \therefore \qquad P_{\text{all}} = \frac{A_{\text{net}}\sigma_{\text{all}}}{K}$$

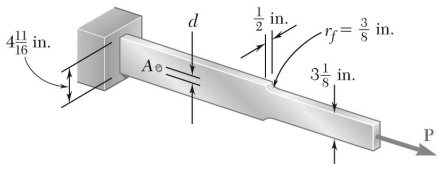
with $D = 4.6875$ in. $\quad t = 0.5$ in. $\quad \sigma_{\text{all}} = 21$ ksi

Hole diam.	r	$d = D - 2r$	$2r/D$	K	A_{net}	P_{all}
0.5 in.	0.25 in.	4.1875 in.	0.107	2.68	2.0938 in^2	16.41 kips
0.75 in.	0.375 in.	3.9375 in.	0.16	2.58	1.96875 in^2	16.02 kips
1 in.	0.5 in.	3.6875 in.	0.213	2.49	1.84375 in^2	15.55 kips
1.25 in.	0.625 in.	3.4375 in.	0.267	2.41	1.71875 in^2	14.98 kips
1.5 in.	0.75 in.	3.1875 in.	0.32	2.34	1.59375 in^2	14.30 kips

(*a*) Largest hole with $P_{\text{all}} > 15.625$ kips is the $\frac{3}{4}$-in. diameter hole. ◀

(*b*) Allowable load $P_{\text{all}} = 15.63$ kips ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.100

(*a*) For $P = 13$ kips and $d = \frac{1}{2}$ in., determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION

Maximum stress at hole:

Use Fig. 2.60*a* for values of *K*.

$$\frac{2r}{D} = \frac{0.5}{4.6875} = 0.017, \qquad K = 2.68$$

$$A_{\text{net}} = (0.5)(4.6875 - 0.5) = 2.0938 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \frac{(2.68)(13)}{2.0938} = 16.64 \text{ ksi},$$

Maximum stress at fillets:

Use Fig. 2.60*b* for values of *K*.

$$\frac{r}{d} = \frac{0.375}{3.125} = 0.12 \qquad \frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad K = 2.10$$

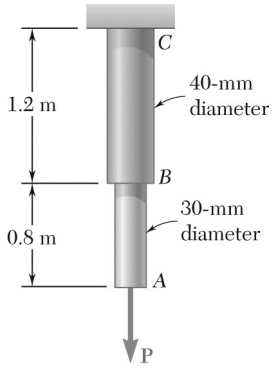
$$A_{\min} = (0.5)(3.125) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\min}} = \frac{(2.10)(13)}{1.5625} = 17.47 \text{ ksi}$$

(*a*) With hole and fillets: 17.47 ksi ◀

(*b*) Without hole: 17.47 ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.101

Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3} \text{ mm}^2 = 1.25664\times10^{-3} \text{ m}^2$$

$$P_{\max} = A_{\min}\sigma_Y = (706.86\times10^{-6})(250\times10^{6}) = 176.715\times10^{3} \text{ N}$$

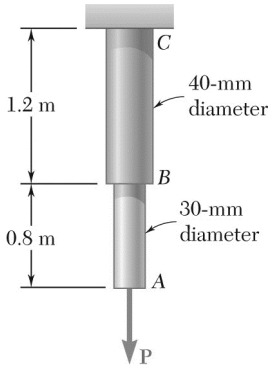
$$P_{\max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times10^{3})(0.8)}{(200\times10^{9})(706.86\times10^{-6})} + \frac{(176.715\times10^{3})(1.2)}{(200\times10^{9})(1.25664\times10^{-3})}$$

$$= 1.84375\times10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' \text{ or } \delta_m = \delta_p + \delta' = 2 + 1.84375 \qquad \delta_m = 3.84 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.102

Rod *ABC* consists of two cylindrical portions *AB* and *BC*; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end *A* has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3} \text{ mm}^2 = 1.25644\times10^{-3} \text{ m}^2$$

$$P_{\max} = A_{\min}\sigma_Y = (706.86\times10^{-6})(250\times10^{6}) = 176.715\times10^{3} \text{ N}$$

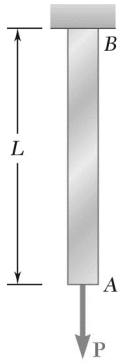
$$P_{\max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times10^{3})(0.8)}{(200\times10^{9})(706.68\times10^{-6})} + \frac{(176.715\times10^{3})(1.2)}{(200\times10^{9})(1.25664\times10^{-3})}$$

$$= 1.84375\times10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm} \qquad \delta_p = 3.16 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.103

The 30-mm square bar AB has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar until end A has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 4.5$ mm, (*b*) $\delta_m = 8$ mm.

SOLUTION

$$A = (30)(30) = 900 \text{ mm}^2 = 900\times10^{-6}\text{ m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.2)(345\times10^6)}{200\times10^9} = 3.795\times10^{-3} = 3.795 \text{ mm}$$

If $\delta_m \geq \delta_Y$, $P_m = A\sigma_Y = (900\times10^{-6})(345\times10^6) = 310.5\times10^3$ N

Unloading: $\delta' = \dfrac{P_m L}{AE} = \dfrac{\sigma_Y L}{E} = \delta_Y = 3.795$ mm

$$\delta_P = \delta_m - \delta'$$

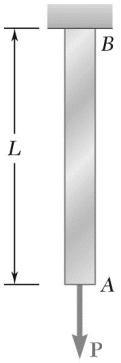
(*a*) $\delta_m = 4.5 \text{ mm} > \delta_Y \quad P_m = 310.5\times10^3$ N $\qquad \delta_m = 310.5$ kN ◀

$\delta_{\text{perm}} = 4.5 \text{ mm} - 3.795 \text{ mm} \qquad \delta_{\text{perm}} = 0.705$ mm ◀

(*b*) $\delta_m = 8 \text{ mm} > \delta_Y \quad P_m = 310.5\times10^3$ N $\qquad \delta_m = 310.5$ kN ◀

$\delta_{\text{perm}} = 8.0 \text{ mm} - 3.795 \text{ mm} \qquad \delta_{\text{perm}} = 4.205$ mm ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.104

The 30-mm square bar AB has a length $L = 2.5\,\text{m}$; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 3.5 mm, (*b*) 6.5 mm.

SOLUTION

$$A = (30)(30) = 900\ \text{mm}^2 = 900\times10^{-6}\ \text{m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.5)(345\times10^6)}{200\times10^9} = 4.3125\times10^{3}\,\text{m} = 4.3125\ \text{mm}$$

When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

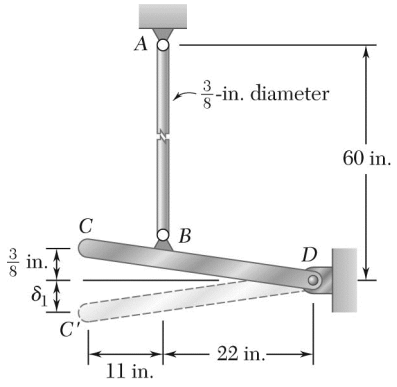
$$P_m = A\sigma_Y = (900\times10^{-6})(345\times10^6) = 310.5\times10^{3}\,\text{N} \qquad = 310.5\ \text{kN} \blacktriangleleft$$

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(*a*) $\delta_p = 3.5$ mm $\quad \delta_m = 3.5\ \text{mm} + 4.3125\ \text{mm} \qquad = 7.81\ \text{mm} \blacktriangleleft$

(*b*) $\delta_p = 6.5$ mm $\quad \delta_m = 6.5\ \text{mm} + 4.3125\ \text{mm} \qquad = 10.81\ \text{mm} \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.105

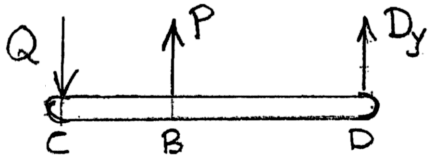
Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is $\frac{3}{8}$ in. too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C′*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

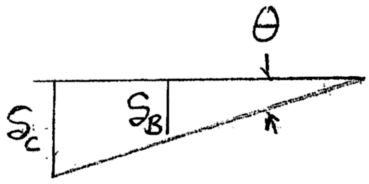
SOLUTION

Since the rod *AB* is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 (36) = 3.976 \text{ kips}$$

Referring to the free body diagram of lever *CD*,





$$\Sigma M_D = 0: \quad 33Q - 22P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(3.976)}{33} = 2.65 \text{ kips} \qquad Q = 2.65 \text{ kips} \blacktriangleleft$$

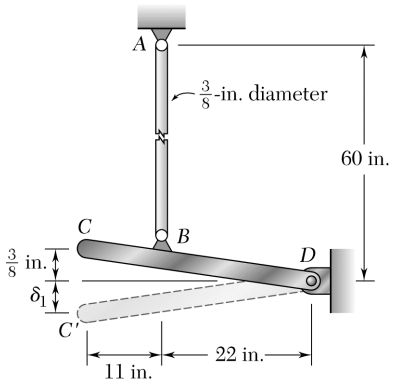
During unloading, the spring back at *B* is

$$\delta_B = L_{AB}\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^6} = 0.0745 \text{ in.}$$

From the deformation diagram,

Slope: $$\theta = \frac{\delta_B}{22} = \frac{\delta_C}{33} \quad \therefore \quad \delta_C = \frac{33}{-22}\delta_B = 0.1117 \text{ in.} \qquad \delta_C = 0.1117 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.106

Solve Prob. 2.105, assuming that the yield point of the mild steel is 50 ksi.

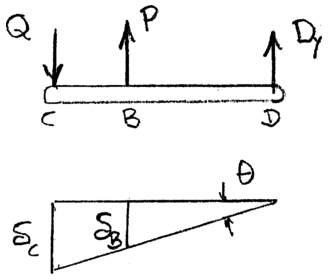
PROBLEM 2.105 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is $\frac{3}{8}$ in. too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C′*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

Since the rod *AB* is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 (50) = 5.522 \text{ kips}$$

Referring to the free body diagram of lever *CD*,



$$\Sigma M_D = 0: \quad 33Q - 22P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(5.522)}{33} = 3.68 \text{ kips} \qquad Q = 3.68 \text{ kips} \blacktriangleleft$$

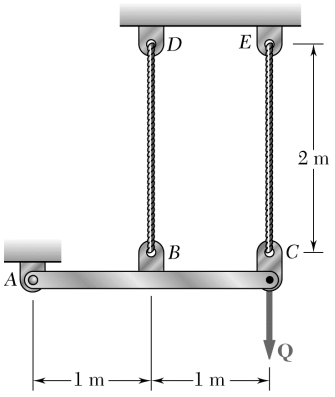
During unloading, the spring back at *B* is

$$\delta_B = L_{AB}\,\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(50\times10^3)}{29\times10^6} = 0.1034 \text{ in.}$$

From the deformation diagram,

Slope: $$\theta = \frac{\delta_B}{22} = \frac{\delta_C}{33} \quad \therefore \quad \delta_C = \frac{33}{22}\delta_B \qquad \delta_C = 0.1552 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.107

Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In Part *c*, cable *CE* is not taut.)

SOLUTION

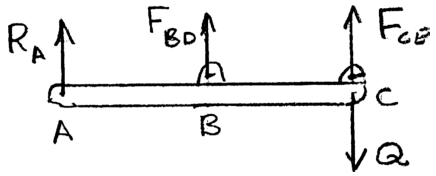
Elongation constraints for taut cables.

Let $\theta =$ rotation angle of rigid bar *ABC*.

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\ \delta_{CE} = \frac{1}{2}\delta_{CE} \tag{1}$$

Equilibrium of bar *ABC*.



$$+\circlearrowleft M_A = 0:\ L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\ Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \tag{2}$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\ \text{N}$

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\ \text{N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\ \text{N},$ cable *BD* is elastic when $Q = 50$ kN.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.107 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^{3}}{100\times10^{-6}} = 310\times10^{6}\text{ Pa} \qquad \sigma_{BD} = 310\text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^{3})(2)}{(200\times10^{9})(100\times10^{-6})} = 3.1\times10^{-3}\text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m $\qquad 6.20\text{ mm}\downarrow \blacktriangleleft$

Permanent elongation of cable *CE*: $(\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

$$(\delta_{CE})_P = (\delta_{CE})_{\max} - \frac{F_{CE}L_{CE}}{EA} = (\delta_{CE})_{\max} - \frac{\sigma_Y L_{CE}}{E}$$

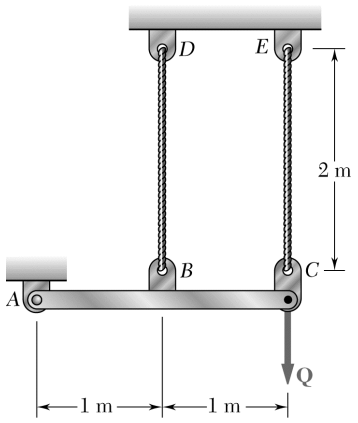
$$= 6.20\times10^{-3} - \frac{(345\times10^{6})(2)}{200\times10^{9}} = 2.75\times10^{-3}\text{ m}$$

(*c*) Unloading. Cable *CE* is slack $(F_{CE} = 0)$ at $Q = 0$.

From (2), $F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$

Since cable *BD* remained elastic, $\delta_{BD} = \dfrac{F_{BD}L_{BD}}{EA} = 0.$ $\blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.108

Solve Prob. 2.107, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

PROBLEM 2.107 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In Part *c*, cable *CE* is not taut.)

SOLUTION

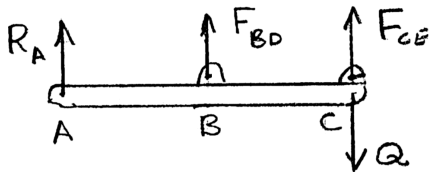
Elongation constraints.

Let $\theta =$ rotation angle of rigid bar *ABC*.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \tag{1}$$

Equilibrium of bar *ABC*.



$$+\circlearrowleft M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\,Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \tag{2}$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\,\text{N}$

From (2), $F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\,\text{N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\,\text{N}$, cable *BD* is elastic when $Q = 50$ kN.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.108 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345$ MPa

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^{3}}{100\times10^{-6}} = 310\times10^{6}\text{ Pa} \qquad \sigma_{BD} = 310\text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^{3})(2)}{(200\times10^{9})(100\times10^{-6})} = 3.1\times10^{-3}\text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3}$ m 6.20 mm ↓ ◀

Unloading. $Q' = 50\times10^{3}$ N, $\delta'_{CE} = \delta'_C$

From (1), $\delta'_{BD} = \frac{1}{2}\delta'_C$

Elastic $F''_{BD} = \dfrac{EA\delta'_{BD}}{L_{BD}} = \dfrac{(200\times10^{9})(100\times10^{-6})(\frac{1}{2}\delta'_C)}{2} = 5\times10^{6}\,\delta'_C$

$$F'_{CE} = \frac{EA\delta'_{CE}}{L_{CE}} = \frac{(200\times10^{9})(100\times10^{-6})(\delta'_C)}{2} = 10\times10^{6}\,\delta'_C$$

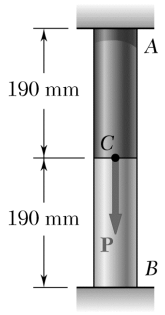
From (2), $Q' = F'_{CE} + \frac{1}{2}F'_{BD} = 12.5\times10^{6}\,\delta'_C$

Equating expressions for Q', $12.5\times10^{6}\,\delta'_C = 50\times10^{3}$

$$\delta'_C = 4\times10^{-3}\text{ m}$$

(*c*) Final displacement. $\delta_C = (\delta_C)_m - \delta'_C = 6.2\times10^{-3} - 4\times10^{-3} = 2.2\times10^{-3}$ m 2.20 mm ↓ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.109

Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

SOLUTION

Displacement at C to cause yielding of AC.

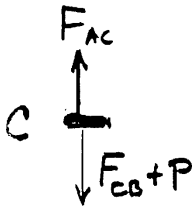
$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\text{ m}$$

Corresponding force. $F_{AC} = A\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6) = 437.5\times10^3\text{ N}$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\text{ N}$$

For equilibrium of element at C,

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\text{ N}$$



Since applied load $P = 975\times10^3\text{ N} > 875\times10^3\text{ N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\text{ N} = -537.5\times10^3\text{ N}$$

(*a*) $$\delta_C = -\frac{F_{CB}L_{CD}}{EA} = \frac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\text{ m}$$

0.292 mm ◀

(*b*) Maximum stresses: $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa 250 MPa ◀

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\text{ Pa} = -307\text{ MPa}$$ −307 MPa ◀

(*c*) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P'_{AC}$$

$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \quad P'_{AC} = 487.5\times10^{-3}\text{ N}$$

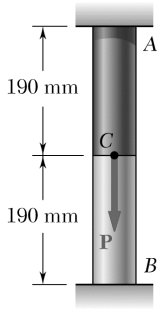
$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^3\text{ m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3}$$

$$= 0.02715\times10^{-3}\text{ m}$$ 0.0272 mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.110



For the composite rod of Prob. 2.109, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3$ mm and then decreased back to zero, determine (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

PROBLEM 2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

SOLUTION

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding:

$$\varepsilon_{Y,\,AC} = \frac{\sigma_{Y,\,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,\,CB} = \frac{\sigma_{Y,\,BC}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(*a*) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^{-3} \text{ N}$$

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CD} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} \qquad = 990 \text{ kN} \blacktriangleleft$$

(*b*) Stresses: AC: $\sigma_{AC} = \sigma_{Y,\,AC}$ $= 250$ MPa ◀

$$CB: \quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} \qquad -316 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.110 *(Continued)*

(*c*) Deflection and forces for unloading.

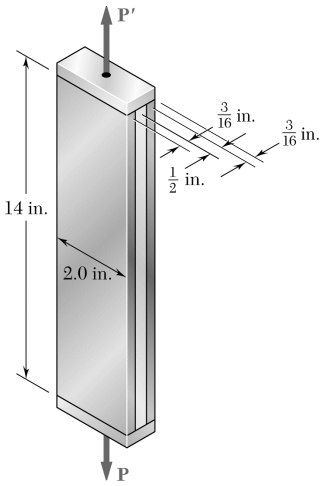
$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1\times10^{3}\text{ N} \quad \therefore \quad P'_{AC} = 495.05\times10^{3}\text{ N}$$

$$\delta' = \frac{(495.05\times10^{3})(0.190)}{(200\times10^{9})(1750\times10^{-6})} = 0.26874\times10^{-3}\text{ m} = 0.26874\text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30\text{ mm} - 0.26874\text{ mm} \qquad 0.031\text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.111

Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$ $\qquad \delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in.}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$ $\qquad \delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^3} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$. The mild steel yields. Tempered steel is elastic.

(*a*) Forces: $P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^3)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

$$P = P_1 + P_2 = 112.14\times10^3 \text{ lb} = 112.1 \text{ kips} \qquad P = 112.1 \text{ kips} \blacktriangleleft$$

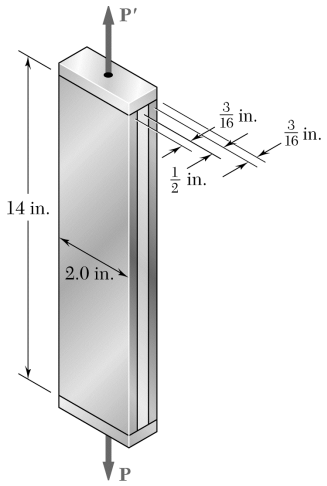
(*b*) Stresses: $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3 \text{ psi} = 50 \text{ ksi}$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi} = 82.86 \text{ ksi} \qquad 82.86 \text{ ksi} \blacktriangleleft$$

Unloading: $\delta' = \frac{PL}{EA} = \frac{(112.14\times10^3)(14)}{(29\times10^6)(1.75)} = 0.03094 \text{ in.}$

(*c*) Permanent set: $\delta_p = \delta_m - \delta' = 0.04 - 0.03094 = 0.00906 \text{ in.} \qquad 0.00906 \text{ in.} \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.112

For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel:

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb}$$

$P > P_Y$, therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_1 = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb}$$

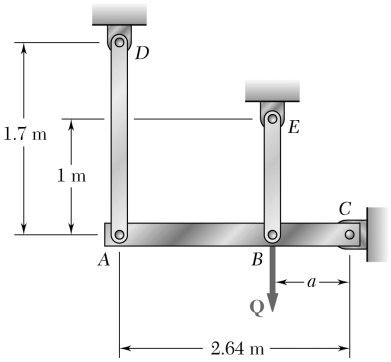
(*a*) $$\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times10^3)(14)}{(29\times10^6)(0.75)} = 0.03090 \text{ in.}$$ ◀

(*b*) $$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi} = 64 \text{ ksi}$$ ◀

Unloading: $$\delta' = \frac{PL}{EA} = \frac{(98\times10^3)(14)}{(29\times10^6)(1.75)} = 0.02703 \text{ in.}$$

(*c*) $$\delta_P = \delta_m - \delta' = 0.03090 - 0.02703 = 0.00387 \text{ in.}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.113

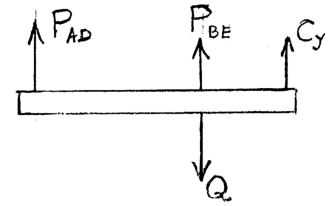
The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

Statics: $\Sigma M_C = 0:\ 0.640(Q - P_{BE}) - 2.64P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta,\ \delta_B = a\theta = 0.640\theta$

Elastic analysis:

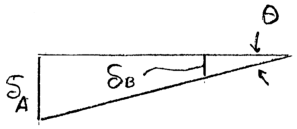


$$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times10^{-6}\ \text{m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$$

$$= (26.47\times10^6)(2.64\theta) = 69.88\times10^6\,\theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$$



$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B$$

$$= (45\times10^6)(0.640\theta) = 28.80\times10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128\times10^9\,\theta$$

From Statics, $Q = P_{BE} + \dfrac{2.64}{0.640}P_{AD} = P_{BE} + 4.125P_{AD}$

$$= [28.80\times10^6 + (4.125)(69.88\times10^6]\,\theta = 317.06\times10^6\,\theta$$

θ_Y at yielding of link *AD*: $\sigma_{AD} = \sigma_Y = 250\times10^6 = 310.6\times10^9\,\theta$

$$\theta_Y = 804.89\times10^{-6}$$

$$Q_Y = (317.06\times10^6)(804.89\times10^{-6}) = 255.2\times10^3\ \text{N}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.113 *(Continued)*

(*a*) Since $Q = 260\times10^3 > Q_Y$, link AD yields. $\sigma_{AD} = 250$ MPa ◀

$$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3 \text{ N}$$

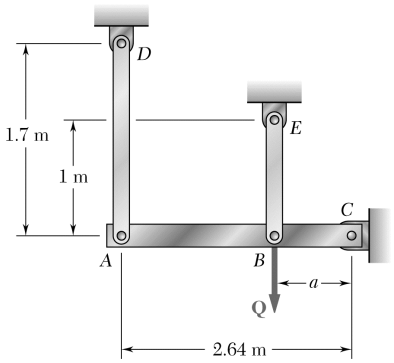
From Statics, $P_{BE} = Q - 4.125P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$$P_{BE} = 27.97\times10^3 \text{ N}$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6 \text{ Pa}$$ $\sigma_{BE} = 124.3$ MPa ◀

(*b*) $$\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6} \text{ m}$$ $\delta_B = 0.622$ mm ↓ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.114

Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

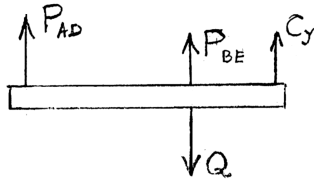
PROBLEM 2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.

SOLUTION

Statics: $\Sigma M_C = 0:\ 1.76(Q - P_{BE}) - 2.64P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta,\ \delta_B = 1.76\theta$

Elastic Analysis:

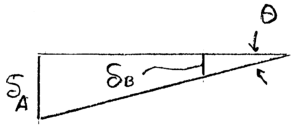


$$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times10^{-6}\ \text{m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\,\delta_A$$

$$= (26.47\times10^6)(2.64\theta) = 69.88\times10^6\,\theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$$



$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B$$

$$= (45\times10^6)(1.76\theta) = 79.2\times10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 352\times10^9\,\theta$$

From Statics, $Q = P_{BE} + \dfrac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$$= [73.8\times10^6 + (1.500)(69.88\times10^6]\,\theta = 178.62\times10^6\,\theta$$

θ_Y at yielding of link BE: $\sigma_{BE} = \sigma_Y = 250\times10^6 = 352\times10^9\,\theta_Y$

$$\theta_Y = 710.23\times10^{-6}$$

$$Q_Y = (178.62\times10^6)(710.23\times10^{-6}) = 126.86\times10^3\ \text{N}$$

Since $Q = 135\times10^3\ \text{N} > Q_Y$, link BE yields. $\sigma_{BE} = \sigma_Y = 250$ MPa ◀

$$P_{BE} = A\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3\ \text{N}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.114 ***(Continued)***

From Statics, $P_{AD} = \dfrac{1}{1.500}(Q - P_{BE}) = 52.5 \times 10^3 \text{ N}$

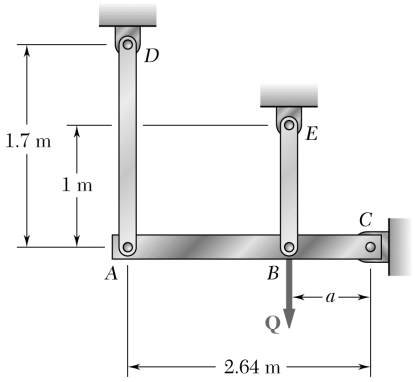
(*a*) $$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5 \times 10^3}{225 \times 10^{-6}} = 233.3 \times 10^6$$ $\sigma_{AD} = 233 \text{ MPa}$ ◀

From elastic analysis of *AD*, $$\theta = \frac{P_{AD}}{69.88 \times 10^6} = 751.29 \times 10^{-3} \text{ rad}$$

(*b*) $$\delta_B = 1.76\theta = 1.322 \times 10^{-3} \text{ m}$$ $\delta_B = 1.322 \text{ mm} \downarrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.115*



Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

PROBLEM 2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

See solution to Problem 2.113 for the normal stresses in each link and the deflection of Point *B* after loading.

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa}$$
$$\sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$
$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to Problem 2.113 applies to the unloading.

$$Q' = 317.06 \times 10^6 \theta'$$
$$Q' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$
$$\sigma'_{BE} = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$
$$\delta'_B = 0.640 \theta' = 524.82 \times 10^{-6} \text{ m}$$

(*a*) Residual stresses.

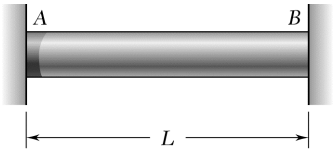
$$\sigma_{AD,\text{res}} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa} \qquad = -4.70 \text{ MPa} \blacktriangleleft$$

$$\sigma_{BE,\text{res}} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \qquad = 19.34 \text{ MPa} \blacktriangleleft$$

(*b*) $\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} = 96.71 \times 10^{-6} \text{ m} \qquad = 0.0967 \text{ mm} \downarrow \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.116



A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of $45\,°\text{F}$. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29\times10^6$ psi. Knowing that $\alpha = 6.5\times10^{-6}/°\text{F}$, determine the stress in the bar (*a*) when the temperature is raised to $320\,°\text{F}$, (*b*) after the temperature has returned to $45\,°\text{F}$.

SOLUTION

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36\times10^3}{(29\times10^6)(6.5\times10^{-6})} = 190.98\,°\text{F}$$

But $\Delta T = 320 - 45 = 275\,°\text{F} > (\Delta T_Y)$

(*a*) Yielding occurs. $\sigma = -\sigma_Y = -36$ ksi ◀

Cooling: $(\Delta\text{T})' = 275\,°\text{F}$

$$\delta' = \delta'_P = \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

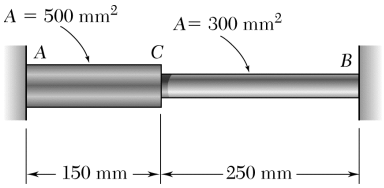
$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(29\times10^6)(6.5\times10^{-6})(275) = -51.8375\times10^3\,\text{psi}$$

(*b*) Residual stress:

$$\sigma_{\text{res}} = -\sigma_Y - \sigma' = -36\times10^3 + 51.8375\times10^3 = 15.84\times10\,\text{psi} \qquad 15.84\text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.117

The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times10^{-6}/°C$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

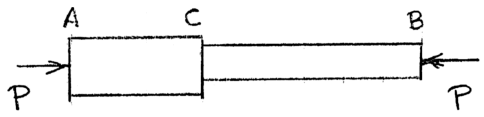
$$A_{AC} = 500\times10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\ \text{m}$$
$$A_{CB} = 300\times10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\ \text{m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$



At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(2.50\times10^{6}) = 75\times10^{3}\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$
$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812\,°C$$

Actual ΔT: $150\,°C - 25\,°C = 125\,°C > (\Delta T)_Y$

Yielding occurs. For $\Delta T > (\Delta T)_Y$, $P = P_Y = 75\times10^{3}\,\text{N}$

(*a*) $\sigma_{AC} = -\dfrac{P_Y}{A_{AC}} = -\dfrac{75\times10^{3}}{500\times10^{-6}} = -150\times10^{6}\ \text{Pa}$ $\qquad \sigma_{AC} = -150\ \text{MPa}$ ◀

$\sigma_{CB} = -\dfrac{P_Y}{A_{CB}} = -\sigma_Y$ $\qquad \sigma_{CB} = -250\ \text{MPa}$ ◀

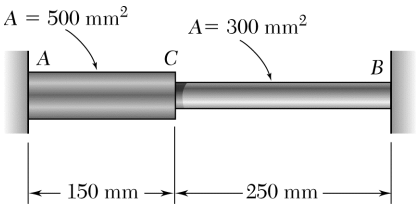
(*b*) For $\Delta T > (\Delta T)_Y$, portion *AC* remains elastic.

$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\alpha(\Delta T)$$
$$= -\frac{(75\times10^{3})(0.150)}{(200\times10^{9})(500\times10^{-6})} + (0.150)(11.7\times10^{-6})(125) = 106.9\times10^{-6}\,\text{m}$$

Since Point *A* is stationary, $\delta_C = \delta_{C/A} = 106.9\times10^{-6}\,\text{m}$ $\qquad \delta_C = 0.1069\ \text{mm}\rightarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.118*



Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150°C and then returned to 25°C.

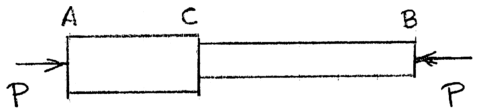
PROBLEM 2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7\times10^{-6}/°\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

$$A_{AC} = 500\times10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\ \text{m} \qquad A_{CB} = 300\times10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\ \text{m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.



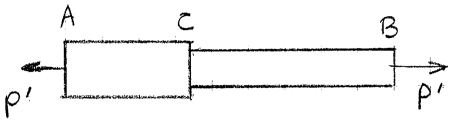
$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(250\times10^{6}) = 75\times10^{3}\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right)$$
$$= 90.812\ °\text{C}$$

Actual ΔT: $150\ °\text{C} - 25\ °\text{C} = 125\ °\text{C} > (\Delta T)_Y$



Yielding occurs. For $\Delta T > (\Delta T)_Y$ $\quad P = P_Y = 75\times10^{3}\,\text{N}$

Cooling: $(\Delta T)' = 125\ °\text{C}$ $\quad P' = \dfrac{EL_{AB}\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)} = \dfrac{(200\times10^{9})(0.400)(11.7\times10^{-6})(125)}{\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}} = 103.235\times10^{3}\,\text{N}$

Residual force: $P_{\text{res}} = P' - P_Y = 103.235\times10^{3} - 75\times10^{3} = 28.235\times10^{3}\,\text{N}$ (tension)

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.118* *(Continued)*

(*a*) Residual stresses.

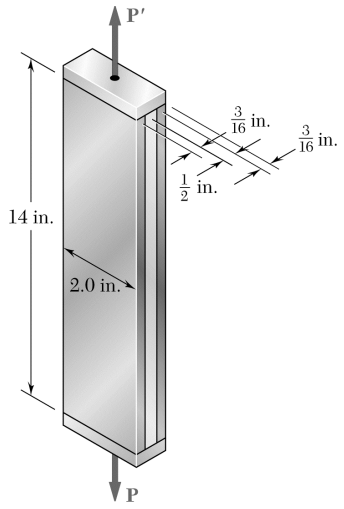
$$\sigma_{AC} = \frac{P_{\text{res}}}{A_{AC}} = \frac{28.235 \times 10^3}{500 \times 10^{-6}} \qquad \sigma_{AC} = 56.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{CB} = \frac{P_{\text{res}}}{A_{CB}} = \frac{28.235 \times 10^3}{300 \times 10^{-6}} \qquad \sigma_{CB} = 9.41 \text{ MPa} \blacktriangleleft$$

(*b*) Permanent deflection of point *C*.

$$\delta_C = \frac{P_{\text{res}} L_{AC}}{E A_{AC}} \qquad \delta_C = 0.0424 \text{ mm} \rightarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.119*

For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = (2)\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel: $\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3 \text{ lb}$

$P > P_Y$; therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$

$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3 \text{ lb}$

$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3 \text{ psi}$

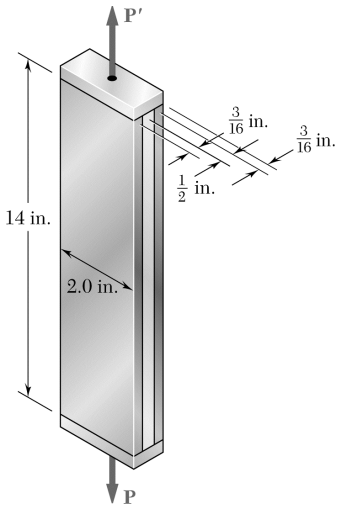
Unloading: $\sigma' = \frac{P}{A} = \frac{98\times10^3}{1.75} = 56\times10^3 \text{ psi}$

Residual stresses.

Mild steel: $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 56\times10^3 = -6\times10^{-3} \text{ psi} = -6 \text{ ksi}$

Tempered steel: $\sigma_{2,\text{res}} = \sigma_2 - \sigma_1 = 64\times10^3 - 56\times10^3 = 8\times10^3 \text{ psi}$ 8.00 ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.120*

For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2 \quad \delta_{Y1} = \frac{L\delta_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2 \quad \delta_{Y2} = \frac{L\delta_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^6} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$$\delta_{Y1} < \delta_m < \delta_{Y2}$$

The mild steel yields. Tempered steel is elastic.

Forces:
$$P_1 = A_1\delta_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$
$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^6)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

Stresses: $\sigma_1 = \frac{P_1}{A_1} = \delta_{Y1} = 50\times10^3 \text{ psi} \quad \sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi}$

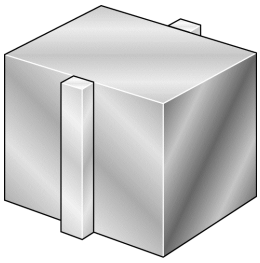
Unloading: $\sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08\times10^3 \text{ psi}$

Residual stresses. $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$

$$\sigma_{2,\text{res}} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.121*



Narrow bars of aluminum are bonded to the two sides of a thick steel plate as shown. Initially, at $T_1 = 70°\text{F}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8\times10^{-6}/°\text{F}$ for the aluminum and $\alpha_s = 6.5\times10^{-6}/°\text{F}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9\times10^{6}$ psi and $\sigma_Y = 58$ ksi. (*Hint:* Neglect the small stresses in the plate.)

SOLUTION

Determine temperature change to cause yielding.

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{58\times10^{3}}{(10.9\times10^{6})(12.8-6.5)(10^{-6})} = 844.62\ °\text{F}$$

(*a*) $T_{2Y} = T_1 + (\Delta T)_Y = 70 + 844.62 = 915\ °\text{F}$ 915 °F ◀

After yielding,

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling:

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{\text{res}} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{\text{res}} = -\sigma_Y$

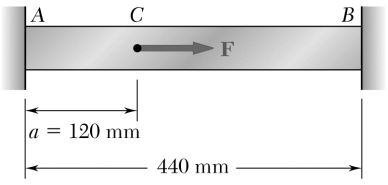
$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(58\times10^{3})}{(10.9\times10^{6})(12.8-6.5)(10^{-6})} = 1689\ °\text{F}$$

(*b*) $T_2 = T_1 + \Delta T = 70 + 1689 = 1759\ °\text{F}$ 1759 °F ◀

If $T_2 > 1759\ °\text{F}$, the aluminum bar will most likely yield in compression.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.122*

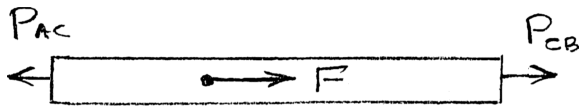
Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion AC:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$$
$$= 300 \times 10^3 \text{ N}$$



For equilibrium, $F + P_{CB} - P_{AC} = 0$.

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$
$$= -220 \times 10^3 \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})}$$
$$= 0.293333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}}$$
$$= -183.333 \times 10^6 \text{ Pa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.122* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.120)}{0.440} = 378.182\times10^3\ \text{N}$$

$$P'_{CB} = P'_{AC} - F = 378.182\times10^3 - 520\times10^3 = -141.818\times10^3\ \text{N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.182\times10^3}{1200\times10^{-6}} = 315.152\times10^6\ \text{Pa}$$

$$\sigma'_{BC} = \frac{P'_{BC}}{A} = -\frac{141.818\times10^3}{1200\times10^{-6}} = -118.182\times10^6\ \text{Pa}$$

$$\delta'_C = \frac{(378.182)(0.120)}{(200\times10^9)(1200\times10^6)} = 0.189091\times10^{-3}\ \text{m}$$

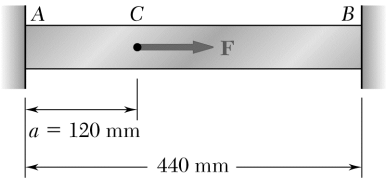
(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.293333\times10^{-3} - 0.189091\times10^{-3} = 0.1042\times10^{-3}\ \text{m}$ $= 0.1042\ \text{mm}$ ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_Y - \sigma'_{AC} = 250\times10^6 - 315.152\times10^6 = -65.2\times10^6\ \text{Pa}$ $= -65.2\ \text{MPa}$ ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 118.182\times10^6 = -65.2\times10^6\ \text{Pa}$ $= -65.2\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.123*



Solve Prob. 2.122, assuming that $a = 180$ mm.

PROBLEM 2.122 Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

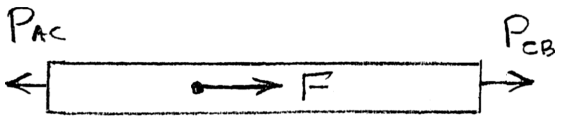
SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion *AC*:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^{6})$$
$$= 300 \times 10^{3} \text{ N}$$

For equilibrium, $F + P_{CB} - P_{AC} = 0$.



$$P_{CB} = P_{AC} - F = 300 \times 10^{3} - 520 \times 10^{3}$$
$$= -220 \times 10^{3} \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^{3})(0.440 - 0.180)}{(200 \times 10^{9})(1200 \times 10^{-6})}$$
$$= 0.238333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^{3}}{1200 \times 10^{-6}}$$
$$= -183.333 \times 10^{6} \text{ Pa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.123* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$= P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.180)}{0.440} = 307.273\times10^3\ \text{N}$$

$$P'_{CB} = P'_{AC} - F = 307.273\times10^3 - 520\times10^3 = -212.727\times10^3\ \text{N}$$

$$\delta'_C = \frac{(307.273\times10^3)(0.180)}{(200\times10^9)(1200\times10^{-6})} = 0.230455\times10^{-3}\ \text{m}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{307.273\times10^3}{1200\times10^{-6}} = 256.061\times10^6\ \text{Pa}$$

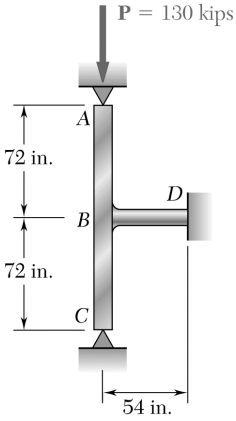
$$\sigma'_{CB} = \frac{P'_{CB}}{A} = \frac{-212.727\times10^3}{1200\times10^{-6}} = -177.273\times10^6\ \text{P}$$

(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.238333\times10^{-3} - 0.230455\times10^{-3} = 0.00788\times10^{-3}\ \text{m}$ $= 0.00788\ \text{mm}$ ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_{AC} - \sigma'_{AC} = 250\times10^6 - 256.061\times10^6 = -6.06\times10^6\ \text{Pa}$ $= -6.06\ \text{MPa}$ ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 177.273\times10^6 = -6.06\times10^6\ \text{Pa}$ $= -6.06\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.124

Rod *BD* is made of steel ($E = 29\times10^6$ psi) and is used to brace the axially compressed member *ABC*. The maximum force that can be developed in member *BD* is 0.02*P*. If the stress must not exceed 18 ksi and the maximum change in length of *BD* must not exceed 0.001 times the length of *ABC*, determine the smallest-diameter rod that can be used for member *BD*.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6\times10^3 \text{lb}$$

Considering stress: $\sigma = 18 \text{ ksi} = 18\times10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \quad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

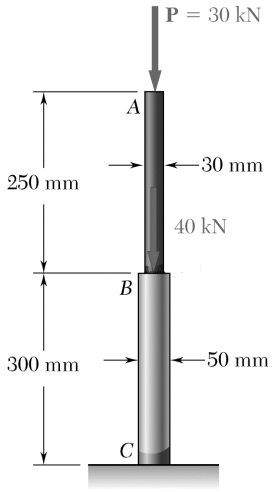
Considering deformation: $\delta = (0.001)(144) = 0.144$ in.

$$\delta = \frac{F_{BD}L_{BD}}{AE} \quad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(2.6\times10^3)(54)}{(29\times10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}} \qquad d = 0.429 \text{in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.125

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel $(E = 200 \text{ GPa})$ and rod BC of brass $(E = 105 \text{ GPa})$. Determine (*a*) the total deformation of the composite rod ABC, (*b*) the deflection of point B.

SOLUTION

Rod AB:

$$F_{AB} = -P = -30\times10^{3}\text{ N}$$

$$L_{AB} = 0.250\text{ m}$$

$$E_{AB} = 200\times10^{9}\text{ GPa}$$

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85\text{ mm}^2 = 706.85\times10^{-6}\text{ m}^2$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{E_{AB}A_{AB}} = -\frac{(30\times10^{3})(0.250)}{(200\times10^{9})(706.85\times10^{-6})} = -53.052\times10^{-6}\text{ m}$$

Rod BC:

$$F_{BC} = 30 + 40 = 70\text{ kN} = 70\times10^{3}\text{ N}$$

$$L_{BC} = 0.300\text{ m}$$

$$E_{BC} = 105\times10^{9}\text{ Pa}$$

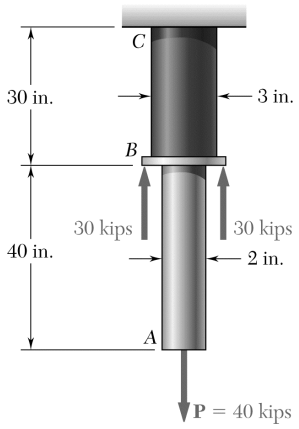
$$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635\times10^{3}\text{ mm}^2 = 1.9635\times10^{-3}\text{ m}^2$$

$$\delta_{BC} = \frac{F_{BC}L_{BC}}{E_{BC}A_{BC}} = -\frac{(70\times10^{3})(0.300)}{(105\times10^{9})(1.9635\times10^{-3})} = -101.859\times10^{-6}\text{ m}$$

(*a*) Total deformation: $\delta_{\text{tot}} = \delta_{AB} + \delta_{BC} = -154.9\times10^{-6}\text{ m} = -0.1549\text{ mm}$ ◀

(*b*) Deflection of Point B. $\delta_B = \delta_{BC}$ $\delta_B = 0.1019\text{ mm}\downarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.126

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 29\times10^6$ psi), and rod BC of brass ($E = 15\times10^6$ psi). Determine (*a*) the total deformation of the composite rod ABC, (*b*) the deflection of point B.

SOLUTION

Portion AB:

$$P_{AB} = 40\times10^3 \text{ lb}$$
$$L_{AB} = 40 \text{ in.}$$
$$d = 2 \text{ in.}$$
$$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$
$$E_{AB} = 29\times10^6 \text{ psi}$$
$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{(40\times10^3)(40)}{(29\times10^6)(3.1416)} = 17.5619\times10^{-3} \text{ in.}$$

Portion BC:

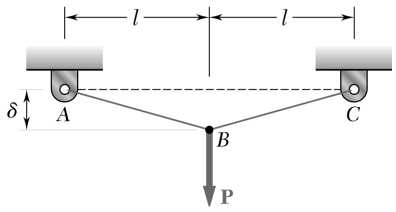
$$P_{BC} = -20\times10^3 \text{ lb}$$
$$L_{BC} = 30 \text{ in.}$$
$$d = 3 \text{ in.}$$
$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$
$$E_{BC} = 15\times10^6 \text{ psi}$$
$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E_{BC}A_{BC}} = \frac{(-20\times10^3)(30)}{(15\times10^6)(7.0686)} = -5.6588\times10^{-3} \text{ in.}$$

(*a*) $\delta = \delta_{AB} + \delta_{BC} = 17.5619\times10^{-6} - 5.6588\times10^{-6}$ $\qquad \delta = 11.90\times10^{-3} \text{ in.} \downarrow$ ◀

(*b*) $\delta_B = -\delta_{BC}$ $\qquad \delta_B = 5.66\times10^{-3} \text{ in.} \uparrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 2.127



The uniform wire *ABC*, of unstretched length 2*l*, is attached to the supports shown and a vertical load **P** is applied at the midpoint *B*. Denoting by *A* the cross-sectional area of the wire and by *E* the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint *B* is

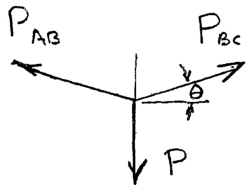
$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

SOLUTION

Use approximation.

$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

Statics: $+\uparrow \Sigma F_Y = 0: \quad 2P_{AB}\sin\theta - P = 0$

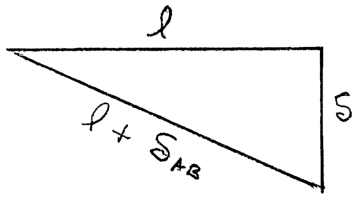


$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation: $\delta_{AB} = \dfrac{P_{AB}l}{AE} = \dfrac{Pl^2}{2AE\delta}$

Deflection:

From the right triangle,



$$(l+\delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = l^2 + 2l\,\delta_{AB} + \delta_{AB}^2 - l^2$$

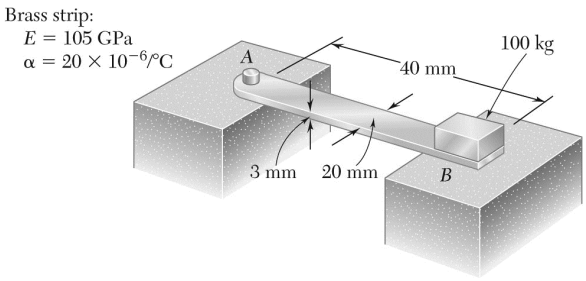
$$= 2l\,\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\,\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}}$$

◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

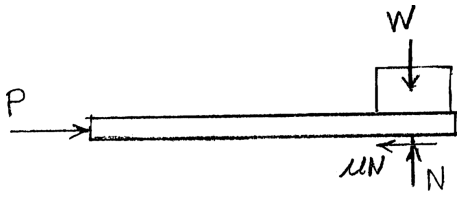


PROBLEM 2.128

The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.

SOLUTION

Brass strip:



$$E = 105 \text{ GPa}$$

$$\alpha = 20 \times 10^{-6}/^{\circ}\text{C}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu mg$$

$$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \quad \Delta T = \frac{P}{EA\alpha} = \frac{\mu mg}{EA\alpha}$$

Data: $\mu = 0.60$

$$A = (20)(3) = 60 \text{ mm}^2 = 60 \times 10^{-6} \text{ m}^2$$

$$m = 100 \text{ kg}$$

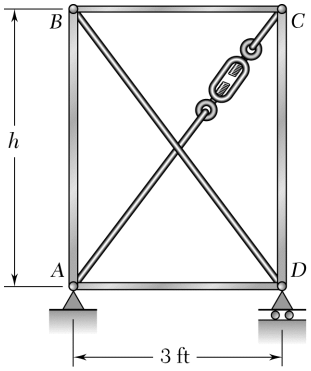
$$g = 9.81 \text{ m/s}^2$$

$$E = 105 \times 10^9 \text{ Pa}$$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105 \times 10^9)(60 \times 10^{-6})(20 \times 10^6)}$$

$$\Delta T = 4.67 \text{ °C} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.129

Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29\times10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

SOLUTION

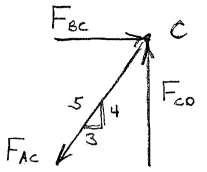
$$\delta_{AB} = \delta_{CD} = 0.04 \text{ in.}$$

$$h = 4\,\text{ft} = 48 \text{ in.} = L_{CD}$$

$$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(29\times10^6)(0.99402)(0.04)}{48} = 24.022\times10^3 \text{ lb}$$



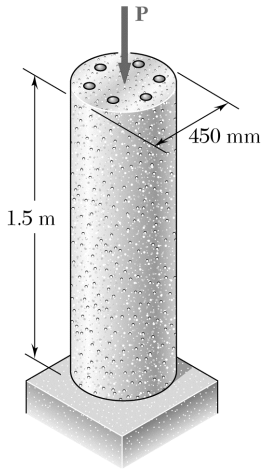
Use joint C as a free body.

$$+\uparrow \Sigma F_y = 0: \; F_{CD} - \frac{4}{5}F_{AC} = 0 \quad \therefore \quad F_{AC} = \frac{5}{4}F_{CD}$$

$$F_{AC} = \frac{5}{4}(24.022\times10^3) = 30.0\times10^3 \text{ lb}$$

$$F_{AC} = 30.0 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.130

The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550-kN axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times10^3 \text{ mm}^2$$

$$= 3.6945\times10^{-3} \text{ m}^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times10^3$$

$$= 155.349\times10^3 \text{ mm}^2$$

$$= 153.349\times10^{-3} \text{ m}^2$$

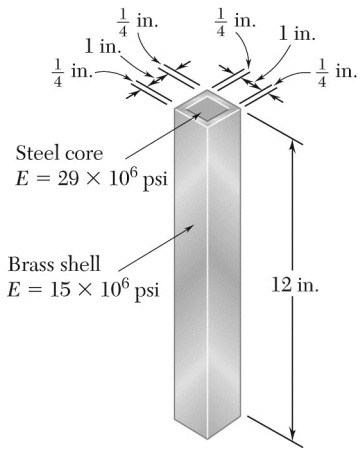
$$L = 1.5 \text{ m}$$

$$\varepsilon = \frac{1550\times10^3}{(25\times10^9)(155.349\times10^{-3}) + (200\times10^9)(3.6945\times10^{-3})} = 335.31\times10^{-6}$$

$$\sigma_s = E_s\varepsilon = (200\times10^9)(335.31\times10^{-6}) = 67.1\times10^6 \text{ Pa} \qquad \sigma_s = 67.1 \text{ MPa} \blacktriangleleft$$

$$\sigma_c = E_c\varepsilon = (25\times10^9)(-335.31\times10^{-6}) = 8.38\times10^6 \text{ Pa} \qquad \sigma_c = 8.38 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.131

The brass shell $(\alpha_b = 11.6 \times 10^{-6}/°\text{F})$ is fully bonded to the steel core $(\alpha_s = 6.5 \times 10^{-6}/°\text{F})$. Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

SOLUTION

Let P_s = axial force developed in the steel core.

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains:
$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T)$$
$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching:
$$\varepsilon_s = \varepsilon_b$$
$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$
$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T) \qquad (1)$$

$$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$$
$$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$$

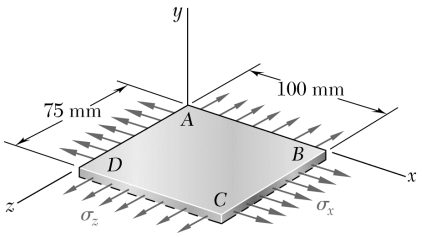
$$\alpha_b - \alpha_s = 5.1 \times 10^{-6}/°\text{F}$$
$$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb}$$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29 \times 10^6)(1.0)} + \frac{1}{(15 \times 10^6)(1.25)} = 87.816 \times 10^{-9} \text{ lb}^{-1}$$

From (1),
$$(87.816 \times 10^{-9})(8 \times 10^3) = (5.1 \times 10^{-6})(\Delta T)$$

$$\Delta T = 137.8°\text{F} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.132

A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $v = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\sigma_x = 120\times10^6 \text{ Pa},$$

$$\sigma_y = 0,$$

$$\sigma_z = 160\times10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{1}{87\times10^9}[120\times10^6 - (0.34)(160\times10^6)] = 754.02\times10^{-6}$$

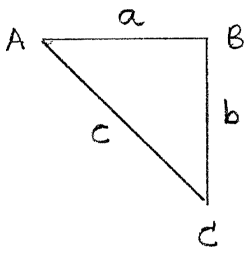
$$\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) = \frac{1}{87\times10^9}[-(0.34)(120\times10^6) + 160\times10^6] = 1.3701\times10^{-3}$$

(*a*) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02\times10^{-6})$ $= 0.0754$ mm ◀

(*b*) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75 \text{ mm})(1.3701\times10^{-6})$ $= 0.1028$ mm ◀

Label sides of right triangle *ABC* as *a*, *b*, and *c*.

$$c^2 = a^2 + b^2$$



Obtain differentials by calculus.

$$2c\ dc = 2a\ da + 2b\ db$$

$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 100$ mm,

$b = 75$ mm,

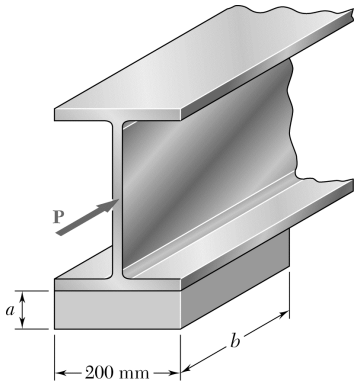
$c = \sqrt{(100^2 + 75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm

$db = \delta_{BC} = 0.1370$ mm

(*c*) $\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028)$ $= 0.1220$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.133

An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22-kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.

SOLUTION

Shearing force: $P = 22\times10^{3}\ \text{N}$

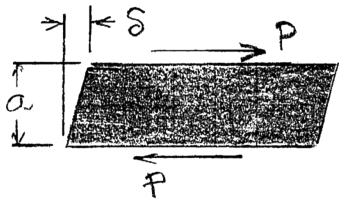
Shearing stress: $\tau = 420\times10^{3}\ \text{Pa}$

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau}$$

$$= \frac{22\times10^{3}}{420\times10^{3}} = 52.381\times10^{-3}\ \text{m}^2$$

$$= 52.381\times10^{3}\ \text{mm}^2$$

$$A = (200\ \text{mm})(b)$$

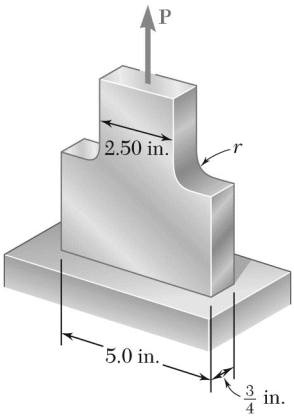


(*a*) $$b = \frac{A}{200} = \frac{52.381\times10^{3}}{200} = 262\ \text{mm}$$ $b = 262$ mm ◀

$$\gamma = \frac{\tau}{G} = \frac{420\times10^{3}}{0.9\times10^{6}} = 466.67\times10^{-3}$$

(*b*) But $\gamma = \frac{\delta}{a}$ $\therefore$ $a = \frac{\delta}{\gamma} = \frac{10\ \text{mm}}{466.67\times10^{-3}} = 21.4\ \text{mm}$ $a = 21.4$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 2.134

Knowing that $P = 10$ kips, determine the maximum stress when (*a*) $r = 0.50$ in., (*b*) $r = 0.625$ in.

SOLUTION

$$P = 10 \times 10^3 \text{ lb} \quad D = 5.0 \text{ in.} \quad d = 2.50 \text{ in.}$$

$$\frac{D}{d} = \frac{5.0}{2.50} = 2.00$$

$$A_{\min} = dt = (2.50)\left(\frac{3}{4}\right) = 1.875 \text{ in}^2$$

(*a*) $r = 0.50$ in. $\quad \frac{r}{d} = \frac{0.50}{2.50} = 0.20$

From Fig. 2.60*b*, $\quad K = 1.94$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.94)(10 \times 10^3)}{1.875} = 10.35 \times 10^3 \text{ psi}$$

10.35 ksi ◀

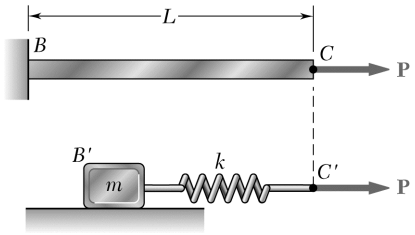
(*b*) $r = 0.625$ in. $\quad \frac{r}{d} = \frac{0.625}{2.50} = 0.25 \quad K = 1.82$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{(1.82)(10 \times 10^3)}{1.875} = 9.71 \times 10^3 \text{ psi}$$

9.71 ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

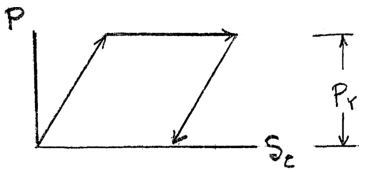
PROBLEM 2.135



The uniform rod BC has a cross-sectional area A and is made of a mild steel that can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_y. Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (*a*) the required mass m of the block, (*b*) the required constant k of the spring.

SOLUTION

Force-deflection diagram for Point C or rod BC.

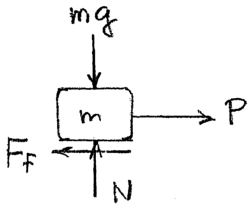


For $P < P_Y = A\sigma_Y$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{\max} = P_Y = A\sigma_Y$$

Force-deflection diagram for Point C' of block-and-spring system.



$$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$$

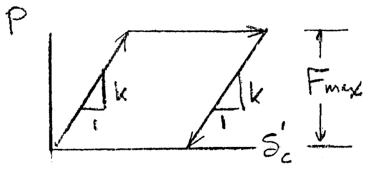
$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$$

If block does not move, i.e., $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $$\delta_c' = \frac{P}{K} \quad \text{or} \quad P = k\delta_c'$$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is the removed, the spring returns to its initial length.

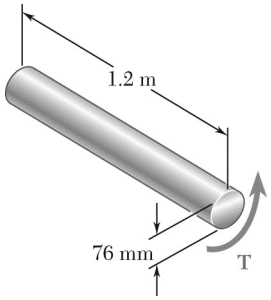


(*a*) Equating P_Y and $F_{\max}$, $A\sigma_Y = \mu mg \qquad m = \dfrac{A\sigma_Y}{\mu g}$ ◀

(*b*) Equating slopes, $k = \dfrac{EA}{L}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

CHAPTER 3



PROBLEM 3.1

(*a*) Determine the maximum shearing stress caused by a 4.6-kN · m torque **T** in the 76-mm-diameter shaft shown. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 24-mm inner diameter.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{d}{2} = 38 \text{ mm} = 0.038 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.038)^4 = 3.2753 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(4.6 \times 10^3)(0.038)}{3.2753 \times 10^{-6}} = 53.4 \times 10^6 \text{ Pa}$$

$\tau = 53.4$ MPa ◀

(*b*) Hollow shaft:

$$c_2 = \frac{d_o}{2} = 0.038 \text{ m}$$

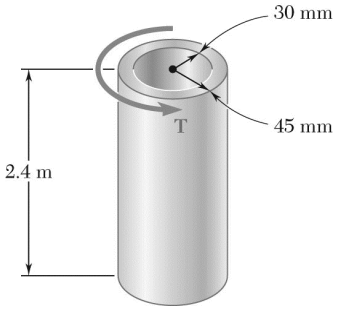
$$c_1 = \frac{1}{2}d_i = 12 \text{ mm} = 0.012 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.038^4 - 0.012^4) = 3.2428 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(4.6 \times 10^3)(0.038)}{3.2428 \times 10^{-6}} = 53.9 \times 10^6 \text{ Pa}$$

$\tau = 53.9$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.2

(*a*) Determine the torque **T** that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (*b*) Determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross-sectional area.

SOLUTION

(*a*) Given shaft:

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

$$J = \frac{\pi}{2}(45^4 - 30^4) = 5.1689 \times 10^6 \text{ mm}^4 = 5.1689 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c}$$

$$T = \frac{(5.1689 \times 10^{-6})(45 \times 10^6)}{45 \times 10^{-3}} = 5.1689 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 5.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Solid shaft of same area:

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(45^2 - 30^2) = 3.5343 \times 10^3 \text{ mm}^2$$

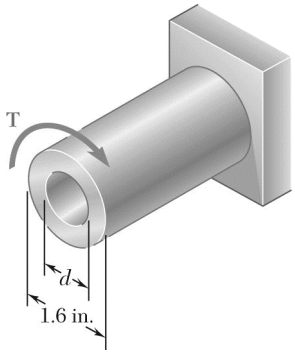
$$\pi c^2 = A \quad \text{or} \quad c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$J = \frac{\pi}{2}c^4, \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{(2)(5.1689 \times 10^3)}{\pi(0.033541)^3} = 87.2 \times 10^6 \text{ Pa}$$

$$\tau = 87.2 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.3

Knowing that $d = 1.2$ in., determine the torque **T** that causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \qquad c = 0.8 \text{ in.}$$

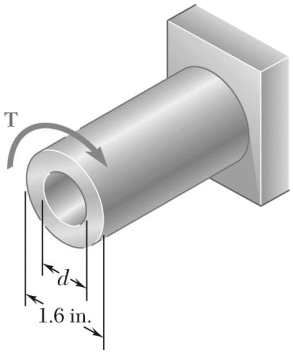
$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(1.2) = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{\max}}{c} = \frac{(0.4398)(7.5)}{0.8} \qquad T = 4.12 \text{ kip} \cdot \text{in} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.4

Knowing that the internal diameter of the hollow shaft shown is $d = 0.9\,\text{in.}$, determine the maximum shearing stress caused by a torque of magnitude $T = 9\,\text{kip} \cdot \text{in.}$

SOLUTION

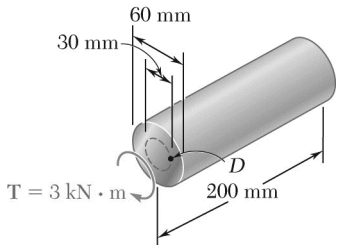
$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \qquad c = 0.8 \text{ in.}$$

$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(0.9) = 0.45 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{(9)(0.8)}{0.5790} \qquad \tau_{\max} = 12.44 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.5

A torque $T = 3\ \text{kN} \cdot \text{m}$ is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point *D* which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15 mm radius.

SOLUTION

(*a*) $c = \frac{1}{2}d = 30\ \text{mm} = 30 \times 10^{-3}\ \text{m}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30 \times 10^{-3})^4 = 1.27235 \times 10^{-6}\ \text{m}^4$$

$$T = 3\ \text{kN} = 3 \times 10^3\ \text{N}$$

$$\tau_m = \frac{Tc}{J} = \frac{(3 \times 10^3)(30 \times 10^{-3})}{1.27235 \times 10^{-6}} = 70.736 \times 10^6\ \text{Pa}$$

$\tau_m = 70.7\ \text{MPa}$ ◀

(*b*) $\rho_D = 15\ \text{mm} = 15 \times 10^{-3}\ \text{m}$

$$\tau_D = \frac{\rho_D}{c}\tau = \frac{(15 \times 10^{-3})(70.736 \times 10^{-6})}{(30 \times 10^{-3})}$$

$\tau_D = 35.4\ \text{MPa}$ ◀

(*c*) $\tau_D = \frac{T_D \rho_D}{J_D} \qquad T_D = \frac{J_D \tau_D}{\rho_D} = \frac{\pi}{2}\rho_D^3 \tau_D$

$$T_D = \frac{\pi}{2}(15 \times 10^{-3})^3 (35.368 \times 10^6) = 187.5\ \text{N} \cdot \text{m}$$

$$\frac{T_D}{T} \times 100\% = \frac{187.5}{3 \times 10^3}(100\%) = 6.25\%$$

6.25% ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.6

(*a*) Determine the torque that can be applied to a solid shaft of 20-mm diameter without exceeding an allowable shearing stress of 80 MPa. (*b*) Solve Part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half of its own outer diameter.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{1}{2}d = \frac{1}{2}(0.020) = 0.010 \text{ m}$$

$$J = \frac{\pi}{2}c^4 - \frac{\pi}{2}(0.10)^4 = 15.7080 \times 10^{-9}\text{m}^4$$

$$T = \frac{J\tau_{\max}}{c} = \frac{(15.7080 \times 10^{-9})(80 \times 10^6)}{0.010} = 125.664 \qquad T = 125.7 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) Hollow shaft: Same area as solid shaft.

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi\left[c_2^2 - \left(\frac{1}{2}c_2\right)^2\right] = \frac{3}{4}\pi c_2^2 = \pi c^2$$

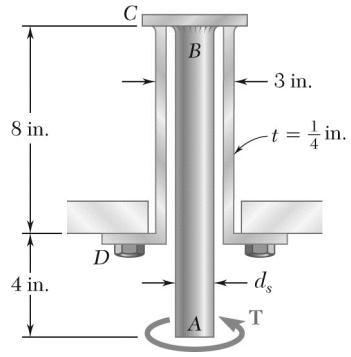
$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.010) = 0.0115470 \text{ m}$$

$$c_1 = \frac{1}{2}c_2 = 0.0057735 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0115470^4 - 0.0057735^4) = 26.180 \times 10^{-9}\text{m}^4$$

$$T = \frac{\tau_{\max} J}{c_2} = \frac{(80 \times 10^6)(26.180 \times 10^{-9})}{0.0115470} = 181.38 \qquad T = 181.4 \text{ N} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.7

The solid spindle *AB* has a diameter $d_s = 1.5$ in. and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve *CD* is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque **T** that can be applied at *A*.

SOLUTION

Analysis of solid spindle *AB*:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2} \tau c^3$$

$$T = \frac{\pi}{2} (12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

Analysis of sleeve *CD*:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2} (3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2} \left(c_2^4 - c_1^4 \right) = \frac{\pi}{2} (1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

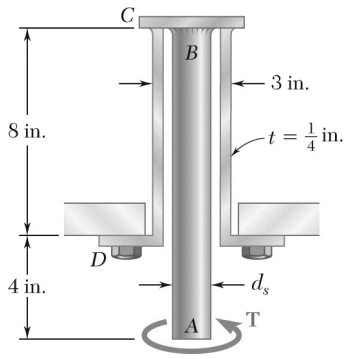
$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^3)}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in}$$

The smaller torque governs:

$$T = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T = 7.95 \text{ kip} \cdot \text{in} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.8

The solid spindle *AB* is made of a steel with an allowable shearing stress of 12 ksi, and sleeve *CD* is made of a brass with an allowable shearing stress of 7 ksi. Determine (*a*) the largest torque **T** that can be applied at *A* if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle *AB*.

SOLUTION

(*a*) Analysis of sleeve *CD*:

$$c_2 = \frac{1}{2}d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7\times10^3)}{1.5} = 19.21\times10^3 \text{ lb}\cdot\text{in}$$

$$T = 19.21 \text{ kip}\cdot\text{in} \blacktriangleleft$$

(*b*) Analysis of solid spindle *AB*:

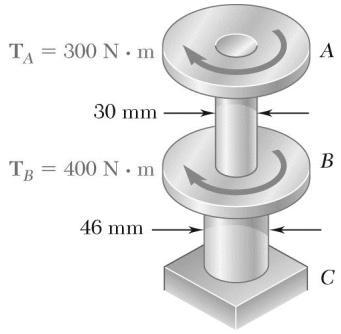
$$\tau = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau} = \frac{19.21\times10^3}{12\times10^3} = 1.601 \text{ in}^3$$

$$c = \sqrt[3]{\frac{(2)(1.601)}{\pi}} = 1.006 \text{ in.} \quad d_s = 2c$$

$$d = 2.01 \text{in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.9

The torques shown are exerted on pulleys *A* and *B*. Knowing that both shafts are solid, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 300 \text{ N} \cdot \text{m}, \quad d = 0.030 \text{ m}, \quad c = 0.015 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 56.6 \text{ MPa} \blacktriangleleft$$

(*b*) Shaft *BC*:

$$T_{BC} = 300 + 400 = 700 \text{ N} \cdot \text{m}$$

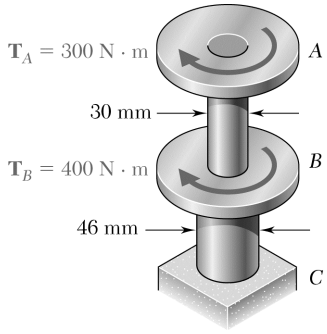
$$d = 0.046 \text{ m}, \quad c = 0.023 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 36.6 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.10

In order to reduce the total mass of the assembly of Prob. 3.9, a new design is being considered in which the diameter of shaft *BC* will be smaller. Determine the smallest diameter of shaft *BC* for which the maximum value of the shearing stress in the assembly will not increase.

SOLUTION

Shaft *AB*:

$$T_{AB} = 300 \text{ N} \cdot \text{m}, \quad d = 0.030 \text{ m}, \quad c = 0.015 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft *BC*:

$$T_{BC} = 300 + 400 = 700 \text{ N} \cdot \text{m}$$

$$d = 0.046 \text{ m}, \quad c = 0.023 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress $(56.588 \times 10^6 \text{ Pa})$ occurs in portion *AB*.

Reduce the diameter of *BC* to provide the same stress.

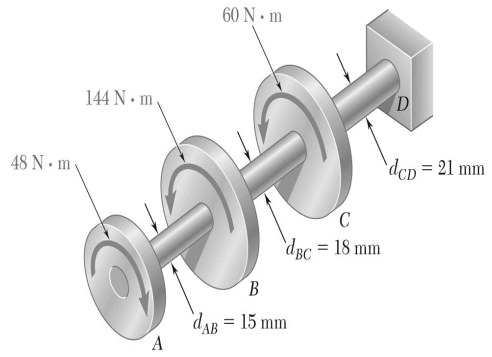
$$T_{BC} = 700 \text{N} \cdot \text{m} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau_{\max}} = \frac{(2)(700)}{\pi(56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{m}^3$$

$$c = 19.895 \times 10^{-3} \text{m} \qquad d = 2c = 39.79 \times 10^{-3} \text{m}$$

$$d = 39.8 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.11

Knowing that each portion of the shafts *AB*, *BC*, and *CD* consist of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Shaft *AB*:

$$T = 48 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 7.5 \text{ mm} = 0.0075 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(48)}{\pi(0.0075)^3} = 72.433 \text{ MPa}$$

Shaft *BC*:

$$T = -48 + 144 = 96 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 9 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(96)}{\pi(0.009)^3} = 83.835 \text{ MPa}$$

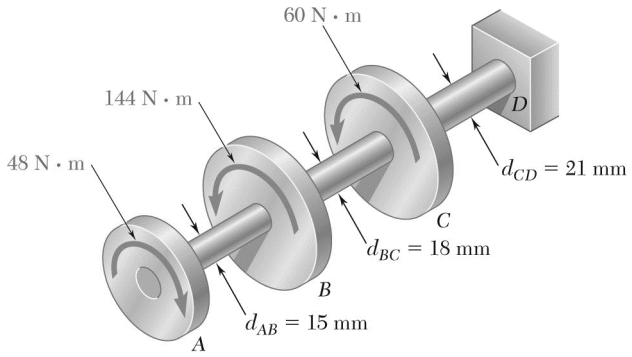
Shaft *CD*:

$$T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 10.5 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2 \times 156)}{\pi(0.0105)^3} = 85.79 \text{ MPa}$$

Answers: (*a*) shaft *CD* (*b*) 85.8 MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.12

Knowing that an 8-mm-diameter hole has been drilled through each of the shafts *AB, BC*, and *CD,* determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Hole: $c_1 = \frac{1}{2} d_1 = 4 \text{ mm}$

Shaft *AB*: $T = 48 \text{ N} \cdot \text{m}$

$$c_2 = \frac{1}{2} d_2 = 7.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0075^4 - 0.004^4) = 4.5679 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(48)(0.0075)}{4.5679 \times 10^{-9}} = 78.810 \text{ MPa}$$

Shaft *BC*: $T = -48 + 144 = 96 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2} d_2 = 9 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.009^4 - 0.004^4) = 9.904 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(96)(0.009)}{9.904 \times 10^{-9}} = 87.239 \text{ MPa}$$

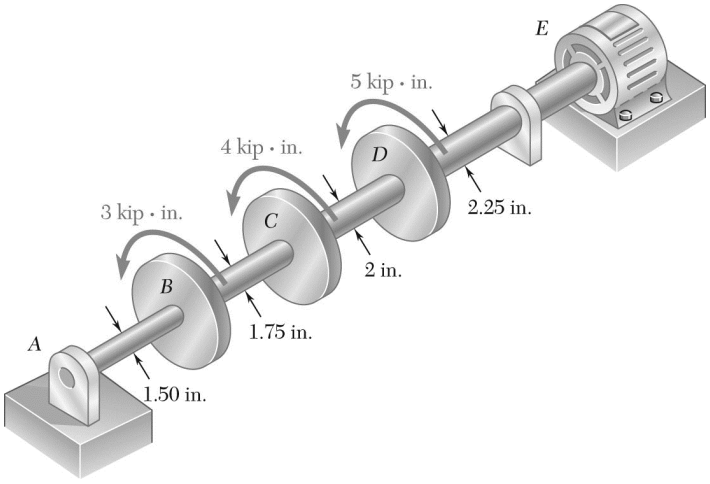
Shaft *CD*: $T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2} d_2 = 10.5 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0105^4 - 0.004^4) = 18.691 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(156)(0.0105)}{18.691 \times 10^{-9}} = 87.636 \text{ MPa}$$

Answers: (*a*) shaft *CD* (*b*) 87.6 MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



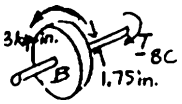
PROBLEM 3.13

Under normal operating conditions, the electric motor exerts a 12-kip · in. torque at *E*. Knowing that each shaft is solid, determine the maximum shearing in (*a*) shaft *BC*, (*b*) shaft *CD*, (*c*) shaft *DE*.

SOLUTION

(*a*) Shaft *BC*: From free body shown:

$$T_{BC} = 3 \text{ kip} \cdot \text{in}$$

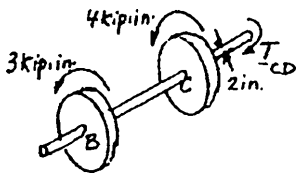


$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2}{\pi}\frac{T}{c^3} \tag{1}$$

$$\tau = \frac{2}{\pi}\frac{3 \text{ kip} \cdot \text{in}}{\left(\frac{1}{2} \times 1.75 \text{ in.}\right)^3} \qquad \tau = 2.85 \text{ ksi} \blacktriangleleft$$

(*b*) Shaft *CD*: From free body shown:

$$T_{CD} = 3 + 4 = 7 \text{ kip} \cdot \text{in}$$

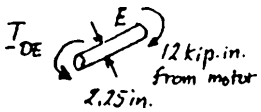


From Eq. (1):

$$\tau = \frac{2}{\pi}\frac{T}{c^3} = \frac{2}{\pi}\frac{7 \text{ kip} \cdot \text{in}}{(1 \text{ in.})^3} \qquad \tau = 4.46 \text{ ksi} \blacktriangleleft$$

(*c*) Shaft *DE*: From free body shown:

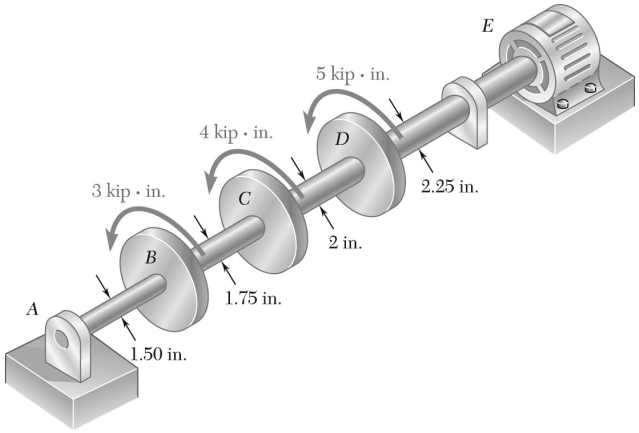
$$T_{DE} = 12 \text{ kip} \cdot \text{in}$$



From Eq. (1):

$$\tau = \frac{2}{\pi}\frac{T}{c^3} = \frac{2}{\pi}\frac{12 \text{ kip} \cdot \text{in}}{\left(\frac{1}{2} \times 2.25 \text{ in.}\right)^3} \qquad \tau = 5.37 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.14

Solve Prob.3.13, assuming that a 1-in.-diameter hole has been drilled into each shaft.

PROBLEM 3.13 Under normal operating conditions, the electric motor exerts a 12-kip · in. torque at *E*. Knowing that each shaft is solid, determine the maximum shearing in (*a*) shaft *BC*, (*b*) shaft *CD*, (*c*) shaft *DE*.

SOLUTION

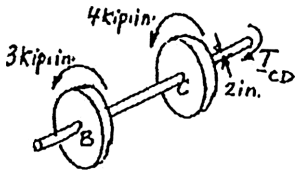
(*a*) Shaft *BC*: From free body shown: $T_{BC} = 3 \text{ kip} \cdot \text{in}$

$$c_2 = \frac{1}{2}(1.75) = 0.875 \text{ in.} \qquad c_1 = \frac{1}{2}(1) = 0.5 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.875^4 - 0.5^4) = 0.82260 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{(3 \text{ kip} \cdot \text{in})(0.875 \text{ in.})}{0.82260 \text{ in}^4} \qquad \tau = 3.19 \text{ ksi} \blacktriangleleft$$

(*b*) Shaft *CD*: From free body shown: $T_{CD} = 3 + 4 = 7 \text{ kip} \cdot \text{in}$

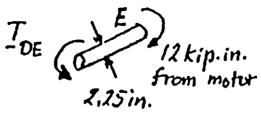


$$c_2 = \frac{1}{2}(2.0) = 1.0 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.0^4 - 0.5^4) = 1.47262 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{(7 \text{ kip} \cdot \text{in})(1.0 \text{ in.})}{1.47262 \text{ in}^4} \qquad \tau = 4.75 \text{ ksi} \blacktriangleleft$$

(*c*) Shaft *DE*: From free body shown: $T_{DE} = 12 \text{ kip} \cdot \text{in}$

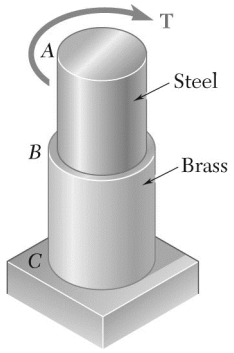


$$c_2 = \frac{2.25}{2} = 1.125 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.125^4 - 0.5^4) = 2.4179 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \text{ kip} \cdot \text{in})(1.125 \text{ in.})}{2.4179 \text{ in}^4} \qquad \tau = 5.58 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.15

The allowable shearing stress is 15 ksi in the 1.5-in.-diameter steel rod *AB* and 8 ksi in the 1.8-in.-diameter brass rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}c^3\tau_{\max}$$

Rod *AB*: $\tau_{\max} = 15 \text{ ksi} \qquad c = \frac{1}{2}d = 0.75 \text{ in.}$

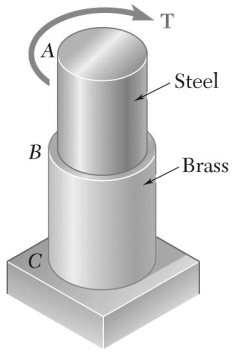
$$T = \frac{\pi}{2}(0.75)^3(15) = 9.94 \text{ kip} \cdot \text{in}$$

Rod *BC*: $\tau_{\max} = 8 \text{ ksi} \qquad c = \frac{1}{2}d = 0.90 \text{ in.}$

$$T = \frac{\pi}{2}(0.90)^3(8) = 9.16 \text{ kip} \cdot \text{in}$$

The allowable torque is the smaller value. $T = 9.16 \text{ kip} \cdot \text{in}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.16

The allowable shearing stress is 15 ksi in the steel rod *AB* and 8 ksi in the brass rod BC. Knowing that a torque of magnitude $T = 10 \text{ kip} \cdot \text{in.}$ is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}, \quad c^3 = \frac{2T}{\pi\tau_{\max}}$$

(*a*) Rod *AB*: $T = 10 \text{ kip} \cdot \text{in}$ $\tau_{\max} = 15 \text{ ksi}$

$$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244 \text{ in}^3$$

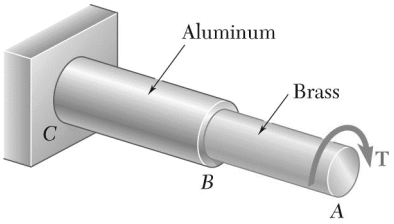
$$c = 0.7515 \text{ in.} \qquad d = 2c = 1.503 \text{ in.} \blacktriangleleft$$

(*b*) Rod *BC*: $T = 10 \text{ kip} \cdot \text{in}$ $\tau_{\max} = 8 \text{ ksi}$

$$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577 \text{ in}^2$$

$$c = 0.9267 \text{ in.} \qquad d = 2c = 1.853 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.17

The allowable stress is 50 MPa in the brass rod *AB* and 25 MPa in the aluminum rod *BC*. Knowing that a torque of magnitude $T = 1250$ N · m is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4 \quad c^3 = \frac{2T}{\pi\tau_{\max}}$$

(*a*) Rod *AB*:

$$c^3 = \frac{(2)(1250)}{\pi(50\times10^6)} = 15.915\times10^{-6}\text{m}^3$$

$$c = 25.15\times10^{-3}\text{m} = 25.15\text{ mm}$$

$$d_{AB} = 2c = 50.3\text{ mm} \blacktriangleleft$$

(*b*) Rod *BC*:

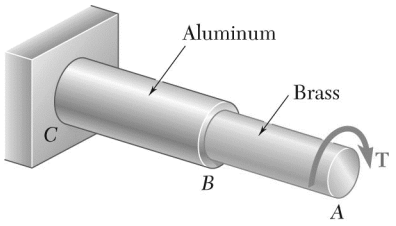
$$c^3 = \frac{(2)(1250)}{\pi(25\times10^6)} = 31.831\times10^{-6}\text{m}^3$$

$$c = 31.69\times10^{-3}\text{m} = 31.69\text{ mm}$$

$$d_{BC} = 2c = 63.4\text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.18



The *solid* rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (*b*) the largest torque that can be applied at *A*.

SOLUTION

Solid rod *BC*:

$$\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4$$

$$\tau_{\text{all}} = 25 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$T_{\text{all}} = \frac{\pi}{2}c^3\tau_{\text{all}} = \frac{\pi}{2}(0.015)^3(25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

Hollow rod *AB*:

$$\tau_{\text{all}} = 50 \times 10^6 \text{ Pa}$$

$$T_{\text{all}} = 132.536 \text{ N} \cdot \text{m}$$

$$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125 \text{ m}$$

$$c_1 = ?$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)\frac{\tau_{\text{all}}}{c_2}$$

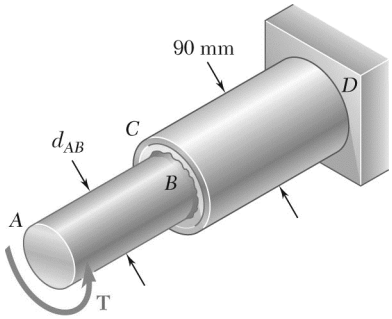
$$c_1^4 = c_2^4 - \frac{2T_{\text{all}}c_2}{\pi\tau_{\text{all}}}$$

$$= 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

(*a*) $c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm}$ $\quad d_1 = 2c_1 = 15.18 \text{ mm}$ ◀

(*b*) Allowable torque. $\quad T_{\text{all}} = 132.5 \text{ N} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.19

The solid rod *AB* has a diameter $d_{AB} = 60$ mm. The pipe *CD* has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of a steel for which the allowable shearing stress is 75 Mpa, determine the largest torque **T** that can be applied at *A*.

SOLUTION

$$\tau_{\text{all}} = 75 \times 10^6 \text{ Pa} \quad T_{\text{all}} = \frac{J\tau_{\text{all}}}{c}$$

Rod *AB*:

$$c = \frac{1}{2}d = 0.030 \text{ m} \quad J = \frac{\pi}{2}c^4$$

$$T_{\text{all}} = \frac{\pi}{2}c^3\tau_{\text{all}} = \frac{\pi}{2}(0.030)^3(75 \times 10^6)$$

$$= 3.181 \times 10^3 \text{ N} \cdot \text{m}$$

Pipe *CD*:

$$c_2 = \frac{1}{2}d_2 = 0.045 \text{ m} \quad c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

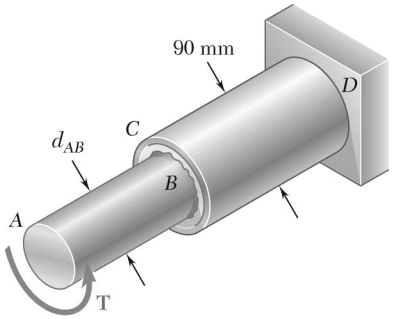
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{\text{all}} = \frac{(2.8073 \times 10^{-6})(75 \times 10^6)}{0.045} = 4.679 \times 10^3 \text{ N} \cdot \text{m}$$

Allowable torque is the smaller value. $T_{\text{all}} = 3.18 \times 10^3 \text{ N} \cdot \text{m}$

$3.18 \text{ kN} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.20

The solid rod *AB* has a diameter d_{AB} = 60 mm and is made of a steel for which the allowable shearing stress is 85 Mpa. The pipe *CD,* which has an outer diameter of 90 mm and a wall thickness of 6 mm, is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque **T** that can be applied at *A*.

SOLUTION

Rod *AB*:

$$\tau_{\text{all}} = 85 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2} d = 0.030 \text{ m}$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2} c^3 \tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.030)^3(85 \times 10^6) = 3.605 \times 10^3 \text{ N} \cdot \text{m}$$

Pipe *CD*:

$$\tau_{\text{all}} = 54 \times 10^6 \text{ Pa} \qquad c_2 = \frac{1}{2} d_2 = 0.045 \text{ m}$$

$$c_1 = c_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

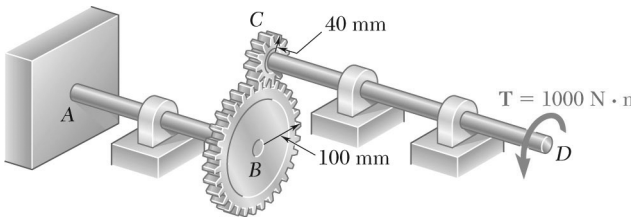
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{(2.8073 \times 10^{-6})(54 \times 10^6)}{0.045} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$$

Allowable torque is the smaller value. $T_{\text{all}} = 3.369 \times 10^3 \text{ N} \cdot \text{m}$

$3.37 \text{ kN} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.21

A torque of magnitude $T = 1000 \text{ N} \cdot \text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.

SOLUTION

$$T_{CD} = 1000 \text{ N} \cdot \text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40} (1000) = 2500 \text{ N} \cdot \text{m}$$

(*a*) Shaft *AB*:

$$c = \frac{1}{2} d = 0.028 \text{ m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi (0.028)^3} = 72.50 \times 10^6$$

72.5 MPa ◀

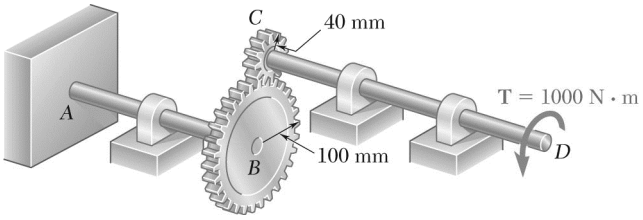
(*b*) Shaft *CD*:

$$c = \frac{1}{2} d = 0.020 \text{ m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi (0.020)^3} = 68.7 \times 10^6$$

68.7 MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.22

A torque of magnitude $T = 1000 \text{ N} \cdot \text{m}$ is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*.

SOLUTION

$$T_{CD} = 1000 \text{ N} \cdot \text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40} (1000) = 2500 \text{ N} \cdot \text{m}$$

(*a*) Shaft *AB*:

$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi (60 \times 10^6)} = 26.526 \times 10^{-6} \text{ m}^3$$

$$c = 29.82 \times 10^{-3} = 29.82 \text{ mm} \qquad d = 2c = 59.6 \text{ mm} \blacktriangleleft$$

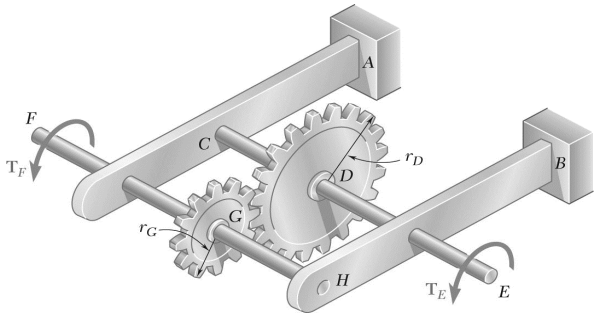
(*b*) Shaft *CD*:

$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(1000)}{\pi (60 \times 10^6)} = 10.610 \times 10^{-6} \text{ m}^3$$

$$c = 21.97 \times 10^{-3} \text{ m} = 21.97 \text{ mm} \qquad d = 2c = 43.9 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.23

Under normal operating conditions, a motor exerts a torque of magnitude $T_F = 1200 \text{ lb} \cdot \text{in}.$ at F. Knowing that $r_D = 8$ in., $r_G = 3$ in., and the allowable shearing stress is 10.5 ksi · in each shaft, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.

SOLUTION

$$T_F = 1200 \text{ lb} \cdot \text{in}$$

$$T_E = \frac{r_D}{r_G} T_F = \frac{8}{3}(1200) = 3200 \text{ lb} \cdot \text{in}$$

$$\tau_{\text{all}} = 10.5 \text{ ksi} = 10500 \text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \text{or} \quad c^3 = \frac{2T}{\pi\tau}$$

(*a*) Shaft *CDE*:

$$c^3 = \frac{(2)(3200)}{\pi(10500)} = 0.194012 \text{ in}^3$$

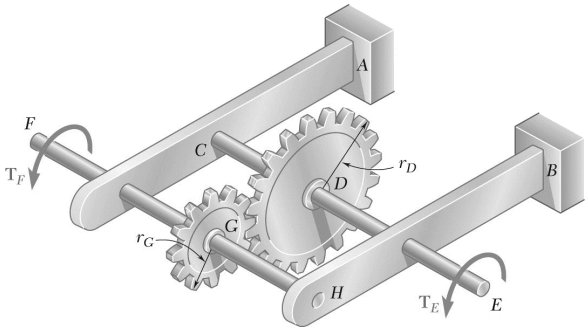
$$c = 0.5789 \text{ in.} \qquad d_{DE} = 2c \qquad d_{DE} = 1.158 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *FGH*:

$$c^3 = \frac{(2)(1200)}{\pi(10500)} = 0.012757 \text{ in}^3$$

$$c = 0.4174 \text{ in.} \qquad d_{FG} = 2c \qquad d_{FG} = 0.835 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.24

Under normal operating conditions, a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters $d_{CDE} = 0.900$ in. and $d_{FGH} = 0.800$ in. Knowing that $r_D = 6.5$ in. and $r_G = 4.5$ in., determine the largest allowable value of T_F.

SOLUTION

$$\tau_{\text{all}} = 12 \text{ ksi}$$

Shaft *FG*:

$$c = \frac{1}{2} d = 0.400 \text{ in.}$$

$$T_{F,\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2} c^3 \tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.400)^3(12) = 1.206 \text{ kip} \cdot \text{in}$$

Shaft *DE*:

$$c = \frac{1}{2} d = 0.450 \text{ in.}$$

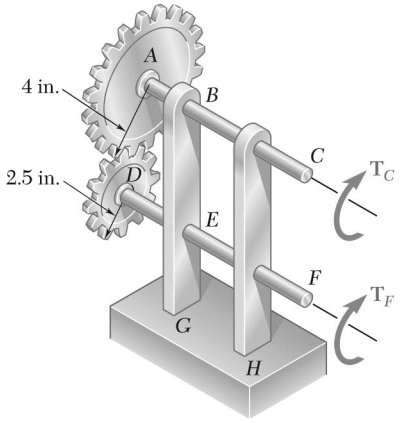
$$T_{E,\text{all}} = \frac{\pi}{2} c^3 \tau_{all}$$

$$= \frac{\pi}{2}(0.450)^3(12) = 1.7177 \text{ kip} \cdot \text{in}$$

$$T_F = \frac{r_G}{r_D} T_E \quad T_{F,\text{all}} = \frac{4.5}{6.5}(1.7177) = 1.189 \text{ kip} \cdot \text{in}$$

Allowable value of T_F is the smaller. $T_F = 1.189 \text{ kip} \cdot \text{in}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.25

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 8500 psi. Knowing that a torque of magnitude $T_C = 5$ kip · in. is applied at C and that the assembly is in equilibrium, determine the required diameter of (*a*) shaft *BC*, (*b*) shaft *EF*.

SOLUTION

$$\tau_{\max} = 8500 \text{ psi} = 8.5 \text{ ksi}$$

(*a*) Shaft *BC*:

$$T_C = 5 \text{ kip} \cdot \text{in}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}}$$

$$c = \sqrt[3]{\frac{(2)(5)}{\pi(8.5)}} = 0.7208 \text{ in.}$$

$$d_{BC} = 2c = 1.442 \text{ in.} \blacktriangleleft$$

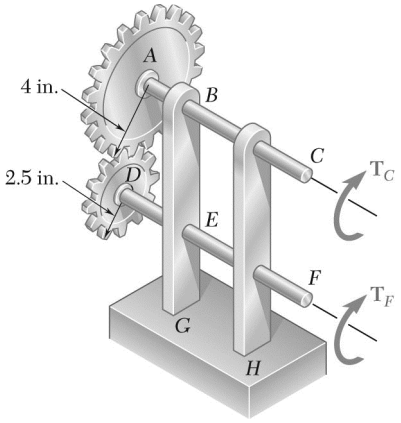
(*b*) Shaft *EF*:

$$T_F = \frac{r_D}{r_A} T_C = \frac{2.5}{4}(5) = 3.125 \text{ kip} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.125)}{\pi(8.5)}} = 0.6163 \text{ in.}$$

$$d_{EF} = 2c = 1.233 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.26

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 7000 psi. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 1.6$ in. and $d_{EF} = 1.25$ in., determine the largest torque $\mathbf{T}_C$ that can be applied at C.

SOLUTION

$$\tau_{\max} = 7000 \text{ psi} = 7.0 \text{ ksi}$$

Shaft *BC*:

$$d_{BC} = 1.6 \text{ in.}$$

$$c = \frac{1}{2} d = 0.8 \text{ in.}$$

$$T_C = \frac{J\tau_{\max}}{c} = \frac{\pi}{2}\tau_{\max} c^3$$

$$= \frac{\pi}{2}(7.0)(0.8)^3 = 5.63 \text{ kip} \cdot \text{in}$$

Shaft *EF*:

$$d_{EF} = 1.25 \text{ in.}$$

$$c = \frac{1}{2} d = 0.625 \text{ in.}$$

$$T_F = \frac{J\tau_{\max}}{c} = \frac{\pi}{2}\tau_{\max} c^3$$

$$= \frac{\pi}{2}(7.0)(0.625)^3 = 2.684 \text{ kip} \cdot \text{in}$$

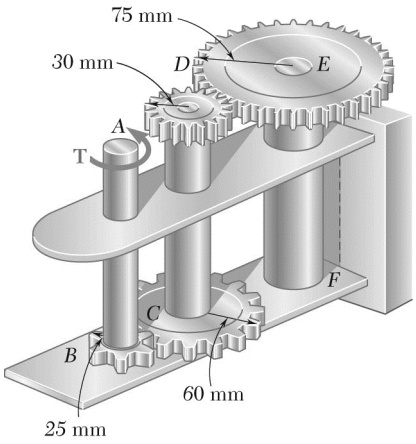
By statics,

$$T_C = \frac{r_A}{r_D} T_F = \frac{4}{2.5}(2.684) = 4.30 \text{ kip} \cdot \text{in}$$

Allowable value of T_C is the smaller.

$$T_C = 4.30 \text{ kip} \cdot \text{in} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.27

A torque of magnitude $T = 100\ \text{N} \cdot \text{m}$ is applied to shaft *AB* of the gear train shown. Knowing that the diameters of the three solid shafts are, respectively, $d_{AB} = 21\ \text{mm}$, $d_{CD} = 30\ \text{mm}$, and $d_{EF} = 40\ \text{mm}$, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *CD*, (*c*) shaft *EF*.

SOLUTION

Statics:

Shaft *AB*: $T_{AB} = T_A = T_B = T$

Gears *B* and *C*: $r_B = 25\ \text{mm}, \quad r_C = 60\ \text{mm}$

Force on gear circles. $F_{BC} = \dfrac{T_B}{r_B} = \dfrac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4T$$

Shaft *CD*: $T_{CD} = T_C = T_D = 2.4T$

Gears *D* and *E*: $r_D = 30\ \text{mm}, \quad r_E = 75\ \text{mm}$

Force on gear circles. $F_{DE} = \dfrac{T_D}{r_D} = \dfrac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4T) = 6T$$

Shaft *EF*: $T_{EF} = T_E = T_F = 6T$

Maximum Shearing Stresses. $\tau_{\max} = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.27 (*Continued*)

(*a*) Shaft *AB*:

$$T = 100 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 10.5 \text{ mm} = 10.5 \times 10^{-3} \text{ m}$$

$$\tau_{\max} = \frac{(2)(100)}{\pi(10.5 \times 10^{-3})^3} = 55.0 \times 10^{6} \text{ Pa} \qquad \tau_{\max} = 55.0 \text{ MPa} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$T = (2.4)(100) = 240 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$\tau_{\max} = \frac{(2)(240)}{\pi(15 \times 10^{-3})^3} = 45.3 \times 10^{6} \text{ Pa} \qquad \tau_{\max} = 45.3 \text{ MPa} \blacktriangleleft$$

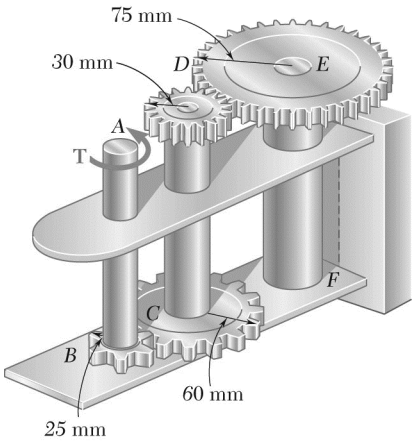
(*c*) Shaft *EF*:

$$T = (6)(100) = 600 \text{ N} \cdot \text{m}$$

$$c = \frac{1}{2}d = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$\tau_{\max} = \frac{(2)(600)}{\pi(20 \times 10^{-3})^3} = 47.7 \times 10^{6} \text{ Pa} \qquad \tau_{\max} = 47.7 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.28

A torque of magnitude $T = 120\,\text{N}\cdot\text{m}$ is applied to shaft *AB* of the gear train shown. Knowing that the allowable shearing stress is 75 MPa in each of the three solid shafts, determine the required diameter of (*a*) shaft *AB*, (*b*) shaft *CD*, (*c*) shaft *EF*.

SOLUTION

Statics:

Shaft *AB*: $T_{AB} = T_A = T_B = T$

Gears *B* and *C*: $r_B = 25$ mm, $r_C = 60$ mm

Force on gear circles. $F_{BC} = \dfrac{T_B}{r_B} = \dfrac{T_C}{r_C}$

$$T_C = \frac{r_C}{r_B} T_B = \frac{60}{25} T = 2.4T$$

Shaft *CD*: $T_{CD} = T_C = T_D = 2.4T$

Gears *D* and *E*: $r_D = 30$ mm, $r_E = 75$ mm

Force on gear circles. $F_{DE} = \dfrac{T_D}{r_D} = \dfrac{T_E}{r_E}$

$$T_E = \frac{r_E}{r_D} T_D = \frac{75}{30}(2.4T) = 6T$$

Shaft *EF*: $T_{EF} = T_E = T_F = 6T$

Required Diameters.

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$d = 2c = 2\sqrt[3]{\frac{2T}{\pi\tau_{\max}}}$$

$$\tau_{\max} = 75 \times 10^6\ \text{Pa}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.28 (*Continued*)

(*a*) Shaft *AB*:

$$T_{AB} = T = 120 \text{ N} \cdot \text{m}$$

$$d_{AB} = 2\sqrt[3]{\frac{2(120)}{\pi(75\times10^{6})}} = 20.1\times10^{-3}\text{ m} \qquad d_{AB} = 20.1 \text{ mm} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$T_{CD} = (2.4)(120) = 288 \text{ N} \cdot \text{m}$$

$$d_{CD} = 2\sqrt[3]{\frac{(2)(288)}{\pi(75\times10^{6})}} = 26.9\times10^{-3}\text{ m} \qquad d_{CD} = 26.9 \text{ mm} \blacktriangleleft$$

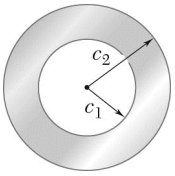
(*c*) Shaft *EF*:

$$T_{EF} = (6)(120) = 720 \text{ N} \cdot \text{m}$$

$$d_{EF} = 2\sqrt[3]{\frac{(2)(720)}{\pi(75\times10^{3})}} = 36.6\times10^{-3}\text{ m} \qquad d_{EF} = 36.6 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.29



(*a*) For a given allowable shearing stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (*b*) Denoting by $(T/w)_0$ the value of this ratio for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.

SOLUTION

w = weight per unit length,

ρg = specific weight,

W = total weight,

L = length

$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi \left(c_2^2 - c_1^2\right)$$

$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2}\frac{c_2^4 - c_1^4}{c_2}\tau_{\text{all}} = \frac{\pi}{2}\frac{\left(c_2^2 + c_1^2\right)\left(c_2^2 - c_1^2\right)}{c_2}\tau_{\text{all}}$$

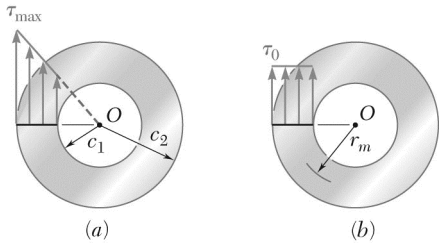
(*a*) $$\frac{T}{W} = \left(c_1^2 + c_2^2\right)\tau_{\text{all}} \qquad \frac{T}{w} = \frac{\left(c_1^2 + c_2^2\right)\tau_{\text{all}}}{2\rho g c_2} \text{ (hollow shaft)} \blacktriangleleft$$

$c_1 = 0$ for solid shaft: $$\left(\frac{T}{w}\right)_0 = \frac{c_2\tau_{\text{all}}}{2pg} \text{ (solid shaft)}$$

(*b*) $$\frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2} \qquad \left(\frac{T}{w}\right) = \left(\frac{T}{w}\right)_0\left(1 + \frac{c_1^2}{c_2^2}\right) \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.30



While the exact distribution of the shearing stresses in a hollow-cylindrical shaft is as shown in Fig. *a*, an approximate value can be obtained for τ_{max} by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in Fig. *b*, and then further assuming that all of the elementary shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2, respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.

SOLUTION

For a hollow shaft:

$$\tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)} = \frac{2Tc_2}{\pi\left(c_2^2 - c_1^2\right)\left(c_2^2 + c_1^2\right)} = \frac{2Tc_2}{A\left(c_2^2 + c_1^2\right)}$$

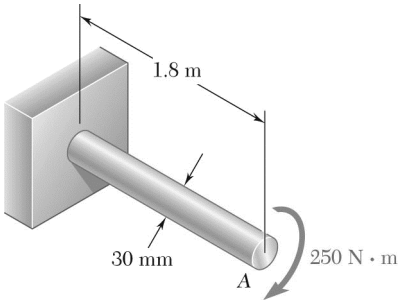
By definition, $$\tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

Dividing, $$\frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$$ ◀

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.31

(*a*) For the solid steel shaft shown ($G = 77$ GPa), determine the angle of twist at *A*. (*b*) Solve part *a*, assuming that the steel shaft is hollow with a 30-mm-outer diameter and a 20-mm-inner diameter.

SOLUTION

(*a*) $c = \frac{1}{2}d = 0.015 \text{ m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4$

$J = 79.522 \times 10^{-9} \text{m}^4, \quad L = 1.8 \text{ m}, \quad G = 77 \times 10^9 \text{Pa}$

$T = 250 \text{ N} \cdot \text{m} \qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$$

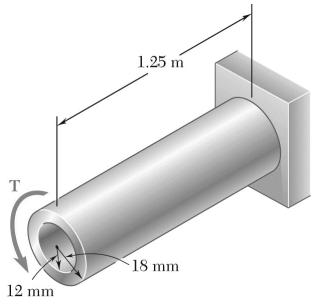
$$\varphi = \frac{(73.49 \times 10^{-3})180}{\pi} \qquad \varphi = 4.21° \blacktriangleleft$$

(*b*) $c_2 = 0.015 \text{ m}, \quad c_1 \frac{1}{2}d_1 = 0.010 \text{ m}, \quad J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$

$J = \frac{\pi}{2}(0.015^4 - 0.010^4) = 63.814 \times 10^{-9} \text{ m}^4 \quad \varphi \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(63.814 \times 10^{-9})} = 91.58 \times 10^{-3} \text{ rad} = \frac{180}{\pi}(91.58 \times 10^{-3}) \qquad \varphi = 5.25° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.32

For the aluminum shaft shown (G = 27 GPa), determine (*a*) the torque **T** that causes an angle of twist of 4°, (*b*) the angle of twist caused by the same torque **T** in a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(*a*)

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$\varphi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.25 \text{ m}$$

$$G = 27 \text{ GPa} = 27 \times 10^9 \text{ Pa}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.018^4 - 0.012^4) = 132.324 \times 10^{-9} \text{ m}^4$$

$$T = \frac{(27 \times 10^9)(132.324 \times 10^9)(69.813 \times 10^{-3})}{1.25}$$

$$= 199.539 \text{ N} \cdot \text{m} \qquad T = 199.5 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) Matching areas:

$$A = \pi c^2 = \pi\left(c_2^2 - c_1^2\right)$$

$$c = \sqrt{c_2^2 - c_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.013416)^4 = 50.894 \times 10^{-9} \text{ m}^4$$

$$\varphi = \frac{TL}{GJ} = \frac{(195.539)(1.25)}{(27 \times 10^9)(50.894 \times 10^9)} = 181.514 \times 10^{-3} \text{ rad} \qquad \varphi = 10.40^\circ \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.33

Determine the largest allowable diameter of a 10-ft-long steel rod ($G = 11.2 \times 10^6$ psi) if the rod is to be twisted through 30° without exceeding a shearing stress of 12 ksi.

SOLUTION

$$L = 10 \text{ ft} = 120 \text{ in.} \qquad \varphi = 30° = \frac{30\pi}{180} = 0.52360 \text{ rad}$$

$$\tau = 12 \text{ ksi} = 12 \times 10^3 \text{psi}$$

$$\varphi = \frac{TL}{GJ}, \quad T = \frac{GJ\varphi}{L}, \quad \tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}, \quad c = \frac{\tau L}{G\varphi}$$

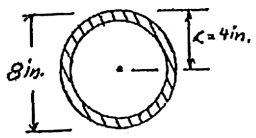
$$c = \frac{(12 \times 10^3)(120)}{(11.2 \times 10^6)(0.52360)} = 0.24555 \text{ in.} \qquad d = 2c = 0.491 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.34

While an oil well is being drilled at a depth of 6000 ft, it is observed that the top of the 8-in.-diameter steel drill pipe rotates though two complete revolutions before the drilling bit starts to rotate. Using $G = 11.2 \times 10^6$ psi, determine the maximum shearing stress in the pipe caused by torsion.

SOLUTION



For outside diameter of 8 in., $c = 4$ in.

For two revolutions, $\varphi = 2(2\pi) = 4\pi$ radians.

$G = 11.2 \times 10^6$ psi

$L = 6000 \text{ ft} = 72000 \text{ in.}$

From text book,

$$\varphi = \frac{TL}{GJ} \tag{1}$$

$$\tau_m = \frac{Tc}{J} \tag{2}$$

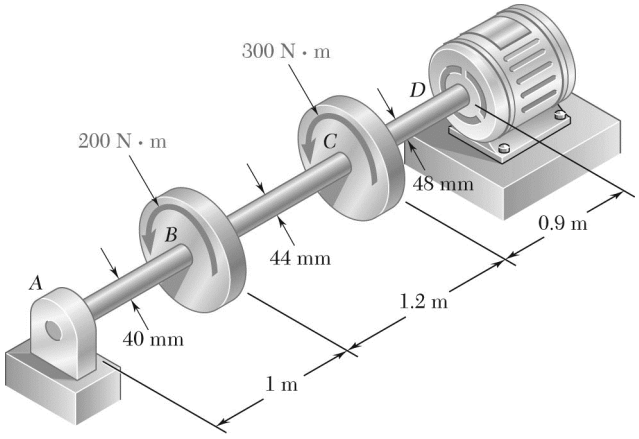
Divide (2) by (1).

$$\frac{\tau_m}{\varphi} = \frac{Gc}{L}$$

$$\tau_m = \frac{Gc\varphi}{L} = \frac{(11 \times 10^6)(4)(4\pi)}{72000} = 7679 \text{ psi}$$

$$\tau_m = 7.68 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.35

The electric motor exerts a 500 N · m torque on the aluminum shaft *ABCD* when it is rotating at a constant speed. Knowing that $G = 27$ GPa and that the torques exerted on pulleys *B* and *C* are as shown, determine the angle of twist between (*a*) *B* and *C*, (*b*) *B* and *D*.

SOLUTION

(*a*) Angle of twist between *B* and *C*.

$$T_{BC} = 200 \text{ N} \cdot \text{m}, \quad L_{BC} = 1.2 \text{ m}$$

$$c = \frac{1}{2}d = 0.022 \text{ m}, \quad G = 27 \times 10^9 \text{ Pa}$$

$$J_{BC} = \frac{\pi}{2}c^4 = 367.97 \times 10^{-9} \text{ m}$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27 \times 10^9)(367.97 \times 10^9)} = 24.157 \times 10^{-3} \text{ rad} \qquad \varphi_{B/C} = 1.384° \blacktriangleleft$$

(*b*) Angle of twist between *B* and *D*.

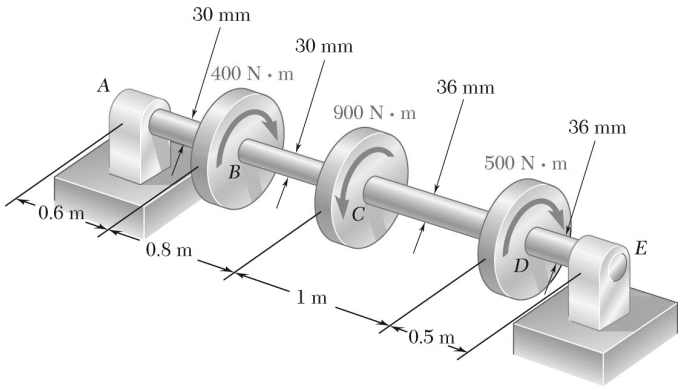
$$T_{CD} = 500 \text{ N} \cdot \text{m}, \quad L_{CD} = 0.9 \text{ m}, \quad c = \frac{1}{2}d = 0.024 \text{ m}, \quad G = 27 \times 10^9 \text{Pa}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.024)^4 = 521.153 \times 10^{-9} \text{ m}^4$$

$$\varphi_{C/D} = \frac{(500)(0.9)}{(27 \times 10^9)(521.153 \times 10^9)} = 31.980 \times 10^{-3} \text{ rad}$$

$$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157 \times 10^{-3} + 31.980 \times 10^{-3} = 56.137 \times 10^{-3} \text{ rad} \qquad \varphi_{B/D} = 3.22° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.36

The torques shown are exerted on pulleys *B*, *C*, and *D*. Knowing that the entire shaft is made of steel ($G = 27$ GPa), determine the angle of twist between (*a*) *C* and *B*, (*b*) *D* and *B*.

SOLUTION

(*a*) Shaft *BC*:

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{BC} = \frac{\pi}{4}c^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$L_{BC} = 0.8 \text{ m}, \quad G = 27 \times 10^9 \text{ Pa}$$

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27 \times 10^9)(79.522 \times 10^{-9})} = 0.149904 \text{ rad} \qquad \varphi_{BC} = 8.54° \blacktriangleleft$$

(*b*) Shaft *CD*:

$$c = \frac{1}{2}d = 0.018 \text{ m} \qquad J_{CD} = \frac{\pi}{4}c^4 = 164.896 \times 10^{-9} \text{ m}^4$$

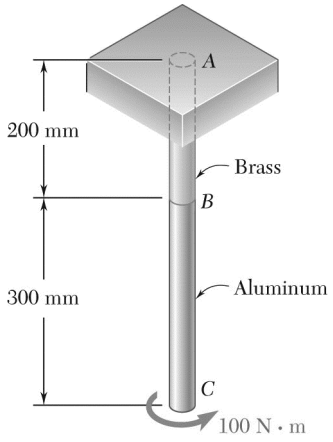
$$L_{CD} = 1.0 \text{ m} \quad T_{CD} = 400 - 900 = -500 \text{ N} \cdot \text{m}$$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27 \times 10^9)(164.896 \times 10^{-9})} = -0.11230 \text{ rad}$$

$$\varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674 \text{ rad}$$

$$\varphi_{BD} = 2.11° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.37

The aluminum rod BC ($G = 26$ GPa) is bonded to the brass rod AB ($G = 39$ GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist (*a*) at B, (*b*) at C.

SOLUTION

Both portions:

$$c = \frac{1}{2}d = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(6 \times 10^{-3})^4 = 2.03575 \times 10^{-9} \text{ m}^4$$

$$T = 100 \text{ N} \cdot \text{m}$$

Rod AB: $G_{AB} = 39 \times 10^9$ Pa, $L_{AB} = 0.200$ m

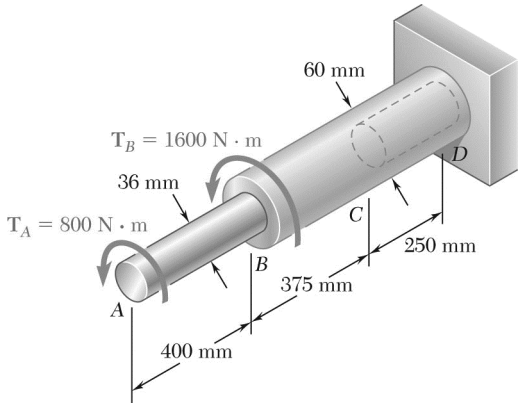
(*a*) $$\varphi_B = \varphi_{AB} = \frac{TL_{AB}}{G_{AB}J} = \frac{(100)(0.200)}{(39 \times 10^9)(2.03575 \times 10^{-9})} = 0.25191 \text{ rad}$$ $\varphi_B = 14.43°$ ◀

Rod BC: $G_{BC} = 26 \times 10^9$ Pa, $L_{BC} = 0.300$ m

$$\varphi_{BC} = \frac{TL_{BC}}{G_{BC}J} = \frac{(100)(0.300)}{(26 \times 10^9)(2.03575 \times 10^{-9})} = 0.56679 \text{ rad}$$

(*b*) $\varphi_C = \varphi_B + \varphi_{BC} = 0.25191 + 0.56679 = 0.81870$ rad $\varphi_C = 46.9°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.38

The aluminum rod AB ($G = 27$ GPa) is bonded to the brass rod BD ($G = 39$ GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at A.

SOLUTION

Rod AB:

$$G = 27 \times 10^9 \text{ Pa}, \quad L = 0.400 \text{ m}$$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part BC:

$$G = 39 \times 10^9 \text{ Pa} \quad L = 0.375 \text{ m}, \quad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part CD:

$$c_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, \quad L = 0.250 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

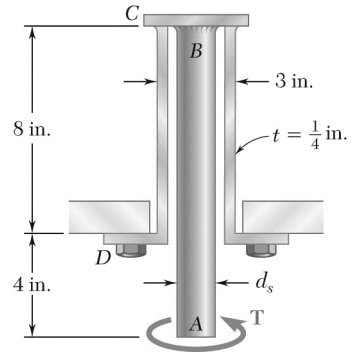
$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

Angle of twist at A.

$$\varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$$
$$= 105.080 \times 10^{-3} \text{ rad}$$

$$\varphi_A = 6.02° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.39

The solid spindle *AB* has a diameter d_s=1.5 in. and is made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{all} = 12$ ksi, while sleeve *CD* is made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{all} = 7$ ksi. Determine the angle through end *A* can be rotated.

SOLUTION

Stress analysis of solid spindle *AB*:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2} \tau c^3$$

$$T = \frac{\pi}{2}(12 \times 10^3)(0.75)^3 = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

Stress analysis of sleeve *CD*:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7 \times 10^{-3})}{1.5} = 19.21 \times 10^3 \text{ lb} \cdot \text{in}$$

The smaller torque governs.

$$T = 7.95 \times 10^3 \text{ lb} \cdot \text{in}$$

Deformation of spindle *AB*:

$$c = 0.75 \text{ in.}$$

$$J = \frac{\pi}{2} c^4 = 0.49701 \text{ in}^4, \quad L = 12 \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(7.95 \times 10^3)(12)}{(11.2 \times 10^6)(0.49701)} = 0.017138 \text{ radians}$$

Deformation of sleeve *CD*:

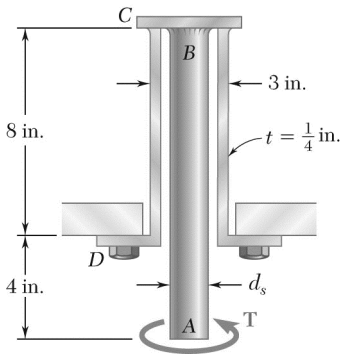
$$J = 4.1172 \text{ in}^4, \quad L = 8 \text{ in.}, \quad G = 5.6 \times 10^6 \text{ psi}$$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(7.95 \times 10^3)(8)}{(5.6 \times 10^6)(4.1172)} = 0.002758 \text{ radians}$$

Total angle of twist:

$$\varphi_{AD} = \varphi_{AB} + \varphi_{CD} = 0.019896 \text{ radians} \qquad \varphi_{AD} = 1.140° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.40

The solid spindle *AB* has a diameter $d_s = 1.75\,\text{in.}$ and is made of a steel with $G = 11.2 \times 10^6\,\text{psi}$ and $\tau_{\text{all}} = 12\,\text{ksi}$, while sleeve *CD* is made of a brass with $G = 5.6 \times 10^6\,\text{psi}$ and $\tau_{\text{all}} = 7\,\text{ksi}$. Determine (*a*) the largest torque **T** that can be applied at *A* if the given allowable stresses are not to be exceeded and if the angle of twist of sleeve *CD* is not to exceed 0.375°, (*b*) the corresponding angle through which end *A* rotates.

SOLUTION

Spindle *AB*: $c = \frac{1}{2}(1.75\,\text{in.}) = 0.875\,\text{in.} \quad L = 12\,\text{in.}, \quad \tau_{\text{all}} = 12\,\text{ksi}, \quad G = 11.2 \times 10^6\,\text{psi}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.875)^4 = 0.92077\,\text{in}^4$$

Sleeve *CD*: $c_1 = 1.25\,\text{in.}, \quad c_2 = 1.5\,\text{in.}, \quad L = 8\,\text{in.}, \quad \tau_{\text{all}} = 7\,\text{ksi}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 4.1172\,\text{in}^4, \quad G = 5.6 \times 10^6\,\text{psi}$$

(*a*) Largest allowable torque *T*.

Ciriterion: Stress in spindle *AB*. $\tau = \frac{Tc}{J} \quad T = \frac{J\tau}{c}$

$$T = \frac{(0.92077)(12)}{0.875} = 12.63\,\text{kip} \cdot \text{in}$$

Critrion: Stress in sleeve *CD*. $T = \frac{J\tau}{c_2} = \frac{4.1172\,\text{in}^4}{1.5\,\text{in.}}(7\,\text{ksi}) \qquad T = 19.21\,\text{kip} \cdot \text{in}$

Criterion: Angle of twist of sleeve CD $\phi = 0.375° = 6.545 \times 10^{-3}\,\text{rad}$

$$\phi = \frac{TL}{JG} \quad T = \frac{JG}{L}\phi = \frac{(4.1172)(5.6 \times 10^6)}{8}(6.545 \times 10^{-3})$$

$$T = 18.86\,\text{kip} \cdot \text{in}$$

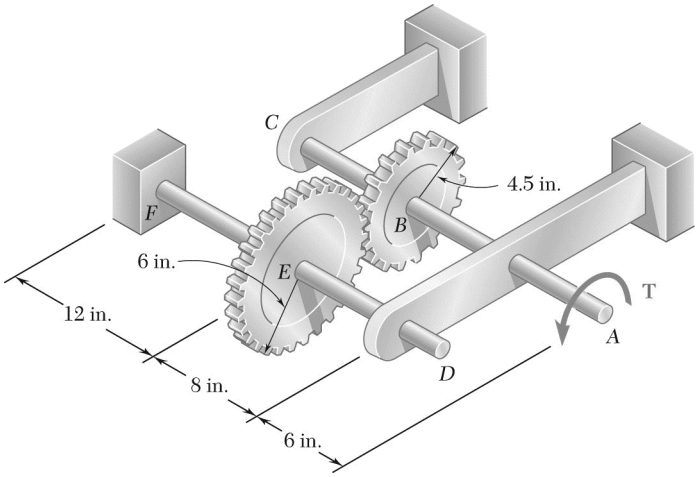
The largest allowable torque is $T = 12.63\,\text{kip} \cdot \text{in}$ ◀

(*b*) Angle of rotation of end A. $\phi_A = \phi_{A/D} = \phi_{A/B} + \phi_{C/D} = \sum \frac{T_i L_i}{J_i G_i} = T \sum \frac{L_i}{J_i G_i}$

$$= (12.63 \times 10^3)\left[\frac{12}{(0.92077)(11.2 \times 10^6)} + \frac{8}{(4.1172)(5.6 \times 10^6)}\right]$$

$$= 0.01908\,\text{radians} \qquad \varphi_A = 1.093° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.41

Two shafts, each of $\frac{7}{8}$-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A.

SOLUTION

Calculation of torques.

Circumferential contact force between gears B and E. $\quad F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{EF}}{r_E} \quad T_{EF} = \dfrac{r_E}{r_B} T_{AB}$

$$T_{AB} = 1.2 \text{ kip} \cdot \text{in} = 1200 \text{ lb} \cdot \text{in}$$

$$T_{EF} = \frac{6}{4.5}(1200) = 1600 \text{ lb} \cdot \text{in}$$

Twist in shaft FE.

$$L = 12 \text{ in.}, \quad c = \frac{1}{2}d = \frac{7}{16} \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{7}{16}\right)^4 = 57.548 \times 10^{-3} \text{ in}^4$$

$$\varphi_{E/F} = \frac{TL}{GJ} = \frac{(1600)(12)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 29.789 \times 10^{-3} \text{ rad}$$

Rotation at E. $\quad \varphi_E = \varphi_{E/F} = 29.789 \times 10^{-3}$ rad

Tangential displacement at gear circle. $\quad \delta = r_E \varphi_E = r_B \varphi_B$

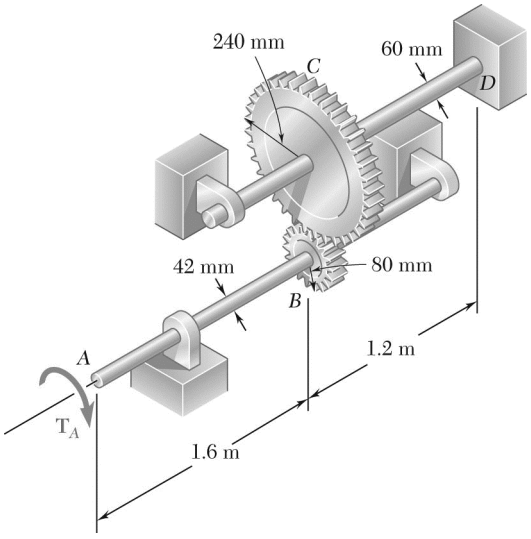
Rotation at B. $\quad \varphi_B = \dfrac{r_E}{r_B}\varphi_E = \dfrac{6}{4.5}(29.789 \times 10^{-3}) = 39.718 \times 10^{-3}$ rad

Twist in shaft BA. $\quad L = 8 + 6 = 14$ in. $\quad J = 57.548 \times 10^{-3}$ in^4

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(14)}{(11.2 \times 10^6)(57.548 \times 10^{-3})} = 26.065 \times 10^{-3} \text{ rad}$$

Rotation at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 65.783 \times 10^{-3}$ rad $\quad \varphi_A = 3.77°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.42

Two solid shafts are connected by gears as shown. Knowing that $G = 77.2$ GPa for each shaft, determine the angle through which end A rotates when $T_A = 1200\,\text{N} \cdot \text{m}$.

SOLUTION

Calculation of torques:

Circumferential contact force between gears B and C. $\quad F = \dfrac{T_{AB}}{r_B} = \dfrac{T_{CD}}{r_C} \quad T_{CD} = \dfrac{r_C}{r_B} T_{AB}$

$$T_{AB} = 1200\ \text{N} \cdot \text{m} \qquad T_{CD} = \frac{240}{80}(1200) = 3600\ \text{N} \cdot \text{m}$$

Twist in shaft CD: $\quad c = \dfrac{1}{2}d = 0.030\ \text{m}, \quad L = 1.2\ \text{m}, \quad G = 77.2 \times 10^9\ \text{Pa}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6}\ \text{m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(3600)(1.2)}{(77.2 \times 10^9)(1.27234 \times 10^{-9})} = 43.981 \times 10^{-3}\ \text{rad}$$

Rotation angle at C. $\quad \varphi_C = \varphi_{C/D} = 43.981 \times 10^{-3}\ \text{rad}$

Circumferential displacement at contact points of gears B and C. $\quad \delta = r_C \varphi_C = r_B \varphi_B$

Rotation angle at B. $\quad \varphi_B = \dfrac{r_C}{r_B}\varphi_C = \dfrac{240}{80}(43.981 \times 10^{-3}) = 131.942 \times 10^{-3}\ \text{rad}$

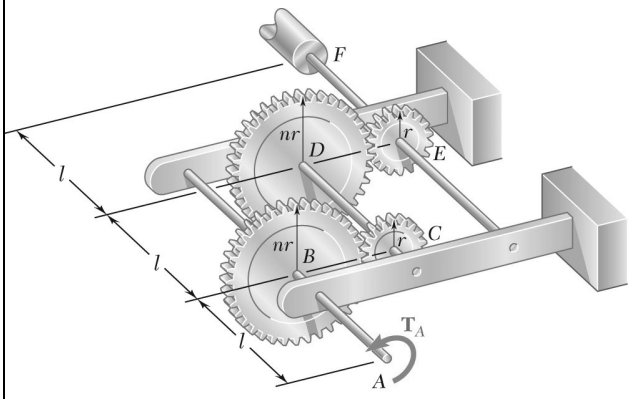
Twist in shaft AB: $\quad c = \dfrac{1}{2}d = 0.021\text{m}, \quad L = 1.6\text{m}, \quad G = 77.2 \times 10^9\ \text{Pa}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.021)^4 = 305.49 \times 10^9\ \text{m}^4$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(1200)(1.6)}{(77.2 \times 10^9)(305.49 \times 10^{-9})} = 81.412 \times 10^{-3}\ \text{rad}$$

Rotation angle at A. $\quad \varphi_A = \varphi_B + \varphi_{A/B} = 213.354 \times 10^{-3}\ \text{rad}$ $\qquad \varphi_A = 12.22°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.43

A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

SOLUTION

$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\varphi_E = \varphi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\varphi_D = \frac{r_E}{r_D} \varphi_E = \frac{\varphi_E}{n} = \frac{T_A l}{n^3 GJ}$$

$$\varphi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{nGJ}$$

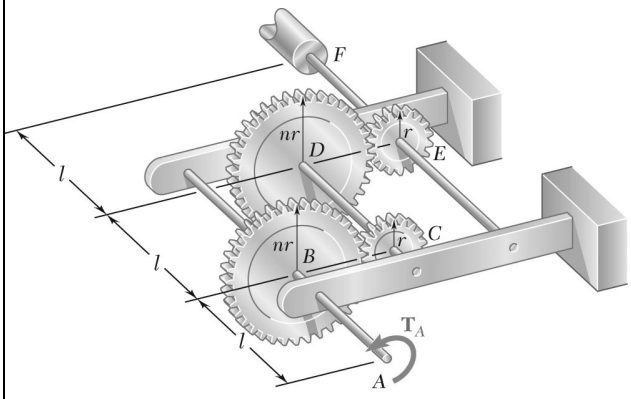
$$\varphi_C = \varphi_D + \varphi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{nGJ} = \frac{T_A l}{GJ}\left(\frac{1}{n^3} + \frac{1}{n}\right)$$

$$\varphi_B = \frac{r_C}{r_B} \varphi_C = \frac{\varphi_C}{n} = \frac{Tl}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2}\right)$$

$$\varphi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\varphi_A = \varphi_B + \varphi_{AB} = \frac{T_A l}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2} + 1\right)$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.44

For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 5\,\text{lb} \cdot \text{in.}$, $l = 2.4\,\text{in.}$, $d = 1/16\,\text{in.}$, $G = 11.2 \times 10^6\,\text{psi}$, and $n = 2$.

PROBLEM 3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

SOLUTION

See solution to Prob. 3.43 for development of equation for φ_A.

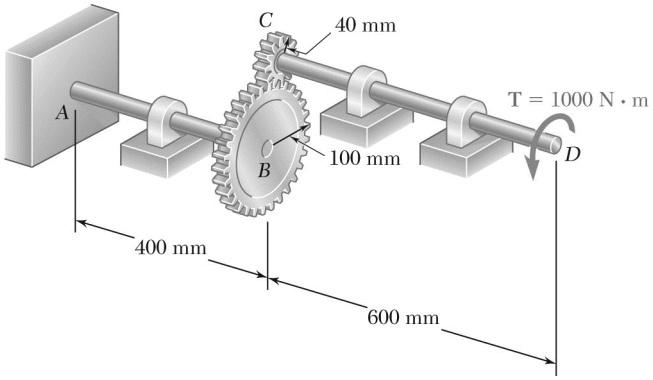
$$\varphi_A = \frac{Tl}{GJ}\left(1 + \frac{1}{n^2} + \frac{1}{n^4}\right)$$

Data: $T = 5\ \text{lb} \cdot \text{in}$, $l = 2.4\ \text{in.}$, $c = \frac{1}{2}d = \frac{1}{32}\ \text{in.}$, $G = 11.2 \times 10^6\,\text{psi}$

$$n = 2, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{1}{32}\right)^4 = 1.49803 \times 10^{-6}\,\text{in}^4$$

$$\varphi_A = \frac{(5)(2.4)}{(11.2 \times 10^6)(1.49803 \times 10^{-6})}\left(1 + \frac{1}{4} + \frac{1}{16}\right) = 938.73 \times 10^{-3}\,\text{rad} \qquad \varphi_A = 53.8^\circ \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.45

The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both *AB* and *CD*. It is further required that $\tau_{\max} \le 60$ MPa, and that the angle ϕ_D through which end *D* of shaft *CD* rotates not exceed 1.5°. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.

SOLUTION

$$T_{CD} = T_D = 1000 \text{ N} \cdot \text{m} \qquad T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500 \text{ N} \cdot \text{m}$$

For design based on stress, use larger torque. $T_{AB} = 2500 \text{ N} \cdot \text{m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6} \text{ m}^3$$

$$c = 29.82 \times 10^{-3} \text{ m} = 29.82 \text{ mm}, \quad d = 2c = 59.6 \text{ mm}$$

Design based on rotation angle. $\varphi_D = 1.5° = 26.18 \times 10^{-3}$ rad

Shaft *AB*: $T_{AB} = 2500 \text{ N} \cdot \text{m}, \quad L = 0.4 \text{ m}$

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$$

$$Gears \begin{cases} \varphi_B = \varphi_{AB} = \dfrac{1000}{GJ} \\ \varphi_C = \dfrac{r_B}{r_C}\varphi_B = \left(\dfrac{100}{40}\right)\left(\dfrac{1000}{GJ}\right) = \dfrac{2500}{GJ} \end{cases}$$

Shaft *CD*: $T_{CD} = 1000 \text{ N} \cdot \text{m}, \quad L = 0.6 \text{ m}$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$$

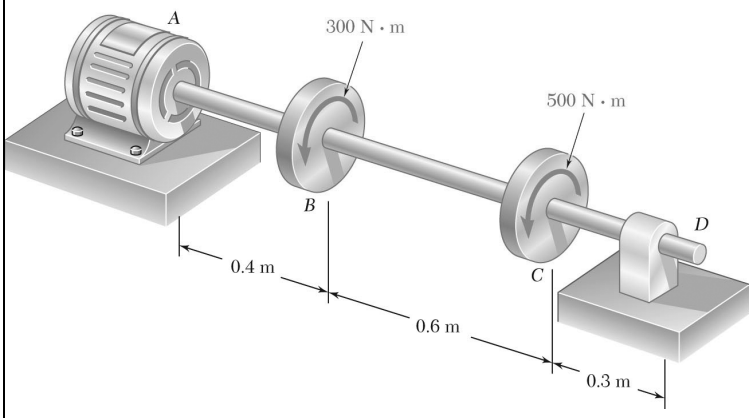
$$\varphi_D = \varphi_C + \varphi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G\frac{\pi}{2}c^4}$$

$$c^4 = \frac{(2)(3100)}{\pi G \varphi_D} = \frac{(2)(3100)}{\pi(77 \times 10^9)(26.18 \times 10^{-3})} = 979.06 \times 10^{-9} \text{ m}^4$$

$$c = 31.46 \times 10^{-3} \text{ m} = 31.46 \text{ mm}, \quad d = 2c = 62.9 \text{ mm}$$

Design must use larger value for *d*. $d = 62.9$ mm ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.46

The electric motor exerts a torque of $800 \text{ N} \cdot \text{m}$ on the steel shaft *ABCD* when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from *A* to *D* and that the angle of twist between *A* to *D* not exceed 1.5°. Knowing that $\tau_{\max} \le 60 \text{ MPa}$ and $G = 77$ GPa, determine the minimum diameter shaft that can be used.

SOLUTION

Torques:

$$T_{AB} = 300 + 500 = 800 \text{ N} \cdot \text{m}$$
$$T_{BC} = 500 \text{ N} \cdot \text{m}, \qquad T_{CD} = 0$$

Design based on stress. $\tau = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(800)}{\pi(60 \times 10^6)} = 8.488 \times 10^{-6} \text{ m}^3$$
$$c = 20.40 \times 10^{-3} \text{ m} = 20.40 \text{ mm}, \qquad d = 2c = 40.8 \text{ mm}$$

Design based on deformation. $\varphi_{D/A} = 1.5° = 26.18 \times 10^{-3} \text{ rad}$

$$\varphi_{D/C} = 0$$
$$\varphi_{C/B} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{300}{GJ}$$
$$\varphi_{B/A} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$$
$$\varphi_{D/A} = \varphi_{D/C} + \varphi_{C/B} + \varphi_{B/A} = \frac{620}{GJ} = \frac{620}{G\frac{\pi}{2}c^4} = \frac{(2)(620)}{\pi G c^4}$$
$$c^4 = \frac{(2)(620)}{\pi G \varphi_{D/A}} = \frac{(2)(620)}{\pi(77 \times 10^9)(26.18 \times 10^{-3})} = 195.80 \times 10^{-9} \text{ m}^4$$
$$c = 21.04 \times 10^{-3} \text{ m} = 21.04 \text{ mm}, \qquad d = 2c = 42.1 \text{ mm}$$

Design must use larger value of *d*. $d = 42.1 \text{ mm}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.47

The design specifications of a 2-m-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 3° when a torque of $9\ \text{kN}\cdot\text{m}$ is applied. Determine the required diameter of the shaft, knowing that the shaft is made of (*a*) a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77 GPa, (*b*) a bronze with an allowable shearing of 35 MPa and a modulus of rigidity of 42 GPa.

SOLUTION

$$\varphi = 3^\circ = 52.360\times10^{-3}\ \text{rad},\quad T = 9\times10^{3}\ \text{N}\cdot\text{m}\quad L = 2.0\ \text{m}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G}\quad\therefore\quad c^4 = \frac{2TL}{\pi G\varphi}\quad\text{based on twist angle.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^2}\quad\therefore\quad c^3 = \frac{2T}{\pi\tau}\quad\text{based on shearing stress.}$$

(*a*) Steel shaft: $\tau = 90\times10^{6}\ \text{Pa},\quad G = 77\times10^{9}\ \text{Pa}$

Based on twist angle, $$c^4 = \frac{(2)(9\times10^{3})(2.0)}{\pi(77\times10^{9})(52.360\times10^{-3})} = 2.842\times10^{-6}\ \text{m}^4$$

$$c = 41.06\times10^{-3}\ \text{m} = 41.06\ \text{mm}\qquad d = 2c = 82.1\ \text{mm}$$

Based on shearing stress, $$c^3 = \frac{(2)(9\times10^{3})}{\pi(90\times10^{6})} = 63.662\times10^{-6}\ \text{m}^3$$

$$c = 39.93\times10^{-3}\ \text{m} = 39.93\ \text{mm}\qquad d = 2c = 79.9\ \text{mm}$$

Required value of *d* is the larger. $d = 82.1\ \text{mm}$ ◀

(*b*) Bronze shaft: $\tau = 35\times10^{6}\ \text{Pa},\quad G = 42\times10^{9}\ \text{Pa}$

Based on twist angle, $$c^4 = \frac{(2)(9\times10^{3})(2.0)}{\pi(42\times10^{9})(52.360\times10^{-3})} = 5.2103\times10^{-6}\ \text{m}^4$$

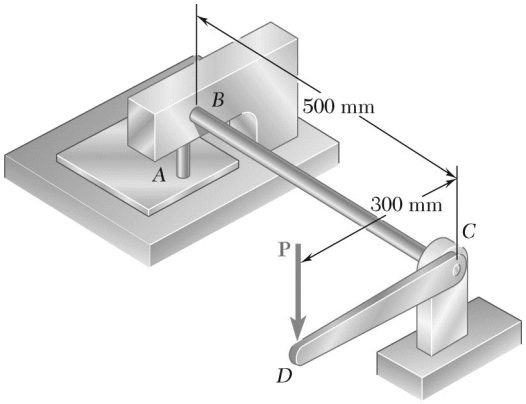
$$c = 47.78\times10^{-3}\ \text{m} = 47.78\ \text{mm}\qquad d = 2c = 95.6\ \text{mm}$$

Based on shearing stress, $$c^3 = \frac{(2)(9\times10^{3})}{\pi(35\times10^{6})} = 163.702\times10^{-6}\ \text{m}^3$$

$$c = 54.70\times10^{-3}\ \text{m} = 54.70\ \text{mm}\qquad d = 2c = 109.4\ \text{mm}$$

Required value of *d* is the larger. $d = 109.4\ \text{mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.48

A hole is punched at *A* in a plastic sheet by applying a 600-N force **P** to end *D* of lever *CD*, which is rigidly attached to the solid cylindrical shaft *BC*. Design specifications require that the displacement of *D* should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft *BC* if the shaft is made of a steel with $G = 77\,\text{GPa}$ and $\tau_{\text{all}} = 80\ \text{MPa}$.

SOLUTION

Torque $\qquad T = rP = (0.300\ \text{m})(600\ \text{N}) = 180\ \text{N}\cdot\text{m}$

Shaft diameter based on displacement limit.

$$\varphi = \frac{\delta}{r} = \frac{15\ \text{mm}}{300\ \text{mm}} = 0.005\ \text{rad}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c^4 = \frac{2TL}{\pi G\varphi} = \frac{(2)(180)(0.500)}{\pi(77\times10^9)(0.05)} = 14.882\times10^{-9}\ \text{m}^4$$

$$c = 11.045\times10^{-3}\ \text{m} = 11.045\ \text{m} \qquad d = 2c = 22.1\ \text{mm}$$

Shaft diameter based on stress.

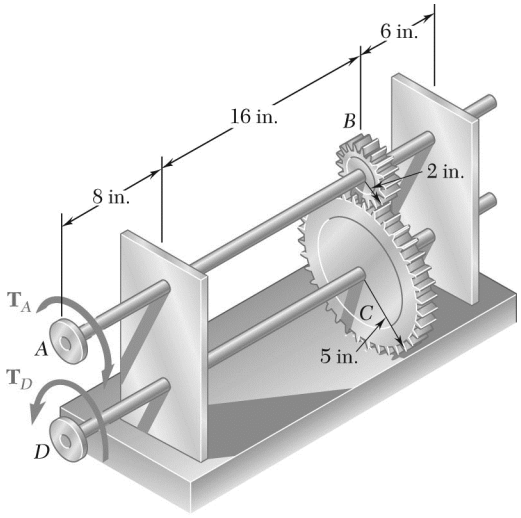
$$\tau = 80\times10^6\ \text{Pa} \qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(180)}{\pi(80\times10^6)} = 1.43239\times10^{-6}\ \text{m}^3$$

$$c = 11.273\times10^{-3}\ \text{m} = 11.273\ \text{mm} \qquad d = 2c = 22.5\ \text{mm}$$

Use the larger value to meet both limits. $\qquad d = 22.5\ \text{mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.49

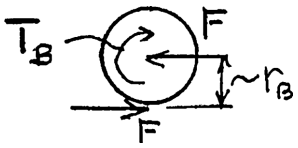
The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts, and that the angle through which pulley A will rotate when subjected to a 2-kip · in. torque $\mathbf{T}_A$ while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{\text{all}} = 12$ ksi.

SOLUTION

Statics:

Gear B.

$$+\circlearrowleft \Sigma M_B = 0: \qquad r_B F - T_A = 0 \quad F = T_B/r_B$$

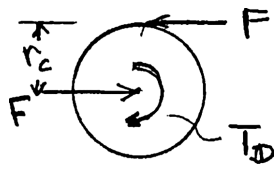


Gear C.

$$+\circlearrowleft \Sigma M_C = 0: \qquad r_C F - T_D = 0$$

$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$

$$n = \frac{r_C}{r_B} = \frac{5}{2} = 2.5$$



Torques in shafts. $\qquad T_{AB} = T_A = T_B \quad T_{CD} = T_C = nT_B = nT_A$

Deformations:

$$\varphi_{C/D} = \frac{T_{CD}L}{GJ} \circlearrowleft = \frac{nT_AL}{GJ} \circlearrowleft$$

$$\varphi_{A/B} = \frac{T_{AB}L}{GJ} \circlearrowright = \frac{T_AL}{GJ} \circlearrowright$$

Kinematics:

$$\varphi_D = 0 \quad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \frac{nT_AL}{GJ} \circlearrowright$$

$$r_B\varphi_B = -r_C\varphi_B \quad \varphi_B = -\frac{r_C}{r_B}\varphi_C = -n\varphi_C \quad \varphi_B \frac{n^2T_AL}{GJ} \circlearrowleft$$

$$\varphi_A = \varphi_C + \varphi_{B/C} = \frac{n^2T_AL}{GJ} + \frac{T_AL}{GJ} = \frac{(n^2+1)T_AL}{GJ} \circlearrowright$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.49 ***(Continued)***

Diameter based on stress.

Largest torque: $T_m = T_{CD} = nT_A$

$$\tau_m = \frac{T_m c}{J} = \frac{2nT_A}{\pi c^3} \quad \tau_m = \tau_{\text{all}} = 12 \times 10^3 \text{ psi}, \quad T_A = 2 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2nT_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2 \times 10^3)}{\pi (12 \times 10^3)}} = 0.6425 \text{ in.}, \quad d = 2c = 1.285 \text{ in.}$$

Diameter based on rotation limit.

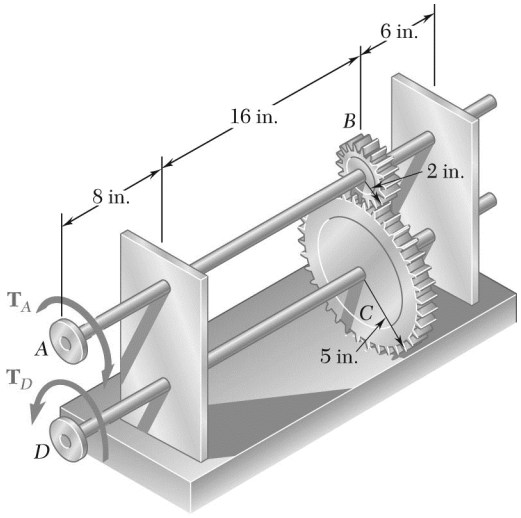
$$\varphi = 7.5^\circ = 0.1309 \text{ rad}$$

$$\varphi = \frac{(n^2 + 1)T_A L}{GJ} = \frac{(2)(7.25)T_A L}{\pi c^4 G} \quad L = 8 + 16 = 24 \text{ in.}$$

$$c = \sqrt[4]{\frac{(2)(7.25)T_A L}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(7.25)(2 \times 10^3)(24)}{\pi (11.2 \times 10^6)(0.1309)}} = 0.62348 \text{ in.}, \quad d = 2c = 1.247 \text{ in.}$$

Choose the larger diameter. $d = 1.285$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.50

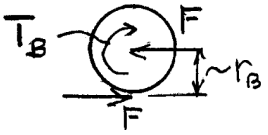
Solve Prob. 3.49, assuming that both shafts are made of a brass with $G = 5.6 \times 10^6$ psi and $\tau_{all} = 8$ ksi.

PROBLEM 3.49 The design specifications for the gear-and-shaft system shown require that the same diameter be used for both shafts, and that the angle through which pulley A will rotate when subjected to a 2-kip · in. torque $\mathbf{T}_A$ while pulley D is held fixed will not exceed 7.5°. Determine the required diameter of the shafts if both shafts are made of a steel with $G = 11.2 \times 10^6$ psi and $\tau_{all} = 12$ ksi.

SOLUTION

Statics:

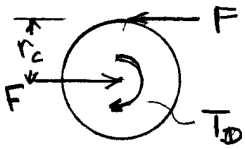
Gear B.



$$+\circlearrowleft \Sigma M_B = 0: \qquad r_B F - T_A = 0 \quad F = T_B / r_B$$

Gear C.

$$+\circlearrowleft \Sigma M_C = 0: \qquad r_C F - T_D = 0$$



$$T_D = r_C F = \frac{r_C}{r_B} T_A = n T_B$$

$$n = \frac{r_C}{r_B} = \frac{5}{2} = 2.5$$

Torques in shafts. $\quad T_{AB} = T_A = T_B \quad T_{CD} = T_C = nT_B = nT_A$

Deformations:

$$\varphi_{C/D} = \frac{T_{CD} L}{GJ} \circlearrowleft = \frac{nT_A L}{GJ} \circlearrowleft$$

$$\varphi_{A/B} = \frac{T_{AB} L}{GJ} \circlearrowright = \frac{T_A L}{GJ} \circlearrowright$$

Kinematics:

$$\varphi_D = 0 \quad \varphi_C = \varphi_D + \varphi_{C/D} = 0 + \frac{nT_A L}{GJ} \circlearrowright$$

$$r_B \varphi_B = -r_C \varphi_B \quad \varphi_B = -\frac{r_C}{r_B} \varphi_C = -n\varphi_C \quad \varphi_B \frac{n^2 T_A L}{GJ} \circlearrowleft$$

$$\varphi_A = \varphi_C + \varphi_{B/C} = \frac{n^2 T_A L}{GJ} + \frac{T_A L}{GJ} = \frac{(n^2 + 1) T_A L}{GJ} \circlearrowright$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.50 *(Continued)*

Diameter based on stress.

Largest torque: $T_m = T_{CD} = nT_A$

$$\tau_m = \frac{T_m c}{J} = \frac{2nT_A}{\pi c^3} \quad \tau_m = \tau_{\text{all}} = 8 \times 10^3 \text{ psi}, \quad T_A = 2 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2nT_A}{\pi \tau_m}} = \sqrt[3]{\frac{(2)(2.5)(2 \times 10^3)}{\pi (8 \times 10^3)}} = 0.7355 \text{ in.}, \quad d = 2c = 1.471 \text{ in.}$$

Diameter based on rotation limit.

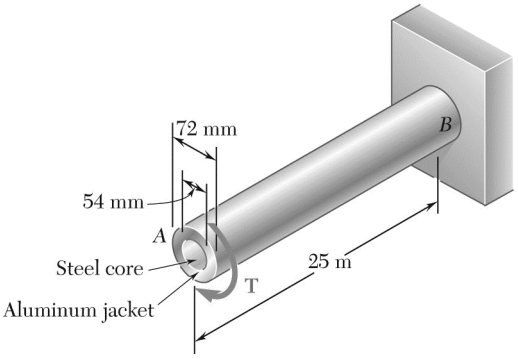
$$\varphi = 7.5^\circ = 0.1309 \text{ rad}$$

$$\varphi = \frac{(n^2 + 1)T_A L}{GJ} = \frac{(2)(7.25)T_A L}{\pi c^4 G} \quad L = 8 + 16 = 24 \text{ in.}$$

$$c = \sqrt[4]{\frac{(2)(7.25)T_A L}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(7.25)(2 \times 10^3)(24)}{\pi (5.6 \times 10^6)(0.1309)}} = 0.7415 \text{ in.}, \quad d = 2c = 1.483 \text{ in.}$$

Choose the larger diameter. $d = 1.483$ in. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.51

A torque of magnitude $T = 4$ kN· m is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.

SOLUTION

Steel core: $c_1 = \frac{1}{2} d_1 = 0.027 \text{ m} \quad J_1 = \frac{\pi}{2} c_1^4 = \frac{\pi}{2} (0.027)^4 = 834.79 \times 10^{-9}$

$$G_1 J_1 = (77 \times 10^9)(834.79 \times 10^{-9}) = 64.28 \times 10^3 \text{ N} \cdot \text{m}^2$$

Torque carried by steel core. $T_1 = G_1 J_1 \varphi / L$

Aluminum jacket: $c_1 = \frac{1}{2} d_1 = 0.027 \text{ m}, \quad c_2 = \frac{1}{2} d_2 = 0.036 \text{ m}$

$$J_2 = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.036^4 - 0.027^4) = 1.80355 \times 10^{-6} \text{ m}^4$$

$$G_2 J_2 = (27 \times 10^9)(1.80355 \times 10^{-6}) = 48.70 \times 10^3 \text{ N} \cdot \text{m}^2$$

Torque carried by aluminum jacket. $T_2 = G_2 J_2 \varphi / L$

Total torque: $T = T_1 + T_2 = (G_1 J_1 + G_2 J_2)\, \varphi / L$

$$\frac{\varphi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{4 \times 10^3}{64.28 \times 10^3 + 48.70 \times 10^3} = 35.406 \times 10^{-3} \text{ rad/m}$$

(*a*) Maximum shearing stress in steel core.

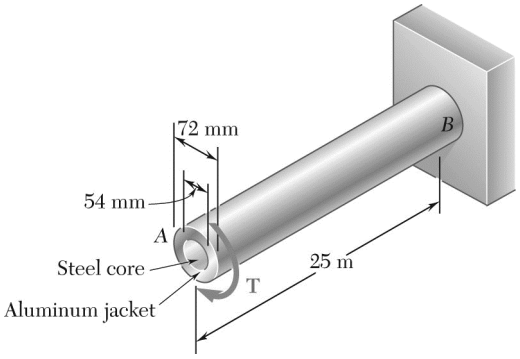
$$\tau = G_1 \gamma = G_1 c_1 \frac{\varphi}{L} = (77 \times 10^9)(0.027)(35.406 \times 10^{-3}) = 73.6 \times 10^6 \text{ Pa} \qquad 73.6 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum shearing stress in aluminum jacket.

$$\tau = G_2 \gamma = G_2 c_2 \frac{\varphi}{L} = (27 \times 10^9)(0.036)(35.406 \times 10^{-3}) = 34.4 \times 10^6 \text{ Pa} \qquad 34.4 \text{ MPa} \blacktriangleleft$$

(*c*) Angle of twist. $\varphi = L \frac{\varphi}{L} = (2.5)(35.406 \times 10^{-3}) = 88.5 \times 10^{-3} \text{ rad} \qquad \varphi = 5.07° \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.52

The composite shaft shown is to be twisted by applying a torque **T** at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* can be rotated if the following allowable stresses are not to be exceeded: $\tau_{\text{steel}} = 60\ \text{MPa}$ and $\tau_{\text{aluminum}} = 45\ \text{MPa}$.

SOLUTION

$$\tau_{\max} = G\gamma_{\max} = Gc_{\max}\frac{\varphi}{L}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{Gc_{\max}} \quad \text{for each material.}$$

Steel core:

$$\tau_{\text{all}} = 60\times10^{6}\ \text{Pa}, \quad c_{\max} = \frac{1}{2}d = 0.027\,\text{m}, \quad G = 77\times10^{9}\ \text{Pa}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{60\times10^{6}}{(77\times10^{9})(0.027)} = 28.860\times10^{-3}\ \text{rad/m}$$

Aluminum Jacket:

$$\tau_{\text{all}} = 45\times10^{6}\ \text{Pa}, \quad c_{\max} = \frac{1}{2}d = 0.036\,\text{m}, \quad G = 27\times10^{9}\ \text{Pa}$$

$$\frac{\varphi_{\text{all}}}{L} = \frac{45\times10^{6}}{(27\times10^{9})(0.036)} = 46.296\times10^{-3}\ \text{rad/m}$$

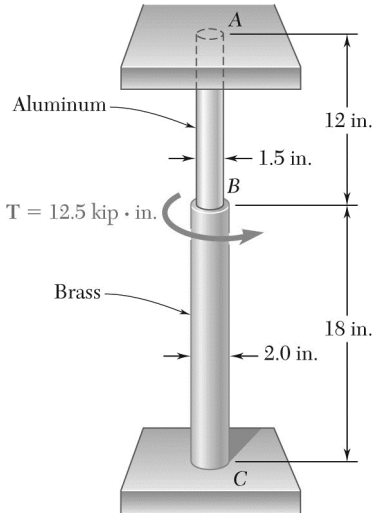
Smaller value governs:

$$\frac{\varphi_{\text{all}}}{L} = 28.860\times10^{-3}\ \text{rad/m}$$

Allowable angle of twist:

$$\varphi_{\text{all}} = L\frac{\varphi_{\text{all}}}{L} = (2.5)\,(28.860\times10^{-3}) = 72.15\times10^{-3}\ \text{rad} \qquad \varphi_{\text{all}} = 4.13^{\circ}\ \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.53

The solid cylinders *AB* and *BC* are bonded together at *B* and are attached to fixed supports at *A* and *C*. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (*a*) in cylinder *AB*, (*b*) in cylinder *BC*.

SOLUTION

The torques in cylinders *AB* and *BC* are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder *AB*: $c = \frac{1}{2}d = 0.75$ in. $L = 12$ in.

$$J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(3.7 \times 10^6)(0.49701)} = 6.5255 \times 10^{-6} T_{AB}$$

Cylinder *BC*: $c = \frac{1}{2}d = 1.0$ in. $L = 18$ in.

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708 \text{ in}^4$$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6 \times 10^6)(1.5708)} = 2.0463 \times 10^{-6} T_{BC}$$

Matching expressions for φ_B $6.5255 \times 10^{-6} T_{AB} = 2.0463 \times 10^{-6} T_{BC}$

$$T_{BC} = 3.1889\, T_{AB} \quad (1)$$

Equilibrium of connection at *B*: $T_{AB} + T_{BC} - T = 0$ $T = 12.5 \times 10^3$ lb · in

$$T_{AB} + T_{BC} = 12.5 \times 10^3 \quad (2)$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.53 *(Continued)*

Substituting (1) into (2), $4.1889\, T_{AB} = 12.5 \times 10^3$

$$T_{AB} = 2.9841 \times 10^3 \text{ lb} \cdot \text{in} \qquad T_{BC} = 9.5159 \times 10^3 \text{ lb} \cdot \text{in}$$

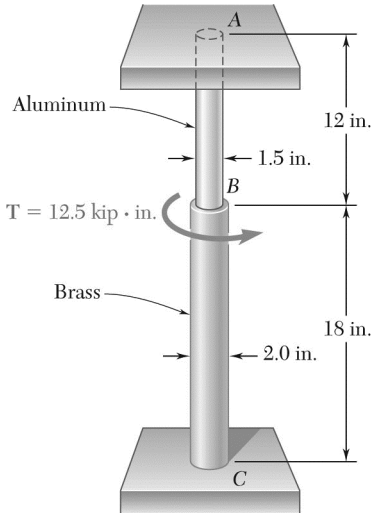
(*a*) Maximum stress in cylinder *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(2.9841 \times 10^3)(0.75)}{0.49701} = 4.50 \times 10^3 \text{ psi} \qquad \tau_{AB} = 4.50 \text{ ksi} \blacktriangleleft$$

(*b*) Maximum stress in cylinder *BC*.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(9.5159 \times 10^3)(1.0)}{1.5708} = 6.06 \times 10^3 \text{ psi} \qquad \tau_{BC} = 6.06 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.54

Solve Prob. 3.53, assuming that cylinder *AB* is made of steel, for which $G = 11.2 \times 10^6$ psi.

PROBLEM 3.53 The solid cylinders *AB* and *BC* are bonded together at *B* and are attached to fixed supports at *A* and *C*. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (*a*) in cylinder *AB*, (*b*) in cylinder *BC*.

SOLUTION

The torques in cylinders *AB* and *BC* are statically indeterminate. Match the rotation φ_B for each cylinder.

Cylinder *AB*: $c = \frac{1}{2}d = 0.75$ in. $L = 12$ in. $J = \frac{\pi}{2}c^4 = 0.49701$ in^4

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(11.2\times10^6)(0.49701)} = 2.1557\times10^{-6}\,T_{AB}$$

Cylinder *BC*. $c = \frac{1}{2}d = 1.0$ in. $L = 18$ in. $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708$ in^4

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6\times10^6)(1.5708)} = 2.0463\times10^{-6}T_{BC}$$

Matching expressions for φ_B $2.1557\times10^{-6}T_{AB} = 2.0463\times10^{-6}T_{BC}$ $T_{BC} = 1.0535\,T_{AB}$ (1)

Equilibrium of connection at *B*: $T_{AB} + T_{BC} - T = 0$ $T_{AB} + T_{BC} = 12.5\times10^3$ (2)

Substituting (1) into (2), $2.0535\,T_{AB} = 12.5\times10^3$

$$T_{AB} = 6.0872\times10^3 \text{ lb}\cdot\text{in} \qquad T_{BC} = 6.4128\times10^3 \text{ lb}\cdot\text{in}$$

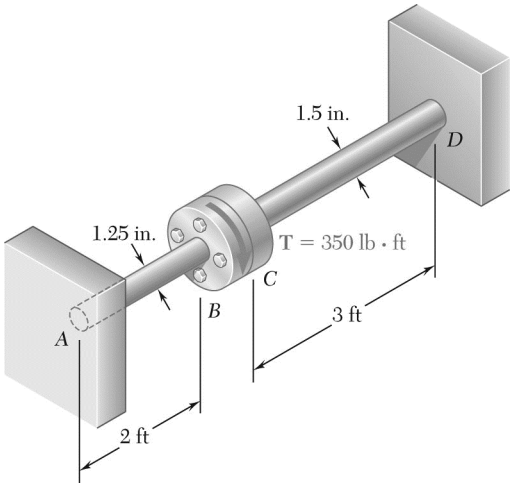
(*a*) Maximum stress in cylinder *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(6.0872\times10^3)(0.75)}{0.49701} = 9.19\times10^3 \text{ psi} \qquad \tau_{AB} = 9.19 \text{ ksi} \blacktriangleleft$$

(*b*) Maximum stress in cylinder *BC*.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(6.4128\times10^3)(1.0)}{1.5708} = 4.08\times10^3 \text{ psi} \qquad \tau_{BC} = 4.08 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.55

Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 11.2 \times 10^6$ psi, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 420 kip · ft is applied to the flange indicated.

PROBLEM 3.55 The torque **T** is applied to flange *B*.

SOLUTION

Shaft *AB*:

$$T = T_{AB}, \quad L_{AB} = 2\,\text{ft} = 24\,\text{in.}, \quad c = \frac{1}{2}d = 0.625\,\text{in.}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968\,\text{in}^4$$

$$\varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}\varphi_B}{L_{AB}} = \frac{(11.2\times10^6)(0.23968)}{24}\varphi_B$$

$$= 111.853\times10^3\varphi_B$$

Shaft *CD*:

Applied torque: $T = 420\,\text{kip}\cdot\text{ft} = 5040\,\text{lb}\cdot\text{in}$

$$T = T_{CD}, \quad L_{CD} = 3\,\text{ft} = 36\,\text{in.}, \quad c = \frac{1}{2}d = 0.75\,\text{in.}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701\,\text{in}^4$$

$$\varphi_C = \frac{T_{CD}L_{CD}}{GJ_{CD}}$$

$$T_{CD} = \frac{GJ_{CD}\varphi_C}{L_{CD}} = \frac{(11.2\times10^6)(0.49701)}{36}\varphi_C = 154.625\times10^3\varphi_C$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.55 *(Continued)*

Clearance rotation for flange B: $\varphi'_B = 1.5° = 26.18 \times 10^{-3}$ rad

Torque to remove clearance: $T'_{AB} = (111.853 \times 10^3)(26.18 \times 10^{-3}) = 2928.3$ lb · in

Torque T'' to cause additional rotation φ'': $T'' = 5040 - 2928.3 = 2111.7$ lb · in

$$T'' = T''_{AB} + T''_{CD}$$

$$2111.7 = (111.853 \times 10^3)\varphi'' + (154.625 \times 10^3)\varphi'' \quad \therefore \quad \varphi'' = 7.923 \times 10^{-3} \text{ rad}$$

$$T''_{AB} = (111.853 \times 10^3)(7.923 \times 10^{-3}) = 886.21 \text{ lb} \cdot \text{in}$$

$$T''_{CD} = (154.625 \times 10^3)(7.923 \times 10^{-3}) = 1225.09 \text{ lb} \cdot \text{in}$$

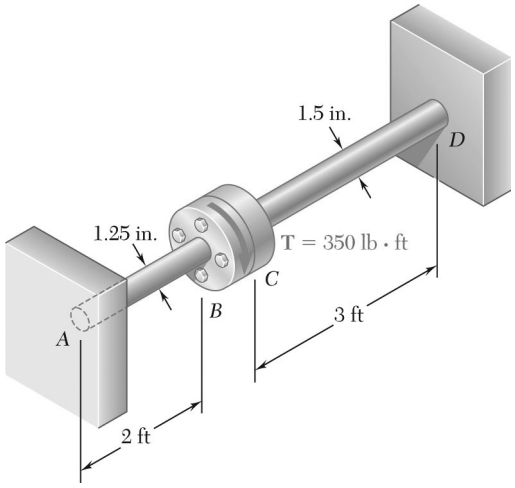
Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(2928.3 + 886.21)(0.625)}{0.23968} = 9950 \text{ psi} \qquad \tau_{AB} = 9.95 \text{ ksi} \blacktriangleleft$$

Maximum shearing stress in CD.

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(1225.09)(0.75)}{0.49701} = 1849 \text{ psi} \qquad \tau_{CD} = 1.849 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.56

Two solid steel shafts are fitted with flanges that are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 11.2 \times 10^6$ psi, determine the maximum shearing stress in each shaft when a torque of **T** of magnitude 420 kip · ft is applied to the flange indicated.

PROBLEM 3.56 The torque **T** is applied to flange *C*.

SOLUTION

Shaft *AB*:

$$T = T_{AB}, \quad L_{AB} = 2\,\text{ft} = 24\,\text{in.}, \quad c = \frac{1}{2}d = 0.625\,\text{in.}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968\,\text{in}^4$$

$$\varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}\varphi_B}{L_{AB}} = \frac{(11.2\times10^6)(0.23968)}{24}\varphi_B$$

$$= 111.853\times10^3\varphi_B$$

Shaft *CD*:

Applied torque: $T = 420\,\text{kip}\cdot\text{ft} = 5040\,\text{lb}\cdot\text{in}$

$$T = T_{CD}, \quad L_{CD} = 3\,\text{ft} = 36\,\text{in.}, \quad c = \frac{1}{2}d = 0.75\,\text{in.}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701\,\text{in}^4$$

$$\varphi_C = \frac{T_{CD}L_{CD}}{GJ_{CD}}$$

$$T_{CD} = \frac{GJ_{CD}\varphi_C}{L_{CD}} = \frac{(11.2\times10^6)(0.49701)}{36}\varphi_C = 154.625\times10^3\varphi_C$$

Clearance rotation for flange *C*: $\varphi'_C = 1.5° = 26.18\times10^{-3}\,\text{rad}$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.56 *(Continued)*

Torque to remove clearance: $T'_{CD} = (154.625 \times 10^3)(26.18 \times 10^{-3}) = 4048.1 \text{ lb} \cdot \text{in}$

Torque T'' to cause additional rotation φ'': $T'' = 5040 - 4048.1 = 991.9 \text{ lb} \cdot \text{in}$

$$T'' = T''_{AB} + T''_{CD}$$

$$991.9 = (111.853 \times 10^3)\varphi'' + (154.625 \times 10^3)\varphi'' \quad \therefore \quad \varphi'' = 3.7223 \times 10^{-3} \text{ rad}$$

$$T''_{AB} = (111.853 \times 10^{-3})(3.7223 \times 10^{-3}) = 416.35 \text{ lb} \cdot \text{in}$$

$$T''_{CD} = (154.625 \times 10^{-3})(3.7223 \times 10^{-3}) = 575.56 \text{ lb} \cdot \text{in}$$

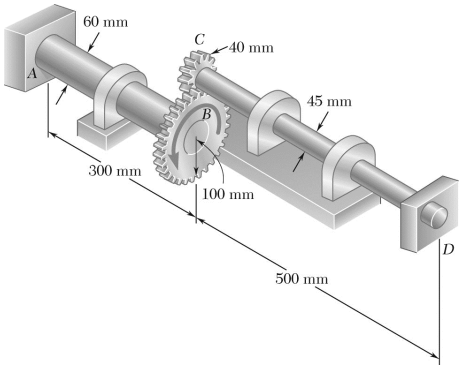
Maximum shearing stress in *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(416.35)(0.625)}{0.23968} = 1086 \text{ psi} \qquad \tau_{AB} = 1.086 \text{ ksi} \blacktriangleleft$$

Maximum shearing stress in *CD*.

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(4048.1 + 575.56)(0.75)}{0.49701} = 6980 \text{ psi} \qquad \tau_{CD} = 6.98 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

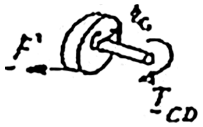


PROBLEM 3.57

Ends *A* and *D* of two solid steel shafts *AB* and *CD* are fixed, while ends *B* and *C* are connected to gears as shown. Knowing that a 4-kN · m torque **T** is applied to gear *B*, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *CD*.

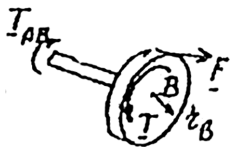
SOLUTION

Gears *B* and *C*:



$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{40}{100}\phi_C \qquad \phi_B = 0.4\,\phi_C \tag{1}$$

$$\Sigma M_C = 0 : T_{CD} = r_C F \tag{2}$$



$$\Sigma M_B = 0 : T - T_{AB} = r_B F \tag{3}$$

Solve (2) for *F* and substitute into (3):

$$T - T_{AB} = \frac{r_B}{r_C}T_{CD} \qquad T = T_{AB} + \frac{100}{40}T_{CD}$$

$$T = T_{AB} + 2.5\,T_{CD} \tag{4}$$

Shaft *AB*: $L = 0.3$ m, $c = 0.030$ m

$$\phi_B = \phi_{B/A} = \frac{T_{AB}L}{JG} = \frac{T_{AB}\,(0.3)}{\frac{\pi}{2}(0.030)^4 6} = 235.79\times10^3\,\frac{T_{AB}}{G} \tag{5}$$

Shaft *CD*: $L = 0.5$ m, $c = 0.0225$ m

$$\phi_C = \phi_{C/D} = \frac{T_{CD}L}{JG} \qquad \frac{T_{CD}\,(0.5)}{\frac{\pi}{2}(0.0225)^4 G} = 1242\times10^3\,\frac{T_{CD}}{G} \tag{6}$$

Substitute from (5) and (6) into (1):

$$\phi_B = 0.4\phi_C: \qquad 235.79\times10^3\,\frac{T_{AB}}{G} = 0.4\times1242\times10^3\,\frac{T_{CD}}{G}$$

$$T_{CD} = 0.47462 = T_{AB} \tag{7}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.57 *(Continued)*

Substitute for T_{CD} from (7) into (4):

$$T = T_{AB} + 2.5\,(0.47462\,T_{AB}) \qquad T = 2.1865\,T_{AB} \quad (8)$$

For $T = 4$ kN · m, Eq. (8) yields

$$4000\,\text{N} \cdot \text{m} = 2.1865 T_{AB} \qquad T_{AB} = 1829.4\,\text{N} \cdot \text{m}$$

Substitute into (7): $T_{CD} = 0.47462(1829.4) = 868.3\,\text{N} \cdot \text{m}$

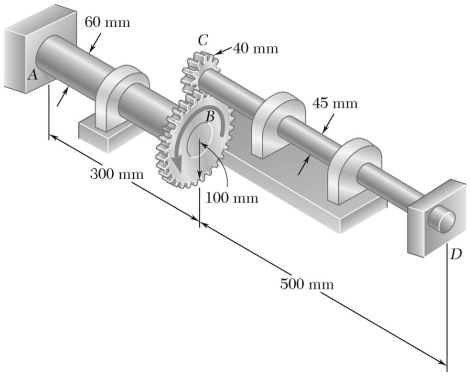
(*a*) Stress in *AB*:

$$\tau_{AB} = \frac{T_{AB} c}{J} = \frac{2}{\pi} \frac{T_{AB}}{c^3} = \frac{2}{\pi} \frac{1829.4}{(0.030)^3} = 43.1 \times 10^6 \qquad \tau_{AB} = 43.1 \text{ MPa} \blacktriangleleft$$

(*b*) Stress in *CD*:

$$\tau_{CD} = \frac{T_{CD} c}{J} = \frac{2}{\pi} \frac{T_{CD}}{c^3} = \frac{2}{\pi} \frac{868.3}{(0.0225)^3} = 48.5 \times 10^6 \qquad \tau_{CD} = 48.5 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



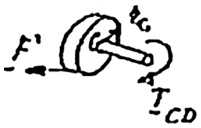
PROBLEM 3.58

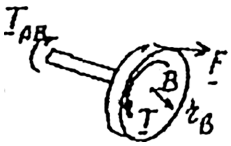
Ends *A* and *D* of the two solid steel shafts *AB* and *CD* are fixed, while ends *B* and *C* are connected to gears as shown. Knowing that the allowable shearing stress is 50 MPa in each shaft, determine the largest torque **T** that may be applied to gear *B*.

SOLUTION

Gears *B* and *C*:

$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{40}{100}\phi_C \qquad \phi_B = 0.4\,\phi_C \tag{1}$$





$$\Sigma M_C = 0: \; T_{CD} = r_C F \tag{2}$$

$$\Sigma M_B = 0: \; T - T_{AB} = r_B F \tag{3}$$

Solve (2) for *F* and substitute into (3):

$$T - T_{AB} = \frac{r_B}{r_C} T_{CD} \qquad T = T_{AB} + \frac{100}{40} T_{CD}$$

$$T = T_{AB} + 2.5\,T_{CD} \tag{4}$$

Shaft *AB*: $L = 0.3$ m, $c = 0.030$ m

$$\phi_B = \phi_{B/A} = \frac{T_{AB}L}{JG} = \frac{T_{AB}\,(0.3)}{\frac{\pi}{2}(0.030)^4\,G} = 235.77 \times 10^3\,\frac{T_{AB}}{G} \tag{5}$$

Shaft *CD*: $L = 0.5$ m, $c = 0.0225$ m

$$\phi_C = \phi_{C/D} = \frac{T_{CD}L}{JG} = \frac{T_{CD}\,(0.5)}{\frac{\pi}{2}(0.0225)^4\,G} = 1242 \times 10^3\,\frac{T_{CD}}{G} \tag{6}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.58 *(Continued)*

Substitute from (5) and (6) into (1) :

$$\phi_B = 0.4\,\phi_C: \quad 235.79 \times 10^3 \frac{T_{AB}}{G} = 0.4 \times 1242 \times 10^3 \frac{T_{CD}}{G}$$

$$T_{CD} = 0.47462\,T_{AB} \tag{7}$$

Substitute for T_{CD} from (7) into (4):

$$T = T_{AB} + 2.5\,(0.47462\,T_{AB}) \qquad T = 2.1865\,T_{AB} \tag{8}$$

Solving (7) for T_{AB} and substituting into (8),

$$T = 2.1865\left(\frac{T_{CD}}{0.47462}\right) \qquad T = 4.6068\,T_{CD} \tag{9}$$

Stress criterion for shaft *AB*:

$$\tau_{AB} = \tau_{\text{all}} = 50 \text{ MPa:}$$

$$\tau_{AB} = \frac{T_{AB}c}{J} \quad T_{AB} = \frac{J}{c}\tau_{AB} = \frac{\pi}{2}c^3\tau_{AB}$$

$$= \frac{\pi}{2}(0.030 \text{ m})^3(50 \times 10^6 \text{ Pa}) = 2120.6 \text{ N} \cdot \text{m}$$

From (8): $T = 2.1865(2120.6 \text{ N} \cdot \text{m}) = 4.64 \text{ kN} \cdot \text{m}$

Stress criterion for shaft *CD*:

$$\tau_{CD} = \tau_{\text{all}} = 50 \text{ MPa:}$$

$$\tau_{CD} = \frac{T_{CD}c}{J} \quad T_{CD} = \frac{\pi}{2}c^3\tau_{CD} = \frac{\pi}{2}(0.0225 \text{ m})^3(50 \times 10^6 \text{ Pa})$$

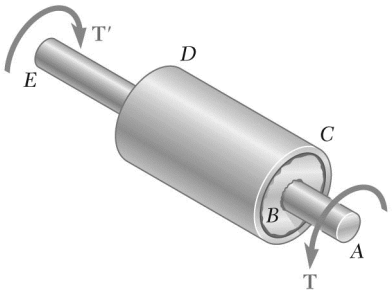
$$= 894.62 \text{ N} \cdot \text{m}$$

From (7): $T = 4.6068(894.62 \text{ N} \cdot \text{m}) = 4.12 \text{ kN} \cdot \text{m}$

The smaller value for *T* governs.

$$T = 4.12 \text{ kN} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.59

The steel jacket *CD* has been attached to the 40-mm-diameter steel shaft *AE* by means of *rigid* flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500 N · m torques are applied as shown, determine the maximum shearing stress in the jacket.

SOLUTION

Solid shaft:

$$c = \frac{1}{2}d = 0.020 \text{ m}$$

$$J_S = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket:

$$c_2 = \frac{1}{2}d = 0.040 \text{ m}$$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.040^4 - 0.036^4)$$

$$= 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft. $T_S = GJ_S\varphi/L$

Torque carried by jacket. $T_J = GJ_J\varphi/L$

Total torque. $T = T_S + T_J = (J_S + J_J)G\varphi/L \quad \therefore \quad \frac{G\varphi}{L} = \frac{T}{J_S + J_J}$

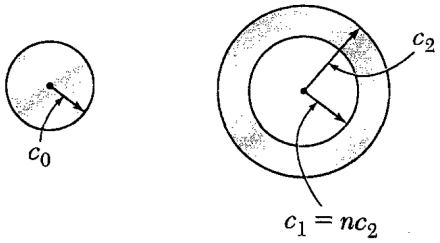
$$T_J = \frac{J_J}{J_S + J_J}T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-6}} = 423.1 \text{ N} \cdot \text{m}$$

Maximum shearing stress in jacket.

$$\tau = \frac{T_J c_2}{J_J} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa} \qquad 12.24 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.60



A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1 / c_2, show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (*a*) $\sqrt{(1-n^2)}/(1+n^2)$ if the maximum shearing stress is the same in each shaft, (*b*) $(1-n)/(1+n^2)$ if the angle of twist is the same for each shaft.

SOLUTION

For equal weight and length, the areas are equal.

$$\pi c_0^2 = \pi\left(c_2^2 - c_1^2\right) = \pi c_2^2(1-n^2) \qquad \therefore \quad c_0 = c_2(1-n^2)^{1/2}$$

$$J_s = \frac{\pi}{2}c_0^4 = \frac{\pi}{2}c_2^4(1-n^2)^2 \qquad J_h = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}c_2^4(1-n^4)$$

(*a*) For equal stresses.

$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

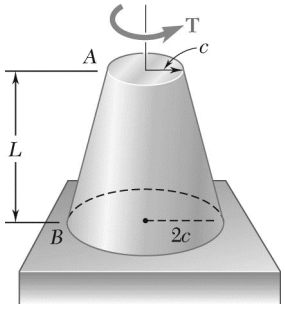
$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2 c_2}{\frac{\pi}{2}c_2^4(1-n^4)c_2(1-n^2)^{1/2}} = \frac{1-n^2}{(1+n^2)(1-n^2)^{1/2}} = \frac{(1-n^2)^{1/2}}{1+n^2} \quad \blacktriangleleft$$

(*b*) For equal angles of twist.

$$\varphi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2}{\frac{\pi}{2}c_2^4(1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2} \quad \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.61

A torque **T** is applied as shown to a solid tapered shaft *AB*. Show by integration that the angle of twist at *A* is

$$\phi = \frac{7TL}{12\pi Gc^4}$$

SOLUTION

Introduce coordinate *y* as shown.

$$r = \frac{cy}{L}$$

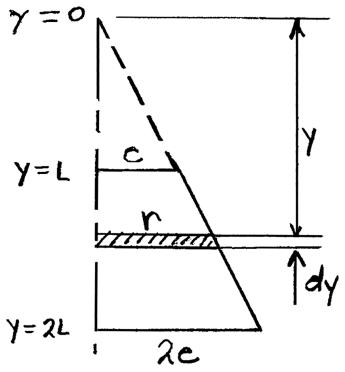
Twist in length *dy*:

$$d\varphi = \frac{T\,dy}{GJ} = \frac{T\,dy}{G\frac{\pi}{2}r^4} = \frac{2TL^4\,dy}{\pi Gc^4 y^4}$$

$$\varphi = \int_L^{2L} \frac{2TL^4}{\pi Gc^4}\frac{dy}{y^4} = \frac{2TL}{\pi Gc^4}\int_L^{2L}\frac{dy}{y^4}$$

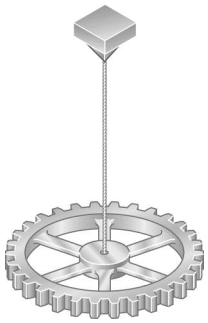
$$= \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{3y^3}\right\}_L^{2L} = \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{24L^3}+\frac{1}{3L^3}\right\}$$

$$= \frac{2TL^4}{\pi Gc^4}\left\{\frac{7}{24L^3}\right\} = \frac{7TL}{12\pi Gc^4}$$



◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.62

The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 6-ft steel wire. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of the wire for which the torsional spring constant will be 4.27 lb · ft/rad.

SOLUTION

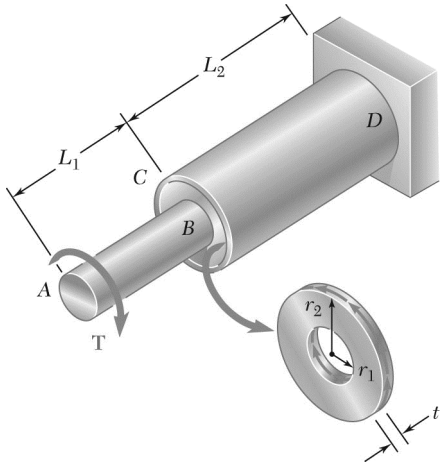
Torsion spring constant $K = 4.27$ lb · ft/rad $= 51.24$ lb · in/rad

$$K = \frac{T}{\varphi} = \frac{T}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(72)(51.24)}{\pi(11.2 \times 10^6)} = 209.7 \times 10^{-6}\ \text{in}^4$$

$$c = 0.1203 \text{ in.} \qquad d = 2c = 0.241 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.63

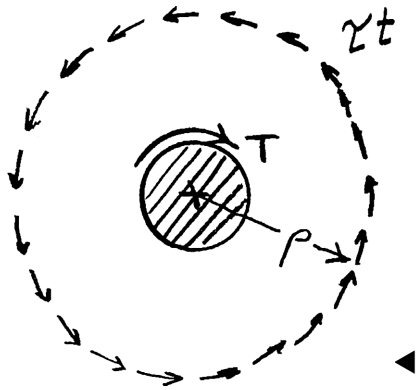
An annular plate of thickness t and modulus G is used to connect shaft AB of radius r_1 to tube CD of radius r_2. Knowing that a torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, (*a*) determine the magnitude and location of the maximum shearing stress in the annular plate, (*b*) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\varphi_{BC} = \frac{T}{4\pi Gt}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

SOLUTION

Use a free body consisting of shaft AB and an inner portion of the plate BC, the outer radius of this portion being ρ.

The force per unit length of circumference is τt.



$$\Sigma M = 0$$

$$\tau t(2\pi\rho)\rho - T = 0$$

$$\tau = \frac{T}{2\pi t\rho^2}$$

(*a*) Maximum shearing stress occurs at $\rho = r_1$ $\qquad \tau_{\max} = \dfrac{T}{2\pi t r_1^2}$ ◀

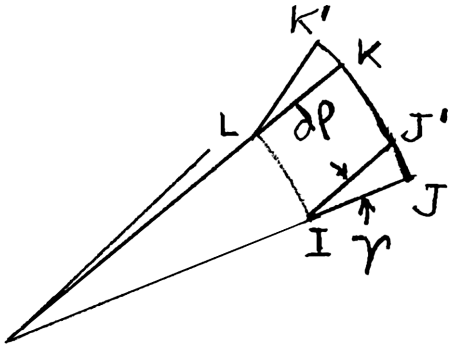
Shearing strain: $\qquad \gamma = \dfrac{\tau}{G} = \dfrac{T}{2\pi GT\rho^2}$

The relative circumferential displacement in radial length $d\rho$ is

$$d\delta = \gamma d\rho = \rho\, d\varphi$$

$$d\varphi = \gamma\frac{d\rho}{\rho}$$

$$d\varphi = \frac{T}{2\pi Gt\rho^2}\frac{d\rho}{\rho} = \frac{T\ d\rho}{2\pi GT\rho^3}$$



(*b*) $$\varphi_{B/C} = \int_{r_1}^{r_2}\frac{T\ d\rho}{2\pi Gt\rho^3} = \frac{T}{2\pi Gt}\int_{r_1}^{r_2}\frac{d\rho}{\rho^3} = \frac{T}{2\pi Gt}\left\{-\frac{1}{2\rho^2}\right\}\Bigg|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi Gt}\left\{-\frac{1}{2r_2^2} + \frac{1}{2r_1^2}\right\} = \frac{T}{4\pi Gt}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.64

Determine the maximum shearing stress in a solid shaft of 12-mm diameter as it transmits 2.5 kW at a frequency of (*a*) 25 Hz, (*b*) 50 Hz.

SOLUTION

$$c = \frac{1}{2}d = 6 \text{ mm} = 0.006 \text{ m} \qquad P = 2.5 \text{ kW} = 2500 \text{ W}$$

(*a*) $\underline{f = 25 \text{ Hz}}$

$$T = \frac{P}{2\pi f} = \frac{2500}{2\pi(25)} = 15.9155 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(15.9155)}{\pi(0.006)^3} = 46.9 \times 10^6 \text{ Pa} \qquad \tau = 46.9 \text{ MPa} \blacktriangleleft$$

(*b*) $\underline{f = 50 \text{ Hz}}$

$$T = \frac{2500}{2\pi(50)} = 7.9577 \text{ N} \cdot \text{m}$$

$$\tau = \frac{2(7.9577)}{\pi(0.006)^3} = 23.5 \times 10^6 \text{ Pa} \qquad \tau = 23.5 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.65

Determine the maximum shearing stress in a solid shaft of 1.5-in. diameter as it transmits 75 hp at a speed of (*a*) 750 rpm, (*b*) 1500 rpm.

SOLUTION

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad P = 75 \text{ hp} = (75)(6600) = 495 \times 10^3 \text{ lb} \cdot \text{in/s}$$

(*a*) $$f = \frac{750}{60} = 12.5 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{495 \times 10^3}{2\pi(12.5)} = 6.3025 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(6.3025 \times 10^3)}{\pi (0.75)^3} = 9.51 \times 10^3 \text{ psi} \qquad \tau = 9.51 \text{ ksi} \blacktriangleleft$$

(*b*) $$f = \frac{1500}{60} = 25 \text{ Hz}$$

$$T = \frac{495 \times 10^3}{2\pi(25)} = 3.1513 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{(2)(3.1513 \times 10^3)}{\pi (0.75)^3} = 4.76 \times 10^3 \text{ psi} \qquad \tau = 4.76 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.66

Design a solid steel shaft to transmit 0.375 kW at a frequency of 29 Hz, if the shearing stress in the shaft is not to exceed 35 MPa.

SOLUTION

$$\tau_{\text{all}} = 35 \times 10^6 \text{ Pa} \quad P = 0.375 \times 10^3 \text{ W} \quad f = 29 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{0.375 \times 10^3}{2\pi(29)} = 2.0580 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(2.0580)}{\pi(35 \times 10^6)} = 37.43 \times 10^{-9} \text{ m}^3$$

$$c = 3.345 \times 10^{-3} \text{ m} = 3.345 \text{ mm}$$

$$d = 2c \qquad d = 6.69 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.67

Design a solid steel shaft to transmit 100 hp at a speed of 1200 rpm, if the maximum shearing stress is not to exceed 7500 psi.

SOLUTION

$\tau_{\text{all}} = 7500\,\text{psi} \quad P = 100\,\text{hp} = 660 \times 10^3\,\text{lb} \cdot \text{in/s}$

$$f = \frac{1200}{60} = 20\,\text{Hz} \qquad T = \frac{P}{2\pi f} = \frac{660 \times 10^3}{2\pi(20)} = 5.2521 \times 10^3\,\text{lb} \cdot \text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(5.2521 \times 10^3)}{\pi(7500)} = 0.4458\ \text{in}^3$$

$c = 0.7639\,\text{in.} \qquad d = 2c$

$d = 1.528\ \text{in.}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.68

Determine the required thickness of the 50-mm tubular shaft of Example 3.07 if is to transmit the same power while rotating at a frequency of 30 Hz.

SOLUTION

From Example 3.07, $P = 100\,\text{kW} = 100 \times 10^3\,\text{W}$

$\tau_{\text{all}} = 60\,\text{MPa} = 60 \times 10^6\,\text{Pa}$ $\quad c_2 = \dfrac{1}{2}d = 25\,\text{mm} = 0.025\,\text{m}$

$f = 30\,\text{Hz}$

$$T = \frac{P}{2\pi f} = 530.52\,\text{N} \cdot \text{m}$$

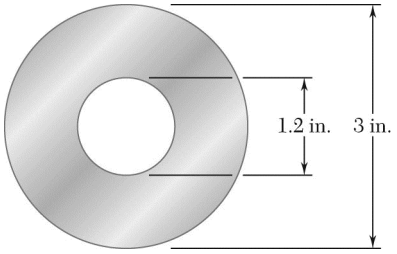
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) \qquad \tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.025^4 - \frac{(2)(530.52)(0.025)}{\pi(60 \times 10^6)} = 249.90 \times 10^{-9}\,\text{m}^4$$

$$c_1 = 22.358 \times 10^{-3}\,\text{m} = 22.358\,\text{mm}$$

$t = c_2 - c_1 = 25\,\text{mm} - 22.358\,\text{mm} = 2642\,\text{mm}$ $\qquad t = 2.64\,\text{mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.69

While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 12-ft length. Using $G = 11.2 \times 10^6$ psi, determine the power being transmitted.

SOLUTION

$$\varphi = 2^\circ = 34.907 \times 10^{-3} \text{ rad} \qquad L = 12 \text{ ft} = 144 \text{ in.}$$

$$c_2 = \frac{1}{2} d_o = 1.5 \text{ in.} \qquad c_1 = \frac{1}{2} d_i = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 0.6^4) = 7.7486 \text{ in}^4$$

$$f = \frac{120}{60} = 2 \text{ Hz}$$

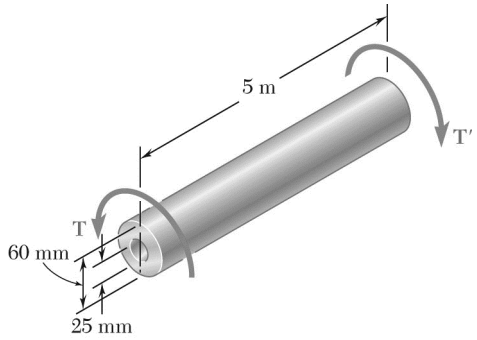
$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(7.7486)(34.907 \times 10^{-3})}{144} = 21.037 \times 10^3 \text{ lb} \cdot \text{in}$$

$$P = 2\pi f T = 2\pi (2)(21.037 \times 10^3) = 264.36 \times 10^3 \text{ lb} \cdot \text{in/s}$$

Since 1 hp = 6600 lb · in/s,

$$P = 40.1 \text{ hp} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.70

The hollow steel shaft shown (G = 77.2 GPa, τ_{all} = 50 MPa) rotates at 240 rpm. Determine (*a*) the maximum power that can be transmitted, (*b*) the corresponding angle of twist of the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6 \text{mm}^4 = 1.234 \times 10^{-6} \text{m}^4$$

$$\tau_m = 50 \times 10^6 \text{ Pa}$$

$$\tau_m = \frac{Tc}{J} \quad T = \frac{\tau_m J}{c} = \frac{(50 \times 10^6)(1.234 \times 10^{-6})}{30 \times 10^{-3}} = 2056.7 \text{ N} \cdot \text{m}$$

Angular speed. $f = 240 \text{ rpm} = 4 \text{ rev/sec} = 4 \text{ Hz}$

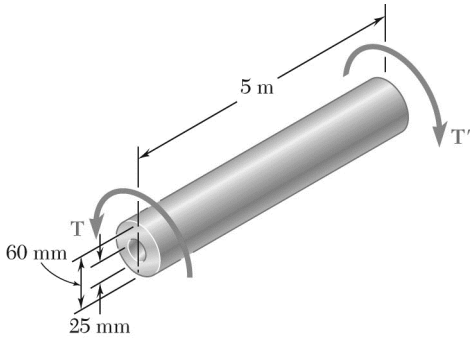
(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(4)(2056.7) = 51.7 \times 10^3 \text{ W}$

$$P = 51.7 \text{ kW} \blacktriangleleft$$

(*b*) Angle of twist. $\varphi = \frac{TL}{GJ} = \frac{(2056.7)(5)}{(77.2 \times 10^9)(1.234 \times 10^{-6})} = 0.1078 \text{ rad}$

$$\varphi = 6.17° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.71

As the hollow steel shaft shown rotates at 180 rpm, a stroboscopic measurement indicates that the angle of twist of the shaft is 3°. Knowing that $G = 77.2$ GPa, determine (*a*) the power being transmitted, (*b*) the maximum shearing stress in the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4)]$$

$$= 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

$$\varphi = 3° = 0.05236 \text{ rad}$$

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(1.234 \times 10^{-6})(0.0536)}{5} = 997.61 \text{ N} \cdot \text{m}$$

Angular speed: $f = 180 \text{ rpm} = 3 \text{ rev/sec} = 3 \text{ Hz}$

(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(3)(997.61) = 18.80 \times 10^3 \text{ W}$

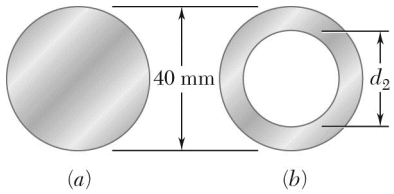
$$P = 18.80 \text{ kW} \blacktriangleleft$$

(*b*) Maximum shearing stress. $\tau_m = \frac{Tc_2}{J} = \frac{(997.61)(30 \times 10^{-3})}{1.234 \times 10^{-6}}$

$$= 24.3 \times 10^6 \text{ Pa} \qquad \tau_m = 24.3 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.72



The design of a machine element calls for a 40-mm-outer-diameter shaft to transmit 45 kW. (*a*) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft *a*. (*b*) If the speed of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft *b* for which the maximum shearing stress will be the same in each shaft.

SOLUTION

(*a*) $f = \dfrac{720}{60} = 12 \text{ Hz}$

$P = 45 \text{ kW} = 45 \times 10^3 \text{ W}$

$T = \dfrac{P}{2\pi f} = \dfrac{45 \times 10^3}{2\pi(12)} = 596.83 \text{ N} \cdot \text{m}$

$c = \dfrac{1}{2} d = 20 \text{ mm} = 0.020 \text{ m}$

$\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3} = \dfrac{(2)(596.83)}{\pi(0.020)^3} = 47.494 \times 10^6 \text{ Pa}$ $\qquad \tau_{\max} = 47.5 \text{ MPa}$ ◀

(*b*) $f = \dfrac{1080}{60} = 18 \text{ Hz}$

$T = \dfrac{45 \times 10^3}{2\pi(18)} = 397.89 \text{ N} \cdot \text{m}$

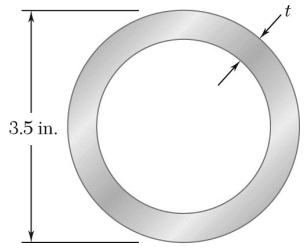
$\tau = \dfrac{Tc_2}{J} = \dfrac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

$c_1^4 = c_2^4 - \dfrac{2Tc_2}{\pi\tau}$

$c_1^4 = 0.020^4 - \dfrac{(2)(397.89)(0.020)}{\pi(47.494 \times 10^6)} = 53.333 \times 10^{-9}$

$c_1 = 15.20 \times 10^{-3} \text{m} = 15.20 \text{ mm}$ $\qquad d_2 = 2c_1 = 30.4 \text{ mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.73

A steel pipe of 3.5-in. outer diameter is to be used to transmit a torque of 3000 lb · ft without exceeding an allowable shearing stress of 8 ksi. A series of 3.5-in.-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 0.25 in. to 0.50 in. in 0.0625-in. increments, choose the lightest pipe that can be used.

SOLUTION

$$T = 3000 \text{ lb} \cdot \text{ft} = 36 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c_2 = \frac{1}{2} d_o = 1.75 \text{ in.}$$

$$\tau = \frac{Tc_2}{J} = \frac{2\text{T}c_2}{\pi\left(c_2^4 - c_1^4\right)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 1.75^4 - \frac{(2)(36 \times 10^3)(1.75)}{\pi(8 \times 10^3)} = 4.3655 \text{ in}^4$$

$$c_1 = 1.4455 \text{ in.}$$

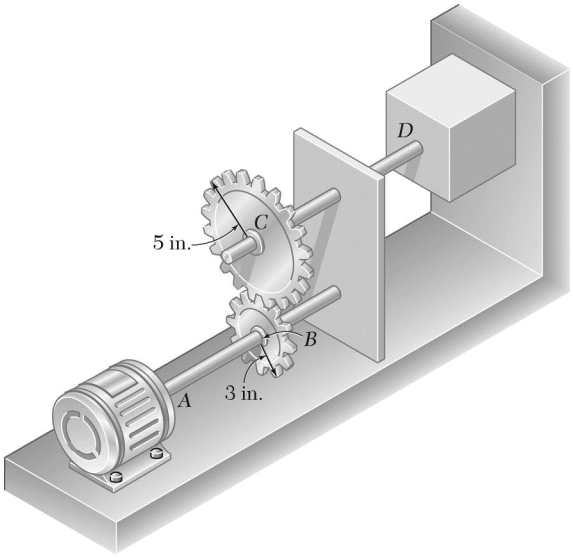
Required minimum thickness: $t = c_2 - c_1$

$$t = 1.75 - 1.4455 = 0.3045 \text{ in.}$$

Available thicknesses: 0.25 in., 0.3125 in., 0.375 in., etc.

Use $t = 0.3125$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.74

The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A,* operating at a speed of 1260 rpm, to a machine tool at *D*. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter (*a*) of shaft *AB*, (*b*) of shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb} \cdot \text{in/sec}$$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$\tau = 8 \text{ ksi} = 8 \times 10^3 \text{ psi}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c = \sqrt[3]{\frac{2T}{\pi\tau}}$$

$$c = \sqrt[3]{\frac{(2)(800.32)}{\pi(8 \times 10^3)}} = 0.399 \text{ in.}$$

$$d_{AB} = 2c = 0.799 \text{ in.} \qquad d_{AB} = 0.799 \text{ in.} \blacktriangleleft$$

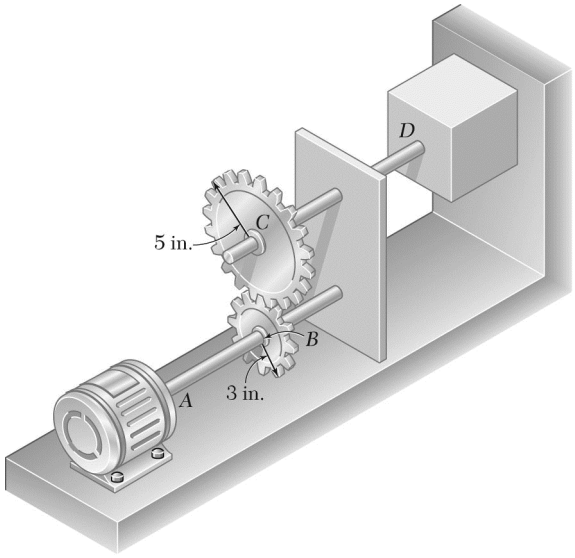
(*b*) Shaft *CD*:

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb} \cdot \text{in}$$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(1.33387 \times 10^3)}{\pi(8 \times 10^3)}} = 0.473 \text{ in.}$$

$$d_{CD} = 2c = 0.947 \text{ in.} \qquad d_{CD} = 0.947 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.75

The two solid shafts and gears shown are used to transmit 16 hp from the motor at *A* operating at a speed of 1260 rpm to a machine tool at *D*. Knowing that each shaft has a diameter of 1 in., determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$P = 16 \text{ hp} = (16)(6600) = 105.6 \times 10^3 \text{ lb} \cdot \text{in/sec}$$

$$f = \frac{1260}{60} = 21 \text{ Hz}$$

$$T_{AB} = \frac{P}{2\pi f} = \frac{105.6 \times 10^3}{2\pi(21)} = 800.32 \text{ lb} \cdot \text{in}$$

$$c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

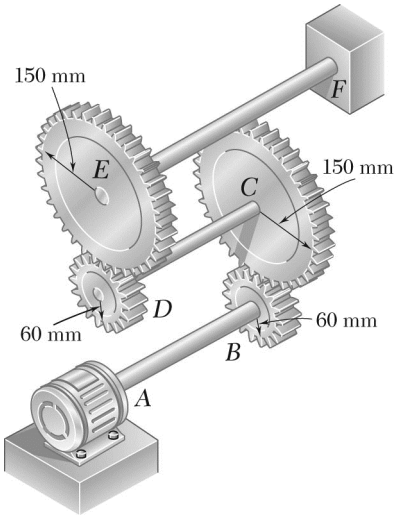
$$= \frac{(2)(800.32)}{\pi(0.5)^3} = 4.08 \times 10^3 \text{ psi} \qquad \tau_{AB} = 4.08 \text{ ksi} \blacktriangleleft$$

(*b*) Shaft *CD*:

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{5}{3}(800.32) = 1.33387 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{2T}{\pi c^3} = \frac{(2)(1.33387 \times 10^3)}{\pi(0.5)^3} = 6.79 \times 10^3 \text{ psi} \qquad \tau_{CD} = 6.79 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.76

Three shafts and four gears are used to form a gear train that will transmit 7.5 kW from the motor at *A* to a machine tool at *F*. (Bearings for the shafts are omitted in the sketch.) Knowing that the frequency of the motor is 30 Hz and that the allowable stress for each shaft is 60 MPa, determine the required diameter of each shaft.

SOLUTION

$$P = 7.5 \text{ kW} = 7.5 \times 10^3 \text{ W} \qquad \tau_{\text{all}} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

Shaft *AB*:

$$f_{AB} = 30 \text{ Hz} \qquad T_{AB} = \frac{P}{2\pi f_{AB}} = \frac{7.5 \times 10^3}{2\pi(30)} = 39.789 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^3} \quad \therefore \quad c_{AB}^3 = \frac{2T}{\pi\tau}$$

$$c_{AB}^3 = \frac{(2)(39.789)}{\pi(60 \times 10^6)} = 422.17 \times 10^{-9} \text{ m}^3$$

$$c_{AB} = 7.50 \times 10^{-3} \text{ m} = 7.50 \text{ mm} \qquad d_{AB} = 2c_{AB} = 15.00 \text{ mm} \blacktriangleleft$$

Shaft *CD*:

$$f_{CD} = \frac{r_B}{r_C} f_{AB} = \frac{60}{150}(30) = 12 \text{ Hz} \qquad T_{CD} = \frac{P}{2\pi f_{CD}} = \frac{7.5 \times 10^3}{2\pi(12)} = 99.472 \text{ N} \cdot \text{m}$$

$$\tau_{CD} = \frac{Tc_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^3} \quad \therefore \quad c_{CD}^3 = \frac{2T_{CD}}{\pi\tau} = \frac{2(99.472)}{\pi(60 \times 10^6)} = 1.05543 \times 10^{-6} \text{ m}^3$$

$$c_{CD} = 10.18 \times 10^{-3} \text{ m} = 10.18 \text{ mm} \qquad d_{CD} = 2c_{CD} = 20.4 \text{ mm} \blacktriangleleft$$

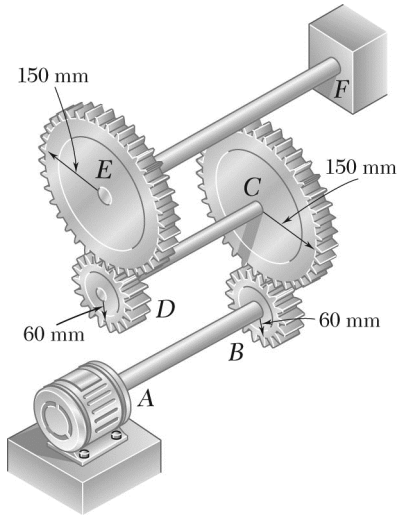
Shaft *EF*:

$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150}(12) = 4.8 \text{ Hz} \qquad T_{EF} = \frac{P}{2\pi f_{EF}} = \frac{7.5 \times 10^3}{2\,\pi\,(4.8)} = 248.68 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc_{EF}}{J_{EF}} = \frac{2T}{\pi c_{EF}^3} \quad \therefore \quad c_{EF}^3 = \frac{(2)(248.68)}{\pi(60 \times 10^6)} = 2.6886 \times 10^{-6} \text{ m}^3$$

$$c_{EF} = 13.82 \times 10^{-3} \text{ m} = 13.82 \text{ mm} \qquad d_{EF} = 2c_{EF} = 27.6 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.77

Three shafts and four gears are used to form a gear train that will transmit power from the motor at A to a machine tool at F. (Bearings for the shafts are omitted in the sketch.) The diameter of each shaft is as follows: $d_{AB}=16$ mm, $d_{CD}=20$ mm, $d_{EF}=28$ mm. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau_{\text{all}}=75\text{ MPa}=75\times10^6\text{ Pa}$$

Shaft AB:

$$c_{AB}=\frac{1}{2}d_{AB}=0.008\text{ m}\qquad \tau=\frac{Tc_{AB}}{J_{AB}}=\frac{2T}{\pi c_{AB}^3}$$

$$T_{\text{all}}=\frac{\pi}{2}c_{AB}^3\tau_{\text{all}}=\frac{\pi}{2}(0.008)^3(75\times10^6)=60.319\text{ N}\cdot\text{m}$$

$$f_{AB}=24\text{ Hz}\quad P_{\text{all}}=2\pi f_{AB}T_{\text{all}}=2\pi(24)(60.319)=9.10\times10^3\text{ W}$$

Shaft CD:

$$c_{CD}=\frac{1}{2}d_{CD}=0.010\text{ m}$$

$$\tau=\frac{Tc_{CD}}{J_{CD}}=\frac{2T}{\pi c_{CD}^3}\quad\therefore\quad T_{\text{all}}=\frac{\pi}{2}c_{CD}^3\tau_{\text{all}}=\frac{\pi}{2}(0.010)^3(75\times10^6)=117.81\text{ N}\cdot\text{m}$$

$$f_{CD}=\frac{r_B}{r_C}f_{AB}=\frac{60}{150}(24)=9.6\text{ Hz}\qquad P_{\text{all}}=2\pi f_{CD}T_{\text{all}}=2\pi(9.6)(117.81)=7.11\times10^3\text{ W}$$

Shaft EF:

$$c_{EF}=\frac{1}{2}d_{EF}=0.014\text{ m}$$

$$T_{\text{all}}=\frac{\pi}{2}c_{EF}^3\tau_{\text{all}}=\frac{\pi}{2}(0.014)^3(75\times10^6)=323.27\text{ N}\cdot\text{m}$$

$$f_{EF}=\frac{r_D}{r_E}f_{CD}=\frac{60}{150}(9.6)=3.84\text{ Hz}$$

$$P_{\text{all}}=2\pi f_{EF}T_{\text{all}}=2\pi(3.84)(323.27)=7.80\times10^3\text{ W}$$

Maximum allowable power is the smallest value. $P_{\text{all}}=7.11\times10^3\text{ W}=7.11\text{ kW}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.78

A 1.5-m-long solid steel of 48 mm diameter is to transmit 36 kW between a motor and a machine tool. Determine the lowest speed at which the shaft can rotate, knowing that $G = 77.2$ GPa, that the maximum shearing stress must not exceed 60 MPa, and the angle of twist must not exceed 2.5°.

SOLUTION

$$P = 36 \times 10^3 \text{ W}, \quad c = \frac{1}{2}d = 0.024 \text{ m}, \quad L = 1.5 \text{ m}, \quad G = 77.2 \times 10^9 \text{ Pa}$$

Torque based on maximum stress: $\tau = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} \quad T = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.024)^3(60 \times 10^6) = 1.30288 \times 10^3 \text{ N} \cdot \text{m}$$

Torque based on twist angle: $\varphi = 2.5° = 43.633 \times 10^{-3} \text{ rad}$

$$\varphi = \frac{TL}{GJ} \quad T = \frac{GJ\varphi}{L} = \frac{\pi c^4 G\varphi}{2L} = \frac{\pi(0.024)^4(77 \times 10^9)(43.633 \times 10^{-3})}{(2)(1.5)}$$

$$= 1.17033 \times 10^3 \text{ N} \cdot \text{m}$$

Smaller torque governs, so $T = 1.17033 \times 10^3 \text{ N} \cdot \text{m}$

$$P = 2\pi fT \qquad f = \frac{P}{2\pi T} = \frac{36 \times 10^3}{2\pi(1.17033 \times 10^3)}$$

$$f = 4.90 \text{ Hz} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.79

A 2.5-m-long steel shaft of 30-mm diameter rotates at a frequency of 30 Hz. Determine the maximum power that the shaft can transmit, knowing that $G = 77.2$ GPa, that the allowable shearing stress is 50 MPa, and that the angle of twist must not exceed 7.5°.

SOLUTION

$$c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m} \quad L = 2.5 \text{ m}$$

Stress requirement.

$$\tau = 50 \times 10^6 \text{ Pa} \quad \tau = \frac{Tc}{J}$$

$$T = \frac{\tau J}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(50 \times 10^6)(0.015)^3 = 265.07 \text{ N} \cdot \text{m}$$

Twist angle requirement.

$$\varphi = 7.5° = 130.90 \times 10^{-3} \text{ rad} \quad G = 77.2 \times 10^9 \text{ Pa}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$T = \frac{\pi}{2}Gc^4\varphi = \frac{\pi}{2}(77.2 \times 10^9)(0.015)^4(130.90 \times 10^{-3}) = 803.60 \text{ N} \cdot \text{m}$$

Smaller value of T is the maximum allowable torque.

$$T = 265.07 \text{ N} \cdot \text{m}$$

Power transmitted at $f = 30$ Hz.

$$P = 2\pi f T = 2\pi(30)(265.07) = 49.96 \times 10^3 \text{ W} \qquad P = 50.0 \text{ kW} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.80

A steel shaft must transmit 210 hp at a speed of 360 rpm. Knowing that $G = 11.2 \times 10^6$ psi, design a solid shaft so that the maximum shearing stress will not exceed 12 ksi, and the angle of twist in a 8.2-ft length must not exceed 3°.

SOLUTION

Power: $P = (210 \text{ hp})(6600 \text{ in} \cdot \text{lb/s/hp}) = 1.336 \times 10^6 \text{ in} \cdot \text{lb/s}$

Angular speed: $f = (360 \text{ rpm}) \dfrac{1 \text{ min.}}{60 \text{ sec}} = 6 \text{ Hz}$

Torque: $T = \dfrac{P}{2\pi f} = \dfrac{1.386 \times 10^6}{(2\pi)(6)} = 36.765 \times 10^3 \text{ lb} \cdot \text{in}$

Stress requirement: $\tau = 12 \text{ ksi}, \quad \tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(36.765 \times 10^3)}{\pi(12 \times 10^3)}} = 1.2494 \text{ in.}$$

Angle of twist requirement: $\varphi = 3° = 52.36 \times 10^{-3} \text{ rad}$

$L = 8.2 \text{ ft} = 98.4 \text{ in.}$

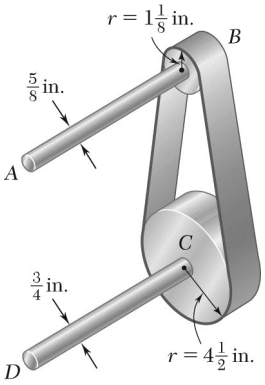
$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c = \sqrt[4]{\frac{2TL}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(36.765 \times 10^3)(98.4)}{\pi(11.2 \times 10^6)(52.36 \times 10^{-3})}} = 1.4077 \text{ in.}$$

The larger value is the required radius. $c = 1.408 \text{ in.}$

$d = 2c$ $\qquad d = 2.82 \text{ in.}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.81

The shaft-disk-belt arrangement shown is used to transmit 3 hp from point *A* to point *D*. (*a*) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft *AB*. (*b*) Solve part *a*, assuming that the diameters of shafts *AB* and *CD* are, respectively, 0.75 in. and 0.625 in.

SOLUTION

$\tau = 9500$ psi $\quad P = 3$ hp $= (3)(6600) = 19800$ lb · in/s

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2} c^3 \tau$$

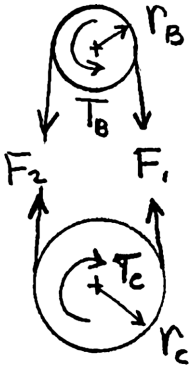
Allowable torques.

$\frac{5}{8}$-in. diameter shaft: $\quad c = \dfrac{5}{16}\text{in.}, \quad T_{\text{all}} = \dfrac{\pi}{2}\left(\dfrac{5}{16}\right)^3 (9500) = 455.4 \text{ lb} \cdot \text{in}$

$\frac{3}{4}$-in. diameter shaft: $\quad c = \frac{3}{8}\text{in.}, \quad T_{\text{all}} = \dfrac{\pi}{2}\left(\dfrac{3}{8}\right)^3 (9500) = 786.9 \text{ lb} \cdot \text{in}$

Statics: $\quad T_B = r_B(F_1 - F_2) \qquad T_C = r_C(F_1 - F_2)$

$$T_B = \frac{r_B}{r_C} T_C = \frac{1.125}{4.5} T_C = 0.25 T_C$$



(*a*) Allowable torques. $\quad T_{B,\text{all}} = 455.4 \text{ lb} \cdot \text{in} \qquad T_{C,\text{all}} = 786.9 \text{ lb} \cdot \text{in}$

Assume $\quad T_C = 786.9 \text{ lb} \cdot \text{in}$

Then $\quad T_B = (0.25)(786.9) = 196.73 \text{ lb} \cdot \text{in} < 455.4 \text{ lb} \cdot \text{in}$ (okay)

$$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(196.73)} \qquad f_{AB} = 16.02 \text{ Hz} \blacktriangleleft$$

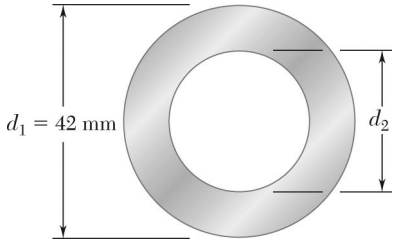
(*b*) Allowable torques. $\quad T_{B,\text{all}} = 786.9 \text{ lb} \cdot \text{in} \qquad T_{C,\text{all}} = 455.4 \text{ lb} \cdot \text{in}$

Assume $\quad T_C = 455.4 \text{ lb} \cdot \text{in}$

Then $\quad T_B = (0.25)(455.4) = 113.85 \text{ lb} \cdot \text{in} < 786.9 \text{ lb} \cdot \text{in}$

$$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(113.85)} \qquad f_{AB} = 27.2 \text{ Hz} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.82

A 1.6-m-long tubular steel shaft of 42-mm outer diameter d_1 is to be made of a steel for which $\tau_{all} = 75\,\text{MPa}$ and $G = 77.2\,\text{GPa}$. Knowing that the angle of twist must not exceed 4° when the shaft is subjected to a torque of 900 N · m, determine the largest inner diameter d_2 that can be specified in the design.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.021\,\text{m} \quad L = 1.6\,\text{m}$$

Based on stress limit: $\tau = 75\,\text{MPa} = 75 \times 10^6\ \text{Pa}$

$$\tau = \frac{Tc_1}{J} \quad \therefore \quad J = \frac{Tc_1}{\tau} = \frac{(900)(0.021)}{75 \times 10^6} = 252 \times 10^{-9}\,\text{m}^4$$

Based on angle of twist limit: $\varphi = 4° = 69.813 \times 10^{-3}\,\text{rad}$

$$\varphi = \frac{TL}{GJ} \quad \therefore \quad J = \frac{TL}{G\varphi} = \frac{(900)(1.6)}{(77 \times 10^9)(69.813 \times 10^{-3})} = 267.88 \times 10^{-9}\ \text{m}^4$$

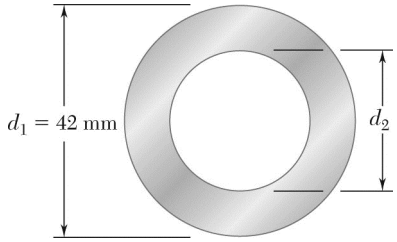
Larger value for J governs. $J = 267.88 \times 10^{-9}\,\text{m}^4$

$$J = \frac{\pi}{2}\left(c_1^4 - c_2^4\right)$$

$$c_2^4 = c_1^4 - \frac{2J}{\pi} = 0.021^4 - \frac{(2)(267.88 \times 10^{-9})}{\pi} = 23.943 \times 10^{-9}\,\text{m}^4$$

$$c_2 = 12.44 \times 10^{-3}\ \text{m} = 12.44\ \text{mm} \qquad d_2 = 2c_2 = 24.9\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.83

A 1.6-m-long tubular steel shaft ($G = 77.2\,\text{GPa}$) of 42-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 120 kW between a turbine and a generator. Knowing that the allowable shearing stress is 65 MPa and that the angle of twist must not exceed 3°, determine the minimum frequency at which the shaft can rotate.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.021\text{ m}, \quad c_2 = \frac{1}{2}d_2 = 0.015\text{ m}$$

$$J = \frac{\pi}{2}\left(c_1^4 - c_2^4\right) = \frac{\pi}{2}(0.021^4 - 0.015^4) = 225.97 \times 10^{-9}\text{ m}^4$$

Based on stress limit: $\tau = 65\,\text{MPa} = 65 \times 10^6\text{ Pa}$

$$\tau = \frac{Tc_1}{J} \quad \text{or} \quad T = \frac{J\tau}{G} = \frac{(225.97 \times 10^{-9})(65 \times 10^6)}{0.021} = 699.43\text{ N} \cdot \text{m}$$

Based on angle of twist limit: $\varphi = 3° = 52.36 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \text{or} \quad T = \frac{GJ\varphi}{L} = \frac{(77 \times 10^9)(225.97 \times 10^{-9})(52.36 \times 10^{-3})}{1.6}$$
$$= 569.40\text{ N} \cdot \text{m}$$

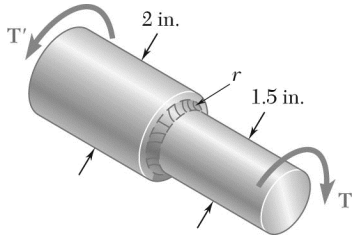
Smaller torque governs. $T = 569.40\text{ N} \cdot \text{m}$

$$P = 120\text{ kW} = 120 \times 10^3\text{ W}$$

$$P = 2\pi fT \quad \text{so} \quad f = \frac{P}{2\pi T} = \frac{120 \times 10^3}{2\pi(569.40)} \qquad f = 33.5\text{ Hz} \blacktriangleleft$$

or 2010 rpm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.84

Knowing that the stepped shaft shown transmits a torque of magnitude $T = 2.50\,\text{kip} \cdot \text{in.}$, determine the maximum shearing stress in the shaft when the radius of the fillet is (*a*) $r = \frac{1}{8}\,\text{in.}$, (*b*) $r = \frac{3}{16}\,\text{in.}$

SOLUTION

$$D = 2\ \text{in.} \quad d = 1.5\ \text{in.} \qquad \frac{D}{d} = \frac{2}{1.5} = 1.33$$

$$c = \frac{1}{2}d = 0.75\ \text{in.} \qquad T = 2.5\ \text{kip} \cdot \text{in}$$

$$\frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.5)}{\pi(0.75)^3} = 3.773\ \text{ksi}$$

(*a*) $r = \frac{1}{8}\,\text{in.} \quad r = 0.125\ \text{in.}$

$$\frac{r}{d} = \frac{0.125}{1.5} = 0.0833$$

From Fig. 3.32, $K = 1.42$

$$\tau_{\max} = K\frac{Tc}{J} = (1.42)(3.773) \qquad \tau_{\max} = 5.36\ \text{ksi} \blacktriangleleft$$

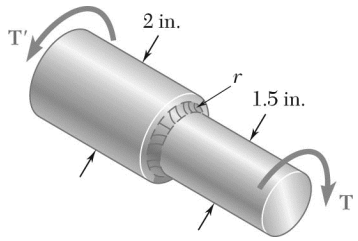
(*b*) $r = \frac{3}{16}\,\text{in.} \qquad r = 0.1875\ \text{in.}$

$$\frac{r}{d} = \frac{0.1875}{1.5} = 0.125$$

From Fig. 3.32, $K = 1.33$

$$\tau_{\max} = K\frac{Tc}{J} = (1.33)(3.773) \qquad \tau_{\max} = 5.02\ \text{ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.85

Knowing that the allowable shearing stress is 8 ksi for the stepped shaft shown, determine the magnitude T of the largest torque that can be transmitted by the shaft when the radius of the fillet is (*a*) $r = \frac{3}{16}$ in., (*b*) $r = \frac{1}{4}$ in.

SOLUTION

$D = 2$ in. $\quad d = 1.5$ in. $\quad \dfrac{D}{d} = 1.33$

$c = \dfrac{1}{2}d = 0.75$ in. $\quad \tau_{\max} = 8$ ksi

$\tau_{\max} = K\dfrac{Tc}{J}$ or $T = \dfrac{J\tau_{\max}}{Kc} = \dfrac{\pi\tau_{\max}c^3}{2K}$

(*a*) $r = \dfrac{3}{16}$ in. $\quad r = 0.1875$ in.

$\dfrac{r}{d} = \dfrac{0.1875}{1.5} = 0.125$

From Fig. 3.32, $K = 1.33$

$T = \dfrac{\pi(8)(0.75)^3}{(2)(1.33)}$ $\quad T = 3.99$ kip · in ◀

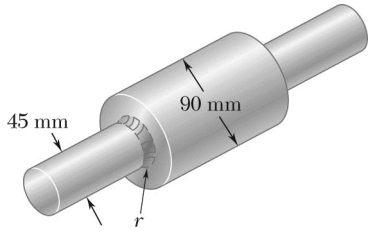
(*b*) $r = \dfrac{1}{4}$ in. $\quad r = 0.25$ in.

$\dfrac{r}{d} = \dfrac{0.25}{1.5} = 0.1667$

From Fig. 3.32, $K = 1.27$

$T = \dfrac{\pi(8)(0.75)^3}{(2)(1.27)}$ $\quad T = 4.17$ kip · in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.86

The stepped shaft shown must transmit 40 kW at a speed of 720 rpm. Determine the minimum radius r of the fillet if an allowable stress of 36 MPa is not to be exceeded.

SOLUTION

Angular speed: $f = (720 \text{ rpm})\left(\dfrac{1 \text{ Hz}}{60 \text{ rpm}}\right) = 12 \text{ Hz}$

Power: $P = 40 \times 10^3 \text{ W}$

Torque: $T = \dfrac{P}{2\pi f} = \dfrac{40 \times 10^3}{2\pi(12)} = 530.52 \text{ N} \cdot \text{m}$

In the smaller shaft, $d = 45 \text{ mm}, \quad c = 22.5 \text{ mm} = 0.0225 \text{ m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(530.52)}{\pi(0.0225)^3} = 29.65 \times 10^6 \text{ Pa}$$

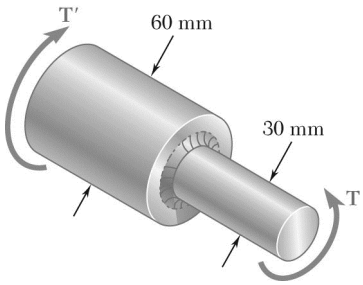
Using $\tau_{\max} = 36 \text{ MPa} = 36 \times 10^6 \text{ Pa}$ results in

$$K = \frac{\tau_{\max}}{\tau} = \frac{36 \times 10^6}{29.65 \times 10^6} = 1.214$$

From Fig 3.32 with $\dfrac{D}{d} = \dfrac{90 \text{ mm}}{45 \text{ mm}} = 2, \quad \dfrac{r}{d} = 0.24$

$r = 0.24\,d = (0.24)(45 \text{ mm})$ $\qquad r = 10.8 \text{ mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.87

The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.

SOLUTION

$$\frac{r}{d} = \frac{6}{30} = 0.2$$

$$\frac{D}{d} = \frac{60}{30} = 2$$

From Fig. 3.32, $K = 1.26$

For smaller side, $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

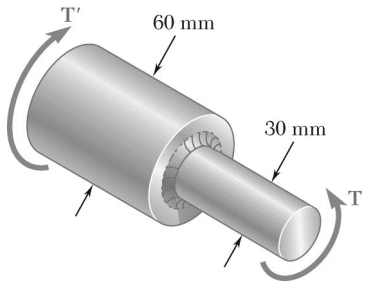
$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.015)^3(40 \times 10^6)}{(2)(1.26)} = 168.30 \text{ N} \cdot \text{m}$$

$$P = 45 \text{ kW} = 45 \times 10^3 \quad P = 2\pi fT$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi(168.30 \times 10^3)} = 42.6 \text{ Hz} \qquad f = 42.6 \text{ Hz} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.88

The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \quad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \quad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$D = 60 \text{ mm}, \quad r = 8 \text{ mm}$$

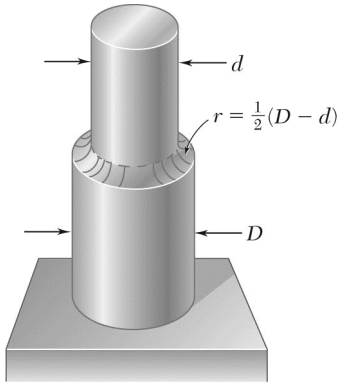
$$\frac{D}{d} = \frac{60}{30} = 2, \quad \frac{r}{d} = \frac{8}{30} = 0.26667$$

From Fig. 3.32, $K = 1.18$

Allowable torque. $$T = \frac{\pi(15 \times 10^{-3})^3(45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N} \cdot \text{m}$$

Maximum power. $$P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$$ $P = 63.5$ kW ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.89

In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.25$ in. and $d = 1$ in. Knowing that the speed of the shaft is 2400 rpm and that the allowable shearing stress is 7500 psi, determine the maximum power that can be transmitted by the shaft.

SOLUTION

$$\frac{D}{d} = \frac{1.25}{1.0} = 1.25$$

$$r = \frac{1}{2}(D - d) = 0.15 \text{ in.}$$

$$\frac{r}{d} = \frac{0.15}{1.0} = 0.15$$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \frac{1}{2}d = 0.5$ in.

$$\tau = \frac{KTc}{J} \quad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$$

$$T = \frac{\pi(0.5)^3(7500)}{(2)(1.31)} = 1.1241 \times 10^3 \text{ lb} \cdot \text{in}$$

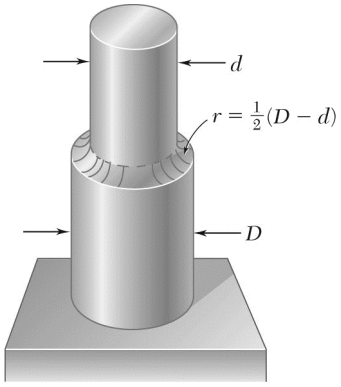
$$f = 2400 \text{ rpm} = 40 \text{ Hz}$$

$$P = 2\pi f T = 2\pi(40)(1.1241 \times 10^3)$$

$$= 282.5 \times 10^3 \text{ lb} \cdot \text{in/s}$$

$$P = 42.8 \text{ hp} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Full quarter-circular fillet extends to edge of larger shaft.

PROBLEM 3.90

A torque of magnitude $T = 200$ lb · in. is applied to the stepped shaft shown, which has a full quarter-circular fillet. Knowing that $D = 1$ in., determine the maximum shearing stress in the shaft when (*a*) $d = 0.8$ in., (*b*) $d = 0.9$ in.

SOLUTION

(*a*) $\dfrac{D}{d} = \dfrac{1.0}{0.8} = 1.25$

$r = \dfrac{1}{2}(D - d) = 0.1$ in.

$\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125$

From Fig. 3.32, $K = 1.31$

For smaller side, $c = \dfrac{1}{2}d = 0.4$ in.

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$= \frac{(2)(1.31)(200)}{\pi(0.4)^3} = 2.61 \times 10^3 \text{ psi} \qquad \tau = 2.61 \text{ ksi} \blacktriangleleft$$

(*b*) $\dfrac{D}{d} = \dfrac{1.0}{0.9} = 1.111$

$r = \dfrac{1}{2}(D - d) = 0.05$

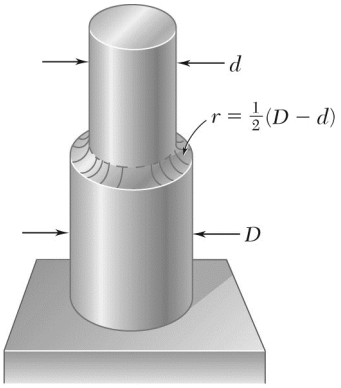
$\dfrac{r}{d} = \dfrac{0.05}{1.0} = 0.05$

From Fig. 3.32, $K = 1.44$

For smaller side, $c = \dfrac{1}{2}d = 0.45$ in.

$$\tau = \frac{2KT}{\pi c^3} = \frac{(2)(1.44)(200)}{\pi(0.45)^3} = 2.01 \times 10^3 \text{ psi} \qquad \tau = 2.01 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Full quarter-circular fillet extends to edge of larger shaft

PROBLEM 3.91

In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 80 MPa. Knowing that $D = 30$ mm, determine the largest allowable torque that can be applied to the shaft if (*a*) $d = 26$ mm, (*b*) $d = 24$ mm.

SOLUTION

$$\tau = 80 \times 10^6 \text{ Pa}$$

(*a*) $\dfrac{D}{d} = \dfrac{30}{26} = 1.154 \quad r = \dfrac{1}{2}(D - d) = 2 \text{ mm} \quad \dfrac{r}{d} = \dfrac{2}{26} = 0.0768$

From Fig. 3.32, $K = 1.36$

Smaller side, $c = \dfrac{1}{2}d = 13 \text{ mm} = 0.013 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.013)^3(80 \times 10^6)}{(2)(1.36)} = 203 \text{ N} \cdot \text{m} \qquad T = 203 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $\dfrac{D}{d} = \dfrac{30}{24} = 1.25 \quad r = \dfrac{1}{2}(D - d) = 3 \text{ mm} \quad \dfrac{r}{d} = \dfrac{3}{24} = 0.125$

From Fig. 3.32, $K = 1.31$

$$c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$$

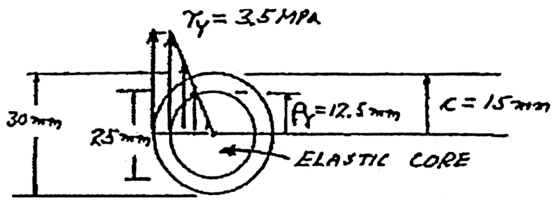
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.012)^3(80 \times 10^6)}{(2)(1.31)} = 165.8 \text{ N} \cdot \text{m} \qquad T = 165.8 \text{ N} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.92

A 30-mm diameter solid rod is made of an elastoplastic material with $\tau_Y = 3.5$ MPa. Knowing that the elastic core of the rod is 25 mm in diameter, determine the magnitude of the applied torque **T**.

SOLUTION



$$\tau_Y = 3.5 \times 10^6 \text{ Pa}, \quad c = \frac{1}{2}(30 \text{ mm}) = 15 \text{ mm} = 0.015 \text{ m}$$

$$\rho_Y = \frac{1}{2}(25 \text{ mm}) = 12.5 \text{ mm} = 0.0125 \text{ m}$$

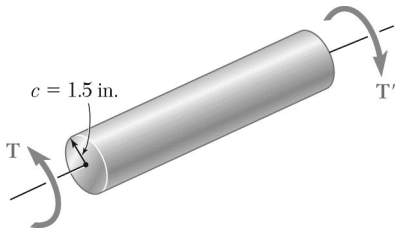
$$T_Y = \frac{J}{c}\tau_Y = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.015)^3(3.5 \times 10^6) = 18.555 \text{ N} \cdot \text{m}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(18.555)\left[1 - \frac{(0.0125)^3}{(0.015)^3}\right]$$

$$= 21.2 \text{ N} \cdot \text{m}$$

$T = 21.2 \text{ N} \cdot \text{m}$ ◀

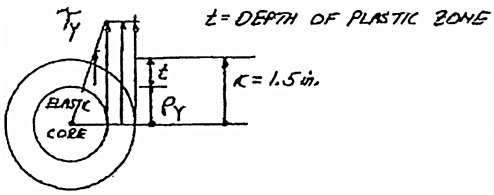
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.93

The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 21\,\text{ksi}$. Determine the magnitude T of the applied torques when the plastic zone is (*a*) 0.8 in. deep, (*b*) 1.2 in. deep.

SOLUTION



$$\tau_Y = 21\,\text{ksi}$$

$$T_Y = \frac{\tau_Y J}{c} = \tau_Y \frac{\pi}{2} c^3$$

$$= (21\,\text{ksi})\frac{\pi}{2}(1.5\,\text{in.})^3$$

$$T_Y = 111.3\,\text{kip}\cdot\text{in}$$

(*a*) For $t = 0.8$ in. $\rho_Y = 1.5 - 0.8 = 0.7$ in.

Eq. (3.32)

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right] = \frac{4}{3}(111.3\,\text{kip}\cdot\text{in})\left[1 - \frac{1}{4}\frac{(0.7\,\text{in.})^3}{(1.5\,\text{in.})^3}\right]$$

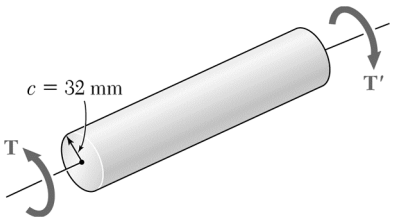
$$T = 144.7\,\text{kip}\cdot\text{in} \blacktriangleleft$$

(*b*) For $t = 1.2$ in. $\rho_Y = 1.5 - 1.2 = 0.3$ in.

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right] = \frac{4}{3}(111.3\,\text{kip}\cdot\text{in})\left[1 - \frac{1}{4}\frac{(0.3\,\text{in.})^3}{(1.5\,\text{in.})^3}\right]$$

$$T = 148.1\,\text{kip}\cdot\text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.94

The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the magnitude T of the applied torques when the plastic zone is (*a*) 16 mm deep, (*b*) 24 mm deep.

SOLUTION

$$c = 32 \text{ mm} = 0.032 \text{ m}$$

$$\tau_Y = 145 \times 10^6 \text{ Pa}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.032)^3(145 \times 10^6)$$

$$= 7.4634 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $t_P = 16 \text{ mm} = 0.016 \text{ m}$

$$\rho_Y = c - t_P = 0.032 - 0.016 = 0.016 \text{ m}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.016^3}{0.032^3}\right)$$

$$= 9.6402 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 9.64 \text{ kN} \cdot \text{m} \blacktriangleleft$$

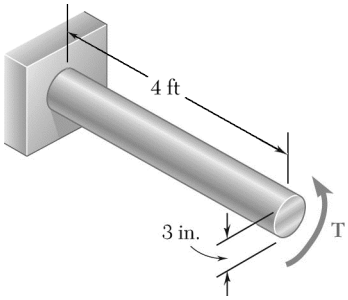
(*b*) $t_P = 24 \text{ mm} = 0.024 \text{ m}$

$$\rho_Y = c - t_P = 0.032 - 0.024 = 0.008 \text{ m}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(7.4634 \times 10^3)\left(1 - \frac{1}{4}\frac{0.008^3}{0.032^3}\right)$$

$$= 9.9123 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 9.91 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.95

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of torque of magnitude (*a*) $T = 100$ kip · in., (*b*) $T = 140$ kip · in.

SOLUTION

$$c = 1.5 \text{ in.}, \quad J = \frac{\pi}{2} c^4 = 7.9522 \text{ in}^4, \quad \tau_Y = 21 \text{ ksi}$$

(*a*) $T = 100$ kip · in

$$\tau_m = \frac{Tc}{J} = \frac{(100 \text{ kip} \cdot \text{in})(1.5 \text{ in.})}{7.9522 \text{ in}^4} \qquad \tau_m = 18.86 \text{ ksi} \blacktriangleleft$$

Since $\tau_m < \tau_Y$, shaft remains elastic.

Radius of elastic core: $c = 1.500$ in. ◀

(*b*) $T = 140$ kip · in

$$\tau_m = \frac{(140)(1.5)}{7.9522} = 26.4 \text{ ksi}.$$

Impossible: $\tau_m = \tau_Y = 21.0$ ksi ◀

Plastic zone has developed. Torque at onset of yield is $T_Y = \frac{J}{c}\tau_Y = \frac{7.9522}{1.5}(21 \text{ ksi}) = 111.33 \text{ kip} \cdot \text{in}$

Eq. (3.32): $T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$

$$\left(\frac{\rho_Y}{c}\right)^3 = 4 - 3\frac{T}{T_Y} = 4 - 3\frac{140}{111.33} = 0.22743 \qquad \frac{\rho_Y}{c} = 0.6104$$

$$\rho_Y = 0.6104c = 0.6104(1.5 \text{ in.}) \qquad \rho_Y = 0.916 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.96

It is observed that a straightened paper clip can be twisted through several revolutions by the application of a torque of approximately 60 mN · m. Knowing that the diameter of the wire in the paper clip is 0.9 mm, determine the approximate value of the yield stress of the steel.

SOLUTION

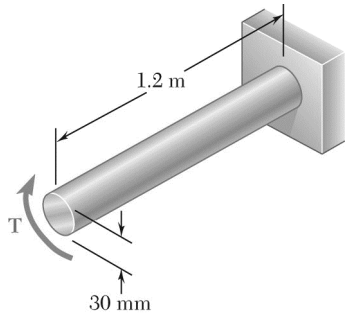
$$c = \frac{1}{2}d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m}$$

$$T_P = 60 \text{ mN} \cdot \text{m} = 60 \times 10^{-3} \text{ N} \cdot \text{m}$$

$$T_P = \frac{4}{3}T_Y = \frac{4}{3}\,\frac{J\tau_Y}{c} = \frac{4}{3} \cdot \frac{\pi}{2}c^3\ T_Y = \frac{2\pi}{3}c^3\tau_Y$$

$$\tau_Y = \frac{3T_P}{2\pi c^3} = \frac{(3)(60 \times 10^{-3})}{2\pi\ (0.45 \times 10^{-3})^3} = 314 \times 10^6 \text{ Pa} \qquad \tau_Y = 314 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.97

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 145\,\text{MPa}$. Determine the radius of the elastic core caused by the application of a torque equal to 1.1 T_Y, where T_Y is the magnitude of the torque at the onset of yield.

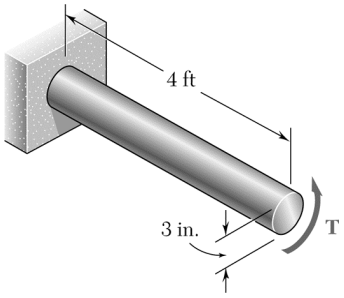
SOLUTION

$$c = \frac{1}{2}d = 15\text{ mm} \qquad T = \frac{4}{3}T_Y\left[1 - \left(\frac{\rho_Y}{c}\right)^3\right]$$

$$\frac{\rho_Y}{c} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt{4 - (3)(1.1)} = 0.88790$$

$$\rho_Y = 0.88790\,c = (0.88790)(15\text{ mm}) \qquad \rho_Y = 13.32\text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.98

For the solid circular shaft of Prob. 3.95, determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 80\ \text{kip} \cdot \text{in.}$, (*b*) $T = 130\ \text{kip} \cdot \text{in.}$

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(3) = 1.5\ \text{in.} \quad \tau_Y = 21 \times 10^3\ \text{psi}$$

$$L = 4\ \text{ft} = 48\ \text{in.} \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.5)^4 = 7.9522\ \text{in}^4$$

Torque at onset of yielding: $\tau = \dfrac{Tc}{J} \qquad T = \dfrac{\tau J}{c}$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(21\times10^3)(7.9522)}{1.5} = 111.330\times10^3\ \text{lb} \cdot \text{in}$$

(*a*) $\underline{\underline{T = 80\ \text{kip} \cdot \text{in}}} = 80\times10^3\ \text{lb} \cdot \text{in}$

Since $T < T_Y$, the shaft is fully elastic. $\varphi = \dfrac{TL}{GJ}$

$$\varphi = \frac{(80\times10^3)(48)}{(11.2\times10^6)(7.9522)} = 43.115\times10^{-3}\ \text{rad} \qquad \varphi = 2.47^\circ \blacktriangleleft$$

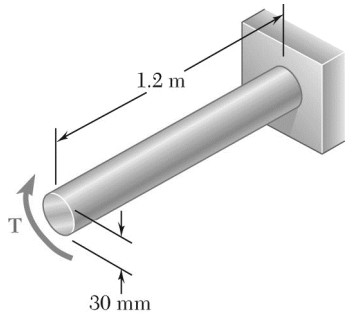
(*b*) $\underline{\underline{T = 130\ \text{kip} \cdot \text{in}}} = 130\times10^3\ \text{lb} \cdot \text{in} \qquad T > T_Y \qquad T = \dfrac{4}{3}T_Y\left[1 - \left(\dfrac{\varphi_Y}{\varphi}\right)^3\right]$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(111.330\times10^3)(48)}{(11.2\times10^6)(7.9522)} = 60.000\times10^{-3}\ \text{rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\frac{T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(130\times10^3)}{111.330\times10^3}} = 0.79205$$

$$\varphi = \frac{\varphi_Y}{0.79205} = \frac{60.000\times10^{-3}}{0.79205} = 75.75\times10^{-3}\ \text{rad} \qquad \varphi = 4.34^\circ \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.99

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600$ N · m, (*b*) $T = 1000$ N · m.

SOLUTION

$$c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

Torque at onset of yielding: $\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi (15 \times 10^{-3})^3 (145 \times 10^6)}{2} = 768.71 \text{ N} \cdot \text{m}$$

(*a*) $T = 600$ N · m. Since $T \le T_Y$, the shaft is elastic.

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(600)(1.2)}{\pi (15 \times 10^{-3})^4 (77.2 \times 10^9)} = 0.11728 \text{ rad} \quad \varphi = 6.72° \blacktriangleleft$$

(*b*) $T = 1000$ N · m. $T > T_Y$ A plastic zone has developed.

$$T = \frac{4}{3} T_Y \left[1 - \left(\frac{\varphi_Y}{\varphi} \right)^3 \right] \qquad \frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - 3\left(\frac{T}{T_Y} \right)}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(768.71)(2.1)}{\pi (15 \times 10^{-3})^4 (77.2 \times 10^9)} = 0.15026 \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{(3)(1000)}{768.71}} = 0.46003$$

$$\varphi = \frac{\varphi_Y}{0.46003} = \frac{0.15026}{0.46003} = 0.32663 \text{ rad} \qquad \varphi = 18.71° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.100

A 3-ft-long solid shaft has a diameter of 2.5 in. and is made of a mild steel that is assumed to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi. Determine the torque required to twist the shaft through an angle of (*a*) 2.5°, (*b*) 5°.

SOLUTION

$$L = 3 \text{ ft} = 36 \text{ in.}, \quad c = \frac{1}{2}d = 1.25 \text{ in.}, \quad \tau_Y = 21 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.25)^4 = 3.835 \text{ in}^4$$

$$\tau_Y = \frac{T_Y c}{J} \qquad T_Y = \frac{J\tau_Y}{c} = \frac{(3.835)(21 \times 10^3)}{1.25} = 64.427 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(64.427 \times 10^3)(36)}{(11.2 \times 10^6)(3.835)} = 53.999 \times 10^{-3} \text{ rad} = 3.0939°$$

(*a*) $\varphi = 2.5° = 43.633 \times 10^{-3}$ rad $\qquad \varphi < \varphi_Y$ The shaft remains elastic.

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(11.2 \times 10^6)(3.835)(43.633 \times 10^{-3})}{36}$$

$$= 52.059 \times 10^3 \text{ lb} \cdot \text{in} \qquad T = 52.1 \text{ kip} \cdot \text{in} \blacktriangleleft$$

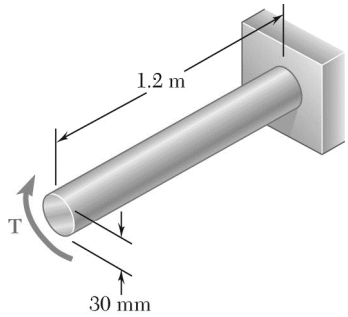
(*b*) $\varphi = 5° = 87.266 \times 10^{-3}$ rad $\qquad \varphi > \varphi_Y$ A plastic zone occurs.

$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right]$$

$$= \frac{4}{3}(64.427 \times 10^3)\left[1 - \frac{1}{4}\left(\frac{53.999 \times 10^{-3}}{87.266 \times 10^{-3}}\right)^3\right]$$

$$= 80.814 \times 10^3 \text{ lb} \cdot \text{in} \qquad T = 80.8 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.101

For the solid shaft of Prob. 3.99, determine (*a*) the magnitude of the torque **T** required to twist the shaft through an angle of 15°, (*b*) the radius of the corresponding elastic core.

PROBLEM 3.99 The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2$ GPa and $\tau_Y = 145$ MPa. Determine the angle of twist caused by the application of a torque of magnitude (*a*) $T = 600\ \text{N} \cdot \text{m}$, (*b*) $T = 1000\ \text{N} \cdot \text{m}$.

SOLUTION

$$c = \frac{1}{2}d = 15\,\text{mm} = 15 \times 10^{-3}\,\text{m}$$

$$\varphi = 15^\circ = 0.2618\ \text{rad}$$

$$\varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(1.2)(145 \times 10^6)}{(15 \times 10^{-3})(77.2 \times 10^9)} = 0.15026\ \text{rad}$$

(*a*) Since $\varphi > \varphi_Y$, there is a plastic zone.

$$\tau_Y = \frac{T_Y c}{J} = \frac{2T}{\pi c^3}$$

$$T_Y = \frac{\pi c^3 \tau_Y}{2} = \frac{\pi(15 \times 10^{-3})^3(145 \times 10^6)}{2} = 768.71\ \text{N} \cdot \text{m}$$

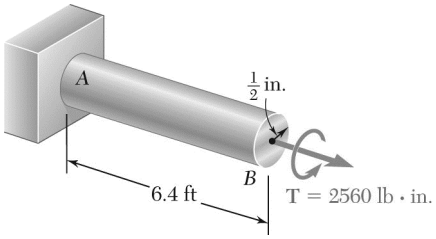
$$T = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\varphi_Y}{\varphi}\right)^3\right] = \frac{4}{3}(768.71)\left[1 - \frac{1}{4}\left(\frac{0.15026}{0.2618}\right)^3\right]$$

$$= 976.5\ \text{N} \cdot \text{m} \qquad T = 977\ \text{N} \cdot \text{m} \blacktriangleleft$$

(*b*) $L\gamma_Y = \rho_Y\varphi = c\varphi_Y$

$$\rho_Y = \frac{c\varphi_Y}{\varphi} = \frac{(15 \times 10^{-3})(0.15026)}{0.2618} = 8.61 \times 10^{-3}\,\text{m} \qquad \rho_Y = 8.61\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.102

The shaft AB is made of a material that is elastoplastic with $\tau_Y = 12\,\text{ksi}$ and $G = 4.5 \times 10^6\,\text{psi}$. For the loading shown, determine (*a*) the radius of the elastic core of the shaft, (*b*) the angle of twist at end B.

SOLUTION

(*a*) Radius of elastic core.

$c = 0.5\,\text{in.} \qquad \tau_Y = 12 \times 10^3\,\text{psi}$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.5)^3(12 \times 10^3)$$
$$= 2356.2\,\text{lb} \cdot \text{in}$$

$T = 2560\,\text{lb} \cdot \text{in} > T_Y$ (plastic region with elastic core)

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y{}^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(2560)}{2356.2} = 0.74051$$

$$\frac{\rho_Y}{c} = 0.9047 \qquad \rho_Y = (0.9047)(0.5) \qquad \rho_Y = 0.452\,\text{in.} \blacktriangleleft$$

(*b*) Angle of twist.

$L = 6.4\,\text{ft} = 76.8\,\text{in.} \qquad G = 4.5 \times 10^6\,\text{psi}$

$$\varphi_Y = \frac{T_Y L}{JG} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(2356.2)(76.8)}{\pi(0.5)^4(4.5 \times 10^6)} = 0.4096\ \text{radians}$$

$$\frac{\varphi_Y}{\varphi} = \frac{\rho_Y}{c} \qquad \varphi = \frac{\varphi_Y c}{\rho_Y} = \frac{0.4096}{0.9047} = 0.4527\ \text{radians}$$

$\varphi = 25.9° \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.103

A 1.25-in.-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 18$ ksi and $G = 11.2 \times 10^6$ psi. For an 8-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 7.5 kip · in. torque.

SOLUTION

$$c = \frac{1}{2}d = 0.625 \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}, \quad \tau_Y = 18 \text{ ksi} = 18000 \text{ psi}$$

$$L = 8 \text{ ft} = 96 \text{ in.} \quad T = 7.5 \text{ kip} \cdot \text{in} = 7.5 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.625)^3(18000) = 6.9029 \times 10^3 \text{ lb} \cdot \text{in}$$

$T > T_Y$: plastic region with elastic core $\therefore \quad \tau_{\max} = \tau_Y = 18$ ksi $\qquad \tau_{\max} = 18 \text{ ksi}$ ◀

$$\gamma_Y = \frac{c\varphi_Y}{L} \quad \therefore \quad \varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(96)(18000)}{(0.625)(11.2 \times 10^6)} = 246.86 \times 10^{-3} \text{ rad}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right)$$

$$\frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \dfrac{3T}{T_Y}}} = \frac{1}{\sqrt[3]{4 - \dfrac{(3)(7.5 \times 10^3)}{6.9029 \times 10^3}}} = 1.10533$$

$$\varphi = 1.10533\varphi_Y = (1.10533)(246.86 \times 10^{-3}) = 272.86 \times 10^{-3} \text{ rad} \qquad \varphi = 15.63° \text{ ◀}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.104

A 18-mm-diameter solid circular shaft is made of a material that is assumed to be elastoplastic with $\tau_Y = 145\,\text{MPa}$ and $G = 77\,\text{GPa}$. For a 1.2-m length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 200 N · m torque.

SOLUTION

$$\tau_Y = 145 \times 10^6\,\text{Pa}, \quad c = \frac{1}{2}d = 0.009\,\text{m}, \quad L = 1.2\,\text{m}, \quad T = 200\,\text{N} \cdot \text{m}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.009)^3(145 \times 10^6) = 166.04\,\text{N} \cdot \text{m}$$

$T > T_Y$ (plastic region with elastic core) $\qquad \tau_{\max} = \tau_Y = 145\,\text{MPa}$ ◀

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(166.04)(1.2)}{\pi(0.009)^4(77 \times 10^9)} = 251.08 \times 10^{-3}\,\text{radians}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi^3}{\varphi_Y^3}\right)$$

$$\left(\frac{\varphi_Y}{\varphi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(200)}{166.04} = 0.38641 \qquad \frac{\varphi_Y}{\varphi} = 0.72837$$

$$\varphi = \frac{\varphi}{0.72837} = \frac{251.08 \times 10^{-3}}{0.72837} = 344.7 \times 10^{-3}\,\text{radians}$$

$\varphi = 19.75°$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.105

A solid circular rod is made of a material that is assumed to be elastoplastic. Denoting by T_Y and φ_Y, respectively, the torque and the angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (*a*) $T = 1.1\,T_Y$, (*b*) $T = 1.25\,T_Y$, (*c*) $T = 1.3\,T_Y$.

SOLUTION

$$T = \frac{4}{3}\,T_Y\left(1 - \frac{1}{4}\frac{{\varphi_Y}^3}{\varphi^3}\right)$$

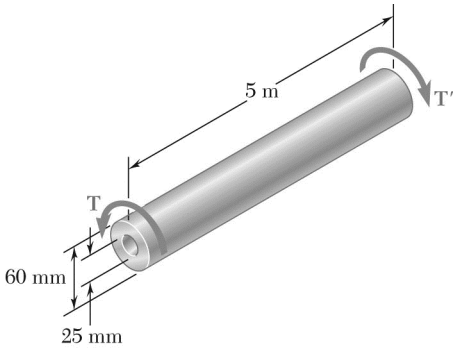
$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}}$$

(*a*) $\dfrac{T}{T_Y} = 1.10 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126$ $\qquad \varphi = 1.126\,\varphi_Y$ ◀

(*b*) $\dfrac{T}{T_Y} = 1.25 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587$ $\qquad \varphi = 1.587\,\varphi_Y$ ◀

(*c*) $\dfrac{T}{T_Y} = 1.3 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15$ $\qquad \varphi = 2.15\,\varphi_Y$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.106

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

SOLUTION

(*a*) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{\max} = \tau_Y$.

$$c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, \quad c_1 = \frac{1}{2}d_1 = 0.0125 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.0125^4) = 1.2340 \times 10^{-6} \text{ m}^4$$

$$\tau_{\max} = \tau_Y = \frac{T_Y c_2}{J} \quad \therefore \quad T_Y = \frac{J\tau_Y}{c_2} = \frac{(1.2340 \times 10^{-6})(145 \times 10^6)}{0.030} = 5.9648 \times 10^3 \text{ N} \cdot \text{m}$$

$$T_Y = 5.96 \text{ kN} \cdot \text{m} \blacktriangleleft$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(5.9643 \times 10^3)(5)}{(77.2 \times 10^9)(2.234010^{-6})} = 313.04 \times 10^{-3} \text{ rad} \qquad \varphi_Y = 17.94° \blacktriangleleft$$

(*b*) $t = 0.010$ m $\qquad \rho_Y = c_2 - t = 0.030 - 0.010 = 0.020$ m

$$\gamma = \frac{\rho\varphi}{L} = \frac{\rho_Y\varphi}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi = \frac{\tau_Y L}{G\rho_Y} = \frac{(145 \times 10^6)(5)}{(77.2 \times 10^9)(0.020)} = 469.56 \times 10^{-3} \text{ rad} \qquad \varphi = 26.9° \blacktriangleleft$$

Torque T_1 carried by elastic portion: $\quad c_1 \le \rho \le \rho_Y$

$$\tau = \tau_Y \text{ at } \rho = \rho_Y. \qquad \tau_Y = \frac{T_1\rho_Y}{J_1} \qquad \text{where} \qquad J_1 = \frac{\pi}{2}\left(\rho_Y^4 - c_1^4\right)$$

$$J_1 = \frac{\pi}{2}(0.020^4 - 0.0125^4) = 212.978 \times 10^{-9} \text{ m}^4$$

$$T_1 = \frac{J_1\tau_Y}{\rho_Y} = \frac{(212.978 \times 10^{-9})(145 \times 10^6)}{0.020} = 1.5441 \times 10^3 \text{ N} \cdot \text{m}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.106 (*Continued*)

Torque T_2 carried by plastic portion:

$$T_2 = 2\pi \int_{\rho_Y}^{c_2} \tau_Y \rho^2 d\rho = 2\pi\, \tau_Y \frac{\rho^3}{3}\bigg|_{\rho_Y}^{c_2} = \frac{2\pi}{3}\tau_Y \left(c_2^3 - \rho_Y^3\right)$$

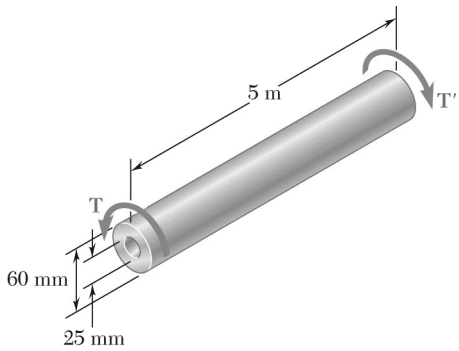
$$= \frac{2\pi}{3}(145 \times 10^6)(0.030^3 - 0.020^3) = 5.7701 \times 10^3 \text{ N} \cdot \text{m}$$

Total torque:

$$T = T_1 + T_2 = 1.5541 \times 10^3 + 5.7701 \times 10^3 = 7.3142 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 7.31 \text{ kN} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.107

For the shaft of Prob. 3.106, determine (*a*) angle of twist at which the section first becomes fully plastic, (*b*) the corresponding magnitude T of the applied torque. Sketch the $T - \phi$ curve for the shaft.

PROBLEM 3.106 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. Determine the magnitude T of the torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.0125 \text{ m} \qquad c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}$$

(*a*) For onset of fully plastic yielding, $\rho_Y = c_1$

$$\tau = \tau_Y \quad \therefore \quad \gamma = \frac{\tau_Y}{G} = \frac{\rho_Y \varphi}{L} = \frac{c_1 \varphi}{L}$$

$$\varphi_f = \frac{L\tau_Y}{c_1 G} = \frac{(25)(145 \times 10^6)}{(0.0125)(77.2 \times 109)} = 751.295 \times 10^{-3} \text{ rad} \qquad \varphi_f = 43.0° \blacktriangleleft$$

(*b*) $$T_P = 2\pi \int_{c_1}^{c_2} \tau_Y \rho^2 d\rho = 2\pi \, \tau_Y \frac{\rho^3}{3}\bigg|_{c_1}^{c_2} = \frac{2\pi}{3}\tau_Y \left(c_2^3 - c_1^3\right)$$

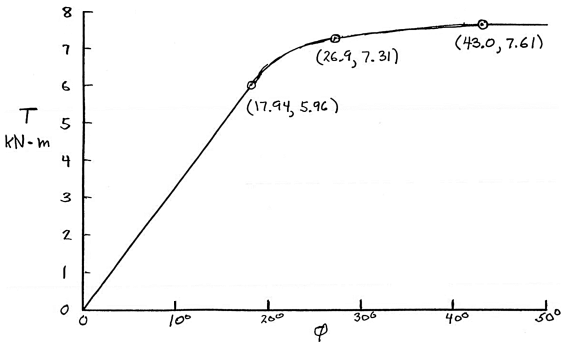
$$= \frac{2\pi}{3}(145 \times 10^6)(0.030^3 - 0.0125^3) = 7.606 \times 10^3 \text{ N} \cdot \text{m} \qquad T_P = 7.61 \text{ kN} \cdot \text{m} \blacktriangleleft$$

From Prob. 3.101, $\varphi_Y = 17.94°$ $T_Y = 5.96$ kN · m

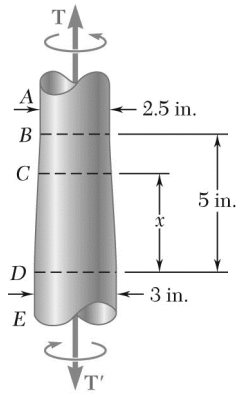
Also from Prob. 3.101, $\varphi = 26.9°$ $T = 7.31$ kN · m

Plot T vs φ using the following data.

φ, deg	0	17.94	26.9	43.0	>43.0
T kN · m	0	5.96	7.31	7.61	7.61



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.108

A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75\ \text{kip} \cdot \text{in.}$ are applied. Assuming the steel to be elastoplastic with $\tau_Y = 21\ \text{ksi}$ and $G = 11.2 \times 10^6\ \text{psi}$, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* that remains fully elastic.

SOLUTION

(*a*) In portion *AB*. $\quad c = \dfrac{1}{2}d = 1.25\ \text{in.}$

$$T_Y = \frac{J_{AB}\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21 \times 10^3) = 64.427 \times 10^3\ \text{lb} \cdot \text{in}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y}{c} = \sqrt[3]{4 - \frac{3T}{T_Y}} = \sqrt[3]{4 - \frac{(3)(75 \times 10^3)}{64.427 \times 10^3}} = 0.79775$$

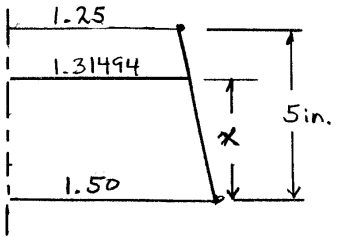
$$\rho_Y = 0.79775c = (0.79775)(1.25) = 0.99718\ \text{in.} \qquad \rho_Y = 0.997\ \text{in.} \blacktriangleleft$$

(*b*) For yielding at point *C*. $\quad \tau = \tau_Y, \quad c = c_x, \quad T = 75 \times 10^3\ \text{lb} \cdot \text{in}$

$$T = \frac{J_C\tau_Y}{c_x} = \frac{\pi}{2}c_x^3\tau_Y$$

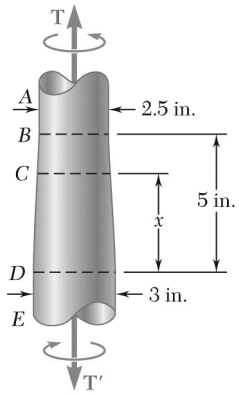
$$c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(75 \times 10^3)}{\pi(21 \times 10^3)}} = 1.31494\ \text{in.}$$

Using proportions from the sketch,



$$\frac{1.50 - 1.31494}{1.50 - 1.25} = \frac{x}{5} \qquad x = 3.70\ \text{in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.109

If the torques applied to the tapered shaft of Prob. 3.108 are slowly increased, determine (*a*) the magnitude T of the largest torques that can be applied to the shaft, (*b*) the length of the portion *CD* that remains fully elastic.

PROBLEM 3.108 A steel rod is machined to the shape shown to form a tapered solid shaft to which torques of magnitude $T = 75$ kip · in. are applied. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi and $G = 11.2 \times 10^6$ psi, determine (*a*) the radius of the elastic core in portion *AB* of the shaft, (*b*) the length of portion *CD* that remains fully elastic.

SOLUTION

(*a*) The largest torque that may be applied is that which makes portion *AB* fully plastic.

In portion *AB*, $c = \frac{1}{2}d = 1.25$ in.

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21\times10^3) = 64.427\times10^3 \text{ lb} \cdot \text{in}$$

For fully plastic shaft, $\rho_Y = 0$ $\quad T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}T$

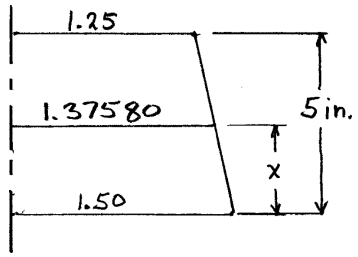
$$T = \frac{4}{3}(64.427\times10^3) = 85.903\times10^3 \text{ lb} \cdot \text{in} \qquad T = 85.9 \text{ kip} \cdot \text{in} \blacktriangleleft$$

(*b*) For yielding at point *C*, $\tau = \tau_Y$, $\quad c = c_x$, $\quad T = 85.903\times10^3$ lb · in

$$\tau_Y = \frac{Tc_x}{J_x} = \frac{2T}{\pi c_x^3}$$

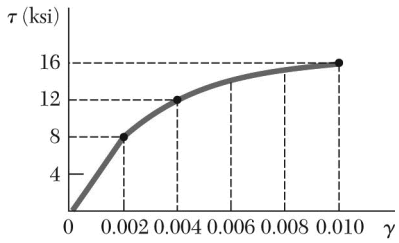
$$c_x = \sqrt[3]{\frac{2T}{\pi\tau_Y}} = \sqrt[3]{\frac{(2)(85.903\times10^3)}{\pi(21\times10^3)}} = 1.37580 \text{ in.}$$

Using proportions from the sketch,



$$\frac{1.50 - 1.37580}{1.50 - 1.25} = \frac{x}{5} \qquad x = 2.48 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.110

A hollow shaft of outer and inner diameters respectively equal to 0.6 in. and 0.2 in. is fabricated from an aluminum alloy for which the stress-strain diagram is given in the diagram shown. Determine the torque required to twist a 9-in. length of the shaft through 10°.

SOLUTION

$$\varphi = 10^\circ = 174.53 \times 10^{-3} \text{ rad}$$

$$c_1 = \frac{1}{2} d_1 = 0.100 \text{ in}, \quad c_2 = \frac{1}{2} d_2 = 0.300 \text{ in}.$$

$$\gamma_{\max} = \frac{c_2 \varphi}{L} = \frac{(0.300)(174.53 \times 10^{-3})}{9} = 0.00582$$

$$\gamma_{\min} = \frac{c_1 \varphi}{L} = \frac{(0.100)(174.53 \times 10^{-3})}{9} = 0.00194$$

Let $$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c_2} \qquad z_1 = \frac{c_1}{c_2} = \frac{1}{3}$$

$$T = 2\pi \int_{c_1}^{c_2} \rho^2 \tau \, d\rho = 2\pi c_2^3 \int_{z_1}^{1} z^2 \tau \, dz = 2\pi c_2^3 I$$

where the integral I is given by $$I = \int_{1/3}^{1} z^2 \tau \, dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \Sigma w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$, we get the values given in the table below.

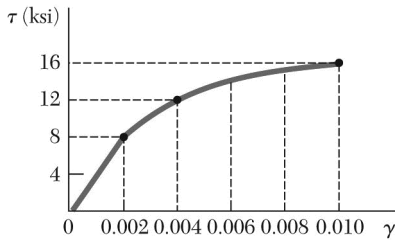
z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
1/3	0.00194	8.0	0.89	1	0.89
1/2	0.00291	10.0	2.50	4	10.00
2/3	0.00383	11.5	5.11	2	10.22
5/6	0.00485	13.0	9.03	4	36.11
1	0.00582	14.0	14.0	1	14.00
					71.22 ⟵ $\Sigma wz^2\tau$

$$I = \frac{(1/6)(71.22)}{3} = 3.96 \text{ ksi}$$

$$T = 2\pi c_2^3 I = 2\pi (0.300)^3 (3.96) = 0.671 \text{ kip} \cdot \text{in} \qquad T = 671 \text{ lb} \cdot \text{in} \blacktriangleleft$$

Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.111

Using the stress-strain diagram shown, determine (*a*) the torque that causes a maximum shearing stress of 15 ksi in a 0.8-in.-diameter solid rod, (*b*) the corresponding angle of twist in a 20-in. length of the rod.

SOLUTION

(*a*) $\tau_{\max} = 15$ ksi $\qquad c = \dfrac{1}{2}d = 0.400$ in.

From the stress-strain diagram, $\gamma_{\max} = 0.008$

Let $$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi\int_0^c \rho^2\tau\, d\rho = 2\pi c^3\int_0^1 z^2\tau\, dz = 2\pi c^3 I$$

where the integral I is given by $$I = \int_0^1 z^2\tau\, dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3}\Sigma w z^2\tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
0	0.000	0	0.000	1	0.00
0.25	0.002	8	0.500	4	2.00
0.5	0.004	12	3.000	2	6.00
0.75	0.006	14	7.875	4	31.50
1.0	0.008	15	15.000	1	15.00
					54.50 ⟵ $\Sigma wz^2\tau$

$$I = \frac{(0.25)(54.50)}{3} = 4.54 \text{ ksi}$$

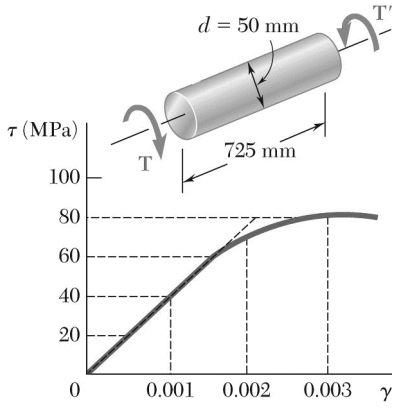
$T = 2\pi c^3 I = 2\pi(0.400)^3(4.54)$ $\qquad T = 1.826$ kip · in ◀

(*b*) $\gamma_{\max} = \dfrac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_m}{c} = \frac{(20)(0.008)}{0.400} = 400\times10^{-3} \text{ rad}$$ $\qquad \varphi = 22.9°$ ◀

Note: Answers may differ slightly due to differences of opinion in reading the stress-strain curve.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.112

A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of torque applied to the shaft

SOLUTION

$$\varphi = 5^\circ = 87.266 \times 10^{-3}\text{rad} \qquad c = \frac{1}{2}d = 0.025\,\text{m}, \quad L = 0.725\,\text{m}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 0.00301$$

Let $$z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi\int_0^c \rho^2 \tau\, d\rho = 2\pi c^3 \int_0^1 z^2 \tau\, dz = 2\pi c^3 I \qquad \text{where the integral I is given by } I = \int_0^1 z^2 \tau\, dz$$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3}\sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25,$ we get the values given in the table below.

z	γ	τ, MPa	$z^2\tau$, MPa	w	$wz^2\tau$, MPa
0	0	0	0	1	0
0.25	0.00075	30	1.875	4	7.5
0.5	0.0015	55	13.75	2	27.5
0.75	0.00226	75	42.19	4	168.75
1.0	0.00301	80	80	1	80
					283.75

$\leftarrow \sum wz^2\tau = 283.75 \times 10^6$ Pa

$$I = \frac{(0.25)(283.75 \times 10^6)}{3} = 23.65 \times 10^6 \text{ Pa}$$

$$T = 2\pi c^3 I = 2\pi(0.025)^3(23.65 \times 10^6) = 2.32 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 2.32\,\text{kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.113

Three points on the nonlinear stress-strain diagram used in Prob. 3.112 are (0,0), (0.0015,55 MPa), and (0.003,80 MPa). By fitting the polynomial $\tau = A + B\gamma + C\gamma^2$ through these points, the following approximate relation has been obtained.

$$T = 46.7 \times 10^9 \gamma - 6.67 \times 10^{12} \gamma^2$$

Solve Prob. 3.113 using this relation, Eq. (3.2), and Eq. (3.26).

PROBLEM 3.112 A 50-mm diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude *T* of torque applied to the shaft.

SOLUTION

$$\varphi = 5° = 87.266 \times 10^{-3} \text{ rad}, \quad c = \frac{1}{2} d = 0.025 \text{ m}, \quad L = 0.725 \text{ m}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 3.009 \times 10^{-3}$$

Let $z = \dfrac{\gamma}{\gamma_{\max}} = \dfrac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz$$

The given stress-strain curve is

$$\tau = A + B\gamma + C\gamma^2 = A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2$$

$$T = 2\pi c^3 \int_0^1 z^2 \left(A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2 \right) dz$$

$$= 2\pi c^3 \left\{ A \int_0^1 z^2 \, dz + B\gamma_{\max} \int_0^1 z^3 dz + C\gamma_{\max}^2 \int_0^1 z^4 dz \right\}$$

$$= 2\pi c^2 \left\{ \frac{1}{3} A + \frac{1}{4} B\gamma_{\max} + \frac{1}{5} C\gamma_{\max}^2 \right\}$$

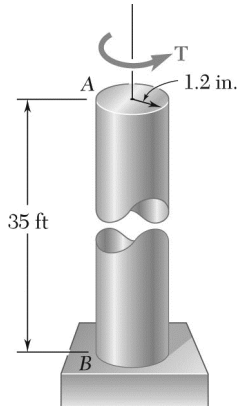
Data: $A = 0, \quad B = 46.7 \times 10^9, \quad C = -6.67 \times 10^{12}$

$$\frac{1}{3} A = 0, \quad \frac{1}{4} B\gamma_{\max} = \frac{1}{4}(46.7 \times 10^9)(3.009 \times 10^{-3}) = 35.13 \times 10^3$$

$$\frac{1}{5} C\gamma_{\max}^2 = -\frac{1}{5}(6.67 \times 10^{12})(3.009 \times 10^{-3})^2 = -12.08 \times 10^3$$

$$T = 2\pi(0.025)^3(0 + (35.13 \times 10^3 - 12.08 \times 10^3) = 2.26 \times 10^3 \text{ N} \cdot \text{m} \qquad T = 2.26 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.114

The solid circular drill rod AB is made of a steel that is assumed to be elastoplastic with $\tau_Y = 22$ ksi and $G = 11.2 \times 10^6$ psi. Knowing that a torque $T = 75$ kip · in. is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

$$c = 1.2 \text{ in.} \qquad L = 35 \text{ ft} = 420 \text{ in.}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(1.2)^4 = 3.2572 \text{ in}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{(3.2572)(22)}{1.2} = 59.715 \text{ kip} \cdot \text{in}$$

Loading: $T = 75$ kip · in

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_Y^3}{c^3} \right)$$

$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(75)}{59.715} = 0.23213$$

$$\frac{\rho_Y}{c} = 0.61458, \qquad \rho_Y = 0.61458c = 0.73749 \text{ in.}$$

Unloading: $\tau' = \dfrac{T\rho}{J}$ where $T = 75$ kip · in

At $\rho = c$: $\tau' = \dfrac{(75)(1.2)}{3.2572} = 27.63$ ksi

At $\rho = \rho_Y$: $\tau' = \dfrac{(75)(0.73749)}{3.2572} = 16.98$ ksi

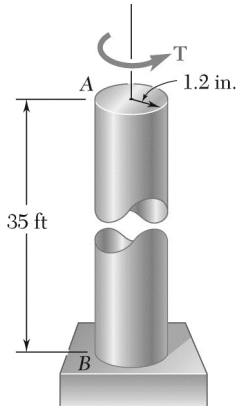
Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$

At $\rho = c$: $\tau_{\text{res}} = 22 - 27.63 = -5.63$ ksi

At $\rho = \rho_Y$: $\tau_{\text{res}} = 22 - 16.98 = 5.02$ ksi

maximum $\tau_{\text{res}} = 5.63$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.115

In Prob. 3.114, determine the permanent angle of twist of the rod.

PROBLEM 3.114 The solid circular drill rod AB is made of steel that is assumed to be elastoplastic with $\tau_Y = 22\,\text{ksi}$ and $G = 11.2 \times 10^6\,\text{psi}$. Knowing that a torque $T = 75\,\text{kip} \cdot \text{in.}$ is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

From the solution to Prob. 3.114,

$$c = 1.2 \text{ in.}$$

$$J = 3.2572 \text{ in}^4$$

$$\frac{\rho_Y}{c} = 0.61458$$

$$\rho_Y = 0.73749 \text{ in.}$$

After loading, $\gamma = \dfrac{\rho\varphi}{L} \quad \therefore \quad \varphi = \dfrac{L\gamma}{\rho} = \dfrac{L\gamma_Y}{\rho_Y} = \dfrac{L\tau_Y}{\rho_Y G} \qquad L = 35 \text{ ft} = 420 \text{ in.}$

$$\varphi_{\text{load}} = \frac{(420)(22 \times 10^3)}{(0.73749)(11.2 \times 10^6)} = 1.11865 \text{ rad} = 64.09°$$

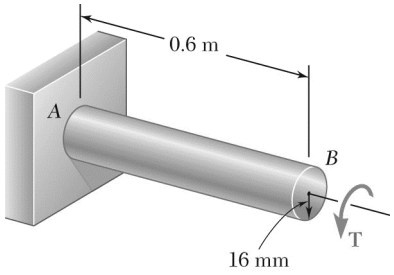
During unloading, $\varphi' = \dfrac{TL}{GJ}$ (elastic) $\qquad T = 5 \times 10^3\,\text{N} \cdot \text{m}$

$$\varphi' = \frac{(75 \times 10^3)(420)}{(11.2 \times 10^6)(3.2572)} = 0.86347 \text{ rad} = 49.47°$$

Permanent angle of twist.

$$\varphi_{\text{perm}} = \varphi_{\text{load}} - \varphi' = 1.11865 - 0.86347 = 0.25518 \qquad \varphi = 14.62° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.116

The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

SOLUTION

$$c = 0.016 \text{ m} \qquad \varphi = 6° = 104.72 \times 10^{-3} \text{ rad}$$

$$\gamma_{\max} = \frac{c\varphi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{145 \times 10^6}{77.2 \times 10^9} = 0.0018782$$

$$\frac{\rho_Y}{c} = \frac{\gamma_Y}{\gamma_{\max}} = \frac{0.0018}{0.0027925} = 0.67260$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.016)^4 = 102.944 \times 10^{-9} \text{ m}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.016)^3 (145 \times 10^6) = 932.93 \text{ N} \cdot \text{m}$$

At end of loading. $T_{\text{load}} = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(932.93)\left[1 - \frac{1}{4}(0.67433)^3\right] = 1.14855 \times 10^3 \text{ N} \cdot \text{m}$

Unloading: elastic $T' = 1.14855 \times 10^3 \text{ N} \cdot \text{m}$

At $\rho = c$ $\tau' = \frac{T'c}{J} = \frac{(1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = 178.52 \times 10^6 \text{ Pa}$

At $\rho = \rho_Y$ $\tau' = \frac{T'c}{J}\frac{\rho_Y}{c} = (178.52 \times 10^6)(0.67433) = 120.38 \times 10^6 \text{ Pa}$

$$\varphi' = \frac{T'L}{GJ} = \frac{(1.14855 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 86.71 \times 10^{-3} \text{ rad} = 4.97°$$

Residual: $\tau_{\text{res}} = \tau_{\text{load}} - \tau' \qquad \varphi_{\text{perm}} = \varphi_{\text{load}} - \varphi'$

(*a*) At $\rho = c$ $\tau_{\text{res}} = 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6 \text{ Pa}$ $\tau_{\text{res}} = -33.5 \text{ MPa}$

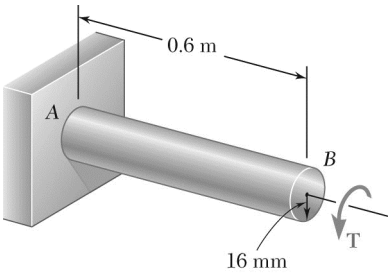
At $\rho = \rho_Y$ $\tau_{\text{res}} = 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6 \text{ Pa}$ $\tau_{\text{res}} = 24.6 \text{ MPa}$

Maximum residual stress: 33.5 MPa at $\rho = 16$ mm ◀

(*b*) $\varphi_{\text{perm}} = 104.72 \times 10^{-3} - 86.71 \times 10^{-3} = 17.78 \times 10^{-3} \text{ rad}$ $\varphi_{\text{perm}} = 1.032°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.117



After the solid shaft of Prob. 3.116 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque $\mathbf{T}$ is applied to the shaft. Assuming no change in the value of φ_Y, determine the angle of twist φ_1 for which yield is initiated in this second loading and compare it with the angle φ_Y for which the shaft started to yield in the original loading.

PROBLEM 3.116 The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The torque is increased in magnitude until the shaft has been twisted through 6°; the torque is then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist.

SOLUTION

From the solution to Prob. 3.116,

$$c = 0.016 \text{ m}, \quad L = 0.6 \text{ m}$$

$$\tau_Y = 145 \times 10^6 \text{ Pa},$$

$$J = 102.944 \times 10^{-9} \text{ m}^4$$

The residual stress at $\rho = c$ is $\tau_{\text{res}} = 33.5$ MPa

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_Y - \tau_{\text{res}} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J\tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717 \text{ N} \cdot \text{m}$$

Angle of twist at yielding under reversed torque.

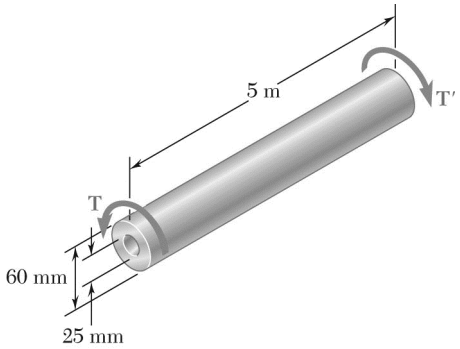
$$\varphi_1 = \frac{T_1 L}{GJ} = \frac{(717 \times 10^3)(0.6)}{(77.2 \times 10^9)(102.944 \times 10^{-9})} = 54.16 \times 10^{-3} \text{ rad} \qquad \varphi_1 = 3.10° \blacktriangleleft$$

Angle of twist for yielding in original loading.

$$\gamma = \frac{\tau_Y}{G} = \frac{c\varphi_Y}{L}$$

$$\varphi_Y = \frac{L\tau_Y}{cG} = \frac{(0.6)(145 \times 10^6)}{(0.016)(77.2 \times 10^9)} = 70.434 \times 10^{-3} \text{ rad} \qquad \varphi_Y = 4.04° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.118

The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho\tau\, dA = \rho\tau_Y(2\pi\rho\, d\rho) = 2\pi\, \tau_Y\, \rho^2 d\rho$$

$$T = 2\pi\, \tau_Y \int_{C_1}^{C_2} \rho^2\, d\rho = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(145\times10^6)[(30\times10^{-3})^3 - (12.5\times10^{-3})^3] = 7.6064\times10^3 \text{ N}\cdot\text{m}$$

Unloading. $T' = 7.6064\times10^3 \text{ N}\cdot\text{m}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234\times10^6 \text{mm}^4 = 1.234\times10^{-6}\text{m}^4$$

$$\tau_1' = \frac{T'c_1}{J} = \frac{(7.6064\times10^3)(12.5\times10^{-3})}{1.234\times10^{-6}} = 77.050\times10^6\text{Pa} = 77.05 \text{ MPa}$$

$$\tau_2' = \frac{T'c_2}{J} = \frac{(7.6064\times10^3)(30\times10^{-3})}{1.234\times10^{-6}} = 192.63\times10^6\text{Pa} = 192.63 \text{ MPa}$$

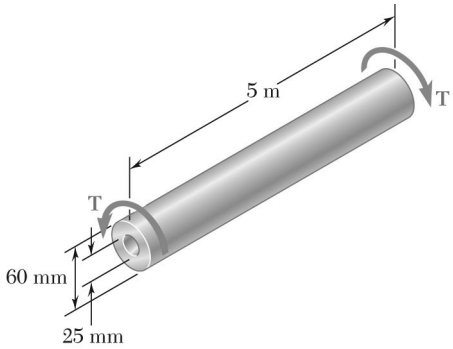
Residual stress.

Inner surface: $\tau_{\text{res}} = \tau_Y - \tau_1' = 145 - 77.05 = 67.95$ MPa

Outer surface: $\tau_{\text{res}} = \tau_Y - \tau_2' = 145 - 192.63 = -47.63$ MPa

Maximum residual stress: 68.0 MPa at inner surface. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.119

In Prob. 3.118, determine the permanent angle of twist of the rod.

PROBLEM 3.118 The hollow shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77.2$ GPa. The magnitude T of the torques is slowly increased until the plastic zone first reaches the inner surface of the shaft; the torques are then removed. Determine the magnitude and location of the maximum residual shearing stress in the rod.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

When the plastic zone reaches the inner surface, the stress is equal to τ_Y. The corresponding torque is calculated by integration.

$$dT = \rho\tau\, dA = \rho\tau_Y(2\pi\rho d\rho) = 2\pi\tau_Y \rho^2 d\rho$$

$$T = 2\pi\tau_Y \int_{c_1}^{c_2} \rho^2\, d\rho = \frac{2\pi}{3}\tau_Y\left(c_2^3 - c_1^3\right)$$

$$= \frac{2\pi}{3}(145\times10^6)[(30\times10^{-3})^3 - (12.5\times10^{-3})^3] = 7.6064\times10^3 \text{ N}\cdot\text{m}$$

Rotation angle at maximum torque.

$$\frac{c_1\varphi_{\max}}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi_{\max} = \frac{\tau_Y L}{Gc_1} = \frac{(145\times10^6)(5)}{(77.2\times10^9)(12.5\times10^{-3})} = 0.75130 \text{ rad}$$

Unloading. $T' = 7.6064\times10^3 \text{ N}\cdot\text{m}$

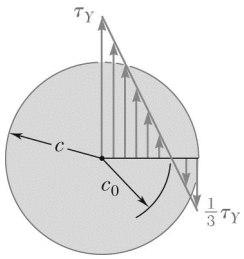
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4] = 1.234\times10^6 \text{mm}^4 = 1.234\times10^{-6}\text{m}^4$$

$$\varphi' = \frac{T'L}{GJ} = \frac{(7.6064\times10^3)(5)}{(77.2\times10^9)(1.234\times10^{-6})} = 0.39922 \text{ rad}$$

Permanent angle of twist.

$$\varphi_{\text{perm}} = \varphi_{\max} - \varphi' = 0.75130 - 0.39922 = 0.35208 \text{ rad} \qquad \varphi_{\text{perm}} = 20.2^\circ \blacktriangleleft$$

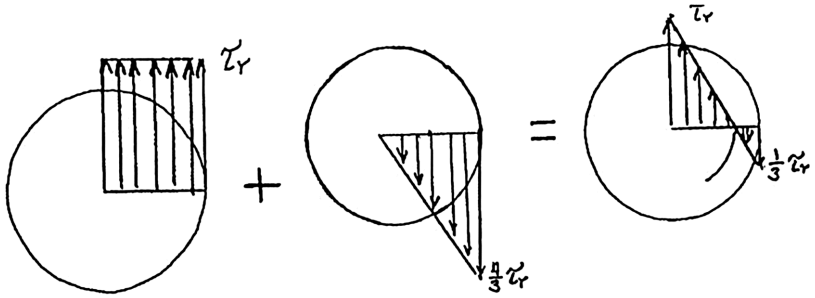
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.120

A torque **T** applied to a solid rod made of an elastoplastic material is increased until the rod is fully plastic and them removed. (*a*) Show that the distribution of residual shearing stresses is as represented in the figure. (*b*) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius c_0.

SOLUTION



Loading Unloading Residual

(*a*)

After loading: $\rho_Y = 0, \quad T_{\text{load}} = \frac{4}{3}T_Y = \frac{4}{3}\frac{\pi}{2}c^3\tau_Y = \frac{2\pi}{3}c^3\tau_Y$

Unloading: $\tau' = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(T_{\text{load}})}{\pi c^3} = \frac{4}{3}\tau_Y \quad \text{at } \rho = c$

$$\tau' = \frac{4}{3}\tau_Y\frac{\rho}{c}$$

Residual: $\tau_{\text{res}} = \tau_Y - \frac{4}{3}\tau_Y\frac{\rho}{c} = \tau_Y\left(1 - \frac{4\rho}{3c}\right)$

To find c_0 set, $\tau_{\text{res}} = 0 \quad \text{and} \quad \rho = c_0$

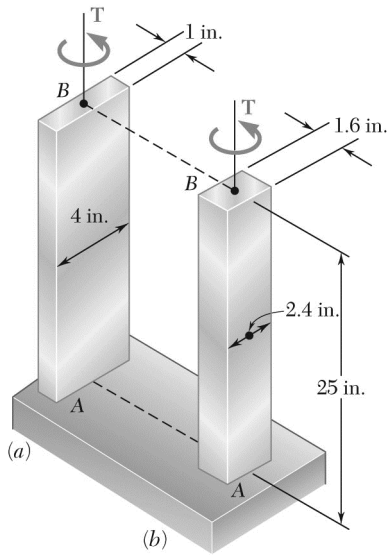
$$0 = 1 - \frac{4c_0}{3c} \quad \therefore \quad c_0 = \frac{3}{4}c \qquad c_0 = 0.150c \blacktriangleleft$$

(*b*) $$T_0 = 2\pi\int_0^{c_0}\rho^2\tau\,d\rho = 2\pi\int_0^{(3/4)c}\rho^2\tau_Y\left(1-\frac{4}{3}\frac{\rho}{c}\right)d\rho$$

$$= 2\pi\tau_Y\left(\frac{\rho^3}{3} - \frac{4}{3}\frac{\rho^4}{4c}\right)\Bigg|_0^{(3/4)c} = 2\pi\tau_Y c^3\left\{\frac{1}{3}\left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right)\frac{1}{4}\left(\frac{3}{4}\right)^4\right\}$$

$$= 2\pi\tau_Y c^3\left\{\frac{9}{64} - \frac{27}{256}\right\} = \frac{9\pi}{128}\tau_Y c^3 = 0.2209\,\tau_Y c^3 \qquad T_0 = 0.221\tau_Y c^3 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.121

Determine the largest torque **T** that can be applied to each of the two brass bars shown and the corresponding angle of twist at B, knowing that $\tau_{\text{all}} = 12$ ksi and $G = 5.6 \times 10^6$ psi.

SOLUTION

$$L = 25 \text{ in.}, \quad G = 5.6 \times 10^6 \text{ psi}, \quad \tau_{all} = 12 \times 10^3 \text{ psi}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \qquad \text{or} \qquad T = c_1 a b^2 \tau_{\max} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \qquad \text{or} \qquad \varphi = \frac{c_1 L \tau_{\max}}{c_2 b G} \tag{2}$$

(a) $a = 4$ in., $b = 1$ in., $\dfrac{a}{b} = 4.0$ From Table 3.1: $c_1 = 0.282$, $c_2 = 0.281$

From (1): $T = (0.282)(4)(1)^2(12 \times 10^3) = 13.54 \times 10^3$ $T = 13.54$ kip · in ◀

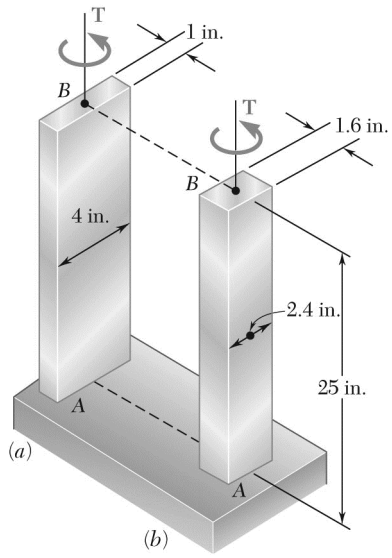
From (2): $\varphi = \dfrac{(0.282)(25)(12 \times 10^3)}{(0.281)(1)(5.6 \times 10^6)} = 0.05376$ radians $\varphi = 3.08°$ ◀

(b) $a = 2.4$ in., $b = 1.6$ in., $\dfrac{a}{b} = 1.5$ From Table 3.1: $c_1 = 0.231$, $c_2 = 0.1958$

From (1): $T = (0.231)(2.4)(1.6)^2(12 \times 10^3) = 17.03 \times 10^3$ $T = 17.03$ kip · in ◀

From (2): $\varphi = \dfrac{(0.231)(25)(12 \times 10^3)}{(0.1958)(1.6)(5.6 \times 10^6)} = 0.0395$ radians $\varphi = 2.26°$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.122

Each of the two brass bars shown is subjected to a torque of magnitude $T = 12.5 \text{ kip} \cdot \text{in}$. Knowing that $G = 5.6 \times 10^6 \text{ psi}$, determine for each bar the maximum shearing stress and the angle of twist at *B*.

SOLUTION

$$L = 25 \text{ in.}, \qquad G = 5.6 \times 10^6 \text{ psi}, \qquad T = 12.5 \times 10^3 \text{ lb} \cdot \text{in}$$

(*a*) $a = 4 \text{ in}, \qquad b = 1 \text{ in}, \qquad \dfrac{a}{b} = 4.0$

From Table 3.1: $c_1 = 0.282, \quad c_2 = 0.281$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{12.5 \times 10^3}{(0.282)(4)(1)^2} = 11.08 \times 10^3$$

$$\tau_{\max} = 11.08 \text{ ksi} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(12.5 \times 10^3)(25)}{(0.282)(4)(1)^3(5.6 \times 10^6)} = 0.04965 \text{ radians}$$

$$\varphi = 2.84° \blacktriangleleft$$

(*b*) $a = 2.4 \text{ in.}, \qquad b = 1.6 \text{ in.}, \qquad \dfrac{a}{b} = 1.5$

From Table 3.1: $c_1 = 0.231, \quad c_2 = 0.1958$

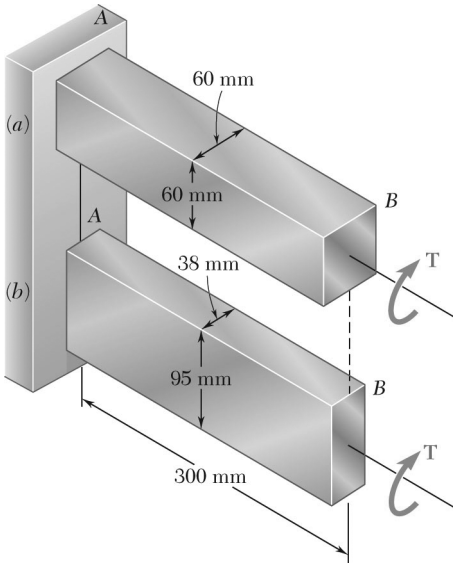
$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{12.5 \times 10^3}{(0.231)(2.4)(1.6)^2} = 8.81 \times 10^3$$

$$\tau_{\max} = 8.81 \text{ ksi} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(12.5 \times 10^6)(25)}{(0.1958)(2.4)(1.6)^3(5.6 \times 10^6)} = 0.02899 \text{ radians}$$

$$\varphi = 1.661° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.123

Each of the two aluminium bars shown is subjected to a torque of magnitude $T = 1800\ \text{N} \cdot \text{m}$. Knowing that $G = 26\ \text{GPa}$, determine for each bar the maximum shearing stress and the angle of twist at B.

SOLUTION

$$T = 1800\ \text{N} \cdot \text{m} \qquad L = 0.300\ \text{m} \qquad G = 26 \times 10^9\ \text{Pa}$$

(*a*) $a = b = 60\,\text{mm} = 0.060\,\text{m} \qquad \dfrac{a}{b} = 1.0$

From Table 3.1: $c_1 = 0.208, \qquad c_2 = 0.1406$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{1800}{(0.208)(0.060)(0.060)^2} = 40.1 \times 10^6\ \text{Pa} \qquad \tau_{\max} = 40.1\ \text{MPa} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1800)(0.300)}{(0.1406)(0.060)(0.060)^3(26 \times 10^9)} = 0.011398\ \text{radians} \qquad \varphi = 0.653° \blacktriangleleft$$

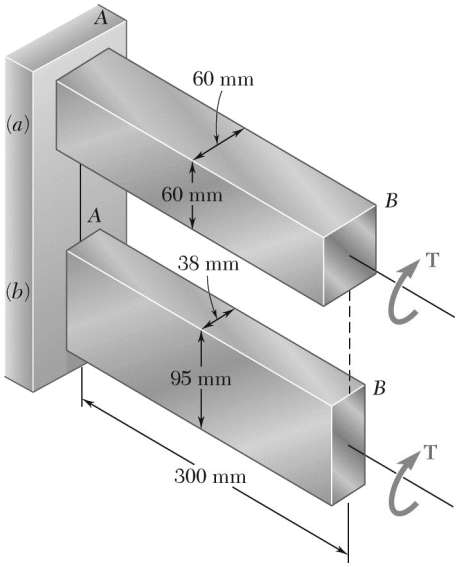
(*b*) $a = 95\ \text{mm} = 0.095\ \text{m}, \qquad b = 38\ \text{mm} = 0.038\ \text{m}, \qquad \dfrac{a}{b} = 2.5$

From Table 3.1: $c_1 = 0.258, \qquad c_2 = 0.249$

$$\tau_{\max} = \frac{T}{c_1 a b^2} = \frac{1800}{(0.258)(0.095)(0.038)^2} = 50.9 \times 10^6\ \text{Pa} \qquad \tau_{\max} = 50.9\ \text{MPa} \blacktriangleleft$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1800)(0.300)}{(0.249)(0.095)(0.038)^3(26 \times 10^9)} = 0.01600\ \text{radians} \qquad \varphi = 0.917° \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.124

Determine the largest torque **T** that can be applied to each of the two aluminium bars shown and the corresponding angle of twist at B, knowing $\tau_{all} = 50$ MPa and $G = 26$ GPa.

SOLUTION

$$L = 0.300 \text{ m}, \quad G = 26 \times 10^9 \text{ Pa}, \quad \tau_{all} = 50 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{T}{c_1 a b^2} \qquad \text{or} \qquad T = c_1 a b^2 \tau_{max} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^4 G} \qquad \text{or} \qquad \varphi = \frac{c_1 L \tau_{max}}{c_2 b G} \tag{2}$$

(*a*) $a = b = 60 \text{ mm} = 0.060 \text{ m}, \quad \dfrac{a}{b} = 1.0$

From Table 3.1: $c_1 = 0.208, \quad c_2 = 0.1406$

From (1): $T = (0.208)(0.060)(0.060)^2(50 \times 10^6) = 2246 \text{ N} \cdot \text{m}$ $\qquad T = 2.25 \text{ kN} \cdot \text{m}$ ◀

From (2): $\varphi = \dfrac{(0.208)(0.300)(50 \times 10^6)}{(0.1406)(0.060)(26 \times 10^9)} = 0.01422 \text{ radians}$ $\qquad \varphi = 0.815°$ ◀

(*b*) $a = 95 \text{ mm} = 0.095 \text{ m}, \quad b = 38 \text{ mm} = 0.038 \text{ m}, \quad \dfrac{a}{b} = 2.5$

From Table 3.1: $c_1 = 0.258, \quad c_2 = 0.249$

From (1): $T = (0.258)(0.095)(0.038)^2(50 \times 10^6) = 1770 \text{ N} \cdot \text{m}$ $\qquad T = 1.770 \text{ kN} \cdot \text{m}$ ◀

From (2): $\varphi = \dfrac{(0.258)(0.300)(50 \times 10^6)}{(0.249)(0.038)(26 \times 10^9)} = 0.01573 \text{ radians}$ $\qquad \varphi = 0.901°$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.125

Determine the largest allowable square cross section of a steel shaft of length 20 ft if the maximum shearing stress is not to exceed 10 ksi when the shaft is twisted through one complete revolution. Use $G = 11.2 \times 10^6$ psi.

SOLUTION

$$L = 20 \text{ ft} = 240 \text{ in.}$$

$$\tau_{\max} = 10 \text{ ksi} = 10 \times 10^3 \text{ psi}$$

$$\varphi = 1 \text{ rev} = 2\pi \text{ radians}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \tag{2}$$

Divide (2) by (1) to eliminate T.

$$\frac{\varphi}{\tau_{\max}} = \frac{c_1 a b^2 L}{c_2 a b^3 G} = \frac{c_1 L}{c_2 b G}$$

Solve for b.

$$b = \frac{c_1 L \tau_{\max}}{c_2 G \varphi}$$

For a square section,

$$\frac{a}{b} = 1.0$$

From Table 3.1,

$$c_1 = 0.208,$$

$$c_2 = 0.1406$$

$$b = \frac{(0.208)(240)(10 \times 10^3)}{(0.1406)(11.2 \times 10^6)(2\pi)}$$

$$b = 0.0505 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.126

Determine the largest allowable length of a stainless steel shaft of $\frac{3}{8} \times \frac{3}{4}$-in. cross section if the shearing stress is not to exceed 15 ksi when the shaft is twisted through 15°. Use $G = 11.2 \times 10^6$ psi.

SOLUTION

$$a = \frac{3}{4} \text{ in.} = 0.75 \text{ in.}$$

$$b = \frac{3}{8} \text{ in.} = 0.375 \text{ in.}$$

$$\tau_{\max} = 15 \text{ ksi} = 15 \times 10^3 \text{ psi}$$

$$\varphi = 15° = \frac{15\pi}{180} \text{ rad} = 0.26180 \text{ rad}$$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \tag{1}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} \tag{2}$$

Divide (2) by (1) to eliminate T. $\dfrac{\varphi}{\tau_{\max}} = \dfrac{c_1 a b^2 L}{c_2 a b^3 G} = \dfrac{c_1 L}{c_2 b G}$

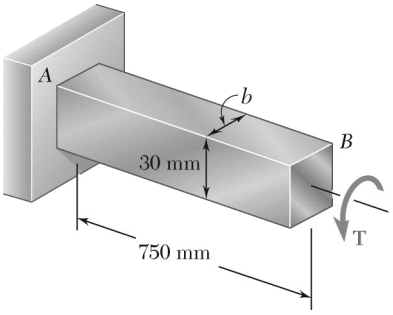
Solve for L. $L = \dfrac{c_2 b G \varphi}{c_1 \tau_{\max}}$

$$\frac{a}{b} = \frac{0.75}{0.375} = 2$$

Table 3.1 gives $c_1 = 0.246, \quad c_2 = 0.229$

$$L = \frac{(0.229)(0.375)(11.2 \times 10^6)(0.26180)}{(0.246)(15 \times 10^3)} = 68.2 \text{ in.} \qquad L = 68.2 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.127

The torque **T** causes a rotation of $2°$ at end B of the stainless steel bar shown. Knowing that $b = 20\,\text{mm}$ and $G = 75\,\text{GPa}$, determine the maximum shearing stress in the bar.

SOLUTION

$$a = 30\text{ mm} = 0.030\text{ m}$$

$$b = 20\text{ mm} = 0.020\text{ m}$$

$$\varphi = 2° = 34.907 \times 10^{-3}\text{ rad}$$

$$\varphi = \frac{TL}{c_2 ab^3 G} \quad \therefore \quad T = \frac{c_2 ab^3 G\varphi}{L}$$

$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{c_2 ab^3 G\varphi}{c_1 ab^2 L} = \frac{c_2 bG\varphi}{c_1 L}$$

$$\frac{a}{b} = \frac{30}{20} = 1.5.$$

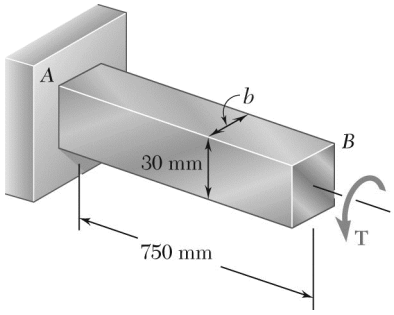
From Table 3.1,

$$c_1 = 0.231$$

$$c_2 = 0.1958$$

$$\tau_{\max} = \frac{(0.1958)(20 \times 10^{-3})(75 \times 10^{9})(34.907 \times 10^{-3})}{(0.231)(750 \times 10^{-3})} = 59.2 \times 10^{6}\text{ Pa} \qquad \tau_{\max} = 59.2\text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.128

The torque **T** causes a rotation of $0.6°$ at end B of the aluminum bar shown. Knowing that $b = 15\,\text{mm}$ and $G = 26\,\text{GPa}$, determine the maximum shearing stress in the bar.

SOLUTION

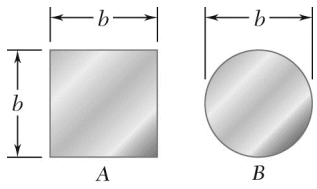
$$a = 30\ \text{mm} = 0.030\ \text{m}$$
$$b = 15\ \text{mm} = 0.015\ \text{m}$$
$$\varphi = 0.6° = 10.472\times10^{-3}\ \text{rad}$$
$$\varphi = \frac{TL}{c_2 ab^3 G} \quad \therefore \quad T = \frac{c_2 ab^3 G\varphi}{c_1 L}$$
$$\tau_{\max} = \frac{T}{c_1 ab^2} = \frac{c_2 ab^3 G\varphi}{c_1 ab^2 L} = \frac{c_2 bG\varphi}{c_1 L}$$
$$\frac{a}{b} = \frac{30}{15} = 2.0$$

From Table 3.1,

$$c_1 = 0.246$$
$$c_2 = 0.229$$
$$\tau_{\max} = \frac{(0.229)(15\times10^{-3})(26\times10^{9})(10.472\times10^{-3})}{(0.246)(750\times10^{-3})}$$
$$= 5.07\times10^{6}\ \text{Pa}$$

$\tau_{\max} = 5.07\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.129

Two shafts are made of the same material. The cross section of shaft A is a square of side b and that of shaft B is a circle of diameter b. Knowing that the shafts are subjected to the same torque, determine the ratio τ_A/τ_B of maximum shearing stresses occurring in the shafts.

SOLUTION

A. Square:

$$\frac{a}{b} = 1, \quad c_1 = 0.208 \quad \text{(Table 3.1)}$$

$$\tau_A = \frac{T}{c_1 a b^2} = \frac{T}{0.208 b^3}$$

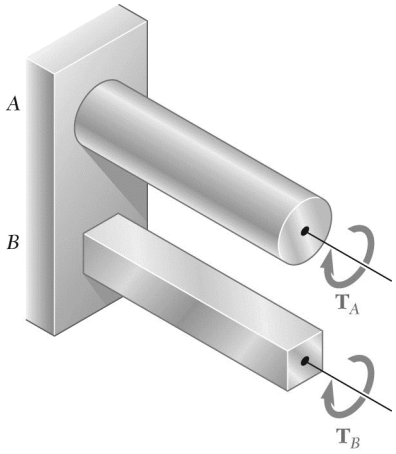
B. Circle:

$$c = \frac{1}{2}b \quad \tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

Ratio:

$$\frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005\pi \qquad \frac{\tau_A}{\tau_B} = 0.944 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.130

Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum shearing stresses occurring in A and B, respectively, when the two shafts are subjected to the same torque $(T_A = T_B)$. Assume both deformations to be elastic.

SOLUTION

Let c be the radius of circular section A and b be the side square section B.

For equal areas, $\pi c^2 = b^2$ $\qquad b = c\sqrt{\pi}$

Circle: $\tau_A = \dfrac{T_{AC}}{J} = \dfrac{2T_A}{\pi c^3}$

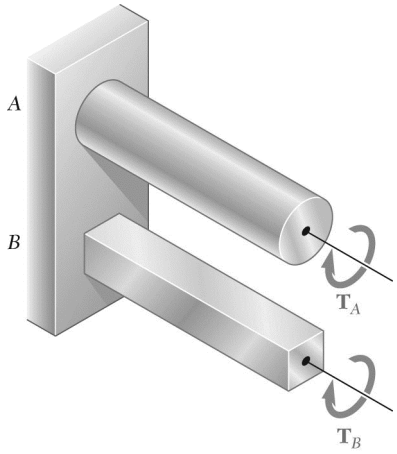
Square: $\dfrac{a}{b} = 1$ $\qquad c_1 = 0.208$ $\qquad$ from Table 3.1

$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{c_1 b^3}$$

Ratio: $$\frac{\tau_A}{\tau_B} = \frac{2T_A}{\pi c^3}\frac{c_1 b^3}{T_B} = \frac{2c_1 b^3}{\pi c^3}\frac{T_A}{T_B} = 2c_1\sqrt{\pi}\,\frac{T_A}{T_B}$$

For $T_A = T_B$, $\dfrac{\tau_A}{\tau_B} = (2)(0.208)\sqrt{\pi}$ $\qquad \dfrac{\tau_A}{\tau_B} = 0.737$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.131

Shafts A and B are made of the same material and have the same cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum torques T_A and T_B that can be safely applied to A and B, respectively.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2$,

$$c = \frac{b}{\sqrt{\pi}}$$

Circle: $$\tau_A = \frac{T_A c}{J} = \frac{2T_A}{\pi c^3} \quad \therefore \quad T_A = \frac{\pi}{2} c^3 \tau_A$$

Square:

From Table 3.1, $c_1 = 0.208$

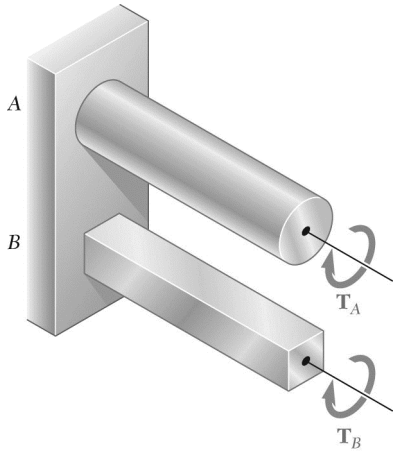
$$\tau_B = \frac{T_A}{c_1 a b^2} = \frac{T_B}{c_1 b^3} \quad \therefore \quad T_B = c_1 b^3 \tau_B$$

Ratio: $$\frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_B}{c_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_B}{c_1 b^3 \tau_B} = \frac{1}{2c_1\sqrt{\pi}} \frac{\tau_A}{\tau_B}$$

For the same stresses, $\tau_B = \tau_A \quad \therefore \quad \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}}$

$$\frac{T_A}{T_B} = 1.356 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.132

Shafts A and B are made of the same material and have the same length and cross-sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum values of the angles φ_A and φ_B through which shafts A and B, respectively, can be twisted.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas, $\pi c^2 = b^2 \quad \therefore \quad b = \sqrt{\pi} c$

Circle: $\gamma_{\max} = \dfrac{\tau_A}{G} = \dfrac{c\varphi_A}{L} \quad \therefore \quad \varphi_A = \dfrac{L\tau_A}{cG}$

Square: From Table 3.1, $c_1 = 0.208, \quad c_2 = 0.1406$

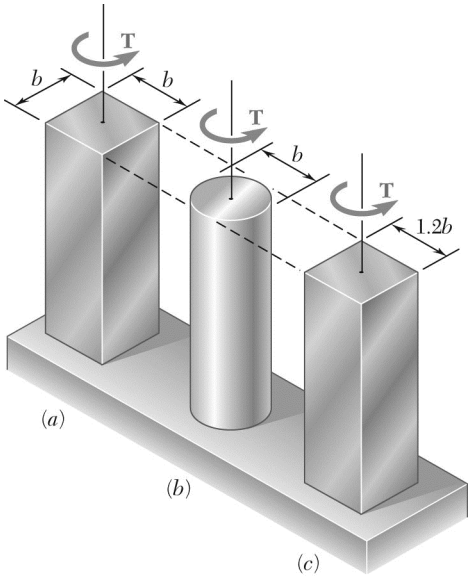
$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{0.208 b^3} \quad \therefore \quad T_B = 0.208\, b^3 \tau_B$$

$$\varphi_B = \frac{T_B L}{c_2 a b^3 G} = \frac{0.208 b^3 \tau_B L}{0.1406 b^4 G} = \frac{1.4794\, L\tau_B}{bG}$$

Ratio: $$\frac{\varphi_A}{\varphi_B} = \frac{L\tau_A}{cG} \cdot \frac{bG}{1.4794 L\tau_B} = 0.676 \frac{b\tau_A}{c\tau_B} = 0.676\sqrt{\pi}\, \frac{\tau_A}{\tau_B}$$

For equal stresses, $\tau_A = \tau_B$ $\quad \dfrac{\varphi_B}{\varphi_A} = 0.676\sqrt{\pi}$ $\quad \dfrac{\varphi_B}{\varphi_A} = 1.198$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.133

Each of the three aluminum bars shown is to be twisted through an angle of 2°. Knowing that $b = 30$ mm, $\tau_{all} = 50$ MPa, and $G = 27$ GPa, determine the shortest allowable length of each bar.

SOLUTION

$\varphi = 2° = 34.907 \times 10^{-3}$ rad, $\tau = 50 \times 10^{6}$ Pa $G = 27 \times 10^{9}$ Pa, $b = 30$ mm $= 0.030$ m

For square and rectangle, $\tau = \dfrac{T}{c_1 a b^2}$ $\varphi = \dfrac{TL}{c_2 a b^3 G}$

Divide to eliminate T; then solve for L. $\dfrac{\varphi}{\tau} = \dfrac{c_1 a b^2 L}{c_2 a b^3 G}$ $L = \dfrac{c_2 b G \varphi}{c_1 \tau}$

(*a*) Square: $\dfrac{a}{b} = 1.0$ From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

$$L = \frac{(0.1406)(0.030)(27 \times 10^{9})(34.907 \times 10^{-3})}{(0.208)(50 \times 10^{6})} = 382 \times 10^{-3} \text{ m}$$

$L = 382$ mm ◀

(*b*) Circle: $c = \dfrac{1}{2} b = 0.015$ m $\tau = \dfrac{Tc}{J}$ $\varphi = \dfrac{TL}{GJ}$

Divide to eliminate T; then solve for L. $\dfrac{\varphi}{\tau} = \dfrac{JL}{cGJ} = \dfrac{L}{cG}$

$$L = \frac{cG\varphi}{\tau} = \frac{(0.015)(27 \times 10^{9})(34.907 \times 10^{-3})}{50 \times 10^{6}} = 283 \times 10^{-3} \text{ m}$$

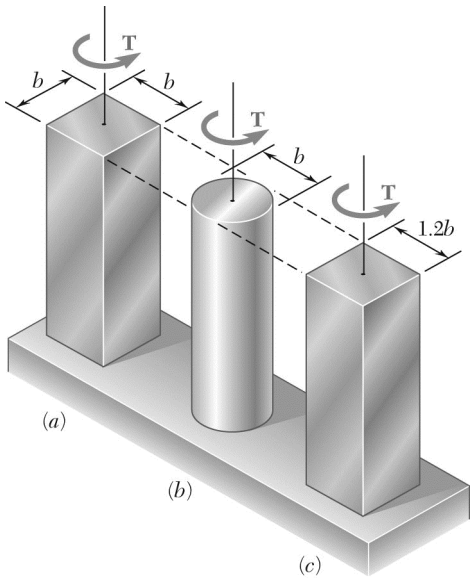
$L = 283$ mm ◀

(*c*) Rectangle: $a = 1.2b$ $\dfrac{a}{b} = 1.2$ From Table 3.1, $c_1 = 0.219$, $c_2 = 0.1661$

$$L = \frac{(0.1661)(0.030)(27 \times 10^{9})(34.907 \times 10^{-3})}{(0.219)(50 \times 10^{6})} = 429 \times 10^{-3} \text{ m}$$

$L = 429$ mm ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.134

Each of the three steel bars is subjected to a torque as shown. Knowing that the allowable shearing stress is 8 ksi and that $b = 1.4$ in., determine the maximum torque **T** that can be applied to each bar.

SOLUTION

$$\tau_{\max} = 8 \text{ ksi}, \quad b = 1.4 \text{ in.}$$

(*a*) Square: $a = b = 1.4$ in. $\quad \dfrac{a}{b} = 1.0$

From Table 3.1, $c_1 = 0.208$

$$\tau_{\max} = \frac{T}{c_1 a b^2} \qquad T = c_1 a b^2 \tau_{\max}$$

$$T = (0.208)(1.4)(1.4)(1.4)^2(8) \qquad T = 4.57 \text{ kip} \cdot \text{in} \blacktriangleleft$$

(*b*) Circle: $c = \dfrac{1}{2} b = 0.7$ in.

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad T = \frac{\pi}{2} c^3 \tau_{\max}$$

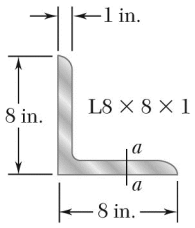
$$T = \frac{\pi}{2}(0.7)^3(8) \qquad T = 4.31 \text{ kip} \cdot \text{in} \blacktriangleleft$$

(*c*) Rectangle: $a = (1.2)(1.4) = 1.68$ in. $\quad \dfrac{a}{b} = 1.2$

From Table 3.1, $c_1 = 0.219$

$$T = c_1 a b^2 \tau_{\max} = (0.219)(1.68)(1.4)^2(8) \qquad T = 5.77 \text{ kip} \cdot \text{in} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.135

A 36-kip · in. torque is applied to a 10-ft-long steel angle with an $L8 \times 8 \times 1$ cross section. From Appendix C, we find that the thickness of the section is 1 in. and that its area is, 15.00 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a*-*a*, (*b*) the angle of twist.

SOLUTION

$$a = \frac{A}{t} = \frac{15 \text{ in}^2}{1 \text{ in.}} = 15 \text{ in.}, \quad b = 1 \text{ in.}, \quad \frac{a}{b} = 15$$

Since $\dfrac{a}{b} > 5, \quad c_1 = c_2 = \dfrac{1}{3}\left(1 - 0.630\dfrac{b}{a}\right)$

or $c_1 = c_2 = \dfrac{1}{3}\left(1 - \dfrac{0.630}{15}\right) = 0.3193$

$$T = 36 \times 10^3 \text{ lb} \cdot \text{in}; \quad L = 120 \text{ in.}; \quad G = 11.2 \times 10^6 \text{ psi}$$

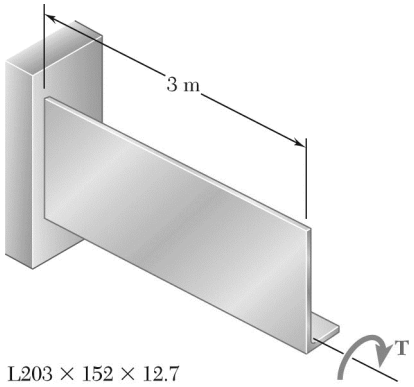
(*a*) Maximum shearing stress: $\tau_{\max} = \dfrac{T}{c_1 a b^2}$

$$\tau_{\max} = \frac{36 \times 10^3}{(0.3193)(15)(1)^2} = 7.52 \times 10^3 \text{ psi} \qquad \tau_{\max} = 7.52 \text{ ksi} \blacktriangleleft$$

(*b*) Angle of twist: $\varphi = \dfrac{TL}{c_2 a b^2 G}$

$$\varphi = \frac{(36 \times 10^3)(120)}{(0.3193)(15)(1)^3(11.2 \times 10^6)} = 0.08052 \text{ radians} \qquad \varphi = 4.61° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.136

A 3-m-long steel angle has an $L203 \times 152 \times 12.7$ cross section. From Appendix C, we find that the thickness of the section is 12.7 mm and that its area is 4350 mm^2. Knowing that $\tau_{all} = 50$ MPa and that $G = 77.2$ GPa, and ignoring the effect of stress concentration, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist.

SOLUTION

$$A = 4350 \text{ mm}^2 \quad b = 12.7 \text{ mm} \quad a = ?$$

Equivalent rectangle.

$$a = \frac{A}{b} = \frac{4350}{12.7} = 342.52 \text{ mm}$$

$$\frac{a}{b} = 26.97$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.32555$$

(*a*) $\tau_{max} = \dfrac{T}{c_1 ab^2} \quad \tau_{max} = 50 \times 10^6 \text{ Pa}$

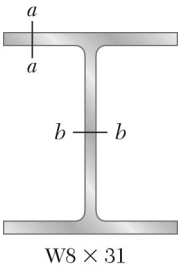
$$T = c_1 ab^2 \tau_{max} = (0.32555)(26.97 \times 10^{-3})(12.7 \times 10^{-3})^2 (50 \times 10^6)$$

$$= 70.807 \text{ N} \cdot \text{m} \qquad T = 70.8 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $$\varphi = \frac{TL}{c_2 ab^3 G} = \frac{(70.807)(3)}{(0.32555)(26.97 \times 10^{-3})(12.7 \times 10^{-3})(77.2 \times 10^9)}$$

$$= 0.15299 \text{ rad} \qquad \varphi = 8.77° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.137

An 8-ft-long steel member with a W8 × 31 cross section is subjected to a 5-kip · in. torque. The properties of the rolled-steel section are given in Appendix C. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a*-*a*, (*b*) the maximum shearing stress along line *b*-*b*, (*c*) the angle of twist. (*Hint*: consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

Flange:

$$a = 7.995 \text{ in.}, \quad b = 0.435 \text{ in.}, \quad \frac{a}{b} = \frac{7.995}{0.435} = 18.38$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3219 \qquad \varphi_f = \frac{T_f L}{c_2 a b^3 G}$$

$$T_f = c_2 a b^3 \frac{G\varphi_f}{L} = K_f \frac{G\varphi}{L} \quad \text{where} \quad K_f = c_2 a b^3$$

$$K_f = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^3$$

Web:

$$a = 8.0 - (2)(0.435) = 7.13 \text{ in.}, \quad b = 0.285 \text{ in.}, \quad \frac{a}{b} = \frac{7.13}{0.285} = 25.02$$

$$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3249 \qquad \varphi_w = \frac{T_w L}{c_2 a b^3 G}$$

$$T_w = c_2 a b^3 \frac{G\varphi_w}{L} = K_w \frac{G\varphi}{L} \quad \text{where} \quad K_w = c_2 a b^3$$

$$K_w = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$$

For matching twist angles:

$$\varphi_f = \varphi_w = \varphi$$

Total torque.

$$T = 2T_f + T_w = (2K_f + K_w)\frac{G\varphi}{L}$$

$$\frac{G\varphi}{L} = \frac{T}{2K_p + K_w}, \quad T_f = \frac{K_f T}{2K_f + K_w}, \quad T_w = \frac{K_w T}{2K_f + K_w}$$

$$T_f = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb} \cdot \text{in}; \quad T_w = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb} \cdot \text{in}$$

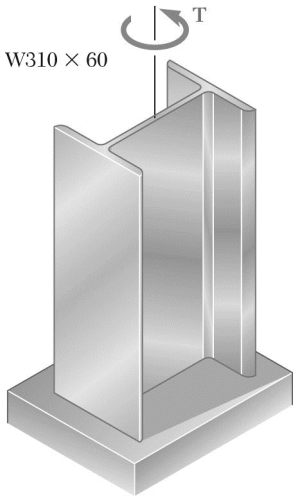
(*a*) $$\tau_f = \frac{T_f}{c_1 a b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi}$$ $\tau_f = 4.57$ ksi ◀

(*b*) $$\tau_w = \frac{T_w}{c_1 a b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi}$$ $\tau_w = 2.96$ ksi ◀

(*c*) $$\frac{G\varphi}{L} = \frac{T}{2K_f + K_w} \quad \therefore \quad \varphi = \frac{TL}{G(2K_f + K_w)} \quad \text{where} \quad L = 8 \text{ ft} = 96 \text{ in.}$$

$$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.563]} = 88.6 \times 10^{-3} \text{ rad}$$ $\varphi = 5.08°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.138

A 4-m-long steel member has a W310 × 60 cross section. Knowing that $G = 77.2$ GPa and that the allowable shearing stress is 40 MPa, determine (*a*) the largest torque **T** that can be applied, (*b*) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.137.)

SOLUTION

$$\text{W310} \times 60, \quad L = 4\text{ m}, \quad G = 77.2\text{ GPa}, \quad \tau_{\text{all}} = 40\text{ MPa}$$

For one flange: From App. C, $a = 203$ mm, $b = 13.1$ mm, $a/b = 15.50$

Eq. (3.45): $$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630}{15.50}\right) = 0.320$$

Eq. (3.44): $$\phi_f = \frac{T_f L}{c_2 a b^3 G} = \frac{T_f(4)}{0.320(0.203)(0.0131)^3(77.2\times10^9)} \tag{1}$$

$$\phi_f = 355.04\times10^{-6}T_f$$

For web: From App. C, $a = 303 - 2(13.1) = 276.8$ mm, $b = 7.5$ mm, $a/b = 36.9$

Eq. (3.45): $$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630}{36.9}\right) = 0.328$$

Eq. (3.44): $$\phi_w = \frac{T_w(4)}{0.328(0.2768)(0.0075)^3(77.2\times10^9)}$$

$$\phi_w = 1.354.2\times10^{-6}T_w \tag{2}$$

Since angle of twist is the same for flanges and web:

$$\phi_f = \phi_w: \quad 355.04\times10^{-6}T_f = 1354.2\times10^{-6}T_w$$

$$T_f = 3.814T_w \tag{3}$$

But the sum of the torques exerted on the two flanges and on the web is equal to the torque **T** applied to the member:

$$2T_f + T_w = T \tag{4}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.138 (*Continued*)

Substituting for T_f from (3) into (4):

$$2(3.814T_w) + T_w = T \qquad T_w = 0.11589T \tag{5}$$

From (3): $T_f = 3.814(0.11589T) \qquad T_f = 0.44205T$ (6)

For one flange:

From Eq. (3.43): $T_f = c_1 ab^2 \tau_{\max} = 0.320(0.203)(0.0131)^2(40 \times 10^6)$

$= 445.91 \text{ N} \cdot \text{m}$

Eq. (6): $445.91 = 0.44205T$ $\qquad T = 1009 \text{ N} \cdot \text{m}$

For web: $T_w = c_1 ab^2 \tau_{\max} = 0.328(0.2768)(0.0075)^2(40 \times 10^6)$

$= 204.28 \text{ N} \cdot \text{m}$

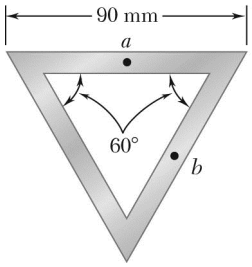
Eq. (5): $204.28 = 0.11589T$ $\qquad T = 1763 \text{ N} \cdot \text{m}$

(*a*) Largest allowable torque: Use the smaller value. $T = 1009 \text{ N} \cdot \text{m}$ ◀

(*b*) Angle of twist: Use T_f, which is critical.

Eq. (1): $\phi = \phi_f = (355.04 \times 10^{-6})(445.91) = 0.15831 \text{ rad}$ $\qquad \phi = 9.07°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

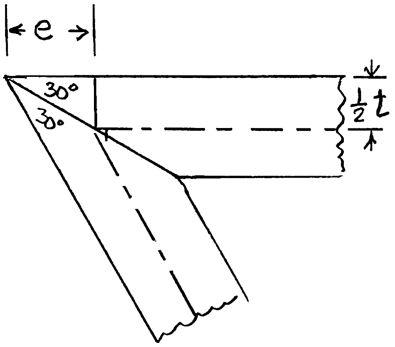


PROBLEM 3.139

A torque $T = 750$ kN · m is applied to a hollow shaft shown that has a uniform 8-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.

SOLUTION

Detail of corner.



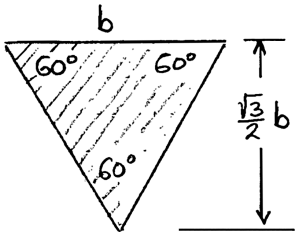
$$\frac{1}{2}t = e \tan 30°$$

$$e = \frac{t}{2 \tan 30°}$$

$$= \frac{8}{2 \tan 30°} = 6.928 \text{ mm}$$

$$b = 90 - 2e = 76.144 \text{ mm}$$

Area bounded by centerline.



$$a = \frac{1}{2}b\frac{\sqrt{3}}{2}b = \frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}(76.144)^2$$

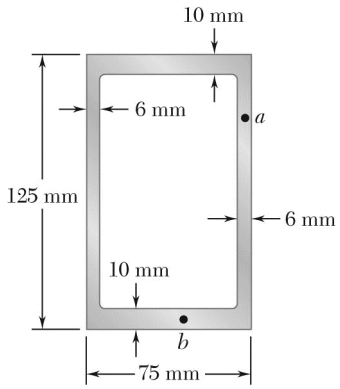
$$= 2510.6 \text{ mm}^2 = 2510.6 \times 10^{-6} \text{ m}^2$$

$$t = 0.008 \text{ m}$$

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.008)(2510 \times 10^{-6})} = 18.67 \times 10^6 \text{ Pa}$$

$$\tau = 18.67 \text{ MPa} \blacktriangleleft$$

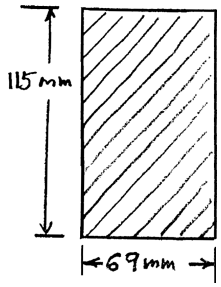
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.140

A torque $T = 5$ kN · m is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION



$$T = 5 \times 10^3 \text{ N} \cdot \text{m}$$

Area bounded by centerline.

$$a = \text{bh} = (69)(115) = 7.935 \times 10^3 \text{ mm}^2$$
$$= 7.935 \times 10^{-3} \text{ m}^2$$

At point *a*: $t = 6 \text{ mm} = 0.006 \text{ m}$

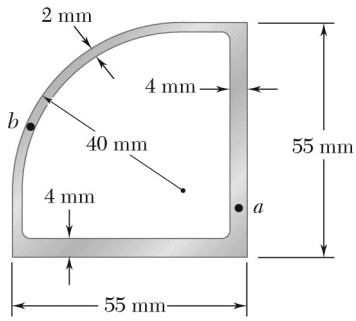
$$\tau = \frac{T}{2ta} = \frac{5 \times 10^3}{(2)(0.006)(7.935 \times 10^{-3})}$$

$$= 52.5 \times 10^6 \text{ Pa} \qquad \tau = 52.5 \text{ MPa} \blacktriangleleft$$

At point *b*: $t = 10 \text{ mm} = 0.010 \text{ m}$

$$\tau = \frac{T}{2ta} = \frac{5 \times 10^3}{(2)(0.010)(7.935 \times 10^{-3})} = 31.5 \times 10^6 \text{ Pa} \qquad \tau = 31.5 \text{ MPa} \blacktriangleleft$$

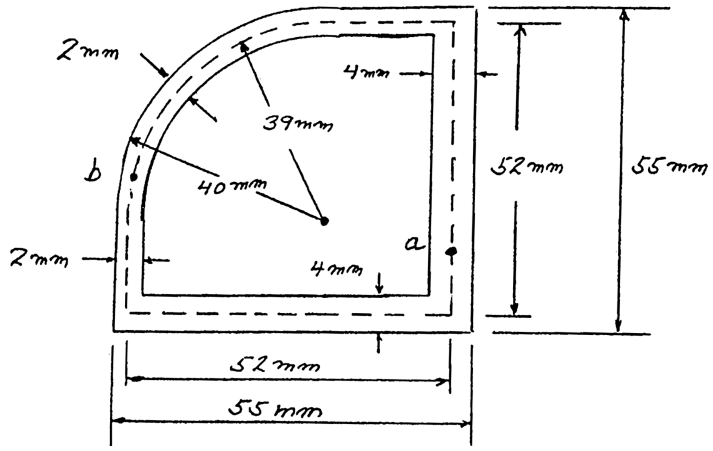
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



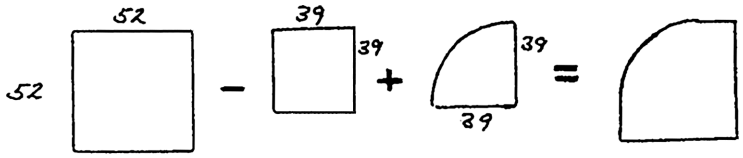
PROBLEM 3.141

A 90-N · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION



Area bounded by centerline.



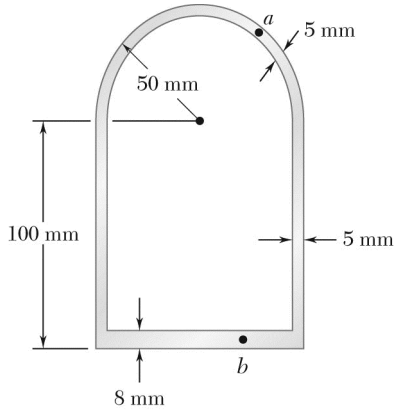
$$a = 52 \times 52 - 39 \times 39 + \frac{\pi}{4}(39)^2 = 2378 \text{ mm}^2 = 2.378 \times 10^{-3} \text{ m}^2$$

$$T = 90 \text{ N} \cdot \text{m}$$

$$\tau_a = \frac{T}{2ta} = \frac{90 \text{ N} \cdot \text{m}}{2(4 \times 10^{-3} \text{ m})(2.378 \times 10^{-3} \text{ m}^2)} \qquad \tau_a = 4.73 \text{ MPa} \blacktriangleleft$$

$$\tau_b = \frac{T}{2ta} = \frac{90 \text{ N} \cdot \text{m}}{2(2 \times 10^{-3} \text{ m})(2.378 \times 10^{-3} \text{m}^2)} \qquad \tau_b = 9.46 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.142

A 5.6 kN·m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION

Area bounded by centerline.

$$a = (96\,\text{mm})(95\,\text{mm}) + \frac{\pi}{2}(47.5\,\text{mm})^2 = 12.664 \times 10^3\,\text{mm}^2$$

$$= 12.664 \times 10^{-3}\,\text{m}^2$$

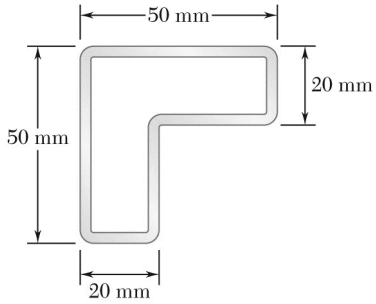
At point *a*, $t = 5\,\text{mm} = 0.005\,\text{m}$

$$\tau = \frac{T}{2at} = \frac{5.6 \times 10^3}{(2)(12.664 \times 10^{-3})(0.005)} = 44.2 \times 10^6\,\text{Pa} \qquad \tau = 44.2\ \text{MPa} \blacktriangleleft$$

At point *b*, $t = 8\,\text{mm} = 0.008\,\text{m}$

$$\tau = \frac{T}{2at} = \frac{5.6 \times 10^3}{(2)(12.664 \times 10^{-3})(0.008)} = 27.6 \times 10^6\,\text{Pa} \qquad \tau = 27.6\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

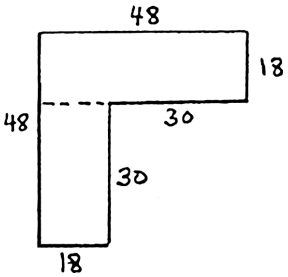


PROBLEM 3.143

A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque that can be applied to the member.

SOLUTION

Area bounded by centerline.



$$a = (48)(18) + (30)(18)$$

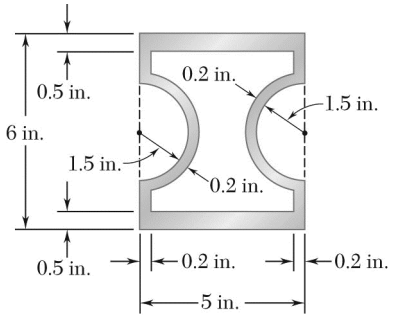
$$= 1404 \text{ mm}^2 = 1404 \times 10^{-6} \text{ m}^2$$

$$t = 0.002 \text{ m}$$

$$\tau = \frac{T}{2ta} \quad \text{or} \quad T = 2ta\tau = (2)(0.002)(1404 \times 10^{-6})(3 \times 10^{6})$$

$$T = 16.85 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

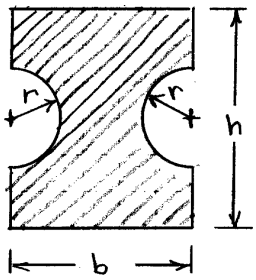


PROBLEM 3.144

A hollow brass shaft has the cross section shown. Knowing that the shearing stress must not exceed 12 ksi and neglecting the effect of stress concentrations, determine the largest torque that can be applied to the shaft.

SOLUTION

Calculate the area bounded by the center line of the wall cross section. The area is a rectangle with two semi-circular cutouts.



$$b = 5 - 0.2 = 4.8 \text{ in.}$$

$$h = 6 - 0.5 = 5.5 \text{ in.}$$

$$r = 1.5 + 0.1 = 1.6 \text{ in.}$$

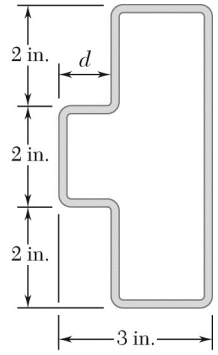
$$a = \text{bh} - 2\left(\frac{\pi}{2}r^2\right) = (4.8)(5.5) - \pi(1.6)^2 = 18.3575 \text{ in}^2$$

$$\tau_{\max} = \frac{T}{2at_{\min}} \qquad \tau_{\max} = 12 \times 10^3 \text{ psi} \qquad t_{\min} = 0.2 \text{ in.}$$

$$T = 2at_{\min}\,\tau_{\max} = (2)(18.3575)(0.2)(12 \times 10^3) = 88.116 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T = 88.1 \text{kip} \cdot \text{in} = 7.34 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

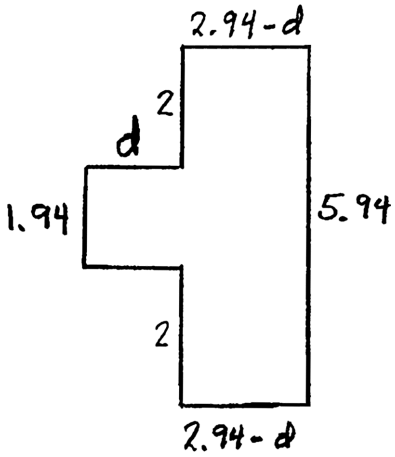
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.145

A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb · in.-torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

SOLUTION



Area bounded by centerline.

$$a = (5.94)(2.94 - d) + 1.94\,d = 17.4636 - 4.00\,d$$

$$t = 0.06 \text{ in.}, \quad \tau = 750 \text{ psi}, \quad T = 1250 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{T}{2ta}$$

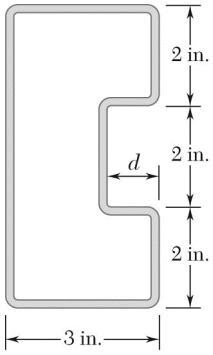
$$a = \frac{T}{2t\tau}$$

$$17.4636 - 4.00d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in.}$$

$d = 0.894$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

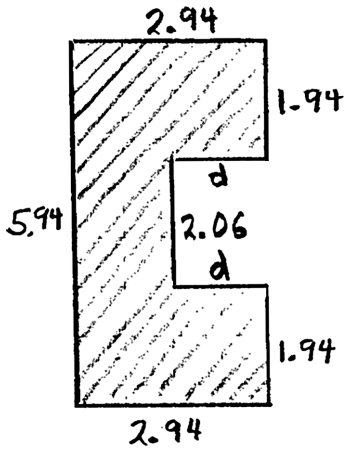


PROBLEM 3.146

A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb · in.-torque will be applied to the member, determine the smallest dimension d that can be used if the shearing stress is not to exceed 750 psi.

SOLUTION

Area bounded by centerline.



$$a = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

$$t = 0.06 \text{ in.}, \quad \tau = 750 \text{ psi}, \quad T = 1250 \text{ lb} \cdot \text{in}$$

$$\tau = \frac{T}{2ta}$$

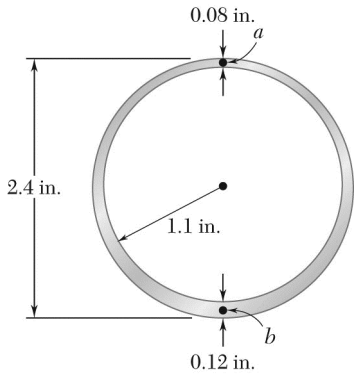
$$a = \frac{T}{2t\tau}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735 \text{ in.}$$

$d = 1.735$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.147

A hollow cylindrical shaft was designed to have a uniform wall thickness of 0.1 in. Defective fabrication, however, resulted in the shaft having the cross section shown. Knowing that a 15 kip · in.-torque is applied to the shaft, determine the shearing stresses at points *a* and *b*.

SOLUTION

Radius of outer circle = 1.2 in.

Radius of inner circle = 1.1 in.

Mean radius = 1.15 in.

Area bounded by centerline.

$$a = \pi r_m^2 = \pi(1.15)^2 = 4.155 \text{ in}^2$$

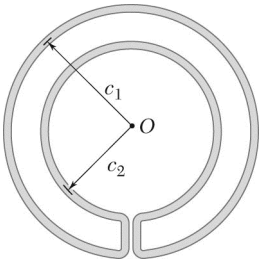
At point *a*, $t = 0.08$ in.

$$\tau = \frac{T}{2ta} = \frac{15}{(2)(0.08)(4.155)} \qquad \tau = 22.6 \text{ ksi} \blacktriangleleft$$

At point *b*, $t = 0.12$ in.

$$\tau = \frac{T}{2ta} = \frac{15}{(2)(0.12)(4.155)} \qquad \tau = 15.04 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.148

A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3-mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the center line of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN · m is applied to the tube, determine (*a*) the maximum shearing stress in the tube, (*b*) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

SOLUTION

Area bounded by centerline.

$$a = \pi\left(c_1^2 - c_2^2\right) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

$$t = 0.003 \text{ m}$$

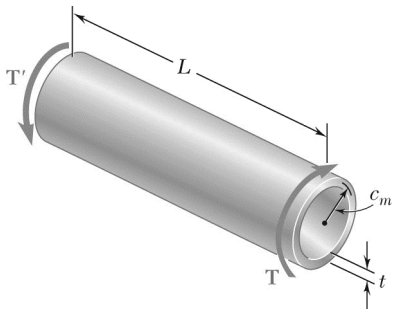
(*a*) $$\tau = \frac{T}{2ta} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa}$$ $\tau = 12.76$ MPa ◀

(*b*) $$T_1 = (2\pi c_1 t\ \tau c_1) = 2\pi c_1^2 t \tau$$

$$= 2\pi(0.150)^2(0.003)(12.73 \times 10^6) = 5.40 \times 10^3 \text{ N} \cdot \text{m}$$ $T_1 = 5.40$ kN · m ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.149



A hollow cylindrical shaft of length L, mean radius c_m, and uniform thickness t is subjected to a torque of magnitude T. Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist φ obtained from the elastic torsion formulas developed in Sections 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled shafts. (*a*) Show that the relative error introduced by using the thin-walled-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and φ and that the relative error is positive and proportional to the ratio t/c_m. (*b*) Compare the percent error corresponding to values of the ratio t/c_m of 0.1, 0.2, and 0.4.

SOLUTION

Let c_2 = outer radius = $c_m + \frac{1}{2}t$ and c_1 = inner radius = $c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}\left(c_2^2 + c_1^2\right)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}\left(c_m^2 + c_m t + \frac{1}{4}t^2 + c_m^2 - c_m t + \frac{1}{4}t^2\right)(2c_m)t$$

$$= 2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m t$$

$$\tau_m = \frac{Tc_m}{J} = \frac{T}{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)t}$$

$$\varphi_1 = \frac{TL}{JG} = \frac{TL}{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m t\,G}$$

Area bounded by centerline. $a = \pi c_m^2$

$$\tau_{\text{ave}} = \frac{T}{2ta} = \frac{T}{2\pi c_m^2 t}$$

$$\varphi_2 = \frac{TL}{4a^2G}\oint\frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3 t\,G}$$

(*a*) Ratios:

$$\frac{\tau_{\text{ave}}}{\tau_m} = \frac{T}{2\pi c_m^2 t} \times \frac{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)t}{T} = 1 + \frac{1}{4}\frac{t^2}{c_m^2} \quad \blacktriangleleft$$

$$\frac{\varphi_2}{\varphi_1} = \frac{TL}{2\pi c_m^3 tG} \times \frac{2\pi\left(c_m^2 + \frac{1}{4}t^2\right)c_m tG}{TL} = 1 + \frac{1}{4}\frac{t^2}{c_m} \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

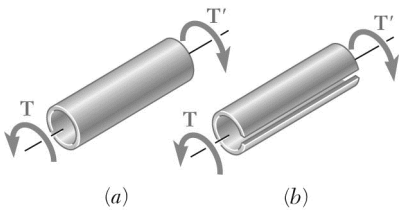
PROBLEM 3.149 (*Continued*)

(*b*) $$\frac{\tau_{\text{ave}}}{\tau_m} - 1 = \frac{\varphi_2}{\varphi_1} - 1 = \frac{1}{4}\frac{t^2}{c_m^2}$$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4}\frac{t^2}{c_m^2}$	0.0025	0.01	0.04
%	0.25%	1%	4%

◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.150

Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (*a*) the ratio τ_b / τ_a of the maximum shearing stresses in the tubes, (*b*) the ratio φ_b/φ_a of the angles of twist of the shafts.

SOLUTION

Without slit:

Area bounded by centerline. $a = \pi c^2$

$$\tau_a = \frac{T}{2ta} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t \qquad \varphi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t\, G}$$

With slit: $a = 2\pi c, \quad b = t, \quad \dfrac{a}{b} = \dfrac{2\pi c}{t} >> 1$

$$c_1 = c_2 = \frac{1}{3}$$

$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\varphi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

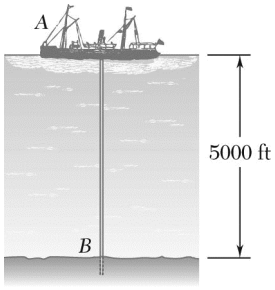
(*a*) Stress ratio:

$$\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t} \qquad \boxed{\frac{\tau_b}{\tau_a} = \frac{3c}{t}} \blacktriangleleft$$

(*b*) Twist ratio:

$$\frac{\varphi_b}{\varphi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t\, G}{TL} = \frac{3c^2}{t^2} \qquad \boxed{\frac{\varphi_b}{\varphi_a} = \frac{3c^2}{t^2}} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.151

The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. Knowing that the top of the 8-in.-diameter steel drill pipe ($G = 11.2 \times 10^6$ psi) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.

SOLUTION

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

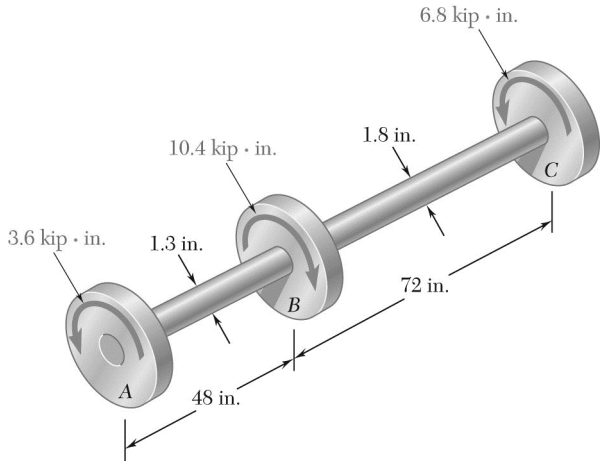
$$\tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}$$

$$\varphi = 2\,\text{rev} = (2)(2\pi) = 12.566\ \text{rad}, \qquad c = \frac{1}{2}d = 4.0\ \text{in.}$$

$$L = 5000\ \text{ft} = 60000\ \text{in.}$$

$$\tau = \frac{(11.2 \times 10^6)(12.566)(4.0)}{60000} = 9.3826 \times 10^3\ \text{psi} \qquad \tau = 9.38\ \text{ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.152

The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 8.5 ksi, determine the smallest allowable diameter of (*a*) shaft *AB*, (*b*) shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau_{\max} = 8.5 \text{ ksi} = 8.5 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2} c^4 \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{AB}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.6 \times 10^3)}{\pi (8.5 \times 10^3)}} = 0.646 \text{ in.} \qquad d_{AB} = 2c = 1.292 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *BC*:

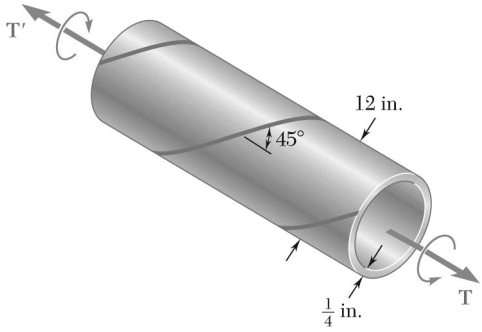
$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\tau_{\max} = 8.5 \times 10^3 \text{ psi}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(6.8 \times 10^3)}{\pi (8.5 \times 10^3)}} = 0.7985 \text{ in.}$$

$$d_{BC} = 2c = 1.597 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



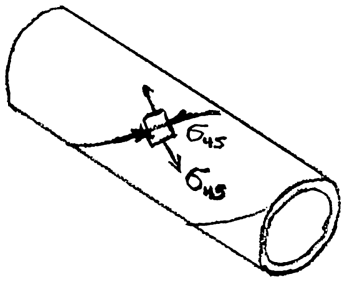
PROBLEM 3.153

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix which forms an angle of 45° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable tensile stress in the weld is 12 ksi, determine the largest torque that can be applied to the pipe.

SOLUTION

From Eq. (3.14) of the textbook,

$$\sigma_{45} = \tau_{\max}$$



hence, $\tau_{\max} = 12\,\text{ksi} = 12 \times 10^3\,\text{psi}$

$$c_2 = \frac{1}{2}d_0 = \frac{1}{2}(12) = 6.00\,\text{in.}$$

$$c_1 = c_2 - t = 6.00 - 0.25 = 5.75\,\text{in.}$$

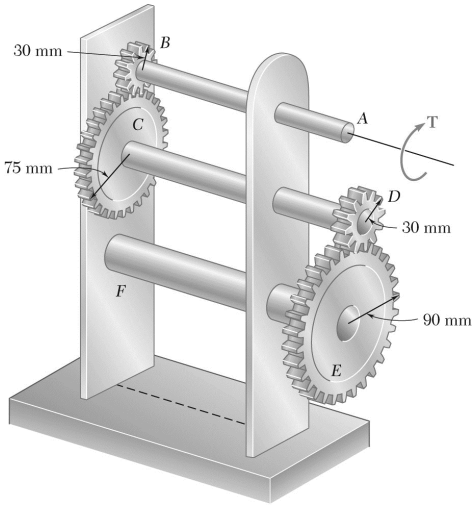
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(6.00)^4 - (5.75)^4] = 318.67\,\text{in.}$$

$$\tau_{\max} = \frac{Tc}{J} \qquad T = \frac{\tau_{\max} J}{c}$$

$$T = \frac{(12 \times 10^3)(318.67)}{6.00} = 637 \times 10^3\,\text{lb} \cdot \text{in}$$

$$T = 637\,\text{kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.154

For the gear train shown, the diameters of the three solid shafts are:

$$d_{AB} = 20\,\text{mm} \quad d_{CD} = 25\,\text{mm} \quad d_{EF} = 40\,\text{mm}$$

Knowing that for each shaft the allowable shearing stress is 60 MPa, determine the largest torque T that can be applied.

SOLUTION

Statics: $T_{AB} = T$

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B} \qquad T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{75}{30} T = 2.5T$$

$$\frac{T_{EF}}{r_F} = \frac{T_{CD}}{r_D} \qquad T_{EF} = \frac{r_F}{r_D} T_{CD} = \frac{90}{30}(2.5T) = 7.5T$$

Determine the magnitude of T so that the stress is $60\ \text{MPa} = 60 \times 10^6\ \text{Pa}$.

$$\tau = \frac{Tc}{J} \qquad T_{\text{shaft}} = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

Shaft *AB*: $c = \frac{1}{2} d_{AB} = 10\ \text{mm} = 0.010\ \text{m}$

$$T_{AB} = T = \frac{\pi}{2}(60 \times 10^6)(0.010)^3 \qquad T = 94.2\ \text{N} \cdot \text{m}$$

Shaft *CD*: $c = \frac{1}{2} d_{CD} = 12.5\ \text{mm} = 0.0125\ \text{m}$

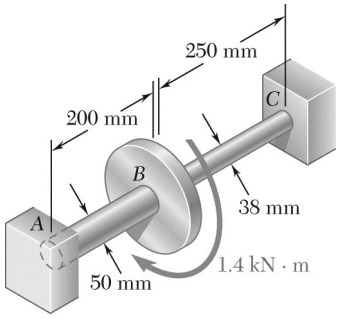
$$T_{CD} = 2.5T = \frac{\pi}{2}(60 \times 10^6)(0.0125)^3 \qquad T = 73.6\ \text{N} \cdot \text{m}$$

Shaft *EF*: $c = \frac{1}{2} d_{EF} = 20\,\text{mm} = 0.020\,\text{m}$

$$T_{EF} = 7.5T = \frac{\pi}{2}(60 \times 10^6)(0.020)^3 \qquad T = 100.5\ \text{N} \cdot \text{m}$$

The smallest value of T is the largest torque that can be applied. $T = 73.6\ \text{N} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.155

Two solid steel shafts ($G = 77.2\,\text{GPa}$) are connected to a coupling disk B and to fixed supports at A and C. For the loading shown, determine (*a*) the reaction at each support, (*b*) the maximum shearing stress in shaft AB, (*c*) the maximum shearing stress in shaft BC.

SOLUTION

Shaft AB: $T = T_{AB}, \quad L_{AB} = 0.200\,\text{m}, \quad c = \dfrac{1}{2}d = 25\,\text{mm} = 0.025\,\text{m}$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.025)^4 = 613.59\times10^{-9}\,\text{m}^4 \qquad \varphi_B = \frac{T_{AB}L_{AB}}{GJ_{AB}}$$

$$T_{AB} = \frac{GJ_{AB}}{L_{AB}}\varphi_B = \frac{(77.2\times10^9)(613.59\times10^{-9})}{0.200}\varphi_B = 236.847\times10^3\,\varphi_B$$

Shaft BC: $T = T_{BC}, \quad L_{BC} = 0.250\,\text{m}, \quad c = \dfrac{1}{2}d = 19\,\text{mm} = 0.019\,\text{m}$

$$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.019)^4 = 204.71\times10^{-9}\,\text{m}^4 \qquad \varphi_B = \frac{T_{BC}L_{BC}}{GJ_{BC}}$$

$$T_{BC} = \frac{GJ_{BC}}{L_{BC}}\varphi_B = \frac{(77.2\times10^9)(204.71\times10^{-9})}{0.250} = 63.214\times10^3\varphi_B$$

Equilibrium of coupling disk. $T = T_{AB} + T_{BC}$

$$1.4\times10^3 = 236.847\times10^3\varphi_B + 63.214\times10^3\varphi_B \qquad \varphi_B = 4.6657\times10^{-3}\,\text{rad.}$$

$$T_{AB} = (236.847\times10^3)(4.6657\times10^{-3}) = 1.10506\times10^3\,\text{N}\cdot\text{m}$$

$$T_{BC} = (63.214\times10^3)(4.6657\times10^{-3}) = 294.94\,\text{N}\cdot\text{m}$$

(*a*) Reactions at supports. $T_A = T_{AB} = 1105\,\text{N}\cdot\text{m}$ ◀

$T_C = T_{BC} = 295\,\text{N}\cdot\text{m}$ ◀

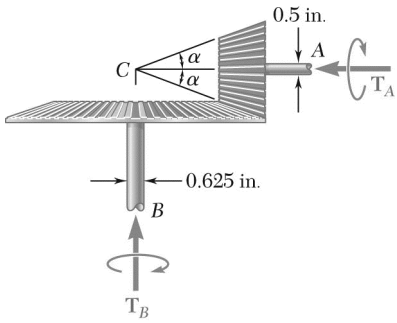
(*b*) Maximum shearing stress in AB.

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(1.10506\times10^3)(0.025)}{613.59\times10^{-9}} = 45.0\times10^6\,\text{Pa} \qquad \tau_{AB} = 45.0\,\text{MPa}$$ ◀

(*c*) Maximum shearing stress in BC.

$$\tau_{BC} = \frac{T_{BC}c}{J_{BC}} = \frac{(294.94)(0.019)}{204.71\times10^{-9}} = 27.4\times10^6\,\text{Pa} \qquad \tau_{BC} = 27.4\,\text{MPa}$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.156

In the bevel-gear system shown, $\alpha = 18.43°$. Knowing that the allowable shearing stress is 8 ksi in each shaft and that the system is in equilibrium, determine the largest torque T_A that can be applied at A.

SOLUTION

Using stress limit for shaft A:

$$\tau = 8 \text{ ksi}, \qquad c = \frac{1}{2}d = 0.25 \text{ in.}$$

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8)(0.25)^3 = 0.1963 \text{ kip} \cdot \text{in}$$

Using stress limit for shaft B:

$$\tau = 8 \text{ ksi}, \quad c = \frac{1}{2}d = 0.3125 \text{ in.}$$

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3 = \frac{\pi}{2}(8)(0.3125)^3 = 0.3835 \text{ kip} \cdot \text{in}$$

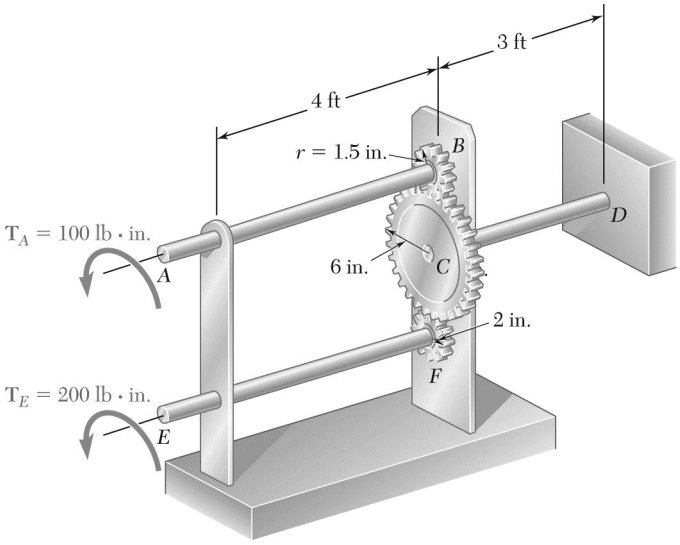
From statics, $$T_A = \frac{r_A}{r_B}T_B = (\tan\alpha)T_B$$

$$T_A = (\tan 18.43°)(0.3835) = 0.1278 \text{ kip} \cdot \text{in}$$

The allowable value of T_A is the smaller.

$$T_A = 0.1278 \text{ kip} \cdot \text{in} \qquad T_A = 127.8 \text{ lb} \cdot \text{in} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.157

Three solid shafts, each of $\frac{3}{4}$-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the angle through which end *A* of shaft *AB* rotates, (*b*) the angle through which end *E* of shaft *EF* rotates.

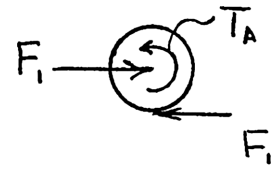
SOLUTION

Geometry:

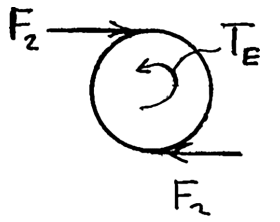
$$r_B = 1.5 \text{ in.}, \quad r_C = 6 \text{ in.}, \quad r_F = 2 \text{ in.}$$
$$L_{AB} = 48 \text{ in.}, \quad L_{CD} = 36 \text{ in.}, \quad L_{EF} = 48 \text{ in.}$$

Statics:
$$T_A = 100 \text{ lb} \cdot \text{in} \circlearrowleft$$
$$T_E = 200 \text{ lb} \cdot \text{in} \circlearrowleft$$

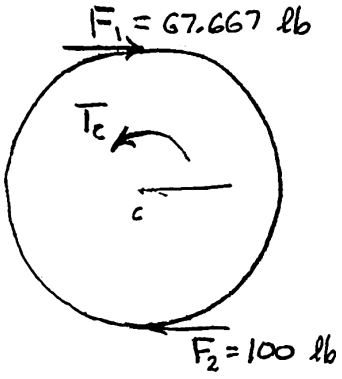
Gear *B*.
$$+\circlearrowleft \Sigma M_B = 0:$$
$$-r_B F_1 + T_A = 0$$
$$-1.5F_1 + 100 = 0$$
$$F_1 = 67.667 \text{ lb}$$



Gear *F*.
$$+\circlearrowleft \Sigma M_F = 0:$$
$$-r_F F_2 + T_E = 0$$
$$-2F_2 + 200 = 0$$
$$F_2 = 100 \text{ lb}$$



Gear *C*.
$$+\circlearrowleft \Sigma M_C = 0:$$
$$-r_C F_1 - r_C F_2 + T_C = 0$$
$$-(6)(66.667) - (6)(100) + T_C = 0$$
$$T_C = 1000 \text{ lb} \cdot \text{in} \circlearrowleft$$



***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.157 (*Continued*)

Deformations:

For all shafts, $c = \frac{1}{2}d = 0.375 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4$$

$$\varphi_{A/B} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(100)(48)}{(11.2 \times 10^6)(0.031063)} = 0.013797 \text{ rad}$$

$$\varphi_{E/F} = \frac{T_{EF}L_{EF}}{GJ} = \frac{(200)(48)}{(11.2 \times 10^6)(0.031063)} = 0.027594 \text{ rad}$$

$$\varphi_{C/D} = \frac{T_{CD}L_{CD}}{GJ} = \frac{(1000)(36)}{(11.2 \times 10^6)(0.031063)} = 0.103476 \text{ rad}$$

Kinematics: $\varphi_C = \varphi_{C/D} = 0.103476 \text{ rad}$

$$r_B\varphi_B = r_C\varphi_C \qquad \varphi_B = \frac{r_C}{r_B}\varphi_C \frac{6}{1.5}(0.103476) = 0.41390 \text{ rad}$$

(*a*) $\varphi_A = \varphi_B + \varphi_{A/B} = 0.41390 + 0.01397 = 0.42788 \text{ rad}$ $\qquad \varphi_A = 24.5°$ ◀

$$r_F\varphi_F = r_C\varphi_C \qquad \varphi_F = \frac{r_C}{r_F}\varphi_C = \frac{6}{2}(0.103476) = 0.31043 \text{ rad}$$

(*b*) $\varphi_E = \varphi_F + \varphi_{E/F} = 0.31043 + 0.027594 = 0.33802 \text{ rad}$ $\qquad \varphi_E = 19.37°$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 3.158

The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

SOLUTION

$$T = 750\ \text{N} \cdot \text{m}, \quad \varphi = 4^\circ = 69.813 \times 10^{-3}\ \text{rad},$$

$$L = 1.2\ \text{m}, \quad J = \frac{\pi}{2} c^4$$

$$\tau = 90\,\text{MPa} = 90 \times 10^6\ \text{Pa} \quad G = 77.2\ \text{GPa} = 77.2 \times 10^9\ \text{Pa}$$

Based on angle of twist. $\varphi = \dfrac{TL}{GJ} = \dfrac{2TL}{\pi G c^4}$

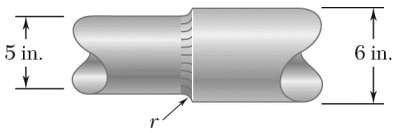
$$c = \sqrt[4]{\frac{2TL}{\pi G \varphi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi (77.2 \times 10^9)(69.813 \times 10^{-3})}} = 18.06 \times 10^{-3}\ \text{m}$$

Based on shearing stress. $\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi \tau}} = \sqrt[3]{\frac{(2)(750)}{\pi (90 \times 10^6)}} = 17.44 \times 10^{-3}\ \text{m}$$

Use larger value. $c = 18.06 \times 10^{-3}\ \text{m} = 18.06\ \text{mm}$ $\qquad d = 2c = 36.1\ \text{mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.159

The stepped shaft shown rotates at 450 rpm. Knowing that $r = 0.5$ in., determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 7500 psi.

SOLUTION

$$d = 5 \text{ in.}$$

$$D = 6 \text{ in.}$$

$$r = 0.5 \text{ in.}$$

$$\frac{D}{d} = \frac{6}{5} = 1.20$$

$$\frac{r}{d} = \frac{0.5}{5} = 0.10$$

From Fig. 3.32, $K = 1.33$

For smaller side, $c = \frac{1}{2}d = 2.5 \text{ in.}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(2.5)^3(7500)}{(2)(1.33)} = 138.404 \times 10^3 \text{ lb} \cdot \text{in}$$

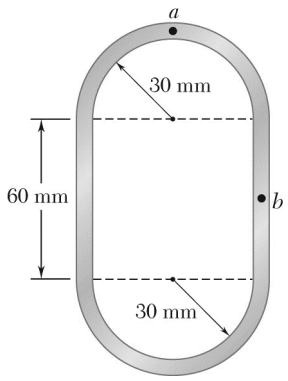
$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

Power. $P = 2\pi f T = 2\pi(7.5)(138.404 \times 10^3) = 6.52 \times 10^6 \text{ in} \cdot \text{lb/s}$

Recalling that $1 \text{ hp} = 6600 \text{ in} \cdot \text{lb/s}$,

$$P = 988 \text{ hp} \blacktriangleleft$$

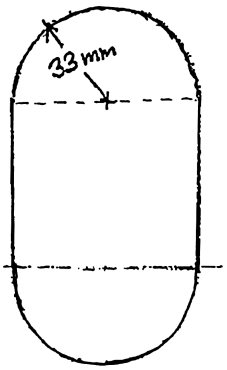
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.160

A 750-N · m torque is applied to a hollow shaft having the cross section shown and a uniform 6-mm wall thickness. Neglecting the effect of stress concentrations, determine the shearing stress at points *a* and *b*.

SOLUTION



Area bounded by centerline.

$$a = 2\frac{\pi}{2}(33)^2 + (60)(66) = 7381 \text{ mm}^2$$

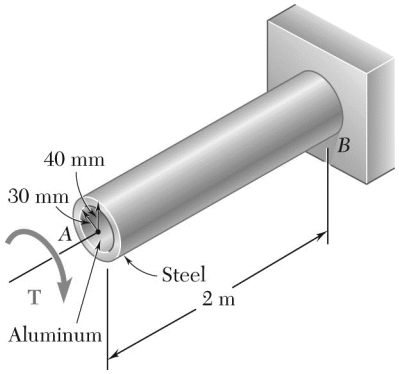
$$= 7381 \times 10^{-6} \text{ m}^2$$

$$t = 0.006 \text{ m at both } a \text{ and } b,$$

Then at points *a* and *b*,

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.006)(7381 \times 10^{-6})} = 8.47 \times 10^6 \text{ Pa} \qquad \tau = 8.47 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.161

The composite shaft shown is twisted by applying a torque **T** at end *A*. Knowing that the maximum shearing stress in the steel shell is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use $G = 77.2$ GPa for steel and $G = 27$ GPa for aluminum.

SOLUTION

Let G_1, J_1, and τ_1 refer to the aluminum core and G_2, J_2, and τ_2 refer to the steel shell.

At the outer surface on the steel shell,

$$\gamma_2 = \frac{c_2\varphi}{L} \quad \therefore \quad \frac{\varphi}{L} = \frac{\gamma_2}{c_2} = \frac{\tau_2}{c_2 G_2}$$

At the outer surface of the aluminum core,

$$\gamma_1 = \frac{c_1\varphi}{L} \quad \therefore \quad \frac{\varphi}{L} = \frac{\gamma_1}{c_1} = \frac{\tau_1}{c_1 G}$$

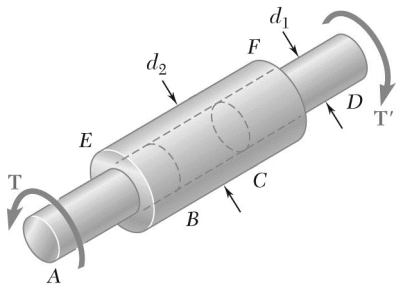
Matching $\frac{\varphi}{L}$ for both components,

$$\frac{\tau_2}{c_2 G_2} = \frac{\tau_1}{c_1 G_1}$$

Solving for τ_2, $\quad \tau_2 = \frac{c_2}{c_1} \cdot \frac{G_2}{G_1}\tau_1 = \frac{0.030}{0.040} \cdot \frac{27 \times 10^9}{77.2 \times 10^9} \cdot 150 \times 10^6 = 39.3 \times 10^6$ Pa

$\tau_2 = 39.3$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 3.162

Two solid brass rods *AB* and *CD* are brazed to a brass sleeve *EF*. Determine the ratio d_2/d_1 for which the same maximum shearing stress occurs in the rods and in the sleeve.

SOLUTION

Let $c_1 = \frac{1}{2}d_1$ and $c_2 = \frac{1}{2}d_2$

Shaft *AB*: $\tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$

Sleeve *EF*. $\tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

For equal stresses, $\frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)}$

$$c_2^4 - c_1^4 = c_1^3 c_2$$

Let $x = \frac{c_2}{c_1}$ $x^4 - 1 = x$ or $x = \sqrt[4]{1 + x}$

Solve by successive approximations starting with $x_0 = 1.0$.

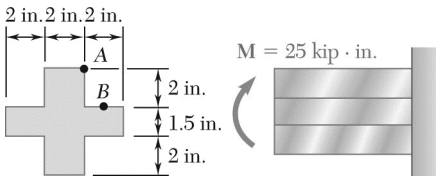
$$x_1 = \sqrt[4]{2} = 1.189, \quad x_2 = \sqrt[4]{2.189} = 1.216, \quad x_3 = \sqrt[4]{2.216} = 1.220$$

$$x_4 = \sqrt[4]{2.220} = 1.221, \quad x_5 = \sqrt[4]{2.221} = 1.221 \quad \text{(converged)}.$$

$x = 1.221$ $\frac{c_2}{c_1} = 1.221$ $\frac{d_2}{d_1} = 1.221$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

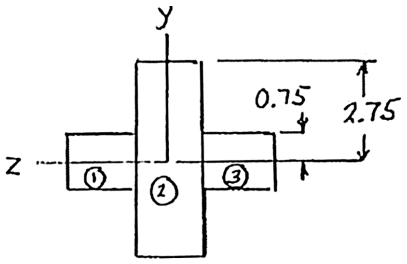
CHAPTER 4



PROBLEM 4.1

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



For rectangle: $I = \frac{1}{12}bh^3$

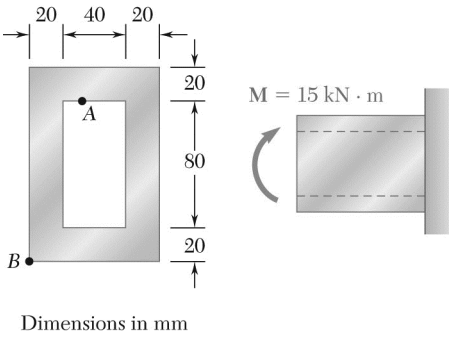
For cross sectional area:

$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(*a*) $y_A = 2.75$ in. $\quad \sigma_A = -\frac{My_A}{I} = -\frac{(25)(2.75)}{28.854}$ $\quad \sigma_A = -2.38$ ksi ◀

(*b*) $y_B = 0.75$ in. $\quad \sigma_B = -\frac{My_B}{I} = -\frac{(25)(0.75)}{28.854}$ $\quad \sigma_B = -0.650$ ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Dimensions in mm

PROBLEM 4.2

Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

For rectangle: $I = \frac{1}{12}bh^3$

Outside rectangle: $I_1 = \frac{1}{12}(80)(120)^3$

$$I_1 = 11.52 \times 10^6 \text{ mm}^4 = 11.52 \times 10^{-6} \text{ m}^4$$

Cutout: $I_2 = \frac{1}{12}(40)(80)^3$

$$I_2 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

Section: $I = I_1 - I_2 = 9.81333 \times 10^{-6} \text{ m}^4$

(*a*) $y_A = 40 \text{ mm} = 0.040 \text{ m}$

$$\sigma_A = -\frac{My_A}{I} = -\frac{(15 \times 10^3)(0.040)}{9.81333 \times 10^{-6}} = -61.6 \times 10^6 \text{ Pa}$$

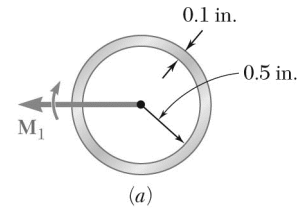
$$\sigma_A = -61.6 \text{ MPa} \blacktriangleleft$$

(*b*) $y_B = -60 \text{ mm} = -0.060 \text{ m}$

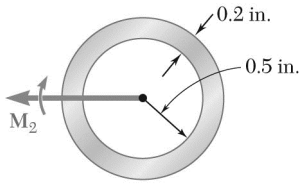
$$\sigma_B = -\frac{My_B}{I} = -\frac{(15 \times 10^3)(-0.060)}{9.81333 \times 10^{-6}} = 91.7 \times 10^6 \text{ Pa}$$

$$\sigma_B = 91.7 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



(*a*)



(*b*)

PROBLEM 4.3

Using an allowable stress of 16 ksi, determine the largest couple that can be applied to each pipe.

SOLUTION

(*a*) $I = \dfrac{\pi}{4}\left(r_o^4 - r_i^4\right) = \dfrac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3}\ \text{in}^4$

$c = 0.6$ in.

$\sigma = \dfrac{Mc}{I}$: $\quad M = \dfrac{\sigma I}{c} = \dfrac{(16)(52.7 \times 10^{-3})}{0.6}$

$M = 1.405\ \text{kip} \cdot \text{in}$ ◀

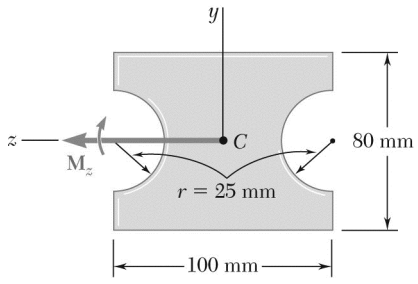
(*b*) $I = \dfrac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3}\ \text{in}^4$

$c = 0.7$ in.

$\sigma = \dfrac{Mc}{I}$: $\quad M = \dfrac{\sigma I}{c} = \dfrac{(16)(139.49 \times 10^{-3})}{0.7}$

$M = 3.19\ \text{kip} \cdot \text{in}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.4

A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple $\mathbf{M_z}$ that can be applied to the bar.

SOLUTION

$$I = I_{\text{rect}} - I_{\text{circle}} = \frac{1}{12}bh^3 - \frac{\pi}{4}r^4$$

$$= \frac{1}{12}(100)(80)^3 - \frac{\pi}{4}(25)^4 = 3.9599 \times 10^6 \text{ mm}^4$$

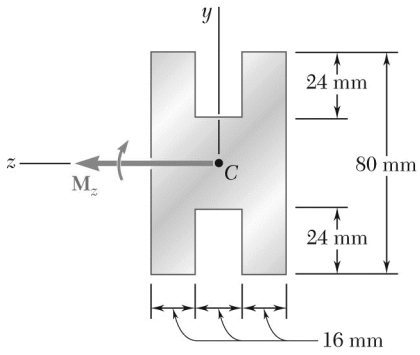
$$= 3.9599 \times 10^{-6} \text{ m}$$

$$c = \frac{80}{2} = 40 \text{ mm} = 0.040 \text{ m}$$

$$\sigma = \frac{Mc}{I}: \qquad M_z = \frac{\sigma I}{c} = \frac{(24 \times 10^6)(3.9599 \times 10^{-6})}{0.040} = 2.38 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_z = 2.38 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



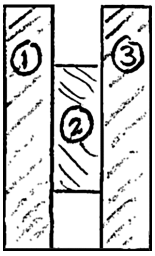
PROBLEM 4.5

A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the *z*-axis.

SOLUTION

Allowable stress. $$= \frac{\sigma_U}{F.S.} = \frac{450}{3} = 150 \text{ MPa}$$

$$= 150 \times 10^6 \text{ Pa}$$



Moment of inertia about *z*-axis.

$$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \text{ mm}^4$$

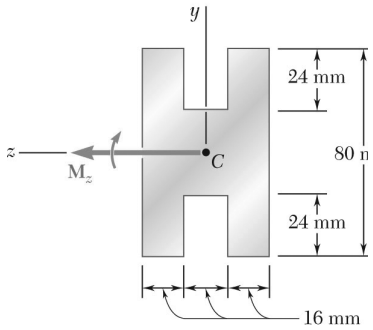
$$I_3 = I_1 = 682.67 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{ mm}^4 = 1.40902 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 5.28 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

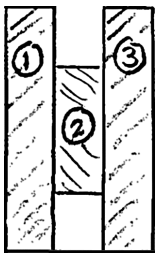


PROBLEM 4.6

Solve Prob. 4.5, assuming that the beam is bent about the *y*-axis.

PROBLEM 4.5 A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the *z*-axis.

SOLUTION



Allowable stress: $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3.00} = 150 \text{ MPa}$

$= 150 \times 10^6 \text{ Pa}$

Moment of inertia about *y*-axis.

$$I_1 = \frac{1}{12}(80)(16)^3 + (80)(16)(16)^2 = 354.987 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(32)(16)^3 = 10.923 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 354.987 \times 10^3 \text{ mm}^4$$

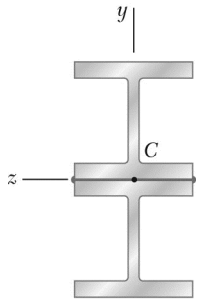
$$I = I_1 + I_2 + I_3 = 720.9 \times 10^3 \text{ mm}^4 = 720.9 \times 10^{-9} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(48) = 24 \text{ mm} = 0.024 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(720.9 \times 10^{-9})(150 \times 10^6)}{0.024} = 4.51 \times 10^3 \text{ N} \cdot \text{m}$$

$M = 4.51 \text{ kN} \cdot \text{m}$ ◀

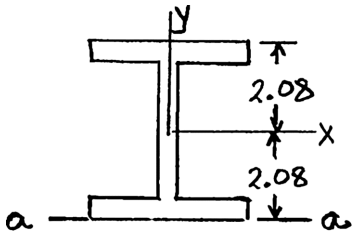
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.7

Two W4×13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

Area $= 3.83$ in^2

Depth $= 4.16$ in.

$I_x = 11.3$ in^4

For one rolled section, moment of inertia about axis *a*-*a* is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections, $I_z = 2I_a = 55.74$ in^4

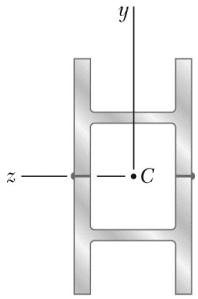
$c =$ depth $= 4.16$ in.

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(55.74)}{4.16}$$

$M_{\text{all}} = 259$ kip · in ◀

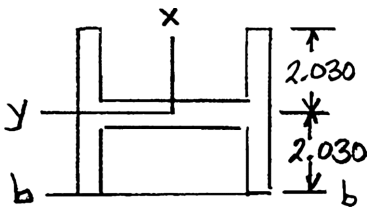
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.8

Two W4×13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the *z* axis.

SOLUTION



Properties of W4 × 13 rolled section.

(See Appendix C.)

$\text{Area} = 3.83 \text{ in}^2$

$\text{Width} = 4.060 \text{ in.}$

$I_y = 3.86 \text{ in}^4$

For one rolled section, moment of inertia about axis *b*-*b* is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections, $I_z = 2I_b = 39.286 \text{ in}^4$

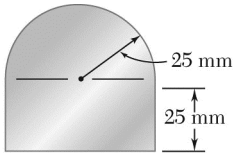
$c = \text{width} = 4.060 \text{ in.}$

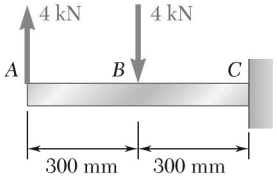
$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{\text{all}} = \frac{\sigma_{\text{all}} I}{c} = \frac{(19.333)(39.286)}{4.060}$$

$$M_{\text{all}} = 187.1 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

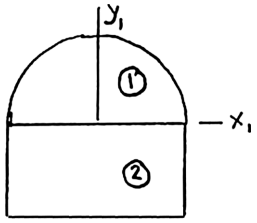




PROBLEM 4.9

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

SOLUTION



$$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2 \quad \bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2 \quad \bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250} = -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$$

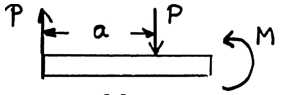
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$$

$$y_{top} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{bot} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$

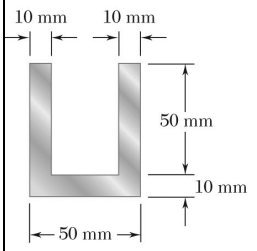


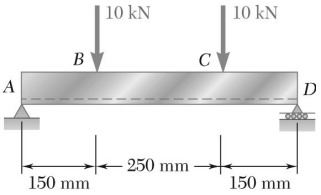
$$M - Pa = 0: \quad M = Pa = (4\times10^3)(300\times10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_{top} = \frac{-My_{top}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa} \qquad \sigma_{top} = -81.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_{bot} = \frac{-My_{bot}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa} \qquad \sigma_{bot} = 67.8 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

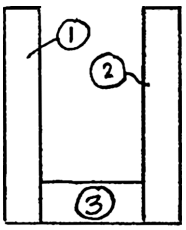




PROBLEM 4.10

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

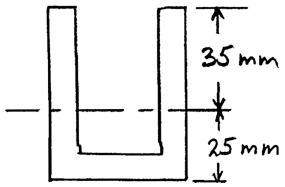
SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_0 = \frac{37.5\times10^3}{1500} = 25\,\text{mm}$$

Neutral axis lies 25 mm above the base.



$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3\,\text{mm}^4 \quad I_2 = I_1 = 195\ \text{mm}^4$$

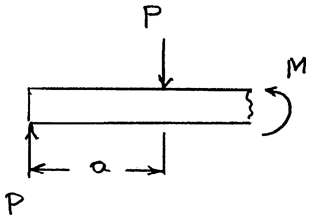
$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3\,\text{mm}^4 = 512.5\times10^{-9}\,\text{m}^4$$

$$y_{\text{top}} = 35\ \text{mm} = 0.035\ \text{m} \quad y_{\text{bot}} = -25\ \text{mm} = -0.025\ \text{m}$$

$$a = 150\ \text{mm} = 0.150\ \text{m} \quad P = 10\times10^3\,\text{N}$$

$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3\,\text{N}\cdot\text{m}$$



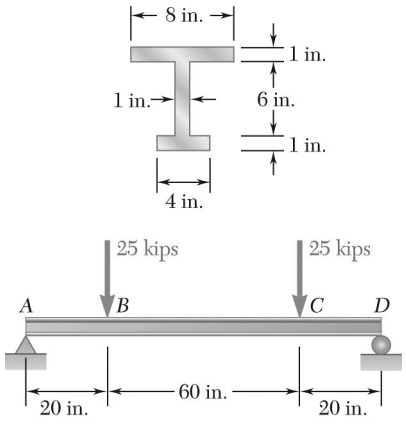
$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6\,\text{Pa}$$

$\sigma_{\text{top}} = -102.4$ MPa (compression) ◀

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6\,\text{Pa}$$

$\sigma_{\text{bot}} = 73.2$ MPa (tension) ◀

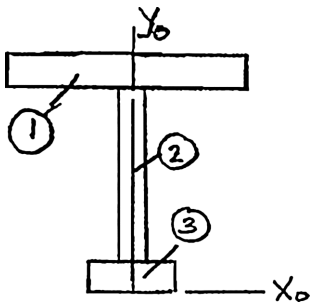
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

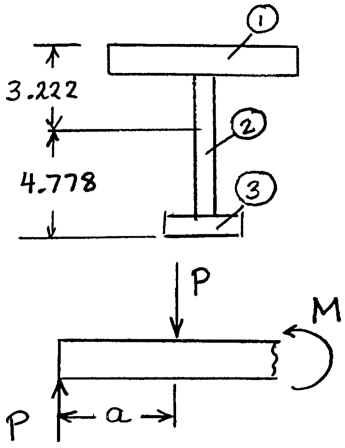
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in.}$$

Neutral axis lies 4.778 in. above the base.



$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.772)^2 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.57 = 155.16 \text{ in}^4$$

$$y_{\text{top}} = 3.222 \text{ in.} \quad y_{\text{bot}} = -4.778 \text{ in.}$$

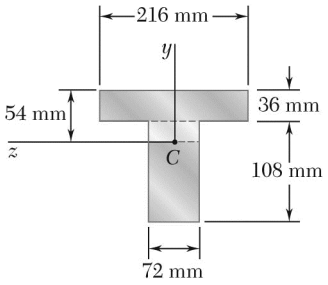
$$M - Pa = 0$$

$$M = Pa = (25)(20) = 500 \text{ kip} \cdot \text{in}$$

$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(500)(3.222)}{155.16} \qquad \sigma_{\text{top}} = -10.38 \text{ ksi (compression)} \blacktriangleleft$$

$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(500)(-4.778)}{155.16} \qquad \sigma_{\text{bot}} = 15.40 \text{ ksi (tension)} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.12

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the top flange.

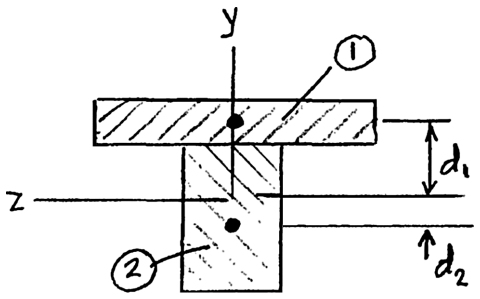
SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$



The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$d_1 = 54 - 18 = 36 \text{ mm}$$
$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

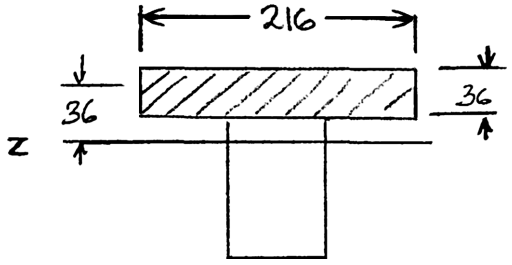
PROBLEM 4.12 *(Continued)*

Moment of inertia of entire cross section:

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$



For the shaded area,

$$A^* = (216)(36) = 7776 \text{ mm}^2$$

$$\bar{y}^* = 36 \text{ mm}$$

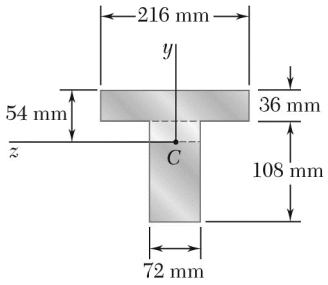
$$A^* \bar{y}^* = 279.936 \times 10^3 \text{ mm}^3 = 279.936 \times 10^{-6} \text{ m}^3$$

$$F = -\left| \frac{MA^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(279.936 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

$$= 58.8 \times 10^3 \text{ N}$$

$$F = 58.8 \text{ kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.13

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kN · m, determine the total force acting on the shaded portion of the web.

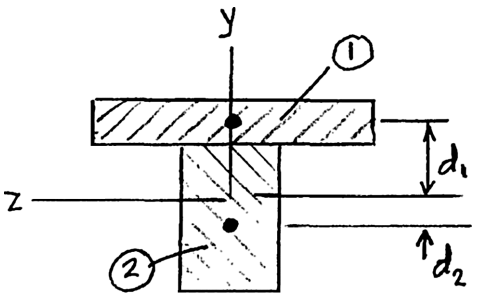
SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$



The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$d_1 = 54 - 18 = 36 \text{ mm}$$
$$d_2 = 54 + 36 - 54 = 36 \text{ mm}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

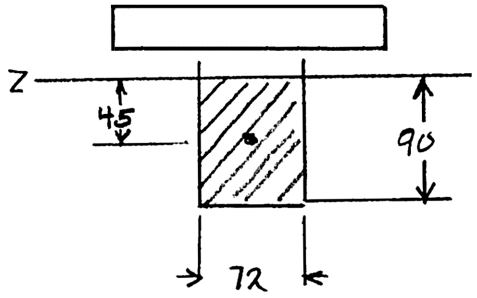
PROBLEM 4.13 *(Continued)*

Moment of inertia of entire cross section:

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(216)(36)^3 + (216)(36)(36)^2 = 10.9175 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(72)(108)^3 + (72)(108)(36)^2 = 17.6360 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.5535 \times 10^6 \text{ mm}^4 = 28.5535 \times 10^{-6} \text{ m}^4$$



For the shaded area,

$$A^* = (72)(90) = 6480 \text{ mm}^2$$

$$\bar{y}^* = 45 \text{ mm}$$

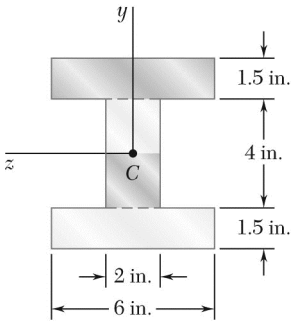
$$A^* \bar{y}^* = 291.6 \times 10^3 \text{ mm}^3 = 291.6 \times 10^{-6} \text{ m}$$

$$F = \left| \frac{MA^* \bar{y}^*}{I} \right| = \frac{(6 \times 10^3)(291.6 \times 10^{-6})}{28.5535 \times 10^{-6}}$$

$$= 61.3 \times 10^3 \text{ N}$$

$$F = 61.3 \text{ kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

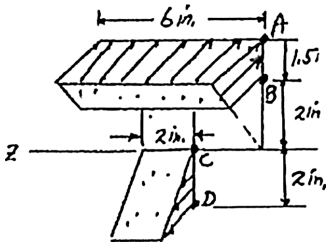


PROBLEM 4.14

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 50 kip · in., determine the total force acting (*a*) on the top flange, (*b*) on the shaded portion of the web.

SOLUTION

The stress distribution over the entire cross-section is given by the bending stress formula: $\sigma_x = -\dfrac{My}{I}$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded portion is calculated from this stress distribution. Over an area element dA, the force is

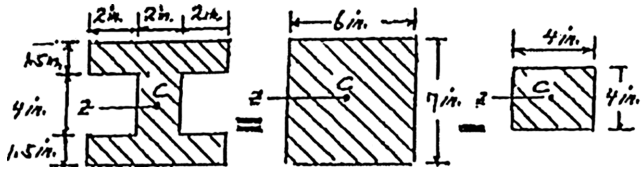
$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

Calculate the moment of inertia.



$$I = \frac{1}{12}(6 \text{ in.})(7 \text{ in.})^3 - \frac{1}{12}(4 \text{ in.})(4 \text{ in.})^3 = 150.17 \text{ in}^4$$

$$M = 50 \text{ kip} \cdot \text{in}$$

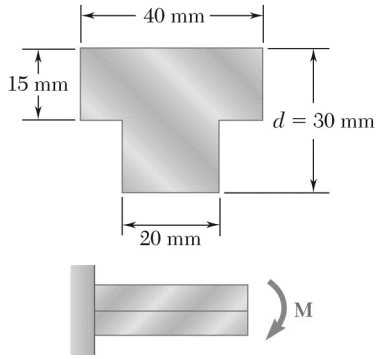
(*a*) Top flange: $A^* = (6 \text{ in.})(1.5 \text{ in.}) = 9 \text{ in}^2$ $\bar{y}^* = 2 \text{ in.} + 0.75 \text{ in.} = 2.75 \text{ in.}$

$$F = \frac{50 \text{ kip} \cdot \text{in}}{150.17 \text{ in}^4}(9 \text{ in}^2)(2.75 \text{ in.}) = 8.24 \text{ kips}$$ $F = 8.24$ kips ◀

(*b*) Half web: $A^* = (2 \text{ in.})(2 \text{ in.}) = 4 \text{ in}^2$ $\bar{y}^* = 1 \text{ in.}$

$$F = \frac{50 \text{ kip} \cdot \text{in}}{150.17 \text{ in}^4}(4 \text{ in}^2)(1 \text{ in.}) = 1.332 \text{ kips}$$ $F = 1.332$ kips ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

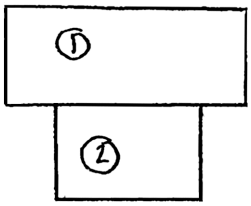


PROBLEM 4.15

The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3



$$\bar{Y}_0 = \frac{15.5\times10^3}{900} = 17.5 \text{ mm}$$ The neutral axis lies 17.5 mm above the bottom.

$$y_{top} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$y_{bot} = -17.5 \text{ mm} = -0.0175 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 61.875\times10^3 \text{ mm}^4 = 61.875\times10^{-9} \text{ m}^4$$

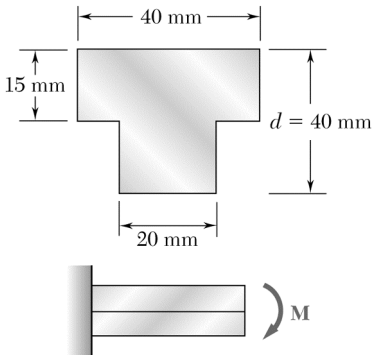
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(61.875\times10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(61.875\times10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 106.1 \text{ N}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



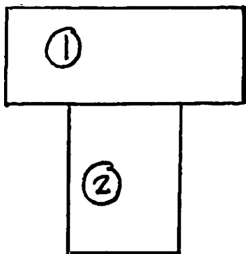
PROBLEM 4.16

Solve Prob. 4.15, assuming that $d = 40$ mm.

PROBLEM 4.15 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	32.5	19.5×10^3
②	500	12.5	6.25×10^3
Σ	1100		25.75×10^3



$$\bar{Y}_0 = \frac{25.75\times10^3}{1100} = 23.41 \text{ mm}$$ The neutral axis lies 23.41 mm above the bottom.

$$y_{\text{top}} = 40 - 23.41 = 16.59 \text{ mm} = 0.01659 \text{ m}$$

$$y_{\text{bot}} = -23.41 \text{ mm} = -0.02341 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(9.09)^2 = 60.827\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^2 + A_2d_2^2 = \frac{1}{12}(20)(25)^3 + (500)(10.91)^2 = 85.556\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 146.383\times10^3 \text{ mm}^4 = 146.383\times10^{-9} \text{ m}^4$$

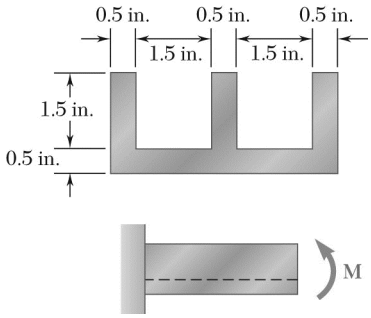
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(146.383\times10^{-9})}{0.01659} = 212 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(146.383\times10^{-9})}{0.02341} = 187.6 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 187.6 \text{ N}\cdot\text{m}$ ◀

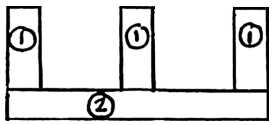
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.17

Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in.}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{\text{top}} = 2.0 - 0.75 = 1.25 \text{ in.}, \qquad y_{\text{bot}} = -0.75 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^2 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

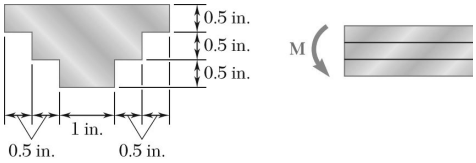
$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

Top: (compression) $$M = \frac{(16)(1.59375)}{1.25} = 20.4 \text{ kip} \cdot \text{in}$$

Bottom: (tension) $$M = \frac{(12)(1.59375)}{0.75} = 25.5 \text{ kip} \cdot \text{in}$$

Choose the smaller as M_{all}. $M_{\text{all}} = 20.4 \text{ kip} \cdot \text{in}$ ◀

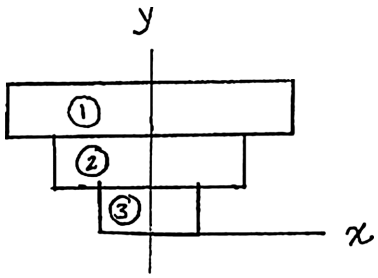
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.18

Knowing that for the casting shown the allowable stress is 5 ksi in tension and 18 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



Locate the neutral axis and compute the moment of inertia.

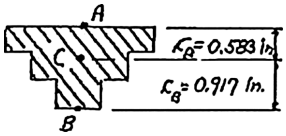
$$\bar{Y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} \qquad \bar{I} = \frac{1}{12} b_i h_i^3 \quad \text{for rectangle}$$

$$d_i = \left| \bar{y}_i - \bar{Y} \right| \qquad I = \sum (A_i d_i^2 + \bar{I})$$

Part	A, in^2	$\bar{y}_i$, in.	$A_i \bar{y}_i$, in^3	d_i, in.	$A_i d_i^2$, in^4	$\bar{I}$, in^4
1	1.5	1.25	1.875	0.3333	0.1667	0.03125
2	1.0	0.75	0.75	0.1667	0.0277	0.02083
3	0.5	0.25	0.125	0.6667	0.2222	0.01042
Σ	3.0		2.75		0.4166	0.0625

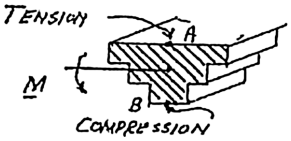
$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{2.75}{3.0} = 0.9167 \text{ in.} \qquad I = \sum (\bar{I} + Ad^2) = 0.4166 + 0.0625 = 0.479 \text{ in}^4$$

Allowable bending moment.



$$\sigma = \frac{Mc}{I} \qquad \text{or} \qquad M = \frac{\sigma I}{c}$$

Tension at A: $\sigma_A \le 5$ ksi



$c_A = 0.583$ in.

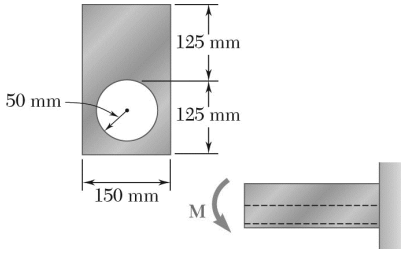
$$M \le \frac{(5)(0.479)}{0.583} = 4.11 \text{ kip} \cdot \text{in}$$

Compression at B: $\sigma_B \le 18$ ksi $\quad c_B = 0.917$ in.

$$M \le \frac{(18)(0.479)}{0.917} = 9.40 \text{ kip} \cdot \text{in}$$

The smaller value is the allowable value of M. $M = 4.11 \text{ kip} \cdot \text{in}$ ◀

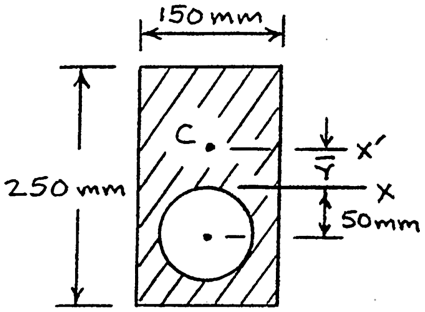
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.19

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION



① rectangle ② circular cutout

$$A_1 = (150)(250) = 37.5\times10^3\ \text{mm}^2$$
$$A_2 = -\pi(50)^2 = -7.85398\times10^3\ \text{mm}^2$$
$$A = A_1 + A_2 = 29.64602\times10^3\ \text{mm}^2$$
$$\bar{y}_1 = 0\ \text{mm}$$
$$\bar{y}_2 = -50\ \text{mm}$$
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$
$$\bar{Y} = \frac{(37.5\times10^3)(0) + (-7.85393\times10^3)(-50)}{29.64602\times10^3}$$
$$= 13.2463\ \text{mm}$$
$$I_{X'} = \Sigma(I + Ad^2) = I_1 - I_2$$
$$= \left[\frac{1}{12}(150)(250)^3 + (37.5\times10^3)(13.2463)^2\right]$$
$$-\left[\frac{\pi}{4}(50)^4 + (7.85398\times10^3)(50+13.2463)^2\right]$$
$$= 201.892\times10^6 - 36.3254\times10^6 = 165.567\times10^6\ \text{mm}^4$$
$$= 165.567\times10^{-6}\ \text{m}^4$$

Top: (tension side) $c = 125 - 13.2463 = 111.7537\ \text{mm} = 0.11175\ \text{m}$

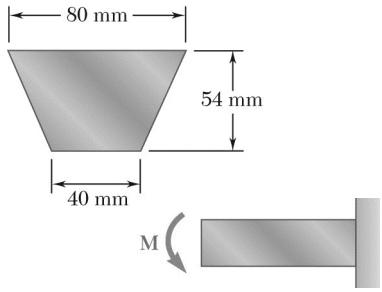
$$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(120\times10^6)}{0.11175}$$
$$= 177.79\times10^3\ \text{N}\cdot\text{m}$$

Bottom: (compression side) $c = 125 + 13.2463 = 138.2463\ \text{mm}$
$$= 0.13825\ \text{m}$$

$$\sigma = \frac{Mc}{I} \qquad M = \frac{I\sigma}{c} = \frac{(165.567\times10^{-6})(150\times10^6)}{0.13825}$$
$$= 179.64\times10^3\ \text{N}\cdot\text{m}$$

Choose the smaller. $M = 177.8\times10^3\ \text{N}\cdot\text{m}$ $M = 177.8\ \text{kN}\cdot\text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

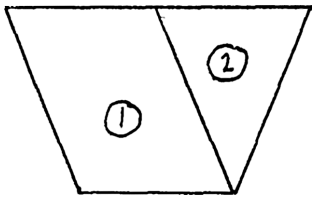


PROBLEM 4.20

Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3	d, mm
①	2160	27	58320	3
②	1080	36	38880	3
Σ	3240		97200	



$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$ The neutral axis lies 30 mm above the bottom.

$$y_{\text{top}} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m} \qquad y_{\text{bot}} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{36} b_2 h_2^2 + A_2 d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

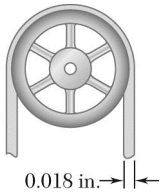
$$|\sigma| = \left| \frac{My}{I} \right| \qquad |M| = \left| \frac{\sigma I}{y} \right|$$

Top: (tension side) $$M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N} \cdot \text{m}$$

Bottom: (compression) $$M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller as M_{all}. $M_{\text{all}} = 3.7908 \times 10^3 \text{ N} \cdot \text{m}$ $M_{\text{all}} = 3.79 \text{ kN} \cdot \text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.21

A steel band saw blade, that was originally straight, passes over 8-in.-diameter pulleys when mounted on a band saw. Determine the maximum stress in the blade, knowing that it is 0.018 in. thick and 0.625 in. wide. Use $E = 29 \times 10^6$ psi.

SOLUTION

Band blade thickness: $t = 0.018$ in.

Radius of pulley: $r = \frac{1}{2} d = 4.000$ in.

Radius of curvature of centerline of blade:

$$\rho = r + \frac{1}{2} t = 4.009 \text{ in.}$$

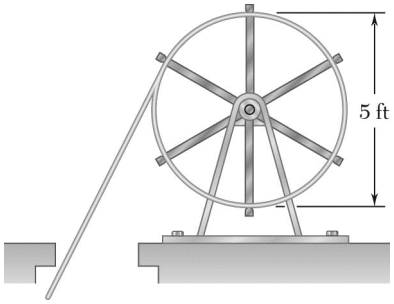
$$c = \frac{1}{2} t = 0.009 \text{ in.}$$

Maximum strain: $\varepsilon_m = \frac{c}{\rho} = \frac{0.009}{4.009} = 0.002245$

Maximum stress: $\sigma_m = E\varepsilon_m = (29 \times 10^6)(0.002245)$

$$\sigma_m = 65.1 \times 10^3 \text{ psi} \qquad \sigma_m = 65.1 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.22

Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a rod, when the rod, which is initially straight, is wrapped on a spool, (*b*) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi .

SOLUTION

Radius of cross section: $r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15$ in.

Moment of inertia: $I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6}$ in^4

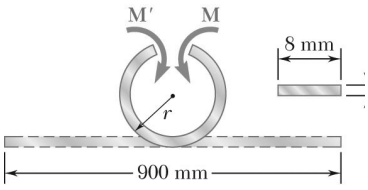
$D = 5$ ft $= 60$ in. $\quad \rho = \frac{1}{2}D = 30$ in.

$c = r = 0.15$ in.

(*a*) $\sigma_{\max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3$ psi $\quad \sigma_{\max} = 145$ ksi ◀

(*b*) $M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30} \quad M = 384$ lb · in ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.23

A 900-mm strip of steel is bent into a full circle by two couples applied as shown. Determine (*a*) the maximum thickness t of the strip if the allowable stress of the steel is 420 MPa, (*b*) the corresponding moment M of the couples. Use $E = 200$ GPa .

SOLUTION

When the rod is bent into a full circle, the circumference is 900 mm. Since the circumference is equal to 2π times ρ, the radius of curvature, we get

$$\rho = \frac{900 \text{ mm}}{2\pi} = 143.24 \text{ mm} = 0.14324 \text{ m}$$

Stress: $\sigma = E\varepsilon = \dfrac{Ec}{\rho}$ or $c = \dfrac{\rho\sigma}{E}$

For $\sigma = 420$ MPa and $E = 200$ GPa,

$$c = \frac{(0.14324)(420 \times 10^6)}{200 \times 10^9} = 0.3008 \times 10^{-3} \text{ m}$$

(*a*) Maximum thickness: $t = 2c = 0.6016 \times 10^{-3}$ m

$$t = 0.602 \text{ mm} \blacktriangleleft$$

Moment of inertia for a rectangular section.

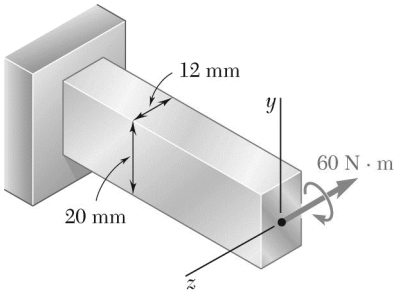
$$I = \frac{bt^3}{12} = \frac{(8 \times 10^{-3})(0.6016 \times 10^{-3})^3}{12} = 145.16 \times 10^{-15} \text{ m}^4$$

(*b*) Bending moment: $M = \dfrac{EI}{\rho}$

$$M = \frac{(200 \times 10^9)(145.16 \times 10^{-15})}{0.14324} = 0.203 \text{ N} \cdot \text{m}$$

$$M = 0.203 \text{ N} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.24

A 60 N · m couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.

SOLUTION

(*a*) Bending about *z*-axis.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(20)^3 = 8\times10^3\ \text{mm}^4 = 8\times10^{-9}\ \text{m}^4$$

$$c = \frac{20}{2} = 10\ \text{mm} = 0.010\ \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.010)}{8\times10^{-9}} = 75.0\times10^6\ \text{Pa} \qquad \sigma = 75.0\ \text{MPa} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(8\times10^{-9})} = 37.5\times10^{-3}\ \text{m}^{-1} \qquad \rho = 26.7\ \text{m} \blacktriangleleft$$

(*b*) Bending about *y*-axis.

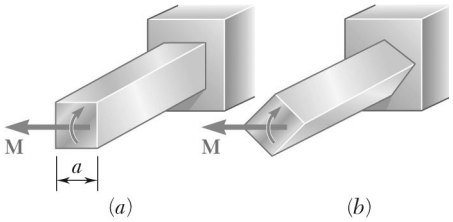
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(20)(12)^3 = 2.88\times10^3\ \text{mm}^4 = 2.88\times10^{-9}\ \text{m}^4$$

$$c = \frac{12}{2} = 6\ \text{mm} = 0.006\ \text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(60)(0.006)}{2.88\times10^{-9}} = 125.0\times10^6\ \text{Pa} \qquad \sigma = 125.0\ \text{MPa} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{(200\times10^9)(2.88\times10^{-9})} = 104.17\times10^{-3}\ \text{m}^{-1} \qquad \rho = 9.60\ \text{m} \blacktriangleleft$$

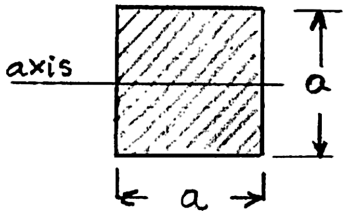
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.25

A couple of magnitude **M** is applied to a square bar of side *a*. For each of the orientations shown, determine the maximum stress and the curvature of the bar.

SOLUTION



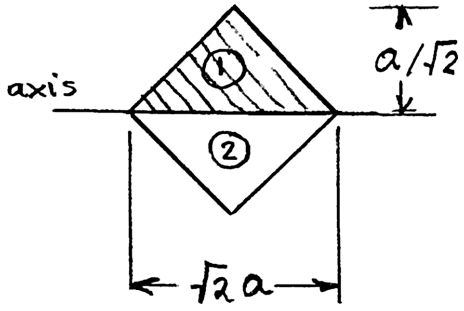
$$I = \frac{1}{12}bh^3 = \frac{1}{12}aa^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M\frac{a}{2}}{\frac{a^4}{12}} \qquad \sigma_{\max} = \frac{6M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

For one triangle, the moment of inertia about its base is



$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}\left(\sqrt{2}a\right)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

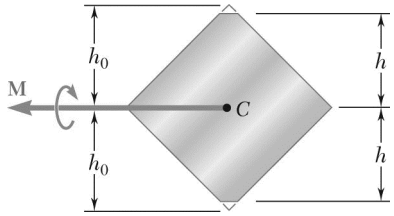
$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}} \qquad \sigma_{\max} = \frac{Mc}{I} = \frac{Ma/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}M}{a^3} \qquad \sigma_{\max} = \frac{8.49M}{a^3} \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} \qquad \frac{1}{\rho} = \frac{12M}{Ea^4} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.26

A portion of a square bar is removed by milling so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

SOLUTION

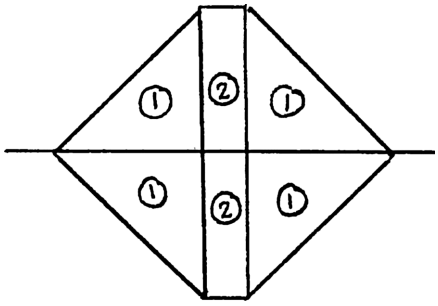
$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h\,h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \frac{1}{3}h^4 + \frac{4}{3}h_0h^3 - \frac{4}{3}h\ h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$



For the original square, $h = h_0,\ c = h_0.$

$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

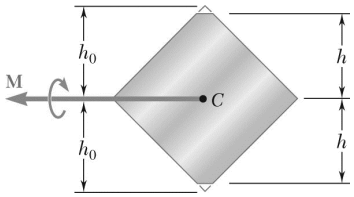
$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9\,h_0^2)} = 0.950$$

$$\sigma = 0.950\,\sigma_0$$

$$k = 0.950 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.27



In Prob. 4.26, determine (*a*) the value of *h* for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of *k*.

PROBLEM 4.26 A portion of a square bar is removed by milling so that its cross section is as shown. The bar is then bent about its horizontal axis by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of *k*.

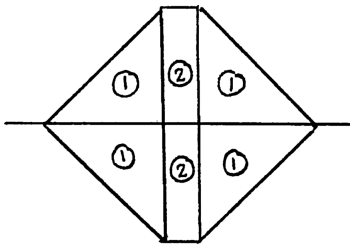
SOLUTION

$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)hh^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)h^3$$

$$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 - \frac{4}{3}h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h \qquad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$$



$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\frac{4}{3}h_0h^2 - h^3\right] = 0.$

$$\frac{8}{3}h_0h - 3h^2 = 0 \qquad h = \frac{8}{9}h_0 \blacktriangleleft$$

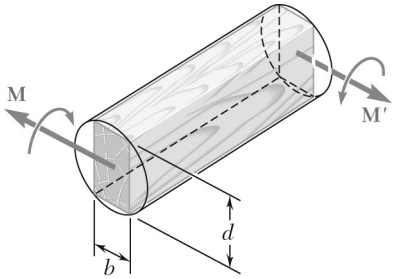
$$\frac{I}{c} = \frac{4}{3}h_0\left(\frac{8}{9}h_0\right)^2 - \left(\frac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729M}{256h_0^3}$$

For the original square, $h = h_0$ $c = h_0$ $\frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.28

A couple M will be applied to a beam of rectangular cross section that is to be sawed from a log of circular cross section. Determine the ratio d/b, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

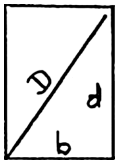
SOLUTION

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$$

$$I = \frac{1}{12}bd^3 \qquad c = \frac{1}{2}d \qquad \frac{I}{c} = \frac{1}{6}bd^2$$

(*a*) σ_m is the minimum when $\dfrac{I}{c}$ is maximum.



$$\frac{I}{c} = \frac{1}{6}b(D^2 - b^2) = \frac{1}{6}D^2b - \frac{1}{6}b^3$$

$$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6}D^2 - \frac{3}{6}b^2 = 0 \qquad b = \frac{1}{\sqrt{3}}D$$

$$d = \sqrt{D^2 - \frac{1}{3}D^2} = \sqrt{\frac{2}{3}}D \qquad \frac{d}{b} = \sqrt{2} \blacktriangleleft$$

(*b*) $\rho = \dfrac{EI}{M}$

ρ is maximum when I is maximum, $\dfrac{1}{12}bd^3$ is maximum, or b^2d^6 is maximum.

$(D^2 - d^2)d^6$ is maximum.

$$6D^2d^5 - 8d^7 = 0 \qquad d = \frac{\sqrt{3}}{2}D$$

$$b = \sqrt{D^2 - \frac{3}{4}D^2} = \frac{1}{2}D \qquad \frac{d}{b} = \sqrt{3} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.29

For the aluminum bar and loading of Sample Prob. 4.1, determine (*a*) the radius of curvature ρ' of a transverse cross section, (*b*) the angle between the sides of the bar that were originally vertical. Use $E = 10.6 \times 10^6$ psi and $v = 0.33$.

SOLUTION

From Sample Prob. 4.1, $I = 12.97 \text{ in}^4$ $M = 103.8 \text{ kip} \cdot \text{in}$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{103.8 \times 10^3}{(10.6 \times 10^6)(12.97)} = 755 \times 10^{-6} \text{ in}^{-1}$$

(*a*) $$\frac{1}{\rho'} = v\frac{1}{\rho} = (0.33)(755 \times 10^{-6}) = 249 \times 10^{-6} \text{ in}^{-1}$$

$\rho' = 4010$ in. $\rho' = 334$ ft ◀

(*b*) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{3.25}{4010} = 810 \times 10^{-6} \text{ rad}$$ $\theta = 0.0464°$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.30

For the bar and loading of Example 4.01, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section, (*c*) the angle between the sides of the bar that were originally vertical. Use $E = 29\times10^6$ psi and $v = 0.29$.

SOLUTION

From Example 4.01, $M = 30 \text{ kip}\cdot\text{in}, \quad I = 1.042 \text{ in}^4$

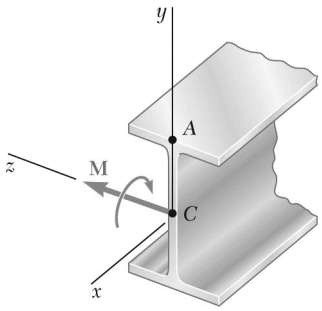
(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{(30\times10^3)}{(29\times10^6)(1.042)} = 993\times10^{-6} \text{ in}^{-1}$$ $\rho = 1007$ in. ◀

(*b*) $$\varepsilon^1 = v\varepsilon = \frac{vc}{\rho} = v\frac{c}{\rho'}$$

$$\frac{1}{\rho'} = v\frac{1}{\rho} = (0.29)(993\times10^{-6})\text{in.}^{-1} = 288\times10^{-6} \text{ in.}^{-1}$$ $\rho' = 3470$ in. ◀

(*c*) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230\times10^{-6} \text{ rad}$$ $\theta = 0.01320°$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.31

A W200×31.3 rolled-steel beam is subjected to a couple **M** of moment 45 kN · m. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.

SOLUTION

For W 200×31.3 rolled steel section,

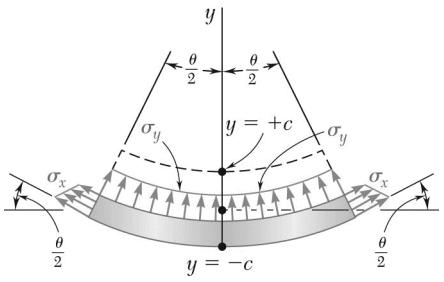
$$I = 31.4\times10^{6}\ \text{mm}^4$$
$$= 31.4\times10^{-6}\ \text{m}^4$$

(*a*) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45\times10^{3}}{(200\times10^{9})(31.4\times10^{-6})} = 7.17\times10^{-3}\ \text{m}^{-1}$$ $\rho = 139.6$ m ◀

(*b*) $$\frac{1}{\rho'} = \nu\frac{1}{\rho} = (0.29)(7.17\times10^{-3}) = 2.07\times10^{-3}\ \text{m}^{-1}$$ $\rho' = 481$ m ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.32



It was assumed in Sec. 4.3 that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of *y*, (*b*) show that $(\sigma_y)_{\max} = -(c/2\rho)(\sigma_x)_{\max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint:* Consider the free-body diagram of the portion of beam located below the surface of ordinate *y* and assume that the distribution of the stress σ_x is still linear.)

SOLUTION

Denote the width of the beam by *b* and the length by *L*.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0: \quad \sigma_y bL + 2\int_{-c}^{y} \sigma_x b dy \sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x dy$$

But, $$\sigma_x = -(\sigma_x)_{\max}\frac{y}{c}$$

(*a*) $$\sigma_y = \frac{(\sigma_x)_{\max}}{\rho c}\int_{-c}^{y} y dy = \frac{(\sigma_x)_{\max}}{\rho c}\frac{y^2}{2}\Big|_{-c}^{y}$$

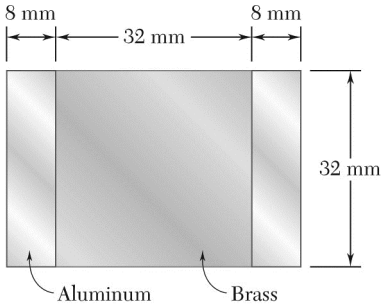
$$\sigma_y = \frac{(\sigma_x)_{\max}}{2\rho c}(y^2 - c^2) \blacktriangleleft$$

The maximum value σ_y occurs at $y = 0$.

(*b*) $$(\sigma_y)_{\max} = -\frac{(\sigma_x)_{\max} c^2}{2\rho c} = -\frac{(\sigma_x)_{\max} c}{2\rho} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

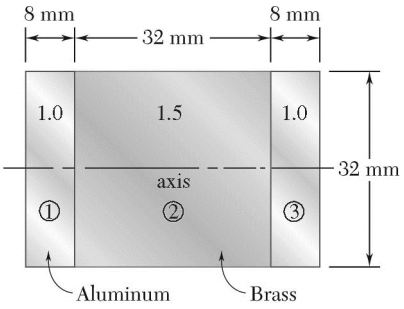
PROBLEM 4.33



A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453\times10^3\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072\times10^3\ \text{mm}^4$$

$$I_3 = I_1 = 21.8453\times10^3\ \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626\times10^3\ \text{mm}^4 = 174.7626\times10^{-9}\ \text{m}^4$$

$$|\sigma| = \left|\frac{n M y}{I}\right| \qquad M = \left|\frac{\sigma I}{n y}\right|$$

Aluminum: $n = 1.0,\ |y| = 16\ \text{mm} = 0.016\ \text{m},\quad \sigma = 100\times10^6\ \text{Pa}$

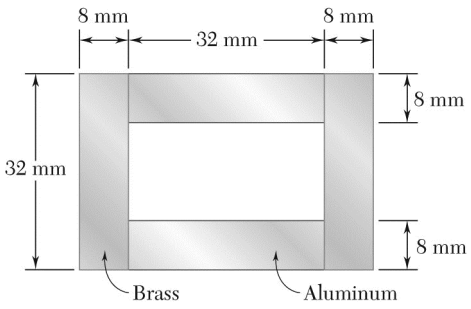
$$M = \frac{(100\times10^6)(174.7626\times10^{-9})}{(1.0)(0.016)} = 1.0923\times10^3\ \text{N}\cdot\text{m}$$

Brass: $n = 1.5,\ |y| = 16\ \text{mm} = 0.016\ \text{m},\quad \sigma = 160\times10^6\ \text{Pa}$

$$M = \frac{(160\times10^6)(174.7626\times10^{-9})}{(1.5)(0.016)} = 1.1651\times10^3\ \text{N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092\times10^3\ \text{N}\cdot\text{m}$ $M = 1.092\ \text{kN}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

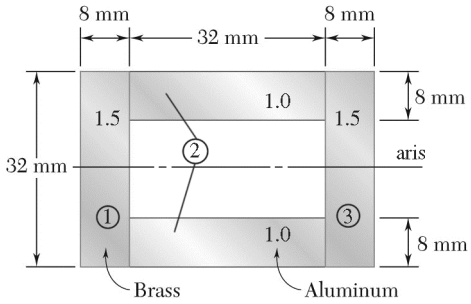


PROBLEM 4.34

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 \left(H_2^3 - h_2^3\right) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N} \cdot \text{m}$$

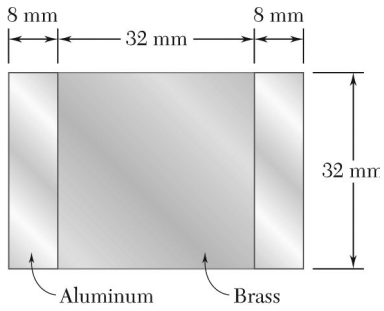
Brass: $n = 1.5, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 887 \text{ N} \cdot \text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.35

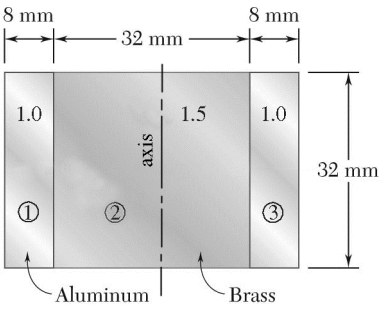


For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.35 Bar of Prob. 4.33.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 b_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(32)(8)^3 + (1.0)[(32)(8)](20)^2 = 103.7653\times10^3\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072\times10^3\ \text{mm}^4$$

$$I_3 = I_1 = 103.7653\times10^3\ \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 338.58\times10^3\ \text{mm}^4 = 338.58\times10^{-9}\ \text{m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0,\ |y| = 24\ \text{mm} = 0.024\ \text{m},\quad \sigma = 100\times10^6\ \text{Pa}$

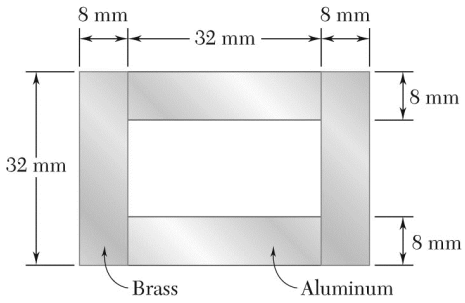
$$M = \frac{(100\times10^6)(338.58\times10^{-9})}{(1.0)(0.024)} = 1.411\times10^3\ \text{N}\cdot\text{m}$$

Brass: $n = 1.5,\ |y| = 16\ \text{mm} = 0.016\ \text{m},\quad \sigma = 160\times10^6\ \text{Pa}$

$$M = \frac{(160\times10^6)(338.58\times10^{-9})}{(1.5)(0.016)} = 2.257\times10^3\ \text{N}\cdot\text{m}$$

Choose the smaller value. $M = 1.411\times10^3\ \text{N}\cdot\text{m}$ $M = 1.411\ \text{kN}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



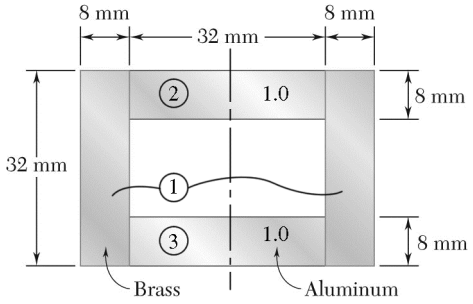
PROBLEM 4.36

For the composite bar indicated, determine the largest permissible bending moment when the bar is bent about a vertical axis.

PROBLEM 4.36 Bar of Prob. 4.34.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} h_1 \left(B_1^3 - b_1^3 \right) = \frac{1.5}{12}(32)(48^3 - 32^3) = 311.296 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} h_2 b_2^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I_3 = I_2 = 21.8453 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 354.99 \times 10^3 \text{ mm}^4 = 354.99 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \left| \frac{\sigma I}{ny} \right|$$

Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(354.99 \times 10^{-9})}{(1.0)(0.016)} = 2.2187 \times 10^3 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5 \quad |y| = 24 \text{ mm} = 0.024 \text{ m} \quad \sigma = 160 \times 10^6 \text{ Pa}$

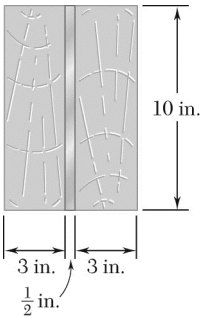
$$M = \frac{(160 \times 10^6)(354.99 \times 10^{-9})}{(1.5)(0.024)} = 1.57773 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value.

$$M = 1.57773 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 1.578 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.37



Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

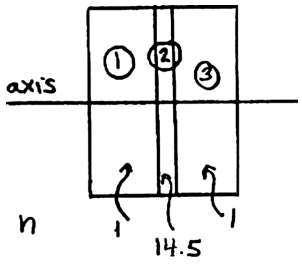
	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$n = 1.0$ in wood

$n = E_s/E_w = 29/2 = 14.5$ in steel



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(3)(10)^3 = 250 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{14.5}{12}\left(\frac{1}{2}\right)(10)^3 = 604.17 \text{ in}^4$$

$$I_3 = I_1 = 250 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 1104.2 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0, \quad y = 5$ in., $\quad \sigma = 2000$ psi

$$M = \frac{(2000)(1104.2)}{(1.0)(5)} = 441.7\times10^3 \text{ lb}\cdot\text{in}$$

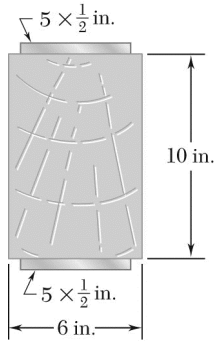
Steel: $n = 14.5, \quad y = 5$ in., $\quad \sigma = 22$ ksi $= 22\times10^3$ psi

$$M = \frac{(22\times10^3)(1104.2)}{(14.5)(5)} = 335.1\times10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value. $M = 335\times10^3$ lb · in $M = 335$ kip · in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.38



Wooden beams and steel plates are securely bolted together to form the composite member shown. Using the data given below, determine the largest permissible bending moment when the member is bent about a horizontal axis.

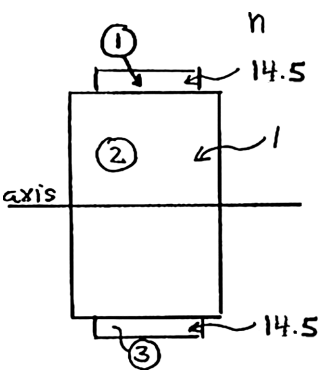
	Wood	Steel
Modulus of elasticity:	2×10^6 psi	29×10^6 psi
Allowable stress:	2000 psi	22 ksi

SOLUTION

Use wood as the reference material.

$n = 1.0$ in wood

$n = E_s/E_w = 29/2 = 14.5$ in steel



For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{14.5}{12}(5)\left(\frac{1}{2}\right)^3 + (14.5)(5)\left(\frac{1}{2}\right)(5.25)^2 = 999.36 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^2 = \frac{1.0}{12}(6)(10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 999.36 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2498.7 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0, \quad y = 5$ in., $\quad \sigma = 2000$ psi

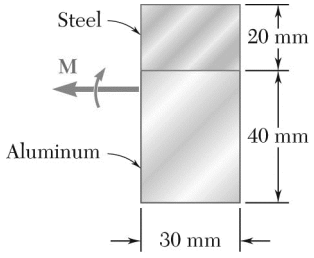
$$M = \frac{(2000)(2499)}{(1.0)(5)} = 999.5\times10^3 \text{ lb}\cdot\text{in}$$

Steel: $n = 14.5, \quad y = 5.5$ in., $\quad \sigma = 22$ ksi $= 22\times10^3$ psi

$$M = \frac{(22\times10^3)(2499)}{(14.5)(5.5)} = 689.3\times10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value. $M = 689\times10^3$ lb · in $M = 689$ kip · in ◀

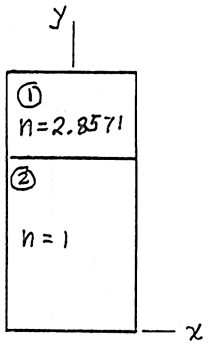
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.39

A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500 \text{ N} \cdot \text{m}$, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 200/70 = 2.8571$

Transformed section:

Part	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3	d, mm
1	600	1714.3	50	85714	12.35
2	1200	1200	20	24000	17.65
Σ		2914.3		109714	

$$\bar{Y}_0 = \frac{109714}{2914.3} = 37.65 \text{ mm} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{2.8571}{12}(30)(20)^3 + (1714.3)(12.35)^2 = 318.61 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(30)(40)^3 + (1200)(17.65)^2 = 533.83 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 852.44 \times 10^3 \text{ mm}^4 = 852.44 \times 10^{-9} \text{ m}^4$$

$$M = 1500 \text{ N} \cdot \text{m}$$

Stress: $\sigma = -\dfrac{nMy}{I}$

(*a*) Aluminum: $n = 1, \quad y = -37.65 \text{ mm} = -0.03765 \text{ m}$

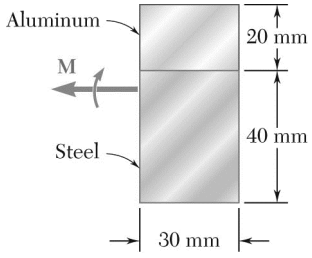
$$\sigma_a = -\frac{(1)(1500)(-0.03765)}{852.44 \times 10^{-9}} = 66.2 \times 10^6 \text{ Pa} \qquad \sigma_a = 66.2 \text{ MPa} \blacktriangleleft$$

(*b*) Steel: $n = 2.8571, \quad y = 60 - 37.65 = 22.35 \text{ mm} = 0.02235 \text{ m}$

$$\sigma_s = -\frac{nMy}{I} = -\frac{(2.8571)(1500)(0.02235)}{852.44 \times 10^{-9}} = -112.4 \times 10^6 \text{ Pa}$$

$$\sigma_s = -112.4 \text{ MPa} \blacktriangleleft$$

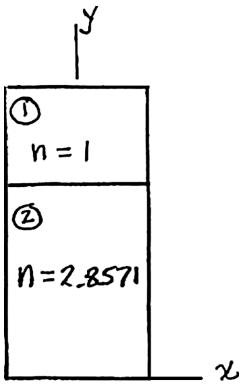
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.40

A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500 \text{ N} \cdot \text{m}$, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 200/70 = 2.8571$

Transformed section:

Part	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3	d, mm
1	600	600	50	30000	25.53
2	1200	3428.5	20	68570	4.47
Σ		4028.5		98570	

$$\bar{Y}_0 = \frac{98570}{4028.5} = 24.47 \text{ mm} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(30)(20)^3 + (600)(25.53)^2 = 411.07 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{2.8571}{12}(30)(40)^3 + (3428.5)(4.47)^2 = 525.64 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 936.71 \times 10^3 \text{ mm}^4 = 936.71 \times 10^{-9} \text{ m}^4$$

$$M = 1500 \text{ N} \cdot \text{m}$$

Stress: $\sigma = -\dfrac{nMy}{I}$

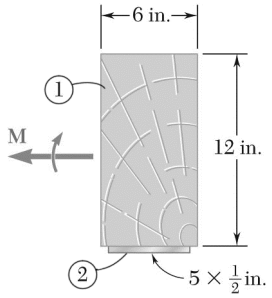
(*a*) Aluminum: $n = 1, \quad y = 60 - 24.47 = 35.53 \text{ mm} = 0.03553 \text{ m}$

$$\sigma_a = -\frac{(1)(1500)(0.03553)}{936.71 \times 10^{-9}} = -56.9 \times 10^6 \text{ Pa} \qquad \sigma_a = -56.9 \text{ MPa} \blacktriangleleft$$

(*b*) Steel: $n = 2.8571, \quad y = -24.47 \text{ mm} = -0.02447 \text{ m}$

$$\sigma_s = -\frac{(2.8571)(1500)(-0.02447)}{936.71 \times 10^{-9}} = 111.9 \times 10^6 \text{ Pa} \qquad \sigma_s = 111.9 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.41

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450\ \text{kip}\cdot\text{in.}$, determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = E_s / E_w = 29/1.8 = 16.1111$

Transformed section: ① = wood ② = steel

$$\bar{Y}_o = \frac{421.931}{112.278} = 3.758 \text{ in.}$$

	A, in^2	nA, in^2	$\bar{y}_o$	$nA\bar{y}_o$, in^3
①	72	72	6	432
②	2.5	40.278	–0.25	–10.069
		112.278		421.931

The neutral axis lies 3.758 in. above the wood-steel interface.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6-3.758)^2 = 1225.91 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{16.1111}{12}(5)(0.5)^3 + (40.278)(3.578+0.25)^2 = 647.87 \text{ in}^4$$

$$I = I_1 + I_2 = 1873.77 \text{ in}^4$$

$$M = 450\ \text{kip}\cdot\text{in} \qquad \sigma = -\frac{nMy}{I}$$

(*a*) Wood: $n = 1, \quad y = 12 - 3.758 = 8.242$ in

$$\sigma_w = -\frac{(1)(450)(8.242)}{1873.77} = -1.979 \text{ ksi}$$

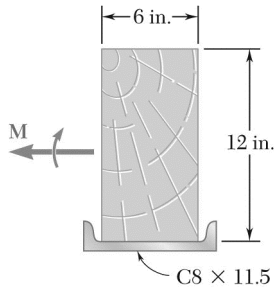
$\sigma_w = -1.979$ ksi ◀

(*b*) Steel: $n = 16.1111, \quad y = -3.758 - 0.5 = -4.258$ in

$$\sigma_s = -\frac{(16.1111)(450)(-4.258)}{1873.77} = 16.48 \text{ ksi}$$

$\sigma_s = 16.48$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

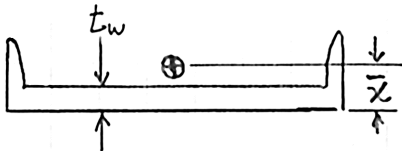


PROBLEM 4.42

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is 1.8×10^6 psi and for steel, 29×10^6 psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

SOLUTION

Use wood as the reference material.



For wood, $n = 1$

For steel, $n = \dfrac{E_s}{E_w} = \dfrac{29\times10^6}{1.8\times10^6} = 16.1111$

For C8 ×11.5 channel section,

$A = 3.38$ in^2, $t_w = 0.220$ in., $\bar{x} = 0.571$ in., $I_y = 1.32$ in^4

For the composite section, the centroid of the channel (part 1) lies 0.571 in. above the bottom of the section. The centroid of the wood (part 2) lies 0.220 + 6.00 = 6.22 in. above the bottom.

Transformed section:

Part	A, in^2	nA, in^2	$\bar{y}$, in.	$nA\bar{y}$, in^3	d, in.
1	3.38	54.456	0.571	31.091	3.216
2	72	72	6.22	447.84	2.433
Σ		126.456		478.93	

$$\bar{Y}_0 = \frac{478.93 \text{ in}^3}{126.456 \text{ in}^2} = 3.787 \text{ in.} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

The neutral axis lies 3.787 in. above the bottom of the section.

$$I_1 = n_1\bar{I}_1 + n_1A_1d_1^2 = (16.1111)(1.32) + (54.456)(3.216)^2 = 584.49 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(2.433)^2 = 1290.20 \text{ in}^4$$

$$I = I_1 + I_2 = 1874.69 \text{ in}^4$$

$$M = 450 \text{ kip·in} \qquad \sigma = -\frac{nMy}{I}$$

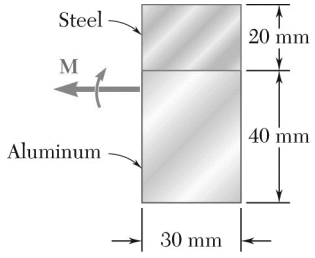
(*a*) Wood: $n = 1, \quad y = 12 + 0.220 - 3.787 = 8.433$ in.

$$\sigma_w = -\frac{(1)(450)(8.433)}{1874.69} = -2.02 \text{ ksi} \qquad \sigma_w = -2.02 \text{ ksi} \blacktriangleleft$$

(*b*) Steel: $n = 16.1111, \quad y = -3.787$ in.

$$\sigma_s = -\frac{(16.1111)(450)(-3.787)}{1874.67} = 14.65 \text{ ksi} \qquad \sigma_s = 14.65 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.43

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 1500 N · m.

Beam of Prob. 4.39.

SOLUTION

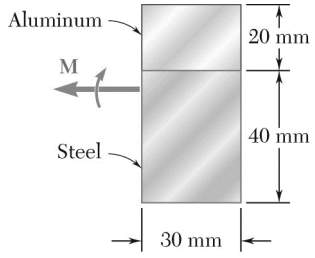
See solution to Prob. 4.39 for the calculation of I.

$$I = 852.44\times10^{-9}\ \text{m}^4 \qquad E_a = 70\times10^{9}\ \text{Pa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{1500}{(70\times10^{9})(852.44\times10^{-9})} = 0.02513\ \text{m}^{-1}$$

$$\rho = 39.8\ \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.44

For the composite bar indicated, determine the radius of curvature caused by the couple of moment 1500 N · m.

Beam of Prob. 4.40.

SOLUTION

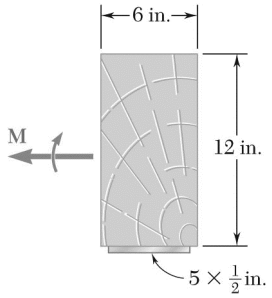
See solution to Prob. 4.40 for calculation of I.

$$I = 936.71\times10^{-9}\ \text{m}^4 \quad E_a = 70\times10^{9}\ \text{Pa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{1500}{(70\times10^{9})(936.71\times10^{-9})} = 0.02288\ \text{m}^{-1}$$

$$\rho = 43.7\ \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.45

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.41.

SOLUTION

See solution to Prob. 4.41 for calculation of I.

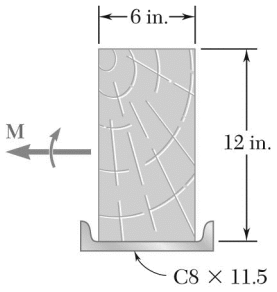
$$I = 1873.77 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in} = 450 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1873.77)} = 133.42 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 7495 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.46

For the composite beam indicated, determine the radius of curvature caused by the couple of moment 450 kip · in.

Beam of Prob. 4.42.

SOLUTION

See solution to Prob. 4.42 for calculation of I.

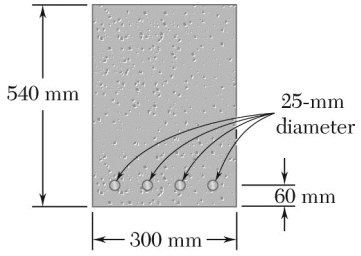
$$I = 1874.69 \text{ in}^4 \qquad E_w = 1.8 \times 10^6 \text{ psi}$$

$$M = 450 \text{ kip} \cdot \text{in} = 450 \times 10^3 \text{ lb} \cdot \text{in}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{450 \times 10^3}{(1.8 \times 10^6)(1874.69)} = 133.36 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 7499 \text{ in.} = 625 \text{ ft} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.47

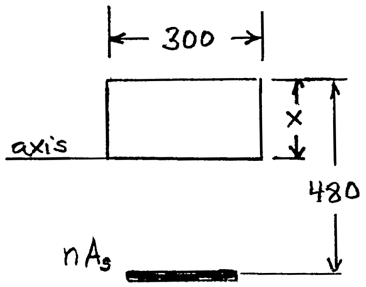
The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$nA_s = 15.708 \times 10^3 \text{ mm}^2$$



Locate the neutral axis:

$$300\, x \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x:

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87 \text{ mm}, \qquad 480 - x = 302.13 \text{ mm}$$

$$I = \frac{1}{3}(300)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9 \text{ mm}^4 = 1.9966 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

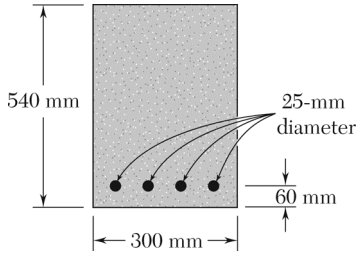
(*a*) Steel: $y = -302.45 \text{ mm} = -0.30245 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa} \qquad \sigma = 212 \text{ MPa} \blacktriangleleft$$

(*b*) Concrete: $y = 177.87 \text{ mm} = 0.17787 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa} \qquad \sigma = -15.59 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

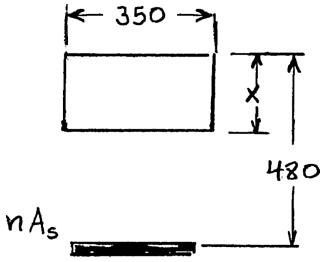


PROBLEM 4.48

Solve Prob. 4.47, assuming that the 300-mm width is increased to 350 mm.

PROBLEM 4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200\ \text{GPa}}{25\ \text{GPa}} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2$$

$$= 1.9635\times10^{3}\ \text{mm}^2$$

$$nA_s = 15.708\times10^{3}\ \text{mm}^2$$

Locate the neutral axis:

$$350\ x\frac{x}{2} - (15.708\times10^{3})(480 - x) = 0$$

$$175x^2 + 15.708\times10^{3}x - 7.5398\times10^{6} = 0$$

Solve for x:

$$x = \frac{-15.708\times10^{3} + \sqrt{(15.708\times10^{3})^2 + (4)(175)(7.5398\times10^{6})}}{(2)(175)}$$

$$x = 167.48\ \text{mm}, \quad 480 - x = 312.52\ \text{mm}$$

$$I = \frac{1}{3}(350)x^3 + (15.708\times10^{3})(480 - x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (15.708\times10^{3})(312.52)^2$$

$$= 2.0823\times10^{9}\ \text{mm}^4 = 2.0823\times10^{-3}\ \text{m}^4$$

$$\sigma = -\frac{nMy}{I}$$

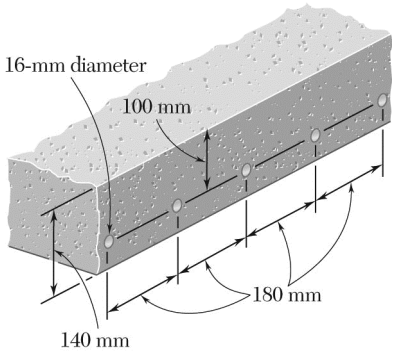
(*a*) Steel: $y = -312.52\ \text{mm} = -0.31252\ \text{m}$

$$\sigma = -\frac{(8.0)(175\times10^{3})(-0.31252)}{2.0823\times10^{-3}} = 210\times10^{6}\ \text{Pa} \qquad \sigma = 210\ \text{MPa} \blacktriangleleft$$

(*b*) Concrete: $y = 167.48\ \text{mm} = 0.16748\ \text{m}$

$$\sigma = -\frac{(1.0)(175\times10^{3})(0.16748)}{2.0823\times10^{-3}} = -14.08\times10^{6}\ \text{Pa} \qquad \sigma = -14.08\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.49

A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPa for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPa for the steel, determine the largest bending moment in a portion of slab 1 m wide.

SOLUTION

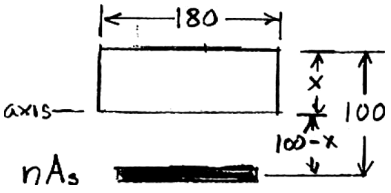
$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$

Consider a section 180-mm wide with one steel rod.

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3 \text{ mm}^2$$

Locate the neutral axis:



$$180\ x\frac{x}{2} - (100 - x)(2.0106 \times 10^3) = 0$$

$$90x^2 + 2.0106 \times 10^3 x - 201.06 \times 10^3 = 0$$

Solving for x:

$$x = \frac{-2.0106 \times 10^3 + \sqrt{(2.0106 \times 10^3)^2 + (4)(90)(201.06 \times 10^3)}}{(2)(90)}$$

$$x = 37.397 \text{ mm} \qquad 100 - x = 62.603 \text{ mm}$$

$$I = \frac{1}{3}(180)x^3 + (2.0106 \times 10^3)(100 - x)^2$$

$$= \frac{1}{3}(180)(37.397)^3 + (2.0106 \times 10^3)(62.603)^2$$

$$= 11.018 \times 10^6 \text{ mm}^4 = 11.018 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left| -\frac{nMy}{I} \right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete:

$$n = 1, \quad y = 37.397 \text{ mm} = 0.037397 \text{ m}, \qquad \sigma = 9 \times 10^6 \text{ Pa}$$

$$M = \frac{(9 \times 10^6)(11.018 \times 10^{-6})}{(1.0)(0.037397)} = 2.6516 \times 10^3 \text{ N} \cdot \text{m}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.49 *(Continued)*

Steel: $n = 10, \quad y = 62.603 \text{ mm} = 0.062603 \text{ m}, \quad \sigma = 120 \times 10^6 \text{ Pa}$

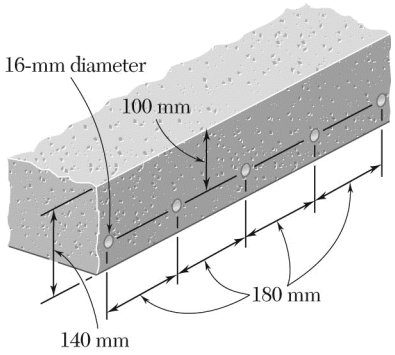
$$M = \frac{(120 \times 10^6)(11.018 \times 10^{-6})}{(10)(0.062603)} = 2.1120 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 2.1120 \times 10^3 \text{ N} \cdot \text{m}$

The above is the allowable positive moment for a 180-mm wide section. For a 1-m = 1000-mm width, mutiply by $\frac{1000}{180} = 5.556$

$$M = (5.556)(2.1120 \times 10^3) = 11.73 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 11.73 \text{ kN} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



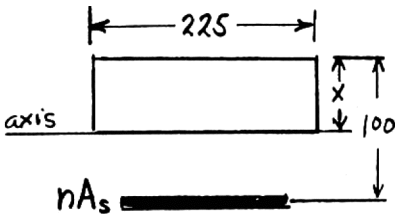
PROBLEM 4.50

A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPa for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPa for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

Solve Prob. 4.49, assuming that the spacing of the 16-mm-diameter rods is increased to 225 mm on centers.

SOLUTION

$$n = \frac{E_s}{E_a} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$



Consider a section 225-mm wide with one steel rod.

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3 \text{ mm}^2$$

Locate the neutral axis:

$$225x \frac{x}{2} - (100 - x)(2.0106 \times 10^3) = 0$$

$$112.5x^2 + 2.0106x - 201.06 \times 10^3 = 0$$

Solving for x:

$$x = \frac{-2.0106 \times 10^3 + \sqrt{(2.0106 \times 10^3)^2 + (4)(112.5)(201.06 \times 10^3)}}{(2)(112.5)}$$

$$x = 34.273 \text{ mm} \qquad 100 - x = 65.727$$

$$I = \frac{1}{3}(225)x^3 + 2.0106 \times 10^3 (100 - x)^2$$

$$= \frac{1}{3}(225)(34.273)^3 + (2.0106 \times 10^3)(65.727)^2$$

$$= 11.705 \times 10^6 \text{ mm}^4 = 11.705 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \qquad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete:

$$n = 1, \quad y = 34.273 \text{ mm} = 0.034273 \text{ m}, \quad \sigma = 9 \times 10^6 \text{ Pa}$$

$$M = \frac{(9 \times 10^6)(11.705 \times 10^{-6})}{(1)(0.034273)} = 3.0738 \times 10^3 \text{ N} \cdot \text{m}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.50 ***(Continued)***

Steel: $n = 10, \quad y = 65.727 \text{ mm} = 0.065727 \text{ m}, \quad \sigma = 120 \times 10^6 \text{ Pa}$

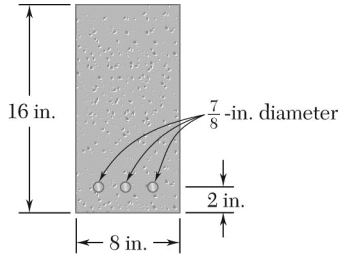
$$M = \frac{(120 \times 10^6)(11.705 \times 10^{-6})}{(10)(0.065727)} = 2.1370 \times 10^3 \text{ N} \cdot \text{m}$$

Choose the smaller value. $M = 2.1370 \times 10^3 \text{ N} \cdot \text{m}$

The above is the allowable positive moment for a 225-mm-wide section. For a 1-m = 1000-mm section, multiply by $\frac{1000}{225} = 4.4444$

$$M = (4.4444)(2.1370 \times 10^3) = 9.50 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 9.50 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.51

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 29×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

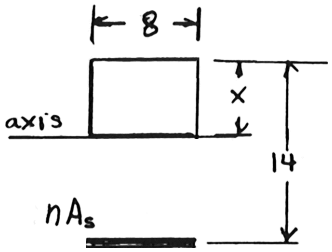
$$n=\frac{E_s}{E_c}=\frac{29\times10^6}{3\times10^6}=9.67$$

$$A_s=3\frac{\pi}{4}d^2=(3)\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2=1.8040\text{ in}^2 \qquad nA_s=17.438\text{ in}^2$$

Locate the neutral axis:

$$8x\frac{x}{2}-(17.438)(14-x)=0$$

$$4x^2+17.438x-244.14=0$$



Solve for x. $$x=\frac{-17.438+\sqrt{17.438^2+(4)(4)(244.14)}}{(2)(4)}=5.6326\text{ in.}$$

$$14-x=8.3674\text{ in.}$$

$$I=\frac{1}{3}8x^3+nA_s(14-x)^2=\frac{1}{3}(8)(5.6326)^3+(17.438)(8.3674)^2=1697.45\text{ in}^4$$

$$|\sigma|=\left|\frac{nMy}{I}\right| \qquad \therefore \quad M=\frac{\sigma I}{ny}$$

Concrete: $n=1.0,\quad |y|=5.6326\text{ in.},\quad |\sigma|=1350\text{ psi}$

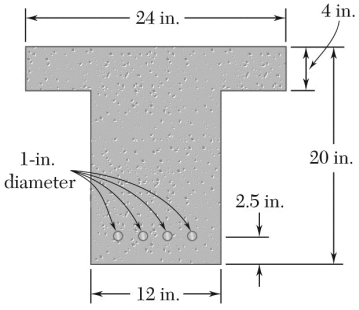
$$M=\frac{(1350)(1697.45)}{(1.0)(5.6326)}=406.835\times10^3\text{ lb}\cdot\text{in}=407\text{ kip}\cdot\text{in}$$

Steel: $n=9.67,\quad |y|=8.3674\text{ in.},\quad \sigma=20\times10^3\text{ psi}$

$$M=\frac{(20\times10^3)(1697.45)}{(9.67)(8.3674)}=419.72\text{ lb}\cdot\text{in}=420\text{ kip}\cdot\text{in}$$

Choose the smaller value. $M=407\text{ kip}\cdot\text{in}$ $M=33.9\text{ kip}\cdot\text{ft}$ ◀

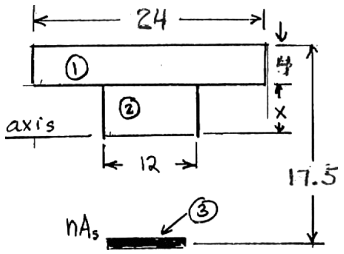
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.52

Knowing that the bending moment in the reinforced concrete beam is +100 kip · ft and that the modulus of elasticity is 3.625×10^6 psi for the concrete and 29×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.625 \times 10^6} = 8.0$$

$$A_s = (4)\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2 \qquad nA_s = 25.133 \text{ in}^2$$

Locate the neutral axis.

$$(24)(4)(x + 2) + (12x)\left(\frac{x}{2}\right) - (25.133)(17.5 - 4 - x) = 0$$

$$96x + 192 + 6x^2 - 339.3 + 25.133x = 0 \quad \text{or} \quad 6x^2 + 121.133x - 147.3 = 0$$

Solve for x.
$$x = \frac{-121.133 + \sqrt{(121.133)^2 + (4)(6)(147.3)}}{(2)(6)} = 1.150 \text{ in.}$$

$$d_3 = 17.5 - 4 - x = 12.350 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(24)(4)^3 + (24)(4)(3.150)^2 = 1080.6 \text{ in}^4$$

$$I_2 = \frac{1}{3} b_2 x^3 = \frac{1}{3}(12)(1.150)^3 = 6.1 \text{ in}^4$$

$$I_3 = nA_3 d_3^2 = (25.133)(12.350)^2 = 3833.3 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 4920 \text{ in}^4$$

$$\sigma = -\frac{nMy}{I} \qquad \text{where } M = 100 \text{ kip} \cdot \text{ft} = 1200 \text{ kip} \cdot \text{in.}$$

(*a*) Steel: $n = 8.0$ $y = -12.350$ in.

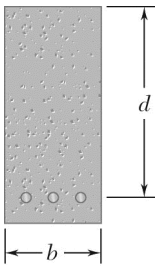
$$\sigma_s = -\frac{(8.0)(1200)(-12.350)}{4920} \qquad \sigma_s = 24.1 \text{ ksi} \blacktriangleleft$$

(*b*) Concrete: $n = 1.0$, $y = 4 + 1.150 = 5.150$ in.

$$\sigma_c = -\frac{(1.0)(1200)(5.150)}{4920} \qquad \sigma_c = -1.256 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.53



The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

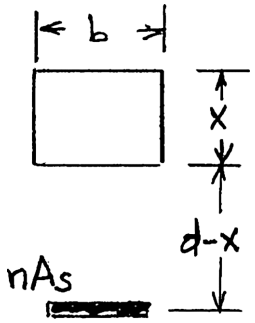
SOLUTION

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

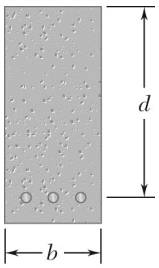
$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

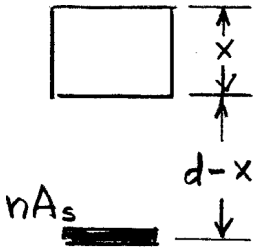
PROBLEM 4.54



For the concrete beam shown, the modulus of elasticity is 3.5×10^6 psi for the concrete and 29×10^6 psi for the steel. Knowing that $b = 8$ in. and $d = 22$ in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (*a*) the required area A_s of the steel reinforcement if the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.53 for definition of a balanced beam.)

The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{3.5\times10^6} = 8.2857$$

$$\sigma_s = \frac{nM(d-x)}{I} \quad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857}\cdot\frac{20\times10^3}{1800} = 2.3410$$

$$x = 0.42717\,d = (0.42717)(22) = 9.398 \text{ in.} \quad d - x = 22 - 9.398 = 12.602 \text{ in.}$$

Locate neutral axis: $bx\frac{x}{2} - nA_s(d-x) = 0$

(*a*) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2$$ $A_s = 3.38 \text{ in}^2$ ◀

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2 = 6665.6 \text{ in}^4$$

$$\sigma = \frac{nMy}{I} \quad M = \frac{\sigma I}{ny}$$

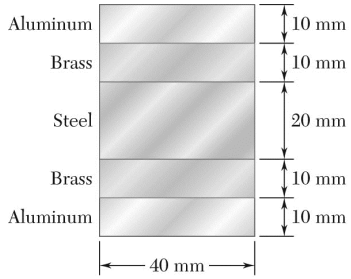
(*b*) Concrete: $n = 1.0$ $y = 9.398$ in. $\sigma_c = 1800$ psi $$M = \frac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277\times10^6 \text{ lb}\cdot\text{in}$$

Steel: $n = 8.2857$ $|y| = 12.602$ in. $\sigma_s = 20\times10^3$ psi

$$M = \frac{(20\times10^3)(6665.6)}{(8.2857)(12.602)} = 1.277\times10^6 \text{ lb}\cdot\text{in}$$

Note that both values are the same for balanced design. $M = 106.4 \text{ kip}\cdot\text{ft}$ ◀

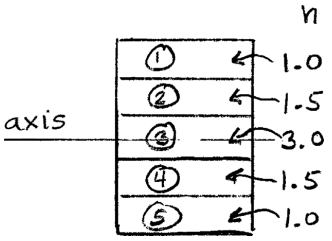
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.55

Five metal strips, each 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

SOLUTION



Use aluminum as the reference material.

$n = 1$ in aluminum.

$n = E_s / E_a = 210/70 = 3$ in steel.

$n = E_b / E_a = 105/70 = 1.5$ in brass.

Due to symmetry of both the material arrangement and the geometry, the neutral axis passes through the center of the steel portion.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(40)(10)^3 + (40)(10)(25)^2 = 253.33\times10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(40)(10)^3 + (1.5)(40)(10)(15)^2 = 140\times10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(40)(20)^3 = 80\times10^3 \text{ mm}^4$$

$$I_4 = I_2 = 140\times10^3 \text{ mm}^4 \qquad I_5 = I_1 = 253.33\times10^3 \text{ mm}^4$$

$$I = \sum I \qquad = 866.66\times10^3 \text{ mm}^4 = 866.66\times10^{-9} \text{ m}^4$$

(*a*) $\sigma = -\dfrac{nMy}{I}$ where $M = 1800$ N · m

Aluminum: $n = 1.0$ $y = -30$ mm $= 0.030$ m

$$\sigma_a = \frac{(1.0)(1800)(0.030)}{866.66\times10^{-9}} = 62.3\times10^6 \text{ Pa} \qquad \sigma_a = 62.3 \text{ MPa} \blacktriangleleft$$

Brass: $n = 1.5$ $y = -20$ mm $= -0.020$ m

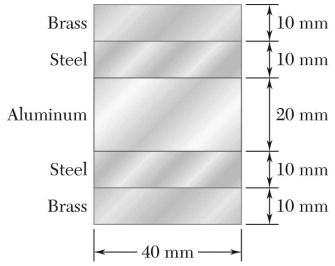
$$\sigma_b = \frac{(1.5)(1800)(0.020)}{866.66\times10^{-9}} = 62.3\times10^6 \text{ Pa} \qquad \sigma_b = 62.3 \text{ MPa} \blacktriangleleft$$

Steel: $n = 3.0$ $y = -10$ mm $= -0.010$ m

$$\sigma_s = \frac{(3.0)(1800)(0.010)}{866.66\times10^{-9}} = 62.3\times10^6 \text{ Pa} \qquad \sigma_s = 62.3 \text{ MPa} \blacktriangleleft$$

(*b*) Radius of curvature. $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{1800}{(70\times10^9)(866.66\times10^{-9})} = 0.02967 \text{ m}^{-1}$ $\rho = 33.7$ m ◀

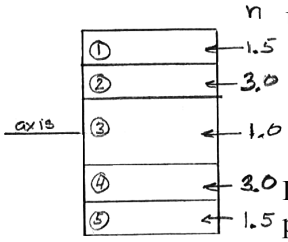
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.56

Five metal strips, each of 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

SOLUTION



Use aluminum as the reference material.

$n = 1.0$ in aluminum.

$n = E_s / E_a = 210/70 = 3$ in steel.

$n = E_b / E_a = 105/70 = 1.5$ in brass.

Due to symmetry of both the material arrangement and the geometry, the neutral axis passes through the center of the aluminum portion.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.5}{12}(40)(10)^3 + (1.5)(40)(10)(25)^2 = 380\times10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{3.0}{12}(40)(10)^3 + (3.0)(40)(10)(15)^2 = 280\times10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.0}{12}(40)(20)^3 = 26.67\times10^3 \text{ mm}^4$$

$$I_4 = I_2 = 280\times10^3 \text{ mm}^4 \qquad I_5 = I_1 = 380\times10^3 \text{ mm}^4$$

$$I = \sum I = 1.3467\times10^6 \text{ mm}^4 = 1.3467\times10^{-6} \text{ m}^4$$

(*a*) $\sigma = -\dfrac{nMy}{I}$ where $M = 1800$ N · m

Aluminum: $n = 1$, $y = -10$ mm $= -0.010$ m

$$\sigma_a = \frac{(1.0)(1800)(0.010)}{1.3467\times10^{-6}} = 13.37\times10^6 \text{ Pa} \qquad \sigma_a = 13.37 \text{ MPa} \blacktriangleleft$$

Brass: $n = 1.5$, $y = -30$ mm $= -0.030$ m

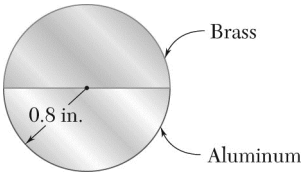
$$\sigma_b = \frac{(1.5)(1800)(0.030)}{1.3467\times10^{-6}} = 60.1\times10^6 \text{ Pa} \qquad \sigma_b = 60.1 \text{ MPa} \blacktriangleleft$$

Steel: $n = 3.0$, $y = -20$ mm $= -0.020$ m

$$\sigma_s = \frac{(3.0)(1800)(0.020)}{1.3467\times10^{-6}} = 80.1\times10^6 \text{ Pa} \qquad \sigma_s = 80.1 \text{ MPa} \blacktriangleleft$$

(*b*) Radius of curvature. $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{1800}{(70\times10^9)(1.3467\times10^{-6})} = 0.01909 \text{ m}^{-1}$ $\qquad \rho = 52.4 \text{ m} \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.57

The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 15×10^6 psi for the brass and 10×10^6 psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 8 kip · in., determine the maximum stress (*a*) in the brass, (*b*) in the aluminum.

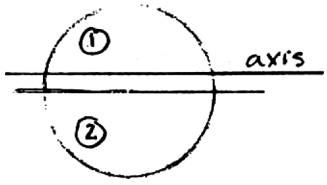
SOLUTION

For each semicircle, $r = 0.8$ in. $A = \dfrac{\pi}{2} r^2 = 1.00531 \text{ in}^2$,

$$\bar{y}_o = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953 \text{ in.} \quad I_{\text{base}} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$$

$$\bar{I} = I_{\text{base}} - A\bar{y}_o^2 = 0.160850 - (1.00531)(0.33953)^2 = 0.044953 \text{ in}^4$$

Use aluminum as the reference material.



$n = 1.0$ in aluminum

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

Locate the neutral axis.

	A, in^2	nA, in^2	$\bar{y}_o$, in.	$nA\bar{y}_o$, in^3
①	1.00531	1.50796	0.33953	0.51200
②	1.00531	1.00531	–0.33953	–0.34133
Σ		2.51327		0.17067

$$\bar{Y}_o = \frac{0.17067}{2.51327} = 0.06791 \text{ in.}$$

The neutral axis lies 0.06791 in. above the material interface.

$d_1 = 0.33953 - 0.06791 = 0.27162$ in., $d_2 = 0.33953 + 0.06791 = 0.40744$ in.

$I_1 = n_1\bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$

$I_2 = n_2\bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$

$I = I_1 + I_2 = 0.39054 \text{ in}^4$

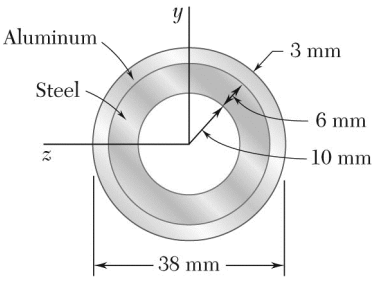
(*a*) Brass: $n = 1.5$, $y = 0.8 - 0.06791 = 0.73209$ in.

$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054} \qquad \sigma = -22.5 \text{ ksi} \blacktriangleleft$$

(*b*) Aluminium: $n = 1.0$, $y = -0.8 - 0.06791 = -0.86791$ in.

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054} \qquad \sigma = 17.78 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.58

A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by a couple of moment 500 N · m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

SOLUTION

Use aluminum as the reference material.

$n = 1.0$ in aluminum

$n = E_s / E_a = 200/70 = 2.857$ in steel

For the transformed section,

Steel: $$I_s = n_s \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (2.857)\left(\frac{\pi}{4}\right)(16^4 - 10^4) = 124.62\times10^3 \text{ mm}^4$$

Aluminium: $$I_a = n_a \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = (1.0)\left(\frac{\pi}{4}\right)(19^4 - 16^4) = 50.88\times10^3 \text{ mm}^4$$

$$I = I_s + I_a = 175.50\times10^3 \text{ mm}^4 = 175.5\times10^{-9} \text{ m}^4$$

(*a*) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma_a = \frac{n_a Mc}{I} = \frac{(1.0)(500)(0.019)}{175.5\times10^{-9}} = 54.1\times10^6 \text{ Pa}$$

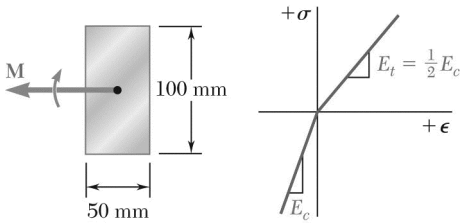
$\sigma_a = 54.1 \text{ MPa}$ ◀

(*b*) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\sigma_s = \frac{n_s Mc}{I} = \frac{(2.857)(500)(0.016)}{175.5\times10^{-9}} = 130.2\times10^6 \text{ Pa}$$

$\sigma_s = 130.2 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



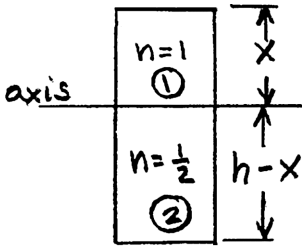
PROBLEM 4.59

The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one-half of its value in compression. For a bending moment $M = 600\ \text{N}\cdot\text{m}$, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$n = \frac{1}{2}$ on the tension side of neutral axis.

$n = 1$ on the compression side.



Locate neutral axis.

$$n_1 bx\frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \quad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421h = 41.421\ \text{mm}$$

$$h - x = 58.579\ \text{mm}$$

$$I_1 = n_1\frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6\ \text{mm}^4$$

$$I_2 = n_2\frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6\ \text{mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6\ \text{mm}^4 = 2.8595\times10^{-6}\ \text{m}^4$$

(*a*) Tensile stress: $n = \frac{1}{2}, \quad y = -58.579\ \text{mm} = -0.058579\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6\ \text{Pa} \qquad \sigma_t = 6.15\ \text{MPa} \blacktriangleleft$$

(*b*) Compressive stress: $n = 1, \quad y = 41.421\ \text{mm} = 0.041421\ \text{m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6\ \text{Pa} \qquad \sigma_c = -8.69\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.60*

A rectangular beam is made of material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

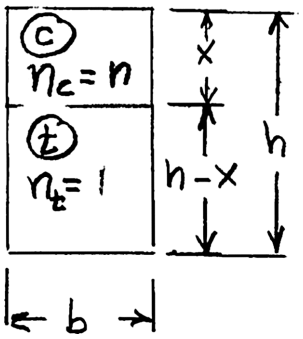
where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION

Use E_t as the reference modulus.

Then $E_c = nE_t$.



Locate neutral axis.

$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \quad \sqrt{n}x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \quad h - x = \frac{\sqrt{n}h}{\sqrt{n}+1}$$

$$I_{\text{trans}} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{h}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right]bh^3$$

$$= \frac{1}{3}\frac{n+n^{3/2}}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\frac{n\left(1+\sqrt{n}\right)}{\left(\sqrt{n}+1\right)^3}bh^3 = \frac{1}{3}\times\frac{n}{\left(\sqrt{n}+1\right)^2}bh^3$$

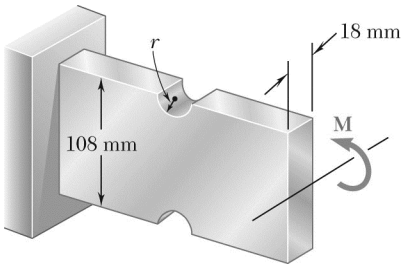
$$\frac{1}{\rho} = \frac{M}{E_t I_{\text{trans}}} = \frac{M}{E_r I} \quad \text{where} \quad I = \frac{1}{12}bh^3$$

$$E_r I = E_t I_{\text{trans}}$$

$$E_r = \frac{E_t I_{\text{trans}}}{I} = \frac{12}{bh^3}\times E_t \times \frac{n}{3\left(\sqrt{n}+1\right)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{\left(\sqrt{E_c/E_t}+1\right)^2} = \frac{4E_t E_c}{\left(\sqrt{E_c}+\sqrt{E_t}\right)^2}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.61

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 60 MPa, determine the largest bending moment that can be applied to the member when (*a*) $r = 9$ mm, (*b*) $r = 18$ mm.

SOLUTION

(*a*) $d = D - 2r = 108 - (2)(9) = 90$ mm

$$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$$

From Fig. 4.32, $K = 2.07$

$$I = \frac{1}{12}(18)(90)^3$$
$$= 1.0935 \times 10^6 \text{ mm}^4$$
$$= 1.0935 \times 10^{-6} \text{ m}^4$$
$$c = \frac{1}{2}d = 45 \text{ mm} = 0.045 \text{ m}$$
$$\sigma = \frac{KMc}{I}$$

$$M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(1.0935 \times 10^{-6})}{(2.07)(0.045)} = 704 \qquad M = 704 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $d = 108 - (2)(18) = 72$ mm

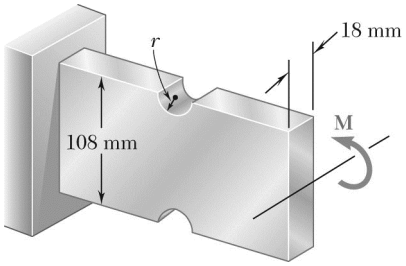
$$\frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$$
$$c = \frac{1}{2}d = 36 \text{ mm} = 0.036 \text{ m}$$

From Fig. 4.32, $K = 1.61$

$$I = \frac{1}{12}(18)(72)^3 = 559.87 \times 10^3 \text{ mm}^4 = 559.87 \times 10^{-9} \text{ m}^4$$

$$M = \frac{\sigma I}{Kc} = \frac{(60 \times 10^6)(559.87 \times 10^{-9})}{(1.61)(0.036)} = 580 \qquad M = 580 \text{ N} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.62

Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 450\ \text{N}\cdot\text{m}$, determine the maximum stress in the member when the radius r of the semicircular grooves is
(*a*) $r = 9$ mm, (*b*) $r = 18$ mm.

SOLUTION

(*a*) $d = D - 2r = 108 - (2)(9) = 90$ mm

$$\frac{D}{d} = \frac{108}{90} = 1.20 \qquad \frac{r}{d} = \frac{9}{90} = 0.1$$

From Fig. 4.32, $K = 2.07$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(90)^3$$
$$= 1.0935\times10^{6}\,\text{mm}^4$$
$$= 1.0935\times10^{-6}\,\text{m}^4$$
$$c = \frac{1}{2}d = 45\ \text{mm} = 0.045\ \text{m}$$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(2.07)(450)(0.045)}{1.0935\times10^{-6}}$$
$$= 38.3\times10^{6}\,\text{Pa} \qquad \sigma_{\max} = 38.3\ \text{MPa} \blacktriangleleft$$

(*b*) $d = D - 2r = 108 - (2)(18) = 72$ mm

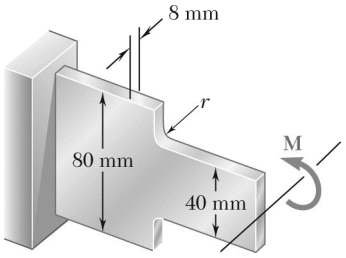
$$\frac{D}{d} = \frac{108}{72} = 1.5 \qquad \frac{r}{d} = \frac{18}{72} = 0.25$$

From Fig. 4.32, $K = 1.61$

$$c = \frac{1}{2}d = 72\ \text{mm} = 0.036\ \text{m}$$
$$I = \frac{1}{12}(18)(72)^3 = 559.87\times10^{3}\ \text{mm}^4 = 559.87\times10^{-9}\,\text{m}^4$$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.61)(450)(0.036)}{559.87\times10^{-9}} = 46.6\times10^{6}\,\text{Pa} \qquad \sigma_{\max} = 46.6\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.63

Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment M when the radius r of the fillets is (*a*) 8 mm, (*b*) 12 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

(*a*) $\dfrac{r}{d} = \dfrac{8 \text{ mm}}{40 \text{ mm}} = 0.2$ From Fig. 4.31, $K = 1.50$

$$\sigma_{\max} = K\frac{Mc}{I} \qquad M = \frac{\sigma_{\max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$$

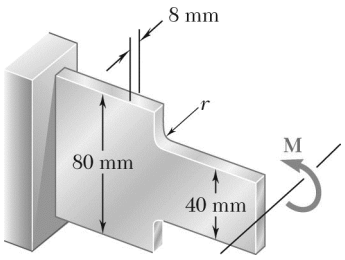
$$M = 128 \text{ N} \cdot \text{m} \blacktriangleleft$$

(*b*) $\dfrac{r}{d} = \dfrac{12 \text{ mm}}{40 \text{ mm}} = 0.3$ From Fig. 4.31, $K = 1.35$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)}$$

$$M = 142 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.64

Knowing that $M = 250\ \text{N}\cdot\text{m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

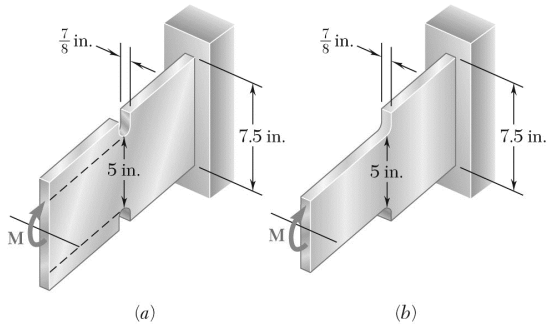
(*a*) $\dfrac{r}{d} = \dfrac{4\ \text{mm}}{40\ \text{mm}} = 0.10$ From Fig. 4.31, $K = 1.87$

$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667\times10^{-9}} = 219\times10^6\ \text{Pa} \qquad \sigma_{\max} = 219\ \text{MPa} \blacktriangleleft$$

(*b*) $\dfrac{r}{d} = \dfrac{8\ \text{mm}}{40\ \text{mm}} = 0.20$ From Fig. 4.31, $K = 1.50$

$$\sigma_{\max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667\times10^{-9}} = 176\times10^6\ \text{Pa} \qquad \sigma_{\max} = 176\ \text{MPa} \blacktriangleleft$$

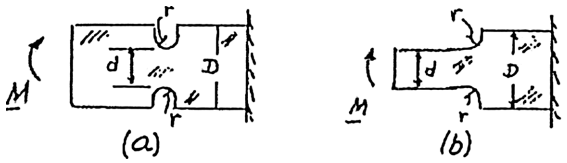
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.65

The allowable stress used in the design of a steel bar is 12 ksi. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = \frac{3}{4}$ in. as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above and below the dashed lines as shown in Fig. *b*.

SOLUTION



Dimensions: $t = \frac{7}{8}$ in. $= 0.875$ in. $\quad r = \frac{3}{4}$ in. $= 0.75$ in.

$D = 7.5$ in. $\quad d = 5$ in.

$\frac{r}{d} = \frac{0.75}{5} = 0.15 \quad \frac{D}{d} = \frac{7.5}{5} = 1.5$

Stress concentration factors: Figs. 4.32 and 4.31

Configuration (*a*): $K = 1.92$

Configuration (*b*): $K = 1.58$

Moment of inertia: $I = \frac{1}{12}td^3 = \frac{1}{12}(0.875)(5)^3 = 9.115 \text{ in}^4$

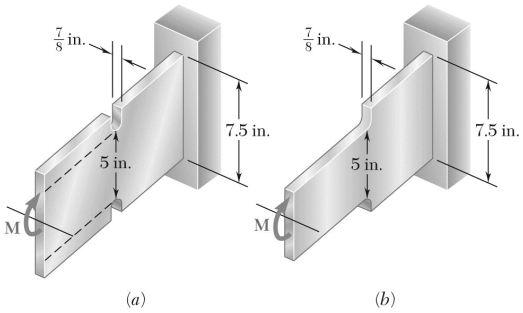
$c = \frac{1}{2}d = 2.5$ in. $\quad \sigma_m = 12$ ksi

$\sigma_m = \frac{KMc}{I} \quad M = \frac{I\sigma_m}{Kc}$

(*a*) $M = \frac{(9.115)(12)}{(1.92)(2.5)}$ $\quad M = 22.8 \text{ kip} \cdot \text{in}$ ◀

(*b*) $M = \frac{(9.115)(12)}{(1.58)(2.5)}$ $\quad M = 27.7 \text{ kip} \cdot \text{in}$ ◀

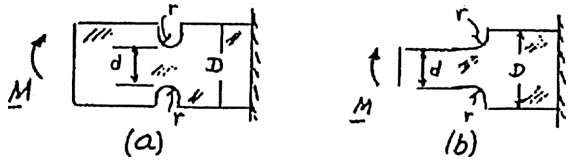
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.66

A couple of moment $M = 20$ kip · in. is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = \frac{1}{2}$ in., as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above and below the dashed line as shown in Fig. *b*.

SOLUTION



Dimensions: $t = \frac{7}{8}$ in. $= 0.875$ in. $\quad r = \frac{1}{2}$ in. $= 0.5$ in.

$D = 7.5$ in. $\quad d = 5$ in.

$\frac{r}{d} = \frac{0.5}{5} = 0.10 \quad \frac{D}{d} = \frac{7.5}{5} = 1.5$

Stress concentration factors: Figs. 4.32 and 4.31

Configuration (*a*): $K = 2.22$

Configuration (*b*): $K = 1.80$

Moment of inertia: $I = \frac{1}{12}td^3 = \frac{1}{12}(0.875)(5)^3 = 9.115 \text{ in}^4$

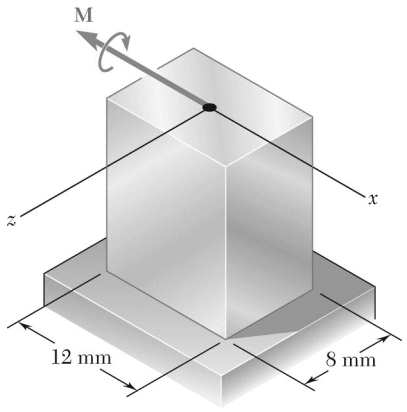
$c = \frac{1}{2}d = 2.5$ in. $\quad M = 20$ kip · in

Maximum stress: $\sigma_m = \frac{KMc}{I}$

(*a*) $\sigma_m = \frac{(2.22)(20)(2.5)}{9.115}$ $\quad \sigma_m = 12.2$ ksi ◀

(*b*) $\sigma_m = \frac{(1.80)(20)(2.5)}{9.115}$ $\quad \sigma_m = 9.9$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.67

The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(8)^3 = 512 \text{ mm}^4$$

$$= 512 \times 10^{-12} \text{ m}^4$$

$$c = \frac{1}{2}h = 4 \text{ mm} = 0.004 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300 \times 10^6)(512 \times 10^{-12})}{0.004}$$

$$= 38.4 \text{ N} \cdot \text{m}$$

$M_Y = 38.4 \text{ N} \cdot \text{m}$ ◀

(*b*) $$y_Y = \frac{1}{2}(4) = 2 \text{ mm} \qquad \frac{y_Y}{c} = \frac{2}{4} = 0.5$$

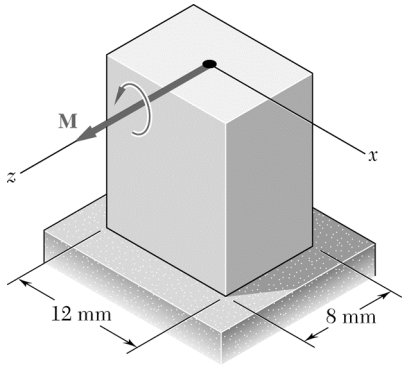
$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(38.4)\left[1 - \frac{1}{3}(0.5)^2\right]$$

$$= 52.8 \text{ N} \cdot \text{m}$$

$M = 52.8 \text{ N} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.68

Solve Prob. 4.67, assuming that the couple **M** is parallel to the *z* axis.

PROBLEM 4.67 The prismatic bar shown is made of a steel that is assumed to be elastoplastic with $\sigma_Y = 300$ MPa and is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 4 mm thick.

SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(12)^3 = 1.152\times10^3 \text{ mm}^4$$

$$= 1.152\times10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}h = 6 \text{ mm} = 0.006 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(300\times10^6)(1.152\times10^{-9})}{0.006}$$

$$= 57.6 \text{ N}\cdot\text{m}$$

$M_Y = 57.6 \text{ N}\cdot\text{m}$ ◀

(*b*) $y_Y = \frac{1}{2}(4) = 2$ mm $\quad \frac{y_Y}{c} = \frac{2}{6} = \frac{1}{3}$

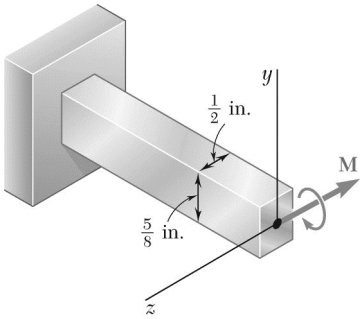
$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(57.6)\left[1 - \frac{1}{3}\left(\frac{1}{3}\right)^2\right]$$

$$= 83.2 \text{ N}\cdot\text{m}$$

$M = 83.2 \text{ N}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.69

The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi, is subjected to a couple of 1350 lb · in. parallel to the z axis. Determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

SOLUTION

(*a*) $$I = \frac{1}{12}\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)^3 = 10.1725\times10^{-3}\ \text{in}^4$$

$$c = \frac{1}{2}\left(\frac{5}{8}\right) = 0.3125\ \text{in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(36\times10^3)(10.1725\times10^{-3})}{0.3125}$$

$$= 1171.872\ \text{lb}\cdot\text{in}$$

$$M = \frac{3}{2}M_Y\left[1-\frac{1}{3}\left(\frac{y_Y}{c}\right)^3\right]$$

$$\frac{y_Y}{c} = \sqrt{3-2\frac{M}{M_Y}} = \sqrt{3-\frac{(2)(1350)}{1171.872}} = 0.83426$$

$$y_Y = (0.83426)(0.3125) = 0.26071\ \text{in.}$$

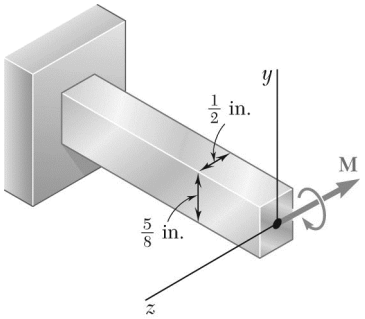
Thickness of elastic core: $2y_Y = 0.521$ in. ◀

(*b*) $$y_Y = \varepsilon_Y\rho = \frac{\sigma_Y}{E}\rho$$

$$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(0.26071)(29\times10^6)}{36\times10^3} = 210.02\ \text{in.}$$

$\rho = 17.50$ ft ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.70

Solve Prob. 4.69, assuming that the 1350-lb · in. couple is parallel to the *y* axis.

PROBLEM 4.69 The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi, is subjected to a couple of 1350 lb · in parallel to the *z* axis. Determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

SOLUTION

(*a*) $$I = \frac{1}{12}\left(\frac{5}{8}\right)\left(\frac{1}{2}\right)^3 = 6.5104\times10^{-3}\text{ in}^4$$

$$c = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 0.25\text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(36\times10^3)(6.5104\times10^{-3})}{0.25}$$

$$= 937.5\text{ lb}\cdot\text{in}$$

$$M = \frac{3}{2}\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(1350)}{937.5}} = 0.34641$$

$$y_Y = (0.34641)(0.25) = 0.086603\text{ in.}$$

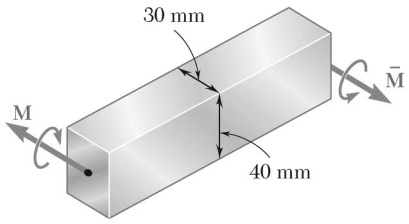
Thickness of elastic core: $2y_Y = 0.1732$ in. ◀

(*b*) $$y_Y = \varepsilon_Y\rho = \frac{\sigma_Y}{E}\rho$$

$$\rho = \frac{y_Y E}{\sigma_Y} = \frac{(0.086603)(29\times10^6)}{36\times10^3} = 69.763\text{ in.}$$

$\rho = 5.81$ ft ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.71

A bar of rectangular cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa. Determine the bending moment M for which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 12 mm thick.

SOLUTION

$$I = \frac{1}{12}(30)(40)^3 = 160 \times 10^3 \text{ mm}^4 = 160 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

(*a*) First yielding: $\sigma = \dfrac{Mc}{I} = \sigma_Y$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(160)(10^{-9})(300 \times 10^6)}{0.020} = 2400 \text{ N} \cdot \text{m}$$

$$M_Y = 2.40 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Plastic zones are 12 mm thick:

$$y_Y = 20 \text{ mm} - 12 \text{ mm} = 8 \text{ mm}$$

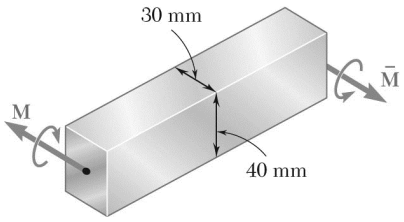
$$\frac{y_Y}{c} = \frac{8 \text{ mm}}{20 \text{ mm}} = 0.4$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3} \left(\frac{y_Y}{c} \right)^2 \right]$$

$$= \frac{3}{2}(2400)\left[1 - \frac{1}{3}(0.4)^2 \right] = 3408 \text{ N} \cdot \text{m}$$

$$M = 3.41 \text{ kN} \cdot \text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.72

Bar AB is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. Determine the bending moment M for which the radius of curvature of the bar will be (*a*) 18 m, (*b*) 9 m.

SOLUTION

$$I = \frac{1}{12}(30)(40)^3 = 160 \times 10^3 \text{ mm}^4 = 160 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}(40) = 20 \text{ mm} = 0.020 \text{ m}$$

$$M_Y = \frac{I\sigma_Y}{c} = \frac{(160 \times 10^{-9})(240 \times 10^6)}{0.020} = 1920 \text{ N} \cdot \text{m}$$

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI} = \frac{1920}{(200 \times 10^9)(160 \times 10^{-9})} = 0.0600 \text{ m}^{-1}$$

(*a*) $\rho = 18$ m: $\dfrac{1}{\rho} = 0.05556 \text{ m}^{-1} < 0.0600 \text{ m}^{-1}$

The bar is fully elastic.

$$M = \frac{EI}{\rho} = \frac{(200 \times 10^9)(160 \times 10^{-9})}{18} = 1778 \text{ N} \cdot \text{m}$$

$M = 1.778 \text{ kN} \cdot \text{m}$ ◀

(*b*) $\rho = 9$ m: $\dfrac{1}{\rho} = 0.11111 \text{ m}^{-1} > 0.0600 \text{ m}^{-1}$

The bar is partially plastic.

$$\sigma_Y = \frac{Ey_Y}{\rho} \qquad y_Y = \frac{\rho\sigma_Y}{E}$$

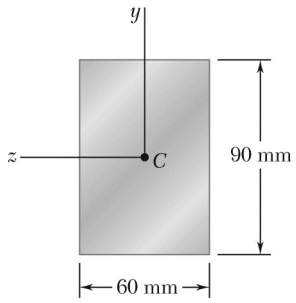
$$y_Y = \frac{(9)(240 \times 10^6)}{200 \times 10^9} = 0.0108 \text{ m} = 10.8 \text{ mm}$$

$$\frac{y_Y}{c} = \frac{10.8 \text{ mm}}{20 \text{ mm}} = 0.54$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right] = \frac{3}{2}(1920)\left[1 - \frac{1}{3}(0.54)^2\right] = 2600 \text{ N} \cdot \text{m}$$

$M = 2.60 \text{ kN} \cdot \text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.73

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z-axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.

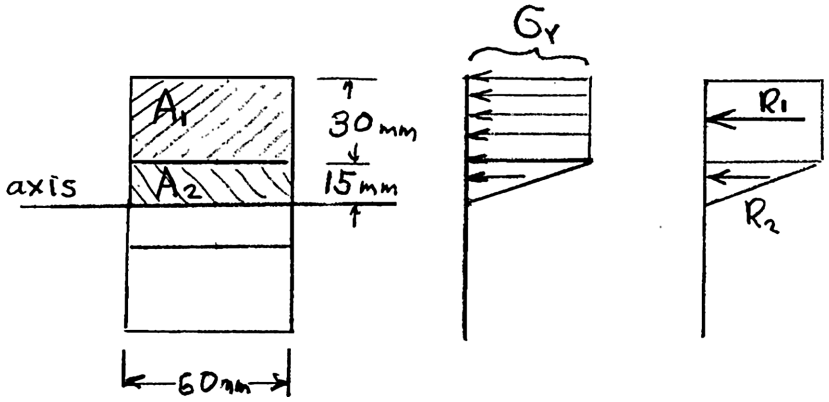
SOLUTION

(*a*) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6\ \text{mm}^4 = 3.645\times10^{-6}\ \text{m}^4$$

$$c = \frac{1}{2}h = 45\ \text{mm} = 0.045\ \text{m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.645\times10^{-6})}{0.045} = 19.44\times10\ \text{N}\cdot\text{m}$$

$$M_Y = 19.44\ \text{kN}\cdot\text{m} \blacktriangleleft$$



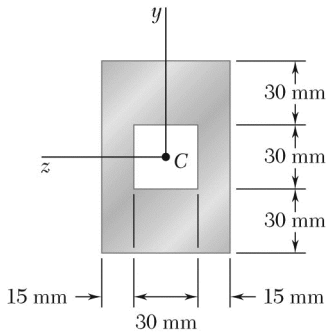
$$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030)$$
$$= 432\times10^3\ \text{N}$$
$$y_1 = 15\ \text{mm} + 15\ \text{mm} = 0.030\ \text{m}$$
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \left(\frac{1}{2}\right)(240\times10^6)(0.060)(0.015)$$
$$= 108\times10^3\ \text{N}$$
$$y_2 = \frac{2}{3}(15\ \text{mm}) = 10\ \text{mm} = 0.010\ \text{m}$$

(*b*) $$M = 2(R_1y_1 + R_2y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.010)]$$

$$= 28.08\times10^3\ \text{N}\cdot\text{m}$$

$$M = 28.1\ \text{kN}\cdot\text{m} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.74

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 30-mm thick.

SOLUTION

(*a*) $$I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6 \text{ mm}^4$$

$$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3 \text{ mm}^4$$

$$I = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^6 \text{ mm}^4$$

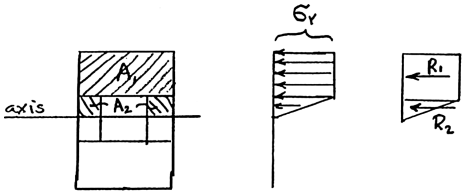
$$= 3.5775\times10^{-6} \text{ mm}^4$$

$$c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-6})}{0.045}$$

$$= 19.08\times10^3 \text{ N}\cdot\text{m}$$

$M_Y = 19.08 \text{ kN}\cdot\text{m}$ ◀



$$R_1 = \sigma_Y A_1 = (240\times10^6)(0.060)(0.030) = 432\times10^3 \text{ N}$$

$$y_1 = 15 \text{ mm} + 15 \text{ mm} = 30 \text{ mm} = 0.030 \text{ m}$$

$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240\times10^6)(0.030)(0.015) = 54\times10^3 \text{ N}$$

$$y_2 = \frac{2}{3}(15 \text{ mm}) = 10 \text{ mm} = 0.010 \text{ m}$$

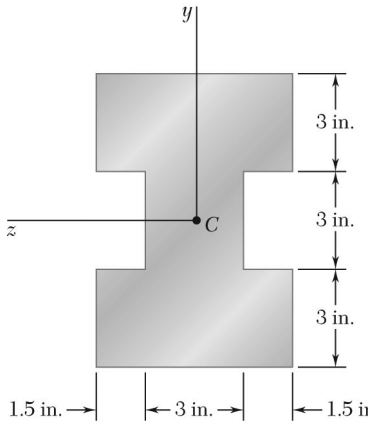
(*b*) $$M = 2(R_1y_1 + R_2y_2)$$

$$= 2[(432\times10^3)(0.030) + (54\times10^3)(0.010)]$$

$$= 27.00\times10^3 \text{ N}\cdot\text{m}$$

$M = 27.0 \text{ kN}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.75

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(*a*) $$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$$

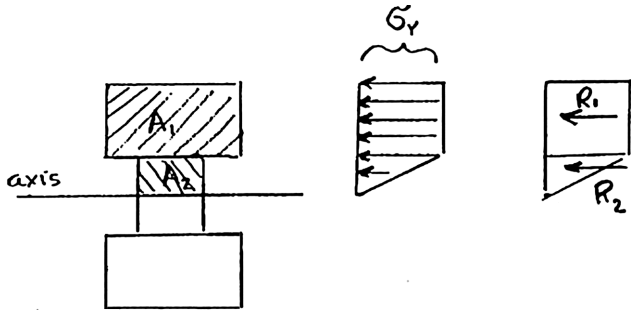
$$I_3 = I_1 = 175.5 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$$

$$c = 4.5 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5}$$

$$M_Y = 3339 \text{ kip}\cdot\text{in} \blacktriangleleft$$



$$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756 \text{ kip}$$

$$y_1 = 1.5 + 1.5 = 3 \text{ in.}$$

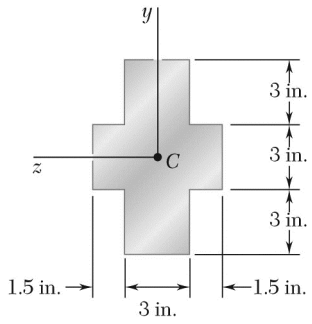
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(3)(1.5) = 94.5 \text{ kip}$$

$$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$$

(*b*) $$M = 2(R_1y_1 + R_2y_2) = 2[(756)(3) + (94.5)(1.0)]$$

$$M = 4725 \text{ kip}\cdot\text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

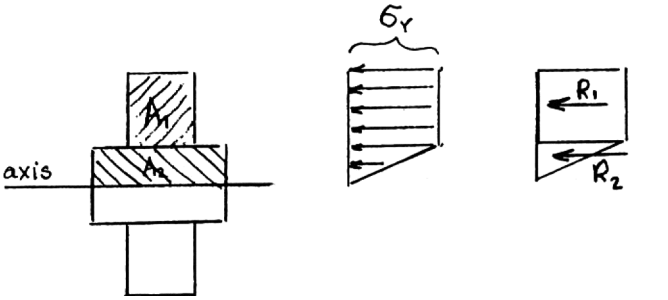


PROBLEM 4.76

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

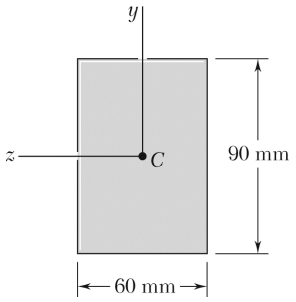
(*a*)
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$$
$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$$
$$I_3 = I_1 = 87.75 \text{ in}^4$$
$$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$$
$$c = 4.5 \text{ in.}$$
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} \qquad M_Y = 1759 \text{ kip}\cdot\text{in} \blacktriangleleft$$



$$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378 \text{ kip}$$
$$y_1 = 1.5 + 1.5 = 3.0 \text{ in.}$$
$$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(6)(1.5) = 189 \text{ kip}$$
$$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$$

(*b*)
$$M = 2(R_1y_1 + R_2y_2) = 2[(378)(3.0) + (189)(1.0)] \qquad M = 2646 \text{ kip}\cdot\text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



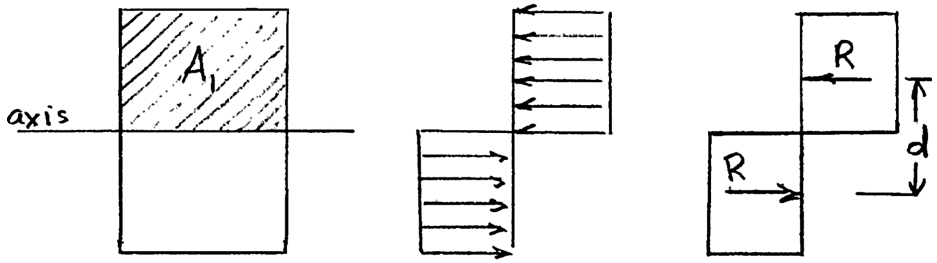
PROBLEM 4.77

For the beam indicated (of Prob. 4.73), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.73, $E = 200$ GPa and $\sigma_Y = 240$ MPa.

$$A_1 = (60)(45) = 2700 \text{ mm}^2$$
$$= 2700\times10^{-6}\text{ m}^2$$
$$R = \sigma_Y A_1$$
$$= (240\times10^6)(2700\times10^{-6})$$
$$= 648\times10^3\text{ N}$$
$$d = 45 \text{ mm} = 0.045 \text{ m}$$



(*a*) $M_p = Rd = (648\times10^3)(0.045) = 29.16\times10^3\text{ N}\cdot\text{m}$ $\quad M_p = 29.2\text{ kN}\cdot\text{m}$ ◀

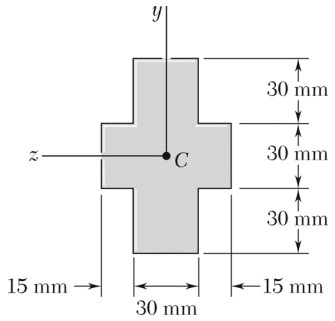
(*b*) $I = \dfrac{1}{12}bh^3 = \dfrac{1}{12}(60)(90)^3 = 3.645\times10^6\text{ mm}^4 = 3.645\times10^{-6}\text{ m}^4$

$c = 45 \text{ mm} = 0.045 \text{ m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.645\times10^{-6})}{0.045} = 19.44\times10^3\text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{29.16}{19.44} \qquad k = 1.500$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.78

For the beam indicated (of Prob. 4.74), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

From Problem 4.74, $E = 200$ GPa and $\sigma_Y = 240$ MPa.

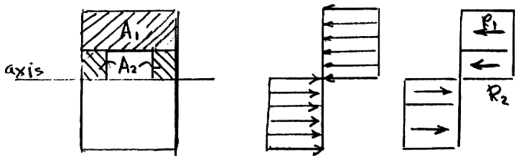
(*a*) $R_1 = \sigma_Y A_1$

$= (240\times10^6)(0.060)(0.030)$

$= 432\times10^3\ \text{N}$

$y_1 = 15\ \text{mm} + 15\ \text{mm} = 30\ \text{mm}$

$= 0.030\ \text{m}$



$R_2 = \sigma_Y A_2$

$= (240\times10^6)(0.030)(0.015)$

$= 108\times10^3\ \text{N}$

$y_2 = \frac{1}{2}(15) = 7.5\ \text{mm} = 0.0075\ \text{m}$

$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(432\times10^3)(0.030) + (108\times10^3)(0.0075)]$

$= 27.54\times10^3\ \text{N}\cdot\text{m}$

$M_p = 27.5\ \text{kN}\cdot\text{m}$ ◀

(*b*) $I_{\text{rect}} = \frac{1}{12}bh^3 = \frac{1}{12}(60)(90)^3 = 3.645\times10^6\ \text{mm}^4$

$I_{\text{cutout}} = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67.5\times10^3\ \text{mm}^4$

$I = I_{\text{rect}} - I_{\text{cutout}} = 3.645\times10^6 - 67.5\times10^3 = 3.5775\times10^3\ \text{mm}^4$

$= 3.5775\times10^{-9}\ \text{m}^4$

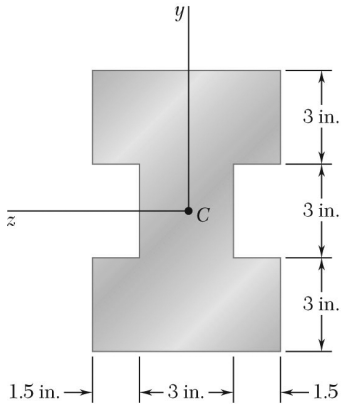
$c = \frac{1}{2}h = 45\ \text{mm} = 0.045\ \text{m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(3.5775\times10^{-9})}{0.045} = 19.08\times10^3\ \text{N}\cdot\text{m}$

$k = \frac{M_p}{M_Y} = \frac{27.54}{19.08}$

$k = 1.443$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.79

For the beam indicated (of Prob. 4.75), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

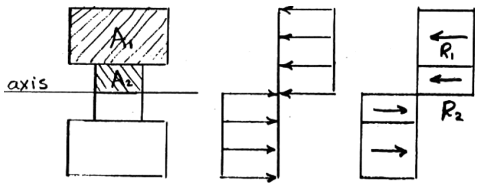
From Problem 4.75, $E = 29\times10^6$ and $\sigma_Y = 42$ ksi.

(*a*) $R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)]$

$M_p = 4819.5$ kip·in ◀

(*b*) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75$ in^4

$I_3 = I_1 = 175.5$ in^4

$I = I_1 + I_2 + I_3 = 357.75$ in^4

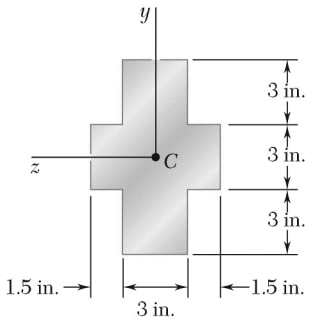
$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339$ kip·in

$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339}$

$k = 1.443$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.80

For the beam indicated (of Prob. 4.76), determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

SOLUTION

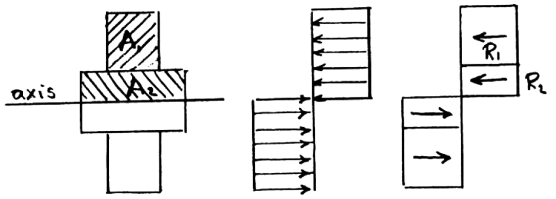
From Problem 4.76, $E = 29\times10^6$ psi and $\sigma_Y = 42$ ksi.

(*a*) $R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in.

$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.



$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)]$

$M_p = 2835$ kip·in ◀

(*b*) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5$ in^4

$I_3 = I_1 = 87.75$ in^4

$I = I_1 + I_2 + I_3 = 188.5$ in^4

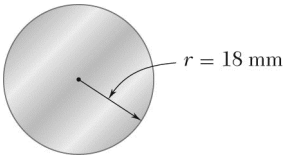
$c = 4.5$ in.

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3$ kip·in

$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3}$

$k = 1.611$ ◀

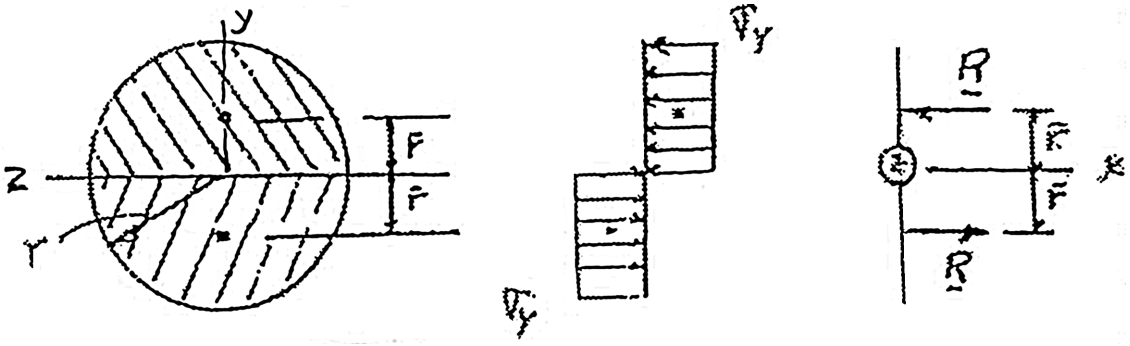
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.81

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION



For a semicircle: $A = \dfrac{\pi}{2} r^2; \qquad \bar{r} = \dfrac{4r}{3\pi}$

Resultant force on semicircular section: $R = \sigma_Y A$

Resultant moment on entire cross section:

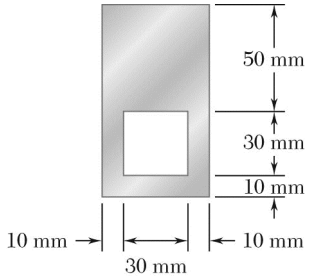
$$M_p = 2R\bar{r} = \frac{4}{3}\sigma_Y r^3$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}, \quad r = 18 \text{ mm} = 0.018 \text{ m}$

$$M_p = \frac{4}{3}(240 \times 10^6)(0.018)^3 = 1866 \text{ N} \cdot \text{m}$$

$$M_p = 1.866 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.82

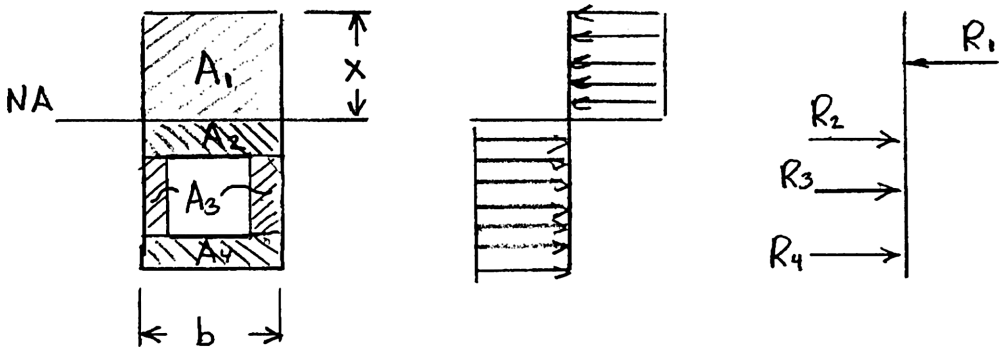
Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$$\frac{1}{2}A = 1800 \text{ mm}^2$$

$$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$$



$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm}, \quad A_1\bar{y}_1 = 32.4\times10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm}, \quad A_2\bar{y}_2 = 4.9\times10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm}, \quad A_3\bar{y}_3 = 17.4\times10^3 \text{ mm}^3$

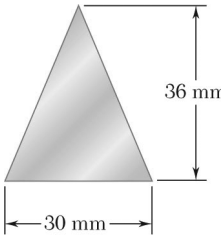
$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm}, \quad A_4\bar{y}_4 = 24.5\times10^3 \text{ mm}^3$

$$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2\times10^3 \text{ mm}^3 = 79.2\times10^{-6} \text{ m}^3$$

$$M_p = \sigma_Y \Sigma A_i\bar{y}_i = (240\times10^6)(79.2\times10^{-6}) = 19.008\times10^3 \text{ N}\cdot\text{m}$$

$$M_p = 19.01 \text{ kN}\cdot\text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

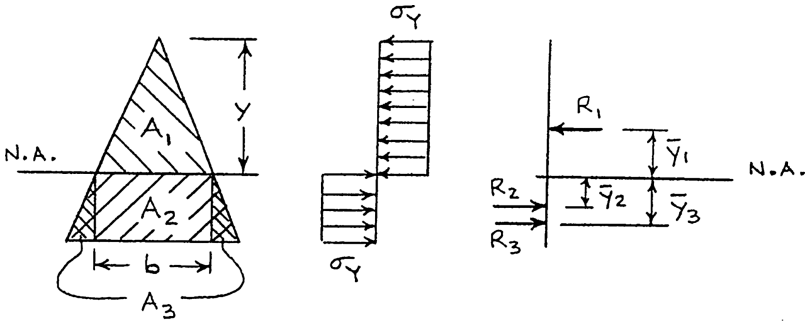


PROBLEM 4.83

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area: $A = \frac{1}{2}(30)(36) = 540 \text{ mm}^2$ Half area: $\frac{1}{2}A = 270 \text{ mm}^2 = A_1$



By similar triangles, $\frac{b}{y} = \frac{30}{36}$ $b = \frac{5}{6}y$

Since $A_1 = \frac{1}{2}by = \frac{5}{12}y^2$, $y^2 = \frac{12}{5}A_1$

$$y = \sqrt{\frac{12}{5}(270)} = 25.4558 \text{ mm}$$

$$b = 21.2132 \text{ mm}$$

$$A_1 = \frac{1}{2}(21.2132)(25.4558) = 270 \text{ mm}^2 = 270\times10^{-6} \text{ m}^2$$

$$A_2 = (21.2132)(36 - 25.4558) = 223.676 \text{ mm}^2 = 223.676\times10^{-6} \text{ m}^2$$

$$A_3 = A - A_1 - A_2 = 46.324 \text{ mm}^2 = 46.324\times10^{-6} \text{ m}^2$$

$$R_i = \sigma_Y A_i = 240\times10^6 A_i$$

$$R_1 = 64.8\times10^3 \text{ N}, \quad R_2 = 53.6822\times10^3 \text{ N}, \quad R_3 = 11.1178\times10^3 \text{ N}$$

$$\bar{y}_1 = \frac{1}{3}y = 8.4853 \text{ mm} = 8.4853\times10^{-3} \text{ m}$$

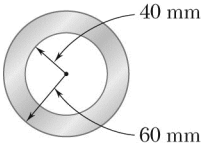
$$\bar{y}_2 = \frac{1}{2}(36 - 25.4558) = 5.2721 \text{ mm} = 5.2721\times10^{-3} \text{ m}$$

$$\bar{y}_3 = \frac{2}{3}(36 - 25.4558) = 7.0295 \text{ mm} = 7.0295\times10^{-3} \text{ m}$$

$$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3 = 911 \text{ N}\cdot\text{m}$$

$$M_p = 911 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

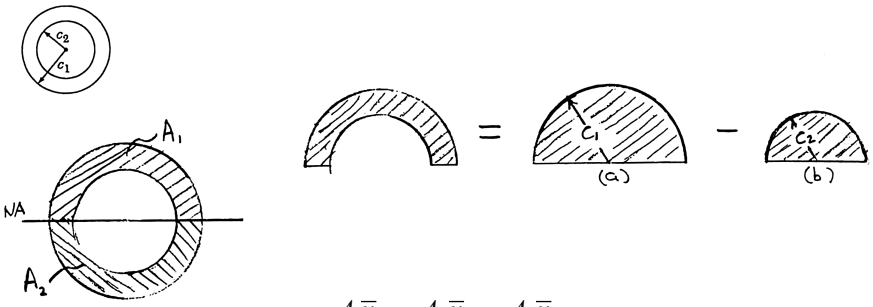


PROBLEM 4.84

Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Let c_1 be the outer radius and c_2 the inner radius.



$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

$$= \frac{2}{3}\left(c_1^3 - c_2\right)^3$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}\left(c_1^3 - c_2^2\right)$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2) = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

Data:

$$\sigma_Y = 240 \text{ MPa} = 240\times10^6 \text{ Pa}$$

$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

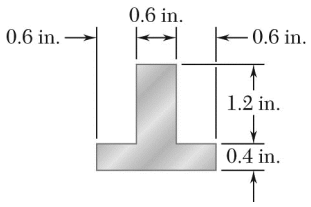
$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$M_p = \frac{4}{3}(240\times10^6)(0.060^3 - 0.040^3)$$

$$= 48.64\times10^3 \text{ N}\cdot\text{m}$$

$$M_p = 48.6 \text{ kN}\cdot\text{m} \blacktriangleleft$$

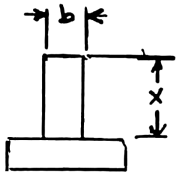
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.85

Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.

SOLUTION

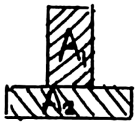


Total area: $$A = (1.8)(0.4) + (0.6)(1.2) = 1.44 \text{ in}^2$$

$$\frac{1}{2}A = 0.72 \text{ in}^2$$

$$x = \frac{\frac{1}{2}A}{b} = \frac{0.72}{0.6} = 1.2 \text{ in.}$$

Neutral axis lies 1.2 in. below the top.



$$A_1 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_1 = \frac{1}{2}(1.2) = 0.6 \text{ in.}$$

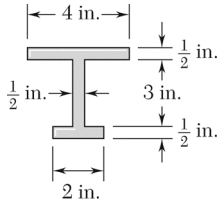
$$A_2 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_2 = \frac{1}{2}(0.4) = 0.2 \text{ in.}$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2)$$

$$= (36)[(0.72)(0.6) + (0.72)(0.2)] = 20.7 \text{ kip} \cdot \text{in}$$

$$M_p = 20.7 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.86

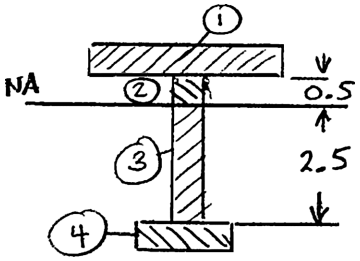
Determine the plastic moment M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.

SOLUTION

Total area:

$$A = (4)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)(3) + (2)\left(\frac{1}{2}\right) = 4.5 \text{ in}^2$$

$$\frac{1}{2}A = 2.25 \text{ in}^2$$

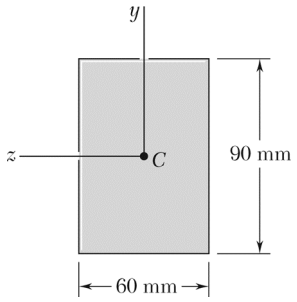


$$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1\bar{y}_1 = 1.50 \text{ in}^3$$
$$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2\bar{y}_2 = 0.0625 \text{ in}^3$$
$$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3\bar{y}_3 = 1.5625 \text{ in}^3$$
$$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4\bar{y}_4 = 2.75 \text{ in}^3$$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$$
$$= (36)(1.50 + 0.0625 + 1.5625 + 2.75)$$

$$M_p = 212 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.87

For the beam indicated (of Prob. 4.73), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

SOLUTION

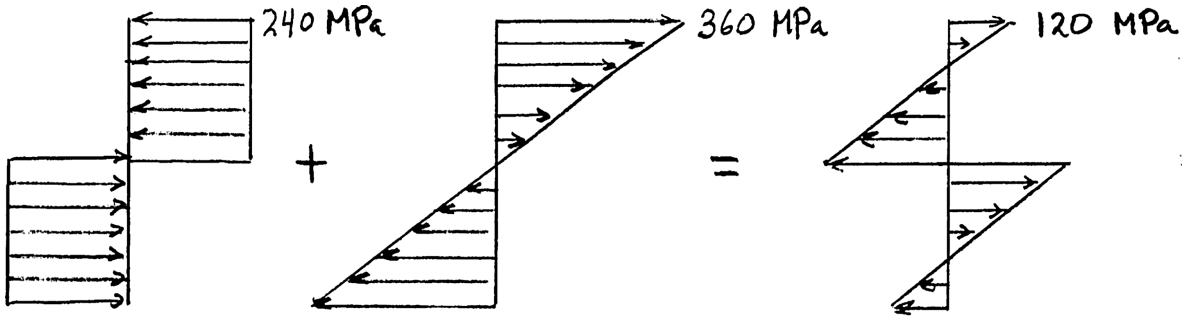
$$M_p = 29.16 \times 10^3\ \text{N} \cdot \text{m}$$

See solutions to Problems 4.73 and 4.77.

$$I = 3.645 \times 10^{-6}\ \text{m}^4$$

$$c = 0.045\ \text{m}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \qquad \text{at } y = c = 45\ \text{mm}$$



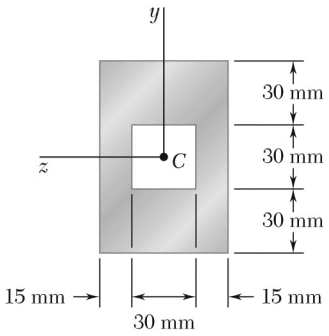
LOADING UNLOADING RESIDUAL STRESSES

$$\sigma' = \frac{(29.16 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 360 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6$$

$$= 120 \times 10^6\ \text{Pa} \qquad \sigma_{\text{res}} = 120.0\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.88

For the beam indicated (of Prob. 4.74), a couple of moment equal to the full plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 45\,\text{mm}$.

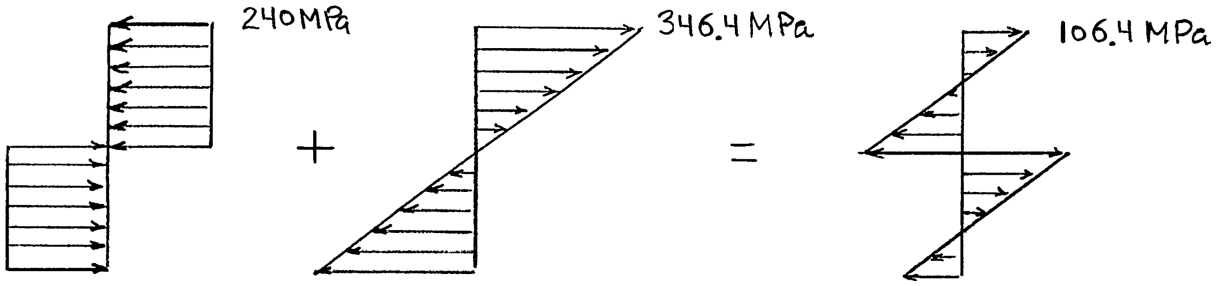
SOLUTION

$M_p = 27.54 \times 10^3\ \text{N} \cdot \text{m}$ (See solutions to Problems 4.74 and 4.78.)

$$I = 3.5775 \times 10^{-6}\ \text{m}^4, \qquad c = 0.045\ \text{m}$$

$$\sigma' = \frac{M_{\max} y}{I} = \frac{M_p c}{I} \quad \text{at} \quad y = c$$

$$\sigma' = \frac{(27.54 \times 10^3)(0.045)}{3.5775 \times 10^{-6}} = 346.4 \times 10^6\ \text{Pa}$$

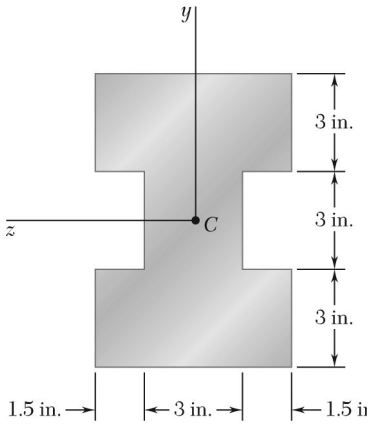


LOADING UNLOADING RESIDUAL STRESSES

$$\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6\ \text{Pa}$$

$$\sigma_{\text{res}} = 106.4\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.89

A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5\,\text{in.}$, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

SOLUTION

See solution to Problem 4.75 for bending couple and stress distribution.

$$M = 4725\ \text{kip}\cdot\text{in} \qquad y_Y = 1.5\ \text{in.} \qquad E = 29\times10^6\ \text{psi} = 29\times10^3\ \text{ksi}$$

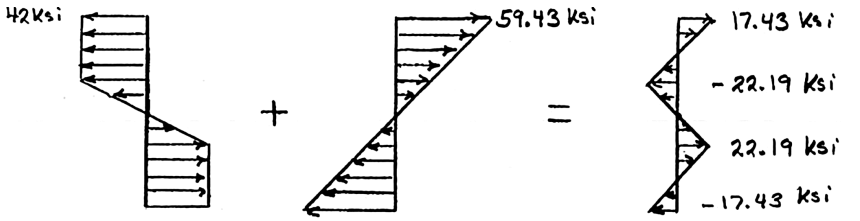
$$\sigma_Y = 42\ \text{ksi} \qquad I = 357.75\ \text{in}^4 \qquad c = 4.5\ \text{in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(4725)(4.5)}{357.75} = 59.43\ \text{ksi}$$ ◀

$$\sigma'' = \frac{My_Y}{I} = \frac{(4725)(1.5)}{357.75} = 19.81\ \text{ksi}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 59.43 - 42 = 17.43\ \text{ksi}$

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 19.81 - 42 = -22.19\ \text{ksi}$



LOADING UNLOADING RESIDUAL STRESSES

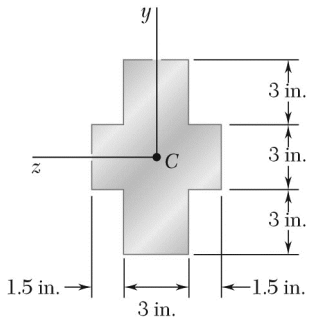
(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(357.75)(42)}{4725} = 3.18\ \text{in.}$$ **Answer:** $y_0 = -3.18\,\text{in.},\ 0,\ 3.18\ \text{in.}$ ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -22.19\,\text{ksi}$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29\times10^3)(1.5)}{22.19} = 1960\ \text{in.}$$ $\rho = 163.4\ \text{ft.}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.90

A bending couple is applied to the bar indicated, causing plastic zones 3 in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

SOLUTION

See solution to Problem 4.76 for bending couple and stress distribution during loading.

$$M = 2646 \text{ kip} \cdot \text{in} \quad y_Y = 1.5 \text{ in.} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

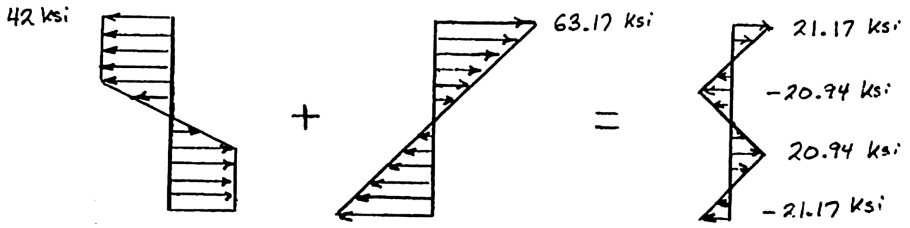
$$\sigma_Y = 42 \text{ ksi} \quad I = 188.5 \text{ in}^4 \quad c = 4.5 \text{ in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(2646)(4.5)}{188.5} = 63.17 \text{ ksi}$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(2646)(1.5)}{188.5} = 21.06 \text{ ksi}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 63.17 - 42 = 21.17$ ksi

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 21.06 - 42$ $\sigma_{\text{res}} = -20.94$ ksi ◀



LOADING UNLOADING RESIDUAL STRESSES

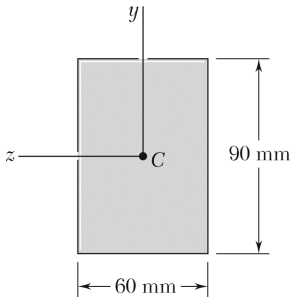
(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} = \sigma_Y$$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(188.5)(42)}{2646} = 2.992 \text{ in.}$$ **Answer:** $y_0 = -2.992$ in., 0, 2.992 in. ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -20.94$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.5)}{20.94} = 2077 \text{ in.}$$ $\rho = 173.1$ ft ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.91

A bending couple is applied to the beam of Prob. 4.73, causing plastic zones 30 mm thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 45$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

See solution to Problem 4.73 for bending couple and stress distribution during loading:

$$M = 28.08 \times 10^3 \text{ N} \cdot \text{m} \quad y_Y = 15 \text{ mm} = 0.015 \text{ m} \quad E = 200 \text{ GPa}$$

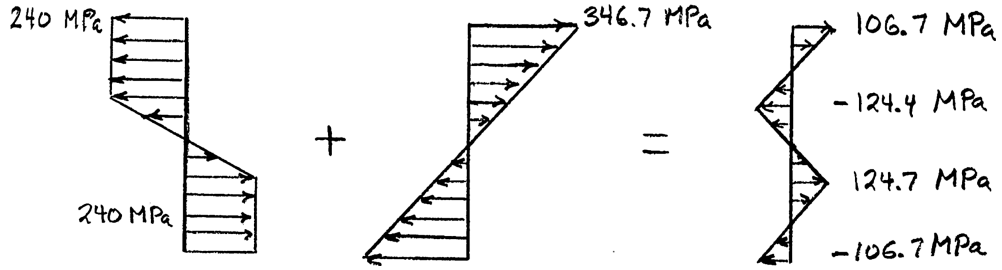
$$\sigma_Y = 240 \text{ MPa} \quad I = 3.645 \times 10^{-6} \text{ m}^4 \quad c = 0.045 \text{ m}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(28.08 \times 10^3)(0.045)}{3.645 \times 10^{-6}} = 346.7 \times 10^6 \text{ Pa} = 346.7 \text{ MPa}$$

$$\sigma'' = \frac{M y_Y}{I} = \frac{(28.08 \times 10^3)(0.015)}{3.645 \times 10^{-6}} = 115.6 \times 10^6 \text{ Pa} = 115.6 \text{ MPa}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 346.7 - 240$ $\qquad \sigma_{\text{res}} = 106.7 \text{ MPa}$ ◀

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 115.6 - 240$ $\qquad \sigma_{\text{res}} = -124.4 \text{ MPa}$



LOADING UNLOADING RESIDUAL STRESSES

(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{M y_0}{I} - \sigma_Y = 0$$

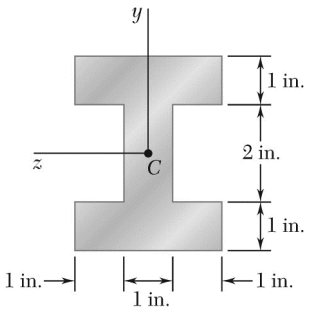
$$y_0 = \frac{I\sigma_Y}{M} = \frac{(3.645 \times 10^{-6})(240 \times 10^6)}{28.08 \times 10^3} = 31.15 \times 10^{-3} \text{ m} = 31.15 \text{ mm}$$

Answer: $y_0 = -31.15 \text{ mm}, 0, 31.15 \text{ mm}$ ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -124.4 \times 10^6 \text{ Pa}$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.015)}{-124.4 \times 10^6}$$ $\qquad \rho = 24.1 \text{ m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.92

A beam of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. A bending couple is applied to the beam about z axis, causing plastic zones 2 in. thick to develop at the top and bottom of the beam. After the couple has been removed, determine (*a*) the residual stress at $y = 2$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the beam.

SOLUTION

See solution to Problem 4.76 for bending couple and stress distribution during loading.

$$M = 406 \text{ kip} \cdot \text{in} \quad y_Y = 1.0 \text{ in.} \quad E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

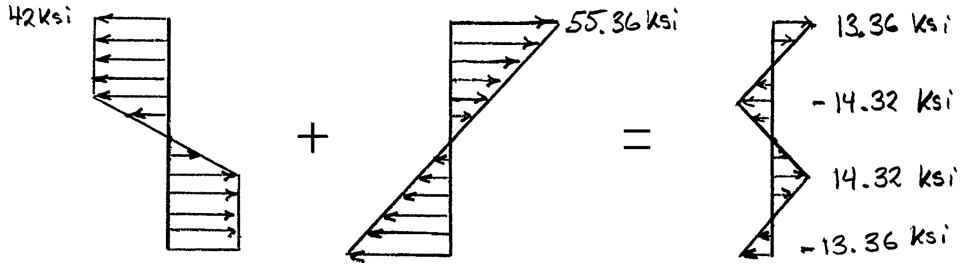
$$\sigma_Y = 42 \text{ ksi} \quad I = 14.6667 \text{ in}^4 \quad c = 2 \text{ in.}$$

(*a*) $$\sigma' = \frac{Mc}{I} = \frac{(406)(2)}{14.6667} = 55.36 \text{ ksi}$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(406)(1.0)}{14.6667} = 27.68 \text{ ksi}$$

At $y = c$, $\sigma_{\text{res}} = \sigma' - \sigma_Y = 55.36 - 42$ $\qquad \sigma_{\text{res}} = 13.36$ ksi ◀

At $y = y_Y$, $\sigma_{\text{res}} = \sigma'' - \sigma_Y = 27.68 - 42$ $\qquad \sigma_{\text{res}} = -14.32$ ksi ◀



LOADING UNLOADING RESIDUAL STRESSES

(*b*) $$\sigma_{\text{res}} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_Y = 0$$

$$y_0 = \frac{I\sigma_Y}{M} = \frac{(14.6667)(42)}{406} = 1.517 \text{ in.}$$ **Answer:** $y_0 = -1.517$ in., $0, 1.517$ in. ◀

(*c*) At $y = y_Y$, $\sigma_{\text{res}} = -14.32$ ksi

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.0)}{14.32} = 2025 \text{ in.}$$ $\qquad \rho = 168.8$ ft ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.93*

A rectangular bar that is straight and unstressed is bent into an arc of circle of radius by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation:

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

SOLUTION

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right),$$

Let m denote $\dfrac{M}{M_Y}$:

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2\,m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\} \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.94

A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.93.

SOLUTION

(*a*) $\dfrac{1}{\rho_Y} = \dfrac{M_Y}{EI}, \qquad M = \dfrac{3}{2}M_Y\left(1 - \dfrac{1}{3}\dfrac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \dfrac{M}{M_Y} = 1.25$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

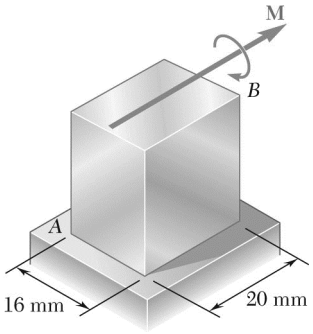
$$\rho = 0.70711\rho_Y \blacktriangleleft$$

(*b*) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y}$$

$$\rho_R = 6.09\rho_Y \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.95

The prismatic bar AB is made of a steel that is assumed to be elastoplastic and for which $E = 200\text{ GPa}$. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 350\text{ N}\cdot\text{m}$ is applied as shown, determine (*a*) the yield strength of the steel, (*b*) the thickness of the elastic core of the bar.

SOLUTION

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \frac{3}{2}\frac{\sigma_Y b(2c)^3}{12c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

$$= \sigma_Y bc^2\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

(*a*) $bc^2\sigma_y\left(1 - \dfrac{\rho^2\sigma_Y^2}{3E^2c^2}\right) = M$ Cubic equation for σ_Y

Data:

$$E = 200\times10^9\text{ Pa}$$

$$M = 420\text{ N}\cdot\text{m}$$

$$\rho = 2.4\text{ m}$$

$$b = 20\text{ mm} = 0.020\text{ m}$$

$$c = \frac{1}{2}h = 8\text{ mm} = 0.008\text{ m}$$

$$(1.28\times10^{-6})\,\sigma_Y\left[1 - 750\times10^{-21}\sigma_Y^2\right] = 350$$

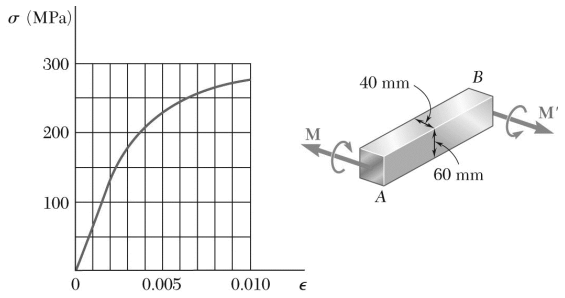
$$\sigma_Y\left[1 - 750\times10^{-21}\sigma_Y^2\right] = 273.44\times10^6$$

Solving by trial, $\sigma_Y = 292\times10^6\text{ Pa}$ $\sigma_Y = 292\text{ MPa}$ ◀

(*b*) $y_Y = \dfrac{\sigma_Y\rho}{E} = \dfrac{(292\times10^6)(2.4)}{200\times10^9} = 3.504\times10^{-3}\text{ m} = 3.504\text{ mm}$

thickness of elastic core $= 2y_Y = 7.01\text{ mm}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.96

The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint:* For part *b*, plot σ versus y and use an approximate method of integration.)

SOLUTION

(*a*) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$c = \frac{1}{2}\text{h} = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1} \qquad \rho = 4.69 \text{ m} \blacktriangleleft$$

(*b*) Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\sigma b dy = 2b\int_0^c y\,|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\Sigma wu|\sigma|$$

where w is a weighting factor.

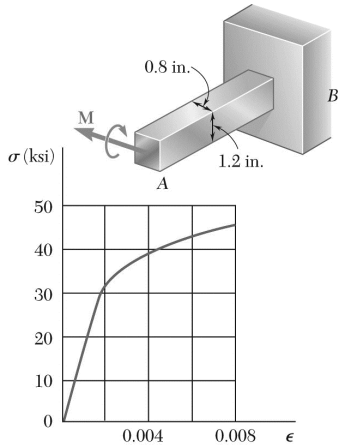
Using $\Delta u = 0.25,$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\,\lvert\sigma\rvert$, (MPa)	w	$wu\,\lvert\sigma\rvert$, (MPa)	
0	0	0	0	1	0	
0.25	0.0016	110	27.5	4	110	
0.5	0.0032	180	90	2	180	
0.75	0.0048	225	168.75	4	675	
1.00	0.0064	250	250	1	250	
					1215	$\leftarrow \Sigma wu\,\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N} \cdot \text{m} \qquad M = 7.29 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.97

The prismatic bar *AB* is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the $\sigma - \varepsilon$ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.96.)

SOLUTION

(*a*) $\rho = 100$ in., $b = 0.8$ in., $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve, $\sigma_m = 43$ ksi ◀

(*b*) Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{\varepsilon}$

Bending couple: $M = -\int_{-c}^{c} y\,\sigma\, b dy = 2b\int_0^c y\,|\sigma| dy = 2bc^2 \int_0^1 u\,|\sigma|\,du = 2bc^2 J$

where the integral J is given by $\int_0^1 u\,|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \Sigma w u |\sigma|$$

where w is a weighting factor.

Using $\Delta u = 0.25$, we get the values given the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\,\lvert\sigma\rvert$, ksi	w	$wu\,\lvert\sigma\rvert$, ksi	
0	0	0	0	1	0	
0.25	0.0015	25	6.25	4	25	
0.5	0.003	36	18	2	36	
0.75	0.0045	40	30	4	120	
1.00	0.006	43	43	1	43	
					224	$\leftarrow \Sigma wu\,\lvert\sigma\rvert$

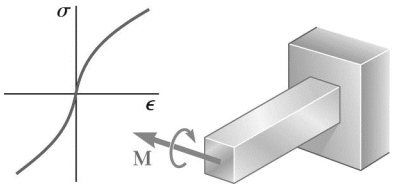
$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67)$$

$M = 10.75$ kip · in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.98



A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\varepsilon = k\sigma^n$ for $\sigma > 0$ and $\varepsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

SOLUTION

Strain distribution: $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad \text{where} \quad u = \frac{y}{c}$

Bending couple:

$$M = -\int_{-c}^{c} y\,\sigma\, b\,dy = 2b\int_0^c y\,|\sigma|\,dy = 2bc^2\int_0^c \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u\,|\sigma|\,du$$

For $\varepsilon = K\sigma^n, \quad \varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \quad \therefore \quad |\sigma| = \sigma_m u^{\frac{1}{n}}$$

Then $M = 2bc^2\int_0^1 u\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}du$

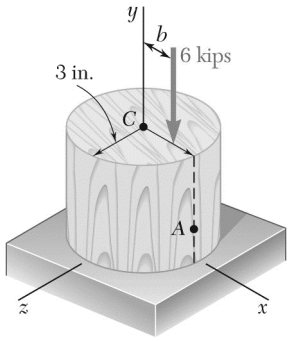
$$= 2bc^2\sigma_m \frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\Big|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall: that $\frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \quad \therefore \quad \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

Then $\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.99

A short wooden post supports a 6-kip axial load as shown. Determine the stress at point A when (a) $b = 0$, (b) $b = 1.5$ in., (c) $b = 3$ in.

SOLUTION

$$A = \pi r^2 = \pi(3)^2 = 28.27 \text{ in}^2$$

$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(3)^4 = 63.62 \text{ in}^4$$

$$S = \frac{I}{c} = \frac{63.62}{3} = 21.206 \text{ in}^3$$

$$P = 6 \text{ kips} \qquad M = Pb$$

(a) $b = 0$ $\quad M = 0$

$$\sigma = -\frac{P}{A} = -\frac{6}{28.27} = -0.212 \text{ ksi}$$

$\sigma = -212$ psi ◀

(b) $b = 1.5$ in. $\quad M = (6)(1.5) = 9$ kip · in

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{9}{21.206} = -0.637 \text{ ksi}$$

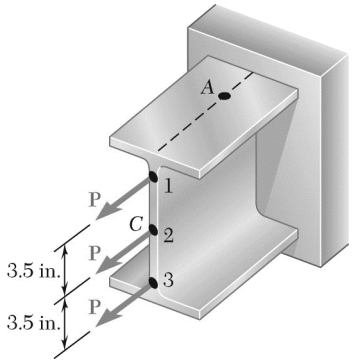
$\sigma = -637$ psi ◀

(c) $b = 3$ in. $\quad M = (6)(3) = 18$ kip · in

$$\sigma = -\frac{P}{A} - \frac{M}{S} = -\frac{6}{28.27} - \frac{18}{21.206} = -1.061 \text{ ksi}$$

$\sigma = -1061$ psi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.100

As many as three axial loads, each of magnitude $P = 10$ kips, can be applied to the end of a $W8 \times 21$ rolled-steel shape. Determine the stress at point A, (*a*) for the loading shown, (*b*) if loads are applied at points 1 and 2 only.

SOLUTION

For $W8 \times 21$ Appendix C gives

$$A = 6.16 \text{ in}^2, \quad d = 8.28 \text{ in.}, \quad I_x = 75.3 \text{ in}^4$$

At point A,

$$y = \frac{1}{2}d = 4.14 \text{ in.}$$

$$\sigma = \frac{F}{A} - \frac{My}{I}$$

(*a*) Centric loading: $F = 30$ kips, $M = 0$

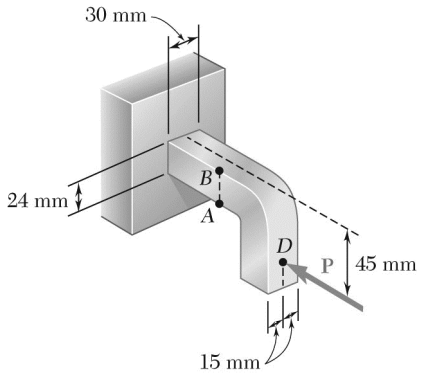
$$\sigma = \frac{30}{6.16} \qquad \sigma = 4.87 \text{ ksi} \blacktriangleleft$$

(*b*) Eccentric loading: $F = 2P = 20$ kips

$$M = -(10)(3.5) = -35 \text{ kip} \cdot \text{in}$$

$$\sigma = \frac{20}{6.16} - \frac{(-35)(4.14)}{75.3} \qquad \sigma = 5.17 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.101

Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = (30)(24) = 720\,\text{mm}^2 = 720\times10^{-6}\,\text{m}^2$$

$$e = 45 - 12 = 33\,\text{mm} = 0.033\,\text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56\times10^3\,\text{mm}^4 = 34.56\times10^{-9}\,\text{m}^4$$

$$c = \frac{1}{2}(24\,\text{mm}) = 12\,\text{mm} = 0.012\,\text{m} \qquad P = 8\times10^3\,\text{N}$$

$$M = Pe = (8\times10^3)(0.033) = 264\,\text{N}\cdot\text{m}$$

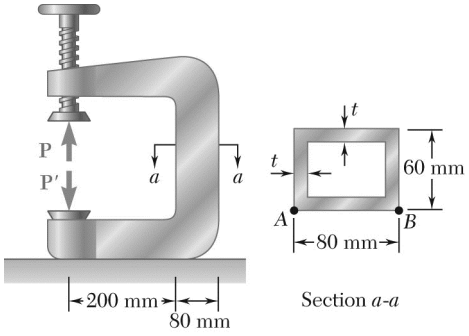
(*a*) $$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8\times10^3}{720\times10^{-6}} = \frac{(264)(0.012)}{34.56\times10^{-9}} = -102.8\times10^6\,\text{Pa}$$

$$\sigma_A = -102.8\,\text{MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8\times10^3}{720\times10^{-6}} + \frac{(264)(0.012)}{34.56\times10^{-9}} = 80.6\times10^6\,\text{Pa}$$

$$\sigma_B = 80.6\,\text{MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.102

The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Rectangular cutout is $60 \text{ mm} \times 40 \text{ mm}$.

$$A = (80)(60) - (60)(40) = 2.4 \times 10^{3} \text{ mm}^2 = 2.4 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^{6} \text{ mm}^4$$

$$= 1.84 \times 10^{-6} \text{ m}^4$$

$$c = 40 \text{ mm} = 0.040 \text{ m} \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$

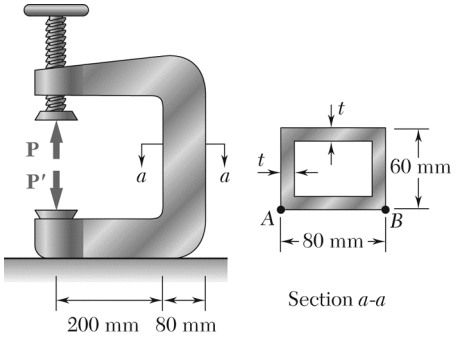
$$P = 20 \times 10^{3} \text{ N}$$

$$M = Pe = (20 \times 10^{3})(0.240) = 4.8 \times 10^{3} \text{ N} \cdot \text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^{3}}{2.4 \times 10^{-3}} + \frac{(4.8 \times 10^{3})(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^{6} \text{ Pa}$$ $\sigma_A = 112.7$ MPa ◀

(*b*) $$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^{3}}{2.4 \times 10^{-3}} - \frac{(4.8 \times 10^{3})(0.040)}{1.84 \times 10^{-6}} = -96.0 \times 10^{6} \text{ Pa}$$ $\sigma_B = -96.0$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.103

Solve Prob. 4.102, assuming that $t = 8$ mm.

PROBLEM 4.102 The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

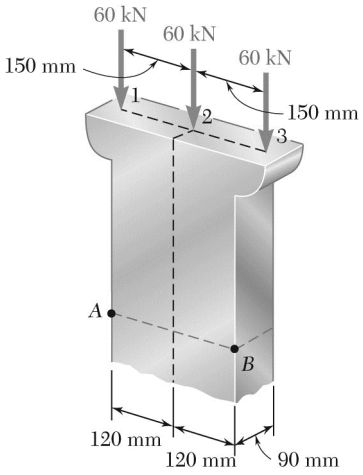
Rectangular cutout is $64 \text{ mm} \times 44 \text{ mm}$.

$$A = (80)(60) - (64)(44) = 1.984 \times 10^3 \text{ mm}^2$$
$$= 1.984 \times 10^{-3} \text{ mm}^2$$
$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(44)(64)^3 = 1.59881 \times 10^6 \text{ mm}^2$$
$$= 1.59881 \times 10^{-6} \text{ m}^4$$
$$c = 40 \text{ mm} = 0.004 \quad e = 200 + 40 = 240 \text{ mm} = 0.240 \text{ m}$$
$$P = 20 \times 10^3 \text{ N}$$
$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} + \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = 130.2 \times 10^6 \text{ Pa}$$ $\sigma_A = 130.2$ MPa ◀

(*b*) $$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{20 \times 10^3}{1.984 \times 10^{-3}} - \frac{(4.8 \times 10^3)(0.040)}{1.59881 \times 10^{-6}} = -110.0 \times 10^6 \text{ Pa}$$ $\sigma_B = -110.0$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.104

Determine the stress at points A and B, (*a*) for the loading shown, (*b*) if the 60-kN loads are applied at points 1 and 2 only.

SOLUTION

(*a*) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-6} \text{ m}^2$$

At A and B:
$$\sigma = -\frac{P}{A} = \frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6 \text{ Pa}$$

$$\sigma_A = \sigma_B = -8.33 \text{ MPa} \blacktriangleleft$$

(*b*) Eccentric loading.

$$P = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

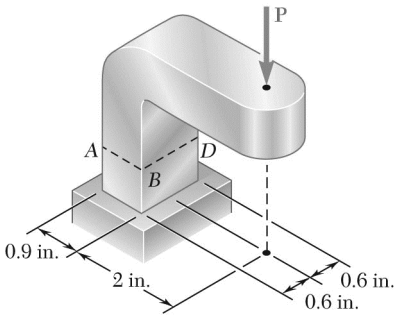
At A:
$$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -15.97 \times 10^6 \text{ Pa}$$

At B:
$$\sigma_B - \frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 4.86 \times 10^6 \text{ Pa}$$

$$\sigma_A = -15.97 \text{ MPa} \blacktriangleleft$$

$$\sigma_B = 4.86 \text{ MPa} \blacktriangleleft$$

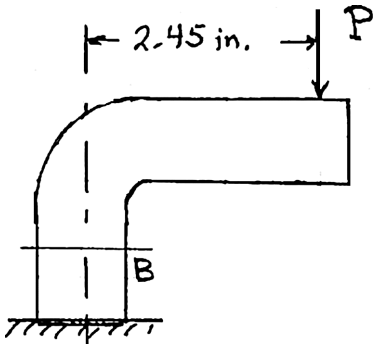
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.105

Knowing that the allowable stress in section *ABD* is 10 ksi, determine the largest force **P** that can be applied to the bracket shown.

SOLUTION

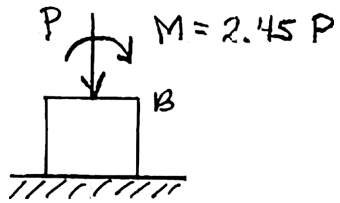


Statics: $M = 2.45P$

Cross section: $A = (0.9)(1.2) = 1.08 \text{ in}^2$

$$c = \frac{1}{2}(0.9) = 0.45 \text{ in.}$$

$$I = \frac{1}{12}(1.2)(0.9)^3 = 0.0729 \text{ in}^4$$



At point *B*: $\sigma = -10$ ksi

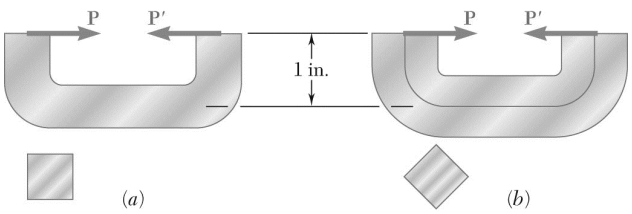
$$\sigma = -\frac{P}{A} - \frac{Mc}{I}$$

$$-10 = -\frac{P}{1.08} - \frac{(2.45P)(0.45)}{0.0729} = -16.049P$$

$$P = 0.623 \text{ kips}$$

$$P = 623 \text{ lb} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.106

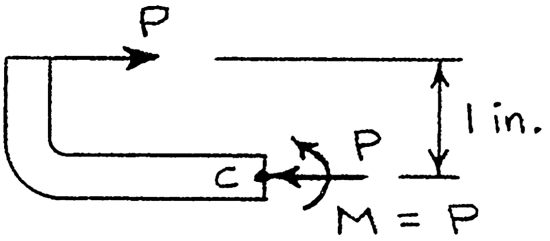
Portions of a $\frac{1}{2} \times \frac{1}{2}$-in. square bar have been bent to form the two machine components shown. Knowing that the allowable stress is 15 ksi, determine the maximum load that can be applied to each component.

SOLUTION

The maximum stress occurs at point B.

$$\sigma_B = -15 \text{ ksi} = -15 \times 10^3 \text{ psi}$$

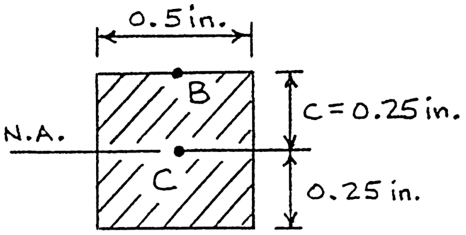
$$\sigma_B = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP$$



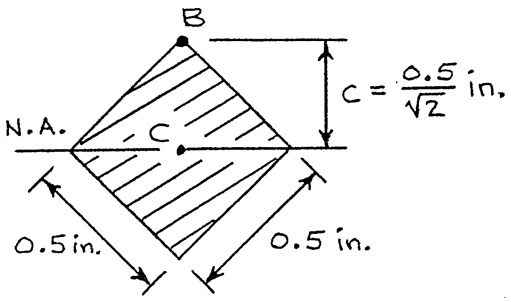
where $K = \dfrac{1}{A} + \dfrac{ec}{I}$ $\quad e = 1.0$ in.

$$A = (0.5)(0.5) = 0.25 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(0.5)^3 = 5.2083 \times 10^{-3} \text{ in}^4 \text{ for all centroidal axes.}$$



(a)



(b)

(*a*) $c = 0.25$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.25)}{5.2083 \times 10^{-3}} = 52 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{52}$$

$P = 288$ lb ◀

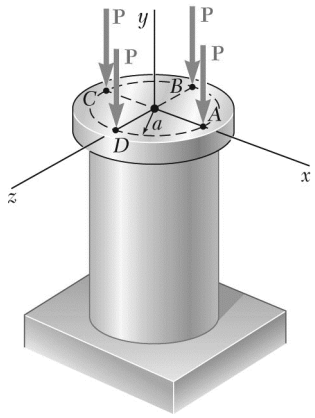
(*b*) $c = \dfrac{0.5}{\sqrt{2}} = 0.35355$ in.

$$K = \frac{1}{0.25} + \frac{(1.0)(0.35355)}{5.2083 \times 10^{-3}} = 71.882 \text{ in}^{-2}$$

$$P = -\frac{\sigma_B}{K} = -\frac{(-15 \times 10^3)}{71.882}$$

$P = 209$ lb ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.107

The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Knowing that $P = 100$ kN and $a = 40$ mm, determine the maximum stress in the post when (a) the force at D is removed, (b) the forces at C and D are removed.

SOLUTION

For a solid circular section of radius a,

$$A = \pi a^2 \qquad I = \frac{\pi}{4} a^4$$

Centric force: $F = 4P, \; M_x = M_z = 0 \qquad \sigma = -\dfrac{F}{A} = -\dfrac{4P}{\pi a^2}$

(a) Force at D is removed:

$$F = 3P, \quad M_x = -Pa, \quad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^2} = -\frac{7P}{\pi a^2}$$

(b) Forces at C and D are removed:

$$F = 2P, \quad M_x = -Pa, \quad M_z = -Pa$$

Resultant bending couple: $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\, Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}\, Pa\, a}{\frac{\pi}{4} a^2} = -\frac{2 + 4\sqrt{2}}{\pi} \frac{P}{a^2} = -2.437\, P/a^2$$

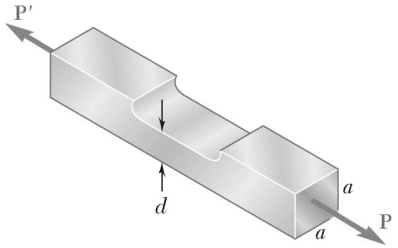
Numerical data: $P = 100 \times 10^3$ N, $\quad a = 0.040$ m

Answers:

(a) $$\sigma = -\frac{(7)(100 \times 10^3)}{\pi (0.040)^2} = -139.3 \times 10^6 \text{ Pa} \qquad \sigma = -139.3 \text{ MPa} \blacktriangleleft$$

(b) $$\sigma = -\frac{(2.437)(100 \times 10^3)}{(0.040)^2} = -152.3 \times 10^6 \text{ Pa} \qquad \sigma = -152.3 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.108

A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm, and $\sigma_{all} = 60$ MPa, determine the magnitude P of the largest forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

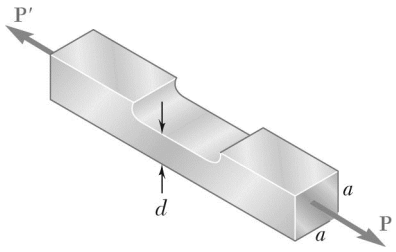
$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP \quad \text{where} \quad K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

Data: $a = 30 \text{ mm} = 0.030 \text{ m} \quad d = 20 \text{ mm} = 0.020 \text{ m}$

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3 \text{ N} \qquad P = 14.40 \text{ kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.109

A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 18$ kN are applied at the centers of the ends of the bar. Knowing that $a = 30$ mm and $\sigma_{all} = 135$ MPa, determine the smallest allowable depth d of the milled portion of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}\,d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \qquad \text{or} \qquad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d,

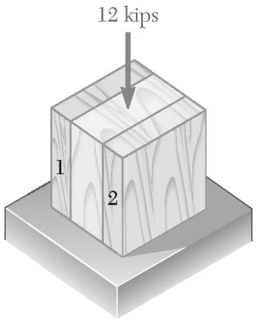
$$d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$$

Data: $a = 0.030$ m, $P = 18\times10^3$ N, $\sigma = 135\times10^6$ Pa

$$d = \frac{1}{(2)(135\times10^6)}\left\{\sqrt{\left[\frac{(2)(18\times10^3)}{0.030}\right]^2 + 12(18\times10^3)(135\times10^6)} - \frac{(2)(18\times10^3)}{0.030}\right\}$$

$$= 16.04\times10^{-3} \text{ m} \qquad d = 16.04 \text{ mm} \blacktriangleleft$$

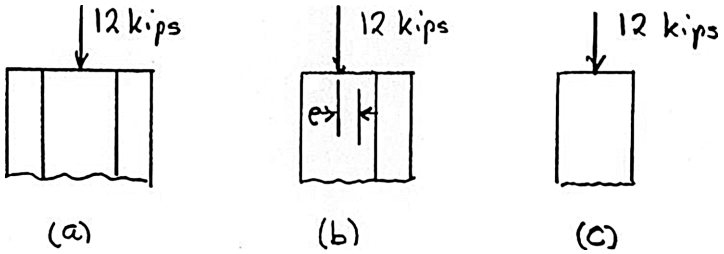
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.110

A short column is made by nailing two 1×4-in. planks to a 2×4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown in the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) both planks are removed.

SOLUTION



(*a*) Centric loading: 4 in. × 4 in. cross section $A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{16} \qquad \sigma = -0.75 \text{ ksi} \blacktriangleleft$$

(*b*) Eccentric loading: 4 in. × 3 in. cross section $A = (4)(3) = 12 \text{ in}^2$

$$c = \left(\frac{1}{2}\right)(3) = 1.5 \text{ in.} \qquad e = 1.5 - 1.0 = 0.5 \text{ in.}$$

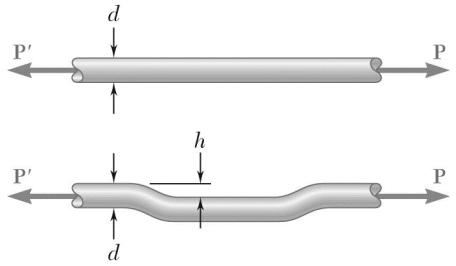
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(3)^3 = 9 \text{ in}^4$$

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} \qquad \sigma = -2.00 \text{ ksi} \blacktriangleleft$$

(*c*) Centric loading: 4 in. × 2 in. cross section $A = (4)(2) = 8 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{8} \qquad \sigma = -1.50 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.111

An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed 5 times the stress in the rod when it is straight, determine the largest offset that can be used.

SOLUTION

For centric loading, $\sigma_c = \dfrac{P}{A}$

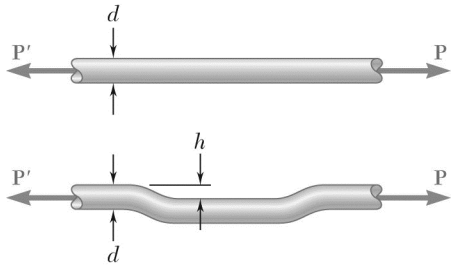
For eccentric loading, $\sigma_e = \dfrac{P}{A} + \dfrac{Phc}{I}$

Given $\sigma_e = 5\,\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 5\frac{P}{A}$$

$$\frac{Phc}{I} = 4\frac{P}{A} \quad \therefore \quad h = \frac{4I}{cA} = \frac{(4)\left(\dfrac{\pi}{64}d^4\right)}{\left(\dfrac{d}{2}\right)\left(\dfrac{\pi}{4}d^2\right)} = \frac{1}{2}d \qquad h = 0.500\,d \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.112

An offset h must be introduced into a metal tube of 0.75-in. outer diameter and 0.08-in. wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed 4 times the stress in the tube when it is straight, determine the largest offset that can be used.

SOLUTION

$$c = \frac{1}{2}d = 0.375 \text{ in.}$$

$$c_1 = c - t = 0.375 - 0.08 = 0.295 \text{ in.}$$

$$A = \pi\left(c^2 - c_1^2\right) = \pi(0.375^2 - 0.295^2)$$

$$= 0.168389 \text{ in}^2$$

$$I = \frac{\pi}{4}\left(c^4 - c_1^4\right) = \frac{\pi}{4}(0.375^4 - 0.295^4)$$

$$= 9.5835 \times 10^{-3} \text{ in}^4$$

For centric loading, $\sigma_{\text{cen}} = \dfrac{P}{A}$

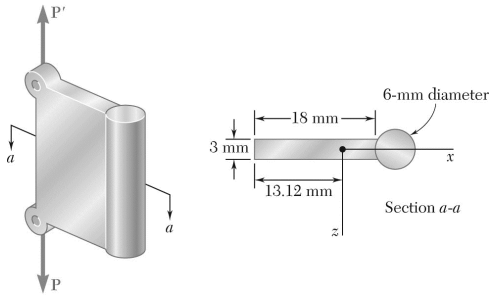
For eccentric loading, $\sigma_{\text{ecc}} = \dfrac{P}{A} + \dfrac{Phc}{I}$

$$\sigma_{\text{ecc}} = 4\sigma_{\text{cen}} \quad \text{or} \quad \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{hc}{I} = \frac{3}{A} \qquad h = \frac{3I}{Ac} = \frac{(3)(9.5835 \times 10^{-3})}{(0.168389)(0.375)}$$

$$h = 0.455 \text{ in.} \blacktriangleleft$$

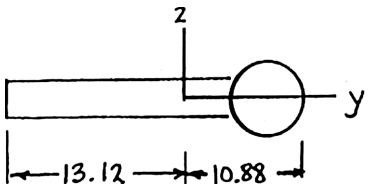
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.113

A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (*a*) the largest force **P** that can be applied to the element, (*b*) the corresponding location of the neutral axis. *Given:* The centroid of the cross section is at *C* and $I_z = 4195 \text{ mm}^4$.

SOLUTION



(*a*) $A = (3)(18) + \dfrac{\pi}{4}(6)^2 = 82.27 \text{ mm}^2 = 82.27 \times 10^{-6} \text{ m}^2$

$I = 4195 \text{ mm}^4 = 4195 \times 10^{-12} \text{ m}^4$

$e = 13.12 \text{ mm} = 0.01312 \text{ m}$

Based on tensile stress at $y = -13.12 \text{ mm} = -0.01312 \text{ m}$

$$\sigma = \frac{P}{A} + \frac{Pec}{I} = \left(\frac{1}{A} + \frac{ec}{I}\right)P = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{82.27 \times 10^{-6}} + \frac{(0.01312)(0.01312)}{4195 \times 10^{-12}} = 53.188 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{135 \times 10^6}{53.188 \times 10^3} = 2.538 \times 10^3 \text{ N} \qquad P = 2.54 \text{ kN} \blacktriangleleft$$

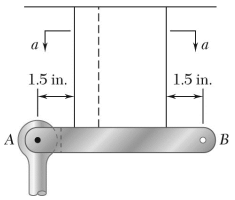
(*b*) Location of neutral axis. $\sigma = 0$

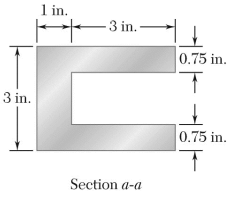
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{4195 \times 10^{-12}}{(82.27 \times 10^{-6})(0.01312)} = 3.89 \times 10^{-3} \text{ m} \qquad y = 3.89 \text{ mm} \blacktriangleleft$$

The neutral axis lies 3.89 mm to the right of the centroid or 17.01 mm to the right of the line of action of the loads.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

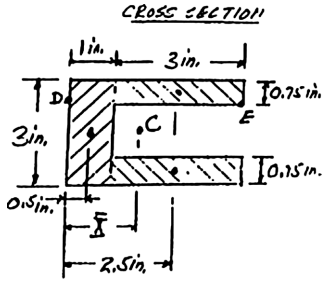




PROBLEM 4.114

A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{all} = +5$ ksi and $\sigma_{all} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1\times 3)(0.5) + 2(3\times 0.25)(2.5)}{(1\times 3) + 2(3\times 0.75)}$$

$$\bar{X} = \frac{12.75 \text{ in}^3}{7.5 \text{ in}^2} = 1.700 \text{ in.}$$

$$A = 7.5 \text{ in}^2$$

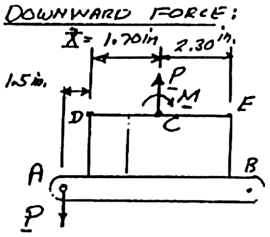
$$\sigma_{all} = +5 \text{ ksi} \qquad \sigma_{all} = -12 \text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3\times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5\times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825 \text{ in}^4$$

Downward Force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\frac{P}{A} + \frac{Mc}{I}$

$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(1.70)}{10.825}$$

$$+5 = P(+0.6359) \qquad \boldsymbol{P} = 7.86 \text{ kips} \downarrow$$

At E: $\sigma_E = +\frac{P}{A} - \frac{Mc}{I}$

$$-12 \text{ ksi} = \frac{P}{7.5} - \frac{(3.20)P(2.30)}{10.825}$$

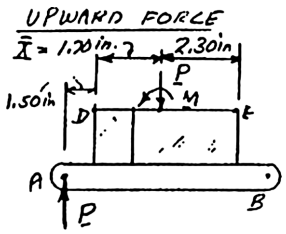
$$-12 = P(-0.5466) \qquad \boldsymbol{P} = 21.95 \text{ kips} \downarrow$$

We choose the smaller value. $\boldsymbol{P} = 7.96 \text{ kips} \downarrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.114 *(Continued)*

Upward Force.



$$M = P(1.5 \text{ in.} + 1.70 \text{ in.}) = (3.20 \text{ in.})P$$

At D: $\sigma_D = +\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.20)P(1.70)}{10.825}$$

$$-12 = P(-0.6359) \qquad \boldsymbol{P} = 18.87 \text{ kips} \uparrow$$

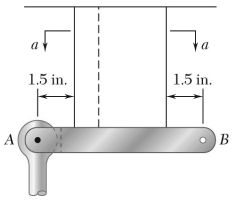
At E: $\sigma_E = +\dfrac{P}{A} + \dfrac{Mc}{I}$

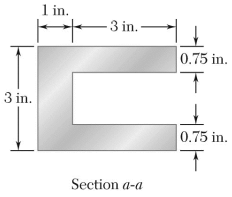
$$+5 \text{ ksi} = \frac{P}{7.5} + \frac{(3.20)P(2.30)}{10.825}$$

$$+5 = P(+0.5466) \qquad \boldsymbol{P} = 9.15 \text{ kips} \uparrow$$

We choose the smaller value. $\boldsymbol{P} = 9.15 \text{ kips} \uparrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



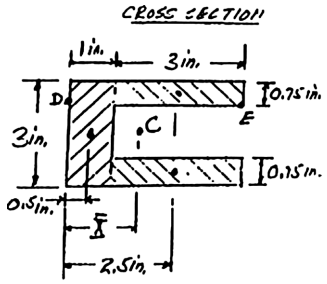


PROBLEM 4.115

Solve Prob. 4.114, assuming that the vertical rod is attached at point B instead of point A.

PROBLEM 4.114 A vertical rod is attached at point A to the cast iron hanger shown. Knowing that the allowable stresses in the hanger are $\sigma_{all} = +5$ ksi and $\sigma_{all} = -12$ ksi, determine the largest downward force and the largest upward force that can be exerted by the rod.

SOLUTION



$$\bar{X} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1\times 3)(0.5) + 2(3\times 0.25)(2.5)}{(1\times 3) + 2(3\times 0.75)}$$

$$\bar{X} = \frac{12.75 \text{ in}^3}{7.5 \text{ in}^2} = 1.700 \text{ in.}$$

$$A = 7.5 \text{ in}^2$$

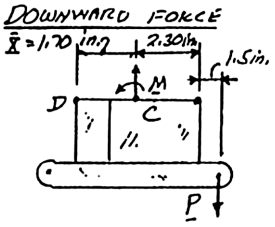
$$\sigma_{all} = +5 \text{ ksi} \qquad \sigma_{all} = -12 \text{ ksi}$$

$$I_c = \sum\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(3)(1)^3 + (3\times 1)(1.70 - 0.5)^2 + \frac{1}{12}(1.5)(3)^3 + (1.5\times 3)(2.5 - 1.70)^2$$

$$I_c = 10.825 \text{ in}^4$$

Downward Force.



$$\sigma_{all} = +5 \text{ ksi} \qquad \sigma_{all} = -12 \text{ ksi}$$

$$M = (2.30 \text{ in.} + 1.5 \text{ in.}) = (3.80 \text{ in.})P$$

At D: $\sigma_D = +\frac{P}{A} - \frac{Mc}{I}$

$$-12 \text{ ksi} = +\frac{P}{7.5} - \frac{(3.80)P(1.70)}{10.825}$$

$$-12 = P(-0.4634) \qquad \boldsymbol{P} = 25.9 \text{ kips} \downarrow$$

At E: $\sigma_E = +\frac{P}{A} + \frac{Mc}{I}$

$$+5 \text{ ksi} = +\frac{P}{7.5} + \frac{(3.80)P(2.30)}{10.825}$$

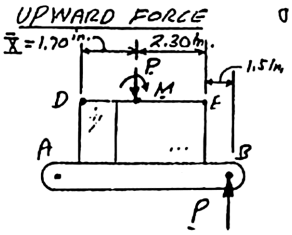
$$+5 = P(+0.9407) \qquad \boldsymbol{P} = 5.32 \text{ kips} \downarrow$$

We choose the smaller value. $\boldsymbol{P} = 5.32 \text{ kips} \downarrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.115 *(Continued)*

Upward Force.



$$\sigma_{\text{all}} = +5 \text{ ksi} \qquad \sigma_{\text{all}} = -12 \text{ ksi}$$

$$M = (2.30 \text{ in.} + 1.5 \text{ in.})P = (3.80 \text{ in.})P$$

At D: $\sigma_D = -\dfrac{P}{A} + \dfrac{Mc}{I}$

$$5 \text{ ksi} = -\frac{P}{7.5} + \frac{(3.80)P(1.70)}{10.825}$$

$$5 = P(+0.4634) \qquad \boldsymbol{P} = 10.79 \text{ kips} \uparrow$$

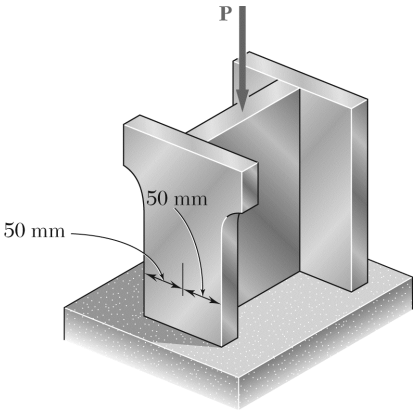
At E: $\sigma_E = -\dfrac{P}{A} - \dfrac{Mc}{I}$

$$-12 \text{ ksi} = -\frac{P}{7.5} - \frac{(3.80)P(2.30)}{10.825}$$

$$-12 = P(-0.9407) \qquad \boldsymbol{P} = 12.76 \text{ kips} \uparrow$$

We choose the smaller value. $\boldsymbol{P} = 10.79 \text{ kips} \uparrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.116

Three steel plates, each of 25×150-mm cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 25-mm strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 100 MPa, determine the largest force **P** (*a*) that could be applied to the original column, (*b*) that can be applied to the modified column.

SOLUTION

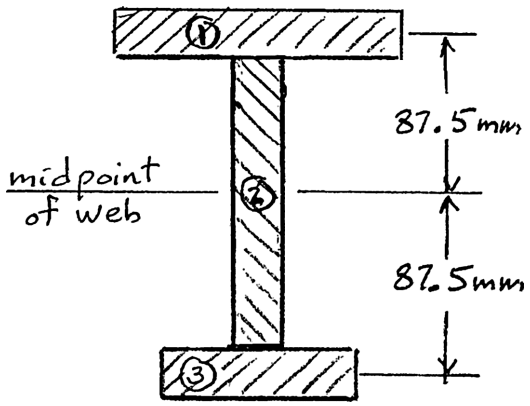
(*a*) Centric loading: $\sigma = -\dfrac{P}{A}$

$$A = (3)(150)(25) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$P = -\sigma A = -(-100 \times 10^6)(11.25 \times 10^{-3})$$

$$= 1.125 \times 10^6 \text{N} \qquad P = 1125 \text{ kN} \blacktriangleleft$$

(*b*) Eccentric loading (reduced cross section):



	A, 10^3 mm^2	$\bar{y}$, mm	$A\bar{y}$ (10^3 mm^3)	d, mm
①	3.75	87.5	328.125	76.5625
②	3.75	0	0	10.9375
③	2.50	–87.5	–218.75	98.4375
Σ	10.00		109.375	

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{109.375 \times 10^3}{10.00 \times 10^3} = 10.9375 \text{ mm}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.116 *(Continued)*

The centroid lies 10.9375 mm from the midpoint of the web.

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(150)(25)^3 + (3.75\times10^3)(76.5625)^2 = 22.177\times10^6\ \text{mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(25)(150)^3 + (3.75\times10^3)(10.9375)^2 = 7.480\times10^6\ \text{mm}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(100)(25)^3 + (2.50\times10^3)(98.4375)^2 = 24.355\times10^6\ \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 54.012\times10^6\ \text{mm}^4 = 54.012\times10^{-6}\ \text{m}^4$$

$$c = 10.9375 + 75 + 25 = 110.9375\ \text{mm} = 0.1109375\ \text{m}$$

$$M = Pe \qquad \text{where} \qquad e = 10.4375\ \text{mm} = 10.4375\times10^{-3}\ \text{m}$$

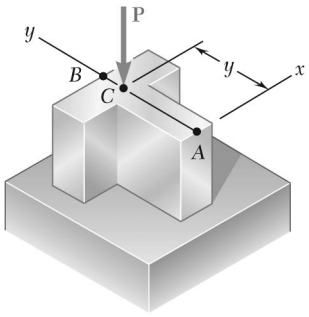
$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} - \frac{Pec}{I} = -KP \qquad A = 10.00\times10^{-3}\ \text{m}^2$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{10.00\times10^{-3}} + \frac{(101.9375\times10^{-3})(0.1109375)}{54.012\times10^{-6}} = 122.465\ \text{m}^{-2}$$

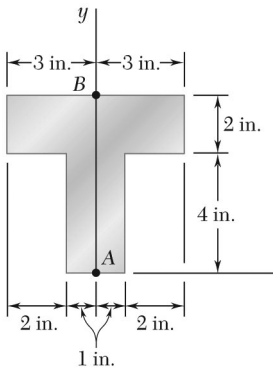
$$P = -\frac{\sigma}{K} = -\frac{(-100\times10^6)}{122.465} = 817\times10^3\ \text{N}$$

$$P = 817\ \text{kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



(*a*)



(*b*)

PROBLEM 4.117

A vertical force **P** of magnitude 20 kips is applied at point *C* located on the axis of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis.

SOLUTION

Locate centroid.

Part	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(*a*) Stress at A: $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{20(1.2)(3.8)}{57.867} \qquad \sigma_A = 0.576 \text{ ksi} \blacktriangleleft$$

(*b*) Stress at B: $c_B = 6 - 3.8 = 2.2$ in.

$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = -\frac{20}{20} - \frac{20(1.2)(2.2)}{57.867} \qquad \sigma_B = -1.912 \text{ ksi} \blacktriangleleft$$

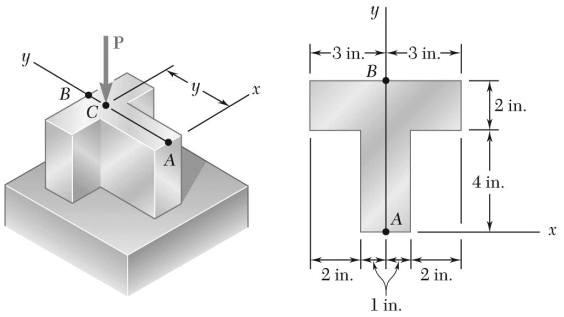
(*c*) Location of neutral axis: $\sigma = 0$ $\quad \sigma = -\dfrac{P}{A} + \dfrac{Pea}{I} = 0 \quad \therefore \quad \dfrac{ea}{I} = \dfrac{1}{A}$

$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in.}$$

Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411 = 1.389$ in. above point *A*.

Answer: 1.389 in. from point *A*. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.118

A vertical force **P** is applied at point *C* located on the axis of symmetry of the cross section of a short column. Determine the range of values of *y* for which tensile stresses do not occur in the column.

SOLUTION

Locate centroid.

	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in.}$$

Eccentricity of load: $e = y - 3.8$ in. $\quad y = e + 3.8$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4 \qquad I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero, $c_A = 3.8$ in.

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \qquad \therefore \qquad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in.} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero. $c_B = 6 - 3.8 = 2.2$ in.

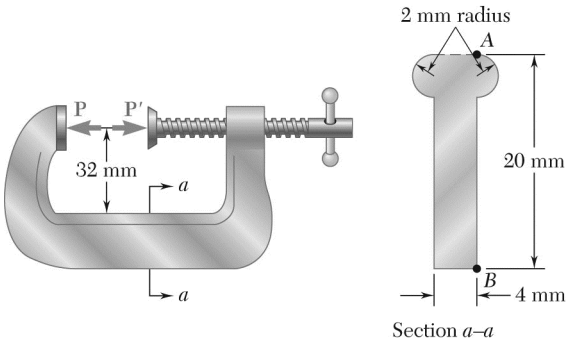
$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \qquad \therefore \qquad \frac{ec_B}{I} = -\frac{1}{A}$$

$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in.}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: 2.485 in. < y < 4.561 in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

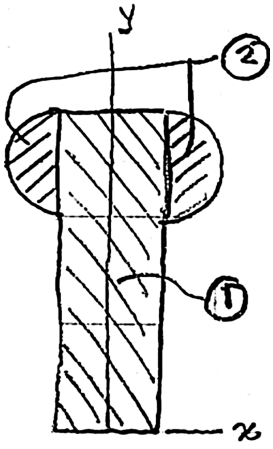


PROBLEM 4.119

Knowing that the clamp shown has been tightened until $P = 400$ N, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the location of the neutral axis of section $a - a$.

SOLUTION

Cross section: Rectangle ① + Circle ②



$$A_1 = (20 \text{ mm})(4 \text{ mm}) = 80 \text{ mm}^2$$

$$\bar{y}_1 = \frac{1}{2}(20 \text{ mm}) = 10 \text{ mm}$$

$$A_2 = \pi(2 \text{ mm})^2 = 4\pi \text{ mm}^2$$

$$\bar{y}_2 = 20 - 2 = 18 \text{ mm}$$

$$c_B = \bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{(80)(10) + (4\pi)(18)}{80 + 4\pi} = 11.086 \text{ mm}$$

$$c_A = 20 - \bar{y} = 8.914 \text{ mm}$$

$$d_1 = 11.086 - 10 = 1.086 \text{ mm}$$

$$d_2 = 18 - 11.086 = 6.914 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = \frac{1}{12}(4)(20)^3 + (80)(1.086)^2 = 2.761 \times 10^3 \text{ mm}^4$$

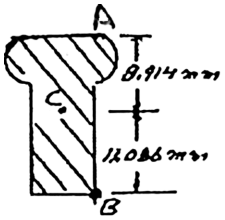
$$I_2 = \bar{I}_2 + A_2 d_2^2 = \frac{\pi}{4}(2)^4 + (4\pi)(6.914)^2 = 0.613 \times 10^3 \text{ mm}^4$$

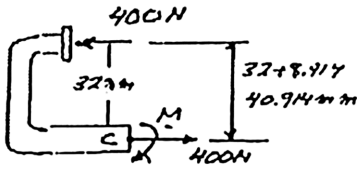
$$I = I_1 + I_2 = 3.374 \times 10^3 \text{ mm}^4 = 3.374 \times 10^{-9} \text{ m}^4$$

$$A = A_1 + A_2 = 92.566 \text{ mm}^2 = 92.566 \times 10^{-6} \text{ m}^2$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.119 *(Continued)*





$e = 32 + 8.914 = 40.914 \text{ mm} = 0.040914 \text{ m}$

$M = Pe = (400 \text{ N})(0.040914 \text{ m}) = 16.3656 \text{ N} \cdot \text{m}$

(*a*) Point *A*:

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} + \frac{(16.3656)(8.914 \times 10^{-3})}{3.374 \times 10^{-9}}$$

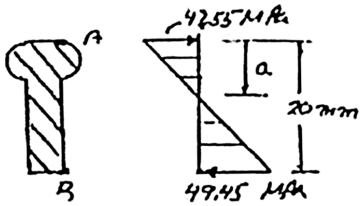
$$= 4.321 \times 10^6 + 43.23 \times 10^6 = 47.55 \times 10^6 \text{ Pa} \qquad \sigma_A = 47.6 \text{ MPa}$$

(*b*) Point *B*:

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{92.566 \times 10^{-6}} - \frac{(16.3656)(11.086)}{3.374 \times 10^{-9}}$$

$$= 4.321 \times 10^6 - 53.72 \times 10^6 = -49.45 \times 10^6 \text{ Pa} \qquad \sigma_B = -49.4 \text{ MPa}$$

(*c*) Neutral axis:



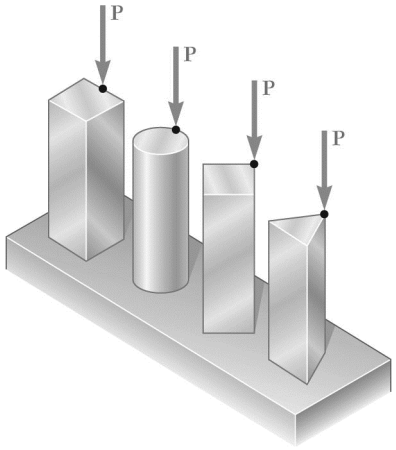
By proportions,

$$\frac{a}{47.55} = \frac{20}{47.55 + 49.45}$$

$$a = 9.80 \text{ mm}$$

9.80 mm below top of section

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.120

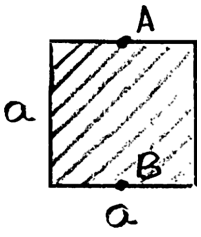
The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note:* the cross section of the triangular bar is an equilateral triangle.)

SOLUTION

Stresses:

At A,
$$\sigma_A = -\frac{P}{A} - \frac{Pec_A}{I} = -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

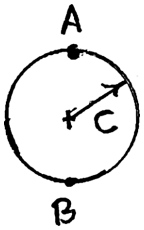
At B,
$$\sigma_B = -\frac{P}{A} + \frac{Pec_B}{I} = \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad c_A = c_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$$

$$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2}\right) \qquad \sigma_A = -4\frac{P}{A_1} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A}\left(\frac{(a^2)\left(\frac{1}{2}a\right)\left(\frac{1}{2}a\right)}{\frac{1}{12}a^2} - 1\right) \qquad \sigma_B = 2\frac{P}{A_1} \blacktriangleleft$$



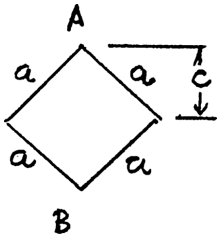
$$A_2 = \pi c^2 = a^2 \quad \therefore \quad c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4, \quad e = c$$

$$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) \qquad \sigma_A = -5\frac{P}{A_2} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) \qquad \sigma_B = 3\frac{P}{A_2} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.120 *(Continued)*



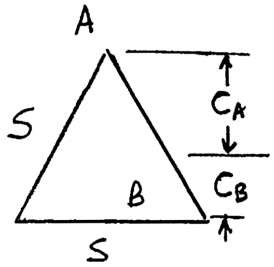
$$A_3 = a^2 \quad c = \frac{\sqrt{2}}{2}a \quad I_3 = \frac{1}{12}a^4 \quad e = c$$

$$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4}\right)$$

$$\sigma_A = -7\frac{P}{A_3} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)\left(\frac{\sqrt{2}}{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)}{\frac{1}{12}a^4} - 1\right)$$

$$\sigma_B = 5\frac{P}{A_3} \blacktriangleleft$$



$$A_4 = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{4}s^2$$

$$I_4 = \frac{1}{36}s\left(\frac{\sqrt{3}}{2}s\right)^3 = \frac{\sqrt{3}}{96}s^4$$

$$c_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad c_B = s$$

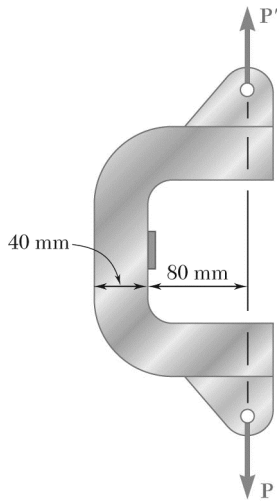
$$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4}\right)$$

$$\sigma_A = -9\frac{P}{A_4} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A_4}\left(\frac{\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s}{\sqrt{3}}\right)\left(\frac{s}{2\sqrt{3}}\right)}{\frac{\sqrt{3}}{96}s^4} - 1\right)$$

$$\sigma_B = 3\frac{P}{A_4} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.121

The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be 450 μ, determine the magnitude P of the forces. Use $E = 200$ GPa.

SOLUTION

At the strain gage location,

$$\sigma = E\varepsilon = (200 \times 10^9)(450 \times 10^{-6}) = 90 \times 10^6 \text{ Pa}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213.33 \times 10^3 \text{ mm}^4 = 213.33 \times 10^{-9} \text{ m}^4$$

$$e = 80 + 20 = 100 \text{ mm} = 0.100 \text{ m}$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

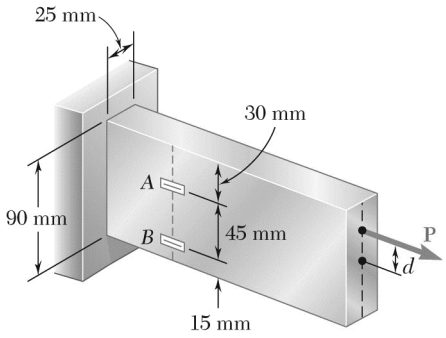
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600 \times 10^{-6}} + \frac{(0.100)(0.020)}{213.33 \times 10^{-9}} = 10.00 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90 \times 10^6}{10.00 \times 10^3} = 9.00 \times 10^3 \text{ N}$$

$P = 9.00$ kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.122

An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at A and B have been measured and found to be

$$\varepsilon_A = +350\,\mu \qquad \varepsilon_B = -70\,\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh - (25)(90) = 2.25\times10^{3}\,\text{mm}^2 = 2.25\times10^{-3}\,\text{m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^{6}\,\text{mm}^4 = 1.51875\times10^{-6}\,\text{m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B: $\sigma_A = E\varepsilon_A = (200\times10^{9})(350\times10^{-6}) = 70\times10^{6}\,\text{Pa}$

$$\sigma_B = E\varepsilon_B = (200\times10^{9})(-70\times10^{-6}) = -14\times10^{6}\,\text{Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(84\times10^{6})}{0.045} = -2835 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

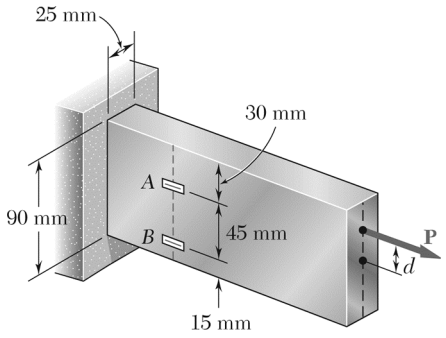
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(-14\times10^{6}) - (-0.030)(70\times10^{6})]}{0.045} = 94.5\times10^{3}\text{ N}$$

(*a*) $M = -Pd \quad \therefore \quad d = -\dfrac{M}{P} = -\dfrac{-2835}{94.5\times10^{3}} = 0.030 \text{ m}$ $\qquad d = 30.0 \text{ mm}$ ◀

(*b*) $\qquad P = 94.5 \text{ kN}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.123



Solve Prob. 4.122, assuming that the measured strains are

$$\varepsilon_A = +600\,\mu \qquad \varepsilon_B = +420\,\mu$$

PROBLEM 4.122 An eccentric force **P** is applied as shown to a steel bar of 25×90-mm cross section. The strains at *A* and *B* have been measured and found to be

$$\varepsilon_A = +350\mu \qquad \varepsilon_B = -70\mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance *d*, (*b*) the magnitude of the force **P**.

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm} \qquad b = 25 \text{ mm} \qquad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25\times10^{3}\,\text{mm}^2 = 2.25\times10^{-3}\,\text{m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^{6}\,\text{mm}^4 = 1.51875\times10^{-6}\,\text{m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m} \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at *A* and *B*: $\sigma_A = E\varepsilon_A = (200\times10^{9})(600\times10^{-6}) = 120\times10^{6}\,\text{Pa}$

$$\sigma_B = E\varepsilon_B = (200\times10^{9})(420\times10^{-6}) = 84\times10^{6}\,\text{Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting, $\sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(36\times10^{6})}{0.045} = -1215 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting, $y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\dfrac{P}{A}$

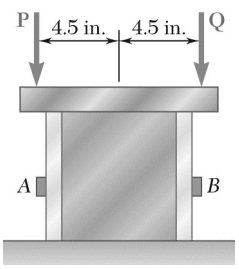
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(84\times10^{6}) - (-0.030)(120\times10^{6})]}{0.045} = 243\times10^{3}\,\text{N}$$

$$M = -Pd$$

(*a*) $\therefore \quad d = -\dfrac{M}{P} = -\dfrac{-1215}{243\times10^{3}} = 5\times10^{-3}\,\text{m}$ $\qquad d = 5.00$ mm ◀

(*b*) $\qquad P = 243$ kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.124

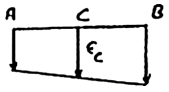
A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

Strains:

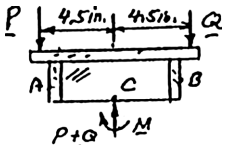


$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_C = \frac{1}{2}(\varepsilon_A + \varepsilon_B) = \frac{1}{2}(-550 - 680)10^{-6} = -615 \times 10^{-6} \text{ in./in.}$$

Stresses: $\sigma_A = E\varepsilon_A = (29 \times 10^6 \text{ psi})(-550 \times 10^{-6} \text{ in./in.}) = -15.95 \text{ ksi}$

$\sigma_C = E\varepsilon_C = (29 \times 10^6 \text{ psi})(-615 \times 10^{-6} \text{ in./in.}) = -17.935 \text{ ksi}$



W8 × 31 $\qquad A = 9.13 \text{ in}^2$

$S = 27.5 \text{ in}^3$

$M = (4.5 \text{ in.})(P - Q)$

At point *C*: $\sigma_C = -\dfrac{P + Q}{A}; \quad -17.835 \text{ ksi} = -\dfrac{P + Q}{9.13 \text{ in}^2}$

$$P + Q = 162.83 \text{ kips} \quad (1)$$

At point *A*: $\sigma_A = -\dfrac{P + Q}{A} - \dfrac{M}{S}$

$$-15.95 \text{ ksi} = -17.835 \text{ ksi} - \frac{(4.5 \text{ in.})(P - Q)}{27.5 \text{ in}^3}; \qquad P - Q = -11.52 \text{ kips} \quad (2)$$

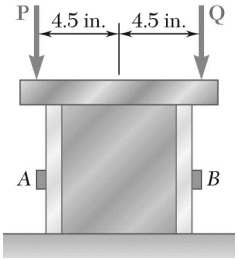
Solve simultaneously, $P = 25.7$ kips $\quad Q = 87.2$ kips

$$\boldsymbol{P} = 25.7 \text{ kips} \downarrow \blacktriangleleft$$

$$\boldsymbol{Q} = 87.2 \text{ kips} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.125



Solve Prob. 4.124, assuming that the measured strains are

$$\varepsilon_A = +35 \times 10^{-6} \text{ in./in.} \quad \varepsilon_B = -450 \times 10^{-6} \text{ in./in.}$$

PROBLEM 4.124 A short length of a W8 × 31 rolled-steel shape supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the centerline of the outer faces of the flanges have been measured and found to be

$$\varepsilon_A = -550 \times 10^{-6} \text{ in./in.} \qquad \varepsilon_B = -680 \times 10^{-6} \text{ in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

See solution and figures of Prob. 4.124.

$$\varepsilon_A = +35 \times 10^{-6} \text{ in./in.}; \qquad \varepsilon_B = -450 \times 10^{-6} \text{ in./in.}$$

$$\varepsilon_C = \frac{1}{2}(\varepsilon_A + \varepsilon_B) = \frac{1}{2}(35 - 450)10^{-6} \text{ in./in.} = -207.5 \times 10^{-6} \text{ in./in.}$$

Stresses: $\sigma_A = E\varepsilon_A = (29 \times 10^6 \text{ psi})(+35 \times 10^{-6} \text{ in./in.}) = +1.015 \text{ ksi}$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6 \text{ psi})(-207.5 \times 10^{-6} \text{ in./in.}) = -6.0175 \text{ ksi}$$

At point *C*: $\sigma_C = -\dfrac{P+Q}{A}; \quad -6.0175 \text{ ksi} = -\dfrac{P+Q}{9.13 \text{ in}^2}$

$$P + Q = 54.94 \text{ kips} \quad (1)$$

At point *A*: $\sigma_A = -\dfrac{P+Q}{A} - \dfrac{M}{S}$

$$+1.015 \text{ ksi} = -6.0175 - \frac{(4.5 \text{ in.})(P - Q)}{27.5 \text{ in}^3}$$

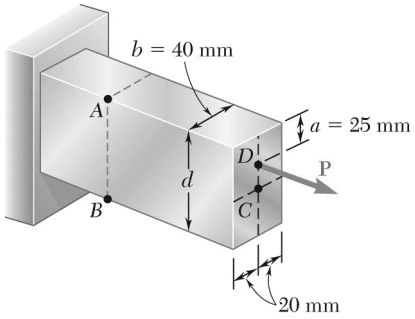
$$P - Q = -42.98 \text{ kips} \quad (2)$$

Solve simultaneously, $P = 5.98$ kips $Q = 49.0$ kips

$$\boldsymbol{P} = 5.98 \text{ kips} \downarrow \blacktriangleleft$$

$$\boldsymbol{Q} = 49.0 \text{ kips} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.126

The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

SOLUTION

$$A = bd \qquad I = \frac{1}{12}bd^3$$

$$c = \frac{1}{2}d \qquad e = \frac{1}{2}d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

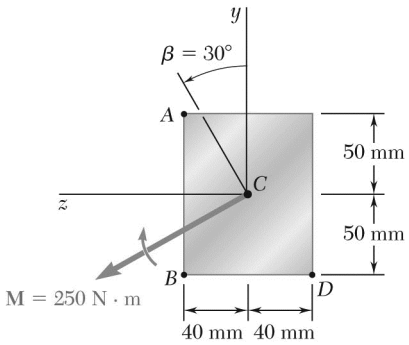
$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(*a*) Depth *d* for maximum σ_A: Differentiate with respect to *d*.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a \qquad d = 75 \text{ mm} \blacktriangleleft$$

(*b*) $$\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} \qquad \sigma_A = 40 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.127

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(80)(100)^3 = 6.6667\times10^6\,\text{mm}^4 = 6.6667\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(100)(80)^3 = 4.2667\times10^6\,\text{mm}^4 = 4.2667\times10^{-6}\,\text{m}^4$$

$$y_A = -y_B = -y_D = 50\ \text{mm}$$

$$z_A = z_B = -z_D = 40\ \text{mm}$$

$$M_y = -250\sin 30^\circ = -125\ \text{N}\cdot\text{m}$$

$$M_z = 250\cos 30^\circ = 216.51\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(0.050)}{6.6667\times10^{-6}} + \frac{(-125)(0.040)}{4.2667\times10^{-6}}$$

$$= -2.80\times10^6\,\text{Pa} \qquad \sigma_A = -2.80\ \text{MPa} \blacktriangleleft$$

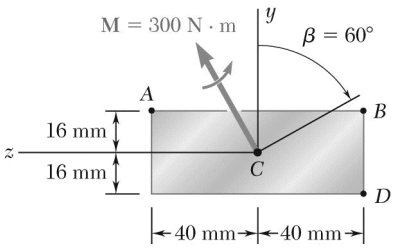
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(-0.050)}{6.6667\times10^{-6}} + \frac{(-125)(0.040)}{4.2667\times10^{-6}}$$

$$= 0.452\times10^3\,\text{Pa} \qquad \sigma_B = 0.452\ \text{MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-0.050)}{6.6667\times10^{-6}} + \frac{(-125)(-0.040)}{4.2667\times10^{-6}}$$

$$= 2.80\times10^6\,\text{Pa} \qquad \sigma_D = 2.80\ \text{MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.128

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times10^3 \text{ mm}^4 = 218.45\times10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times10^6 \text{ mm}^4 = 1.36533\times10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 16 \text{ mm}$$

$$z_A = -z_B = -z_D = 40 \text{ mm}$$

$$M_y = 300\cos 30° = 259.81 \text{ N}\cdot\text{m} \qquad M_z = 300\sin 30° = 150 \text{ N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -3.37\times10^6 \text{ Pa} \qquad \sigma_A = -3.37 \text{ MPa} \blacktriangleleft$$

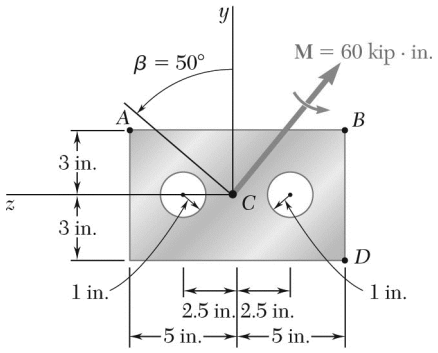
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -18.60\times10^6 \text{ Pa} \qquad \sigma_B = -18.60 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= 3.37\times10^6 \text{ Pa} \qquad \sigma_D = 3.37 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.129

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$M_z = -60 \sin 40^\circ = -38.567 \text{ kip} \cdot \text{in}$$
$$M_y = 60 \cos 40^\circ = 45.963 \text{ kip} \cdot \text{in}$$
$$y_A = y_B = -y_D = 3 \text{ in.}$$
$$z_A = -z_B = -z_D = 5 \text{ in.}$$
$$I_z = \frac{1}{12}(10)(6)^3 - 2\left[\frac{\pi}{4}(1)^2\right] = 178.429 \text{ in}^4$$
$$I_y = \frac{1}{12}(6)(10)^3 - 2\left[\frac{\pi}{4}(1)^4 + \pi(1)^2(2.5)^2\right] = 459.16 \text{ in}^4$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(5)}{459.16}$$

$$= 1.149 \text{ ksi} \qquad \sigma_A = 1.149 \text{ ksi} \blacktriangleleft$$

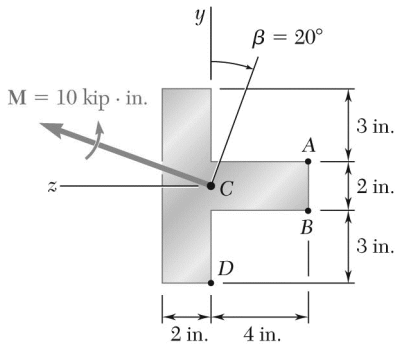
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-38.567)(3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= 0.1479 \qquad \sigma_B = 0.1479 \text{ ksi} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-38.567)(-3)}{178.429} + \frac{(45.963)(-5)}{459.16}$$

$$= -1.149 \text{ ksi} \qquad \sigma_D = -1.149 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

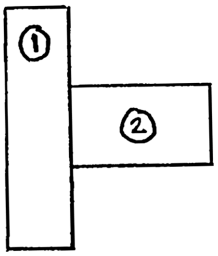


PROBLEM 4.130

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

Locate centroid.



	A, in^2	$\bar{z}$, in.	$A\bar{z}$, in^3
①	16	−1	−16
②	8	2	16
Σ	24		0

The centroid lies at point *C*.

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in.}, \qquad y_D = -4 \text{ in.}$$

$$z_A = z_B = -4 \text{ in.}, \qquad z_D = 0$$

$$M_z = 10 \cos 20° = 9.3969 \text{ kip} \cdot \text{in}$$

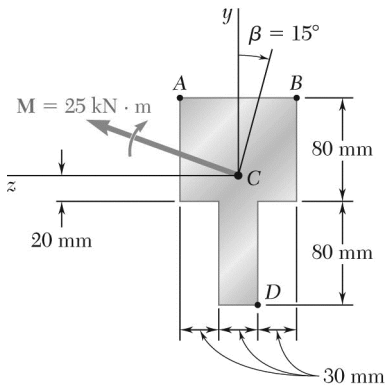
$$M_y = 10 \sin 20° = 3.4202 \text{ kip} \cdot \text{in}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(-4)}{64}$$ $\sigma_A = 0.321$ ksi ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64}$$ $\sigma_B = -0.107$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64}$$ $\sigma_D = 0.427$ ksi ◀

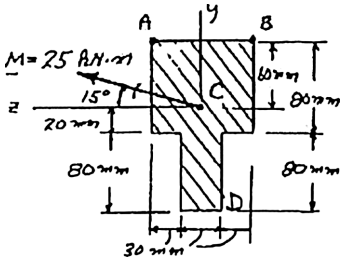
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.131

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION



$$M_y = 25\sin 15° = 6.4705 \text{ kN}\cdot\text{m}$$

$$M_z = 25\cos 15° = 24.148 \text{ kN}\cdot\text{m}$$

$$I_y = \frac{1}{12}(80)(90)^3 + \frac{1}{12}(80)(30)^3 = 5.04\times10^6 \text{ mm}^4$$

$$I_y = 5.04\times10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{3}(90)(60)^3 + \frac{1}{3}(60)(20)^3 + \frac{1}{3}(30)(100)^3 = 16.64\times10^6 \text{ mm}^4 = 16.64\times10^{-6} \text{ m}^4$$

Stress: $\sigma = \dfrac{M_y z}{I_y} - \dfrac{M_z y}{I_z}$

(*a*) $$\sigma_A = \frac{(6.4705 \text{ kN}\cdot\text{m})(0.045 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= 57.772 \text{ MPa} - 87.072 \text{ MPa}$$

$$\sigma_A = -29.3 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_B = \frac{(6.4705 \text{ kN}\cdot\text{m})(-0.045 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(0.060 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= -57.772 \text{ MPa} - 87.072 \text{ MPa}$$

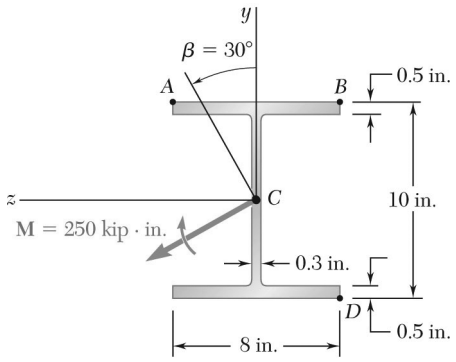
$$\sigma_B = -144.8 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = \frac{(6.4705 \text{ kN}\cdot\text{m})(-0.015 \text{ m})}{5.04\times10^{-6} \text{ m}^4} - \frac{(24.148 \text{ kN}\cdot\text{m})(-0.100 \text{ m})}{16.64\times10^{-6} \text{ m}^4}$$

$$= -19.257 \text{ MPa} + 145.12 \text{ MPa}$$

$$\sigma_D = -125.9 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.132

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

Flange:

$$I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2$$
$$= 90.333 \text{ in}^4$$
$$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$$

Web:

$$I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$$
$$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$$

Total:

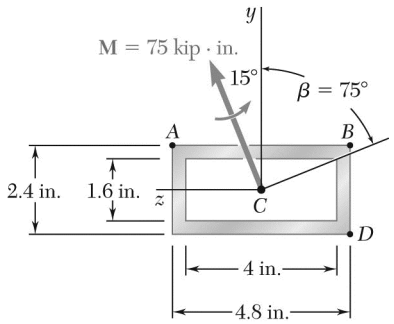
$$I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$$
$$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$$
$$y_A = y_B = -y_D = 5 \text{ in.}$$
$$z_A = -z_B = -z_C = 4 \text{ in.}$$
$$M_z = 250 \cos 30° = 216.51 \text{ kip} \cdot \text{in}$$
$$M_y = -250 \sin 30° = -125 \text{ kip} \cdot \text{in}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(4)}{42.687}$$ $\sigma_A = -17.16$ ksi ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(-4)}{42.687}$$ $\sigma_B = 6.27$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-5)}{198.89} + \frac{(-125)(-4)}{42.687}$$ $\sigma_D = 17.16$ ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.133

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(4.8)(2.4)^3 - \frac{1}{12}(4)(1.6)^3 = 4.1643 \text{ in}^4$$

$$I_y = \frac{1}{12}(2.4)(4.8)^3 - \frac{1}{12}(1.6)(4)^3 = 13.5851 \text{ in}^4$$

$$y_A = y_B = -y_D = 1.2 \text{ in.}$$

$$z_A = -z_B = -z_D = 2.4 \text{ in.}$$

$$M_z = 75\sin 15° = 19.4114 \text{ kip} \cdot \text{in}$$

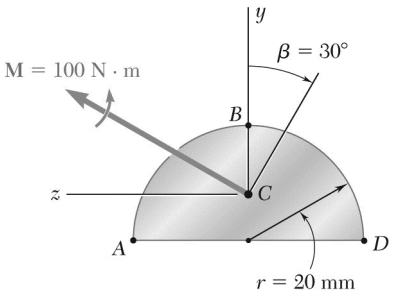
$$M_y = 75\cos 15° = 72.444 \text{ kip} \cdot \text{in}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(2.4)}{13.5851}$$ $\sigma_A = 7.20$ ksi ◀

(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(19.4114)(1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851}$$ $\sigma_B = -18.39$ ksi ◀

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(19.4114)(-1.2)}{4.1643} + \frac{(72.444)(-2.4)}{13.5851}$$ $\sigma_D = -7.20$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.134

The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(20)^4 = 17.5611\times10^{-3}\ \text{mm}^4 = 17.5611\times10^{-9}\ \text{m}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi(20)^4}{8} = 62.832\times10^{-3}\ \text{mm}^4 = 62.832\times10^{-9}\ \text{m}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(20)}{3\pi} = -8.4883\ \text{mm}$$

$$y_B = 20 - 8.4883 = 11.5117\ \text{mm}$$

$$z_A = -z_D = 20\ \text{mm} \qquad z_B = 0$$

$$M_z = 100\cos 30° = 86.603\ \text{N}\cdot\text{m}$$

$$M_y = 100\sin 30° = 50\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(20\times10^{-3})}{62.832\times10^{-9}}$$

$$= 57.8\times10^6\ \text{Pa} \qquad \sigma_A = 57.8\ \text{MPa} \blacktriangleleft$$

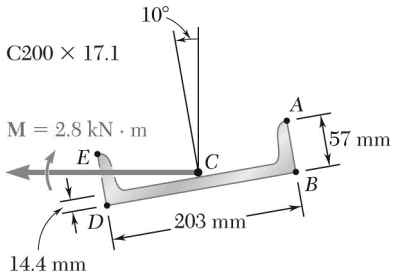
(*b*) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(86.603)(11.5117\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(0)}{62.832\times10^{-9}}$$

$$= -56.8\times10^6\ \text{Pa} \qquad \sigma_B = -56.8\ \text{MPa} \blacktriangleleft$$

(*c*) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(86.603)(-8.4883\times10^{-3})}{17.5611\times10^{-9}} + \frac{(50)(-20\times10^{3})}{62.832\times10^{-9}}$$

$$= 25.9\times10^6\ \text{Pa} \qquad \sigma_D = 25.9\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.135

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

For C200×17.1 rolled steel shape, $I_z = 0.538\times10^{6}\,\text{mm}^4 = 0.538\times10^{-6}\,\text{m}^4$

$I_y = 13.4\times10^{6}\,\text{mm}^4 = 13.4\times10^{-6}\,\text{m}^4$

$$z_E = z_D = -z_A = -z_B = \frac{1}{2}(203) = 101.5\ \text{mm}$$

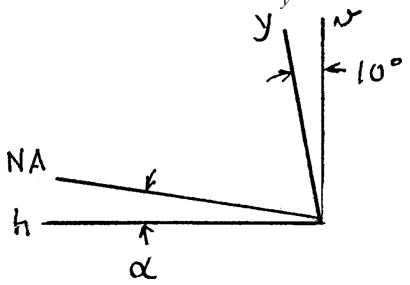
$$y_D = y_B = -14.4\ \text{mm} \qquad y_E = y_A = 57 - 14.4 = 42.6\ \text{mm}$$

$$M_z = (2.8\times10^{3})\cos10^\circ = 2.7575\times10^{3}\ \text{N}\cdot\text{m}$$

$$M_y = (2.8\times10^{3})\sin10^\circ = 486.21\ \text{N}\cdot\text{m}$$

(*a*) Angle of neutral axis.

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{0.538}{13.4}\tan10^\circ = 0.007079 \qquad \varphi = 0.4056^\circ$$



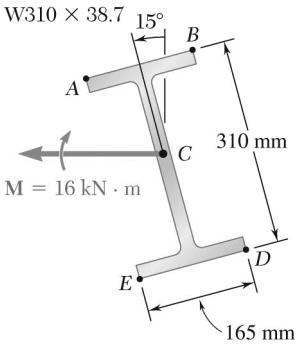
$$\alpha = 10^\circ - 0.4056^\circ \qquad \alpha = 9.59^\circ \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.7575\times10^{3})(-14.4\times10^{-3})}{0.538\times10^{-6}} + \frac{(486.21)(0.1015)}{13.4\times10^{-6}}$$

$$= 73.807\times10^{6} + 3.682\times10^{6} = 77.5\times10^{6}\ \text{Pa} \qquad \sigma_D = 77.5\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



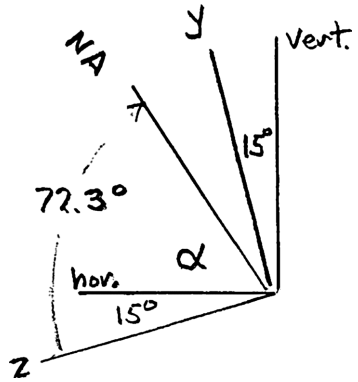
PROBLEM 4.136

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

For W310×38.7 rolled steel shape,

$$I_z = 85.1\times10^6\,\text{mm}^4 = 85.1\times10^{-6}\,\text{m}^4$$
$$I_y = 7.27\times10^6\,\text{mm}^4 = 7.27\times10^{-6}\,\text{m}^4$$
$$y_A = y_B = -y_D = -y_E = \left(\frac{1}{2}\right)(310) = 155\ \text{mm}$$
$$z_A = z_E = -z_B = -z_D = \left(\frac{1}{2}\right)(165) = 82.5\ \text{mm}$$
$$M_z = (16\times10^3)\cos 15^\circ = 15.455\times10^3\,\text{N}\cdot\text{m}$$
$$M_y = (16\times10^3)\sin 15^\circ = 4.1411\times10^3\,\text{N}\cdot\text{m}$$



(*a*) Angle of neutral axis.

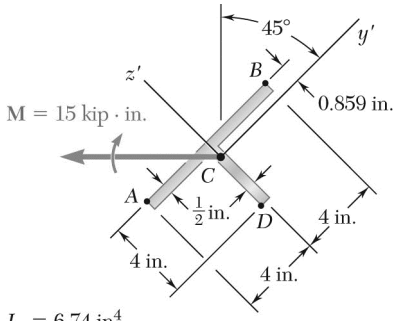
$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{85.1\times10^{-6}}{7.27\times10^{-6}}\tan 15^\circ = 3.1365$$

$$\varphi = 72.3^\circ$$
$$\alpha = 72.3^\circ - 15^\circ \qquad \alpha = 57.3^\circ \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point *E*.

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y}$$
$$= -\frac{(15.455\times10^3)(-155\times10^{-3})}{85.1\times10^{-6}} + \frac{(4.1411\times10^3)(82.5\times10^{-3})}{7.27\times10^{-6}}$$
$$= 75.1\times10^6\,\text{Pa} \qquad \sigma_E = 75.1\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.137

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 21.4 \text{ in}^4 \qquad I_{y'} = 6.74 \text{ in}^4$$

$$z'_A = z'_B = 0.859 \text{ in.} \qquad z'_D = -4 + 0.859 \text{ in.} = -3.141 \text{ in.}$$

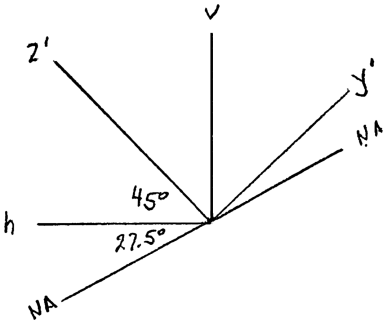
$$y_A = -4 \text{ in.} \qquad y_{B'} = 4 \text{ in.} \qquad y_{D'} = -0.25 \text{ in.}$$

$$M_{y'} = -15 \sin 45° = -10.6066 \text{ kip} \cdot \text{in}$$

$$M_{z'} = 15 \cos 45° = 10.6066 \text{ kip} \cdot \text{in}$$

(*a*) Angle of neutral axis.

$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{21.4}{6.74} \tan(-45°) = 3.1751$$



$$\varphi = -72.5°$$

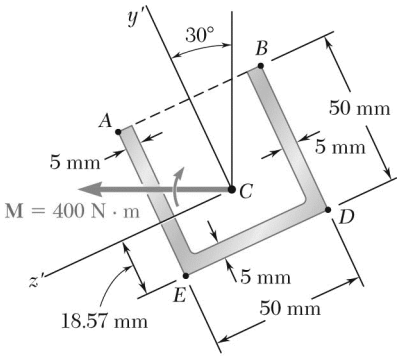
$$\alpha = 72.5° - 45° \qquad \alpha = 27.5° \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *D*.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(10.6066)(-0.25)}{21.4} + \frac{(-10.6066)(-3.141)}{6.74}$$

$$= 0.12391 + 4.9429 \qquad \sigma_D = 5.07 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$I_{y'} = 281 \times 10^3 \text{ mm}^4$
$I_{z'} = 176.9 \times 10^3 \text{ mm}^4$

PROBLEM 4.138

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 176.9 \times 10^3 \text{ mm}^4 = 176.9 \times 10^{-9} \text{ m}^4$$
$$I_{y'} = 281 \times 10^3 \text{ mm}^4 = 281 \times 10^{-9} \text{ m}^4$$
$$y'_E = -18.57 \text{ mm}, \quad z_E = 25 \text{ mm}$$
$$M_{z'} = 400 \cos 30° = 346.41 \text{ N} \cdot \text{m}$$
$$M_{y'} = 400 \sin 30° = 200 \text{ N} \cdot \text{m}$$

(*a*) $$\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$$

$$\varphi = 19.97°$$

$$\alpha = 30° - 19.97° \qquad \alpha = 10.03° \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point *E*.

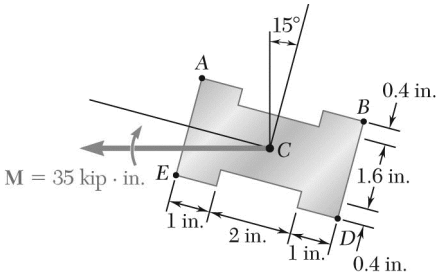
$$\sigma_E = -\frac{M_{z'} y'_E}{I_{z'}} + \frac{M_{y'} z'_E}{I_{y'}} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6 \text{ Pa}$$

$$\sigma_E = 54.2 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

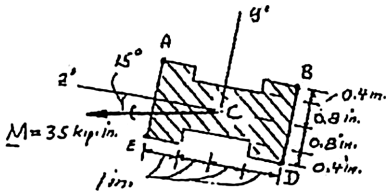
PROBLEM 4.139



The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

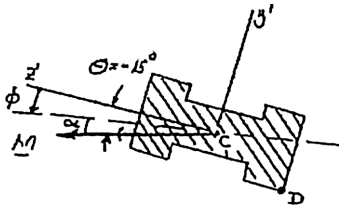
Bending moments:



$$M_{y'} = -35\sin 15° = -9.059 \text{ kip} \cdot \text{in}$$

$$M_{z'} = -35\cos 15° = 33.807 \text{ kip} \cdot \text{in}$$

Moments of inertia:



$$I_{y'} = \frac{1}{12}(2.4)(4)^3 - \frac{1}{12}(2 \times 0.4)(2)^3 = 12.267 \text{ in}^4$$

$$I_{z'} = \frac{1}{12}(2)(2.4)^3 + \frac{1}{12}(2)(1.6)^3 = 2.987 \text{ in}^4$$

(*a*) Neutral axis:

$$\tan\phi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{2.987 \text{ in}^4}{12.267 \text{ in}^4}\tan(-15°) = -0.06525$$

$$\phi = 3.73°$$

$$\alpha = 15° - \phi = 15° - 3.73° = 11.27°$$

$$\alpha = 11.3° \text{ ◀}$$

(*b*) Maximum tensile stress at *D*:

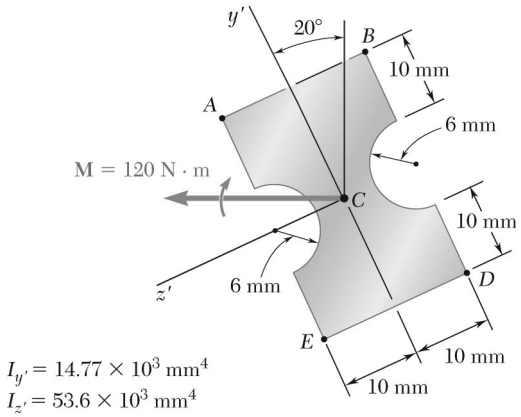
$$y'_D = -1.2 \text{ in.} \qquad z'_D = -2 \text{ in.}$$

$$\sigma_D = \frac{M_{y'}z_D}{I_{y'}} - \frac{M_{z'}y'_D}{I_{z'}} = \frac{(-9.059 \text{ kip} \cdot \text{in})(-2 \text{ in.})}{12.267 \text{ in}^4} - \frac{(33.807 \text{ kip} \cdot \text{in})(-1.2 \text{ in.})}{2.987 \text{ in}^4}$$

$$= 1.477 \text{ ksi} + 13.582 \text{ ksi} = 15.059 \text{ ksi}$$

$$\sigma_D = 15.06 \text{ ksi} \text{ ◀}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$I_{y'} = 14.77 \times 10^3\ \text{mm}^4$
$I_{z'} = 53.6 \times 10^3\ \text{mm}^4$

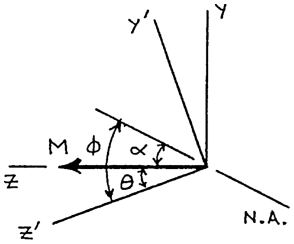
PROBLEM 4.140

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_{z'} = 53.6\times10^3\,\text{mm}^4 = 53.6\times10^{-9}\,\text{m}^4$$
$$I_{y'} = 14.77\times10^3\,\text{mm}^4 = 14.77\times10^{-9}\,\text{m}^4$$
$$M_{z'} = 120\sin 70^\circ = 112.763\ \text{N}\cdot\text{m}$$
$$M_{y'} = 120\cos 70^\circ = 41.042\ \text{N}\cdot\text{m}$$

(*a*) Angle of neutral axis. $\theta = 20^\circ$.

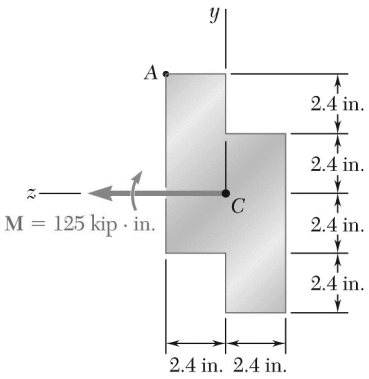


$$\tan\varphi = \frac{I_{z'}}{I_{y'}} \quad \tan\theta = \frac{53.6\times10^{-9}}{14.77\times10^{-9}}\tan 20^\circ = 1.32084$$
$$\varphi = 52.871^\circ$$
$$\alpha = 52.871^\circ - 20^\circ \qquad \alpha = 32.9^\circ \blacktriangleleft$$

(*b*) The maximum tensile stress occurs at point *E*.

$$y'_E = -16\ \text{mm} = -0.016\ \text{m}$$
$$z_E = 10\ \text{mm} = 0.010\ \text{m}$$
$$\sigma_E = -\frac{M_{z'}y'_E}{I_{z'}} + \frac{M_{y'}z'_E}{I_{y'}}$$
$$= -\frac{(112.763)(-0.016)}{53.6\times10^{-9}} + \frac{(41.042)(0.010)}{14.77\times10^{-9}}$$
$$= 61.448\times10^6\,\text{Pa} \qquad \sigma_E = 61.4\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.141

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

SOLUTION

$$I_y = 2\left\{\frac{1}{3}(7.2)(2.4)^3\right\} = 66.355 \text{ in}^4$$

$$I_z = 2\left\{\frac{1}{12}(2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2\right\} = 199.066 \text{ in}^4$$

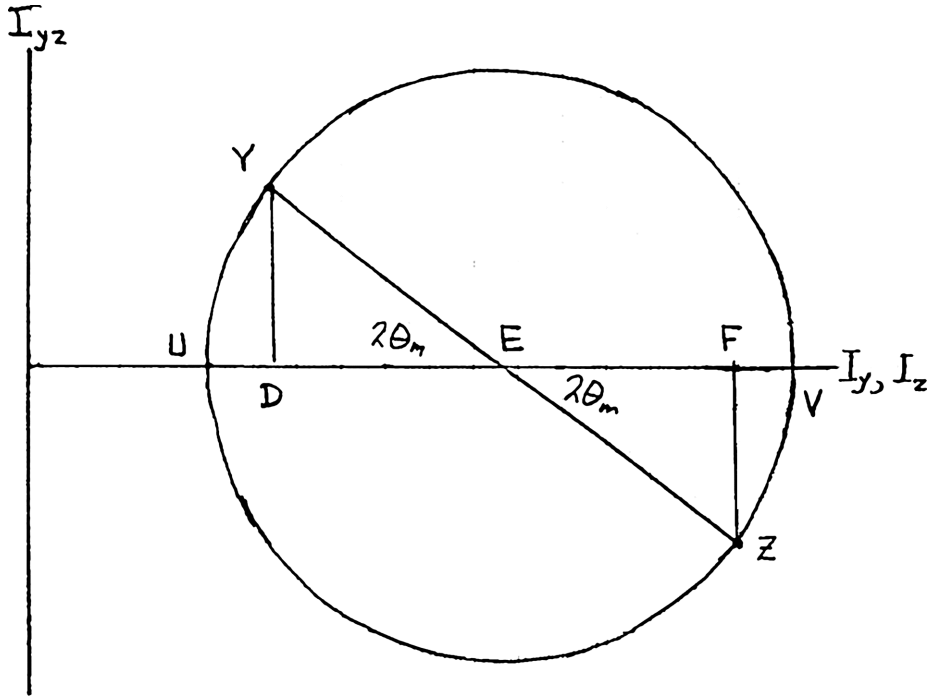
$$I_{yz} = 2\{(2.4)(7.2)(1.2)(1.2)\} = 49.766 \text{ in}^4$$

Using Mohr's circle determine the principal axes and principal moments of inertia.

$$Y: (66.355 \text{ in}^4,\ 49.766 \text{ in}^4)$$

$$Z: (199.066 \text{ in}^4,\ -49.766 \text{ in}^4)$$

$$E: (132.710 \text{ in}^4,\ 0)$$

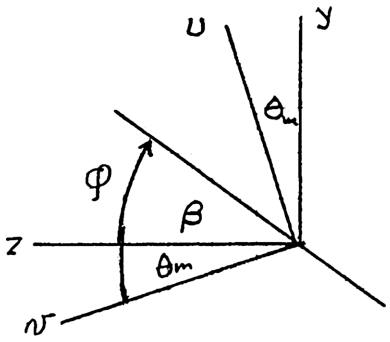


PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.141 *(Continued)*

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{49.766}{66.355}$$

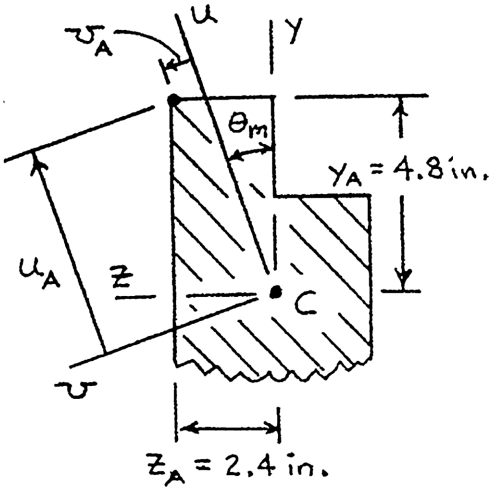
$$2\theta_m = 36.87^\circ \quad \theta_m = 18.435^\circ$$



$$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 82.944 \text{ in}^4$$

$$I_u = 132.710 - 82.944 = 49.766 \text{ in}^4$$

$$I_v = 132.710 + 82.944 = 215.654 \text{ in}^4$$



$$M_u = 125 \sin 18.435^\circ = 39.529 \text{ kip} \cdot \text{in}$$

$$M_v = 125 \cos 18.435^\circ = 118.585 \text{ kip} \cdot \text{in}$$

$$u_A = 4.8 \cos 18.435^\circ + 2.4 \sin 18.435^\circ = 5.3126 \text{ in.}$$

$$v_A = -4.8 \sin 18.435^\circ + 2.4 \cos 18.435^\circ = 0.7589 \text{ in.}$$

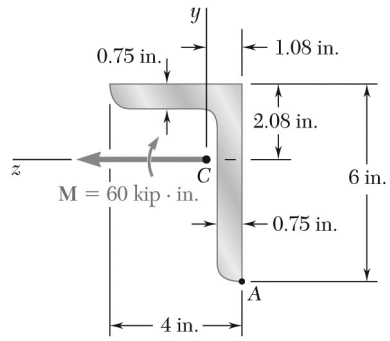
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$

$$= -\frac{(118.585)(5.3126)}{215.654} + \frac{(39.529)(0.7589)}{49.766}$$

$$= -2.32 \text{ ksi}$$

$$\sigma_A = -2.32 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$I_y = 8.7 \text{ in}^4$
$I_z = 24.5 \text{ in}^4$
$I_{yz} = +8.3 \text{ in}^4$

PROBLEM 4.142

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.

SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.

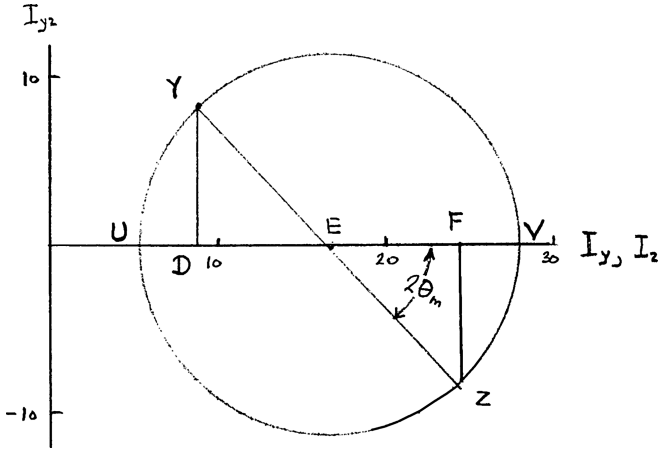
$Y: (8.7,\ 8.3) \text{ in}^4$

$Z: (24.5,\ -8.3) \text{ in}^4$

$E: (16.6,\ 0) \text{ in}^4$

$EF = 7.9 \text{ in}^4$

$FZ = 8.3 \text{ in}^4$



$$R = \sqrt{7.9^2 + 8.3^2} = 11.46 \text{ in}^4 \qquad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$$

$$\theta_m = 23.2^\circ \quad I_u = 16.6 - 11.46 = 5.14 \text{ in}^4 \quad I_v = 16.6 + 11.46 = 28.06 \text{ in}^4$$

$$M_u = M \sin\theta_m = (60) \sin 23.2^\circ = 23.64 \text{ kip} \cdot \text{in}$$

$$M_v = M \cos\theta_m = (60) \cos 23.2^\circ = 55.15 \text{ kip} \cdot \text{in}$$

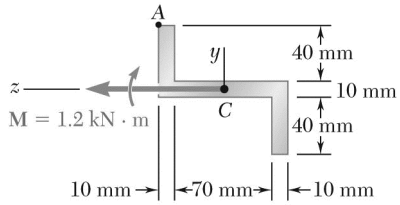
$$u_A = y_A \cos\theta_m + z_A \sin\theta_m = -3.92 \cos 23.2^\circ - 1.08 \sin 23.2^\circ = -4.03 \text{ in.}$$

$$v_A = z_A \cos\theta_m - y_A \sin\theta_m = -1.08 \cos 23.2^\circ + 3.92 \sin 23.2^\circ = 0.552 \text{ in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$$\sigma_A = 10.46 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$I_y = 1.894 \times 10^6\ \text{mm}^4$
$I_z = 0.614 \times 10^6\ \text{mm}^4$
$I_{yz} = +0.800 \times 10^6\ \text{mm}^4$

PROBLEM 4.143

The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point A.

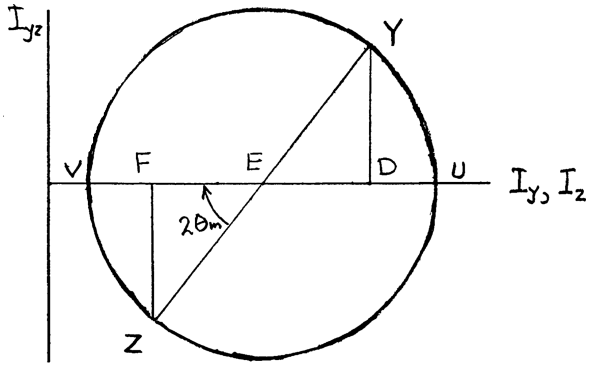
SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.

$Y: (1.894, 0.800)\times 10^6\,\text{mm}^4$

$Z: (0.614, 0.800)\times 10^6\,\text{mm}^4$

$E: (1.254, 0)\times 10^6\,\text{mm}^4$



$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2}\times 10^{-6} = 1.0245\times 10^6\,\text{mm}^4$$

$$I_v = (1.254 - 1.0245)\times 10^6\,\text{mm}^4 = 0.2295\times 10^6\,\text{mm}^4 = 0.2295\times 10^{-6}\,\text{m}^4$$

$$I_u = (1.254 + 1.0245)\times 10^6\,\text{mm}^4 = 2.2785\times 10^6\,\text{mm}^4 = 2.2785\times 10^{-6}\,\text{m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800\times 10^6}{0.640\times 10^6} = 1.25 \qquad \theta_m = 25.67^\circ$$

$$M_v = M\cos\theta_m = (1.2\times 10^3)\cos 25.67^\circ = 1.0816\times 10^3\,\text{N}\cdot\text{m}$$

$$M_u = -M\sin\theta_m = -(1.2\times 10^3)\sin 25.67^\circ = -0.5198\times 10^3\,\text{N}\cdot\text{m}$$

$$u_A = y_A\cos\theta_m - z_A\sin\theta_m = 45\cos 25.67^\circ - 45\sin 25.67^\circ = 21.07\ \text{mm}$$

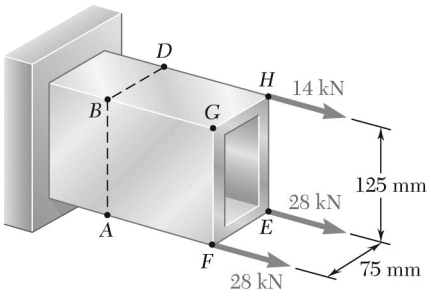
$$v_A = z_A\cos\theta_m + y_A\sin\theta_m = 45\cos 25.67^\circ + 45\sin 25.67^\circ = 60.05\ \text{mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816\times 10^3)(21.07\times 10^{-3})}{0.2295\times 10^{-6}} + \frac{(-0.5198\times 10^3)(60.05\times 10^{-3})}{2.2785\times 10^{-6}}$$

$$= 113.0\times 10^6\,\text{Pa}$$

$$\sigma_A = 113.0\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

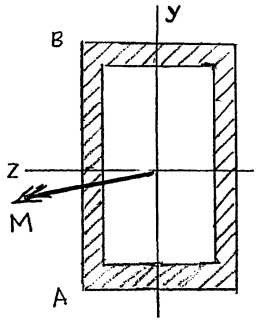


PROBLEM 4.144

The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

SOLUTION

Add *y*- and *z*-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^{6}\,\text{mm}^4 = 7.8283\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^{3}\,\text{mm}^4 = 3.2781\times10^{-6}\,\text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^{3}\,\text{mm}^2 = 4.224\times10^{-3}\,\text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 + 28 = 70\ \text{kN} = 70\times10^{3}\ \text{N}$$

$$M_z = -(62.5\ \text{mm})(14\ \text{kN}) + (62.5\ \text{mm})(28\text{kN}) + (62.5\ \text{mm})(28\ \text{kN}) = 2625\ \text{N}\cdot\text{m}$$

$$M_y = -(37.5\ \text{mm})(14\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) = -525\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{70\times10^{3}}{4.224\times10^{-3}} - \frac{(2625)(-0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 31.524\times10^{6}\,\text{Pa} \qquad \sigma_A = 31.5\ \text{MPa} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{70\times10^{3}}{4.224\times10^{-3}} - \frac{(2625)(0.0625)}{7.8283\times10^{-6}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}$$

$$= -10.39\times10^{6}\,\text{Pa} \qquad \sigma_B = -10.39\ \text{MPa} \blacktriangleleft$$

(*b*) Let point *H* be the point where the neutral axis intersects *AB*.

$$z_H = 0.0375\ \text{m}, \quad y_H = ?, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

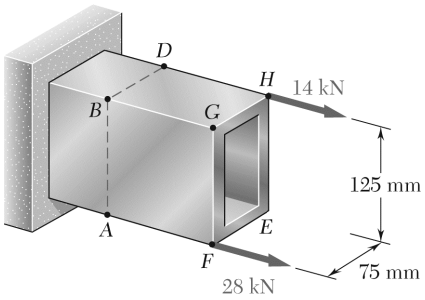
$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{Mz_H}{I_y}\right) = \frac{7.8283\times10^{-6}}{2625}\left[\frac{70\times10^{3}}{4.224\times10^{-3}} + \frac{(-525)(0.0375)}{3.2781\times10^{-6}}\right]$$

$$= 0.03151\ \text{m} = 31.51\ \text{mm}$$

$$31.51 + 62.5 = 94.0\ \text{mm}$$

Answer: 94.0 mm above point *A*. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



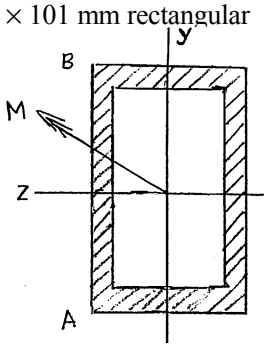
PROBLEM 4.145

Solve Prob. 4.144, assuming that the 28-kN force at point *E* is removed.

PROBLEM 4.144 The tube shown has a uniform wall thickness of 12 mm. For the loading given, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

SOLUTION

Add *y*- and *z*-axes as shown. Cross section is a 75 mm × 125 mm rectangle with a 51 mm × 101 mm rectangular cutout.



$$I_z = \frac{1}{12}(75)(125)^3 - \frac{1}{12}(51)(101)^3 = 7.8283\times10^{6}\,\text{mm}^4 = 7.8283\times10^{-6}\,\text{m}^4$$

$$I_y = \frac{1}{12}(125)(75)^3 - \frac{1}{12}(101)(51)^3 = 3.2781\times10^{6}\,\text{mm}^4 = 3.2781\times10^{-6}\,\text{m}^4$$

$$A = (75)(125) - (51)(101) = 4.224\times10^{3}\,\text{mm}^2 = 4.224\times10^{-3}\,\text{m}^2$$

Resultant force and bending couples:

$$P = 14 + 28 = 42\ \text{kN} = 42\times10^{3}\ \text{N}$$

$$M_z = -(62.5\ \text{mm})(14\ \text{kN}) + (62.5\ \text{mm})(28\ \text{kN}) = 875\ \text{N}\cdot\text{m}$$

$$M_y = -(37.5\ \text{mm})(14\ \text{kN}) + (37.5\ \text{mm})(28\ \text{kN}) = 525\ \text{N}\cdot\text{m}$$

(*a*) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{42\times10^{3}}{4.224\times10^{-3}} - \frac{(875)(-0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 22.935\times10^{6}\,\text{Pa} \qquad \sigma_A = 22.9\ \text{MPa} \blacktriangleleft$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{42\times10^{3}}{4.224\times10^{-3}} - \frac{(875)(0.0625)}{7.8283\times10^{-6}} + \frac{(525)(0.0375)}{3.2781\times10^{-6}}$$

$$= 8.9631\times10^{6}\,\text{Pa} \qquad \sigma_B = 8.96\ \text{MPa} \blacktriangleleft$$

(*b*) Let point *K* be the point where the neutral axis intersects *BD*.

$$z_K = ?, \quad y_K = 0.0625\ \text{m}, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

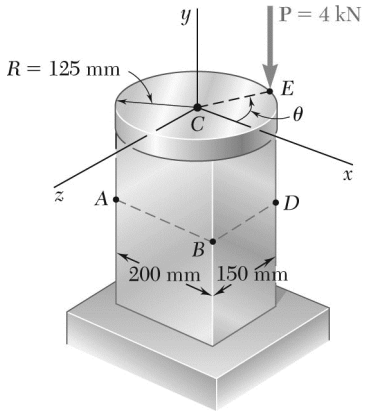
$$z_H = \frac{I_y}{M_y}\left(\frac{M_z y_H}{I_z} - \frac{P}{A}\right) = \frac{3.2781\times10^{-6}}{525}\left[\frac{(875)(0.0625)}{7.8283\times10^{-6}} - \frac{42\times10^{3}}{4.224\times10^{-3}}\right]$$

$$= -0.018465\ \text{m} = -18.465\ \text{mm}$$

$$37.5 + 18.465 = 56.0\ \text{mm}$$

Answer: 56.0 mm to the right of point *B*. ◀

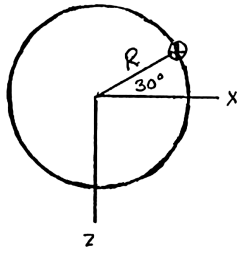
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.146

A rigid circular plate of 125-mm radius is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at E with $\theta = 30°$, determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the point where the neutral axis intersects line ABD.

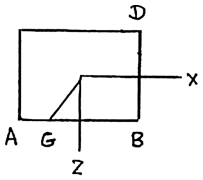
SOLUTION



$$P = 4 \times 10^3 \text{ N (compression)}$$

$$M_x = -PR\sin 30° = -(4 \times 10^3)(125 \times 10^{-3})\sin 30° = -250 \text{ N} \cdot \text{m}$$

$$M_z = -PR\cos 30° = -(4 \times 10^3)(125 \times 10^{-3})\cos 30° = -433 \text{ N} \cdot \text{m}$$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm} \qquad z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$\sigma_A = 633 \times 10^3 \text{ Pa} = 633 \text{ kPa} \blacktriangleleft$$

(*b*) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(100 \times 10^{-3})}{100 \times 10^{-6}}$$

$$\sigma_B = -233 \times 10^3 \text{ Pa} = -233 \text{ kPa} \blacktriangleleft$$

(*c*) Let G be the point on AB where the neutral axis intersects.

$$\sigma_G = 0 \qquad z_G = 75 \text{ mm} \qquad x_G = ?$$

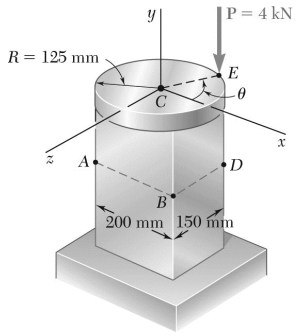
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x Z_G}{I_x}\right\} = \frac{100 \times 10^{-6}}{-433}\left\{\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}}\right\}$$

$$= 46.2 \times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



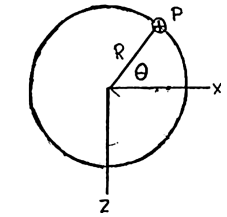
PROBLEM 4.147

4.147 In Prob. 4.146, determine (*a*) the value of θ for which the stress at *D* reaches it largest value, (*b*) the corresponding values of the stress, at *A*, *B*, *C*, and *D*.

4.146 A rigid circular plate of 125-mm radius is attached to a solid 150×200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with $\theta = 30°$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.

SOLUTION

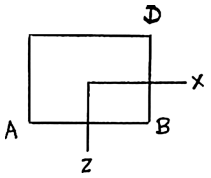
(*a*)



$$P = 4 \times 10^3 \text{ N} \qquad PR = (4 \times 10^3)(125 \times 10^{-3}) = 500 \text{ N} \cdot \text{m}$$

$$M_x = -PR\sin\theta = -500\sin\theta \qquad M_x = -PR\cos\theta = -500\cos\theta$$

$$I_x = \frac{1}{2}(200)(150)^3 = 56.25 \times 10^6 \text{mm}^4 = 56.25 \times 10^{-6} \text{m}^4$$



$$I_z = \frac{1}{2}(150)(200)^3 = 100 \times 10^6 \text{mm}^4 = 100 \times 10^{-6} \text{m}^4$$

$$x_D = 100 \text{ mm} \qquad z_D = -75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum, $\dfrac{d\sigma}{d\theta} = 0$ with $z = z_D, \quad x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 + \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_Z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100 \times 10^{-6})(-75 \times 10^{-3})}{(56.25 \times 10^{-6})(100 \times 10^{-3})} = \frac{4}{3}$$

$$\sin\theta = 0.8, \qquad \cos\theta = 0.6, \qquad \theta = 53.1° \blacktriangleleft$$

(*b*) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(500)(0.8)(75 \times 10^{-3})}{56.25 \times 10^{-6}} - \frac{(500)(0.6)(-100 \times 10^{-3})}{100 \times 10^{-6}}$$

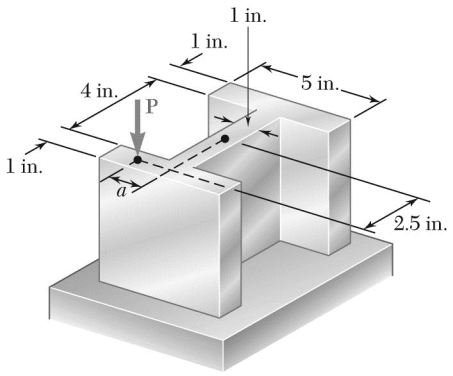
$$= (-0.13333 + 0.53333 + 0.300) \times 10^6 \text{ Pa} = 0.700 \times 10^6 \text{ Pa} \qquad \sigma_A = 700 \text{kPa} \blacktriangleleft$$

$$\sigma_B = (-0.13333 + 0.53333 - 0.300) \times 10^6 \text{ Pa} = 0.100 \times 10^6 \text{ Pa} \qquad \sigma_B = 100 \text{kPa} \blacktriangleleft$$

$$\sigma_C = (-0.13333 + 0 + 0) \times 10^6 \text{ Pa} \qquad \sigma_C = -133.3 \text{kPa} \blacktriangleleft$$

$$\sigma_D = (-0.13333 - 0.53333 - 0.300) \times 10^6 \text{ Pa} = -0.967 \times 10^6 \text{ Pa} \qquad \sigma_D = -967 \text{kPa} \blacktriangleleft$$

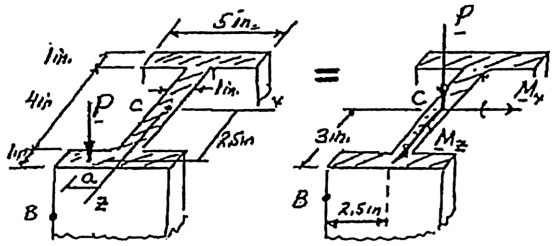
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.148

Knowing that $P = 90$ kips, determine the largest distance a for which the maximum compressive stress dose not exceed 18 ksi.

SOLUTION



$$A = (5\text{ in.})(6\text{ in.}) - 2(2\text{ in.})(4\text{ in.}) = 14\text{ in}^2$$

$$I_x = \frac{1}{12}(5\text{ in.})(6\text{ in.})^3 - 2\frac{1}{12}(2\text{ in.})(4\text{ in.})^3 = 68.67\text{ in}^4$$

$$I_z = 2\frac{1}{12}(1\text{ in.})(5\text{ in.})^3 + \frac{1}{12}(4\text{ in.})(1\text{ in.})^3 = 21.17\text{ in}^4$$

Force-couple system at C: $\quad P = P \quad M_x = P(2.5\text{ in.}) \quad M_z = Pa$

For $P = 90$ kips: $\quad P = 90\text{ kips} \quad M_x = (90\text{ kips})(2.5\text{ in.}) = 225\text{ kip}\cdot\text{in} \quad M_z = (90\,\text{kips})\,a$

Maximum compressive stress at B: $\quad \sigma_B = -18\text{ ksi}$

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3\text{ in.})}{I_x} - \frac{M_z(2.5\text{ in.})}{I_z}$$

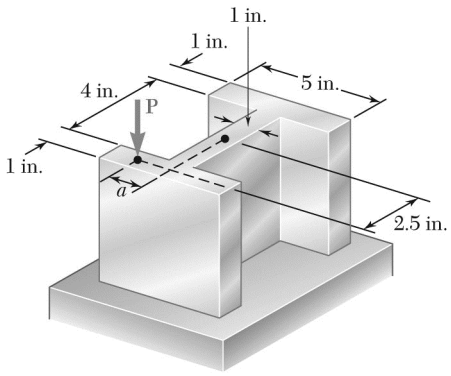
$$-18\text{ ksi} = -\frac{90\text{ kips}}{14\text{ in}^2} - \frac{(225\text{ kip}\cdot\text{in})(3\text{ in.})}{68.67\text{ in}^4} - \frac{(90\text{ kips})\,a\,(2.5\text{ in.})}{21.17\text{ in}^4}$$

$$-18 = -6.429 - 9.830 - 10.628a$$

$$-1.741 = -10.628\,a$$

$$a = 0.1638\text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.149

Knowing that $a = 1.25$ in., determine the largest value of **P** that can be applied without exceeding either of the following allowable stresses:

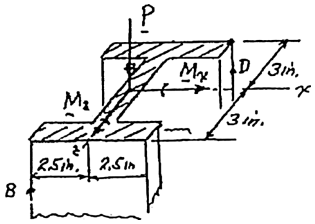
$$\sigma_{ten} = 10 \text{ ksi} \qquad \sigma_{comp} = 18 \text{ ksi}$$

SOLUTION

$$A = (5 \text{ in.})(6 \text{ in.}) - (2)(2 \text{ in.})(4 \text{ in.}) = 14 \text{ in}^2$$

$$I_x = \frac{1}{12}(5 \text{ in.})(6 \text{ in.})^3 - 2\frac{1}{12}(2 \text{ in.})(4 \text{ in.})^3 = 68.67 \text{ in}^4$$

$$I_z = 2\frac{1}{12}(1 \text{ in.})(5 \text{ in.})^3 + \frac{1}{12}(4 \text{ in.})(1 \text{ in.})^3 = 21.17 \text{ in}^4$$



Force-couple system at *C*: For $a = 1.25$ in.,

$$P = P \quad M_x = P(2.5 \text{ in.})$$

$$M_y = Pa = (1.25 \text{ in.})$$

Maximum compressive stress at *B*: $\sigma_B = -18$ ksi

$$\sigma_B = -\frac{P}{A} - \frac{M_x(3 \text{ in.})}{I_x} - \frac{M_z(2.5 \text{ in.})}{I_z}$$

$$-18 \text{ ksi} = -\frac{P}{14 \text{ in}^2} - \frac{P(2.5 \text{ in.})(3 \text{ in.})}{68.67 \text{ in}^4} - \frac{P(1.25 \text{ in.})(2.5 \text{ in.})}{21.17 \text{ in}^4}$$

$$-18 = -0.0714P - 0.1092P - 0.1476P$$

$$-18 = 0.3282P \qquad P = 54.8 \text{ kips}$$

Maximum tensile stress at *D*: $\sigma_D = +10$ ksi

$$\sigma_D = -\frac{P}{A} + \frac{M_x(3 \text{ in.})}{I_x} + \frac{M_z(2.5 \text{ in.})}{I_z}$$

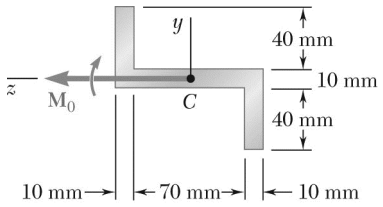
$$+10 \text{ ksi} = -0.0714P + 0.1092P + 0.1476P$$

$$10 = 0.1854P \qquad P = 53.9 \text{ kips}$$

The smaller value of **P** is the largest allowable value.

$P = 53.9$ kips ◀

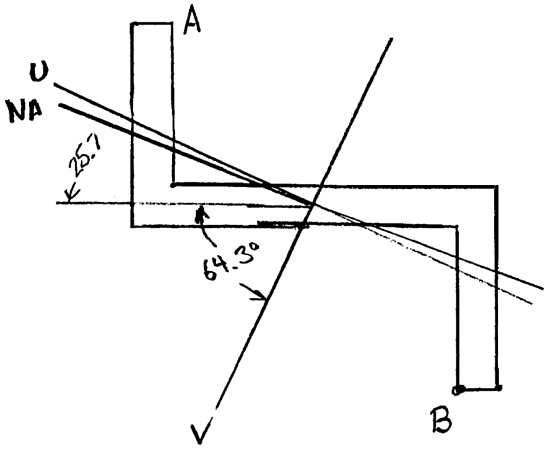
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.150

The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{\max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{\min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes $25.7°$ and $64.3°$.

SOLUTION



$$I_v = I_{\max} = 2.28 \times 10^{6}\ \text{mm}^4 = 2.28 \times 10^{-6}\ \text{m}^4$$

$$I_u = I_{\min} = 0.23 \times 10^{6}\ \text{mm}^4 = 0.23 \times 10^{-6}\ \text{m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3°$$

$$= 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05\ \text{mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37\ \text{mm}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

$$80 \times 10^{6} = -\frac{(M_0 \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

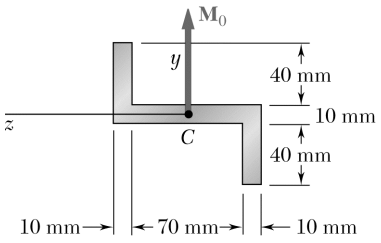
$$= 109.1 \times 10^{3} M_0$$

$$M_0 = \frac{80 \times 10^{6}}{109.1 \times 10^{3}}$$

$$M_0 = 733\ \text{N} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.151

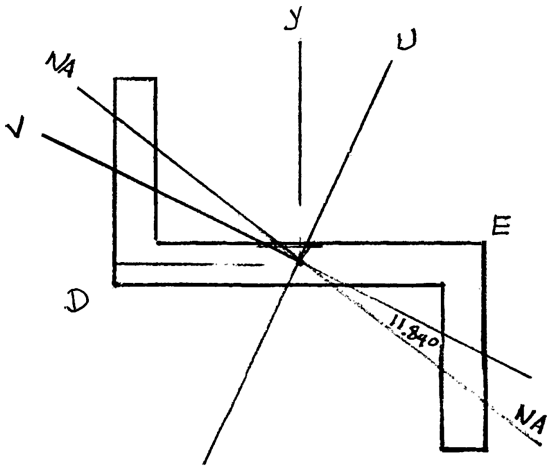


Solve Prob. 4.150 assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.150 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{\max} = 2.28 \times 10^{-6}\ \text{mm}^4$, $I_{\min} = 0.23 \times 10^{-6}\ \text{mm}^4$, principal axes $25.7°$ ↘ and $64.3°$ ↗.

SOLUTION

$$I_v = I_{\min} = 0.23 \times 10^6\ \text{mm}^4 = 0.23 \times 10^6\ \text{m}^4$$
$$I_u = I_{\max} = 2.28 \times 10^6\ \text{mm}^4 = 2.28 \times 10^6\ \text{m}^4$$
$$M_v = M_0 \cos 64.3°$$
$$M_u = M_0 \sin 64.3°$$
$$\theta = 64.3°$$
$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$
$$= \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3°$$
$$= 0.20961$$
$$\varphi = 11.84°$$

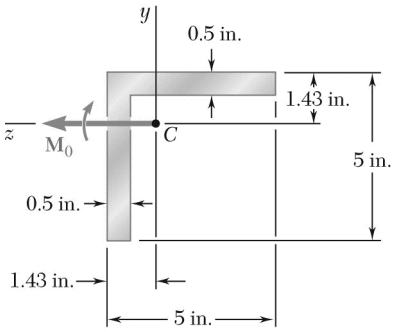


Points D and E are farthest from the neutral axis.

$$u_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45 \sin 25.7°$$
$$= -24.02\ \text{mm}$$
$$v_D = z_D \cos 25.7° + y_D \sin 25.7° = 45\cos 25.7° + (-5)\sin 25.7°$$
$$= 38.38\ \text{mm}$$
$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -\frac{(M_D \cos 64.3°)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$
$$80 \times 10^6 = 60.48 \times 10^3 M_0$$
$$M_0 = 1.323 \times 10^3\ \text{N} \cdot \text{m}$$

$$M_0 = 1.323\ \text{kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.152

A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3$ in.^{4}, $A = 4.75$ in.^{2}, $k_{\min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$)

SOLUTION

$$M_u = M_0 \sin 45° = 0.70711\, M_0$$

$$M_v = M_0 \cos 45° = 0.7071\, M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_B = y_B \cos 45° + z_B \sin 45° = -3.57 \cos 45° + 0.93 \sin 45° = -1.866 \text{ in.}$$

$$v_B = z_B \cos 45° - y_B \sin 45° = 0.93 \cos 45° - (-3.57) \sin 45° = 3.182 \text{ in.}$$

$$\sigma_B = -\frac{M_v u_B}{Iv} + \frac{M_u v_B}{Iu} = -0.70711\, M_0 \left[-\frac{u_B}{I_{\min}} + \frac{v_B}{I_{\max}} \right]$$

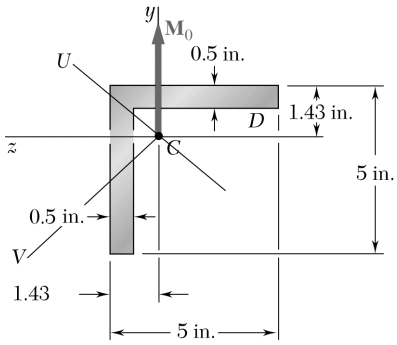
$$= 0.70711\, M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124}$$

$$M_0 = 29.1 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.153



Solve Prob. 4.152, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

PROBLEM 4.152 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ that acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3$ in.^{4}, $A = 4.75$ in.^{2}, $k_{\min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{\min} = Ak_{\min}^2$ and $I_{\min} + I_{\max} = I_y + I_z$)

SOLUTION

$$M_u = M_0 \cos 45° = 0.70711\, M_0$$

$$M_v = -M_0 \sin 45° = -0.70711\, M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_D = y_D \cos 45° + z_D \sin 45° = -0.93 \cos 45° + (-3.57 \sin 45°) = -1.866 \text{ in.}$$

$$v_D = z_D \cos 45° - y_D \sin 45° = (-3.57) \cos 45° - (0.93) \sin 45° = 3.182 \text{ in.}$$

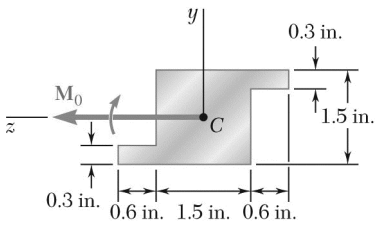
$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = -0.70711\, \mathrm{M}_0 \left[-\frac{u_D}{I_{\min}} + \frac{v_D}{I_{\max}} \right]$$

$$= 0.70711\, M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124}$$

$$M_0 = 29.1 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.154

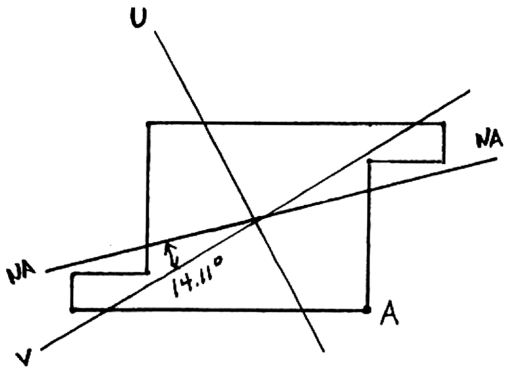
An extruded aluminum member having the cross section shown is subjected to a couple acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 12 ksi. *Given:* $I_{\max} = 0.957\ \text{in}^4$, $I_{\min} = 0.427\ \text{in}^4$, principal axes 29.4° and 60.6°.

SOLUTION

$$I_u = I_{\max} = 0.957\,\text{in}^4$$

$$I_v = I_{\min} = 0.427\,\text{in}^4$$

$$M_u = M_0 \sin 29.4°, \quad M_v = M_0 \cos 29.4°$$



$$\theta = 29.4°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.427}{0.957}\tan 29.4°$$

$$= 0.2514 \qquad \varphi = 14.11°$$

Point A is farthest from the neutral axis.

$$y_A = -0.75\ \text{in.}, \quad z_A = -0.75\ \text{in.}$$

$$u_A = y_A \cos 29.4° + z_A \sin 29.4° = -1.0216\ \text{in.}$$

$$v_A = z_A \cos 29.4° - y_A \sin 29.4° = -0.2852\ \text{in.}$$

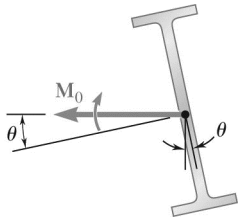
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(M_0 \cos 29.4°)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4°)(-0.2852)}{0.957}$$

$$= 1.9381 M_0$$

$$M_0 = \frac{\sigma_A}{1.9381} = \frac{12}{1.9381}$$

$$M_0 = 6.19\ \text{kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

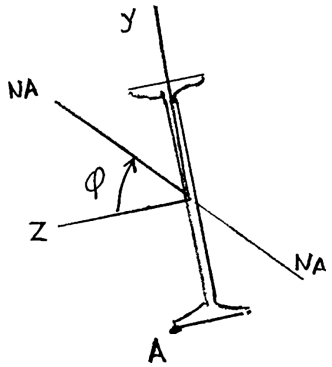


PROBLEM 4.155

A couple $\mathbf{M}_0$ acting in a vertical plane is applied to a $W12 \times 16$ rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.

SOLUTION

For $W12 \times 16$ rolled steel section,



$$I_z = 103 \text{ in}^4 \qquad I_y = 2.82 \text{ in}^4$$

$$d = 11.99 \text{ in.} \qquad b_f = 3.990 \text{ in.}$$

$$y_A = -\frac{d}{2} \qquad z_A = \frac{b_f}{2}$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{103}{2.82}\tan\theta = 36.52\tan\theta$$

Point A is farthest from the neutral axis.

$$M_y = M_0\sin\theta \qquad M_z = M_0\cos\theta$$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta = \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)\cos\theta$$

For $\theta = 0$, $\sigma_0 = \dfrac{M_0 d}{2I_z}$

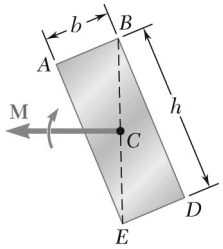
$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)\cos\theta = 2\sigma_0$$

$$\frac{I_2 b_f}{I_y d}\tan\theta = \frac{(103)(3.990)}{(2.82)(11.99)}\left(\frac{2}{\cos\theta} - 1\right)$$

$$\tan\theta = 0.082273\left(\frac{2}{\cos\theta} - 1\right)$$

Assuming $\cos\theta \approx 1$, we get $\theta = 4.70°$ ◀

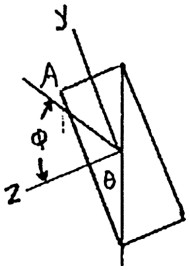
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.156

Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of a rectangular cross section, the neutral axis will lie along the other diagonal.

SOLUTION



$$\tan\theta = \frac{b}{h}$$

$$M_z = M\cos\theta, \quad M_z = M\sin\theta$$

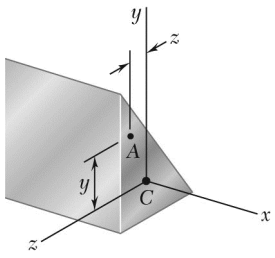
$$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

The neutral axis passes through corner A of the diagonal AD. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.157



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal plane *xz*. Show that the stress at point *A*, of coordinates *y* and *z*, is

$$\sigma_A = \frac{zI_z - yI_{yz}}{I_yI_z - I_{yz}^2}M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_y the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates *y* and *z*. Since the axial force is zero, the *y*- and *z*-axes are centroidal axes:

$$\sigma_A = C_1y + C_2z$$

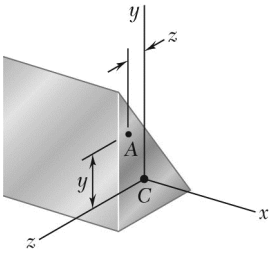
where C_1 and C_2 are constants.

$$\begin{aligned}
M_z &= -\int y\sigma_A dA = -C_1\int y^2 dA - C_2\int yz dA \\
&= -I_zC_1 - I_{yz}C_2 = 0 \\
C_1 &= -\frac{I_{yz}}{I_z}C_2 \\
M_y &= \int z\sigma_A dA = C_1\int yz\, dA + C_2\int z^2 dA \\
&= I_{yz}C_1 + I_yC_2 \\
&\quad - I_{yz}\frac{I_{yz}}{I_z}C_2 + I_yC_2 \\
I_zM_y &= \left(I_yI_z - I_{yz}^2\right)C_2 \\
C_2 &= \frac{I_zM_y}{I_yI_z - I_{yz}^2} \\
C_1 &= -\frac{I_{yz}M_y}{I_yI_z - I_{yz}^2} \\
\sigma_A &= \frac{I_zz - I_{yz}y}{I_yI_z - I_{yz}^2}M_y
\end{aligned}$$

◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.158



A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical plane *xy*. Show that the stress at point *A*, of coordinates *y* and *z*, is

$$\sigma_A = \frac{yI_y - zI_{yz}}{I_yI_z - I_{yz}^2}M_z$$

where $I_y, I_z,$ and I_{yz} denote the moments and product of inertia of the cross section with respect to the coordinate axes, and M_z the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates *y* and *z*. Since the axial force is zero, the *y*- and *z*-axes are centroidal axes:

$$\sigma_A = C_1y + C_2z$$

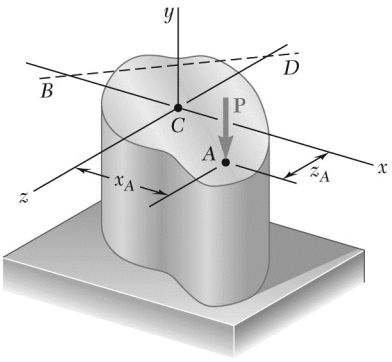
where C_1 and C_2 are constants.

$$\begin{aligned}
M_y &= \int z\sigma_A dA = C_1\int yzdA + C_2\int z^2dA \\
&= I_{yz}C_1 + I_yC_2 = 0 \\
C_2 &= -\frac{I_{yz}}{I_y}C_1 \\
M_z &= -\int y\sigma_A dz = -C_1\int y^2dA + C_2\int yzdA \\
&= -I_ZC_1 - I_{yz}\frac{I_{yz}}{I_y}C_1 \\
I_yM_z &= -\left(I_yI_z - I_{yz}^2\right)C_1 \\
C_1 &= -\frac{I_yM_z}{I_yI_z - I_{yz}^2} \\
C_2 &= +\frac{I_{yz}M_z}{I_yI_z - I_{yz}^2} \\
\sigma_A &= -\frac{I_y y - I_{yz^2}}{I_yI_z - I_{yz}^2}M_z
\end{aligned}$$

◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.159



(*a*) Show that, if a vertical force **P** is applied at point *A* of the section shown, the equation of the neutral axis *BD* is

$$\left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1$$

where r_z and r_x denote the radius of gyration of the cross section with respect to the *z* axis and the *x* axis, respectively. (*b*) Further show that, if a vertical force **Q** is applied at any point located on line *BD*, the stress at point *A* will be zero.

SOLUTION

Definitions:

$$r_x^2 = \frac{I_x}{A}, \quad r_z^2 = \frac{I_z}{A}$$

(*a*) $M_x = Pz_A \qquad M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ar_z^2} - \frac{Pz_A z_E}{Ar_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{r_z^2}\right)x_E + \left(\frac{z_A}{r_x^2}\right)z_E\right] = 0$$

if *E* lies on neutral axis.

$$1 + \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = 0, \qquad \left(\frac{x_A}{r_z^2}\right)x + \left(\frac{z_A}{r_x^2}\right)z = -1 \quad \blacktriangleleft$$

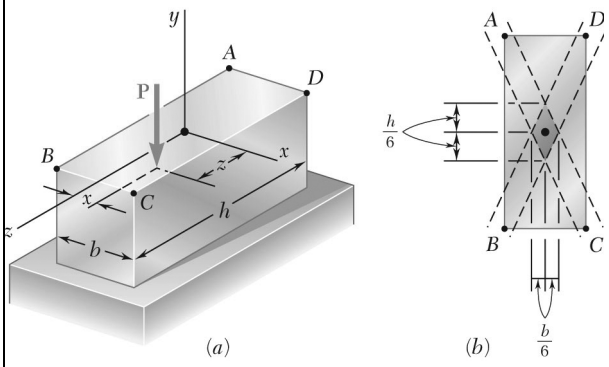
(*b*) $M_x = Pz_E \quad M_z = -Px_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_y} = -\frac{P}{A} - \frac{Px_E x_A}{Ar_z^2} - \frac{Pz_E z_A}{Ar_x^2}$$

$= 0$ by equation from part (*a*). $\blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.160



(*a*) Show that the stress at corner *A* of the prismatic member shown in part *a* of the figure will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$

(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and three similar lines corresponding to the condition of zero stress at *B*, *C*, and *D*, respectively. This area, shown in part *b* of the figure, is known as the *kern* of the cross section.

SOLUTION

$$I_z = \frac{1}{12}hb^3 \quad I_x = \frac{1}{12}bh^3 \quad A = bh$$

$$z_A = -\frac{h}{2} \quad x_A = -\frac{b}{2}$$

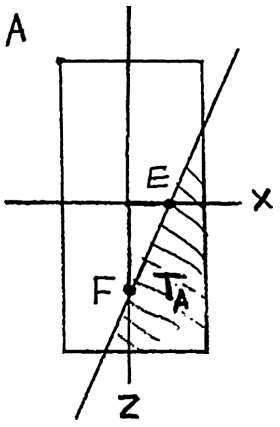
Let *P* be the load point.

$$M_z = -Px_P \quad M_x = Pz_P$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

$$= -\frac{P}{bh} + \frac{(-Px_P)\left(-\frac{b}{2}\right)}{\frac{1}{12}hb^3} - \frac{(Pz_P - \frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$



(*a*) For $\sigma_A = 0$, $1 - \frac{x}{b/6} - \frac{z}{h/6} = 0$ $\frac{x}{b/6} + \frac{z}{h/6} = 1$

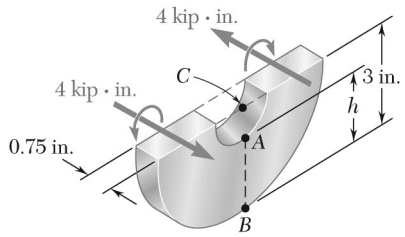
(*b*) At point *E*: $z = 0 \quad \therefore \quad x_E = b/6$

At point *F*: $x = 0 \quad \therefore \quad z_F = h/6$

If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile stress will occur at corner *A*.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.161

For the machine component and loading shown, determine the stress at point A when (a) $h = 2$ in., (b) $h = 2.6$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in}$$

Rectangular cross section: $A = bh \quad r_2 = 3 \text{ in.} \quad r_1 = r_2 - h$

$$\bar{r} = \frac{1}{2}(r_1 + r_2), \quad R = \frac{h}{\ln \frac{r_2}{r_1}}, \quad e = \bar{r} - R$$

(a) $h = 2$ in. $\quad A = (0.75)(2) = 1.5 \text{ in}^2$

$$r_1 = 3 - 2 = 1 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 1) = 2 \text{ in.}$$

$$R = \frac{2}{\ln \frac{3}{1}} = 1.8205 \text{ in.} \quad e = 2 - 1.8205 = 0.1795 \text{ in.}$$

At point A: $\quad r = r_1 = 1$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(1 - 1.8205)}{(1.5)(0.1795)(1)} = 12.19 \text{ ksi}$$

$$\sigma_A = 12.19 \text{ ksi} \blacktriangleleft$$

(b) $h = 2.6$ in. $\quad A = (0.75)(2.6) = 1.95 \text{ in}^2$

$$r_1 = 3 - 2.6 = 0.4 \text{ in.} \qquad \bar{r} = \frac{1}{2}(3 + 0.4) = 1.7 \text{ in.}$$

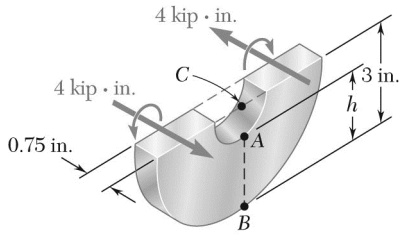
$$R = \frac{2.6}{\ln \frac{3}{0.4}} = 1.2904 \text{ in.} \quad e = 1.7 - 1.2904 = 0.4906 \text{ in.}$$

At point A: $\quad r = r_1 = 0.4$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4)(0.4 - 1.2904)}{(1.95)(0.4096)(0.4)} = 11.15 \text{ ksi}$$

$$\sigma_A = 11.15 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.162

For the machine component and loading shown, determine the stress at points A and B when $h = 2.5$ in.

SOLUTION

$$M = -4 \text{ kip} \cdot \text{in}$$

Rectangular cross section: $h = 2.5$ in. $b = 0.75$ in. $A = 1.875$ in.^{2}

$$r_2 = 3 \text{ in.} \quad r_1 = r_2 - h = 0.5 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = \frac{1}{2}(0.5 + 3.0) = 1.75 \text{ in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{3.0}{0.5}} = 1.3953 \text{ in.}$$

$$e = \bar{r} - R = 1.75 - 1.3953 = 0.3547 \text{ in.}$$

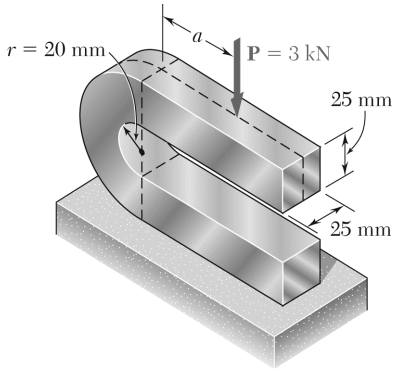
At point A: $r = r_1 = 0.5$ in.

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in})(0.5 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.})(2.5 \text{ in.})(0.3547 \text{ in.})(0.5 \text{ in.})} \qquad \sigma_A = 10.77 \text{ ksi} \blacktriangleleft$$

At point B: $r = r_2 = 3$ in.

$$\sigma_B = \frac{M(r - R)}{Aer} = \frac{(-4 \text{ kip} \cdot \text{in})(3 \text{ in.} - 1.3953 \text{ in.})}{(0.75 \text{ in.} \times 2.5 \text{ in.})(0.3547 \text{ in.})(3 \text{ in.})} \qquad \sigma_B = -3.22 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.163

The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.

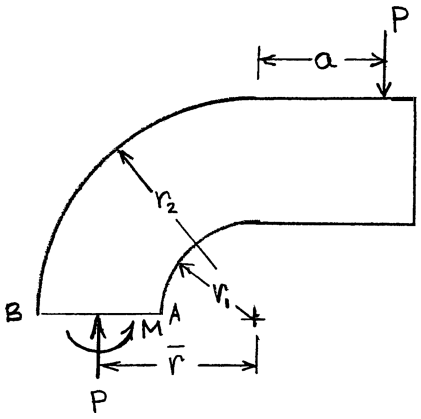
SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}$$



Also $e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1} = -K\frac{P}{A}$$

with $y_A = R - r_1$

Thus, $K = 1 + \dfrac{(a+\bar{r})(R-r_1)}{er_1}$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$b = 25$ mm, $A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$ $R - r_1 = 10.8288$ mm

$P = 3 \times 10^3 \text{ N} \cdot \text{m}$, $\sigma_A = -150 \times 10^6$ Pa

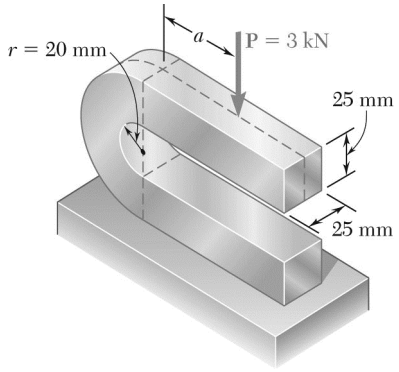
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)er_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$a = 93.37 - 32.5$

$a = 60.9$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.164

The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.

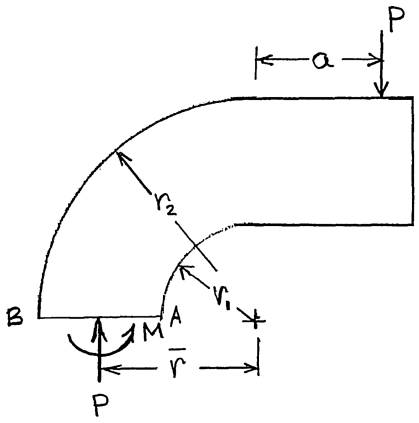
SOLUTION

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}.$$



Also $e = \bar{r} - R$

The maximum compressive stress occurs at point *A*. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1} = -K\frac{P}{A}$$

with $y_A = R - r_1$

Thus, $K = 1 + \dfrac{(a + \bar{r})(R - r_1)}{er_1}$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

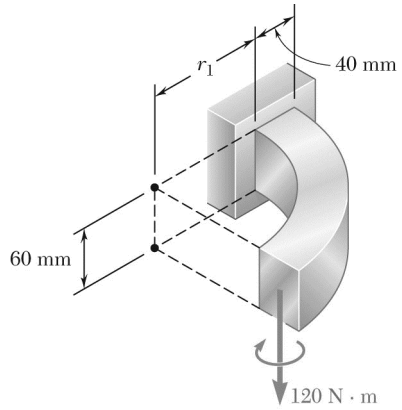
$b = 25$ mm, $A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{m}^2$

$a = 60$ mm, $a + \bar{r} = 92.5$ mm, $R - r_1 = 10.8288$ mm

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968 \quad P = 30 \times 10^3 \text{ N}$$

$$\sigma_A = \frac{KP}{A} = -\frac{(30.968)(3 \times 10^3)}{625 \times 10^{-6}} = -148.6 \times 10^6 \text{ Pa} \qquad \sigma_A = -148.6 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.165

The curved bar shown has a cross section of 40×60 mm and an inner radius $r_1 = 15$ mm. For the loading shown, determine the largest tensile and compressive stresses.

SOLUTION

$h = 40$ mm, $r_1 = 15$ mm, $r_2 = 55$mm

$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{m}^2$

$$R = \frac{h}{\ln \dfrac{r_2}{r_1}} = \frac{40}{\ln \dfrac{55}{40}} = 30.786 \text{ mm}$$

$$\overline{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$e = \overline{r} - R = 4.214$ mm $\qquad \sigma = -\dfrac{My}{Aer}$

At $r = 15$mm, $y = 30.786 - 15 = 15.756$ mm

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^{-6} \text{ Pa}$$

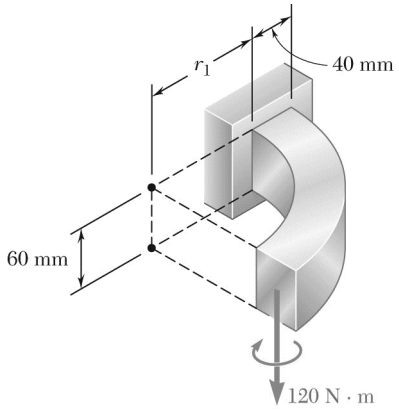
$\sigma = -12.49$MPa ◀

At $r = 55$mm, $y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^{6} \text{ Pa}$$

$\sigma = 5.22$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.166

For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (*a*) $r_1 = 20$ mm, (*b*) $r_1 = 200$ mm, (*c*) $r_1 = 2$ m.

SOLUTION

$$h = 40 \text{ mm}, \quad A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2, \quad M = 120 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12} bh^3 = \frac{1}{12}(60)(40)^3 = 0.32 \times 10^6 \text{ mm}^4 = 0.32 \times 10^{-6} \text{ mm}^4, \quad c = \frac{1}{2} h = 20 \text{ mm}$$

Assuming that the bar is straight

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20 \times 10^{-8})}{(0.32 \times 10^{-6})} = 7.5 \times 10^6 \text{ Pa} = 7.5 \text{ MPa}$$

(*a*) $r_1 = 20$ mm $\quad r_2 = 60$ mm

$$R = \frac{h}{\ln \dfrac{r_2}{r_1}} = \frac{40}{\ln \dfrac{60}{20}} = 36.4096 \text{ mm} \qquad r_1 - R = -16.4096 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm} \qquad e = \bar{r} - R = 3.5904 \text{ mm}$$

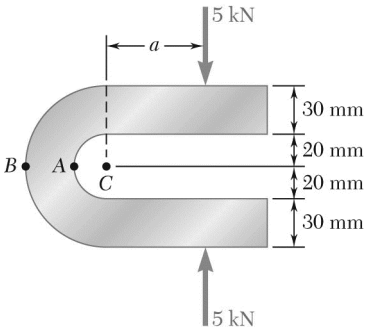
$$\sigma_a = \frac{M(r_1 - R)}{Aer} = \frac{(120)(-16.4096 \times 10^{-3})}{(2400 \times 10^{-6})(3.5904 \times 10^{-3})(20 \times 10^{-3})} = -11.426 \times 10^6 \text{ Pa} = -11.426 \text{ MPa}$$

$$\% \text{ error} = \frac{-11.426 - (-7.5)}{-11.426} \times 100\% = -34.4\% \quad \blacktriangleleft$$

For parts (*b*) and (*c*), we get the values in the table below:

	r_1, mm	r_2, mm	R, mm	$\bar{r}$, mm	e, mm	σ, MPa	% error
(*a*)	20	60	36.4096	40	3.5904	−11.426	−34.4 %
(*b*)	200	240	219.3926	220	0.6074	−7.982	6.0 % ◀
(*c*)	2000	2040	2019.9340	2020	0.0660	−7.546	0.6 % ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



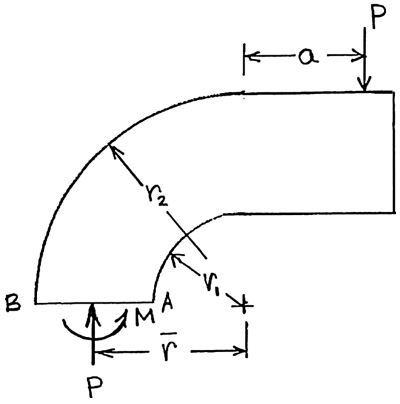
PROBLEM 4.167

The curved bar shown has a cross section of 30×30 mm. Knowing that $a = 60$ mm, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \dfrac{r_2}{r_1}}.$$

Also $e = \bar{r} - R$

The maximum compressive stress occurs at point *A*. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with} \qquad y_A = R - r_1$$

Thus, $$K = 1 + \frac{(a + \bar{r})(R - r_1)}{er_1}$$

Data: $h = 30$ mm, $r_1 = 20$ mm, $r_2 = 50$ mm, $\bar{r} = 35$ mm

$$R = \frac{30}{\ln \dfrac{50}{20}} = 32.7407 \text{ mm}, \quad e = 35 - 32.7407 = 2.2593 \text{ mm}$$

$b = 30$ mm, $A = bh = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{m}^2$

$a = 60$ mm, $a + \bar{r} = 95$ mm, $R - r_1 = 12.7407$ mm

$$K = 1 + \frac{(95)(12.7407)}{(2.2593)(20)} = 27.786$$

$P = 5 \times 10^3$ N

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.167 *(Continued)*

(*a*) $$\sigma_A = -\frac{KP}{A} = -\frac{(27.786)(5\times10^3)}{900\times10^{-6}} = -154.4\times10^6 \text{ Pa}$$

$$\sigma_A = -154.4 \text{ MPa} \blacktriangleleft$$

(*b*) At point *B*, $y_B = r_2 - R = 50 - 32.7407 = 17.2953$ mm

$$\sigma_B = -\frac{P}{A} + \frac{My_B}{Aer_2} = \frac{P}{A}\left[-1 + \frac{(a+\bar{r})y_B}{er_2}\right]$$

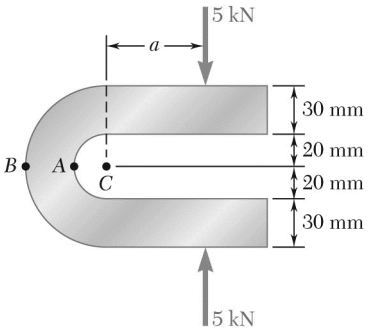
$$= \frac{K'P}{A} \quad \text{where} \quad K' = \frac{(a+\bar{r})y_B}{er_2} - 1$$

$$K' = \frac{(95)(17.2953)}{(2.2593)(50)} - 1 = 13.545$$

$$\sigma_B = \frac{(13.545)(5\times10^3)}{900\times10^{-6}} = 75.2\times10^6 \text{ Pa}$$

$$\sigma_B = 75.2 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.168

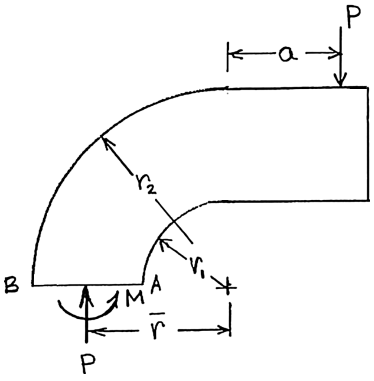
The curved bar shown has a cross section of 30×30 mm. Knowing that the allowable compressive stress is 175 MPa, determine the largest allowable distance a.

SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at $R = \dfrac{h}{\ln \dfrac{r_2}{r_1}}$.



Also $e = \bar{r} - R$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \quad \text{with} \quad y_A = R - r_1$$

Thus, $K = 1 + \dfrac{(a + \bar{r})(R - r_1)}{er_1}$ (1)

Data: $h = 30$ mm, $r_1 = 20$ mm, $r_2 = 50$ mm, $\bar{r} = 35$ mm, $R = \dfrac{30}{\ln \frac{50}{20}} = 32.7407$ mm

$e = 35 - 32.7407 = 2.2593$ mm, $b = 30$ mm, $R - r_1 = 12.7407$ mm, $a = ?$

$\sigma_A = -175$ MPa $= -175 \times 10^6$ Pa, $P = 5$ kN $= 5 \times 10^3$ N

$$K = -\frac{A\sigma_A}{P} = -\frac{(900 \times 10^{-6})(-175 \times 10^6)}{5 \times 10^3} = 31.5$$

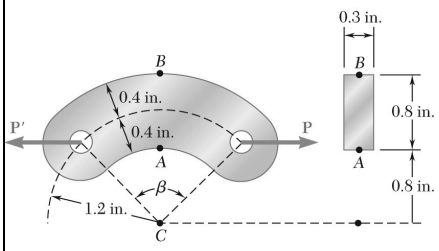
Solving (1) for $a + \bar{r}$, $a + \bar{r} = \dfrac{(K - 1)er_1}{R - r_1}$

$$a + \bar{r} = \frac{(30.5)(2.2593)(20)}{12.7407} = 108.17 \text{ mm}$$

$a = 108.17$ mm $- 35$ mm

$a = 73.2$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.169

Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

SOLUTION

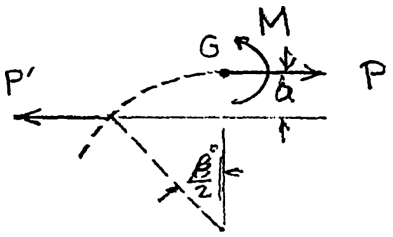
Reduce section force to a force-couple system at G; the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$



Also $e = \bar{r} - R$

At point A the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24$ in^2 $\qquad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

$e = 1.2 - 1.154156 = 0.045844$ in., $\qquad y_A = 1.154156 - 0.8 = 0.35416$ in.

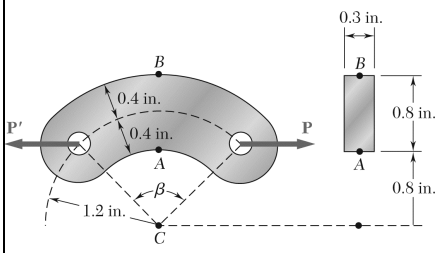
$a = 1.2(1 - \cos 45°) = 0.35147$ in.

$$K = 1 + \frac{(0.35147)(0.35416)}{(0.045844)(0.8)} = 4.3940$$

$$P = \frac{(0.24)(12)}{4.3940} = 0.65544 \text{ kips}$$

$P = 655$ lb ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.170

Solve Prob. 4.169, assuming that $\beta = 60°$.

PROBLEM 4.169 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 12 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

SOLUTION

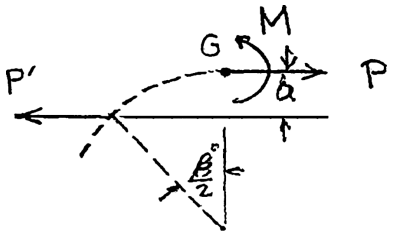
Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$.

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}.$$



Also $e = \bar{r} - R$

At point A, the tensile stress is $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{P}{A} + \dfrac{Pay_A}{Aer_1} = \dfrac{P}{A}\left(1 + \dfrac{ay_A}{er_1}\right) = K\dfrac{P}{A}$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 1.2$ in., $r_1 = 0.8$ in., $r_2 = 1.6$ in., $h = 0.8$ in., $b = 0.3$ in.

$A = (0.3)(0.8) = 0.24$ in^2 $\qquad R = \dfrac{0.8}{\ln\frac{1.6}{0.8}} = 1.154156$ in.

$e = 1.2 - 1.154156 = 0.045844$ in. $\qquad y_A = 1.154156 - 0.8 = 0.35416$ in.

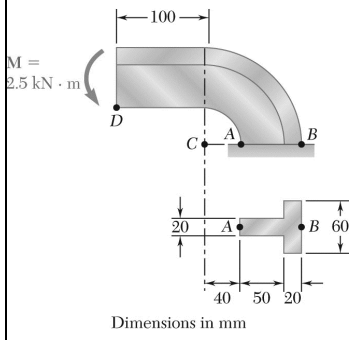
$a = (1.2)(1 - \cos 30°) = 0.160770$ in.

$$K = 1 + \frac{(0.160770)(0.35416)}{(0.045844)(0.8)} = 2.5525$$

$$P = \frac{(0.24)(12)}{2.5525} = 1.128 \text{ kips}$$

$P = 1128$ lb ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.171

A machine component has a T-shaped cross section that is orientated as shown. Knowing that $M = 2500\ \text{N} \cdot \text{m}$, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Properties of the cross section.

$$R = \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} \qquad \bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$

r, mm	Part	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}_i$, mm	$A_i \bar{r}_i$, mm^3
40	1	20	50	1000	16.2186	65	65,000
90	2	60	20	1200	12.0402	100	120,000
110	$\sum$			2200	28.2588		185,000

$$R = \frac{2200}{28.2588} = 77.852\ \text{mm}, \qquad \bar{r} = \frac{185000}{2200} = 84.091\ \text{mm}, \qquad e = \bar{r} - R = 6.239\ \text{mm}$$

Stresses.

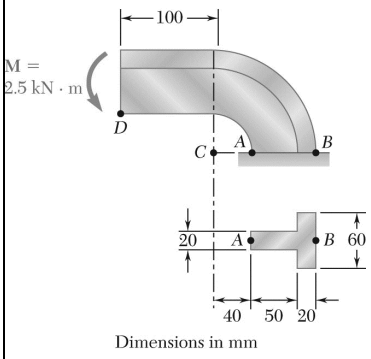
(*a*) Point *A*: $r = r_A = 40\ \text{mm}$

$$\sigma_A = \frac{M(r - R)}{Aer} = \frac{(2.5\ \text{kN} \cdot \text{m})(0.040\ \text{m} - 0.0778517\ \text{m})}{(2.2 \times 10^{-3}\ \text{m}^2)(6.2392 \times 10^{-3}\ \text{m})(0.040\ \text{m})} \qquad \sigma_A = -172.4\ \text{MPa} \blacktriangleleft$$

(*b*) Point *B*: $r = r_B = 110\ \text{mm}$

$$\sigma_B = \frac{M(r - R)}{Aer} = \frac{(2.5\ \text{kN} \cdot \text{m})(0.110\ \text{m} - 0.0778517\ \text{m})}{(2.2 \times 10^{-3}\ \text{m}^2)(6.2392 \times 10^{-3}\ \text{m})(0.110\ \text{m})} \qquad \sigma_B = 53.2\ \text{MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.172

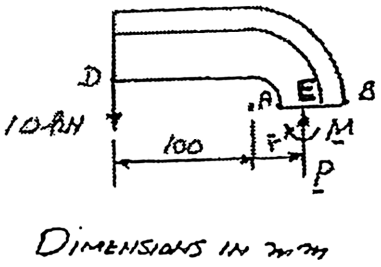
Assuming that the couple shown is replaced by a vertical 10-kN force attached at point D and acting downward, determine the stress at (*a*) point A, (*b*) point B.

SOLUTION

Properties of the cross section. $R = \dfrac{\Sigma A}{\Sigma \int \dfrac{1}{r} dA} = \dfrac{\Sigma b_i h_i}{\Sigma b_i \ln \dfrac{r_{i+1}}{r_i}} = \dfrac{\Sigma A_i}{\Sigma b_i \ln \dfrac{r_{i+1}}{r_i}} \qquad \bar{r} = \dfrac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$

r, mm	Part	b, mm	h, mm	A, mm^2	$b_i \ln \dfrac{r_{i+1}}{r_i}$, mm	$\bar{r}_i$, mm	$A_i \bar{r}_i$, mm^3
40							
90	1	20	50	1000	16.2186	65	65,000
110	2	60	20	1200	12.0402	100	120,000
	Σ			2200	28.2588		185,000

$$R = \frac{2200}{28.2588} = 77.852 \text{ mm}, \qquad \bar{r} = \frac{185000}{2200} = 84.091 \text{ mm}, \qquad e = \bar{r} - R = 6.239 \text{ mm}$$



Force-couple system at the centroid E.

$P = 10$ kN

$M = (10 \text{ kN})(100 \text{ mm} + 84.0909 \text{ mm}) = 1840.9 \text{ N} \cdot \text{m}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.172 *(Continued)*

Stresses.

(*a*) Point *A*: $r = r_A = 40$ mm

$$\sigma_A = -\frac{P}{A} + \frac{M(r-R)}{Aer} = -\frac{10\text{ kN}}{2.2\times10^{-3}\text{ m}^2} + \frac{(1840.9\text{ N}\cdot\text{m})(0.040\text{ m} - 0.0778517\text{ m})}{(2.2\times10^{-3}\text{ m}^2)(6.2392\times10^{-3}\text{ m})(0.040\text{ m})}$$

$$= -4.545\text{ MPa} - 126.913\text{ MPa}$$

$$\sigma_A = -131.5\text{ MPa} \blacktriangleleft$$

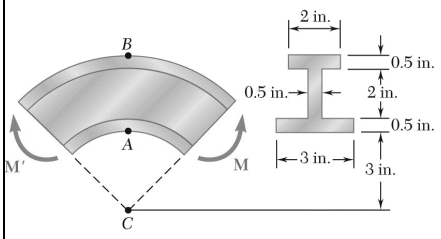
(*b*) Point *B*: $r = r_B = 110$ mm

$$\sigma_B = -\frac{P}{A} + \frac{M(r-R)}{Aer} = -\frac{10\text{ kN}}{2.2\times10^{-3}\text{ m}^2} + \frac{(1840.9\text{ N}\cdot\text{m})(0.110\text{ m} - 0.0778517\text{ m})}{(2.2\times10^{-3}\text{ m}^2)(6.2392\times10^{-3}\text{ m})(0.110\text{ m})}$$

$$= -4.545\text{ MPa} + 39.196\text{ MPa}$$

$$\sigma_B = 34.7\text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



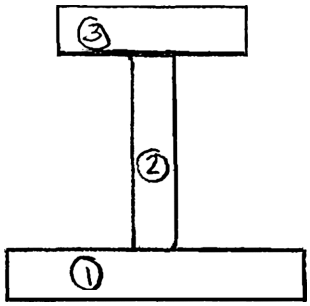
PROBLEM 4.173

Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\,dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



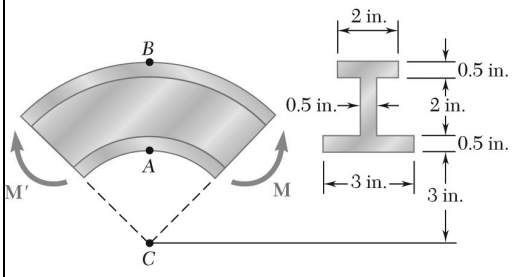
r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3							
3.5	①	3	0.5	1.5	0.462452	3.25	4.875
5.5	②	0.5	2	1.0	0.225993	4.5	4.5
6	③	2	0.5	1.0	0.174023	5.75	5.75
	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.}$$

$e = 0.263$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



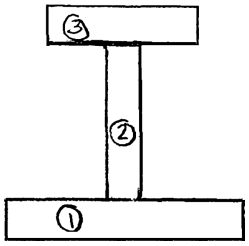
PROBLEM 4.174

Three plates are welded together to form the curved beam shown. For $M = 8$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	Part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \qquad M = -8 \text{ kip} \cdot \text{in}$$

(*a*) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} \qquad \sigma_A = 3.06 \text{ ksi} \blacktriangleleft$$

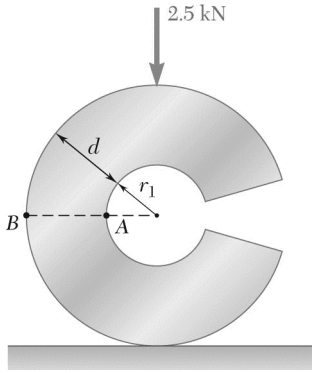
(*b*) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} \qquad \sigma_B = -2.81 \text{ ksi} \blacktriangleleft$$

(*c*) $y_C = R - \bar{r} = -e$

$$\sigma_C = -\frac{My_C}{Ae\bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)} \qquad \sigma_C = 0.529 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.175

The split ring shown has an inner radius $r_1 = 20$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 20 \text{ mm}, \quad r_2 = r_1 + d = 52 \text{ mm}$$

$$\bar{r} = r_1 + c = 36 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\right]$$

$$= 34.1245 \text{ mm}$$

$$e = \bar{r} - R = 1.8755 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{m}^2$$

$$P = 2.5 \times 10^3 \text{N} \qquad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N} \cdot \text{m}$$

(*a*) Point *A*: $y_A = R - r_1 = 34.1245 - 20 = 14.125$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

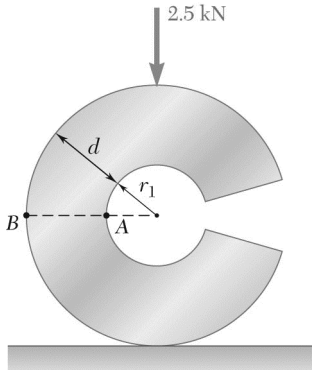
$$= -45.2 \times 10^6 \text{ Pa} \qquad \sigma_A = -45.2 \text{ MPa} \blacktriangleleft$$

(*b*) Point *B*: $y_B = R - r_2 = 34.1245 - 52 = -17.8755$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$$= 17.40 \times 10^6 \text{ Pa} \qquad \sigma_B = 17.40 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.176

The split ring shown has an inner radius $r_1 = 16$ mm and a *circular* cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 16 \text{ mm}, \quad r_2 = r_1 + d = 48 \text{ mm}$$

$$\bar{r} = r_1 + c = 32 \text{ mm}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$$

$$= 29.8564 \text{ mm}$$

$$e = \bar{r} - R = 2.1436 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{m}^2$$

$$P = 2.5 \times 10^3 \text{N} \qquad M = P\bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80 \text{ N} \cdot \text{m}$$

(*a*) Point *A*: $y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})}$$

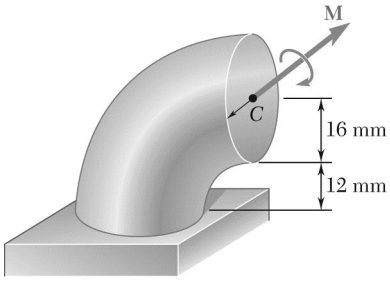
$$= -43.3 \times 10^6 \text{Pa} \qquad \sigma_A = -43.3 \text{ MPa} \blacktriangleleft$$

(*b*) Point *B*: $y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^6}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})}$$

$$= 14.43 \times 10^6 \text{Pa} \qquad \sigma_B = 14.43 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.177

The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple **M** that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.

SOLUTION

$c = 16$ mm $\qquad \bar{r} = 12 + 16 = 28$ mm

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right]$$

$$= \frac{1}{2}\left[28 + \sqrt{28^2 - 16^2}\right] = 25.4891 \text{ mm}$$

$$e = \bar{r} - R = 28 - 25.4891 = 2.5109 \text{ mm}$$

$\sigma_{\max}$ occurs at A, which lies at the inner radius.

It is given by $|\sigma_{\max}| = \left|\dfrac{My_A}{Aer_1}\right|$ from which $\qquad M = \dfrac{Aer_1|\sigma_{\max}|}{y_A}$.

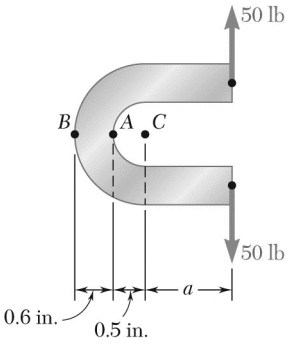
Also, $\qquad A = \pi c^2 = \pi(16)^2 = 804.25 \text{ mm}^2$

Data: $\qquad y_A = R - r_1 = 25.4891 - 12 = 13.4891 \text{ mm}$

$$M = \frac{(804.25 \times 10^{-6})(2.5109 \times 10^{-3})(12 \times 10^{-3})(60 \times 10^{6})}{13.4891 \times 10^{-3}}$$

$M = 107.8 \text{ N} \cdot \text{m}$ ◀

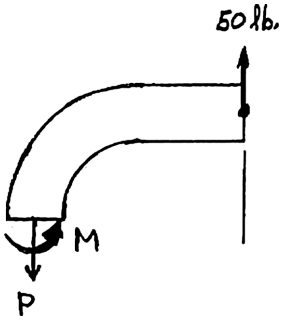
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.178

The bar shown has a circular cross section of 0.6 in. diameter. Knowing that $a = 1.2$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



$$c = \frac{1}{2}d = 0.3 \text{ in.} \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in.}$$

$$e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$P = 50 \text{ lb}$$

$$M = -P(a + \bar{r}) = -50(1.2 + 0.8) = -100 \text{ lb} \cdot \text{in}$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919 \text{ in.}$$

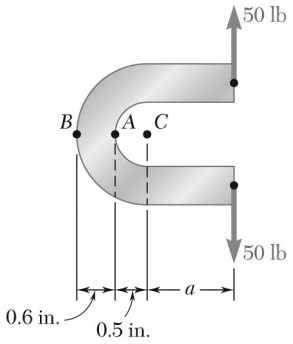
(*a*) $$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{50}{0.28274} - \frac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3 \text{ psi}$$

$$\sigma_A = 6.74 \text{ ksi} \blacktriangleleft$$

(*b*) $$\sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2} = \frac{50}{0.28274} - \frac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3 \text{ psi}$$

$$\sigma_B = -3.45 \text{ ksi} \blacktriangleleft$$

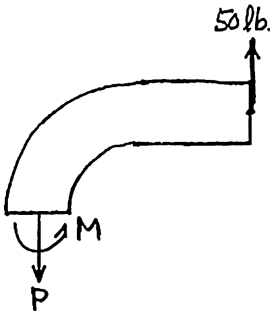
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.179

The bar shown has a circular cross section of 0.6 in. diameter. Knowing that the allowable stress is 8 ksi, determine the largest permissible distance a from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.

SOLUTION



$$c = \frac{1}{2}d = 0.3 \text{ in.}, \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in.} \qquad e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{Aer_1} = \frac{P}{A}\left[1 + \frac{(a+\bar{r})y_A}{er_1}\right]$$

$$= \frac{KP}{A} \qquad \text{where} \qquad K = 1 + \frac{(a+\bar{r})y_A}{er_1}$$

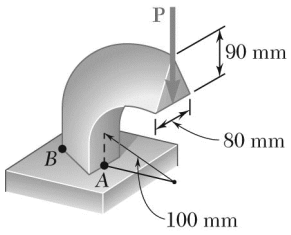
$$K = \frac{\sigma_A A}{P} = \frac{(8\times10^3)(0.28274)}{50} = 45.238$$

$$a + \bar{r} = \frac{(K-1)er_1}{y_A} = \frac{(44.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in.}$$

$$a = 2.384 - 0.8$$

$$a = 1.584 \text{ in.} \blacktriangleleft$$

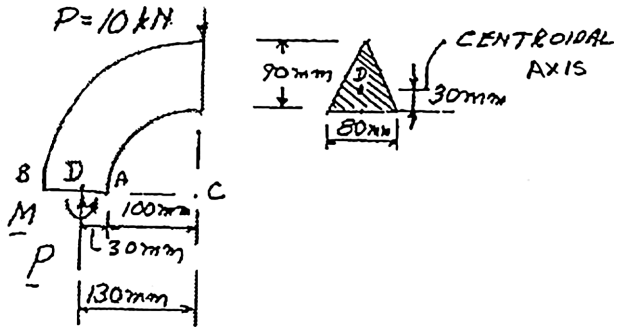
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.180

Knowing that $P = 10$ kN , determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



Locate the centroid *D* of the cross section.

$$\bar{r} = 100 \text{ mm} + \frac{90 \text{ mm}}{3} = 130 \text{ mm}$$

Force-couple system at *D*.

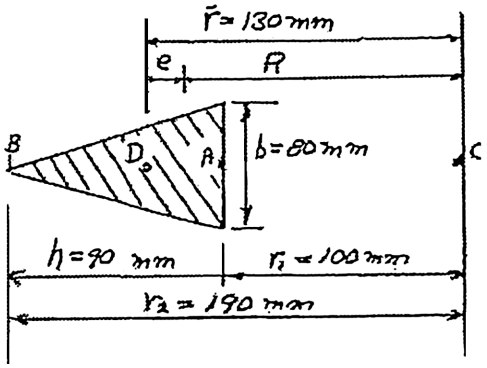
$$P = 10 \text{ kN}$$

$$M = P\bar{r} = (10 \text{ kN})(130 \text{ mm}) = 1300 \text{ N} \cdot \text{m}$$

Triangular cross section.

$$A = \frac{1}{2}bh = \frac{1}{2}(90 \text{ mm})(80 \text{ mm})$$

$$= 3600 \text{ mm}^2 = 3600 \times 10^{-6} \text{ m}^2$$



$$R = \frac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1} = \frac{\frac{1}{2}(90)}{\frac{190}{90}\ln\frac{190}{100} - 1} = \frac{45 \text{ mm}}{0.355025}$$

$$R = 126.752 \text{ mm}$$

$$e = \bar{r} - R = 130 \text{ mm} - 126.752 \text{ mm} = 3.248 \text{ mm}$$

(*a*) Point *A*: $r_A = 100 \text{ mm} = 0.100 \text{ m}$

$$\sigma_A = -\frac{P}{A} + \frac{M(r_A - R)}{Aer_A} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.100 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3248 \times 10^{-3} \text{ m})(0.100 \text{ m})}$$

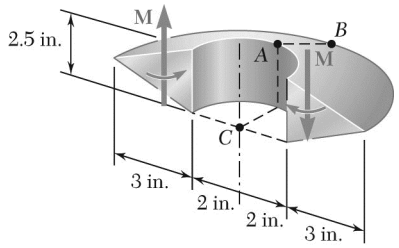
$$= -2.778 \text{ MPa} - 29.743 \text{ MPa} \qquad \sigma_A = -32.5 \text{ MPa} \blacktriangleleft$$

(*b*) Point *B*: $r_B = 190 \text{ mm} = 0.190 \text{ m}$

$$\sigma_B = -\frac{P}{A} + \frac{M(r_B - R)}{Aer_B} = -\frac{10 \text{ kN}}{3600 \times 10^{-6} \text{ m}^2} + \frac{(1300 \text{ N} \cdot \text{m})(0.190 \text{ m} - 0.126752 \text{ m})}{(3600 \times 10^{-6} \text{ m}^2)(3.248 \times 10^{-3} \text{ m})(0.190 \text{ m})}$$

$$= -2.778 \text{ MPa} + 37.01 \text{ MPa} \qquad \sigma_B = +34.2 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.181

Knowing that $M = 5$ kip · in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 1 = 3.00000 \text{ in.}$$

$$b_1 = 2.5 \text{ in.}, \quad r_1 = 2 \text{ in.}, \quad b_2 = 0, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid with $b_2 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5 + 0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5 - 0)} = 2.84548 \text{ in.}$$

$$e = \bar{r} - R = 0.15452 \text{ in.} \qquad M = 5 \text{ kip} \cdot \text{in}$$

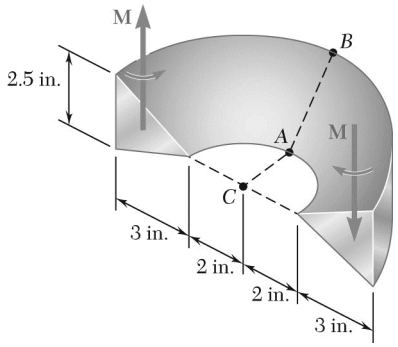
(*a*) $y_A = R - r_1 = 0.84548$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)} \qquad \sigma_A = -3.65 \text{ ksi} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = -2.15452$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)} \qquad \sigma_B = 3.72 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.182

Knowing that $M = 5\,\text{kip} \cdot \text{in}$, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = \frac{1}{2}(2.5)(3) = 3.75\ \text{in}^2$$

$$\bar{r} = 2 + 2 = 4.00000\ \text{in.}$$

$$b_1 = 0, \quad r_1 = 2\ \text{in.}, \quad b_2 = 2.5\ \text{in.}, \quad r_2 = 5\ \text{in.}$$

Use formula for trapezoid with $b_1 = 0$.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466\ \text{in.}$$

$$e = \bar{r} - R = 0.14534\ \text{in.} \qquad M = 5\ \text{kip} \cdot \text{in}$$

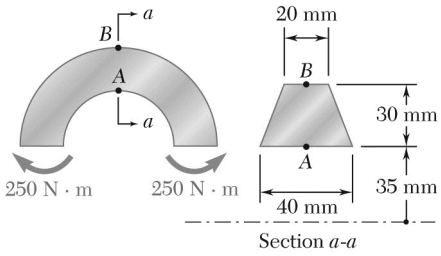
(*a*) $y_A = R - r_1 = 1.85466\ \text{in.}$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} \qquad \sigma_A = -8.51\ \text{ksi} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = -1.14534\ \text{in.}$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} \qquad \sigma_B = 2.10\ \text{ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

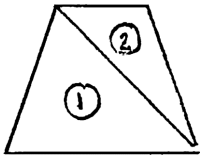


PROBLEM 4.183

For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	600	45	27×10^3
②	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5\times10^3}{900} = 48.333 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N}\cdot\text{m}$$

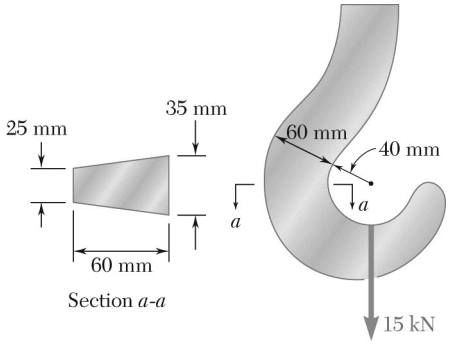
(*a*) $y_A = R - r_1 = 11.8608$ mm

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608\times10^{-3})}{(900\times10^{-6})(1.4725\times10^{-3})(35\times10^{-3})} = 63.9\times10^6 \text{Pa} \qquad \sigma_A = 63.9 \text{ MPa} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = -18.1392$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392\times10^{-3})}{(900\times10^{-6})(1.4725\times10^{-3})(65\times10^{-3})} = -52.6\times10^6 \text{Pa} \qquad \sigma_B = -52.6 \text{ MPa} \blacktriangleleft$$

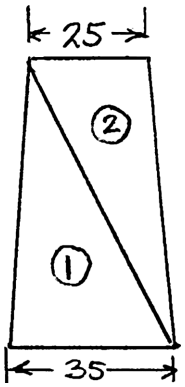
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.184

For the crane hook shown, determine the largest tensile stress in section *a-a*.

SOLUTION



Locate centroid.

	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103\times10^3}{1800} = 63.333 \text{ mm}$$

Force-couple system at centroid: $P = 15\times10^3 \text{ N}$

$$M = -P\bar{r} = -(15\times10^3)(68.333\times10^{-3}) = -1.025\times10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1r_2 - b_2r_1)\ln\frac{r_2}{r_1} - h(b_1-b_2)}$$

$$= \frac{(0.5)(60)^2(35+25)}{[(35)(100)-(25)(40)]\ln\frac{100}{40} - (60)(35+25)} = 63.878 \text{ mm}$$

$$e = \bar{r} - R = 4.452 \text{ mm}$$

Maximum tensile stress occurs at point A.

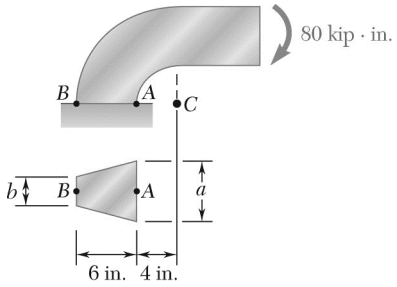
$$y_A = R - r_1 = 23.878 \text{ mm}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1}$$

$$= \frac{15\times10^3}{1800\times10^{-6}} - \frac{-(1.025\times10^3)(23.878\times10^{-3})}{(1800\times10^{-6})(4.452\times10^{-3})(40\times10^{-3})}$$

$$= 84.7\times10^6 \text{ Pa} \qquad \sigma_A = 84.7 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

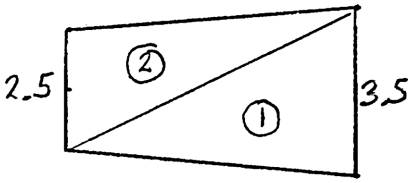


PROBLEM 4.185

Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	10.5	6	63
②	7.5	8	60
Σ	18		123

$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in.}$$

$$e = \bar{r} - R = 0.4452 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in}$$

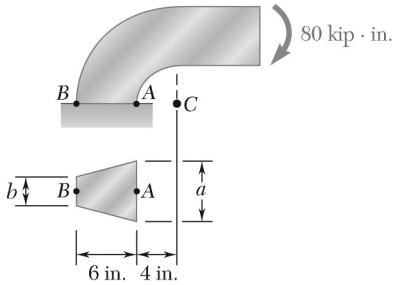
(*a*) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)} \qquad \sigma_A = -5.96 \text{ ksi} \blacktriangleleft$$

(*b*) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)} \qquad \sigma_B = 3.61 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

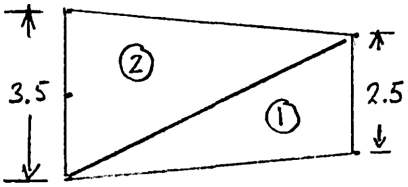


PROBLEM 4.186

Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Locate centroid.



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	7.5	6	45
②	10.5	8	84
Σ	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5+3.5)}{[(2.5)(10) - (3.5)(4)]\ln\frac{10}{4} - (6)(2.5-3.5)} = 6.7168 \text{ in.}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \qquad M = 80 \text{ kip} \cdot \text{in}$$

(*a*) $y_A = R - r_1 = 2.7168$ in.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)}$$

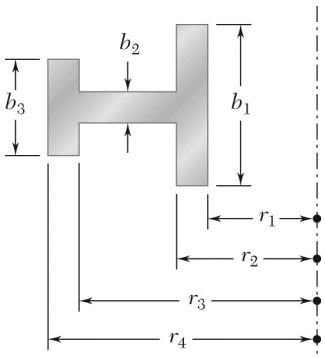
$\sigma_A = -6.71$ ksi ◀

(*b*) $y_B = R - r_2 = -3.2832$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)}$$

$\sigma_B = 3.24$ ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.187

Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln\left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$ ◀

Note that for each rectangle, $\int \frac{1}{r} d A = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

$$= b_i \int_{r_2}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

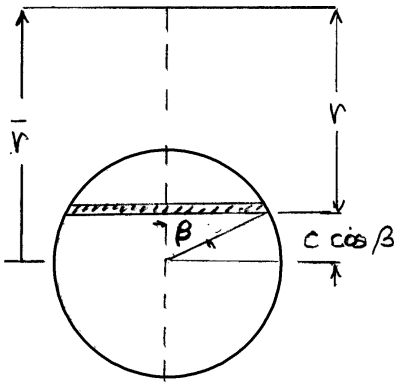
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.188

Using Eq. (4.66), derive the expression for R given in Fig. 4.73 for a circular cross section.

SOLUTION

Use polar coordinate β as shown. Let w be the width as a function of β



$$w = 2c\,\sin\beta$$

$$r = \bar{r} - c\cos\beta$$

$$dr = c\,\sin\,\beta\,d\beta$$

$$dA = w\,dr = 2c^2\sin^2\beta\,d\beta$$

$$\int\frac{dA}{r} = \int_0^\pi \frac{2c^2\sin\beta}{\bar{r} - c\cos\beta}\,d\beta$$

$$\int\frac{dA}{r} = \int_0^\pi \frac{c^2(1-\cos^2\beta)}{\bar{r} - c\cos\beta}d\beta$$

$$= 2\int_0^\pi \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\,\cos\beta}d\beta$$

$$= 2\int_0^\pi (\bar{r} + c\,\cos\beta)\,d\beta - 2(\bar{r}^2 - c^2)\int_0^\pi \frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\,\beta\Big|_0^\pi + 2c\,\sin\beta\Big|_0^\pi$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}}\tan^{-1}\,\frac{\sqrt{\bar{r}^2 - c^2}\,\tan\left(\frac{1}{2}\beta\right)}{\bar{r} + c}\Big|_0^\pi$$

$$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2}\cdot\left(\frac{\pi}{2} - 0\right)$$

$$= 2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int\frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}} = \frac{1}{2}\frac{c^2}{\bar{r} - \sqrt{r^2 - c^2}}\times\frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\frac{c^2\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\frac{c^2\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)}{c^2} = \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)$$ ◀

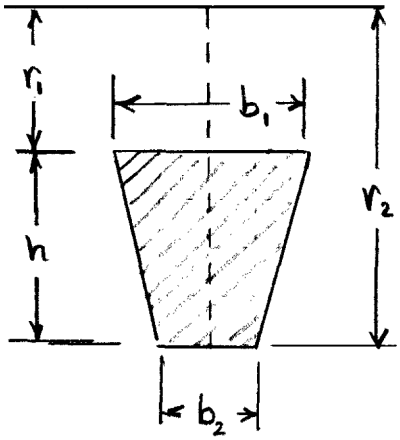
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.189

Using Eq. (4.66), derive the expression for R given in Fig. 4.73 for a trapezoidal cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \quad \text{at} \quad r = r_2$$

$$b_1 = c_0 + c_1 r_1$$

$$b_2 = c_0 + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1)c_0 = h c_0$$

$$c_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

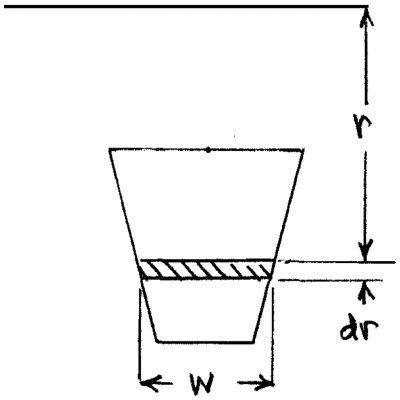
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r}\,dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$



$$A = \frac{1}{2}(b_1 + b_2)h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(r_2 b_1 - r_1 b_2)\ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

◀

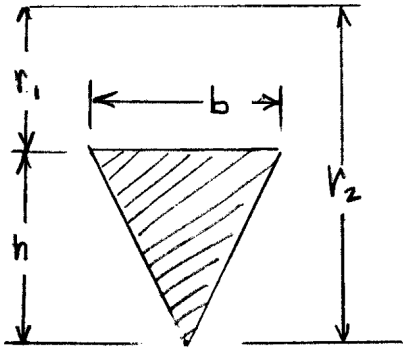
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.190

Using Equation (4.66), derive the expression for R given in Fig. 4.73 for a triangular cross section.

SOLUTION

The section width w varies linearly with r.



$$w = c_0 + c_1 r$$

$$w = b \text{ at } r = r_1 \quad \text{and} \quad w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{b r_2}{h}$$

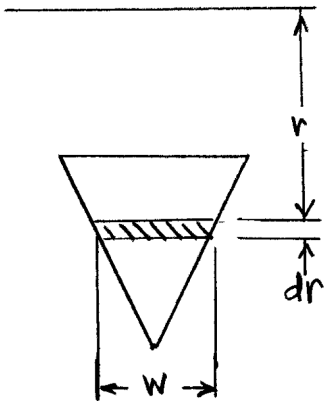
$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1 (r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)$$

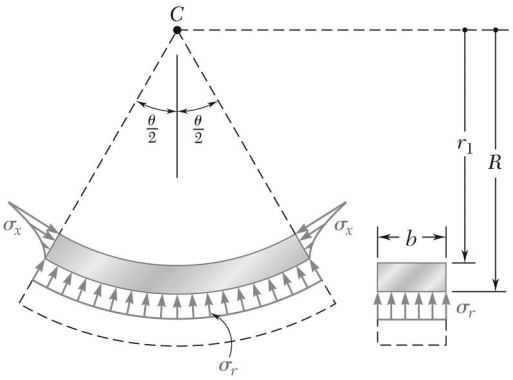


$$A = \frac{1}{2} b h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} bh}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.191

For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)

SOLUTION

At radial distance r, $\sigma_r = \dfrac{M(r-R)}{Aer} = \dfrac{M}{Ae} - \dfrac{MR}{Aer}$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_r dA = \int_{r_1}^{R} \sigma_r b dr$$

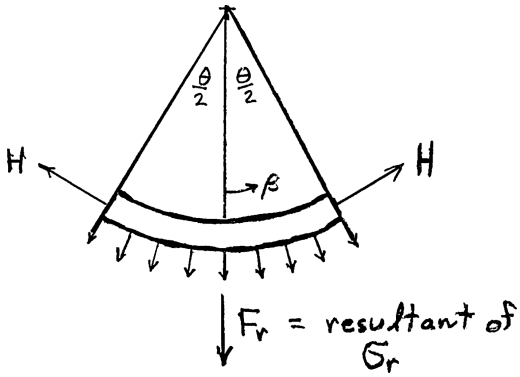
$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_n: $F_r = \int \sigma_r \cos\beta \, dA$

$$= \int_{-\theta/2}^{\theta/2} \sigma_r \cos\beta (bR d\beta) = \sigma_r bR \int_{-\theta/2}^{\theta/2} \cos\beta \, d\beta$$

$$= \sigma_r \, bR \ \sin\beta\Big|_{-\theta/2}^{\theta/2} = 2\sigma_r \, bR \sin\frac{\theta}{2}$$



For equilibrium: $F_r - 2H \sin\dfrac{\theta}{2} = 0$

$$2\sigma_r bR \sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)\sin\frac{\theta}{2} = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$ ◀

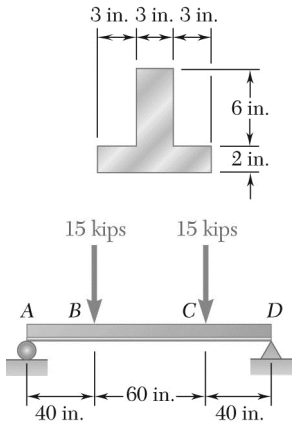
Using results of Examples 4.10 and 4.11 as data,

$M = 8$ kip · in, $A = 3.75$ in^2, $R = 5.9686$ in., $e = 0.0314$ in., $r_1 = 5.25$ in.

$$\sigma_r = \frac{8}{(3.75)(0.0314)}\left[1 - \frac{5.25}{5.9686} - \ln\frac{5.9686}{5.25}\right]$$

$\sigma_r = -0.54$ ksi ◀

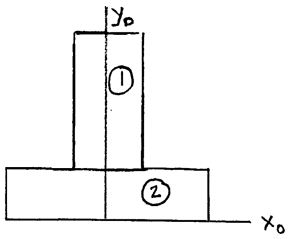
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.192

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

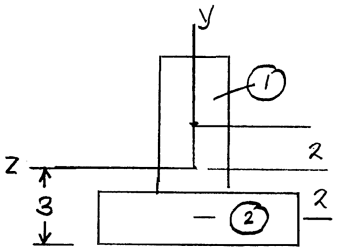
SOLUTION



	A	$\bar{y}_0$	$A\,\bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in.}$$

Neutral axis lies 3 in. above the base.

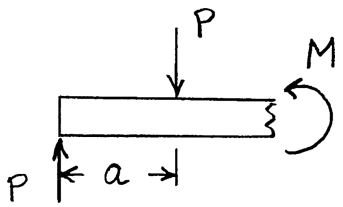


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{\text{top}} = 5 \text{ in.} \quad y_{\text{bot}} = -3 \text{ in.}$$



$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip} \cdot \text{in}$$

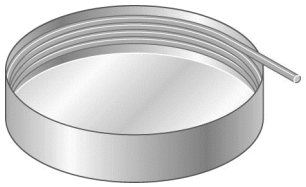
$$\sigma_{\text{top}} = -\frac{M y_{\text{top}}}{I} = -\frac{(600)(5)}{204}$$

$$\sigma_{\text{top}} = -14.71 \text{ ksi (compression)} \blacktriangleleft$$

$$\sigma_{\text{bot}} = -\frac{M y_{\text{bot}}}{I} = -\frac{(600)(-3)}{204}$$

$$\sigma_{\text{bot}} = 8.82 \text{ ksi (tension)} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.193

Straight rods of 6-mm diameter and 30-m length are stored by coiling the rods inside a drum of 1.25-m inside diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a coiled rod, (*b*) the corresponding bending moment in the rod. Use $E = 200$ GPa.

SOLUTION

Let $D =$ inside diameter of the drum.

$d =$ diameter of rod, $\quad c = \dfrac{1}{2}d,$

$\rho =$ radius of curvature of centerline of rods when bent.

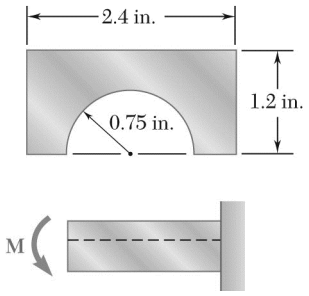
$$\rho = \frac{1}{2}D - \frac{1}{2}d = \frac{1}{2}(1.25) - \frac{1}{2}(6\times10^{-3}) = 0.622 \text{ m}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.003)^4 = 63.617\times10^{-12}\,\text{m}^4$$

(*a*) $$\sigma_{\max} = \frac{Ec}{\rho} = \frac{(200\times10^9)(0.003)}{0.622} = 965\times10^6\,\text{Pa}$$ $\sigma = 965$ MPa ◀

(*b*) $$M = \frac{EI}{\rho} = \frac{(200\times10^9)(63.617\times10^{-12})}{0.622} = 20.5\ \text{N}\cdot\text{m}$$ $M = 20.5\ \text{N}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.194

Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION

① = rectangle ② = semi-circular cutout

$$A_1 = (2.4)(1.2) = 2.88 \text{ in}^2$$

$$A_2 = \frac{\pi}{2}(0.75)^2 = 0.8836 \text{ in}^2$$

$$A = 2.88 - 0.8836 = 1.9964 \text{ in}^2$$

$$\overline{y}_1 = 0.6 \text{ in.}$$

$$\overline{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.75)}{3\pi} = 0.3183 \text{ in.}$$

$$\overline{Y} = \frac{\Sigma A\overline{y}}{\Sigma A} = \frac{(2.88)(0.6) - (0.8836)(0.3183)}{1.9964} = 0.7247 \text{ in.}$$

Neutral axis lies 0.7247 in. above the bottom.

Moment of inertia about the base:

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(2.4)(1.2)^3 - \frac{\pi}{8}(0.75)^4 = 1.25815 \text{ in}^4$$

Centroidal moment of inertia:

$$\overline{I} = I_b - A\overline{Y}^2 = 1.25815 - (1.9964)(0.7247)^2 = 0.2097 \text{ in}^4$$

$$y_{\text{top}} = 1.2 - 0.7247 = 0.4753 \text{ in.},$$

$$y_{\text{bot}} = -0.7247 \text{ in.}$$

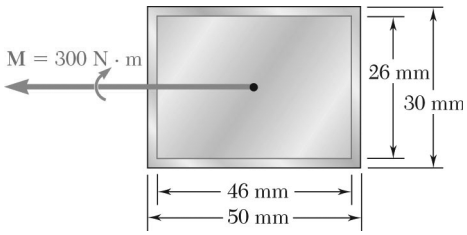
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(12)(0.2097)}{0.4753} = 5.29 \text{ kip} \cdot \text{in}$$

Bottom: (compression) $$M = \frac{(16)(0.2097)}{0.7247} = 4.63 \text{ kip} \cdot \text{in}$$

Choose the smaller value. $M = 4.63 \text{ kip} \cdot \text{in}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.195

In order to increase corrosion resistance, a 2-mm-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is 200 GPa for steel and 70 GPa for aluminum. For a bending moment of 300 N · m, determine (*a*) the maximum stress in the steel, (*b*) the maximum stress in the aluminum, (*c*) the radius of curvature of the bar.

SOLUTION

Use aluminum as the reference material.

$n = 1$ in aluminum. $\qquad n = \dfrac{E_s}{E_a} = \dfrac{200}{70} = 2.857$ in steel.

Cross section geometry:

Steel: $A_s = (46\,\text{mm})(26\,\text{mm}) = 1196\,\text{mm}^2 \qquad I_s = \dfrac{1}{12}(46\,\text{mm})(26\,\text{mm})^3 = 67{,}375\,\text{mm}^4$

Aluminum: $A_a = (50\,\text{mm})(30\,\text{mm}) - 1196\,\text{mm}^2 = 304\,\text{mm}^2$

$$I_a = \frac{1}{12}(50\,\text{mm})(30\,\text{mm}^3) - 67{,}375\,\text{mm}^4 = 45{,}125\,\text{mm}^4$$

Transformed section.

$$I = n_a I_a + n_s I_s = (1)(45{,}125) + (2.857)(67{,}375) = 237{,}615\,\text{mm}^4 = 237.615 \times 10^{-9}\,\text{m}^4$$

Bending moment. $M = 300\,\text{N} \cdot \text{m}$

(*a*) Maximum stress in steel: $\qquad n_s = 2.857 \qquad y_s = 13\,\text{mm} = 0.013\,\text{m}$

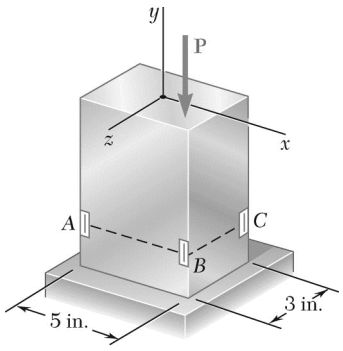
$$\sigma_s = \frac{n_s M y_s}{I} = \frac{(2.857)(300)(0.013)}{237.615 \times 10^{-9}} = 46.9 \times 10^6\,\text{Pa} \qquad \sigma_s = 46.9\,\text{MPa} \blacktriangleleft$$

(*b*) Maximum stress in aluminum: $\qquad n_a = 1, \qquad y_a = 15\,\text{mm} = 0.015\,\text{m}$

$$\sigma_a = \frac{n_a M y_a}{I} = \frac{(1)(300)(0.015)}{237.615 \times 10^{-9}} = 18.94 \times 10^6\,\text{Pa} \qquad \sigma_a = 18.94\,\text{MPa} \blacktriangleleft$$

(*c*) Radius of curvative: $\rho = \dfrac{EI}{M} \qquad \rho = \dfrac{(70 \times 10^9)(237.615 \times 10^{-9})}{300} \qquad \rho = 55.4\,\text{m} \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.196

A single vertical force **P** is applied to a short steel post as shown. Gages located at *A, B,* and *C* indicate the following strains:

$$\in_A = -500\mu \qquad \in_B = -1000\mu \qquad \in_C = -200\mu$$

Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4 \qquad I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in}^4 \qquad A = (5)(3) = 15 \text{ in}^2$$

$$M_x = Pz \qquad M_z = -Px$$

$$x_A = -2.5 \text{ in.}, \quad x_B = 2.5 \text{ in.}, \quad x_C = 2.5 \text{ in.}, \quad x_D = -2.5 \text{ in.}$$

$$z_A = 1.5 \text{ in.}, \quad z_B = 1.5 \text{ in.}, \quad z_C = -1.5 \text{ in.}, \quad z_D = -1.5 \text{ in.}$$

$$\sigma_A = E\varepsilon_A = (29 \times 10^6)(-500 \times 10^{-6}) = -14{,}500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\varepsilon_B = (29 \times 10^6)(-1000 \times 10^{-6}) = -29{,}000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29 \times 10^6)(-200 \times 10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667\,\text{P} - 0.13333\,M_x - 0.08\,M_z \tag{1}$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667\,\text{P} - 0.13333\,M_x + 0.08\,M_z \tag{2}$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667\,\text{P} + 0.13333\,M_x + 0.08\,M_z \tag{3}$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87 \text{ kip} \cdot \text{in}, \quad M_z = -90.625 \text{ kip} \cdot \text{in}, \qquad (a)\ P = 152.25 \text{ kips} \blacktriangleleft$$

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25} \qquad (b)\ x = 0.595 \text{ in.} \blacktriangleleft$$

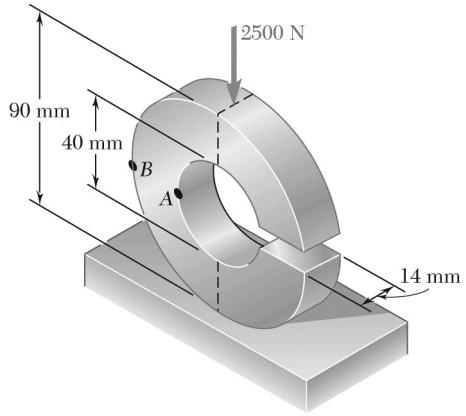
$$z = \frac{M_x}{P} = \frac{87}{152.25} \qquad z = 0.571 \text{ in.} \blacktriangleleft$$

$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667\,\text{P} + 0.13333\,M_x - 0.08\,M_z$$

$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625) = 8.70 \text{ ksi}$$

(*c*) Strain at hidden edge: $\varepsilon = \dfrac{\sigma_D}{E} = \dfrac{8.70 \times 10^3}{29 \times 10^6}$ $\varepsilon = 300\,u$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.197

For the split ring shown, determine the stress at (*a*) point *A*, (*b*) point *B*.

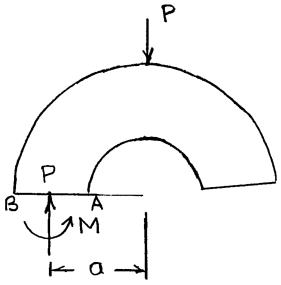
SOLUTION

$$r_1 = \frac{1}{2}40 = 20 \text{ mm}, \qquad r_2 = \frac{1}{2}(90) = 45 \text{ mm} \qquad h = r_2 - r_1 = 25 \text{ mm}$$

$$A = (14)(25) = 350 \text{ mm}^2 \qquad R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 32.5 \text{ mm} \qquad e = \bar{r} - R = 1.6712 \text{ mm}$$

Reduce the internal forces transmitted across section *AB* to a force-couple system at the centroid of the cross section. The bending couple is



$$M = Pa = P\bar{r} = (2500)(32.5\times10^{-3}) = 81.25 \text{ N}\cdot\text{m}$$

(*a*) Point *A*: $r_A = 20$ mm $\quad y_A = 30.8288 - 20 = 10.8288$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{AeR} = -\frac{2500}{350\times10^{-6}} - \frac{(81.25)(10.8288\times10^{-3})}{(350\times10^{-6})(1.6712\times10^{-3})(20\times10^{-3})}$$

$$= -82.4\times10^6 \text{ Pa}$$

$\sigma_A = -82.4$ MPa ◀

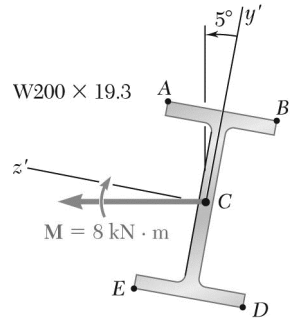
(*b*) Point *B*: $r_B = 45$ mm $\quad y_B = 30.8288 - 45 = -14.1712$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_B} = -\frac{2500}{350\times10^{-6}} - \frac{(81.25)(-14.1712\times10^{-3})}{(350\times10^{-6})(1.6712\times10^{-3})(45\times10^{-3})}$$

$$= 36.6\times10^6 \text{ Pa}$$

$\sigma_B = 36.6$ MPa ◀

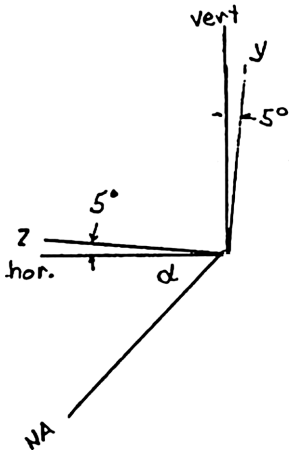
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.198

A couple **M** of moment 8 kN · m acting in a vertical plane is applied to a W200 × 19.3 rolled-steel beam as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum stress in the beam.

SOLUTION



For W200 × 19.3 rolled steel sector,

$$I_{z'} = 16.6 \times 10^6 \text{ mm}^4 = 16.6 \times 10^{-6} \text{ m}^4$$

$$I_{y'} = 1.15 \times 10^6 \text{ mm}^4 = 1.15 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = \frac{203}{2} = 101.5 \text{ mm}$$

$$z_A = -z_B = -z_D = z_E = \frac{102}{2} = 51 \text{ mm}$$

$$M_z = (8 \times 10^3)\cos 5° = 7.9696 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_y = -(8 \times 10^3)\sin 5° = -0.6972 \times 10^3 \text{ N} \cdot \text{m}$$

(*a*) $$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{16.6 \times 10^{-6}}{1.15 \times 10^{-6}}\tan(-5°) = -1.2629$$

$$\varphi = -51.6°$$

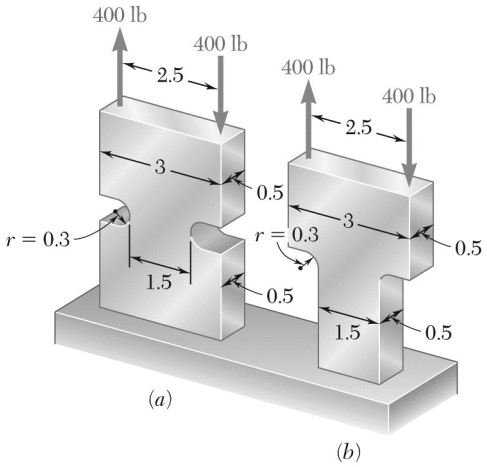
$$\alpha = 51.6° - 5° \qquad \alpha = 46.6° \blacktriangleleft$$

(*b*) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(7.9696 \times 10^3)(-101.5 \times 10^{-3})}{16.6 \times 10^{-6}} + \frac{(-0.6972 \times 10^3)(-51 \times 10^{-3})}{1.15 \times 10^{-6}}$$

$$= 79.6 \times 10^6 \text{ Pa} \qquad \sigma_D = 79.6 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.199

Determine the maximum stress in each of the two machine elements shown.

SOLUTION

For each case, $M = (400)(2.5) = 1000 \text{ lb} \cdot \text{in}$

At the minimum section,

$$I = \frac{1}{12}(0.5)(1.5)^3 = 0.140625 \text{ in}^4$$

$$c = 0.75 \text{ in.}$$

(*a*) $D/d = 3/1.5 = 2$

$r/d = 0.3/1.5 = 0.2$

From Fig 4.32, $K = 1.75$

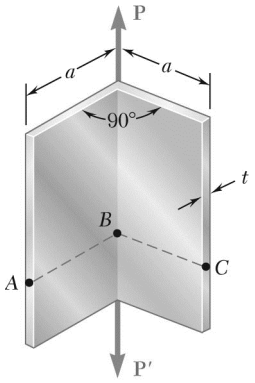
$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3 \text{ psi} \qquad \sigma_{\max} = 9.33 \text{ ksi} \blacktriangleleft$$

(*b*) $D/d = 3/1.5 = 2$ $r/d = 0.3/1.5 = 0.2$

From Fig. 4.31, $K = 1.50$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.50)(1000)(0.75)}{0.140625} = 8.00 \times 10^3 \text{ psi} \qquad \sigma_{\max} = 8.00 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

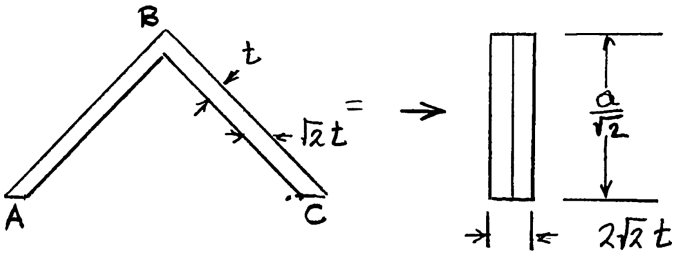


PROBLEM 4.200

The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.

SOLUTION

Moment of inertia about centroid:



$$I = \frac{1}{12}\left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right)^3$$

$$= \frac{1}{12}ta^3$$

Area: $A = \left(2\sqrt{2}t\right)\left(\frac{a}{\sqrt{2}}\right) = 2at, \quad c = \frac{a}{2\sqrt{2}}$

(a) $$\sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3}$$

$$\sigma_A = -\frac{P}{2at} \blacktriangleleft$$

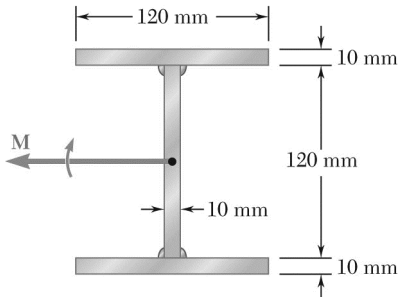
(b) $$\sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}ta^3}$$

$$\sigma_B = -\frac{2P}{at} \blacktriangleleft$$

(c) $$\sigma_C = \sigma_A$$

$$\sigma_C = -\frac{P}{2at} \blacktriangleleft$$

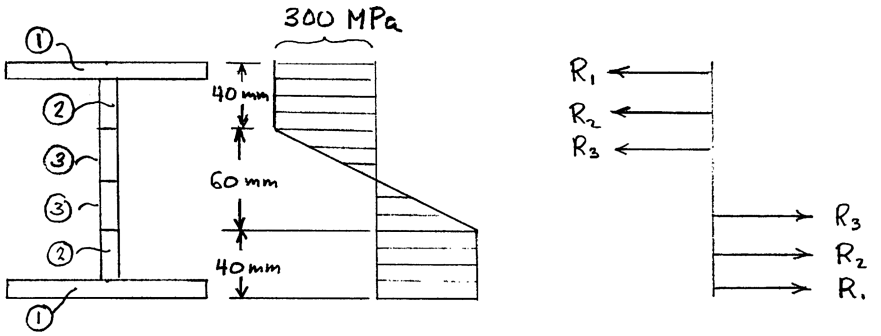
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 4.201

Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.

SOLUTION



$$A_1 = (120)(10) = 1200 \text{ mm}^2$$

$$R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3 \text{ N}$$

$$A_2 = (30)(10) = 300 \text{ mm}^2$$

$$R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3 \text{ N}$$

$$A_3 = (30)(10) = 300 \text{ mm}^2$$

$$R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6}) = 45\times10^3 \text{ N}$$

$$y_1 = 65 \text{ mm} = 65\times10^{-3} \text{ m} \qquad y_2 = 45 \text{ mm} = 45\times10^{-3} \text{ m} \qquad y_3 = 20 \text{ mm} = 20\times10^{-3} \text{ m}$$

(*a*) $$M = 2(R_1y_1 + R_2y_2 + R_3y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$$

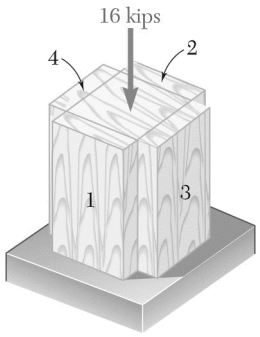
$$= 56.7\times10^3 \text{ N}\cdot\text{m}$$

$$M = 56.7 \text{ kN}\cdot\text{m} \blacktriangleleft$$

(*b*) $$\frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \qquad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200\times10^9)(30\times10^{-3})}{300\times10^6}$$

$$\rho = 20 \text{ m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

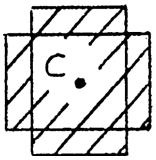


PROBLEM 4.202

A short column is made by nailing four 1×4-in. planks to a 4×4-in. timber. Determine the largest compressive stress created in the column by a 16-kip load applied as shown in the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) planks 1 and 2 are removed, (*d*) planks 1, 2, and 3 are removed, (*e*) all planks are removed.

SOLUTION

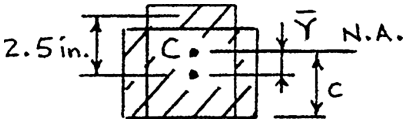
(*a*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$



$$A = (4)(4) + (4)(1)(4) = 32 \text{ in}^2$$

$$\sigma = -\frac{16 \times 10^3}{32} \qquad \sigma = -500 \text{ psi} \blacktriangleleft$$

(*b*) Eccentric loading: $M = Pe \qquad \sigma = -\dfrac{P}{A} - \dfrac{Pec}{I}$



$$A = (4)(4) + (3)(1)(3) = 28 \text{ in}^2 \qquad e = \bar{y}$$

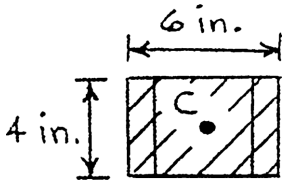
$$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{(1)(4)(2.5)}{28} = 0.35714 \text{ in.}$$

$$I = \Sigma(\bar{I} + Ad^2) = \frac{1}{12}(6)(4)^3 + (6)(4)(0.35714)^2$$

$$+ \frac{1}{12}(4)(1)^3 + (4)(1)(2.14286)^2 = 53.762 \text{ in}^4$$

$$\sigma = -\frac{16 \times 10^3}{28} - \frac{(16 \times 10^3)(0.35714)(2.35714)}{53.762} \qquad \sigma = -822 \text{ psi} \blacktriangleleft$$

(*c*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$



$$A = (6)(4) = 24 \text{ in}^2$$

$$\sigma = -\frac{16 \times 10^3}{24} \qquad \sigma = -667 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

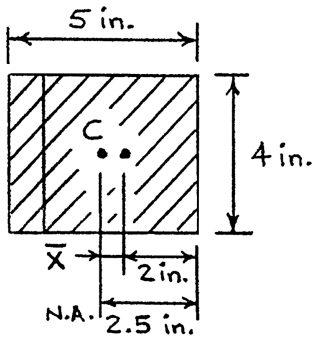
PROBLEM 4.202 *(Continued)*

(*d*) Eccentric loading: $M = Pe \qquad \sigma = -\dfrac{P}{A} - \dfrac{Pec}{I}$

$A = (4)(1) = (1)(4)(1) = 20 \text{ in}^2 \qquad e = \bar{x}$

$\bar{x} = 2.5 - 2 = 0.5 \text{ in.}$

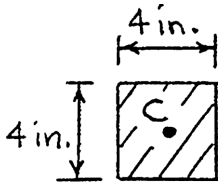
$I = \dfrac{1}{12}(4)(5)^3 = 41.667 \text{ in}^4$



$$\sigma = -\frac{16\times10^3}{20} - \frac{(16\times10^3)(0.5)(2.5)}{41.667}$$

$\sigma = -1280 \text{ psi}$ ◀

(*e*) Centric loading: $M = 0 \qquad \sigma = -\dfrac{P}{A}$



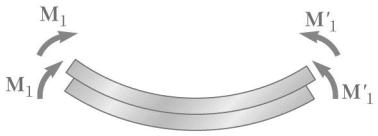
$A = (4)(4) = 16 \text{ in}^2$

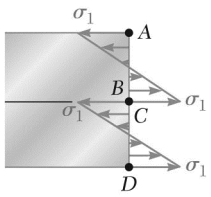
$$\sigma = -\frac{16\times10^3}{16}$$

$\sigma = -1000 \text{ psi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.203





Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by σ_1 the maximum stress and by ρ_1 the radius of curvature of each strip while the couples were applied, determine (*a*) the final stresses at points *A, B, C,* and *D,* (*b*) the final radius of curvature.

SOLUTION

Let b = width and t = thickness of one strip.

Loading one strip, $M = M_1$

$$I_1 = \frac{1}{12}bt^3, \quad c = \frac{1}{2}t$$

$$\sigma_1 = \frac{M_1 c}{I} = \frac{\sigma M_1}{bt^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{EI_1} = \frac{12M_1}{Et^3}$$

After M_1 is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is $2M_1$.

After the strips are glued together, this couple is removed.

$$M' = 2M_1, \quad I' = \frac{1}{12}b(2t)^3 = \frac{2}{3}bt^3 \quad c = t$$

The stresses removed are

$$\sigma' = -\frac{M'y}{I} = -\frac{2M_1 y}{\frac{2}{3}bt^3} = -\frac{3M_1 y}{bt^2}$$

$$\sigma'_A = -\frac{3M_1}{bt^2} = -\frac{1}{2}\sigma_1, \quad \sigma'_B = \sigma'_C = 0, \quad \sigma'_D = \frac{3M_1}{bt^2} = \frac{1}{2}\sigma_1$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 4.203 *(Continued)*

(*a*) Final stresses: $\sigma_A = -\sigma_1 - (-\frac{1}{2}\sigma_1)$ $\sigma_A = -\frac{1}{2}\sigma_1$ ◀

$\sigma_B = \sigma_1$ ◀

$\sigma_C = -\sigma_1$ ◀

$\sigma_D = \sigma_1 - \frac{1}{2}\sigma_1$ $\sigma_D = \frac{1}{2}\sigma_1$ ◀

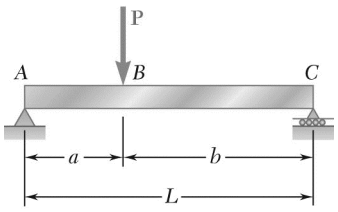
$$\frac{1}{\rho'} = \frac{M'}{EI'} = \frac{2M_1}{E\frac{2}{3}bt^3} = \frac{3M_1}{Et^3} = \frac{1}{4}\frac{1}{\rho'}$$

(*b*) Final radius: $\frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4}\frac{1}{\rho_1} = \frac{3}{4}\frac{1}{\rho_1}$

$\rho = \frac{4}{3}\rho_1$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

CHAPTER 5



PROBLEM 5.1

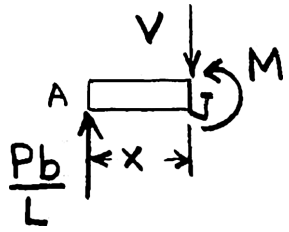
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:

$$(+\Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

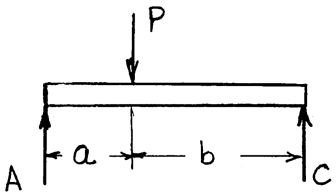
$$+)\Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



From *A* to *B*: $0 < x < a$

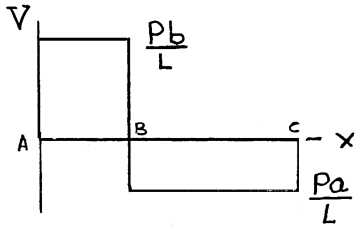
$$+\uparrow \Sigma F_y = 0: \quad \frac{Pb}{L} - V = 0$$

$$V = \frac{Pb}{L} \blacktriangleleft$$

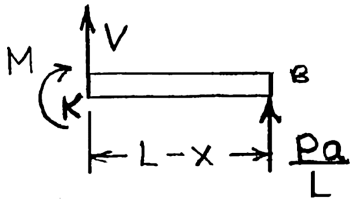


$$+)\Sigma M_J = 0: \quad M - \frac{Pb}{L}x = 0$$

$$M = \frac{Pbx}{L} \blacktriangleleft$$

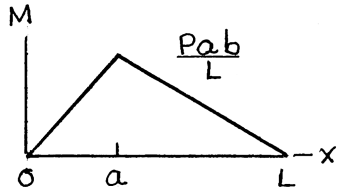


From *B* to *C*: $a < x < L$



$$+\uparrow \Sigma F_y = 0: \quad V + \frac{Pa}{L} = 0$$

$$V = -\frac{Pa}{L} \blacktriangleleft$$



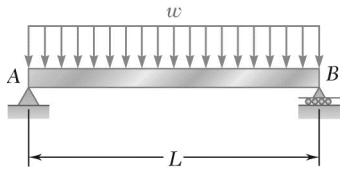
$$+)\Sigma M_K = 0: \quad -M + \frac{Pa}{L}(L - x) = 0$$

$$M = \frac{Pa(L - x)}{L} \blacktriangleleft$$

At section *B*:

$$M = \frac{Pab}{L^2} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

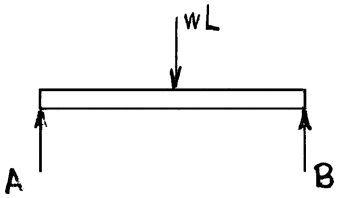


PROBLEM 5.2

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

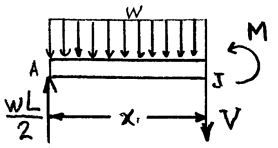
Reactions:



$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

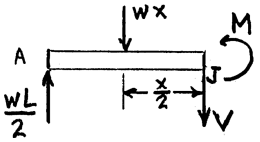
Free body diagram for determining reactions.



Over whole beam, $0 < x < L$

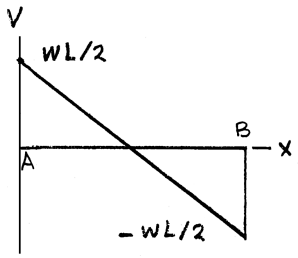
Place section at x.

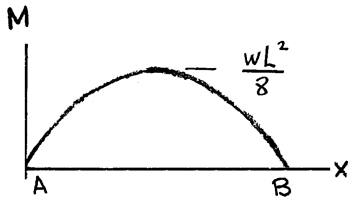
Replace distributed load by equivalent concentrated load.



$$+\uparrow \Sigma F_y = 0: \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

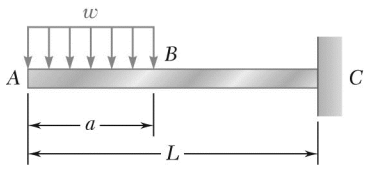
$$M = \frac{w}{2}(Lx - x^2)$$

$$M = \frac{w}{2}x(L - x) \blacktriangleleft$$

Maximum bending moment occurs at $x = \frac{L}{2}$.

$$M_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

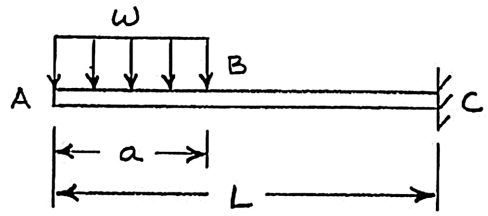
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

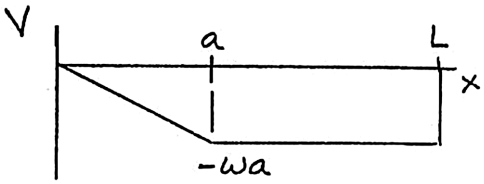


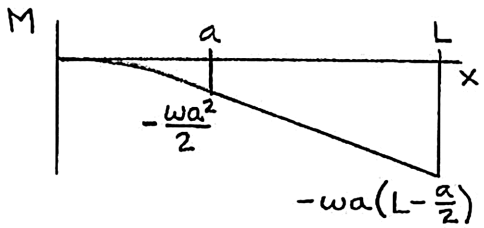
PROBLEM 5.3

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

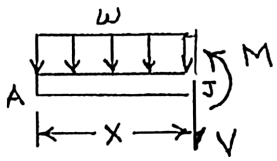
SOLUTION







From A to B $(0 < x < a)$:



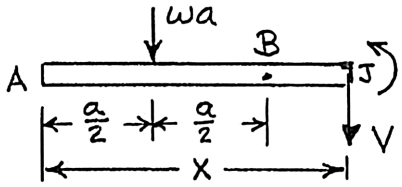
$+\uparrow \sum F_y = 0: \quad -wx - V = 0$

$$V = -wx \blacktriangleleft$$

$+\circlearrowleft \sum M_J = 0: \quad (wx)\frac{x}{2} + M = 0$

$$M = -\frac{wx^2}{2} \blacktriangleleft$$

From B to C $(a < x < L)$:



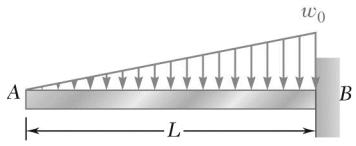
$+\uparrow \sum F_y = 0: \quad -wa - V = 0$

$$V = -wa \blacktriangleleft$$

$+\circlearrowleft \sum M_J = 0: \quad (wa)\left(x - \frac{a}{2}\right) + M = 0$

$$M = -wa\left(x - \frac{a}{2}\right) \blacktriangleleft$$

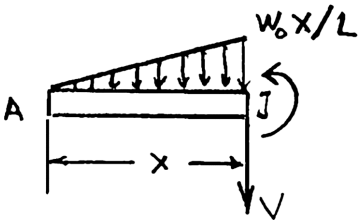
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

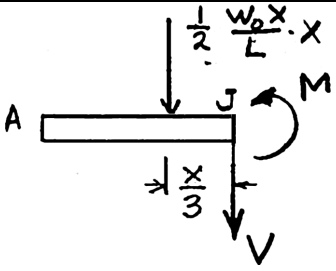


PROBLEM 5.4

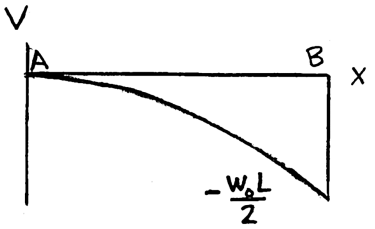
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION





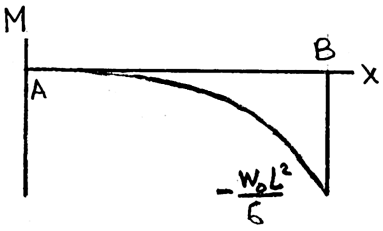
$$+\uparrow \Sigma F_y = 0: \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$$



$$V = -\frac{w_0 x^2}{2L} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x\cdot\frac{x}{3} + M = 0$$

$$M = -\frac{w_0 x^3}{6L} \blacktriangleleft$$

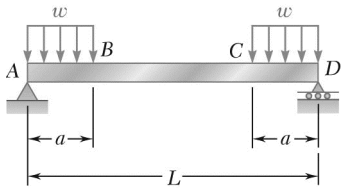


At $x = L$,

$$V = -\frac{w_0 L}{2} \qquad |V|_{\max} = \frac{w_0 L}{2} \blacktriangleleft$$

$$M = -\frac{w_0 L^2}{6} \qquad |M|_{\max} = \frac{w_0 L^2}{6} \blacktriangleleft$$

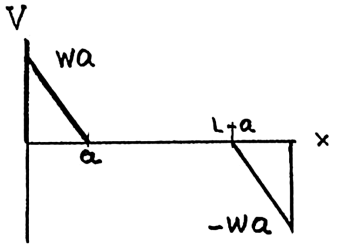
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

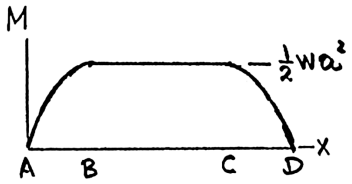


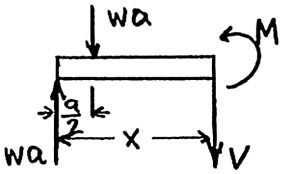
PROBLEM 5.5

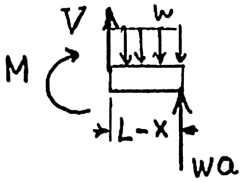
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

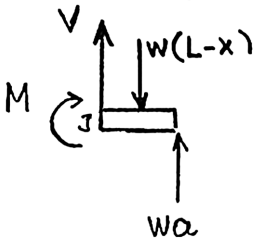
SOLUTION





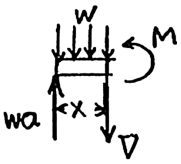


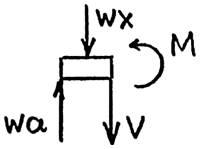




Reactions: $A = D = wa$

From A to B: $0 < x < a$





$+\uparrow \sum F_y = 0: \quad wa - wx - V = 0$

$$V = w(a - x) \blacktriangleleft$$

$+\circlearrowleft \sum M_J = 0: \quad -wax + (wx)\frac{x}{2} + M = 0$

$$M = w\left(ax - \frac{x^2}{2}\right) \blacktriangleleft$$

From B to C: $a < x < L - a$

$\sum F_y = 0: \quad wa - wa - V = 0$

$$V = 0 \blacktriangleleft$$

$+\circlearrowleft \sum M_J = 0: \quad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2 \blacktriangleleft$

From C to D: $L - a < x < L$

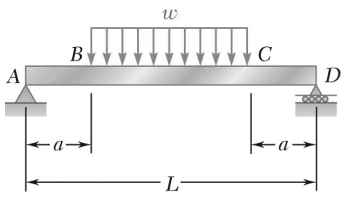
$+\uparrow \sum F_y = 0: \quad V - w(L - x) + wa = 0$

$$V = w(L - x - a) \blacktriangleleft$$

$+\circlearrowleft \sum M_J = 0: \quad -M - w(L - x)\left(\frac{L - x}{2}\right) + wa(L - x) = 0$

$$M = wa[(L - x) - \frac{1}{2}(L - x)^2] \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.6

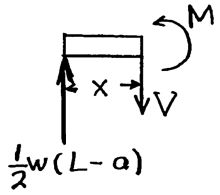
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Calculate reactions after replacing distributed load by an equivalent concentrated load.

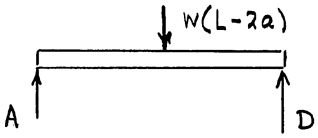
Reactions are

$$A = D = \frac{1}{2}w(L - 2a)$$



From *A* to *B*: $0 < x < a$

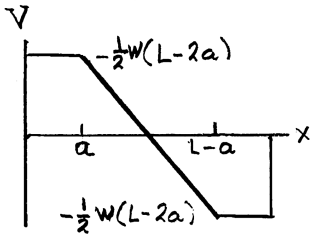
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L - 2a) - V = 0$$



$$V = \frac{1}{2}w(L - 2a) \blacktriangleleft$$

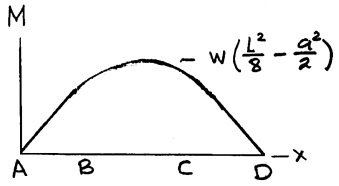
$$+\circlearrowleft \Sigma M = 0: \quad -\frac{1}{2}w(L - 2a) + M = 0$$

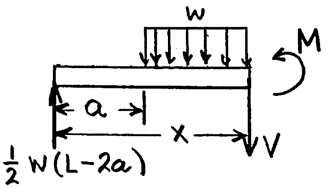
$$M = \frac{1}{2}w(L - 2a)x \blacktriangleleft$$

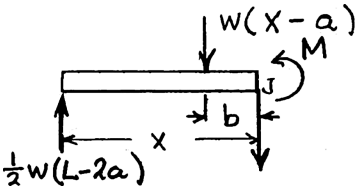


From *B* to *C*: $a < x < L - a$

$$b = \frac{x - a}{2}$$







Place section cut at *x*. Replace distributed load by equivalent concentrated load.

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L - 2a) - w(x - a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$

$$+\circlearrowleft M_J = 0: \quad -\frac{1}{2}w(L - 2a)x + w(x - a)\left(\frac{x - a}{2}\right) + M = 0$$

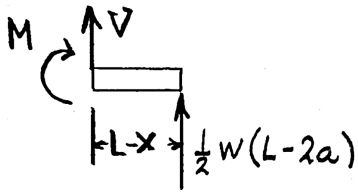
$$M = \frac{1}{2}w[(L - 2a)x - (x - a)^2] \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.6 ***(Continued)***

From C to D: $\quad L - a < x < L$

$$+\uparrow \Sigma F_y = 0: \quad V + \frac{1}{2}w(L - 2a) = 0$$



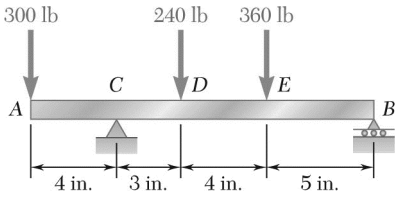
$$V = -\frac{w}{2}(L - 2a) \blacktriangleleft$$

$$+) \Sigma M_J = 0: \quad -M + \frac{1}{2}w(L - 2a)(L - x) = 0$$

$$M = \frac{1}{2}w(L - 2a)(L - x) \blacktriangleleft$$

At $x = \dfrac{L}{2}$, $\qquad M_{\max} = w\left(\dfrac{L^2}{8} - \dfrac{a^2}{2}\right) \blacktriangleleft$

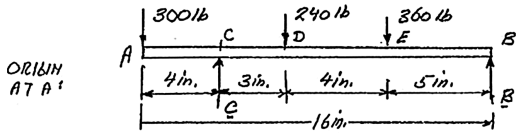
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.7

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

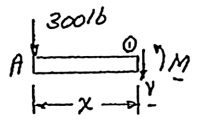
SOLUTION



Reactions:

$$+\circlearrowleft \sum M_C = 0: \quad (300)(4) - (240)(3) - (360)(7) + 12B = 0 \qquad \underline{\boldsymbol{B}} = 170 \text{ lb} \uparrow$$

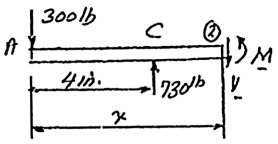
$$+\uparrow \sum F_y = 0: \quad -300 + C - 240 - 360 + 170 = 0 \qquad \underline{\boldsymbol{C}} = 730 \text{ lb} \uparrow$$



From A to C:

$$+\uparrow \sum F_y = 0: \quad -300 - V = 0 \qquad V = -300 \text{ lb}$$

$$+\circlearrowleft \sum M_1 = 0: \quad (300)(x) + M = 0 \qquad M = -300x$$

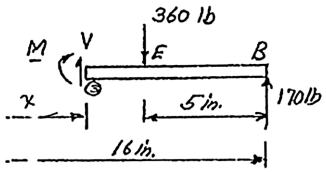


From C to D:

$$+\uparrow \sum F_y = 0: \quad -300 + 730 - V = 0 \qquad V = +430 \text{ lb}$$

$$+\circlearrowleft \sum M_2 = 0: \quad (300)x - (730)(x - 4) + M = 0$$

$$M = -2920 + 430x$$

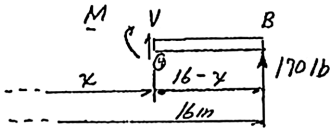


From D to E:

$$+\uparrow \sum F_y = 0: \quad V - 360 + 170 = 0 \qquad V = +190 \text{ lb}$$

$$+\circlearrowleft \sum M_3 = 0: \quad (170)(16 - x) - (360)(11 - x) - M = 0$$

$$M = -1240 + 190x$$

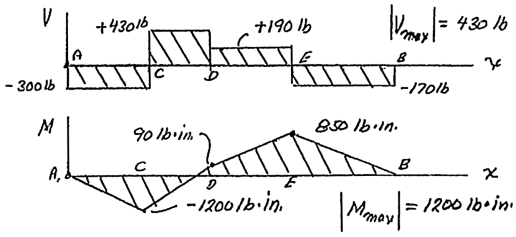


From E to B:

$$+\uparrow \sum F_y = 0: \quad V + 170 = 0 \qquad V = -170 \text{lb}$$

$$+\circlearrowleft \sum M_4 = 0: \quad (170)(16 - x) - M = 0$$

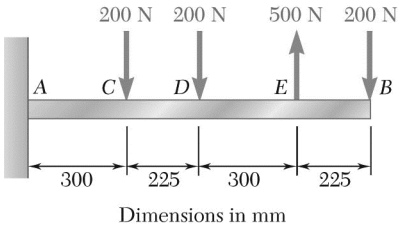
$$M = 2720 - 170x$$



(*a*) $\quad |V|_{\max} = 430 \text{ lb}$ ◀

(*b*) $\quad |M|_{\max} = 1200 \text{ lb} \cdot \text{in}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Dimensions in mm

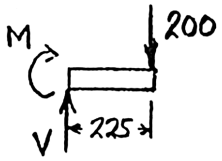
PROBLEM 5.8

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

At B, $V = 200\text{N}, \quad M = 0$

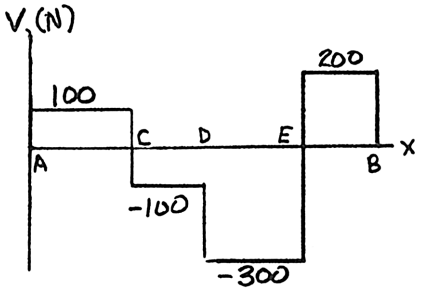
At E^+,



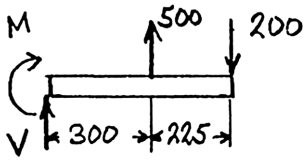
$+\uparrow \Sigma F_y = 0: \quad V - 200 = 0 \quad V = 200\text{N}$

$+\circlearrowleft \Sigma M_E = 0: \quad -M - (0.225)(200) = 0$

$M = -45\ \text{N} \cdot \text{m}$

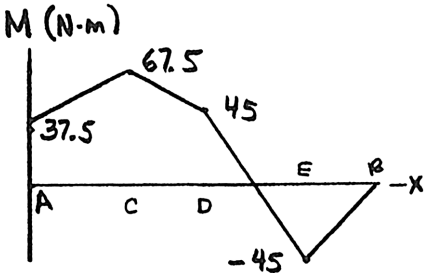


At D^+,



$+\uparrow \Sigma F_y = 0: \quad V + 500 - 200 = 0$

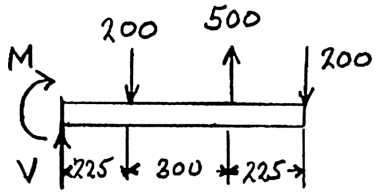
(*a*) $V = -300\ \text{N}$ ◀



$+\circlearrowleft \Sigma M_D = 0: \quad -M + (0.3)(500) - (0.525)(200) = 0$

$M = 45\ \text{N} \cdot \text{m}$

At C^+,



$+\uparrow \Sigma F_y = 0: \quad V - 200 + 500 - 200 = 0 \quad V = -100\text{N}$

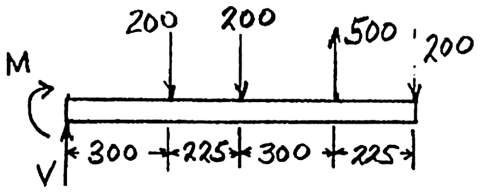
$+\circlearrowleft \Sigma M_C = 0: \quad -M - (0.225)(200) + (0.525)(500) - (0.75)(200) = 0$

(*b*) $M = 67.5\ \text{N} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.8 *(Continued)*

At A,

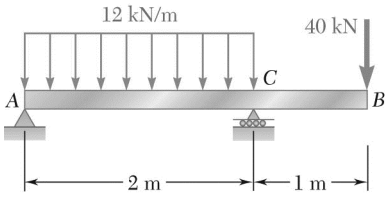


$$+\uparrow \sum F_y = 0: \quad V - 200 - 200 + 500 - 200 = 0 \qquad V = 100 \text{ N}$$

$$+\circlearrowleft \sum M_A = 0: \quad -M - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$$

$$M = 37.5 \text{ N} \cdot \text{m}$$

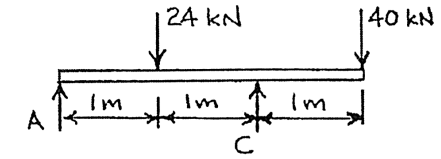
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

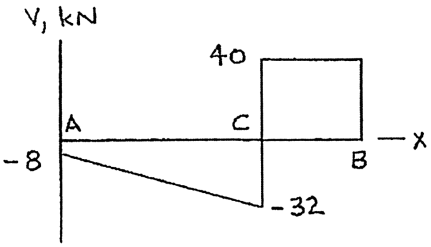


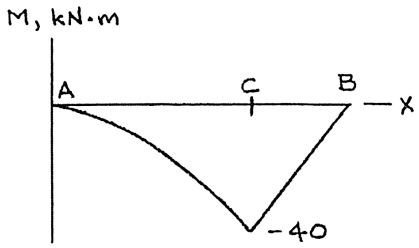
PROBLEM 5.9

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION







Reactions:

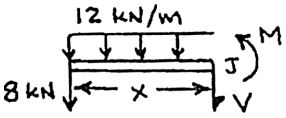
$$+\circlearrowleft \Sigma M_C = 0: \quad -2A + (1)(24) - (1)(40) = 0$$

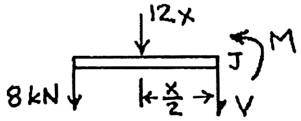
$$\boldsymbol{A} = -8\,\text{kN} = 8\,\text{kN} \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2C - (1)(24) - (3)(40) = 0$$

$$\boldsymbol{C} = 72\,\text{kN} = 72\,\text{kN} \uparrow$$

<u>*A* to *C*.</u> $0 < x < 2\text{m}$



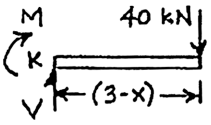


$$+\uparrow \Sigma F_y = 0: \quad -8 - 12x - V = 0 \quad V = (-8 - 12x)\,\text{kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -8x - (12x)\left(\frac{x}{2}\right) - M = 0$$

$$M = (-8x - 6x^2)\,\text{kN} \cdot \text{m}$$

<u>*C* to *B*.</u> $2\text{m} < x < 3\text{m}$



$$+\uparrow \Sigma F_y = 0: \quad V - 40 = 0$$

$$V = 40\,\text{kN}$$

$$+\circlearrowleft \Sigma M_K = 0: \quad -M - (3 - x)(40) = 0$$

$$M = (40x - 120)\,\text{kN} \cdot \text{m}$$

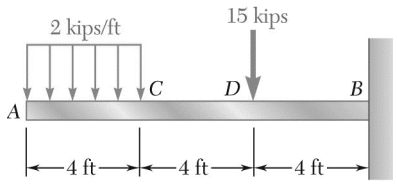
From the diagrams,

(*a*) $|V|_{\max} = 40.0\,\text{kN}$ ◀

(*b*) $|M|_{\max} = 40.0\,\text{kN} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.10

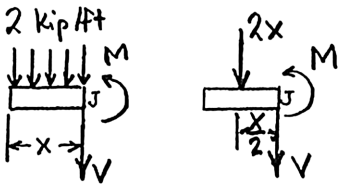


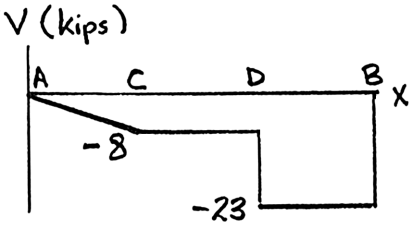
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

A to *C*:

$0 < x < 4$ ft



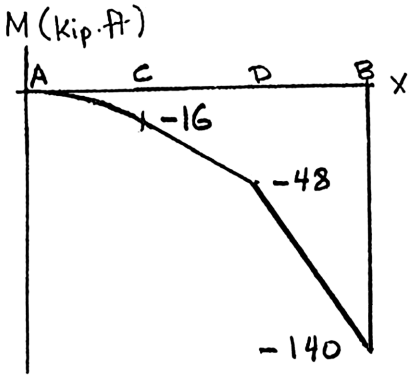


$+\uparrow \sum F_y = 0: \quad -V - 2x = 0 \quad V = -2x$ kips

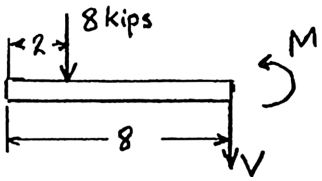
$+\circlearrowleft \sum M_J = 0: \quad M + (2x)\left(\frac{x}{2}\right) = 0$

$M = -x$ kip · ft

At *C*, $V = -8$ kips $\quad M = -16$ kip · ft



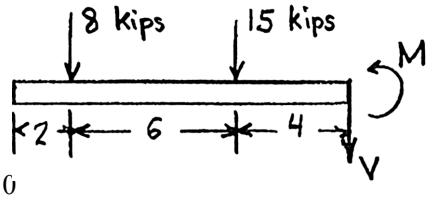
At D^-,



$+\uparrow \sum F_y = 0: \quad -8 - V = 0 \quad V = -8$ kips

$+\circlearrowleft \sum M_D = 0: \quad (6)(8) - M = 0 \quad M = -48$ kip · ft

At B^-,



$+\uparrow \sum F_y = 0: \quad -8 - 15 - V = 0$

$V = -23$ kips

$+\circlearrowleft \sum M_B = 0: \quad -(10)(8) - (4)(15) - M = 0$

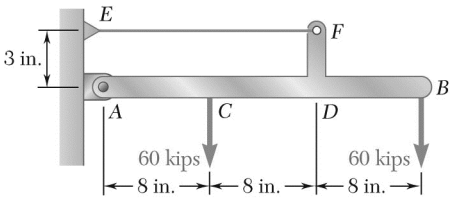
$M = -140$ kip · ft

From the diagrams:

(*a*) $|V|_{\max} = 23.0$ kips ◀

(*b*) $|M|_{\max} = 140.0$ kip · ft ◀

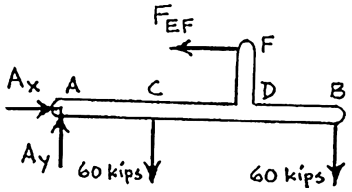
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.11

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



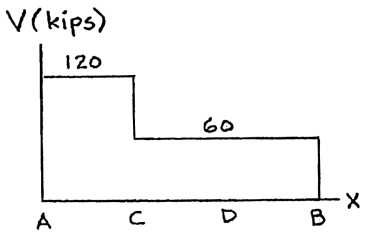
Reactions:

$$+\circlearrowleft \Sigma M_A = 0: \quad 3F_{EF} - (8)(60) - (24)(60) = 0$$

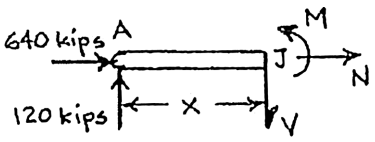
$$F_{EF} = 640 \text{ kips}$$

$$\pm \Sigma F_x = 0: \quad A_x - 640 = 0 \quad A_x = 640 \text{ kips} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 60 - 60 = 0 \quad A_y = 120 \text{ kips} \uparrow$$



From *A* to *C*: $(0 < x < 8 \text{ in.})$

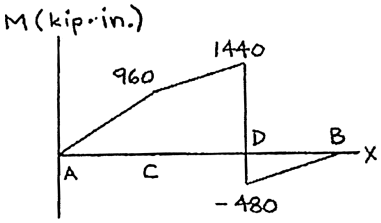


$$+\uparrow \Sigma F_y = 0:$$

$$120 - V = 0$$

$$V = 120 \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - 120x = 0 \quad M = 120x \text{ kip} \cdot \text{in}$$

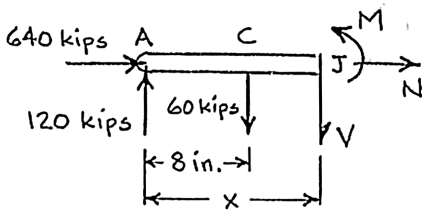


From *C* to *D*: $(8 \text{ in.} < x < 16 \text{ in.})$

$$+\uparrow \Sigma F_Y = 0: \quad 120 - 60 - V = 0 \quad V = 60 \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - 120x + 60(x - 8) = 0$$

$$M = (60x + 480) \text{ kips} \cdot \text{in}$$

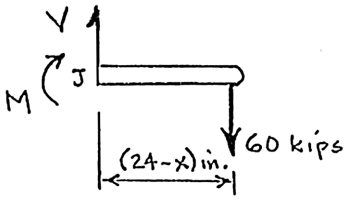


From *D* to *B*: $(16 \text{ in.} < x < 24 \text{ in.})$

$$+\uparrow \Sigma F_y = 0: \quad V - 60 = 0 \quad V = 60 \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - 60(24 - x) = 0$$

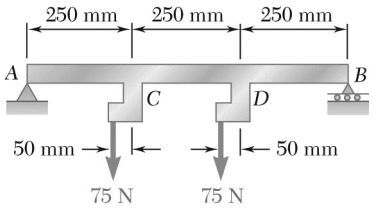
$$M = (60x - 1440) \text{ kip} \cdot \text{in}$$



(*a*) $|V|_{\max} = 120.0 \text{ kips}$ ◀

(*b*) $|M|_{\max} = 1440 \text{ kip} \cdot \text{in} = 120.0 \text{ kip} \cdot \text{ft}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.12

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

Reaction at A: $+\circlearrowleft M_B = 0$: $-0.750\,R_A + (0.550)(75) + (0.300)(75) = 0$

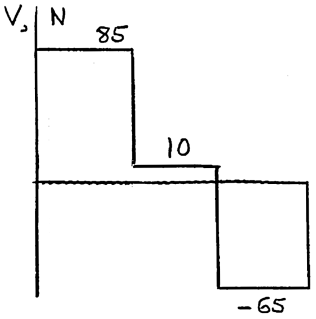
$$\boldsymbol{R}_A = 85\,\text{N}\uparrow$$

Also, $$\boldsymbol{R}_B = 65\,\text{N}\uparrow$$

A to C: $V = 85$ N

C to D: $V = 10$ N

D to B: $V = -65$ N

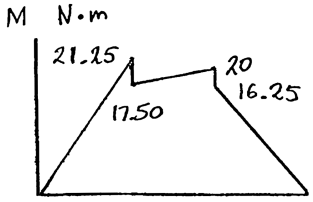


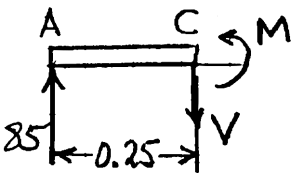
At A and B, $M = 0$

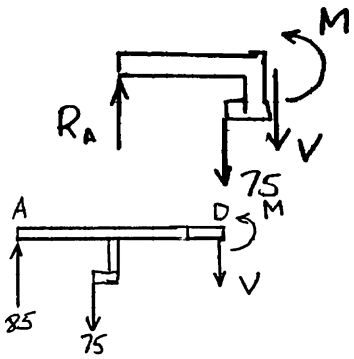
Just to the left of C,

$$+\circlearrowleft \Sigma M_C = 0: \quad -(0.25)(85) + M = 0$$

$$M = 21.25\ \text{N}\cdot\text{m}$$







Just to the right of C,

$$+\circlearrowleft \Sigma M_C = 0: \quad -(0.25)(85) + (0.050)(75) + M = 0$$

$$M = 17.50\ \text{N}\cdot\text{m}$$

Just to the left of D,

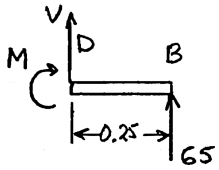
$$+\circlearrowleft \Sigma M_D = 0: \quad -(0.50)(85) + (0.300)(75) + M = 0$$

$$M = 20\ \text{N}\cdot\text{m}$$

Just to the right of D,

$$+\circlearrowleft \Sigma M_D = 0: \quad -M + (0.25)(65) = 0$$

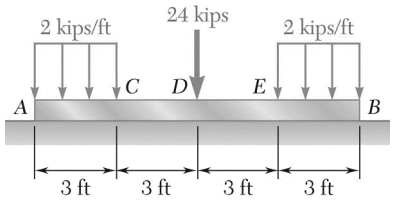
$$M = 16.25\ \text{kN}$$



(*a*) $|V|_{\max} = 85.0$ N ◀

(*b*) $|M|_{\max} = 21.25\ \text{N}\cdot\text{m}$ ◀

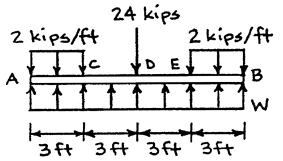
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.13

Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

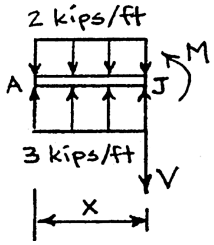
SOLUTION



Over the whole beam,

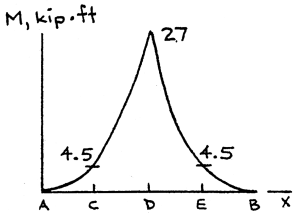
$$+\uparrow \Sigma F_y = 0: \quad 12w - (3)(2) - 24 - (3)(2) = 0 \qquad w = 3 \text{ kips/ft}$$

A to C: $\quad (0 \le x < 3 \text{ ft})$



$$+\uparrow \Sigma F_y = 0: \quad 3x - 2x - V = 0 \qquad V = (x) \text{ kips}$$

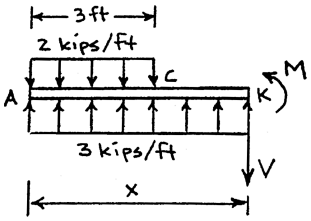
$$+\Sigma M_J = 0: \quad -(3x)\frac{x}{2} + (2x)\frac{x}{2} + M = 0 \qquad M = (0.5x^2) \text{ kip} \cdot \text{ft}$$



At C, $\quad x = 3$ ft

$V = 3$ kips, $\quad M = 4.5$ kip · ft

C to D: $\quad (3 \text{ ft} \le x < 6 \text{ ft})$



$$+\uparrow \Sigma F_y = 0: \quad 3x - (2)(3) - V = 0 \qquad V = (3x - 6) \text{ kips}$$

$$+\circlearrowleft \Sigma MK = 0: \quad -(3x)\left(\frac{x}{2}\right) + (2)(3)\left(x - \frac{3}{2}\right) + M = 0$$

$$M = (1.5x^2 - 6x + 9) \text{ kip} \cdot \text{ft}$$

At D^-, $\quad x = 6$ ft

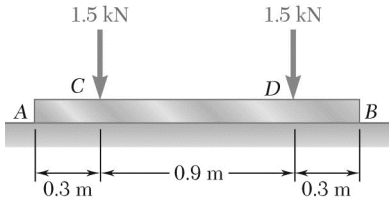
$V = 12$ kips, $\quad M = 27$ kip · ft

D to B: $\quad$ Use symmetry to evaluate.

(a) $\quad |V|_{\max} = 12.00$ kips ◀

(b) $\quad |M|_{\max} = 27.0$ kip · ft ◀

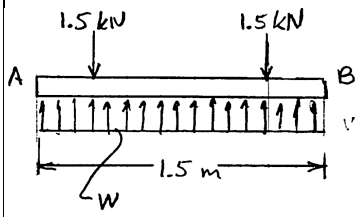
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.14

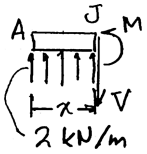
Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

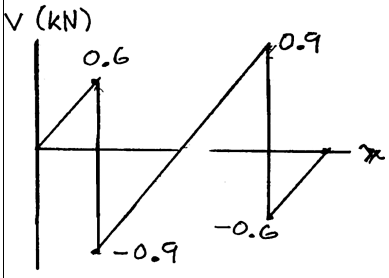
SOLUTION



Over the whole beam,

$$+\uparrow \Sigma F_y = 0: \quad 1.5w - 1.5 - 1.5 = 0 \qquad w = 2 \text{ kN/m}$$

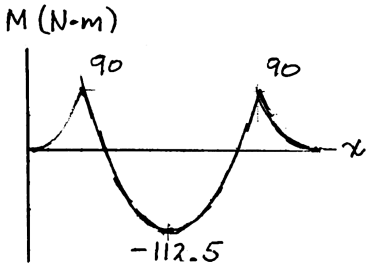




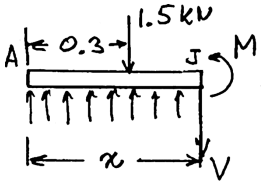
A to C: $\quad 0 \le x < 0.3$ m

$$+\uparrow \Sigma F_y = 0: \quad 2x - V = 0 \qquad V = (2x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -(2x)\left(\frac{x}{2}\right) + M = 0 \qquad M = (x^2) \text{ kN} \cdot \text{m}$$



At C^-: $\quad x = 0.3$ m



$$V = 0.6 \text{ kN}, \quad M = 0.090 \text{ kN} \cdot \text{m} = 90 \text{ N} \cdot \text{m}$$

C to D: $\quad 0.3 \text{ m} < x < 1.2$ m

$$+\uparrow \Sigma F_y = 0: \quad 2x - 1.5 - V = 0 \qquad V = (2x - 1.5) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -(2x)\left(\frac{x}{2}\right) + (1.5)(x - 0.3) + M = 0$$

$$M = (x^2 - 1.5x + 0.45) \text{ kN} \cdot \text{m}$$

At the center of the beam: $x = 0.75$ m

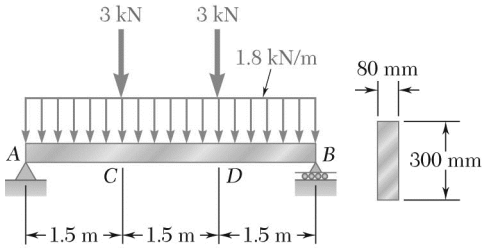
$$V = 0 \qquad M = -0.1125 \text{ kN} \cdot \text{m} = -112.5 \text{ N} \cdot \text{m}$$

At C^+: $\quad x = 0.3$ m, $\quad V = -0.9$ kN

(a) Maximum $|V| = 0.9$ kN $= 900$ N ◀

(b) Maximum $|M| = 112.5$ N · m ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.15

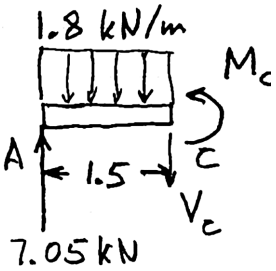
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Reaction at *A*:

$$+\circlearrowleft M_B = 0: \quad -4.5A + (3.0)(3) + (1.5)(3) + (1.8)(4.5)(2.25) = 0 \qquad A = 7.05 \text{ kN} \uparrow$$

Use *AC* as free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (7.05)(1.5) + (1.8)(1.5)(0.75) = 0$$

$$M_C = 8.55 \text{ kN} \cdot \text{m} = 8.55 \times 10^3 \text{ N} \cdot \text{m}$$

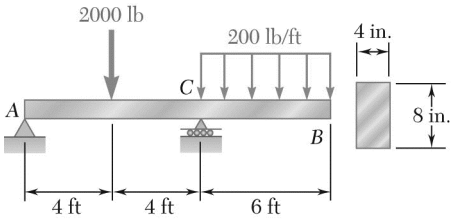
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(80)(300)^3 = 180 \times 10^6 \text{ mm}^4$$

$$= 180 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}(300) = 150 \text{ mm} = 0.150 \text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(8.55 \times 10^3)(0.150)}{180 \times 10^{-6}} = 7.125 \times 10^6 \text{ Pa} \qquad \sigma = 7.13 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

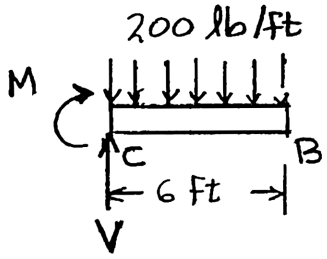


PROBLEM 5.16

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Use *CB* as free body.



$$+\circlearrowleft M_C = 0: \quad -M - (200)(6)\left(\frac{6}{2}\right) = 0$$

$$M = -3600 \text{ lb} \cdot \text{ft}$$

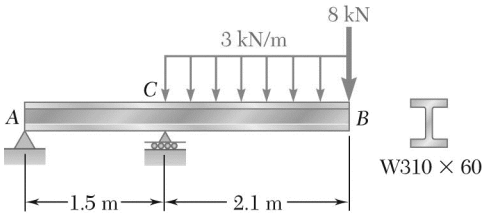
$$= -43.2 \times 10^3 \text{ lb} \cdot \text{in}$$

For rectangular section, $$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(8)^3 = 170.667 \text{ in}^4$$

$$c = \frac{1}{2}h = 4 \text{ in.}$$

$$\sigma = \frac{|M|c}{I} = \frac{(43.2 \times 10^3)(4)}{170.667} = 1.0125 \times 10^3 \text{ psi} \qquad \sigma = 1.013 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.17

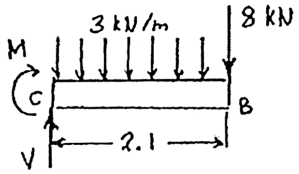
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Use portion CB as free body.

$$+\circlearrowleft \sum M_C = 0: \quad -M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

$$M = 23.415\ \text{kN} \cdot \text{m} = 23.415 \times 10^3\ \text{N} \cdot \text{m}$$



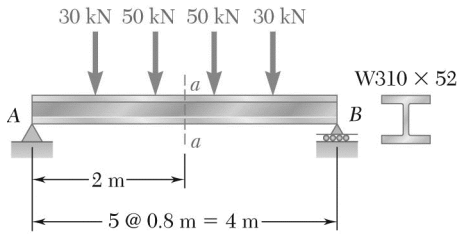
For W310 × 60 : $S = 844 \times 10^3\ \text{mm}^3$

$$= 844 \times 10^{-6}\ \text{m}^3$$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{23.415 \times 10^3}{844 \times 10^{-6}} = 27.7 \times 10^6\ \text{Pa}$

$\sigma = 27.7\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.18

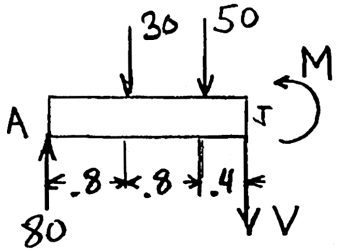
For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION

Reactions: By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad \mathbf{A} = \mathbf{B} = 80\ \text{kN}\uparrow$$

Using left half of beam as free body,



$$+\circlearrowleft \Sigma M_J = 0:$$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

$$M = 104\ \text{kN} \cdot \text{m} = 104 \times 10^3\ \text{N} \cdot \text{m}$$

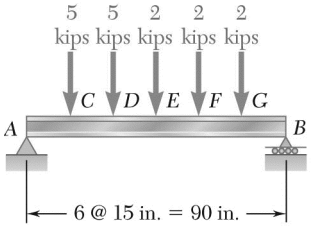
For W310 × 52: $S = 747 \times 10^3\ \text{mm}^3$

$$= 747 \times 10^{-6}\ \text{m}^3$$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6\ \text{Pa}$

$$\sigma = 139.2\ \text{MPa} \blacktriangleleft$$

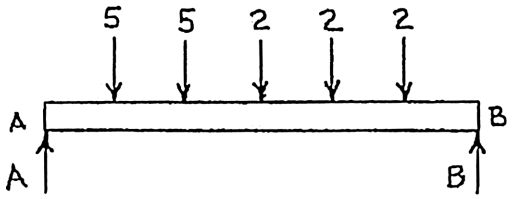
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.19

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



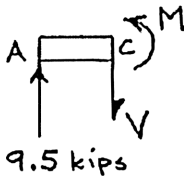
Use entire beam as free body.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$A = 9.5 \text{ kips} \uparrow$$

Use portion AC as free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad M - (15)(9.5) = 0$$

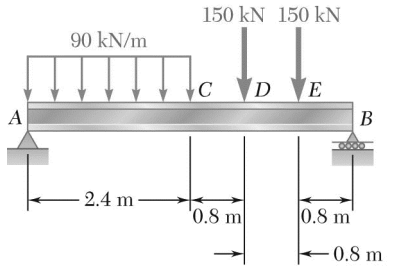
$$M = 142.5 \text{ kip} \cdot \text{in}$$

For $S8 \times 18.4$, $S = 14.4 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$

$$\sigma = 9.90 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

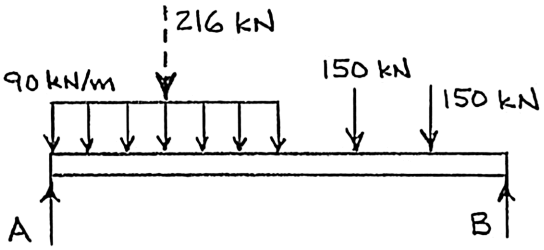


W460 × 113

PROBLEM 5.20

For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

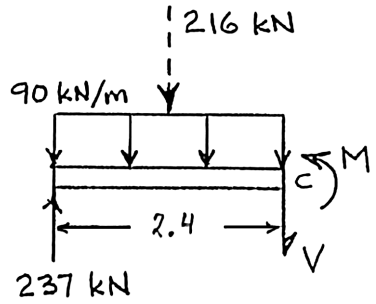


Use entire beam as free body.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-4.8A + (3.6)(216) + (1.6)(150) + (0.8)(150) = 0$$

$$A = 237 \text{ kN} \uparrow$$



Use portion *AC* as free body.

$$+\circlearrowleft \Sigma M_C = 0:$$

$$M - (2.4)(237) + (1.2)(216) = 0$$

$$M = 309.6 \text{ kN} \cdot \text{m}$$

For W460 × 113, $S = 2390 \times 10^6 \text{mm}^3$

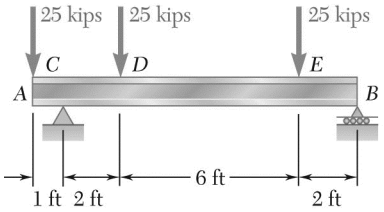
Normal stress:

$$\sigma = \frac{M}{S} = \frac{309.6 \times 10^3 \text{ N} \cdot \text{m}}{2390 \times 10^{-6} \text{m}^3}$$

$$= 129.5 \times 10^6 \text{ Pa}$$

$$\sigma = 129.5 \text{ MPa} \blacktriangleleft$$

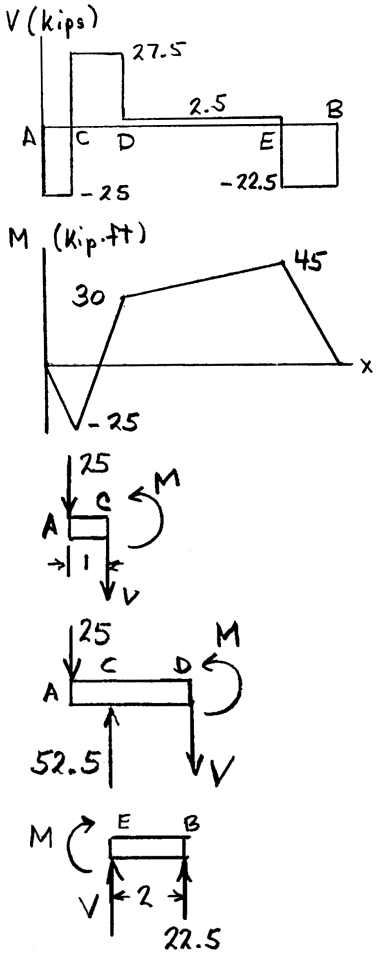
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.21

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



$+\circlearrowleft \Sigma M_B = 0$:

$(11)(25) - 10C + (8)(25) + (2)(25) = 0 \quad \boldsymbol{C} = 52.5 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_C = 0$:

$(1)(25) - (2)(25) - (8)(25) + 10B = 0 \quad \boldsymbol{B} = 22.5 \text{ kips}$

Shear:

A to C^-: $V = -25$ kips

C^+ to D^-: $V = 27.5$ kips

D^+ to E^-: $V = 2.5$ kips

E^+ to B: $V = -22.5$ kips

Bending moments:

At C, $+\circlearrowleft \Sigma M_C = 0$: $(1)(25) + M = 0$

$M = -25 \text{ kip} \cdot \text{ft}$

At D, $+\circlearrowleft \Sigma M_D = 0$: $(3)(25) - (2)(52.5) + M = 0$

$M = 30 \text{ kip} \cdot \text{ft}$

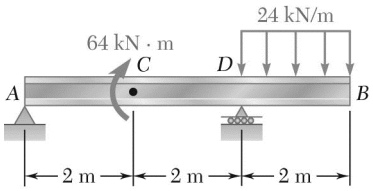
At E, $+\circlearrowleft \Sigma M_E = 0$: $-M + (2)(22.5) = 0 \quad M = 45 \text{ kip} \cdot \text{ft}$

$\max|M| = 45 \text{ kip} \cdot \text{ft} = 540 \text{ kip} \cdot \text{in}$

For $S12 \times 35$ rolled steel section: $S = 38.1 \text{ in}^3$

Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{540}{38.1} = 14.17 \text{ ksi} \qquad \sigma = 14.17 \text{ ksi} \blacktriangleleft$

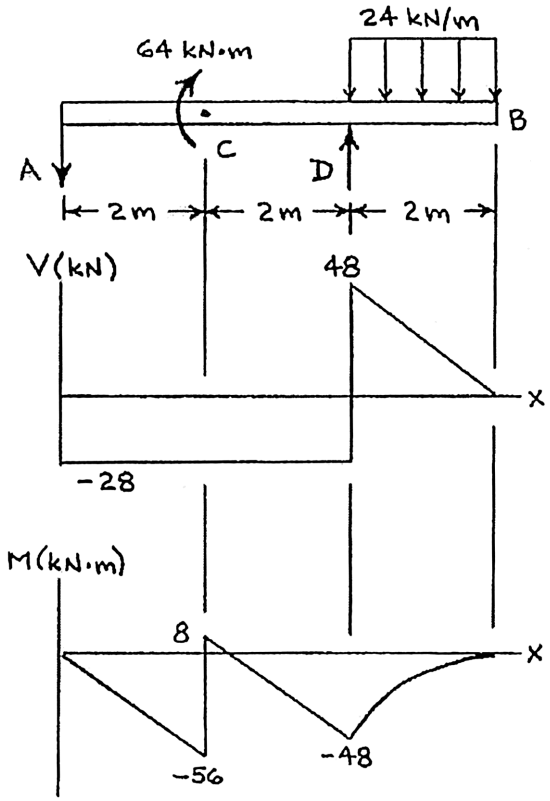
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.22

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

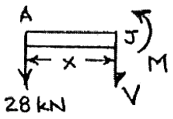


Reactions:

$$+\circlearrowleft \sum M_D = 0: \quad 4A - 64 - (24)(2)(1) = 0 \quad \boldsymbol{A} = 28\ \text{kN}\downarrow$$

$$+\uparrow \sum F_y = 0: \quad -28 + D - (24)(2) = 0 \quad \boldsymbol{D} = 76\ \text{kN}\uparrow$$

A to *C*: $0 < x < 2\text{m}$



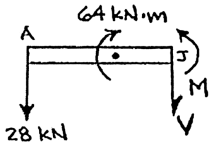
$$+\uparrow \sum F_y = 0: \quad -V - 28 = 0$$

$$V = -28\ \text{kN}$$

$$+\circlearrowleft \sum M_J = 0: \quad M + 28x = 0$$

$$M = (-28x)\ \text{kN} \cdot \text{m}$$

C to *D*: $2\text{m} < x < 4\text{m}$



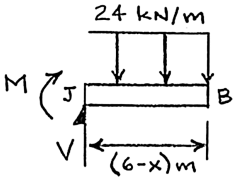
$$+\uparrow \sum F_y = 0: \quad -V - 28 = 0$$

$$V = -28\ \text{kN}$$

$$+\circlearrowleft \sum M_J = 0: \quad M + 28x - 64 = 0$$

$$M = (-28x + 64)\ \text{kN} \cdot \text{m}$$

D to *B*: $4\text{m} < x < 6\text{m}$



$$+\uparrow \sum F_y = 0:$$

$$V - 24(6 - x) = 0$$

$$V = (-24x + 144)\ \text{kN}$$

$$+\circlearrowleft \sum M_J = 0:$$

$$-M - 24(6 - x)\left(\frac{6 - x}{2}\right) = 0$$

$$M = -12(6 - x)^2\ \text{kN} \cdot \text{m}$$

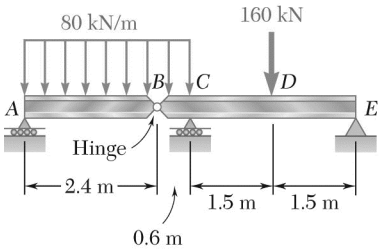
$$\max|M| = 56\ \text{kN} \cdot \text{m} = 56 \times 10^3\ \text{N} \cdot \text{m}$$

For $\text{S250} \times 52$ section, $S = 482 \times 10^3\ \text{mm}^3$

$$\text{Normal Stress: } \sigma = \frac{|M|}{S} = \frac{56 \times 10^3\ \text{N} \cdot \text{m}}{482 \times 10^{-6}\ \text{m}^3} = 116.2 \times 10^6\ \text{Pa}$$

$$\sigma = 116.2\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

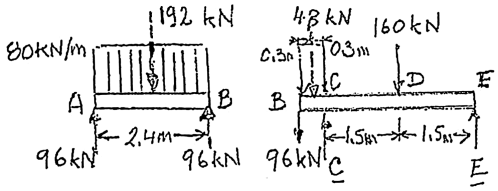


PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

Statics: Consider portion AB and BE separately.



Portion *BE*:

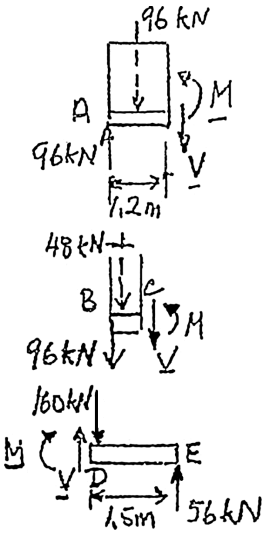
$+\circlearrowleft \Sigma M_E = 0$:

$(96)(3.6) + (48)(3.3) - C(3) + (160)(1.5) = 0$

$\boldsymbol{C} = 248\,\text{kN} \uparrow$

$\boldsymbol{E} = 56\,\text{kN} \uparrow$

$M_A = M_B = M_E = 0$



At midpoint of *AB*:

$\Sigma F_y = 0$: $V = 0$

$\Sigma M = 0$: $M = (96)(1.2) - (96)(0.6) = 57.6\ \text{kN} \cdot \text{m}$

Just to the left of *C*:

$\Sigma F_y = 0$: $V = -96 - 48 = -144\ \text{kN}$

$\Sigma M_C = 0$: $M = -(96)(0.6) - (48)(0.3) = -72\ \text{kN}$

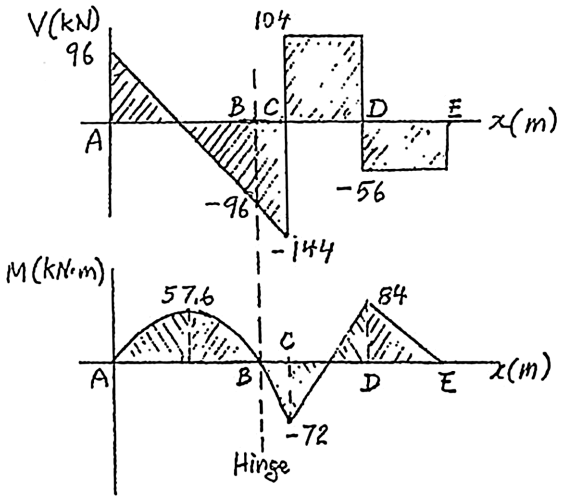
Just to the left of *D*:

$\Sigma F_y = 0$: $V = 160 - 56 = +104\ \text{kN}$

$\Sigma M_D = 0$: $M = (56)(1.5) = +84\ \text{kN} \cdot \text{m}$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.23 *(Continued)*



From the diagram:

$$|M|_{\max} = 84 \text{ kN} \cdot \text{m} = 84 \times 10^3 \text{ N} \cdot \text{m}$$

For W310 × 60 rolled steel shape,

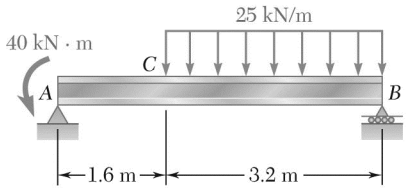
$$S_x = 844 \times 10^3 \text{mm}^3$$
$$= 844 \times 10^{-6} \text{m}^3$$

Stress: $\sigma_m = \dfrac{|M|_{\max}}{S}$

$$\sigma_m = \frac{84 \times 10^3}{844 \times 10^{-6}} = 99.5 \times 10^6 \text{ Pa}$$

$$\sigma_m = 99.5 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

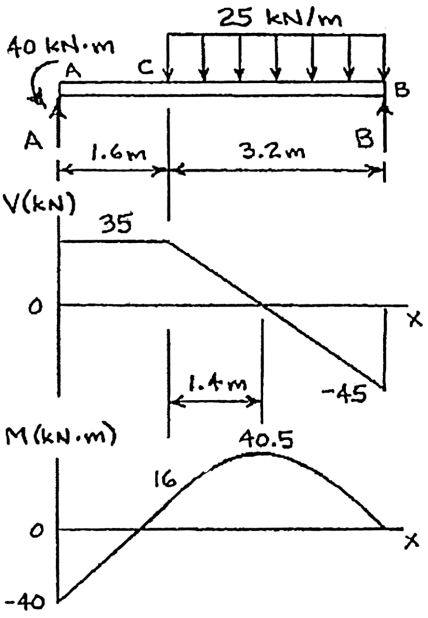


W200 × 31.3

PROBLEM 5.24

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

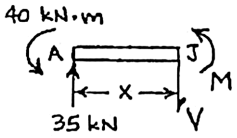


Reaction at A:

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.8A + 40 + (25)(3.2)(1.6) = 0$$

$$A = 35\ \text{kN} \uparrow$$

A to C: $\quad 0 < x < 1.6\,\text{m}$



$$+\uparrow \Sigma F_y = 0: \quad 35 - V = 0 \quad V = 35\ \text{kN}$$

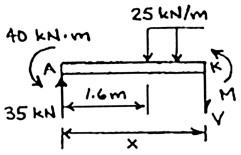
$$+\circlearrowleft \Sigma M_J = 0: \quad M + 40 - 35x = 0$$

$$M = (30x - 40)\ \text{kN} \cdot \text{m}$$

C to B: $\quad 1.6\,\text{m} < x < 4.8\,\text{m}$

$$+\uparrow \Sigma F_y = 0: \quad 35 - 25(x - 1.6) - V = 0$$

$$V = (-25x + 75)\ \text{kN}$$



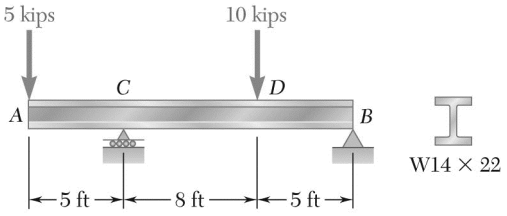
$$+\circlearrowleft \Sigma M_K = 0: \quad M + 40 - 35x + (25)(x - 1.6)\left(\frac{x - 1.6}{2}\right) = 0$$

$$M = (-12.5x^2 + 75x - 72)\ \text{kN} \cdot \text{m}$$

Normal stress: $\quad$ For $\text{W200} \times 31.3$, $S = 298 \times 10^3\ \text{mm}^3$

$$\sigma = \frac{|M|}{S} = \frac{40.5 \times 10^3\ \text{N} \cdot \text{m}}{298 \times 10^{-6}\ \text{m}^3} = 135.9 \times 10^6\ \text{Pa} \qquad \sigma = 135.9\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.25

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

Reaction at C: $+\circlearrowleft \Sigma M_B = 0$: $(18)(5) - 13C + (5)(10) = 0$

$$C = 10.769 \text{ kips}$$

Reaction at B: $+\circlearrowleft M_C = 0$: $(5)(5) - (8)(10) + 13B = 0$

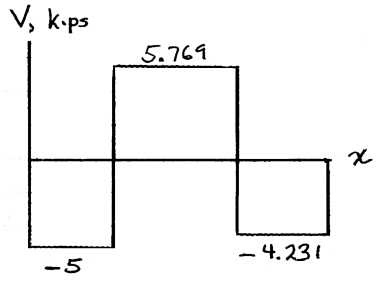
$$B = 4.231 \text{ kips}$$

Shear diagram:

A to C^-: $V = -5$ kips

C^+ to D^-: $V = -5 + 10.769 = 5.769$ kips

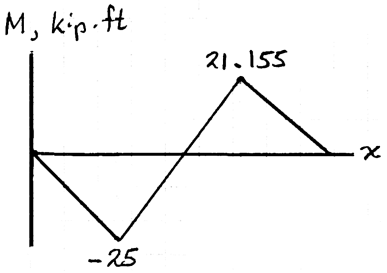
D^+ to B: $V = 5.769 - 10 = -4.231$ kips

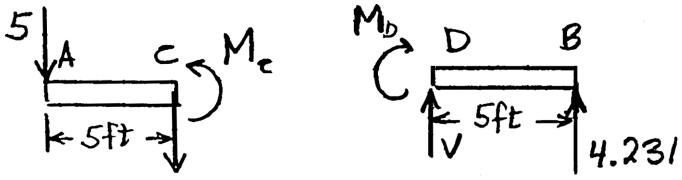


At A and B, $M = 0$

At C, $+\circlearrowleft M_C = 0$: $(5)(5) + M_C = 0$

$$M_C = -25 \text{ kip} \cdot \text{ft}$$





At D, $+\circlearrowleft \Sigma M_D = 0$: $-M_D + (5)(4.231)$

$$M_D = 21.155 \text{ kip} \cdot \text{ft}$$

$|V|_{\max} = 5.77$ kips ◀

$|M|_{\max}$ occurs at C.

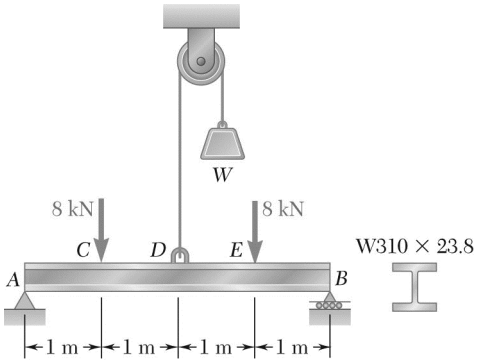
$|M|_{\max} = 25 \text{ kip} \cdot \text{ft} = 300 \text{ kip} \cdot \text{in}$ ◀

For W14 × 22 rolled steel section, $S = 29.0 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{300}{29.0}$

$\sigma = 10.34$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.26

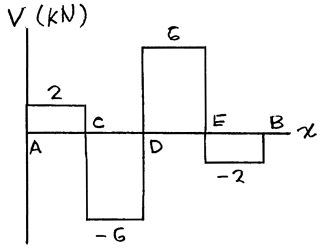
Knowing that $W = 12\,\text{kN}$, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.

SOLUTION

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - 8 + 12 - 8 + B = 0$$

$$A = B = 2\ \text{kN}$$

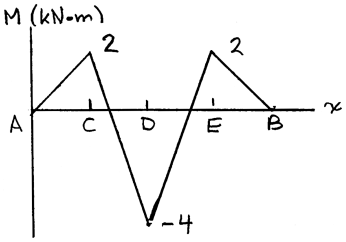


Shear: A to C^-: $V = 2$ kN ◀

C^+ to D^-: $V = -6$ kN ◀

D^+ to E^-: $V = 6$ kN ◀

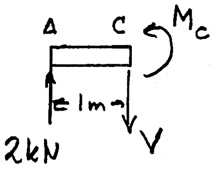
E^+ to B: $V = -2$ kN ◀

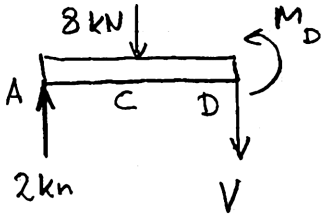


Bending moment:

At C, $+\circlearrowleft \Sigma M_C = 0: \quad M_C - (1)(2) = 0$

$M_C = 2\ \text{kN} \cdot \text{m}$ ◀





At D, $+\Sigma M_D = 0: \quad M_D - (2)(2) + (8)(1) = 0$

$M_D - 4\ \text{kN} \cdot \text{m}$ ◀

By symmetry, $M = 2\ \text{kN} \cdot \text{m}$ at E. $M_E = 2\ \text{kN} \cdot \text{m}$ ◀

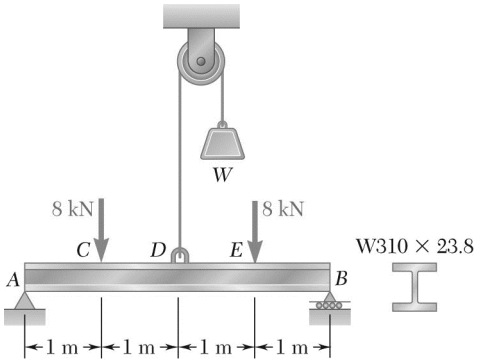
$\max |M| = 4\ \text{kN} \cdot \text{m}$ occurs at E.

For W310 × 23.8, $S_x = 280 \times 10^3\,\text{mm}^3 = 280 \times 10^{-6}\,\text{m}^3$

Normal stress: $\sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{4 \times 10^3}{280 \times 10^{-6}}$

$= 14.29 \times 10^6\ \text{Pa}$ $\sigma_{\max} = 14.29\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.27

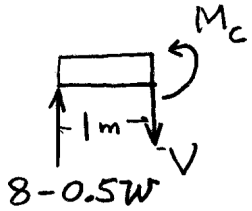
Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - 8 + W - 8 + B = 0$$

$$A = B = 8 - 0.5W$$

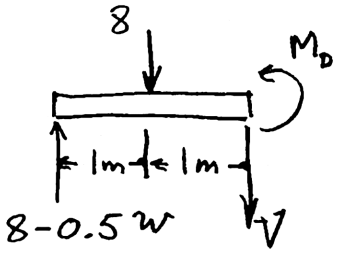


Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad -(8 - 0.5W)(1) + M_C = 0$$

$$M_C = (8 - 0.5W) \text{ kN} \cdot \text{m}$$

Bending moment at D:



$$+\circlearrowleft \Sigma M_D = 0: \quad -(8 - 0.5W)(2) + (8)(1) + M_D = 0$$

$$M_D = (8 - W) \text{ kN} \cdot \text{m}$$

Equate: $\quad -M_D = M_C \qquad W - 8 = 8 - 0.5W$

$$W = 10.67 \text{ kN} \blacktriangleleft$$

(*a*) $W = 10.6667 \text{ kN}$

$M_C = -2.6667 \text{ kN} \cdot \text{m}$

$M_D = 2.6667 \text{ kN} \cdot \text{m} = 2.6667.10^3 \text{ N} \cdot \text{m}$

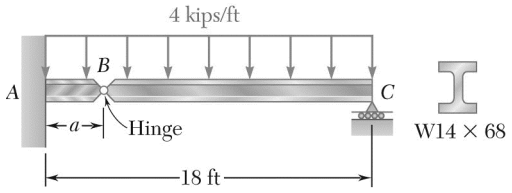
$|M|_{\max} = 2.6667 \text{ kN} \cdot \text{m}$

For W310 × 23.8 rolled steel shape,

$$S_x = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{m}^3$$

(*b*) $\sigma_{\max} = \dfrac{|M|_{\max}}{S_x} = \dfrac{2.6667 \times 10^3}{280 \times 10^{-6}} = 9.52 \times 10^6 \text{ Pa}$ $\qquad \sigma_{\max} = 9.52 \text{ MPa} \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.28

Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

For W14 × 68, $S_x = 103 \text{ in}^3$

Let $b = (18 - a)$ ft

Segment *BC*:

By symmetry, $V_B = C$

$$+\uparrow \Sigma F_y = 0: \quad V_B + C - 4b = 0$$

$$V_B = 2b$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -V_B x + (4x)\left(\frac{x}{2}\right) - M = 0$$

$$M = V_B x - 2x^2 = 2bx - 2x^2 \text{ lb} \cdot \text{ft}$$

$$\frac{dM}{dx} = 2b - x_m = 0 \qquad x_m = \frac{1}{2}b$$

$$M_{\max} = b^2 - \frac{1}{2}b^2 = \frac{1}{2}b^2$$

Segment *AB*:

$$+\circlearrowleft \Sigma M_K = 0: \quad -4(a - x)\frac{(a - x)}{2} - V_B(a - x) - M = 0$$

$$M = -2(a - x)^2 + 2b(a - x)$$

$|M_{\max}|$ occurs at $x = 0$.

$$|M_{\max}| = -2a^2 - 2ab = -2a^2 - 2a(18 - a) = 36a$$

(*a*) Equate the two values of $|M_{\max}|$: $\quad 36a = \frac{1}{2}b^2 = \frac{1}{2}(18 - a)^2 = 162 - 18a + \frac{1}{2}a^2$

$$\frac{1}{2}a^2 - 54a + 162 = 0 \qquad a = 54 \pm \sqrt{(54)^2 - (4)\left(\tfrac{1}{2}\right)(162)}$$

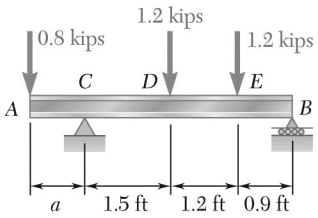
$$a = 54 \pm 50.9118 = 3.0883 \text{ ft} \qquad a = 3.09 \text{ ft} \blacktriangleleft$$

(*b*) $|M|_{\max} = 36a = 111.179 \text{ kip} \cdot \text{ft} = 1334.15 \text{ kip} \cdot \text{in}$

$$\sigma = \frac{|M|_{\max}}{S_x} = \frac{1334.15}{103} = 12.95 \text{ kips/in}^2 \qquad \sigma_m = 12.95 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.29

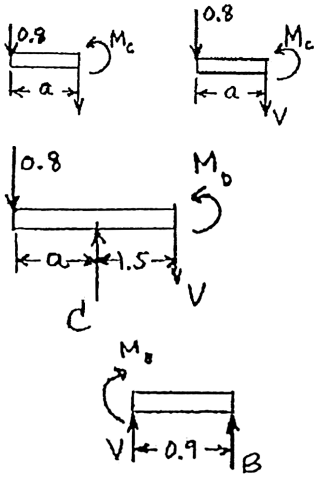


Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$+\circlearrowleft \Sigma M_C = 0 : 0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0 \qquad \boldsymbol{B} = 1.4 - 0.22222\,a \uparrow$

$+\circlearrowleft \Sigma M_B = 0 : (0.8)(3.6 + a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0 \qquad \boldsymbol{C} = 1.8 + 0.22222a \uparrow$



Bending moment at *C*: $+\circlearrowleft \Sigma M_C = 0 : M_C + (0.8)(a) = 0$

$M_C = -0.8a$

Bending moment at *D*: $\Sigma M_D = 0 :$

$M_D + (0.8)(a + 1.5) - 1.5C = 0$

$M_D = 1.5 - 0.46667a$

Bending moment at *E*: $+\circlearrowleft \Sigma M_E = 0 : -M_E + 0.9B = 0$

$M_E = 1.26 - 0.2a$

Assume $-M_C = M_E$: $\quad 0.8a = 1.26 - 0.2a \qquad a = 1.26 \text{ ft}$ ◀

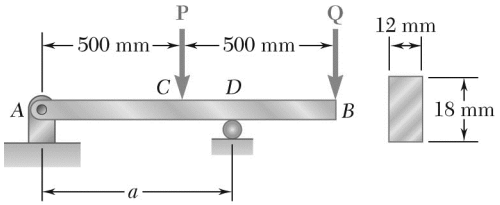
$M_C = -1.008 \text{ kip} \cdot \text{ft} \qquad M_E = 1.008 \text{ kip} \cdot \text{ft} \qquad M_D = 0.912 \text{ kip} \cdot \text{ft}$

Note that $|M_D| < 1.008 \text{ kip} \cdot \text{ft} \qquad \max|M| = 1.008 \text{ kip} \cdot \text{ft} = 12.096 \text{ kip} \cdot \text{in}$

For rolled steel section S3 × 5.7 : $\quad S = 1.67 \text{ in}^3$

Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{12.096}{1.67} \qquad \sigma = 7.24 \text{ ksi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

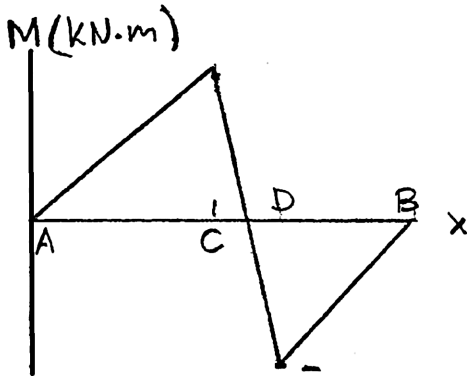


PROBLEM 5.30

Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 480 \text{ N}$$



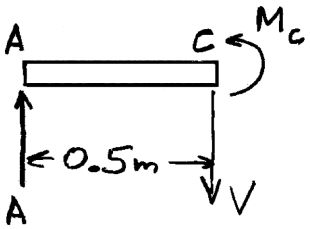
Reaction at *A*: $+\circlearrowleft \Sigma M_D = 0: \quad -Aa + 480(a - 0.5)$

$$-480(1 - a) = 0$$

$$A = \left(960 - \frac{720}{a}\right) \text{N}$$

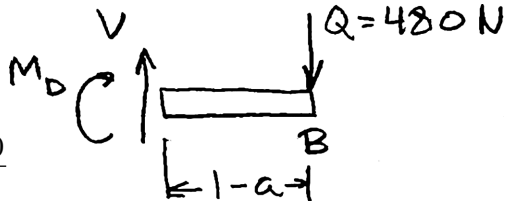
Bending moment at *C*: $+\circlearrowleft \Sigma M_C = 0: \quad -0.5A + M_C = 0$

$$M_C = 0.5A = \left(480 - \frac{360}{a}\right) \text{N} \cdot \text{m}$$



Bending moment at *D*: $+\circlearrowleft \Sigma M_D = 0: \quad -M_D - 480(1 - a) = 0$

$$M_D = -480(1 - a) \text{ N} \cdot \text{m}$$



(*a*) Equate: $-M_D = M_C \qquad 480(1 - a) = 480 - \dfrac{360}{a}$

$$a = 0.86603 \text{ m} \qquad a = 866 \text{ mm} \blacktriangleleft$$

$$A = 128.62 \text{ N} \quad M_C = 64.31 \text{ N} \cdot \text{m} \qquad M_D = -64.31 \text{ N} \cdot \text{m}$$

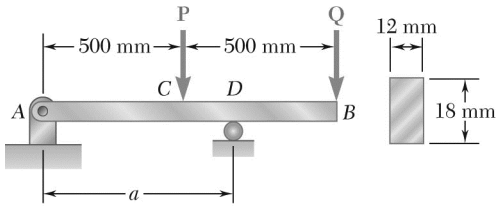
(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(13)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{64.31}{6.48 \times 10^{-9}} = 99.2 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 99.2 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.31

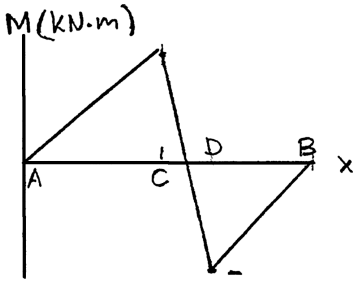


Solve Prob. 5.30, assuming that $P = 480$ N and $Q = 320$ N.

PROBLEM 5.30 Knowing that $P = Q = 480$ N, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

$$P = 480 \text{ N} \qquad Q = 320 \text{ N}$$

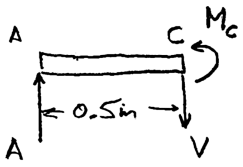


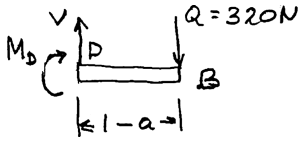
Reaction at A: $+\circlearrowleft \Sigma M_D = 0$: $Aa + 480(a - 0.5) - 320(1 - a) = 0$

$$A = \left(800 - \frac{560}{a}\right) \text{N}$$

Bending moment at C: $+\circlearrowleft \Sigma M_C = 0$: $-0.5A + M_C = 0$

$$M_C = 0.5A = \left(400 - \frac{280}{a}\right) \text{ N} \cdot \text{m}$$





Bending moment at D: $+\circlearrowleft \Sigma M_D = 0$: $-M_D - 320(1 - a) = 0$

$$M_D = (-320 + 320a) \text{ N} \cdot \text{m}$$

(*a*) Equate: $-M_D = M_C \qquad 320 - 320a = 400 - \dfrac{280}{a}$

$$320a^2 + 80a - 280 = 0 \qquad a = 0.81873 \text{ m}, -1.06873 \text{ m}$$

Reject negative root. $a = 819$ mm ◀

$A = 116.014$ N $\qquad M_C = 58.007$ N · m $\qquad M_D = -58.006$ N · m

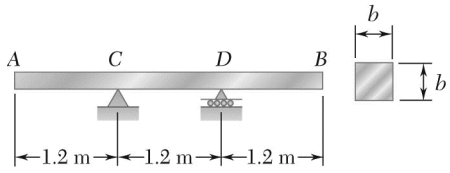
(*b*) For rectangular section, $S = \dfrac{1}{6}bh^2$

$$S = \frac{1}{6}(12)(18)^2 = 648 \text{ mm}^3 = 648 \times 10^{-9} \text{ m}^3$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{58.0065}{648 \times 10^{-9}} = 89.5 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 89.5 \text{ MPa} \blacktriangleleft$$

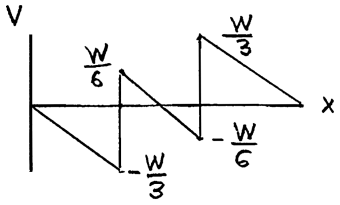
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.32



A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.

SOLUTION

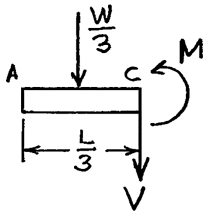


Weight density: $\gamma = \rho g$

Let L = total length of beam.

$$W = AL\rho g = b^2 L\rho g$$

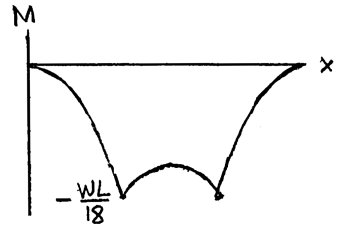
Reactions at C and D: $\quad C = D = \dfrac{W}{2}$

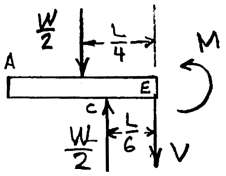


Bending moment at C:

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$





Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0 \qquad M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2L^2\rho g}{18}$$

For a square section, $\quad S = \dfrac{1}{6}\, b^3$

Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{b^2L^2\rho g/18}{b^3/6} = \dfrac{L^2\rho g}{3b}$

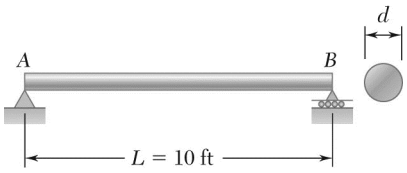
Solve for b: $\quad b = \dfrac{L^2\rho g}{3\sigma}$

Data: $L = 3.6\ \text{m} \quad \rho = 7860\ \text{kg/m}^3 \quad g = 9.81\ \text{m/s}^2 \quad (a)\ \sigma = 10\times10^6\ \text{Pa} \quad (b)\ \sigma = 50\times10^6\ \text{Pa}$

(*a*) $\quad b = \dfrac{(3.6)^2(7860)(9.81)}{(3)(10\times10^6)} = 33.3\times10^{-3}\ \text{m}$ $\qquad b = 33.3\ \text{mm}$ ◀

(*b*) $\quad b = \dfrac{(3.6)^2(7860)(9.81)}{(3)(50\times10^6)} = 6.66\times10^{-3}\ \text{m}$ $\qquad b = 6.66\ \text{mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.33

A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490$ lb/ft^3, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

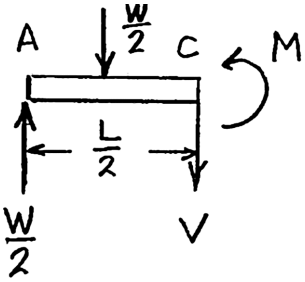
SOLUTION

Let W = total weight.

$$W = AL\gamma = \frac{\pi}{4} d^2 L\gamma$$

Reaction at A:

$$A = \frac{1}{2} W$$



Bending moment at center of beam:

$$+\circlearrowleft \Sigma M_C = 0: \quad -\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section, $\left(c = \frac{1}{2} d\right)$

$$I = \frac{\pi}{4} c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for d,

$$d = \frac{L^2 \gamma}{\sigma}$$

Data:

$$L = 10 \text{ ft} = (12)(10) = 120 \text{ in.}$$

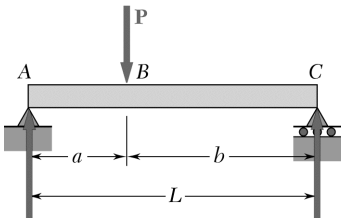
$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

$$d = \frac{(120)^2 (0.28356)}{4000}$$

$$d = 1.021 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

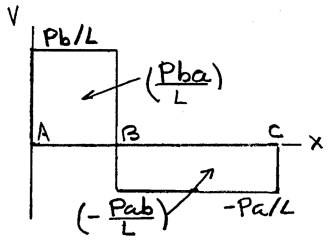


PROBLEM 5.34

Using the method of Sec. 5.3, solve Prob. 5.1*a*.

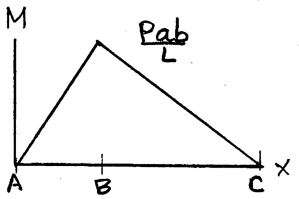
PROBLEM 5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



At A^+: $\quad V = A = \dfrac{Pb}{L} \qquad M = 0$

A to B^-: $\quad 0 < x < a$

$$w = 0 \qquad \int_0^x w\,dx = 0$$

$$V - V_A = 0 \qquad V = \frac{Pb}{L} \blacktriangleleft$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{Pb}{L}\,dx = \frac{Pba}{L} \qquad M_B = \frac{Pba}{L} \blacktriangleleft$$

At B^+: $\quad V = A - P = \dfrac{Pb}{L} - P = -\dfrac{Pa}{L}$

B^+ to C: $\quad a < x < L$

$$w = 0 \qquad \int_a^x w\,dx = 0$$

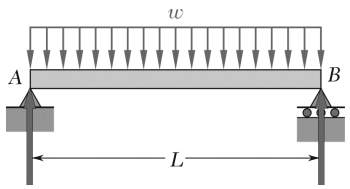
$$V_C - V_B = 0 \qquad V = -\frac{Pa}{L} \blacktriangleleft$$

$$M_C - M_B = \int_a^L V\,dx = -\frac{Pa}{L}(L - a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

$$|M|_{\max} = \frac{Pab}{L} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.35

Using the method of Sec. 5.3, solve Prob. 5.2*a*.

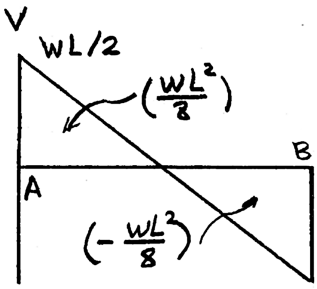
PROBLEM 5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$



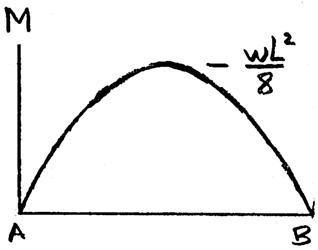
$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = A - wx \qquad V = \frac{wL}{2} - wx \blacktriangleleft$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right) dx$$

$$= \frac{wLx}{2} - \frac{wx^2}{2}$$



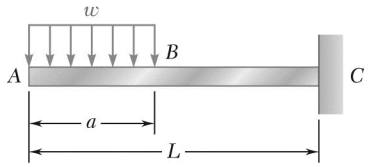
$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} \qquad M = \frac{w}{2}(Lx - x^2) \blacktriangleleft$$

Maximum M occurs at $x = \frac{1}{2}$, where

$$V = \frac{dM}{dx} = 0 \qquad |M|_{\max} = \frac{wL^2}{8} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

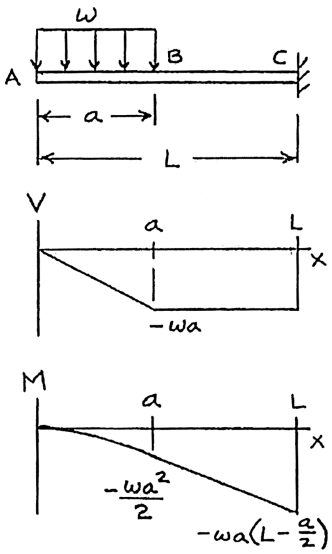
PROBLEM 5.36



Using the method of Sec. 5.3, solve Prob. 5.3*a*.

PROBLEM 5.3 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



Over *AB*: $V_A = 0 \quad M_A = 0$

$$V = -\int_0^x w\,dx = -wx$$

$$\frac{dM}{dx} = V = -wx$$

$$M = \int_0^x V\,dx = -\frac{wx^2}{2}\bigg|_0^x \qquad M = -\frac{wx^2}{2} \blacktriangleleft$$

At *B*: $x = a \qquad V_B = -wa \qquad M_B = -\dfrac{wa^2}{2} \blacktriangleleft$

Over *BC*: $w = 0$

$$\frac{dV}{dx} = 0 \qquad V = \text{constant} = V_B \qquad V = -wa \blacktriangleleft$$

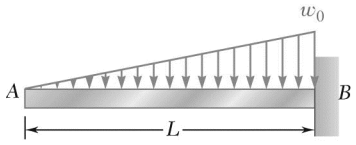
$$\frac{dM}{dx} = V = -wa$$

$$M - M_B = \int_a^x V\,dx = -wax\Big|_a^x = -wa(x - a)$$

$$M = -wa(x - a) - \frac{wa^2}{2} \qquad M = -wa\left(x - \frac{a}{2}\right) \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

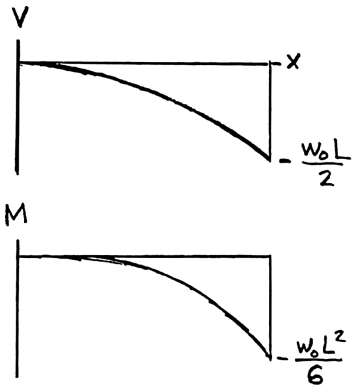
PROBLEM 5.37



Using the method of Sec. 5.3, solve Prob. 5.4*a*.

PROBLEM 5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{W_0 x}{L}$$

$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

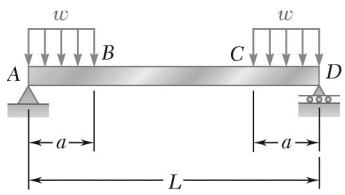
$$V = -\frac{w_0 x^2}{2L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L} dx$$

$$M = -\frac{w_0 x^3}{6L} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.38

Using the method of Sec. 5.3, Solve Prob. 5.5a.

PROBLEM 5.5 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

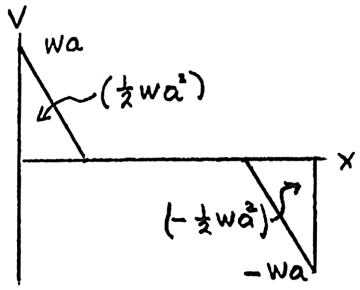
Reactions: $A = D = wa$

A to *B*: $0 < x < a \qquad w = w$

$V_A = A = wa, \qquad M_A = 0$

$V - V_A = -\int_0^x w\,dx = -wx$

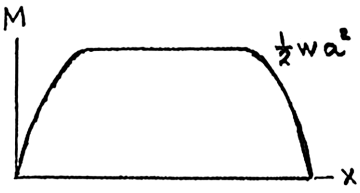
$$V = w(a - x) \blacktriangleleft$$



$$\frac{dM}{dx} = V = wa - wx$$

$M - M_A = \int_0^x V dx = \int_0^x (wa - wx)dx$

$$M = wax - \frac{1}{2}wx^2 \blacktriangleleft$$



$V_B = 0 \qquad M_B = \frac{1}{2}wa^2$

B to *C*: $a < x < L - a$ $\qquad V = 0 \blacktriangleleft$

$$\frac{dM}{dx} = V = 0$$

$M - M_B = \int_a^x V\,dx = 0$

$M = M_B$ $\qquad M = \frac{1}{2}wa^2 \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.38 ***(Continued)***

C to D: $V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L - a)]$

$$V = -w[x - (L - a)] \blacktriangleleft$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -[wx - (L - a)]dx$$

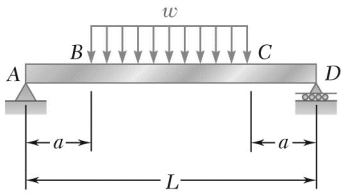
$$= -w\left[\frac{x^2}{2} - (L - a)x\right]\Bigg|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L - a)x - \frac{(L - a)^2}{2} + (L - a)^2\right]$$

$$= -w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right]$$

$$M = \frac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L - a)x + \frac{(L - a)^2}{2}\right] \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

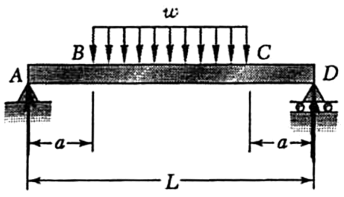


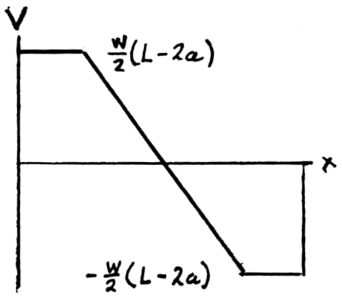
PROBLEM 5.39

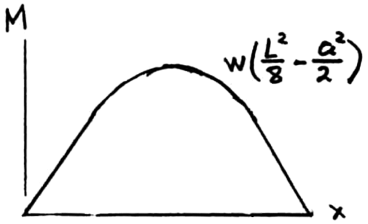
Using the method of Sec. 5.3, solve Prob. 5.6*a*.

PROBLEM 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION







Reactions. $A = D = \frac{1}{2}w(L - 2a)$

At A. $V_A = A = \frac{1}{2}w(L - 2a), \quad M_A = 0$

A to B. $0 < x < a \qquad w = 0$

$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L - 2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)dx$$

$$M_B = \frac{1}{2}w(L - 2a)a$$

B to C. $a < x < L - a \qquad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x - a)$$

$$V = \frac{1}{2}w(L - 2a) - w(x - a) = \frac{1}{2}w(L - 2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2}w(L - 2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$= \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L - 2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.39 ***(Continued)***

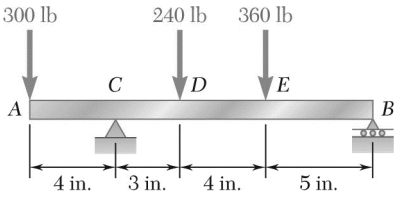
At C. $x = L - a$ $V_C = -\frac{1}{2}w(L - 2a)$ $M_C = \frac{1}{2}(L - 2a)a$

C to D. $V = V_C = -\frac{1}{2}w(L - 2a)$

$M_D = 0$

$$At\ x = \frac{L}{2}, \qquad M_{\max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right) \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

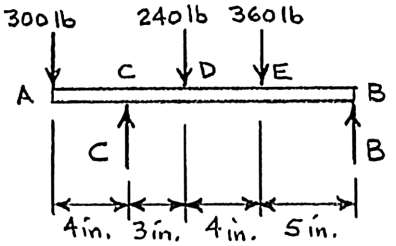


PROBLEM 5.40

Using the method of Sec. 5.3 solve Prob. 5.7.

PROBLEM 5.7 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



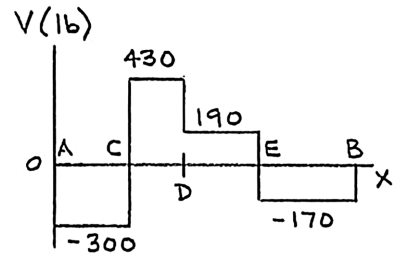
Reaction at *C*:

$$+\circlearrowright \sum M_B = 0: \quad (16)(300) - 12C + (9)(240) + (5)(360) = 0$$

$$\boldsymbol{C} = 730\,\text{lb} \uparrow$$

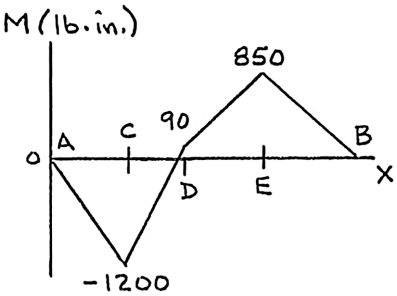
Shear diagram:

A to *C*:	$V = -300$ lb
C to *D*:	$V = -300 + 730 = 430$ lb
D to *E*:	$V = 430 - 240 = 190$ lb
E to *B*:	$V = 190 - 360 = -170$ lb



Areas of shear diagram:

A to *C*:	$A_{AC} = (-300)(4) = -1200$ lb · in
C to *D*:	$A_{CD} = (430)(3) = 1290$ lb · in
D to *E*:	$A_{DE} = (190)(4) = 760$ lb · in
E to *B*:	$A_{EB} = (-170)(5) = -850$ lb · in



Bending moments:

$M_A = 0$

$M_C = 0 - 1200 = -1200$ lb · in

$M_D = -1200 + 1290 = 90$ lb · in

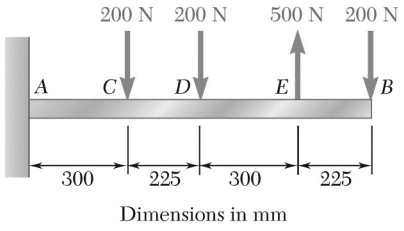
$M_E = 90 + 760 = 850$ lb · in

$M_B = 850 - 850 = 0$

(*a*) Maximum $|V| = 430$ lb ◀

(*b*) Maximum $|M| = 1200$ lb · in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



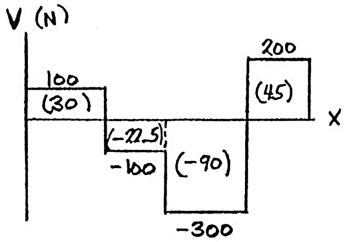
Dimensions in mm

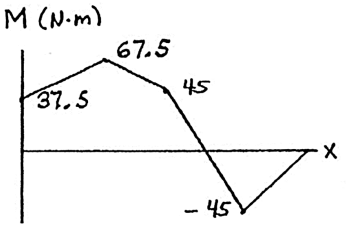
PROBLEM 5.41

Using the method of Sec. 5.3, Solve Prob. 5.8

PROBLEM 5.8 Draw the shear and bending-moment diagram for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION





$+\circlearrowleft \sum M_A = 0: \quad -M_A - (0.3)(200) - (0.525)(200)$
$+ (0.825)(500) - (1.05)(200) = 0$

$M_A = 37.5\,\text{N} \cdot \text{m}$

$+\uparrow \sum F_y = 0: \quad V_A - 200 - 200 + 500 - 200 = 0$

$V_A = 100\,\text{N}$

Shear:

A to C: $V = 100\,\text{N}$

C to D: $V = 100 - 200 = -100\,\text{N}$

D to E: $V = -100 - 200 = -300\,\text{N}$

E to B: $V = -300 + 500 = 200\,\text{N}$

Areas under shear diagram:

A to C: $\int V dx = (100)(0.3) = 30\,\text{N} \cdot \text{m}$

C to D: $\int V dx = (-100)(0.225) = -22.5\,\text{N} \cdot \text{m}$

D to E: $\int V dx = (-300)(0.3) = -90\,\text{N} \cdot \text{m}$

E to B: $\int V dx = (200)(0.225) = 45\,\text{N} \cdot \text{m}$

Bending moments:

$M_A = 37.5\,\text{N} \cdot \text{m}$

$M_C = M_A + \int_A^C V\,dx = 37.5 + 30 = 67.5\,\text{N} \cdot \text{m}$

$M_D = M_C + \int_C^D V\,dx = 67.5 - 22.5 = 45\,\text{N} \cdot \text{m}$

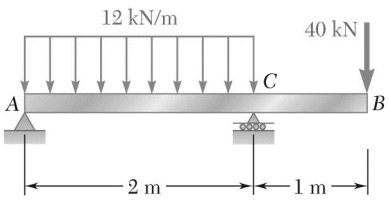
$M_E = M_D + \int_D^E V\,dx = 45 - 90 = -45\,\text{N} \cdot \text{m}$

$M_B = M_E + \int_E^D V\,dx = -45 + 45 = 0$

(*a*) Maximum $|V| = 300\,\text{N}$ ◀

(*b*) Maximum $|M| = 67.5\,\text{N} \cdot \text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

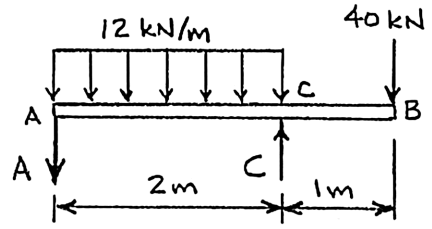


PROBLEM 5.42

Using the method of Sec. 5.3, Solve Prob. 5.9

PROBLEM 5.9 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



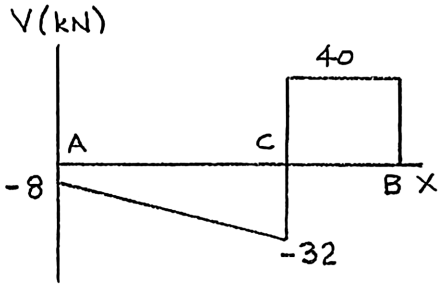
Reactions:

$$+\circlearrowleft \Sigma M_C = 0: \; 2A + (12)(2)(1) - (40)(1) = 0$$

$$\boldsymbol{A} = 8\,\text{kN} \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \; 2C - (12)(2)(1) - (40)(3) = 0$$

$$\boldsymbol{C} = 72\,\text{kN} \uparrow$$



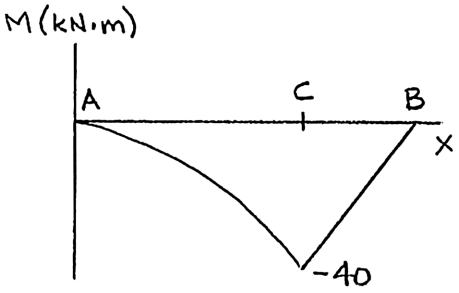
Shear diagram: $V_A = -8$ kN

A to *C*: $0 < x < 2\,\text{m}$ $w = 12\text{kN/m}$

$$V_C - V_A = -\int_0^2 w\,dx = -\int_0^2 12\,dx = -24\text{ kN}$$

$$V_C = -24 - 8 = -32\text{ kN}$$

C to *B*: $V_B = -32 + 72 = 40$ kN



Areas of shear diagram:

A to *C*: $\int V dx = \frac{1}{2}(-8 - 32)(2) = -40\text{ kN}\cdot\text{m}$

C to *B*: $\int V dx = (1)(40) = 40\text{ kN}\cdot\text{m}$

Bending moments:

$M_A = 0$

$M_C = M_A + \int V dx = 0 - 40 = -40\text{ kN}\cdot\text{m}$

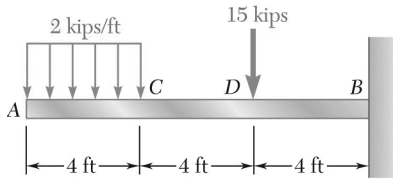
$M_B = M_C + \int V dx = -40 + 40 = 0$

(*a*) Maximum $|V| = 40.0$ kN ◀

(*b*) Maximum $|M| = 40.0\text{ kN}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

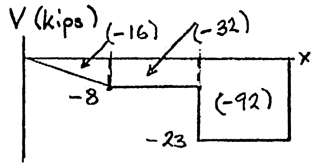
PROBLEM 5.43

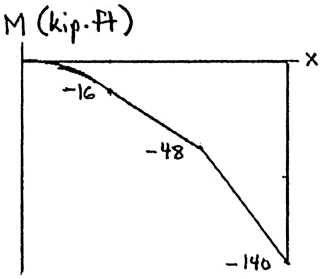


Using the method of Sec. 5.3, solve Prob. 5.10

PROBLEM 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION





Shear:

$V_A = 0$

$V_B = V_A - \int_A^B w\,dx = 0 - (4)(2) = -8$ kips

C to D: $V = -8$ kips

D to B: $V = -8 - 15 = -23$ kips

Areas under shear diagram:

A to C: $\int V\,dx = \left(\frac{1}{2}\right)(4)(-8) = -16$ kip · ft

C to D: $\int V\,dx = (4)(-8) = -32$ kip · ft

D to B: $\int V\,dx = (4)(-23) = -92$ kip · ft

Bending moments:

$M_A = 0$

$M_C = M_A + \int V\,dx = 0 - 16 = -16$ kip · ft

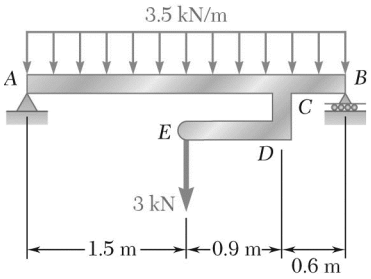
$M_D = M_C + \int V\,dx = -16 - 32 = -48$ kip · ft

$M_B = M_D + \int V\,dx = -48 - 92 = -140$ kip · ft

(*a*) Maximum $|V| = 23$ kips ◀

(*b*) Maximum $|M| = 140$ kip · ft ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.44

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

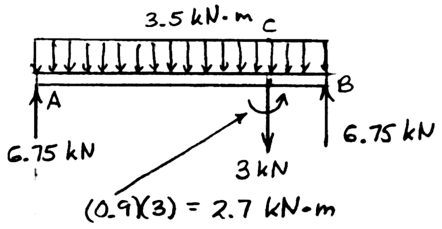
SOLUTION

Reaction at *A*:

$$+\circlearrowleft \Sigma M_B = 0: \quad -3.0A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$$

$$\boldsymbol{A} = 6.75 \text{ kN} \uparrow$$

Reaction at *B*: $\boldsymbol{B} = 6.75 \text{ kN} \uparrow$



Beam *ACB* and loading: (See sketch.)

Areas of load diagram:

A to *C*: $(2.4)(3.5) = 8.4$ kN

C to *B*: $(0.6)(3.5) = 2.1$ kN

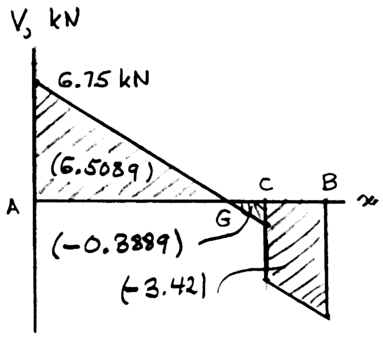
Shear diagram:

$$V_A = 6.75 \text{ kN}$$

$$V_{C^-} = 6.75 - 8.4 = -1.65 \text{ kN}$$

$$V_{C^+} = -1.65 - 3 = -4.65 \text{ kN}$$

$$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$$



Over *A* to *C*, $V = 6.75 - 3.5x$

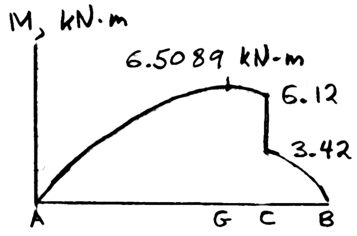
At *G*, $V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286$ m

Areas of shear diagram:

A to *G*: $\frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN} \cdot \text{m}$

G to *C*: $\frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN} \cdot \text{m}$

C to *B*: $\frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN} \cdot \text{m}$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.44 *(Continued)*

Bending moments:

$$M_A = 0$$

$$M_G = 0 + 6.5089 = 6.5089 \text{ kN} \cdot \text{m}$$

$$M_{C^-} = 6.5089 - 0.3889 = 6.12 \text{ kN} \cdot \text{m}$$

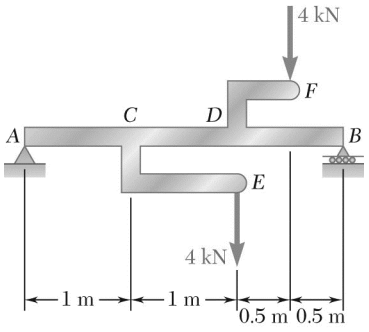
$$M_{C^+} = 6.12 - 2.7 = 3.42 \text{ kN} \cdot \text{m}$$

$$M_B = 3.42 - 3.42 = 0$$

(*a*) $|V|_{\max} = 6.75 \text{ kN}$ ◀

(*b*) $|M|_{\max} = 6.51 \text{ kN} \cdot \text{m}$ ◀

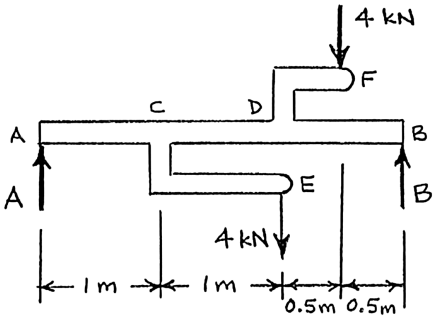
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.45

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION



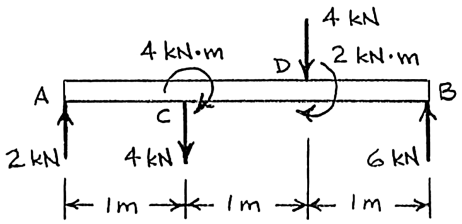
$+\circlearrowleft \Sigma M_B = 0$:

$$-3A + (1)(4) + (0.5)(4) = 0$$

$$\boldsymbol{A} = 2 \text{ kN} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0$: $3B - (2)(4) - (2.5)(4) = 0$

$$\boldsymbol{B} = 6 \text{ kN} \uparrow$$



Shear diagram:

A to *C*: $V = 2$ kN

C to *D*: $V = 2 - 4 = -2$ kN

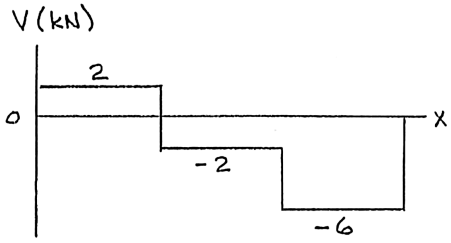
D to *B*: $V = -2 - 4 = -6$ kN

Areas of shear diagram:

A to *C*: $\int V dx = (1)(2) = 2$ kN · m

C to *D*: $\int V dx = (1)(-2) = -2$ kN · m

D to *E*: $\int V dx = (1)(-6) = -6$ kN · m



Bending moments:

$M_A = 0$

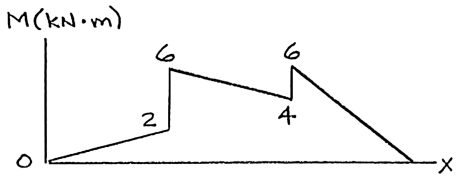
$M_{C^-} = 0 + 2 = 2$ kN · m

$M_{C^+} = 2 + 4 = 6$ kN · m

$M_{D^-} = 6 - 2 = 4$ kN · m

$M_{D^+} = 4 + 2 = 6$ kN · m

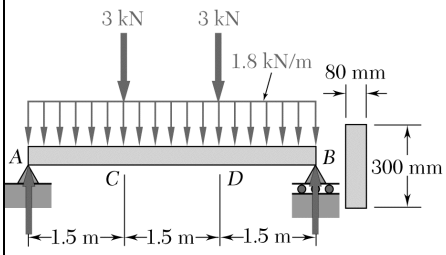
$M_B = 6 - 6 = 0$



(*a*) $|V|_{\max} = 6.00$ kN ◀

(*b*) $|M|_{\max} = 6.00$ kN · m ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.46

Using the method of Sec. 5.3, solve Prob. 5.15

PROBLEM 5.15 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

By symmetry, $A = B$.

$$+\uparrow \Sigma F_y = 0: \quad A + B - 3 - 3 - (4.5)(1.8) = 0$$

$$A = B = 7.05 \text{ kN}$$

Shear diagram: $V_A = 7.05$ kN

A to C^-: $w = 1.8$ kN/m

At C^-, $V = 7.05 - (1.8)(1.5) = 4.35$ kN

At C^+, $V = 4.35 - 3 = 1.35$ kN

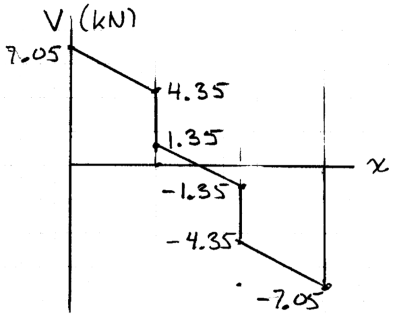
C^+ to D^-: $w = 1.8$ kN/m

At D^-, $V = 1.35 - (1.5)(1.8) = -1.35$ kN

At D^+, $V = -1.35 - 3 = -4.35$ kN

D^+ to B: $w = 1.8$ kN

At B, $V = -4.35 - (1.5)(1.8) = -7.05$ kN



Draw the shear diagram:

$V = 0$ at point E, the midpoint of CD.

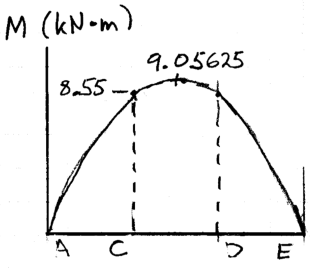
Areas of the shear diagram:

A to C: $\frac{1}{2}(7.05 + 4.35)(1.5) = 8.55 \text{ kN} \cdot \text{m}$

C to E: $\frac{1}{2}(1.35)(0.75) = 0.50625 \text{ kN} \cdot \text{m}$

E to D: $\frac{1}{2}(-1.35)(0.75) = -0.50625 \text{ kN} \cdot \text{m}$

D to B: $\frac{1}{2}(-4.35 - 7.05)(1.5) = -8.55 \text{ kN} \cdot \text{m}$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.46 *(Continued)*

Bending moments:

$$M_A = 0$$
$$M_C = 0 + 8.55 = 8.55 \text{ kN} \cdot \text{m}$$
$$M_E = 8.55 + 0.50625 = 9.05625 \text{ kN} \cdot \text{m}$$
$$M_D = 9.05625 - 0.50625 = 8.55 \text{ kN} \cdot \text{m}$$
$$M_B = 8.55 - 8.55 = 0$$
$$M_C = 8.55 \times 10^3 \text{ N} \cdot \text{m}$$

For a rectangular section,

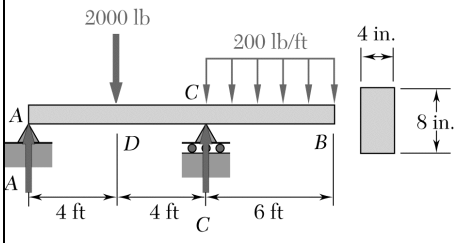
$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(80)(300)^2$$
$$= 1.2 \times 10^6 \text{ mm}^3 = 1.2 \times 10^{-3} \text{ m}^3$$

Maximum normal stress at C:

$$\sigma = \frac{M_C}{S} = \frac{8.55 \times 10^3}{1.2 \times 10^{-3}}$$
$$= 7.125 \times 10^6 \text{ Pa}$$

$$\sigma = 7.13 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.47

Using the method of Sec. 5.3, solve Prob. 5.16.

PROBLEM 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -8A + (4)(2000) - (3)(6)(200) = 0$$

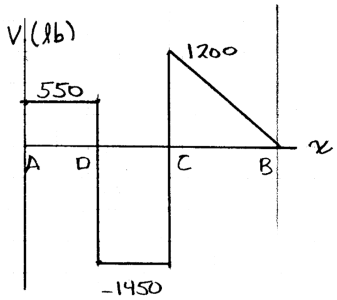
$$A = 550 \text{ lb} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 8C - (4)(2000) - (11)(6)(200) = 0$$

$$C = 2650 \text{ lb} \uparrow$$

Check: $+\uparrow \Sigma F_y = 550 - 2000 + 2650 - (6)(200) = 0$

Shear diagram:



A to D: $V = 550$ lb

D to C^-: $V = 550 - 2000 = -1450$ lb

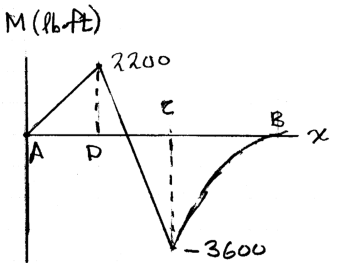
At C^+, $V = -1450 + 2650 = 1200$ lb

C^+ to B: $w = 200$ lb/ft

$$\int_8^{14} 200\, dx = 1200 \text{ lb}$$

At B, $V = 0$ as expected.

Areas of shear diagram:



A to D: $A_{AD} = (550)(4) = 2200 \text{ lb} \cdot \text{ft}$

D to C: $A_{DC} = (-1450)(4) = -5800 \text{ lb} \cdot \text{ft}$

C to B: $A_{CB} = \frac{1}{2}(1200)(6) = 3600 \text{ lb} \cdot \text{ft}$

Bending moments:

$M_A = 0$

$M_D = 0 + 2200 = 2200 \text{ lb} \cdot \text{ft}$

$M_C = 2200 + (-5800) = -3600 \text{ lb} \cdot \text{ft}$

$M_B = -3600 + 3600 = 0$ as expected

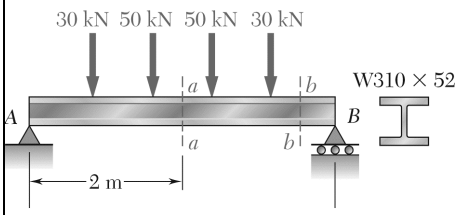
$$|M|_{\max} = 3600 \text{ lb} \cdot \text{ft} = 43200 \text{ lb} \cdot \text{in}$$

For a rectangular section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(4)(8)^2 = 42.667 \text{ in}^3$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} = \frac{43200}{42.667} = 1012.5 \text{ psi}$$

$\sigma_{\max} = 1013$ psi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

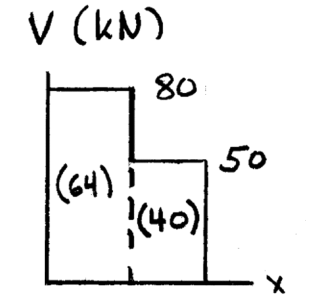


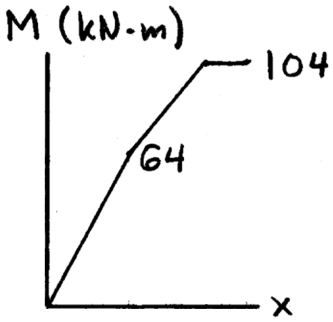
PROBLEM 5.48

Using the method of Sec. 5.3, solve Prob. 5.18.

PROBLEM 5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.

SOLUTION





Reactions: By symmetry, $A = B$.

$+\uparrow \sum F_y = 0: \quad \boldsymbol{A} = \boldsymbol{B} = 80\ \text{kN} \uparrow$

Shear diagram:

A to C: $V = 80\ \text{kN}$

C to D: $V = 80 - 30 = 50\ \text{kN}$

D to E: $V = 50 - 50 = 0$

Areas of shear diagram:

A to C: $\int V dx = (80)(0.8) = 64\ \text{kN} \cdot \text{m}$

C to D: $\int V dx = (50)(0.8) = 40\ \text{kN} \cdot \text{m}$

D to E: $\int V dx = 0$

Bending moments:

$M_A = 0$

$M_C = 0 + 64 = 64\ \text{kN} \cdot \text{m}$

$M_D = 64 + 40 = 104\ \text{kN} \cdot \text{m}$

$M_E = 104 + 0 = 104\ \text{kN} \cdot \text{m}$

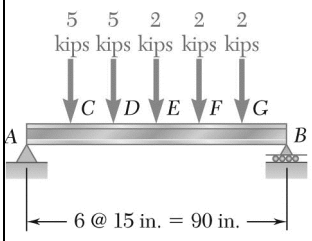
$$|M|_{\max} = 104\ \text{kN} \cdot \text{m} = 104 \times 10^3\ \text{N} \cdot \text{m}$$

For W310 × 52, $S = 747 \times 10^3\ \text{mm}^3 = 747 \times 10^{-6}\ \text{m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{104 \times 10^3}{747 \times 10^{-6}} = 139.2 \times 10^6\ \text{Pa}$$

$$\sigma = 139.2\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

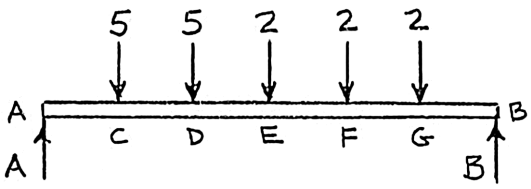


PROBLEM 5.49

Using the method of Sec. 5.3, solve Prob.5.19.

PROBLEM 5.19 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION



Use entire beam as free body.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-90A + (75)(5) + (60)(5) + (45)(2) + (30)(2) + (15)(2) = 0$$

$$\boldsymbol{A} = 9.5 \text{ kips} \uparrow$$

Shear *A* to *C*: $V = 9.5$ kips

Area under shear curve *A* to *C*: $\int V dx = (15)(9.5)$

$$= 142.5 \text{ kip} \cdot \text{in}$$

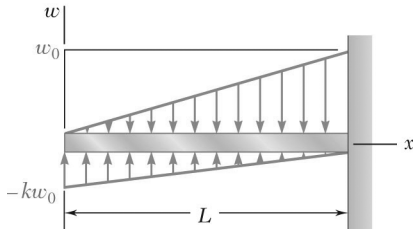
$$M_A = 0$$

$$M_C = 0 + 142.5 = 142.5 \text{ kip} \cdot \text{in}$$

For $S8 \times 18.4$, $S = 14.4 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{142.5}{14.4}$ $\sigma = 9.90$ ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.50

For the beam and loading shown, determine the equations of the shear and bending-moment curves, and the maximum absolute value of the bending moment in the beam, knowing that (*a*) $k = 1$, (*b*) $k = 0.5$.

SOLUTION

$$w = \frac{w_0 x}{L} - \frac{k w_0 (L - x)}{L} = (1 + k)\frac{w_0 x}{L} - k w.$$

$$\frac{dV}{dx} = -w = k w_0 - (1 + k)\frac{w_0 x}{L}$$

$$V = k w_0 x - (1 + k)\frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \quad C_1 = 0$$

$$\frac{dM}{dx} = V = k w_0 x - (1 + k)\frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1 + k)\frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \quad \text{at} \quad x = 0 \quad C_2 = 0$$

$$M = \frac{k w_0 x^2}{2} - \frac{(1 + k) w_0 x^3}{6L}$$

(*a*) $\underline{k = 1}$.

$$V = w_0 x - \frac{w_0 x^2}{L} \blacktriangleleft$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L} \blacktriangleleft$$

Maximum M occurs at $x = L$.

$$|M|_{\max} = \frac{w_0 L^2}{6} \blacktriangleleft$$

(*b*) $\underline{k = \frac{1}{2}}$.

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L} \blacktriangleleft$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L} \blacktriangleleft$$

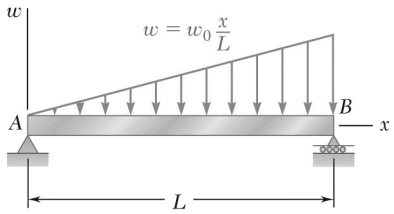
$$V = 0 \quad \text{at} \quad x = \frac{2}{3} L$$

At $x = \frac{2}{3} L$, $M = \frac{w_0 \left(\frac{2}{3} L\right)^2}{4} - \frac{w_0 \left(\frac{2}{3} L\right)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704\, w_0 L^2$

At $x = L$, $M = 0$

$$|M|_{\max} = \frac{w_0 L^2}{27} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.51

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\quad C_2 = 0$

$M = 0$ at $x = L$ $\quad 0 = -\frac{1}{6} w_0 L^2 + C_1 L \quad C_1 = \frac{1}{6} w_0 L$

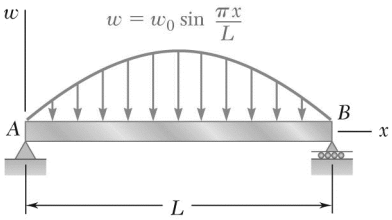
(*a*) $V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L^2$ $\qquad V = \frac{1}{6} w_0 (L^2 - 3x^2)/L$ ◀

$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x$ $\qquad M = \frac{1}{6} w_0 (Lx - x^3/L)$ ◀

(*b*) $M_{\max}$ occurs when $\frac{dM}{dx} = V = 0$. $\quad L^2 - 3x_m^2 = 0$

$$x_m = \frac{L}{\sqrt{3}} \qquad M_{\max} = \frac{1}{6} w_0 \left(\frac{L^2}{\sqrt{3}} - \frac{L^2}{3\sqrt{3}} \right) \qquad M_{\max} = 0.0642 w_0 L^2 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.52

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0 \qquad C_2 = 0$$

$$M = 0 \text{ at } x = L \qquad 0 = 0 + C_1 L + 0$$

$$C_1 = 0$$

(*a*) $$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$ ◀

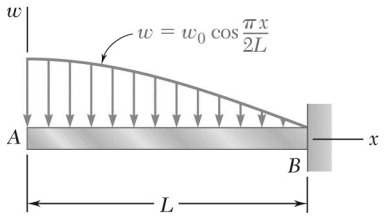
$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$ ◀

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

(*b*) $$M_{\max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2}$$

$$M_{\max} = \frac{w_0 L^2}{\pi^2}$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.53

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \cos\frac{\pi x}{2L}$$

$$V = -\frac{2Lw_0}{\pi}\sin\frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 w_0}{\pi^2}\cos\frac{\pi x}{2L} + C_1 x + C_2$$

$V = 0$ at $x = 0$. Hence, $C_1 = 0$.

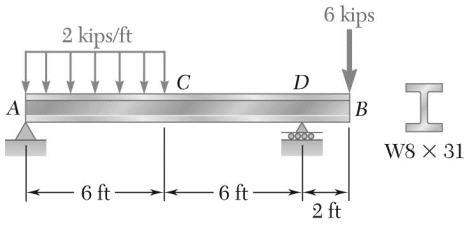
$M = 0$ at $x = 0$. Hence, $C_2 = -\dfrac{4L^2 w_0}{\pi^2}$.

(*a*) $V = -(2Lw_0 / \pi)\sin(\pi x / 2L)$ ◀

$M = -(4L^2 w_0/\pi^2)[1 - \cos(\pi x/2L)]$ ◀

(*b*) $|M|_{\max} = 4w_0 L^2/\pi^2$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.54

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

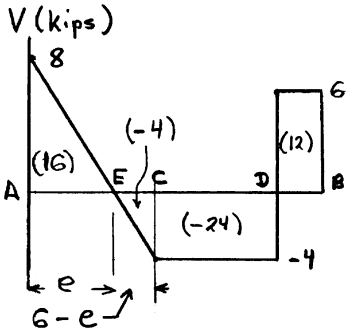
$$+\circlearrowleft M_D = 0: \; -12A + (9)(6)(2) - (2)(6) = 0$$
$$A = 8 \text{ kips}$$
$$+\circlearrowleft M_A = 0: \; -(3)(6) + 12D - (14)(6) = 0$$
$$D = 10 \text{ kips}$$

Shear: $V_A = 8$ kips

$V_C = 8 - (6)(2) = -4$ kips

C to D: $V = -4$ kips

D to B: $V = -4 + 10 = 6$ kips



Locate point E where $V = 0$.

$$\frac{e}{8} = \frac{6-e}{4} \quad 12e = 48$$
$$e = 4 \text{ ft} \qquad 6 - e = 2 \text{ ft}$$

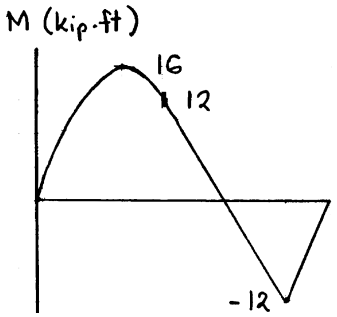
Areas of the shear diagram:

A to E: $\int V dx = \left(\frac{1}{2}\right)(4)(8) = 16 \text{ kip} \cdot \text{ft}$

E to C: $\int V dx = \left(\frac{1}{2}\right)(2)(-4) = -4 \text{ kip} \cdot \text{ft}$

C to D: $\int V dx = (6)(-4) = -24 \text{ kip} \cdot \text{ft}$

D to B: $\int V dx = (2)(6) = 12 \text{ kip} \cdot \text{ft}$



Bending moments: $M_A = 0$

$M_E = 0 + 16 = 16 \text{ kip} \cdot \text{ft}$

$M_C = 16 - 4 = 12 \text{ kip} \cdot \text{ft}$

$M_D = 12 - 24 = -12 \text{ kip} \cdot \text{ft}$

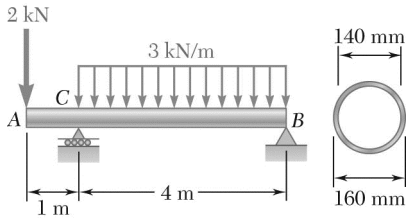
$M_B = -12 + 12 = 0$

Maximum $|M| = 16 \text{ kip} \cdot \text{ft} = 192 \text{ kip} \cdot \text{in}$

For W8× 31 rolled steel section, $S = 27.5 \text{ in}^3$

Normal stress: $\sigma = \frac{|M|}{S} = \frac{192}{27.5}$ $\sigma = 6.98$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.55

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \ (2)(1) - (3)(4)(2) + 4B = 0$$
$$B = 5.5 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0: \ (5)(2) + (3)(4)(2) - 4C = 0$$
$$C = 8.5 \text{ kN}$$

Shear:

A to C: $V = -2$ kN

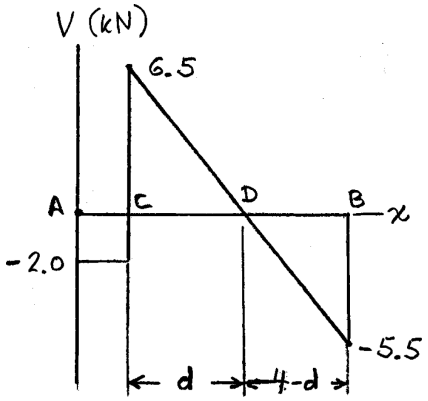
C^+: $V = -2 + 8.5 = 6.5$ kN

B: $V = 6.5 - (3)(4) = -5.5$ kN

Locate point D where $V = 0$.

$$\frac{d}{6.5} = \frac{4-d}{5.5} \qquad 12d = 26$$

$$d = 2.1667 \text{ m} \qquad 4 - d = 3.8333 \text{ m}$$

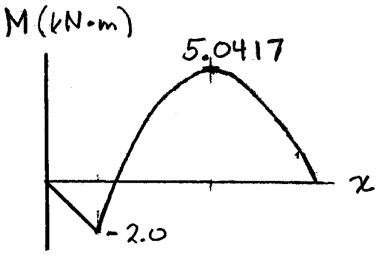


Areas of the shear diagram:

A to C: $\int V dx = (-2.0)(1) = -2.0 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = \frac{1}{2}(2.16667)(6.5) = 7.0417 \text{ kN} \cdot \text{m}$

D to B: $\int V dx = \frac{1}{2}(3.83333)(-5.5) = -5.0417 \text{ kN} \cdot \text{m}$



Bending moments:

$M_A = 0$

$M_C = 0 - 2.0 = -2.0 \text{ kN} \cdot \text{m}$

$M_D = -2.0 + 7.0417 = 5.0417 \text{ kN} \cdot \text{m}$

$M_B = 5.0417 - 5.0417 = 0$

Maximum $|M| = 5.0417 \text{ kN} \cdot \text{m} = 5.0417 \times 10^3 \text{ N} \cdot \text{m}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.55 ***(Continued)***

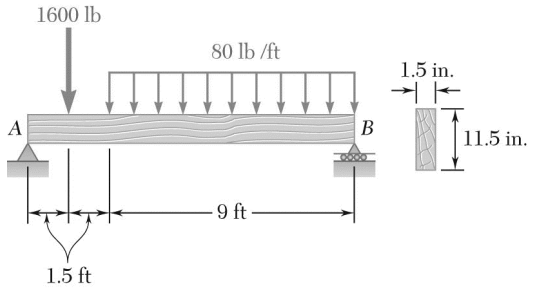
For pipe: $c_o = \frac{1}{2}d_o = \frac{1}{2}(160) = 80 \text{ mm}, \quad c_i = \frac{1}{2}d_i = \frac{1}{2}(140) = 70 \text{ mm}$

$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = \frac{\pi}{4}\left[(80)^4 - (70)^4\right] = 13.3125 \times 10^6 \text{ mm}^4$$

$$S = \frac{I}{c_o} = \frac{13.3125 \times 10^6}{80} = 166.406 \times 10^3 \text{ mm}^3 = 166.406 \times 10^{-6} \text{ m}^3$$

Normal stress: $\sigma = \frac{M}{S} = \frac{5.0417 \times 10^3}{166.406 \times 10^{-6}} = 30.3 \times 10^6 \text{ Pa}$ $\sigma = 30.3 \text{ MPa}$ ◀

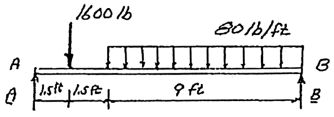
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.56

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

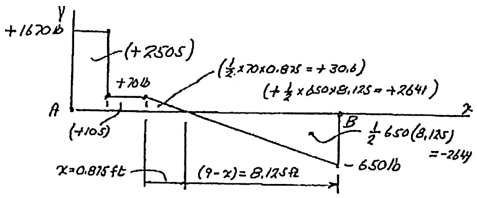


$$\circlearrowright \sum M_A = 0 : (1600 \text{ lb})(1.5 \text{ ft}) + \big[(80 \text{ lb/ft})(9 \text{ ft})\big](7.5 \text{ ft}) - 12B = 0$$

$$B = +650 \text{ lb} \qquad \underline{\boldsymbol{B}} = 650 \text{ lb} \uparrow$$

$$+\uparrow \sum F_y = 0 : A - 1600 \text{ lb} - \big[(80 \text{ lb/ft})(9 \text{ ft})\big] + 650 \text{ lb} = 0$$

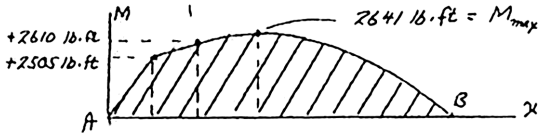
$$A = +1670 \text{ lb} \qquad \underline{\boldsymbol{A}} = 1670 \text{ lb} \uparrow$$



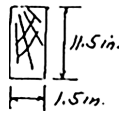
$$\frac{x}{70 \text{ lb}} = \frac{9 - x}{650 \text{ lb}} \qquad x = 0.875 \text{ ft} \qquad 2641 \text{ lb} \cdot \text{ft} = M_{\max}$$

$$c = \frac{1}{2}(11.5 \text{ in.}) = 5.75 \text{ in.}$$

$$I = \frac{1}{12}(1.5 \text{ in.})(11.5 \text{ in.})^3 = 190.1 \text{ in}^4$$



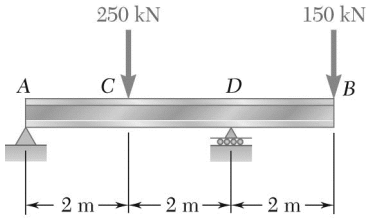
◀



$$M_{\max} = 2641 \text{ lb} \cdot \text{ft} = 31{,}690 \text{ lb} \cdot \text{in}$$

$$\sigma_m = \frac{M_{\max} c}{I} = \frac{(31{,}690 \text{ lb} \cdot \text{in})(5.75 \text{ in.})}{190.1 \text{ in}^4} \qquad \sigma_m = 959 \text{ psi}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



W410 × 114

PROBLEM 5.57

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$$w = 0$$

$$+\circlearrowright \Sigma M_D = 0:$$

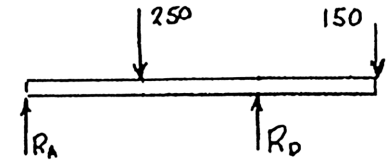
$$-4R_A + (2)(250) - (2)(150) = 0$$

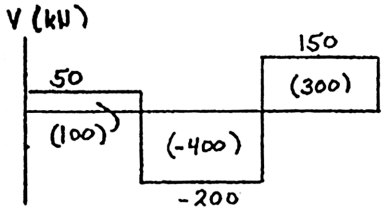
$$\boldsymbol{R}_A = 50 \text{ kN} \uparrow$$

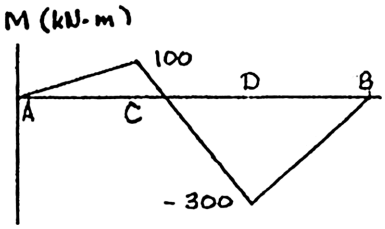
$$+\circlearrowright \Sigma M_A = 0:$$

$$4R_D - (2)(250) - (6)(150) = 0$$

$$\boldsymbol{R}_D = 350 \text{ kN} \uparrow$$







Shear: $V_A = 50 \text{ kN}$

A to C: $V = 50 \text{ kN}$

C to D: $V = 50 - 250 = -200 \text{ kN}$

D to B: $V = -200 + 350 = 150 \text{ kN}$

Areas of shear diagram:

A to C: $\int V dx = (50)(2) = 100 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = (-200)(2) = -400 \text{ kN} \cdot \text{m}$

D to B: $\int V dx = (150)(2) = 300 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_C = M_A + \int V dx = 0 + 100 = 100 \text{ kN} \cdot \text{m}$$

$$M_D = M_C + \int V dx = 100 - 400 = -300 \text{ kN} \cdot \text{m}$$

$$M_B = M_D + \int V dx = -300 + 300 = 0$$

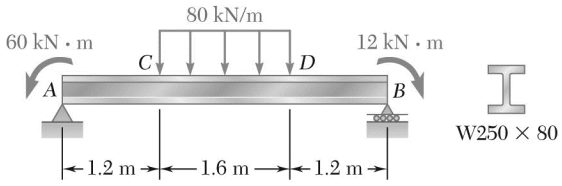
Maximum $|M| = 300 \text{ kN} \cdot \text{m} = 300 \times 10^3 \text{ N} \cdot \text{m}$

For W410 × 114 rolled steel section, $S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa}$$

$$\sigma_m = 136.4 \text{ MPa} \blacktriangleleft$$

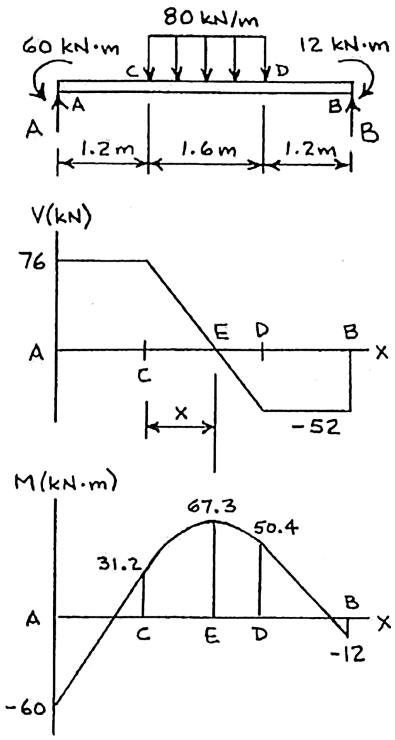
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.58

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION



Reaction:

$$+\circlearrowleft \Sigma M_B = 0 : -4A + 60 + (80)(1.6)(2) - 12 = 0$$

$$A = 76\,\text{kN} \uparrow$$

Shear: $V_A = 76\,\text{kN}$

A to C: $V = 76\,\text{kN}$ ◀

$$V_D = 76 - (80)(1.6) = -52\,\text{kN}$$

D to C: $V = -52\,\text{kN}$

Locate point where $V = 0$:

$$V(x) = -80x + 76 = 0 \qquad x = 0.95\,\text{m}$$

Areas of shear diagram:

$$A \text{ to } C: \int V dx = (1.2)(76) = 91.2\,\text{kN} \cdot \text{m}$$

$$C \text{ to } E: \int V dx = \frac{1}{2}(0.95)(76) = 36.1\,\text{kN} \cdot \text{m}$$

$$E \text{ to } D: \int V dx = \frac{1}{2}(0.65)(-52) = -16.9\,\text{kN} \cdot \text{m}$$

$$D \text{ to } B: \int V dx = (1.2)(-52) = -62.4\,\text{kN} \cdot \text{m}$$

Bending moments: $M_A = -60\,\text{kN} \cdot \text{m}$

$$M_C = -60 + 91.2 = 31.2\,\text{kN} \cdot \text{m}$$

$$M_E = 31.2 + 36.1 = 67.3\,\text{kN} \cdot \text{m}$$ ◀

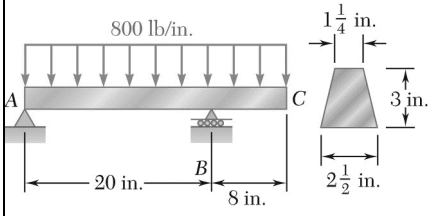
$$M_D = 67.3 - 16.9 = 50.4\,\text{kN} \cdot \text{m}$$

$$M_B = 50.4 - 62.4 = -12\,\text{kN} \cdot \text{m}$$

For W250 × 80, $S = 983 \times 10^3\,\text{mm}^3$

Normal stress: $$\sigma_{\max} = \frac{|M|}{S} = \frac{67.3 \times 10^3\,\text{N} \cdot \text{m}}{983 \times 10^{-6}\,\text{m}^3} = 68.5 \times 10^6\,\text{Pa} \qquad \sigma_m = 68.5\,\text{MPa}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.59

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -20A + (6)(28)(800) = 0$$
$$A = 6.72\times10^3 \text{ lb}$$

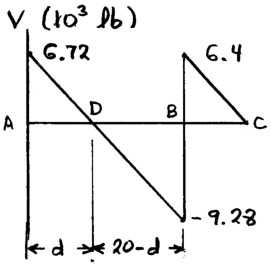
$$+\circlearrowleft \Sigma M_A = 0: \quad 20B - (14)(28)(800) = 0$$
$$B = 15.68\times10^3 \text{ lb}$$

Shear: $V_A = 6.72\times10^3 \text{ lb}$

B^-: $V_{B^-} = 6.72\times10^3 - (20)(800) = -9.28\times10^3 \text{ lb}$

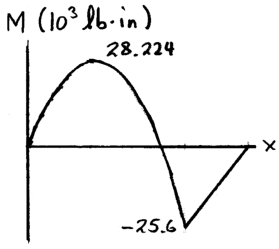
B^+: $V_{B^+} = -9.28\times10^3 + (15.68\times10^3) = 6.4\times10^3 \text{ lb}$

C: $V_C = 6.4\times10^3 - (8)(800) = 0$



Locate point D where $V = 0$.

$$\frac{d}{6.72} = \frac{20-d}{9.28} \qquad 16d = 134.4$$
$$d = 8.4 \text{ in.} \qquad 20 - d = 11.6 \text{ in.}$$



Areas of the shear diagram:

A to D: $\int V dx = \left(\frac{1}{2}\right)(8.4)(6.72\times10^3) = 28.224\times10^3 \text{ lb}\cdot\text{in}$

D to B: $\int V dx = \left(\frac{1}{2}\right)(11.6)(-9.28\times10^3) = -53.824\times10^3 \text{ lb}\cdot\text{in}$

B to C: $\int V dx = \left(\frac{1}{2}\right)(8)(6.4\times10^3) = 25.6\times10^3 \text{ lb}\cdot\text{in}$

Bending moments:

$$M_A = 0$$
$$M_D = 0 + 28.224\times10^3 = 28.224\times10^3 \text{ lb}\cdot\text{in}$$
$$M_B = 28.224\times10^3 - 53.824\times10^3 = -25.6\times10^3 \text{ lb}\cdot\text{in}$$
$$M_C = -25.6\times10^3 + 25.6\times10^3 = 0$$

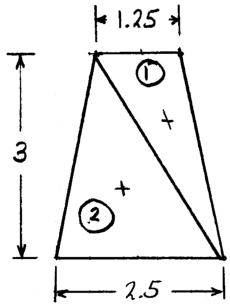
Maximum $|M| = 28.224\times10^3 \text{ lb}\cdot\text{in}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.59 *(Continued)*

Locate centroid of cross section. See table below.

$$\bar{Y} = \frac{7.5}{5.625} = 1.3333 \text{ in. from bottom.}$$



For each triangle, $\bar{I} = \frac{1}{36}bh^3$

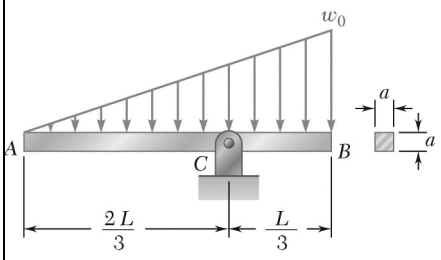
Moment of inertia:

$$I = \Sigma\bar{I} + \Sigma Ad^2$$
$$= 1.25 + 2.8125 = 4.0625 \text{ in}^4$$

Normal stress: $\sigma = \frac{Mc}{I} = \frac{(28.224\times10^3)(1.6667)}{4.0625} = 11.58\times10^3 \text{ psi}$ $\sigma = 11.58 \text{ ksi}$ ◀

Part	A, in^2	$\bar{y}$, in.	$A\bar{y}$, in^3	d, in.	Ad^2 in^4	$\bar{I}$ in^4
①	1.875	2	3.75	0.6667	0.8333	0.9375
②	3.75	1	3.75	0.3333	0.4167	1.875
Σ	5.625		7.5		1.25	2.8125

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.60

Beam AB, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (*a*) Check that the beam is in equilibrium. (*b*) Show that the maximum stress due to bending occurs at C and is equal to $w_0L^2/(1.5a)^3$.

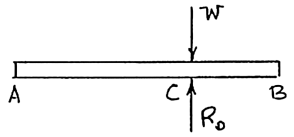
SOLUTION

(*a*) Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution, the centroid lies at $x = \dfrac{2L}{3}$. $\quad W = \dfrac{1}{2}w_0L$

(*a*) $+\uparrow \sum F_y = 0: \; R_D - W = 0 \qquad R_D = \dfrac{1}{2}w_0L \qquad +\circlearrowleft \sum M_C = 0: \; 0 = 0$ equilibrium ◀

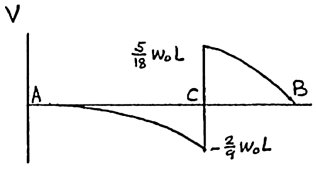
$$V = 0, \; M = 0, \text{ at } x = 0$$



$$0 < x < \frac{2L}{3}, \quad \frac{dV}{dx} = -w = -\frac{w_0x}{L}$$

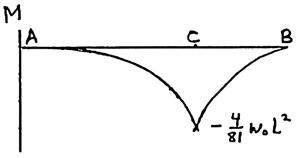
$$\frac{dM}{dx} = V = -\frac{w_0x^2}{2L} + C_1 = -\frac{w_0x^2}{2L}$$

$$M = -\frac{w_0x^3}{6L} + C_2 = -\frac{w_0x^3}{6L}$$



Just to the left of C, $\quad V = -\dfrac{w_0(2L/3)^2}{2L} = -\dfrac{2}{9}w_0L$

Just to the right of C, $\quad V = -\dfrac{2}{9}w_0L + R_D = \dfrac{5}{18}w_0L$



Note sign change. Maximum $|M|$ occurs at C. $\quad M_C = -\dfrac{w_0(2L/3)^3}{6L} = -\dfrac{4}{81}w_0L^2$

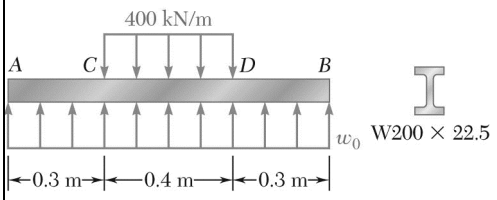
Maximum $|M| = \dfrac{4}{81}w_0L^2$

For square cross section, $\quad I = \dfrac{1}{12}a^4 \qquad c = \dfrac{1}{2}a$

(*b*) $\quad \sigma_m = \dfrac{|M|_{\max} c}{I} = \dfrac{4}{81}\dfrac{w_0L^2 6}{a^3} = \dfrac{8}{27}\dfrac{w_0L^2}{a^3} = \left(\dfrac{2}{3}\right)^3 \dfrac{w_0L^2}{a^3}$

$$\sigma_m = \frac{w_0L^2}{(1.5a)^3}$$ ◀

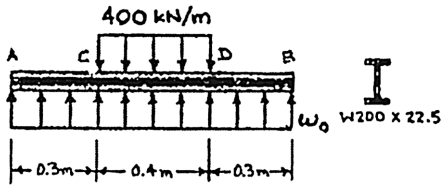
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.61

Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION



$$+\uparrow \sum F_y = 0: \ (1)(w_0) - (0.4)(400) = 0$$

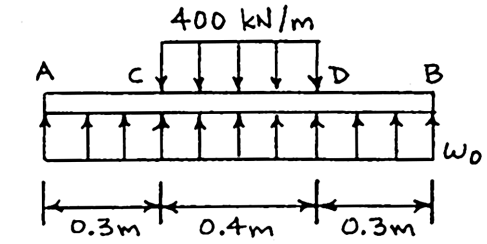
$$w_0 = 160 \text{ kN/m}$$

Shear diagram: $V_A = 0$

$$V_C = 0 + (0.3)(160) = 48 \text{ kN}$$

$$V_D = 48 - (0.3)(400) + (0.3)(160) = -48 \text{ kN}$$

$$V_B = -48 + (0.3)(160) = 0$$



Locate point E where $V = 0$.

By symmetry, E is the midpoint of CD.

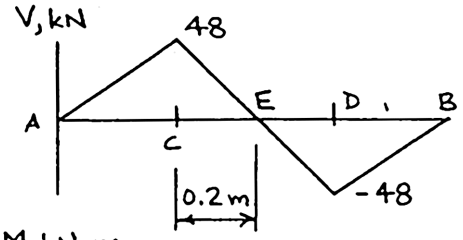
Areas of shear diagram:

$$A \text{ to } C: \quad \frac{1}{2}(0.3)(48) = 7.2 \text{ kN} \cdot \text{m}$$

$$C \text{ to } E: \quad \frac{1}{2}(0.2)(48) = 4.8 \text{ kN} \cdot \text{m}$$

$$E \text{ to } D: \quad \frac{1}{2}(0.2)(-48) = -4.8 \text{ kN} \cdot \text{m}$$

$$D \text{ to } B: \quad \frac{1}{2}(0.3)(-48) = -7.2 \text{ kN} \cdot \text{m}$$



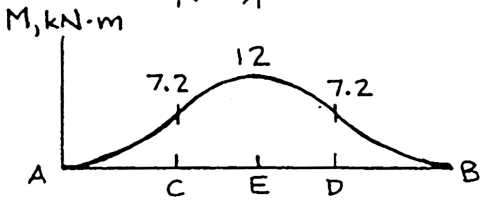
Bending moments: $M_A = 0$

$$M_C = 0 + 7.2 = 7.2 \text{ kN}$$

$$M_E = 7.2 + 4.8 = 12.0 \text{ kN}$$

$$M_D = 12.0 - 4.8 = 7.2 \text{ kN}$$

$$M_B = 7.2 - 7.2 = 0$$

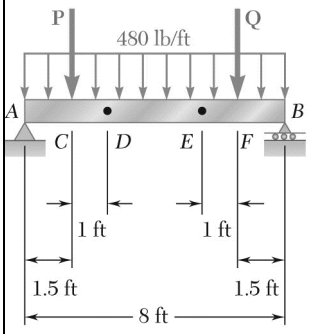


$$|M|_{\max} = 12.0 \text{ kN} \cdot \text{m} = 12.0 \times 10^3 \text{ N} \cdot \text{m}$$

For W200 × 22.5 rolled steel shape, $S_x = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{12.0 \times 10^3}{193 \times 10^{-6}} = 62.2 \times 10^6 \text{ Pa}$$ $\sigma = 62.2$ MPa ◀

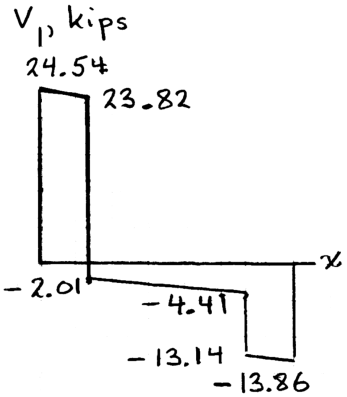
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

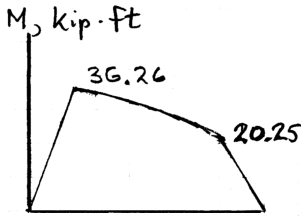


PROBLEM 5.62

The beam *AB* supports a uniformly distributed load of 480 lb/ft and two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at *D* and +10.65 ksi at *E*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION





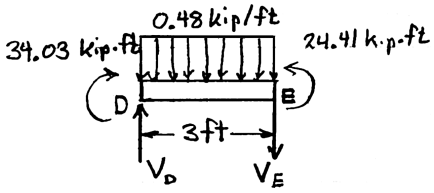
(*a*) For W8 × 31 rolled steel section, $S = 27.5 \text{ in}^3$

$$M = S\sigma$$

At *D*, $M_D = (27.5)(14.85) = 408.375 \text{ kip} \cdot \text{in}$

At *E*, $M_E = (27.5)(10.65) = 292.875 \text{ kip} \cdot \text{in}$

$$M_D = 34.03 \text{ kip} \cdot \text{ft} \quad M_E = 24.41 \text{ kip} \cdot \text{ft}$$



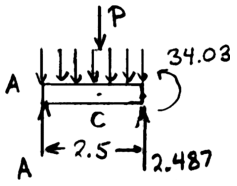
Use free body *DE*:

$$+\circlearrowleft \Sigma M_E = 0 : -34.03 + 24.41 + (1.5)(3)(0.48) - 3V_D = 0$$

$$V_D = -2.487 \text{ kips}$$

$$+\circlearrowleft M_D = 0 : -34.03 + 24.41 - (1.5)(3)(0.48) - 3V_E = 0$$

$$V_E = -3.927 \text{ kips}$$



Use free body *ACD*:

$$+\circlearrowleft \Sigma M_A = 0 : -1.5P - (1.25)(2.5)(0.48) + (2.5)(2.487) + 34.03 = 0$$

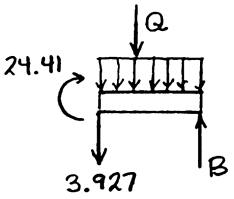
$$\boldsymbol{P} = 25.83 \text{ kips} \downarrow$$

$$+\uparrow \Sigma F_y = 0 : A - (2.5)(0.48) + 2.487 - 25.83 = 0$$

$$\boldsymbol{A} = 24.54 \text{ kips} \uparrow$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.62 *(Continued)*



Use free body *EFB*:

$$+\circlearrowleft \sum M_B = 0: \; 1.5Q + (1.25)(2.5)(0.48) + (2.5)(3.927) - 24.41 = 0$$

$$Q = 8.728 \text{ kips}$$

$$+\uparrow \sum F_y = 0: \; B - 3.927 - (2.5)(0.48) - 8.7 = 0$$

$$B = 13.855 \text{ kips}$$

Areas of load diagram:

A to C: $(1.5)(0.48) = 0.72 \text{ kip} \cdot \text{ft}$

C to F: $(5)(0.48) = 2.4 \text{ kip} \cdot \text{ft}$

F to B: $(1.5)(0.48) = 0.72 \text{ kip} \cdot \text{ft}$

Shear diagram: $V_A = 24.54 \text{ kips}$

$V_C^- = 24.54 - 0.72 = 23.82 \text{ kips}$

$V_C^+ = 23.82 - 25.83 = -2.01 \text{ kips}$

$V_F^- = -2.01 - 2.4 = 4.41 \text{ kips}$

$V_F^+ = -4.41 - 8.728 = -13.14 \text{ kips}$

$V_B = -13.14 - 0.72 = -13.86 \text{ kips}$

Areas of shear diagram:

A to C: $\frac{1}{2}(1.5)(24.52 + 23.82) = 36.23 \text{ kip} \cdot \text{ft}$

C to F: $\frac{1}{2}(5)(-2.01 - 4.41) = -16.05 \text{ kip} \cdot \text{ft}$

F to B: $\frac{1}{2}(1.5)(-13.14 - 13.86) = 20.25 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$M_C = 0 + 36.26 = 36.26 \text{ kip} \cdot \text{ft}$

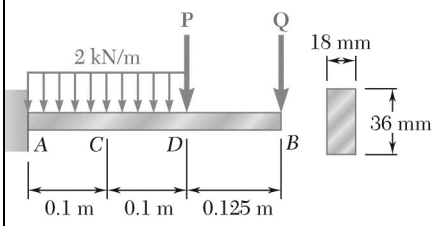
$M_F = 36.26 - 16.05 = 20.21 \text{ kip} \cdot \text{ft}$

$M_B = 20.21 - 20.25 \approx 0$

Maximum $|M|$ occurs at C: $|M|_{\max} = 36.26 \text{ kip} \cdot \text{ft} = 435.1 \text{ kip} \cdot \text{in}$

(*b*) Maximum stress: $\sigma = \dfrac{|M|_{\max}}{S} = \dfrac{435.1}{27.5}$ $\sigma = 15.82 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.63*

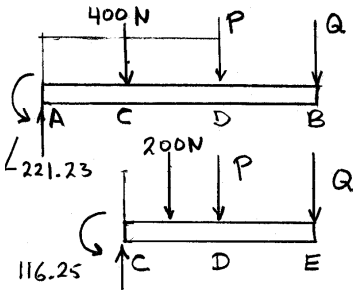
Beam AB supports a uniformly distributed load of 2 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending in bottom edge of the beam is −56.9 MPa at A and −29.9 MPa at C. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.

SOLUTION

$$I = \frac{1}{12}(18)(36)^3 = 69.984\times10^3\ \text{mm}^4$$

$$c = \frac{1}{2}d = 18\ \text{mm}$$

$$S = \frac{I}{c} = 3.888\times10^3\ \text{mm}^3 = 3.888\times10^{-6}\ \text{m}^3$$



At A, $M_A = S\sigma_A = (3.888\times10^{-6})(-56.9) = -221.25\ \text{N}\cdot\text{m}$

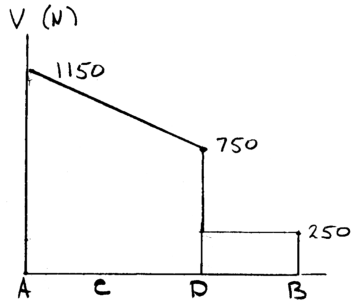
At C, $M_C = S\sigma_C = (3.888\times10^{-6})(-29.9) = -116.25\ \text{N}\cdot\text{m}$

$$+\circlearrowright \Sigma M_A = 0:\quad 221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \qquad (1)$$

$$+\circlearrowright \Sigma M_C = 0:\quad 116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \qquad (2)$$



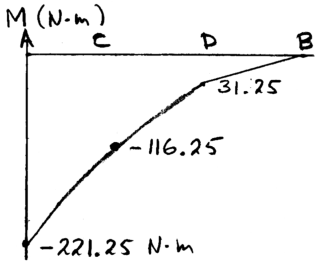
Solving (1) and (2) simultaneously, $P = 500\ \text{N}$ ◀

$Q = 250\ \text{N}$ ◀

Reaction force at A: $R_A - 400 - 500 - 250 = 0 \quad R_A = 1150\ \text{N}\cdot\text{m}$

$V_A = 1150\ \text{N} \quad V_D = 250$

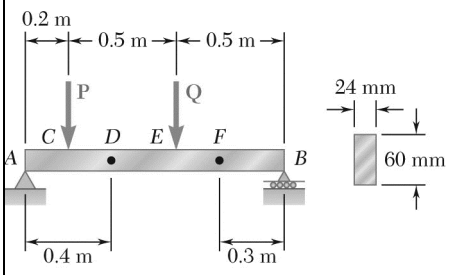
$M_A = -221.25\ \text{N}\cdot\text{m} \quad M_C = -116.25\ \text{N}\cdot\text{m} \quad M_D = -31.25\ \text{N}\cdot\text{m}$



$|V|_{\max} = 1150\ \text{N}$ ◀

$|M|_{\max} = 221\ \text{N}\cdot\text{m}$ ◀

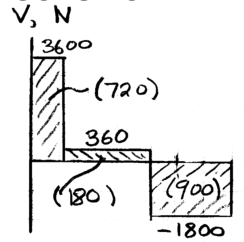
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.64*

The beam AB supports two concentrated loads **P** and **Q**. The normal stress due to bending on the bottom edge of the beam is +55 MPa at D and +37.5 MPa at F. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending that occurs in the beam.

SOLUTION

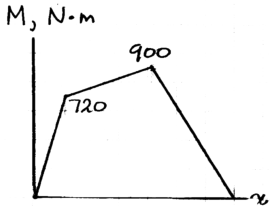


(*a*) $I = \dfrac{1}{12}(24)(60)^3 = 432\times10^3\ \text{mm}^4 \qquad c = 30\ \text{mm}$

$S = \dfrac{I}{c} = 14.4\times10^3\ \text{mm}^3 = 14.4\times10^{-6}\ \text{m}^3 \qquad M = S\sigma$

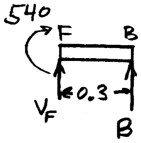
At D, $M_D = (14.4\times10^{-6})(55\times10^6) = 792\ \text{N}\cdot\text{m}$

At F, $M_F = (14.4\times10^{-6})(37.5\times10^6) = 540\ \text{N}\cdot\text{m}$



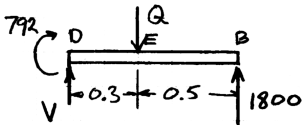
Using free body FB, $+\circlearrowleft \Sigma M_F = 0$: $-540 + 0.3B = 0$

$B = \dfrac{540}{0.3} = 1800\ \text{N}$



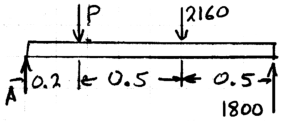
Using free body $DEFB$, $+\circlearrowleft \Sigma M_D = 0$: $-792 - 3Q + (0.8)(1800) = 0$

$Q = 2160\ \text{N}$



Using entire beam, $+\circlearrowleft \Sigma M_A = 0$: $-0.2P - (0.7)(2160) + (1.2)(1800) = 0$

$P = 3240\ \text{N}$



$+\uparrow \Sigma F_y = 0$: $A - 3240 - 2160 + 1800 = 0$

$A = 3600\ \text{N}$

Shear diagram and its areas:

A to C^-:	$V = 3600\ \text{N}$	$A_{AC} = (0.2)(3600) = 720\ \text{N}\cdot\text{m}$
C^+ to E^-:	$V = 3600 - 3240 = 360\ \text{N}$	$A_{CE} = (0.5)(360) = 180\ \text{N}\cdot\text{m}$
E^+ to B:	$V = 360 - 2160 = -1800\ \text{N}$	$A_{EB} = (0.5)(-1800) = -900\ \text{N}\cdot\text{m}$

Bending moments:

$M_A = 0$

$M_C = 0 + 720 = 720\ \text{N}\cdot\text{m}$

$M_E = 720 + 180 = 900\ \text{N}\cdot\text{m}$

$M_B = 900 - 900 = 0$

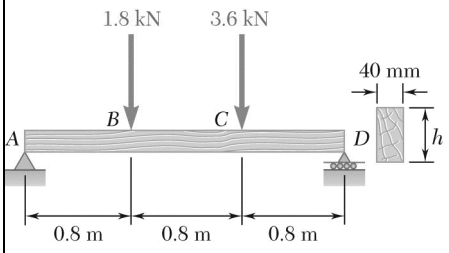
$|M|_{\max} = 900\ \text{N}\cdot\text{m}$

(*b*) Normal stress.

$\sigma_{\max} = \dfrac{|M|_{\max}}{S} = \dfrac{900}{14.4\times10^{-6}} = 62.5\times10^6\ \text{Pa}$

$\sigma_{\max} = 62.5\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.65

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

Reactions:

$$+\circlearrowleft \Sigma M_D = 0: \quad -2.4A + (1.6)(1.8) + (0.8)(3.6) = 0 \qquad A = 2.4 \text{ kN}$$

$$+\circlearrowright \Sigma M_A = 0: \quad -(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0 \qquad D = 3 \text{ kN}$$

Construct shear and bending moment diagrams:

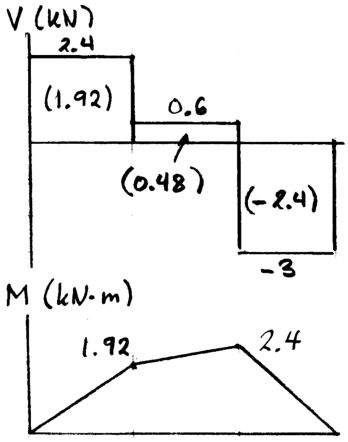
$$|M|_{\max} = 2.4 \text{ kN} \cdot \text{m} = 2.4 \times 10^3 \text{ N} \cdot \text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3 = 200 \times 10^3 \text{ mm}^3$$

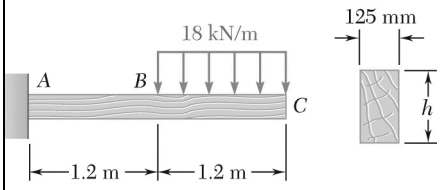
$$S = \frac{1}{6} bh^2 = \frac{1}{6}(40)h^2 = 200 \times 10^3$$

$$h^2 = \frac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$$



$$h = 173.2 \text{ mm} \blacktriangleleft$$

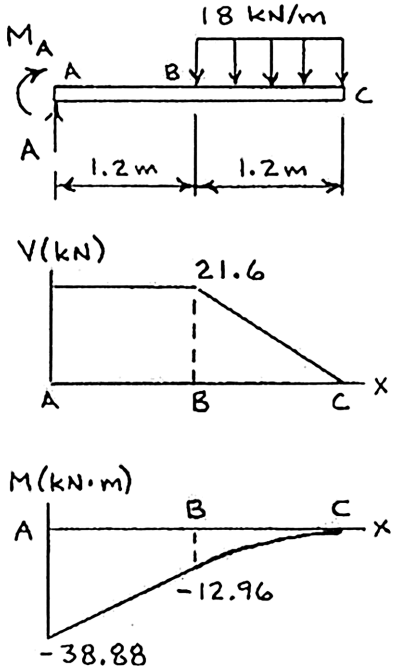
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.66

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION



Reactions: $+\uparrow \Sigma F_y = 0: \quad A - (1.2)(18) = 0$

$$\boldsymbol{A} = 21.6 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (1.8)(1.2)(18) = 0$$

$$M_A = -38.88 \text{ kN} \cdot \text{m}$$

Shear diagram: $V_A = V_B = 21.6 \text{ kN}$

$$V_C = 21.6 - (1.2)(18) = 0$$

Areas of shear diagram:

$$A \text{ to } B: (1.2)(21.6) = 25.92 \text{ kN} \cdot \text{m}$$

$$B \text{ to } C: \frac{1}{2}(1.2)(21.6) = 12.96 \text{ kN} \cdot \text{m}$$

Bending moments: $M_A = -38.88 \text{ kN} \cdot \text{m}$

$$M_B = -38.88 + 25.92 = -12.96 \text{ kN} \cdot \text{m}$$

$$M_C = -12.96 + 12.96 = 0$$

$$|M|_{\max} = 38.88 \text{ kN} \cdot \text{m} = 38.8 \times 10^3 \text{ N} \cdot \text{m}$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S}$$

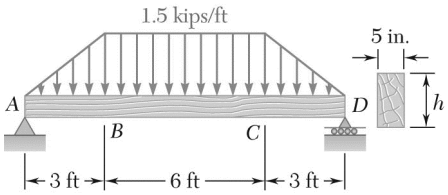
$$S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{38.8 \times 10^3 \text{ N} \cdot \text{m}}{12 \times 10^6 \text{ Pa}} = 3240 \times 10^{-6} \text{ m}^3 = 3240 \times 10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(3240 \times 10^3)}{125}} = 394 \text{ mm}$$

$$h = 394 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.67

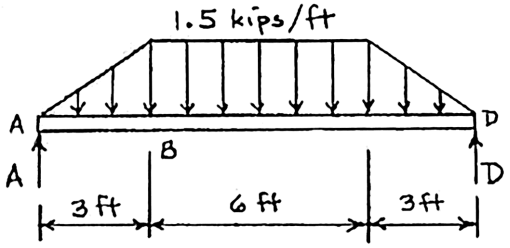
For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION

Reactions: By symmetry, $A = D$.

$$+\uparrow \Sigma F_y = 0: \; A - \frac{1}{2}(3)(1.5) - (6)(1.5) - \frac{1}{2}(3)(1.5) - D = 0$$

$$A = D = 6.75 \text{ kips} \uparrow$$

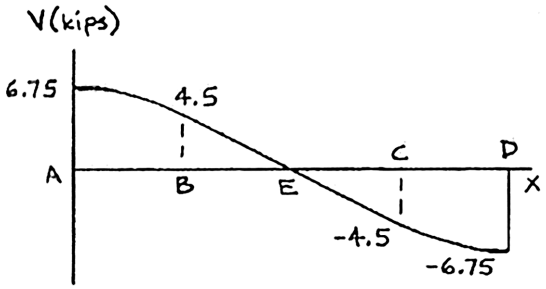


Shear diagram: $V_A = 6.75$ kips

$$V_B = 6.75 - \frac{1}{2}(3)(1.5) = 4.5 \text{ kips}$$

$$V_C = 4.5 - (6)(1.5) = -4.5 \text{ kips}$$

$$V_D = -4.5 - \frac{1}{2}(3)(1.5) = -6.75 \text{ kips}$$



Locate point E where $V = 0$:

By symmetry, E is the midpoint of BC.

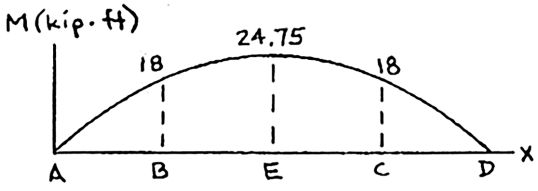
Areas of the shear diagram:

$$A \text{ to } B: \; (3)(4.5) + \frac{2}{3}(3)(2.25) = 18 \text{ kip} \cdot \text{ft}$$

$$B \text{ to } E: \; \frac{1}{2}(3)(4.5) = 6.75 \text{ kip} \cdot \text{ft}$$

$$E \text{ to } C: \; \frac{1}{2}(3)(-4.5) = -6.75 \text{ kip} \cdot \text{ft}$$

C to D: By antisymmetry, -18 kip · ft



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.67 *(Continued)*

Bending moments: $M_A = 0$

$$M_B = 0 + 18 = 18 \text{ kip} \cdot \text{ft}$$

$$M_E = 18 + 6.75 = 24.75 \text{ kip} \cdot \text{ft}$$

$$M_C = 24.75 - 6.75 = 18 \text{ kip} \cdot \text{ft}$$

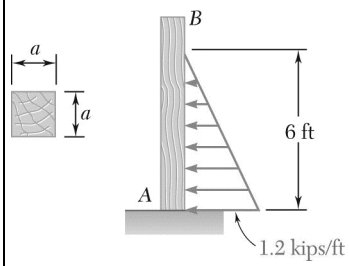
$$M_D = 18 - 18 = 0$$

$$\sigma_{\max} = \frac{|M|_{\max}}{S} \qquad S = \frac{|M|_{\max}}{\sigma_{\max}} = \frac{(24.75 \text{ kip} \cdot \text{ft})(12 \text{ in/ft})}{1.750 \text{ ksi}} = 169.714 \text{ in}^3$$

For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(169.714)}{5}} = 14.27 \text{ in.} \qquad h = 14.27 \text{ in.} \blacktriangleleft$$

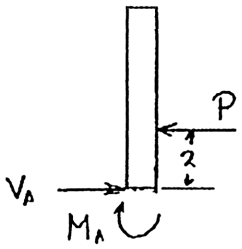
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.68

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION



Equivalent concentrated load:

$$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A:

$$M_A = (2)(3.6) = 7.2 \text{ kip} \cdot \text{ft} = 86.4 \text{ kip} \cdot \text{in}$$

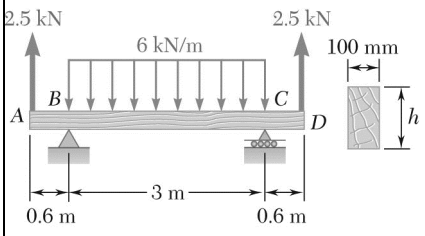
$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

For a square section, $S = \frac{1}{6}a^3$

$$a = \sqrt[3]{6S}$$

$$a_{\min} = \sqrt[3]{(6)(49.37)} \qquad a_{\min} = 6.67 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.69

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0: \quad B + C + 2.5 + 2.5 - (3)(6) = 0 \qquad B = C = 6.5 \text{ kN}$$

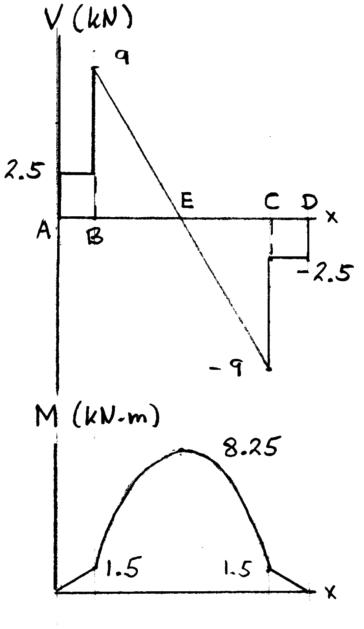
Shear:

A to B: $V = 2.5$ kN

$V_{B^+} = 2.5 + 6.5 = 9$ kN

$V_{C^-} = 9 - (3)(6) = -9$ kN

C to D: $V = -9 + 6.5 = -2.5$ kN



Areas of the shear diagram:

A to B: $\int V dx = (0.6)(2.5) = 1.5 \text{ kN} \cdot \text{m}$

B to E: $\int V dx = \left(\frac{1}{2}\right)(1.5)(9) = 6.75 \text{ kN} \cdot \text{m}$

E to C: $\int V dx = -6.75 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = -1.5 \text{ kN} \cdot \text{m}$

Bending moments:

$M_A = 0$

$M_B = 0 + 1.5 = 1.5 \text{ kN} \cdot \text{m}$

$M_E = 1.5 + 6.75 = 8.25 \text{ kN} \cdot \text{m}$

$M_C = 8.25 - 6.75 = 1.5 \text{ kN} \cdot \text{m}$

$M_D = 1.5 - 1.5 = 0$

Maximum $|M| = 8.25 \text{ kN} \cdot \text{m} = 8.25 \times 10^3 \text{ N} \cdot \text{m}$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

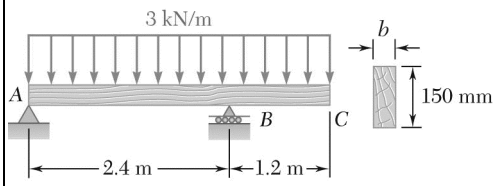
$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

For a rectangular section, $S = \frac{1}{6} b h^2$

$$687.5 \times 10^3 = \left(\frac{1}{6}\right)(100) h^2$$

$$h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2 \qquad h = 203 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.70

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

$$+\circlearrowleft M_B = 0: \quad -2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$$

$$+\circlearrowleft M_A = 0: \quad -(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$$

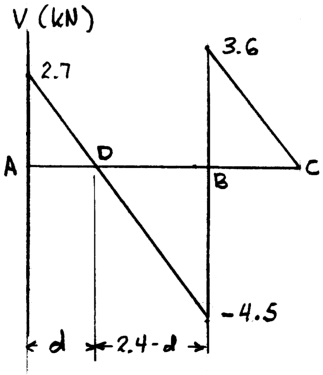
Shear:

$$V_A = 2.7 \text{ kN}$$

$$V_{B^-} = 2.7 - (2.4)(3) = -4.5 \text{ kN}$$

$$V_{B^+} = -4.5 + 8.1 = 3.6 \text{ kN}$$

$$V_C = 3.6 - (1.2)(3) = 0$$



Locate point D where $V = 0$. $\dfrac{d}{2.7} = \dfrac{2.4 - d}{4.5}$ $\qquad 7.2d = 6.48$

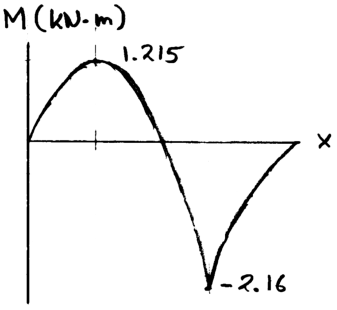
$$d = 0.9 \text{ m} \qquad 2.4 - d = 1.5 \text{ m}$$

Areas of the shear diagram:

A to D: $\displaystyle\int V dx = \left(\frac{1}{2}\right)(0.9)(2.7) = 1.215 \text{ kN} \cdot \text{m}$

D to B: $\displaystyle\int V dx = \left(\frac{1}{2}\right)(1.5)(-4.5) = -3.375 \text{ kN} \cdot \text{m}$

B to C: $\displaystyle\int V dx = \left(\frac{1}{2}\right)(1.2)(3.6) = 2.16 \text{ kN} \cdot \text{m}$



Bending moments:

$$M_A = 0$$

$$M_D = 0 + 1.215 = 1.215 \text{ kN} \cdot \text{m}$$

$$M_B = 1.215 - 3.375 = -2.16 \text{ kN} \cdot \text{m}$$

$$M_C = -2.16 + 2.16 = 0$$

Maximum $|M| = 2.16 \text{ kN} \cdot \text{m} = 2.16 \times 10^3 \text{ N} \cdot \text{m}$ $\qquad \sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

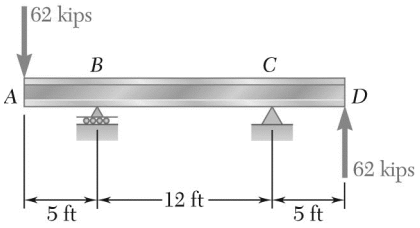
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

For rectangular section, $S = \dfrac{1}{6}bh^2 = \dfrac{1}{6}b(150)^2 = 180 \times 10^3$

$$b = \frac{(6)(180 \times 10^3)}{150^2}$$

$b = 48.0 \text{ mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.71

Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$: $(17)(62) - 12B + (5)(62) = 0$ $\qquad B = 113.667 \text{ kips} \uparrow$

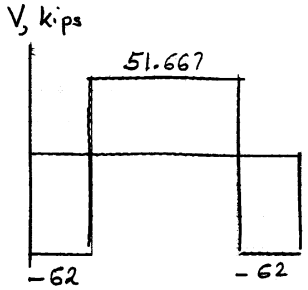
$+\circlearrowleft \Sigma M_B = 0$: $(5)(62) + 12C + (17)(62) = 0$ $\qquad C = -113.667 \text{ kips}$ or $C = 113.667 \text{ kips} \downarrow$

Shear diagram:

A to B^-: $V = -62$ kips

B^+ to C^-: $V = -62 + 113.667 = 51.667$ kips

C^+ to D: $V = 51.667 - 113.667 = -62$ kips.



Areas of shear diagram:

A to B: $(5)(-62) = -310 \text{ kip} \cdot \text{ft}$

B to C: $(12)(51.667) = 620 \text{ kip} \cdot \text{ft}$

C to D: $(5)(-62) = -310 \text{ kip} \cdot \text{ft}$

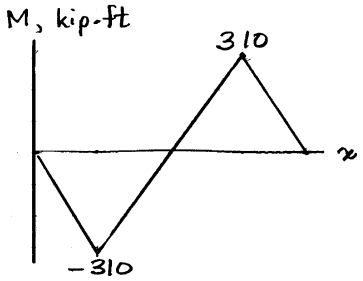
Bending moments:

$M_A = 0$

$M_B = 0 - 310 = -310 \text{ kip} \cdot \text{ft}$

$M_C = -310 + 620 = 310 \text{ kip} \cdot \text{ft}$

$M_D = 310 - 310 = 0$



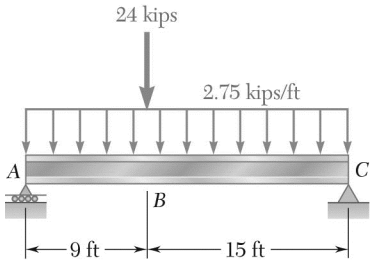
$|M|_{\max} = 310 \text{ kip} \cdot \text{ft} = 3.72 \times 10^3 \text{ kip} \cdot \text{in}$

Required $S_{\min}$: $S_{\min} = \dfrac{|M|_{\max}}{\sigma_{\text{all}}} = \dfrac{3.72 \times 10^3}{24} = 155 \text{ in}^3$

Shape	$S(\text{in}^3)$
$W27 \times 84$	213
$W21 \times 101$	227
$W18 \times 106$	204
$W14 \times 232$	232

Use $W27 \times 84$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.72

Knowing that the allowable stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -24A + (12)(24)(2.75) + (15)(24) = 0 \qquad A = 48 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 24C - (12)(24)(2.75) - (9)(24) = 0 \qquad C = 42 \text{ kips}$$

Shear:

$$V_A = 48$$
$$V_{B^-} = 48 - (9)(2.75) = 23.25 \text{ kips}$$
$$V_{B^+} = 23.25 - 24 = -0.75 \text{ kips}$$
$$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$$

Areas of the shear diagram:

A to B: $\int V dx = \left(\frac{1}{2}\right)(9)(48 + 23.25) = 320.6 \text{ kip} \cdot \text{ft}$

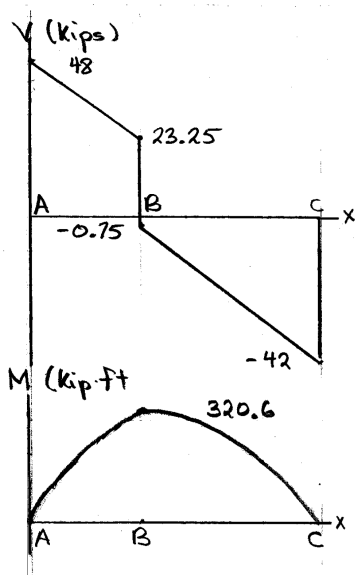
B to C: $\int V dx = \left(\frac{1}{2}\right)(15)(-0.75 - 42) = -320.6 \text{ kip} \cdot \text{ft}$

Bending moments:

$$M_A = 0$$
$$M_B = 0 + 320.6 = 320.6 \text{ kip} \cdot \text{ft}$$
$$M_C = 320.6 - 320.6 = 0$$

Maximum $|M| = 320.6 \text{ kip} \cdot \text{ft} = 3848 \text{ kip} \cdot \text{in}$

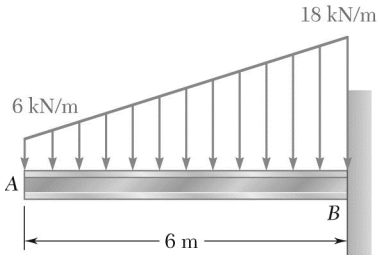
$$\sigma_{all} = 24 \text{ ksi}$$
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$$



Shape	S, (in^3)
W30×99	269
W27×84	213 ←
W24×104	258
W21×101	227
W18×106	204

Lightest wide flange beam: W27×84 @ 84 lb/ft ◀

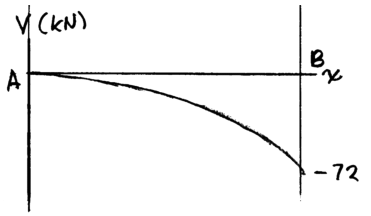
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

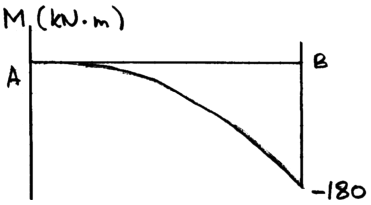


PROBLEM 5.73

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION





$$w = \left(6 + \frac{18-6}{6}x\right) = (6 + 2x)\ \text{kN/m}$$

$$\frac{dV}{dx} = -w = -6 - 2x$$

$$V = -6x - x^2 + C_1$$

$$V = 0 \quad \text{at} \quad x = 0, \quad C_1 = 0$$

$$\frac{dM}{dx} = V = -6x - x^2$$

$$M = -3x^2 - \frac{1}{3}x^3 + C_2$$

$$M = 0 \quad \text{at} \quad x = 0, \quad C_2 = 0$$

$$M = -3x^2 - \frac{1}{3}x^3$$

$|M|_{\max}$ occurs at $x = 6$ m.

$$|M|_{\max} = \left|-(3)(6)^2 - \left(\frac{1}{3}\right)(6)^3\right| = 80\ \text{kN} \cdot \text{m} = 180 \times 10^3\ \text{N} \cdot \text{m}$$

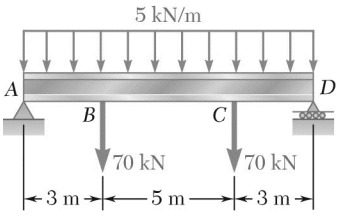
$$\sigma_{\text{all}} = 160\ \text{MPa} = 160 \times 10^6\ \text{Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{180 \times 10^3}{160 \times 10^6} = 1.125 \times 10^{-3}\ \text{m}^3 = 1125 \times 10^3\ \text{mm}^3$$

Shape	S, (10^3mm^3)
W530 × 66	1340 ←
W460 × 74	1460
W410 × 85	1510
W360 × 79	1270
W310 × 107	1600
W250 × 101	1240

Lightest acceptable wide flange beam: W530 × 66 ◀

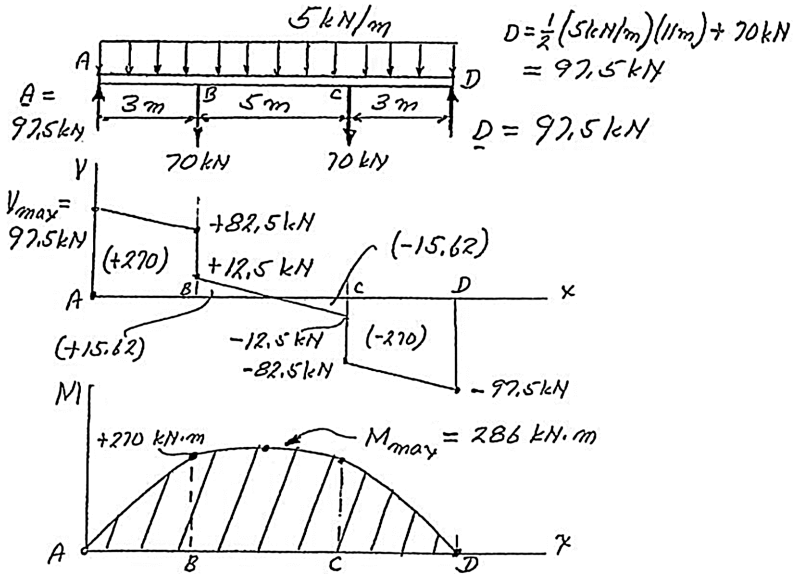
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.74

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION



$$\frac{\text{Section modulus}}{\sigma_{all} = 160 \text{ Mpa}}$$

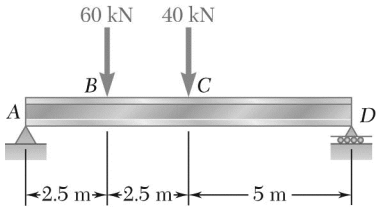
$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{286 \text{ kN} \cdot \text{m}}{160 \text{ MPa}} = 1787 \times 10^{-6} \text{ m}^3$$

$$= 1787 \times 10^{3} \text{ mm}^3$$

Shape	S, (10^3mm^3)	
W610 × 101	2520	
W530 × 92	2080 ←	◀
W460 × 113	2390	
W410 × 114	2200	
W360 × 122	2020	
W310 × 143	2150	

Use W530 × 92 ◀

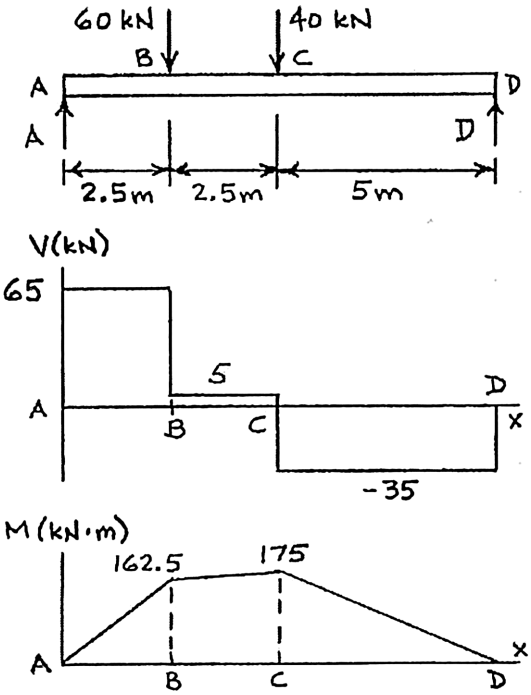
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.75

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION



Reaction: $+\circlearrowleft \sum M_D = 0 : -10A + (7.5)(60) + (5)(40) = 0$

$$A = 65\,\text{kN} \uparrow$$

Shear diagram:

A to B: $V = 65$ kN

B to C: $V = 65 - 60 = 5$ kN

C to D: $V = 5 - 40 = -35$ kN

Areas of shear diagram:

A to B: $(2.5)(65) = 162.5\ \text{kN} \cdot \text{m}$

B to C: $(2.5)(5) = 12.5\ \text{kN} \cdot \text{m}$

C to D: $(5)(-35) = -175\ \text{kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$M_B = 0 + 162.5 = 162.5\ \text{kN} \cdot \text{m}$

$M_C = 162.5 + 12.5 = 175\ \text{kN} \cdot \text{m}$

$M_D = 175 - 175 = 0$

$|M|_{\max} = 175\ \text{kN} \cdot \text{m} = 175 \times 10^3\ \text{N} \cdot \text{m}$

$\sigma_{\text{all}} = 160\ \text{MPa} = 160 \times 10^6\ \text{Pa}$

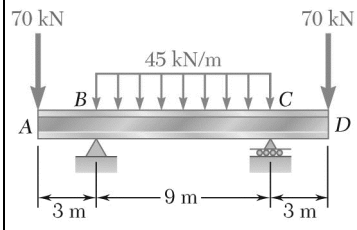
Shape	S_x, $(10^3\,\text{mm}^3)$
S610 × 119	2870
S510 × 98.2	1950
S460 × 81.4	1460 ←

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{175 \times 10^3}{160 \times 10^6} = 1093.75 \times 10^{-6}\ \text{m}^3$$

$$= 1093.75 \times 10^3\ \text{mm}^3$$

Lightest S-section: S460 × 81.4 ◀

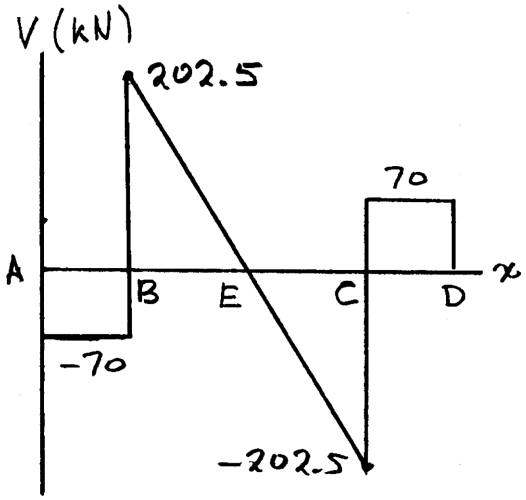
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.76

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical S-shape beam to support the loading shown.

SOLUTION



Reactions: By symmetry, $B = C$.

$$+\uparrow \sum F_y = 0 : -70 + B - (9)(45) + C - 70 = 0$$

$$\boldsymbol{B} = \boldsymbol{C} = 272.5\,\text{kN}\uparrow$$

Shear: $V_A = -70$ kN

$V_{B^-} = -70 + 0 = -70$ kN

$V_{B^+} = -70 + 272.5 = 202.5$ kN

$V_{C^-} = 202.5 - (9)(45) = -202.5$ kN

$V_{C^+} = -202.5 + 272.5 = 70$ kN

$V_D = 70$ kN

Draw shear diagram. Locate point E where $V = 0$.

E is the midpoint of BC.

Areas of the shear diagram:

A to B: $\int V dx = (3)(-70) = -210\ \text{kN} \cdot \text{m}$

B to E: $\int V dx = \frac{1}{2}(4.5)(202.5) = 455.625\ \text{kN} \cdot \text{m}$

E to C: $\int V dx = \frac{1}{2}(4.5)(-202.5) = -455.625\ \text{kN} \cdot \text{m}$

C to D: $\int V dx = (3)(70) = 210\ \text{kN} \cdot \text{m}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.76 *(Continued)*

Bending moments: $M_A = 0$

$M_B = 0 - 210 = -210.5\,\text{kN} \cdot \text{m}$

$M_E = -210 + 455.625 = 245.625\,\text{kN}$

$M_C = 245.625 - 455.625 = -210\,\text{kN}$

$M_D = -210 + 210 = 0$

$|M|_{\max} = 245.625\,\text{kN} \cdot \text{m} = 245.625 \times 10^3\,\text{N} \cdot \text{m}$

$\sigma_{\text{all}} = 160\,\text{MPa} = 160 \times 10^6\,\text{Pa}$

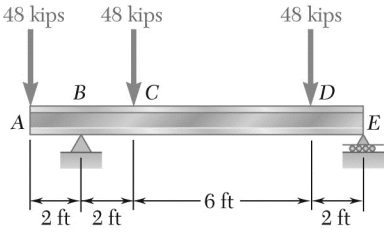
$$\sigma = \frac{|M|}{S}$$

$$S = \frac{|M|}{\sigma} = \frac{245.625 \times 10^3}{160 \times 10^6} = 1.5352 \times 10^{-3}\,\text{m}^3$$
$$= 1535.2 \times 10^3\,\text{mm}^3$$

Shape	$S(10^3\,\text{mm}^3)$
$S610 \times 119$	2870
$S510 \times 98.2$	1950 ←
$S460 \times 104$	1690

Lightest S-shape $S510 \times 98.2$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



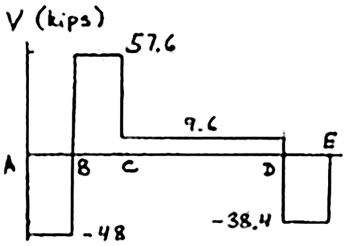
PROBLEM 5.77

Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \sum M_E = 0: (12)(48) - 10B + (8)(48) + (2)(48) = 0 \qquad \boldsymbol{B} = 105.6 \text{ kips} \uparrow$$

$$+\circlearrowleft \sum M_B = 0: (2)(48) - (2)(48) - (8)(48) + 10E = 0 \qquad \boldsymbol{E} = 38.4 \text{ kips} \uparrow$$

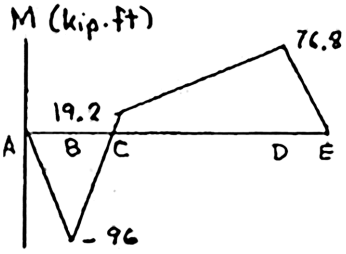


Shear: A to B: $V = -48$ kips

B to C: $V = -48 + 105.6 = 57.6$ kips

C to D: $V = 57.6 - 48 = 9.6$ kips

D to E: $V = 9.6 - 48 = -38.4$ kips



Areas: A to B: $(2)(-48) = -96$ kip · ft

B to C: $(2)(57.6) = 115.2$ kip · ft

C to D: $(6)(9.6) = 57.6$ kip · ft

D to E: $(2)(-38.4) = 76.8$ kip · ft

Bending moments: $M_A = 0$

$M_B = 0 - 96 = -96$ kip · ft

$M_C = -96 + 115.2 = 19.2$ kip · ft

$M_D = 19.2 + 57.2 = 76.8$ kip · ft

$M_E = 76.8 - 76.8 = 0$

Maximum $|M| = 96$ kip · ft $= 1152$ kip · in

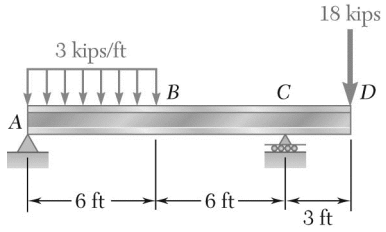
$\sigma_{\text{all}} = 24$ ksi

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	S (in^3)
S15 × 42.9	59.4
S12 × 50	50.6

Lightest S-shaped beam: S15 × 42.9 ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



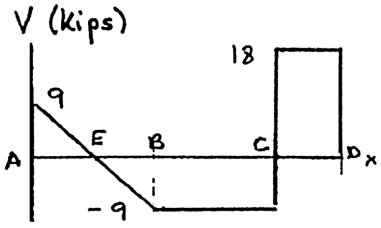
PROBLEM 5.78

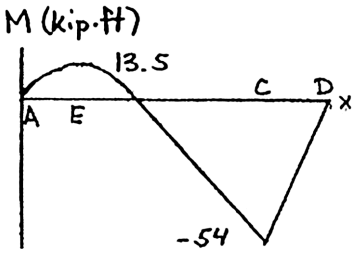
Knowing that the allowable stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

$+\circlearrowleft \sum M_C = 0 : -12A + (9)(6)(3) - (3)(18) = 0 \quad A = 9 \text{ kips}$

$+\circlearrowleft \sum M_A = 0 : 12C - (3)(6)(3) - (15)(18) = 0 \quad C = 27 \text{ kips}$





Shear: $V_A = 9 \text{ kips}$

B to C: $V = 9 - (6)(3) = -9 \text{ kips}$

C to D: $V = -9 + 27 = 18 \text{ kips}$

Areas:

A to E: $(0.5)(3)(9) = 13.5 \text{ kip} \cdot \text{ft}$

E to B: $(0.5)(3)(-9) = -13.5 \text{ kip} \cdot \text{ft}$

B to C: $(6)(-9) = -54 \text{ kip} \cdot \text{ft}$

C to D: $(3)(18) = 54 \text{ kip} \cdot \text{ft}$

Bending moments: $M_A = 0$

$M_E = 0 + 13.5 = 13.5 \text{ kip} \cdot \text{ft}$

$M_B = 13.5 - 13.5 = 0$

$M_C = 0 + 54 = 54 \text{ kip} \cdot \text{ft}$

$M_D = 54 - 54 = 0$

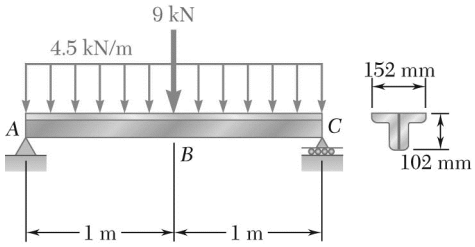
Maximum $|M| = 54 \text{ kip} \cdot \text{ft} = 648 \text{ kip} \cdot \text{in} \quad \sigma_{\text{all}} = 24 \text{ ksi}$

$$S_{\min} = \frac{648}{24} = 27 \text{ in}^3$$

Shape	$S(\text{in}^3)$
S12 × 31.8	36.2
S10 × 35	29.4

Lightest S-shaped beam: S12 × 31.8 ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.79

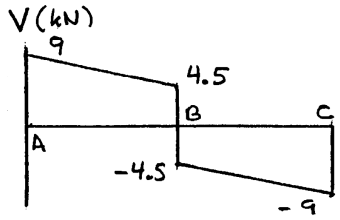
Two $L102 \times 76$ rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine the maximum angle of thickness that can be used.

SOLUTION

Reactions: By symmetry, $A = C$

$$+\uparrow \Sigma F_y = 0: \quad A - (2)(4.5) - 9 + C = 0$$

$$\boldsymbol{A} = \boldsymbol{C} = 9 \text{ kN} \uparrow$$



Shear:

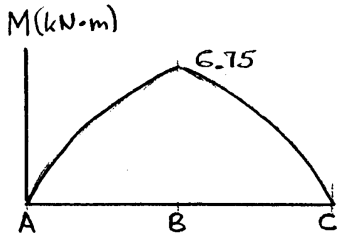
$$V_A = 9 \text{ kN}$$

$$V_{B^-} = 9 - (1)(4.5) = 4.5 \text{ kN}$$

$$V_{B^+} = 4.5 - 9 = -4.5 \text{ kN}$$

$$V_C = -4.5 - (1)(4.5) = -9 \text{ kN}$$

Areas of shear diagram:



A to B: $\int V dx = \frac{1}{2}(1)(9 + 4.5) = 6.75 \text{ kN} \cdot \text{m}$

B to C: $\int V dx = \frac{1}{2}(1)(-9 - 4.5) = -6.75 \text{ kN} \cdot \text{m}$

Bending moments:

$$M_A = 0$$

$$M_B = 0 + 6.75 = 6.75 \text{ kN} \cdot \text{m}$$

$$M_C = 6.75 - 6.75 = 0$$

Maximum $|M| = 6.75 \text{ kN} \cdot \text{m} = 6.75 \times 10^3 \text{ N} \cdot \text{m}$

$$\sigma_{\text{all}} = 140 \text{ MPa} = 140 \times 10^6 \text{ Pa}$$

For the section of two angles, $S_{\min} = \dfrac{|M|}{\sigma_{\text{all}}} = \dfrac{6.75 \times 10^3}{140 \times 10^6} = 48.21 \times 10^{-6} \text{ m}^3$

$$= 48.21 \times 10^3 \text{ mm}^3$$

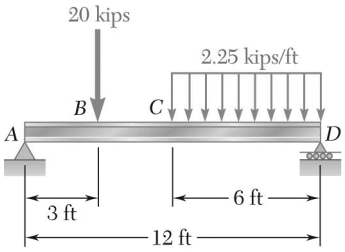
For each angle, $S_{\min} = \frac{1}{2}(48.21) = 24.105 \times 10^3 \text{ mm}^3$

Shape	$S(10^3 \text{ mm}^3)$	
L102×76×12.7	31.1	←
L102×76×9.5	24.0	
L102×76×6.4	16.6	

Lightest angle is L102×76×12.7 ◀

$t_{\min} = 12.7$ mm ◀

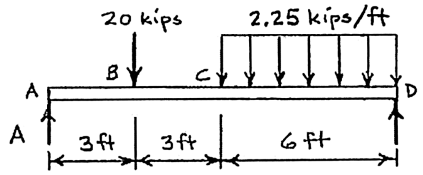
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

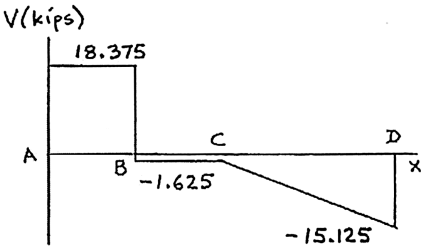


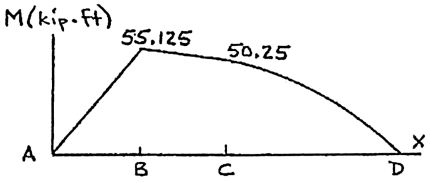
PROBLEM 5.80

Two rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 30 ksi, determine the most economical channels that can be used.

SOLUTION







Shape	$S(\text{in})^3$
C10 × 15.3	13.5
C9 × 15	11.3 ←
C8 × 18.7	11.0

Reaction: $+\circlearrowleft \Sigma M_D = 0$: $-12A + 9(20) + (6)(2.25)(3) = 0$

$$A = 18.375 \text{ kips} \uparrow$$

Shear diagram:

A to B: $V = 18.375$ kips

B to C: $V = 18.375 - 20 = -1.625$ kips

$V_D = -1.625 - (6)(2.25) = -15.125$ kips

Areas of shear diagram:

A to B: $(3)(18.375) = 55.125$ kip · ft

B to C: $(3)(-1.625) = -4.875$ kip · ft

C to D: $0.5(6)(-1.625 - 15.125) = -50.25$ kip · ft

Bending moments: $M_A = 0$

$M_B = 0 + 55.125 = 55.125$ kip · ft

$M_C = 55.125 - 4.875 = 50.25$ kip · ft

$M_D = 50.25 - 50.25 = 0$

$|M|_{\max} = 55.125 \text{ kip} \cdot \text{ft} = 661.5 \text{ kip} \cdot \text{in}$

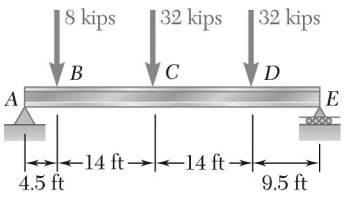
$\sigma_{\text{all}} = 30$ ksi

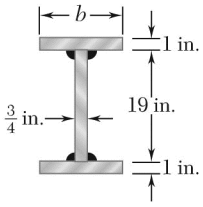
For double channel, $S_{\min} = \dfrac{|M|}{\sigma_{\text{all}}} = \dfrac{661.5}{30} = 22.05 \text{ in}^3$

For single channel, $S_{\min} = 0.5(22.05) = 11.025 \text{ in}^3$

Lightest channel section: C9 × 15 ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 5.81

Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 22 ksi, determine the minimum flange width b that can be used.

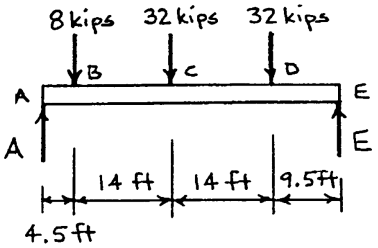
SOLUTION

Reactions: $+\circlearrowleft \Sigma M_E = 0: \quad -42A + (37.5)(8) + (23.5)(32) - (9.5)(32) = 0$

$$\boldsymbol{A} = 32.2857 \text{ kips} \uparrow$$

$+\circlearrowleft \Sigma M_A = 0: \quad 42E - (4.5)(8) - (18.5)(32) - (32.5)(32) = 0$

$$\boldsymbol{E} = 39.7143 \text{ kips} \uparrow$$



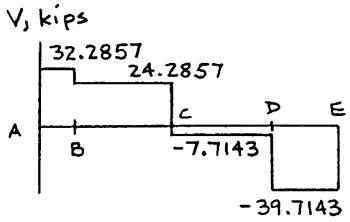
Shear:

A to B: 32.2857 kips

B to C: $32.2857 - 8 = 24.2857$ kips

C to D: $24.2857 - 32 = -7.7143$ kips

D to E: $-7.7143 - 32 = -39.7143$ kips



Areas:

A to B: $(4.5)(32.2857) = 145.286$ kip · ft

B to C: $(14)(24.2857) = 340$ kip · ft

C to D: $(14)(-7.7143) = -108$ kip · ft

D to E: $(9.5)(-39.7143) = -377.286$ kip · ft

Bending moments:

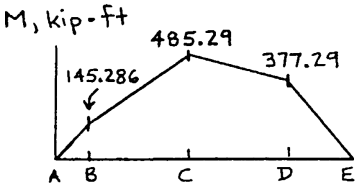
$M_A = 0$

$M_B = 0 + 145.286 = 145.286$ kip · ft

$M_C = 145.286 + 340 = 485.29$ kip · ft

$M_D = 485.29 - 108 = 377.29$ kip · ft

$M_E = 377.29 - 377.286 = 0$



Maximum $|M| = 485.29 \text{ kip} \cdot \text{ft} = 5.2834 \times 10^3 \text{ kip} \cdot \text{in} \qquad \sigma_{\text{all}} = 22 \text{ ksi}$

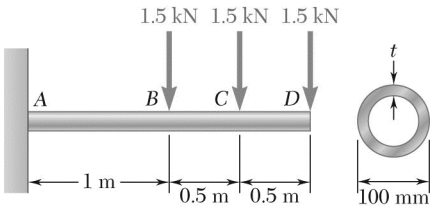
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{5.2834 \times 10^3}{22} = 264.70 \text{ in}^3$$

$$I = \frac{1}{12}\left(\frac{3}{4}\right)(19)^3 + 2\left[\frac{1}{12}(b)(1)^3 + (b)(1)(10)^2\right] = 428.69 + 200.17b$$

$$c = 9.5 + 1 = 10.5 \text{ in.}$$

$$S_{\min} = \frac{I}{c} = 40.828 + 19.063b = 264.70 \qquad b = 11.74 \text{ in.} \blacktriangleleft$$

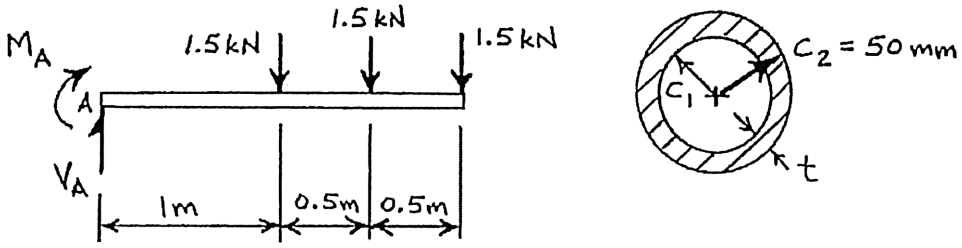
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.82

A steel pipe of 100-mm diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in 3-mm increments, and that the allowable normal stress for the steel used is 150 MPa, determine the minimum wall thickness t that can be used.

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (1)(1.5) - (1.5)(1.5) - (2)(1.5) = 0 \qquad M_A = -6.75 \text{ kN} \cdot \text{m}$$

$$|M|_{\max} = |M_A| = 6.75 \text{ kN} \cdot \text{m}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{6.75 \times 10^3 \text{ N} \cdot \text{m}}{150 \times 10^6 \text{ Pa}} = 45 \times 10^{-6} \text{ m}^3 = 45 \times 10^3 \text{ mm}^3$$

$$S_{\min} = \frac{\text{I}_{\min}}{c_2} \qquad \text{I}_{\min} = c_2 S_{\min} = (50)(45 \times 10^3) = 2.25 \times 10^6 \text{ mm}^4$$

$$\text{I}_{\min} = \frac{\pi}{4}\left(c_2^4 - c_{1\max}^4\right)$$

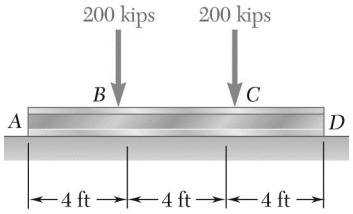
$$c_{1\max}^4 = c_2^4 - \frac{4}{\pi}\text{I}_{\min} = (50)^4 - \frac{4}{\pi}(2.25 \times 10^6) = 3.3852 \times 10^6 \text{ mm}^4$$

$$c_{1\max} = 42.894 \text{ mm}$$

$$t_{\min} = c_2 - c_{1\max} = 50 - 42.894 = 7.106 \text{ mm}$$

$$t = 9 \text{ mm} \blacktriangleleft$$

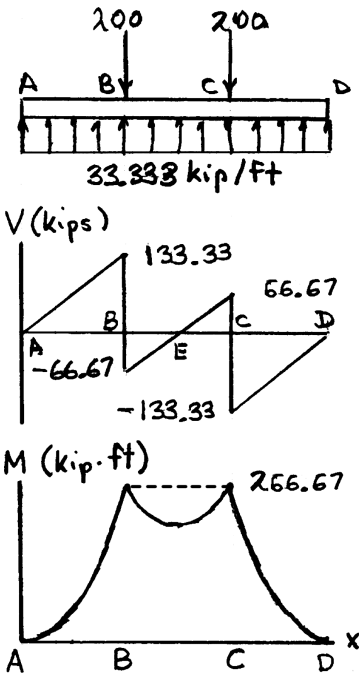
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.83

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.

SOLUTION



Distributed reaction: $q = \dfrac{400}{12} = 33.333 \text{ kip/ft}$

Shear:

$V_A = 0$

$V_{B^-} = 0 + (4)(33.333) = 133.33 \text{ kips}$

$V_{B^+} = 133.33 - 200 = -66.67 \text{ kips}$

$V_{C^-} = -66.67 + 4(33.333) = 66.67 \text{ kips}$

$V_{C^+} = 66.67 - 200 = -133.33 \text{ kips}$

$V_D = -133.33 + (4)(33.333) = 0 \text{ kips}$

Areas:

A to B: $\left(\dfrac{1}{2}\right)(4)(133.33) = 266.67 \text{ kip} \cdot \text{ft}$

B to E: $\left(\dfrac{1}{2}\right)(2)(-66.67) = -66.67 \text{ kip} \cdot \text{ft}$

E to C: $\left(\dfrac{1}{2}\right)(2)(66.67) = 66.67 \text{ kip} \cdot \text{ft}$

C to D: $\left(\dfrac{1}{2}\right)(4)(-133.33) = -266.67 \text{ kip} \cdot \text{ft}$

Bending moments:

$M_A = 0$

$M_B = 0 + 266.67 = 266.67 \text{ kip} \cdot \text{ft}$

$M_E = 266.67 - 66.67 = 200 \text{ kip} \cdot \text{ft}$

$M_C = 200 + 66.67 = 266.67 \text{ kip} \cdot \text{ft}$

$M_D = 266.67 - 266.67 = 0$

Maximum $|M| = 266.67 \text{ kip} \cdot \text{ft} = 3200 \text{ kip} \cdot \text{in.}$

$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3200}{24} = 133.3 \text{ in}^3$$

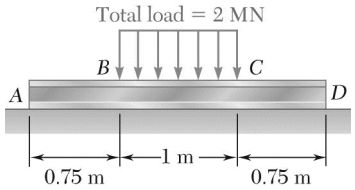
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.83 ***(Continued)***

Shape	$S(\text{in}^3)$	
W 27 × 84	213	
W 24 × 68	154	←
W 21 × 101	227	
W 18 × 76	146	
W 16 × 77	134	
W 14 × 145	232	

Lightest W-shaped section: W 24 × 68 ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.84

Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

Downward distributed load: $w = \dfrac{2}{1.0} = 2$ MN/m

Upward distributed reaction: $q = \dfrac{2}{2.5} = 0.8$ MN/m

Net distributed load over BC: 1.2 MN/m

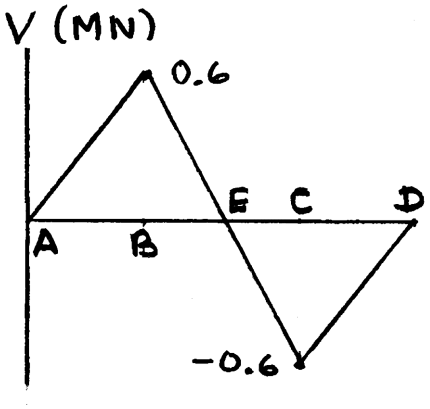
Shear:

$V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6$ MN

$V_C = 0.6 - (1.0)(1.2) = -0.6$ MN

$V_D = -0.6 + (0.75)(0.8) = 0$



Areas:

A to B: $\left(\dfrac{1}{2}\right)(0.75)(0.6) = 0.225$ MN · m

B to E: $\left(\dfrac{1}{2}\right)(0.5)(0.6) = 0.150$ MN · m

E to C: $\left(\dfrac{1}{2}\right)(0.5)(-0.6) = -0.150$ MN · m

C to D: $\left(\dfrac{1}{2}\right)(0.75)(-0.6) = -0.225$ MN · m

Bending moments:

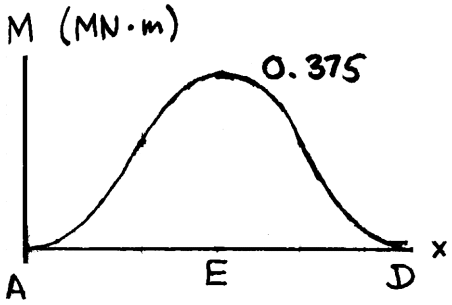
$M_A = 0$

$M_B = 0 + 0.225 = 0.225$ MN · m

$M_E = 0.225 + 0.150 = 0.375$ MN · m

$M_C = 0.375 - 0.150 = 0.225$ MN · m

$M_D = 0.225 - 0.225 = 0$



Maximum $|M| = 0.375$ MN · m $= 375 \times 10^3$ N · m

$$\sigma_{\text{all}} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$$

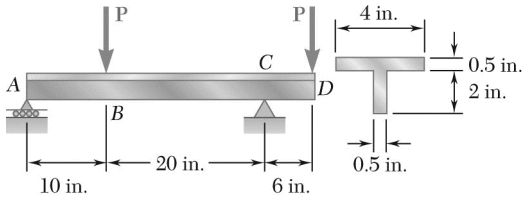
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.84 ***(Continued)***

Shape	$S(10^3\ \text{mm}^3)$		
$W\,690 \times 125$	3510		
$W\,610 \times 101$	2530	$\leftarrow$	Lightest wide flange section: $W\,610 \times 101$ ◀
$W\,530 \times 150$	3720		
$W\,460 \times 113$	2400		

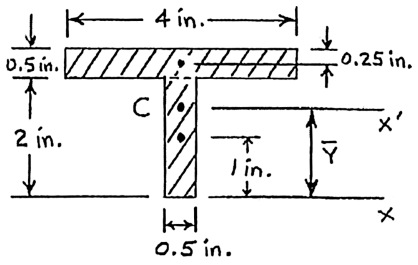
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.85

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +6 ksi in tension and −18 ksi in compression.

SOLUTION



$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{(1)(0.5)(2) + (2.25)(4)(0.5)}{(0.5 \times 2) + (4 \times 0.5)}$$

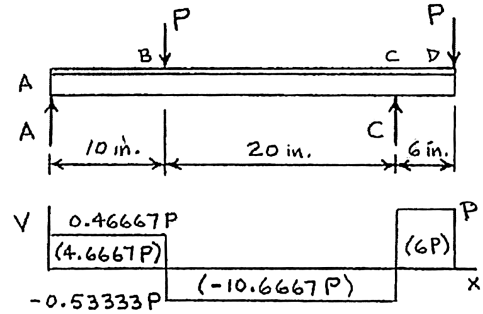
$$= 1.83333 \text{ in.}$$

$$I_{x'} = \Sigma\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \left[\frac{1}{12}(0.5)(2)^3 + (0.5)(2)(1.83333 - 1)^2\right]$$

$$+ \left[\frac{1}{12}(4)(0.5)^3 + (4)(0.5)(2.25 - 1.83333)^2\right]$$

$$= 1.41667 \text{ in}^4$$

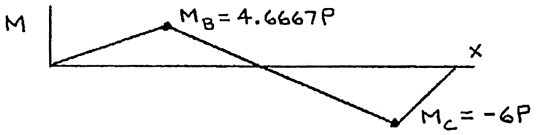


$$+\circlearrowleft \Sigma M_C = 0: \quad -30A + (20)P - (6)P = 0$$

$$\boldsymbol{A} = 0.46667P \uparrow$$

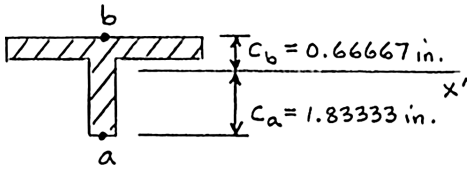
$$+\uparrow \Sigma F_y = 0: \quad 0.46667P - 2P + \boldsymbol{C} = 0$$

$$\boldsymbol{C} = 1.53333P \uparrow$$

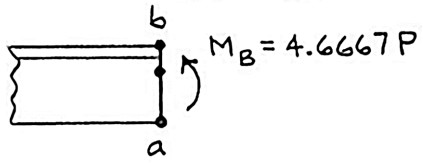


$\sigma_{\text{all}} = +6$ ksi

$\sigma_{\text{all}} = -18$ ksi



At section *B*: For $\sigma_b = 18$ ksi (compression):



$$\sigma_b = M_B \frac{c_b}{I_{x'}}$$

$$18 = (4.6667P)\frac{0.66667}{1.41667} \qquad P = 8.20 \text{ kips}$$

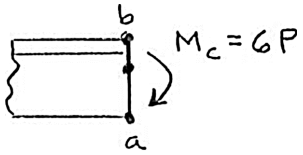
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.85 *(Continued)*

For $\sigma_a = 6$ ksi (tension), $\sigma_a = M_B \dfrac{c_a}{I_{x'}}$

$$6 = (4.6667P)\frac{1.83333}{1.41667} \qquad P = 0.994 \text{ kips}$$

At section *C*: For $\sigma_b = 6$ ksi (tension),



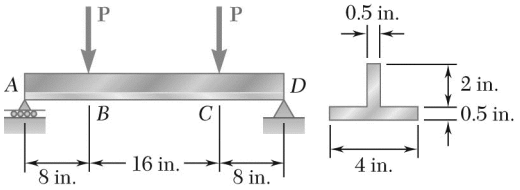
$$\sigma_b = M_C \frac{c_b}{I_{x'}} \quad 6 = (6P)\frac{0.66667}{1.41667} \qquad P = 2.13 \text{ kips}$$

For $\sigma_a = 18$ ksi (Compression),

$$\sigma_a = M_C \frac{c_a}{I_{x'}} \quad 18 = (6P)\frac{1.83333}{1.41667} \qquad P = 2.32 \text{ kips}$$

Choose smallest value of *P*: $P = 0.994$ kips ◀

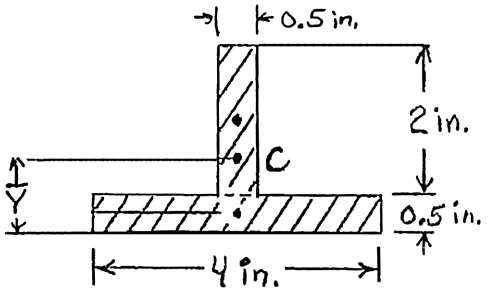
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.86

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +6 ksi in tension and −18 ksi in compression.

SOLUTION



$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{(1.5)(0.5)(2) + (0.25)(4)(0.5)}{(0.5)(2) + (4)(0.5)}$$

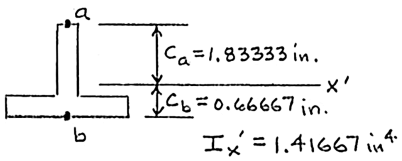
$$= 0.66667 \text{ in.}$$

$$I = \Sigma\left(\frac{1}{12}bh^3 + Ad^2\right)$$

$$= \frac{1}{12}(0.5)(2)^3 + (0.5)(2)(1.5 - 0.66667)^2$$

$$+ \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(0.66667 - 0.25)^2$$

$$= 1.41667 \text{ in}^4$$



$\sigma_{\text{all}} = +6 \text{ ksi}, \quad -18 \text{ ksi}$

For $\sigma_a = 18$ ksi (compression),

$$\sigma_a = M_{\max}\frac{c_a}{I_{x'}}$$

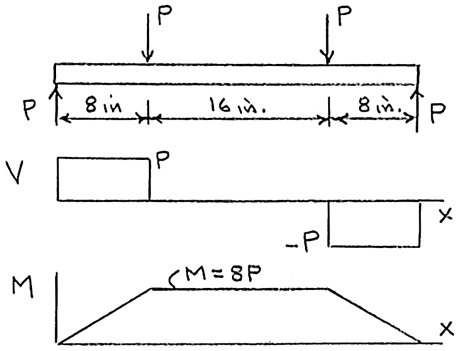
$$18 = (8P)\frac{1.83333}{1.41667} \qquad P = 1.739 \text{ kips}$$

For $\sigma_b = 6$ ksi (tension),

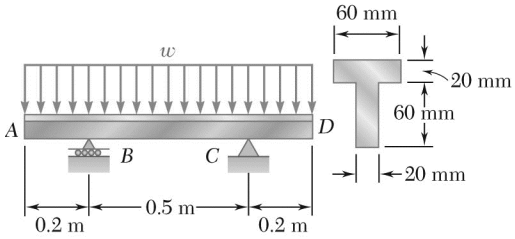
$$\sigma_b = M_{\max}\frac{c_b}{I_{x'}}$$

$$6 = (8P)\frac{0.66667}{1.41667} \qquad P = 1.594 \text{ kips}$$

Choose smallest value of P: $P = 1.594$ kips ◀



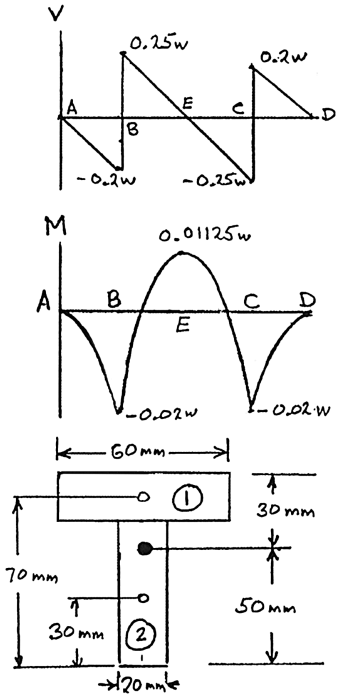
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.87

Determine the largest permissible distributed load w for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

SOLUTION



Reactions. By symmetry, $B = C$

$$+\uparrow \quad \sum F_y = 0: \; B + C - 0.9w = 0$$

$$B = C = 0.45w \uparrow$$

Shear: $V_A = 0$

$$V_{B^-} = 0 - 0.2w = -0.2w$$

$$V_{B^+} = -0.2w + 0.45w = 0.25w$$

$$V_{C^-} = 0.25w - 0.5w = -0.25w$$

$$V_{C^+} = -0.25w + 0.45w = 0.2w$$

$$V_D = 0.2w - 0.2w = 0$$

Areas: A to B. $\frac{1}{2}(0.2)(-0.2w) = -0.02w$

B to E $\frac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments: $M_A = 0$

$$M_B = 0 - 0.02w = -0.02w$$

$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}(10^3$ mm$^3)$	d, mm.	$Ad^2(10^3$ mm$^4)$	$\bar{I}(10^3$ mm$^4)$
①	1200	70	84	20	480	40
②	1200	30	36	20	480	360
Σ	2400		120		960	400

$$\bar{Y} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 1360 \times 10^3 \text{ mm}^4$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.87 *(Continued)*

Top: $I/y = (1360 \times 10^3)/30 = 45.333 \times 10^3 \text{ mm}^3 = 45.333 \times 10^{-6} \text{ m}^3$

Bottom: $I/y = (1360 \times 10^3)/(-50) = -27.2 \times 10^3 \text{ mm}^3 = -27.2 \times 10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80 \times 10^6)(45.333 \times 10^{-6})$ $w = 181.3 \times 10^3$ N/m

Compression at B and C: $-0.02w = -(-130 \times 10^6)(27.2 \times 10^{-6})$ $w = 176.8 \times 10^3$ N/m

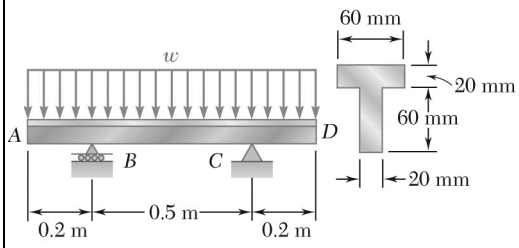
Tension at E: $0.01125w = -(80 \times 10^6)(27.2 \times 10^{-6})$ $w = 193.4 \times 10^3$ N/m

Compression at E: $0.01125w = -(-130 \times 10)(45.333 \times 10^{-6})$ $w = 523.8 \times 10^3$ N/m

The smallest allowable load controls: $w = 176.8 \times 10^3$ N/m

$w = 176.8$ kN/m ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

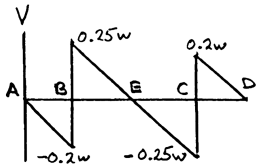


PROBLEM 5.88

Solve Prob. 5.87, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at *B* and *C*.

PROBLEM 5.87 Determine the largest permissible distributed load *w* for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and −130 MPa in compression.

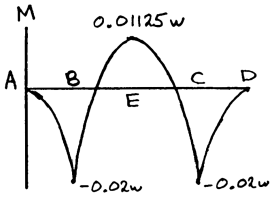
SOLUTION



Reactions: By symmetry, $B = C$

$$+\uparrow \Sigma F_y = 0: \; B + C - 0.9w = 0$$

$$\boldsymbol{B = C = 0.45w\uparrow}$$



Shear:

$$V_A = 0$$

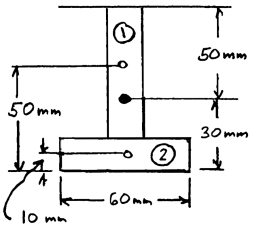
$$V_{B^-} = 0 - 0.2w = -0.2w$$

$$V_{B^+} = -0.2w + 0.45w = 0.25w$$

$$V_{C^-} = 0.25w - 0.5w = -0.25w$$

$$V_{C^+} = -0.25w + 0.45w = 0.2w$$

$$V_D = 0.2w - 0.2w = 0$$



Areas:

A to *B*: $\dfrac{1}{2}(0.2)(-0.2w) = -0.02w$

B to *E*: $\dfrac{1}{2}(0.25)(0.25w) = 0.03125w$

Bending moments:

$$M_A = 0$$

$$M_B = 0 - 0.02w = -0.02w$$

$$M_E = -0.02w + 0.03125w = 0.01125w$$

Centroid and moment of inertia:

Part	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, (10^3 mm^3)	d, mm	Ad^2(10^3 mm^4)	$\bar{I}$, (10^3 mm^4)
①	1200	50	60	20	480	360
②	1200	10	12	20	480	40
Σ	2400		72		960	400

$$\bar{Y} = \frac{72 \times 10^3}{2400} = 30\,\text{mm} \qquad I = \Sigma Ad^2 + \Sigma\bar{I} = 1360 \times 10^3\ \text{mm}^3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.88 *(Continued)*

Top: $I/y = (1360\times10^3)/(50) = 27.2\times10^3 \text{ mm}^3 = 27.2\times10^{-6} \text{ m}^3$

Bottom: $I/y = (1360\times10^3)/(-30) = -45.333\times10^8 \text{ mm}^3 = -45.333\times10^{-6} \text{ m}^3$

Bending moment limits $(M = -\sigma I/y)$ and load limits w.

Tension at B and C: $-0.02w = -(80\times10^6)(27.2\times10^{-6})$ $\quad w = 108.8\times10^3$ N/m

Compression at B and C: $-0.02w = -(-130\times10^6)(-45.333\times10^{-6})$ $\quad w = 294.7\times10^3$ N/m

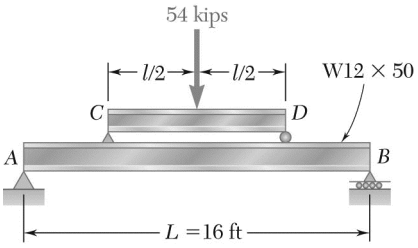
Tension at E: $0.01125w = -(80\times10^6)(-45.333\times10^{-6})$ $\quad w = 322.4\times10^3$ N/m

Compression at E: $0.01125w = -(-130\times10^6)(27.2\times10^{-6})$ $\quad w = 314.3\times10^3$ N/m

The smallest allowable load controls: $w = 108.8\times10^3$ N/m

$w = 108.8$ kN/m ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

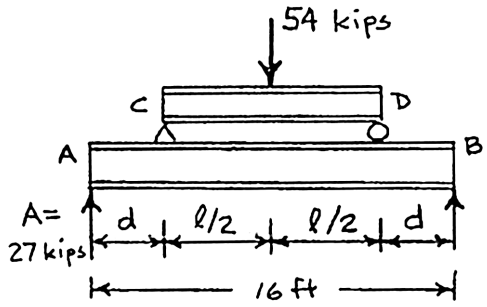


PROBLEM 5.89

A 54-kip is load is to be supported at the center of the 16-ft span shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine (*a*) the smallest allowable length l of beam CD if the $W12 \times 50$ beam AB is not to be overstressed, (*b*) the most economical W shape that can be used for beam CD. Neglect the weight of both beams.

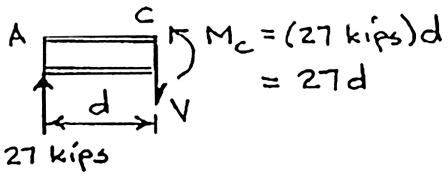
SOLUTION

(*a*)



$$d = 8\,\text{ft} - \frac{l}{2} \qquad l = 16\,\text{ft} - 2d \tag{1}$$

Beam AB (Portion AC):

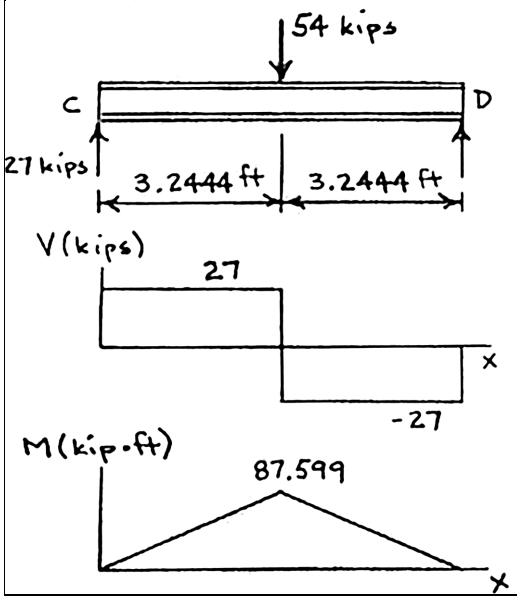


For $W12 \times 50$, $S_x = 64.2\,\text{in}^3$ $\sigma_{\text{all}} = 24\,\text{ksi}$

$$M_{\text{all}} = \sigma_{\text{all}} S_x = (24)(64.2) = 1540.8\,\text{kip} \cdot \text{in} = 128.4\,\text{kip} \cdot \text{ft}$$

$$M_C = 27d = 128.4\,\text{kip} \cdot \text{ft} \qquad d = 4.7556\,\text{ft}$$

Using (1), $l = 16 - 2d = 16 - 2(4.7556) = 6.4888\,\text{ft}$ $\quad l = 6.49\,\text{ft}$ ◀



(*b*)

Beam CD: $\quad l = 6.4888\,\text{ft} \quad \sigma_{\text{all}} = 24\,\text{ksi}$

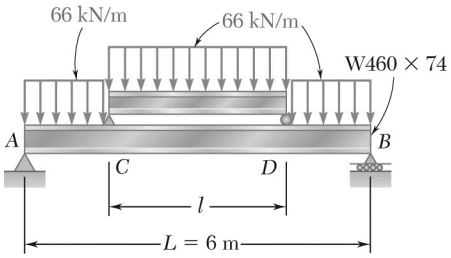
$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{(87.599 \times 12)\,\text{kip} \cdot \text{in}}{24\,\text{ksi}}$$

$$= 43.800\,\text{in}^3$$

Shape	$S(\text{in}^3)$	
$W18 \times 35$	57.6	
$W16 \times 31$	47.2	←
$W14 \times 38$	54.6	
$W12 \times 35$	45.6	
$W10 \times 45$	49.1	

$W16 \times 31$. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.90

A uniformly distributed load of 66 kN/m is to be supported over the 6-m span shown. Knowing that the allowable normal stress for the steel used is 140 MPa, determine (*a*) the smallest allowable length *l* of beam *CD* if the W460×74 beam *AB* is not to be overstressed, (*b*) the most economical *W* shape that can be used for beam *CD*. Neglect the weight of both beams.

SOLUTION

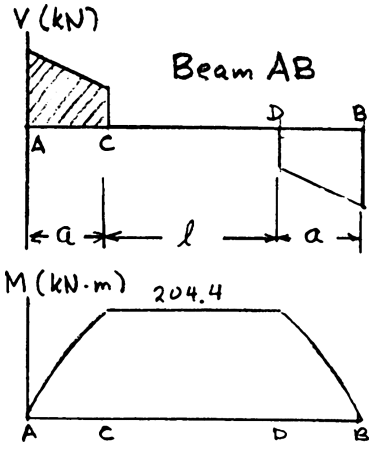
For W460×74,

$$S = 1460\times10^{3}\ \text{mm}^3 = 1460\times10^{-6}\ \text{m}^3$$

$$\sigma_{\text{all}} = 140\ \text{MPa} = 140\times10^{6}\ \text{Pa}$$

$$M_{\text{all}} = S\sigma_{\text{all}} = (1460\times10^{-6})(140\times10^{6})$$

$$= 204.4\times10^{3}\ \text{N}\cdot\text{m} = 204.4\ \text{kN}\cdot\text{m}$$



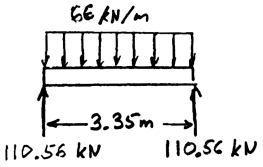
Reactions: By symmetry, $A = B, \quad C = D$

$$+\uparrow \Sigma F_y = 0:\ A + B - (6)(66) = 0$$

$$A = B = 198\ \text{kN} = 198\times10^{3}\ \text{N}$$

$$+\Sigma F_y = 0:\ C + D - 66l = 0$$

$$C = D = (33l)\ \text{kN} \qquad (1)$$



Shear and bending moment in beam *AB*:

$$0 < x < a, \qquad V = 198 - 66x\ \text{kN}$$

$$M = 198x - 33x^2\ \text{kN}\cdot\text{m}$$

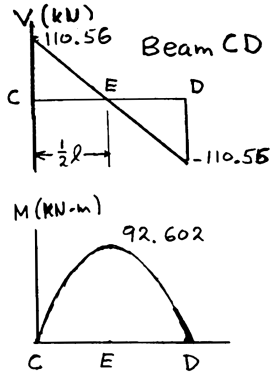
At *C*, $x = a$. $\qquad M = M_{\max}$

$$M = 198a - 33a^2\ \text{kN}\cdot\text{m}$$

Set $M = M_{\text{all}}$. $\qquad 198a - 33a^2 = 204.4$

$$33a^2 - 198a + 204.4 = 0$$

$$a = \cancel{4.6751\ \text{m}}, \qquad 1.32487\ \text{m}$$



(*a*) By geometry, $l = 6 - 2a = 3.35\ \text{m}$ $\qquad l = 3.35\ \text{m}$ ◀

From (1), $C = D = 110.56\ \text{kN}$

Draw shear and bending moment diagrams for beam *CD*. $V = 0$ at point *E*, the midpoint of *CD*.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.90 *(Continued)*

Area from A to E:

$$\int V dx = \frac{1}{2}(110.560)\left(\frac{1}{2}l\right) = 92.602 \text{ kN} \cdot \text{m}$$

$$M_E = 92.602 \text{ kN} \cdot \text{m} = 92.602 \times 10^3 \text{ N} \cdot \text{m}$$

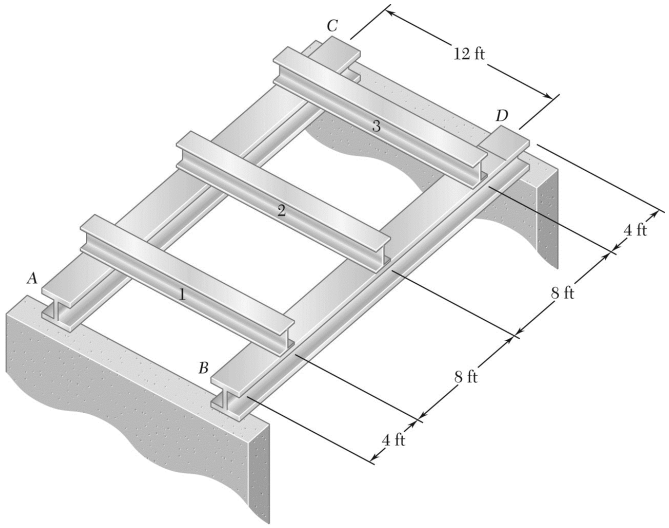
$$S_{\min} = \frac{M_E}{\sigma_{\text{all}}} = \frac{92.602 \times 10^3}{140 \times 10^6} = 661.44 \times 10^{-6} \text{ m}^3$$

$$= 661.44 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$	
W410×46.1	774	
W360×44	693	←
W310×52	748	
W250×58	693	
W200×71	709	

(*b*) Use W360×44. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



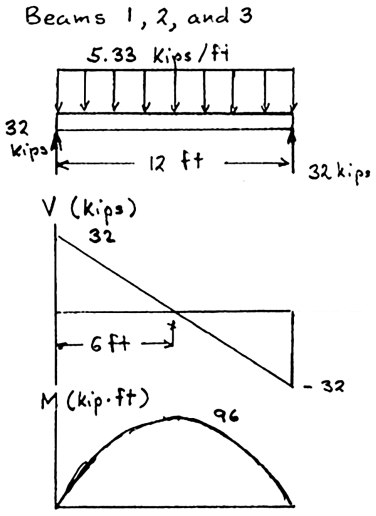
PROBLEM 5.91

Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders AC and BD. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S shape for the three beams, (*b*) the most economical W shape for the two girders.

SOLUTION

For beams 1, 2, and 3

$$\text{Maximum } M = \left(\frac{1}{2}\right)(6)(32) = 96 \text{ kip} \cdot \text{ft} = 1152 \text{ kip} \cdot \text{in}$$



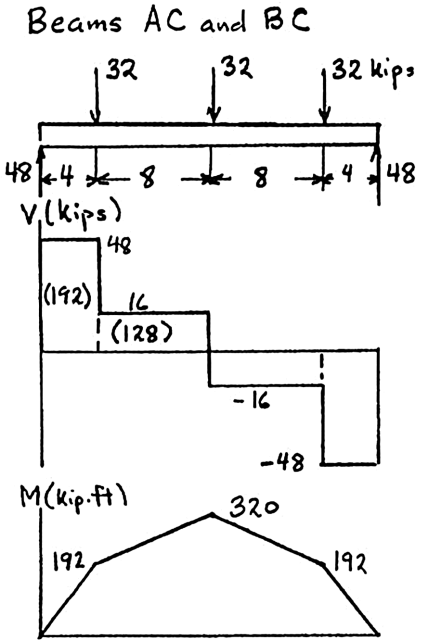
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$$

Shape	$S(\text{in}^3)$
S15 × 42.9	59.4
S12 × 50	50.6

(*a*) Use S15 × 42.9. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.91 *(Continued)*



For beams AC and BC

Areas under shear digram:

$(4)(48) = 192 \text{ kip} \cdot \text{ft}$

$(8)(16) = 128 \text{ kip} \cdot \text{ft}$

Maximum $M = 192 + 128 = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in}$

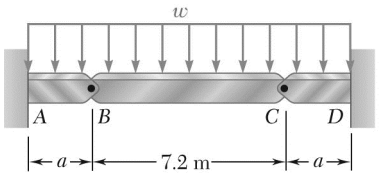
$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3840}{24} = 160 \text{ in}^3$$

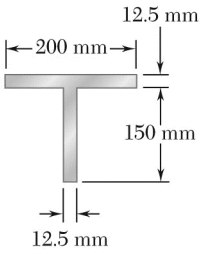
Shape	$S(\text{in}^3)$
$W30 \times 99$	269
$W27 \times 84$	213
$W24 \times 104$	258
$W21 \times 101$	227
$W18 \times 106$	204

(*b*) Use $W27 \times 84$. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.92



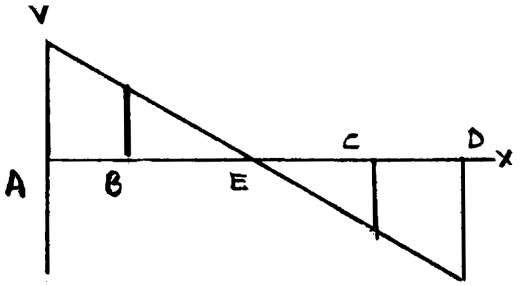


Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of *w* if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

SOLUTION

$M_B = M_C = 0$

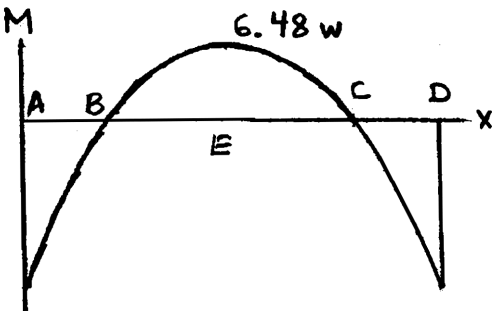
$V_B = -V_C = \left(\frac{1}{2}\right)(7.2)w = 3.6w$



Area *B* to *E* of shear diagram:

$$\left(\frac{1}{2}\right)(3.6)(3.6w) = 6.48w$$

$M_E = 0 + 6.48w = 6.48w$

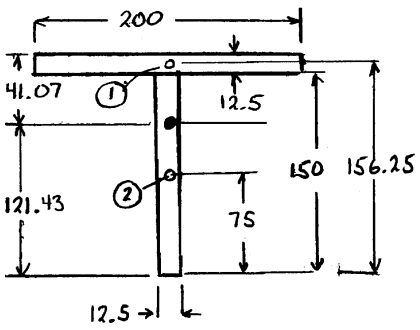


Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$\bar{Y} = \dfrac{531250}{4375} = 121.43 \text{ mm}$

$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6 \text{ mm}^4$



Location	$y\,(\text{mm})$	$I/y\,(10^3\text{ mm}^3)$ ← also (10^{-6} m^3)
Top	41.07	258.6
Bottom	−121.43	−87.47

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.92 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E: $-(110\times10^{6})(-87.47\times10^{-6}) = 9.622\times10^{3}\ \text{N}\cdot\text{m}$

Compression at E: $-(-150\times10^{-6})(258.6\times10^{-6}) = 38.8\times10^{3}\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times10^{6})(258.6\times10^{-6}) = -28.45\times10^{3}\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times10^{6})(-87.47\times10^{-6}) = -13.121\times10^{3}\ \text{N}\cdot\text{m}$

(a) Allowable load w: $6.48\,w = 9.622\times10^{3}$ $\quad w = 1.485\times10^{3}$ N/m $\quad w = 1.485$ kN/m ◀

Shear at A: $V_A = (a+3.6)\,w$

Area A to B of shear diagram: $\dfrac{1}{2}a\,(V_A + V_B) = \dfrac{1}{2}a(a+7.2)\,w$

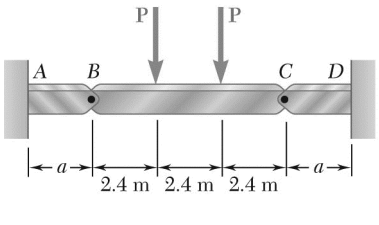
Bending moment at A (also D): $M_A = -\dfrac{1}{2}a(a+7.2)\,w$

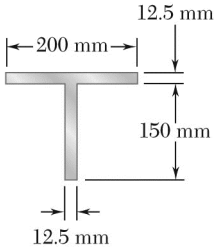
$$-\frac{1}{2}a(a+7.2)(4.485\times10^{3}) = -13.121\times10^{3}$$

(b) Distance a: $\dfrac{1}{2}a^2 + 3.6a - 8.837 = 0$ $\quad a = 1.935$ m ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.93

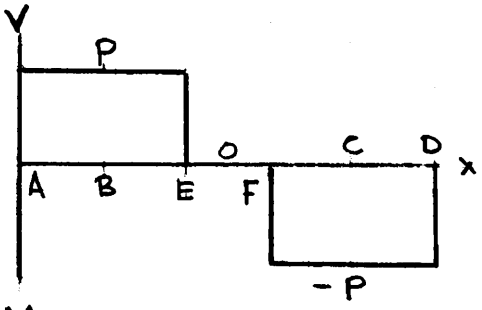




Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.

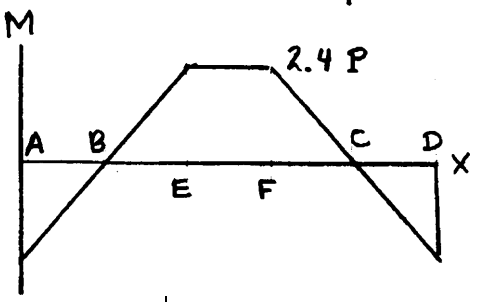
SOLUTION

$$M_B = M_C = 0$$
$$V_B = -V_C = P$$



Area *B* to *E* of shear diagram: 2.4 P

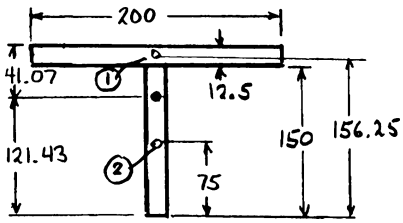
$$M_E = 0 + 2.4\text{ P} = 2.4\text{ P} = M_F$$



Centroid and moment of inertia:

Part	$A\,(\text{mm}^2)$	$\bar{y}\,(\text{mm})$	$A\bar{y}\,(\text{mm}^3)$	$d\,(\text{mm})$	$Ad^2\,(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43\text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6\text{ mm}^4$$



Location	$y\,(\text{mm})$	$I/y\,(10^3\text{ mm}^3)$ ← also (10^{-6} m^3)
Top	41.07	258.6
Bottom	−121.43	−87.47

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.93 *(Continued)*

Bending moment limits: $M = -\sigma I/y$

Tension at E and F: $-(110\times10^{6})(-87.47\times10^{-6}) = 9.622\times10^{3}\ \text{N}\cdot\text{m}$

Compression at E and F: $-(-150\times10^{6})(258.6\times10^{-6}) = 38.8\times10^{3}\ \text{N}\cdot\text{m}$

Tension at A and D: $-(110\times10^{6})(258.6\times10^{-6}) = -28.45\times10^{3}\ \text{N}\cdot\text{m}$

Compression at A and D: $-(-150\times10^{6})(-87.47\times10^{-6}) = -13.121\times10^{3}\ \text{N}\cdot\text{m}$

(*a*) Allowable load P: $2.4\ \text{P} = 9.622\times10^{3}$ $P = 4.01\times10^{3}\ \text{N}$ $P = 4.01\ \text{kN}$ ◀

Shear at A: $V_A = P$

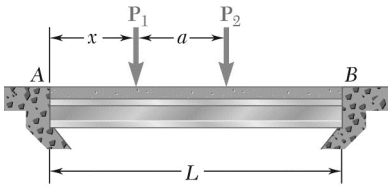
Area A to B of shear diagram: $aV_A = aP$

Bending moment at A: $M_A = -aP = -4.01\times10^{3}a$

(*b*) Distance a: $-4.01\times10^{3}a = -13.121\times10^{3}$ $a = 3.27\ \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

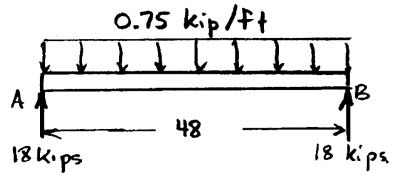
PROBLEM 5.94*



A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

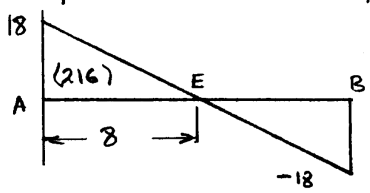
SOLUTION

$$L = 48 \text{ ft} \quad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$



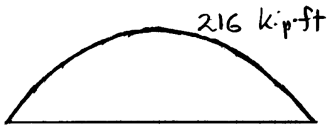
Dead load: $$R_A = R_B = \left(\frac{1}{2}\right)(48)(0.75) = 18 \text{ kips}$$

Area A to E of shear diagram:



$$\left(\frac{1}{2}\right)(8)(18) = 216 \text{ kip} \cdot \text{ft}$$

$$M_{\max} = 216 \text{ kip} \cdot \text{ft} = 2592 \text{ kip} \cdot \text{in} \text{ at point } E.$$



Live load: $$u = \frac{aP_2}{2(P_1 + P_2)} = \frac{(14)(6)}{(2)(30)} = 1.4 \text{ ft}$$

$$x = \frac{L}{2} - u = 24 - 1.4 = 22.6 \text{ ft}$$

$$x + a = 22.6 + 14 = 36.6 \text{ ft}$$

$$L - x - a = 48 - 36.6 = 11.4 \text{ ft}$$

$$+\circlearrowleft \Sigma M_B = 0: -48\,R_A + (25.4)(24) + (11.4)(6) = 0$$

$$R_A = 14.125 \text{ kips}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

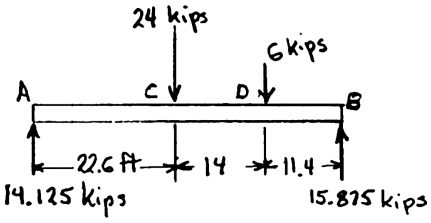
PROBLEM 5.94* *(Continued)*

Shear:

A to *C*: $V = 14.125$ kips

C to *D*: $V = 14.125 - 24 = -9.875$ kips

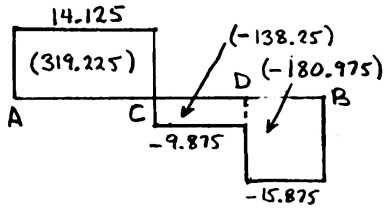
D to *B*: $V = -15.875$ kips



Area:

A to *C*: $(22.6)(14.125) = 319.225$ kip · ft

Bending moment: $M_C = 319.225 \text{ kip} \cdot \text{ft} = 3831 \text{ kip} \cdot \text{in}$



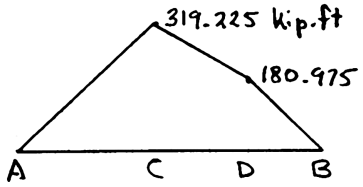
Design:

$$\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S_{\min}$$

$$S_{\min} = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U}$$

$$= \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$$

$$= 184.2 \text{ in}^3$$

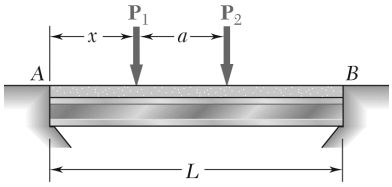


Shape	$S(\text{in}^3)$	
W 30×99	269	
W 27×84	213	←
W 24×104	258	
W 21×101	227	
W 18×106	204	

Use W 27×84. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.95



Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.94, determine how much heavier a truck could safely cross the bridge designed in that problem.

PROBLEM 5.94* A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength of $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kips/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

SOLUTION

$$L = 48 \text{ ft} \qquad a = 14 \text{ ft} \quad P_1 = 24 \text{ kips}$$
$$P_2 = 6 \text{ kips} \quad W = 0.75 \text{ kip/ft}$$

See solution to Prob. 5.94 for calculation of the following:

$$M_D = 2592 \text{ kip} \cdot \text{in} \qquad M_L = 3831 \text{ kip} \cdot \text{in}$$

For rolled steel section W27 × 84, $S = 213 \text{ in}^3$

Allowable live load moment M_L^*:

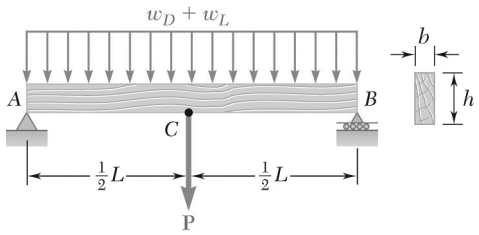
$$\gamma_D M_D + \gamma_L M_L^* = \varphi M_U = \varphi \sigma_U S$$
$$M_L^* = \frac{\varphi \sigma_U S - \gamma_D M_D}{\gamma_L}$$
$$= \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75}$$
$$= 4721 \text{ kip} \cdot \text{in}$$

Ratio:
$$\frac{M_L^*}{M_L} = \frac{4721}{3831} = 1.232 = 1 + 0.232$$

Increase 23.2%. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

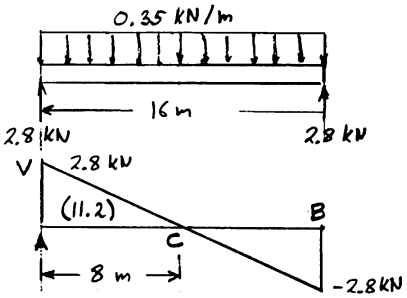
PROBLEM 5.96



A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint *C* of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth *h* of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION

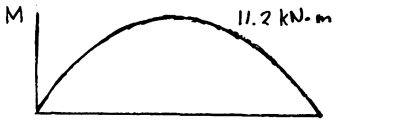
$$L = 16 \text{ m}, \quad w_D = 350 \text{ N/m} = 0.35 \text{ kN/m}$$
$$w_L = 600 \text{ N/m} = 0.6 \text{ kN/m}, \quad P = 6 \text{ kN}$$



Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8 \text{ kN}$

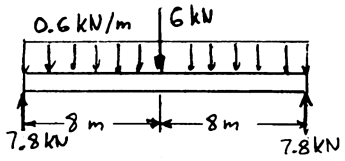
Area *A* to *C* of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2 \text{ kN} \cdot \text{m}$

Bending moment at *C*: $11.2 \text{ kN} \cdot \text{m} = 11.2 \times 10^3 \text{ N} \cdot \text{m}$



Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8 \text{ kN}$

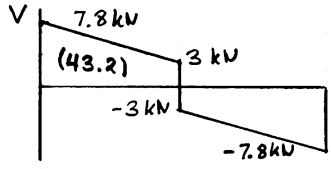
Shear at C^-: $V = 7.8 - (8)(0.6) = 3 \text{ kN}$



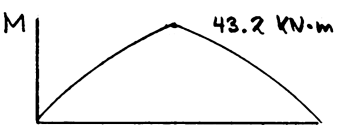
Area *A* to *C* of shear diagram: $\left(\frac{1}{2}\right)(8)(7.8 + 3) = 43.2 \text{ kN} \cdot \text{m}$

Bending moment at *C*: $43.2 \text{ kN} \cdot \text{m} = 43.2 \times 10^3 \text{ N} \cdot \text{m}$

Design: $\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$



$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$
$$= 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$

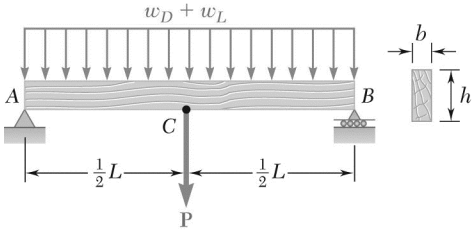


For a rectangular section, $S = \frac{1}{6}bh^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} \quad h = 383 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

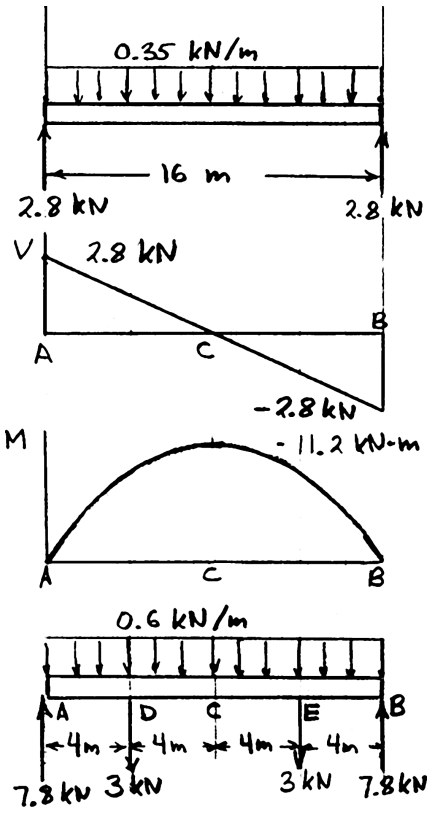
PROBLEM 5.97*



Solve Prob. 5.96, assuming that the 6-kN concentrated load **P** applied to each beam is replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.

PROBLEM 5.96* A roof structure consists of plywood and roofing material supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live load consists of a snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beam is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION



$L = 16$ m, $a = 4$ m, $w_D = 350$ N/m $= 0.35$ kN/m

$w_L = 600$ N/m $= 0.6$ kN/m, $P = 3$ kN

Dead load: $R_A = \left(\frac{1}{2}\right)(16)(0.35) = 2.8$ kN

Area A to C of shear diagram: $\left(\frac{1}{2}\right)(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2\times10^3$ N·m

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8$ kN

Shear at D^-: $7.8 - (4)(0.6) = 5.4$ kN

Shear at D^+: $5.4 - 3 = 2.4$ kN

Area A to D:

$\left(\frac{1}{2}\right)(4)(7.8 + 5.4) = 26.4$ kN·m

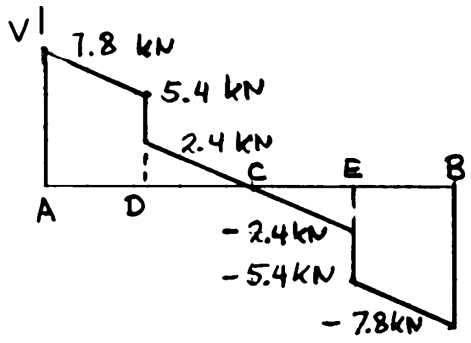
Area D to C: $\left(\frac{1}{2}\right)(4)(2.4) = 4.8$ kN·m

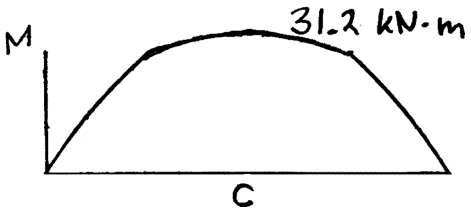
Bending moment at C: $26.4 + 4.8 = 31.2$ kN·m

$= 31.2\times10^3$ N·m

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.97* *(Continued)*





Design: $\gamma_D M_D + \gamma_L M_L = \varphi M_U = \varphi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\varphi \sigma_U} = \frac{(1.2)(11.2\times10^3) + (1.6)(31.2\times10^3)}{(0.9)(50\times10^6)}$$

$$= 1.408\times10^{-3}\,\text{m}^3 = 1.408\times10^6\,\text{mm}^3$$

For a rectangular section,

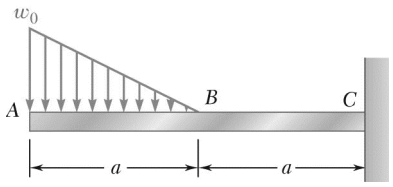
$$S = \frac{1}{6}bh^2$$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408\times10^6)}{75}}$$

$$h = 336\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.98



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1$$

$$= -\frac{dV}{dx}$$

(*a*) $$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

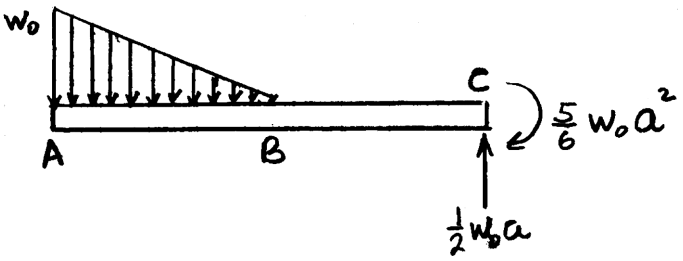
At point *C*, $x = 2a$

(*b*) $$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a}$$

$$M_C = -\frac{5}{6} w_0 a^2$$ ◀

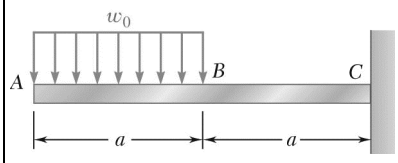
Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{5}{6} w_0 a^2$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.99



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - w_0\langle x-a\rangle^0$$

$$= -\frac{dV}{dx}$$

(*a*) $V = -w_0 x + w_0\langle x-a\rangle^1 = \dfrac{dM}{dx}$ ◀

$$M = -\frac{1}{2}w_0 x^2 + \frac{1}{2}w_0\langle x-a\rangle^2$$ ◀

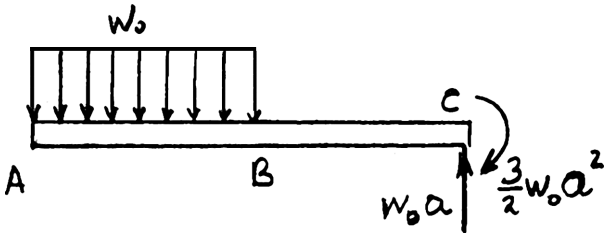
At point C, $x = 2a$

(*b*) $M_C = -\dfrac{1}{2}w_0(2a)^2 + \dfrac{1}{2}w_0 a^2$

$$M_C = -\frac{3}{2}w_0 a^2$$ ◀

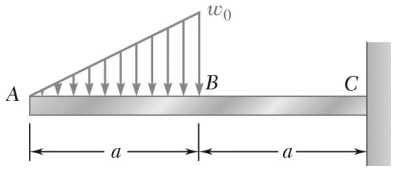
Check: $+\circlearrowleft \Sigma M_C = 0$: $\left(\dfrac{3a}{2}\right)(w_0 a) + M_C = 0$

$$M_C = -\frac{3}{2}w_0 a^2$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.100



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = \frac{w_0 x}{a} - w_0\langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1$$

$$= -\frac{dV}{dx}$$

(*a*) $$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$ ◀

$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$ ◀

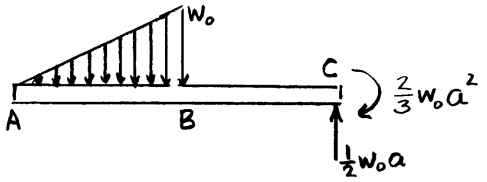
At point C, $x = 2a$

(*b*) $$M_C = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a}$$

$$M_C = -\frac{2}{3} w_0 a^2$$ ◀

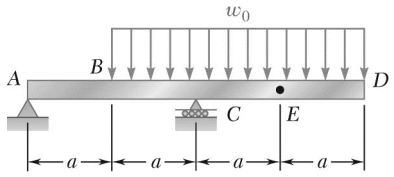
Check: $$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{4a}{3}\right)\left(\frac{1}{2} w_0 a\right) + M_C = 0$$

$$M_C = -\frac{2}{3} w_0 a^2$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.101



(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -2aA - \left(\frac{a}{2}\right) + (3aw_0) = 0 \qquad A = -\frac{3}{4}w_0 a$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2aC - \left(\frac{5a}{2}\right) + (3aw_0) = 0 \qquad C = \frac{15}{4}w_0 a$$

$$w = w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$$

(*a*) $$V = -w_0\langle x-a\rangle^1 - \frac{3}{4}w_0 a + \frac{15}{4}w_0 a\langle x-2a\rangle^0 = \frac{dM}{dx}$$ ◀

$$M = -\frac{1}{2}w_0\langle x-a\rangle^2 - \frac{3}{4}w_0 ax + \frac{15}{4}w_0 a\langle x-2a\rangle^1 + 0$$ ◀

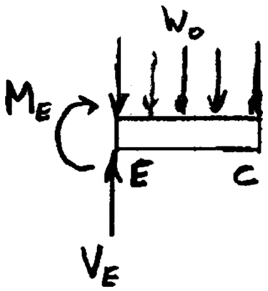
At point *E*, $x = 3a$

(*b*) $$M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0 a(3a) + \frac{15}{4}w_0 a(a)$$

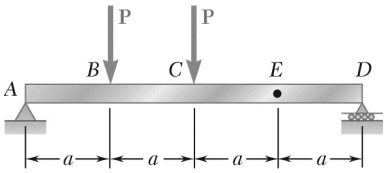
$$M_E = -\frac{1}{2}w_0 a^2$$ ◀

Check: $$+\circlearrowleft \Sigma M_E = 0: \quad -M_E - \frac{a}{2}(w_0 a) = 0$$

$$M_E = -\frac{1}{2}w_0 a^2$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.102

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4aA + 3aP + 2aP = 0 \qquad A = 1.25P$$

(*a*) $V = 1.25P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$ ◀

$$M = 1.25Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 \blacktriangleleft$$

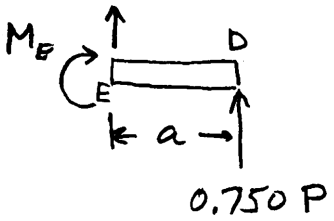
(*b*) At point *E*, $x = 3a$

$$M_E = 1.25P(3a) - P(2a) - P(a) = 0.750\,Pa \blacktriangleleft$$

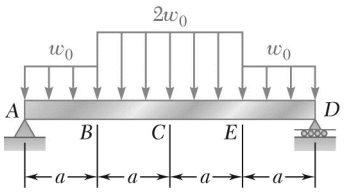
Reaction: $+\uparrow \Sigma F_y = 0: \quad A - P - P + D = 0 \qquad \boldsymbol{D} = 0.750P \uparrow$

$$+\circlearrowleft \Sigma M_E = 0: \quad -M_E + 0.750\,Pa = 0$$

$$M_E = 0.750\,Pa$$



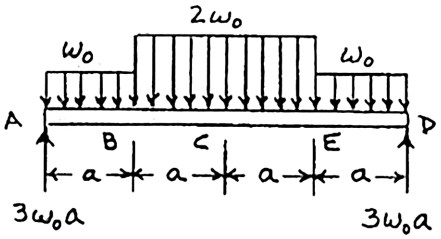
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.103

(*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of the beam to the right of *E*.

SOLUTION



$$w = w_0 + w_0\langle x-a\rangle^0 - w_0\langle x-3a\rangle^0$$

(*a*) $$V = 3w_0a - \int w\,dx$$

$$= 3w_0a - w_0x - w_0\langle x-a\rangle^1 + w_0\langle x-3a\rangle^1 \quad \blacktriangleleft$$

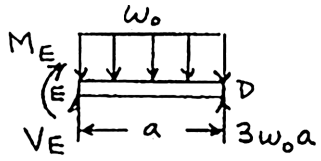
$$M = \int V\,dx = 3w_0ax - w_0x^2/2 - w_0\langle x-a\rangle^2/2$$

$$+w_0\langle x-3a\rangle^2/2 \quad \blacktriangleleft$$

(*b*) At point *E*, $x = 3a$

$$M_E = 3w_0a(3a) - w_0(3a)^2/2 - w_0(2a)^2/2$$

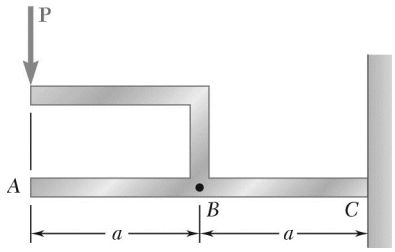
$$M_E = 5w_0a^2/2 \quad \blacktriangleleft$$



$$+\circlearrowleft \sum M_E = 0: \quad 3w_0a(a) - (w_0a)(\tfrac{a}{2}) - M_E = 0$$

$$M_E = 5w_0a^2/2 \text{ (checks)}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.104

(*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

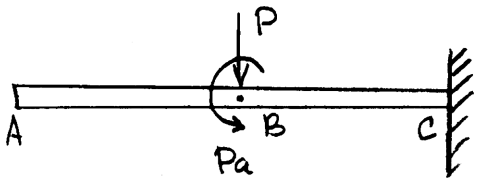
(*a*) $V = -P\langle x-a\rangle^0$ ◀

$$\frac{dM}{dx} = -P\langle x-a\rangle^0$$

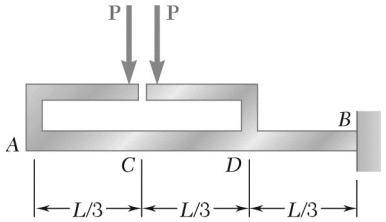
$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0$ ◀

Just to the right of B, $x = a^1$.

(*b*) $M = -\,0 - Pa$ $M = -Pa$ ◀



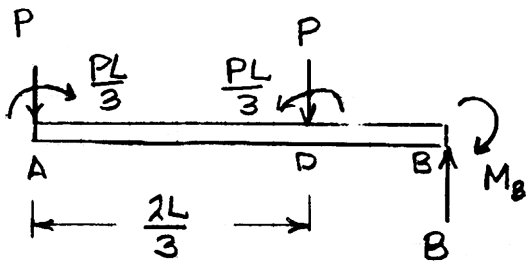
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.105

(*a*) Using singularity functions, write the equations for the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *D*.

SOLUTION



(*a*) $$V = -P - P\left\langle x - \frac{2L}{3}\right\rangle^0 = \frac{dM}{dx}$$ ◀

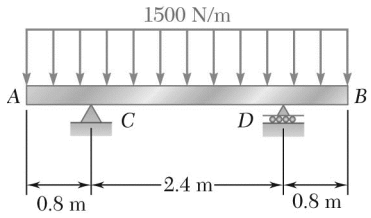
$$M = -Px + \frac{PL}{3} - P\left\langle x - \frac{2L}{3}\right\rangle^1 - \frac{PL}{3}\left\langle x - \frac{2L}{3}\right\rangle^0$$ ◀

Just to the right of D, $x = \frac{2L}{3}$.

(*b*) $$M_D^+ = -P\left(\frac{2L}{3}\right) + \frac{PL}{3} - P(0) - \frac{PL}{3}$$

$$M_D^+ = -\frac{4PL}{3}$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.106

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$w = 1.5 \text{ kN/m}$$

By statics,

$$\boldsymbol{C} = \boldsymbol{D} = 3 \text{ kN} \uparrow$$

(*a*) $V = -1.5x + 3\langle x - 0.8\rangle^0 + 3\langle x - 3.2\rangle^0 \text{ kN}$ ◀

$$M = -0.75x^2 + 3\langle x - 0.8\rangle^1 + 3\langle x - 3.2\rangle^1 \text{ kN} \cdot \text{m}$$ ◀

Locate point E where $V = 0$. Assume $x_C < x_E < x_D$

$$0 = -1.5x_E + 3(x_E - 0.8) + 0 \qquad x_E = 2.0 \text{ m}$$

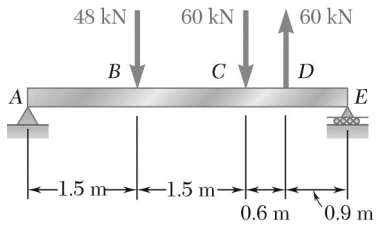
$$M_C = -(0.75)(0.8)^2 + 0 + 0 = -0.480 \text{ kN} \cdot \text{m}$$

$$M_E = -(0.75)(2.0)^2 + (3)(1.2) + 0 = 0.600 \text{ kN} \cdot \text{m}$$

$$M_D = -(0.75)(3.2)^2 + (3)(2.4) + 0 = -0.480 \text{ kN} \cdot \text{m}$$

(*b*) $|M|_{\max} = 0.600 \text{ kN} \cdot \text{m}$ $\qquad |M|_{\max} = 600 \text{ N} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.107

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

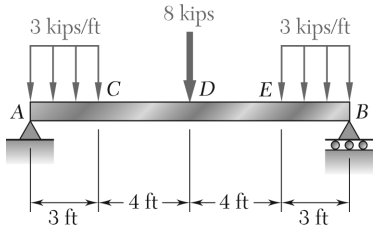
(*a*) $V = 40 - 48\langle x - 1.5\rangle^0 - 60\langle x - 3.0\rangle^0 + 60\langle x - 3.6\rangle^0 \text{ kN}$ ◀

$M = 40x - 48\langle x - 1.5\rangle^1 - 60\langle x - 3.0\rangle^1 + 60\langle x - 3.6\rangle^1 \text{ kN} \cdot \text{m}$ ◀

Pt.	x(m)	M(kN · m)
A	0	0
B	1.5	$(40)(1.5) = 60$ kN · m
C	3.0	$(40)(3.0) - (48)(1.5) = 48$ kN · m
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2$ kN · m
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

(*b*) $M_{\max} = 60 \text{ kN} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.108

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -14A + (12.5)(3)(3) + (7)(8) + (1.5)(3)(3) = 0$$

$$A = 13 \text{ kips} \uparrow$$

$$w = 3 - 3\langle x-3\rangle^0 + 3\langle x-11\rangle^0 = -\frac{dV}{dx}$$

(*a*) $$V = 13 - 3x + 3\langle x-3\rangle^1 - 8\langle x-7\rangle^0 - 3\langle x-11\rangle^1 \text{ kips}$$ ◀

$$M = 13x - 1.5x^2 + 1.5\langle x-3\rangle^2 - 8\langle x-7\rangle^1 - 1.5\langle x-11\rangle^2 \text{ kip}\cdot\text{ft}$$ ◀

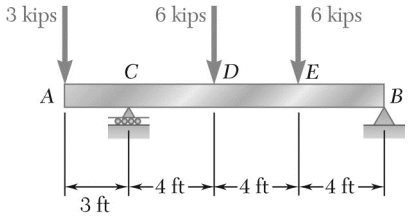
$$V_C = 13 - (3)(3) = 4 \text{ kips}$$
$$V_{D^-} = 13 - (3)(7) + (3)(4) = 4 \text{ kips}$$
$$V_{D^+} = 13 - (3)(7) + (3)(4) - 8 = -4 \text{ kips}$$
$$V_E = 13 - (3)(11) + (3)(8) - 8 = -4 \text{ kips}$$
$$V_B = 13 - (3)(14) + (3)(11) - 8 - (3)(3) = -13 \text{ kips}$$

(*b*) Note that V changes sign at D.

$$|M|_{\max} = M_D = (13)(7) - (1.5)(7)^2 + (1.5)(4)^2 - 0 - 0$$

$$|M|_{\max} = 41.5 \text{ kip}\cdot\text{ft}$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.109

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \ (15)(3) - 12C + (8)(6)C + (4)(6) = 0$$

$$\boldsymbol{C} = 9.75 \text{ kips} \uparrow$$

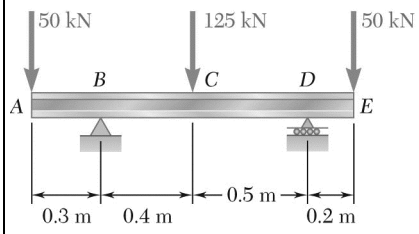
(*a*) $V = -3 + 9.75\langle x-3\rangle^0 - 6\langle x-7\rangle^0 - 6\langle x-11\rangle^0$ kips ◀

$M = -3x + 9.75\langle x-3\rangle^1 - 6\langle x-7\rangle^1 - 6\langle x-11\rangle^1$ kip · ft ◀

Pt.	x(ft)	M(kip · ft)
A	0	0
C	3	$-(3)(3) = -9$
D	7	$-(3)(7) + (9.75)(4) = 18$
E	11	$-(3)(11) + (9.75)(8) - (6)(4) = 21$ ← maximum
B	15	$-(3)(15) + (9.75)(12) - (6)(8) - (6)(4) = 0$

(*b*) $|M|_{\max} = 21.0$ kip · ft ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.110

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum stress due to bending.

SOLUTION

$$+\circlearrowright \Sigma M_D = 0:$$

$$(1.2)(50) - 0.9B + (0.5)(125) - (0.2)(50) = 0$$

$$\boldsymbol{B} = 125 \text{ kN} \uparrow$$

$$+\circlearrowright \Sigma M_B = 0:$$

$$(0.3)(50) - (0.4)(125) + 0.9D - (1.1)(50) = 0$$

$$\boldsymbol{D} = 100 \text{ kN} \uparrow$$

(*a*) $V = -50 + 125\langle x - 0.3\rangle^0 - 125\langle x - 0.7\rangle^0 + 100\langle x - 1.2\rangle^0 \text{ kN}$ ◀

$M = -50x + 125\langle x - 0.3\rangle^1 - 125\langle x - 0.7\rangle^1 + 100\langle x - 1.2\rangle^1 \text{ kN} \cdot \text{m}$ ◀

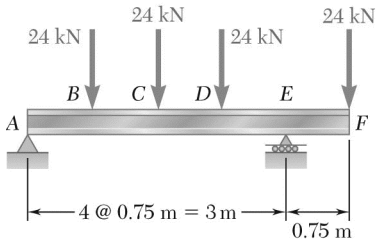
Point	x(m)	M(kN · m)
B	0.3	$-(50)(0.3) + 0 - 0 + 0 = -15$ kN · m
C	0.7	$-(50)(0.7) + (125)(0.4) - 0 + 0 = 15$ kN · m
D	1.2	$-(50)(1.2) + (125)(0.9) - (125)(0.5) + 0 = -10$ kN · m
E	1.4	$-(50)(1.4) + (125)(1.1) - (125)(0.7) + (100)(0.2) = 0$ (checks)

Maximum $|M| = 15 \text{ kN} \cdot \text{m} = 15 \times 10^3 \text{ N} \cdot \text{m}$

For S150×18.6 rolled steel section, $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{15 \times 10^3}{120 \times 10^{-6}} = 125 \times 10^6 \text{ Pa}$ $\sigma = 125.0 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.111

(*a*) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_E = 0: \quad -3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$$

$$R_A = 30 \text{ kips}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$$

$$R_E = 66 \text{ kips}$$

(*a*) $V = 30 - 24\langle x - 0.75\rangle^0 - 24\langle x - 1.5\rangle^0 - 24\langle x - 2.25\rangle^0 + 66\langle x - 3\rangle^0 \text{ kN}$ ◀

$M = 30x - 24\langle x - 0.75\rangle^1 - 24\langle x - 1.5\rangle^1 - 24\langle x - 2.25\rangle^1 + 66\langle x - 3\rangle^1 \text{ kN} \cdot \text{m}$ ◀

Point	x(m)	M(kN · m)
B	0.75	$(30)(0.75) = 22.5 \text{ kN} \cdot \text{m}$
C	1.5	$(30)(1.5) - (24)(0.75) = 27 \text{ kN} \cdot \text{m}$
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5 \text{ kN} \cdot \text{m}$
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18 \text{ kN} \cdot \text{m}$
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

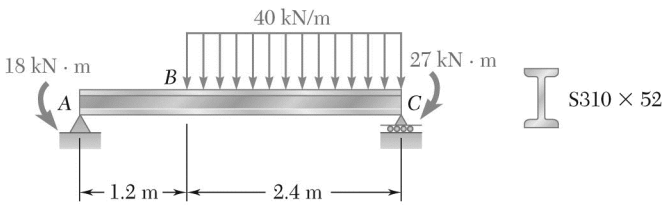
Maximum $|M| = 27 \text{ kN} \cdot \text{m} = 27 \times 30^3 \text{ N} \cdot \text{m}$

For rolled steel section W250×28.4, $S = 308 \times 10^3 \text{ mm}^3 = 308 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress:

$$\sigma = \frac{|M|}{S} = \frac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa} \qquad \sigma = 87.7 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.112

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_c = 0: \quad 18 - 3.6A + (1.2)(2.4)(40) - 27 = 0$$

$$\boldsymbol{A} = 29.5\ \text{kN} \uparrow$$

$$V = 29.5 - 40\langle x - 1.2\rangle^1\ \text{kN}$$

Point D. $\quad V = 0 \qquad 29.5 - 40(x_D - 1.2) = 0$

$$x_D = 1.9375\ \text{m}$$

$$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2\ \text{kN} \cdot \text{m}$$

$$M_A = -18\ \text{kN} \cdot \text{m}$$

$$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278\ \text{kN} \cdot \text{m}$$

$$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27\ \text{kN} \cdot \text{m}$$

(*a*) Maximum $\quad |M| = 28.278\ \text{kN} \cdot \text{m}$ at $x = 1.9375\ \text{m}$ ◀

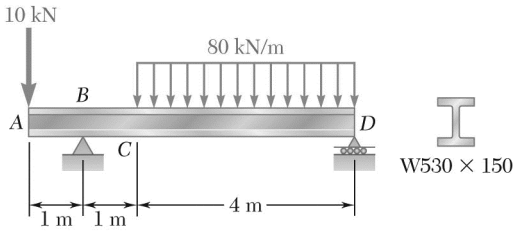
For S310 × 52 rolled steel section, $\quad S = 624 \times 10^3\ \text{mm}^3$

$$= 624 \times 10^{-6}\ \text{m}^3$$

(*b*) Normal stress: $\quad \sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{624 \times 10^{-6}} = 45.3 \times 10^6\ \text{Pa}$

$\sigma = 45.3\ \text{MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.113

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft M_D = 0: \quad (6)(10) - 5R_B + (2)(4)(80) = 0$$

$$R_B = 140 \text{ kN}$$

$$w = 80\langle x-2\rangle^0 \text{ kN/m} = -dV/dx$$

$$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1 \text{ kN}$$

A to *B*: $V = -10 \text{ kN}$

B to *C*: $V = -10 + 140 = 130 \text{ kN}$

D: $(x = 6)$ $V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at *B* and at point *E* $(x = x_E)$ between *C* and *D*.

$$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$$

$$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625 \text{ m}$$

$$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2 \text{ kN} \cdot \text{m}$$

At pt. *B*, $x = 1$ $M_B = -(10)(1) = -10 \text{ kN} \cdot \text{m}$

At pt. *E*, $x = 3.625$

$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN} \cdot \text{m}$$

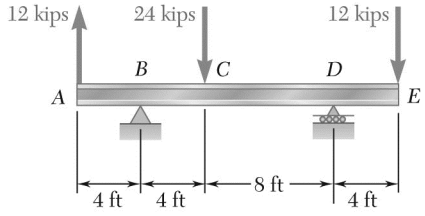
(*a*) $|M|_{\max} = 225.6 \text{ kN} \cdot \text{m}$ at $x = 3.625 \text{ m}$ ◀

For W530 × 150, $S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$

$\sigma = 60.6 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.114

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowright \Sigma M_D = 0: \quad -(16)(12) - 12B + (8)(24) - (4)(12) = 0$$

$$B = -4 \text{ kips} \quad \boldsymbol{B} = 4 \text{ kips} \downarrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(4)(12) - (4)(24) + 12D - (16)(12) = 0$$

$$\boldsymbol{D} = 28 \text{ kips} \uparrow$$

Check: $+\uparrow \Sigma F_y = 12 - 4 - 24 + 28 - 12 = 0$ ✓

$$V = 12 - 4\langle x-4\rangle^0 - 24\langle x-8\rangle^0 + 28\langle x-16\rangle^0$$

$$M = 12x - 4\langle x-4\rangle^1 - 24\langle x-8\rangle^1 + 28\langle x-16\rangle^1$$

At A, $x = 0$, $M = 0$

At B, $x = 4$ ft, $M = (12)(4) = 48 \text{ kip} \cdot \text{ft}$

At C, $x = 8$ ft, $M = (12)(8) - (4)(4) = 80 \text{ kip} \cdot \text{ft}$

At D, $x = 16$ ft, $M = (12)(16) - (4)(12) - (24)(8) = -48 \text{ kip} \cdot \text{ft}$

At E, $x = 20$ ft, $M = (12)(20) - (4)(16) - (24)(12) + (28)(4) = 0$ (checks)

(*a*) $|M|_{\max} = 80 \text{ kip} \cdot \text{ft}$ at C. ◀

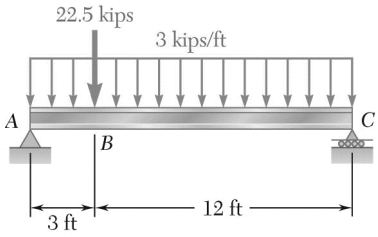
(*b*) $|M|_{\max} = 960 \text{ kip} \cdot \text{in}$ $\sigma_{\text{all}} = 24 \text{ ksi}$

$$\sigma = \frac{|M|}{S} \qquad S_{\min} = \frac{|M|_{\max}}{\sigma} = \frac{960}{24} = 40 \text{ in}^3$$

Shape	S_x (in^3)	
W18×35	57.6	
W16×31	47.2	
W14×30	42.0	←
W12×35	45.6	
W10×39	42.1	
W8×48	43.5	

Lightest wide flange shape: W14×30 ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.115

A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable normal stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that can be used.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -15R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$\boldsymbol{R}_A = 40.5 \text{ kips} \uparrow$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3\rangle^0 \text{ kips}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3\rangle^1 \text{ kip}\cdot\text{ft}$$

(*a*) Location of point *D* where $V = 0$. Assume $3 \text{ ft} < x_D < 12 \text{ ft}$.

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

At point *D*, $(x = 6 \text{ ft})$. $M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$

$$= 121.5 \text{ kip}\cdot\text{ft} = 1458 \text{ kip}\cdot\text{in}$$

Maximum $|M|$: $|M|_{\max} = 121.5 \text{ kip}\cdot\text{ft}$ at $x = 6.00 \text{ ft}$ ◀

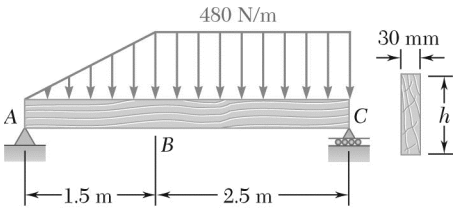
$$S_{\min} = \frac{M}{\sigma_{\text{all}}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

(*b*)

Shape	$S(\text{in}^3)$	
W21×44	81.6	
W18×50	88.9	
W16×40	64.7	←
W14×43	62.6	
W12×50	64.2	
W10×68	75.7	

Wide-flange shape: W16×40 ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.116

A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3)\left(\frac{1}{2}\right)(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$$

$$\boldsymbol{R}_A = 0.645 \text{ kN} \uparrow$$

$$w = \frac{0.48}{1.5}x - \frac{0.48}{1.5}\langle x - 1.5\rangle^1 = 0.32x - 0.32\langle x - 1.5\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16x^2 + 0.16\langle x - 1.5\rangle^2 \text{ kN}$$

$$M = 0.645x - 0.05333x^3 + 0.05333\langle x - 1.5\rangle^3 \text{ kN} \cdot \text{m}$$

(*a*) Locate point D where $V = 0$.

Assume $1.5 \text{ m} < x_D < 4 \text{ m}$.

$$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$$

$$= 0.645 - \cancel{0.16x_D^2} + \cancel{0.16x_D^2} - 0.48x_D + 0.36$$

$$x_D = 2.09375 \text{ m} \blacktriangleleft$$

At point D, $M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$

$$M_D = 0.87211 \text{ kN} \cdot \text{m} \blacktriangleleft$$

$$S_{\min} = \frac{M_D}{\sigma_{\text{all}}} = \frac{0.87211 \times 10^3}{12 \times 10^6} = 72.6758 \times 10^{-6} \text{ m}^3 = 72.6758 \times 10^3 \text{ mm}^3$$

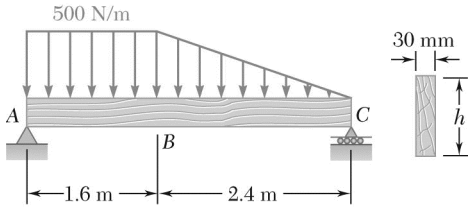
For a rectangular cross section, $S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{\min} = \sqrt{\frac{(6)(72.6758 \times 10^3)}{30}} = 120.56 \text{ mm}$$

(*b*) At next larger 10-mm increment, $h = 130 \text{ mm} \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.117



A timber beam is being designed with supports and loads as shown. (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of 30-mm width and depth h varying from 80 mm to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$$500 \text{ N/m} = 0.5 \text{ kN/m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -4R_A + (3.2)(1.6)(0.5) + (1.6)\left(\frac{1}{2}\right)(2.4)(0.5) = 0 \qquad R_A = 0.880 \text{ kN} \uparrow$$

$$w = 0.5 - \frac{0.5}{2.4}\langle x - 1.6\rangle^1 = 0.5 - 0.20833\langle x - 1.6\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167\langle x - 1.6\rangle^2 \text{ kN}$$

$$V_A = 0.880 \text{ kN}$$

$$\left.\begin{aligned} V_B &= 0.880 - (0.5)(1.6) = 0.080 \text{ kN} \\ V_C &= 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN} \end{aligned}\right\} \text{ Sign change}$$

Locate point D (between B and C) where $V = 0$. $\qquad 0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$

$$0.104167\,x_D^2 - 0.83333\,x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)} = \cancel{6.2342}, \quad 1.7658 \text{ m} \qquad \blacktriangleleft$$

$$M = 0.880x - 0.25x^2 + 0.347222\langle x - 1.6\rangle^3 \text{ kN} \cdot \text{m}$$

$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.776 \text{ kN} \cdot \text{m}$$

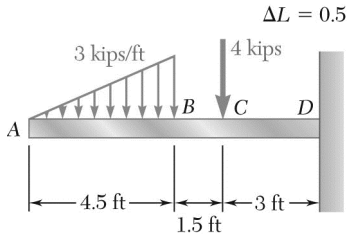
(*a*) $\qquad M_{\max} = 0.776 \text{ kN} \cdot \text{m} \quad \text{at} \quad x = 1.7658 \text{ m} \qquad \blacktriangleleft$

$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{0.776 \times 10^3}{12 \times 10^6} = 64.66 \times 10^{-6} \text{ m}^3 = 64.66 \times 10^3 \text{ mm}^3$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}} \qquad h_{\min} = \sqrt{\frac{(6)(64.66 \times 10^3)}{30}} = 113.7 \text{ mm}$

(*b*) At next higher 10-mm increment, $\qquad h = 120 \text{ mm} \qquad \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.118

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$w = \frac{3}{4.5}x - 3\langle x-4.5\rangle^0 - \frac{3}{4.5}\langle x-4.5\rangle^1$$

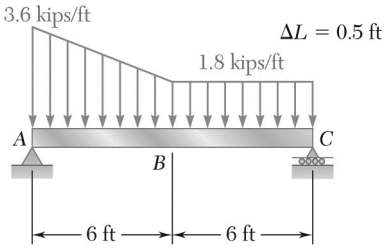
$$= \frac{2}{3}x - 3\langle x-4.5\rangle^0 - \frac{2}{3}\langle x-4.5\rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{1}{3}x^2 + 3\langle x-4.5\rangle^1 + \frac{1}{3}\langle x-4.5\rangle^2 - 4\langle x-6\rangle^0$$

$$M = -\frac{1}{9}x^3 + \frac{3}{2}\langle x-4.5\rangle^2 + \frac{1}{9}\langle x-4.5\rangle^3 - 4\langle x-6\rangle^1$$

x	V	M
ft	kips	kip · ft
0.0	0.00	0.00
0.5	–0.08	–0.01
1.0	–0.33	–0.11
1.5	–0.75	–0.38
2.0	–1.33	–0.89
2.5	–2.08	–1.74
3.0	–3.00	–3.00
3.5	–4.08	–4.76
4.0	–5.33	–7.11
4.5	–6.75	–10.13
5.0	–6.75	–13.50
5.5	–6.75	–16.88
6.0	–10.75	–20.25
6.5	–10.75	–25.63
7.0	–10.75	–31.00
7.5	–10.75	–36.38
8.0	–10.75	–41.75
8.5	–10.75	–47.13
9.0	–10.75	–52.50

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.119

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_C = 0: \quad -12R_A + (6)(12)(1.8) + (10)\left(\frac{1}{2}\right)(6)(1.8) = 0$$

$$R_A = 15.3 \text{ kips}$$

$$w = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}\langle x-6\rangle^1$$

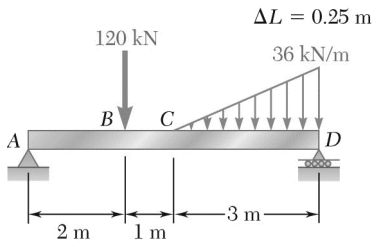
$$= 3.6 - 0.3x + 0.3\langle x-6\rangle^1$$

$$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6\rangle^2 \text{ kips} \quad \blacktriangleleft$$

$$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6\rangle^3 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

x	V	M
ft	kips	kip · ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	–0.90	37.8
6.5	–1.80	37.1
7.0	–2.70	36.0
7.5	–3.60	34.4
8.0	–4.50	32.4
8.5	–5.40	29.9
9.0	–6.30	27.0
9.5	–7.20	23.6
10.0	–8.10	19.8
10.5	–9.00	15.5
11.0	–9.90	10.8
11.5	–10.80	5.6
12.0	–11.70	0.0

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.120

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -6R_A + (4)(120) + (1)\left(\frac{1}{2}\right)(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$$

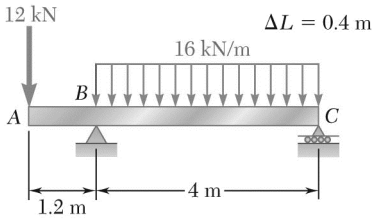
$$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2 \text{ kN} \quad \blacktriangleleft$$

$$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	–31.0	178.0
2.3	–31.0	170.3
2.5	–31.0	162.5
2.8	–31.0	154.8
3.0	–31.0	147.0
3.3	–31.4	139.2
3.5	–32.5	131.3
3.8	–34.4	122.9
4.0	–37.0	114.0
4.3	–40.4	104.3
4.5	–44.5	93.8
4.8	–49.4	82.0

x	V	M
m	kN	kN · m
5.0	–55.0	69.0
5.3	–61.4	54.5
5.5	–68.5	38.3
5.8	–76.4	20.2
6.0	–85.0	–0.0

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.121

Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$$+\circlearrowright \Sigma M_C = 0: \quad (5.2)(12) - 4B + (2)(4)(16) = 0$$

$$\boldsymbol{B} = 47.6 \text{ kN} \uparrow$$

$$+\circlearrowright \Sigma M_B = 0: \quad (1.2)(12) - (2)(4)(16) + 4C = 0$$

$$\boldsymbol{C} = 28.4 \text{ kN} \uparrow$$

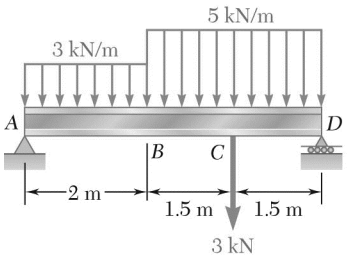
$$w = 16\langle x - 1.2\rangle^0 = -\frac{dV}{dx}$$

$$V = -16\langle x - 1.2\rangle^1 - 12 + 47.6\langle x - 1.2\rangle^0 \quad \blacktriangleleft$$

$$M = -8\langle x - 1.2\rangle^2 - 12x + 47.6\langle x - 1.2\rangle^1 \quad \blacktriangleleft$$

x	V	M
m	kN	kN · m
0.0	–12.0	0.00
0.4	–12.0	–4.80
0.8	–12.0	–9.60
1.2	35.6	–14.40
1.6	29.2	–1.44
2.0	22.8	8.96
2.4	16.4	16.80
2.8	10.0	22.08
3.2	3.6	24.80
3.6	–2.8	24.96
4.0	–9.2	22.56
4.4	–15.6	17.60
4.8	–22.0	10.08
5.2	–28.4	–0.00

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



W200 × 22.5
$L = 5$ m
$\Delta L = 0.25$ m

PROBLEM 5.122

For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the *x*-axis at end *A* of the beam.

SOLUTION

$$+\circlearrowright \Sigma M_D = 0:$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$w = 3 + 2\langle x - 2\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

(*a*) $$V = 10.2 - 3x - 2\langle x - 2\rangle^1 - 3\langle x - 3.5\rangle^0 \text{ kN}$$ ◀

(*b*) $$M = 10.2x - 1.5x^2 - \langle x - 2\rangle^2 - 3\langle x - 3.5\rangle^1 \text{ kN} \cdot \text{m}$$ ◀

For rolled steel section W200 × 22.5,

$$S = 193 \times 10^3 \text{ mm}^3 = 193 \times 10^{-6} \text{ m}^3$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{16.164 \times 10^3}{193 \times 10^{-6}} = 83.8 \times 10^6 \text{ Pa}$$

$$\sigma = 83.8 \text{ MPa}$$ ◀

x	V	M	σ
m	kN	kN · m	MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.73	24.5
0.75	7.95	6.81	35.3
1.00	7.20	8.70	45.1
1.25	6.45	10.41	53.9
1.50	5.70	11.93	61.8
1.75	4.95	13.26	68.7
2.00	4.20	14.40	74.6
2.25	2.95	15.29	79.2
2.50	1.70	15.88	82.3
2.75	0.45	16.14	83.6

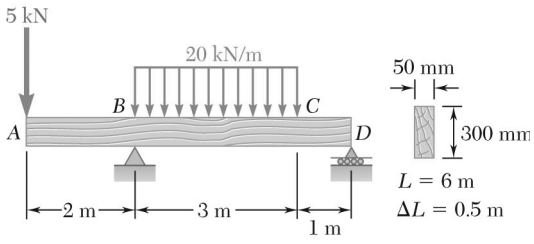
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.122 ***(Continued)***

x	V	M	σ	
m	kN	kN · m	MPa	
3.00	–0.80	16.10	83.4	
3.25	–2.05	15.74	81.6	
3.50	–6.30	15.08	78.1	
3.75	–7.55	13.34	69.1	
4.00	–8.80	11.30	58.5	
4.25	–10.05	8.94	46.3	
4.50	–11.30	6.28	32.5	
4.75	–12.55	3.29	17.1	
5.00	–13.80	0.00	0.0	
2.83	0.05	16.164	83.8	
2.84	0.00	16.164	83.8	←
2.85	–0.05	16.164	83.8	

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.123



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the *x*-axis at end *A* of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -4R_B + (6)(5) + (2.5)(3)(20) = 0 \qquad R_B = 45 \text{ kN}$$

$$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

(*a*)

x	*V*	*M*	stress	
m	kN	kN · m	MPa	
0.00	–5	0.00	0.0	
0.50	–5	–2.50	–3.3	
1.00	–5	–5.00	–6.7	
1.50	–5	–7.50	–10.0	
2.00	40	–10.00	–13.3	
2.50	30	7.50	10.0	
3.00	20	20.00	26.7	
3.50	10	27.50	36.7	
4.00	0	30.00	40.0	←
4.50	–10	27.50	36.7	
5.00	–20	20.00	26.7	
5.50	–20	10.00	13.3	
6.00	–20	0.00	0.0	

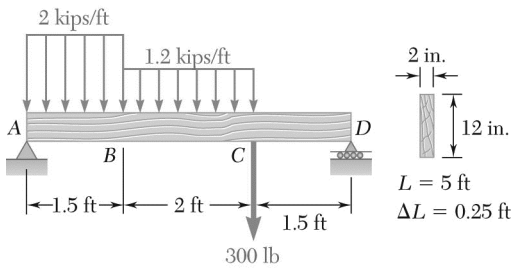
(*b*) Maximum $|M| = 30 \text{ kN}\cdot\text{m}$ at $x = 4.0 \text{ m}$

For rectangular cross section, $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(50)(300)^2 = 750\times10^3 \text{ mm}^3 = 750\times10^{-6} \text{ m}^3$

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30\times10^3}{750\times10^{-6}} = 40\times10^6 \text{ Pa} \qquad \sigma_{\max} = 40.0 \text{ MPa} \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.124



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$300 \text{ lb} = 0.3 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$$

$$R_A = 3.84 \text{ kips}$$

$$w = 2 - 0.8\langle x - 1.5\rangle^0 - 1.2\langle x - 3.5\rangle^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8\langle x - 1.5\rangle^1 + 1.2\langle x - 3.5\rangle^1 - 0.3\langle x - 3.5\rangle^0 \text{ kips} \quad \blacktriangleleft$$

$$M = 3.84x - x^2 + 0.4\langle x - 1.5\rangle^2 + 0.6\langle x - 3.5\rangle^2 - 0.3\langle x - 3.5\rangle^1 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

x	V	M	stress
ft	kips	kip · ft	ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	−0.06	3.80	0.951
2.50	−0.36	3.75	0.937
2.75	−0.66	3.62	0.906
3.00	−0.96	3.42	0.855
3.25	−1.26	3.14	0.786
3.50	−1.86	2.79	0.697
3.75	−1.86	2.32	0.581

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.124 *(Continued)*

x	V	M	stress	
ft	kips	kip · ft	ksi	
4.00	–1.86	1.86	0.465	
4.25	–1.86	1.39	0.349	
4.50	–1.86	0.93	0.232	
4.75	–1.86	0.46	0.116	
5.00	–1.86	–0.00	–.000	
2.10	0.12	3.80	0.949	
2.20	0.00	3.80	0.951	←
2.30	–0.12	3.80	0.949	

Maximum $|M| = 3.804 \text{ kip} \cdot \text{ft} = 45.648 \text{ kip} \cdot \text{in}$ at $x = 2.20 \text{ ft}$

Rectangular section:

$$2 \text{ in.} \times 12 \text{ in.}$$

$$S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(2)(12)^2$$

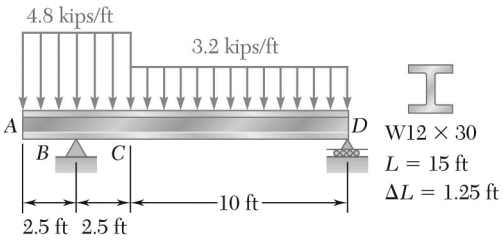
$$= 48 \text{ in}^3$$

$$\sigma = \frac{M}{S} = \frac{45.648}{48}$$

$\sigma = 0.951 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.125



For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2% accuracy the maximum normal stress in the beam. Place the origin of the x axis at end A of the beam.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -12.5R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$$

$$R_B = 36.8 \text{ kips}$$

$$w = 4.8 - 1.6\langle x - 5\rangle^0 \text{ kips/ft}$$

$$V = -4.8x + 36.8\langle x - 2.5\rangle^0 + 1.6\langle x - 5\rangle^1 \text{ kips} \quad \blacktriangleleft$$

$$M = -2.4x^2 + 36.8\langle x - 2.5\rangle^1 + 0.8\langle x - 5\rangle^2 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

x	V	M	stress	
ft	kips	kip · ft	ksi	
0.00	0.00	0.00	0.00	
1.25	–6.0	–3.75	–1.17	
2.50	24.8	–15.00	–4.66	
3.75	18.8	12.25	3.81	
5.00	12.8	32.00	9.95	
6.25	8.8	45.50	14.15	
7.50	4.8	54.00	16.79	
8.75	0.8	57.50	17.88	
10.00	–3.2	56.00	17.41	
11.25	–7.2	49.50	15.39	
12.50	–11.2	38.00	11.81	
13.75	–15.2	21.50	6.68	
15.00	–19.2	0.00	0.00	
8.90	0.32	57.58	17.90	
9.00	–0.00	57.60	17.91	←
9.10	–0.32	57.58	17.90	

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.125 ***(Continued)***

Maximum $|M| = 57.6 \text{ kip} \cdot \text{ft} = 691.2 \text{ kip} \cdot \text{in}$ at $x = 9.0 \text{ ft}$

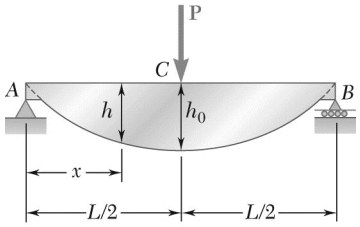
For rolled steel section W12×30,

$$S = 38.6 \text{ in}^3$$

Maximum normal stress:

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6} \qquad \sigma = 17.91 \text{ ksi} \blacktriangleleft$$

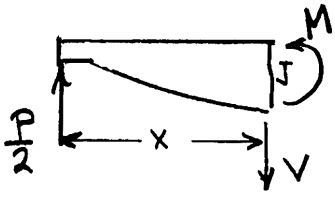
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.126

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2}\uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

$$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \qquad h = \sqrt{\frac{3Px}{\sigma_{all}b}}$

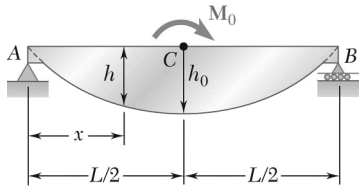
(*a*) At $x = \frac{L}{2}$, $h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all}b}}$ $\qquad h = h_0\sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$ ◀

For $x > \frac{L}{2}$, replace x by $L - x$.

(*b*) Solving for P, $P = \frac{2\sigma_{all}bh_0^2}{3L} = \frac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60\times10^3 \text{ N}$ $\qquad P = 60 \text{ kN}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

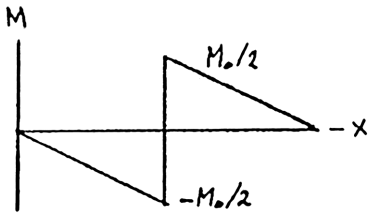
PROBLEM 5.127

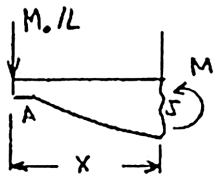


The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$\boldsymbol{A} = M_0/L \downarrow \qquad \boldsymbol{B} = M_0/L \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0}{L}x + M = 0$$

$$M = -\frac{M_0 x}{L} \quad \left(0 < x < \frac{L}{2}\right)$$

For $x > \dfrac{L}{2}$,

$$M = \frac{M_0(L - x)}{L} \quad \left(\frac{L}{2} < x < L\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{M_0 x}{\sigma_{all} L} \qquad \text{for} \qquad \left(0 < x \frac{L}{2}\right)$$

For $x > \dfrac{L}{2}$, replace x by $L - x$.

For a rectangular cross section, $S = \dfrac{1}{6}bh^2$

Equating,

$$\frac{1}{6}bh^2 = \frac{M_0 x}{\sigma_{all} L} \qquad h = \sqrt{\frac{6M_0 x}{\sigma_{all} bL}}$$

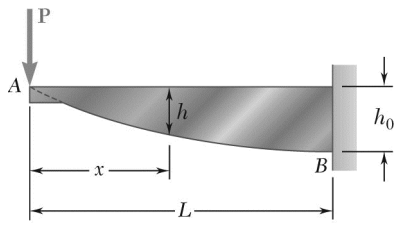
(*a*) At $x = \dfrac{L}{2}$,

$$h = h_0 = \sqrt{\frac{3M_0}{\sigma_{all} b}} \qquad h = h_0\sqrt{2x/L} \blacktriangleleft$$

(*b*) Solving for M_0, $M_0 = \dfrac{\sigma_{all} b h_0^2}{3} = \dfrac{(72 \times 10^6)(0.025)(0.200)^2}{3} = 24 \times 10^3 \text{ N} \cdot \text{m}$

$$M_0 = 24 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.128

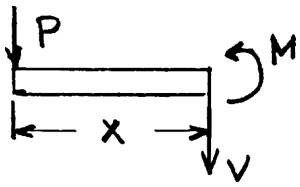
The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$V = -P$$

$$M = -Px \qquad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P}{\sigma_{all}} x$$



For a rectangular cross section, $S = \frac{1}{6} bh^2$

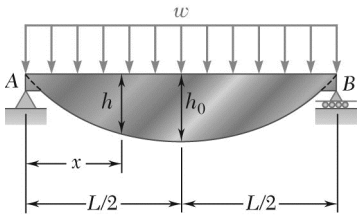
Equating, $$\frac{1}{6} bh^2 = \frac{Px}{\sigma_{all}} \qquad h = \left(\frac{6Px}{\sigma_{all} b} \right)^{1/2} \tag{1}$$

At $x = L$, $$h = h_0 = \left\{ \frac{6PL}{\sigma_{all} b} \right\}^{1/2} \tag{2}$$

(*a*) Divide Eq. (1) by Eq. (2) and solve for h. $h = h_0 (x/L)^{1/2}$ ◀

(*b*) Solving for P, $$P = \frac{\sigma_{all} b h_0^2}{6L} = \frac{(24)(1.25)(12)^2}{(6)(36)}$$ $P = 20.0$ kips ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



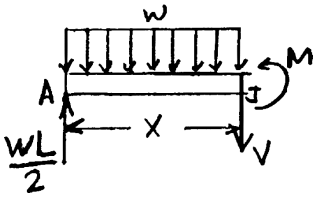
PROBLEM 5.129

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0:$$



$$\frac{wL}{2}x - wx\frac{x}{2} + M = 0 \qquad M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

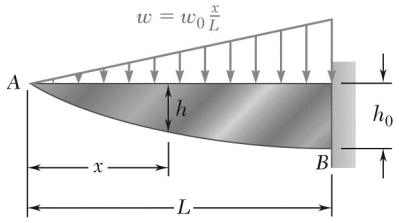
For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}}$ $\qquad h = \left\{\frac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$

(*a*) At $x = \frac{L}{2}$, $\qquad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all}b}\right\}^{1/2}$ $\qquad h = h_0\left[\frac{x}{L}\left(1-\frac{x}{L}\right)\right]^{1/2}$ ◀

(*b*) Solving for w, $\qquad w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(24)(1.25)(12)^2}{(3)(36)^2}$ $\qquad w = 4.44$ kip/in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.130

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section, $S = \frac{1}{6} b h^2$

Equating, $\frac{1}{6} b h^2 = \frac{w_0 x^3}{6L\sigma_{all}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} b L}}$

At $x = L$, $h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

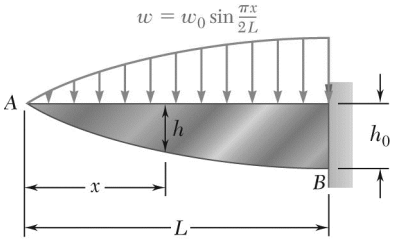
(*a*) $h = h_0 \left(\frac{x}{L} \right)^{3/2}$ ◀

Data: $L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$

(*b*) $h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 167.7 \times 10^{-3} \text{ m}$ $\qquad h_0 = 167.7 \text{ mm}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.131

The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 200$ MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{2L}$$

$$V = \frac{2w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \rightarrow C_1 = \frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\left(1 - \cos \frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0 L}{\pi \sigma_{all}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

For a rectangular cross section, $S = \frac{1}{6} bh^2$

Equating, $$\frac{1}{6} bh^2 = \frac{2w_0 L}{\pi \sigma_{all}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

$$h = \left\{\frac{12 w_0 L}{\pi \sigma_{all} b}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)\right\}^{1/2}$$

At $x = L$, $$h = h_0 = \left\{\frac{12 w_0 L^2}{\pi \sigma_{all} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178 \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$$

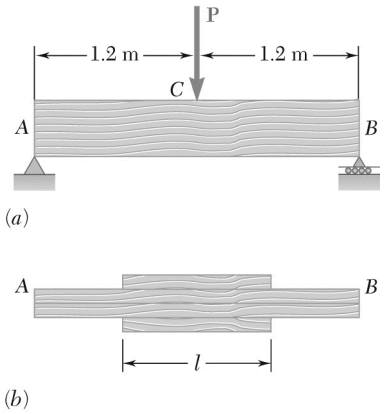
(*a*) $$h = h_0 \left[\left(\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L}\right) \Big/ \left(1 - \frac{2}{\pi}\right)\right]^{1/2} \qquad h = 1.659\, h_0 \left[\frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L}\right]^{1/2} \blacktriangleleft$$

Data: $L = 750 \text{ mm} = 0.75 \text{ m}, \quad b = 30 \text{ mm} = 0.030 \text{ m}$

$w_0 = 300 \text{ kN/m} = 300 \times 10^3 \text{ N/m}, \quad \sigma_{all} = 200 \text{ MPa} = 200 \times 10^6 \text{ Pa}$

(*b*) $$h_0 = 1.178 \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(200 \times 10^6)(0.030)}} = 197.6 \times 10^{-3} \text{ m} \qquad h_0 = 197.6 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



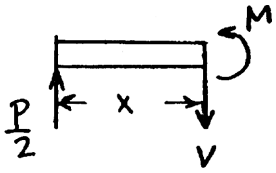
PROBLEM 5.132

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$

$$0 < x < \frac{1}{2}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{\max}x}{1.2}$$

Bending moment diagram is two straight lines.

At C, $\quad S_C = \frac{1}{6}bh_C^2 \qquad M_C = M_{\max}$

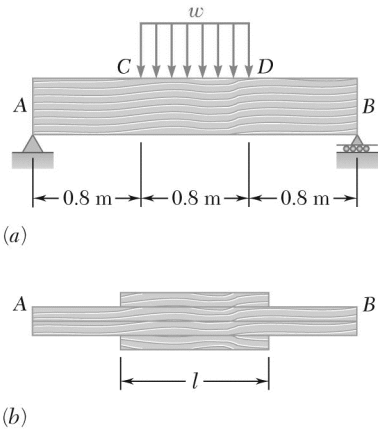
Let D be the point where the thickness changes.

At D, $\quad S_D = \frac{1}{6}bh_D^2 \qquad M_D = \frac{M_{\max}x_D}{1.2}$

$$\frac{S_D}{S_C} = \frac{h_D^2}{h_C^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3\text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9 \qquad l = 1.800\text{ m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.133

A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50×50-mm cross section. Determine the length l of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{0.8\text{ N}}{2} = 0.4w$$

Shear:

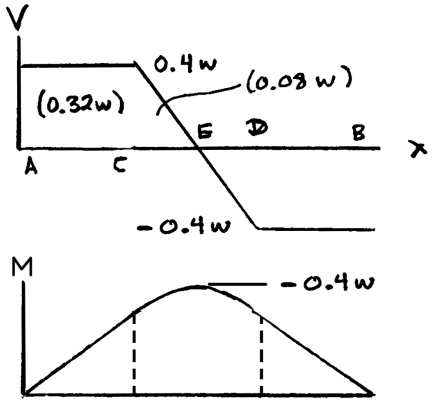
A to C: $V = 0.4w$

D to B: $V = -0.4w$

Areas:

A to C: $(0.8)(0.4)w = 0.32w$

C to E: $\left(\frac{1}{2}\right)(0.4)(0.4)w = 0.08w$



Bending moments:

At C, $M_C = 0.40w$

A to C: $M = 0.40wx$

At C, $S_C = \frac{1}{6}bh_C^2$ $\qquad M_C = M_{\max} = 0.40w$

Let F be the point were the thickness changes.

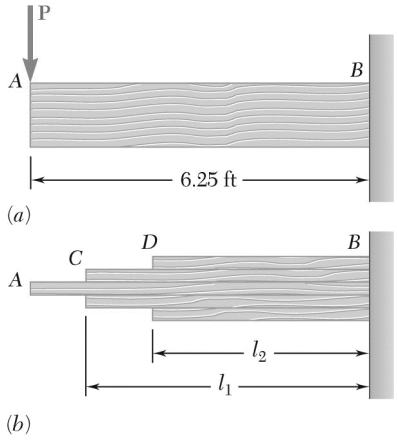
At F, $S_F = \frac{1}{6}bh_F^2$ $\qquad M_F = 0.40wx_F$

$$\frac{S_F}{S_C} = \frac{h_F^2}{h_C^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_C} = \frac{0.40wx_F}{0.40w}$$

$$x_F = 0.25\text{ m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95\text{ m}$$

$$l = 1.900\text{ m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



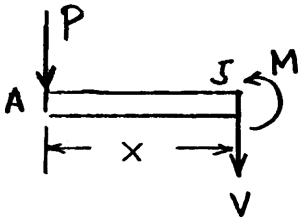
PROBLEM 5.134

A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0 \qquad M = -Px$$

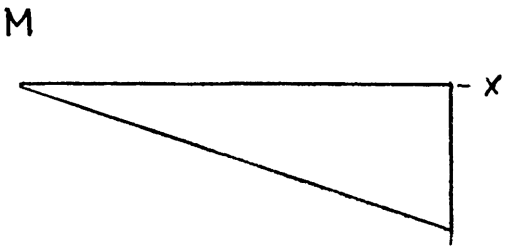
$$|M| = Px$$



At B, $\quad |M|_B = M_{\max}$

At C, $\quad |M|_C = M_{\max} x_C/6.25$

At D, $\quad |M|_D = M_{\max} x_D/6.25$



$$S_B = \frac{1}{6}bh^2 = \frac{1}{6}\cdot b(5b)^2 = \frac{25}{6}b^3$$

A to C: $\quad S_C = \frac{1}{6}\cdot b(b)^2 = \frac{1}{6}b^3$

C to D: $\quad S_D = \frac{1}{6}b\,(3b)^2 = \frac{9}{6}b^3$

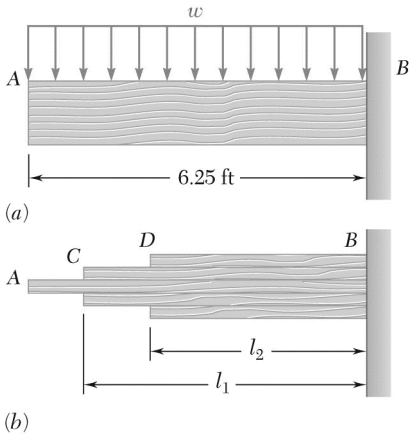
$$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

$$l_1 = 6.25 - 0.25 \qquad l_1 = 6.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

$$l_2 = 6.25 - 2.25 \qquad l_2 = 4.00 \text{ ft} \blacktriangleleft$$

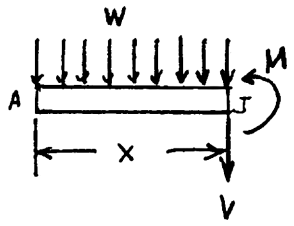
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.135

A preliminary design on the use of a cantilever prismatic timber beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2×2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



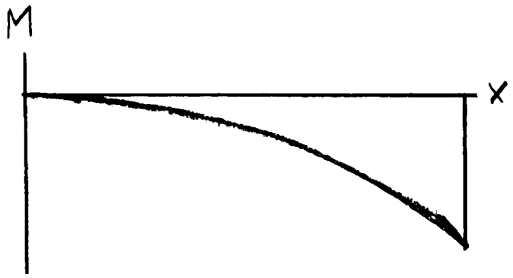
$$+\circlearrowleft \Sigma M_J = 0: \quad wx\frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$$

At B, $\quad |M|_B = |M|_{\max}$

At C, $\quad |M|_C = |M|_{\max}\ (x_C/6.25)^2$

At D, $\quad |M|_D = |M|_{\max}\ (x_D/6.25)^2$



At B, $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C: $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to D: $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

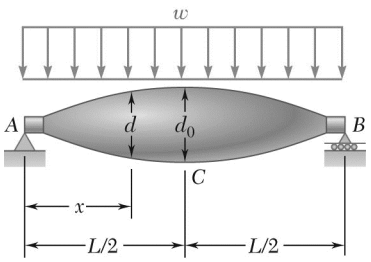
$$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

$$l_1 = 6.25 - 1.25 \text{ ft} \qquad l_1 = 5.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

$$l_2 = 6.25 - 3.75 \text{ ft} \qquad l_2 = 2.50 \text{ ft} \blacktriangleleft$$

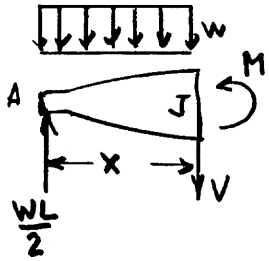
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.136

A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION



$$R_A = R_B = \frac{wL}{2}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

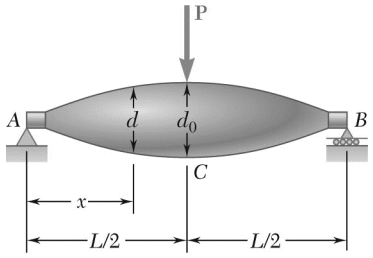
$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{wx(L-x)}{2\sigma_{\text{all}}}$$

For a solid circular cross section, $c = \dfrac{d}{2}$ $I = \dfrac{\pi}{4}c^3$ $S = \dfrac{I}{c} = \dfrac{\pi d^3}{32}$

Equating, $\dfrac{\pi d^3}{32} = \dfrac{wx(L-x)}{2\sigma_{\text{all}}}$ $d = \left\{\dfrac{16\,wx(L-x)}{\pi\sigma_{\text{all}}}\right\}^{1/3}$

At $x = \dfrac{L}{2}$, $d = d_0 = \left\{\dfrac{4wL^2}{\pi\sigma_{\text{all}}}\right\}^{1/3}$ $d = d_0\left\{4\dfrac{x}{L}\left(1-\dfrac{x}{L}\right)\right\}^{1/3}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

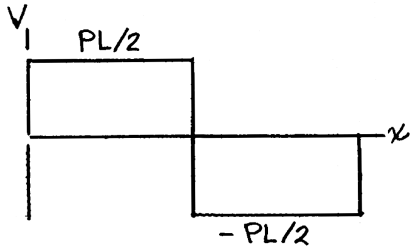


PROBLEM 5.137

A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support the load shown. Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0.

SOLUTION

Draw shear and bending moment diagrams.

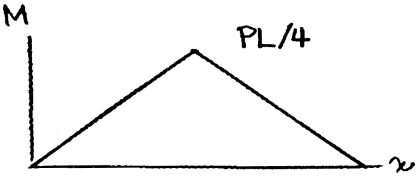


$$0 \le x \le \frac{L}{2}, \qquad M = \frac{Px}{2}$$

$$\frac{L}{2} \le x \le L, \qquad M = \frac{P(L-x)}{2}$$

For a solid circular section, $c = \frac{1}{2}d$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4 \qquad S = \frac{I}{c} = \frac{\pi}{32}d^3$$



For constant strength design, $\sigma = \text{constant}$. $\qquad S = \frac{M}{\sigma}$

For $0 \le x \le \frac{L}{2}$, $\qquad \frac{\pi}{32}d^3 = \frac{Px}{2}$ (1a)

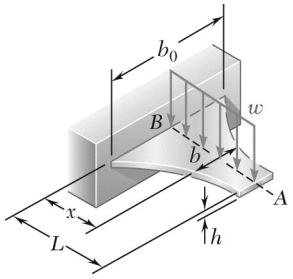
For $\frac{L}{2} \le x \le L$, $\qquad \frac{\pi}{32}d^3 = \frac{P(L-x)}{2}$ (1b)

At point C, $\qquad \frac{\pi}{32}d_0^3 = \frac{PL}{4}$ (2)

Dividing Eq. (1a) by Eq. (2), $\quad 0 \le x \le \frac{L}{2}$, $\quad \frac{d^3}{d_0^3} = \frac{2x}{L}$ $\qquad d = d_0(2x/L)^{1/3}$ ◀

Dividing Eq. (1b) by Eq. (2), $\quad \frac{L}{2} \le x \le L$, $\quad \frac{d^3}{d_0^3} = \frac{2(L-x)}{L}$ $\qquad d = d_0[2(L-x)/L]^{1/3}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

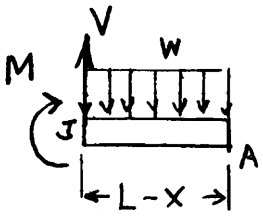


PROBLEM 5.138

A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support the distributed load *w* along its centerline *AB*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the maximum allowable value of *w* if $L = 15$ in., $b_0 = 8$ in., $h = 0.75$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L-x)\frac{L-x}{2} = 0$$



$$M = -\frac{w(L-x)^2}{2} \qquad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w(L-x)^2}{2\sigma_{all}}$$

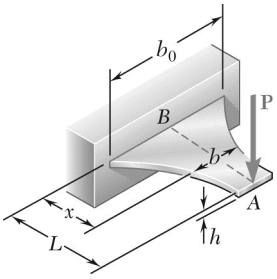
For a rectangular cross section, $S = \frac{1}{6}bh^2$

$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{all}} \qquad b = \frac{3w(L-x)^2}{\sigma_{all}h^2}$$

(*a*) At $x = 0$, $b = b_0 = \dfrac{3wL^2}{\sigma_{all}h^2}$ $\qquad b = b_0\left(1 - \dfrac{x}{L}\right)^2$ ◀

(*b*) Solving for *w*, $w = \dfrac{\sigma_{all}b_0h^2}{3L^2} = \dfrac{(24)(8)(0.75)^2}{(3)(15)^2} = 0.160$ kip/in $\qquad w = 160.0$ lb/in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.139

A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support the concentrated load **P** at point *A*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the smallest allowable value of *h* if $L = 300$ mm, $b_0 = 375$ mm, $P = 14.4$ kN, and $\sigma_{\text{all}} = 160$ MPa.

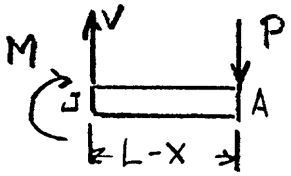
SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{\text{all}}} = \frac{P(L-x)}{\sigma_{\text{all}}}$$



For a rectangular cross section, $S = \frac{1}{6}bh^2$

Equating, $\frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{\text{all}}} \qquad b = \frac{6P(L-x)}{\sigma_{\text{all}}h^2}$

(*a*) At $x = 0$, $b = b_0 = \frac{6PL}{\sigma_{\text{all}}h^2}$ $\qquad b = b_0\left(1 - \frac{x}{L}\right)$ ◀

Solving for *h*, $h = \sqrt{\frac{6PL}{\sigma_{\text{all}}b_0}}$

Data: $L = 300 \text{ mm} = 0.300 \text{ m}, \quad b_0 = 375 \text{ mm} = 0.375 \text{ m}$

$P = 14.4 \text{ kN} = 14.4 \times 10^3 \text{ N} \cdot \text{m}, \quad \sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

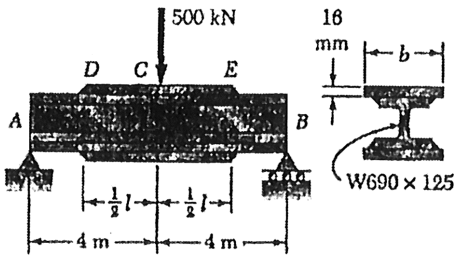
(*b*) $h = \sqrt{\frac{(6)(14.4 \times 10^3)(0.300)}{(160 \times 10^6)(0.375)}} = 20.8 \times 10^{-3} \text{ m}$ $\qquad h = 20.8 \text{ mm}$ ◀

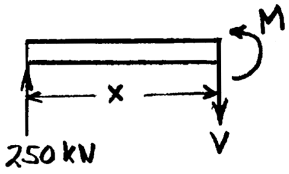
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.140

Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION





$A = B = 250$ kN ↑

$$+\circlearrowleft \Sigma M_J = 0: \quad -250x + M = 0$$

$$M = 250x \text{ kN} \cdot \text{m}$$

At center of beam, $x = 4$ m $\quad M_C = (250)(4) = 1000$ kN · m

At *D*, $x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2$ m $\qquad M_0 = 500$ kN · m

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 1190 \times 10^6 + 2\left\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

$$c = \frac{678}{2} + 16 = 355 \text{ mm} \qquad S = \frac{I}{c} = 6445 \times 10^3 \text{mm}^3$$

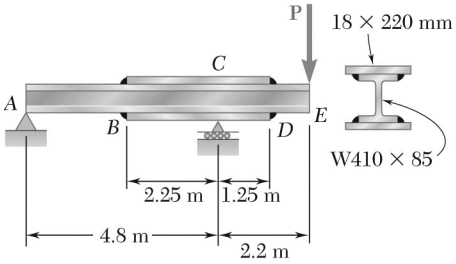
$$= 6445 \times 10^{-6} \text{m}^3$$

(*a*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6 \text{Pa}$ $\qquad \sigma = 155.2$ MPa ◀

At *D*, $S = 3490 \times 10^3 \text{mm}^3 = 3510 \times 10^{-6} \text{m}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{500 \times 10^3}{3490 \times 10^{-6}} = 143.3 \times 10^6$ Pa $\qquad \sigma = 143.3$ MPa ◀

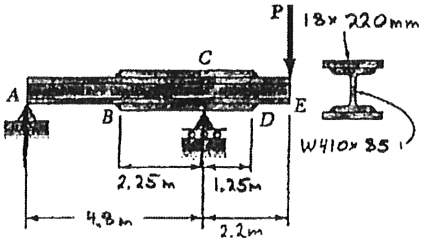
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.141

Knowing that $\sigma_{all} = 150$ MPa, determine the largest concentrated load **P** that can be applied at end *E* of the beam shown.

SOLUTION



$+\circlearrowleft \Sigma M_C = 0$: $-4.8A - 2.2P = 0$

$A = -0.45833P$ $\quad \boldsymbol{A} = 0.45833P \downarrow$

$+\circlearrowleft \Sigma M_A = 0$: $4.8D - 7.0P = 0$

$\boldsymbol{D} = 1.45833P \uparrow$

Shear: A to C: $V = -0.45833P$

C to E: $V = P$

Bending moments: $M_A = 0$

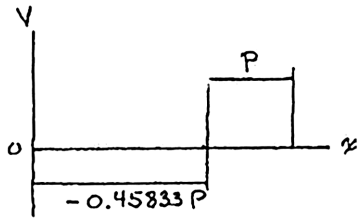
$M_C = 0 + (4.8)(-0.45833P) = -2.2P$

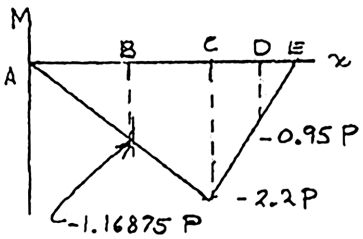
$M_E = -2.2P + 2.2P = 0$

$M_B = \left(\dfrac{4.8 - 2.25}{48}\right)(-2.2P) = -1.16875P$

$M_D = \left(\dfrac{2.2 - 1.25}{2.2}\right)(-2.2P) = -0.95P$

$|M_D| < |M_B|$





For W410 × 85, $S = 1510 \times 10^3 \text{mm}^3 = 1510 \times 10^{-6} \text{m}^3$

Allowable value of *P* based on strength at *B*. $\sigma = \dfrac{|M_B|}{S}$

$$150 \times 10^6 = \frac{1.16875P}{1510 \times 10^{-6}} \qquad P = 193.8 \times 10^3 \text{N}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.141 ***(Continued)***

Section properties over portion *BCD*:

$$\text{W}410\times 85: \quad d = 417 \text{ mm}, \quad \frac{1}{2}d = 208.5 \text{ mm}, \quad I_x = 316\times 10^6 \text{mm}^4$$

Plate: $A = (18)(220) = 3960 \text{ mm}^2 \qquad d = 208.5 + \left(\frac{1}{2}\right)(18) = 217.5 \text{ mm}$

$$\bar{I} = \frac{1}{12}(220)(18)^3 = 106.92\times 10^3 \text{ mm}^4 \quad Ad^2 = 187.333\times 10^6 \text{mm}^4$$

$$I_x = \bar{I} + Ad^2 = 187.440\times 10^6 \text{ mm}^4$$

For section, $I = 316\times 10^6 + (2)(187.440\times 10^6) = 690.88\times 10^6 \text{ mm}^4$

$$c = 208.5 + 18 = 226.5 \text{ mm}$$

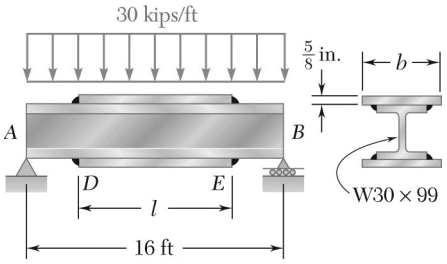
$$S = \frac{I}{c} = \frac{690.88\times 10^6}{226.5} = 3050.2\times 10^3 \text{mm}^3 = 3050.2\times 10^{-6} \text{ m}^3$$

Allowable load based on strength at *C*: $\sigma = \dfrac{|M_C|}{S}$

$$150\times 10^6 = \frac{2.2P}{3050.2\times 10^{-6}} \qquad P = 208.0\times 10^3 \text{ N}$$

The smaller allowable load controls. $P = 193.8\times 10^3 \text{N}$ $P = 193.8 \text{ kN}$ ◀

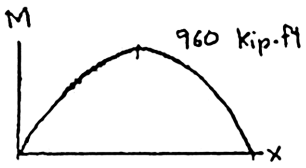
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



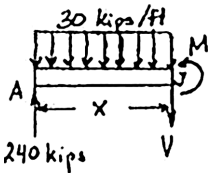
PROBLEM 5.142

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30×99 beam as shown. Knowing that $l = 9$ ft and $b = 12$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION



$$\boldsymbol{A} = \boldsymbol{B} = 240 \text{ kips} \uparrow$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

At center of beam, $x = 8$ ft

$$M_C = 960 \text{ kip} \cdot \text{ft} = 11{,}520 \text{ kip} \cdot \text{in}$$

At point *D*, $x = \frac{1}{2}(16 - 9) = 3.5$ ft

$$M_D = 656.25 \text{ kip} \cdot \text{ft} = 7875 \text{ kip} \cdot \text{in}$$

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7439 \text{ in}^4$$

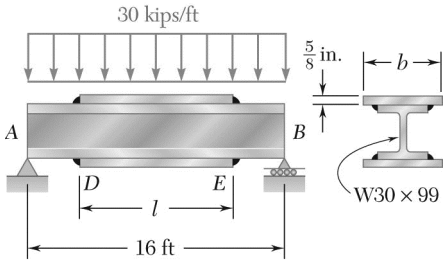
$$c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

(*a*) Normal stress: $\sigma = \dfrac{Mc}{I} = \dfrac{(11{,}520)(15.475)}{7439}$ $\sigma = 24.0$ ksi ◀

At point *D*, $S = 269 \text{ in}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{7875}{269}$ $\sigma = 29.3$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

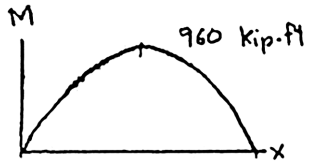


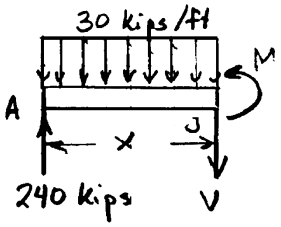
PROBLEM 5.143

Two cover plates, each $\frac{5}{8}$ in. thick, are welded to a W30×99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 240 \text{ kips} \uparrow$$





$$+\circlearrowleft \Sigma M_J = 0: \quad -240x + 30x\frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip} \cdot \text{ft}$$

For W30×99 rolled steel section, $S = 269 \text{ in}^3$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (22)(269) = 5918 \text{ kip} \cdot \text{in} = 493.167 \text{ kip} \cdot \text{ft}$$

To locate points *D* and *E*, set $M = M_{all}$.

$$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, \quad 13.58 \text{ ft}$$

(*a*) $l = x_E - x_D = 13.58 - 2.42$ $\qquad l = 11.16 \text{ ft}$ ◀

Center of beam: $M = 960 \text{ kip} \cdot \text{ft} = 11520 \text{ kip} \cdot \text{in}$

$$S = \frac{M}{\sigma_{all}} = \frac{11520}{22} = 523.64 \text{ in}^3 \qquad c = \frac{29.7}{2} + 0.625 = 15.475 \text{ in.}$$

Required moment of inertia: $I = Sc = 8103.3 \text{ in}^4$

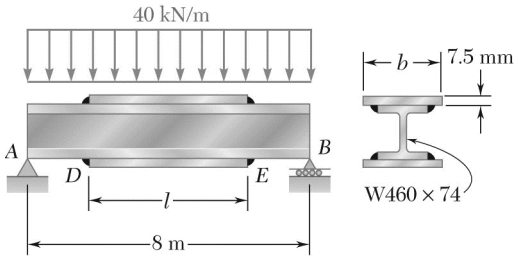
But $I = I_{beam} + 2I_{plate}$

$$8103.3 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.7}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$$= 3990 + 287.42b$$

(*b*) $b = 14.31 \text{ in.}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

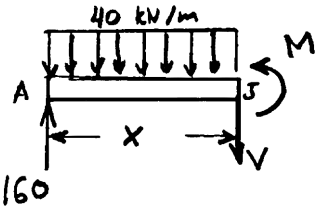


PROBLEM 5.144

Two cover plates, each 7.5 mm thick, are welded to a W460×74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN} \cdot \text{m}$$

At center of beam, $x = 4\text{m} \qquad M_C = 320 \text{ kN} \cdot \text{m}$

At *D*, $x = \frac{1}{2}(8 - l) = 1.5 \text{ m} \qquad M_D = 195 \text{ kN} \cdot \text{m}$

At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 333 \times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

$$S = \frac{I}{c} = 2097 \times 10^3 \text{ mm}^3 = 2097 \times 10^{-6} \text{ m}^3$$

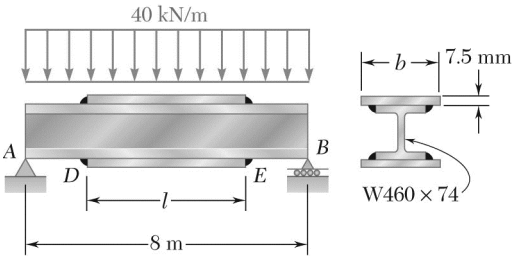
(*a*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{320 \times 10^3}{2097 \times 10^{-6}} = 152.6 \times 10^6 \text{ Pa}$

$\sigma = 152.6 \text{ MPa}$ ◀

At *D*, $S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa}$ $\sigma = 133.6 \text{ MPa}$ ◀

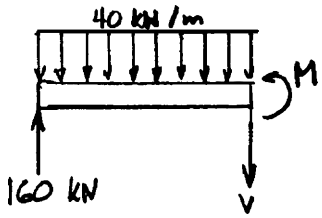
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.145

Two cover plates, each 7.5 mm thick, are welded to a W460×74 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

SOLUTION

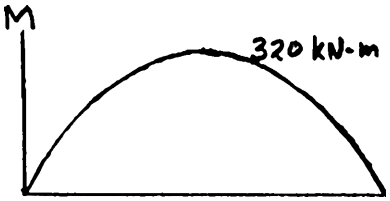


$$\boldsymbol{R}_A = \boldsymbol{R}_B = 160 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -160x + (40x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 160x - 20x^2 \text{ kN} \cdot \text{m}$$

For W460×74 rolled steel beam,



$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$$
$$= 219 \times 10^3 \text{ N} \cdot \text{m} = 219 \text{ kN} \cdot \text{m}$$

To locate points *D* and *E*, set $M = M_{all}$

$$160x - 20x^2 = 219 \qquad 20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} \qquad x = 1.753 \text{ m and } x = 6.247 \text{ m}$$

(*a*) $x_D = 1.753$ ft $\quad x_E = 6.247$ ft $\qquad l = x_E - x_D = 4.49$ m ◀

At center of beam, $M = 320 \text{ kN} \cdot \text{m} = 320 \times 10^3 \text{ N} \cdot \text{m} \quad c = \frac{457}{2} + 7.5 = 236 \text{ mm}^4$

$$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{ m}^3 = 2133 \times 10^3 \text{ mm}^3$$

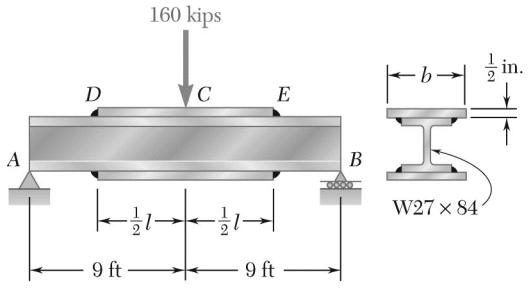
Required moment of inertia: $I = Sc = 503.4 \times 10^6 \text{ mm}^4$

But $I = I_{beam} + 2I_{plate}$

$$503.4 \times 10^6 = 333 \times 10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$$

(*b*) $$= 333 \times 10^6 + 809.2 \times 10^3 b \qquad b = 211 \text{ mm}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

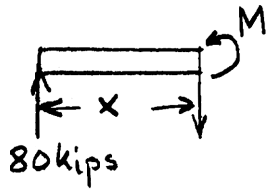


PROBLEM 5.146

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W 27×84 beam as shown. Knowing that $l = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.

SOLUTION

$$\boldsymbol{R}_A = \boldsymbol{R}_B = 80 \text{ kips} \uparrow$$



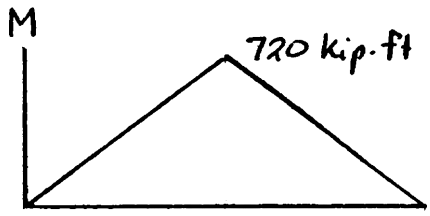
$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$

At *C*, $x = 9$ ft $\quad M_C = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in}$

At *D*, $x = 9 - 5 = 4$ ft

$$M_D = (80)(4) = 320 \text{ kip} \cdot \text{ft} = 3840 \text{ kip} \cdot \text{in}$$



At center of beam, $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$$

$$= 4794 \text{ in}^3$$

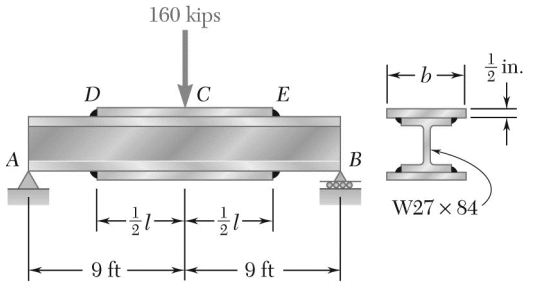
$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in.}$$

(*a*) Normal stress: $\sigma = \dfrac{Mc}{I} = \dfrac{(8640)(13.855)}{4794}$ $\sigma = 25.0$ ksi ◀

At point *D*, $S = 213 \text{ in}^3$

(*b*) Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{3840}{213}$ $\sigma = 18.03$ ksi ◀

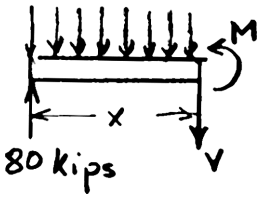
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.147

Two cover plates, each $\frac{1}{2}$ in. thick, are welded to a W27×84 beam as shown. Knowing that $\sigma_{all} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.

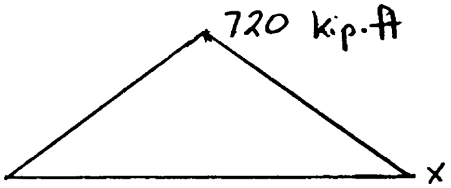
SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = 80 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -80x + M = 0$$

$$M = 80x \text{ kip} \cdot \text{ft}$$



At D, $S = 213 \text{ in}^3$

Allowable bending moment:

$$M_{all} = \sigma_{all} S = (24)(213) = 5112 \text{ kip} \cdot \text{in}$$

$$= 426 \text{ kip} \cdot \text{ft}$$

Set $M_D = M_{all}$. $80x_D = 426$ $x_D = 5.325$ ft

(*a*) $l = 18 - 2x_D$ $l = 7.35$ ft ◀

At center of beam, $M = (80)(9) = 720 \text{ kip} \cdot \text{ft} = 8640 \text{ kip} \cdot \text{in}$

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.7}{2} + 0.500 = 13.85 \text{ in.}$$

Required moment of inertia: $I = Sc = 4986 \text{ in}^4$

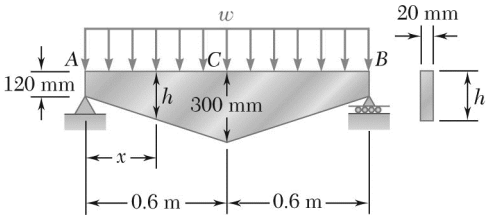
But $I = I_{beam} + 2I_{plate}$

$$4986 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.7}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

$$= 2850 + 184.981b$$

(*b*) $b = 11.55$ in. ◀

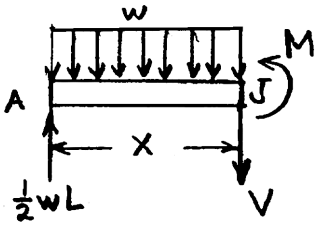
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.148

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 140$ MPa.

SOLUTION

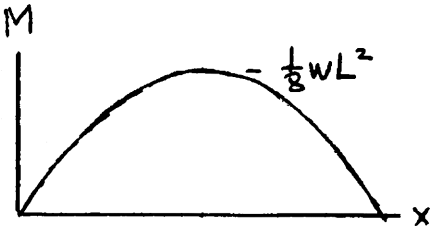


$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow \qquad L = 1.2 \text{ m}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wL + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $h = a + kx$

$a = 120$ mm

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a + kx)^3}\right\} = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.148 *(Continued)*

(*a*) $x_m = \dfrac{aL}{2a + kL} = \dfrac{(120)(1.2)}{(2)(120) + (300)(1.2)}$ $\qquad x_m = 0.24 \text{ m}$ ◀

$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$

$S_m = \dfrac{1}{6} b h_m^2 = \dfrac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$

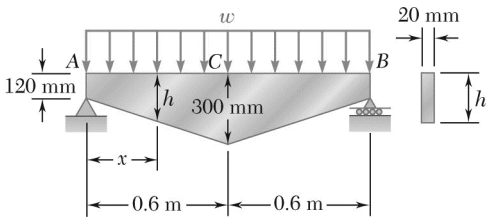
Allowable value of M_m: $M_m = S_m \sigma_{\text{all}} = (122.88 \times 10^{-6})(140 \times 10^6)$

$= 17.2032 \times 10^3 \text{ N} \cdot \text{m}$

(*b*) Allowable value of *w*: $w = \dfrac{2M_m}{x_m(L - x_m)} = \dfrac{(2)(17.2032 \times 10^3)}{(0.24)(0.96)}$

$= 149.3 \times 10^3 \text{ N/m}$ $\qquad w = 149.3 \text{ kN/m}$ ◀

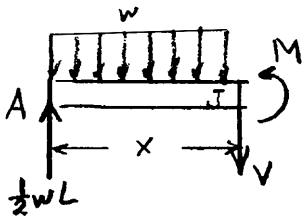
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.149

For the tapered beam shown, knowing that w = 160 kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.

SOLUTION

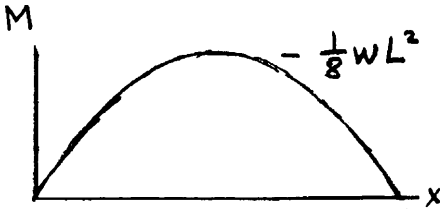


$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



where w = 160 kN/m and L = 1.2 m.

For the tapered beam, $h = a + kx$

$$a = 120 \text{ mm}$$

$$k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress: $$\sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax + kLx}{(a + kx)^3}\right\} = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.149 *(Continued)*

(*a*) $$x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} \qquad x_m = 0.240 \text{ m} \blacktriangleleft$$

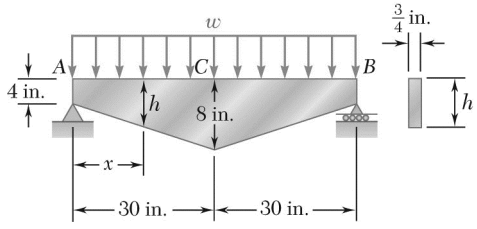
$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

$$M_m = \frac{w}{2} x_m (L - x_m) = \frac{160 \times 10^3}{2}(0.24)(0.96) = 18.432 \times 10^3 \text{ N} \cdot \text{m}$$

(*b*) Maximum bending stress: $$\sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6 \text{ Pa} \qquad \sigma_m = 150.0 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

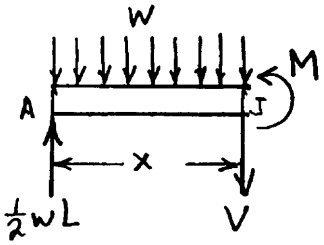


PROBLEM 5.150

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{\text{all}} = 24$ ksi.

SOLUTION

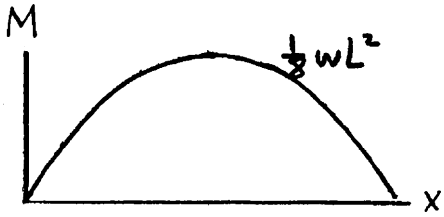
$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{1}{2}wL \uparrow \quad L = 60 \text{ in.}$$



$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$



For the tapered beam, $h = a + kx$

$$a = 4 \text{ in.} \quad k = \frac{8-4}{30} = \frac{2}{15} \text{ in./in.}$$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.150 *(Continued)*

(*a*) $$x_m = \frac{aL}{2a + kL} = \frac{(4)(60)}{(2)(4) + \left(\frac{2}{15}\right)(6.0)}$$ $x_m = 15$ in. ◀

$$h_m = a + kx_m = 4 + \left(\frac{2}{15}\right)(15) = 6.00 \text{ in.}$$

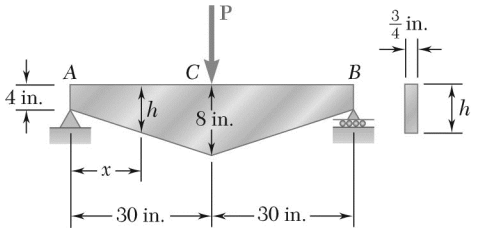
$$S_m = \frac{1}{6} b h_m^3 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(6.00)^2 = 4.50 \text{ in}^3$$

Allowable value of M_m: $M_m = S_m \, \sigma_{\text{all}} = (4.50)(24) = 180.0 \text{ kip} \cdot \text{in}$

(*b*) Allowable value of w: $$w = \frac{2M_m}{x_m (L - x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320 \text{ kip/in}$$

$w = 320$ lb/in ◀

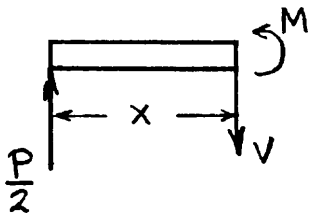
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.151

For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 24$ ksi.

SOLUTION



$$\boldsymbol{R}_A = \boldsymbol{R}_B = \frac{P}{2} \uparrow$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam, $h = a + kx$

For a rectangular cross section, $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress: $\sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress, set $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\frac{(a+kx)^2 - x - 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $a = 4$ in., $k = \frac{8-4}{30} = 0.13333$ in/in

(*a*) $x_m = \frac{4}{0.13333} = 30$ in. $\qquad x_m = 30.0$ in. ◀

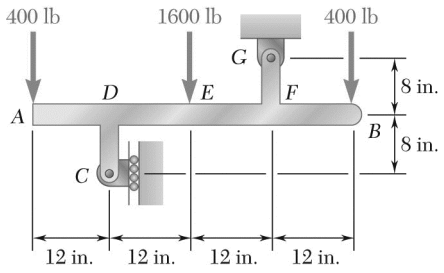
$h_m = a + kx_m = 8$ in.

$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$

$M_m = \sigma_{all} S_m = (24)(8) = 192 \text{ kip} \cdot \text{in}$

(*b*) $P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8$ kips $\qquad P = 12.80$ kips ◀

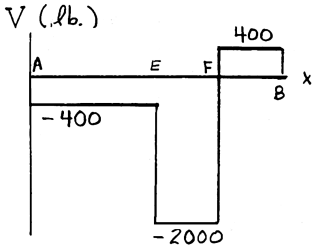
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.152

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

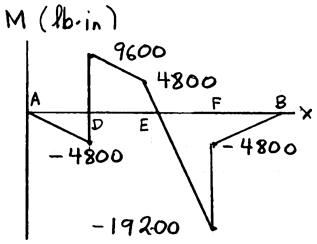


$+\circlearrowleft \Sigma M_G = 0$: $-16C + (36)(400) + (12)(1600)$

$- (12)(400) = 0$ $\quad C = 1800$ lb

$\xrightarrow{+} \Sigma F_x = 0$: $-C + G_x = 0$ $\quad G_x = 1800$ lb

$+\uparrow \Sigma F_y = 0$: $-400 - 1600 + G_y - 400 = 0$ $\quad G_y = 2400$ lb

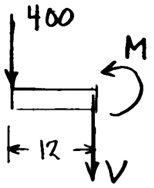


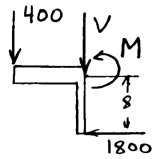
A to E: $\quad V = -400$ lb

E to F: $\quad V = -2000$ lb

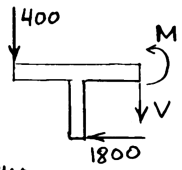
F to B: $\quad V = 400$ lb

At A and B, $\quad M = 0$





At D^-, $+\circlearrowleft \Sigma M_D = 0$: $(12)(400) + M = 0$ $\quad M = -4800$ lb · in

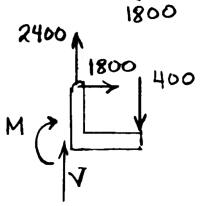


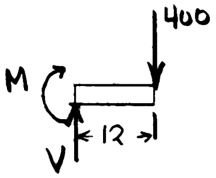
At D^+, $+\circlearrowleft \Sigma M_D = 0$: $(12)(400) - (8)(1800) + M = 0$ $\quad M = 9600$ lb · in

At E, $+\circlearrowleft \Sigma M_E = 0$: $(24)(400) - (8)(1800) + M = 0$ $\quad M = 4800$ lb · in

At F^-, $+\circlearrowleft \Sigma M_F = 0$: $-M - (8)(1800) - (12)(400) = 0$ $\quad M = -19200$ lb · in

At F^+, $+\circlearrowleft \Sigma M_F = 0$: $-M - (12)(400) = 0$ $\quad M = -4800$ lb · in

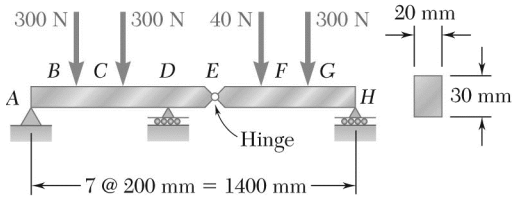




(*a*) Maximum $|V| = 2000$ lb ◀

(*b*) Maximum $|M| = 19200$ lb · in ◀

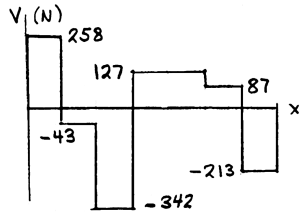
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

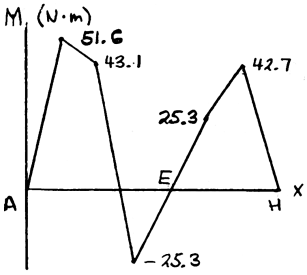


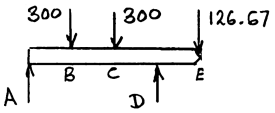
PROBLEM 5.153

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION







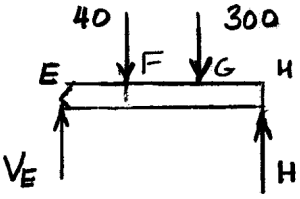
Free body *EFGH*. Note that $M_E = 0$ due to hinge.

$$+\circlearrowright \Sigma M_E = 0: \quad 0.6\,H - (0.2)(40) - (0.40)(300) = 0$$

$$H = 213.33 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad V_E - 40 - 300 + 213.33 = 0$$

$$V_E = 126 \cdot 67 \text{ N}$$

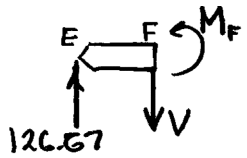


Shear:

E to F: $V = 126.67 \text{ N} \cdot \text{m}$

F to G: $V = 86.67 \text{ N} \cdot \text{m}$

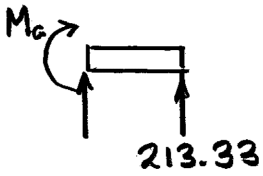
G to H: $V = -213.33 \text{ N} \cdot \text{m}$



Bending moment at F:

$$+\circlearrowright \Sigma M_F = 0: \quad M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N} \cdot \text{m}$$



Bending moment at G:

$$+\circlearrowright \Sigma M_G = 0: \quad -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N} \cdot \text{m}$$

Free body *ABCDE*.

$$+\circlearrowright \Sigma M_B = 0: \quad 0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.63) = 0$$

$$A = 257.78 \text{ N}$$

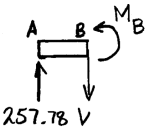
$$+\circlearrowright \Sigma M_A = 0: \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$D = 468.89 \text{ N}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.153 *(Continued)*

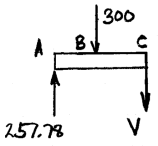
Bending moment at B.



$$+\circlearrowleft \Sigma M_B = 0: \quad -(0.2)(257.78) + M_B = 0$$

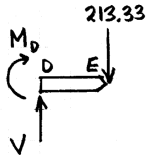
$$M_B = 51.56 \text{ N} \cdot \text{m}$$

Bending moment at C.



$$+\circlearrowleft \Sigma M_C = 0: \quad -(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N} \cdot \text{m}$$



Bending moment at D.

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N} \cdot \text{m}$$

$$\max|M| = 51.56 \text{ N} \cdot \text{m}$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(20)(30)^2$$

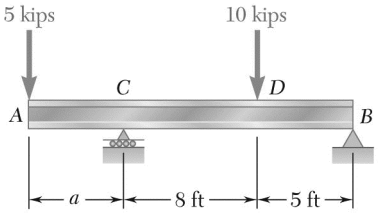
$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

Normal stress.

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



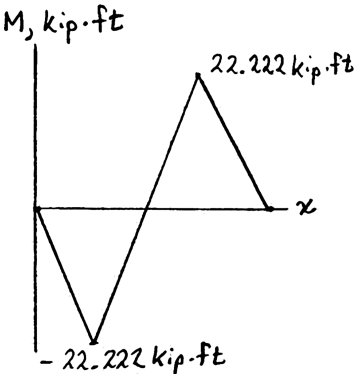
PROBLEM 5.154

Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)

SOLUTION

Reaction at *B*: $+\circlearrowleft \Sigma M_C = 0: \quad 5a - (8)(10) + 13R_B = 0$

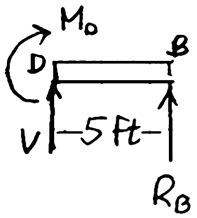
$$R_B = \frac{1}{18}(80 - 5a)$$



Bending moment at *D*:

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + 5R_B = 0$$

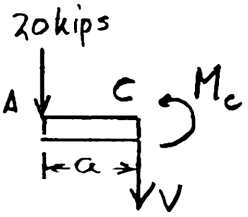
$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$



Bending moment at *C*:

$$+\circlearrowleft M_C = 0 \quad 5a + M_C = 0$$

$$M_C = -5a$$



Equate: $-M_C = M_D$

$$5a = \frac{5}{13}(80 - 5a)$$

$$a = 4.4444 \text{ ft}$$

(*a*) $a = 4.44$ ft ◀

Then $-M_C = M_D = (5)(4.4444) = 22.222 \text{ kip} \cdot \text{ft}$

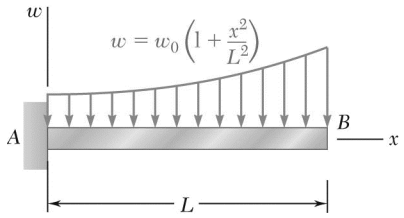
$$|M|_{\max} = 22.222 \text{ kip} \cdot \text{ft} = 266.67 \text{ kip} \cdot \text{in}$$

For W14 × 22 rolled steel section, $S = 29.0 \text{ in}^3$

Normal stress: $\sigma = \dfrac{M}{S} = \dfrac{266.67}{29.0} = 9.20 \text{ ksi}$

(*b*) 9.20 ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.155

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

(*a*)

$$\frac{dV}{dx} = -w = -w_0\left(1 + \frac{x^2}{L^2}\right)$$

$$V = -w_0\left(x + \frac{x^3}{3L^2}\right) + C_1$$

$V = 0 \quad \text{at} \quad x = L.$

$$0 = -w_0\left(L + \frac{1}{3}L\right) + C_1 \quad C_1 = \frac{4}{3}w_0 L$$

$$\frac{dM}{dx} = V = w_0\left(\frac{4}{3}L - x - \frac{1}{3}\frac{x^3}{L^2}\right) \blacktriangleleft$$

$$M = w_0\left(\frac{4}{3}Lx - \frac{1}{2}x^2 - \frac{1}{12}\frac{x^4}{L^2}\right) + C_2$$

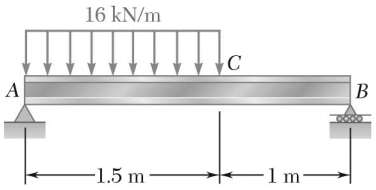
$M = 0 \quad \text{at} \quad x = L. \quad w_0\left(\frac{4}{3}L^2 - \frac{1}{2}L^2 - \frac{1}{12}L^2\right) + C_2 = 0$

$$C_2 = -\frac{3}{4}w_0 L^2$$

$$M = w_0\left(\frac{4}{3}Lx - \frac{1}{2}x^2 - \frac{1}{12}\frac{x^4}{L^2} - \frac{3}{4}L^2\right) \blacktriangleleft$$

(*b*) $|M|_{\max}$ occurs at $x = 0$. $|M|_{\max} = \frac{3}{4}w_0 L^2$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.156

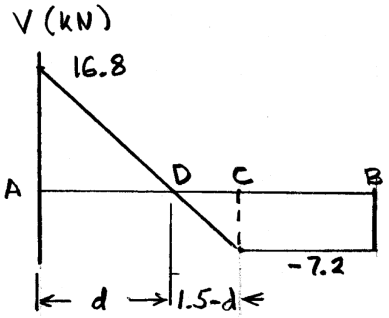
Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.5A + (1.75)(1.5)(16) = 0$$
$$A = 16.8 \text{ kN}$$
$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.75) + (1.5)(16) + 2.5B = 0$$
$$B = 7.2 \text{ kN}$$

Shear diagram:

$$V_A = 16.8 \text{ kN}$$
$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$
$$V_B = -7.2 \text{ kN}$$



Locate point D where $V = 0$.

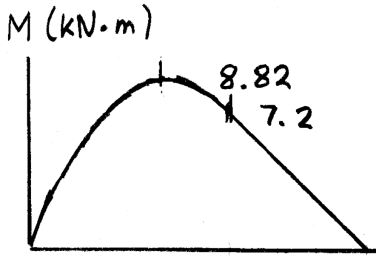
$$\frac{d}{16.8} = \frac{1.5 - d}{7.2} \qquad 24d = 25.2$$
$$d = 1.05 \text{ m} \quad 1.5 - d = 0.45 \text{ m}$$

Areas of the shear diagram:

A to D: $$\int V dx = \left(\frac{1}{2}\right)(1.05)(16.8) = 8.82 \text{ kN} \cdot \text{m}$$

D to C: $$\int V dx = \left(\frac{1}{2}\right)(0.45)(-7.2) = -1.62 \text{ kN} \cdot \text{m}$$

C to B: $$\int V dx = (1)(-7.2) = -7.2 \text{ kN} \cdot \text{m}$$



Bending moments:

$$M_A = 0$$
$$M_D = 0 + 8.82 = 8.82 \text{ kN} \cdot \text{m}$$
$$M_C = 8.82 - 1.62 = 7.2 \text{ kN} \cdot \text{m}$$
$$M_B = 7.2 - 7.2 = 0$$

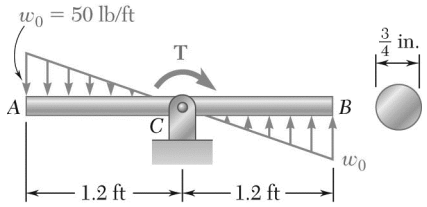
Maximum $|M| = 8.82 \text{ kN} \cdot \text{m} = 8.82 \times 10^3 \text{ N} \cdot \text{m}$

For S150×18.6 rolled steel section, $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

Normal stress: $$\sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa}$$

$\sigma = 73.5$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.157

Knowing that beam AB is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION

A to C: $\quad 0 < x < 1.2$ ft

$$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

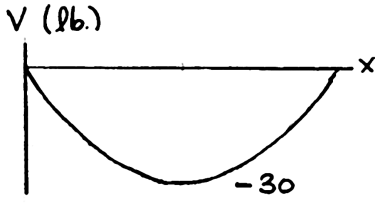
$$V = V_A + \int_0^x (41.667x - 50)dx$$

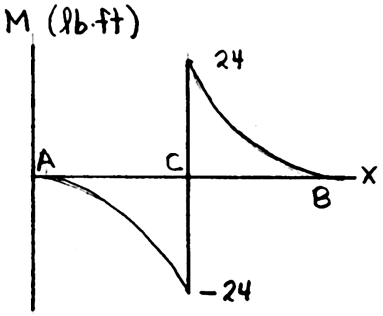
$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V\,dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x)dx$$

$$= 6.944x^3 - 25x^2$$





At $\quad x = 1.2$ ft, $\quad V = -30$ lb

$\qquad\qquad\qquad\quad M = -24$ lb · in

C to B: Use symmetry conditions.

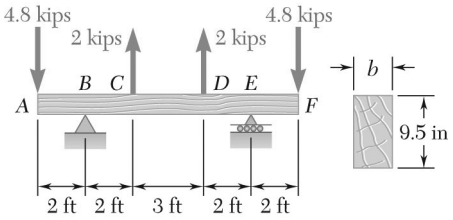
Maximum $|M| = 24$ lb · ft $= 288$ lb · in

Cross section: $\quad c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375$ in.

$$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532 \times 10^{-3}\,\text{in}^4$$

Normal stress: $\quad \sigma = \frac{|M|c}{I} = \frac{(2.88)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3$ psi $\qquad \sigma = 6.95$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.158

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION

For equilibrium, $B = E = 2.8$ kips

Shear diagram:

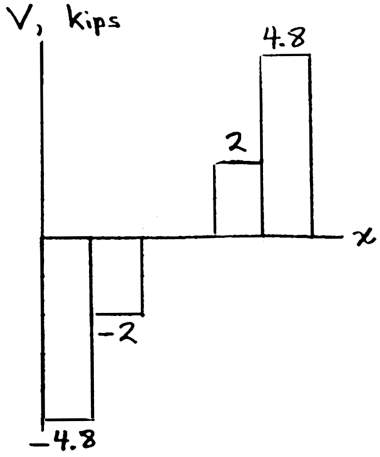
A to B^-: $V = -4.8$ kips

B^+ to C^-: $V = -4.8 + 2.8 = -2$ kips

C^+ to D^-: $V = -2 + 2 = 0$

D^+ to E^-: $V = 0 + 2 = 2$ kips

E^+ to F: $V = 2 + 2.8 = 4.8$ kips



Areas of shear diagram:

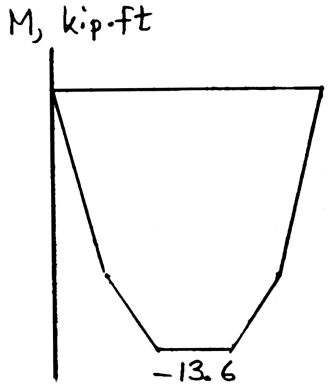
A to B: $(2)(-4.8) = -9.6$ kip · ft

B to C: $(2)(-2) = -4$ kip · ft

C to D: $(3)(0) = 0$

D to E: $(2)(2) = 4$ kip · ft

E to F: $(2)(4.8) = 9.6$ kip · ft



Bending moments:

$M_A = 0$

$M_B = 0 - 9.6 = -9.6$ kip · ft

$M_C = -9.6 - 4 = -13.6$ kip · ft

$M_D = -13.6 + 0 = -13.6$ kip · ft

$M_E = -13.6 + 4 = -9.6$ kip · ft

$M_F = -9.6 + 9.6 = 0$

$$|M|_{\max} = 13.6 \text{ kip} \cdot \text{ft} = 162.3 \text{ kip} \cdot \text{in} = 162.3 \times 10^3 \text{ lb} \cdot \text{in}$$

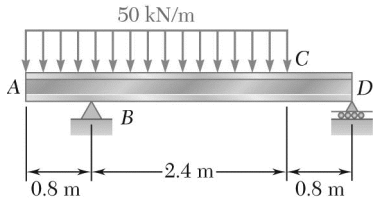
Required value for S:

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{162.3 \times 10^3}{1750} = 93.257 \text{ in}^3$$

For a rectangular section, $I = \frac{1}{12} bh^3$, $c = \frac{1}{2} h$ $S = \frac{I}{c} = \frac{bh^2}{6} = \frac{(b)(9.5)^2}{6} = 15.0417b$

Equating the two expressions for S, $15.0417\, b = 93.257$ $b = 6.20$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.159

Knowing that the allowable stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \quad -3.2B + (24)(3.2)(50) = 0 \qquad B = 120 \text{ kN}$$

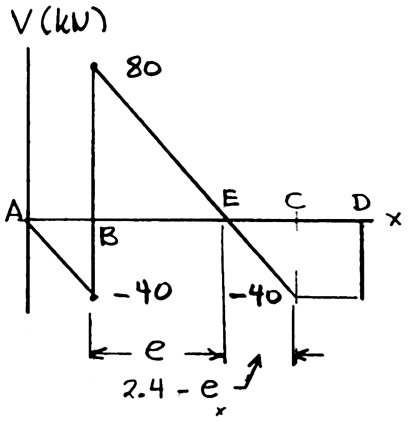
$$+\circlearrowleft \Sigma M_B = 0: \quad 3.2D - (0.8)(3.2)(50) = 0 \qquad D = 40 \text{ kN}$$

Shear:

$$V_A = 0$$
$$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$$
$$V_{B^+} = -40 + 120 = 80 \text{ kN}$$
$$V_C = 80 - (2.4)(50) = -40 \text{ kN}$$
$$V_D = -40 + 0 = -40 \text{ kN}$$

Locate point E where $V = 0$.

$$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$$
$$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$$



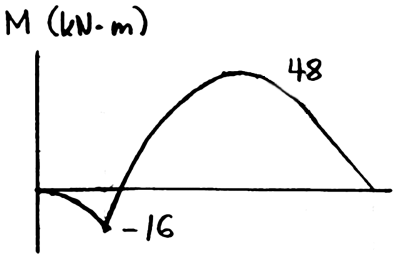
Areas:

A to B: $\int V dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$

B to E: $\int V dx = \left(\frac{1}{2}\right)(1.6)(80) = 64 \text{ kN} \cdot \text{m}$

E to C: $\int V dx = \left(\frac{1}{2}\right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m}$

C to D: $\int V dx = (0.8)(-40) = -32 \text{ kN} \cdot \text{m}$



Bending moments:

$$M_A = 0$$
$$M_B = 0 - 16 = -16 \text{ kN} \cdot \text{m}$$
$$M_E = -16 + 64 = 48 \text{ kN} \cdot \text{m}$$
$$M_C = 48 - 16 = 32 \text{ kN} \cdot \text{m}$$
$$M_D = 32 - 32 = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.159 *(Continued)*

Maximum $|M| = 48 \text{ kN} \cdot \text{m} = 48 \times 10^3 \text{ N} \cdot \text{m}$

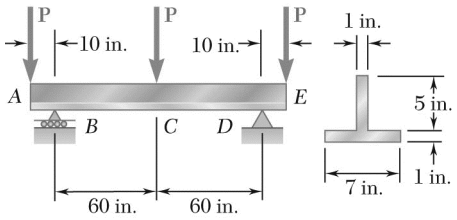
$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 \text{ mm}^3)$
W 310×32.7	415
W 250×28.4	308 ←
W 200×35.9	342

Lightest wide flange beam: W 250×28.4 @ 28.4 kg/m ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

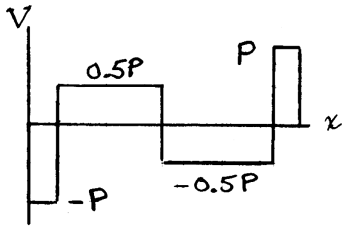


PROBLEM 5.160

Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and −18 ksi in compression.

SOLUTION

Reactions: $\boldsymbol{B} = \boldsymbol{D} = 1.5\text{ P} \uparrow$



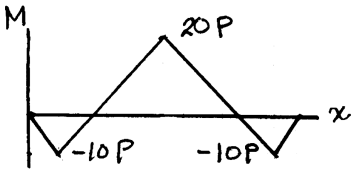
Shear diagram:

A to B^-: $V = -P$

B^+ to C^-: $V = -P + 1.5\text{ P} = 0.5\text{ P}$

C^+ to D^-: $V = 0.5\text{ P} - P = -0.5\text{ P}$

D^+ to E: $V = -0.5\text{ P} + 1.5\text{ P} = P$



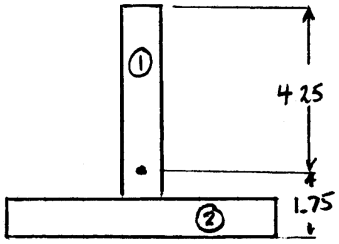
Areas:

A to B: $(10)(-P) = -10\text{ P}$

B to C: $(60)(0.5\text{ P}) = 30\text{ P}$

C to D: $(60)(-0.5\text{ P}) = -30\text{ P}$

D to E: $(10)(P) = 10\text{ P}$



Bending moments:

$M_A = 0$

$M_B = 0 - 10\text{ P} = -10\text{ P}$

$M_C = -10\text{ P} + 30\text{ P} = 20\text{ P}$

$M_D = 20\text{ P} - 30\text{ P} = -10\text{ P}$

$M_E = -10\text{ P} + 10\text{ P} = 0$

Largest positive bending moment: 20 P

Largest negative bending moment: −10 P

Centroid and moment of inertia:

Part	A, in^2	$\bar{y}_0$, in.	$A\bar{y}_0$, in^3	d, in.	Ad^2, in^4	$\bar{I}$, in^4
①	5	3.5	17.5	1.75	15.3125	10.417
②	7	0.5	3.5	1.25	10.9375	0.583
Σ	12		21		26.25	11.000

$$\bar{Y} = \frac{21}{12} = 1.75\text{ in.} \qquad I = \Sigma Ad^2 + \Sigma I = 37.25\text{ in}^4$$

Top: $y = 4.25$ in. Bottom: $y = -1.75$ in. $\sigma = -\dfrac{My}{I}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 5.160 ***(Continued)***

Top, tension: $8 = -\dfrac{(-10\text{ P})(4.25)}{37.25}$ $P = 7.01$ kips

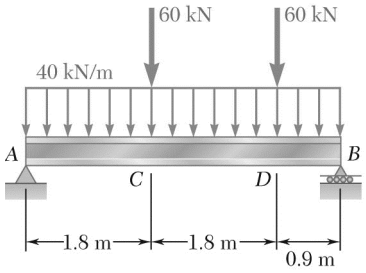
Top, comp.: $-18 = -\dfrac{(20\text{ P})(4.25)}{37.25}$ $P = 7.89$ kips

Bottom. tension: $8 = -\dfrac{(20\text{ P})(-1.75)}{37.25}$ $P = 8.51$ kips

Bottom. comp.: $-18 = -\dfrac{(-10\text{ P})(-1.75)}{37.25}$ $P = 38.3$ kips

Smallest value of P is the allowable value. $P = 7.01$ kips ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.161

(*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.5A + (2.25)(4.5)(40) + (2.7)(60) + (0.9)(60) = 0$$

$$\boldsymbol{A} = 138 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2.25)(4.5)(40) - (1.8)(60) - (3.6)(60) + 4.5B = 0$$

$$\boldsymbol{B} = 162 \text{ kN} \uparrow$$

$$w = 40 \text{ kN/m} = \frac{dV}{dx}$$

$$V = -40x + 138 - 60\langle x - 1.8\rangle^0 - 60\langle x - 3.6\rangle^0 = \frac{dM}{dx}$$

$$M = -20x^2 - 138x - 60\langle x - 1.8\rangle^1 - 60\langle x - 3.6\rangle^1$$

$$V_C^+ = -(40)(1.8) + 138 - 60 = 6 \text{ kN}$$

$$V_D^- = -(40)(3.6) + 138 - 60 = -66 \text{ kN}$$

Locate point E where $V = 0$. It lies between C and D.

$$V_E = -40x_E + 138 - 60 + 0 = 0 \qquad x_E = 1.95 \text{ m}$$

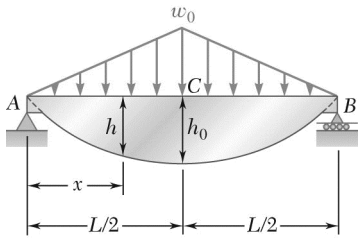
$$M_E = -(20)(1.95)^2 + (138)(1.95) - (60)(1.95 - 1.8) = 184 \text{ kN} \cdot \text{m}$$

(*a*) $$|M|_{\max} = 184 \text{ kN} \cdot \text{m} = 184 \times 10^3 \text{ N} \cdot \text{m} \quad \text{at} \quad x = 1.950 \text{ m} \blacktriangleleft$$

For W 530×66 rolled steel section, $S = 1340 \times 10^3 \text{ mm}^3 = 1340 \times 10^{-6} \text{ m}^3$

(*b*) Normal stress: $$\sigma = \frac{|M|_{\max}}{S} = \frac{184 \times 10^3}{1340 \times 10^{-6}} = 137.3 \times 10^6 \text{ Pa} \qquad \sigma = 137.3 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



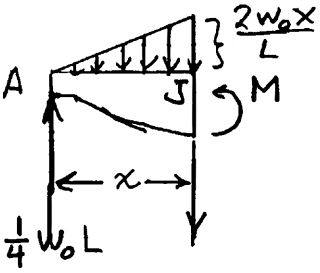
PROBLEM 5.162

The beam AB, consisting of an aluminum plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

By symmetry, $A = B$

$$+\uparrow \Sigma F_y = 0: \quad A - \frac{1}{2} w_0 L + B = 0 \qquad \boldsymbol{A} = \boldsymbol{B} = \frac{1}{4} w_0 L \uparrow$$



For $0 \le x \le \frac{L}{2}$, $\quad w = \frac{2w_0 x}{L} \quad \frac{dV}{dx} = -w = -\frac{2w_0 x}{L} \quad V = C_1 - \frac{w_0 x^2}{L}$

At $x = 0$, $\quad V = \frac{1}{4} w_0 L: \quad C_1 = \frac{1}{4} w_0 L$

$$\frac{dM}{dx} = V = \frac{1}{4} w_0 L - \frac{w_0 x^2}{L} \qquad M = C_2 + \frac{1}{4} w_0 L x - \frac{1}{3} \frac{w_0 x^3}{L}$$

At $x = 0$, $\quad M = 0: \quad C_2 = 0$

$$M = \frac{1}{2} \frac{w_0}{L} (3L^2 - 4x^3)$$

(*a*) At $x = \frac{L}{2}$, $\quad M = M_C = \frac{1}{12} \frac{w_0}{L} \left[3L^2 \left(\frac{L}{2} \right) - 4 \left(\frac{L}{2} \right)^3 \right] = \frac{1}{12} w_0 L^2$

For constant strength, $\quad S = \frac{M}{\sigma_{all}}, \quad S_0 = \frac{M_0}{\sigma_{all}} = \frac{M_C}{\sigma_{all}} \quad \frac{S}{S_0} = \frac{M}{M_0} = \frac{1}{L^3} (3L^2 x - 4x^3)$

For a rectangular section, $\quad S = \frac{1}{6} b h^2 \quad S_0 = \frac{1}{6} b h_0^2 \quad \frac{S}{S_0} = \left(\frac{h}{h_0} \right)^2$

$$h = h_0 \sqrt{\frac{3L^2 x - 4x^3}{L^3}} \quad \blacktriangleleft$$

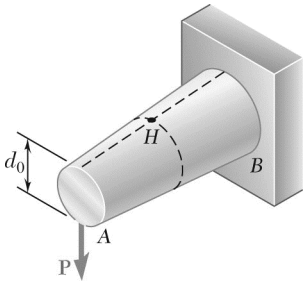
(*b*) Data: $\quad L = 800$ mm $\quad h_0 = 200$ mm $\quad b = 25$ mm $\quad \sigma_{all} = 72$ MPa

$$S_0 = \frac{1}{6} b h_0^2 = \frac{1}{6} (25)(200)^2 = 166.667 \times 10^3 \text{ mm}^3 = 166.667 \times 10^{-6} \text{ m}^3$$

$$M_C = \sigma_{all} S_0 = (72 \times 10^6)(166.667 \times 10^{-6}) = 12 \times 10^3 \text{ N} \cdot \text{m}$$

$$w_0 = \frac{12 M_C}{L^2} = \frac{(12)(12 \times 10^3)}{(0.800)^2} = 225 \times 10^3 \text{ N/m} \qquad w_0 = 225 \text{ kN/m} \quad \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 5.163

A transverse force **P** is applied as shown at end *A* of the conical taper *AB*. Denoting by d_0 the diameter of the taper at *A*, show that the maximum normal stress occurs at point *H*, which is contained in a transverse section of diameter $d = 1.5d_0$.

SOLUTION

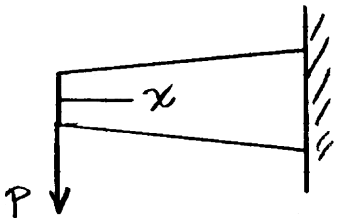
$$V = -P = \frac{dM}{dx} \quad M = -Px$$

Let $d = d_0 + kx$

For a solid circular section, $I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^3$

$$c = \frac{d}{2} \quad S = \frac{I}{c} = \frac{\pi}{32}d^3 = \frac{\pi}{32}(d_0 + kx)^3$$

$$\frac{dS}{dx} = \frac{3\pi}{32}(d_0 + kx)^2 k = \frac{3\pi}{32}d^2 k$$



Stress: $\sigma = \frac{|M|}{S} = \frac{Px}{S}$

At *H*, $\frac{d\sigma}{dx} = \frac{1}{S^2}\left(PS - Px_H \frac{dS}{dx}\right) = 0$

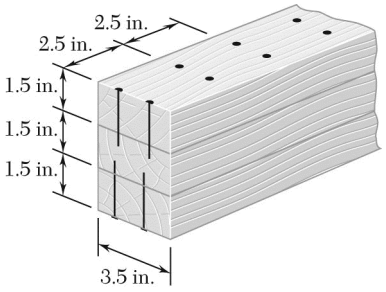
$$S - x_H \frac{dS}{dx} = \frac{\pi}{32}d^3 - x_H \frac{3\pi}{32}d^2 k$$

$$kx_H = \frac{1}{3}d = \frac{1}{3}(d_0 + k_H x_H) \qquad kx_H = \frac{1}{2}d_0$$

$$d = d_0 + \frac{1}{2}d_0 = \frac{3}{2}d_0 \qquad d = 1.5d_0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

CHAPTER 6



PROBLEM 6.1

Three boards, each of 1.5×3.5-in. rectangular cross section, are nailed together to form a beam that is subjected to a vertical shear of 250 lb. Knowing that the spacing between each pair of nails is 2.5 in., determine the shearing force in each nail.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(3.5)(4.5)^3 = 26.578 \text{ in}^4$$

$$A = (3.5)(1.5) = 5.25 \text{ in}^2$$

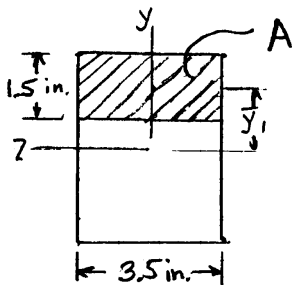
$$\bar{y}_1 = 1.5 \text{ in.}$$

$$Q = Ay_1 = 7.875 \text{ in}^3$$

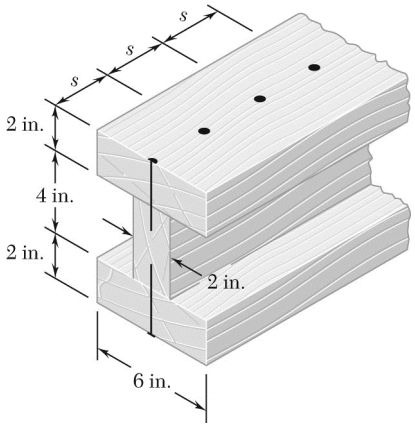
$$q = \frac{VQ}{I} = \frac{(250)(7.875)}{26.578} = 74.074 \text{ lb/in}$$

$$qs = 2F_{\text{nail}} \qquad F_{\text{nail}} = \frac{qs}{2} = \frac{(74.074)(2.5)}{2}$$

$$F_{\text{nail}} = 92.6 \text{ lb} \blacktriangleleft$$



***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.2

Three boards, each 2 in. thick, are nailed together to form a beam that is subjected to a vertical shear. Knowing that the allowable shearing force in each nail is 150 lb, determine the allowable shear if the spacing s between the nails is 3 in.

SOLUTION

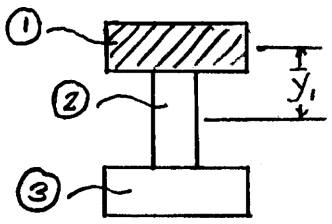
$$I_1 = \frac{1}{12}bh^3 + Ad^2$$

$$= \frac{1}{12}(6)(2)^3 + (6)(2)(3)^2 = 112 \text{ in}^4$$

$$I_2 = \frac{1}{12}bh^3 = \frac{1}{12}(2)(4)^3 = 10.667 \text{ in}^4$$

$$I_3 = I_1 = 112 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 234.667 \text{ in}^4$$



$$Q = A_1\bar{y}_1 = (6)(2)(3) = 36 \text{ in}^3$$

$$qs = F_{\text{nail}} \quad (1)$$

$$q = \frac{VQ}{I} \quad (2)$$

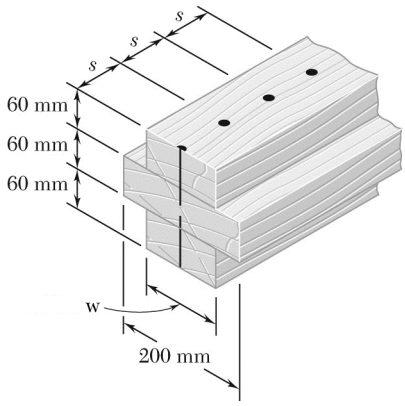
Dividing Eq. (2) by Eq. (1),

$$\frac{1}{s} = \frac{VQ}{F_{\text{nail}}I}$$

$$V = \frac{F_{\text{nail}}I}{Qs} = \frac{(150)(234.667)}{(36)(3)}$$

$$V = 326 \text{ lb} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

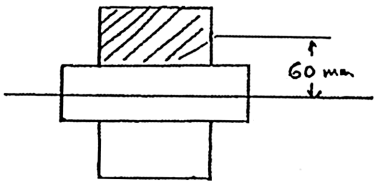


PROBLEM 6.3

Three boards are nailed together to form a beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 75\,\text{mm}$ and that the allowable shearing force in each nail is 400 N, determine the allowable shear when $w = 120$ mm.

SOLUTION

Part	$A\,(\text{mm}^2)$	$d\,(\text{mm})$	$Ad^2\,(10^6\,\text{mm}^4)$	$\bar{I}\,(10^6\,\text{mm}^4)$
Top plank	7200	60	25.92	2.16
Middle Plank	12,000	0	0	3.60
Bottom Plank	7200	60	25.92	2.16
Σ			51.84	7.92



$$I = \sum Ad^2 + \sum \bar{I} = 59.76 \times 10^6\,\text{mm}^4 = 59.76 \times 10^{-6}\,\text{m}^4$$

$$Q = (7200)(60) = 432 \times 10^3\,\text{mm}^3 = 432 \times 10^{-6}\,\text{m}^3$$

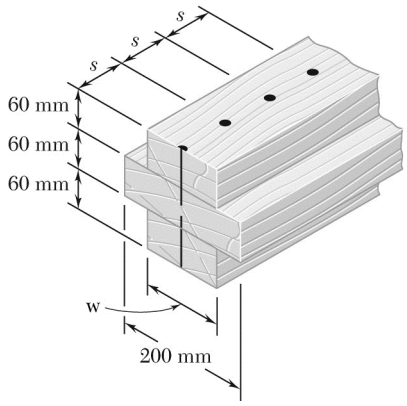
$$q = \frac{VQ}{I} \qquad F_{\text{nail}} = qs$$

$$q = \frac{F_{\text{nail}}}{s} \qquad V = \frac{Iq}{Q} = \frac{IF_{\text{nail}}}{Qs}$$

$$V = \frac{(59.76 \times 10^{-6})(400)}{(432 \times 10^{-6})(75 \times 10^{-3})}$$

$$V = 738\ \text{N} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



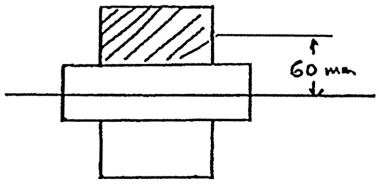
PROBLEM 6.4

Solve Prob. 6.3, assuming that the width of the top and bottom boards is changed to $w = 100$ mm.

PROBLEM 6.3 Three boards are nailed together to form a beam shown, which is subjected to a vertical shear. Knowing that the spacing between the nails is $s = 75$ mm and that the allowable shearing force in each nail is 400 N, determine the allowable shear when $w = 120$ mm.

SOLUTION

Part	A (mm^2)	d (mm)	Ad^2 (10^6 mm^4)	$\bar{I}$ (10^6 mm^4)
Top plank	6000	60	21.6	1.80
Middle Plank	12,000	0	0	3.60
Bottom Plank	6000	60	21.6	1.80
Σ			43.2	7.20



$$I = \sum Ad^2 + \sum \bar{I} = 50.4 \times 10^6 \text{ mm}^4 = 50.4 \times 10^{-6} \text{ m}^4$$

$$Q = (6000)(60) = 360 \times 10^3 \text{ mm}^3 = 360 \times 10^{-6} \text{ m}^3$$

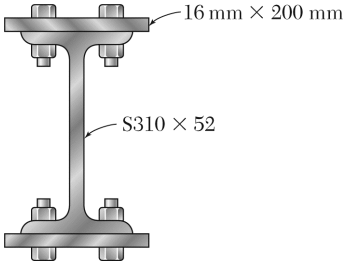
$$q = \frac{VQ}{I} \qquad F_{\text{nail}} = qs$$

$$q = \frac{F_{\text{nail}}}{s} \qquad V = \frac{Iq}{Q} = \frac{IF_{\text{nail}}}{Qs}$$

$$V = \frac{(50.4 \times 10^{-6})(400)}{(360 \times 10^{-6})(75 \times 10^{-3})}$$

$$V = 747 \text{ N} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.5

The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16×200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia:

Part	$A\,(\text{mm}^2)$	$d\,(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}(10^6\,\text{mm}^4)$
Top plate	3200	$^*160.5$	82.43	0.07
S310×52	6650	0		95.3
Bot. plate	3200	$^*160.5$	82.43	0.07
Σ			164.86	95.44

$$^*d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}}\, d_{\text{plate}} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

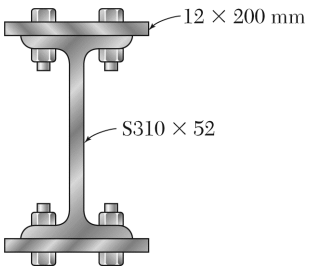
$$F_{\text{bolt}} = \tau_{\text{all}}\, A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

$$qs = 2F_{\text{bolt}} \qquad q = \frac{2F_{\text{bolt}}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$V = 193.5 \text{ kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.6

Solve Prob. 6.5, assuming that the reinforcing plates are only 12 mm thick.

PROBLEM 6.5 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16×200-mm plates, using 18-mm-diameter bolts spaced longitudinally every 120 mm. Knowing that the average allowable shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia:

Part	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(10^6\,\text{mm}^4)$	$\bar{I}\ (10^6\,\text{mm}^4)$
Top plate	2400	$^*158.5$	60.29	0.03
S310×52	6650	0	0	95.3
Bot. plate	2400	$^*158.5$	60.29	0.03
Σ			120.58	95.36

$$^*d = \frac{305}{2} + \frac{12}{2} = 158.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 215.94 \times 10^6 \text{mm}^4 = 215.94 \times 10^{-6} \text{m}^4$$

$$Q = A_{\text{plate}}\, d_{\text{plate}} = (200)(12)(158.5) = 380.4 \times 10^3 \text{mm}^3 = 380.4 \times 10^{-6} \text{m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{m}^2$$

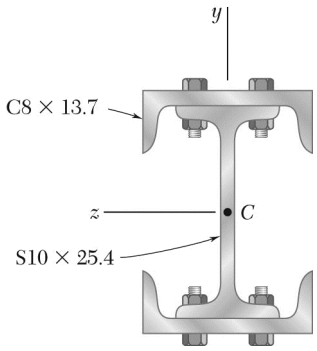
$$F_{\text{bolt}} = \tau_{\text{all}}\, A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.902 \times 10^3 \text{N}$$

$$qs = 2F_{\text{bolt}} \qquad q = \frac{2F_{\text{bolt}}}{s} = \frac{(2)(22.903 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(215.94 \times 10^{-6})(381.7 \times 10^3)}{380.4 \times 10^{-6}} = 217 \times 10^3 \text{N}$$

$$V = 217 \text{ kN} \blacktriangleleft$$

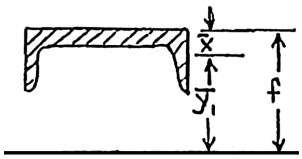
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.7

A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.

SOLUTION



Geometry:

$$f = \left(\frac{d}{2}\right)_s + (t_w)_C$$

$$= \frac{10.0}{2} + 0.303 = 5.303 \text{ in.}$$

$$\bar{x} = 0.534 \text{ in.}$$

$$\bar{y}_1 = f - \bar{x} = 5.303 - 0.534 = 4.769 \text{ in.}$$

Determine moment of inertia.

Part	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}\ (\text{in}^4)$
$C8\times13.7$	4.04	4.769	91.88	1.52
$S10\times25.4$	7.4	0	0	123
$C8\times13.7$	4.04	4.769	91.88	1.52
Σ			183.76	126.04

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 183.76 + 126.04 = 309.8 \text{ in}^4$$

$$Q = A\,\bar{y}_1 = (4.04)(4.769) = 19.267 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(19.267)}{309.8} = 1.8658 \text{ kip/in}$$

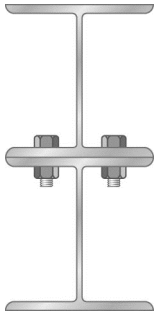
$$F_{\text{bolt}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(1.8658)(5) = 4.664 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{4.664}{0.4418} = 10.56 \text{ ksi}$$

$\tau_{\text{bolt}} = 10.56 \text{ ksi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.8

The composite beam shown is fabricated by connecting two $W6 \times 20$ rolled-steel members, using bolts of $\frac{5}{8}$-in. diameter spaced longitudinally every 6 in. Knowing that the average allowable shearing stress in the bolts is 10.5 ksi, determine the largest allowable vertical shear in the beam.

SOLUTION

$W6 \times 20$: $A = 5.87 \text{ in}^2$, $d = 6.20 \text{ in.}$, $I_x = 41.4 \text{ in}^4$

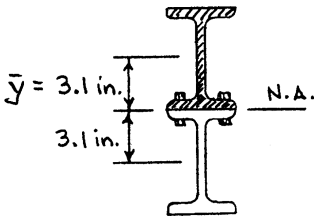
$$\bar{y} = \frac{1}{2}d = 3.1 \text{ in.}$$

Composite:

$$I = 2[41.4 + (5.87)(3.1)^2]$$

$$= 195.621 \text{ in}^4$$

$$Q = A\bar{y} = (5.87)(3.1) = 18.197 \text{ in}^3$$



Bolts:

$$d = \frac{5}{8} \text{ in.}, \quad \tau_{all} = 10.5 \text{ ksi}, \quad s = 6 \text{ in.}$$

$$A_{bolt} = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (10.5)(0.30680) = 3.2214 \text{ kips}$$

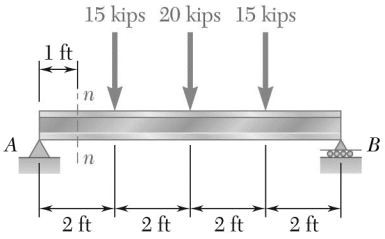
$$q = \frac{2F_{bolt}}{s} = \frac{(2)(3.2214)}{6} = 1.07380 \text{ kips/in}$$

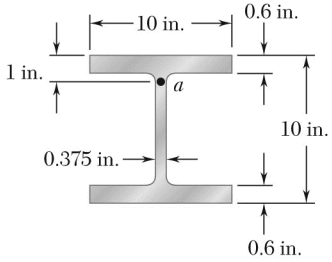
Shear:

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(195.621)(1.0780)}{18.197}$$

$$V = 11.54 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 6.9

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

By symmetry, $R_A = R_B$.

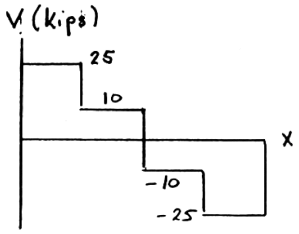
$$+\uparrow \sum F_y = 0:$$

$$R_A + R_B - 15 - 20 - 15 = 0$$

$$R_A = R_B = 25 \text{ kips}$$

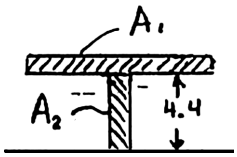
From shear diagram, $V = 30$ kips at *n-n*.

Determine moment of inertia.



Part	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}\ (\text{in}^4)$
Top Flng	6	4.7	132.54	0.18
Web	3.30	0	0	21.30
Bot. Flng	6	4.7	132.54	0.18
Σ			265.08	21.66

(*a*)



$$I = \sum Ad^2 + \sum \bar{I} = 286.74 \text{ in}^4$$

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$
①	6	4.7	28.2
②	1.65	2.2	3.63
Σ			31.83

$$Q = \sum A\bar{y}$$

$$= 31.83 \text{ in}^3$$

$$t = 0.375 \text{ in.}$$

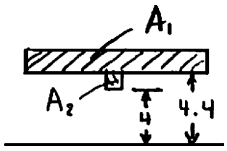
$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{(25)(31.83)}{(286.74)(0.375)}$$

$$\tau_{\max} = 7.40 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.9 *(Continued)*

(*b*)



Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$
①	6	4.7	28.2
②	0.15	4.2	0.63
Σ			28.83

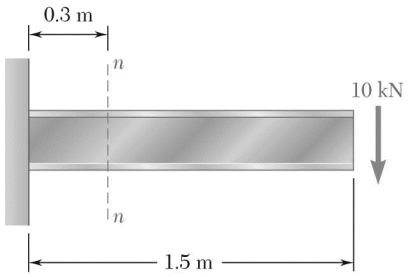
$$Q = \sum A\bar{y} = 28.83 \text{ in}^3$$

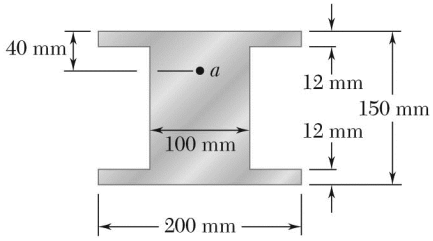
$$t = 0.375 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(23)(28.83)}{(286.74)(0.375)}$$

$$\tau = 6.70 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 6.10

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

At section *n-n*, $V = 10$ kN.

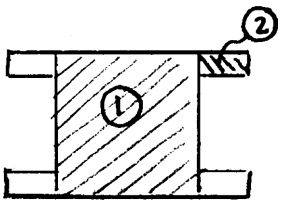
$$I = I_1 + 4I_2$$

$$= \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right)$$

$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$

$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$

$$= 39.58\times10^6\ \text{mm}^4 = 39.58\times10^{-6}\ \text{m}^4$$

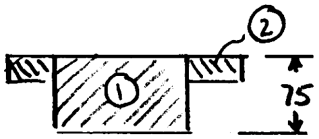


(*a*) $$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$

$$= (100)(75)(37.5) + (2)(50)(12)(69)$$

$$= 364.05\times10^3\ \text{mm}^3 = 364.05\times10^{-6}\ \text{m}^3$$

$$t = 100\ \text{mm} = 0.100\ \text{m}$$



$$\tau_{\max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3\ \text{Pa}$$

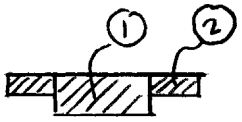
$\tau_{\max} = 920$ kPa ◀

(*b*) $$Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$$

$$= (100)(40)(55) + (2)(50)(12)(69)$$

$$= 302.8\times10^3\ \text{mm}^3 = 302.8\times10^{-6}\ \text{m}^3$$

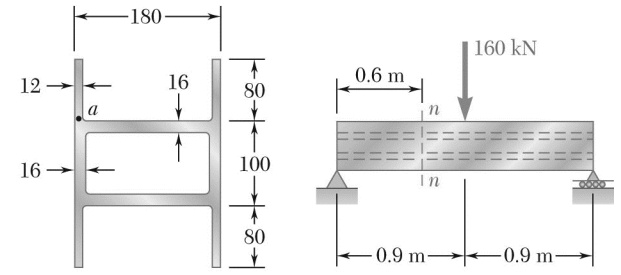
$$t = 100\ \text{mm} = 0.100\ \text{m}$$



$$\tau_a = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3\ \text{Pa}$$

$\tau_a = 765$ kPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

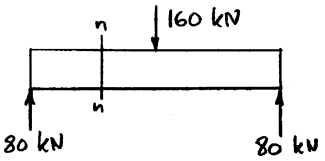


Dimensions in mm

PROBLEM 6.11

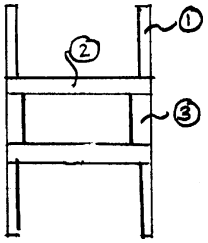
For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



At section *n-n*, $V = 80$ kN

Consider cross section as composed of rectangles of types ①, ②, and ③.



$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.288 \times 10^6 \text{ mm}^4$$

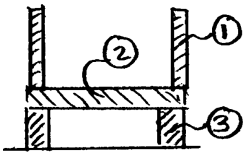
$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24 \times 10^3 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_3 = 44.274 \times 10^6 \text{ mm}^4$$

$$= 44.274 \times 10^{-6} \text{ m}^4$$

(*a*) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^3 \text{ mm}^4$$

$$Q_2 = (180)(16)(42) = 120.96 \times 10^3 \text{ mm}^4$$

$$Q_3 = (16)(34)(17) = 9.248 \times 10^3 \text{ mm}^4$$

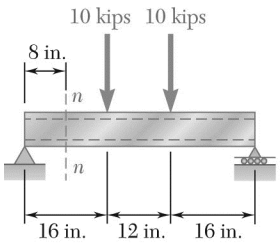
$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3 \text{ mm}^3 = 312.256 \times 10^{-6} \text{ m}^3$$

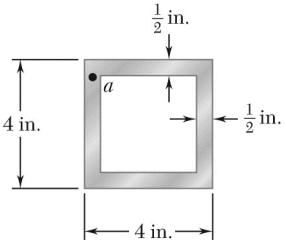
$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-6})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6 \text{ Pa} \qquad \tau = 17.63 \text{ MPa} \blacktriangleleft$$

(*b*) At point *a*, $Q = Q_1 = 86.4 \times 10^3 \text{ mm}^4 = 86.4 \times 10^{-6} \text{ m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-6})(12 \times 10^{-3})} = 13.01 \times 10^6 \text{ Pa} \qquad \tau = 13.01 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 6.12

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

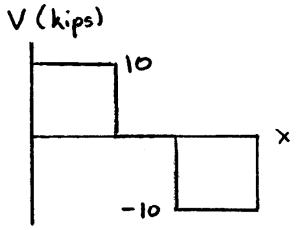
SOLUTION

By symmetry, $R_A = R_B$.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 10 - 10 = 0$$

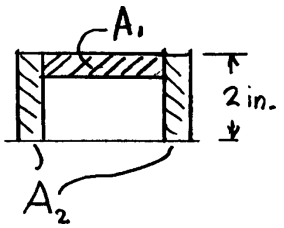
$$R_A = R_B = 10 \text{ kips}$$

From the shear diagram, $V = 10$ kips at *n-n*.



$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4$$

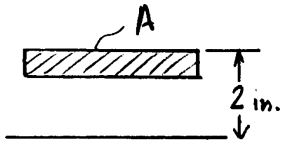


(*a*) $$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1) = 4.625 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)}$$

$\tau_{\max} = 3.17$ ksi ◀



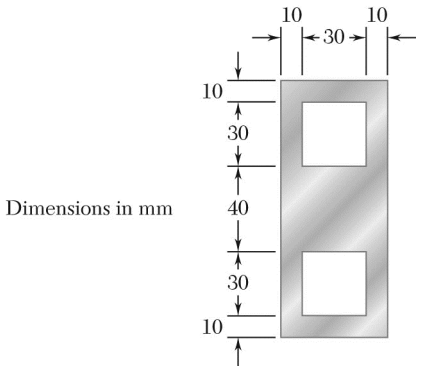
(*b*) $$Q = A\bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)}$$

$\tau_a = 2.40$ ksi ◀

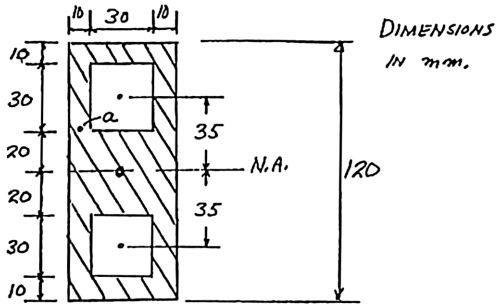
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.13

For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

SOLUTION



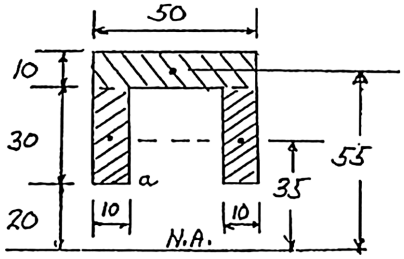
Calculate moment of inertia.

$$I = \frac{1}{12}(50\,\text{mm})(120\,\text{mm})^3 - 2\left[\frac{1}{12}(30\,\text{mm})^4 + (30\,\text{mm} \times 30\,\text{mm})(35\,\text{mm})^2\right]$$

$$I = 7.2 \times 10^6\,\text{mm}^4 - 2[1.170 \times 10^6\,\text{mm}^4] = 4.86 \times 10^6\,\text{mm}^4$$

$$= 4.86 \times 10^{-6}\,\text{m}^4$$

Assume that τ_m occurs at point *a*.



$$t = 2(10\,\text{mm}) = 0.02\,\text{m}$$

$$Q = (10\,\text{mm} \times 50\,\text{mm})(55\,\text{mm}) + 2[(10\,\text{mm} \times 30\,\text{mm})(35\,\text{mm})]$$

$$= 48.5 \times 10^3\,\text{mm}^3 = 48.5 \times 10^{-6}\,\text{m}^3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.13 *(Continued)*

For $\tau_{\text{all}} = 60$ MPa, $\tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

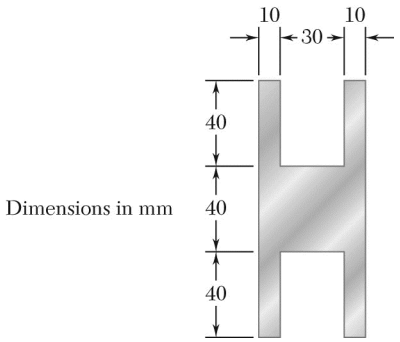
$$60 \times 10^6 \text{ Pa} = \frac{V(48.5 \times 10^{-6} \text{ m}^3)}{(4.86 \times 10^{-6} \text{ m}^4)(0.02 \text{ m})} \qquad V = 120.3 \text{ kN} \blacktriangleleft$$

Check τ at neutral axis: $t = 50 \text{ mm} = 0.05 \text{ m}$

$$Q = (50 \times 60)(30) - (30 \times 30)(35) = 58.5 \times 10^3 \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(120.3 \text{ kN})(58.5 \times 10^{-6} \text{ m}^3)}{(4.86 \times 10^{-6} \text{ m}^4)(0.05 \text{ m})} = 29.0 \text{ MPa} < 60 \text{ MPa} \quad \underline{\text{OK}}$$

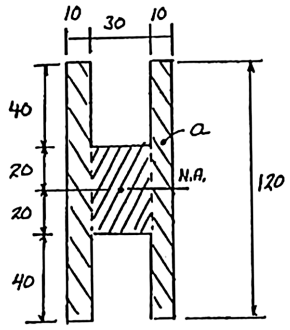
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.14

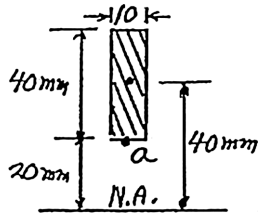
For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

SOLUTION



Calculate moment of inertia.

$$I = 2\left[\frac{1}{12}(10\text{ mm})(120\text{ mm})^3\right] + \frac{1}{12}(30\text{ mm})(40\text{ mm})^3$$
$$= 2[1.440\times10^6\text{ mm}^4] + 0.160\times10^6\text{ mm}^4$$
$$= 3.04\times10^6\text{ mm}^4$$
$$I = 3.04\times10^{-6}\text{ m}^4$$

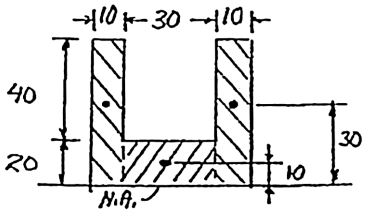


Assume that τ_m occurs at point *a*.

$$t = 10\text{ mm} = 0.01\text{ m}$$
$$Q = (10\text{ mm}\times40\text{ mm})(40\text{ mm})$$
$$= 16\times10^3\text{ mm}^3 \quad Q = 16\times10^{-6}\text{ m}^3$$

For $\tau_{\text{all}} = 60$ MPa, $\tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

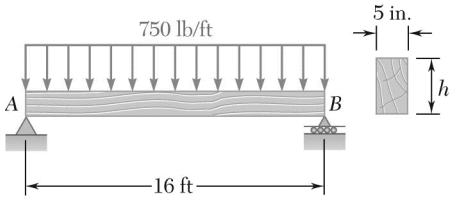
$$60\times10^6\text{ Pa} = \frac{V(16\times10^{-6}\text{ m}^3)}{(3.04\times10^{-6}\text{ m}^4)(0.01\text{m})} \qquad V = 114.0\text{ kN} \blacktriangleleft$$



Check τ at neutral axis:

$$t = 50\text{ mm} = 0.05\text{ m}$$
$$Q = 2[(10\times60)(30)] + (30\times20)(10) = 42\times10^3\text{ m}^3 = 42\times10^{-6}\text{ m}^3$$
$$\tau_{NA} = \frac{VQ}{It} = \frac{(114.0\text{ kN})(42\times10^{-6}\text{ m}^3)}{(3.04\times10^{-6}\text{ m}^4)(0.05\text{ m})} = 31.5\text{ MPa} < 60\text{ MPa} \quad \underline{\text{OK}}$$

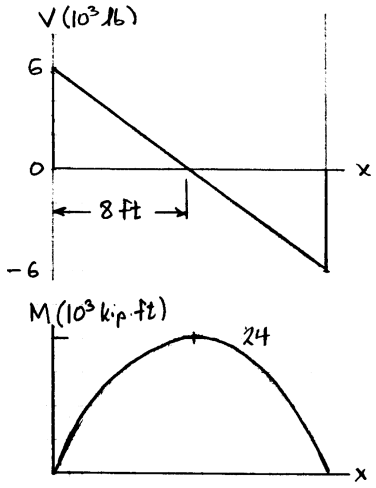
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.15

For the beam and loading shown, determine the minimum required depth h, knowing that for the grade of timber used, $\sigma_{all} = 1750$ psi and $\tau_{all} = 130$ psi.

SOLUTION



Total load: $(750 \text{ lb/ft})(16 \text{ ft}) = 12 \times 10^3 \text{ lb}$

Reaction at A: $\boldsymbol{R}_A = 6 \times 10^3 \text{ lb} \uparrow$

$$V_{max} = 6 \times 10^3 \text{ lb}$$

$$M_{max} = \frac{1}{2}(8 \text{ ft})(6 \times 10^3) = 24 \times 10^3 \text{ lb} \cdot \text{ft}$$

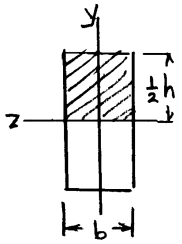
$$= 288 \times 10^3 \text{ lb} \cdot \text{in}$$

Bending: $S = \frac{1}{6}bh^2$ for rectangular section.

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{288 \times 10^3}{1750} = 164.57 \text{ in}^3$$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(164.57)}{5}} = 14.05 \text{ in.}$$

Shear: $I = \frac{1}{12}bh^3$ for rectangular section.



$$A = \frac{1}{2}bh$$

$$\bar{y} = \frac{1}{4}h$$

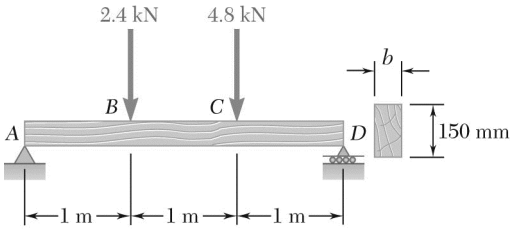
$$Q = A\bar{y} = (b)\left(\frac{1}{2}h\right)\left(\frac{1}{4}h\right) = \frac{1}{8}bh^2$$

$$\tau_{max} = \frac{VQ}{Ib} = \frac{3V_{max}}{2bh}$$

$$h = \frac{3V_{max}}{2b\tau_{max}} = \frac{(3)(6 \times 10^3)}{(2)(5)(130)} = 13.85 \text{ in.}$$

The larger value of h is the minimum required depth. $h = 14.05$ in. ◀

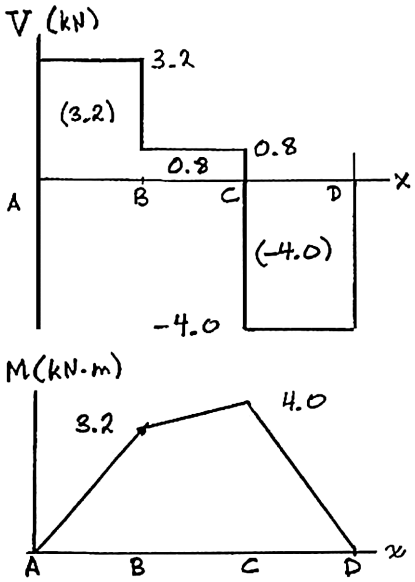
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

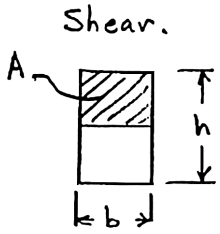


PROBLEM 6.16

For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.

SOLUTION





$$+\circlearrowleft M_D = 0: \quad -3R_A + (2)(2.4) + (1)(4.8) = 0$$

$$R_A = 3.2 \text{ kN}$$

Draw shear and bending moment diagrams.

$$|V|_{max} = 4.0 \text{ kN} \qquad |M|_{max} = 4.0 \text{ kN} \cdot \text{m}$$

Bending: $$S = \frac{|M|_{max}}{\sigma_{all}} = \frac{4.0 \times 10^3}{12 \times 10^6}$$

$$= 333.33 \times 10^{-6} \text{ m}^3 = 333.33 \times 10^3 \text{ mm}^3$$

For a rectangular cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{\frac{1}{2}h} = \frac{1}{6}bh^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(333.33 \times 10^3)}{150^2} = 88.9 \text{ mm}$$

$$A = \frac{1}{2}bh, \quad \bar{y} = \frac{1}{4}h$$

$$Q = A\bar{y} = \frac{1}{8}bh^2, \quad I = \frac{1}{12}bh^3$$

$$\tau = \frac{VQ}{It} = \frac{3V}{2bh}$$

$$bh = \frac{3}{2}\frac{V}{\tau} = \frac{3}{2}\frac{4.0 \times 10^3}{825 \times 10^3}$$

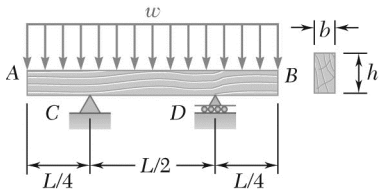
$$= 7.2727 \times 10^{-3} \text{ m}^2 = 7.2727 \times 10^3 \text{ mm}^2$$

$$b = \frac{bh}{h} = \frac{7.2727 \times 10^3}{150} = 48.5 \text{ mm}$$

The required value for b is the larger one. $b = 88.9$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.17

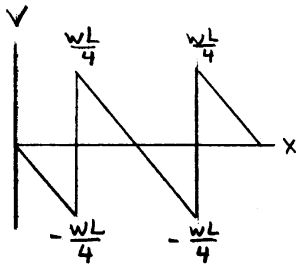


A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$

From shear diagram, $$|V|_m = \frac{wL}{4} \tag{1}$$

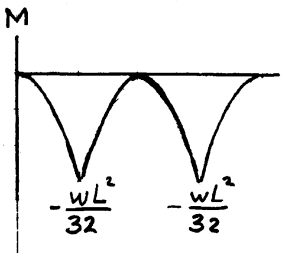


For rectangular section, $$A = bh \tag{2}$$

$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \tag{3}$$

From bending moment diagram, $$|M|_m = \frac{wL^2}{32} \tag{4}$$

For a rectangular cross section,



$$S = \frac{1}{6}bh^2 \tag{5}$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \tag{6}$$

(*a*) Dividing Eq. (3) by Eq. (6), $$\frac{\tau_m}{\sigma_m} = \frac{2h}{L}$$ ◀

(*b*) Solving for h:

$$h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08\times10^6)}{(2)(12\times10^6)} = 225\times10^{-3}\text{m} \qquad h = 225\text{ mm}$$ ◀

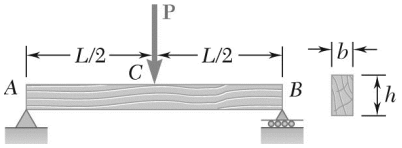
Solving Eq. (3) for b:

$$b = \frac{3wL}{8h\tau_m} = \frac{(3)(8\times10^3)(5)}{(8)(225\times10^{-3})(1.08\times10^6)}$$

$$= 61.7\times10^{-3}\text{m} \qquad b = 61.7\text{ mm}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.18



A timber beam AB of length L and rectangular cross section carries a single concentrated load **P** at its midpoint C. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.

SOLUTION

Reactions: $R_A = R_B = P/2 \uparrow$

(1) $V_{\max} = R_A = \dfrac{P}{2}$

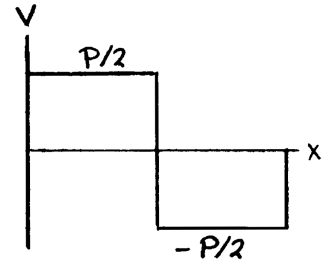
(2) $A = bh$ for rectangular section.

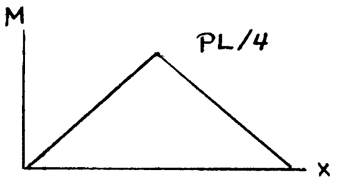
(3) $\tau_m = \dfrac{3}{2}\dfrac{V_{\max}}{A} = \dfrac{3P}{4bh}$ for rectangular section.

(4) $M_{\max} = \dfrac{PL}{4}$

(5) $S = \dfrac{1}{6}bh^2$ for rectangular section.

(6) $\sigma_m = \dfrac{M_{\max}}{S} = \dfrac{3PL}{2bh^2}$





(*a*) $$\frac{\tau_m}{\sigma_m} = \frac{h}{2L}$$ ◀

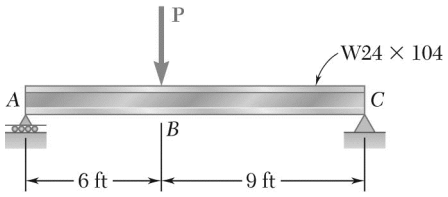
(*b*) Solving for h,

$$h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960\times10^3)}{12\times10^6} = 320\times10^{-3}\text{ m} \qquad h = 320\text{ mm}$$ ◀

Solving Eq. (3) for b,

$$b = \frac{3P}{4h\tau_m} = \frac{(3)(40\times10^3)}{(4)(320\times10^{-3})(960\times10^3)} = 97.7\times10^{-3}\text{ m} \qquad b = 97.7\text{ mm}$$ ◀

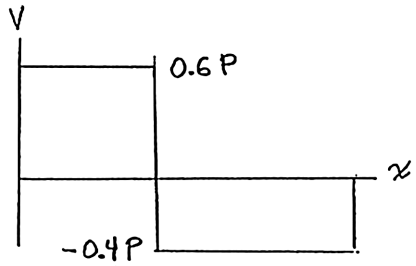
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.19

For the wide-flange beam with the loading shown, determine the largest load **P** that can be applied, knowing that the maximum normal stress is 24 ksi and the largest shearing stress using the approximation $\tau_m = V/A_{\text{web}}$ is 14.5 ksi.

SOLUTION



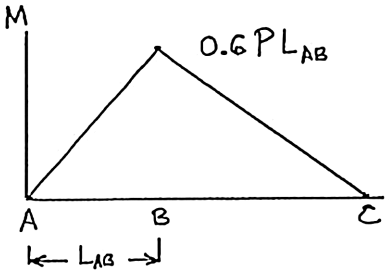
$$+\circlearrowleft \sum M_C = 0: -15R_A + qP = 0$$

$$R_A = 0.6P$$

Draw shear and bending moment diagrams.

$$|V|_{\max} = 0.6P \qquad |M|_{\max} = 0.6PL_{AB}$$

$$L_{AB} = 6\,\text{ft} = 72\,\text{in.}$$



Bending. For W 24 × 104, $S = 258\,\text{in}^3$

$$S = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{0.6PL_{AB}}{\sigma_{\text{all}}}$$

$$P = \frac{\sigma_{\text{all}} S}{0.6L_{AB}} = \frac{(24)(258)}{(0.6)(72)} = 143.3\,\text{kips}$$

Shear.

$$A_{\text{web}} = dt_w$$

$$= (24.1)(0.500)$$

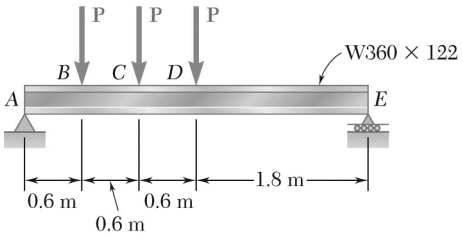
$$= 12.05\,\text{in}^2$$

$$\tau = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{0.6P}{A_{\text{web}}}$$

$$P = \frac{\tau A_{\text{web}}}{0.6} = \frac{(14.5)(12.05)}{0.6} = 291\,\text{kips}$$

The smaller value of P is the allowable value. $P = 143.3$ kips ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.20

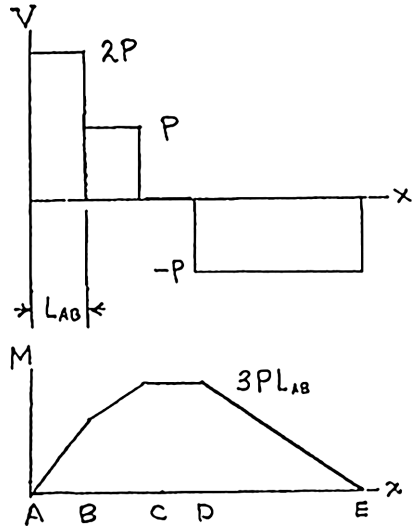
For the wide-flange beam with the loading shown, determine the largest load P that can be applied, knowing that the maximum normal stress is 160 MPa and the largest shearing stress using the approximation $\tau_m = V/A_{\text{web}}$ is 100 MPa.

SOLUTION

$$+\circlearrowleft \sum M_E = 0: -3.6R_A + 3.0P + 2.4P + 1.8P = 0$$

$$R_A = 2P \uparrow$$

Draw shear and bending moment diagrams.



$$M_B = 2PL_{AB}, \quad M_C = M_D = 3PL_{AB}$$

$$|V|_{\max} = 2P \qquad |M|_{\max} = 3PL_{AB}$$

Bending. For W 360 × 122, $S = 2020 \times 10^3 \text{ mm}^3$

$$= 2020 \times 10^{-6} \text{ m}^3$$

$$\frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{3PL_{AB}}{\sigma_{\text{all}}} = S$$

$$P = \frac{\sigma_{\text{all}} S}{3L_{AB}} = \frac{(160 \times 10^6)(2020 \times 10^3)}{(3)(0.6)} = 179.6 \times 10^3 \text{ N}$$

Shear. $A_{\text{web}} = dt_w = (363)(13.0)$

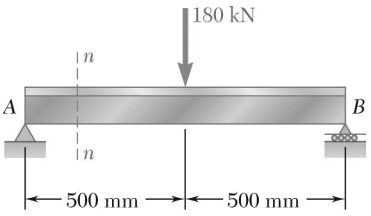
$$= 4.719 \times 10^3 \text{ mm}^2 = 4.719 \times 10^{-3} \text{ m}^2$$

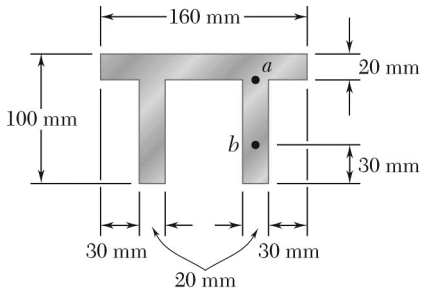
$$\tau = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{2P}{A_{\text{web}}}$$

$$P = \frac{\tau A_{\text{web}}}{2} = \frac{(100 \times 10^6)(4.719 \times 10^{-3})}{2} = 236 \times 10^3 \text{ N}$$

The smaller value of P is the allowable one. $P = 179.6 \text{ kN}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





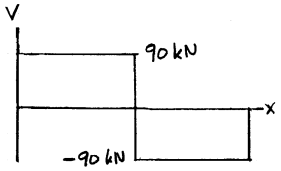
PROBLEM 6.21

For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Draw the shear diagram. $|V|_{\max} = 90$ kN

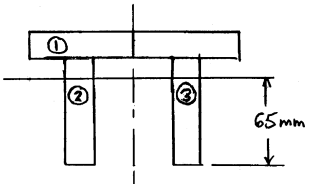
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\ \text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\text{mm}^4)$	$\bar{I}(10^6\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	–25	1.000	0.8533
③	1600	40	64	–25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times10^3}{6400} = 65\ \text{mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133)\times10^6\text{mm}^4$$

$$= 5.8133\times10^6\text{mm}^4 = 5.8133\times10^{-6}\text{m}^4$$



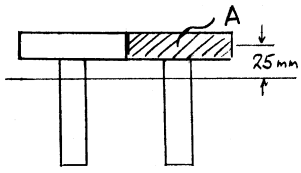
(*a*) $A = (80)(20) = 1600\ \text{mm}^2$

$\bar{y} = 25$ mm

$$Q_a = A\bar{y} = 40\times10^3\text{mm}^3 = 40\times10^{-6}\text{m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90\times10^3)(40\times10^{-6})}{(5.8133\times10^{-6})(20\times10^{-3})} = 31.0\times10^6\text{Pa}$$

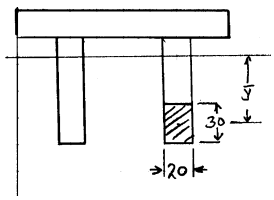
$\tau_a = 31.0$ MPa ◀



(*b*) $A = (30)(20) = 600\ \text{mm}^2$ $\bar{y} = 65 - 15 = 50$ mm

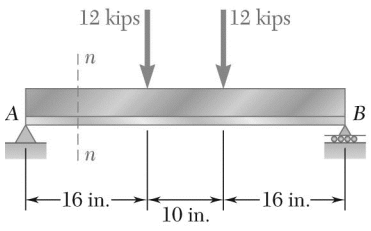
$$Q_b = A\bar{y} = 30\times10^3\text{mm}^3 = 30\times10^{-6}\text{m}^3$$

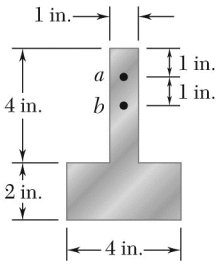
$$\tau_b = \frac{VQ_b}{It} = \frac{(90\times10^3)(30\times10^{-6})}{(5.8133\times10^{-6})(20\times10^{-3})} = 23.2\times10^6\text{Pa}$$



$\tau_b = 23.2$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 6.22

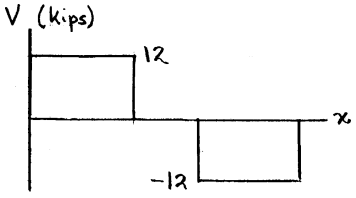
For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$R_A = R_B = 12 \text{ kips}$$

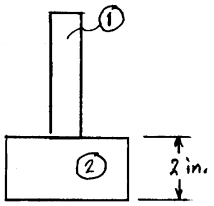
Draw shear diagram.

$$V = 12 \text{ kips}$$



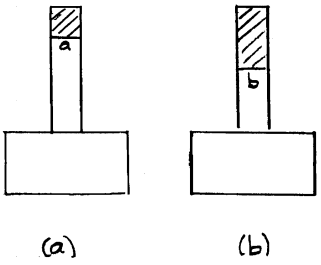
Determine section properties.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4	4	16	2	16	5.333
②	8	1	8	−1	8	2.667
Σ	12		24		24	8.000



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 32 \text{ in}^4$$



(*a*) $A = 1 \text{ in}^2 \quad \bar{y} = 3.5 \text{ in.} \quad Q_a = A\bar{y} = 3.5 \text{ in}^3$

$t = 1 \text{ in.}$

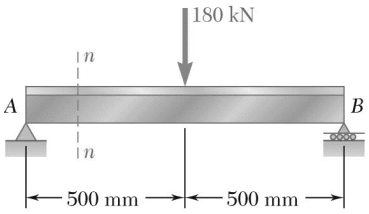
$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} \qquad \tau_a = 1.3125 \text{ ksi} \blacktriangleleft$$

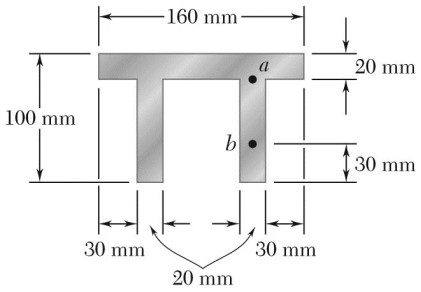
(*b*) $A = 2 \text{ in}^2 \quad \bar{y} = 3 \text{ in.} \quad Q_b = A\bar{y} = 6 \text{ in}^3$

$t = 1 \text{ in.}$

$$\tau_b = \frac{VQ_b}{It} = \frac{(12)(6)}{(32)(1)} \qquad \tau_b = 2.25 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

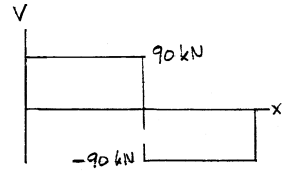




PROBLEM 6.23

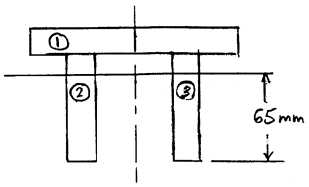
For the beam and loading shown, determine the largest shearing stress in section *n-n*.

SOLUTION



Draw the shear diagram. $|V|_{\max} = 90$ kN

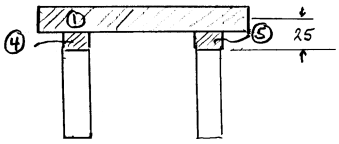
Part	$A(\text{mm}^2)$	x'	$A\bar{y}(10^3\text{mm}^3)$	$d(\text{mm})$	$Ad^2(10^6\text{mm}^4)$	$\bar{I}(10^6\text{mm}^4)$
①	3200	90	288	25	2.000	0.1067
②	1600	40	64	−25	1.000	0.8533
③	1600	40	64	−25	1.000	0.8533
Σ	6400		416		4.000	1.8133



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times10^3}{6400} = 65 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4.000 + 1.8133)\times10^6\text{mm}^4$$

$$= 5.8133\times10^6\text{mm}^4 = 5.8133\times10^{-6}\text{m}^4$$



Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(10^3\text{mm}^3)$
①	3200	25	80
④	300	7.5	2.25
⑤	300	7.5	2.25
Σ			84.5

$$Q = \Sigma A\bar{y} = 84.5\times10^3\text{mm}^3 = 84.5\times10^{-6}\text{m}^3$$

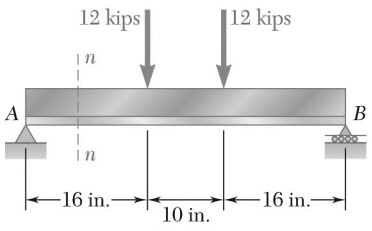
$$t = (2)(20) = 40 \text{ mm} = 40\times10^{-3}\text{m}$$

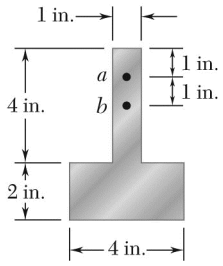
$$\tau_{\max} = \frac{VQ}{It} = \frac{(90\times10^3)(84.5\times10^{-6})}{(5.8133\times10^{-6})(40\times10^{-3})}$$

$$= 32.7\times10^6\text{Pa}$$

$$\tau_m = 32.7 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

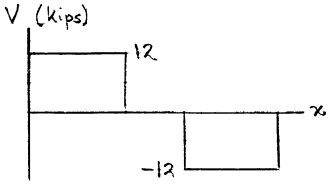




PROBLEM 6.24

For the beam and loading shown, determine the largest shearing stress in section *n-n*.

SOLUTION

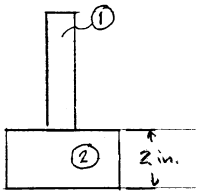


$$R_A = R_B = 12 \text{ kips}$$

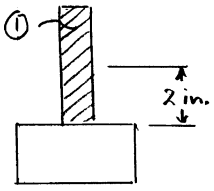
Draw shear diagram.

$$V = 12 \text{ kips}$$

Determine section properties.



Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4	4	16	2	16	5.333
②	8	1	8	−1	8	2.667
Σ	12		24		24	8.000



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 32 \text{ in}^4$$

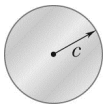
$$Q = A_1\bar{y}_1 = (4)(2) = 8 \text{ in}^3$$

$$t = 1 \text{ in.}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(12)(8)}{(32)(1)} \qquad \tau_{\max} = 3.00 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.25



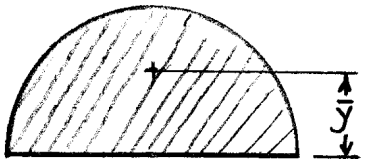
A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$I = \frac{\pi}{4}c^4 \quad \text{and} \quad A = \pi c^2$$



For semicircle, $A_s = \frac{\pi}{2}c^2 \qquad \bar{y} = \frac{4c}{3\pi}$

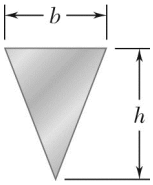
$$Q = A_s\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$$

(*a*) $\tau_{\max}$ occurs at center where $t = 2c.$ ◀

(*b*) $\tau_{\max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3}\frac{V}{A}$ $\qquad k = \frac{4}{3} = 1.333$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.26



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$A = \frac{1}{2}bh \qquad I = \frac{1}{36}bh^3$$

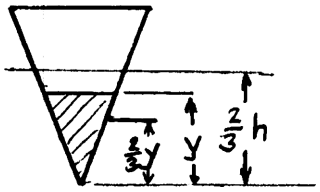
For a cut at location *y*,

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h - y)$$

$$t(y) = \frac{by}{h}$$



$$\tau(y) = \frac{VQ}{It} = \frac{V\frac{by^2}{3}(h - y)}{(\frac{1}{36}bh^3)\frac{by}{h}} = \frac{12\,Vy(h - y)}{bh^3} = \frac{12V}{bh^3}(hy - y^2)$$

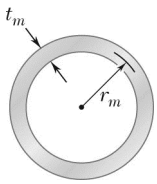
(*a*) To find location of maximum of τ, set $\dfrac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h - 2y_m) = 0 \qquad y_m = \frac{1}{2}h, \text{ i.e., at mid-height} \blacktriangleleft$$

(*b*) $$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\frac{1}{2}h^2 - \left(\frac{1}{2}h\right)^2\right] = \frac{3V}{bh} = \frac{3}{2}\frac{V}{A} \qquad k = \frac{3}{2} = 1.500 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.27



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

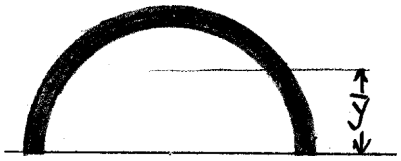
$$\tau_{\max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

SOLUTION

For a thin-walled circular section, $A = 2\pi r_m t_m$

$$J = Ar_m^2 = 2\pi r_m^3 t_m, \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$$



For a semicircular arc, $\bar{y} = \dfrac{2r_m}{\pi}$

$$A_s = \pi r_m t_m$$

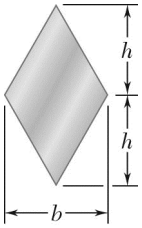
$$Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$$

(*a*) $t = 2t_m$ at neutral axis where maximum occurs. ◀

(*b*) $$\tau_{\max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A}$$ $k = 2.00$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.28



A beam having the cross section shown is subjected to a vertical shear **V.** Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$$A = 2\left(\frac{1}{2}bh\right) = bh \qquad I = 2\left(\frac{1}{12}bh^3\right) = \frac{1}{6}bh^3$$

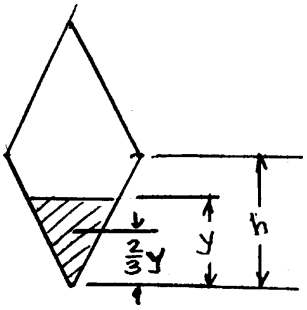
For a cut at location *y*, where $y \le h$,

$$A(y) = \frac{1}{2}\left(\frac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = h - \frac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{2} - \frac{by^3}{3h}$$

$$t(y) = \frac{by}{h}$$



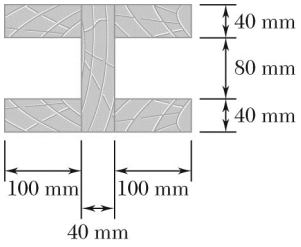
$$\tau(y) = \frac{VQ}{It} = V\frac{6}{bh^3}\cdot\frac{h}{by}\cdot\frac{by^2}{2} - \frac{by^3}{3h} = \frac{V}{bh}\left[3\left(\frac{y}{h}\right) - 2\left(\frac{y}{h}\right)^2\right]$$

(*a*) To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$.

$$\frac{d\tau}{dy} = \frac{V}{bh^2}\left[3 - 4\frac{y_m}{h}\right] = 0 \qquad \frac{y_m}{h} = \frac{3}{4}, \text{ i.e., } \pm\frac{1}{4}h \text{ from neutral axis.} \blacktriangleleft$$

(*b*) $$\tau(y_m) = \frac{V}{bh}\left[3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2\right] = \frac{9}{8}\frac{V}{bh} = 1.125\frac{V}{A} \qquad k = 1.125 \blacktriangleleft$$

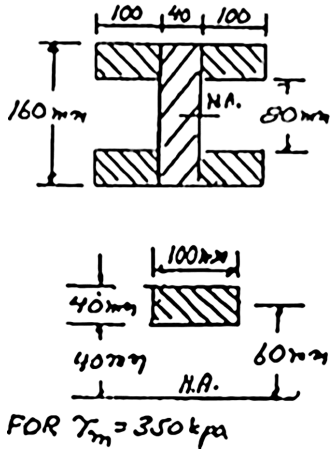
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.29

The built-up beam shown is made by gluing together five planks. Knowing that in the glued joints the average allowable shearing stress is 350 kPa, determine the largest permissible vertical shear in the beam.

SOLUTION



$$I = \frac{1}{12}(240\text{ mm})(160\text{ mm})^3 - \frac{1}{12}(200\text{ mm})(80\text{ mm})^3$$

$$= 73.4 \times 10^6\text{ mm}^4$$

$$I = 73.4 \times 10^{-6}\text{ m}^4$$

$$t = 40\text{ mm} = 0.04\text{ m}$$

$$Q = (100\text{ mm} \times 40\text{ mm})(60\text{ mm})$$

$$= 240 \times 10^3\text{ mm}^3$$

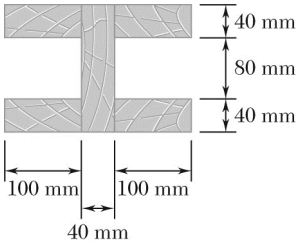
$$Q = 240 \times 10^{-6}\text{ m}^3$$

For $\tau_m = 350\text{ kPa}$,

$$\tau_m = \frac{VQ}{It}: \quad 350 \times 10^3\text{ Pa} = \frac{V(240 \times 10^{-6}\text{ m}^3)}{(73.4 \times 10^{-6}\text{ m}^4)0.04\text{ m}}$$

$$V = 4.28\text{ kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

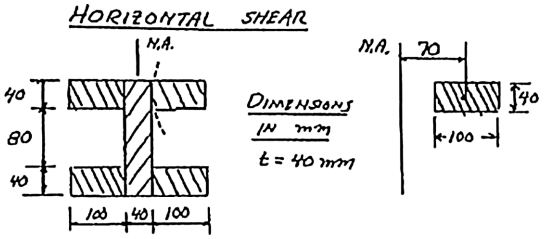


PROBLEM 6.30

For the beam of Prob. 6.29, determine the largest permissible horizontal shear.

PROBLEM 6.29 The built-up beam shown is made by gluing together five planks. Knowing that in the glued joints the average allowable shearing stress is 350 kPa, determine the largest permissible vertical shear in the beam.

SOLUTION



$$I = 2\frac{1}{12}(40)(240)^3 + \frac{1}{12}(80)(40)^3 \qquad Q = (40 \times 100)70$$
$$= 280 \times 10^3 \text{ mm}^3$$

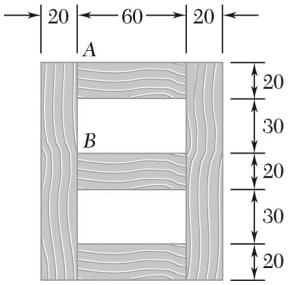
$$I = 92.6 \times 10^{+6} \text{ mm}^4 = 92.6 \times 10^{-6} \text{ m}^4 \qquad Q = (280 \times 10^{-6}) \text{ m}^3$$

For $\tau = 350$ kPa,

$$\tau = \frac{VQ}{It}; \qquad 350 \times 10^3 \text{ Pa} = \frac{V(280 \times 10^{-6} \text{ m}^3)}{(92.6 \times 10^{-6} \text{ m}^4)(0.04 \text{ m})}$$

$$V = 4630 \text{ N} \qquad V = 4.63 \text{ kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Dimensions in mm

PROBLEM 6.31

Several wooden planks are glued together to form the box beam shown. Knowing that the beam is subjected to a vertical shear of 3 kN, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

SOLUTION

$$I_A = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(60)(20)^3 + (60)(20)(50)^2$$
$$= 3.04 \times 10^6 \text{mm}^4$$

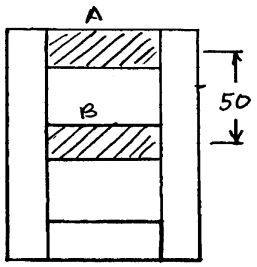
$$I_B = \frac{1}{12}bh^3 = \frac{1}{12}(60)(20)^3 = 0.04 \times 10^6 \text{mm}^4$$

$$I_C = \frac{1}{12}bh^3 = \frac{1}{12}(20)(120)^3 = 2.88 \times 10^6 \text{mm}^4$$

$$I = 2I_A + I_B + 2I_C = 11.88 \times 10^6 \text{mm}^4 = 11.88 \times 10^{-6} \text{m}^4$$

$$Q_A = A\bar{y} = (60)(20)(50) = 60 \times 10^3 \text{mm}^3 = 60 \times 10^{-6} \text{m}^3$$

$$t = 20 \text{ mm} + 20 \text{ mm} = 40 \text{ mm} = 40 \times 10^{-3} \text{m}$$



(*a*) $$\tau_A = \frac{VQ_A}{It} = \frac{(3 \times 10^3)(60 \times 10^{-6})}{(11.88 \times 10^{-6})(40 \times 10^{-3})} = 379 \times 10^3 \text{Pa}$$

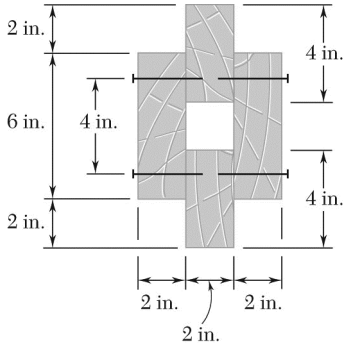
$Q_B = 0$

$\tau_A = 379$ kPa ◀

(*b*) $$\tau_B = \frac{VQ_B}{It} = 0$$

$\tau_B = 0$ ◀

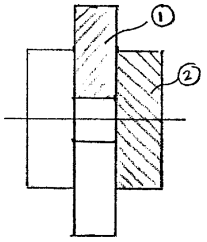
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.32

The built-up timber beam is subjected to a 1500-lb vertical shear. Knowing that the longitudinal spacing of the nails is $s = 2.5$ in. and that each nail is 3.5 in. long, determine the shearing force in each nail.

SOLUTION



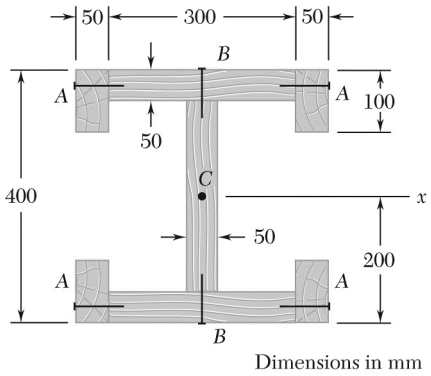
$$I_1 = \frac{1}{12}(2)(4)^3 + (2)(4)(3)^2$$
$$= 82.6667 \text{ in}^4$$
$$I_2 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$
$$I = 2I_1 + 2I_2$$
$$= 237.333 \text{ in}^4$$

$$Q = A_1\bar{y}_1 = (2)(4)(3) = 24 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1500)(24)}{237.333} = 151.685 \text{ lb/in}$$

$$2F_{\text{nail}} = qs \qquad F_{\text{nail}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(151.685)(2.5) \qquad F_{\text{nail}} = 189.6 \text{ lb} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.33

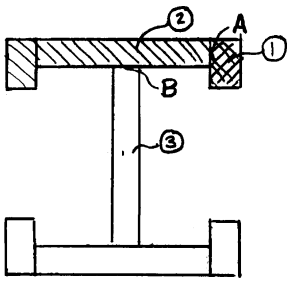
The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (a) at A, (b) at B. (Given: $I_x = 1.504 \times 10^9$ mm^4.)

SOLUTION

$$I_x = 1.504 \times 10^9 \text{mm}^4 = 1504 \times 10^{-6} \text{m}^4$$

$$s_A = 60 \text{ mm} = 0.060 \text{ m}$$

$$s_B = 25 \text{ mm} = 0.025 \text{ m}$$



(a) $$Q_A = Q_1 = A_1\bar{y}_1 = (50)(100)(150) = 750 \times 10^3 \text{mm}^3$$
$$= 750 \times 10^{-6} \text{m}^3$$

$$F_A = q_A s_A$$
$$= \frac{VQ_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}}$$

$$F_A = 239 \text{ N} \blacktriangleleft$$

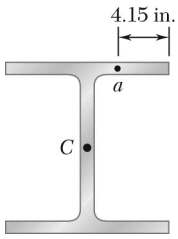
(b) $$Q_2 = A_2\bar{y}_2 = (300)(50)(175) = 2625 \times 10^3 \text{mm}^3$$

$$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3 \text{mm}^3$$
$$= 4125 \times 10^{-6} \text{m}^3$$

$$F_B = q_B s_B = \frac{VQ_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}}$$

$$F_B = 549 \text{ N} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



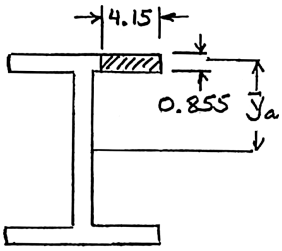
PROBLEM 6.34

Knowing that a vertical shear **V** of 50 kips is exerted on $W14 \times 82$ rolled-steel beam, determine the shearing stress (*a*) at point *a*, (*b*) at the centroid *C*.

SOLUTION

For $W14 \times 82$, $d = 14.3$ in., $b_f = 10.1$ in., $t_f = 0.855$ in., $t_w = 0.510$ in., $I = 881$ in^4

(*a*)



$$A_a = (4.15)(0.855) = 3.5482 \text{ in}^2$$

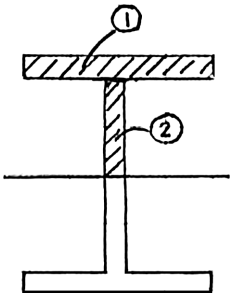
$$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{14.3}{2} - \frac{0.855}{2} = 6.7225 \text{ in.}$$

$$Q_a = A_a \bar{y}_a = 23.853 \text{ in}^3$$

$$t_a = t_f = 0.855 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(50)(23.853)}{(881)(0.855)} \qquad \tau_a = 1.583 \text{ ksi} \blacktriangleleft$$

(*b*)



$$A_1 = b_f t_f \ (10.1)(0.855) = 8.6355 \text{ in}^2$$

$$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = 6.7225 \text{ in.}$$

$$A_2 = t_w\left(\frac{d}{2} - t_f\right) = (0.510)(6.295) = 3.2105 \text{ in}^2$$

$$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_f\right) = \frac{1}{2}(6.295) = 3.1475 \text{ in.}$$

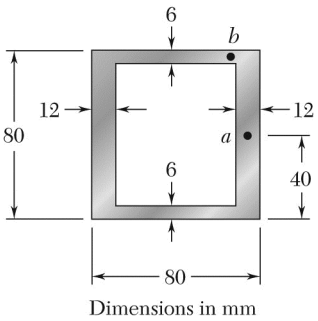
$$Q_C = A_1\bar{y}_1 + A_2\bar{y}_2 \ (8.6355)(6.7225) + (3.2105)(3.1475)$$

$$= 68.157 \text{ in}^3$$

$$t_C = t_w = 0.510 \text{ in.}$$

$$\tau_C = \frac{VQ_C}{It_C} = \frac{(50)(68.157)}{(881)(0.510)} \qquad \tau_C = 7.59 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

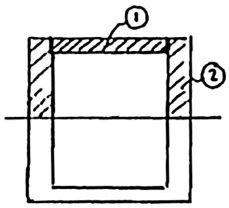


PROBLEM 6.35

An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460 \times 10^6 \text{ mm}^4$$
$$= 1.946 \times 10^{-6} \text{ m}^4$$



(*a*) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3$$
$$= 31.632 \times 10^{-6} \text{ m}^3$$
$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(31.632 \times 10^{-6})}{(1.946 \times 10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa}$$

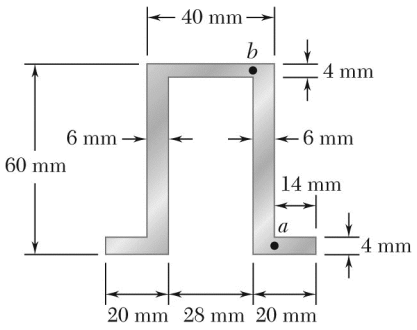
$\tau_a = 101.6$ MPa ◀

(*b*) $Q_b = A_1\bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3$

$$= 12.432 \times 10^{-6} \text{ m}^3$$
$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(12.432 \times 10^{-6})}{(1.946 \times 10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa}$$

$\tau_b = 79.9$ MPa ◀

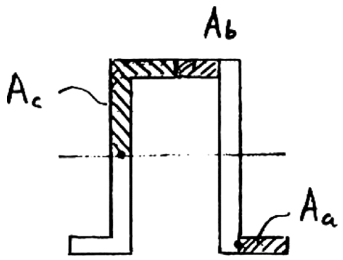
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.36

Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*.

SOLUTION



Neutral axis lies 30 mm above bottom.

$$\tau_c = \frac{VQ_c}{It} \qquad \tau_a = \frac{VQ_a}{It_a} \qquad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a} \qquad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$

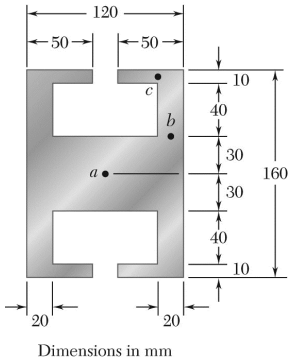
$$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$$
$$t_c = 6 \text{ mm}$$
$$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$$
$$t_a = 4 \text{ mm}$$
$$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$$
$$t_b = 4 \text{ mm}$$

$$\tau_c = 75 \text{ MPa}$$

(*a*) $$\tau_a = \frac{Q_a}{Q_c} \cdot \frac{t_c}{t_a} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75$$ $\tau_a = 41.4 \text{ MPa}$ ◀

(*b*) $$\tau_b = \frac{Q_b}{Q_c} \cdot \frac{t_c}{t_b} \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75$$ $\tau_b = 41.4 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.37

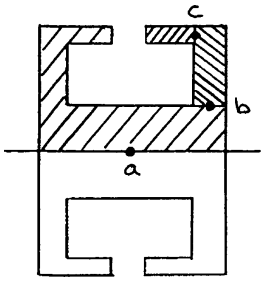
Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.

SOLUTION

$$\tau = \frac{VQ}{It} \qquad \tau \text{ is proportional to } Q/t.$$

Point *c*:

$$Q_c = (30)(10)(75)$$
$$= 22.5 \times 10^3 \text{ mm}^3$$
$$t_c = 10 \text{ mm}$$
$$Q_c/t_c = 2250 \text{ mm}^2$$



Point *b*:

$$Q_b = Q_c + (20)(50)(55)$$
$$= 77.5 \times 10^3 \text{ mm}^3$$
$$t_b = 20 \text{ mm}$$
$$Q_b/t_b = 3875 \text{ mm}^2$$

Point *a*:

$$Q_a = 2Q_b + (120)(30)(15)$$
$$= 209 \times 10^3 \text{ mm}^3$$
$$t_a = 120 \text{ mm}$$
$$Q_a/t_a = 1741.67 \text{ mm}^2$$

$(Q/t)_m$ occurs at *b*.

$$\tau_m = \tau_b = 75 \text{ MPa}$$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

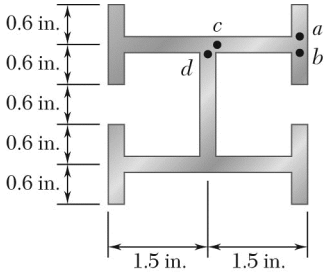
$$\frac{\tau_a}{1741.67 \text{ mm}^2} = \frac{75 \text{ MPa}}{3875 \text{ mm}^2} = \frac{\tau_a}{2250 \text{ mm}^2}$$

$\tau_a = 33.7$ MPa ◀

$\tau_b = 75.0$ MPa ◀

$\tau_c = 43.5$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.38

An extruded beam has the cross section shown and a uniform wall thickness of 0.20 in. Knowing that a given vertical shear **V** causes a maximum shearing stress $\tau = 9$ ksi, determine the shearing stress at the four points indicated.

SOLUTION

$$Q_a = (0.2)(0.5)(0.5 - 0.25) = 0.125 \text{ in}^3$$

$$Q_b = (0.2)(0.5)(0.3 + 0.25) = 0.055 \text{ in}^3$$

$$Q_c = Q_a + Q_b + (1.4)(0.2)(0.9) = 0.432 \text{ in}^3$$

$$Q_d = 2Q_a + 2Q_b + (3.0)(0.2)(0.9) = 0.900 \text{ in.}^3$$

$$Q_m = Q_d + (0.2)(0.8)(0.4) = 0.964 \text{ in}^3$$

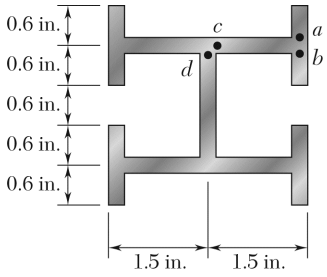
$$\tau = \frac{VQ}{It}$$

Since V, I, and t are constant, τ is proportional to Q.

$$\frac{\tau_a}{0.125} = \frac{\tau_b}{0.055} = \frac{\tau_c}{0.432} = \frac{\tau_d}{0.900} = \frac{\tau_m}{0.964} = \frac{9}{0.964}$$

$$\tau_a = 1.167 \text{ ksi}; \quad \tau_b = 0.513 \text{ ksi}; \quad \tau_c = 4.03 \text{ ksi}; \quad \tau_d = 8.40 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.39

Solve Prob. 6.38, assuming that the beam is subjected to a horizontal shear **V.**

PROBLEM 6.38 An extruded beam has the cross section shown and a uniform wall thickness of 0.20 in. Knowing that a given vertical shear **V** causes a maximum shearing stress $\tau = 9$ ksi, determine the shearing stress at the four points indicated.

SOLUTION

$$Q_a = (0.5)(0.2)(1.4) = 0.140 \text{ in}^3$$

$$Q_b = (0.5)(0.2)(1.4) = 0.140 \text{ in}^3$$

$$Q_c = Q_a + Q_b + (0.2)(1.4)(0.8) = 0.504 \text{ in}^3$$

$$Q_d = 0$$

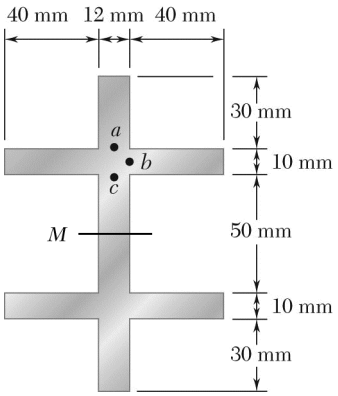
$$Q_m = Q_c = 0.504 \text{ in}^3$$

$\tau = \dfrac{VQ}{It}$ Since V, I, and t are constant, τ is proportional to Q.

$$\frac{\tau_a}{0.140} = \frac{\tau_b}{0.140} = \frac{\tau_c}{0.504} = \frac{\tau_d}{0} = \frac{\tau_m}{Q_m} = \frac{9}{0.504}$$

$\tau_a = 2.50$ ksi; $\tau_b = 2.50$ ksi, $\tau_c = 9.00$ ksi, $\tau_d = 0$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.40

Knowing that a given vertical shear **V** causes a maximum shearing stress of 50 MPa in a thin-walled member having the cross section shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*, (*c*) at point *c*.

SOLUTION

$$Q_a = (12)(30)(25 + 10 + 15) = 18 \times 10^3 \text{ mm}^3$$

$$Q_b = (40)(10)(25 + 5) = 12 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_a + 2Q_b + (12)(10)(25 + 5) = 45.6 \times 10^3 \text{ mm}^3$$

$$Q_m = Q_c + (12)(25)\left(\frac{25}{2}\right) = 49.35 \times 10^3 \text{ mm}^3$$

$$t_a = t_c = t_m = 12 \text{ mm}$$

$$t_b = 10 \text{ mm}$$

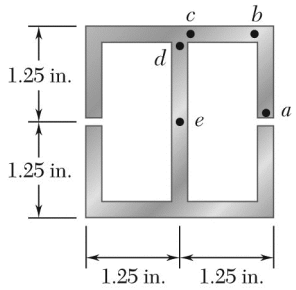
$$\tau_m = 50 \text{ MPa}$$

(*a*) $$\frac{\tau_a}{\tau_m} = \frac{Q_a}{Q_m} \cdot \frac{t_m}{t_a} = \frac{18}{49.35} \cdot \frac{12}{12} = 0.3647$$ $\tau_a = 18.23$ MPa ◀

(*b*) $$\frac{\tau_b}{\tau_m} = \frac{Q_b}{Q_m} \cdot \frac{t_m}{t_b} = \frac{12}{49.35} \cdot \frac{12}{10} = 0.2918$$ $\tau_b = 14.59$ MPa ◀

(*c*) $$\frac{\tau_c}{\tau_m} = \frac{Q_c}{Q_m} \cdot \frac{t_m}{t_c} = \frac{45.6}{49.35} \cdot \frac{12}{12} = 0.9240$$ $\tau_c = 46.2$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.41

The extruded aluminum beam has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the corresponding shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

$t = 0.125$ in. at all sections.

$V = 2$ kips

$Q_a = 0$ $\qquad \tau_a = \dfrac{VQ_a}{It}$ $\qquad \tau_a = 0$ ◀

$$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.09766 \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} \qquad \tau_b = 1.26 \text{ ksi} \blacktriangleleft$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in.}^2$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} \qquad \tau_c = 3.30 \text{ ksi} \blacktriangleleft$$

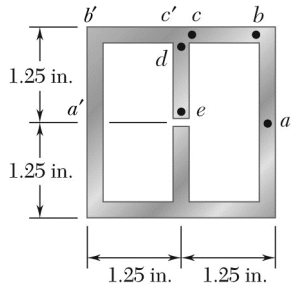
$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} \qquad \tau_d = 6.84 \text{ ksi} \blacktriangleleft$$

$$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$$

$$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} \qquad \tau_e = 7.86 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.42

The extruded aluminum beam has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the corresponding shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{2}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

Add symmetric points *c*', *b*', and *a*'.

$Q_e = 0$

$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.07910 \text{ in}^3$ $\qquad t_d = 0.125$ in.

$Q_c = Q_e = (0.125)^2(1.1875) = 0.09765 \text{ in}^4$ $\qquad t_c = 0.25$ in.

$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3$ $\qquad t_b = 0.25$ in.

$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60839 \text{ in}^3$ $\qquad t_a = 0.25$ in.

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60839)}{(1.2382)(0.25)} \qquad \tau_a = 3.93 \text{ ksi} \blacktriangleleft$$

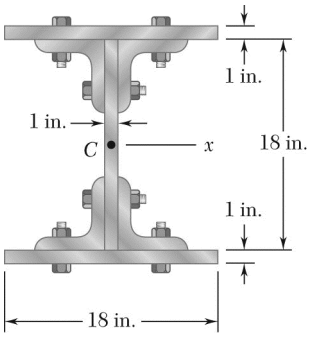
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} \qquad \tau_b = 2.67 \text{ ksi} \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} \qquad \tau_c = 0.63 \text{ ksi} \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.25)} \qquad \tau_d = 1.02 \text{ ksi} \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It_e} \qquad \tau_e = 0 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.43

Three 1×18-in. steel plates are bolted to four $L6 \times 6 \times 1$ angles to form a beam with the cross section shown. The bolts have a $\frac{7}{8}$-in. diameter and are spaced longitudinally every 5 in. Knowing that the allowable average shearing stress in the bolts is 12 ksi, determine the largest permissible vertical shear in the beam. (*Given:* $I_x = 6123 \text{ in}^4$)

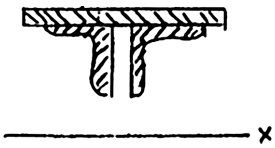
SOLUTION

Flange: $$I_f = \frac{1}{12}(18)(1)^3 + (18)(1)(9.5)^2 = 1626 \text{ in}^4$$

Web: $$I_w = \frac{1}{12}(1)(18)^3 = 486 \text{ in}^4$$

Angle: $$\bar{I} = 35.5 \text{ in}^4, \quad A = 11.0 \text{ in}^2$$
$$\bar{y} = 1.86 \text{ in.} \quad d = 9 - 1.86 = 7.14 \text{ in.}$$
$$I_a = \bar{I} + Ad^2 = 596.18 \text{ in}^4$$

$$I = 2I_f + I_w + 4I_a = 6123 \text{ in}^4, \text{ which agrees with the given value.}$$

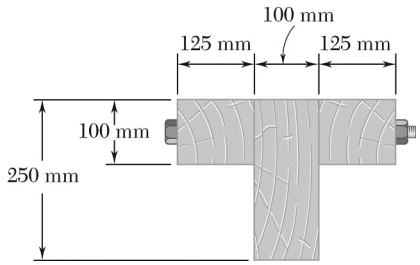


Flange: $$Q_f = (18)(1)(9.5) = 171 \text{ in}^3$$

Angle: $$Q_a = Ad = (11.0)(7.14) = 78.54 \text{ in}^3$$
$$Q = Q_f + 2Q_a = 328.08 \text{ in}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$
$$F_{\text{bolt}} = 2\tau_{\text{bolt}} A_{\text{bolt}} = (2)(12)(0.60132) = 14.4317 \text{ kips}$$
$$q_{\text{all}} = \frac{F_{\text{bolt}}}{s} = \frac{14.4317}{5} = 2.8863 \text{ kip/s}$$
$$q = \frac{VQ}{I} \qquad V_{\text{all}} = \frac{q_{\text{all}} I}{Q} = \frac{(2.8863)(6123)}{328.08} \qquad V_{\text{all}} = 53.9 \text{ kips} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.44

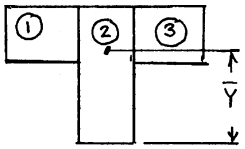
Three planks are connected as shown by bolts of 14-mm diameter spaced every 150 mm along the longitudinal axis of the beam. For a vertical shear of 10 kN, determine the average shearing stress in the bolts.

SOLUTION

Locate neutral axis and compute moment of inertia.

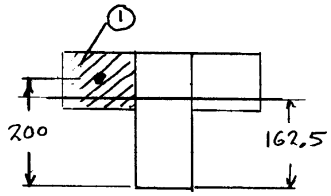
Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}(\text{mm}^3)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
①	12500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
②	25000	125	3.125×10^6	37.5	35.156×10^6	130.208×10^6
③	12500	200	2.5×10^6	37.5	17.5781×10^6	10.4167×10^6
Σ	50000		8.125×10^6		70.312×10^6	151.04×10^6

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.125\times10^6}{50\times10^3} = 162.5 \text{ mm}$$



$$I = \Sigma Ad^2 + \Sigma\bar{I} = 221.35\times10^6 \text{mm}^4$$
$$= 221.35\times10^{-6} \text{m}^4$$

$$Q = A_1\bar{y}_1 = (12500)(37.5) = 468.75\times10^3 \text{mm}^3$$
$$= 468.75\times10^{-6} \text{m}^3$$



$$q = \frac{VQ}{I} = \frac{(10\times10^3)(468.75\times10^{-6})}{221.35\times10^{-6}}$$
$$= 21.177\times10^3 \text{N/m}$$

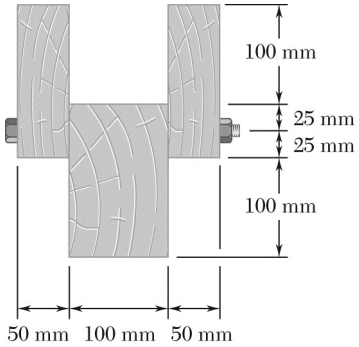
$$F_{\text{bolt}} = qs = (21.177\times10^3)(150\times10^{-3}) = 3.1765\times10^3 \text{N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}(14)^2 = 153.938 \text{ mm}^2 = 153.938\times10^{-6} \text{m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{3.1765\times10^3}{153.938\times10^{-6}} = 20.6\times10^6 \text{Pa}$$

$$\tau_{\text{bolt}} = 20.6 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

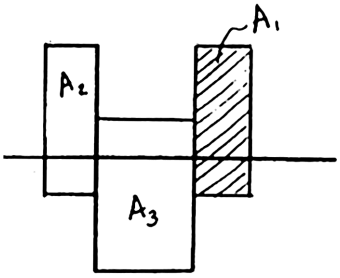


PROBLEM 6.45

A beam consists of three planks connected as shown by steel bolts with a longitudinal spacing of 225 mm. Knowing that the shear in the beam is vertical and equal to 6 kN and that the allowable average shearing stress in each bolt is 60 MPa, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}^2(10^6\text{mm}^4)$	$\bar{I}(10^6\text{mm}^4)$
①	7500	50	18.75	14.06
②	7500	50	18.75	14.06
③	15,000	–50	37.50	28.12
Σ			75.00	56.25



$$I = \Sigma A\bar{y}^2 + \Sigma \bar{I} = 131.25 \times 10^6\ \text{mm}^4 = 131.25 \times 10^{-6}\ \text{m}^4$$

$$Q = A_1\bar{y}_1 = (7500)(50) = 375 \times 10^3\ \text{mm}^3$$

$$= 375 \times 10^{-6}\ \text{m}^3$$

$$F_{\text{bolt}} = \tau_{\text{bolt}} A_{\text{bolt}} = qs = \frac{VQs}{I}$$

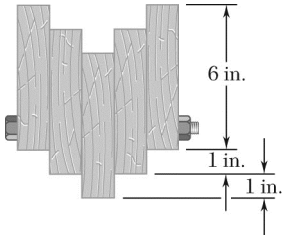
$$A_{\text{bolt}} = \frac{VQs}{\tau_{\text{bolt}} I} = \frac{(6 \times 10^3)(375 \times 10^{-6})(0.225)}{(6 \times 10^6)(131.25 \times 10^6)} = 64.286 \times 10^{-6}\ \text{m}^2$$

$$= 64.286\ \text{mm}^2$$

$$d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(64.286)}{\pi}}$$

$$d_{\text{bolt}} = 9.05\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.46

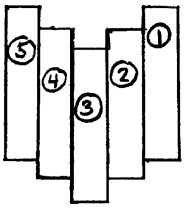
A beam consists of five planks of 1.5×6-in. cross section connected by steel bolts with a longitudinal spacing of 9 in. Knowing that the shear in the beam is vertical and equal to 2000 lb and that the allowable average shearing stress in each bolt is 7500 psi, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{in}^2)$	$\bar{y}_0(\text{in.})$	$A\bar{y}_0(\text{in}^3)$	$\bar{y}(\text{in.})$	$A\bar{y}^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	9	5	45	0.8	5.76	27
②	9	4	36	–0.2	0.36	27
③	9	3	27	–1.2	12.96	27
④	9	4	36	–0.2	0.36	27
⑤	9	5	45	0.8	5.76	27
Σ	45		189		25.20	135

$$\bar{Y}_0 = \frac{\Sigma Ay}{\Sigma A} = \frac{189}{45} = 4.2 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 160.2 \text{ in}^4$$



Between ① and ②: $Q_{12} = Q_1 = A\bar{y}_1 = (9)(0.8) = 7.2 \text{ in}^3$

Between ② and ③: $Q_{23} = Q_1 + A\bar{y}_2 = 7.2 + (9)(-0.2) = 5.4 \text{ in}^3$

$$q = \frac{VQ}{I}$$

Maximum q is based on $Q_{12} = 7.2 \text{ in}^3$.

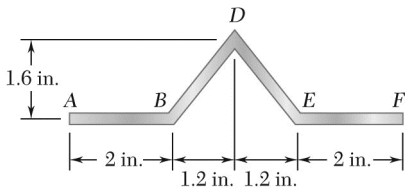
$$q = \frac{(2000)(7.2)}{160.2} = 89.888 \text{ lb/in}$$

$$F_{\text{bolt}} = qs = (89.888)(9) = 809 \text{ lb}$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} \qquad A_{\text{bolt}} = \frac{F_{\text{bolt}}}{\tau_{\text{bolt}}} = \frac{809}{7500} = 0.1079 \text{ in}^2$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 \qquad d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(0.1079)}{\pi}} \qquad d_{\text{bolt}} = 0.371 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.47

A plate of $\frac{1}{4}$-in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (*a*) the maximum shearing stress in the section, (*b*) the shearing stress at point *B*. Also, sketch the shear flow in the cross section.

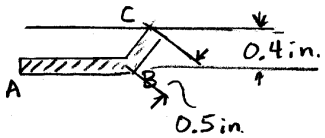
SOLUTION

$$L_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in.} \quad A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}\ (\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
AB	0.5	0	0	0.4	0.080	neglect
BD	0.5	0.8	0.4	0.4	0.080	*0.1067
DE	0.5	0.8	0.4	0.4	0.080	*0.1067
EF	0.5	0	0	0.4	0.080	neglect
Σ	2.0		0.8		0.320	0.2133

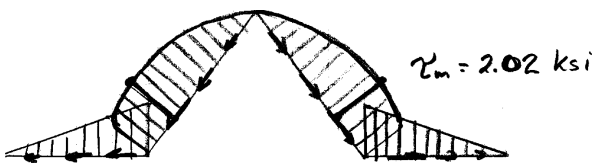
$$^*\frac{1}{12}A_{BD}h^2 = \frac{1}{12}(0.5)(1.6)^2 = 0.1067 \text{ in}^4 \qquad \bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{0.8}{2.0} = 0.4 \text{ in.}$$



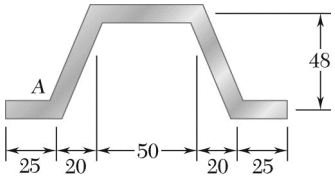
$$I = \Sigma Ad^2 + \bar{\Sigma}I = 0.5333 \text{ in}^4$$

(*a*)
$$Q_m = Q_{AB} + Q_{BC}$$
$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$
$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$
$$Q_m = 0.225 \text{ in}^3$$
$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5333)(0.25)} \qquad \tau_m = 2.03 \text{ ksi} \blacktriangleleft$$

(*b*)
$$Q_B = Q_{AB} = 0.2 \text{ in}^3$$
$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5333)(0.25)} \qquad \tau_B = 1.800 \text{ ksi} \blacktriangleleft$$
$$\tau_D = 0$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



Dimensions in mm

PROBLEM 6.48

A plate of 4-mm thickness is bent as shown and then used as a beam. For a vertical shear of 12 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also, sketch the shear flow in the cross section.

SOLUTION

$$\tan\alpha = \frac{20}{48} \qquad \alpha = 22.62^\circ$$

Slanted side: $A_s = (4 \sec\alpha)(48) = 208 \text{ mm}^2$

$$\bar{I}_s = \frac{1}{12}(4 \sec\alpha)(48)^3 = 39.936 \times 10^3 \text{ mm}^4$$

Top: $I_T = \frac{1}{12}(50)(4)^3 + (50)(4)(24)^2 = 115.46 \times 10^3 \text{ mm}^4$

Bottom: $I_B = I_T = 115.46 \times 10^3 \text{ mm}^4$

$$I = 2\bar{I}_s + I_T + I_B = 310.8 \times 10^3 \text{ mm}^4 = 310.8 \times 10^{-9} \text{ m}^4$$

(*a*) $Q_A = (25)(4)(24) = 2.4 \times 10^3 \text{ mm}^3 = 2.4 \times 10^{-6} \text{ m}^3$

$t = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

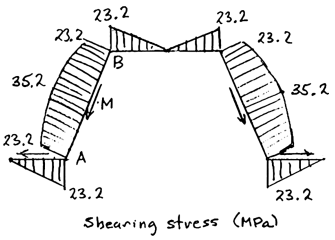
$$\tau_A = \frac{VQ_A}{It} = \frac{(12 \times 10^3)(2.4 \times 10^{-6})}{(310.8 \times 10^{-9})(4 \times 10^{-3})} = 23.2 \times 10^6 \text{ Pa} \qquad \tau_A = 23.2 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum shearing occurs at point *M*, 24 mm above the bottom

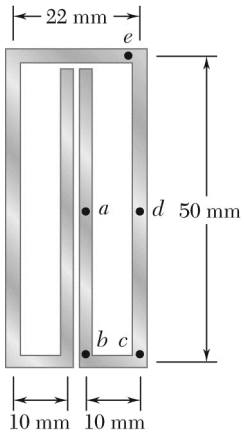
$$Q_M = Q_A + (4 \sec\alpha)(24)(12) = 2.4 \times 10^3 + 1.248 \times 10^3 = 3.648 \times 10^3 \text{ mm}^3$$
$$= 3.648 \times 10^{-6} \text{ m}^3$$

$$\tau_M = \frac{VQ_M}{It} = \frac{(12 \times 10^3)(3.648 \times 10^{-6})}{(310.8 \times 10^{-9})(4 \times 10^{-3})} = 35.2 \times 10^6 \text{ Pa} \qquad \tau_M = 35.2 \text{ MPa} \blacktriangleleft$$

$Q_B = Q_A \quad \tau_B = \tau_A = 23.2 \text{ MPa}$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.49

A plate of 2-mm thickness is bent as shown and then used as a beam. For a vertical shear of 5 kN, determine the shearing stress at the five points indicated and sketch the shear flow in the cross section.

SOLUTION

$$I = 2\left[\frac{1}{12}(2)(48)^3 + \frac{1}{12}(2)(52)^3 + \frac{1}{12}(20)(2)^3 + (20)(2)(25)^2\right]$$

$$= 133.75 \times 10^3 \text{ mm}^4 = 133.75 \times 10^{-9} \text{ mm}^4$$

$$Q_a = (2)(24)(12) = 576 \text{ mm}^3 = 576 \times 10^{-9} \text{ mm}^3$$

$$Q_a = 0$$

$$Q_c = Q_b - (12)(2)(25) = -600 \text{ mm}^3 = -600 \times 10^{-9} \text{ m}^3$$

$$Q_d = Q_c - (2)(24)(12) = -1.176 \times 10^3 \text{ mm}^3 = -1.176 \times 10^{-6} \text{ m}^3$$

$$Q_e = Q_d + (2)(26)(13) = -600 \text{ mm}^3 = -500 \times 10^{-9} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(5 \times 10^3)(576 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 10.77 \times 10^6 \text{ Pa} \qquad \tau_a = 10.76 \text{ MPa} \blacktriangleleft$$

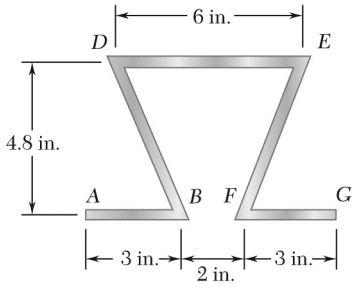
$$\tau_b = \frac{VQ_b}{It} \qquad \tau_b = 0 \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(5 \times 10^3)(600 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 11.21 \times 10^6 \text{ Pa} \qquad \tau_c = 11.21 \text{ MPa} \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(5 \times 10^3)(1.176 \times 10^{-6})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 22.0 \times 10^6 \text{ Pa} \qquad \tau_d = 22.0 \text{ MPa} \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(5 \times 10^3)(500 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 9.35 \times 10^6 \text{ Pa} \qquad \tau_e = 9.35 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.50

A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (*a*) the thickness t for which the maximum shearing stress is 300 psi, (*b*) the corresponding shearing stress at point E. Also, sketch the shear flow in the cross section.

SOLUTION

$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above AB.

Calculate I.

$$I_{AB} = (3t)(2.4)^2 = 17.28t$$

$$I_{BD} = \frac{1}{12}(5.2t)(4.8)^2 = 9.984t$$

$$I_{DE} = (6t)(2.4)^2 = 34.56t$$

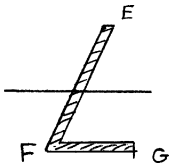
$$I_{EF} = I_{DB} = 9.984t$$

$$I_{FG} = I_{AB} = 17.28t$$

$$I = \Sigma I = 89.09t$$

(*a*) At point C, $Q_C = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6t)(1.2) = 10.32t$

$$\tau = \frac{VQ_C}{It} \quad \therefore \quad t = \frac{VQ}{\tau I} = \frac{(600)(10.32t)}{(300)(89.09t)} = 0.23168 \text{ in.} \qquad t = 0.232 \text{ in.} \blacktriangleleft$$

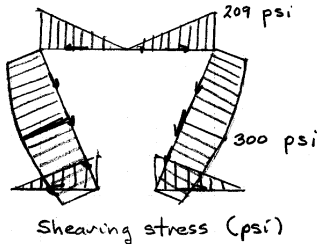


(*b*) $I = (89.09)(0.23168) = 20.64 \text{ in}^4$

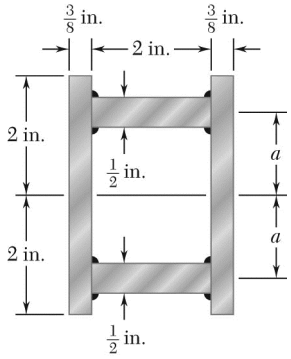
$$Q_E = Q_{EF} + Q_{FG}$$

$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^3$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)} \qquad \tau_E = 209 \text{ psi} \blacktriangleleft$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.51

The design of a beam calls for connecting two vertical rectangular $\frac{3}{8}\times 4$-in. plates by welding them to horizontal $\frac{1}{2}\times 2$-in. plates as shown. For a vertical shear **V**, determine the dimension a for which the shear flow through the welded surface is maximum.

SOLUTION

$$I = (2)\left(\frac{1}{12}\right)\left(\frac{3}{8}\right)(4)^3 + (2)\left(\frac{1}{12}\right)(2)\left(\frac{1}{2}\right)^3 + (2)(2)\left(\frac{1}{2}\right)a^2$$

$$= 4.041667 + 2a^2 \text{ in}^4$$

$$Q = (2)\left(\frac{1}{2}\right)a = a \text{ in}^3$$

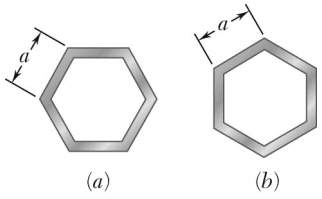
$$q = \frac{VQ}{I} = \frac{Va}{4.041667 + 2a^2} \qquad \text{Set } \frac{dq}{da} = 0.$$

$$\frac{dq}{da} = \left[\frac{(4.041667 + 2a^2) - (a)(4a)}{(4.041667 + 2a^2)^2}\right]V = 0$$

$$2a^2 = 4.041667$$

$$a = 1.422 \text{ in.} \blacktriangleleft$$

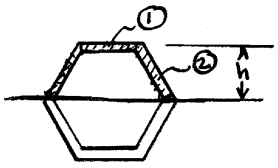
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.52

An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{\max} = k(V/A)$ and determine the constant k for each of the two orientations shown.

SOLUTION



(*a*) $h = \dfrac{\sqrt{3}}{2}a$

$A_1 = A_2 = at$

$I_1 = A_1h^2 = ath^2 = \dfrac{3}{4}a^3t$

$I_2 = \dfrac{1}{3}A_2h^2 = \dfrac{1}{3}at\dfrac{3}{4}a^2 = \dfrac{1}{4}a^3t$

$I = 2I_1 + 4I_2 = \dfrac{5}{2}a^3t$

$Q_1 = A_1h = \dfrac{\sqrt{3}}{2}a^2t$

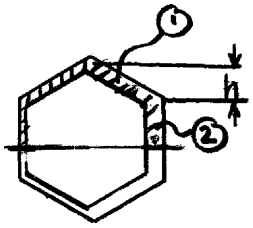
$Q_2 = A_2\dfrac{h}{2} = \dfrac{\sqrt{3}}{4}a^2t$

$Q_m = Q_1 + 2Q_2 = \sqrt{3}a^2t$

$\tau_m = \dfrac{VQ}{I(2t)} = \dfrac{V\sqrt{3}a^2t}{\left(\frac{5}{2}a^3t\right)2t} = \dfrac{\sqrt{3}}{5}\dfrac{V}{at}$

$= \dfrac{6\sqrt{3}}{5}\dfrac{V}{6at} = \dfrac{6\sqrt{3}}{5}\dfrac{V}{A} = k\dfrac{V}{A}$

$k = \dfrac{6\sqrt{3}}{5}$ $\qquad k = 2.08$ ◀



(*b*) $h = \dfrac{a}{2}$

$A_1 = at$

$A_2 = \dfrac{1}{2}at$

$I_1 = \bar{I}_1 + A_1d^2$

$= \dfrac{1}{12}ath^2 + at\left(\dfrac{a}{2} + \dfrac{h}{2}\right)^2$

$= \dfrac{1}{48}a^3t + \dfrac{9}{16}a^3t = \dfrac{7}{12}a^3t$

$I_2 = \dfrac{1}{3}t\left(\dfrac{a}{2}\right)^3 = \dfrac{1}{24}a^3t$

$I = 4I_1 + 4I_2 = \dfrac{5}{2}a^3t$

$Q_1 = at\left(\dfrac{a}{2} + \dfrac{h}{2}\right) = \dfrac{3}{4}a^2t$

$Q_2 = \left(\dfrac{1}{2}at\right)\left(\dfrac{a}{4}\right) = \dfrac{1}{8}a^2t$

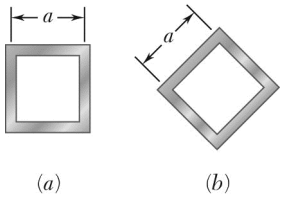
$Q = 2Q_1 + 2Q_2 = \dfrac{7}{4}a^3t$

$\tau_m = \dfrac{VQ}{I(2t)} = \dfrac{V\cdot\frac{7}{4}a^3t}{\left(\frac{5}{2}a^3t\right)(2t)}$

$= \dfrac{7}{20}\dfrac{V}{at} = \dfrac{42}{20}\dfrac{V}{6at} = \dfrac{21}{10}\dfrac{V}{A}$

$= k\dfrac{V}{A}$ $\qquad k = \dfrac{21}{10} = 2.10$ ◀

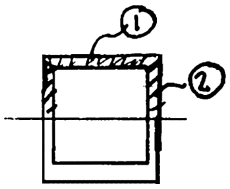
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.53

An extruded beam has a uniform wall thickness t. Denoting by **V** the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{\max} = k(V/A)$ and determine the constant k for each of the two orientations shown.

SOLUTION



(*a*)
$$I_1 = (at)\left(\frac{a}{2}\right)^2 = \frac{1}{4}a^3t$$

$$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3t$$

$$I = 2I_1 + 4I_2 = \frac{2}{3}a^3t$$

$$Q_1 = (at)\left(\frac{a}{2}\right) = \frac{1}{2}a^2t$$

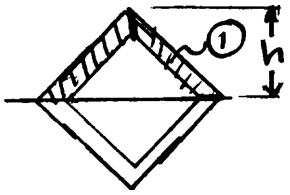
$$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2t$$

$$Q = Q_1 + 2Q_2 = \frac{3}{4}a^2t$$

$$\tau_{\max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{3}{4}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)}$$

$$= \frac{9}{16}\frac{V}{at} = \frac{9}{4}\frac{V}{4at} = \frac{9}{4}\frac{V}{A}$$

$$= k\frac{V}{A} \qquad k = \frac{9}{4} = 2.25 \blacktriangleleft$$



(*b*)
$$h = \frac{1}{2}\sqrt{2}a$$

$$I_1 = \frac{1}{3}A_1h^2 = \left(\frac{1}{3}at\right)\left(\frac{\sqrt{2}}{2}a\right)^2$$

$$= \frac{1}{6}a^3t$$

$$I = 4I_1 = \frac{2}{3}a^3t$$

$$Q_1 = at\left(\frac{h}{2}\right) = \frac{1}{4}\sqrt{2}a^2t$$

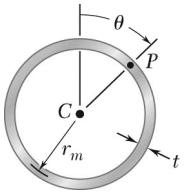
$$Q = 2Q_1 = \frac{1}{2}\sqrt{2}a^2t$$

$$\tau_{\max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{1}{2}\sqrt{2}a^2t\right)}{\left(\frac{2}{3}a^3t\right)(2t)}$$

$$= \frac{3\sqrt{2}}{8}\frac{V}{at} = \frac{3\sqrt{2}}{2}\frac{V}{4at}$$

$$= \frac{3\sqrt{2}}{2}\frac{V}{A} = k\frac{V}{A} \qquad k = \frac{3\sqrt{2}}{2} = 2.12 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



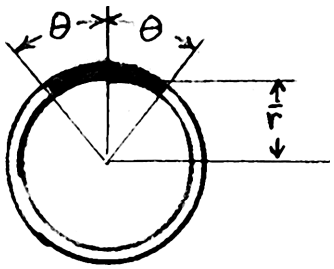
PROBLEM 6.54

(*a*) Determine the shearing stress at point *P* of a thin-walled pipe of the cross section shown caused by a vertical shear **V.** (*b*) Show that the maximum shearing stress occurs for $\theta = 90°$ and is equal to $2V/A$, where *A* is the cross-sectional area of the pipe.

SOLUTION

$$A = 2\pi r_m t \quad J = Ar_m^2 = 2\pi r_m^3 t \quad I = \frac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin\theta}{\theta} \text{ for a circular arc.}$$



$$A_P = 2r\theta t$$

$$Q_P = A_P\bar{r} = 2rt\sin\theta$$

(*a*) $$\tau_P = \frac{VQ_P}{I(2t)} = \frac{(V)(2rt\sin\theta)}{\left(\pi r_m^3 t\right)(2t)}$$

$$\tau_P = \frac{V\sin\theta}{\pi r_m t} \blacktriangleleft$$

(*b*) $$\tau_m = \frac{2V\sin\frac{\pi}{2}}{2\pi r_m t}$$

$$\tau_m = \frac{2V}{A} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.55

For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

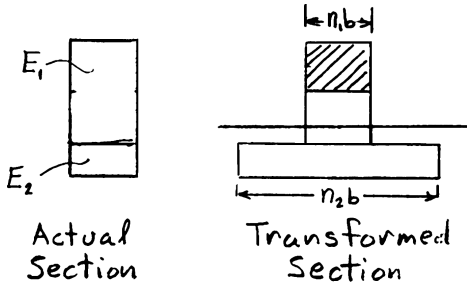
$$\tau_{\text{ave}} = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed by using the transformed section of the beam (see Sec. 4.6), and provided further that t is the actual width of the beam where τ_{ave} is computed.

SOLUTION

Let E_{ref} be a reference modulus of elasticity.

$$n_1 = \frac{E_1}{E_{\text{ref}}}, \quad n_2 = \frac{E_2}{E_{\text{ref}}}, \text{ etc.}$$



Widths b of actual section are multiplied by n's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

$$\Delta H = -\int (\Delta\sigma_x)\, dA = \int \frac{n(\Delta M)y}{I}\, dA = \frac{(\Delta M)}{I}\int ny\, dA = \frac{Q(\Delta M)}{I}$$

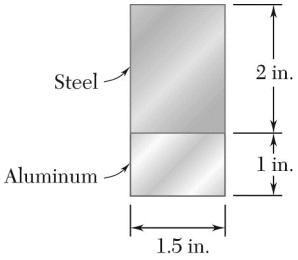
$$Q = \int ny\, dA = \text{ first moment of transformed section.}$$

Shear flow: $$q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x}\frac{Q}{I} = \frac{VQ}{I}$$

q is distributed over actual width t, thus $$\tau = \frac{q}{t}.$$

$$\tau = \frac{VQ}{It}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.56

A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

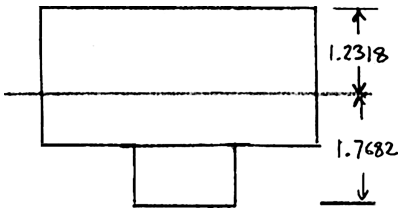
$n = 1$ in aluminum.

$$n = \frac{29\times10^6 \text{ psi}}{10.6\times10^6 \text{ psi}} = 2.7358 \text{ in steel.}$$

Part	$nA(\text{in}^2)$	$\bar{y}(\text{in.})$	$nA\bar{y}(\text{in}^3)$	$d(\text{in.})$	$nAd^2(\text{in}^2)$	$n\bar{I}(\text{in}^4)$
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.2682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608

$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in.}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$



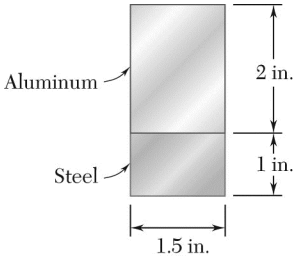
(*a*) At the bonded surface, $Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} \qquad \tau = 0.888 \text{ ksi} \blacktriangleleft$$

(*b*) At the neutral axis, $Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} \qquad \tau_{\max} = 1.453 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.57

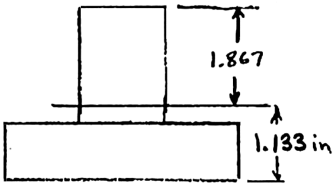
A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum shearing stress in the beam. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

$n = 1$ in aluminum.

$$n = \frac{29\times10^6 \text{ psi}}{10.6\times10^6 \text{ psi}} = 2.7358 \text{ in steel.}$$

Part	$nA(\text{in}^2)$	$\bar{y}(\text{in.})$	$nA\bar{y}(\text{in}^3)$	$d(\text{in.})$	$nAd^2(\text{in}^2)$	$n\bar{I}(\text{in}^4)$
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335 \text{ in.}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$$

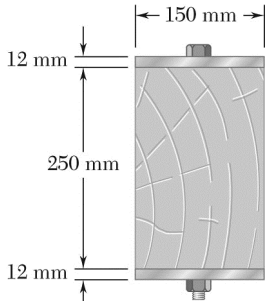
(*a*) At the bonded surface, $Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} \qquad \tau = 1.323 \text{ ksi} \blacktriangleleft$$

(*b*) At the neutral axis, $Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^3$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2814)(1.5)} \qquad \tau_{\max} = 1.329 \text{ ksi} \blacktriangleleft$$

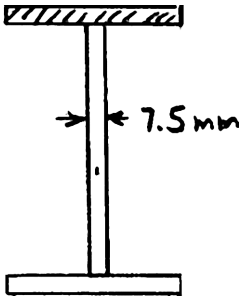
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.58

A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION



Let $E_{\text{ref}} = E_s = 200 \text{ GPa}$

$$n_s = 1 \qquad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$$

Widths of transformed section:

$$b_s = 150 \text{ mm} \qquad b_w = \left(\frac{1}{20}\right)(150) = 7.5 \text{ mm}$$

$$I = 2\left[\frac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right] + \frac{1}{12}(7.5)(250)^3$$

$$= 2[0.0216\times10^6 + 30.890\times10^6] + 9.766\times10^6$$

$$= 71.589\times10^6 \text{ mm}^4 = 71.589\times10^{-6} \text{ m}^4$$

(*a*) $$Q_1 = (150)(12)(125+6) = 235.8\times10^3 \text{ mm}^3 = 235.8\times10^{-6} \text{ m}^3$$

$$q = \frac{VQ_1}{I} = \frac{(4\times10^3)(235.8\times10^{-6})}{71.589\times10^{-6}} = 13.175\times10^3 \text{ N/m}$$

$$F_{\text{bolt}} = qs = (23.187\times10^3)(200\times10^{-3}) = 2.635\times10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \left(\frac{\pi}{4}\right)(12)^2 = 113.1 \text{ mm}^2 = 113.1\times10^{-6} \text{ m}^2$$

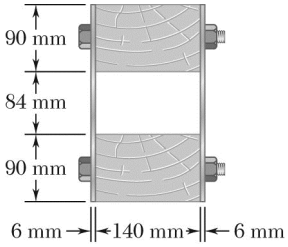
$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{2.635\times10^3}{113.1\times10^{-6}} = 23.3\times10^6 \text{ Pa} \qquad \tau_{\text{bolt}} = 23.3 \text{ MPa} \blacktriangleleft$$

(*b*) $$Q_2 = Q_1 + (7.5)(125)(62.5) = 235.8\times10^3 + 58.594\times10^3 = 294.4\times10^3 \text{ mm}^3 = 294.4\times10^{-6} \text{ m}^3$$

$$t = 150 \text{ mm} = 150\times10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_2}{It} = \frac{(4\times10^3)(294.4\times10^{-6})}{(71.589\times10^{-6})(150\times10^{-3})} = 109.7\times10^3 \text{ Pa} \qquad \tau_c = 109.7 \text{ kPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.59

A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. For a vertical shear of 4 kN, determine (*a*) the average shearing stress in the bolts, (*b*) the shearing stress at the center of the cross section. (*Hint:* Use the method indicated in Prob. 6.55.)

SOLUTION

Let steel be the reference material.

$$n_s = 1.0 \qquad n_w = \frac{E_w}{E_s} = \frac{10\text{ GPa}}{200\text{ GPa}} = 0.05$$

Depth of section: $d = 90 + 84 + 90 = 264$ mm

For steel portion: $I_s = 2\dfrac{1}{12}bd^3 = (2)\left(\dfrac{1}{12}\right)(6)(264)^3 = 18.400\times10^6\text{ mm}^4$

For the wooden portion: $I_w = \dfrac{1}{12}b\left(d_1^3 - d_2^3\right) = \dfrac{1}{12}(140)(264^3 - 84^3) = 207.75\times10^6\text{ mm}^4$

For the transformed section: $I = n_sI_s + n_wI_w$

$$I = (1.0)(18.400\times10^6) + (0.05)(207.75\times10^6) = 28.787\times10^6\text{ mm}^4 = 28.787\times10^{-6}\text{ m}^4$$

(*a*) Shearing stress in the bolts.

For the upper wooden portion $Q_w = (90)(140)(42 + 45) = 1.0962\times10^6\text{ mm}^3$

For the transformed wooden portion

$$Q = n_wQ_w = (0.05)(1.0962\times10^6) = 54.81\times10^3\text{ mm}^3 = 54.81\times10^{-6}\text{ m}^3$$

Shear flow on upper wooden portion.

$$q = \frac{VQ}{I} = \frac{(4000)(54.81\times10^{-6})}{28.787\times10^{-6}} = 7616\text{ N/m}$$

$$F_{\text{bolt}} = qs = (7616)(0.200) = 1523.2\text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4}d_{\text{bolt}}^2 = \frac{\pi}{4}(12)^2 = 113.1\text{ mm}^2 = 113.1\times10^{-6}\text{ m}^2$$

Double shear: $\tau_{\text{bolt}} = \dfrac{F_{\text{bolt}}}{2A_{\text{bolt}}} = \dfrac{1523.2}{(2)(113.1\times10^{-6})}$

$$= 6.73\times10^6\text{ Pa} \qquad \tau_{\text{bolt}} = 6.73\text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.59 *(Continued)*

(*b*) Shearing stress at the center of the cross section.

For two steel plates $Q_s = (2)(6)(90 + 42)(90 - 42) = 76.032 \times 10^3 \text{ mm}^3 = 76.032 \times 10^{-6} \text{ m}^3$

For the neutral axis $Q = 54.81 \times 10^{-6} + 76.032 \times 10^{-6} = 130.842 \times 10^{-6} \text{ m}^3$

Shear flow across the neutral axis

$$q = \frac{VQ}{I} = \frac{(4000)(130.842 \times 10^{-6})}{28.787 \times 10^{-6}} = 18.181 \times 10^3 \text{ N/m}$$

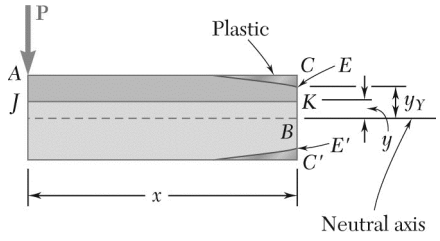
Double thickness $2t = 12 \text{ mm} = 0.012 \text{ m}$

Shearing stress $$\tau = \frac{q}{2t} = \frac{18.181 \times 10^3}{0.012} = 1.515 \times 10^6 \text{ Pa}$$

$\tau = 1.515 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.60



Consider the cantilever beam *AB* discussed in Sec. 6.8 and the portion *ACKJ* of the beam that is located to the left of the transverse section *CC′* and above the horizontal plane *JK*, where *K* is a point at a distance $y < y_Y$ above the neutral axis. (See Figure). (*a*) Recalling that $\sigma_x = \sigma_Y$ between *C* and *E* and $\sigma_x = (\sigma_Y/y_Y)y$ between *E* and *K*, show that the magnitude of the horizontal shearing force **H** exerted on the lower face of the portion of beam *ACKJ* is

$$H = \frac{1}{2} b\sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y} \right)$$

(*b*) Observing that the shearing stress at *K* is

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \to 0} \frac{1}{b} \frac{\Delta H}{\Delta x} = \frac{1}{b} \frac{\partial H}{\partial x}$$

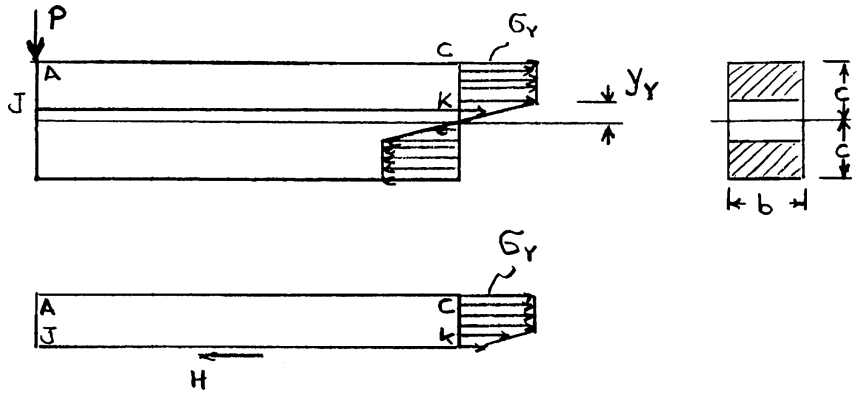
and recalling that y_Y is a function of *x* defined by Eq. (6.14), derive Eq. (6.15).

SOLUTION

Point *K* is located a distance *y* above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_Y \frac{y}{y_Y} \quad \text{for} \quad 0 \le y < y_Y \quad \text{and} \quad \sigma = \sigma_Y \quad \text{for} \quad y_Y \le y \le c.$$



***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.60 *(Continued)*

For equilibrium of horizontal forces acting on *ACKJ*,

$$H = \int \sigma \, dA = \int_{y}^{y_Y} \frac{\sigma_Y y b}{y_Y} dy + \int_{y_Y}^{c} \sigma_Y b dy$$

$$= \frac{\sigma_Y b}{y_Y}\left(\frac{y_Y^2 - y^2}{2}\right) + \sigma_y b(c - y_Y)$$

$$H = \frac{1}{2} b \sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y}\right) \quad \blacktriangleleft \quad (a)$$

Note that y_Y is a function of x.

$$\tau_{xy} = \frac{1}{b}\frac{\partial H}{\partial x} = \frac{1}{2}\sigma_Y \left(-\frac{\partial y_Y}{\partial x} + \frac{y^2}{y_{Y^2}}\frac{dy_Y}{dx}\right)$$

$$= -\frac{1}{2}\sigma_Y \left(1 - \frac{y^2}{y_{Y^2}}\right)\frac{dy_Y}{dx}$$

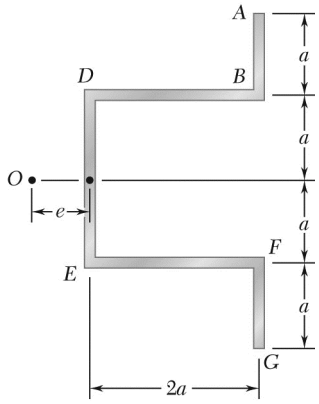
But $$M = Px = \frac{3}{2} M_y \left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right)$$

Differentiating, $$\frac{dM}{dx} = P = \frac{3}{2} M_Y \left(-\frac{2}{3}\frac{y_Y}{c^2}\frac{dy_Y}{dx}\right)$$

$$\frac{dy_Y}{dx} = -\frac{Pc^2}{y_Y M_Y} = -\frac{Pc^2}{y_Y \frac{2}{3}\sigma_Y b c^2} = -\frac{3}{2}\frac{P}{\sigma_Y b y_Y}$$

Then $$\tau_{xy} = \frac{1}{2}\sigma_Y \left(1 - \frac{y^2}{y_{Y^2}}\right)\frac{3}{2}\frac{P}{\sigma_Y b \sigma_Y} = \frac{3P}{4by_Y}\left(1 - \frac{y^2}{y_{Y^2}}\right) \quad \blacktriangleleft \quad (b)$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.61

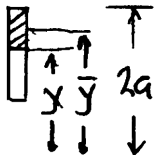
Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

$$I_{AB} = I_{FG} = \frac{1}{12}ta^3 + (ta)\left(\frac{3a}{2}\right)^2 = \frac{7}{3}ta^3$$

$$I_{DB} = I_{EF} = (2at)\,a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3t$$

$$I_{DE} = \frac{1}{12}t(2a)^3 = \frac{2}{3}ta^3 \qquad I = \Sigma I = \frac{28}{3}ta^3$$



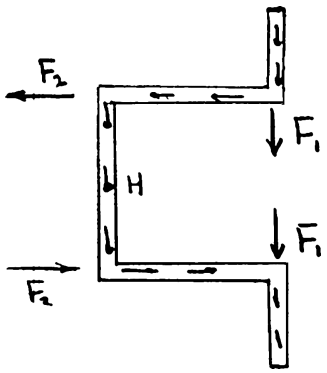
Part AB:

$$A = t(2a - y); \qquad \bar{y} = \frac{2a + y}{3}$$

$$Q = A\bar{y} = \frac{1}{2}t(2a - y)(2a + y)$$

$$= \frac{1}{2}t(4a^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$



$$F_1 = \int \tau dA = \int_a^{2a} \frac{V}{2I}(4a^2 - y^2)\,tdy$$

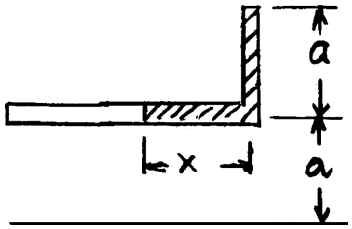
$$= \frac{Vt}{2I}\left(4a^2y - \frac{y^3}{3}\right)\Bigg|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\,\frac{Vta^3}{I} = \frac{5}{56}V$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.61 *(Continued)*

Part *DB*:



$$Q = (ta)\frac{3a}{2} + txa$$

$$= ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right) tdx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right)dx$$

$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Bigg|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

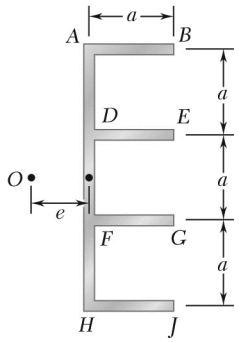
$$= 5\frac{Vta^3}{I} = \frac{15}{28}V$$

$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H: \quad Ve = F_2(2a) - 2F_1(2a)$$

$$= \frac{30}{28}Va - \frac{20}{56}Va = \frac{5}{7}Va$$

$$e = \frac{5}{7}a = 0.714a \blacktriangleleft$$

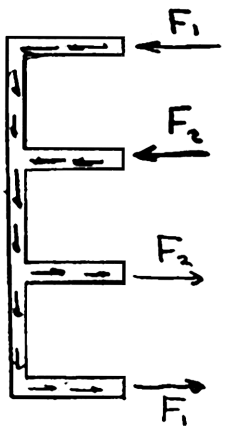
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.62

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



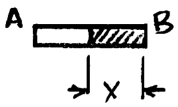
$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^2$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3$$

$$I = \Sigma I = \frac{29}{4}ta^3$$

Part AB:

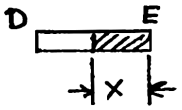


$$A = tx \qquad \bar{y} = \frac{3a}{2} \qquad Q = \frac{3}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2}atx}{\frac{29}{4}ta^3 t} = \frac{6Vx}{29a^2 t}$$

$$F_1 = \int \tau dA = \int_0^a \frac{6Vx}{29a^2 t}\, tdx = \frac{6V}{29a^2}\int_0^a x\, dx = \frac{3}{29}V$$

Part DE:



$$A = tx \qquad \bar{y} = \frac{a}{2} \qquad Q = \frac{1}{2}atx$$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}atx}{\frac{29}{4}ta^3 t} = \frac{2Vx}{29a^2 t}$$

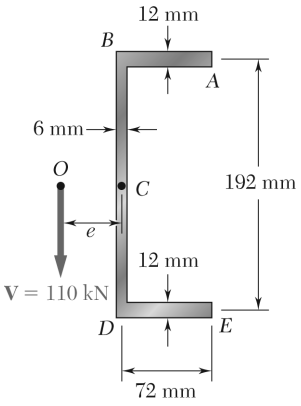
$$F_2 = \int \tau dA = \int_0^a \frac{2Vx}{29a^2 t}\, t\, dx = \frac{2V}{29a^2}\int_0^a x dx = \frac{1}{29}V$$

$$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K: \quad Ve = F_1(3a) + F_2(a)$$

$$= \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a \qquad e = 0.345a \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



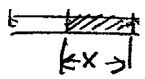
PROBLEM 6.63

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

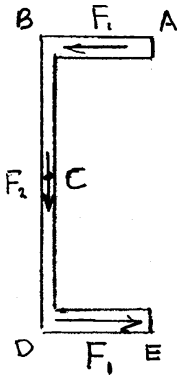
$$I = 2\left[\left(\frac{1}{12}\right)(72)(12)^3 + (72)(12)\left(\frac{192}{2}\right)^2\right] + \frac{1}{12}(6)(192)^3$$

$$= 19.4849\times10^{6}\,\text{mm}^4 = 19.4849\times10^{-6}\,\text{m}^4$$



Part *AB*: $A = 12x \qquad Q = A\bar{y} = (12x)\left(\frac{192}{2}\right) = 1152x$

$$q = \frac{VQ}{I} = \frac{1152Vx}{I}$$



$x = 0$ at point *A*. $x = l_{AB} = 72$ mm at point *B*.

$$F_1 = \int_{x_A}^{x_B} q\,dx = \int_0^{72} \frac{1152Vx}{I}\,dx = \frac{1152V}{I}\frac{(72)^2}{2}$$

$$= \frac{(576)(72)^2}{19.4849\times10^{6}}V = 0.153246V$$

$$+\circlearrowright \Sigma M_C = +\circlearrowright M_C: \quad Ve = (0.153246V)(192)$$

(*a*) $e = 29.423$ mm $\qquad e = 29.4$ mm ◀

(*b*) Point *A*: $x = 0 \quad Q = 0, \quad q = 0$ $\qquad \tau_A = 0$ ◀

Point *B* in part *AB*: $x = 72$ mm

$$Q = (1152)(72) = 82.944\times10^{3}\,\text{mm}^3 - 82.944\times10^{-6}\,\text{m}^3$$

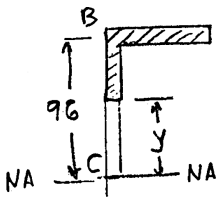
$$t = 12\text{ mm} = 0.012\text{ m}$$

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^{3})(82.944\times10^{-6})}{(19.4849\times10^{6})(0.012)}$$

$$= 39.0\times10^{6}\,\text{Pa} \qquad \tau_B = 39.0\text{ MPa in } AB \text{ ◀}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

Part *BD*:



Point *B*: $y = 96 \text{ mm} \quad Q = 82.944\times10^{3}\,\text{mm}^3 = 82.944\times10^{-6}\,\text{m}^3$

$t = 6 \text{ mm} = 0.006 \text{ m}$

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^{3})(82.944\times10^{-6})}{(19.4849\times10^{-6})(0.006)}$$

$$= 78.0\times10^{6}\,\text{Pa}$$

$\tau_B = 78.0$ MPa in *BD* ◀

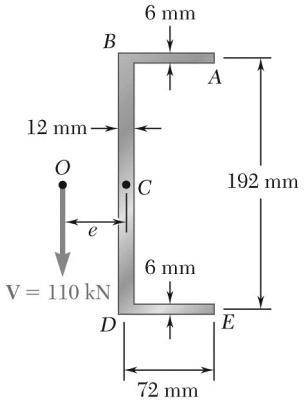
Point *C*: $y = 0, \quad t = 6 \text{ mm} = 0.006 \text{ m}$

$$Q = 82.944\times10^{3} + (6)(96)\left(\frac{96}{2}\right) = 110.592\times10^{3}\,\text{mm}^3 = 110.592\times10^{-6}\,\text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(110.10^{3})(110.592\times10^{-6})}{(19.4849\times10^{-6})(0.006)} = 104.1\times10^{6}\,\text{Pa}$$

$\tau_C = 104.1$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



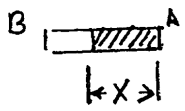
PROBLEM 6.64

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

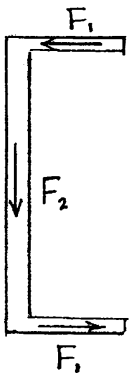
$$I = 2\left[\left(\frac{1}{12}\right)(72)(6)^3 + (72)(6)\left(\frac{192}{2}\right)^2\right] + \frac{1}{12}(12)(192)^3$$

$$= 15.0431\times10^6\ \text{mm}^4 = 15.0431\times10^{-6}\ \text{m}^4$$



Part AB: $A = 6x \qquad Q = A\bar{y} = (6x)\left(\frac{192}{2}\right) = 576x$

$$q = \frac{VQ}{I} = \frac{576Vx}{I}$$



$x = 0$ at point A. $x = l_{AB} = 72$ mm at point B.

$$F_1 = \int_{x_A}^{x_B} q\,dx = \int_0^{72} \frac{576Vx}{I}\,dx = \frac{576V}{I}\frac{(72)^2}{2}$$

$$= \frac{(288)(72)^2}{15.0431\times10^6}V = 0.099247V$$

$$+\circlearrowright M_C = +\circlearrowright M_C:\ Ve = (0.099247)V(192)$$

(*a*) $e = 19.0555$ mm $\qquad e = 19.06$ mm ◀

(*b*) Point A: $x = 0 \qquad Q = 0, \quad q = 0$ $\qquad \tau_A = 0$ ◀

Point B in part AB: $\qquad x = 72$ mm

$$Q = (576)(72) = 41.472\times10^3\ \text{mm}^3 = 41.472\times10^{-6}\ \text{m}^3$$

$$t = 6\ \text{mm} = 0.006\ \text{m}$$

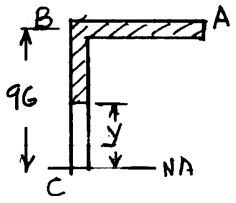
$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^3)(41.472\times10^{-6})}{(15.0431\times10^{-6})(0.006)}$$

$$= 50.5\times10^6\ \text{Pa} \qquad \tau_B = 50.5\ \text{MPa}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.64 *(Continued)*

Part *BD*:



Point *B*: $y = 96$ mm $\quad Q = 41.472\times10^{3}\,\text{mm}^3 = 41.472\times10^{-6}\,\text{m}^3$

$t = 12$ mm $= 0.012$ m

$$\tau_B = \frac{VQ}{It} = \frac{(110\times10^{3})(41.472\times10^{-6})}{(15.0431\times10^{-6})(0.012)}$$

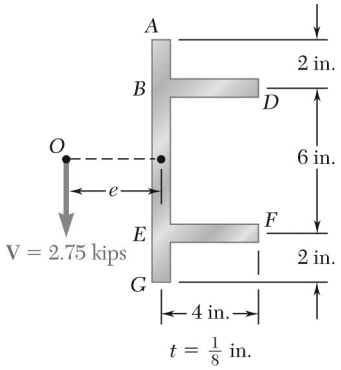
$$= 25.271\times10^{6}\,\text{Pa} \qquad \tau_B = 25.3\ \text{MPa} \blacktriangleleft$$

Point *C*: $y = 0, \quad t = 0.012$ m

$$Q = 41.472\times10^{3} + (12)(96)\left(\frac{96}{2}\right) = 96.768\times10^{3}\,\text{mm}^3 = 96.768\times10^{-6}\,\text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(110\times10^{3})(96.768\times10^{-6})}{(15.0431\times10^{-6})(0.012)} = 58.967\times10^{6}\,\text{Pa} \qquad \tau_C = 59.0\ \text{MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.65

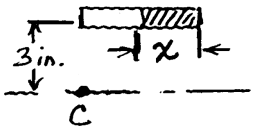
An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

Part	A (in^2)	d(in.)	Ad^2(in^4)	$\bar{I}$ (in^4)
BD	0.50	3	4.50	≈ 0
ABEG	1.25	0	0	10.417
EF	0.50	3	4.50	≈ 0
Σ	2.25		9.00	10.417

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 19.417 \text{ in}^4$$

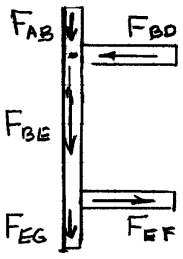
(*a*) Part *BD*: $Q(x) = 3tx$



$$q(x) = \frac{VQ(x)}{I} = \frac{V}{I}(3tx)$$

$$F_{BD} = \frac{3Vt}{I}\int_0^4 x\,dx = \frac{3Vt}{I}(8) = \frac{24Vt}{I}$$

Its moment about *H*: $(M_{BD})_H = 3F_{BD} = \dfrac{72Vt}{I}$



Part *EF*: By same method, $F_{EF} = \dfrac{24Vt}{I}$ $\quad (M_{EF})_H = \dfrac{72Vt}{I}$

Moments of F_{AB}, F_{BE}, and F_{EG} about *H* are zero.

$$Ve = \Sigma M_H = \frac{72Vt}{I} + \frac{72Vt}{I} = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(144)(0.125)}{19.417}$$

$e = 0.927$ in. ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.65 *(Continued)*

(*b*) At A, D, F, and G: $Q = 0$ $\tau_A = \tau_D = \tau_F = \tau_G = 0$ ◀

Just above B: $Q_1 = Q_{AB} = (2t)(4) = 8t$

$$\tau_1 = \frac{VQ_1}{It} = \frac{(2.75)(8t)}{(19.417)t} \qquad \tau_1 = 1.133 \text{ ksi} \blacktriangleleft$$

Just to the right of B: $Q_2 = Q_{BD} = (3)t(4) = 12t$

$$\tau_2 = \frac{VQ_2}{It} = \frac{(2.75)(12t)}{(19.417)t} \qquad \tau_2 = 1.700 \text{ ksi} \blacktriangleleft$$

Just below B: $Q_3 = Q_1 + Q_2 = 20t$

$$\tau_3 = \frac{VQ_3}{It} = \frac{(2.75)(20t)}{(19.417)t} \qquad \tau_3 = 2.83 \text{ ksi} \blacktriangleleft$$

At H (neutral axis): $Q_H = Q_3 + Q_{BH} = 20t + t(3)(1.5) = 24.5t$

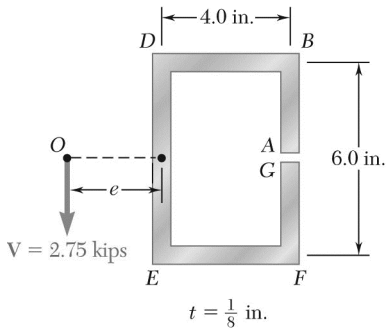
$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(24.5t)}{(19.417)t} \qquad \tau_H = 3.47 \text{ ksi} \blacktriangleleft$$

By symmetry: $\tau_4 = \tau_3 = 2.83$ ksi ◀

$\tau_5 = \tau_2 = 1.700$ ksi ◀

$\tau_6 = \tau_1 = 1.133$ ksi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.66

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

SOLUTION

$$I_{AB} = \frac{1}{3}(0.125)(3)^3 = 1.125 \text{ in}^4$$

$$I_{BD} = \frac{1}{12}(4)(0.125)^3 + (4)(0.125)(3)^2 = 4.50065 \text{ in}^4$$

$$I_{DE} = \frac{1}{12}(0.125)(6)^3 = 2.25 \text{ in}^4$$

$$I_{EF} = I_{BD} = 4.50065 \text{ in}^4$$

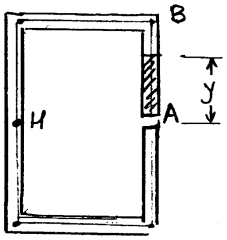
$$I_{FG} = I_{AB} = 1.125 \text{ in}^4$$

$$I = \Sigma I = 13.50 \text{ in}^4$$

(*a*) Part *AB*: $Q(y) = ty\frac{y}{2} = 0.5ty^2$

$$q(y) = \frac{VQ(y)}{I} = \frac{0.5Vt}{I}y^2$$

$$F_{AB} = \int_0^3 q(y)\,dy = \frac{0.5Vt}{I}\int_0^3 y^2\,dy = 4.5\frac{Vt}{I} \uparrow$$



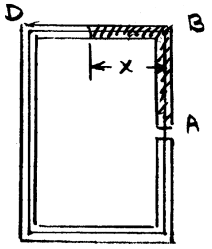
Its moment about *H* is $4F_{AB} = 18\ \frac{Vt}{I}\circlearrowright$

$$Q_B = (0.5)(t)(3)^2 = 4.5t$$

Part *BD*: $Q(x) = Q_B + xt(3) = (4.5 + 3x)t$

$$q(x) = \frac{Vq(x)}{I} = \frac{Vt}{I}(4.5 + 3x)$$

$$F_{BD} = \int_0^4 q(x)\,dx = \frac{\nabla t}{I}\int_0^4 (4.5 + 3x)\,dx = 42\frac{Vt}{I} \leftarrow$$



Its moment about *H*: $3F_{BD} = 126\frac{Vt}{I}\circlearrowright$

$$Q_D = [4.5 + (3)(4)]t = 16.5t$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.66 *(Continued)*

Part *EF*: By symmetry with part *BD*, $F_{EF} = 42\ \dfrac{Vt}{I} \rightarrow$

Its moment about *H* is $3F_{EF} = 126\ \dfrac{Vt}{I}$ ↻

Part *FG*: By symmetry with part *AB*, $F_{FG} = 4.5\dfrac{VT}{I} \uparrow$

Its moment about *H* is 4. $F_{FG} = 18\ \dfrac{Vt}{I}$ ↻

Moment about *H* of force in part *DE* is zero.

$$Ve = \Sigma M_H = \frac{Vt}{I}(18 + 126 + 0 + 126 + 18) = \frac{144Vt}{I}$$

$$e = \frac{144t}{I} = \frac{(2.88)(0.125)}{13.50} \qquad e = 2.67 \text{ in.} \blacktriangleleft$$

(*b*) $Q_A = Q_G = 0$ $\qquad \tau_A = \tau_G = 0 \blacktriangleleft$

$Q_B = Q_F = 4.5t$

$$\tau_B = \tau_F = \frac{VQ_B}{It} = \frac{(2.75)(4.5t)}{13.50t} \qquad \tau_B = \tau_F = 0.917 \text{ ksi} \blacktriangleleft$$

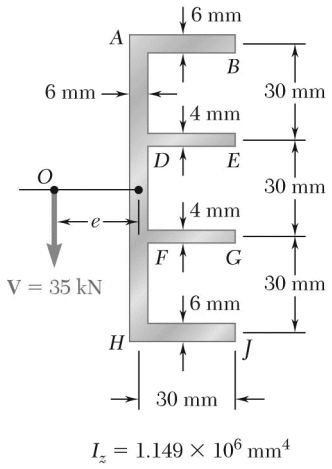
$Q_D = Q_E = 16.5t$

$$\tau_D = \tau_E = \frac{VQ_0}{It} = \frac{(2.75)(16.5t)}{13.50t} \qquad \tau_D = \tau_E = 3.36 \text{ ksi} \blacktriangleleft$$

At *H* (neutral axis): $Q_H = Q_D + t(3)(1.5) = 21t$

$$\tau_H = \frac{VQ_H}{It} = \frac{(2.75)(21t)}{13.50t} \qquad \tau_H = 4.28 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$I_z = 1.149 \times 10^6\ \text{mm}^4$

PROBLEM 6.67

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

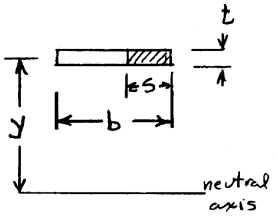
SOLUTION

$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(6)^3 + (30)(6)(45)^2 = 0.365\times10^6\,\text{mm}^4$$

$$I_{DE} = I_{FG} = \frac{1}{12}(30)(4)^3 + (30)(4)(15)^2 = 0.02716\times10^6\,\text{mm}^4$$

$$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645\times10^6\,\text{mm}^4$$

$$I = \Sigma I = 1.14882\times10^6\,\text{mm}^4$$

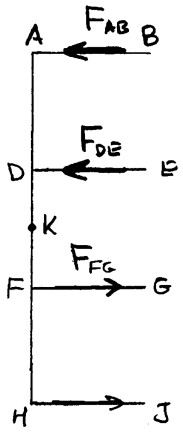


(*a*) For a typical flange, $A(s) = ts$

$$Q(s) = yts$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$$

$$F = \int_0^b q(s)ds = \frac{Vytb^2}{2I}$$



Flange *AB*: $$F_{AB} = \frac{V(45)(6)(30^2)}{(2)(1.14882\times10^{-6})} = 0.10576V \leftarrow$$

Flange *DE*: $$F_{DE} = \frac{V(15)(4)(30)^2}{(2)(1.14882\times10^{6})} = 0.023502V \leftarrow$$

Flange *FG*: $$F_{FG} = 0.023502V \rightarrow$$

Flange *HJ*: $$F_{HJ} = 0.10576V \rightarrow$$

$$+\circlearrowright \Sigma M_K = +\circlearrowright \Sigma M_K: \quad Ve = 45\,F_{AB} + 15F_{DE} + 15F_{FG} + 45\,F_{HJ} = 10.223V$$

Dividing by *V*, $$e = 10.22\ \text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.67 *(Continued)*

(*b*) Calculation of shearing stresses.

$$V = 35\times10^{3}\,\text{N} \qquad I = 1.14882\times10^{-6}\,\text{m}^4$$

At *B*, *E*, *G*, and *J*, $\tau = 0$ ◀

At *A* and *H*,

$$Q = (30)(6)(45) = 8.1\times10^{3}\,\text{mm}^3 = 8.1\times10^{-6}\,\text{m}^3$$

$$t = 6\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(8.1\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 41.1\times10^{6}\,\text{Pa} \qquad \tau = 41.1\ \text{MPa}$$ ◀

Just above *D* and just below *F*:

$$Q = 8.1\times10^{3} + (6)(30)(30) = 13.5\times10^{3}\,\text{mm}^3 = 13.5\times10^{-6}\,\text{m}^3$$

$$t = 6\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(13.5\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 68.5\times10^{6}\,\text{Pa} \qquad \tau = 68.5\ \text{MPa}$$ ◀

Just to right of *D* and just to the right of *F*:

$$Q = (30)(4)(15) = 1.8\times10^{3}\,\text{mm}^3 = 1.8\times10^{-6}\,\text{m}^3 \quad t = 4\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(1.8\times10^{-6})}{(1.14882\times10^{-6})(4\times10^{-3})} = 13.71\times10^{6}\,\text{Pa} \qquad \tau = 13.71\ \text{MPa}$$ ◀

Just below *D* and just above *F*:

$$Q = 13.5\times10^{3} + 1.8\times10^{3} = 15.3\times10^{3}\,\text{mm}^3 = 15.3\times10^{-6}\,\text{m}^3$$

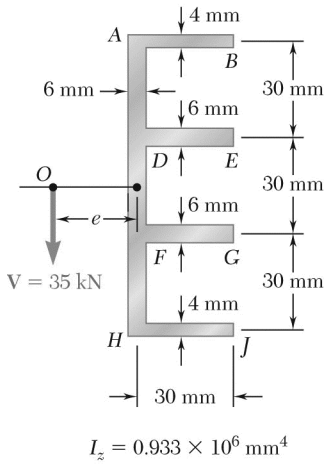
$$t = 6\times10^{-3}\,\text{m}$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(15.3\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 77.7\times10^{6}\,\text{Pa} \qquad \tau = 77.7\ \text{MPa}$$ ◀

At *K*, $Q = 15.3\times10^{3} + (6)(15)(7.5) = 15.975\times10^{3}\,\text{mm}^3 = 15.975\times10^{-6}\,\text{m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(15.975\times10^{-6})}{(1.14882\times10^{-6})(6\times10^{-3})} = 81.1\times10^{6}\,\text{Pa} \qquad \tau = 81.1\ \text{MPa}$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.68

An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by the vertical shearing force **V** shown applied at *O*.

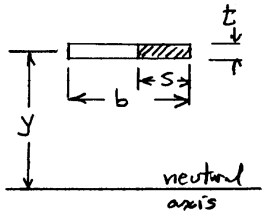
SOLUTION

$$I_{AB} = I_{HJ} = \frac{1}{12}(30)(4)^3 + (30)(4)(45)^2 = 0.24316\times10^6\,\text{mm}^4$$

$$I_{DE} = I_{FG} = \frac{1}{12}(30)(6)^3 + (30)(6)(15)^2 = 0.04104\times10^6\,\text{mm}^4$$

$$I_{AH} = \frac{1}{12}(6)(90)^3 = 0.3645\times10^6\,\text{mm}^4$$

$$I = \Sigma I = 0.9329\times10^6\,\text{mm}^4$$

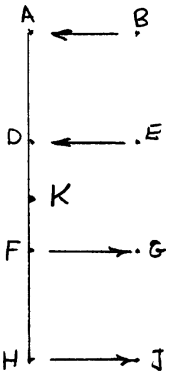


(*a*) For a typical flange, $A(s) = ts$

$$Q(s) = yts$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vyts}{I}$$

$$F = \int_0^b q(s)ds = \frac{Vytb^2}{2I}$$



Flange *AB*: $$F_{AB} = \frac{V(45)(4)(30)^2}{(2)(0.9329\times10^6)} = 0.086826V \leftarrow$$

Flange *DE*: $$F_{DE} = \frac{V(15)(6)(30)^2}{(2)(0.9329\times10^6)} = 0.043413\ V \leftarrow$$

Flange *FG*: $$F_{FG} = 0.043413\ V \rightarrow$$

Flange *HJ*: $$F_{HJ} = 0.086826\ V \rightarrow$$

$$+\circlearrowleft \Sigma M_K = +\circlearrowleft \Sigma M_K: \quad Ve = 45\,F_{AB} + 15F_{DE} + 15F_{FG} + 45\,F_{HJ} = 9.1167\ V$$

Dividing by *V*, $$e = 9.12\ \text{mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.68 *(Continued)*

(*b*) Calculation of shearing stresses.

$$V = 35\times10^{3}\text{ N} \quad I = 0.9329\times10^{-6}\text{ m}^4$$

At *B*, *E*, *G*, and *J*, $\tau = 0$ ◀

At *A* and *H*, $Q = (30)(4)(45) = 5.4\times10^{3}\text{ mm}^3 = 5.4\times10^{-6}\text{ m}^3$

$$q = \frac{VQ}{I} = \frac{(35\times10^{3})(5.4\times10^{-6})}{0.9329\times10^{-6}} = 202.59\times10^{3}\text{ N/m}$$

Just to the right of *A* and *H*: $t = 4\times10^{-3}\text{ m}$

$$\tau = \frac{q}{t} = \frac{202.59\times10^{3}}{4\times10^{-3}} = 50.6\times10^{6}\text{ Pa} \qquad \tau = 50.6\text{ MPa}$$ ◀

Just below *A* and just above *H*: $t = 6\times10^{-3}\text{ m}$

$$\tau = \frac{q}{t} = \frac{202.59\times10^{3}}{6\times10^{-3}} = 33.8\times10^{6}\text{ Pa} \qquad \tau = 33.8\text{ MPa}$$ ◀

Just above *D* and just below *F*: $t = 6\times10^{-3}\text{ m}$

$$Q = 5.4\times10^{3} + (6)(30)(30) = 10.8\times10^{3}\text{ mm}^3 = 10.8\times10^{-6}\text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(10.8\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-6})} = 67.5\times10^{6}\text{ Pa} \qquad \tau = 67.5\text{ MPa}$$ ◀

Just to the right of *D* and *E*: $t = 6\times10^{-3}\text{ m}$

$$Q = (30)(6)(15) = 2.7\times10^{3}\text{ mm}^2 = 2.7\times10^{-6}\text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(2.7\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 16.88\times10^{6}\text{ Pa} \qquad \tau = 16.88\text{ MPa}$$ ◀

Just below *D* and just above *F*: $t = 6\times10^{-3}$

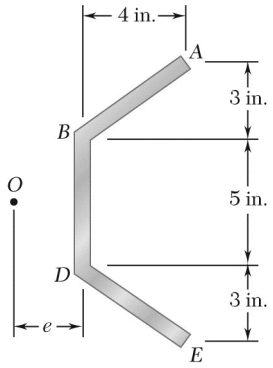
$$Q = 10.8\times10^{3} + 2.7\times10^{3} = 13.5\times10^{3}\text{ mm}^3 = 13.5\times10^{-6}\text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(13.5\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-6})} = 84.4\times10^{6}\text{ Pa} \qquad \tau = 84.4\text{ MPa}$$ ◀

At *K*, $Q = 13.5\times10^{3} + (6)(15)(7.5) = 14.175\times10^{3}\text{ mm}^3 = 14.175\times10^{-6}\text{ m}^3$

$$\tau = \frac{VQ}{It} = \frac{(35\times10^{3})(14.175\times10^{-6})}{(0.9329\times10^{-6})(6\times10^{-3})} = 88.6\times10^{6}\text{ Pa} \qquad \tau = 88.6\text{ MPa}$$ ◀

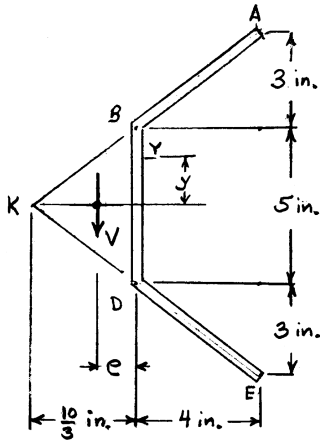
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.69

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



$$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{AB} = 5t$$

$$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2 = 83.75\, t \text{ in}^4$$

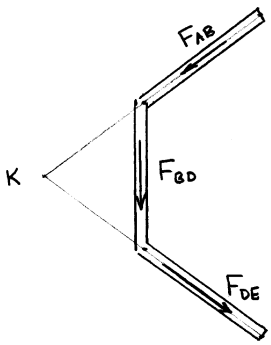
$$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417t \text{ in}^4$$

$$I = 2I_{AB} + I_{BD} = 177.917t \text{ in}^4$$

In part BD, $\quad Q = Q_{AB} + Q_{BY}$

$$Q = (5t)(4) + (2.5 - y)t\left(\frac{1}{2}\right)(2.5 + y)$$

$$= 20t + 3.125t - \frac{1}{2}ty^2 = \left(23.125 - \frac{1}{2}y^2\right)t$$



$$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau\, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{It} \cdot t\, dy$$

$$= \frac{Vt}{I}\int_{-2.5}^{2.5}\left(23.125 - \frac{1}{2}y^2\right)dy = \frac{Vt}{I}\left[23.125y - \frac{1}{6}y^3\right]_{-2.5}^{2.5}$$

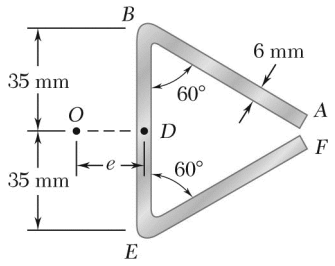
$$= \frac{Vt}{I} \cdot 2\left[(23.125)(2.5) - \frac{(2.5)^3}{6}\right] = \frac{Vt(110.417)}{177.917t} = 0.62061V$$

$$+\circlearrowleft \Sigma M_K = +\circlearrowleft \Sigma M_K: \quad -V\left(\frac{10}{3} - e\right) = -\frac{10}{3}(0.62061V)$$

$$e = \frac{10}{3}[1 - 0.62061] \qquad e = 1.265 \text{ in.} \blacktriangleleft$$

Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.70

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

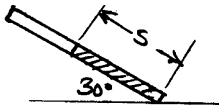
$$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75\times10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \qquad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AB} = \frac{1}{3}A_{AB}h^2 = \left(\frac{1}{3}\right)(420)(35)^2 = 171.5\times10^3 \text{ mm}^4$$

$$I = (2)(85.75\times10^3) + (2)(171.5\times10^3) = 514.5\times10^3 \text{ mm}^4$$

Part AB:

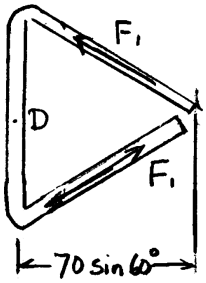


$$A = ts = 6s$$

$$\bar{y} = \frac{1}{2}s \sin 30° = \frac{1}{4}s$$

$$Q = A\bar{y} = \frac{3}{2}s^2$$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$



$$F_1 = \int \tau dA = \int_0^{70} \frac{3Vs^2}{2It} tds = \frac{3V}{I}\int_0^{70} s^2 ds$$

$$= \frac{(3)(70)^3}{(2)(3)I}V = \frac{1}{3}V$$

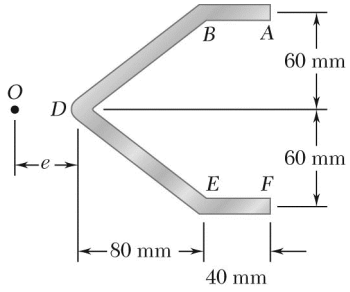
$$+\circlearrowright \Sigma M_D = +\circlearrowright \Sigma M_D: \quad Ve = 2[(F_1 \cos 60°)(70 \sin 60°)]$$

$$= 20.2V$$

Dividing by V,

$$e = 20.2 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.71

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

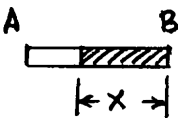
SOLUTION

$$I_{AB} = (40t)(60)^2 = 144\times10^3 t$$

$$L_{DB} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \qquad A_{DB} = 100t$$

$$I_{DB} = \frac{1}{3} A_{DB} h^2 = \frac{1}{3}(100t)(60)^2 = 120\times10^3 t$$

$$I = 2I_{AB} + 2I_{DB} = 528\times10^3 t$$



Part AB:

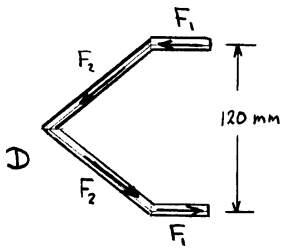
$$A = tx \quad \bar{y} = 60 \text{ mm}$$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60\,Vx}{I}$$

$$F_1 = \int \tau dA = \int_0^{40} \frac{60Vx}{I} t dx = \frac{60Vt}{I}\int_0^{40} x dx$$

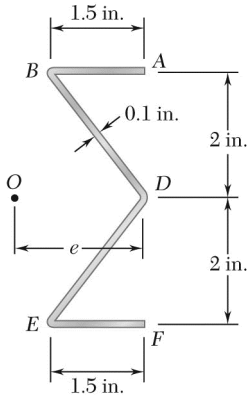
$$= \frac{60Vt}{I}\left.\frac{x^2}{2}\right|_0^{30} = \frac{(60)(30)^2 Vt}{(2)(528\times10^3)t} = 0.051136V$$



$$+\circlearrowright \Sigma M_D = +\circlearrowright \Sigma M_D: \quad Ve = (0.051136V)(120)$$

$$e = 6.14 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.72

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

$$I_{AB} = \frac{1}{12}(1.5)(0.1)^3 + (1.5)(0.1)(2)^2 = 0.600125 \text{ in}^4$$

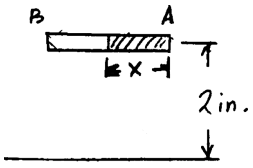
$$L_{BD} = \sqrt{1.5^2 + 2^2} = 2.5 \text{ in.} \quad A_{BD} = (2.5)(0.1) = 0.25 \text{ in}^2$$

$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3}(0.25)(2)^2 = 0.33333 \text{ in}^4$$

$$I = 2I_{AB} + 2I_{BD} = 1.86692 \text{ in}^4$$

Part AB:

$$A(x) = tx = 0.1x, \qquad \bar{y} = 2 \text{ in.}$$



$$Q(x) = A(x)\bar{y} = 0.2x \text{ in}^3$$

$$q(x) = \frac{VQ(x)}{I} = \frac{0.2Vx}{I}$$

$$F_1 = \int_0^{1.5} q(x)\,dx = \frac{0.2V}{I}\int_0^{1.5} x\,dx$$

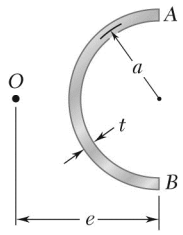
$$= \frac{(0.2)(1.5)^2}{2}\frac{V}{I} = 0.225\,\frac{V}{I}$$

Likewise, by symmetry in part EF: $F_1 = 0.225\,\dfrac{V}{I}$

$$+\circlearrowright \Sigma M_D = +\circlearrowright \Sigma M_D: \quad Ve = 4F_1 = 0.9\frac{V}{I} = 0.482V$$

Dividing by V, $e = 0.482$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.73

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

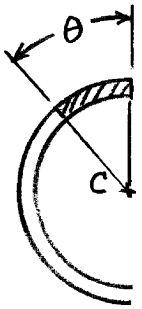
SOLUTION

For a thin-walled hollow circular cross section, $A = 2\pi at$

$$J = a^2 A = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$$

For the half-pipe section, $I = \frac{\pi}{2} a^3 t$

Use polar coordinate θ for partial cross section.



$$A = st = a\theta t \qquad s = \text{arc length}$$

$$\bar{r} = a\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{\theta}{2}$$

$$\bar{y} = \bar{r}\cos\alpha = a\frac{\sin\alpha\cos\alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t\ a\frac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\ (2\sin\alpha\cos\alpha)$$

$$= a^2 t\ \sin 2\alpha = a^2 t\ \sin\theta$$

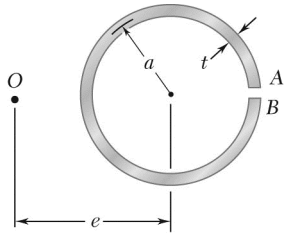
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I}\sin\theta$$

$$M_H = \int a\tau\, dA = \int_0^\pi a\ \frac{Va^2}{I}\ \sin\theta\ tad\theta = \frac{Va^4 t}{I} - \cos\theta\Big|_0^\pi$$

$$= 2\frac{Va^4 t}{I} = \frac{4}{\pi} Va$$

But $M_H = Ve$, hence $e = \frac{4}{\pi} a = 1.273a$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.74

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

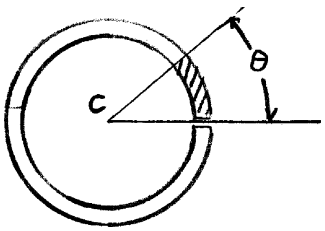
For whole cross section, $A = 2\pi at$

$$J = Aa^2 = 2\pi a^3 t \qquad I = \frac{1}{2}J = \pi a^3 t$$

Use polar coordinate θ for partial cross section.

$A = st = a\theta t$ $\qquad$ s = arc length

$$\bar{r} = a\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{1}{2}\theta$$



$$\bar{y} = \bar{r}\,\sin\alpha = a\frac{\sin^2\alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t\; a\frac{\sin^2\alpha}{\alpha} = a^2 t\;\; 2\sin^2\alpha$$

$$= a^2 t\;\; 2\sin^2\frac{\theta}{2} = a^2 t(1-\cos\theta)$$

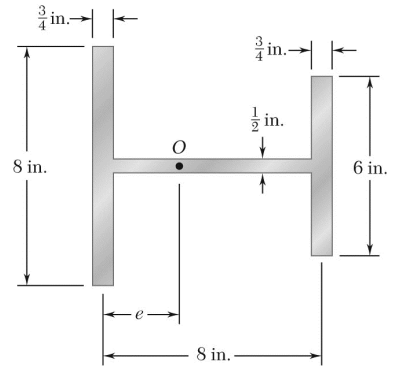
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1-\cos\theta)$$

$$M_C = \int a\tau\, dA = \int_0^{2\pi}\frac{Va^3}{I}(1-\cos\theta)\,tad\theta = \frac{Va^4 t}{I}(\theta-\sin\theta)\Big|_0^{2\pi}$$

$$= \frac{2\pi Va^4 t}{\pi a^3 t} = 2aV$$

But $M_C = Ve$, hence $e = 2a$ ◀

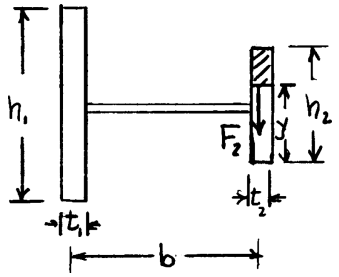
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.75

A thin-walled beam has the cross section shown. Determine the location of the shear center *O* of the cross section.

SOLUTION



$$I = \frac{1}{12}t_1h_1^3 + \frac{1}{12}t_2h_2^3$$

Right flange:

$$A = \left(\frac{1}{2}h_2 - y\right)t_2$$

$$\bar{y} = \frac{1}{2}\left(\frac{1}{2}h_2 + y\right)t_2$$

$$Q = A\bar{y}$$

$$= \frac{1}{2}\left(\frac{1}{2}h_2 - y\right)\left(\frac{1}{2}h_2 + y\right)t_2$$

$$= \frac{1}{2}\left(\frac{1}{4}h_2^2 - y^2\right)t_2$$

$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2}\left(\frac{1}{4}h_2^2 - y^2\right)t_2$$

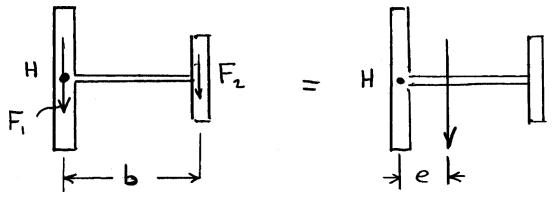
$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2I\cancel{t_2}}\left(\frac{1}{4}h_2^2 - y^2\right)\cancel{t_2}\, dy = \frac{Vt_2}{2I}\left(\frac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Bigg|_{-h_2/2}^{h_2/2}$$

$$= \frac{Vt_2}{2I}\left\{\frac{1}{4}h_2^2\frac{h_2}{2} - \frac{1}{3}\left(\frac{h_2}{2}\right)^3 + \frac{1}{4}h_2^2\frac{h_2}{2} - \frac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{Vt_2h_2^3}{12I} = \frac{V\,t_2h_2^3}{t_1h_1^3 + t_2h_2^3}$$

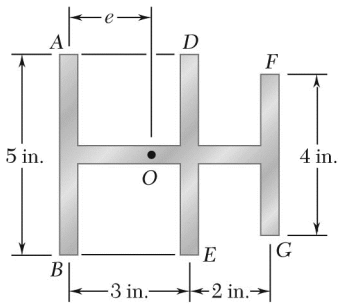
$$+\circlearrowleft M_H = +\circlearrowleft M_H: \quad -Ve = -F_2b = -V\frac{t_2h_2^3b}{t_1h_1^3 + t_2h_2^3}$$

$$e = \frac{t_2h_2^3b}{t_1h_1^3 + t_2h_2^3} = \frac{(0.75)(6)^3(8)}{(0.75)(8)^3 + (0.75)(6)^3}$$

$$e = 2.37 \text{ in.} \blacktriangleleft$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

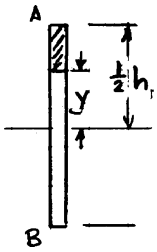


PROBLEM 6.76

A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.

SOLUTION

Let $h_1 = \overline{AB}$, $h_2 = \overline{DE}$, and $h_3 = \overline{FG}$. $I = \dfrac{1}{12}t(h_1^3 + h_2^3 + h_3^3)$



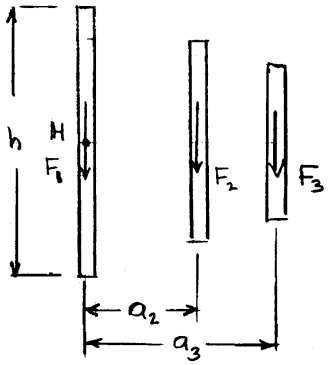
Part AB: $A = \left(\dfrac{1}{2}h_1 - y\right)t \quad \bar{y} = \dfrac{1}{2}\left(\dfrac{1}{2}h_1 + y\right)$

$$Q = A\bar{y} = \frac{1}{2}t\left(\frac{1}{2}h_1 - y\right)\left(\frac{1}{2}h_1 + y\right) = \frac{1}{2}t\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$F_1 = \int \tau dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)tdy = \frac{Vt}{2I}\left(\frac{1}{4}h_1^2 y - \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left(\frac{1}{4}h_1^2\frac{1}{2}h_1 - \frac{1}{3}\left(\frac{h_1}{2}\right)^3\right) = \frac{Vt\,h_1^3}{12I} = \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$



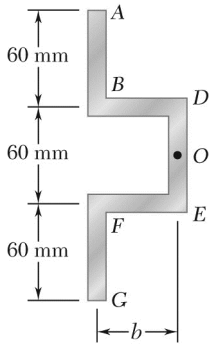
Likewise, for part DE, $F_2 = \dfrac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$

and for part FG, $F_3 = \dfrac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H: \quad Ve = a_2F_2 + a_3F_3 = \frac{a_2h_2^3 + a_3h_3^3}{h_1^3 + h_2^3 + h^3}V$$

Dividing by V, $e = \dfrac{a_2h_2^3 + a_3h_3^3}{h_1^3 + h_2^3 + h_3^2} = \dfrac{(3)(5)^3 + (5)(4)^3}{5^3 + 5^3 + 4^3}$ $\qquad e = 2.21$ in. ◀

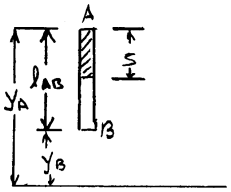
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.77

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION



Part AB:

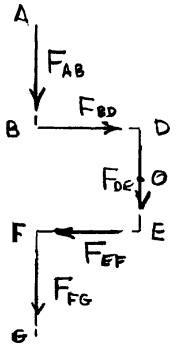
$$A(s) = ts \qquad \bar{y}(s) = y_A - \frac{1}{2}s$$

$$Q(s) = A(s)\bar{y}(s) = ty_A s - \frac{1}{2}ts^2$$

$$q(s) = \frac{VQ(s)}{I} = \frac{Vt}{I}\left(y_A s - \frac{1}{2}s^2\right)$$

$$F_{AB} = \int_0^{l_{AB}} q(s)\,ds$$

$$= \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right)\downarrow$$



At B:

$$Q_B = ty_A l_{AB} - \frac{1}{2}tl_{AB}^2$$

By symmetry,

$$F_{FG} = F_{AB}$$

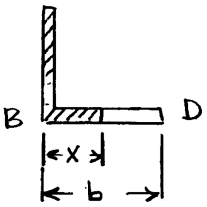
Part BD:

$$A(x) = tx$$

$$Q(x) = Q_B + y_B A(x) = Q_B + t\,y_B x$$

$$q(x) = \frac{V\,Q(x)}{I} = \frac{V}{I}(Q_B + ty_B x)$$

$$F_{BD} = \int_0^b q(x)\,dx = \frac{V}{I}\left(Q_B b + \frac{1}{2}t\,y_B b^2\right)\rightarrow$$



By symmetry,

$$F_{EF} = F_{BD}$$

F_{DE} is not required, since its moment about O is zero.

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.77 *(Continued)*

$$\circlearrowleft \Sigma M_O = 0: \quad b(F_{AB} + F_{FG}) - y_B F_{BD} + y_F F_{EF} = 0$$

$$2b\,F_{AB} - 2\,y_B F_{BD} = 0$$

$$2b \cdot \frac{Vt}{I}\left(\frac{y_A l_{AB}^2}{2} - \frac{l_{AB}^3}{6}\right) - 2y_B \frac{V}{I}\left(Q_B b + \frac{1}{2} t y_B b^2\right)$$

$$\frac{2Vt}{I}\left\{\frac{1}{2} y_A\, l_{AB}^2 - \frac{1}{6} l_{AB}^3\right\} b - \frac{2Vt}{I}\left\{\left(y_A l_{AB} - \frac{1}{2} l_{AB}^2\right) y_B b - \frac{1}{2} y_B^2 b^2 = 0\right.$$

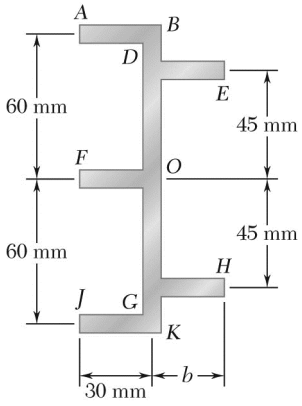
Dividing by $\frac{2Vt}{I}$ and substituting numerical data,

$$\left\{\frac{1}{2}(90)(60)^2 - \frac{1}{6}(60)^3\right\} b - \left\{(90)(60) - \frac{1}{2}(60)^2\right\}\ (30)b + \frac{1}{2}(30)^2 b^2 = 0$$

$$126 \times 10^3 b - 108 \times 10^3 b + 450 b^2 = 0$$

$$18 \times 10^3 b - 450 b^2 = 0 \qquad b = 0 \quad \text{and} \quad b = 40.0 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.78

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION

Part AB:

$$A = tx \qquad \bar{y} = 60 \text{ mm}$$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{60\,Vx}{I}$$

$$F_1 = \int \tau\, dA = \int_0^{30} \frac{60\,Vx}{I} t\, dx = \frac{60Vt}{I}\int_0^{30} x\, dx$$

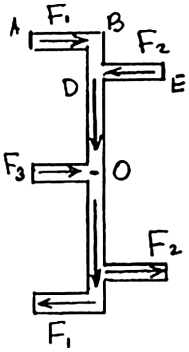
$$= \frac{60\,Vt}{I}\,\frac{x^2}{2}\bigg|_0^{30} = \frac{(60)(30)^2}{2}\,\frac{Vt}{I} = 27\times10^3\,\frac{Vt}{I}$$

Part DE:

$$A = tx \qquad \bar{y} = 45 \text{ mm}$$

$$Q = A\bar{y} = 45tx$$

$$\tau = \frac{VQ}{It} = \frac{45\,Vx}{I}$$



$$F_2 = \int \tau\, dA = \int_O^b \frac{45Vx}{I} t\, dx = \frac{45Vt}{I}\int_O^b x dx = \frac{45b^2Vt}{2I}$$

$$+\circlearrowright \Sigma M_O = +\circlearrowright \Sigma M_O: \qquad 0 = (2)(45)F_2 - (2)(60)F_1$$

$$\left[(45)^2b^2 - (2)(60)(27\times10^3)\right]\frac{Vt}{I} = 0$$

$$b^2 = \frac{(2)(60)(27\times10^3)}{45^2} = 1600 \text{ mm}^2 \qquad b = 40 \text{ mm} \blacktriangleleft$$

Note that the pair of F_1 forces form a couple. Likewise, the pair of F_2 forces form a couple. The lines of action of the forces in $BDOGK$ pass through point O.

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.79

For the angle shape and loading of Sample Prob. 6.6, check that $\int q dz = 0$ along the horizontal leg of the angle and $\int q dy = P$ along its vertical leg.

SOLUTION

Refer to Sample Prob. 6.6.

Along horizontal leg: $$\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3}(a^2 - 4az + 3z^2)$$

$$\int q dz = \int_0^a \tau_f t\, dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2) dz$$

$$= \frac{3P}{4a^3}\left(a^2 z - 4a\frac{z^2}{2} + 3\frac{z^2}{3} \right)\Bigg|_0^a$$

$$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0 \quad \blacktriangleleft$$

Along vertical leg: $$\tau_e = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$$

$$\int q dy = \int_0^a \tau_e t\, dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2) dy$$

$$= \frac{3P}{4a^3}\left(a^2 y + 4a\frac{y^2}{2} - 5\frac{y^3}{3} \right)\Bigg|_0^a$$

$$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3 \right) = \frac{3P}{4a^3}\cdot\frac{4}{3}a^3 = P \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.80

For the angle shape and loading of Sample Prob. 6.6, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

SOLUTION

Refer to Sample Prob. 6.6.

(*a*) Along vertical leg: $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \qquad y = \frac{2}{5}a \blacktriangleleft$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 + (4a)\left(\frac{2}{5}a\right) - (5)\left(\frac{2}{5}a\right)^2\right] = \frac{3P}{4ta^3}\left(\frac{9}{5}a^2\right) \qquad \tau_m = \frac{27}{20}\frac{P}{ta} \blacktriangleleft$$

Along horizontal leg: $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \qquad z = \frac{2}{3}a \blacktriangleleft$$

$$\tau_{\text{m}} = \frac{3P}{4ta^3}\left[a^2 - (4a)\left(\frac{2}{3}a\right) + (3)\left(\frac{2}{3}a\right)^2\right] = \frac{3P}{4ta^3}\left(-\frac{5}{3}a^2\right) \qquad \tau_{\text{m}} = -\frac{1}{4}\frac{P}{ta} \blacktriangleleft$$

At the corner: $y = 0, \quad z = 0,$ $\qquad \tau = \dfrac{3}{4}\dfrac{P}{ta} \blacktriangleleft$

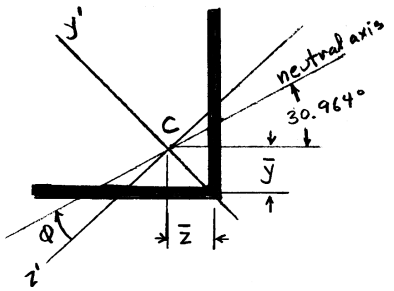
(*b*) $I_{y'} = \dfrac{1}{3}ta^3 \quad I_{z'} = \dfrac{1}{12}ta^3 \quad \theta = 45°$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \quad \varphi = 14.036°$$

$$\theta - \varphi = 45 - 14.036 = 30.964°$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.80 ***(Continued)***

Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z} \tan 30.964°$$

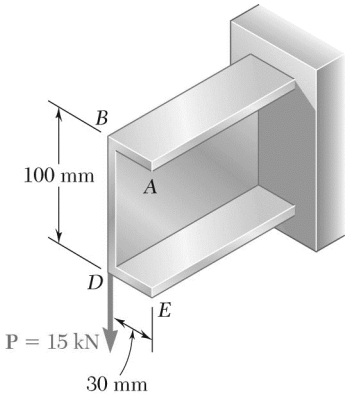
$$= \left(\frac{1}{4} + \frac{1}{4} \tan 30.964° \right) a = 0.400a \qquad y = \frac{2}{5} a \blacktriangleleft$$

Neutral axis intersects horizontal leg at

$$z = \bar{z} + \bar{y} \tan (45° + \varphi)$$

$$= \left(\frac{1}{4} + \frac{1}{4} \tan 59.036° \right) a = 0.667a \qquad z = \frac{2}{3} a \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.81*

A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P,** (*b*) the maximum shearing stress in the channel caused by the load **P.**

SOLUTION

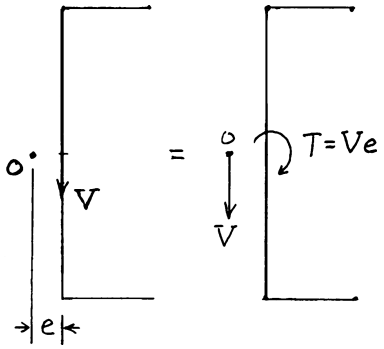
Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 8$ mm.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^2 (6b + h) = \frac{1}{12}(8)(100)^2[(16)(30) + 100] = 1.86667 \times 10^6 \text{ mm}^4 = 1.86667 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$

(*a*) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3})$ $\qquad T = 144.64 \text{ N} \cdot \text{m}$ ◀



Stress at neutral axis due to V:

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{1}{8} th(h + 4b)$$

$$= \frac{1}{8}(8)(100)\left[100 + (4)(30)\right] = 22 \times 10^3 \text{ mm}^3 = 22 \times 10^{-6} \text{ m}^3$$

$$t = 8 \times 10^{-3} \text{ m}$$

$$\tau_V = \frac{VQ}{It} = \frac{(15 \times 10^3)(22 \times 10^{-6})}{(1.86667 \times 10^{-6})(8 \times 10^{-3})} = 22.10 \times 10^6 \text{ Pa} = 22.10 \text{ MPa}$$

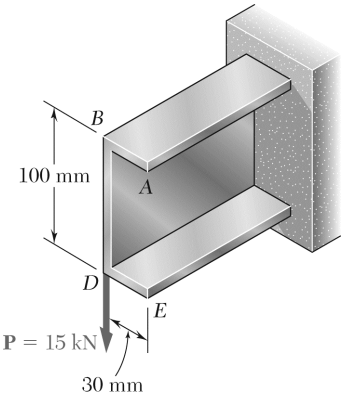
Stress due to T: $\qquad a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\frac{8}{160}\right] = 0.3228$$

$$\tau_V = \frac{T}{c_1 a t^2} = \frac{144.64}{(0.3228)(0.160)(8 \times 10^{-3})^2} = 43.76 \times 10^6 \text{ Pa} = 43.76 \text{ MPa}$$

(*b*) By superposition, $\qquad \tau_{\max} = \tau_V + \tau_T$ $\qquad \tau_{\max} = 65.9 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.82*

Solve Prob. 6.81, assuming that a 6-mm-thick plate is bent to form the channel shown.

PROBLEM 6.81* A steel plate, 160 mm wide and 8 mm thick, is bent to form the channel shown. Knowing that the vertical load **P** acts at a point in the midplane of the web of the channel, determine (*a*) the torque **T** that would cause the channel to twist in the same way that it does under the load **P,** (*b*) the maximum shearing stress in the channel caused by the load **P.**

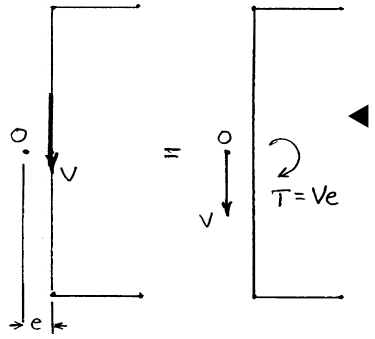
SOLUTION

Use results of Example 6.06 with $b = 30$ mm, $h = 100$ mm, and $t = 6$ mm.

$$e = \frac{b}{2 + \frac{h}{3b}} = \frac{30}{2 + \frac{100}{(3)(30)}} = 9.6429 \text{ mm} = 9.6429 \times 10^{-3} \text{ m}$$

$$I = \frac{1}{12} t h^2 (6b + h) = \frac{1}{12}(6)(100)^2[(6)(30) + 100] = 1.400 \times 10^6 \text{ mm}^4 = 1.400 \times 10^{-6} \text{ m}^4$$

$$V = 15 \times 10^3 \text{ N}$$



(*a*) $T = Ve = (15 \times 10^3)(9.6429 \times 10^{-3})$

Stress at neutral axis due to V:

$$Q = bt\frac{h}{2} + t\left(\frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{1}{8} th(h + 4b)$$

$$= \frac{1}{8}(6)(100)\left[100 + (4)(30)\right] = 16.5 \times 10^3 \text{ mm}^3 = 16.5 \times 10^{-6} \text{ m}^3$$

$$t = 6 \times 10^{-3} \text{ m}$$

$$\tau_V = \frac{VQ}{It} = \frac{(15 \times 10^3)(16.5 \times 10^{-6})}{(1.400 \times 10^{-6})(6 \times 10^{-6})} = 29.46 \times 10^6 \text{ Pa} = 29.46 \text{ MPa}$$

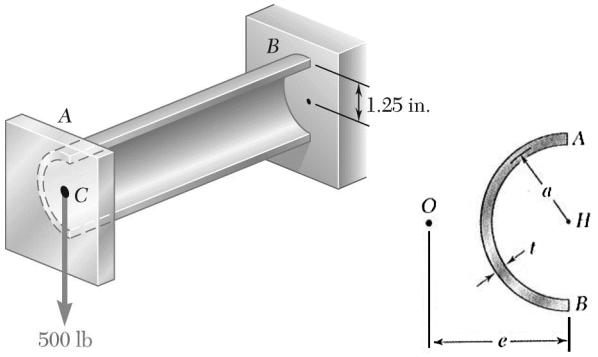
Stress due to T: $a = 2b + h = 160 \text{ mm} = 0.160 \text{ m}$

$$c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{a}\right) = \frac{1}{3}\left[1 - (0.630)\left(\frac{6}{160}\right)\right] = 0.32546$$

$$\tau_V = \frac{T}{c_1 a\, t^2} = \frac{144.64}{(0.32546)(0.160)(6 \times 10^{-3})^2} = 77.16 \times 10^6 \text{ Pa} = 77.16 \text{ MPa}$$

(*b*) By superposition, $\tau_{\max} = \tau_V + \tau_T$ $\tau_{\max} = 106.6 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.83*

The cantilever beam *AB*, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid *C* of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center *O* of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid *C*.)

SOLUTION

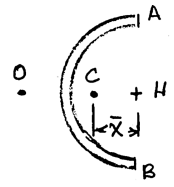
From the solution to Prob. 6.73,

$$I = \frac{\pi}{2}a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi}a \qquad Q_{\max} = a^2 t$$

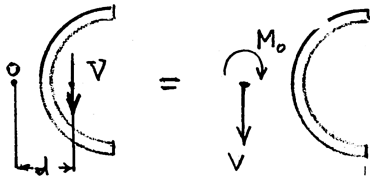
For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi}a$$



At each section of the beam, the shearing force *V* is equal to P. Its line of action passes through the centroid *C*. The moment arm of its moment about the shear center *O* is

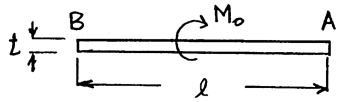
$$d = e - \bar{x} = \frac{4}{\pi}a - \frac{2}{\pi}a = \frac{2}{\pi}a$$



(*a*) <u>Equivalent force-couple system at *O*.</u>

$$V = P \qquad M_0 = Vd = \frac{2}{\pi}\text{Pa}$$

Data: $P = 500$ lb $\quad a = 1.25$ in.



$$V = 500 \text{ lb} \blacktriangleleft$$

$$M_0 = \left(\frac{2}{\pi}\right)(500)(1.25) \qquad M_0 = 398 \text{ lb}\cdot\text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.83* *(Continued)*

(*b*) Shearing stresses.

(1) Due to V: $\tau_V = \dfrac{VQ_{\max}}{It}$

$$\tau_V = \frac{(P)(a^2t)}{\left(\dfrac{\pi}{2}a^3t\right)(t)} = \frac{2P}{\pi at} = \frac{(2)(500)}{\pi(1.25)(0.375)} = 679 \text{ psi}$$

(2) Due to the torque. M_0,

For a long rectangular section of length *l* and width *t*, the shearing stress due to torque M_0 is

$$\tau_M = \frac{M_0}{c_1 l t^2} \quad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

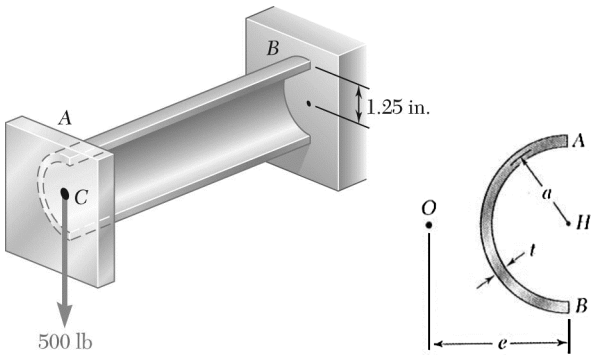
Data: $l = \pi a = \pi(1.25) = 3.927$ in. $t = 0.375$ in. $c_1 = 0.31328$

$$\tau_M = \frac{397.9}{(0.31328)(3.927)(0.375)^2} = 2300 \text{ psi}$$

By superposition, $\tau = \tau_V + \tau_M = 679 \text{ psi} + 2300 \text{ psi}$ $\tau = 2980$ psi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.84*



Solve Prob. 6.83, assuming that the thickness of the beam is reduced to $\frac{1}{4}$ in.

PROBLEM 6.83* The cantilever beam *AB*, consisting of half of a thin-walled pipe of 1.25-in. mean radius and $\frac{3}{8}$-in. wall thickness, is subjected to a 500-lb vertical load. Knowing that the line of action of the load passes through the centroid *C* of the cross section of the beam, determine (*a*) the equivalent force-couple system at the shear center of the cross section, (*b*) the maximum shearing stress in the beam. (*Hint:* The shear center *O* of this cross section was shown in Prob. 6.73 to be located twice as far from its vertical diameter as its centroid *C*.)

SOLUTION

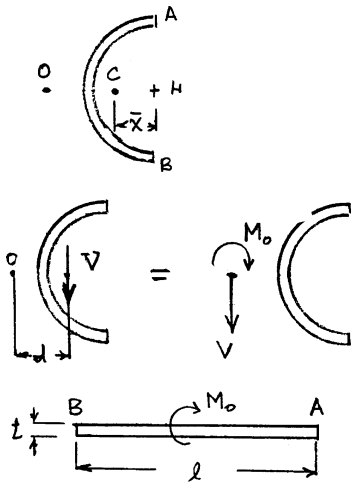
From the solution to Prob. 6.73,

$$I = \pi a^3 t \qquad Q = a^2 t \sin\theta$$

$$e = \frac{4}{\pi} a \qquad Q_{\max} = a^2 t$$

For a half-pipe section, the distance from the center of the semi-circle to the centroid is

$$\bar{x} = \frac{2}{\pi} a$$



At each section of the beam, the shearing force *V* is equal to *P*. Its line of action passes through the centroid *C*. The moment arm of its moment about the shear center *O* is

$$d = e - \bar{x} = \frac{4}{\pi} a - \frac{2}{\pi} a = \frac{2}{\pi} a$$

(*a*) Equivalent force-couple system at *O*.

$$V = P \qquad M_0 = Vd = \frac{2}{\pi} Pa$$

Data: $P = 500$ lb $\quad a = 1.25$ in.

$V = 500$ lb ◀

$M_0 = 398$ lb · in ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.84* *(Continued)*

(*b*) Shearing stresses.

(1) Due to V, $\tau_V = \dfrac{VQ_{\max}}{It}$

$$\tau_V = \frac{(P)(a^2 t)}{\left(\dfrac{\pi}{2}a^3 t(t)\right)} = \frac{2P}{\pi at} = \frac{(2)(500)}{\pi(1.25)(0.250)} = 1019 \text{ psi}$$

(2) Due to the torque. M_0,

For a long rectangular section of length *l* and width *t*, the shearing stress due to torque M_0 is

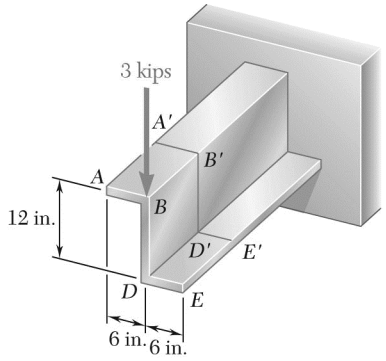
$$\tau_M = \frac{M_0}{c_1 l t^2} \quad \text{where} \quad c_1 = \frac{1}{3}\left(1 - 0.630\frac{t}{l}\right)$$

Data: $l = \pi a = \pi(1.25) = 3.927$ in. $t = 0.250$ in. $c_1 = 0.31996$

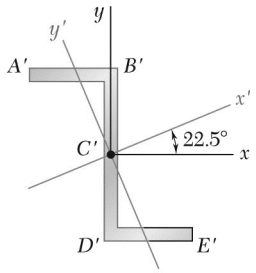
$$\tau_M = \frac{397.9}{(0.31996)(3.927)(0.250)^2} = 5067 \text{ psi}$$

By superposition $\tau = \tau_V + \tau_M = 1019 \text{ psi} + 5067 \text{ psi}$ $\tau = 6090$ psi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



(a)



(b)

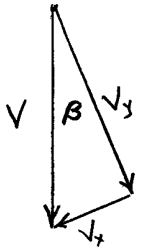
PROBLEM 6.85*

The cantilever beam shown consists of a *Z*-shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z-shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and $I_{y'} = 13.61 \text{ in}^4$.

SOLUTION

$$V = 3 \text{ kips} \quad \beta = 22.5°$$

$$V_{x'} = V \sin\beta \quad V_{y'} = V \cos\beta$$



In upper horizontal leg, use coordinate x: $(-6 \text{ in} \le x \le 0)$

$$A = \frac{1}{4}(6 + x) \text{ in.}$$

$$\bar{x} = \frac{1}{2}(-6 + x) \text{ in.}$$

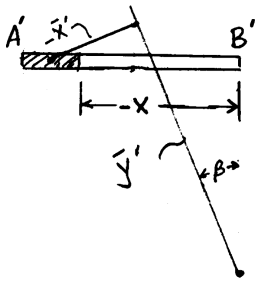
$$\bar{y} = 6 \text{ in.}$$

$$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$$

$$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'} A \bar{x}'}{I_y t}$$



$$\tau_1 = \frac{(V\sin\beta)\left(\frac{1}{4}\right)(6+x)\left[\frac{1}{2}(-6+x)\cos\beta + 6\sin\beta\right]}{(13.61)\left(\frac{1}{4}\right)}$$

$$= 0.084353(6 + x)(-0.47554 + 0.46194x)$$

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'} A \bar{y}'}{I_x t} = \frac{(V\cos\beta)\left(\frac{1}{4}\right)(6+x)\left[6\cos\beta - \frac{1}{2}(-6+x)\sin\beta\right]}{(166.3)\left(\frac{1}{4}\right)}$$

$$= 0.0166665(6 + x)[6.69132 - 0.19134x]$$

Total:

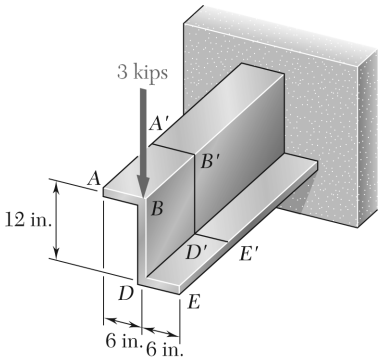
$$\tau_1 + \tau_2 = (6 + x)[-0.07141 + 0.035396x]$$

x (in)	−6	−5	−4	−3	−2	−1	0
τ (ksi)	0	−0.105	−0.140	−0.104	0.003	0.180	0.428

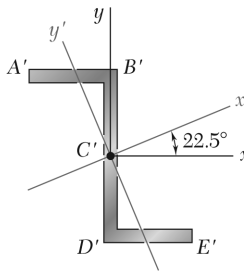
◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.86*



(a)



(b)

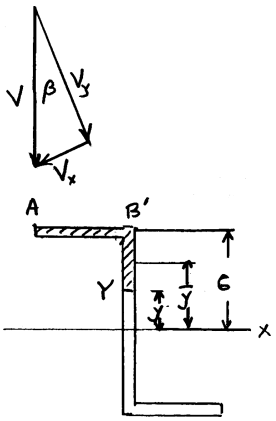
For the cantilever beam and loading of Prob. 6.85, determine the distribution of the shearing stress along line $B'D'$ in the vertical web of the *Z*-shape.

PROBLEM 6.85* The cantilever beam shown consists of a *Z*-shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the *Z*-shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3 \text{ in}^4$ and $I_{y'} = 13.61 \text{ in}^4$.

SOLUTION

$$V = 3 \text{ kips} \quad \beta = 22.5°$$

$$V_{x'} = V \sin\beta \quad V_{y'} = V \cos\beta$$



For part AB'

$$A = \left(\frac{1}{4}\right)(6) = 1.5 \text{ in}^2$$

$$\bar{x} = -3 \text{ in.}, \quad y = 6 \text{ in.}$$

For part $B'Y$,

$$A = \frac{1}{4}(6 - y)$$

$$\bar{x} = 0 \quad \bar{y} = \frac{1}{2}(6 + y)$$

$$x' = x\cos\beta + y\sin\beta$$

$$y' = y\cos\beta - x\sin\beta$$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'}(A_{AB}\bar{x}_{AB} + A_{BY}\bar{x}'_{BY})}{I_{y'}t}$$

$$\tau_1 = \frac{(V\sin\beta)[(1.5)(-3\cos\beta + 6\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\sin\beta]}{(13.61)\left(\frac{1}{4}\right)}$$

$$= \frac{(V\sin\beta)[-0.7133 + 1.7221 - 0.047835\,y^2]}{3.4025} = 0.3404 - 0.01614\,y^2$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.86* *(Continued)*

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'}(A_{AB}\overline{y}'_{AB} + A_{BY}\overline{y}')}{I_{x'}t}$$

$$\tau_2 = \frac{(V\cos\beta)[(1.5)(6\cos\beta + 3\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\cos\beta]}{(166.3)\left(\frac{1}{4}\right)}$$

$$= \frac{(V\cos\beta)[10.037 + 4.1575 - 0.11548\,y^2]}{(166.3)\left(\frac{1}{4}\right)} = 0.9463 - 0.00770\,y^2$$

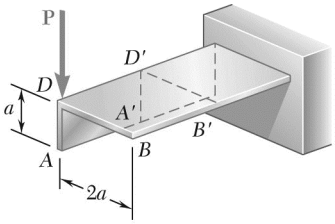
Total:

$$\tau_1 + \tau_2 = 1.2867 - 0.02384\,y^2$$

y (in.)	0	± 2	±4	±6
τ (ksi)	1.287	1.191	0.905	0.428

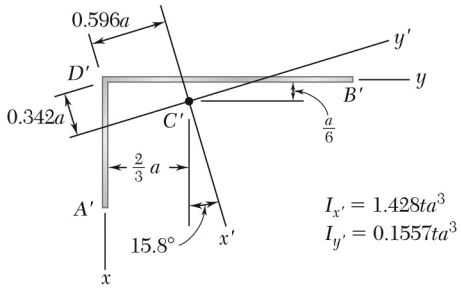
◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.87*

Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.



SOLUTION

$$\beta = 15.8^\circ \qquad V_x' = P\cos\beta \qquad V_y' = -P\sin\beta$$

$$A(y) = (2a - y)t \qquad \bar{y} = \frac{1}{2}(2a + y), \qquad \bar{x} = 0$$

Coordinate transformation.

$$y' = \left(y - \frac{2}{3}a\right)\cos\beta - \left(x - \frac{1}{6}a\right)\sin\beta$$

$$x' = \left(x - \frac{1}{6}a\right)\cos\beta + \left(y - \frac{2}{3}a\right)\sin\beta$$

In particular,

$$\bar{y}' = \left(\bar{y} - \frac{2}{3}a\right)\cos\beta - \left(\bar{x} - \frac{1}{6}a\right)\sin\beta$$

$$= \left(\frac{1}{2}y + \frac{1}{3}a\right)\cos\beta - \left(-\frac{1}{6}a\right)\sin\beta$$

$$= 0.48111y + 0.36612a$$

$$\bar{x}' = \left(\bar{x} - \frac{1}{6}a\right)\cos\beta + \left(\bar{y} - \frac{2}{3}a\right)\sin\beta$$

$$= \left(-\frac{1}{6}a\right)\cos\beta + \left(\frac{1}{2}y + \frac{1}{3}a\right)\sin\beta$$

$$= 0.13614y - 0.06961a$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

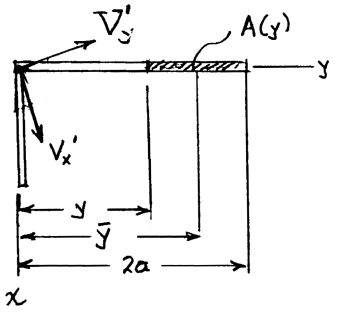
PROBLEM 6.87* *(Continued)*

$$+\rightarrow \tau = \frac{V_{x'}A\bar{x}'}{I_{y'}t} + \frac{V_{y'}A\bar{y}'}{I_{x'}t}$$

$$= \frac{(P\cos\beta)(2a-y)(t)(0.13614y - 0.06961a)}{(0.1557\ ta^3)(t)}$$

$$+ \frac{(-P\sin\beta)(2a-y)(0.48111y + 0.36612a)}{(1.428a^3t)(t)}$$

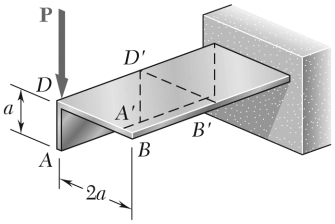
$$= \frac{P(2a-y)(0.750y - 0.500a)}{ta^3}$$

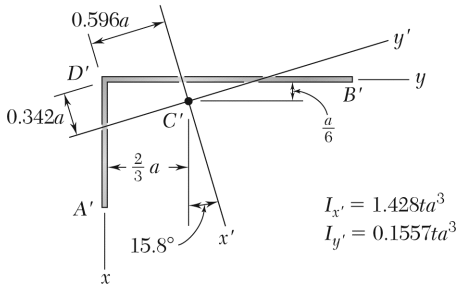


$y(a)$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$-\tau\left(\frac{P}{at}\right)$	–1.000	–0.417	0	0.250	0.333	0.250	0

◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 6.88*

For the angle shape and loading of Prob. 6.87, determine the distribution of the shearing stresses along line $D'A'$ in the vertical leg.

PROBLEM 6.87* Determine the distribution of the shearing stresses along line $D'B'$ in the horizontal leg of the angle shape for the loading shown. The x' and y' axes are the principal centroidal axes of the cross section.

SOLUTION

$$\beta = 15.8^\circ \qquad V_{x'} = P\cos\beta$$
$$V_{x'} = -P\sin\beta \qquad A(x) = (a-x)t$$
$$\bar{x} = \frac{1}{2}(a+x), \quad \bar{y} = 0$$

Coordinate transformation.

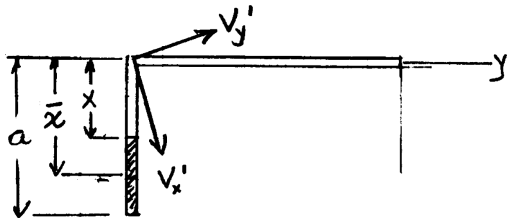
$$y' = \left(y - \frac{2}{3}a\right)\cos\beta - \left(x - \frac{1}{6}a\right)\sin\beta$$
$$x' = \left(x - \frac{1}{6}a\right)\cos\beta + \left(y - \frac{2}{3}a\right)\sin\beta$$

In particular,

$$\bar{y}' = \left(\bar{y} - \frac{2}{3}a\right)\cos\beta - \left(\bar{x} - \frac{1}{6}a\right)\sin\beta$$
$$= \left(-\frac{2}{3}a\right)\cos\beta - \left(\frac{1}{2}x + \frac{1}{3}a\right)\sin\beta$$
$$= -0.13614x - 0.73224a$$
$$\bar{x}' = \left(\bar{x} - \frac{1}{6}a\right)\cos\beta + \left(\bar{y} - \frac{2}{3}a\right)\sin\beta$$
$$= \left(\frac{1}{2}x + \frac{1}{3}a\right)\cos\beta + \left(-\frac{2}{3}a\right)\sin\beta$$
$$= 0.48111x + 0.13922a$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.88* (*Continued*)



$$+\downarrow \tau = \frac{V_{x'}A(x)\bar{x}'}{I_y' t} - \frac{V_{y'}A(x)\bar{y}'}{I_{x'} t}$$

$$= \frac{(P\cos\beta)(a-x)(t)(0.48111x + 0.13922a)}{(0.1557\,ta^3)(t)}$$

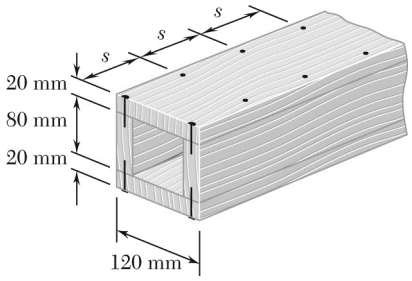
$$+ \frac{(-P\sin\beta)(a-x)(t)(-0.13614x - 0.73224a)}{(1.428\,ta^3)(t)}$$

$$= \frac{P(a-x)(3.00x + 1.000a)}{ta^3}$$

$x(a)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$\tau \downarrow \left(\frac{P}{at}\right)$	1.000	1.250	1.333	1.250	1.000	0.583	0

◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.89

A square box beam is made of two 20×80-mm planks and two 20×120-mm planks nailed together as shown. Knowing that the spacing between the nails is $s=30$ mm and that the vertical shear in the beam is $V=1200$ N, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.

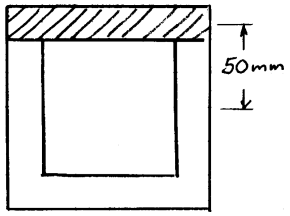
SOLUTION

$$I=\frac{1}{12}b_2h_2^3-\frac{1}{12}b_1h_1^3$$
$$=\frac{1}{12}(120)(120)^3-\frac{1}{12}(80)(80)^3=13.8667\times10^6\,\text{mm}^4$$
$$=13.8667\times10^{-6}\,\text{m}^4$$

(*a*) $A_1=(120)(20)=2400\ \text{mm}^2$

$\bar{y}_1=50$ mm

$Q_1=A_1\bar{y}_1=120\times10^3\ \text{mm}^3=120\times10^{-6}\,\text{m}^3$



$$q=\frac{VQ}{I}=\frac{(1200)(120\times10^{-6})}{13.8667\times10^{-6}}=10.385\times10^3\ \text{N/m}$$

$qs=2F_{\text{nail}}$

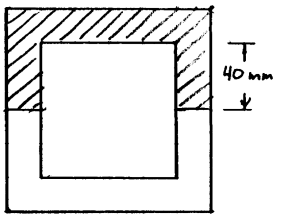
$$F_{\text{nail}}=\frac{qs}{2}=\frac{(10.385\times10^3)(30\times10^{-3})}{2}$$

$F_{\text{nail}}=155.8$ N ◀

(*b*) $Q=Q_1+(2)(20)(40)(20)$

$=120\times10^3+32\times10^3=152\times10^3\ \text{mm}^3$

$=152\times10^{-6}\ \text{m}^3$

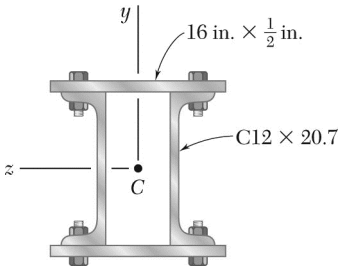


$$\tau_{\max}=\frac{VQ}{It}=\frac{(1200)(152\times10^{-6})}{(13.8667\times10^{-6})(2\times20\times10^{-3})}$$

$=329\times10^3\,\text{Pa}$

$\tau_{\max}=329$ kPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.90

The beam shown is fabricated by connecting two channel shapes and two plates, using bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 7.5 in. Determine the average shearing stress in the bolts caused by a shearing force of 25 kips parallel to the *y*-axis.

SOLUTION

C12×20.7: $d = 12.00$ in., $I_x = 129$ in^4

For top plate, $\bar{y} = \dfrac{12.00}{2} + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = 6.25$ in.

$$I_t = \frac{1}{12}(16)\left(\frac{1}{2}\right)^3 + (16)\left(\frac{1}{2}\right)(6.25)^2 = 312.667 \text{ in}^4$$

For bottom plate, $I_b = 312.667$ in^4

Moment of inertia of fabricated beam:

$$I = (2)(129) + 312.667 + 312.667$$
$$= 883.33 \text{ in}^4$$

$$Q = A_{\text{plate}}\,\bar{y}_{\text{plate}} = (16)\left(\frac{1}{2}\right)(6.25) = 50 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(25)(50)}{883.33} = 1.41510 \text{ kips/in}$$

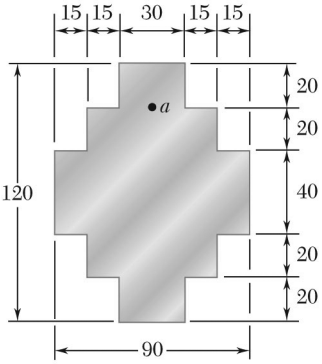
$$F_{\text{bolt}} = \frac{1}{2}qs = \left(\frac{1}{2}\right)(1.41510)(7.5) = 5.3066 \text{ kips}$$

$$A_{\text{bolt}} = \frac{\pi}{4}(d_{\text{bolt}})^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

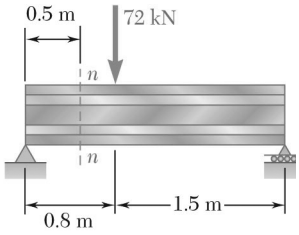
$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{5.3066}{0.44179} = 12.01 \text{ ksi}$$

$\tau_{\text{bolt}} = 12.01$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



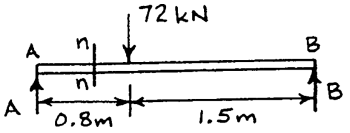
Dimensions in mm



PROBLEM 6.91

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad -2.3A + (1.5)(72) = 0$$

$$A = 46.957 \text{ kN} \uparrow$$

At section *n-n*, $V = A = 46.957$ kN

Calculate moment of inertia:

$$I = 2\left[\frac{1}{12}(15)(40)^3\right] + 2\left[\frac{1}{12}(15)(80)^3\right] + \frac{1}{12}(30)(120^3)$$

$$= 5.76\times10^{6}\,\text{mm}^4 = 5.76\times10^{-6}\,\text{m}^4$$

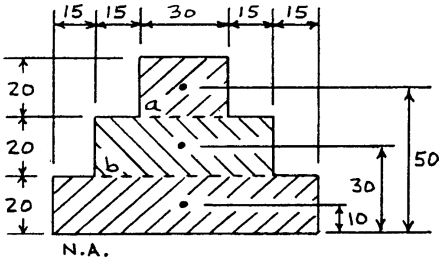
At *a*, $t_a = 30 \text{ mm} = 0.030 \text{ m}$

$$Q_a = (30\times20)(50) = 30\times10^{3}\,\text{mm}^3$$

$$= 30\times10^{-6}\,\text{m}^3$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(46.957\times10^{3})(30\times10^{-6})}{(5.76\times10^{-6})(0.030)}$$

$$= 8.15\times10^{6} \text{ Pa} = 8.15 \text{ MPa}$$



At *b*, $t_b = 60 \text{ mm} = 0.060 \text{ m}$

$$Q_b = Q_a + (60\times20)(30) = 30\times10^{3} + 36\times10^{3} = 66\times10^{3}\,\text{mm}^3 = 66\times10^{-6}\,\text{m}^4$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(46.957\times10^{3})(66\times10^{-6})}{(5.76\times10^{-6})(0.060)} = 8.97\times10^{6}\,\text{Pa} = 8.97 \text{ MPa}$$

At *NA*, $t_{\text{NA}} = 90 \text{ mm} = 0.090 \text{ m}$

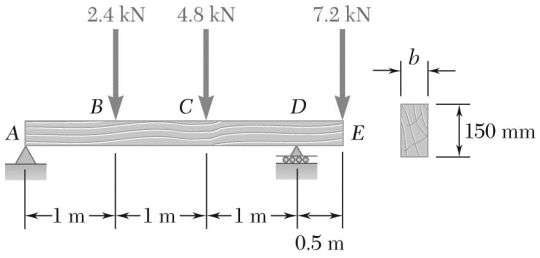
$$Q_{\text{NA}} = Q_b + (90\times20)(10) = 66\times10^{3} + 18\times10^{3} = 84\times10^{3}\,\text{mm}^3 = 84\times10^{-6}\,\text{m}^3$$

$$\tau_{\text{NA}} = \frac{VQ_{\text{NA}}}{It_{\text{NA}}} = \frac{(46.957\times10^{3})(84\times10^{-6})}{(5.76\times10^{-6})(0.090)} = 7.61\times10^{6}\,\text{Pa} = 7.61 \text{ MPa}$$

(*a*) $\tau_{\max}$ occurs at *b*. $\tau_{\max} = 8.97$ MPa ◀

(*b*) $\tau_a = 8.15$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.92

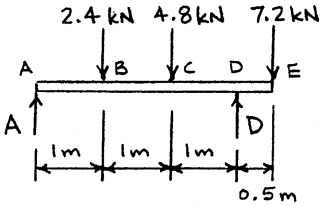
For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.

SOLUTION

$$+\circlearrowleft \Sigma M_D = 0: \; -3A + (2)(2.4) + (1)(4.8) - (0.5)(7.2) = 0$$

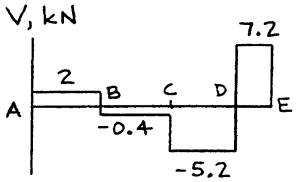
$$A = 2 \text{ kN} \uparrow$$

Draw the shear and bending moment diagrams.



$$|V|_{\max} = 7.2 \text{ kN} = 7.2 \times 10^3 \text{ N}$$

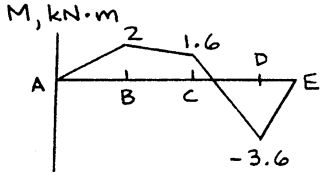
$$|M|_{\max} = 3.6 \text{ kN} \cdot \text{m} = 3.6 \times 10^3 \text{ N} \cdot \text{m}$$



Bending:

$$\sigma = \frac{M}{S}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma} = \frac{3.6 \times 10^3}{12 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$$



For a rectangular section,

$$S = \frac{1}{6} bh^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(300 \times 10^3)}{(150)^2} = 80 \text{ mm}$$

Shear: Maximum shearing stress occurs at the neutral axis of bending for a rectangular section.

$$A = \frac{1}{2} bh, \quad \bar{y} = \frac{1}{4} h, \quad Q = A\bar{y} = \frac{1}{8} bh^2$$

$$I = \frac{1}{12} bh^3 \quad t = b$$

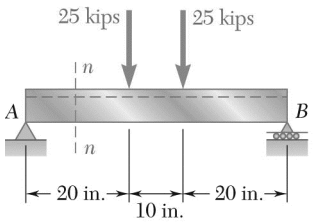
$$\tau = \frac{VQ}{It} = \frac{V(\frac{1}{8} bh^2)}{(\frac{1}{12} bh^3)(b)} = \frac{3}{2} \frac{V}{bh}$$

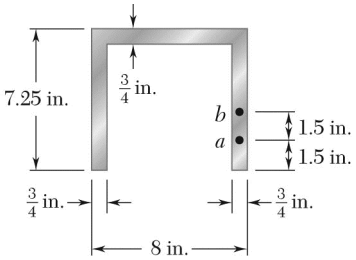
$$b = \frac{3V}{2h\tau} = \frac{(3)(7.2 \times 10^3)}{(2)(150 \times 10^{-3})(825 \times 10^3)} = 87.3 \times 10^{-3} \text{ m}$$

The required value of b is the larger one.

$$b = 87.3 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



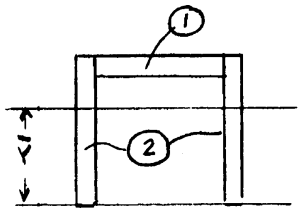


PROBLEM 6.93

For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$R_A = R_B = 25 \text{ kips}$$



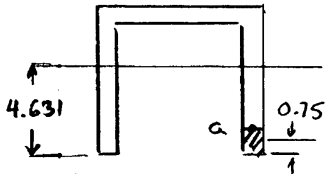
At section *n-n*, $V = 25$ kips

Locate centroid and calculate moment of inertia.

Part	$A(\text{in}^2)$	$\bar{y}(\text{in.})$	$A\bar{y}(\text{in}^3)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

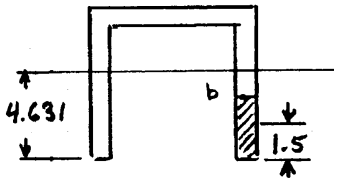
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$



(*a*) $$Q_a = A\bar{y} = \left(\frac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} \qquad \tau_a = 1.745 \text{ ksi} \blacktriangleleft$$

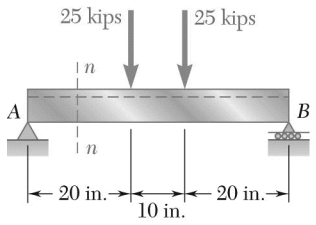


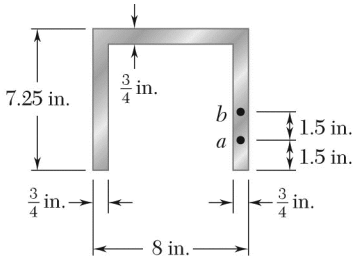
(*b*) $$Q_b = A\bar{y} = \left(\frac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

$$t = 0.75 \text{ in.}$$

$$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} \qquad \tau_b = 2.82 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 6.94

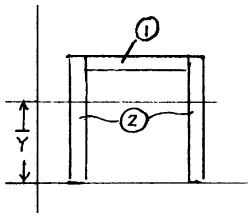
For the beam and loading shown, determine the largest shearing stress in section *n-n*.

SOLUTION

$$R_A = R_B = 25 \text{ kips}$$

At section *n-n*, $V = 25$ kips.

Locate centroid and calculate moment of inertia.

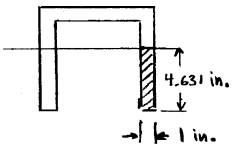


Part	$A(\text{in}^2)$	$\bar{y}$(in.)	$A\bar{y}(\text{in}^3)$	d(in.)	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



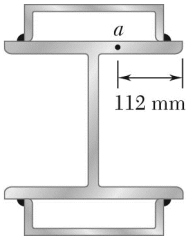
$$Q = Ay = \left(\frac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)}$$

$$\tau_m = 3.21 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.95

The composite beam shown is made by welding C200×17.1 rolled-steel channels to the flanges of a W250×80 wide-flange rolled-steel shape. Knowing that the beam is subjected to a vertical shear of 200 kN, determine (*a*) the horizontal shearing force per meter at each weld, (*b*) the shearing stress at point *a* of the flange of the wide-flange shape.

SOLUTION

For W250 × 80, $d = 257\,\text{mm}$, $t_f = 15.6\ \text{mm}$, $I_x = 126 \times 10^6\ \text{mm}^4$

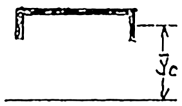
For C200 × 17.1, $A = 2170\,\text{mm}^2$, $b_f = 57.4\ \text{mm}$, $t_f = 9.91\ \text{mm}$

$$I_y = 0.545 \times 10^6\,\text{mm}^4, \quad \bar{x} = 14.5\ \text{mm}$$

For the channel in the composite beam,

$$\bar{y}_c = \frac{257}{2} + 57.4 - 14.5 = 171.4\ \text{mm}$$

For the composite beam,



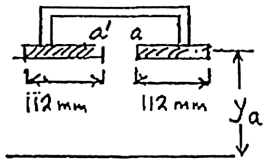
$$I = 126 \times 10^6 + 2\left[0.545 \times 10^6 + (2170)(171.4)^2\right]$$

$$= 254.59 \times 10^6\ \text{mm}^4 = 254.59 \times 10^{-6}\ \text{m}^4$$

(*a*) For the two welds,

$$Q_w = A\bar{y}_c = (2170)\,(171.4) = 371.94 \times 10^3\ \text{mm}^3 = 371.94 \times 10^{-6}\ \text{m}^3$$

$$q = \frac{VQ}{I} = \frac{(200 \times 10^3)\,(371.94 \times 10^{-6})}{254.59 \times 10^{-6}} = 292.2 \times 10^3\ \text{N/m}$$



For one weld, $\dfrac{q}{2} = 146.1 \times 10^3\ \text{N/m}$

Shearing force per meter of weld: 146.1 kN/m ◀

(*b*) For cuts at *a* and *a'* together,

$$A_a = 2(112)\,(15.6) = 3494.4\ \text{mm}^2 \qquad \bar{y}_a = \frac{257}{2} - \frac{15.6}{2} = 120.7\ \text{mm}$$

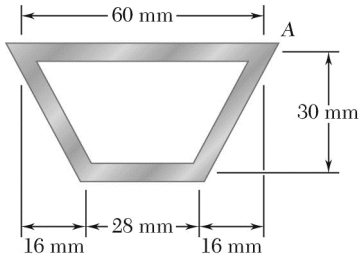
$$Q_a = 371.94 \times 10^3 + (3494.4)(120.7) = 793.71 \times 10^3\ \text{mm}^3 = 793.71 \times 10^{-6}\ \text{m}^3$$

Since there are cuts at *a* and *a'*, $t = 2t_f = (2)(15.6) = 31.2\ \text{mm} = 0.0312\ \text{m}$.

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(793.71 \times 10^{-6})}{(254.59 \times 10^6)(0.0312)} = 19.99 \times 10^6\ \text{Pa}$$

$\tau_a = 19.99\ \text{MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.96

An extruded beam has the cross section shown and a uniform wall thickness of 3 mm. For a vertical shear of 10 kN, determine (*a*) the shearing stress at point *A*, (*b*) the maximum shearing stress in the beam. Also, sketch the shear flow in the cross section.

SOLUTION

$$\tan\alpha = \frac{16}{30} \qquad \alpha = 28.07^\circ$$

Side:
$$A = (3\sec\alpha)(30) = 102 \text{ mm}^2$$
$$\bar{I} = \frac{1}{12}(3\sec\alpha)(30)^3 = 7.6498\times10^3 \text{ mm}^4$$

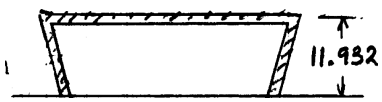
Part	A (mm^2)	$\bar{y}_0$ (mm)	$A\bar{y}$ (10^3 mm^3)	d (mm)	Ad^2 (10^3 mm^4)	$\bar{I}$ (10^3 mm^4)
Top	180	30	5.4	11.932	25.627	neglect
Side	102	15	1.53	3.077	0.966	7.6498
Side	102	15	1.53	3.077	0.966	7.6498
Bot	84	0	0	18.077	27.449	neglect
Σ	468		8.46		55.008	15.2996

$$\bar{Y}_0 = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8.46\times10^3}{468} = 18.077 \text{ mm}$$
$$I = \Sigma Ad^2 + \Sigma\bar{I} = 70.31\times10^3 \text{ mm}^4 = 70.31\times10^{-9} \text{ m}^4$$

(*a*) $$Q_A = (180)(11.932) = 2.14776\times10^3 \text{ mm}^3 = 2.14776\times10^{-6} \text{ m}^3$$

$$t = (2)(3\times10^{-3}) = 6\times10^{-3} \text{ m}$$
$$\tau_A = \frac{VQ}{It} = \frac{(10\times10^3)(2.14776\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-6})} = 50.9\times10^6 \text{ Pa}$$
$$\tau_A = 50.9 \text{ MPa} \blacktriangleleft$$

(*b*) $$Q_m = Q_A + (2)(3\sec\alpha)(11.932)\left(\frac{1}{2}\times11.932\right)$$
$$= 2.14776\times10^3 + 484.06 = 2.6318\times10^3 \text{ mm}^3$$
$$= 2.6318\times10^{-6} \text{ m}^3$$



$$t = 6\times10^{-3} \text{ m}$$
$$\tau_m = \frac{VQ_m}{It} = \frac{(10\times10^3)(2.6318\times10^{-6})}{(70.31\times10^{-9})(6\times10^{-3})} = 62.4\times10^6 \text{ Pa}$$
$$\tau_m = 62.4 \text{ MPa} \blacktriangleleft$$

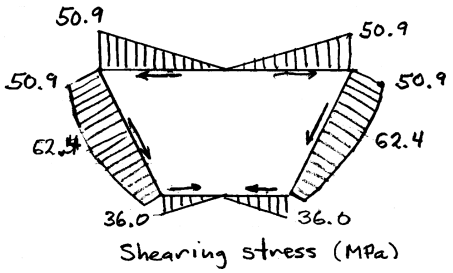
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 6.96 *(Continued)*

$$Q_B = (28)(3)(18.077) = 1.51847 \times 10^3 \text{ mm}^3$$

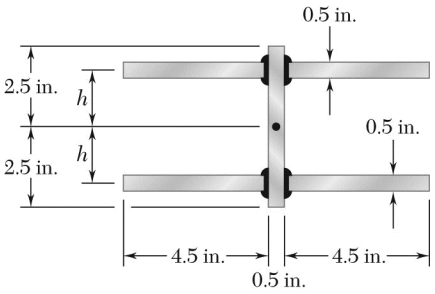
$$\tau_B = \frac{Q_B}{Q_A}\tau_A = \frac{1.51847 \times 10^3}{2.14776 \times 10^3}(50.9)$$

$$= 36.0 \text{ MPa}$$



Multiply shearing stresses by $t(3 \text{ mm} = 0.003 \text{ m})$ to get shear flow.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.97

The design of a beam requires welding four horizontal plates to a vertical 0.5×5-in. plate as shown. For a vertical shear **V,** determine the dimension h for which the shear flow through the welded surface is maximum.

SOLUTION

Horizontal plate:

$$I_h = \frac{1}{12}(4.5)(0.5)^3 + (4.5)(0.5)h^2$$
$$= 0.046875 + 2.25h^2$$

Vertical plate: $I_v = \frac{1}{12}(0.5)(5)^3 = 5.2083 \text{ in}^4$

Whole section: $I = 4I_h + I_v = 9h^2 + 5.39583 \text{ in}^4$

For one horizontal plate, $Q = (4.5)(0.5)h = 2.25h \text{ in}^3$

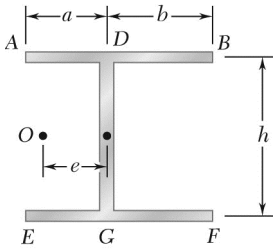
$$q = \frac{VQ}{I} = \frac{2.25Vh}{9h^2 + 5.39583}$$

To maximize q, set $\frac{dq}{dh} = 0.$

$$2.25V\frac{(9h^2 + 5.39583) - 18h^2}{(9h^2 + 5.39583)^2} = 0$$

$h = 0.774$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



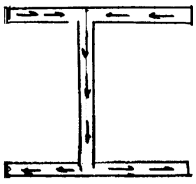
PROBLEM 6.98

Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION

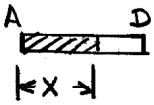
$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$



Part *AD*:

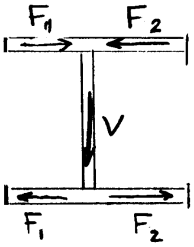
$$Q = tx\frac{h}{2} = \frac{1}{2}thx$$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

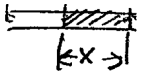
$$F_1 = \int \tau dA = \int_0^a \frac{Vhx}{2I}tdx = \frac{Vht}{2I}\int_0^a xdx$$

$$= \frac{Vht}{2I}\frac{x^2}{2}\bigg|_0^a = \frac{Vhta^2}{4I}$$



Part *BD*:

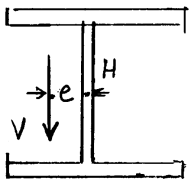
$$Q = tx\frac{h}{2} = \frac{1}{2}thx$$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau dA = \int_0^b \frac{Vhx}{2I}tdx = \frac{Vht}{2I}\int_0^b xdx$$

$$= \frac{Vht}{2I}\frac{x^2}{2}\bigg|_0^b = \frac{Vhtb^2}{4I}$$



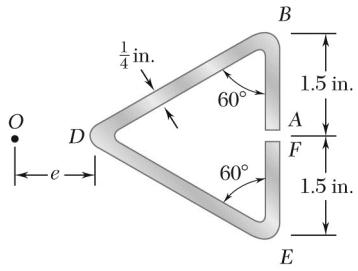
$$+\circlearrowright \Sigma M_H = +\circlearrowright \Sigma M_H:$$

$$Ve = F_2 h - F_1 h = \frac{Vh^2 t(b^2 - a^2)}{4I}$$

$$= \frac{Vh^2 t(b^2 - a^2)}{4 \cdot \frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2 - a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2 - a^2)}{6(a+b)+h} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 6.99

Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

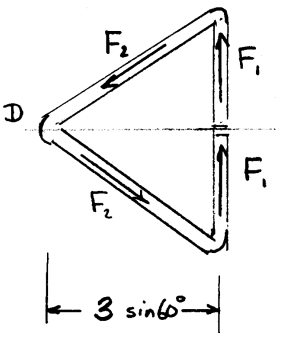
SOLUTION

$$I_{AB} = \frac{1}{3}\left(\frac{1}{4}\right)(1.5)^3 = 0.28125 \text{ in}^4$$

$$L_{BD} = 3 \text{ in.} \quad A_{BD} = (3)\left(\frac{1}{4}\right) = 0.75 \text{ in}^2$$

$$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$



Part AB:

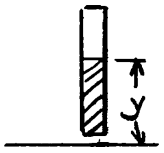
$$A = \frac{1}{4}y \quad \bar{y} = \frac{1}{2}y \quad Q = A\bar{y} = \frac{1}{8}y^2$$

$$\tau = \frac{VQ}{It} = \frac{Vy^2}{(8)(1.6875)(0.25)} = \frac{Vy^2}{3.375}$$

$$F_1 = \int \tau dA = \int_0^{1.5} \frac{Vy^2}{3.375}(0.25dy)$$

$$= \frac{(0.25)V}{3.375}\left.\frac{y^3}{3}\right|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.08333V$$

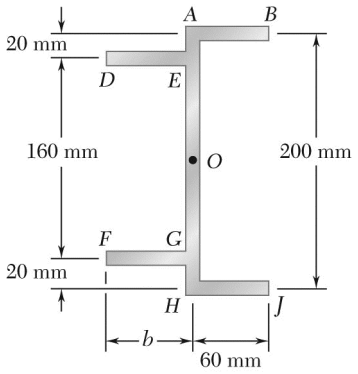


$$+\circlearrowright M_D = +\circlearrowright M_D: \quad Ve = 2F_1(3 \sin 60°)$$

$$Ve = (2)(0.08333)V(3 \sin 60°)$$

$$e = (2)(0.08333)(3 \sin 60°) \qquad e = 0.433 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



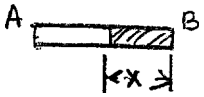
PROBLEM 6.100

A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.

SOLUTION

Part AB:

$$A = tx, \quad \bar{y} = 100 \text{ mm}, \quad Q = 100\ tx$$



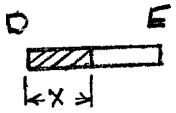
$$q = \frac{VQ}{I} = \frac{100Vtx}{I}$$

$$F_1 = \int_{x_A}^{x_B} q\,dx = 100\frac{Vt}{I}\int_0^{60} x\,dx$$

$$= \frac{(100)(60)^2}{2}\frac{Vt}{I} = 180\times10^3\frac{Vt}{I}$$

Part DE:

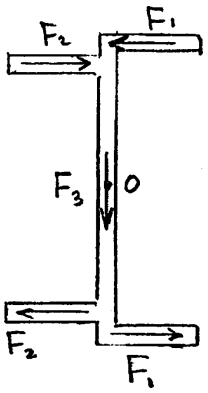
$$A = tx, \quad \bar{y} = 80 \text{ mm}, \quad Q = 80\ tx$$



$$q = \frac{VQ}{I} = \frac{80\ Vtx}{I}$$

$$F_2 = \int_{x_D}^{x} q\,dx = 80\frac{Vt}{I}\int_0^{b} x\,dx$$

$$= (40b^2)\frac{Vt}{I}$$



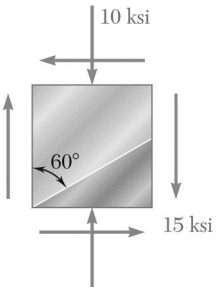
$$+\circlearrowleft \Sigma M_0 = 0: \quad (200)(180\times10^3)\frac{Vt}{I} - (160)(40b^2)\frac{Vt}{I} = 0$$

$$b^2 = \frac{(200)(180\times10^3)}{(160)(40)} = 5625 \text{ mm}^2$$

$$b = 75.0 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

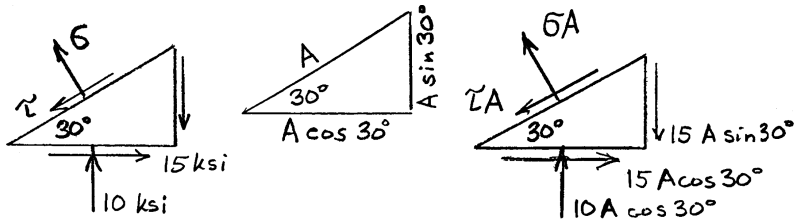
CHAPTER 7



PROBLEM 7.1

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



Stresses Areas Forces

$+\nwarrow \Sigma F = 0: \quad \sigma A - 15A \sin 30° \cos 30° - 15A \cos 30° \sin 30° + 10A \cos 30 \cos 30° = 0$

$$\sigma = 30 \sin 30° \cos 30° - 10 \cos^2 30°$$

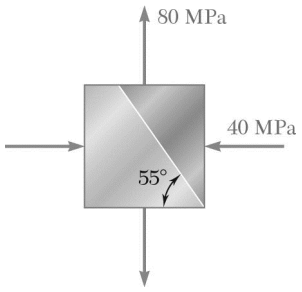
$$\sigma = 5.49 \text{ ksi} \blacktriangleleft$$

$+\nearrow \Sigma F = 0: \quad \tau A + 15A \sin 30° \sin 30° - 15A \cos 30° \cos 30° - 10A \cos 30° \sin 30° = 0$

$$\tau = 15(\cos^2 30° - \sin^2 30°) + 10 \cos 30° \sin 30°$$

$$\tau = 11.83 \text{ ksi} \blacktriangleleft$$

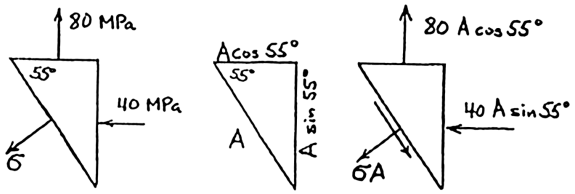
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



Stresses Areas Forces

$+\swarrow \Sigma F = 0$:

$$\sigma A - 80A\cos 55^\circ \cos 55^\circ + 40A \sin 55^\circ \sin 55^\circ = 0$$

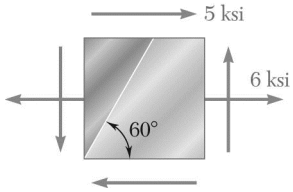
$$\sigma = 80\cos^2 55^\circ - 40\sin^2 55^\circ \qquad \sigma = -0.521 \text{ MPa} \blacktriangleleft$$

$+\searrow \Sigma F = 0$:

$$\tau A - 80A \cos 55^\circ \sin 55^\circ - 40A \sin 55^\circ \cos 55^\circ$$

$$\tau = 120 \cos 55^\circ \sin 55^\circ \qquad \tau = 56.4 \text{ MPa} \blacktriangleleft$$

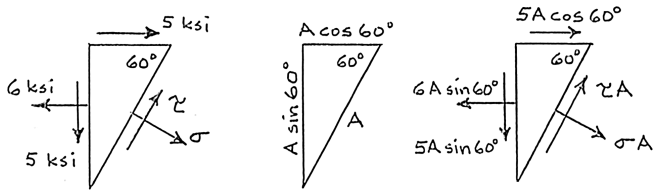
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.3

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



Stresses Areas Forces

$+\searrow \Sigma F = 0$:

$$\sigma A + 5A\cos 60^\circ \sin 60^\circ - 6A\sin 60^\circ \sin 60^\circ + 5A\sin 60^\circ \cos 60^\circ = 0$$

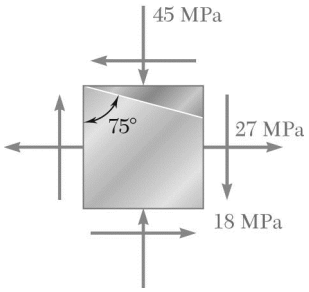
$$\sigma = 6\sin^2 60^\circ - 10\cos 60^\circ \sin 60^\circ \qquad \sigma = 0.1699 \text{ ksi} \blacktriangleleft$$

$+\nearrow \Sigma F = 0$:

$$\tau A + 5A\cos 60^\circ \cos 60^\circ - 6A\sin 60^\circ \cos 60^\circ - 5A\sin 60^\circ \sin 60^\circ = 0$$

$$\tau = 5(\sin^2 60^\circ - \cos^2 60^\circ) + 6\sin 60^\circ \cos 60^\circ \qquad \tau = 5.10 \text{ ksi} \blacktriangleleft$$

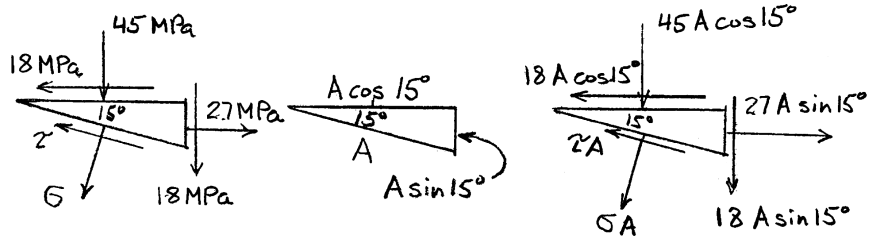
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.4

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



Stresses Areas Forces

$$+\swarrow \Sigma F = 0: \quad \sigma A + 18A \cos 15^\circ \sin 15^\circ + 45A \cos 15^\circ \cos 15^\circ - 27A \sin 15^\circ \sin 15^\circ + 18A \sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -18 \cos 15^\circ \sin 15^\circ - 45 \cos^2 15^\circ + 27 \sin^2 15^\circ - 18 \sin 15^\circ \cos 15^\circ$$

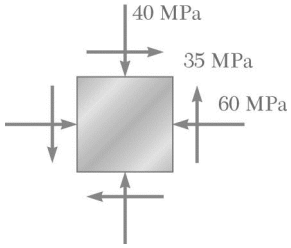
$$\sigma = -49.2 \text{ MPa} \blacktriangleleft$$

$$+\nwarrow \Sigma F = 0: \quad \tau A + 18A \cos 15^\circ \cos 15^\circ - 45A \cos 15^\circ \sin 15^\circ - 27A \sin 15^\circ \cos 15^\circ - 18A \sin 15^\circ \sin 15^\circ = 0$$

$$\tau = -18(\cos^2 15^\circ - \sin^2 15^\circ) + (45 + 27) \cos 15^\circ \sin 15^\circ$$

$$\tau = 2.41 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.5

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60+40} = -3.50$$

$$2\theta_p = -74.05^\circ$$

$$\theta_p = -37.0^\circ, \quad 53.0^\circ \blacktriangleleft$$

(*b*) $$\sigma_{\max,\,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

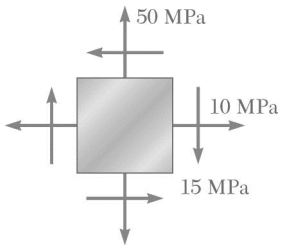
$$= \frac{-60-40}{2} \pm \sqrt{\left(\frac{-60+40}{2}\right)^2 + (35)^2}$$

$$= -50 \pm 36.4 \text{ MPa}$$

$$\sigma_{\max} = -13.60 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -86.4 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.6

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 10\,\text{MPa} \quad \sigma_y = 50\ \text{MPa} \quad \tau_{xy} = -15\,\text{MPa}$$

(*a*) $$\tan 2\theta_p = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-15)}{10 - 50} = 0.750$$

$$2\theta_p = 36.8699°$$

$$\theta_p = 18.4°,\ 108.4° \blacktriangleleft$$

(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

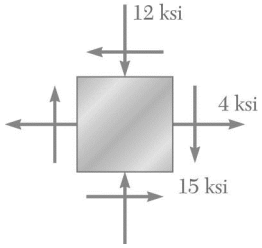
$$= \frac{10 + 50}{2} \pm \sqrt{\left(\frac{10 - 50}{2}\right)^2 + (-15)^2}$$

$$= 30 \pm 25$$

$$\sigma_{\max} = 55.0\ \text{ksi} \blacktriangleleft$$

$$\sigma_{\min} = 5.00\ \text{ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.7

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 4 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -15 \text{ ksi}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-15)}{4+12} = -1.875$$

$$2\theta_p = -61.93°$$

$$\theta_p = -31.0°, \quad 59.0° \blacktriangleleft$$

(*b*) $$\sigma_{\max,\,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

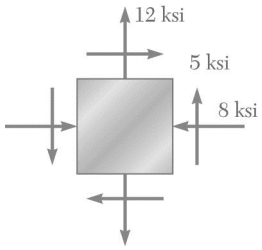
$$= \frac{4-12}{2} \pm \sqrt{\left(\frac{4+12}{2}\right)^2 + 15^2}$$

$$= -4 \pm 17$$

$$\sigma_{\max} = 13.00 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = -21.0 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.8

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = -8 \text{ ksi} \quad \sigma_y = 12 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(5)}{-8 - 12} = -0.5$$

$$2\theta_p = -26.5651°$$

$$\theta_p = -13.3°, \quad 76.7° \blacktriangleleft$$

(*b*) $$\sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

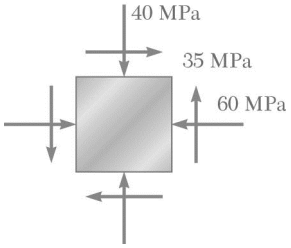
$$= \frac{-8 + 12}{2} \pm \sqrt{\left(\frac{-8 - 12}{2}\right)^2 + (5)^2}$$

$$= 2 \pm 11.1803$$

$$\sigma_{\max} = 13.18 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = -9.18 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.9

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-60 + 40}{(2)(35)} = 0.2857$$

$$2\theta_s = 15.95°$$

$$\theta_s = 8.0°, \quad 98.0° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

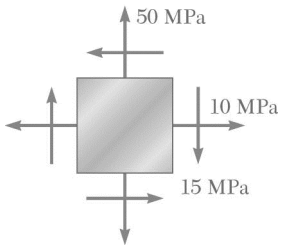
$$= \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2}$$

$$\tau_{\max} = 36.4 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60 - 40}{2}$$

$$\sigma' = -50.0 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.10

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 10 \text{ MPa} \quad \sigma_y = 50 \text{ MPa} \quad \tau_{xy} = -15 \text{ MPa}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{10 - 50}{2(-15)} = -1.33333$$

$$2\theta_s = -53.130° \qquad \theta_s = -26.6°, \quad 63.4° \blacktriangleleft$$

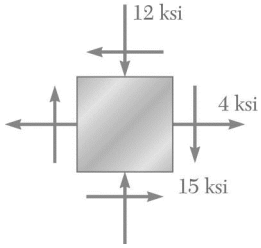
(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{10 - 50}{2}\right)^2 + (-15)^2} \qquad \tau_{\max} = 25.0 \text{ MPa} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{10 + 50}{2} \qquad \sigma' = 30.0 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.11

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 4 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -15 \text{ ksi}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{4+12}{(2)(-15)} = 0.53333$$

$$2\theta_s = 28.07°$$

$$\theta_s = 14.0°, \quad 104.0° \blacktriangleleft$$

(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

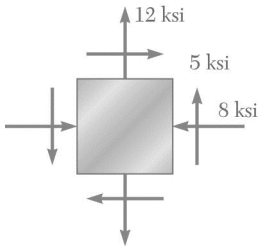
$$= \sqrt{\left(\frac{4+12}{2}\right)^2 + (-15)^2}$$

$$\tau_{\max} = 17.00 \text{ ksi} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{4-12}{2}$$

$$\sigma' = -4.00 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.12

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -8 \text{ ksi} \quad \sigma_y = 12 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

(*a*) $$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-8 - 12}{2(5)} = +2.0$$

$$2\theta_s = 63.435^\circ \qquad \theta_s = 31.7^\circ, \quad 121.7^\circ \blacktriangleleft$$

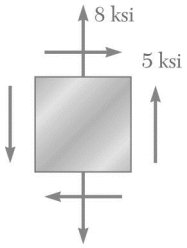
(*b*) $$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-8 - 12}{2}\right)^2 + (5)^2} \qquad \tau_{\max} = 11.18 \text{ ksi} \blacktriangleleft$$

(*c*) $$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{-8 + 12}{2} \qquad \sigma' = 2.00 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.13

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = 8 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = -4 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$$\sigma_{x'} = 4 - 4 \cos(-50°) + 5 \sin(-50°) \qquad \sigma_{x'} = -2.40 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = 4 \sin(-50°) + 5 \cos(-50°) \qquad \tau_{x'y'} = 0.15 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = 4 + 4 \cos(-50°) - 5 \sin(-50) \qquad \sigma_{y'} = 10.40 \text{ ksi} \blacktriangleleft$$

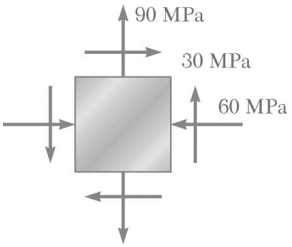
(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$$\sigma_{x'} = 4 - 4 \cos(20°) + 5 \sin(20°) \qquad \sigma_{x'} = 1.95 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = 4 \sin(20°) + 5 \cos(20°) \qquad \tau_{x'y'} = 6.07 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = 4 + 4 \cos(20°) - 5 \cos(20°) \qquad \sigma_{y'} = 6.05 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.14

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = 90 \text{ MPa} \quad \tau_{xy} = 30 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = -75 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$$\sigma_{x'} = 15 - 75 \cos(-50°) + 30 \sin(-50°) \qquad \sigma_{x'} = -56.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = +75 \sin(-50°) + 30 \cos(-50°) \qquad \tau_{x'y'} = -38.2 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75 \cos(-50°) - 30 \sin(-50°) \qquad \sigma_{y'} = 86.2 \text{ MPa} \blacktriangleleft$$

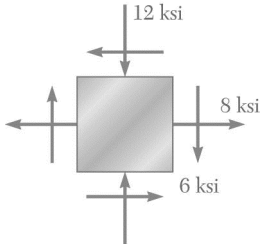
(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$$\sigma_{x'} = 15 - 75 \cos(20°) + 30 \sin(20°) \qquad \sigma_{x'} = -45.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = +75 \sin(20°) + 30 \cos(20°) \qquad \tau_{x'y'} = 53.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = 15 + 75 \cos(20°) - 30 \sin(20°) \qquad \sigma_{y'} = 75.2 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.15

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 8 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = 10 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25^\circ}$ $2\theta = -50^\circ$

$$\sigma_{x'} = -2 + 10 \cos(-50^\circ) - 6 \sin(-50^\circ) \qquad \sigma_{x'} = 9.02 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = -10 \sin(-50^\circ) - 6 \cos(-50^\circ) \qquad \tau_{x'y'} = 3.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = -2 - 10 \cos(-50^\circ) + 6 \sin(-50^\circ) \qquad \sigma_{y'} = -13.02 \text{ ksi} \blacktriangleleft$$

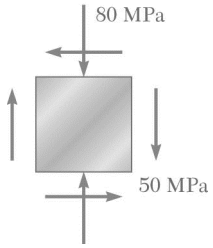
(*b*) $\underline{\theta = 10^\circ}$ $2\theta = 20^\circ$

$$\sigma_{x'} = -2 + 10 \cos(20^\circ) - 6 \sin(20^\circ) \qquad \sigma_{x'} = 5.34 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = -10 \sin(20^\circ) - 6 \cos(20^\circ) \qquad \tau_{x'y'} = -9.06 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = -2 - 10 \cos(20^\circ) + 6 \sin(20^\circ) \qquad \sigma_{y'} = -9.34 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.16

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25^\circ}$ $2\theta = -50^\circ$

$$\sigma_{x'} = -40 + 40\cos(-50^\circ) - 50\sin(-50^\circ) \qquad \sigma_{x'} = 24.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -40\sin(-50^\circ) - 50\cos(-50^\circ) \qquad \tau_{x'y'} = -1.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40\cos(-50^\circ) + 50\sin(-50^\circ) \qquad \sigma_{y'} = -104.0 \text{ MPa} \blacktriangleleft$$

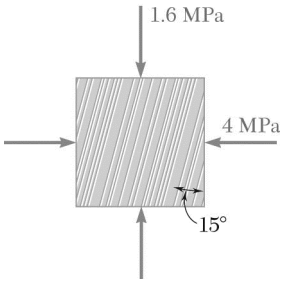
(*b*) $\underline{\theta = 10^\circ}$ $2\theta = 20^\circ$

$$\sigma_{x'} = -40 + 40\cos(20^\circ) - 50\sin(20^\circ) \qquad \sigma_{x'} = -19.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -40\sin(20^\circ) - 50\cos(20^\circ) \qquad \tau_{x'y'} = -60.7 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40\cos(20^\circ) + 50\sin(20^\circ) \qquad \sigma_{y'} = -60.5 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.17

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -4 \text{ MPa} \quad \sigma_y = -1.6 \text{ MPa} \quad \tau_{xy} = 0$$
$$\theta = -15° \qquad 2\theta = -30°$$

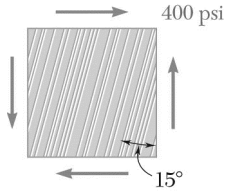
(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{-4 - (-1.6)}{2} \sin(-30°) + 0 \qquad \tau_{x'y'} = -0.600 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-4 + (-1.6)}{2} + \frac{-4 - (-1.6)}{2} \cos(-30°) + 0 \qquad \sigma_{x'} = -3.84 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.18

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = 400 \text{ psi}$$
$$\theta = -15° \quad 2\theta = -30°$$

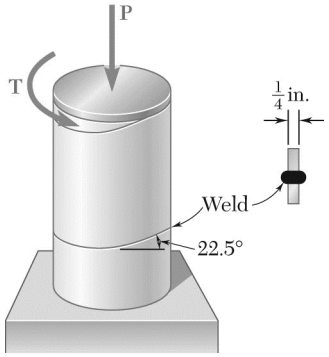
(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= 0 + 400\cos(-30°) \qquad \tau_{x'y'} = 346 \text{ psi} \blacktriangleleft$$

(*b*) $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= 0 + 0 + 400\sin(-30°) \qquad \sigma_{x'} = -200 \text{ psi} \blacktriangleleft$$

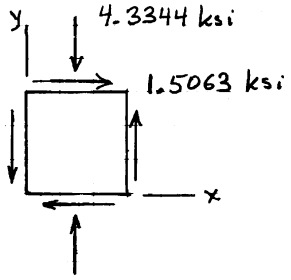
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.19

A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix that forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 40-kip axial force **P** and an 80-kip · in. torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

SOLUTION



$$d_2 = 12 \text{ in.}, \quad c_2 = \frac{1}{2} d_2 = 6 \text{ in.}, \quad t = 0.25 \text{ in.}$$

$$c_1 = c_2 - t = 5.75 \text{ in.}$$

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(6^2 - 5.75^2) = 9.2284 \text{ in}^2$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(6^4 - 5.75^4) = 318.67 \text{ in}^4$$

Stresses:

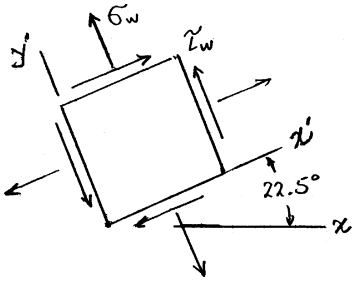
$$\sigma = -\frac{P}{A}$$

$$= -\frac{40}{9.2284} = -4.3344 \text{ ksi}$$

$$\tau = \frac{Tc_2}{J}$$

$$= \frac{(80)(6)}{318.67} = 1.5063 \text{ ksi}$$

$$\sigma_x = 0, \quad \sigma_y = -4.3344 \text{ ksi}, \quad \tau_{xy} = 1.5063 \text{ ksi}$$



Choose the x' and y' axes, respectively, tangential and normal to the weld.

Then $\sigma_w = \sigma_{y'}$ and $\tau_w = \tau_{x'y'}$ $\theta = 22.5°$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{(-4.3344)}{2} - \frac{[-(-4.3344)]}{2}\cos 45° - 1.5063 \sin 45°$$

$$= -4.76 \text{ ksi}$$

$\sigma_w = -4.76$ ksi ◀

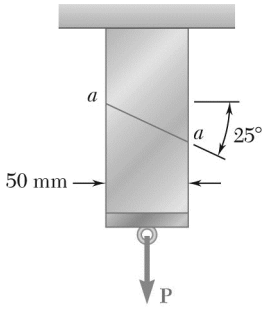
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{[-(-4.3344)]}{2}\sin 45° + 1.5063 \cos 45°$$

$$= -0.467 \text{ ksi}$$

$\tau_w = -0.467$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

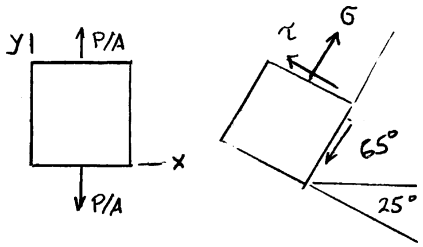


PROBLEM 7.20

Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION

For plane *a-a*, $\theta = 65°$.



$$\sigma_x = 0, \quad \tau_{xy} = 0, \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

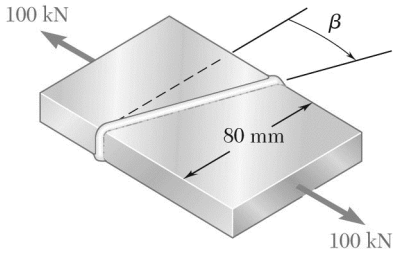
$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^{3})}{\sin^2 65°} = 3.90\times10^{3}\ \text{N}$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^{3})}{\sin 65°\cos 65°} = 6.27\times10^{3}\ \text{N}$$

Allowable value of *P* is the smaller one. $P = 3.90$ kN ◀

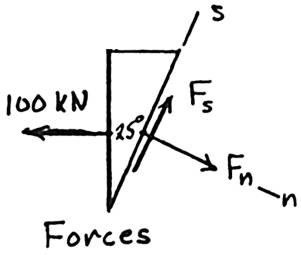
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.21

Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.

SOLUTION



Area of weld:

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos 25°}$$
$$= 882.7 \times 10^{-6}\ \text{m}^2$$

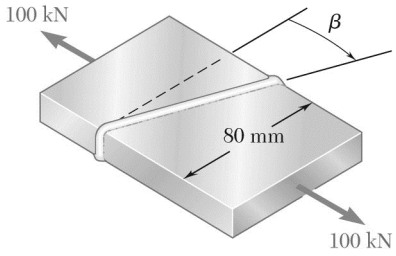
(*a*) $\sum F_s = 0$: $F_s - 100 \sin 25° = 0$ $\quad F_s = 42.26$ kN

$$\tau_w = \frac{F_s}{A_w} = \frac{42.26 \times 10^3}{882.7 \times 10^{-6}} = 47.9 \times 10^6\ \text{Pa} \qquad \tau_w = 47.9\ \text{MPa} \blacktriangleleft$$

(*b*) $\sum F_n = 0$: $F_n - 100 \cos 25° = 0$ $\quad F_n = 90.63$ kN

$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63 \times 10^3}{882.7 \times 10^{-6}} = 102.7 \times 10^6\ \text{Pa} \qquad \sigma_w = 102.7\ \text{MPa} \blacktriangleleft$$

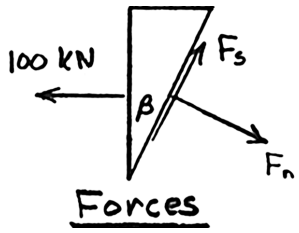
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.22

Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b)* the corresponding normal stress perpendicular to the weld.

SOLUTION



Area of weld:

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos\beta} = \frac{800 \times 10^{-6}}{\cos\beta}\ \text{m}^2$$

(*a*) $\sum F_s = 0$: $F_s - 100\sin\beta = 0$ $\quad F_s = 100\sin\beta\ \text{kN} = 100 \times 10^3 \sin\beta\ \text{N}$

$$\tau_w = \frac{F_s}{A_w} \quad 30 \times 10^6 = \frac{100 \times 10^3 \sin\beta}{800 \times 10^{-6} / \cos\beta} = 125 \times 10^6 \sin\beta\cos\beta$$

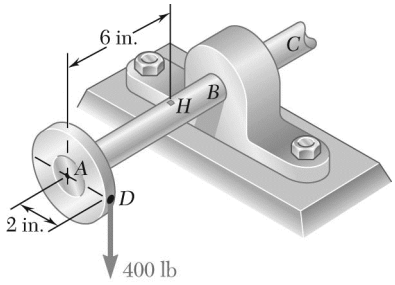
$$\sin\beta\cos\beta = \frac{1}{2}\sin 2\beta = \frac{30 \times 10^6}{125 \times 10^6} = 0.240 \qquad \beta = 14.34° \blacktriangleleft$$

(*b*) $\sum F_n = 0$: $F_n - 100\cos\beta = 0$ $\quad F_n = 100\cos 14.34° = 96.88\ \text{kN}$

$$A_w = \frac{800 \times 10^{-6}}{\cos 14.34} = 825.74 \times 10^{-6}\ \text{m}^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88 \times 10^3}{825.74 \times 10^{-6}} = 117.3 \times 10^6\ \text{Pa} \qquad \sigma = 117.3\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.23

A 400-lb vertical force is applied at *D* to a gear attached to the solid l-in. diameter shaft *AB*. Determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*:

$$V = 400 \text{ lb} \qquad M = (400)(6) = 2400 \text{ lb} \cdot \text{in}$$
$$T = (400)(2) = 800 \text{ lb} \cdot \text{in}$$

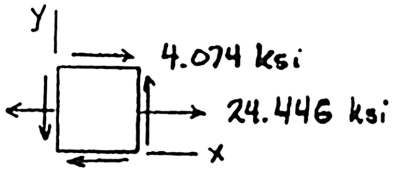
Shaft cross section: $d = 1 \text{ in.} \quad c = \frac{1}{2}d = 0.5 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion: $$\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$$

Bending: $$\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$$

Transverse shear: Stress at point *H* is zero.



$$\sigma_x = 24.446 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 4.074 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

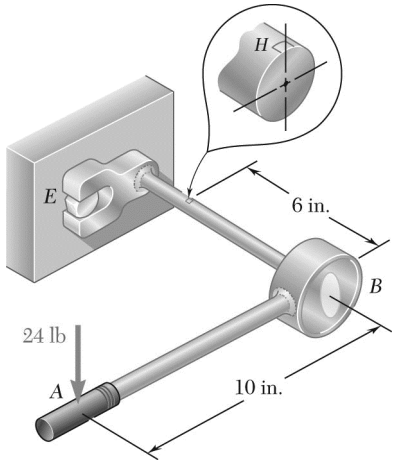
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2}$$
$$= 12.884 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.1 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.661 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 12.88 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.24

A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 24-lb force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown as on top of the $\frac{3}{4}$-in. diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*:

$$V = 24 \text{ lb} \quad M = (24)(6) = 144 \text{ lb} \cdot \text{in}$$
$$T = (24)(10) = 240 \text{ lb} \cdot \text{in}$$

Shaft cross section: $d = 0.75 \text{ in.}, \quad c = \frac{1}{2}d = 0.375 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \quad I = \frac{1}{2}J = 0.015532 \text{ in}^4$$

Torsion: $$\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$$

Bending: $$\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$$

Transverse shear: At point *H*, the stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 2.897 \text{ ksi}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738 \text{ ksi}$$

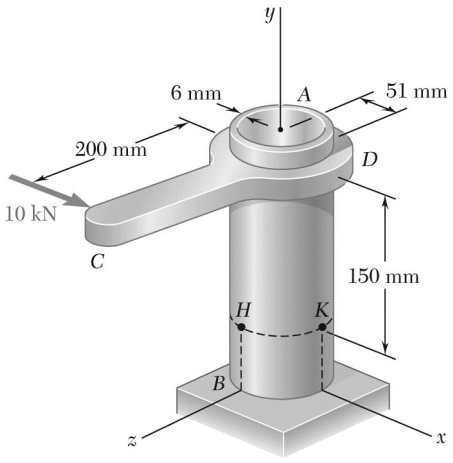
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 5.12 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -1.640 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 3.38 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.25

The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855\times10^{6}\,\text{mm}^4$$

$$= 4.1855\times10^{-6}\,\text{m}^4$$

$$I = \frac{1}{2}J = 2.0927\times10^{-6}\,\text{m}^4$$

Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10 \text{ kN}$$

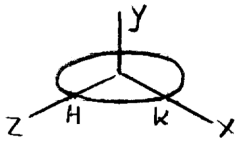
$$= 10\times10^{3}\,\text{N}$$

$$M_y = (10\times10^{3})(200\times10^{-3})$$

$$= 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10\times10^{3})(150\times10^{-3})$$

$$= -1500 \text{ N}\cdot\text{m}$$



Torsion: At point *K*, place local *x*-axis in negative global *z*-direction.

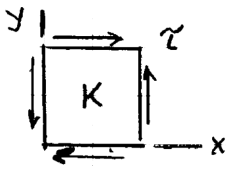
$$T = M_y = 2000 \text{ N}\cdot\text{m}$$

$$c = r_o = 51\times10^{-3}\,\text{m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times10^{-3})}{4.1855\times10^{6}}$$

$$= 24.37\times10^{6}\,\text{Pa}$$

$$= 24.37 \text{ MPa}$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.25 *(Continued)*

Transverse shear: Stress due to transverse shear $V = F_x$ is zero at point *K*.

Bending:

$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^{6}\ \text{Pa} = 36.56\ \text{MPa}$$

Point *K* lies on compression side of neutral axis:

$$\sigma_y = -36.56\ \text{MPa}$$

Total stresses at point *K*:

$$\sigma_x = 0, \quad \sigma_y = -36.56\ \text{MPa}, \quad \tau_{xy} = 24.37\ \text{MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\ \text{MPa}$$

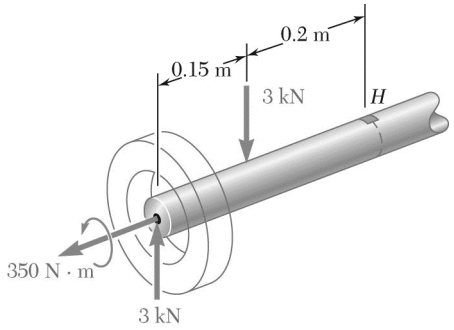
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\ \text{MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46 \qquad \sigma_{\max} = 12.18\ \text{MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46 \qquad \sigma_{\min} = -48.7\ \text{MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 30.5\ \text{MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.26

The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16 \text{ mm} = 16 \times 10^{-3} \text{ m}$$

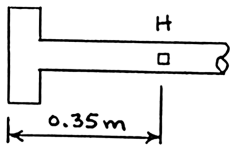
Torsion:
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(350 \text{ N} \cdot \text{m})}{\pi(16 \times 10^{-3} \text{ m})^3} = 54.399 \times 10^6 \text{ Pa} = 54.399 \text{ MPa}$$

Bending:
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16 \times 10^{-3})^4 = 51.472 \times 10^{-9} \text{ m}^4$$

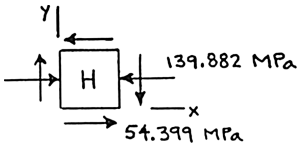
$$M = (0.15 \text{ m})(3 \times 10^3 \text{ N}) = 450 \text{ N} \cdot \text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16 \times 10^{-3})}{51.472 \times 10^{-9}} = -139.882 \times 10^6 \text{ Pa} = -139.882 \text{ MPa}$$

Top view:



Stresses:



$$\sigma_x = -139.882 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -54.399 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(-139.882 + 0) = -69.941 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-69.941)^2 + (-54.399)^2} = 88.606 \text{ MPa}$$

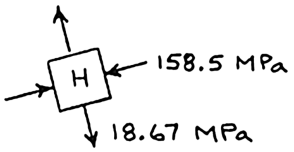
(*a*)
$$\sigma_{\max} = \sigma_{\text{ave}} + R = -69.941 + 88.606 \qquad \sigma_{\max} = 18.67 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -69.941 - 88.606 \qquad \sigma_{\min} = -158.5 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.26 *(Continued)*

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882} = 0.77778 \qquad 2\theta_p = 37.88°$$

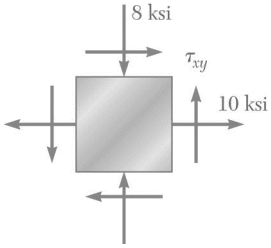


$\theta_p = 18.9°$ and $108.9°$ ◀

(*b*) $\tau_{\max} = R = 88.6$ MPa

$\tau_{\max} = 88.6$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.27

For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

$$\sigma_x = 10 \text{ ksi}, \quad \sigma_y = -8 \text{ ksi}, \quad \tau_{xy} = ?$$

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{z}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10-(-8)}{z}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{9^2 + \tau_{xy}^2} = 12 \text{ ksi}$$

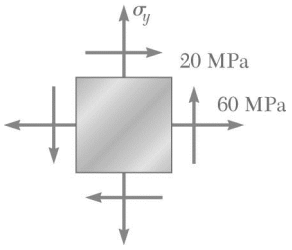
(*a*) $\tau_{xy} = \sqrt{12^2 - 9^2}$ $\qquad \tau_{xy} = 7.94 \text{ ksi}$ ◀

(*b*) $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 1 \text{ ksi}$

$\sigma_a = \sigma_{\text{ave}} + R = 1 + 12 = 13 \text{ ksi}$ $\qquad \sigma_a = 13.00 \text{ ksi}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R = 1 - 12 = -11 \text{ ksi}$ $\qquad \sigma_b = -11.00 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.28

For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$

Let

$$u = \frac{\sigma_x - \sigma_y}{2}.$$

Then

$$\sigma_y = \sigma_x - 2u$$

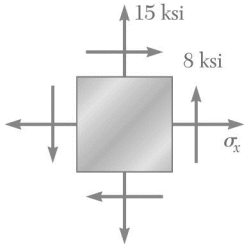
$$R = \sqrt{u^2 + \tau_{xy}^2} = 75 \text{ MPa}$$

$$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{75^2 - 20^2} = 72.284 \text{ MPa}$$

$$\sigma_y = \sigma_x - 2u = 60 \mp (2)(72.284) = -84.6 \text{ MPa} \quad \text{or} \quad 205 \text{ MPa}$$

Largest value of σ_y is required. $\sigma_y = 205$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.29

Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.

SOLUTION

$$\sigma_x = ?, \quad \sigma_y = 15 \text{ ksi}, \quad \tau_{xy} = 8 \text{ ksi}$$

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_x = \sigma_y + 2u$

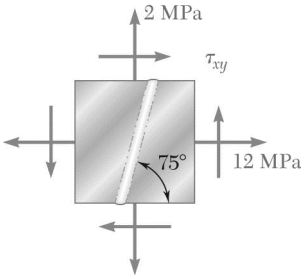
$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{\max} = 10 \text{ ksi}$$

$$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{10^2 - 8^{2z}} = \pm 6 \text{ ksi}$$

$$\sigma_x = \sigma_y + 2u = 15 \pm (2)(6) = 27 \text{ ksi} \quad \text{or} \quad 3 \text{ ksi}$$

Allowable range: $3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



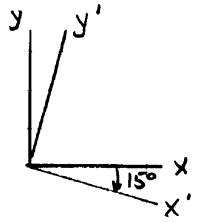
PROBLEM 7.30

For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

$$\sigma_x = 12 \text{ MPa}, \quad \sigma_y = 2 \text{ MPa}, \quad \tau_{xy} = ?$$

Since $\tau_{x'y'} = 0$, x'-direction is a principal direction.



$$\theta_p = -15°$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

(*a*) $$\tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y) \tan 2\theta_p = \frac{1}{2}(12-2)\tan(-30°)$$

$\tau_{xy} = -2.89$ MPa ◀

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 7 \text{ MPa}$$

(*b*) $$\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.7735$$

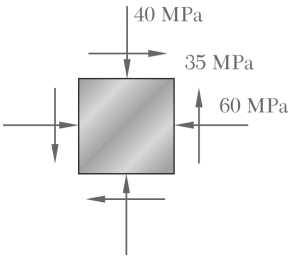
$\sigma_a = 12.77$ MPa ◀

$$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.7735$$

$\sigma_b = 1.226$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.31



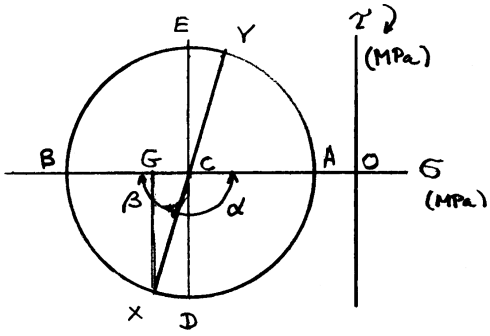
Solve Probs. 7.5 and 7.9, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = -40 \text{ MPa},$$
$$\tau_{xy} = 35 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$



Plotted points for Mohr's circle:

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$
$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$
$$C: (\sigma_{\text{ave}}, 0) = (-50 \text{ MPa}, 0)$$

(*a*) $\tan\beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_b = -\dfrac{1}{2}\beta = -37.03°$ $\qquad \theta_b = -37.0°$ ◀

$\alpha = 180° - \beta = 105.95°$

$\theta_a = \dfrac{1}{2}\alpha = 52.97°$ $\qquad \theta_a = 53.0°$ ◀

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$

(*b*) $\sigma_{\min} = \sigma_{\text{ave}} - R = -50 - 36.4$ $\qquad \sigma_{\min} = -86.4 \text{ MPa}$ ◀

$\sigma_{\max} = \sigma_{\text{ave}} + R = -50 + 36.4$ $\qquad \sigma_{\max} = -13.6 \text{ MPa}$ ◀

(*a*′) $\theta_d = \theta_B + 45° = 7.97°$ $\qquad \theta_d = 8.0°$ ◀

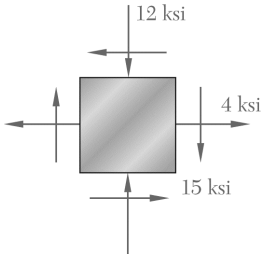
$\theta_e = \theta_A + 45° = 97.97°$ $\qquad \theta_e = 98.0°$ ◀

$\tau_{\max} = R = 36.4 \text{ MPa}$ $\qquad \tau_{\max} = 36.4 \text{ MPa}$ ◀

(*b*′) $\sigma' = \sigma_{\text{ave}} = -50 \text{ MPa}$ $\qquad \sigma' = -50.0 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.32



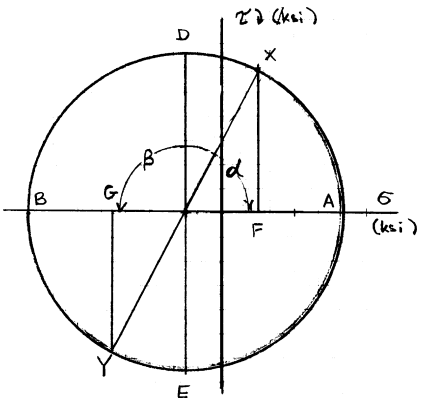
Solve Probs 7.7 and 7.11, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 4 \text{ ksi},$$
$$\sigma_y = -12 \text{ ksi},$$
$$\tau_{xy} = -15 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -4 \text{ ksi}$$



Plotted points for Mohr's circle:

$$X: (\sigma_x, -\tau_{xy}) = (4 \text{ ksi}, 15 \text{ ksi})$$
$$Y: (\sigma_y, \tau_{xy}) = (-12 \text{ ksi}, -15 \text{ ksi})$$
$$C: (\sigma_{\text{ave}}, 0) = (-4 \text{ ksi}, 0)$$

(*a*) $\tan \alpha = \dfrac{FX}{CF} = \dfrac{15}{8} = 1.875$

$$\alpha = 61.93°$$

$$\theta_a = -\frac{1}{2}\alpha = -30.96° \qquad \theta_a = -31.0° \blacktriangleleft$$

$$\beta = 180° - \alpha = 118.07°$$

$$\theta_b = \frac{1}{2}\beta = 59.04° \qquad \theta_b = 59.0° \blacktriangleleft$$

$$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(8)^2 + (15)^2} = 17 \text{ ksi}$$

(*b*) $\sigma_a = \sigma_{\max} = \sigma_{\text{ave}} + R = -4 + 17$ $\qquad \sigma_{\max} = 13.00 \text{ ksi} \blacktriangleleft$

$\sigma_{\min} = \sigma_{\min} = \sigma_{\text{ave}} - R = -4 - 17$ $\qquad \sigma_{\min} = -21.0 \text{ ksi} \blacktriangleleft$

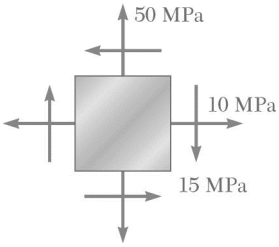
(*a*′) $\theta_d = \theta_a + 45° = 14.04°$ $\qquad \theta_d = 14.0° \blacktriangleleft$

$\theta_e = \theta_b + 45° = 104.04°$ $\qquad \theta_e = 104.0° \blacktriangleleft$

$\tau_{\max} = R$ $\qquad \tau_{\max} = 17.00 \text{ ksi} \blacktriangleleft$

(*b*′) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = -4.00 \text{ ksi} \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.33

Solve Problem 7.10, using Mohr's circle.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 10 \text{ MPa} \quad \sigma_y = 50 \text{ MPa} \quad \tau_{xy} = -15 \text{ MPa}$$

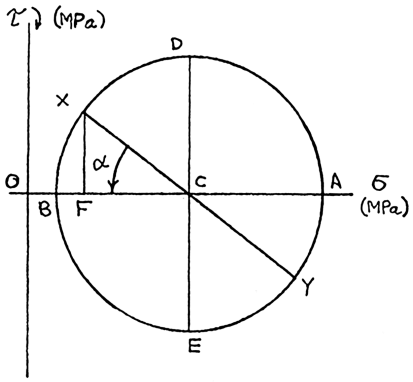
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(10 + 50) = 30 \text{ MPa}$$

Plotted points for Mohr's circle:

X: $(\sigma_x, -\tau_{xy}) = (10 \text{ MPa}, 15 \text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (50 \text{ MPa}, -15 \text{ MPa})$

C: $(\sigma_{\text{ave}}, 0) = (30 \text{ MPa}, 0)$



$$\tan\alpha = \frac{FX}{FC} = \frac{15}{20} = 0.75$$

$$\alpha = 36.87°$$

$$\theta_b = \frac{1}{2}\alpha = 18.43°$$

(*a*) $\theta_d = \theta_b - 45°$ $\qquad \theta_d = -26.6°$ ◀

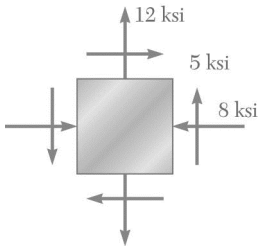
$\theta_e = \theta_b + 45°$ $\qquad \theta_e = 63.4°$ ◀

(*b*) $R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{20^2 + 15^2} = 25 \text{ MPa}$

$\tau_{\text{max (in-plane)}} = R$ $\qquad \tau_{\text{max (in-plane)}} = 25.0 \text{ MPa}$ ◀

(*c*) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = 30.0 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.34

Solve Prob. 7.12, using Mohr's circle.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

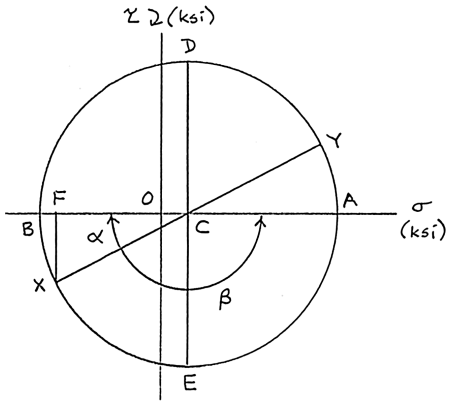
$$\sigma_x = -8 \text{ ksi} \quad \sigma_y = 12 \text{ ksi} \quad \tau_{xy} = 5 \text{ ksi}$$

Plotted points for Mohr's circle:

X: $(\sigma_x, -\tau_{xy}) = (-8 \text{ ksi}, -5 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (12 \text{ ksi}, 5 \text{ ksi})$

C: $(\sigma_{\text{ave}}, 0) = (2 \text{ ksi}, 0)$



$$\tan \alpha = \frac{FX}{FC} = \frac{5}{10} = 0.5$$

$$\alpha = 26.565°$$

$$\beta = 180 - \alpha = 153.435°$$

$$\theta_a = \frac{1}{2}\beta = 76.718°$$

(*a*) $\theta_d = \theta_a + 45°$ $\qquad \theta_d = 121.7° \blacktriangleleft$

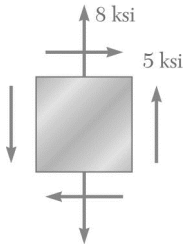
$\theta_e = \theta_a - 45°$ $\qquad \theta_e = 31.7° \blacktriangleleft$

(*b*) $R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{10^2 + 5^2} = 11.1803 \text{ ksi}$

$\tau_{\text{max (in-plane)}} = R$ $\qquad \tau_{\text{max (in-plane)}} = 11.18 \text{ ksi} \blacktriangleleft$

(*c*) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = 2.00 \text{ ksi} \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



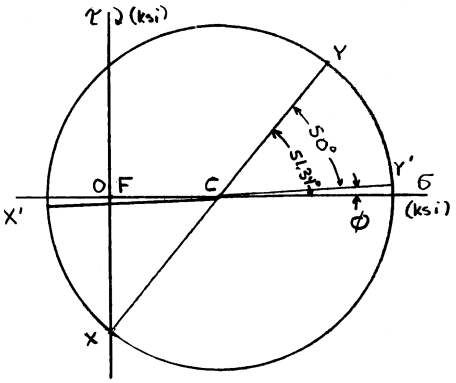
PROBLEM 7.35

Solve Prob. 7.13, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0,$$
$$\sigma_y = 8 \text{ ksi},$$
$$\tau_{xy} = 5 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 4 \text{ ksi}$$

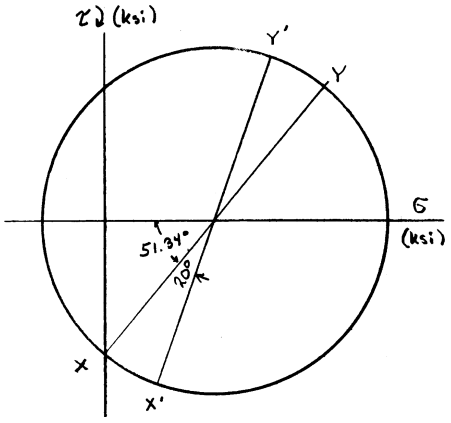


Plotted points for Mohr's circle:

$$X: (0, -5 \text{ ksi})$$
$$Y: (8 \text{ ksi}, 5 \text{ ksi})$$
$$C: (4 \text{ ksi}, 0)$$

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{5}{4} = 1.25$$
$$2\theta_p = 51.34°$$
$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{4^2 + 5^2} = 6.40 \text{ ksi}$$

(*a*) $\underline{\theta = 25°}$ ↷. $2\theta = 50°$ ↷



$$\varphi = 51.34° - 50° = 1.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{x'} = -2.40 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = R\sin\varphi \qquad \tau_{x'y'} = 0.15 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{y'} = 10.40 \text{ ksi} \blacktriangleleft$$

(*b*) $\underline{\theta = 10°}$ ↶. $2\theta = 20°$ ↶

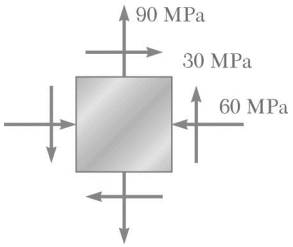
$$\varphi = 51.34° + 20° = 71.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{x'} = 1.95 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = R\sin\varphi \qquad \tau_{x'y'} = 6.07 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{y'} = 6.05 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.36

Solve Prob 7.14, using Mohr's circle.

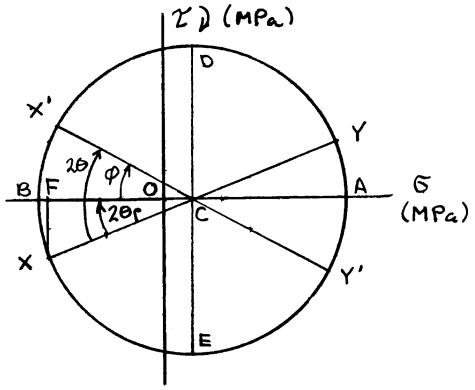
PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = 90 \text{ MPa},$$
$$\tau_{xy} = 30 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$

Plotted points for Mohr's circle:

X: (–60 MPa, – 30 MPa)
Y: (90 MPa, 30 MPa)
C: (15 MPa, 0)



$$\tan 2\theta_p = \frac{FX}{FC} = \frac{30}{75} = 0.4$$
$$2\theta_p = 21.80° \quad \theta_P = 10.90° \circlearrowright$$
$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$

(*a*) $\theta = 25° \circlearrowright$. $2\theta = 50° \circlearrowright$

$$\varphi = 2\theta - 2\theta_P = 50° - 21.80° = 28.20° \circlearrowright$$

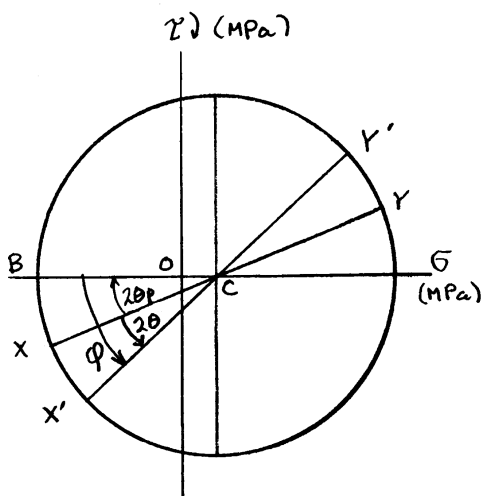
$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi$ $\quad \sigma_{x'} = -56.2 \text{ MPa}$ ◀

$\tau_{x'y'} = -R\sin\varphi$ $\quad \tau_{x'y'} = -38.2 \text{ MPa}$ ◀

$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi$ $\quad \sigma_{y'} = 86.2 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.36 *(Continued)*



(*b*) $\underline{\theta = 10^\circ}$ ↻. $2\theta = 20^\circ$ ↻

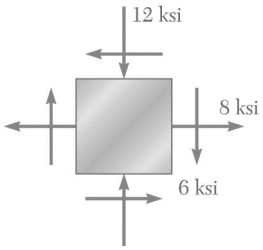
$$\varphi = 2\theta_p + 2\theta = 21.80^\circ + 20^\circ = 41.80^\circ \text{ ↻}$$

$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi$ $\sigma_{x'} = -45.2$ MPa ◀

$\tau_{x'y'} = R\sin\varphi$ $\tau_{x'y'} = 53.8$ MPa ◀

$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi$ $\sigma_{y'} = 75.2$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

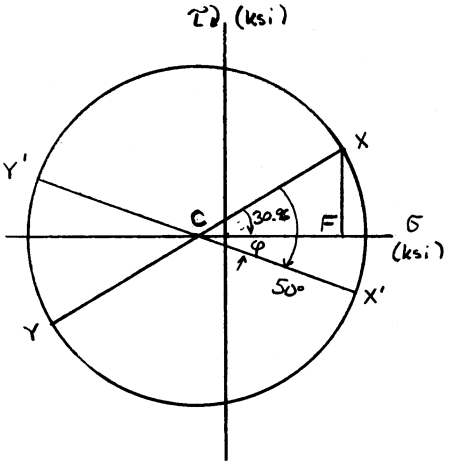


PROBLEM 7.37

Solve Prob. 7.15, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

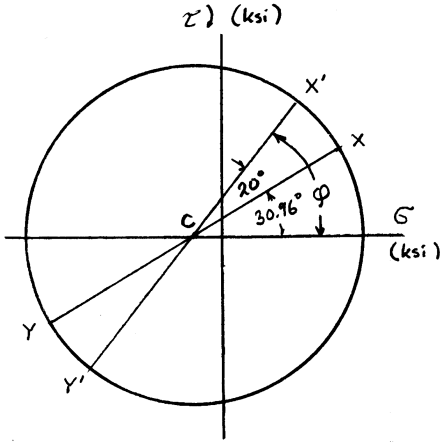


$$\sigma_x = 8 \text{ ksi},$$
$$\sigma_y = -12 \text{ ksi},$$
$$\tau_{xy} = -6 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi}$$

Plotted points for Mohr's circle:

X: (8 ksi, 6 ksi)
Y: (−12 ksi, −6 ksi)
C: (−2 ksi, 0)

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{6}{10} = 0.6$$
$$2\theta_p = 30.96°$$
$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{10^2 + 6^2} = 11.66 \text{ ksi}$$



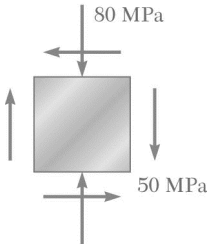
(*a*) $\underline{\theta = 25°}$ ↻. $2\theta = 50°$ ↻

$$\varphi = 50° - 30.96° = 19.04°$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{x'} = 9.02 \text{ ksi} \blacktriangleleft$$
$$\tau_{x'y'} = R\sin\varphi \qquad \tau_{x'y'} = 3.80 \text{ ksi} \blacktriangleleft$$
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{y'} = -13.02 \text{ ksi} \blacktriangleleft$$

(*b*) $\underline{\theta = 10°}$ ↺. $2\theta = 20°$ ↺

$$\varphi = 30.96° + 20° = 50.96°$$
$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{x'} = 5.34 \text{ ksi} \blacktriangleleft$$
$$\tau_{x'y'} = -R\sin\varphi \qquad \tau_{x'y'} = -9.06 \text{ ksi} \blacktriangleleft$$
$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{y'} = -9.34 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

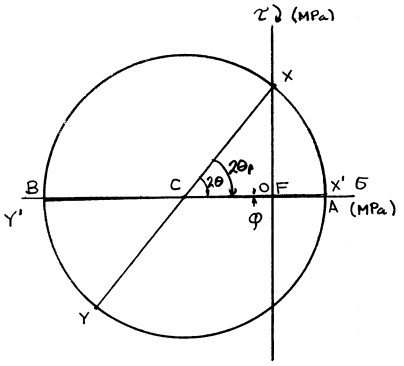


PROBLEM 7.38

Solve Prob. 7.16, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

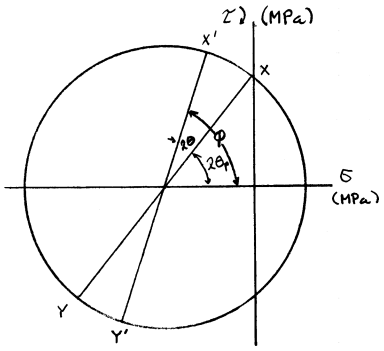
SOLUTION



$$\sigma_x = 0,$$
$$\sigma_y = -80 \text{ MPa},$$
$$\tau_{xy} = -50 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$

Plotted points for Mohr's circle:

X: (0, 50 MPa)
Y: (−80 MPa, −50 MPa)
C: (−40 MPa, 0)



$$\tan 2\theta_p = \frac{FX}{CF} = \frac{50}{40} = 1.25$$
$$2\theta_p = 51.34°$$
$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2}$$
$$= 64.03 \text{ MPa}$$

(*a*) $\theta = 25°$ ↻. $2\theta = 50°$ ↻

$$\varphi = 51.34° - 50° = 1.34° \text{ ↺}$$

$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{x'} = 24.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -R\sin\varphi \qquad \tau_{x'y'} = -1.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{y'} = -104.0 \text{ MPa} \blacktriangleleft$$

(*b*) $\theta = 10°$ ↺. $2\theta = 20°$ ↺

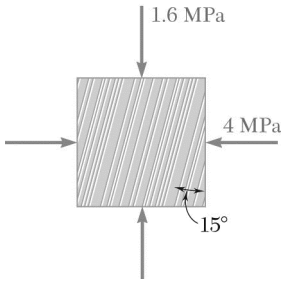
$$\varphi = 51.34° + 20° = 71.34°$$

$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos\varphi \qquad \sigma_{x'} = -19.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{x'y'} = -R\sin\varphi \qquad \tau_{x'y'} = -60.7 \text{ MPa} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos\varphi \qquad \sigma_{y'} = -60.5 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.39

Solve Prob. 7.17, using Mohr's circle.

PROBLEM 7.17 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -4 \text{ MPa} \quad \sigma_y = -1.6 \text{ MPa} \quad \tau_{xy} = 0$$

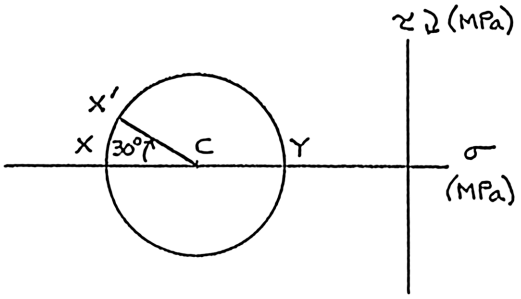
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -2.8 \text{ MPa}$$

Plotted points for Mohr's circle:

$$X\text{: } (\sigma_x, -\tau_{xy}) = (-4 \text{ MPa}, 0)$$

$$Y\text{: } (\sigma_y, \tau_{xy}) = (-1.6 \text{ MPa}, 0)$$

$$C\text{: } (\sigma_{\text{ave}}, 0) = (-2.8 \text{ MPa}, 0)$$



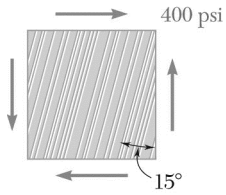
$\theta = -15°$. $2\theta = -30°$

$\overline{CX} = 1.2 \text{ MPa} \quad R = 1.2 \text{ MPa}$

(*a*) $\tau_{x'y'} = -\overline{CX'} \sin 30° = -R \sin 30° = -1.2 \sin 30°$ $\tau_{x'y'} = -0.600 \text{ MPa}$ ◀

(*b*) $\sigma_{x'} = \sigma_{\text{ave}} - \overline{CX'} \cos 30° = -2.8 - 1.2 \cos 30°$ $\sigma_{x'} = -3.84 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.40

Solve Prob. 7.18, using Mohr's circle.

PROBLEM 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = \sigma_y = 0, \quad \tau_{xy} = 400\,\text{psi}$$

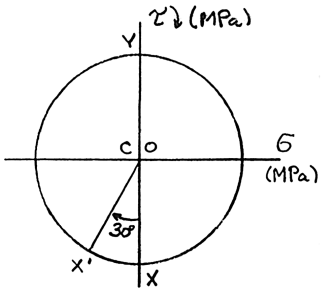
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Points:

X: $(\sigma_x, -\tau_{xy}) = (0, -400\ \text{psi})$

Y: $(\sigma_y, \tau_{xy}) = (0, 400\ \text{psi})$

C: $(\sigma_{\text{ave}}, 0) = (0, 0)$



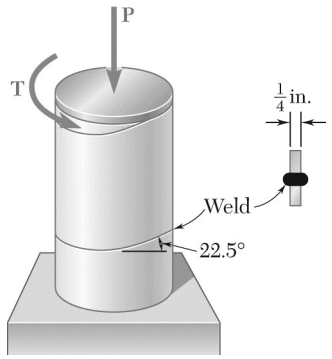
$\underline{\theta = -15°}$, $\quad 2\theta = -30°$

$\overline{CX} = R = 400\ \text{psi}$

(*a*) $\tau_{x'y'} = R\cos 30° = 400\cos 30°$ $\qquad \tau_{x'y'} = 346\ \text{psi}$ ◀

(*b*) $\sigma_{x'} = \sigma_{\text{ave}} - R\sin 30° = -400\sin 30°$ $\qquad \sigma_{x'} = -200\ \text{psi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

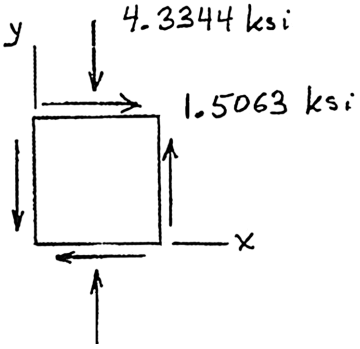


PROBLEM 7.41

Solve Prob. 7.19, using Mohr's circle.

PROBLEM 7.19 A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 40-kip axial force **P** and an 80-kip · in. torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

SOLUTION



$$d_2 = 12 \text{ in.} \quad c_2 = \frac{1}{2} d_2 = 6 \text{ in.}, \quad t = 0.25 \text{ in.}$$

$$c_1 = c_2 - t = 5.75 \text{ in.}$$

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(6^2 - 5.75^2) = 9.2284 \text{ in}^2$$

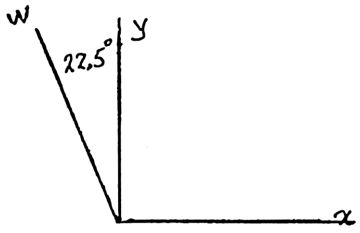
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(6^4 - 5.75^4) = 318.67 \text{ in}^4$$

Stresses:

$$\sigma = -\frac{P}{A} = -\frac{40}{9.2284} = -4.3344 \text{ ksi}$$

$$\tau = -\frac{Tc_2}{J} = -\frac{(80)(6)}{318.67} = 1.5063 \text{ ksi}$$

$$\sigma_x = 0, \quad \sigma_y = -4.3344 \text{ ksi}, \quad \tau_{xy} = 1.5063 \text{ ksi}$$

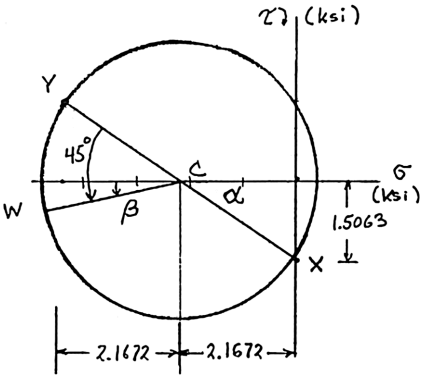


Draw the Mohr's circle.

$$X\text{: } (0, -1.5063 \text{ ksi})$$

$$Y\text{: } (-4.3344 \text{ ksi}, 1.5063 \text{ ksi})$$

$$C\text{: } (-2.1672 \text{ ksi}, 0)$$



$$\tan\alpha = \frac{1.5063}{2.1672} = 0.69504 \quad \alpha = 34.8°$$

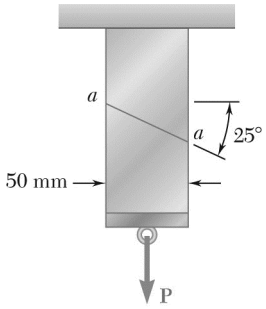
$$\beta = (2)(22.5°) - \alpha = 10.8°$$

$$R = \sqrt{(2.1672)^2 + (1.5063)^2} = 2.6393 \text{ ksi}$$

$$\sigma_w = -2.1672 - 2.6393 \cos 10.8° \qquad \sigma_w = -4.76 \text{ ksi} \blacktriangleleft$$

$$\tau_w = -2.6393 \sin 10.2° \qquad \tau_w = -0.467 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

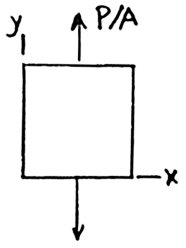


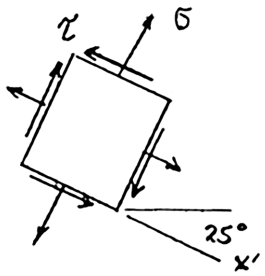
PROBLEM 7.42

Solve Prob. 7.20, using Mohr's circle.

PROBLEM 7.20 Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION





$$\sigma_x = 0$$

$$\tau_{xy} = 0$$

$$\sigma_y = P/A$$

$$A = (50\times10^{-3})(80\times10^{-3})$$

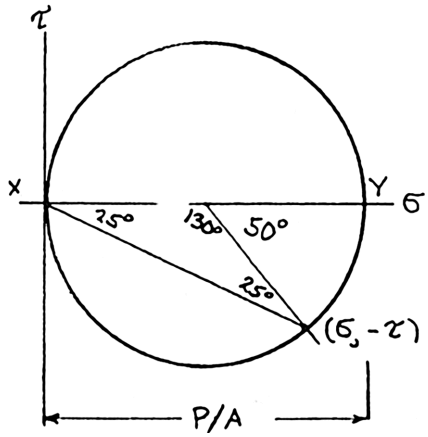
$$= 4\times10^{-3}\,\text{m}^2$$

$$\sigma = \frac{P}{2A}(1+\cos 50°)$$

$$P = \frac{2A\sigma}{1+\cos 50°}$$

$$P \le \frac{(2)(4\times10^{-3})(800\times10^{3})}{1+\cos 50°}$$

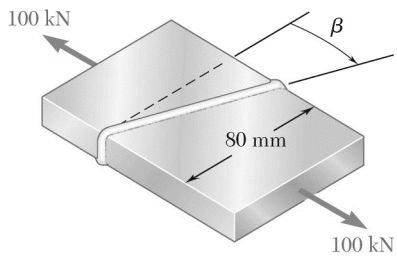
$$P \le 3.90\times10^{3}\,\text{N}$$



$$\tau = \frac{P}{2A}\sin 50° \qquad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4\times10^{-3})(600\times10^{3})}{\sin 50°} = 6.27\times10^{3}\,\text{N}$$

Choosing the smaller value, $P = 3.90$ kN ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

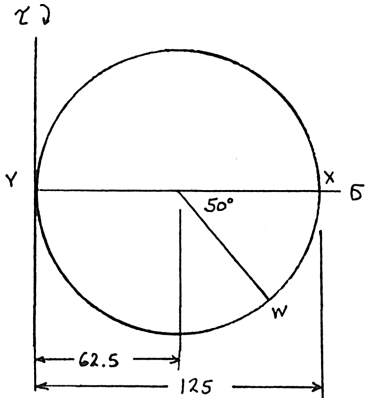


PROBLEM 7.43

Solve Prob. 7.21, using Mohr's circle.

PROBLEM 7.21 Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.

SOLUTION



$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

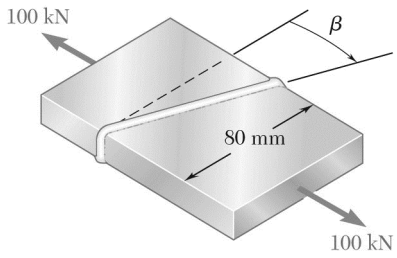
From Mohr's circle:

(*a*) $\tau_w = 62.5 \sin 50°$ $\tau_w = 47.9$ MPa ◀

(*b*) $\sigma_w = 62.5 + 62.5 \cos 50°$

$\sigma_w = 102.7$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

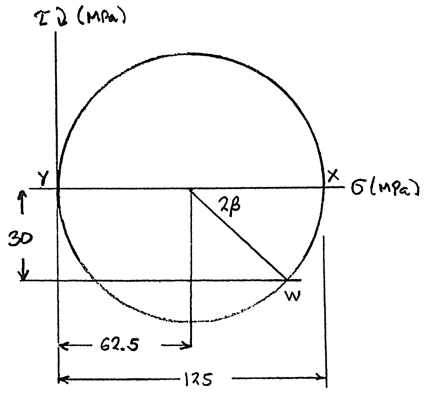


PROBLEM 7.44

Solve Prob. 7.22, using Mohr's circle.

PROBLEM 7.22 Two steel plates of uniform cross section 10×80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.

SOLUTION



$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

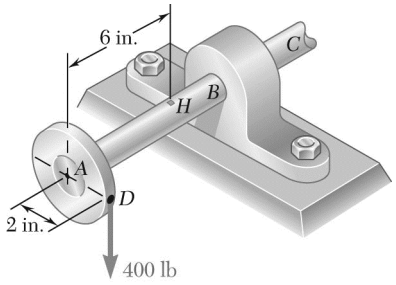
From Mohr's circle:

(*a*) $$\sin 2\beta = \frac{30}{62.5} = 0.48 \qquad \beta = 14.3° \blacktriangleleft$$

(*b*) $$\sigma = 62.5 + 62.5 \cos 2\beta$$

$$\sigma = 117.3 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.45

Solve Prob. 7.23, using Mohr's circle.

PROBLEM 7.23 A 400-lb vertical force is applied at D to a gear attached to the solid 1-in.-diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point H:

$$V = 400 \text{ lb} \qquad M = (400)(6) = 2400 \text{ lb} \cdot \text{in}$$

$$T = (400)(2) = 800 \text{ lb} \cdot \text{in}$$

Shaft cross section

$$d = 1 \text{ in.} \qquad c = \frac{1}{2}d = 0.5 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion:

$$\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$$

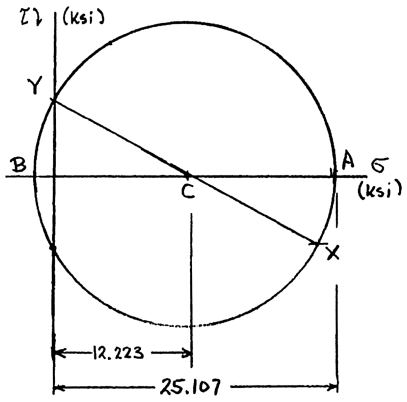
Bending:

$$\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$$

Transverse shear: Stress at point H is zero.

Resultant stresses:

$$\sigma_x = 24.446 \text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 4.074 \text{ ksi}$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

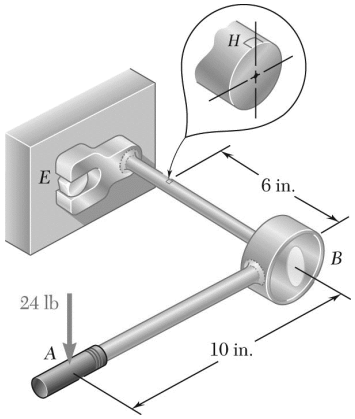
$$= \sqrt{(12.223)^2 + (4.074)^2} = 12.884 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.107 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.661 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 12.88 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.46

Solve Prob. 7.24 using Mohr's circle.

PROBLEM 7.24 A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 24-lb force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown as on top of the $\frac{3}{4}$-in.-diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*:

$V = 24$ lb $\qquad M = (24)(6) = 144$ lb · in

$T = (24)(10) = 240$ lb · in

Shaft cross section: $\qquad d = 0.75$ in. $\qquad c = \dfrac{1}{2}d = 0.375$ in.

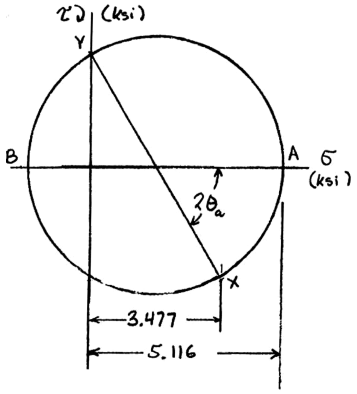
$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.015532 \text{ in}^4$$

Torsion:
$$\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$$

Bending:
$$\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$$

Transverse shear: At point *H*, stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\quad \sigma_y = 0, \quad \tau_{xy} = 2.897$ ksi



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

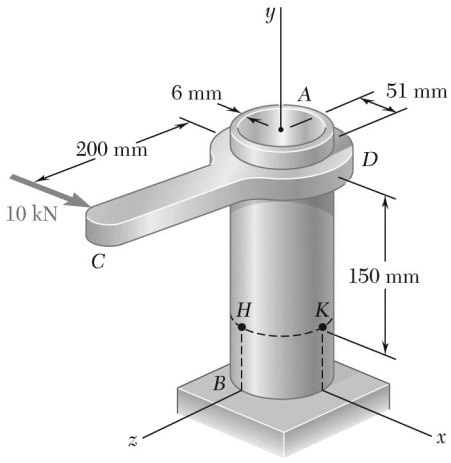
$$= \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad \sigma_a = 5.116$ ksi ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad \sigma_b = -1.640$ ksi ◀

$\tau_{\max} = R$ $\qquad \tau_{\max} = 3.378$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.47

Solve Prob. 7.25, using Mohr's circle.

PROBLEM 7.25 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\text{ mm} \qquad r_i = r_o - t = 45\text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6\text{ mm}^4 = 4.1855 \times 10^{-6}\text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927 \times 10^{-6}\text{m}^4$$

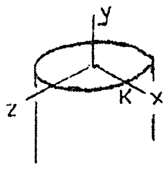
Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10 \times 10^3\text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000\text{ N} \cdot \text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500\text{ N} \cdot \text{m}$$

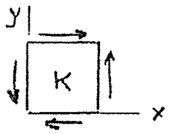
Torsion:



$$T = M_y = 2000\text{ N} \cdot \text{m}$$

$$c = r_o = 51 \times 10^{-3}\text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37\text{ MPa}$$



Note that the local *x*-axis is taken along a negative global *z*-direction.

Transverse shear: Stress due to $V = F_x$ is zero at point *K*.

Bending:
$$\left|\sigma_y\right| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56\text{ MPa}$$

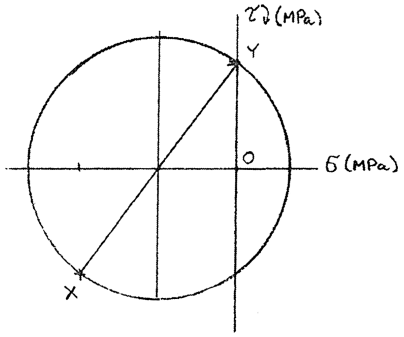
Point *K* lies on compression side of neutral axis. $\sigma_y = -36.56\text{ MPa}$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.47 *(Continued)*

Total stresses at point K: $\sigma_x = 0, \quad \sigma_y = -36.56 \text{ MPa}, \quad \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46$$

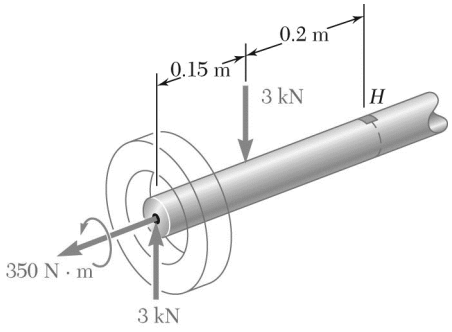
$$\sigma_{\max} = 12.18 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46$$

$$\sigma_{\min} = -48.74 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 30.46 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.48

Solve Prob. 7.26, using Mohr's circle.

PROBLEM 7.26 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point *H* located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16 \text{ mm} = 16 \times 10^{-3} \text{ m}$$

Torsion: $$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

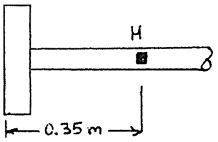
$$\tau = \frac{2(350 \text{ N} \cdot \text{m})}{\pi(16 \times 10^{-3} \text{m})^3} = 54.399 \times 10^6 \text{ Pa} = 54.399 \text{ MPa}$$

Bending: $$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16 \times 10^{-3})^4 = 51.472 \times 10^{-9} \text{m}^4$$

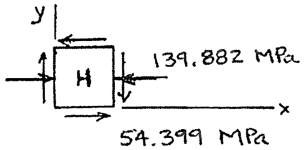
$$M = (0.15 \text{m})(3 \times 10^3 \text{ N}) = 450 \text{ N} \cdot \text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16 \times 10^{-3})}{51.472 \times 10^{-9}} = -139.882 \times 10^6 \text{ Pa} = -139.882 \text{ MPa}$$

Top view

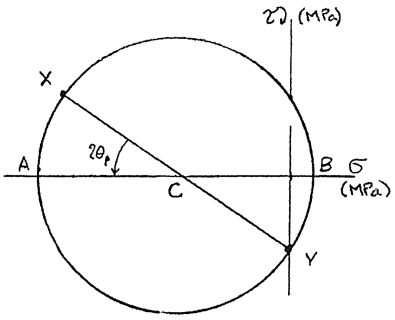


Stresses



$$\sigma_x = -139.882 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = -54.399 \text{ MPa}$$

Plotted points: $X: (-139.882, 54.399)$; $Y: (0, -54.399)$; $C: (-69.941, 0)$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -69.941 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-139.882}{2}\right)^2 + (54.399)^2} = 88.606 \text{ MPa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.48 *(Continued)*

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882}$$

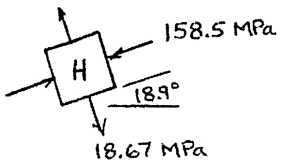
$$= 0.77778$$

(*a*) $\theta_a = 18.9°\ \curvearrowleft, \quad \theta_b = 108.9°\ \curvearrowright$ ◀

$$\sigma_a = \sigma_{\text{ave}} - R = -69.941 - 88.606 \qquad \sigma_a = -158.5 \text{ MPa}$$ ◀

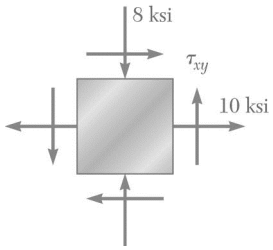
$$\sigma_b = \sigma_{\text{ave}} + R = -69.941 + 88.606 \qquad \sigma_b = 18.67 \text{ MPa}$$ ◀

(*b*)



$$\tau_{\max} = R \qquad \tau_{\max} = 88.6 \text{ MPa}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.49

Solve Prob. 7.27, using Mohr's circle.

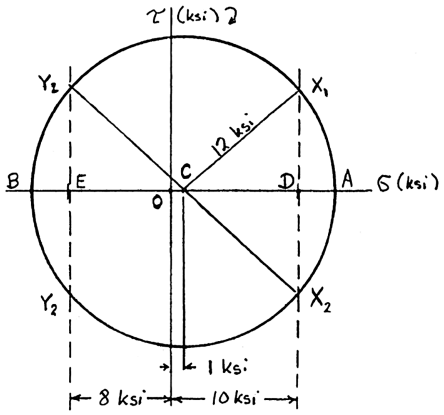
PROBLEM 7.27 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point *C* with coordinates

$$\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)=\left(\frac{10-8}{2}, 0\right)=(1\text{ ksi}, 0).$$

The radius of the circle is $\tau_{\max\,(\text{in-plane})}=12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lies along the line X_1X_2 of the Mohr circle diagram. The extreme points with $R \le 12$ ksi are X_1 and X_2.

(*a*) The largest allowable value of τ_{xy} is obtained from triangle *CDX*:

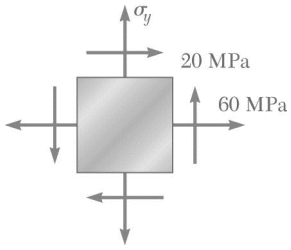
$$\overline{DX_1}^2=\overline{DX_2}^2=\sqrt{\overline{CX_1}^2-\overline{CD}^2}$$

$$\tau_{xy}=\sqrt{12^2-9^2} \qquad \tau_{xy}=7.94\text{ ksi} \blacktriangleleft$$

(*b*) The principal stresses are $\sigma_a=1+12$ $\qquad \sigma_a=13.00\text{ ksi} \blacktriangleleft$

$$\sigma_b=1-12 \qquad \sigma_b=-11.00\text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



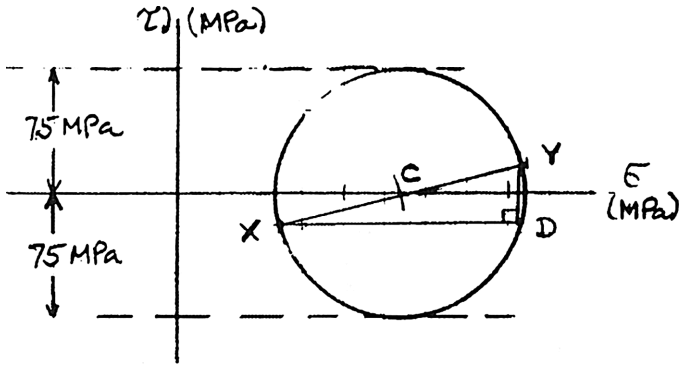
PROBLEM 7.50

Solve Prob. 7.28, using Mohr's circle.

PROBLEM 7.28 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 75 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = ?, \quad \tau_{xy} = 20 \text{ MPa}$$



Given:

$$\tau_{\max} = R = 75 \text{ MPa}$$

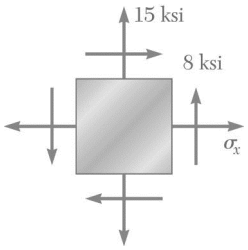
$$\overline{XY} = 2R = 150 \text{ MPa}$$

$$\overline{DY} = (2)(\tau_{xy}) = 40 \text{ MPa}$$

$$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{150^2 - 40^2} = 144.6 \text{ MPa}$$

$$\sigma_y = \sigma_x + \overline{XD} = 60 + 144.6 \qquad \sigma_y = 205 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.51

Solve Prob. 7.29, using Mohr's circle.

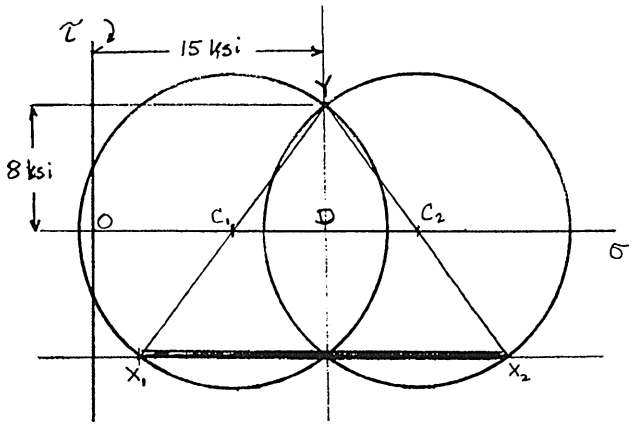
PROBLEM 7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 10 ksi.

SOLUTION

For the Mohr's circle, point Y lies at (15 ksi, 8 ksi). The radius of limiting circles is $R = 10$ ksi.

Let C_1 be the location of the leftmost limiting circle and C_2 be that of the rightmost one.

$$\overline{C_1Y} = 10 \text{ ksi}$$
$$\overline{C_2Y} = 10 \text{ ksi}$$



Noting right triangles C_1DY and C_2DY,

$$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 + 8^2 = 10^2 \qquad \overline{C_1D} = 6 \text{ ksi}$$

Coordinates of point C_1 are $(0, 15-6) = (0, 9 \text{ ksi})$.

Likewise, coordinates of point C_2 are $= (0, 15+6) = (0, 21 \text{ ksi})$.

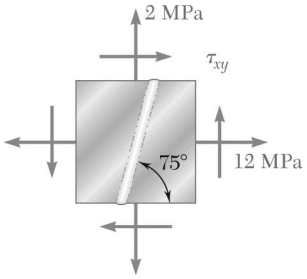
Coordinates of point X_1: $(9-6, -8) = (3 \text{ ksi}, -8 \text{ ksi})$

Coordinates of point X_2: $(21+6, -8) = (27 \text{ ksi}, -8 \text{ ksi})$

The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2.

Thus, $3 \text{ ksi} \le \sigma_x \le 27 \text{ ksi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.52

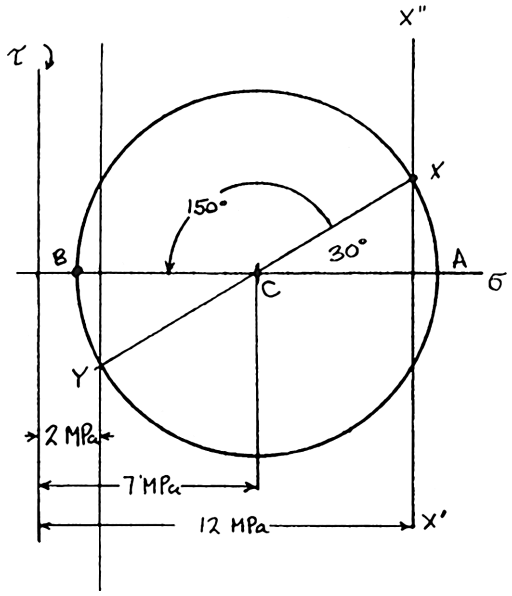
Solve Prob. 7.30, using Mohr's circle.

PROBLEM 7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

Point *X* of Mohr's circle must lie on $X'X''$ so that $\sigma_x = 12$ MPa. Likewise, point *Y* lies on line $Y'Y''$ so that $\sigma_y = 2$ MPa. The coordinates of *C* are

$$\frac{2+12}{2}, \quad 0 = (7 \text{ MPa}, 0).$$



Counterclockwise rotation through 150° brings line *CX* to *CB*, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2} \sec 30° = \frac{12-2}{2} \sec 30° = 5.77 \text{ MPa}$$

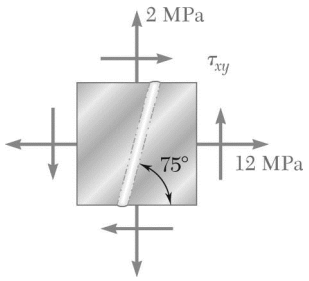
(*a*) $$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2} \tan 30°$$

$$= -\frac{12-2}{2} \tan 30° \qquad \tau_{xy} = -2.89 \text{ MPa} \blacktriangleleft$$

(*b*) $$\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.77 \qquad \sigma_a = 12.77 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.77 \qquad \sigma_b = 1.23 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

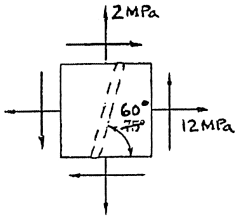


PROBLEM 7.53

Solve Problem 7.30, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

PROBLEM 7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

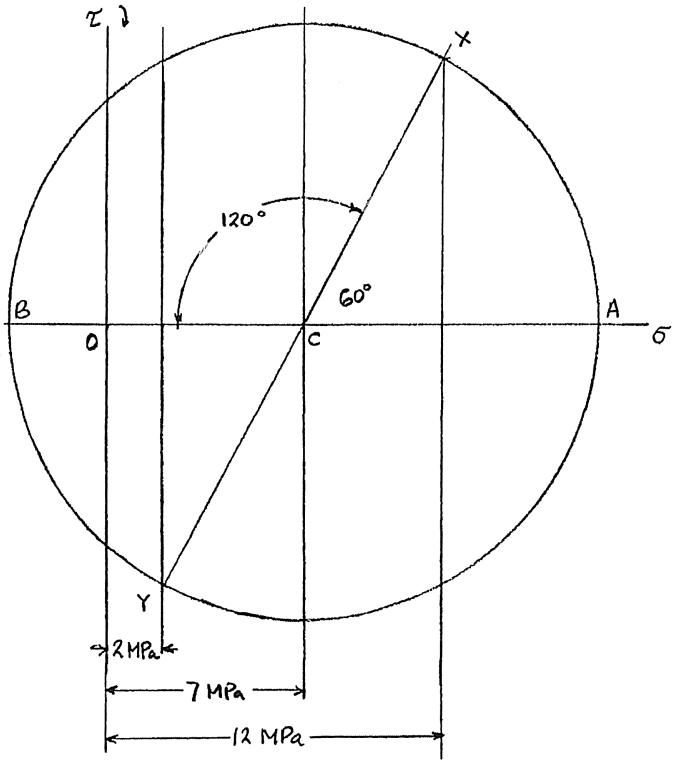
SOLUTION



Locate point C at $\sigma = \dfrac{12+2}{2} = 7$ MPa with $\tau = 0$.

Angle $XCB = 120°$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2}$$
$$= 5 \text{ MPa}$$



$$R = 5\sec 60°$$
$$= 10 \text{ MPa}$$

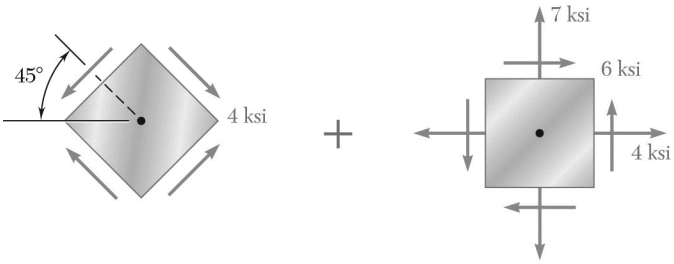
$$\tau_{xy} = -5\tan 60°$$

$$\tau_{xy} = -8.66 \text{ MPa} \blacktriangleleft$$

$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 7 + 10 \qquad \sigma_a = 17 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R$$
$$= 7 - 10 \qquad \sigma_b = -3 \text{ MPa} \blacktriangleleft$$

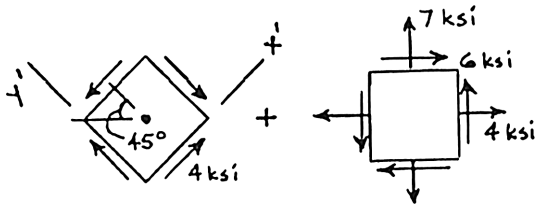
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



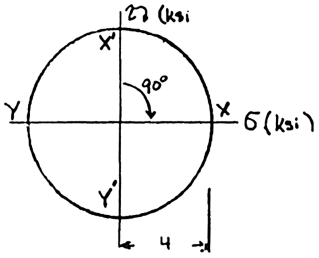
PROBLEM 7.54

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION



Mohr's circle for 1st stress state.



$\sigma_x = 4$ ksi

$\sigma_y = -4$ ksi

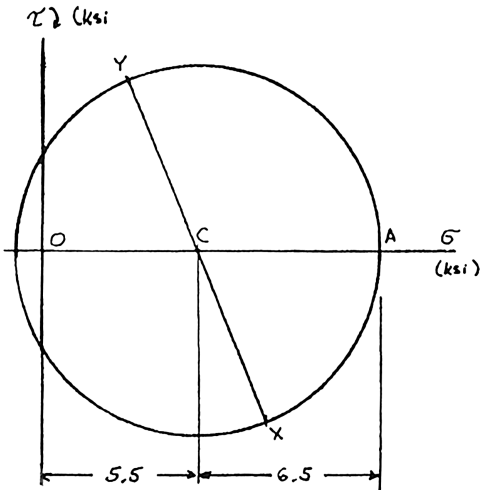
$\tau_{xy} = 0$

Resultant stresses:

$\sigma_x = 4 + 4 = 8$ ksi

$\sigma_y = -4 + 7 = 3$ ksi

$\tau_{xy} = 6 + 0 = 6$ ksi



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{5} = 2.4$$

$$2\theta_p = 67.38°$$

$\theta_a = 33.69°$ ◀

$\theta_b = 123.69°$ ◀

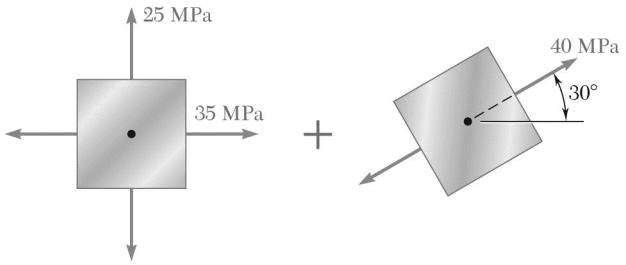
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + 6^2} = 6.5 \text{ ksi}$$

$\sigma_a = \sigma_{\text{ave}} + R$ $\quad \sigma_a = 12$ ksi ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\quad \sigma_b = -1$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

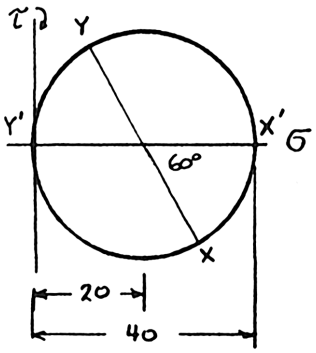


PROBLEM 7.55

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd stress state:



$$\sigma_x = 20 + 20\cos 60° = 30 \text{ MPa}$$

$$\sigma_y = 20 - 20\cos 60° = 10 \text{ MPa}$$

$$\tau_{xy} = 20 \sin 60° = 17.32 \text{ MPa}$$

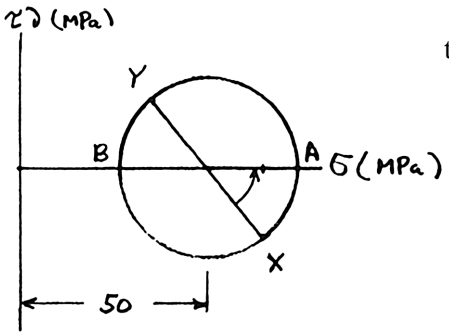
Resultant stresses:

$$\sigma_x = 35 + 30 = 65 \text{ MPa}$$

$$\sigma_y = 25 + 10 = 35 \text{ MPa}$$

$$\tau_{xy} = 0 + 17.32 = 17.32 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(65 + 35) = 50 \text{ MPa}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(17.32)}{65 - 35} = 1.1547$$

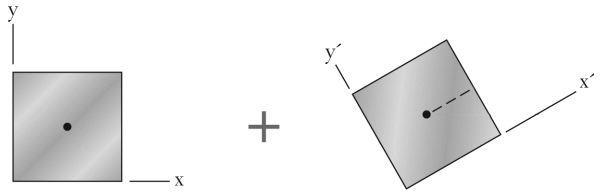
$$2\theta_p = 49.11°, \qquad \theta_a = 24.6°, \quad \theta_b = 114.6° \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22.91 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 72.91 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = 27.09 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

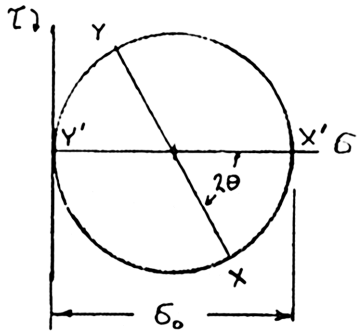


PROBLEM 7.56

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd stress state:



$$\sigma_x = \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = \frac{1}{2}\sigma_0 \sin 2\theta$$

Resultant stresses:

$$\sigma_x = \sigma_0 + \frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta = \frac{3}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta$$

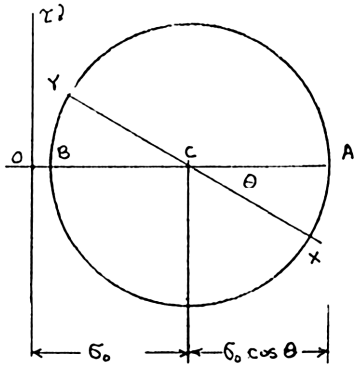
$$\sigma_y = 0 + \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta = \frac{1}{2}\sigma_0 - \frac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = 0 + \frac{1}{2}\sigma_0 \sin 2\theta = \frac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$$



$$\theta_p = \frac{1}{2}\theta \quad \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{1}{2}\sigma_0 + \frac{1}{2}\sigma_0 \cos 2\theta\right)^2 + \left(\frac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

$$= \frac{1}{2}\sigma_0\sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta} = \frac{\sqrt{2}}{2}\sigma_0\sqrt{1 + \cos 2\theta} = \sigma_0 |\cos\theta|$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = \sigma_0 + \sigma_0 \cos\theta \quad \blacktriangleleft$$

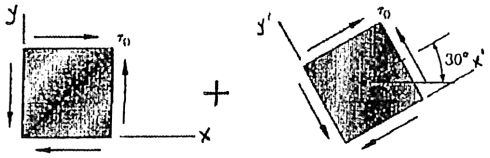
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = \sigma_0 - \sigma_0 \cos\theta \quad \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.57

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

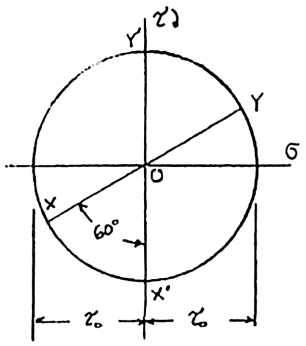


Mohr's circle for 2nd state of stress:

$$\sigma_{x'} = 0$$

$$\sigma_{y'} = 0$$

$$\tau_{x'y'} = \tau_0$$



$$\sigma_x = -\tau_0 \sin 60° = -\frac{\sqrt{3}}{2}\tau_0 \qquad \sigma_y = \tau_0 \sin 60° = \frac{\sqrt{3}}{2}\tau_0$$

$$\tau_{xy} = \tau_0 \cos 60° = \frac{1}{2}\tau_0$$

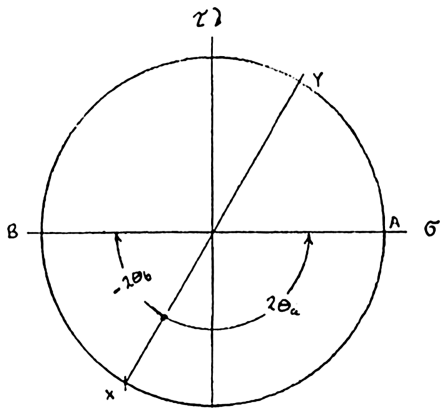
Resultant stresses:

$$\sigma_x = 0 - \frac{\sqrt{3}}{2}\tau_0 = -\frac{\sqrt{3}}{2}\tau_0 \qquad \sigma_y = 0 + \frac{\sqrt{3}}{2}\tau_0 = \frac{\sqrt{3}}{2}\tau_0$$

$$\tau_{xy} = \tau_0 + \frac{1}{2}\tau_0 = \frac{3}{2}\tau_0$$

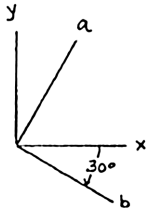
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\tau_0\right)^2 + \left(\frac{3}{2}\tau_0\right)^2} = \sqrt{3}\,\tau_0$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)\left(\frac{3}{2}\right)}{-\sqrt{3}} = -\sqrt{3}$$

$$2\theta_p = -60°$$

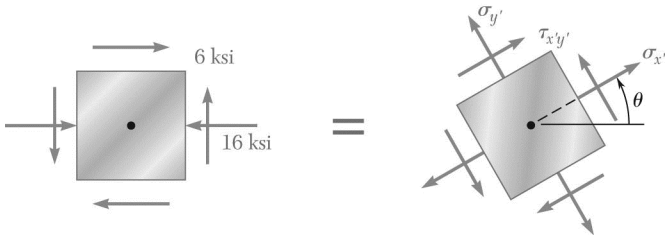


$$\theta_b = -30° \qquad \theta_a = 60° \blacktriangleleft$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = \sqrt{3}\,\tau_0 \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -\sqrt{3}\,\tau_0 \blacktriangleleft$$

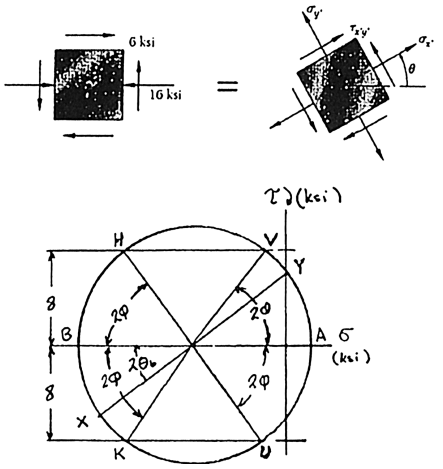
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.58

For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 8 ksi

SOLUTION



$$\sigma_x = -16 \text{ ksi}, \quad \sigma_y = 0$$

$$\tau_{xy} = 6 \text{ ksi}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -8 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-8)^2 + (6)^2} = 10 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{-16} = -0.75$$

$$2\theta_p = -36.870°$$

$$\theta_b = -18.435°$$

$|\tau_{x'y'}| \le 8$ ksi for states of stress corresponding to arcs *HBK* and *UAV* of Mohr's circle. The angle φ is calculated from

$$R \sin 2\varphi = 8 \qquad \sin 2\varphi = \frac{8}{10} = 0.8$$

$$2\varphi = 53.130° \qquad \varphi = 26.565°$$

$$\theta_k = \theta_b - \varphi = -18.435° - 26.565° = -45°$$

$$\theta_k = \theta_b + \varphi = -18.435 + 26.565° = 8.13°$$

$$\theta_u = \theta_h + 90° = 45°$$

$$\theta_v = \theta_k + 90° = 98.13°$$

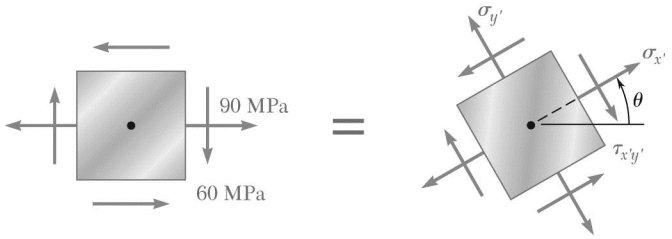
Permissible range of θ: $\theta_h \le \theta \le \theta_k$ $\quad -45° \le \theta \le 8.13°$ ◀

$\theta_u \le \theta \le \theta_v$ $\quad 45° \le \theta \le 98.13°$ ◀

Also, $135° \le \theta \le 188.13°$ and $225° \le \theta \le 278.13°$

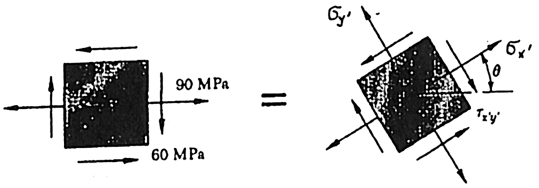
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.59



For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 50 MPa.

SOLUTION



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

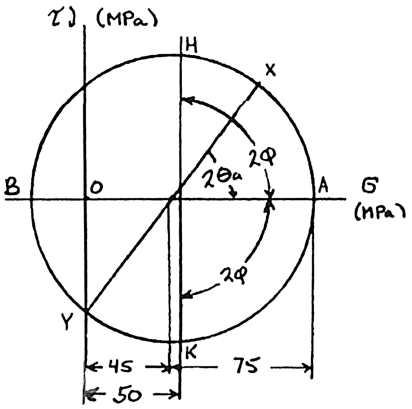
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13°$$

$$\theta_a = -26.565°$$



$\sigma_{x'} \le 50$ MPa for states of stress corresponding to the arc *HBK* of Mohr's circle. From the circle,

$$R\cos 2\varphi = 50 - 45 = 5 \text{ MPa}$$

$$\cos 2\varphi = \frac{5}{75} = 0.066667$$

$$2\varphi = 86.177° \qquad \varphi = 43.089°$$

$$\theta_h = \theta_a + \varphi = -26.565° + 43.089° = 16.524°$$

$$2\theta_k = 2\theta_h + 360° - 4\varphi = 32.524° + 360° - 172.355° = 220.169°$$

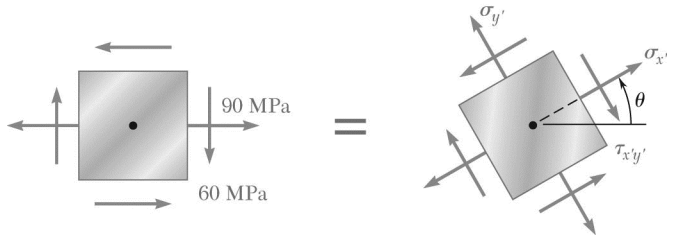
$$\theta_k = 110.085°$$

Permissible range of θ: $\theta_h \le \theta \le \theta_k$

$$16.524° \le \theta \le 110.085° \blacktriangleleft$$

Also, $196.524° \le \theta \le 290.085°$

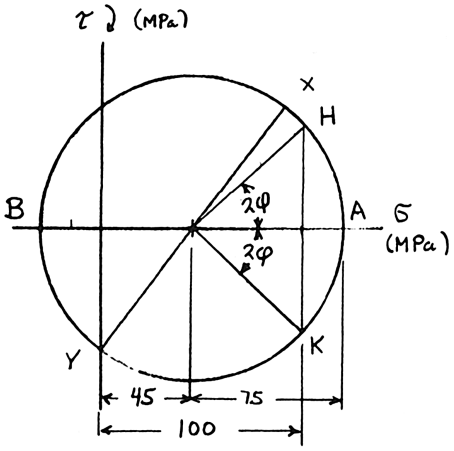
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.60

For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 100 MPa.

SOLUTION



$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13°$$

$$\theta_a = -26.565°$$

$\sigma_{x'} \le 100$ MPa for states of stress corresponding to arc *HBK* of Mohr's circle. From the circle,

$$R\cos 2\varphi = 100 - 45 = 55 \text{ MPa}$$

$$\cos 2\varphi = \frac{55}{75} = 0.73333$$

$$2\varphi = 42.833° \qquad \varphi = 21.417°$$

$$\theta_h = \theta_a + \varphi = -26.565° + 21.417° = -5.15°$$

$$2\theta_k = 2\theta_h + 360° - 4\varphi = -10.297° + 360° - 85.666° = 264.037°$$

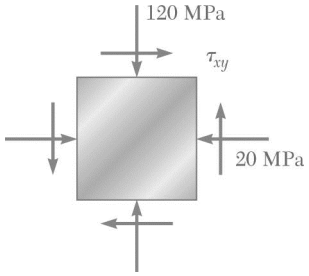
$$\theta_k = 132.02°$$

Permissible range of θ is $\theta_h \le \theta \le \theta_k$

$$-5.15° \le \theta \le 132.02°$$ ◀

Also, $174.85° \le \theta \le 312.02°$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.61

For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.

SOLUTION

$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -70 \text{ MPa}$$

Set

$$\sigma_{\max} = 60 \text{ MPa} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 130 \text{ MPa}$$

But

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_x}{2}\right)^2}$$

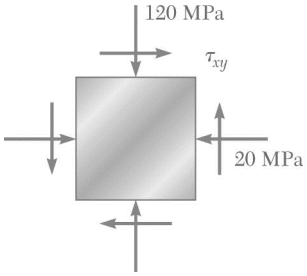
$$= \sqrt{130^2 - 50^2}$$

$$= 120 \text{ MPa}$$

Range of τ_{xy} :

$$-120 \text{ MPa} \le \tau_{xy} \le 120 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.62

For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.

SOLUTION

$$\sigma_x = -20\,\text{MPa} \quad \sigma_y = -120\ \text{MPa}$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = 50\ \text{MPa}$$

Set $$\tau_{\max\,(\text{in-plane})} = R = 150\ \text{MPa}$$

But $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

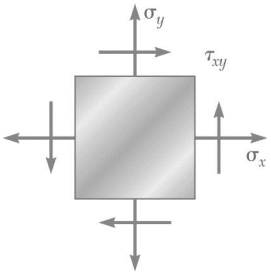
$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$= \sqrt{150^2 - 50^2}$$

$$= 141.4\ \text{MPa}$$

Range of τ_{xy}: $$-141.4\ \text{MPa} \le \tau_{xy} \le 141.4\ \text{MPa}$$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.63

For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, and $\sigma_{\min} = 5$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress $\sigma_{\max}$, (*c*) the maximum in-plane shearing stress.

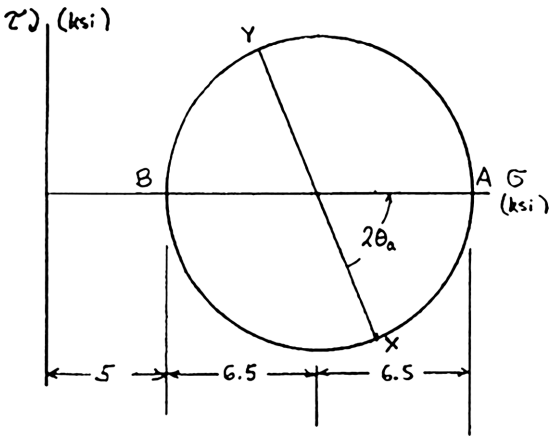
SOLUTION

$$\sigma_x = 14 \text{ ksi}, \quad \sigma_y = 9 \text{ ksi}, \quad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.5 \text{ ksi}$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \quad \therefore \quad R = \sigma_{\text{ave}} - \sigma_{\min}$$
$$= 11.5 - 5 = 6.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$



But it is given that τ_{xy} is positive, thus $\tau_{xy} = +6$ ksi.

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$
$$= \frac{(2)(6)}{5} = 2.4$$
$$2\theta_p = 67.38°$$

$$\theta_a = 33.7° \blacktriangleleft$$

$$\theta_b = 123.7° \blacktriangleleft$$

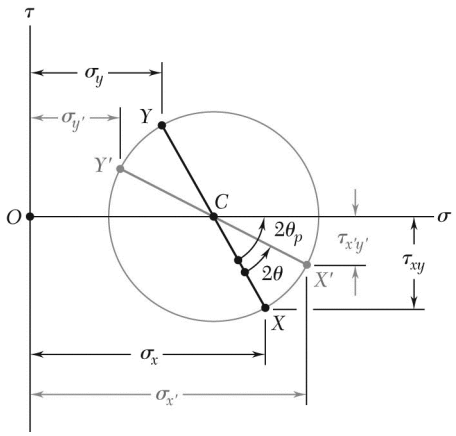
(*b*) $$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$\sigma_{\max} = 18.00 \text{ ksi} \blacktriangleleft$$

(*c*) $$\tau_{\max(\text{in-plane})} = R$$

$$\tau_{\max(\text{in-plane})} = 6.50 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.64

The Mohr's circle shown corresponds to the state of stress given in Fig. 7.5*a* and *b*. Noting that $\sigma_{x'} = OC + (CX')\cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (CX')\sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.5) and (7.6), respectively. [*Hint:* Use $\sin(A+B) = \sin A\cos B + \cos A\sin B$ and $\cos(A+B) = \cos A\cos B - \sin A \sin B$.]

SOLUTION

$$\overline{OC} = \frac{1}{2}(\sigma_x + \sigma_y) \qquad \overline{CX'} = \overline{CX}$$

$$\overline{CX'}\cos 2\theta_p = \overline{CX}\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'}\sin 2\theta_p = \overline{CX}\sin 2\theta_p = \tau_{xy}$$

$$\begin{aligned}\sigma_{x'} &= \overline{OC} + \overline{CX'}\cos(2\theta_p - 2\theta)\\ &= \overline{OC} + \overline{CX'}(\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta)\\ &= \overline{OC} + \overline{CX'}\cos 2\theta_p \cos 2\theta + \overline{CX'}\sin 2\theta_p \sin 2\theta\\ &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\end{aligned}$$ ◀

$$\begin{aligned}\tau_{x'y'} &= \overline{CX'}\sin(2\theta_p - 2\theta) = \overline{CX'}(\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta)\\ &= \overline{CX'}\sin 2\theta_p \cos 2\theta - \overline{CX'}\cos 2\theta_p \sin 2\theta\\ &= \tau_{xy}\cos 2\theta - \frac{\sigma_x - \sigma_y}{2}\sin 2\theta\end{aligned}$$ ◀

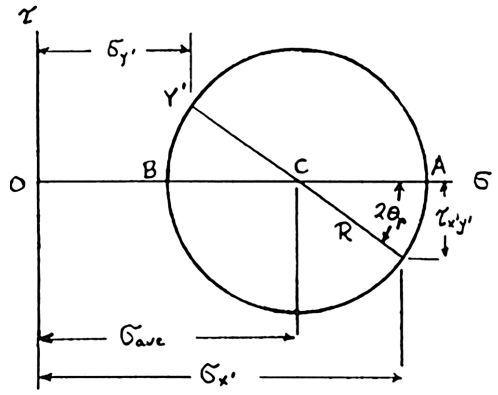
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.65

(*a*) Prove that the expression $\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$, where $\sigma_{x'}, \sigma_{y'}$, and $\tau_{x'y'}$ are components of the stress along the rectangular axes x' and y', is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (*b*) Using the invariance property established in part *a*, express the shearing stress τ_{xy} in terms of σ_x, σ_y, and the principal stresses $\sigma_{\max}$ and $\sigma_{\min}$.

SOLUTION

(*a*) From Mohr's circle,



$$\tau_{x'y'} = R\sin 2\theta_p$$

$$\sigma_{x'} = \sigma_{\text{ave}} + R\cos 2\theta_p$$

$$\sigma_{y'} = \sigma_{\text{ave}} - R\cos 2\theta_p$$

$$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$

$$= \sigma_{\text{ave}}^2 - R^2\cos^2 2\theta_p - R^2\sin^2 2\theta_p$$

$$= \sigma_{\text{ave}}^2 - R^2;\ \text{independent of } \theta_p.$$

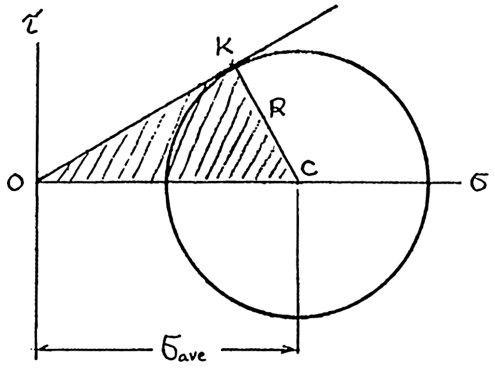
Draw line $\overline{OK}$ from origin tangent to the circle at *K*. Triangle *OCK* is a right triangle.

$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$

$$\overline{OK}^2 = \overline{OC}^2 - \overline{CK}^2$$

$$= \sigma_{\text{ave}}^2 - R^2$$

$$= \sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$



(*b*) Applying above to σ_x, σ_y, and τ_{xy}, and to σ_a, σ_b,

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_a\sigma_b - \tau_{ab}^2 = \sigma_{\text{ave}}^2 - R^2$$

But $\tau_{ab} = 0, \quad \sigma_a = \sigma_{\max}, \quad \sigma_b = \sigma_{\min}$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_{\max}\sigma_{\min}$$

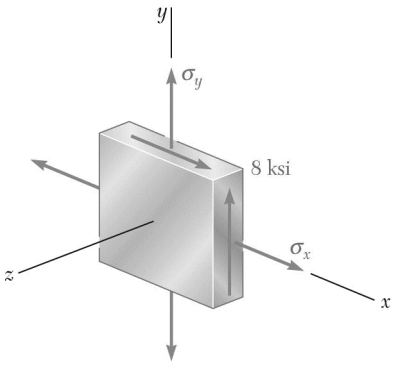
$$\tau_{xy}^2 = \sigma_x\sigma_y - \sigma_{\max}\sigma_{\min}$$

$$\tau_{xy} = \pm\sqrt{\sigma_x\sigma_y - \sigma_{\max}\sigma_{\min}}$$

◀

The sign cannot be determined from above equation.

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

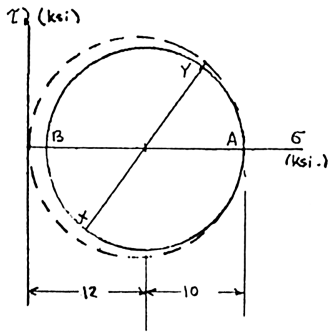


PROBLEM 7.66

For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 6$ ksi and $\sigma_y = 18$ ksi, (*b*) $\sigma_x = 14$ ksi and $\sigma_y = 2$ ksi. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 6\,\text{ksi}, \quad \sigma_y = 18\ \text{ksi}, \quad \tau_{xy} = 8\ \text{ksi}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 12\ \text{ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{6^2 + 8^2} = 10\ \text{ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 12 + 10 = 22\ \text{ksi (max)}$$

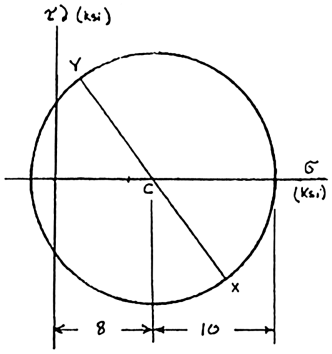
$$\sigma_b = \sigma_{\text{ave}} - R = 12 - 10 = 2\ \text{ksi}$$

$$\sigma_c = 0 \qquad \text{(min)}$$

$$\tau_{\text{max(in-plane)}} = R = 10\ \text{ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 11\ \text{ksi} \blacktriangleleft$$

(*b*) $\sigma_x = 14\ \text{ksi}, \quad \sigma_y = 2\ \text{ksi}, \quad \tau_{xy} = 8\ \text{ksi}$



$$\sigma_a = \frac{1}{2}(\sigma_x + \sigma_y) = 8\ \text{ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{6^2 + 8^2} = 10\ \text{ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 18\ \text{ksi (max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -2\ \text{ksi (min)}$$

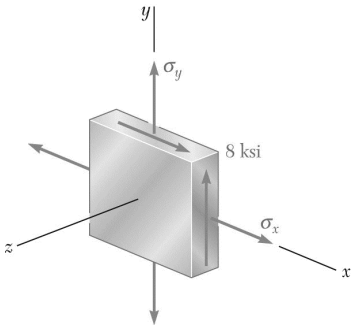
$$\sigma_c = 0$$

$$\sigma_{\max} = 18\ \text{ksi}$$

$$\sigma_{\min} = -2\ \text{ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 10\ \text{ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.67

For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 12$ ksi, (*b*) $\sigma_x = 21$ ksi and $\sigma_y = 9$ ksi. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 0, \quad \sigma_y = 12 \text{ ksi}, \quad \tau_{xy} = 8 \text{ ksi}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y)$$

$$= 6 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-6)^2 + 8^2}$$

$$= 10 \text{ ksi}$$

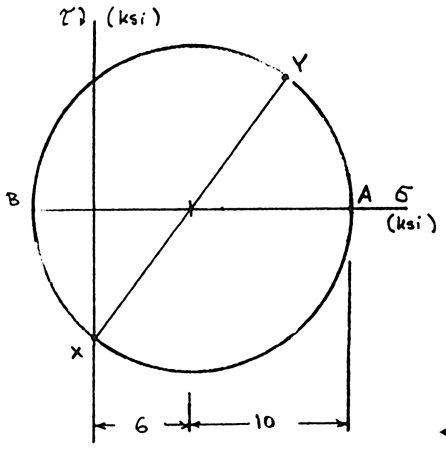
$$\sigma_a = \sigma_{\text{ave}} + R = 16 \text{ ksi} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -4 \text{ ksi} \quad \text{(min)}$$

$$\sigma_c = 0$$

$$\sigma_{\max} = 16 \text{ ksi} \quad \sigma_{\min} = -4 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 10 \text{ ksi}$$



◀

(*b*) $\sigma_x = 21 \text{ ksi} \qquad \sigma_y = 9 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$

$$\sigma_{\text{ave}} = 15 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-6)^2 + 8^2} = 10 \text{ ksi}$$

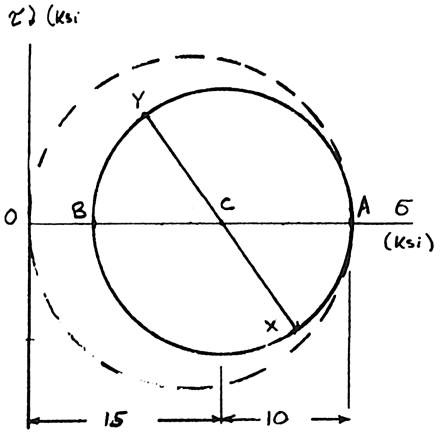
$$\sigma_a = \sigma_{\text{ave}} + R = 25 \text{ ksi} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 5 \text{ ksi}$$

$$\sigma_c = 0 \qquad \text{(min)}$$

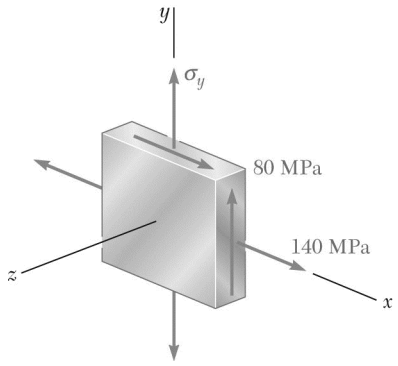
$$\sigma_{\max} = 25 \text{ ksi}, \quad \sigma_{\min} = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 12.5 \text{ ksi}$$



◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

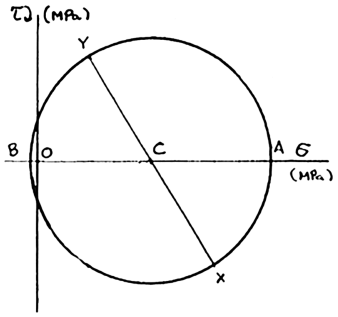


PROBLEM 7.68

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 140$ MPa, $\sigma_y = 40$ MPa, $\tau_{xy} = 80$ MPa



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 184.34 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -4.34 \text{ MPa} \quad \text{(min)}$$

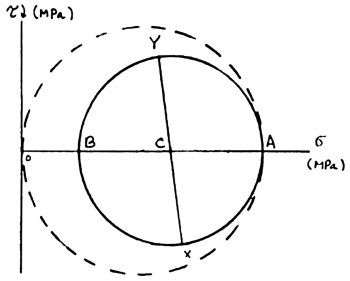
$$\sigma_c = 0$$

$$\tau_{\max(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.3 \text{ MPa}$$

$\tau_{\max} = 94.3$ MPa ◀

(*b*) $\sigma_x = 140$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 210.62 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 49.38 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

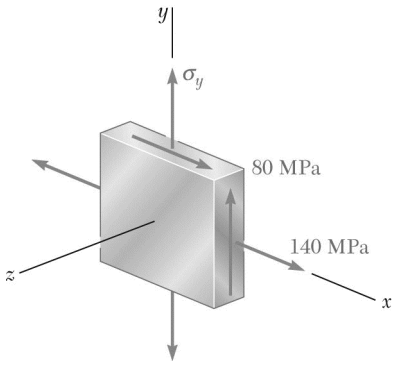
$$\sigma_{\max} = \sigma_a = 210.62 \text{ MPa} \quad \sigma_{\min} = \sigma_c = 0$$

$$\tau_{\max(\text{in-plane})} = R = 86.62 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 105.3 \text{ MPa}$$

$\tau_{\max} = 105.3$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.69

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 20 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$

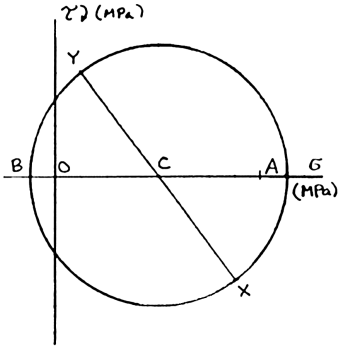
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 80 + 100 = 180 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 80 - 100 = -20 \text{ MPa} \quad \text{(min)}$$

$$\sigma_c = 0$$



$$\tau_{\max\,(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = 100 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 100 \text{ MPa}$$

$\tau_{\max} = 100 \text{ MPa}$ ◀

(*b*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 140 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$

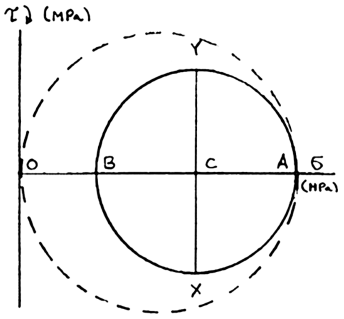
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 220 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 60 \text{ MPa}$$

$$\sigma_c = 0 \quad \text{(min)}$$

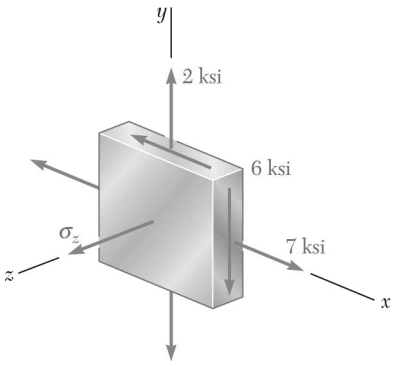


$$\tau_{\max\,(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = 80 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 110 \text{ MPa}$$

$\tau_{\max} = 110 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.70

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 7 \text{ ksi}, \qquad \sigma_y = 2 \text{ ksi}, \qquad \tau_{xy} = -6 \text{ ksi}$$

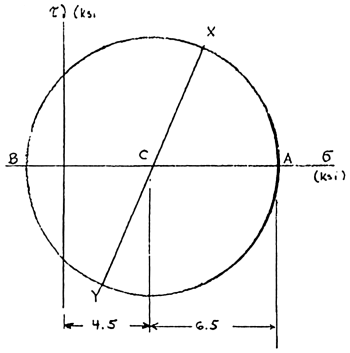
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 4.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 11 \text{ ksi}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -2 \text{ ksi}$$



(*a*) $\sigma_z = 4 \text{ ksi}, \quad \sigma_a = 11 \text{ ksi}, \quad \sigma_b = -2 \text{ ksi}$

$$\sigma_{\max} = 11 \text{ ksi}, \quad \sigma_{\min} = -2 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 6.5 \text{ ksi} \blacktriangleleft$$

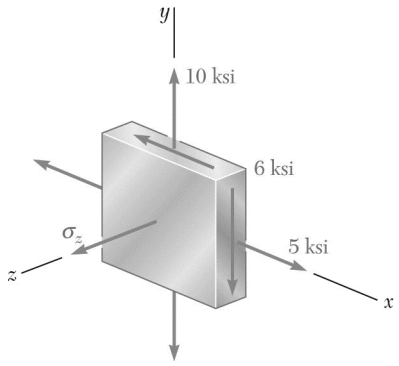
(*b*) $\sigma_z = -4 \text{ ksi}, \quad \sigma_a = 11 \text{ ksi}, \quad \sigma_b = -2 \text{ ksi}$

$$\sigma_{\max} = 11 \text{ ksi}, \quad \sigma_{\min} = -4 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 7.5 \text{ ksi} \blacktriangleleft$$

(*c*) $\sigma_z = 0, \quad \sigma_a = 11 \text{ ksi}, \quad \sigma_b = -2 \text{ ksi}$

$$\sigma_{\max} = 11 \text{ ksi}, \quad \sigma_{\min} = -2 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 6.5 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.71

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 5 \text{ ksi}, \qquad \sigma_y = 10 \text{ ksi}, \qquad \tau_{xy} = -6 \text{ ksi}$$

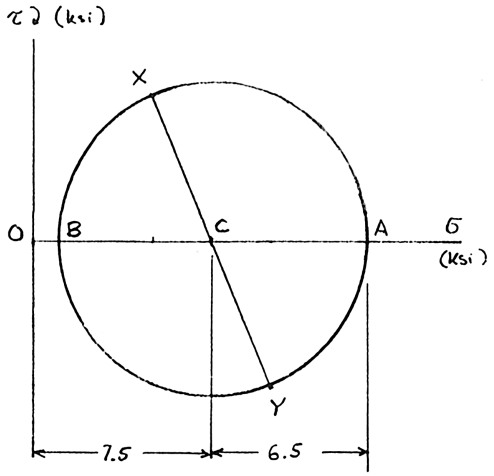
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 7.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-2.5)^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 14 \text{ ksi}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 1 \text{ ksi}$$



(*a*) $\sigma_z = +4 \text{ ksi}, \quad \sigma_a = 14 \text{ ksi}, \quad \sigma_b = 1 \text{ ksi}$

$$\sigma_{\max} = 14 \text{ ksi}, \quad \sigma_{\min} = 1 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$\tau_{\max} = 6.5 \text{ ksi}$ ◀

(*b*) $\sigma_z = -4 \text{ ksi}, \quad \sigma_a = 14 \text{ ksi}, \quad \sigma_b = 1 \text{ ksi}$

$$\sigma_{\max} = 14 \text{ ksi}, \quad \sigma_{\min} = -4 \text{ ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

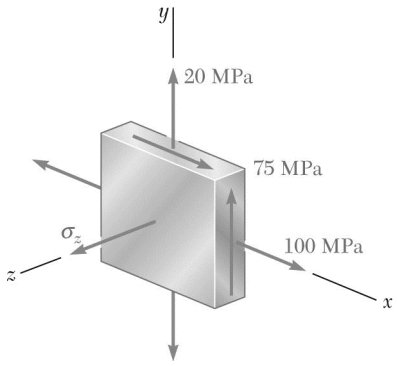
$\tau_{\max} = 9 \text{ ksi}$ ◀

(*c*) $\sigma_z = 0, \quad \sigma_a = 14 \text{ ksi}, \quad \sigma_b = 1 \text{ ksi}$

$$\sigma_{\max} = 14 \text{ ksi}, \qquad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$

$\tau_{\max} = 7 \text{ ksi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.72

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.

SOLUTION

$$\sigma_x = 100 \text{ MPa}, \qquad \sigma_y = 20 \text{ MPa}, \qquad \tau_{xy} = 75 \text{ MPa}$$

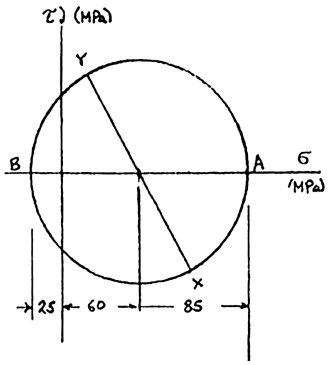
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 60 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -25 \text{ MPa}$$



(*a*) $\sigma_z = 0, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{\max} = 145 \text{ MPa}, \quad \sigma_{\min} = -25 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 85 \text{ MPa} \blacktriangleleft$$

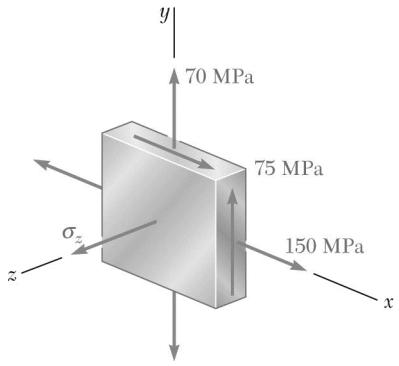
(*b*) $\sigma_z = +45 \text{ MPa}, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{\max} = 145 \text{ MPa}, \quad \sigma_{\min} = -25 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 85 \text{ MPa} \blacktriangleleft$$

(*c*) $\sigma_z = -45 \text{ MPa}, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{\max} = 145 \text{ MPa}, \quad \sigma_{\min} = -45 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 95 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.73

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.

SOLUTION

$$\sigma_x = 150 \text{ MPa}, \qquad \sigma_y = 70 \text{ MPa}, \qquad \tau_{xy} = 75 \text{ MPa}$$

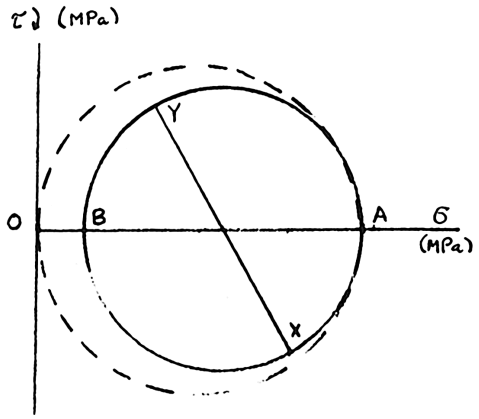
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 110 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 195 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 25 \text{ MPa}$$



(*a*) $\sigma_z = 0, \quad \sigma_a = 195 \text{ MPa}, \quad \sigma_b = 25 \text{ MPa}$

$$\sigma_{\max} = 195 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 97.5 \text{ MPa} \blacktriangleleft$$

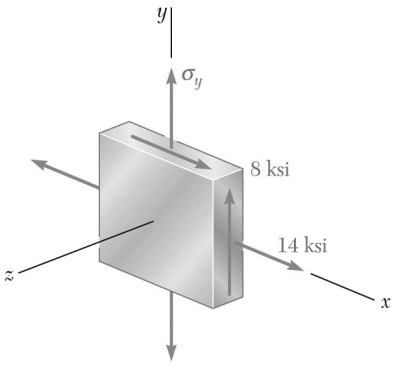
(*b*) $\sigma_z = +45 \text{ MPa}, \quad \sigma_a = 195 \text{ MPa}, \quad \sigma_b = 25 \text{ MPa}$

$$\sigma_{\max} = 195 \text{ MPa}, \quad \sigma_{\min} = 25 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 85 \text{ MPa} \blacktriangleleft$$

(*c*) $\sigma_z = -45 \text{ MPa}, \quad \sigma_a = 195 \text{ MPa}, \quad \sigma_b = 25 \text{ MPa}$

$$\sigma_{\max} = 195 \text{ MPa}, \quad \sigma_{\min} = -45 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 120 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.74

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 10 ksi.

SOLUTION

$$\sigma_x = 14\,\text{ksi}, \quad \tau_{xy} = 8\ \text{ksi}, \quad \tau_{\max} = 10\ \text{ksi}$$

Let $$u = \frac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2u + \sigma_x$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $\tau_{\max} = R = 10$ ksi, $\quad u = \pm 6$ ksi

(1a) $u = +6\,\text{ksi} \quad \sigma_y = 2u + \sigma_x = 26$ ksi (reject)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 20\ \text{ksi}, \quad \sigma_a = \sigma_{\text{ave}} + R = 30\ \text{ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 10\ \text{ksi}$$

$$\sigma_{\max} = 30\ \text{ksi}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 15\ \text{ksi} \neq 7.5\ \text{ksi}$$

(1b) $u = -6$ ksi $\quad \sigma_y = 2u + \sigma_x = 2$ ksi

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 8\ \text{ksi}, \quad \sigma_a = \sigma_{\text{ave}} + R = 18\ \text{ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = -2\ \text{ksi}$$

$$\sigma_{\max} = 18\ \text{ksi}, \quad \sigma_{\min} = -2\ \text{ksi}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 10\ \text{ksi} \quad \text{(o.k.)}$$

$\sigma_y = 2.00$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.74 *(Continued)*

Case (2) Assume $\sigma_{\min} = 0$. $\sigma_{\max} = 2\tau_{\max} = 20 \text{ ksi} = \sigma_a$

$$\sigma_a = \sigma_{\text{ave}} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$$

$$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + u^2 = u^2 + \tau_{xy}^2$$

$$2u = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(20 - 14)^2 - 8^2}{20 - 14} = -4.6667 \text{ ksi}$$

$$u = -2.3333 \text{ ksi} \quad \sigma_y = 2u + \sigma_x = 9.3333 \text{ ksi}$$

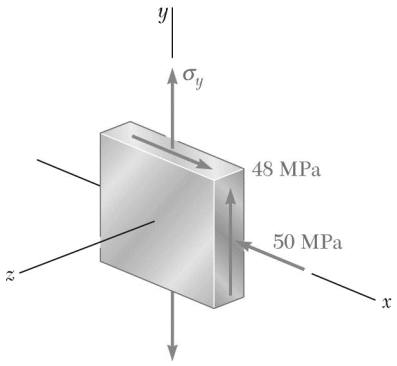
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.6667 \text{ ksi} \quad R = \sqrt{u^2 + \tau_{xy}^2} = 8.3333 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 20 \text{ ksi} \qquad \sigma_b = \sigma_{\text{ave}} - R = 3.3334 \text{ ksi}$$

$$\sigma_{\max} = 20 \text{ ksi}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = 10 \text{ ksi}$$

$\sigma_y = 9.33 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.75

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 73 MPa.

SOLUTION

$$\sigma_x = -50 \text{ MPa}, \quad \tau_{xy} = 48 \text{ MPa}$$

Let $$u = \frac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2u + \sigma_x$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$$

Case (1) $\tau_{\max} = R = 73$ MPa, $\quad u = \pm\sqrt{73^2 - 48^2} = \pm 55$ MPa

(1a) $u = +55$ MPa $\quad \sigma_y = 2u + \sigma_x = 60$ MPa

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 5 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 78 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = -68 \text{ MPa}$$

$\sigma_a = 0 \quad \sigma_{\max} = 78$ MPa, $\quad \sigma_{\min} = -68$ MPa, $\quad \tau_{\max} = 73$ MPa

(1b) $u = -55$ MPa $\quad \sigma_y = 2u + \sigma_x = -160$ MPa (reject)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -105 \text{ MPa}, \quad \sigma_a = \sigma_{\text{ave}} + R = -32 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -178 \text{ MPa}, \quad \sigma_c = 0, \quad \sigma_{\max} = 0$$

$$\sigma_{\min} = -178 \text{ MPa}, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 89 \text{ MPa} \neq 73 \text{ MPa}$$

$\sigma_y = 60.0$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.75 *(Continued)*

Case (2) Assume $\sigma_{\max} = 0$. $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 73$ MPa

$$\sigma_{\min} = -146 \text{ MPa} = \sigma_b$$

$$\sigma_b = \sigma_{\text{ave}} - R = \sigma_x + u - \sqrt{u^2 + \tau_{xy}^2}$$

$$\sqrt{u^2 + \tau_{xy}^2} = -\sigma_x + u - \sigma_b$$

$$u^2 + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + u^2$$

$$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} = \frac{(48)^2 - (-50 + 146)^2}{-50 + 146} = -72 \text{ MPa}$$

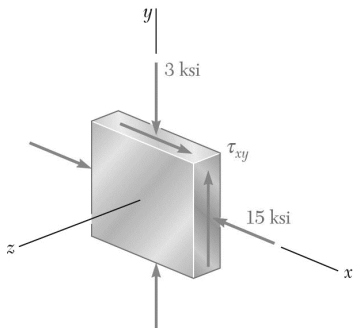
$$u = -36 \text{ MPa} \qquad \sigma_y = 2u + \sigma_x = -122 \text{ MPa}$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = 60 \text{ MPa}$$

$$\sigma_a = \sigma_b + 2R = -146 + 120 = -26 \text{ MPa} \quad \text{(o.k.)}$$

$$\sigma_y = -122.0 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.76

For the state of plane stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 10 ksi, (*b*) 8.25 ksi.

SOLUTION

$$\sigma_x = -15 \text{ ksi} \quad \sigma_y = -3 \text{ ksi} \quad \sigma_z = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -9 \text{ ksi}$$

(*a*) $\underline{\tau_{\max} = 10 \text{ ksi}}$:

If $\tau_{\max}$ is the in-plane maximum shearing stress, then

$$R = \tau_{\max} = 10 \text{ ksi}$$
$$\sigma_{\max} = \sigma_{\text{ave}} + R = 1 \text{ ksi}$$
$$\sigma_{\min} = \sigma_{\text{ave}} - R = -19 \text{ ksi}$$

σ_z is the intermediate principal stress; the assumption $R = \tau_{\max}$ is correct.

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + \tau_{xy}^2} = 10$$

$$\tau_{xy} = \sqrt{10^2 - 6^2} \qquad \tau_{xy} = 8 \text{ ksi} \blacktriangleleft$$

(*b*) $\underline{\tau_{\max} = 8.25 \text{ ksi.}}$

If $\tau_{\max}$ is the in-plane maximum shearing stress, then

$$R = \tau_{\max} = 8.25 \text{ ksi.}$$
$$\sigma_{\max} = \sigma_{\text{ave}} + R = -9 + 8.25 = -0.75 \text{ ksi}$$
$$\sigma_{\min} = \sigma_{\text{ave}} - R = -9 - 8.25 = -17.25 \text{ ksi}$$

$\sigma_z = 0$ is <u>not</u> the intermediate principal stress.

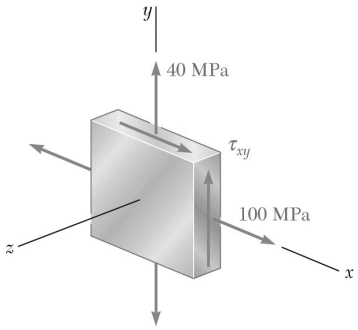
Let $\sigma_z = 0 = \sigma_{\max}$.

$$\sigma_{\min} = \sigma_{\max} - 2\tau_{\max} = 0 - (2)(8.25) = -16.5 \text{ ksi}$$
$$R = \sigma_{\text{ave}} - \sigma_{\min} = -9 - (-16.5) = 7.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + \tau_{xy}^2} = 7.5 \text{ ksi}$$

$$\tau_{xy} = \sqrt{7.5^2 - 6^2} = 4.5 \text{ ksi} \qquad \tau_{xy} = 4.5 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.77

For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 60 MPa, (*b*) 78 MPa.

SOLUTION

$$\sigma_x = 100 \text{ MPa}, \quad \sigma_y = 40 \text{ MPa}, \quad \sigma_z = 0$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 70 \text{ MPa}$$

(*a*) $\underline{\tau_{\max} = 60 \text{ MPa}.}$

If σ_z is $\sigma_{\min}$, then $\sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$.

$$\sigma_{\max} = 0 + (2)(60) = 120 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 120 - 70 = 50 \text{ MPa}$$

$$\sigma_b = \sigma_{\max} - 2R = 20 \text{ MPa} > 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\tau_{xy} = \sqrt{50^2 - 30^2} \qquad \tau_{xy} = 40 \text{ MPa} \blacktriangleleft$$

(*b*) $\underline{\tau_{\max} = 78 \text{ MPa}.}$

If σ_z is $\sigma_{\min}$, then $\sigma_{\max} = \sigma_{\min} + 2\tau_{\max} = 0 + (2)(78) = 156$ MPa.

$$\sigma_{\max} = \sigma_{\text{ave}} + R$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 156 - 70 = 86 \text{ MPa} > \tau_{\max} = 78 \text{ MPa}$$

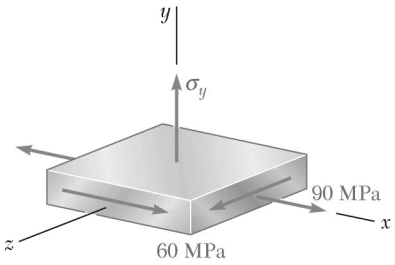
Set

$$R = \tau_{\max} = 78 \text{ MPa}. \quad \sigma_{\min} = \sigma_{\text{ave}} - R = -8 \text{ MPa} < 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{78^2 - 30^2} \qquad \tau_{xy} = 72 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.78

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.

SOLUTION

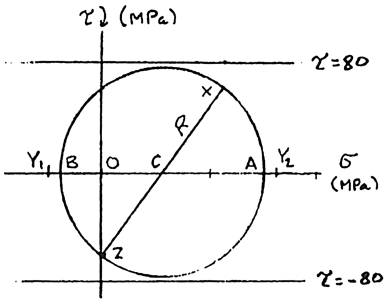
$$\sigma_x = 90 \text{ MPa}, \quad \sigma_z = 0, \quad \tau_{xz} = 60 \text{ MPa}$$

Mohr's circle of stresses in *zx*-plane:

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^2} = \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 120 \text{ MPa}, \quad \sigma_b = \sigma_{\text{ave}} - R = -30 \text{ MPa}$$



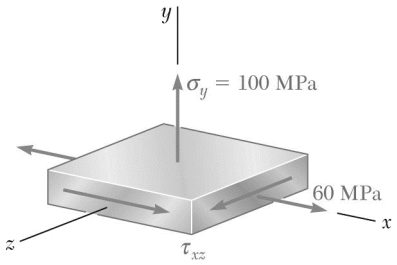
Assume $\sigma_{\max} = \sigma_a = 120$ MPa.

$$\sigma_y = \sigma_{\min} = \sigma_{\max} - 2\tau_{\max}$$
$$= 120 - (2)(80) \qquad \sigma_y = -40 \text{ MPa} \blacktriangleleft$$

Assume $\sigma_{\min} = \sigma_b = -30$ MPa.

$$\sigma_y = \sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$$
$$= -30 + (2)(8) \qquad \sigma_y = 130 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.79

For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 60 MPa.

SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_z = 0, \quad \sigma_y = 100 \text{ MPa}$$

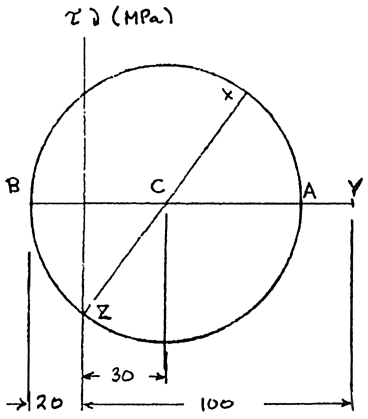
For Mohr's circle of stresses in *zx*-plane

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 30 \text{ MPa}$$

$$u = \frac{\sigma_x - \sigma_z}{2} = 30 \text{ MPa}$$

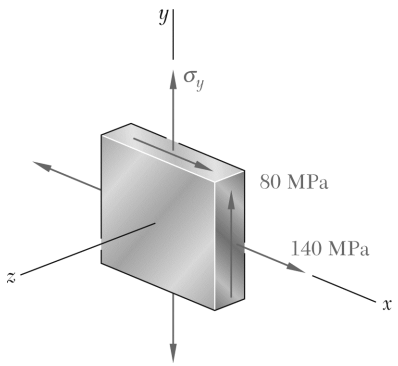
Assume $\sigma_{\max} = \sigma_y = 100$ MPa

$$\sigma_{\min} = \sigma_b = \sigma_{\max} - 2\tau_{\max}$$
$$= 100 - (2)(60) = -20 \text{ MPa}$$
$$R = \sigma_{\text{ave}} - \sigma_b$$
$$= 30 - (-20) = 50 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R$$
$$= 30 + 50 = 80 \text{ MPa} < \sigma_y$$
$$R = \sqrt{u^2 + \tau_{xz}^2}$$
$$\tau_{xz} = \pm\sqrt{R^2 - u^2}$$
$$= \pm\sqrt{50^2 - 30^2} = \pm 40 \text{ MPa}$$



$$-40 \text{ MPa} \le \tau_{xz} \le 40 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.80*

For the state of stress of Prob. 7.69, determine (*a*) the value of σ_y for which the maximum shearing stress is as small as possible, (*b*) the corresponding value of the shearing stress.

SOLUTION

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\sigma_y = \sigma_x - 2u$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{\text{ave}} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume $\tau_{\max}$ is the in-plane shearing stress. $\tau_{\max} = R$

Then $\tau_{\max\,(\text{in-plane})}$ is minimum if $u = 0$.

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \quad \sigma_{\text{ave}} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{\max} = 220 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 110 \text{ MPa}$$

Assumption is incorrect.

Assume $\sigma_{\max} = \sigma_a = \sigma_{\text{ave}} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$

$$\sigma_{\min} = 0 \qquad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \qquad \text{(no minimum)}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.80* *(Continued)*

Optimum value for u occurs when $\tau_{\max\,(\text{out-of-plane})} = \tau_{\max\,(\text{in-plane})}$

$$\frac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

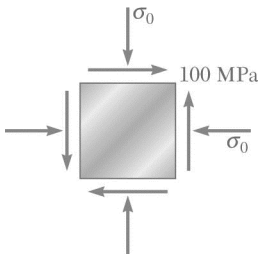
$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$$

$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \qquad u = 47.14 \text{ MPa}$$

(*a*) $\sigma_y = \sigma_x - 2u = 140 - 94.3$ $\qquad \sigma_y = 45.7 \text{ MPa}$ ◀

(*b*) $R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{\max} = 92.9 \text{ MPa}$ $\qquad \tau_{\max} = 92.9 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.81

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_{\text{ave}} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(*a*) $\underline{\sigma_0 = 200 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -200$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -100 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -300 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa} \qquad \text{(No yielding)}$$

$$F.S. = \frac{325}{264.56} \qquad F.S. = 1.228 \blacktriangleleft$$

(*b*) $\underline{\sigma_0 = 240 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -240$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -340 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa} \qquad \text{(No yielding)}$$

$$F.S. = \frac{325}{295.97} \qquad F.S. = 1.098 \blacktriangleleft$$

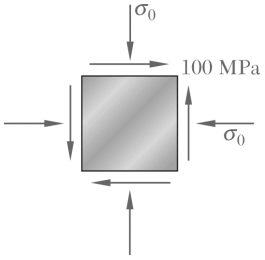
(*c*) $\underline{\sigma_0 = 280 \text{ MPa}}$ $\quad \sigma_{\text{ave}} = -280$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -380 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.82



Solve Prob. 7.81, using the maximum-shearing-stress criterion.

PROBLEM 7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_{\text{ave}} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(*a*) $\underline{\sigma_0 = 200 \text{ MPa}}$, $\sigma_{\text{ave}} = -200$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -100 \text{ MPa} \qquad \sigma_b = \sigma_{\text{ave}} - R = -300 \text{ MPa}$$

$$\sigma_{\max} = 0, \quad \sigma_{\min} = -300 \text{ MPa}$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 300 \text{ MPa} < 325 \text{ MPa} \qquad \text{(No yielding)}$$

$$F.S. = \frac{325}{300} \qquad F.S. = 1.083 \blacktriangleleft$$

(*b*) $\underline{\sigma_0 = 240 \text{ MPa}}$, $\sigma_{\text{ave}} = -240$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -140 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -340 \text{ MPa}$$

$$\sigma_{\max} = 0, \qquad \sigma_{\min} = -340 \text{ MPa}$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 340 \text{ MPa} > 325 \text{ MPa} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

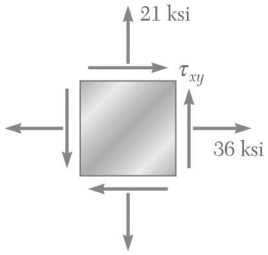
(*c*) $\underline{\sigma_0 = 280 \text{ MPa}}$, $\sigma_{\text{ave}} = -280$ MPa

$$\sigma_a = \sigma_{\text{ave}} + R = -180 \text{ MPa}, \qquad \sigma_b = \sigma_{\text{ave}} - R = -380 \text{ MPa}$$

$$\sigma_{\max} = 0, \quad \sigma_{\min} = -380 \text{ MPa}$$

$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 380 \text{ MPa} > 325 \text{ MPa} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.83

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 36 \text{ ksi}, \quad \sigma_y = 21 \text{ ksi}, \quad \sigma_z = 0$$

For stresses in *xy*-plane, $\quad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5 \text{ ksi}$

$$\frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$$

(*a*) $\tau_{xy} = 9$ ksi $\qquad R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (9)^2} = 11.715$ ksi

$\sigma_a = \sigma_{\text{ave}} + R = 40.215$ ksi, $\quad \sigma_b = \sigma_{\text{ave}} - R = 16.875$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 34.977 \text{ ksi} < 45 \text{ ksi}$ (No yielding)

$F.S. = \dfrac{45}{39.977}$ $\qquad F.S. = 1.287$ ◀

(*b*) $\tau_{xy} = 18$ ksi $\quad R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (18)^2} = 19.5$ ksi

$\sigma_a = \sigma_{\text{ave}} + R = 48$ ksi, $\quad \sigma_b = \sigma_{\text{ave}} - R = 9$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 44.193 \text{ ksi} < 45 \text{ ksi}$ (No yielding)

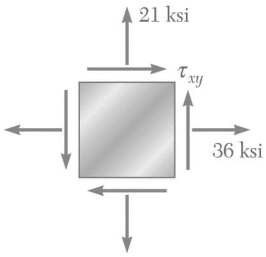
$F.S. = \dfrac{45}{44.193}$ $\qquad F.S. = 1.018$ ◀

(*c*) $\tau_{xy} = 20$ ksi $\qquad R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (20)^2} = 21.36$ ksi

$\sigma_a = \sigma_{\text{ave}} + R = 49.86$ ksi, $\quad \sigma_b = \sigma_{\text{ave}} - R = 7.14$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 46.732 \text{ ksi} > 45 \text{ ksi}$ (Yielding occurs) ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.84

Solve Prob. 7.83, using the maximum-shearing-stress criterion.

PROBLEM 7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 36 \text{ ksi}, \quad \sigma_y = 21 \text{ ksi}, \quad \sigma_z = 0$$

For stress in *xy*-plane, $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5 \text{ ksi}$ $\quad \frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$

(*a*) $\tau_{xy} = 9$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 11.715$ ksi

$$\sigma_a = \sigma_{\text{ave}} + R = 40.215 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 16.875 \text{ ksi}$$

$$\tau_{\max} = 34.977 \text{ ksi}, \quad \sigma_{\min} = 0$$

$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 40.215 \text{ ksi} < 45 \text{ ksi}$ (No yielding)

$F.S. = \frac{45}{40.215}$ $F.S. = 1.119$ ◀

(*b*) $\tau_{xy} = 18$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 19.5$ ksi

$$\sigma_a = \sigma_{\text{ave}} + R = 48 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 9 \text{ ksi}$$

$$\sigma_{\max} = 48 \text{ ksi} \quad \sigma_{\min} = 0$$

$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 48 \text{ ksi} > 45 \text{ ksi}$ (Yielding occurs) ◀

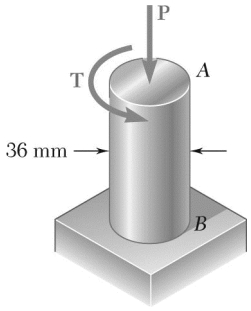
(*c*) $\tau_{xy} = 20$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.36$ ksi

$$\sigma_a = \sigma_{\text{ave}} + R = 49.86 \text{ ksi} \quad \sigma_b = \sigma_{\text{ave}} - R = 7.14 \text{ ksi}$$

$$\tau_{\max} = 49.86 \text{ ksi} \quad \sigma_{\min} = 0$$

$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = 49.86 \text{ ksi} > 45 \text{ ksi}$ (Yielding occurs) ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.85

The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 200$ kN.

SOLUTION

$$P = 200 \text{ kN} = 200 \times 10^3 \text{ N} \qquad c = \frac{1}{2}d = 18 \text{ mm} = 18 \times 10^{-3} \text{m}$$

$$A = \pi c^2 = \pi (18 \times 10^{-3})^2 = 1.01788 \times 10^{-3} \text{m}^2$$

$$\sigma_y = -\frac{P}{A} = -\frac{200 \times 10^3}{1.01788 \times 10^{-3}} = 196.488 \times 10^6 \text{ Pa}$$

$$= 196.488 \text{ MPa}$$

$$\sigma_x = 0 \qquad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R \quad \text{(positive)}$$

$$\sigma_b = \sigma_{\text{ave}} - R \quad \text{(negative)}$$

$$|\sigma_a - \sigma_b| = 2R \qquad |\sigma_a - \sigma_b| > |\sigma_a| \qquad |\sigma_a - \sigma_b| > |\sigma_b|$$

Maximum shear stress criterion under the above conditions:

$$|\sigma_a - \sigma_b| = 2R = \sigma_Y = 250 \text{ MPa} \qquad R = 125 \text{ MPa}$$

Equating expressions for R,

$$125 = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{(125)^2 - (98.244)^2} = 77.286 \text{ MPa} = 77.286 \times 10^6 \text{Pa}$$

Torsion:

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(18 \times 10^{-3})^4 = 164.896 \times 10^{-9} \text{m}^4$$

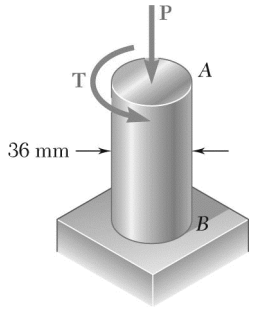
$$\tau_{xy} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(164.846 \times 10^{-9})(77.286 \times 10^6)}{18 \times 10^{-3}}$$

$$= 708 \text{ N} \cdot \text{m}$$

$T = 708 \text{ N} \cdot \text{m}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.86

Solve Prob. 7.85, using the maximum-distortion-energy criterion.

PROBLEM 7.85 The 36-mm-diameter shaft is made of a grade of steel with a 250-MPa tensile yield stress. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 200$ kN.

SOLUTION

$$P = 200 \text{ kN} = 200\times10^{3} \text{ N} \qquad c = \frac{1}{2}d = 18 \text{ mm} = 18\times10^{-3} \text{ m}$$

$$A = \pi c^2 = \pi(18\times10^{-3})^2 = 1.01788\times10^{-3} \text{ m}^2$$

$$\sigma_y = -\frac{P}{A} = -\frac{200\times10^{3}}{1.01788\times10^{-3}} = 196.488\times10^{6} \text{ Pa}$$

$$= 196.448 \text{ MPa}$$

$$\sigma_x = 0 \qquad \sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_y = 98.244 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(98.244)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_b = \sigma_{\text{ave}} - R$$

Distortion energy criterion:

$$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = \sigma_Y^2$$

$$(\sigma_{\text{ave}} + R)^2 + (\sigma_{\text{ave}} - R)^2 - (\sigma_{\text{ave}} + R)(\sigma_{\text{ave}} - R) = \sigma_Y^2$$

$$\sigma_{\text{ave}}^2 + 3R^2 = \sigma_Y^{-2}$$

$$(98.244)^2 + (3)[(98.244)^2 + \tau_{xy}^2] = (250)^2$$

$$\tau_{xy} = \pm 89.242 \text{ MPa}$$

Torsion:

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(18\times10^{-3})^4 = 164.846\times10^{-9} \text{ m}^4$$

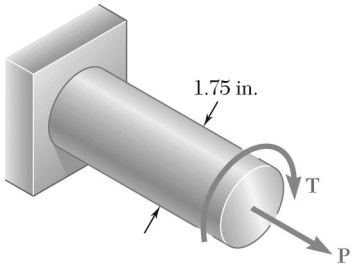
$$\tau_{xy} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(164.846\times10^{-9})(89.242\times10^{6})}{18\times10^{-3}}$$

$$= 818 \text{ N}\cdot\text{m}$$

$$T = 818 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.87

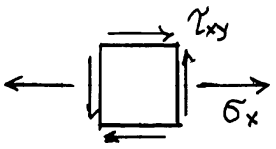
The 1.75-in.-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 36$ ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the force **P** for which yield occurs when $T = 15$ kip · in.

SOLUTION

Let the x-axis lie along the shaft axis.

$$\sigma_y = 0, \quad \sigma_z = 0$$

$$\sigma_x = \frac{P}{A}, \ \tau_{xy} = \frac{Tc}{J}$$



Section properties: $c = \frac{1}{2}d = 0.875$ in.

$$A = \pi c^2 = \pi(0.875)^2 = 2.4053 \text{ in}^2, \ J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

From torsion, $\tau_{xy} = \frac{Tc}{J} = \frac{(15)(0.875)}{0.92077} = 14.254$ ksi

From Mohr's circle, $\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{\text{ave}} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Maximum shearing stress criterion:

$$\sigma_{\max} = \sigma_a \qquad \sigma_{\min} = \sigma_b$$

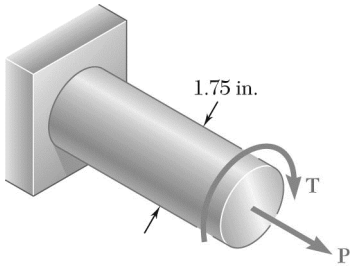
$$2\tau_{\max} = \sigma_{\max} - \sigma_{\min} = \sigma_a - \sigma_b = 2\sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} = \sigma_Y$$

$$\sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y \qquad \sigma_x = \sqrt{\sigma_Y^2 - 4\tau_{xy}^2}$$

$$\sigma_x = \sqrt{(36)^2 - 4(14.254)^2} = 21.984 \text{ ksi}$$

$$P = \sigma_x A = (21.984)(2.4053) \qquad P = 52.9 \text{ kips} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.88

Solve Prob. 7.87, using the maximum-distortion-energy criterion.

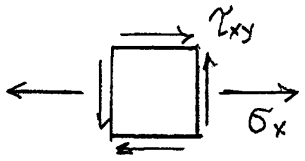
PROBLEM 7.87 The 1.75-in.-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_Y = 36$ ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the force **P** for which yield occurs when $T = 15$ kip · in.

SOLUTION

Let the *x*-axis lie along the shaft axis.

$$\sigma_y = 0, \qquad \sigma_z = 0$$

$$\sigma_x = \frac{P}{A}, \qquad \tau_{xy} = \frac{Tc}{J}$$



Section properties: $c = \frac{1}{2}d = 0.875$ in.

$$A = \pi c^2 = \pi(0.875)^2 = 2.4053 \text{ in}^2, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

From torsion, $$\tau_{xy} = \frac{Tc}{J} = \frac{(15)(0.875)}{0.92077} = 14.254 \text{ ksi}$$

From Mohr's circle, $$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{\text{ave}} + R$$

$$\sigma_b = \sigma_{\text{ave}} - R$$

Distortion energy criterion:

$$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = \sigma_Y^2$$

$$(\sigma_{\text{ave}} + R)^2 + (\sigma_{\text{ave}} - R)^2 - (\sigma_{\text{ave}} + R)(\sigma_{\text{ave}} - R) = \sigma_Y^2$$

$$\sigma_{\text{ave}}^2 + 3R^2 = \sigma_Y^2$$

$$\left(\frac{\sigma_x}{2}\right)^2 + 3\left[\frac{\sigma_x^2}{4} + \tau_{xy}^2\right] = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2$$

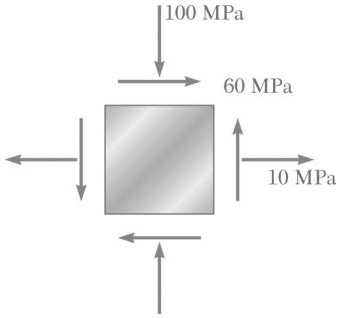
$$\sigma_x = \sqrt{\sigma_Y^2 - 3\tau_{xy}^2}$$

$$\sigma_x = \sqrt{(36)^2 - (3)(14.254)^2}$$

$$= 26.200 \text{ ksi}$$

$$P = \sigma_x A = (26.200)(2.4053) \qquad P = 63.0 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.89

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

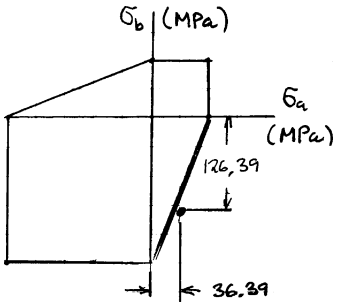
SOLUTION

$$\sigma_x = 10 \text{ MPa},$$
$$\sigma_y = -100 \text{ MPa},$$
$$\tau_{xy} = 60 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{10 - 100}{2} = -45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(55)^2 + (60)^2} = 81.39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = -45 + 81.39 = 36.39 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -45 - 81.39 = -126.39 \text{ MPa}$$

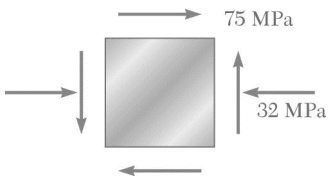


Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{36.39}{80} - \frac{(-126.39)}{200} = 1.087 > 1$$

Rupture will occur. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

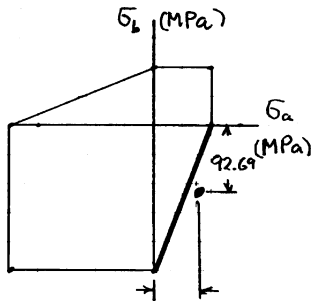


PROBLEM 7.90

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 80$ MPa and $\sigma_{UC} = 200$ MPa and using Mohr's criterion, determine whether rupture of the casting will occur.

SOLUTION

$$\sigma_x = -32 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = 75 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -16 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(16)^2 + (75)^2} = 76.69 \text{ MPa}$$
$$\sigma_a = \sigma_{\text{ave}} + R = -16 + 76.69 = 60.69 \text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -16 - 76.69 = -92.69 \text{ MPa}$$

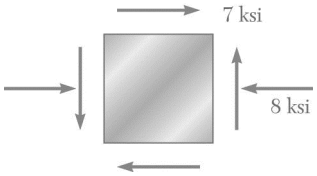


Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{60.69}{80} - \frac{(-92.69)}{200} = 1.222 > 1$$

Rupture will occur. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

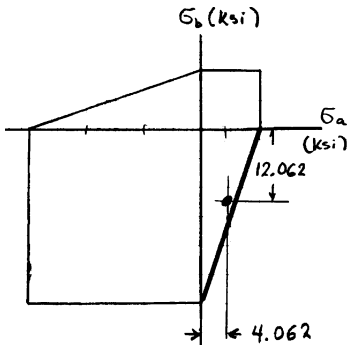


PROBLEM 7.91

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the casting will occur.

SOLUTION

$$\sigma_x = -8 \text{ ksi},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = 7 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -4 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062 \text{ ksi}$$
$$\sigma_a = \sigma_{\text{ave}} + R = -4 + 8.062 = 4.062 \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -4 - 8.062 = -12.062 \text{ ksi}$$

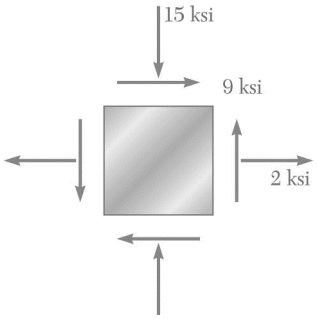


Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

No rupture. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

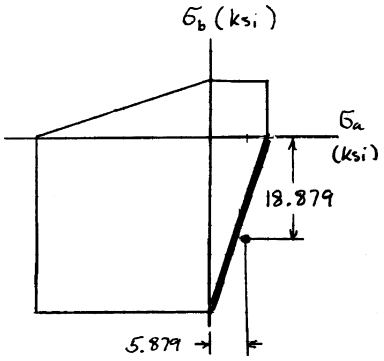


PROBLEM 7.92

The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the casting will occur.

SOLUTION

$$\sigma_x = 2 \text{ ksi},$$
$$\sigma_y = -15 \text{ ksi},$$
$$\tau_{xy} = 9 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379 \text{ ksi}$$
$$\sigma_a = \sigma_{\text{ave}} + R = 5.879 \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R = -18.879 \text{ ksi}$$

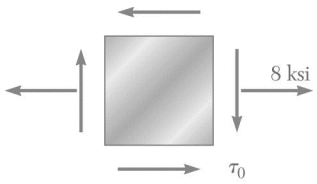


Equation of 4th quadrant of boundary:

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

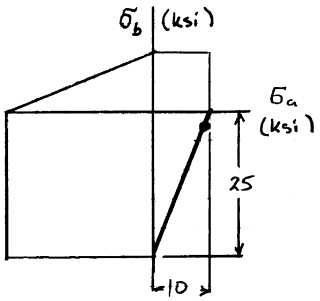


PROBLEM 7.93

The state of plane stress shown will occur at a critical point in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 25$ ksi. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

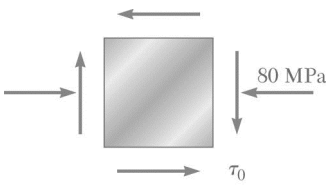
$$\sigma_x = 8 \text{ ksi},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = \tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 4 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_0^2}$$
$$\tau_0 = \pm\sqrt{R^2 - 4^2}$$
$$\sigma_a = \sigma_{\text{ave}} + R = (4 + R) \text{ ksi}$$
$$\sigma_b = \sigma_{\text{ave}} - R = (4 - R) \text{ ksi}$$



Since $|\sigma_{\text{ave}}| < R,$ stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{4 + R}{10} - \frac{4 - R}{25} = 1$$
$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$
$$R = 5.429 \text{ ksi} \qquad \tau_0 = \pm\sqrt{5.429^2 - 4^2} \qquad \tau_0 = \pm 3.67 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

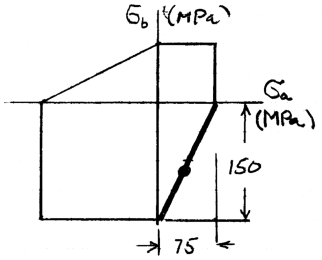


PROBLEM 7.94

The state of plane stress shown will occur at a critical point in a pipe made of an aluminum alloy for which $\sigma_{UT} = 75$ MPa and $\sigma_{UC} = 150$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

$$\sigma_x = -80 \text{ MPa},$$
$$\sigma_y = 0,$$
$$\tau_{xy} = -\tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R$$
$$\sigma_b = \sigma_{\text{ave}} - R$$
$$\tau_0 = \pm\sqrt{R^2 - 40^2}$$

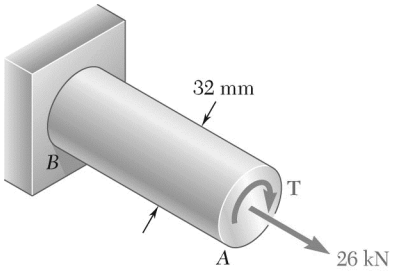
Since $|\sigma_{\text{ave}}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$
$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$$R = 63.33 \text{ MPa}, \qquad \tau_0 = \pm\sqrt{63.33^2 - 40^2}$$

$$\tau_0 = \pm 8.49 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.95

The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 60$ MPa and $\sigma_{UC} = 120$ MPa. Using Mohr's criterion, determine the magnitude of the torque **T** for which failure should be expected.

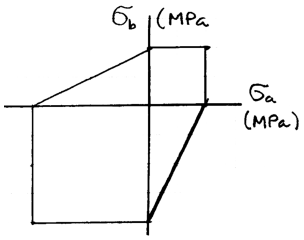
SOLUTION

$$P = 26\times10^{3}\text{ N} \quad A = \frac{\pi}{4}(32)^2 = 804.25\text{ mm}^2 = 804.25\times10^{-6}\text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{26\times10^{3}}{804.25\times10^{-6}} = 32.328\times10^{6}\text{ Pa} = 32.328\text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(32.328 + 0) = 16.164\text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(32.328 - 0) = 16.164\text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R = 16.164 + R\text{ MPa}$$
$$\sigma_b = \sigma_{\text{ave}} - R = 16.164 - R\text{ MPa}$$

Since $|\sigma_{\text{ave}}| < R$, stress point lies in the 4th quadrant. Equation of the 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1 \quad \frac{16.164 + R}{60} - \frac{16.164 - R}{120} = 1$$

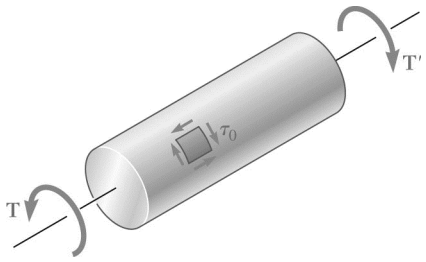
$$\left(\frac{1}{60} + \frac{1}{120}\right)R = 1 - \frac{16.164}{60} + \frac{16.164}{120} \qquad R = 34.612\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + \tau_{xy}^2} \quad \tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{34.612^2 - 16.164^2} = 30.606\text{ MPa}$$
$$= 30.606\times10^{6}\text{ Pa}$$

For torsion, $\tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ where $c = \frac{1}{2}d = 16\text{ mm} = 16\times10^{-3}\text{ m}$

$$T = \frac{\pi}{2}c^3\tau_{xy} = \frac{\pi}{2}(16\times10^{-3})^3(30.606\times10^{6}) \qquad T = 196.9\text{ N}\cdot\text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

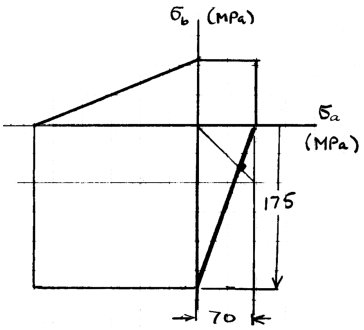


PROBLEM 7.96

The cast-aluminum rod shown is made of an alloy for which σ_{UT} = 70 MPa and σ_{UC} = 175 MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 that should be expected at rupture.

SOLUTION

$$\sigma_x = 0$$
$$\sigma_y = 0$$
$$\tau_{xy} = -\tau_0$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$
$$\sigma_a = \sigma_{\text{ave}} + R = R$$
$$\sigma_b = \sigma_{\text{ave}} - R = -R$$

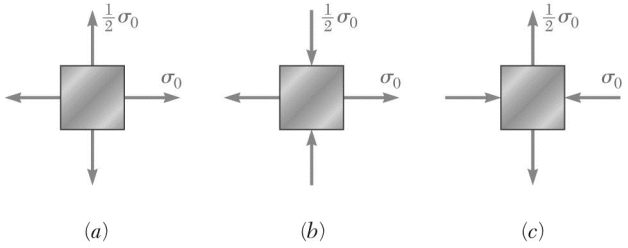


Since $|\sigma_{\text{ave}}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$
$$\frac{R}{70} - \frac{-R}{175} = 1$$
$$\left(\frac{1}{70} + \frac{1}{175}\right)R = 1$$
$$R = 50 \text{ MPa}$$
$$\tau_0 = R$$

$\tau_0 = 50.0$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.97

A machine component is made of a grade of cast iron for which σ_{UT} = 8 ksi and σ_{UC} = 20 ksi. For each of the states of stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.

SOLUTION

(*a*) $\sigma_a = \sigma_0$

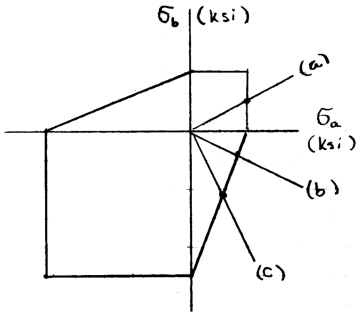
$\sigma_b = \dfrac{1}{2}\sigma_0$

Stress point lies in 1st quadrant.

$$\sigma_a = \sigma_0 = \sigma_{UT} \qquad \sigma_0 = 8 \text{ ksi} \blacktriangleleft$$

(*b*) $\sigma_a = \sigma_0$

$\sigma_b = -\dfrac{1}{2}\sigma_0$



Stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{8} - \frac{-\frac{1}{2}\sigma_0}{20} = 1 \qquad \sigma_0 = 6.67 \text{ ksi} \blacktriangleleft$$

(*c*) $\sigma_a = \dfrac{1}{2}\sigma_0, \quad \sigma_b = -\sigma_0, \quad$ 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{8} - \frac{-\sigma_0}{20} = 1 \qquad \sigma_0 = 8.89 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.98

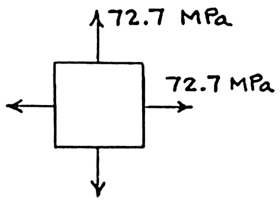
A spherical gas container made of steel has a 5-m outer diameter and a wall thickness of 6 mm. Knowing that the internal pressure is 350 kPa, determine the maximum normal stress and the maximum shearing stress in the container.

SOLUTION

$$d = 5\text{ m} \quad t = 6\text{ mm} = 0.006\text{ m}, \quad r = \frac{d}{2} - t = 2.494\text{ m}$$

$$\sigma = \frac{pr}{2t} = \frac{(350 \times 10^3\text{ Pa})(2.494\text{ m})}{2(0.006\text{ m})} = 72.742 \times 10^6\text{ Pa}$$

$$\sigma = 72.7\text{ MPa} \blacktriangleleft$$



$$\sigma_{\max} = 72.742\text{ MPa}$$

$$\sigma_{\min} \approx 0 \quad (\text{Neglecting small radial stress})$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 36.4\text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.99

The maximum gage pressure is known to be 8 MPa in a spherical steel pressure vessel having a 250-mm outer diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400$ MPa, determine the factor of safety with respect to tensile failure.

SOLUTION

$$r = \frac{d}{2} - t = \frac{250}{2} - 6 = 119 \text{ mm} = 119 \times 10^{-3} \text{ m}, \quad t = 6 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(8 \times 10^6 \text{ Pa})(119 \times 10^{-3} \text{ m})}{2(6 \times 10^{-3} \text{ m})} = 79.333 \times 10^6 \text{ Pa}$$

$$F.S. = \frac{\sigma_U}{\sigma_{\max}} = \frac{400 \times 10^6}{79.333 \times 10^6} \qquad F.S. = 5.04 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.100

A basketball has a 9.5-in. outer diameter and a 0.125-in. wall thickness. Determine the normal stress in the wall when the basketball is inflated to a 9-psi gage pressure.

SOLUTION

$$r = \frac{d}{2} - t = \frac{9.5}{2} - 0.125 = 4.625 \text{ in.} \qquad p = 9 \text{ psi}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(9)(4.625)}{2(0.125)} = 166.5 \text{ psi} \qquad \sigma = 166.5 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.101

A spherical pressure vessel of 900-mm outer diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of 4.0 is desired and that the gage pressure can reach 3.5 MPa, determine the smallest wall thickness that should be used.

SOLUTION

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{400}{4.0} = 100 \text{ MPa} \qquad r = \frac{d}{2} - t = (0.45 - t) \text{ m}$$

$$\sigma_{\text{all}} = \frac{pr}{2t} \qquad 2\sigma_{\text{all}} t = pr$$

$$2(100)t = 3.5(0.45 - t)$$

$$203.5t = 1.575$$

$$t = 7.74 \times 10^{-3} \text{ m} \qquad t_{\min} = 7.74 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.102

A spherical pressure vessel has an outer diameter of 10 ft and a wall thickness of 0.5 in. Knowing that for the steel used $\sigma_{all} = 12$ ksi, $E = 29 \times 10^6$ psi, and $\nu = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

SOLUTION

$$d = 10 \text{ ft} = 120 \text{ in.} \quad r = \frac{d}{2} - t = \frac{120}{2} - 0.5 = 59.5 \text{ in.}$$

$$\sigma_1 = \sigma_2 = \sigma_{all} = 12 \text{ ksi} = 12 \times 10^3 \text{ psi}$$

(*a*) $$\sigma_1 = \sigma_2 = \frac{pr}{2t} \qquad p = \frac{2t\sigma_1}{r} = \frac{2(0.5)(12 \times 10^3)}{59.5}$$

$$p = 202 \text{ psi} \blacktriangleleft$$

(*b*) $$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \frac{1 - 0.29}{29 \times 10^6}(12 \times 10^{-3})$$

$$= 293.79 \times 10^{-6}$$

$$\Delta d = d\varepsilon_1 = (120)(293.79 \times 10^{-6}) \qquad \Delta d = 0.0353 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.103

A spherical gas container having an outer diameter of 5 m and a wall thickness of 22 mm is made of steel for which $E = 200$ GPa and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.7 MPa, determine (*a*) the maximum normal stress in the container, (*b*) the corresponding increase in the diameter of the container.

SOLUTION

$$r = \frac{d}{2} - t = \frac{5}{2} - 22 \times 10^{-3} = 2.478 \text{ m}, \quad t = 22 \times 10^{-3} \text{ m}$$

(*a*) $$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.7 \times 10^6 \text{ Pa})(2.478 \text{ m})}{2(22 \times 10^{-3} \text{ m})} = 95.741 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = 95.7 \text{ MPa} \blacktriangleleft$$

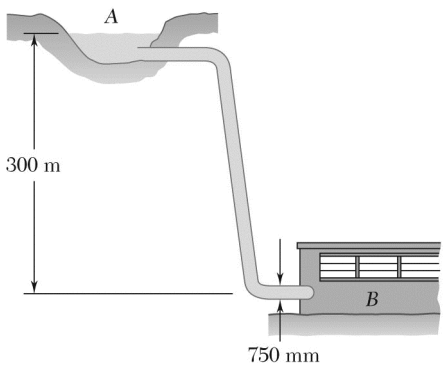
(*b*) $$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1 - \nu}{E}\sigma_1$$

$$= \frac{(1 - 0.29)(95.741 \times 10^6 \text{ Pa})}{200 \times 10^9 \text{ Pa}} = 339.88 \times 10^{-6}$$

$$\Delta d = d\varepsilon_1 = (5 \times 10^3 \text{ mm})(339.88 \times 10^{-6})$$

$$\Delta d = 1.699 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.104

A steel penstock has a 750-mm outer diameter, a 12-mm wall thickness, and connects a reservoir at *A* with a generating station at *B*. Knowing that the density of water is 1000 kg/m^3, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(750) - 12 = 363 \text{ mm} = 363\times10^{-3}\text{m}$$

$$t = 12 \text{ mm} = 12\times\ 10^{-3}\text{m}$$

$$p = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(300 \text{ m})$$

$$= 2.943\times10^{6}\text{ Pa}$$

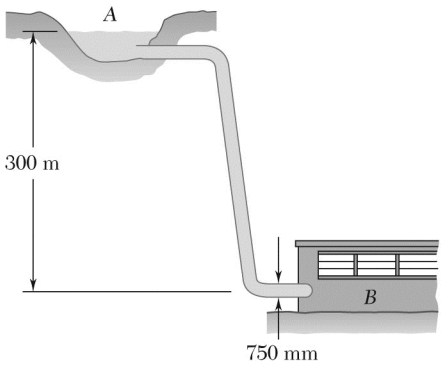
$$\sigma_1 = \frac{pr}{t} = \frac{(2.943\times10^{6})(363\times10^{-3})}{12\times10^{-3}} = 89.0\times10^{6}\text{ Pa}$$

$$\sigma_{\max} = \sigma_1 \qquad \sigma_{\max} = 89.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 44.5 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.105

A steel penstock has a 750-mm outer diameter and connects a reservoir at A with a generating station at B. Knowing that the density of water is $1000\ \text{kg/m}^3$ and that the allowable normal stress in the steel is 85 MPa, determine the smallest thickness that can be used for the penstock.

SOLUTION

$$p = \rho g h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(300\ \text{m})$$
$$= 2.943 \times 10^6\ \text{Pa}$$
$$\sigma_1 = 85\ \text{MPa} = 85 \times 10^6\ \text{Pa}$$
$$r = \frac{1}{2}d - t = \frac{1}{2}(750 \times 10^{-3}) - t = 0.375 - t$$
$$\sigma_1 = \frac{pr}{t}$$
$$85 \times 10^6 = \frac{(2.943 \times 10^6)(0.375 - t)}{t}$$
$$(87.943 \times 10^6)t = 1.103625 \times 10^6 \quad t = 12.549 \times 10^{-3}\ \text{m}$$

$$t = 12.55\ \text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.106

The bulk storage tank shown in Photo 7.3 has an outer diameter of 3.3 m and a wall thickness of 18 mm. At a time when the internal pressure of the tank is 1.5 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$r = \frac{d}{2} - t = \frac{3.3}{2} - 18 \times 10^{-3} = 1.632 \text{ m}, \qquad t = 18 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa})(1.632 \text{ m})}{18 \times 10^{-3} \text{ m}} = 136 \times 10^6 \text{ Pa}$$

$$\sigma_{\max} = \sigma_1 = 136 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 136.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 68 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 68.0 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.107

Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft outer diameter and $\frac{5}{8}$-in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

SOLUTION

$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{65\text{ ksi}}{5.0} = 13\text{ ksi} = 13 \times 10^3\text{ psi}$$

$$r = \frac{d}{2} - t = \frac{(5.5)(12)}{2} - 0.625 = 32.375\text{ in.}$$

$$\sigma_1 = \frac{pr}{t} \quad p = \frac{t\sigma_1}{r} = \frac{(0.625)(13 \times 10^3)}{32.375}$$

$$p = 251\text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.108

A cylindrical storage tank contains liquefied propane under a pressure of 1.5 MPa at a temperature of 38°C. Knowing that the tank has an outer diameter of 320 mm and a wall thickness of 3 mm, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$r = \frac{d}{2} - t = \frac{320}{2} - 3 = 157 \text{ mm} = 157 \times 10^{-3} \text{ m}$$

$$t = 3 \times 10^{-3} \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa})(157 \times 10^{-3} \text{m})}{3 \times 10^{-3} \text{ m}} = 78.5 \times 10^6 \text{ Pa}$$

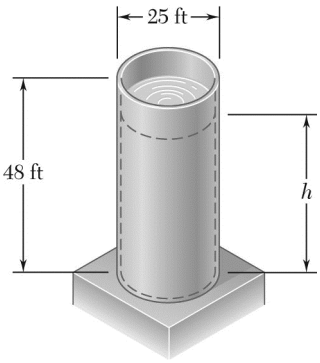
$$\sigma_{\max} = \sigma_1 = 78.5 \times 10^6 \text{ Pa} \qquad \sigma_{\max} = 78.5 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = -p \approx 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39.25 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 39.3 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.109

The unpressurized cylindrical storage tank shown has a $\frac{3}{16}$-in. wall thickness and is made of steel having a 60-ksi ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 62.4 lb/ft^3.)

SOLUTION

$$d_0 = (25)(12) = 300 \text{ in.}$$

$$r = \frac{1}{2}d - t = 150 - \frac{3}{16} = 149.81 \text{ in.}$$

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{60 \text{ ksi}}{4.0} = 15 \text{ ksi} = 15 \times 10^3 \text{psi}$$

$$\sigma_{\text{all}} = \frac{pr}{t}$$

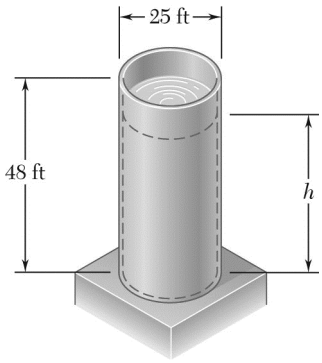
$$p = \frac{t\sigma_{\text{all}}}{r} = \frac{\left(\frac{3}{16}\right)(15 \times 10^3)}{149.81} = 18.77 \text{ psi} = 2703 \text{ lb/ft}^2$$

But $p = \gamma h$,

$$h = \frac{p}{\gamma} = \frac{2703 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3}$$

$h = 43.3$ ft ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.110

For the storage tank of Prob. 7.109, determine the maximum normal stress and the maximum shearing stress in the cylindrical wall when the tank is filled to capacity (h = 48 ft).

PROBLEM 7.109 The unpressurized cylindrical storage tank shown has a $\frac{3}{16}$-in. wall thickness and is made of steel having a 60-ksi ultimate strength in tension. Determine the maximum height h to which it can be filled with water if a factor of safety of 4.0 is desired. (Specific weight of water = 62.4 lb/ft^3.)

SOLUTION

$$d_0 = (25)(12) = 300 \text{ in.} \quad t = \frac{3}{16} \text{ in.} = 0.1875 \text{ in.}$$

$$r = \frac{1}{2}d - t = 149.81 \text{ in.}$$

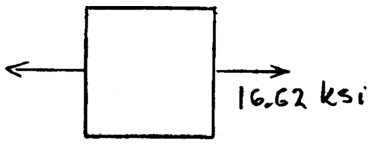
$$p = \gamma h = (62.4 \text{ lb/ft}^3)(48 \text{ ft}) = 2995.2 \text{ lb/ft}^2$$

$$= 20.8 \text{ lb/in}^2 = 20.8 \text{ psi}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(20.8)(149.81)}{0.1875} = 16.62 \times 10^3 \text{ psi}$$

$$\sigma_{\max} = \sigma_1 \qquad \sigma_{\max} = 16.62 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} \approx 0 \quad \tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 8.31 \text{ ksi} \blacktriangleleft$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.111

A standard-weight steel pipe of 12-in. nominal diameter carries water under a pressure of 400 psi. (*a*) Knowing that the outside diameter is 12.75 in. and the wall thickness is 0.375 in., determine the maximum tensile stress in the pipe. (*b*) Solve part *a*, assuming an extra-strong pipe is used of 12.75-in. outside diameter and 0.5-in. wall thickness.

SOLUTION

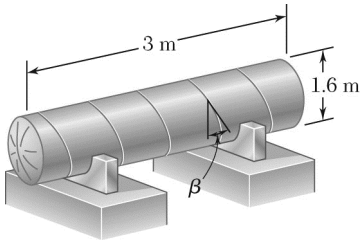
(*a*) $d_0 = 12.75$ in. $t = 0.375$ in. $r = \frac{1}{2}d_0 - t = 6.00$ in.

$$\sigma = \frac{pr}{t} = \frac{(400)(6.00)}{0.375} = 6400 \text{ psi} \qquad \sigma = 6.40 \text{ ksi} \blacktriangleleft$$

(*b*) $d_0 = 12.75$ in. $t = 0.500$ in. $r = \frac{1}{2}d_0 - t = 5.875$ in.

$$\sigma = \frac{pr}{t} = \frac{(400)(5.875)}{0.500} = 4700 \text{ psi} \qquad \sigma = 4.70 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.112

The pressure tank shown has an 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

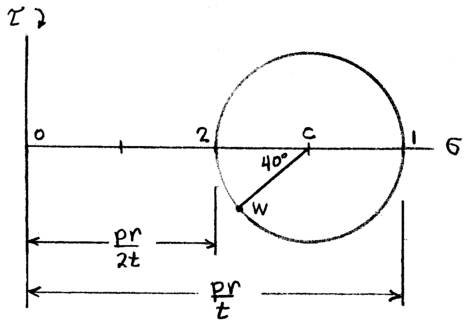
$$d = 1.6 \text{ m} \quad t = 8\times10^{-3}\text{ m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}} = 59.4\times10^6 \text{ Pa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(600\times10^3)(0.792)}{(2)(8\times10^{-3})} = 29.7\times10^6 \text{ Pa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 44.56\times10^6 \text{ Pa}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = 14.85\times10^6 \text{ Pa}$$

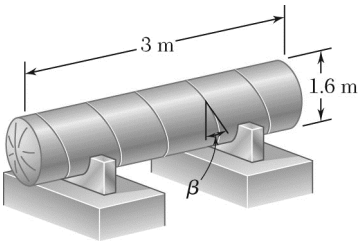


(*a*) $\sigma_w = \sigma_{\text{ave}} - R\cos 40° = 33.17\times10^6 \text{ Pa}$ $\sigma_w = 33.2 \text{ MPa}$ ◀

(*b*) $\tau_w = R\sin 40° = 9.55\times10^6 \text{ Pa}$ $\tau_w = 9.55 \text{ MPa}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.113



For the tank of Prob. 7.112, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 120 MPa and the allowable shearing stress parallel to the weld is 80 MPa.

PROBLEM 7.112 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3} \text{ m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

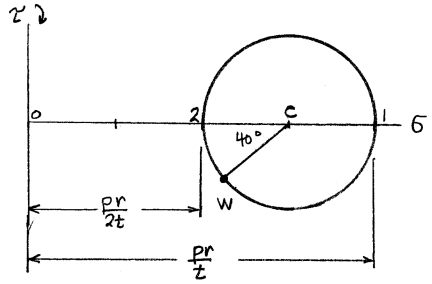
$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 40°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 40°\right)\frac{pr}{t} = 0.5585\frac{pr}{t}$$



$$p = \frac{\sigma_w t}{0.5585r} = \frac{(120\times10^6)(8\times10^{-3})}{(0.5585)(0.792)} = 2.17\times10^6 \text{ Pa} = 2.17 \text{ MPa}$$

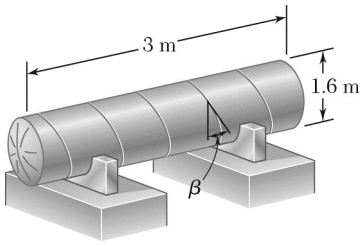
$$\tau_w = R\sin 40° = \left(\frac{1}{4}\sin 40°\right)\frac{pr}{t} = 0.1607\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.1607r} = \frac{(80\times10^6)(8\times10^{-3})}{(0.1607)(0.792)} = 5.03\times10^6 \text{ Pa} = 5.03 \text{ MPa}$$

The largest allowable pressure is the smaller value. $p = 2.17$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.114



For the tank of Prob. 7.112, determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 12 MPa when the gage pressure is 600 kPa.

PROBLEM 7.112 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3} \text{ mm} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

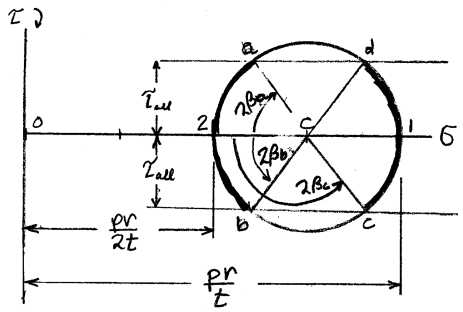
$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}}$$

$$= 59.4\times10^6 \text{ Pa} = 59.4 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 29.7 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 14.85 \text{ MPa}$$

$$\tau_w = R|\sin 2\beta|$$



$$|\sin 2\beta_a| = \frac{\tau_N}{R} = \frac{12}{14.85} = 0.80808$$

$$2\beta_a = -53.91° \qquad \beta_a = +27.0°$$

$$2\beta_b = +53.91° \qquad \beta_b = 27.0°$$

$$2\beta_c = 180° - 53.91° = +126.09° \text{ (□)} \qquad \beta_c = 63.0°$$

$$2\beta_d = 180° + 53.91° = +233.91° \text{ (↻)} \qquad \beta_d = 117.0°$$

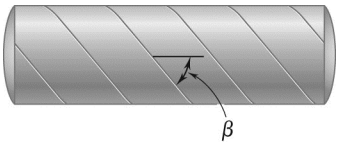
Let the total range of values for β be $-180° < \beta \le 180°$

Safe ranges for β: $-22.0° \le \beta \le 27.0°$ ◀

and $63.0° \le \beta \le 117.0°$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.115



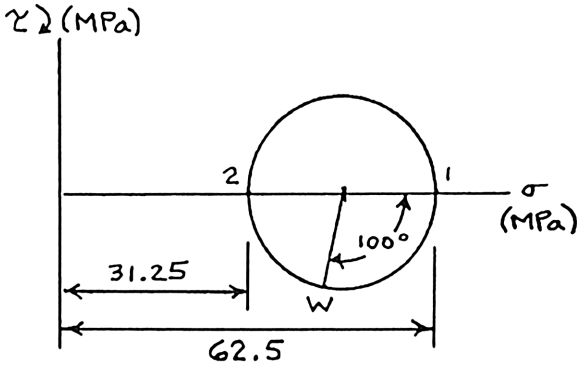
The steel pressure tank shown has a 750-mm inner diameter and a 9-mm wall thickness. Knowing that the butt-welded seams form an angle $\beta = 50°$ with the longitudinal axis of the tank and that the gage pressure in the tank is 1.5 MPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{d}{2} = 375 \text{ mm} = 0.375 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6 \text{ Pa} \times 0.375 \text{ m})}{0.009 \text{ m}} = 62.5 \times 10^6 \text{ Pa} = 62.5 \text{ MPa}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 31.25 \text{ MPa} \qquad 2\beta = 100°$$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 46.875 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 15.625 \text{ MPa}$$

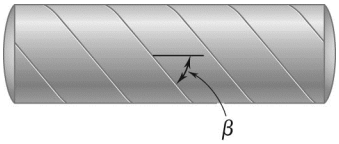
(*a*) $\sigma_w = \sigma_{\text{ave}} + R\cos 100°$

$$\sigma_w = 44.2 \text{ MPa} \blacktriangleleft$$

(*b*) $\tau_w = R\sin 100°$

$$\tau_w = 15.39 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.116

The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the tank.

SOLUTION

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

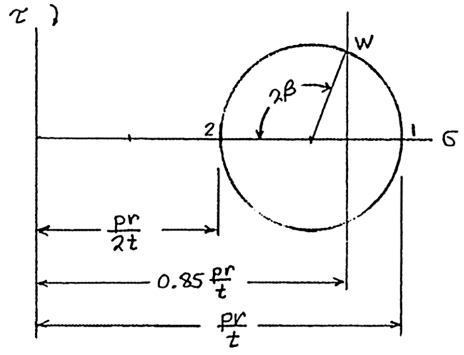
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 2\beta$$

$$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$$

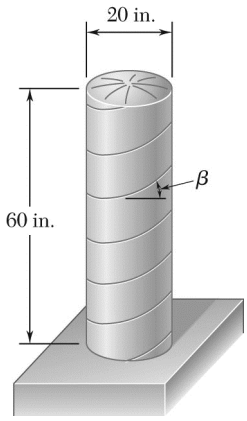
$$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$$

$$2\beta = 113.6°$$



$$\beta = 56.8° \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.117

The cylindrical portion of the compressed-air tank shown is fabricated of 0.25-in.-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 10.5 ksi, determine the largest gage pressure that can be used in the tank.

SOLUTION

$$r = \frac{1}{2}d - t = 10 - 0.25 = 9.75 \text{ in.}$$

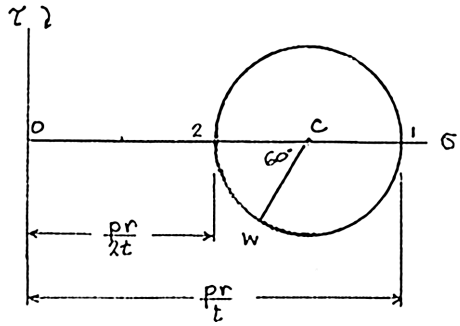
$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} + R\cos 60°$$

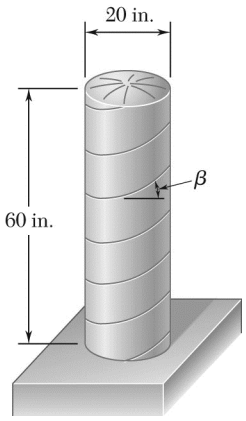
$$= \frac{5}{8}\frac{pr}{t}$$



$$p = \frac{8}{5}\frac{\sigma_w t}{r} = \frac{(8)(10.5)(0.25)}{(5)(9.75)} = 0.43077 \text{ ksi}$$

$p = 431$ psi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.118

For the compressed-air tank of Prob. 7.117, determine the gage pressure that will cause a shearing stress parallel to the weld of 4 ksi.

PROBLEM 7.117 The cylindrical portion of the compressed-air tank shown is fabricated of 0.25-in.-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 10.5 ksi, determine the largest gage pressure that can be used in the tank.

SOLUTION

$$r = \frac{1}{2}d - t = 10 - 0.25 = 9.75 \text{ in.}$$

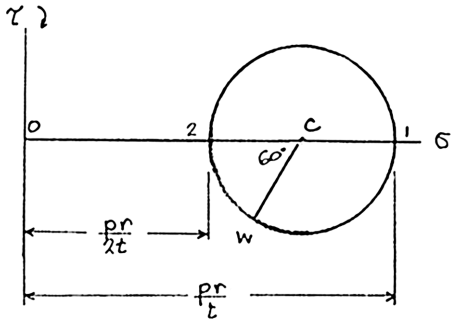
$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\tau_w = R \sin 60°$$

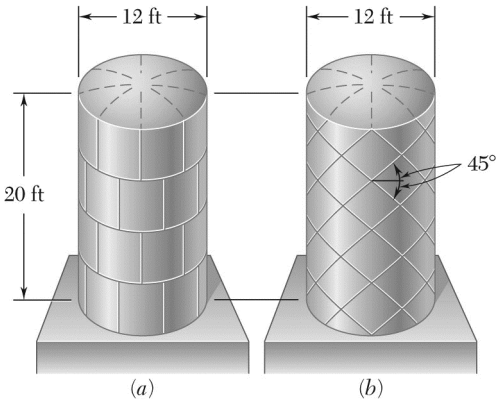
$$= \frac{\sqrt{3}}{8}\frac{pr}{t}$$



$$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{r} = \frac{8}{\sqrt{3}} \cdot \frac{(4)(0.25)}{9.75} = 0.47372 \text{ ksi}$$

$$p = 474 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.119

Square plates, each of 0.5-in. thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed-air tank. Knowing that the allowable normal stress perpendicular to the weld is 12 ksi, determine the largest allowable gage pressure in each case.

SOLUTION

$$d = 12\,\text{ft} = 144\,\text{in.} \quad r = \frac{1}{2}d - t = 71.5\,\text{in.}$$

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

(*a*)

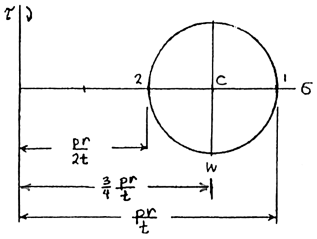
$$\sigma_1 = 12\,\text{ksi}$$

$$p = \frac{\sigma_1 t}{r} = \frac{(12)(0.5)}{71.5} = 0.0839\,\text{ksi}$$

$$p = 83.9\,\text{psi} \blacktriangleleft$$

(*b*)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$



$$R = \frac{\sigma_1 + \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

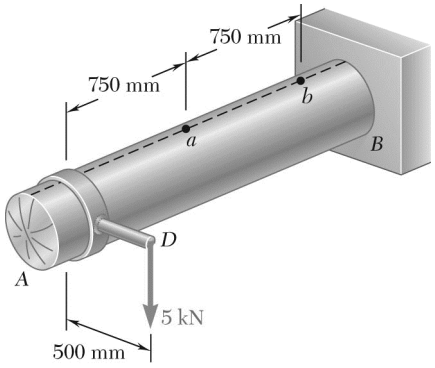
$$\beta = \pm 45°$$

$$\sigma_w = \sigma_{\text{ave}} + R\cos\beta$$

$$= \frac{3}{4}\frac{pr}{t}$$

$$p = \frac{4}{3}\frac{\sigma_w t}{r} = \frac{4}{3}\cdot\frac{(12)(0.5)}{71.5} = 0.1119\,\text{ksi} \qquad p = 111.9\,\text{psi} \blacktriangleleft$$

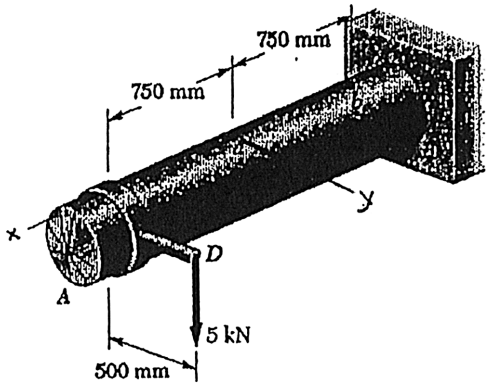
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.120

The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.

SOLUTION



Internal pressure:

$$r = \frac{1}{2}d = 225\,\text{mm} \quad t = 6\,\text{mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45\ \text{MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5\ \text{MPa}$$

Torsion: $c_1 = 225\ \text{mm}, \quad c_2 = 225 + 6 = 231\ \text{mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9 \times 10^6\ \text{mm}^4 = 446.9 \times 10^{-6}\ \text{m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500\ \text{N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6\ \text{Pa} = 1.292\ \text{MPa}$$

Transverse shear: $\tau = 0$ at point *a*.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6}\ \text{m}^4, \quad c = 231 \times 10^{-3}\ \text{m}$

At point *a*, $M = (5 \times 10^3)(750 \times 10^{-3}) = 3750\ \text{N} \cdot \text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88\ \text{MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 3.88 = 26.38\ \text{MPa}$

Circumferential: $\sigma_y = 45\ \text{MPa}$

Shear: $\tau_{xy} = 1.292\ \text{MPa}$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.120 ***(Continued)***

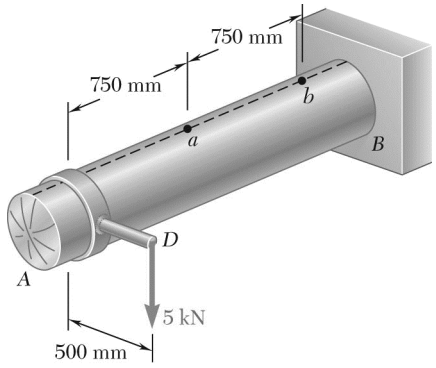
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1 \text{ MPa} \qquad \sigma_{\max} = 45.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max\,(\text{in-plane})} = R = 9.40 \text{ MPa} \qquad \tau_{\max\,(\text{in-plane})} = 9.40 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

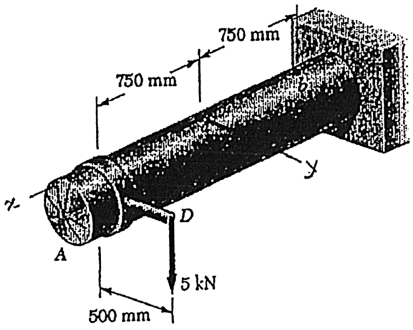


PROBLEM 7.121

For the compressed-air tank and loading of Prob. 7.120, determine the maximum normal stress and the maximum in-plane shearing stress at point *b* on the top of the tank.

PROBLEM 7.120 The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point *a* on the top of the tank.

SOLUTION



Internal pressure:

$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$$

Torsion: $c_1 = 225$ mm, $c_2 = 225 + 6 = 231$ mm

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9 \times 10^6 \text{mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear: $\tau = 0$ at point *b*.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4, \quad c = 231 \times 10^{-3} \text{ m}$

At point *b*, $M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500 \text{ N} \cdot \text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(7500)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 7.75 \text{ MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 7.75 = 30.25$ MPa

Circumferential: $\sigma_y = 45$ MPa

Shear: $\tau_{xy} = 1.292$ MPa

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.121 ***(Continued)***

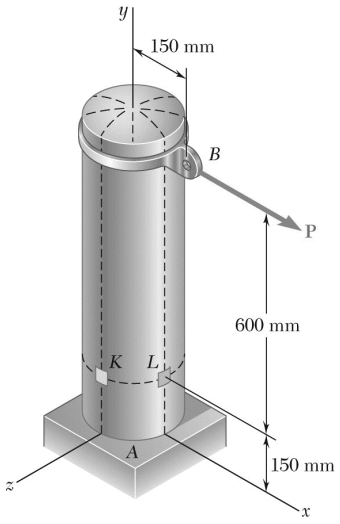
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1 \text{ MPa} \qquad \sigma_{\max} = 45.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max \text{ (in-plane)}} = R = 7.49 \text{ MPa} \qquad \tau_{\max \text{ (in-plane)}} = 7.49 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

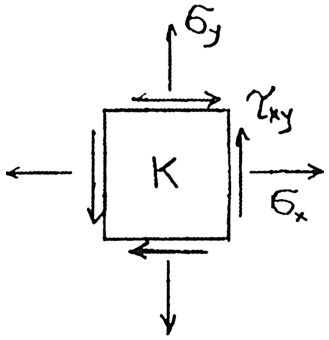


PROBLEM 7.122

The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.

SOLUTION

Consider element at point *K*.



Stresses due to internal pressure:

$$p = 5 \text{ MP} = 5\times10^{6} \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5\times10^{6})(117\times10^{-3})}{(8\times10^{-3})} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5\times10^{6})(117\times10^{-3})}{(2)(8\times10^{-3})} = 36.563 \text{ MPa}$$

Stress due to bending moment: Point *K* is on the neutral axis.

$$\sigma_y = 0$$

Stress due to transverse shear:

$$V = P = 40\times10^{3} \text{ N}$$

$$c_2 = \frac{1}{2}d = 125 \text{ mm}$$

$$c_1 = c_2 - t = 117 \text{ mm}$$

$$Q = \frac{2}{3}\left(c_2^3 - c_1^3\right) = \frac{2}{3}(125^3 - 117^3)$$

$$= 234.34\times10^{3} \text{ mm}^3 = 234.34\times10^{-6} \text{ m}^3$$

$$I = \frac{\pi}{4}\left(c_2^4 - c_1^4\right) = \frac{\pi}{4}(125^4 - 117^4)$$

$$= 44.573\times10^{6} \text{ mm}^4 = 44.573\times10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{PQ}{I(2t)} = \frac{(40\times10^{3})(234.34\times10^{-6})}{(44.573\times10^{-6})(16\times10^{-3})}$$

$$= 13.144\times10^{6} \text{ Pa} = 13.144 \text{ MPa}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.122 *(Continued)*

Total stresses: $\sigma_x = 73.125 \text{ MPa}, \quad \sigma_y = 36.563 \text{ MPa}, \quad \tau_{xy} = 13.144 \text{ MPa}$

Mohr's circle: $\sigma_{\text{ave}} = \dfrac{1}{2}(\sigma_x + \sigma_y) = 54.844 \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(18.281)^2 + (13.144)^2} = 22.516 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 77.360 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 32.328 \text{ MPa}$$

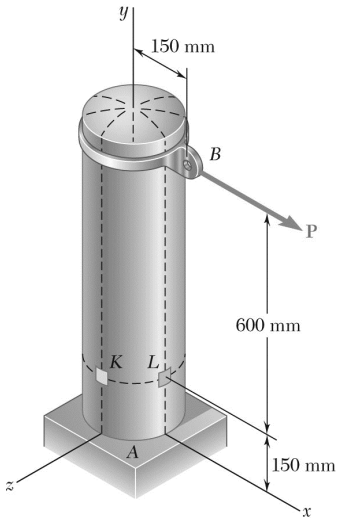
Principal stresses: $\sigma_a = 77.4 \text{ MPa}, \quad \sigma_b = 32.3 \text{ MPa}$ ◀

The 3rd principal stress is the radial stress. $\sigma_z \approx 0$ ◀

$$\sigma_{\max} = 77.4 \text{ MPa}, \quad \sigma_{\min} = 0$$

Maximum shearing stress: $\tau_{\max} = \dfrac{1}{2}(\sigma_{\max} - \sigma_{\min})$ $\tau_{\max} = 38.7 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.123

In Prob. 7.122, determine the maximum normal stress and the maximum shearing stress at point *L*.

PROBLEM 7.122 The compressed-air tank *AB* has a 250-mm outside diameter and an 8-mm wall thickness. It is fitted with a collar by which a 40-kN force **P** is applied at *B* in the horizontal direction. Knowing that the gage pressure inside the tank is 5 MPa, determine the maximum normal stress and the maximum shearing stress at point *K*.

SOLUTION

Consider element at point *L*.

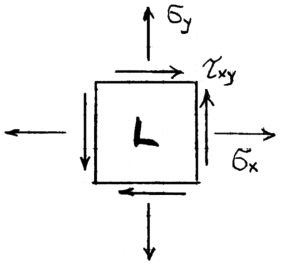
Stresses due to internal pressure:

$$p = 5 \text{ MPa} = 5\times10^{6} \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{250}{2} - 8 = 117 \text{ mm}$$

$$\sigma_x = \frac{pr}{t} = \frac{(5\times10^{6})(117\times10^{-3})}{8\times10^{-3}} = 73.125 \text{ MPa}$$

$$\sigma_y = \frac{pr}{2t} = \frac{(5\times10^{3})(117\times10^{-3})}{(2)(8\times10^{-3})} = 36.563 \text{ MPa}$$



Stress due to bending moment: $M = (40 \text{ kN})(600 \text{ mm}) = 24000 \text{ N}\cdot\text{m}$

$$c_2 = \frac{1}{2}d = 125 \text{ mm}$$

$$c_1 = c_2 - t = 125 - 8 = 117 \text{ mm}$$

$$I = \frac{\pi}{4}\left(c_2^4 - c_1^4\right) = \frac{\pi}{4}(125^4 - 117^4)$$

$$= 44.573\times10^{6} \text{ mm}^4 = 44.573\times10^{-6} \text{ m}^4$$

$$\sigma_y = -\frac{Mc}{I} = -\frac{(24000)(125\times10^{-3})}{44.573\times10^{-6}} = -67.305 \text{ MPa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.123 *(Continued)*

Stress due to transverse shear: Point L lies in a plane of symmetry.

$$\tau_{xy} = 0$$

Total stresses: $\sigma_x = 73.125 \text{ MPa}, \quad \sigma_y = -30.742 \text{ MPa}, \quad \tau_{xy} = 0$

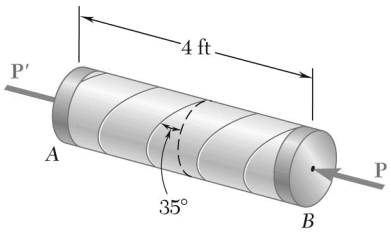
Principal stresses: Since $\tau_{xy} = 0$, σ_x and σ_y are principal stresses. The 3rd principal stress is in the radial direction, $\sigma_z \approx 0$.

$$\sigma_{\max} = 73.125 \text{ MPa}, \quad \sigma_{\min} = 0, \quad \sigma_a = 73.1 \text{ MPa}, \quad \sigma_b = -30.7 \text{ MPa}, \quad \sigma_z = 0$$

Maximum stress: $\sigma_{\max} = 73.1 \text{ MPa}$ ◀

Maximum shearing stress: $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$ $\tau_{\max} = 51.9 \text{ MPa}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.124

A pressure vessel of 10-in. inner diameter and 0.25-in. wall thickness is fabricated from a 4-ft section of spirally-welded pipe *AB* and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces **P** and **P′** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \qquad t = 0.25 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$$

$$r_0 = r + t = 5 + 0.25 = 5.25 \text{ in.}$$

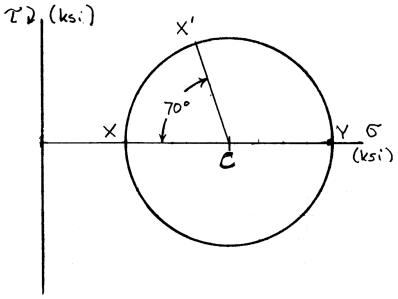
$$A = \pi\left(r_0^2 - r^2\right) = \pi(5.25^2 - 5.00^2) = 8.0503 \text{ in}^2$$

$$\sigma = -\frac{P}{A} = -\frac{10\times10^3}{8.0803} = -1242 \text{ psi} = -1.242 \text{ ksi}$$

Total stresses. Longitudinal: $\sigma_x = 3 - 1.242 = 1.758$ ksi

Circumferential: $\sigma_y = 6$ ksi

Shear: $\tau_{xy} = 0$



Plotted points for Mohr's circle:

X: (1.758, 0)

Y: (6, 0)

C: (3.879)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 3.879 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

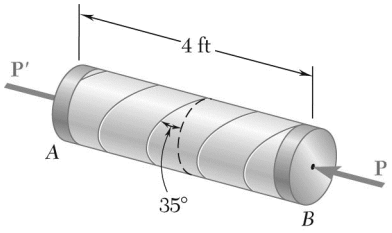
$$= \sqrt{\left[\frac{(1.758 - 6)}{2}\right]^2} + 0 = 2.121 \text{ ksi}$$

(*a*) $\sigma_{x'} = \sigma_{\text{ave}} + R\cos 70° = 3.879 - 2.121\cos 70°$ $\qquad \sigma_{x'} = 3.15$ ksi ◀

(*b*) $|\tau_{xy}| = R\sin 70° = 2.121\sin 70°$ $\qquad |\tau_{x'y'}| = 1.993$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.125



Solve Prob. 7.124, assuming that the magnitude P of the two forces is increased to 30 kips.

PROBLEM 7.124 A pressure vessel of 10-in. inner diameter and 0.25-in. wall thickness is fabricated from a 4-ft section of spirally-welded pipe AB and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric axial forces **P** and **P′** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(10) = 5 \text{ in.} \qquad t = 0.25 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(300)(5)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(300)(5)}{(2)(0.25)} = 3000 \text{ psi} = 3 \text{ ksi}$$

$$r_0 = r + t = 5 + 0.25 = 5.25 \text{ in.}$$

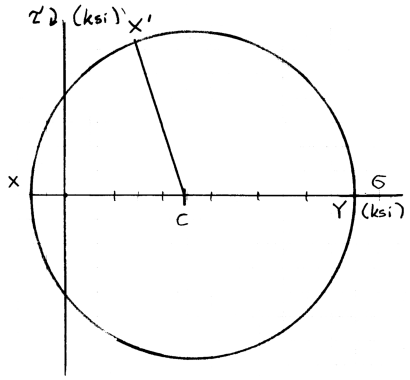
$$A = \pi\left(r_0^2 - r^2\right) = \pi(5.25^2 - 5^2) = 8.0503 \text{ in}^2$$

$$\sigma = -\frac{P}{A} = -\frac{30\times10^3}{8.0503} = -3727 \text{ psi} = -3.727 \text{ ksi}$$

Total stresses: Longitudinal: $\sigma_x = 3 - 3.727 = -0.727$ ksi

Circumferential: $\sigma_y = 6$ ksi

Shear: $\tau_{xy} = 0$



Plotted points for Mohr's circle:

X: (−0.727, 0)

Y: (6, 0)

C: (2.6365, 0)

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.6365 \text{ ksi}$$

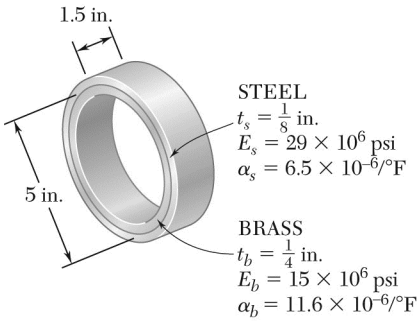
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-0.727 - 6}{2}\right)^2 + 0} = 3.3635 \text{ ksi}$$

(*a*) $\sigma_{x'} = \sigma_{\text{ave}} - R\cos 70° = 2.6365 - 3.3635\cos 70°$ $\qquad \sigma_{x'} = 1.486$ ksi ◀

(*b*) $|\tau_{x'y'}| = R\sin 70° = 3.3635\sin 70°$ $\qquad |\tau_{x'y'}| = 3.16$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.126

A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50°F. Knowing that the temperature of both rings is then raised to 125°F, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring. Internal pressure p: $\sigma_s = \dfrac{pr}{t_s}$ (1)

Corresponding strain: $\varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change: $\varepsilon_{sT} = \alpha_s \Delta T$

Total strain: $\varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference:

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring. External pressure p: $\sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains: $\varepsilon_{bp} = -\dfrac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference:

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b, $\dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \quad (2)$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.126 *(Continued)*

Data: $\Delta T = 125\ ^\circ\text{F} - 50\ ^\circ\text{F} = 75\ ^\circ\text{F}$

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5 \text{ in.}$$

From Equation (2), $$\left[\frac{2.5}{(29\times10^6)(0.125)} + \frac{2.5}{(15\times10^6)(0.25)}\right] p = (11.6 - 6.5)(10^{-6})(75)$$

$$1.35632\times10^{-6}\, p = 382.5\times10^{-6}$$

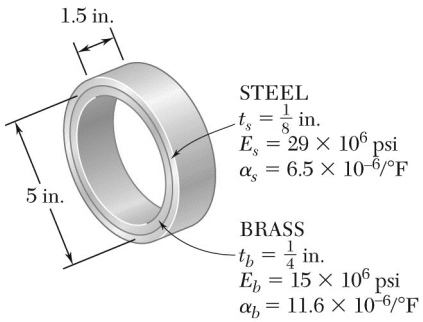
$$p = 282.0 \text{ psi}$$

From Equation (1), $$\sigma_s = \frac{pr}{t_s} = \frac{(282.0)(2.5)}{0.125} = 5.64\times10^3 \text{ psi}$$

(*a*) $\sigma_s = 5.64$ ksi ◀

(*b*) $p = 282$ psi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.127

Solve Prob. 7.126, assuming that the brass ring is 0.125 in. thick and the steel ring is 0.25 in. thick.

PROBLEM 7.126 A brass ring of 5-in. outer diameter and 0.25-in. thickness fits exactly inside a steel ring of 5-in. inner diameter and 0.125-in. thickness when the temperature of both rings is 50 °F. Knowing that the temperature of both rings is then raised to 125 °F, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring. Internal pressure p:
$$\sigma_s = \frac{pr}{t_s} \tag{1}$$

Corresponding strain:
$$\varepsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$$

Strain due to temperature change:
$$\varepsilon_{sT} = \alpha_s \Delta T$$

Total strain:
$$\varepsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$$

Change in length of circumference:
$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring. External pressure p:
$$\sigma_b = -\frac{pr}{t_b}$$

Corresponding strains:
$$\varepsilon_{bp} = -\frac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$$

Change in length of circumference:
$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b,
$$\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \tag{2}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.127 *(Continued)*

Data: $\Delta T = 125°\text{F} - 50°\text{F} = 75°\text{F}$

$$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5 \text{ in.}$$

From Equation (2),
$$\left[\frac{2.5}{(29\times10^6)(0.25)} + \frac{2.5}{(15\times10^6)(0.125)}\right] p = (11.6 - 6.5)(10^{-6})(75)$$

$$1.67816\times10^{-6}\, p = 382.5\times10^{-6}$$

$$p = 227.93 \text{ psi}$$

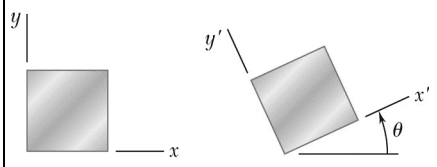
From Equation (1), $\sigma_s = \dfrac{pr}{t_s} = \dfrac{(227.93)(2.5)}{0.25} = 2279 \text{ psi}$

(*a*) $\sigma_s = 2.28$ ksi ◀

(*b*) $p = 228$ psi ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.128



For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -500\mu, \quad \varepsilon_y = +250\mu, \quad \gamma_{xy} = 0, \quad \theta = 15^\circ \text{ ↻}$$

SOLUTION

$$\theta = +15^\circ$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -125\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -375\mu \qquad \frac{\gamma_{xy}}{2} = 0$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= -125\mu + (-375\mu)\cos 30^\circ + 0 \qquad \varepsilon_{x'} = -450\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= -125\mu - (-375\mu)\cos 30^\circ - 0 \qquad \varepsilon_{y'} = 200\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(-500\mu - 250\mu)\sin 30^\circ + 0 \qquad \gamma_{x'y'} = 375\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.129

For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = +240\mu, \quad \varepsilon_y = +160\mu, \quad \gamma_{xy} = +150\mu, \quad \theta = 60^\circ \circlearrowleft$$

SOLUTION

$$\theta = -60^\circ$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = +200\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = 40\mu \qquad \frac{\gamma_{xy}}{2} = 75\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 200 + 40\cos(-120^\circ) + 75\sin(-120^\circ) \qquad \varepsilon_{x'} = 115.0\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 200 - 40\cos(-120^\circ) - 75\sin(-120^\circ) \qquad \varepsilon_{y'} = 285\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(240 - 160)\sin(-120^\circ) + 150\cos(-120^\circ) \qquad \gamma_{x'y'} = -5.72\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.130

For the given state of plane strain, use the method of Sec. 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -800\mu, \quad \varepsilon_y = +450\mu, \quad \gamma_{xy} = +200\mu, \quad \theta = 25^\circ \circlearrowleft$$

SOLUTION

$$\theta = -25^\circ$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -175\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -625\mu \qquad \frac{\gamma_{xy}}{2} = +100\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 175\mu + (-625\mu)\cos(-50^\circ) + (100\mu)\sin(-50^\circ) \qquad \varepsilon_{x'} = -653\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= -175\mu - (-625\mu)\cos(-50^\circ) - (100\mu)\sin(-50^\circ) \qquad \varepsilon_{y'} = 303\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(-800\mu - 450\mu)\sin(-50^\circ) + (+200\mu)\cos(-50^\circ) \qquad \gamma_{x'y'} = -829\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.131

For the given state of plane strain, use the method of Sec 7.10 to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = 0, \quad \varepsilon_y = +320\mu, \quad \gamma_{xy} = -100\mu, \quad \theta = 30^\circ \text{ ↺}$$

SOLUTION

$$\theta = +30^\circ$$

$$\frac{\varepsilon_x + \varepsilon_y}{2} = 160\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -160\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 160 - 160\cos 60^\circ - \frac{100}{2}\sin 60^\circ \qquad \varepsilon_{x'} = +36.7\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 160 + 160\cos 60^\circ + \frac{100}{2}\sin 60^\circ \qquad \varepsilon_{y'} = +283\mu \blacktriangleleft$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= -(0 - 320)\sin 60^\circ - 100\cos 60^\circ \qquad \gamma_{x'y'} = +227\mu \blacktriangleleft$$

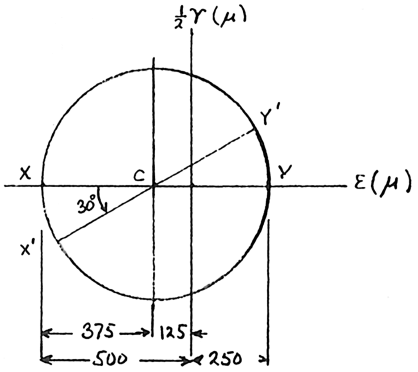
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.132

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -500\mu, \quad \varepsilon_y = +250\mu, \quad \gamma_{xy} = 0, \quad \theta = 15^\circ \text{)}$$

SOLUTION



Plotted points:

X: $(-500\mu, 0)$

Y: $(+250\mu, 0)$

C: $(-125\mu, 0)$

$R = 375\mu$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\theta = -125 - 375\cos 30^\circ \qquad \varepsilon_{x'} = -450\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos 2\theta = -125 + 375\cos 30^\circ \qquad \varepsilon_{y'} = 200\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = R\sin 2\theta = 375\sin 30^\circ \qquad \gamma_{x'y'} = 375\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.133

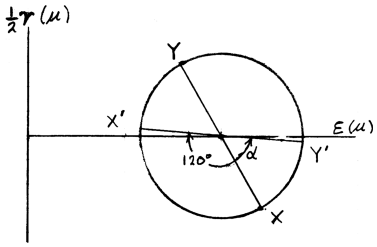
For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = +240\mu, \quad \varepsilon_y = +160\mu, \quad \gamma_{xy} = +150\mu, \quad \theta = 60^\circ \circlearrowleft$$

SOLUTION

Plotted points for Mohr's circle:

$$X\text{: } (+240\,\mu, -75\mu)$$
$$Y\text{: } (+160\mu, 75\mu)$$
$$C\text{: } (+200\mu, 0)$$



$$\tan\alpha = \frac{75}{40} = 1.875 \quad \alpha = 61.93^\circ$$

$$R = \sqrt{(40\mu)^2 + (75\mu)^2} = 85\mu$$

$$\beta = 2\theta - \alpha = -120^\circ - 61.93^\circ = -181.93^\circ$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos\beta = 200\mu + (85\mu)\cos(-181.93^\circ) \qquad \varepsilon_{x'} = 115.0\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} - R\cos\beta = 200\mu - (85\mu)\cos(-181.93^\circ) \qquad \varepsilon_{y'} = 285\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -85\sin(-181.93^\circ) = -2.86\mu \qquad \gamma_{x'y'} = -5.72\mu \blacktriangleleft$$

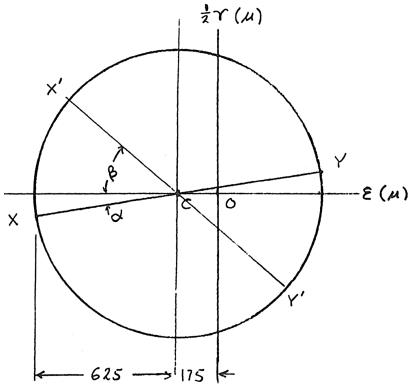
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.134

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -800\mu, \quad \varepsilon_y + = 450\mu, \quad \gamma_{xy} = +200\mu, \quad \theta = 25^\circ \circlearrowleft$$

SOLUTION



Plotted points:

X: $(-800\mu,\ -100\mu)$

Y: $(+450\mu,\ +100\mu)$

C: $(-175\mu,\ 0)$

$$\tan\alpha = \frac{100}{625} \qquad \alpha = 9.09^\circ$$

$$R = \sqrt{(625\mu)^2 + (100\mu)^2} = 632.95\mu$$

$$\beta = 2\theta - \alpha = 50^\circ - 9.09^\circ = 40.91^\circ$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = -175\mu - 632.95\mu\cos 40.91^\circ$$

$$\varepsilon_{x'} = -653\mu \blacktriangleleft$$

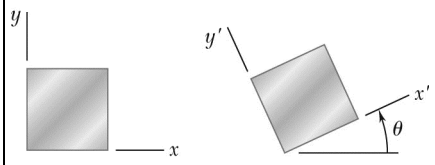
$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\beta = -175\mu + 632.95\mu\cos 40.91^\circ$$

$$\varepsilon_{y'} = 303\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = -632.95\mu\sin 40.91^\circ \qquad \gamma_{x'y'} = -829\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.135

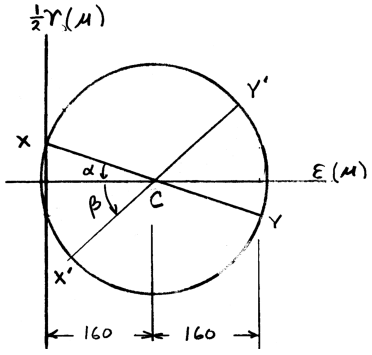


For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = 0, \quad \varepsilon_y = +320\mu, \quad \gamma_{xy} = -100\mu, \quad \theta = 30^\circ \circlearrowleft$$

SOLUTION

Plotted points for Mohr's circle:



$$X\text{: } (0, 50\mu)$$
$$Y\text{: } (320\mu, -50\mu)$$
$$C\text{: } (160\mu, 0)$$

$$\tan\alpha = \frac{50}{160} \quad \alpha = 17.35^\circ$$

$$R = \sqrt{(160\mu)^2 + (50\mu)^2} = 167.63\mu$$

$$\beta = 2\theta - \alpha = 60^\circ - 17.35^\circ = 42.65^\circ$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} - R\cos\beta = 160\mu - (167.63\mu)\cos 42.65^\circ \qquad \varepsilon_{x'} = 36.7\mu \blacktriangleleft$$

$$\varepsilon_{y'} = \varepsilon_{\text{ave}} + R\cos\beta = 160\mu + (167.63\mu)\cos 42.65^\circ \qquad \varepsilon_{y'} = 283\mu \blacktriangleleft$$

$$\frac{1}{2}\gamma_{x'y'} = -R\sin\beta = (167.63\mu)\sin 42.65^\circ \qquad \gamma_{x'y'} = 227\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.136

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = -260\mu, \quad \varepsilon_y = -60\mu, \quad \gamma_{xy} = +480\mu$$

SOLUTION

For Mohr's circle of strain, plot points:

$$X\text{: } (-260\mu, -240\mu)$$
$$Y\text{: } (-60\mu, 240\mu)$$
$$C\text{: } (-160\mu, 0)$$

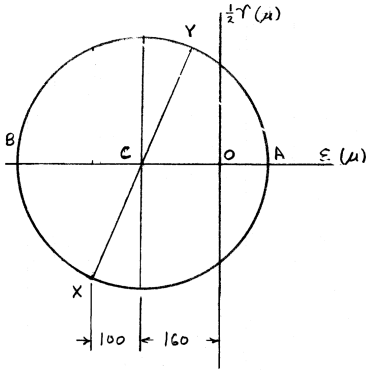
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{-260 + 60} = -2.4$$

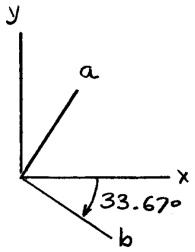
$$2\theta_p = -67.38°$$

$\theta_b = -33.67°$ ◀

$\theta_a = 56.31°$ ◀

$$R = \sqrt{(100\mu)^2 + (240\mu)^2}$$
$$R = 260\mu$$





(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = -160\mu + 260\mu$

$\varepsilon_a = 100\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -160\mu - 260\mu$

$\varepsilon_b = -420\mu$ ◀

(*b*) $\frac{1}{2}\gamma_{\max\,(\text{in-plane})} = R \quad \gamma_{\max\,(\text{in-plane})} = 2R$

$\gamma_{\max\,(\text{in-plane})} = 520\mu$ ◀

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-260 - 60)$$
$$= 160\mu$$

$$\varepsilon_{\max} = 160\mu \quad \varepsilon_{\min} = -420\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 160\mu + 420\mu$

$\gamma_{\max} = 580\mu$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.137

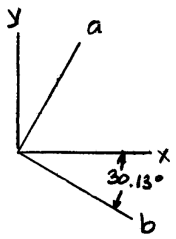
The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = -600\mu, \quad \varepsilon_y = -400\mu, \quad \gamma_{xy} = +350\mu$$

SOLUTION

Plotted points for Mohr's circle:

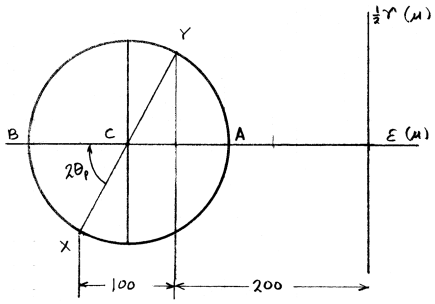
$$X\text{: } (-600\mu, -175\mu)$$
$$Y\text{: } (-400\mu, +175\mu)$$
$$C\text{: } (-500\mu, 0)$$



$$\tan 2\theta_p = -\frac{175}{100}$$
$$2\theta_p = -60.26°$$

$$\theta_b = -30.13° \blacktriangleleft$$
$$\theta_a = 59.87° \blacktriangleleft$$

$$R = \sqrt{(100\mu)^2 + (175\mu)^2}$$
$$= 201.6\mu$$



(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = -500\mu + 201.6\mu$ $\varepsilon_a = -298\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -500\mu - 201.6\mu$ $\varepsilon_b = -702\mu \blacktriangleleft$

(*b*) $\gamma_{\max\,(\text{in-plane})} = 2R$ $\gamma_{\max\,(\text{in-plane})} = 403\mu \blacktriangleleft$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-600\mu - 400\mu)$$

$$\varepsilon_c = 500\mu \blacktriangleleft$$

$$\varepsilon_{\max} = 500\mu \quad \varepsilon_{\min} = -702\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 500\mu + 702\mu$ $\gamma_{\max} = 1202\mu \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.138

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$$\varepsilon_x = +160\mu, \quad \varepsilon_y = -480\mu, \quad \gamma_{xy} = -600\mu$$

SOLUTION

(*a*) For Mohr's circle of strain, plot points:

$$X\text{: } (160\mu,\ 300\mu)$$
$$Y\text{: } (-480\mu, -300\mu)$$
$$C\text{: } (-160\mu,\ 0)$$

(*a*) $$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-300}{320} = -0.9375$$

$$2\theta_p = -43.15^\circ \quad \theta_p = -21.58^\circ \quad \text{and} \quad -21.58 + 90 = 68.42^\circ$$

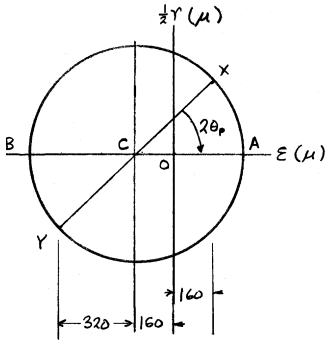
$\theta_a = -21.58^\circ$ ◀

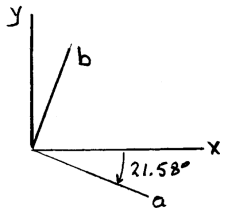
$\theta_b = 68.42^\circ$ ◀

$$R = \sqrt{(320\mu)^2 + (300\mu)^2} = 438.6\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = -160\mu + 438.6\mu \qquad \varepsilon_a = +278.6\mu \text{ ◀}$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = -160\mu - 438.6\mu \qquad \varepsilon_b = -598.6\mu \text{ ◀}$$





(*b*) $$\frac{1}{2}\gamma_{(\text{max, in-plane})} = R \quad \gamma_{(\text{max, in-plane})} = 2R$$

$\gamma_{(\text{max, in-plane})} = 877\mu$ ◀

(*c*) $$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(160\mu - 480\mu)$$

$\varepsilon_c = 160.0\mu$ ◀

$$\varepsilon_{\max} = 278.6\mu \quad \varepsilon_{\min} = -598.6\mu$$

$$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 278.6\mu + 598.6\mu \qquad \gamma_{\max} = 877\mu \text{ ◀}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.139

The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

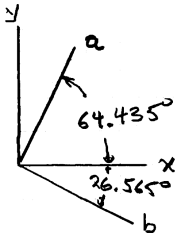
$$\varepsilon_x = +30\mu, \quad \varepsilon_y = +570\mu, \quad \gamma_{xy} = +720\mu$$

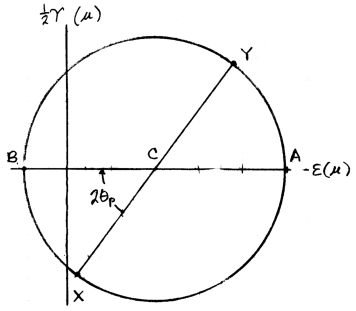
SOLUTION

Plotted points for Mohr's circle:

$$X\text{: } (30\mu, -360\mu)$$
$$Y\text{: } (570\mu, +360\mu)$$
$$C\text{: } (300\mu, 0)$$

$$\tan 2\theta_p = \frac{-360}{270} = -1.3333$$
$$2\theta_p = -53.13°$$





(*a*) $\theta_b = -26.565°$ ◀

$\theta_a = 64.435°$ ◀

$$R = \sqrt{(270\mu)^2 + (360\mu)^2} = 450\mu$$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = 300\mu + 450\mu$ $\quad \varepsilon_a = 750\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 300\mu - 450\mu$ $\quad \varepsilon_b = -150\mu$ ◀

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\quad \gamma_{\text{max (in-plane)}} = 900\mu$ ◀

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{1/3}{2/3}(750\mu - 150\mu) \qquad \varepsilon_c = -300\mu \blacktriangleleft$$

$$\varepsilon_{\max} = \varepsilon_a = 750\mu, \quad \varepsilon_{\min} = \varepsilon_c = -300\mu$$

(*c*) $\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 750\mu - (-300\mu)$ $\quad \gamma_{\max} = 1050\mu$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.140

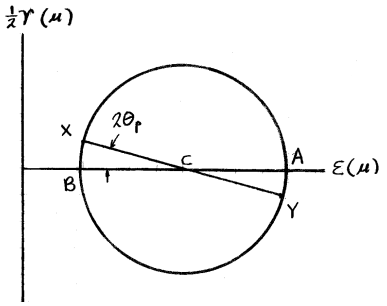
For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +60\mu, \quad \varepsilon_y = +240\mu, \quad \gamma_{xy} = -50\mu$$

SOLUTION

Plotted points:

$$X\colon\ (60\mu, 25\mu)$$
$$Y\colon\ (240\mu, -25\mu)$$
$$C\colon\ (150\mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60 - 240} = 0.277778$$

$$2\theta_p = 15.52°$$

$$\theta_a = 97.76° \blacktriangleleft$$

$$\theta_b = 7.76° \blacktriangleleft$$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 150\mu + 93.4\mu$

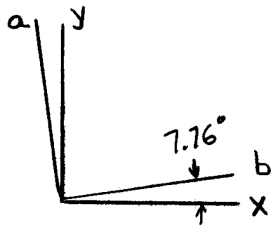
$$\varepsilon_a = 243.4\mu \blacktriangleleft$$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 150\mu - 93.4\mu$

$$\varepsilon_b = 56.6\mu \blacktriangleleft$$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$

$$\gamma_{\text{max (in-plane)}} = 186.8\mu \blacktriangleleft$$



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 243.4\mu, \quad \varepsilon_{\min} = 0$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$

$$\gamma_{\max} = 243.4 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

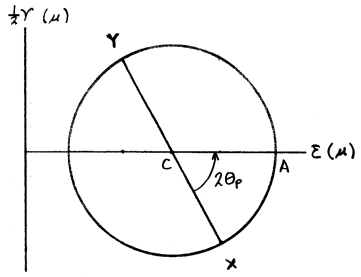
PROBLEM 7.141

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +400\mu, \quad \varepsilon_y = +200\mu, \quad \gamma_{xy} = 375\mu$$

SOLUTION

Plotted points for Mohr's circle:



$$X\text{: } (+400\mu, -187.5\mu)$$
$$Y\text{: } (+200\mu, +187.5\mu)$$
$$C\text{: } (+300\mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400 - 200} = 1.875$$

$$2\theta_p = 61.93^\circ$$

$$\theta_a = 30.96^\circ \blacktriangleleft$$

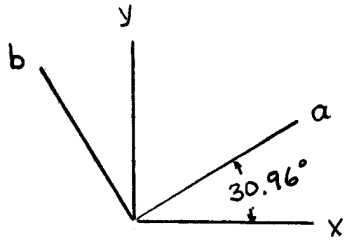
$$\theta_b = 120.96^\circ \blacktriangleleft$$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 300\mu + 212.5\mu$ $\qquad \varepsilon_a = 512.5\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 300\mu - 212.5\mu$ $\qquad \varepsilon_b = 87.5\mu \blacktriangleleft$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 425\mu \blacktriangleleft$



(*c*) $\varepsilon_c = 0 \quad \varepsilon_{\max} = 512.5\mu \quad \varepsilon_{\min} = 0$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\qquad \gamma_{\max} = 512.5\mu \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

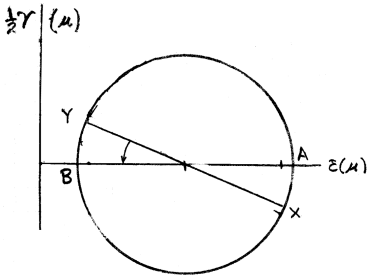
PROBLEM 7.142

For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = +300\mu, \quad \varepsilon_y = +60\mu, \quad \gamma_{xy} = +100\mu$$

SOLUTION

$$X\text{: } (300\mu, -50\mu)$$
$$Y\text{: } (60\mu, 50\mu)$$
$$C\text{: } (180\mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{100}{300 - 60}$$
$$2\theta_p = 22.62°$$

$$\theta_a = 11.31° \blacktriangleleft$$

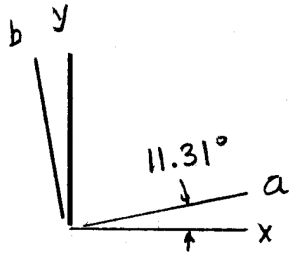
$$\theta_b = 101.31° \blacktriangleleft$$

$$R = \sqrt{(120\mu)^2 + (50\mu)^2} = 130\mu$$

(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 180\mu + 130\mu$ $\qquad \varepsilon_a = 310\mu \blacktriangleleft$

$\varepsilon_b = \varepsilon_{\text{ave}} - R = 180\mu - 130\mu$ $\qquad \varepsilon_b = 50\mu \blacktriangleleft$

(*b*) $\gamma_{\text{max (in-plane)}} = 2R$ $\qquad \gamma_{\text{max (in-plane)}} = 260\mu \blacktriangleleft$



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 310\mu, \quad \varepsilon_{\min} = 0$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$ $\qquad \gamma_{\max} = 310\mu \blacktriangleleft$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.143

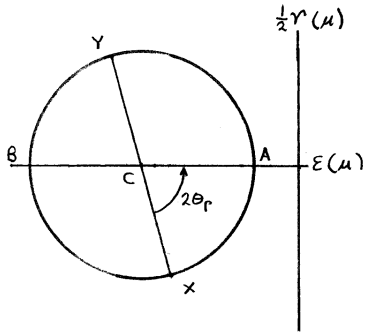
For the given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

$$\varepsilon_x = -180\mu, \quad \varepsilon_y = -260\mu, \quad \gamma_{xy} = +315\mu$$

SOLUTION

Plotted points for Mohr's circle:

$$X\text{: } (-180\mu, -157.5\mu)$$
$$Y\text{: } (-260\mu, +157.5\mu)$$
$$C\text{: } (-220\mu, 0)$$



(*a*) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{315}{80} = 3.9375$

$2\theta_p = 75.75°$

$\theta_a = 37.87°$ ◀

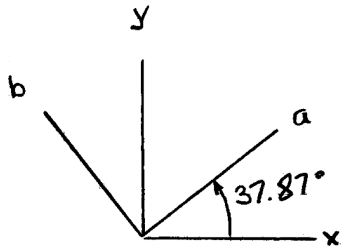
$\theta_b = 127.87°$ ◀

$R = \sqrt{(40\mu)^2 + (157.5\mu)^2} = 162.5\mu$

$\varepsilon_a = \varepsilon_{\text{ave}} + R = -220\mu + 162.5\mu$ $\quad \varepsilon_a = -57.5\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -220\mu - 162.5$ $\quad \varepsilon_b = -382.5\mu$ ◀

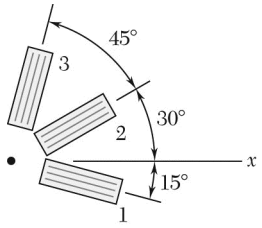
(*b*) $\gamma_{\text{max (in-plane)}} = 2R = 325\mu$ ◀



(*c*) $\varepsilon_c = 0, \quad \varepsilon_{\max} = 0, \quad \varepsilon_{\min} = -382.5\mu$

$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min} = 0 + 382.5\mu$ $\quad \gamma_{\max} = 382.5\mu$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

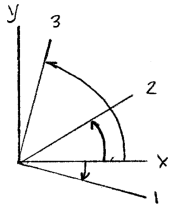


PROBLEM 7.144

Determine the strain ε_x knowing that the following strains have been determined by use of the rosette shown:

$$\varepsilon_1 = +480\mu \quad \varepsilon_2 = -120\mu \quad \varepsilon_3 = +80\mu$$

SOLUTION



$$\theta_1 = -15°$$
$$\theta_2 = 30°$$
$$\theta_3 = 75°$$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.9330\varepsilon_x + 0.06699\varepsilon_y - 0.25\gamma_{xy} = 480\mu \tag{1}$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\varepsilon_x + 0.25\varepsilon_y + 0.4330\gamma_{xy} = -120\mu \tag{2}$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$0.06699\varepsilon_x + 0.9330\varepsilon_y + 0.25\gamma_{xy} = 80\mu \tag{3}$$

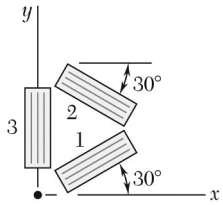
Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 253\mu, \qquad \varepsilon_y = 307\mu, \qquad \gamma_{xy} = -893\mu$$

$$\varepsilon_x = 253\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.145



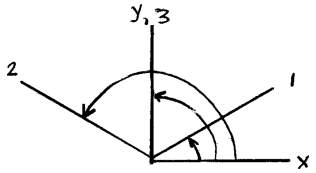
The strains determined by the use of the rosette shown during the test of a machine element are

$$\varepsilon_1 = +600\mu \quad \varepsilon_2 = +450\mu \quad \varepsilon_3 = -75\mu$$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain.

SOLUTION

$$\theta_1 = 30°$$
$$\theta_2 = 150°$$
$$\theta_3 = 90°$$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.75\varepsilon_x + 0.25\varepsilon_y + 0.43301\gamma_{xy} = 600\mu \tag{1}$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\varepsilon_x + 0.25\varepsilon_y - 0.43301\gamma_{xy} = 450\mu \tag{2}$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$0 + \varepsilon_y + 0 = -75\mu \tag{3}$$

Solving (1), (2), and (3) simultaneously,

$$\varepsilon_x = 725\mu, \quad \varepsilon_y = -75\mu, \quad \gamma_{xy} = 173.21\mu$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 325\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725+75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\mu$$

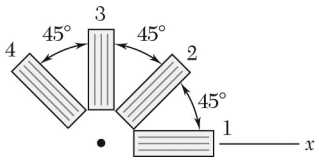
(*a*) $\varepsilon_a = \varepsilon_{\text{ave}} + R = 734\mu$ $\qquad \varepsilon_a = 734\mu$ ◀

$\varepsilon_b = \varepsilon_{\text{ave}} - R = -84.3\mu$ $\qquad \varepsilon_b = -84.3\mu$ ◀

(*b*) $\gamma_{\text{max (in-plane)}} = 2R = 819\mu$ $\qquad \gamma_{\text{max (in-plane)}} = 819\mu$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.146



The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$$\varepsilon_1 = +420\times10^{-6}\text{ in./in.}\quad \varepsilon_2 = -45\times10^{-6}\text{ in./in.}\quad \varepsilon_4 = +165\times10^{-6}\text{ in./in.}$$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

SOLUTION

(*a*) Gages 2 and 4 are 90° apart.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(-45\times10^{-6} + 165\times10^{-6}) = 60\times10^{-6}\text{ in/in}$$

Gages 1 and 3 are also 90° apart.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$\varepsilon_3 = 2\varepsilon_{\text{ave}} - \varepsilon_1 = (2)(60\times10^{-6}) - 420\times10^{-6}$$

$$\varepsilon_3 = -300\times10^{-6}\text{ in/in} \blacktriangleleft$$

(*b*) $\varepsilon_x = \varepsilon_1 = 420\times10^{-6}\text{ in/in}\quad \varepsilon_y = \varepsilon_3 = -300\times10^{-6}\text{ in/in}$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = (2)(-45\times10^{-6}) - 420\times10^{-6} - (-300\times10^{-6})$$

$$= -210\times10^{-6}\text{ in/in}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\times10^{-6} - (-300\times10^{-6})}{2}\right)^2 + \left(\frac{-210\times10^{-6}}{2}\right)^2}$$

$$= 375\times10^{-6}\text{ in/in}$$

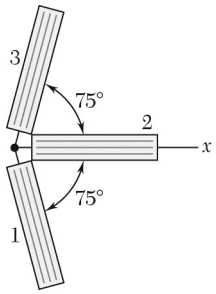
$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 60\times10^{-6} + 375\times10^{-6} \qquad \varepsilon_a = 435\times10^{-6}\text{ in/in} \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 60\times10^{-6} - 375\times10^{-6} \qquad \varepsilon_b = -315\times10^{-6}\text{ in/in} \blacktriangleleft$$

$$\gamma_{\text{max (in-plane)}} = 2R \qquad \gamma_{\text{max (in-plane)}} = 750\times10^{-6}\text{ in/in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.147



The strains determined by the use of the rosette attached as shown during the test of a machine element are

$$\varepsilon_1 = -93.1\times10^{-6}\ \text{in./in.}$$
$$\varepsilon_2 = +385\times10^{-6}\ \text{in./in.}$$
$$\varepsilon_3 = +210\times10^{-6}\ \text{in./in.}$$

Determine (*a*) the orientation and magnitude of the principal strains in the plane of the rosette, (*b*) the maximum in-plane shearing strain.

SOLUTION

Use $\varepsilon_{x'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$

where $\theta = -75°$ for gage 1,

$\theta = 0$ for gage 2,

and $\theta = +75°$ for gage 3.

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(-150°) + \frac{\gamma_{xy}}{2}\sin(-150°) \tag{1}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0 + \frac{\gamma_{xy}}{2}\sin 0 \tag{2}$$

$$\varepsilon_3 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(150°) + \frac{\gamma_{xy}}{2}\sin(150°) \tag{3}$$

From Eq. (2), $\varepsilon_x = \varepsilon_z = 385\times10^{-6}$ in/in

Adding Eqs. (1) and (3),

$$\varepsilon_1 + \varepsilon_3 = (\varepsilon_x + \varepsilon_y) + (\varepsilon_x - \varepsilon_y)\cos 150°$$
$$= \varepsilon_x(1 + \cos 150°) + \varepsilon_y(1 - \cos 150°)$$
$$\varepsilon_y = \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_x(1 + \cos 150°)}{(1 - \cos 150°)}$$
$$= \frac{-93.1\times10^{-6} + 210\times10^{-6} - 385\times10^{-6}(1 + \cos 150°)}{1 - \cos 150°}$$
$$= 35.0\times10^{-6}\ \text{in/in}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.147 *(Continued)*

Subtracting Eq. (1) from Eq. (3),

$$\varepsilon_3 - \varepsilon_1 = \gamma_{xy} \sin 150°$$

$$\gamma_{xy} = \frac{\varepsilon_3 - \varepsilon_1}{\sin 150°} = \frac{210\times10^{-6} - (-93.1\times10^{-6})}{\sin 150°}$$

$$= 606.2\times10^{-6} \text{ in/in}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{606.2\times10^{-6}}{385\times10^{-6} - 35.0\times10^{-6}} = 1.732$$

(*a*) $\theta_a = 30.0°, \quad \theta_b = 120.0°$ ◀

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = \frac{1}{2}(385\times10^{-6} + 35.0\times10^{-6})$$

$$= 210\times10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

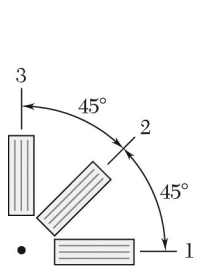
$$= \sqrt{\left(\frac{385\times10^{-6} - 35.0\times10^{-6}}{2}\right)^2 + \left(\frac{606.2}{2}\right)^2} = 350.0\times10^{-6}$$

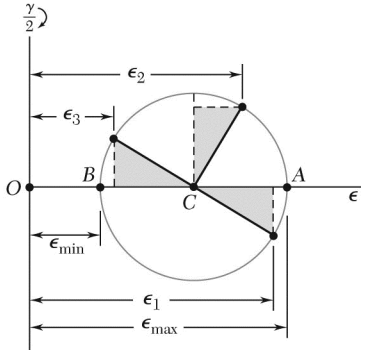
$$\varepsilon_a = \varepsilon_{\text{ave}} + R = 210\times10^{-6} + 350.0\times10^{-6} \qquad \varepsilon_a = 560\times10^{-6} \text{ in/in} \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R = 210\times10^{-6} - 350.0\times10^{-6} \qquad \varepsilon_b = -140.0\times10^{-6} \text{ in/in} \blacktriangleleft$$

(*b*) $$\frac{\gamma_{\text{max (in-plane)}}}{2} = R = 350.0\times10^{-6} \text{ in/in} \qquad \gamma_{\text{max (in-plane)}} = 700\times10^{-6} \text{ in/in} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 7.148

Using a 45° rosette, the strains ε_1, ε_2, and ε_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are:

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \pm \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{\frac{1}{2}}$$

(*Hint:* The shaded triangles are congruent.)

SOLUTION

Since gage directions 1 and 3 are 90° apart,

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

Let $\quad \text{u} = \varepsilon_1 - \varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 - \varepsilon_3).$

$$\text{v} = \varepsilon_2 - \varepsilon_{\text{ave}} = \varepsilon_2 - \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$R^2 = \text{u}^2 + \text{v}^2$$

$$= \frac{1}{4}(\varepsilon_1 - \varepsilon_3)^2 + \varepsilon_2^2 - \varepsilon_2(\varepsilon_1 + \varepsilon_3) + \frac{1}{4}(\varepsilon_1 + \varepsilon_3)^2$$

$$= \frac{1}{4}\varepsilon_1^2 - \cancel{\frac{1}{2}\varepsilon_1\varepsilon_3} + \frac{1}{4}\varepsilon_3^2 + \varepsilon_2^2 - \varepsilon_2\varepsilon_1 - \varepsilon_2\varepsilon_3 + \frac{1}{4}\varepsilon_1^2 + \cancel{\frac{1}{2}\varepsilon_1\varepsilon_3} + \frac{1}{4}\varepsilon_3^2$$

$$= \frac{1}{2}\varepsilon_1^2 - \varepsilon_2\varepsilon_1 + \varepsilon_2^2 - \varepsilon_2\varepsilon_3 + \frac{1}{2}\varepsilon_3^2$$

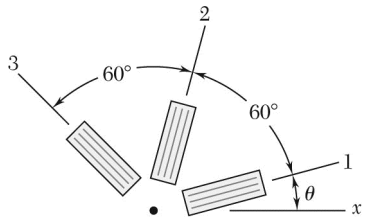
$$= \frac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 + \frac{1}{2}(\varepsilon_2 - \varepsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

$$\varepsilon_{\max,\,\min} = \varepsilon_{\text{ave}} \pm R$$

gives the required formula.

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.149

Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{avg}}$$

where ε_{avg} is the abscissa of the center of the corresponding Mohr's circle.

SOLUTION

$$\varepsilon_1 = \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \tag{1}$$

$$\begin{aligned}\varepsilon_2 &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120°\cos 2\theta - \sin 120°\sin 2\theta)\\ &\quad + \frac{\gamma_{xy}}{2}(\cos 120°\sin 2\theta + \sin 120°\cos 2\theta)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right)\\ &\quad + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\varepsilon_3 &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240°\cos 2\theta - \sin 240°\sin 2\theta)\\ &\quad + \frac{\gamma_{xy}}{2}(\cos 240°\sin 2\theta + \sin 240°\cos 2\theta)\\ &= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right)\\ &\quad + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right)\end{aligned} \tag{3}$$

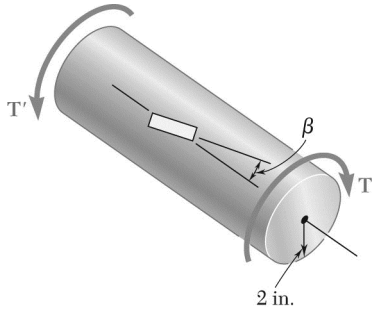
Adding (1), (2), and (3),

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{ave}} + 0 + 0$$

$$3\varepsilon_{\text{ave}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

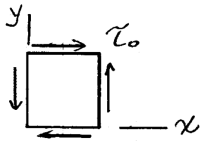


PROBLEM 7.150

A single strain gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque **T** indicated by a gage reading of 300×10^{-6} in./in.

SOLUTION

For torsion, $\sigma_x = \sigma_y = 0, \quad \tau = \tau_0$



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0$$

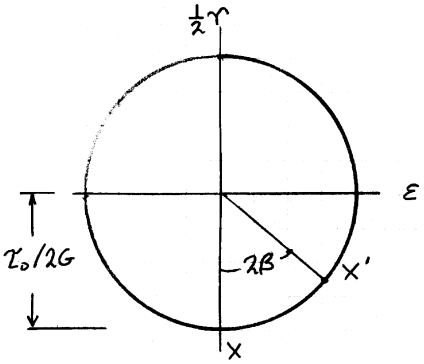
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2}\gamma_{xy} = \frac{\tau_0}{2G}$$

Draw the Mohr's circle for strain.

$$R = \frac{\tau_0}{2G}$$

$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$



But $$\tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$$

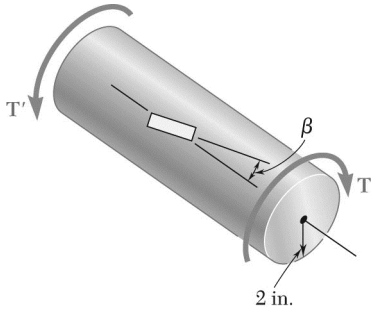
$$T = \frac{\pi c^3 G \varepsilon_{x'}}{\sin 2\beta}$$

$$= \frac{\pi(2)^3(11.5\times10^6)(300\times10^{-6})}{\sin 50°}$$

$$= 113.2 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T = 113.2 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



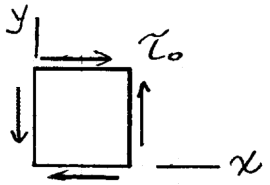
PROBLEM 7.151

Solve Prob. 7.150, assuming that the gage forms an angle $\beta = 35°$ with a line parallel to the axis of the shaft.

PROBLEM 7.150 A single gage is cemented to a solid 4-in.-diameter steel shaft at an angle $\beta = 25°$ with a line parallel to the axis of the shaft. Knowing that $G = 11.5 \times 10^6$ psi, determine the torque **T** indicated by a gage reading of 300×10^{-6} in./in.

SOLUTION

For torsion, $\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0$$

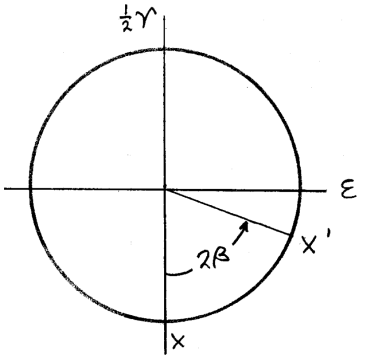
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

$$\gamma_{xy} = \frac{\tau_0}{G} \quad \frac{1}{2}\gamma_{xy} = \frac{\tau_0}{2G}$$

Draw Mohr's circle for strain.

$$R = \frac{\tau_0}{2G}$$

$$\varepsilon_{x'} = R \sin 2\beta = \frac{\tau_0}{2G} \sin 2\beta$$



But

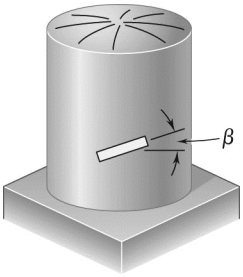
$$\tau_0 = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2G\varepsilon_{x'}}{\sin 2\beta}$$

$$T = \frac{\pi c^3 G\varepsilon_{x'}}{\sin 2\beta} = \frac{\pi(2)^3(11.5\times10^6)(300\times10^{-6})}{\sin 70°}$$

$$= 92.3 \times 10^3 \text{ lb} \cdot \text{in}$$

$$T = 92.3 \text{ kip} \cdot \text{in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.152

A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.

SOLUTION

$$\sigma_x = \sigma_1 = \frac{pr}{t}$$

$$\sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

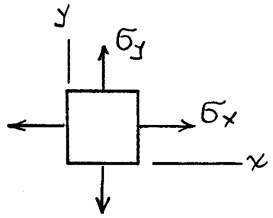
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E}$$

$$= 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E}$$

$$= 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



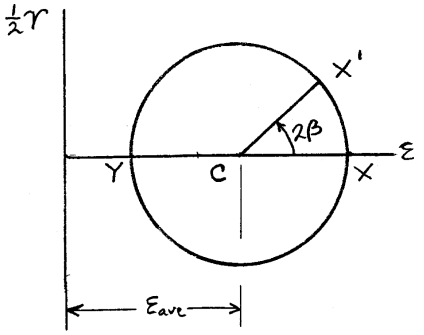
Draw Mohr's circle for strain.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta = (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$



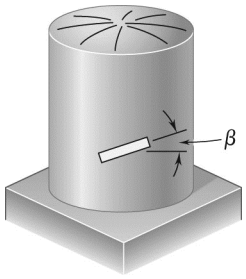
Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

$$t = 6\times10^{-3}\text{ m} \quad E = 200\times10^{9}\text{ Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 18°$$

$$p = \frac{(6\times10^{-3})(200\times10^{9})(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 36°)} = 1.421\times10^{6}\text{ Pa}$$

$$p = 1.421 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.153

Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 35°$ with a horizontal plane.

PROBLEM 7.152 A single strain gage forming an angle $\beta = 18°$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6 mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200\,\text{GPa}$ and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280μ.

SOLUTION

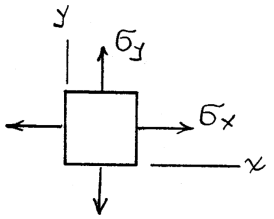
$$\sigma_x = \sigma_1 = \frac{pr}{t}$$

$$\sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E} = 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E} = 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



Draw Mohr's circle for strain.

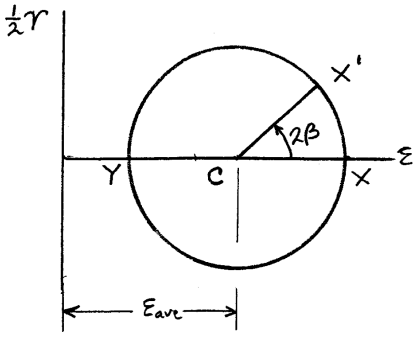
$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta$$

$$= (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$



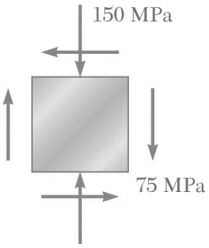
Data:

$$r = \frac{1}{2}d = \frac{1}{2}(600) = 300\ \text{mm} = 0.300\ \text{m}$$

$$t = 6\times10^{-3}\,\text{m} \quad E = 200\times10^{9}\,\text{Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 35°$$

$$p = \frac{(6\times10^{-3})(200\times10^{9})(280\times10^{-6})}{(0.300)(0.525 + 0.325\cos 70°)} = 1.761\times10^{6}\,\text{Pa} \qquad p = 1.761\,\text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.154

The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200\,\text{GPa}$ and $G = 77.2\,\text{GPa}$, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43) and Eq. (2.38)] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

SOLUTION

(*a*) $\sigma_x = 0, \quad \sigma_y = -150\times10^6\,\text{Pa}, \quad \tau_{xy} = -75\times10^6\,\text{Pa}$

$E = 200\times10^9\,\text{Pa} \quad G = 77\times10^9\,\text{Pa}$

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200\times10^9}[0 + (0.2987)(150\times10^6)]$$
$$= 224\mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200\times10^9}[(-150\times10^6) - 0]$$
$$= -750\mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-75\times10^6}{77\times10^9} = -974\mu$$

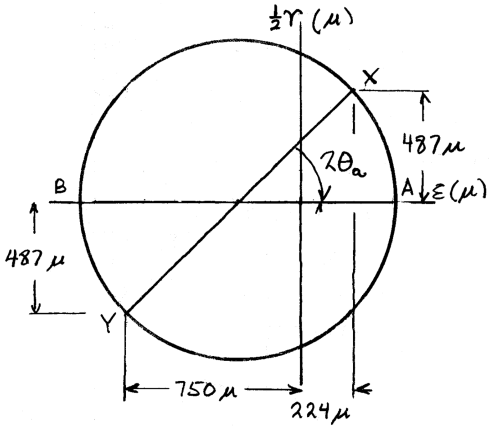
$$\frac{\gamma_{xy}}{2} = -487.0\mu$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -263\mu$$

$$\varepsilon_x - \varepsilon_y = 974\mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-974}{974} = -1.000$$

$$2\theta_a = -45.0° \qquad \theta_a = -22.5° \blacktriangleleft$$



$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 689\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R \qquad \varepsilon_a = 426\mu \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{\text{ave}} - R \qquad \varepsilon_b = -952\mu \blacktriangleleft$$

$$\varepsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 150\times10^6)}{200\times10^9} \qquad \varepsilon_c = -224\mu \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.154 *(Continued)*

(*b*) $$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0+150}{2}\right)^2 + 75^2}$$

$$= 106.07 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 31.07 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -181.07 \text{ MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b)$$

$$= \frac{1}{200\times10^9}[31.07\times10^6 - (0.2987)(-181.07\times10^6)]$$

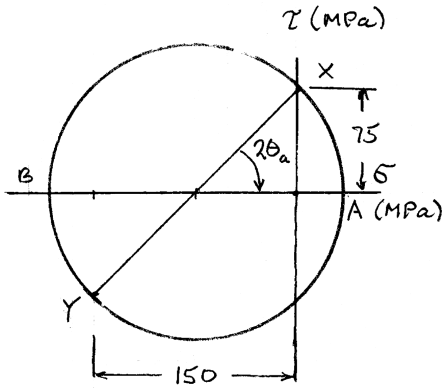
$$= 426\times10^{-6}$$

$\varepsilon_a = 426\mu$ ◀

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.000$$

$2\theta_a = -45°$

$\theta_a = -22.5°$ ◀



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.155

The following state of strain has been determined on the surface of a cast-iron machine part:

$$\varepsilon_x = -720\mu \quad \varepsilon_y = -400\mu \quad \gamma_{xy} = +660\mu$$

Knowing that $E = 69$ GPa and $G = 28$ GPa, determine the principal planes and principal stresses (*a*) by determining the corresponding state of plane stress [use Eq. (2.36), Eq. (2.43), and the first two equations of Prob. 2.72] and then using Mohr's circle for stress, (*b*) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determining the corresponding stresses.

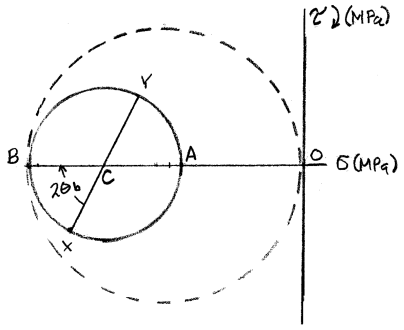
SOLUTION

The 3rd principal stress is $\sigma_z = 0$.

$$G = \frac{E}{2(1+v)} \quad v = \frac{E}{2G} - 1 = \frac{69}{56} - 1 = 0.2321$$

$$\frac{E}{1-v^2} = \frac{69}{1-(0.232)^2} = 72.93 \text{ GPa}$$

(*a*)
$$\sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y)$$
$$= (72.93\times10^9)[-720\times10^{-6} + (0.232)(-400\times10^{-6})]$$
$$= -59.28 \text{ MPa}$$
$$\sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x)$$
$$= (72.93\times10^9)[-400\times10^{-6} + (0.2321)(-720\times10^{-6})]$$
$$= -41.36 \text{ MPa}$$
$$\tau_{xy} = G\gamma_{xy} = (28\times10^9)(660\times10^{-6})$$
$$= 18.48 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -50.32 \text{ MPa}$$
$$\tan 2\theta_b = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.0625$$
$$2\theta_b = -64.1°, \quad \theta_b = -32.1°, \quad \theta_a = 57.9°$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.54 \text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = -29.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -70.9 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.155 *(Continued)*

(*b*) $$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = -560\mu$$

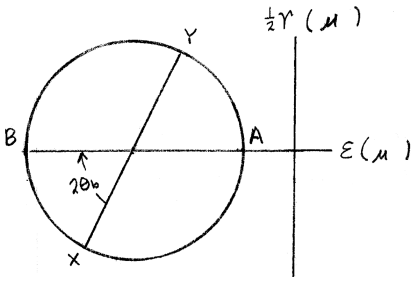
$$\tan 2\theta_b = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -2.0625$$

$$2\theta_b = -64.1°, \quad \theta_b = -32.1°, \quad \theta_a = 57.9°$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 366.74\mu$$

$$\varepsilon_a = \varepsilon_{\text{ave}} + R = -193.26\mu$$

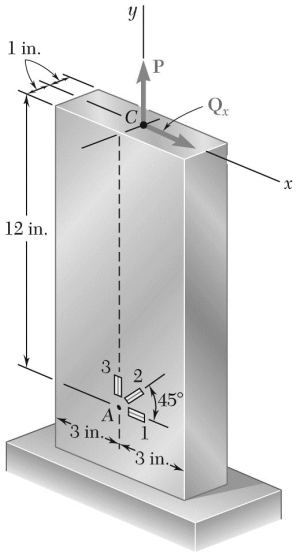
$$\varepsilon_b = \varepsilon_{\text{ave}} - R = -926.74\mu$$



$$\sigma_a = \frac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_b) \qquad \sigma_a = -29.8 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \frac{E}{1-\nu^2}(\varepsilon_b + \nu\varepsilon_a) \qquad \sigma_b = -70.9 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.156

A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point *C* of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point *A* indicates the following strains:

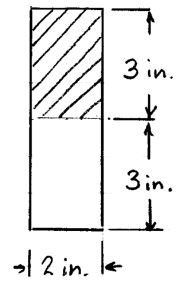
$$\varepsilon_1 = -60 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_2 = +240 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_3 = +200 \times 10^{-6} \text{in./in.}$$

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -60 \times 10^{-6}$$
$$\varepsilon_y = \varepsilon_3 = 200 \times 10^{-6}$$
$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 340 \times 10^{-6}$$
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-(0.3)^2}[-60 + (0.3)(200)] = 0$$
$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-(0.3)^2}[200 + (0.3)(-60)] = 5.8 \times 10^3 \text{ psi}$$
$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (2)(6)(5.8 \times 10^3)$$
$$= 69.6 \times 10^3 \text{ lb} \qquad P = 69.6 \text{ kips} \blacktriangleleft$$

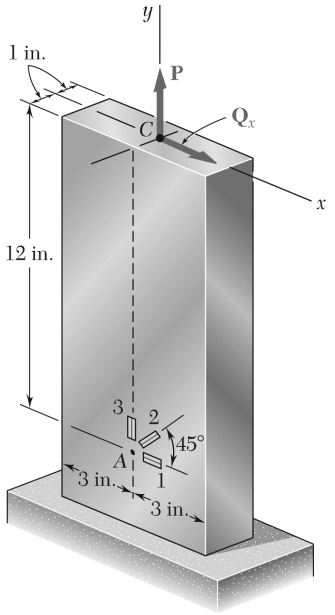
$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.30)} = 11.1538 \times 10^6 \text{ psi}$$
$$\tau_{xy} = G\gamma_{xy} = (11.1538)(340) = 3.7923 \times 10^3 \text{ psi}$$
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$
$$\hat{Q} = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in.}$$
$$\tau_{xy} = \frac{V\hat{Q}}{It}$$
$$V = \frac{It\tau_{xy}}{\hat{Q}} = \frac{(36)(2)(3.7923 \times 10^3)}{9} = 30.338 \times 10^3 \text{ lb}$$



$$Q = V \qquad Q = 30.3 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.157



Solve Prob. 7.156, assuming that the rosette at point *A* indicates the following strains:

$$\varepsilon_1 = -30 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_2 = +250 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_3 = +100 \times 10^{-6} \text{in./in.}$$

PROBLEM 7.156 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point *C* of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point *A* indicates the following strains:

$$\varepsilon_1 = -60 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_2 = +240 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_3 = +200 \times 10^{-6} \text{in./in.}$$

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -30 \times 10^{-6}$$
$$\varepsilon_y = \varepsilon_3 = +100 \times 10^{-6}$$
$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 430 \times 10^{-6}$$
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-(0.3)^2}[-30 + (0.3)(100)]$$
$$= 0$$
$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-(0.3)^2}[100 + (0.3)(-30)]$$
$$= 2.9 \times 10^3 \text{ psi}$$
$$\frac{P}{A} = \sigma_y \quad P = A\sigma_y = (2)(6)(2.9 \times 10^3)$$
$$= 34.8 \times 10^3 \text{ lb} \qquad P = 34.8 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.157 *(Continued)*

$$G = \frac{E}{2(1+\nu)} = \frac{29\times10^6}{(2)(1.30)} = 11.1538\times10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.1538)(430) = 4.7962\times10^3 \text{ psi}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

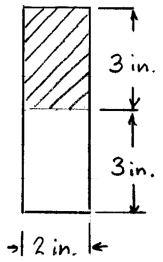
$$\hat{Q} = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3$$

$$t = 2 \text{ in.}$$

$$\tau_{xy} = \frac{V\hat{Q}}{It}$$

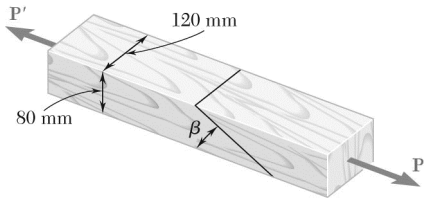
$$V = \frac{It\tau_{xy}}{\hat{Q}} = \frac{(36)(2)(4.7962\times10^3)}{9} = 38.37\times10^3 \text{ lb}$$

$$Q = V$$



$Q = 38.4$ kips ◀

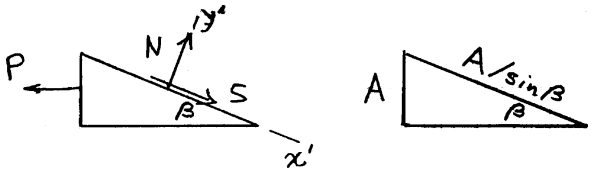
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.158

Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 22°$ and that the maximum allowable stresses in the joint are, respectively, 400 kPa in tension (perpendicular to the splice and 600 kPa in shear (parallel to the splice), determine the largest centric load **P** that can be applied.

SOLUTION



Forces Areas

$$A = (80)(120) = 9.6\times10^{3}\,\text{mm}^2 = 9.6\times10^{-3}\,\text{m}^2$$

$$N_{\text{all}} = \sigma_{\text{all}}\ A/\sin\beta = \frac{(400\times10^{3})(9.6\times10^{-3})}{\sin 22°} = 10.251\times10^{3}\,\text{N}$$

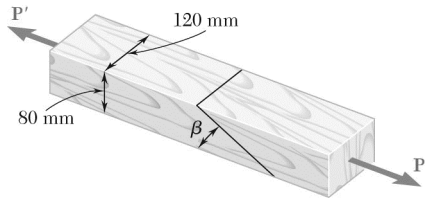
$$+\nearrow\ \Sigma F_{y'} = 0:\quad N - P\sin\beta = 0 \qquad P = \frac{N}{\sin\beta} = \frac{10.251\times10^{3}}{\sin 22°} = 27.4\times10^{3}\,\text{N}$$

$$S_{\text{all}} = \tau_{\text{all}}\ A/\sin\beta = \frac{(600\times10^{3})(9.6\times10^{-3})}{\sin 22°} = 15.376\times10^{3}\,\text{N}$$

$$+\searrow\ \Sigma F_{x'} = 0:\quad S - P\cos\beta = 0 \qquad P = \frac{S}{\cos\beta} = \frac{15.376\times10^{3}}{\cos 22°} = 16.58\times10^{3}\,\text{N}$$

The smaller value for P governs. $P = 16.58$ kN ◀

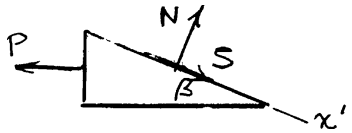
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.159

Two wooden members of 80 × 120-mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $\beta = 25°$ and that centric loads of magnitude $P = 10$ kN are applied to the members as shown, determine (*a*) the in-plane shearing stress parallel to the splice, (*b*) the normal stress perpendicular to the splice.

SOLUTION



Forces

Areas

$$A = (80)(120) = 9.6\times10^{3}\,\text{mm}^2 = 9.6\times10^{-3}\,\text{m}^2$$

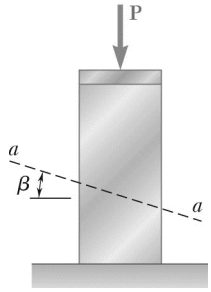
(*a*) $+\nearrow \quad \Sigma F_{y'} = 0: \quad N - P\sin\beta = 0 \qquad N = P\sin\beta = (10\times10^{3})\sin 25° = 4.226\times10^{3}\,\text{N}$

$$\sigma = \frac{N}{A/\sin\beta} = \frac{(4.226\times10^{3})\sin 25°}{9.6\times10^{3}} = 186.0\times10^{3}\,\text{Pa} \qquad \sigma = 186.0\ \text{kPa} \blacktriangleleft$$

(*b*) $+\searrow \quad \Sigma F_{x'} = 0: \quad S - P\cos\beta = 0 \qquad S = P\cos\beta = (10\times10^{3})\cos 25° = 9.063\times10^{3}\,\text{N}$

$$\tau = \frac{N}{A/\sin\beta} = \frac{(9.063\times10^{3})\sin 25°}{9.6\times10^{-3}} = 399\times10^{3}\,\text{Pa} \qquad \tau = 399\ \text{kPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

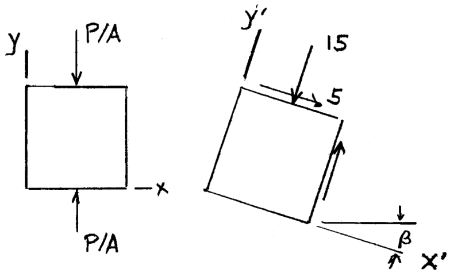


PROBLEM 7.160

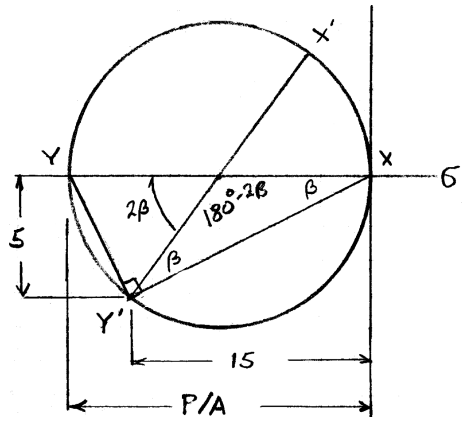
The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

SOLUTION

$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = -P/A$$



(*a*) From the Mohr's circle,

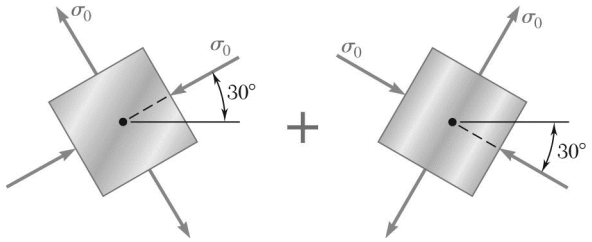


$$\tan\beta = \frac{5}{15} = 0.3333 \qquad \beta = 18.4° \blacktriangleleft$$

$$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$$

(*b*) $$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(15)}{1+\cos 2\beta} \qquad \frac{P}{A} = 16.67 \text{ ksi} \blacktriangleleft$$

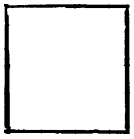
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



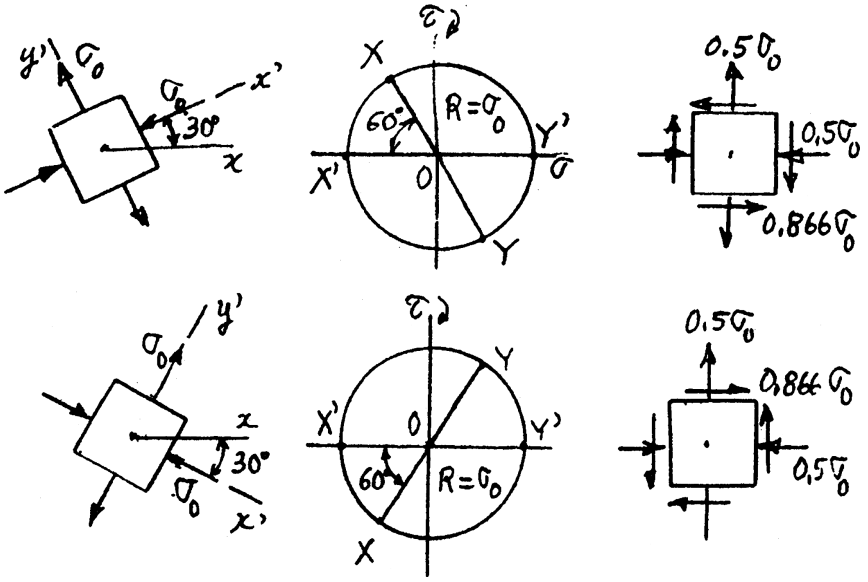
PROBLEM 7.161

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

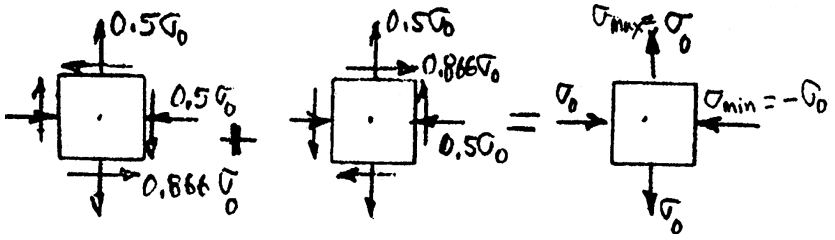
SOLUTION



Express each state of stress in terms of components acting on the element shown above.



Add like components of the two states of stress.

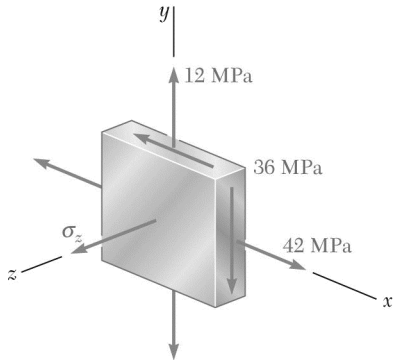


$\theta_p = 0$ and $90°$ ◀

$\sigma_{max} = \sigma_0$ ◀

$\sigma_{min} = -\sigma_0$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.162

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +24$ MPa, (*b*) $\sigma_z = -24$ MPa, (*c*) $\sigma_z = 0$.

SOLUTION

$$\sigma_x = 42 \text{ MPa}, \quad \sigma_y = 12 \text{ MPa}, \quad \tau_{xy} = -36 \text{ MPa}$$

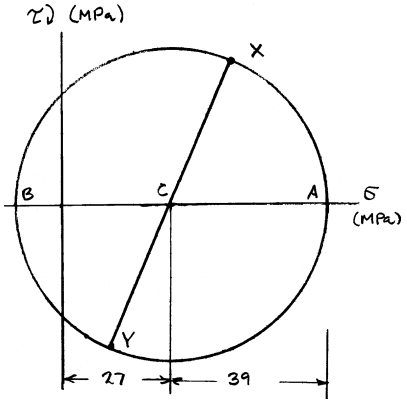
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 27 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15)^2 + (-36)^2} = 39 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 66 \text{ MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12 \text{ MPa}$$



(*a*) $\underline{\sigma_z = +24 \text{ MPa}}$ $\sigma_a = 66$ MPa $\sigma_b = -12$ MPa

$\sigma_{\max} = 66$ MPa $\sigma_{\min} = -12$ MPa $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39$ MPa ◀

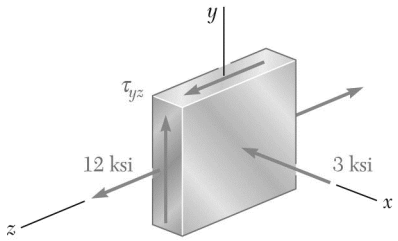
(*b*) $\underline{\sigma_z = -24 \text{ MPa}}$ $\sigma_a = 66$ MPa $\sigma_b = -12$ MPa

$\sigma_{\max} = 66$ MPa $\sigma_{\min} = -24$ MPa $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 45$ MPa ◀

(*c*) $\underline{\sigma_z = 0}$ $\sigma_a = 66$ MPa $\sigma_b = -12$ MPa

$\sigma_{\max} = 66$ MPa $\sigma_{\min} = -12$ MPa $\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 39$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

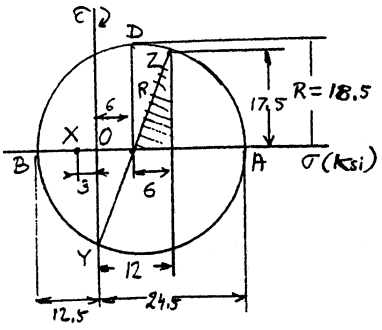


PROBLEM 7.163

For the state of stress shown, determine the maximum shearing stress when (*a*) $\tau_{yz} = 17.5$ ksi, (*b*) $\tau_{yz} = 8$ ksi, (*c*) $\tau_{yz} = 0$.

SOLUTION

(*a*) $\underline{\tau_{yz} = 17.5 \text{ ksi}}$ $\sigma_x = -3$ ksi



$$R = \sqrt{(6)^2 + (17.5)^2} = 18.5$$

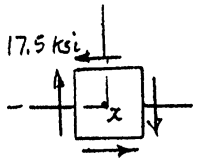
$$\sigma_A = 6 + 18.5 = 24.5$$

$$\sigma_B = 6 - 18.5 = -12.5$$

$$\sigma_{\max} = \sigma_A = 24.5 \text{ ksi}$$

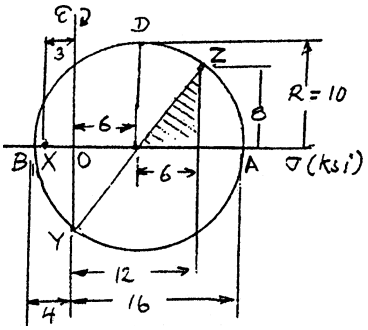
$$\sigma_{\min} = \sigma_B = -12.5 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$



$\tau_{\max} = 18.5$ ksi ◀

(*b*) $\underline{\tau_{yz} = 8 \text{ ksi}}$ $\sigma_x = -3$ ksi



$$R = \sqrt{(6)^2 + (8)^2} = 10$$

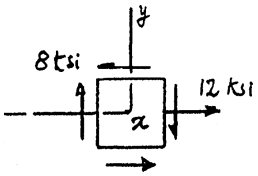
$$\sigma_A = 6 + 10 = 16$$

$$\sigma_B = 6 - 10 = -4$$

$$\sigma_{\max} = \sigma_A = 16 \text{ ksi}$$

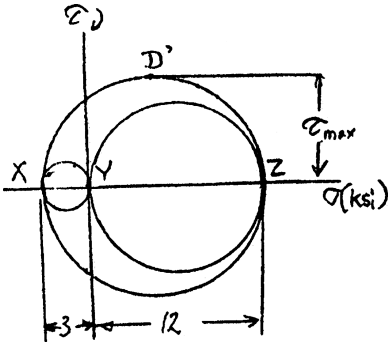
$$\sigma_{\min} = \sigma_B = -4 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$



$\tau_{\max} = 10.0$ ksi ◀

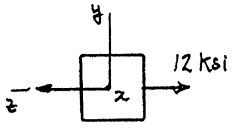
(*c*) $\underline{\tau_{yz} = 0}$ $\sigma_x = -3$ ksi



$$\sigma_{\max} = \sigma_z = 12 \text{ ksi}$$

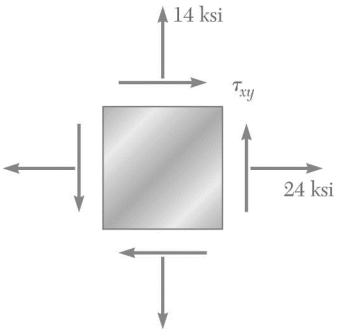
$$\sigma_{\min} = \sigma_x = -3 \text{ ksi}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min})$$



$\tau_{\max} = 7.5$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.164

The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 30$ ksi. Using the maximum-distortion-energy criterion, determine whether yield will occur when (*a*) $\tau_{xy} = 6$ ksi, (*b*) $\tau_{xy} = 12$ ksi, (*c*) $\tau_{xy} = 14$ ksi. If yield does not occur, determine the corresponding factor of safety.

SOLUTION

$$\sigma_x = 24 \text{ ksi} \quad \sigma_y = 14 \text{ ksi} \quad \sigma_z = 0$$

For stresses in *xy*-plane,

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 19 \text{ ksi} \qquad \frac{\sigma_x - \sigma_y}{2} = 5 \text{ ksi}$$

(*a*) $\underline{\tau_{xy} = 6 \text{ ksi}}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (6)^2} = 7.810 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 26.810 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 11.190 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 23.324 \text{ ksi} < 30 \text{ ksi} \qquad \text{(No yielding)}$$

$$F.S. = \frac{30}{23.324} \qquad F.S. = 1.286 \blacktriangleleft$$

(*b*) $\underline{\tau_{xy} = 12 \text{ ksi}}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (12)^2} = 13 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 32 \text{ ksi}, \quad \sigma_b = \sigma_{\text{ave}} - R = 6 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 29.462 \text{ ksi} < 30 \text{ ksi} \qquad \text{(No yielding)}$$

$$F.S. = \frac{30}{29.462} \qquad F.S. = 1.018 \blacktriangleleft$$

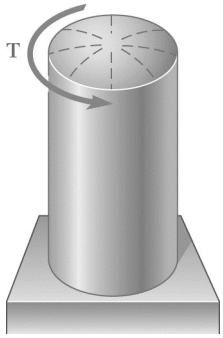
(*c*) $\underline{\tau_{xy} = 14 \text{ ksi}}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(5)^2 + (14)^2} = 14.866 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 33.866, \quad \sigma_b = \sigma_{\text{ave}} - R = 4.134 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 32.00 \text{ ksi} > 30 \text{ ksi} \qquad \text{(Yielding occurs)} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.165

A torque of magnitude $T = 12\ \text{kN} \cdot \text{m}$ is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inner diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$d = 180\ \text{mm} \quad r = \frac{1}{2}d = 90\ \text{mm} \quad t = 12\ \text{mm}$$

Torsion:

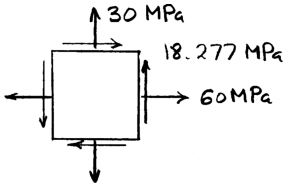
$$c_1 = 90\ \text{mm} \quad c_2 = 90 + 12 = 102\ \text{mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 66.968 \times 10^6\,\text{mm}^4 = 66.968 \times 10^{-6}\,\text{m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277\ \text{MPa}$$

Pressure:

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60\ \text{MPa} \quad \sigma_2 = \frac{pr}{2t} = 30\ \text{MPa}$$



Summary of stresses:

$$\sigma_x = 60\ \text{MPa}, \quad \sigma_y = 30\ \text{MPa}, \quad \tau_{xy} = 18.277\ \text{MPa}$$

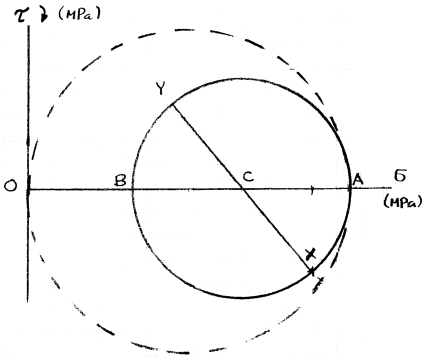
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 45\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64\ \text{MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 68.64\ \text{MPa}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 21.36\ \text{MPa}$$

$$\sigma_c \approx 0$$

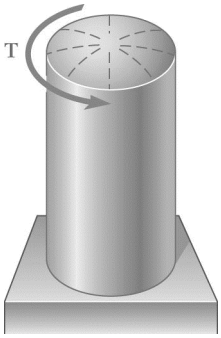


$$\sigma_{\max} = 68.6\ \text{MPa} \blacktriangleleft$$

$$\sigma_{\min} = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 34.3\ \text{MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.166

The tank shown has a 180-mm inner diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress is 75 MPa.

SOLUTION

$$r = \frac{1}{2}d = \left(\frac{1}{2}\right)(180) = 90 \text{ mm} \qquad t = 12 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 30 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_y) = 45 \text{ MPa}$$

$$\sigma_{\max} = 75 \text{ MPa}$$

$$R = \sigma_{\max} - \sigma_{\text{ave}} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa}$$

$$= 25.98 \times 10^6 \text{ Pa}$$

Torsion:

$$c_1 = 90 \text{ mm}$$

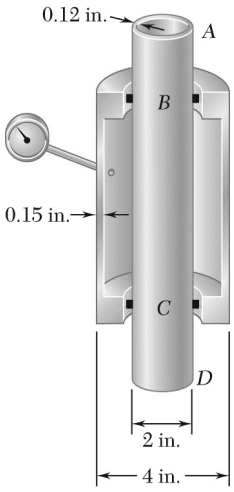
$$c_2 = 90 + 12 = 102 \text{ mm}$$

$$J = \frac{T}{2}\left(c_2^4 - c_1^4\right) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 17.06 \text{ kN} \cdot \text{m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.167

The brass pipe *AD* is fitted with a jacket used to apply a hydrostatic pressure of 500 psi to portion *BC* of the pipe. Knowing that the pressure inside the pipe is 100 psi, determine the maximum normal stress in the pipe.

SOLUTION

The only stress to be considered is the hoop stress. This stress can be obtained by applying

$$\sigma_1 = \frac{pr}{t}$$

Using successively the inside and outside pressures (the latter of which causes a compressive stress),

$$p_i = 100 \text{ psi}, \quad r_i = 1 - 0.12 = 0.88 \text{ in.}, \quad t = 0.12 \text{ in.}$$

$$(\sigma_{\max})_i = \frac{P_i r_i}{t} = \frac{(100)(0.88)}{0.12} = +733.33 \text{ psi}$$

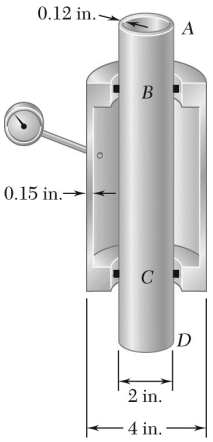
$$p_o = 500 \text{ psi}, \quad r_o = 1 \text{ in.}, \quad t = 0.12 \text{ in.}$$

$$(\sigma_{\max})_o = -\frac{P_o r_o}{t} = -\frac{(500)(1)}{0.12} = -4166.7 \text{ psi}$$

$$\sigma_{\max} = +733.33 - 4166.7 = -3433.4 \text{ psi}$$

$$\sigma_{\max} = 3.43 \text{ ksi (compression)} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 7.168

For the assembly of Prob. 7.167, determine the normal stress in the jacket (*a*) in a direction perpendicular to the longitudinal axis of the jacket, (*b*) in a direction parallel to that axis.

PROBLEM 7.167 The brass pipe *AD* is fitted with a jacket used to apply a hydrostatic pressure of 500 psi to portion *BC* of the pipe. Knowing that the pressure inside the pipe is 100 psi, determine the maximum normal stress in the pipe.

SOLUTION

(*a*) Hoop stress.

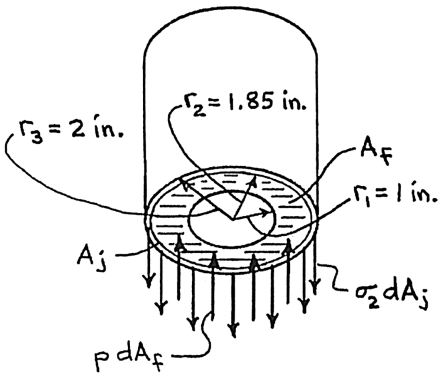
$$p = 500 \text{ psi}, \quad t = 0.15 \text{ in.}, \quad r = 2 - 0.15 = 1.85 \text{ in.}$$

$$(\sigma_1) = \frac{pr}{t} = \frac{(500)(1.85)}{0.15} = 6166.7 \text{ psi}$$

$$\sigma_1 = 6.17 \text{ ksi} \blacktriangleleft$$

(*b*) Longitudinal stress.

Free body of portion of jacket above a horizontal section, considering vertical forces only:



$$+\uparrow \sum F_y = 0:$$

$$\int_{A_f} p\,dA_f - \int_{A_j} \sigma_2\,dA_j = 0$$

$$pA_f - \sigma_2 A_j = 0$$

$$\sigma_2 = p\frac{A_f}{A_j} \qquad (1)$$

Areas :

$$A_f = \pi\left(r_2^2 - r_1^2\right) = \pi[(1.85)^2 - (1)^2] = 7.6105 \text{ in}^2$$

$$A_j = \pi\left(r_3^2 - r_2^2\right) = \pi[(2)^2 - (1.85)^2] = 1.81427 \text{ in}^2$$

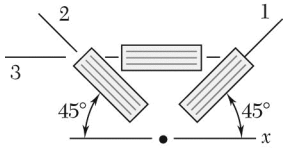
Recalling Eq (1):

$$\sigma_2 = p\frac{A_f}{A_j} = (500)\frac{7.6105}{1.81427} = 2097.4 \text{ psi}$$

$$\sigma_2 = 2.10 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 7.169



Determine the largest in-plane normal strain, knowing that the following strains have been obtained by the use of the rosette shown:

$$\varepsilon_1 = -50 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_2 = +360 \times 10^{-6} \text{in./in.}$$
$$\varepsilon_3 = +315 \times 10^{-6} \text{in./in.}$$

SOLUTION

$$\theta_1 = 45°, \quad \theta_2 = -45°, \quad \theta_3 = 0$$

$$\varepsilon_x \cos^2 \theta_1 + \varepsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \varepsilon_1$$
$$0.5\varepsilon_x + 0.5\varepsilon_y + 0.5\gamma_{xy} = -50 \times 10^{-6} \tag{1}$$

$$\varepsilon_x \cos^2 \theta_2 + \varepsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \varepsilon_2$$
$$0.5\varepsilon_x + 0.5\varepsilon_y - 0.5\gamma_{xy} = 360 \times 10^{-6} \tag{2}$$

$$\varepsilon_x \cos^2 \theta_3 + \varepsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \varepsilon_3$$
$$\varepsilon_x + 0 + 0 = 315 \times 10^{-6} \tag{3}$$

From (3), $\varepsilon_x = 315 \times 10^{-6}$ in/in

Eq. (1) – Eq. (2): $\gamma_{xy} = -50 \times 10^{-6} - 360 \times 10^{-6} = -410 \times 10^{-6}$ in/in

Eq. (1) + Eq. (2): $\varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2$

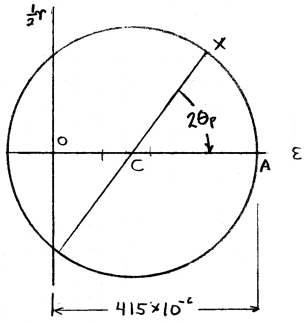
$$\varepsilon_y = \varepsilon_1 + \varepsilon_2 - \varepsilon_x = -50 \times 10^{-6} + 360 \times 10^{-6} - 315 \times 10^{-6} = -5 \times 10^{-6} \text{ in/in}$$

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 155 \times 10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \sqrt{\left(\frac{315 \times 10^{-6} + 5 \times 10^{-6}}{2}\right)^2 + \left(\frac{-410 \times 10^{-6}}{2}\right)^2}$$

$$= 260 \times 10^{-6} \text{ in/in}$$



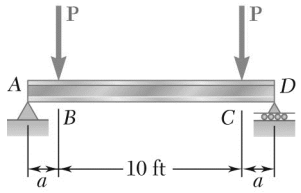
$$\varepsilon_{\max} = \varepsilon_{\text{ave}} + R = 155 \times 10^{-6} + 260 \times 10^{-6}$$

$$\varepsilon_{\max} = 415 \times 10^{-6} \text{ in/in} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

CHAPTER 8

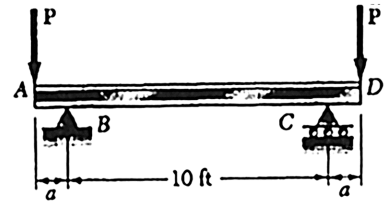
PROBLEM 8.1

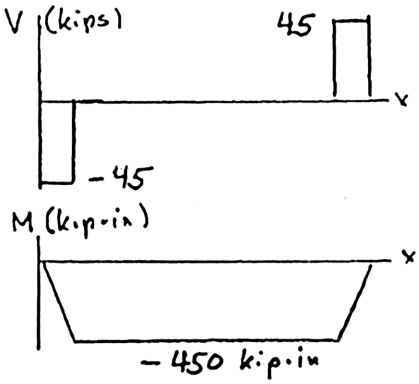


A W10×39 rolled-steel beam supports a load **P** as shown. Knowing that $P = 45$ kips, $a = 10$ in., and $\sigma_{all} = 18$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION

$$|V|_{max} = 90 \text{ kips}$$
$$|M|_{max} = (45)(10) = 450 \text{ kip} \cdot \text{in}$$





For W10 × 39 rolled steel section,

$$d = 9.92 \text{ in.}, \quad b_f = 7.99 \text{ in.}, \quad t_f = 0.530 \text{ in.},$$
$$t_w = 0.315 \text{ in.}, \quad I_x = 209 \text{ in}^4 \quad S_x = 42.1 \text{ in}^3$$
$$c = \frac{1}{2}d = 4.96 \text{ in.} \qquad y_b = c - t_f = 4.43 \text{ in.}$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450}{42.1}$$ $$\sigma_m = 10.69 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(10.69) = 9.55 \text{ ksi}$$
$$A_f = b_f t_f = 4.2347 \text{ in}^2$$
$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$
$$Q_b = A_f \bar{y}_f = 19.8819 \text{ in}^3$$
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(45)(19.8819)}{(209)(0.315)} = 13.5898 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 14.4043 \text{ ksi}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 19.18 \text{ ksi}$$ $$\sigma_{max} = 19.18 \text{ ksi} \blacktriangleleft$$

(*c*) Since $\sigma_{max} > \sigma_{all} (= 18 \text{ ksi})$, W10 × 39 is not acceptable. ◀

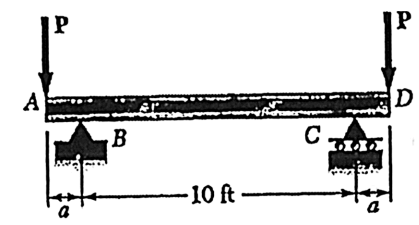
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

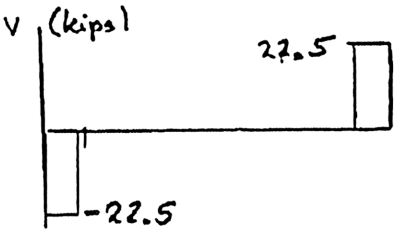
PROBLEM 8.2

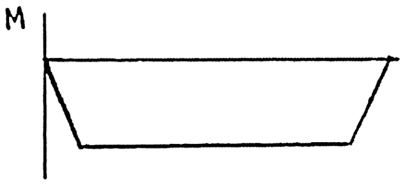
Solve Prob. 8.1, assuming that $P = 22.5$ kips and $a = 20$ in.

PROBLEM 8.1 A W10×39 rolled-steel beam supports a load **P** as shown. Knowing that $P = 45$ kips, $a = 10$ in., and $\sigma_{all} = 18$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION







$$|V|_{max} = 22.5 \text{ kips}$$
$$|M|_{max} = (22.5)(20) = 450 \text{ kip} \cdot \text{in}$$

For W10×39 rolled steel section,

$$d = 9.92 \text{ in.}, \quad b_f = 7.99 \text{ in.}, \quad t_f = 0.530 \text{ in.},$$
$$t_w = 0.315 \text{ in.}, \quad I_x = 209 \text{ in}^4, \quad S_x = 42.1 \text{ in}^3$$
$$c = \frac{1}{2}d = 4.96 \text{ in.} \quad y_b = c - t_f = 4.43 \text{ in.}$$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450}{42.1} \qquad \sigma_m = 10.69 \text{ ksi} \blacktriangleleft$$

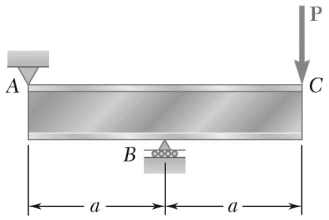
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.43}{4.96}\right)(10.69) = 9.55 \text{ ksi}$$
$$A_f = b_f t_f = 4.2347 \text{ in}^2$$
$$\bar{y}_f = \frac{1}{2}(c + y_b) = 4.695 \text{ in.}$$
$$Q_b = A_f \bar{y}_f = 19.8819 \text{ in}^3$$
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(22.5)(19.8819)}{(209)(0.315)} = 6.7949 \text{ ksi}$$
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 8.3049 \text{ ksi}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 13.08 \text{ ksi} \qquad \sigma_{max} = 13.08 \text{ ksi} \blacktriangleleft$$

(*c*) Since $\sigma_{max} < \sigma_{all} (= 18 \text{ ksi})$, W10×39 is acceptable. ◀

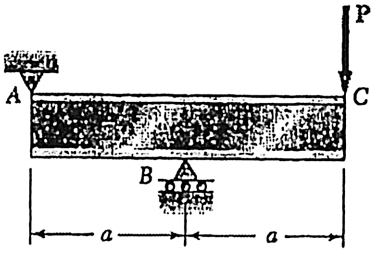
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.3



An overhanging W920×449 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{max} = 700 \text{ kN} = 700\times10^3 \text{ N}$$

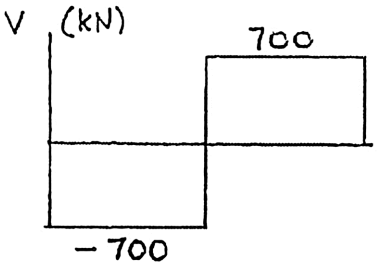
$$|M|_{max} = (700\times10^3)(2.5) = 1.75\times10^6 \text{ N}\cdot\text{m}$$

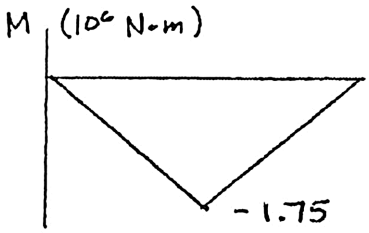
For W920×449 rolled steel beam,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780\times10^6 \text{ mm}^4, \quad S_x = 18{,}500\times10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 473.5 \text{ mm}, \quad y_b = c - t_f = 430.8 \text{ mm}$$





(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.75\times10^6}{18500\times10^{-6}}$$

$$\sigma_m = 94.595 \text{ MPa}$$

$$\sigma_m = 94.6 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{430.8}{473.5}(94.595) = 86.064 \text{ MPa}$$

$$A_f = b_f t_t = 18.1048\times10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1\times10^3 \text{ mm}^3 = 8186.1\times10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(700\times10^3)(8186.1\times10^{-6})}{(8780\times10^{-6})(24.0\times10^{-3})} = 27.194 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{86.064}{2}\right)^2 + 27.194^2} = 50.904 \text{ MPa}$$

(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 93.9 \text{ MPa}$$

$$\sigma_{max} = 93.9 \text{ MPa} \blacktriangleleft$$

(*c*) Since 94.6 MPa < σ_{all} (= 100 MPa), W920×449 is acceptable. ◀

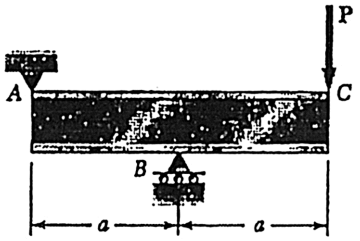
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.4

Solve Prob. 8.3, assuming that $P = 850$ kN and $a = 2.0$ m.

PROBLEM 8.3 An overhanging W920×449 rolled-steel beam supports a load **P** as shown. Knowing that $P = 700$ kN, $a = 2.5$ m, and $\sigma_{all} = 100$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

SOLUTION



$$|V|_{max} = 850 \text{ kN} = 850 \times 10^3 \text{ N}$$

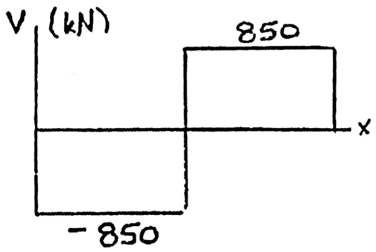
$$|M|_{max} = (850 \times 10^3)(2.0) = 1.70 \times 10^6 \text{ N} \cdot \text{m}$$

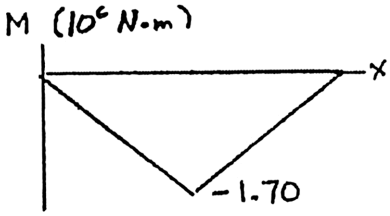
For W920×449 rolled steel section,

$$d = 947 \text{ mm}, \quad b_f = 424 \text{ mm}, \quad t_f = 42.7 \text{ mm},$$

$$t_w = 24.0 \text{ mm}, \quad I_x = 8780 \times 10^6 \text{ mm}^4, \quad S_x = 18500 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2} d = 473.5 \text{ mm} \qquad y_b = c - t_f = 430.8 \text{ mm}$$





(*a*) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.70 \times 10^6}{18,500 \times 10^{-6}}$$

$$\sigma_m = 91.892 \text{ MPa}$$

$$\sigma_m = 91.9 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{430.8}{473.5}(91.892) = 83.605 \text{ MPa}$$

$$A_f = b_f t_f = 18.1048 \times 10^3 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 452.15 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 8186.1 \times 10^3 \text{ mm}^3 = 8186.1 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(850 \times 10^3)(8186.1 \times 10^{-6})}{(8780 \times 10^{-6})(24.0 \times 10^{-3})} = 33.021 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{83.605}{2}\right)^2 + 33.021^2} = 53.271 \text{ MPa}$$

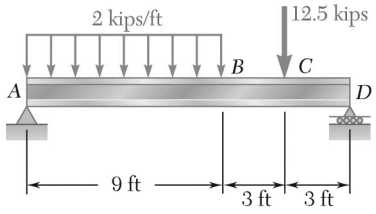
(*b*) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 95.1 \text{ MPa}$$

$$\sigma_{max} = 95.1 \text{ MPa} \quad \blacktriangleleft$$

(*c*) Since 95.1 MPa < σ_{all} (= 100 MPa), W920×449 is acceptable. ◀

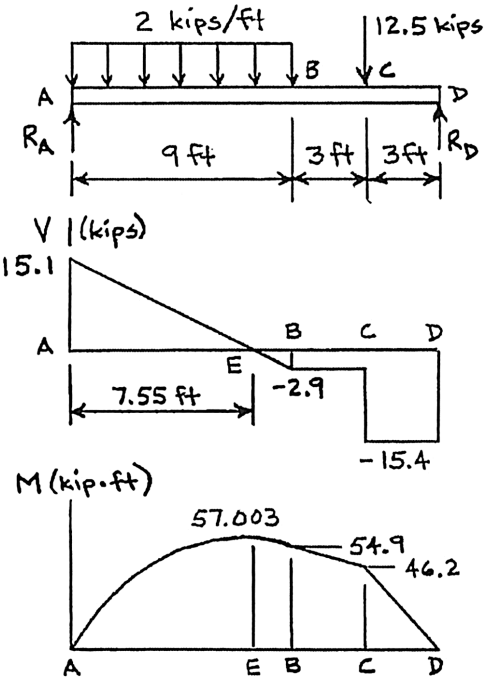
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.5



(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$+\circlearrowleft \Sigma M_D = 0: \; -15R_A + (10.5)(9)(2) + (3)(12.5) = 0$$

$$R_A = 15.1 \text{ kips} \uparrow \qquad R_D = 15.4 \text{ kips} \uparrow$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{(57.003 \times 12 \text{ kip} \cdot \text{in})}{24 \text{ ksi}} = 28.502 \text{ in}^3$$

	Shape	S (in^3)
	W16×26	38.4
→	W14×22	29.0
	W12×26	33.4
	W10×30	32.4
	W8×35	31.2

(*a*) Use W14 × 22 ◀

For W14 × 22, $A_{web} = dt_w = (13.7)(0.230) = 3.151$ in^2

Point *E*: $\sigma_m = \dfrac{M_E}{S} = \dfrac{(57.003)(12)}{29.0}$ (*b*) $\sigma_m = 23.6$ ksi ◀

Point *C*: $\sigma_m = \dfrac{M_C}{S} = \dfrac{(46.2)(12)}{29.0} = 19.1172$ ksi $\sigma_m = 19.12$ ksi ◀

$\tau_m = \dfrac{|V|}{A_{web}} = \dfrac{15.4}{3.151} = 4.8873$ ksi $\tau_m = 4.89$ ksi ◀

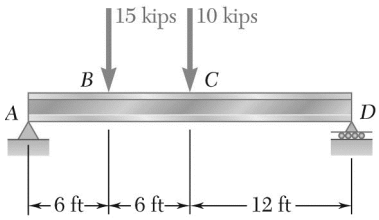
$$c = \frac{1}{2}d = 6.85 \text{ in.} \qquad y_b = c - t_f = 6.515 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{6.515}{6.85}\right)(19.1172) = 18.1823 \text{ ksi} \qquad R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = 10.3216 \text{ ksi}$$

$\sigma_{max} = \dfrac{\sigma_b}{2} + R$ $\sigma_{max} = 19.41$ ksi ◀

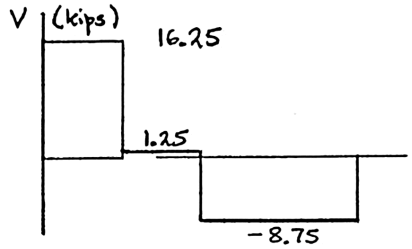
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

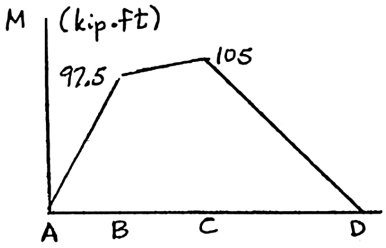
PROBLEM 8.6



(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for $\sigma_m, \tau_m,$ and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION





$$+\circlearrowleft \Sigma M_D = 0: \quad -24R_A + (18)(15) + (12)(10) = 0$$

$$R_A = 16.25 \text{ kips} \uparrow$$

$$|V|_{max} = 16.25 \text{ kips}$$

$$|M|_{max} = 105 \text{ kip} \cdot \text{ft} = 1260 \text{ kip} \cdot \text{in.}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1260}{24} = 52.5 \text{ in}^3$$

Shape	S_x (in^3)
W 21×44	81.6
W18×35	57.6
W16×40	64.7
W14×38	54.6
W12×50	64.7
W10×54	60.0

(*a*) Use W18×35. ◀

$$d = 17.70 \text{ in.}$$

$$t_f = 0.425 \text{ in.}$$

$$t_w = 0.300 \text{ in.}$$

(*b*) For W18×35, $S = 57.6$ in^3

$$c = \frac{1}{2}d = 8.85 \text{ in.}$$

$$y_b = c - t_f = 8.425 \text{ in.}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.6 *(Continued)*

At C: $M = (105)(12) = 1260 \text{ kip} \cdot \text{in}$

$$\sigma_m = \frac{M}{S} = \frac{1260}{57.6} \qquad \sigma_m = 21.9 \text{ ksi} \blacktriangleleft$$

$$|V| = 8.75 \text{ kips} \quad \tau_m = \frac{V}{d\,t_w} = \frac{8.75}{(17.70)(0.300)} \qquad \tau_m = 1.65 \text{ ksi}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{8.425}{8.85}\right)(21.875) = 20.82 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{19.33}{2}\right)^2 + 1.65^2} = 10.54 \text{ ksi}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 21.0 \text{ ksi} \blacktriangleleft$$

At B: $M = (97.5)(12) = 1170 \text{ kip} \cdot \text{in}$ $\qquad \sigma_m = \dfrac{1170}{57.6} \qquad \sigma_m = 20.3 \text{ ksi}$

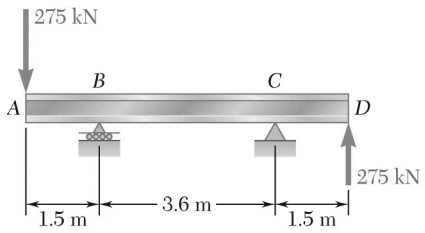
$$|V| = 16.25 \text{ kips} \quad \tau_m = \frac{V}{dt_w} = \frac{16.25}{(17.70)(0.300)} \qquad \tau_m = 3.06 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{8.425}{8.85}\right)(20.31) = 19.34 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{19.34}{2}\right)^2 + 3.06^2} = 10.14 \text{ ksi}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 19.81 \text{ ksi} \quad \sigma_{\max} = 19.81 \text{ ksi} \blacktriangleleft$$

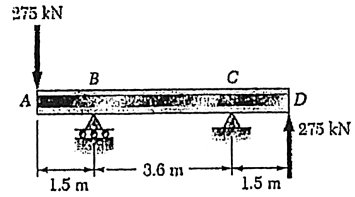
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.7

(*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION

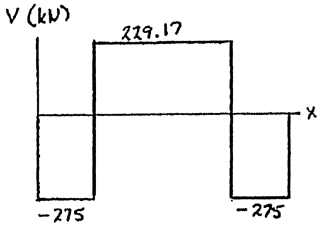


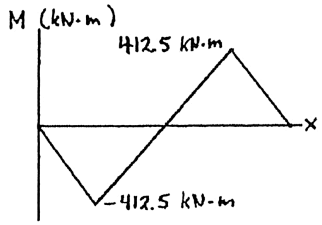
$R_B = 504.17$ kN ↑ $\quad R_C = 504.17$ ↓

$|V|_{max} = 275$ kN $\quad |M|_{max} = 412.5$ kN · m

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5 \times 10^2}{160 \times 10^6} = 2578 \times 10^{-6} \text{ m}^3$$

$$= 2578 \times 10^3 \text{ mm}^3$$





→

Shape	$S_x(10^3 \text{ mm}^3)$
W760×147	4410
W690×125	3490
W530×150	3720
W460×158	3340
W360×216	3800

(*a*) Use W690 × 125. ◀

$d = 678$ mm $\quad t_f = 16.3$ mm $\quad t_w = 11.7$ mm

(*b*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5 \times 10^3}{3490 \times 10^{-6}} = 118.195 \times 10^6 \text{ Pa}$$ $\sigma_m = 118.2$ MPa ◀

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{dt_w} = \frac{275 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 34.667 \times 10^6 \text{ Pa}$$ $\tau_m = 34.7$ MPa ◀

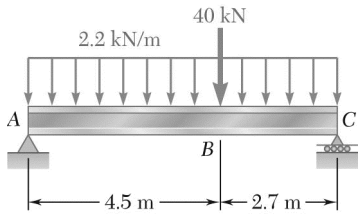
$$c = \frac{1}{2}d = \frac{67.8}{2} = 339 \text{ mm}, \quad t_f = 16.3 \text{ mm}, \quad y_b = c - t_f = 339 - 16.3 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(118.195) = 112.512 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(56.256)^2 + (34.667)^2} = 66.080 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 56.256 + 66.080$$ $\sigma_{max} = 122.3$ MPa ◀

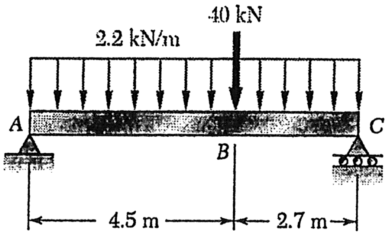
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

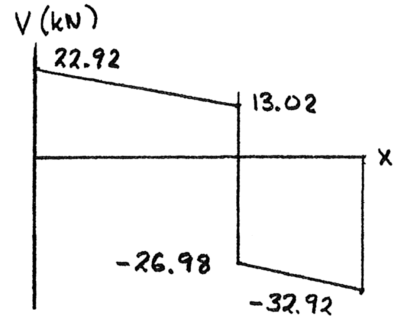


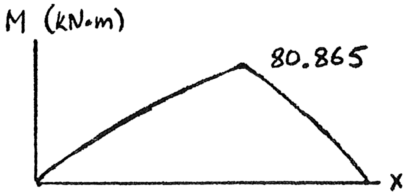
PROBLEM 8.8

(*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION







$+\sum M_C = 0$:

$-7.2\,R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$

$R_A = 22.92$ kN

$V_A = R_A = 22.92$ kN

$V_B^- = 22.92 - (2.2)(4.5) = 13.02$ kN

$V_B^+ = 13.02 - 40 = -26.98$ kN

$V_C = -26.98 - (2.2)(2.7) = -32.92$ kN

$M_A = 0$

$M_B = 0 + \frac{1}{2}(22.92 + 13.02)(4.5) = 80.865$ kN · m

$M_C = 0$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{165 \times 10^6} = 490 \times 10^{-6}\ \text{m}^3$$

$$= 490 \times 10^3\ \text{mm}^3$$

Shape	$S\ (10^3\ \text{mm}^3)$	
W360 × 39	578	
W310 × 38.7	547	←
W250 × 44.8	531	
W200 × 52	511	

(*a*) Use W310 × 38.7. ◀

$d = 310$ mm $\quad t_f = 9.65$ mm

$t_w = 5.84$ mm

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.8 *(Continued)*

(*b*) $$\sigma_m = \frac{M_B}{S} = \frac{80.865 \times 10^3}{547 \times 10^{-6}} = 147.834 \times 10^6 \text{ Pa}$$

$$\sigma_m = 147.8 \text{ MPa} \blacktriangleleft$$

$$\tau_m = \frac{|V|_{\max}}{dt_w} = \frac{32.92 \times 10^3}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 18.1838 \times 10^6 \text{ Pa}$$

$$\tau_m = 18.18 \text{ MPa} \blacktriangleleft$$

$$c = \frac{1}{2}d = 155 \text{ mm} \qquad y_b = c - t_f = 155 - 9.65 = 145.35 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{145.35}{155}\right)(147.834) = 138.630 \text{ MPa}$$

At point *B*, $$\tau_w = \frac{V}{dt_w} = \frac{(26.98 \times 10^3)}{(310 \times 10^{-3})(5.84 \times 10^{-3})} = 14.9028 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.315)^2 + (14.9028)^2} = 70.899 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 69.315 + 70.899$$

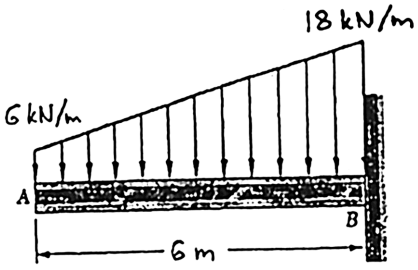
$$\sigma_{\max} = 140.2 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.9

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.73 and selected W530 × 66 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

SOLUTION



From Prob. 5.73, $\sigma_{\text{all}} = 160$ MPa

$|M|_{\max} = 180 \text{ kN} \cdot \text{m}$ at section *B*.

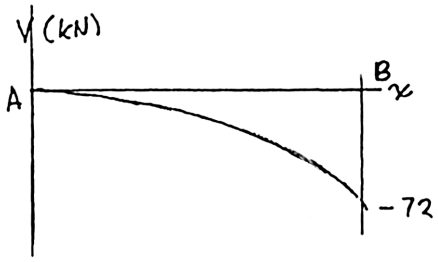
$|V| = 72$ kN at section *B*.

For W530×66 rolled steel section,

$d = 526$ mm, $b_f = 165$ mm, $t_f = 11.4$ mm,

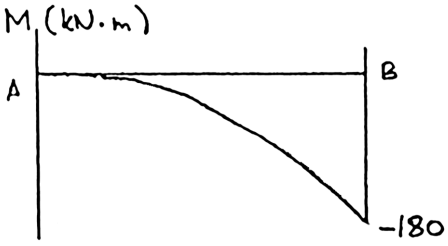
$t_w = 8.89$ mm, $I = 351 \times 10^6 \text{ mm}^4$, $S = 1340 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 263$ mm



(*a*) $|M|_{\max} = 180 \times 10^3 \text{ N} \cdot \text{m}$ $S = 1340 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S} = 134.328 \times 10^6, \qquad \sigma_m = 134.3 \text{ MPa} \blacktriangleleft$$



(*b*) $y_b = c - t_f = 251.6$ mm $\quad \bar{y} = \frac{1}{2}(c + y_b) = 257.3$ mm

$A_f = b_f t_f = 1881 \text{ mm}^2$

At section *B*, $V = 72 \times 10^3$ N

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It} = \frac{(72 \times 10^3)(1881 \times 10^{-6})(257.3 \times 10^{-3})}{(351 \times 10^{-6})(8.89 \times 10^{-3})}$$

$$= 11.1674 \times 10^6 = 11.1674 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{257.3}{263}(134.328) = 131.417 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 66.651 \text{ MPa}$$

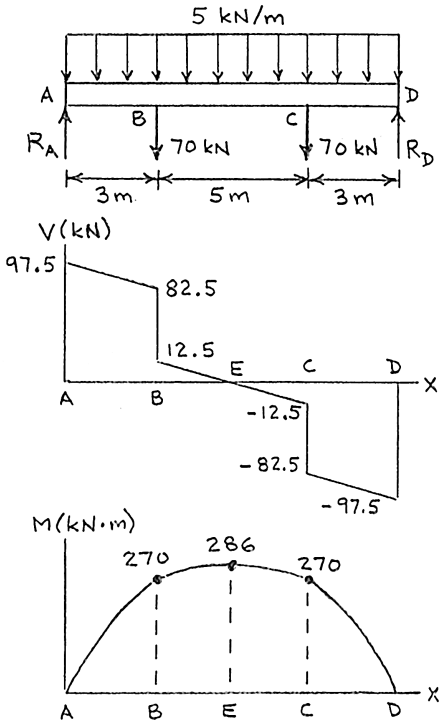
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = \frac{131.417}{2} + 66.651 \qquad \sigma_{\max} = 132.4 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.10

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.74 and selected W530 × 92 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



Reactions: $R_A = 97.5 \text{ kN} \uparrow \quad R_D = 97.5 \text{ kN} \uparrow$

$|V|_{max} = 97.5 \text{ kN}$

$|M|_{max} = 286 \text{ kN} \cdot \text{m}$

For W530 × 92 rolled steel section,

$d = 533 \text{ mm}, \quad b_f = 209 \text{ mm}, \quad t_f = 15.6 \text{ mm},$

$t_w = 10.2 \text{ mm}, \quad c = \frac{1}{2}d = 266.5 \text{ mm}$

$I = 554 \times 10^6 \text{ mm}^4 \quad S = 2080 \times 10^3 \text{ mm}^3$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{286 \times 10^3}{2080 \times 10^{-6}} = 137.5 \times 10^6 \text{ Pa}$$

$$\sigma_m = 137.5 \text{ MPa} \blacktriangleleft$$

$y_b = c - t_f = 250.9 \text{ mm}$

$A_f = b_f t_f = 3260.4 \text{ mm}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 258.7 \text{ mm}$

$Q = A_f \bar{y} = 843.47 \times 10^3 \text{mm}^3$

At midspan: $V = 0 \quad \tau_b = 0$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(137.5) = 129.5 \text{ MPa}$$

$$\sigma_{max} = 129.5 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.10 ***(Continued)***

At sections *B* and *C*:

$$\sigma_m = \frac{M}{S} = \frac{270 \times 10^3}{2080 \times 10^{-6}} = 129.808 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{250.9}{266.5}(129.808) = 122.209 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(82.5 \times 10^3)(3260.4 \times 10^{-6})(258.7 \times 10^{-3})}{(554 \times 10^{-6})(10.2 \times 10^{-3})}$$

$$= 12.3143 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 62.333 \text{ MPa}$$

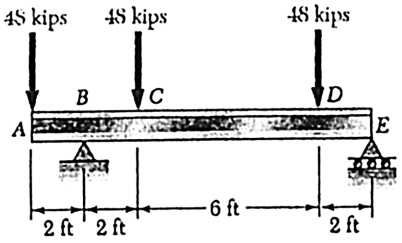
$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 123.4 \text{ MPa} \blacktriangleleft$$

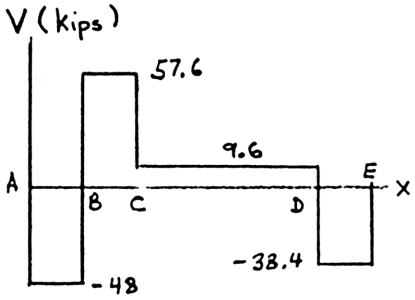
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

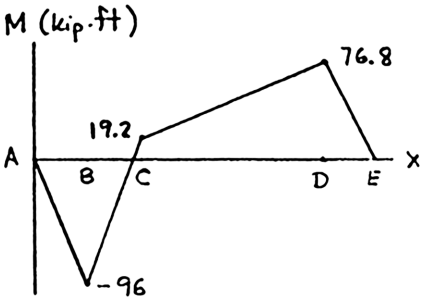
PROBLEM 8.11

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.77 and selected S15 × 42.9 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION







From Prob. 5.77, $\sigma_{all} = 24$ ksi

$|M|_{max} = 96 \text{ kip} \cdot \text{ft} = 1152 \text{ kip} \cdot \text{in}$ at D.

At D, $|V| = 38.4$ kips

For S15×42.9 shape,

$d = 15.0$ in., $b_f = 5.50$ in. $t_f = 0.622$ in.,

$t_w = 0.411$ in., $I_z = 446 \text{ in}^4$, $S_z = 59.4 \text{ in}^3$

$$c = \frac{1}{2}d = 7.5 \text{ in.}$$

$$\sigma_m = \frac{|M|}{S} = \frac{1152}{59.4} = 19.3939 \text{ ksi} \qquad (a) \quad \sigma_m = 19.39 \text{ ksi} \blacktriangleleft$$

$$y_b = c - t_f = 6.878 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = 17.7855 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.8928 \text{ ksi}$$

$$A_f = b_f t_f = 3.421 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 7.189 \text{ in.}$$

$$Q = A_f \bar{y} = 24.594 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(57.6)(24.594)}{(446)(0.411)} = 7.7281 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.8928^2 + 7.7281^2} = 11.7816 \text{ ksi}$$

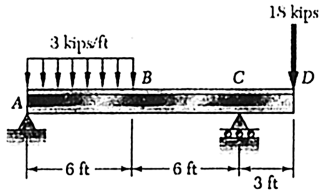
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.8928 + 11.7816 \qquad (b) \quad \sigma_{max} = 20.7 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.12

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{\text{all}}$. For the selected design (use the loading of Prob. 5.78 and selected S12 × 31.8 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

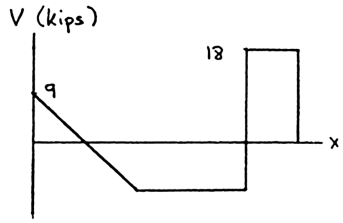
SOLUTION



From Prob. 5.78, $\sigma_{\text{all}} = 24$ ksi

$|M|_{\max} = 54 \text{ kip} \cdot \text{ft} = 648 \text{ kip} \cdot \text{in}$ at C.

At C $\quad |V| = 18$ kips



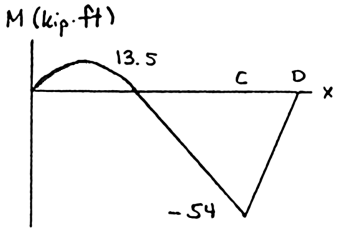
For S12×31.8 rolled steel shape,

$d = 12.0$ in., $\quad b_f = 5.00$ in., $\quad t_f = 0.544$ in.,

$t_w = 0.350$ in. $\quad I_z = 217 \text{ in}^4$, $\quad S_z = 36.2 \text{ in}^3$

$$c = \frac{1}{2} d = 6.00 \text{ in.}$$

$$\sigma_m = \frac{|M|}{S_z} = \frac{648}{36.2} = 17.9006 \text{ ksi}$$ (*a*) $\sigma_m = 17.90$ ksi ◀



$$y_b = c - t_f = 5.456 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 16.2776 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.1388 \text{ ksi}$$

$$A_f = b_f t_f = 2.72 \text{ in}^2$$

$$\overline{y} = \frac{1}{2}(c + y_b) = 5.728 \text{ in.}$$

$$Q = A_f \overline{y} = 15.5802 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(15.5802)}{(217)(0.350)} = 3.6925 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.1388^2 + 3.6925^2} = 8.9373 \text{ ksi}$$

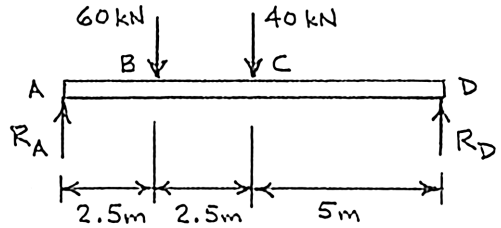
$$\sigma_{\max} = \frac{\sigma_b}{2} + R = 8.1388 + 8.9373$$ (*b*) $\sigma_{\max} = 17.08$ ksi ◀

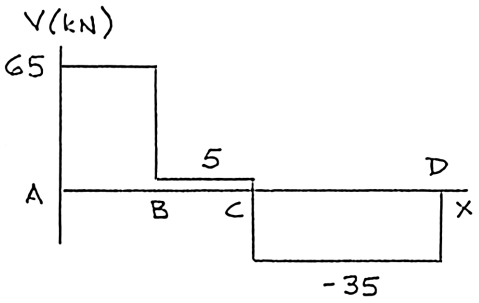
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

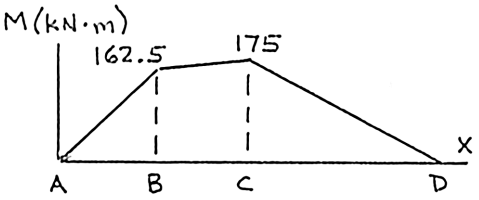
PROBLEM 8.13

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.75 and selected S460 × 81.4 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION







Reactions: $R_A = 65 \text{ kN} \uparrow \qquad R_D = 35 \text{ kN} \uparrow$

$|V|_{max} = 65 \text{ kN}$

$|M|_{max} = 175 \text{ kN} \cdot \text{m}$

For S460×81.4 rolled steel section,

$d = 457 \text{ mm}, \qquad b_f = 152 \text{ mm}, \qquad t_f = 17.6 \text{ mm},$

$t_w = 11.7 \text{ mm}, \qquad c = \frac{1}{2} d = 228.5 \text{ mm}$

$I = 333 \times 10^6 \text{ mm}^4 \qquad S = 1460 \times 10^3 \text{ mm}^3$

(*a*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{175 \times 10^3}{1460 \times 10^{-6}} = 119.863 \times 10^6 \text{ Pa}$$

$$\sigma_m = 119.9 \text{ MPa} \blacktriangleleft$$

(*b*) $$y_b = c - t_f = 210.9 \text{ mm}$$

$$A_f = b_f t_f = 2675.2 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 219.7 \text{ mm}$$

$$Q = A_f \bar{y} = 587.74 \times 10^3 \text{ mm}^3$$

At section *C*: $$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{210.9}{228.5}(119.863) = 110.631 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f \bar{y}}{It_w} = \frac{(35 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 5.2799 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.567 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R \qquad \sigma_{max} = 110.9 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.13 ***(Continued)***

At section B:

$$\sigma_m = \frac{M}{S} = \frac{162.5 \times 10^3}{1460 \times 10^{-6}} = 111.301 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{210.9}{228.5}(111.301) = 102.728 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(65 \times 10^3)(2675.2 \times 10^{-6})(219.7 \times 10^{-3})}{(333 \times 10^{-6})(11.7 \times 10^{-3})} = 9.8055 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 52.292 \text{ MPa}$$

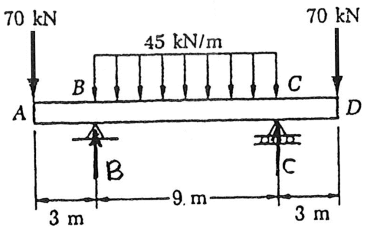
$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 103.7 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.14

The following problem refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design (use the loading of Prob. 5.76 and selected S510 × 98.2 shape), determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION



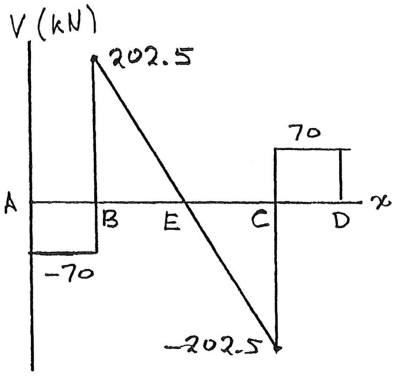
From Prob. 5.76, $\sigma_{all} = 160$ MPa

Bending moments:

-210 kN · m at sections *B* and *C*.

245.625 kN · m at midspan *E*.

$|V|_{max} = 202.5$ kN at sections *B* and *C*.



For S510×98.2 rolled steel section,

$d = 508$ mm, $b_f = 159$ mm, $t_f = 20.2$ mm,

$t_w = 12.8$ mm, $I = 495\times10^6$ mm^4, $S = 1950\times10^3$ mm^3

$c = \frac{1}{2}d = 254$ mm

(*a*) $|M|_{max} = 245.625\times10^3$ N · m $S = 1960\times10^{-6}$ m^3

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{245.625\times10^3}{1950\times10^{-6}} = 125.962\times10^6 \text{ Pa} \qquad \sigma_m = 126.0 \text{ MPa} \blacktriangleleft$$

(*b*) $y_b = c - t_f = 233.8$ mm, $A_f = b_f t_f = 3.212\times10^3$ mm^2, $\bar{y} = \frac{1}{2}(c + y_b) = 243.9$ mm

At midspan: $V = 0$ $\tau_b = 0$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{233.8}{254}(125.962) = 115.9 \text{ MPa} \qquad \sigma_{max} = 115.9 \text{ MPa} \blacktriangleleft$$

At sections *B* and *C*:

$$\sigma_m = \frac{M}{S} = \frac{210\times10^3}{1950\times10^{-6}} = 107.69 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{233.8}{254}(107.69) = 99.126 \text{ MPa}$$

$$\tau_b = \frac{VQ}{It} = \frac{VA_f\bar{y}}{It_w} = \frac{(202.5\times10^3)(3.212\times10^{-3})(243.9\times10^{-3})}{(495\times10^{-6})(12.8\times10^{-3})}$$

$$= 25.04 \text{ MPa}$$

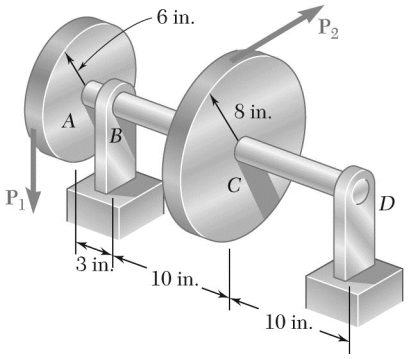
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.14 ***(Continued)***

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = 55.53 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R \qquad \sigma_{\max} = 105.1 \text{ MPa} \blacktriangleleft$$

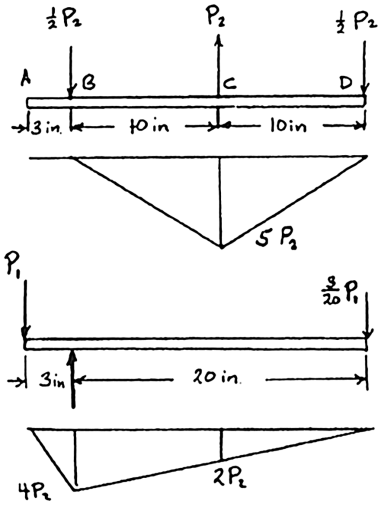
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.15

The vertical force $\mathbf{P_1}$ and the horizontal force $\mathbf{P_2}$ are applied as shown to disks welded to the solid shaft *AD*. Knowing that the diameter of the shaft is 1.75 in. and that $\tau_{all} = 8$ ksi, determine the largest permissible magnitude of the force $\mathbf{P_2}$.

SOLUTION



Let P_2 be in kips.

$$\sum M_{\text{shaft}} = 0: \qquad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$$

Torque over portion *ABC*: $T = 8P_2$

Bending in horizontal plane: $M_{Cy} = 10 \cdot \frac{1}{2}P_2 = 5P_2$

Bending in vertical plane:

$$M_{Bz} = 3P_1$$
$$= 3 \cdot \frac{4}{3}P_2$$
$$= 4P_1$$

Critical point is just to the left of point *C*.

$$T = 8P_2 \qquad M_y = 5P_2 \qquad M_z = 2P_2$$

$$d = 1.75 \text{ in.} \qquad c = \frac{1}{2}d = 0.875 \text{ in.}$$

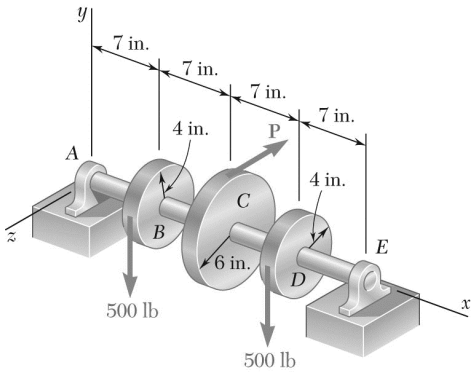
$$J = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{T^2 + M_y^2 + M_z^2}$$

$$8 = \frac{0.875}{0.92077}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 9.164\,P_2$$

$$P_2 = 0.873 \text{ kips} \qquad P_2 = 873 \text{ lb} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.16

The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE.

SOLUTION

$$\Sigma M_x = 0: \quad (4)(500) - 6P + (4)(500) = 0$$
$$P = 666.67 \text{ lb}$$

Torques:

$$AB: \quad T = 0$$
$$BC: \quad T = -(4)(500) = -2000 \text{ in} \cdot \text{lb}$$
$$CD: \quad T = 4(500) = 2000 \text{ in} \cdot \text{lb}$$
$$DE: \quad T = 0$$

Critical sections are either side of disk C.

$$T = 2000 \text{ in} \cdot \text{lb}$$
$$M_z = 3500 \text{ in} \cdot \text{lb}$$
$$M_y = 4667 \text{ in} \cdot \text{lb}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3$$

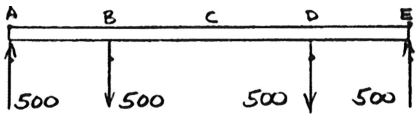
$$= \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}}$$

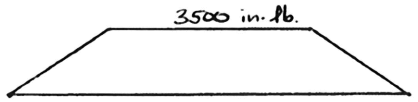
$$= \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3}$$

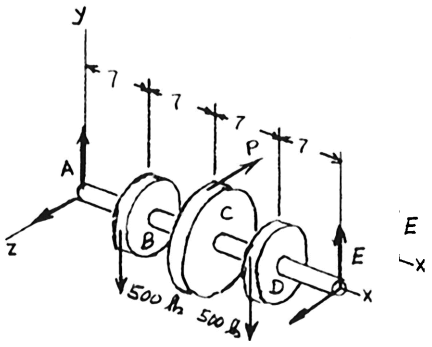
$$= 0.77083 \text{ in}^3$$

$$c = 0.7888 \text{ in.} \qquad d = 2c$$

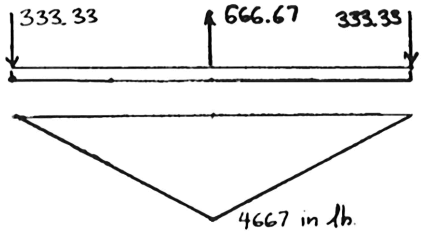
Forces in vertical plane:





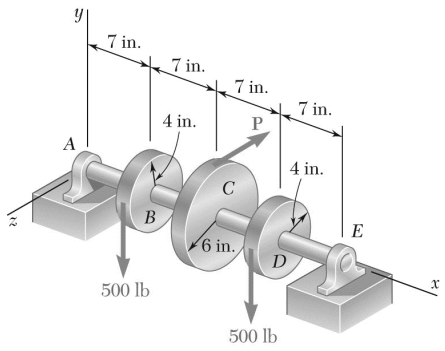


Forces in horizontal plane:



$$d = 1.578 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.17

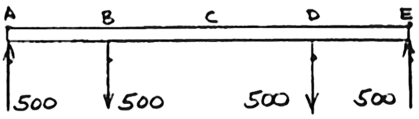
For the gear-and-shaft system and loading of Prob. 8.16, determine the smallest permissible diameter of shaft *AE*, knowing that the shaft is hollow and has an inner diameter that is $\frac{2}{3}$ the outer diameter.

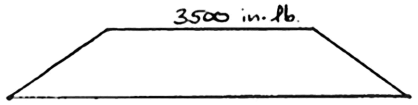
PROBLEM 8.16 The two 500-lb forces are vertical and the force **P** is parallel to the *z* axis. Knowing that $\tau_{\text{all}} = 8$ ksi, determine the smallest permissible diameter of the solid shaft *AE*.

SOLUTION

$$\Sigma M_x = 0: \quad (4)(500) - 6P + (4)(500) = 0$$
$$P = 666.67 \text{ lb}$$

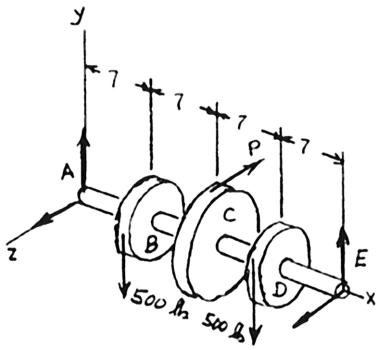
Forces in vertical plane:





Torques:

AB: $T = 0$
BC: $T = -(4)(500) = -2000 \text{ in} \cdot \text{lb}$
CD: $T = 4(500) = 2000 \text{ in} \cdot \text{lb}$
DE: $T = 0$



Critical sections are either side of disk *C*.

$$T = 2000 \text{ in} \cdot \text{lb}$$
$$M_z = 3500 \text{ in} \cdot \text{lb}$$
$$M_y = 4667 \text{ in} \cdot \text{lb}$$

$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2c}\left[c_o^4 - c_i^4\right] = \frac{\pi}{2c}\left[c^4 - \left(\frac{2}{3}c\right)^4\right]$$

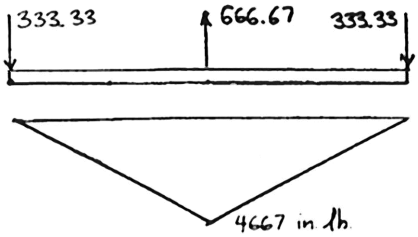
$$= \frac{\pi}{2c}\frac{65c^4}{81} = \frac{65\pi}{162}c^3$$

$$\frac{65\pi}{162}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\text{all}}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8\times10^3}$$

$$= 0.77083 \text{ in}^3$$

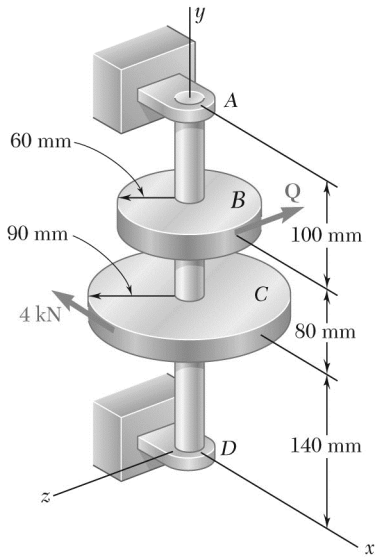
$$c = 0.8488 \text{ in.} \qquad d = 2c$$

Forces in horizontal plane:



$$d = 1.698 \text{ in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.18

The 4-kN force is parallel to the *x* axis, and the force **Q** is parallel to the *z* axis. The shaft *AD* is hollow. Knowing that the inner diameter is half the outer diameter and that $\tau_{all} = 60$ MPa, determine the smallest permissible outer diameter of the shaft.

SOLUTION

$$\Sigma M_y = 0: \quad 60\times10^{-3}\,\mathrm{Q} - (90\times10^{-3})(4\times10^{3}) = 0$$

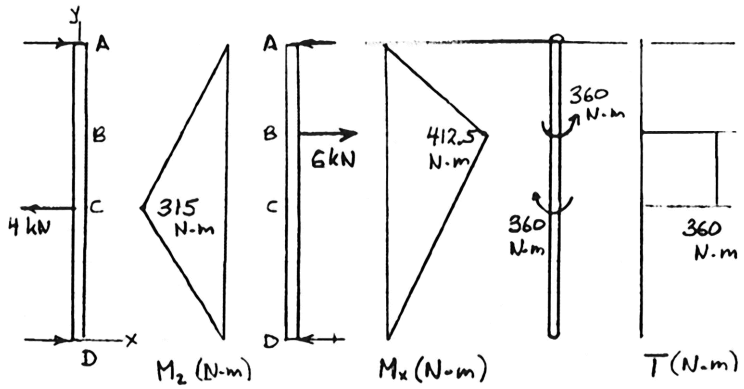
$$\mathrm{Q} = 6\times10^{3}\ \mathrm{N} = 6\ \mathrm{kN}$$

Bending moment and torque diagrams.

In *xy* plane: $(M_z)_{\max} = 315\ \mathrm{N\cdot m}$ at *C*.

In *yz* plane: $(M_x)_{\max} = 412.5\ \mathrm{N\cdot m}$ at *B*.

About *z*-axis: $T_{\max} = 360\ \mathrm{N\cdot m}$ between *B* and *C*.



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.18 *(Continued)*

At B:

$$M_z = \left(\frac{100}{180}\right)(315) = 175 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{175^2 + 412.5^2 + 360^2} = 574.79 \text{ N} \cdot \text{m}$$

At C:

$$M_x = \left(\frac{140}{220}\right)(412.5) = 262.5 \text{ N} \cdot \text{m}$$

$$\sqrt{M_x^2 + M_z^2 + T^2} = \sqrt{315^2 + 262.5^2 + 360^2} = 545.65 \text{ N} \cdot \text{m}$$

Largest value is $574.79 \text{ N} \cdot \text{m}$.

$$\tau_{\max} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}\, c}{J}$$

$$\frac{J}{c} = \frac{\max\sqrt{M_x^2 + M_z^2 + T^2}}{\tau_{\max}} = \frac{574.79}{60 \times 10^6}$$

$$= 9.5798 \times 10^{-6} \text{ m}^3 = 9.5798 \times 10^3 \text{ mm}^3$$

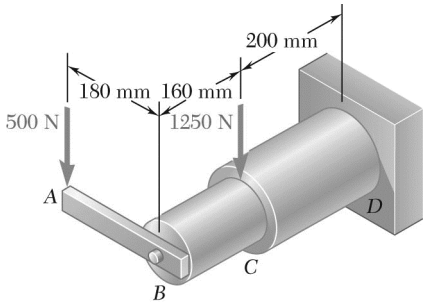
$$\frac{J}{c} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{\pi}{2} c_o^3 \left(1 - \frac{c_i^4}{c_o^4}\right) = \frac{\pi}{2} c_o^3 \left[1 - \left(\frac{1}{2}\right)^4\right]$$

$$= 1.47262 c_o^3$$

$$1.47262 c_o^3 = 9.5798 \times 10^3 \quad c_o = 18.67 \text{ mm}$$

$$d_o = 2c_o \qquad d_o = 37.3 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.19

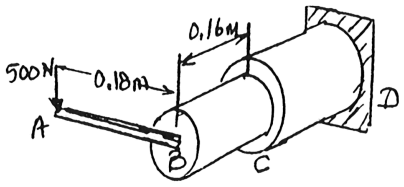
Neglecting the effect of fillets and of stress concentrations, determine the smallest permissible diameters of the solid rods *BC* and *CD*. Use $\tau_{all} = 60$ MPa.

SOLUTION

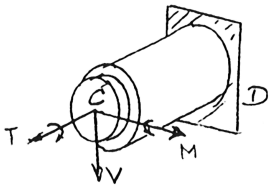
$$\tau_{all} = 60 \times 10^6 \text{ Pa}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} \qquad d = 2c$$



Bending moments and torques.

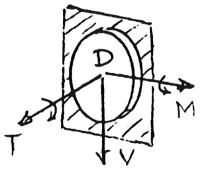


Just to the left of *C*:

$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$

$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$

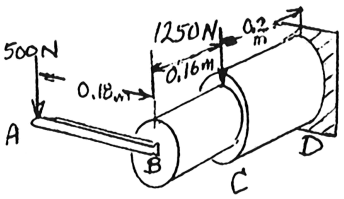


Just to the left of *D*:

$$T = 90 \text{ N} \cdot \text{m}$$

$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$



Smallest permissible diameter d_{BC}.

$$c^3 = \frac{(2)(120.416)}{\pi(60 \times 10^6)} = 1.27765 \times 10^{-6} \text{m}^3$$

$$c = 0.01085 \text{ m} = 10.85 \text{ mm}$$

$$d_{BC} = 21.7 \text{ mm} \blacktriangleleft$$

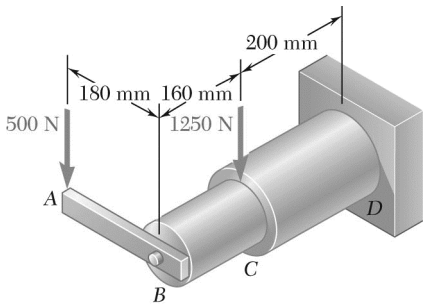
Smallest permissible diameter d_{CD}.

$$c^3 = \frac{(2)(439.32)}{\pi(60 \times 10^6)} = 4.6613 \times 10^{-6} \text{m}^3$$

$$c = 0.01670 \text{ m} = 16.7 \text{ mm}$$

$$d_{CD} = 33.4 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.20

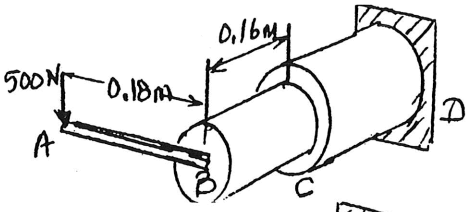
Knowing that rods *BC* and *CD* are of diameter 24 mm and 36 mm, respectively, determine the maximum shearing stress in each rod. Neglect the effect of fillets and of stress concentrations.

SOLUTION

Over *BC*: $c = \frac{1}{2}d = 12 \text{ mm} = 0.012 \text{ m}$

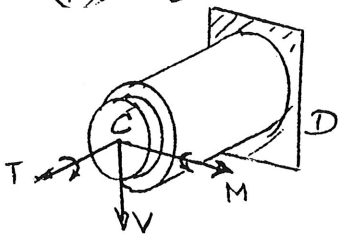
Over *CD*: $c = \frac{1}{2}d = 18 \text{ mm} = 0.018 \text{ m}$

$$\tau = \frac{\sqrt{M^2 + T^2}\,c}{J} \qquad \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{c^3}$$

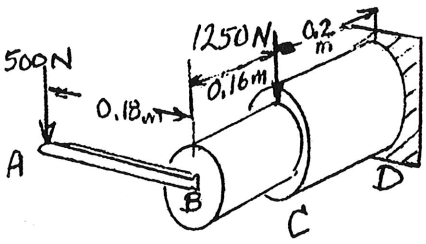


Bending moments and torques.

Just to the left of *C*:

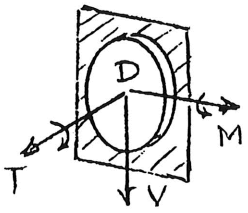


$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$
$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$
$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$



Just to the left of *D*:

$$T = 90 \text{ N} \cdot \text{m}$$
$$M = (500)(0.36) + (1250)(0.2)$$
$$= 430 \text{ N} \cdot \text{m}$$
$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$



Maximum shearing stress in portion *BC*.

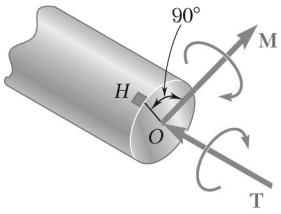
$$\tau_{\max} = \frac{(2)(120.416)}{\pi(0.012)^3} = 44.36 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 44.4 \text{ MPa} \blacktriangleleft$$

Maximum shearing stress in portion *CD*.

$$\tau_{\max} = \frac{(2)(439.32)}{\pi(0.018)^3} = 47.96 \times 10^6 \text{ Pa} \qquad \tau_{\max} = 48.0 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

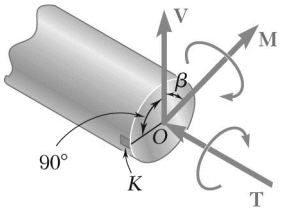
PROBLEM 8.21



(*a*)

It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems, their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point *H* (Fig. P8.21*a*) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$



(*b*)

Show that the maximum shearing stress at point *K* (Fig. P8.21*b*), where the effect of the shear *V* is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\frac{2}{3}cV + T\right)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear *V* cannot be ignored when $\tau_K \geq \tau_H$. (*Hint:* Only the component of **M** along **V** contributes to the shearing stress at *K*.)

SOLUTION

Shearing stress at point *K*.

Due to *V:* For a semicircle, $Q = \frac{2}{3}c^3$

For a circle cut across its diameter, $t = d = 2c$

For a circular section, $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)\left(\frac{2}{3}c^3\right)}{\left(\frac{1}{2}J\right)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

Due to *T*: $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation,

$$\tau_{xy} = \frac{c}{J}\left(\frac{2}{3}Vc + T\right)$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.21 *(Continued)*

Bending stress at point K: $\sigma_x = \dfrac{Mu}{I} = \dfrac{2Mu}{J}$

where u is the distance between point K and the neutral axis,

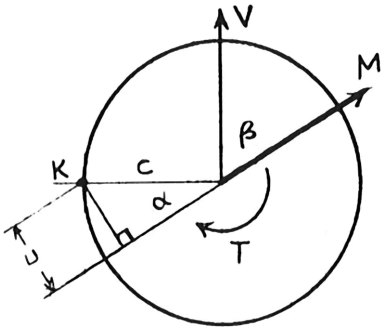
$$u = c \sin \alpha = c \sin\left(\frac{\pi}{2} - \beta\right) = c \cos \beta$$

$$\sigma_x = \frac{2Mc \cos \beta}{J}$$

Using Mohr's circle,

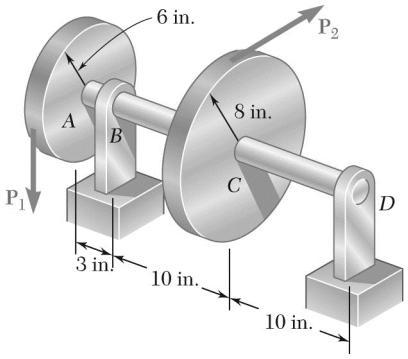
$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M \cos \beta)^2 + \left(\frac{2}{3}Vc + T\right)^2}$$



Cross section

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.22

Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.15 are, respectively, $P_1 = 1080$ lb and $P_2 = 810$ lb, and using the expressions given in Prob. 8.21, determine the values τ_H and τ_K in a section (*a*) just to the left of B, (*b*) just to the left of C.

SOLUTION

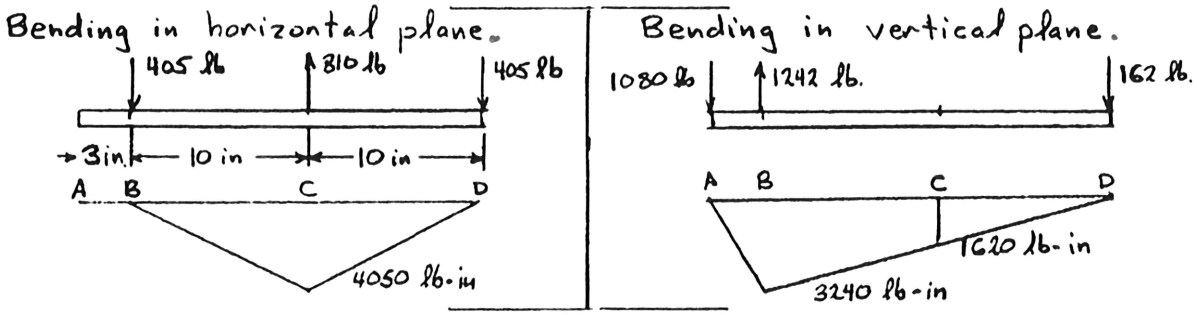
From Prob. 8.15, shaft diameter = 1.75 in.

$$c = \frac{1}{2}d = 0.875 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.92077$$

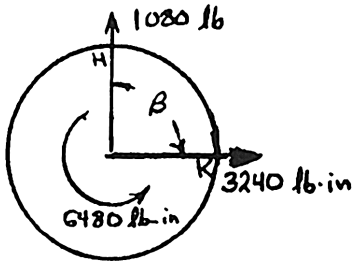
Torque over portion ABC:

$$T = (6)(1080) = (8)(810) = 6480 \text{ lb} \cdot \text{in}$$



(*a*) Just to the left of point B: $V = 1080$ lb $M = 3240$ lb · in

$\beta = 90°$ $T = 6480$ lb · in



$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{0.875}{0.92077}\sqrt{(3240)^2 + (6480)^2}$$

$$\tau_H = 6880 \text{ psi} \blacktriangleleft$$

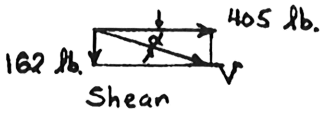
$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\tfrac{2}{3}V_C + T\right)^2} = \frac{c}{J}\left[\frac{2}{3}V_C + T\right]$$

$$= \frac{0.875}{0.92077}\left[\left(\frac{2}{3}\right)(1080)(0.875) + 6480\right] \qquad \tau_K = 6760 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

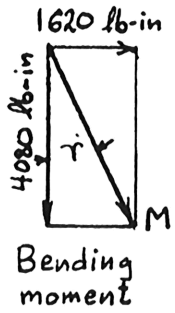
PROBLEM 8.22 *(Continued)*

(*b*) Just to the left of point *C*:

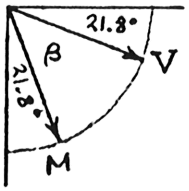


$$V = \sqrt{(162)^2 + (405)^2} = 436.2 \text{ lb}$$

$$\alpha = \tan^{-1}\frac{162}{405} = 21.80°$$



$$M = \sqrt{(1620)^2 + (4050)^2} = 4362 \text{ lb} \cdot \text{in}$$



$$\gamma = \tan^{-1}\frac{1620}{4050} = 21.80°$$

$$\beta = 90° - 21.8° - 21.8° = 46.4°$$

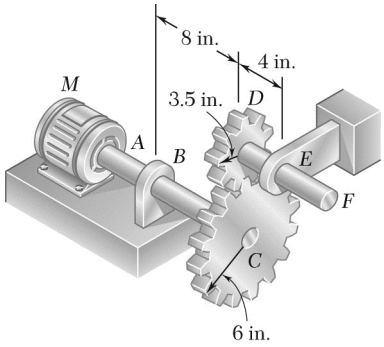
$$\tau_H = \frac{0.875}{0.92077}\sqrt{(6480)^2 + (4362)^2} \qquad \tau_H = 7420 \text{ psi} \blacktriangleleft$$

$$\frac{2}{3}V_C + T = \left(\frac{2}{3}\right)(436.2)(0.875) + 6480 = 6734 \text{ lb} \cdot \text{in}$$

$$M\cos\beta = 4362\cos 46.4° = 3008 \text{ lb} \cdot \text{in}$$

$$\tau_K = \frac{0.875}{0.92077}\sqrt{(3008)^2 + (6734)^2} \qquad \tau_K = 7010 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.23

The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in} \cdot \text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(*a*) Shaft *ABC*:

$$T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ lb} \cdot \text{in}$$

Gear *C*:

$$F_{CD} = \frac{T}{r_C} = \frac{5252}{6} = 875.4 \text{ lb}$$

Bending moment at *B*:

$$M_B = (8)(875.4) = 7003 \text{ lb} \cdot \text{in}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$$c = 0.9057 \text{ in.} \quad d = 2c \qquad d = 1.811 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *DEF*:

$$T = r_D F_{CD} = (3.5)(875.4) = 3064 \text{ lb} \cdot \text{in}$$

Bending moment at *E*:

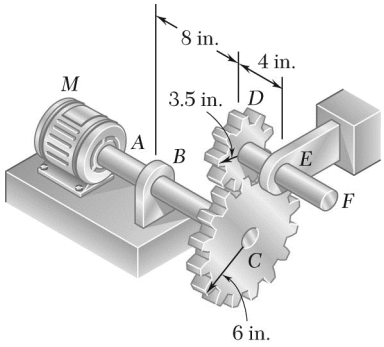
$$M_E = (4)(875.4) = 3502 \text{ lb} \cdot \text{in}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$$c = 0.7337 \text{ in.} \quad d = 2c \qquad d = 1.467 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.24

Solve Prob. 8.23, assuming that the motor rotates at 360 rpm.

PROBLEM 8.23 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in} \cdot \text{lb/s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(*a*) Shaft *ABC*:

$$T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ lb} \cdot \text{in}$$

Gear *C*:

$$F_{CD} = \frac{T}{r_C} = \frac{3501}{6} = 583.6 \text{ lb}$$

Bending moment at *B*:

$$M_B = (8)(583.6) = 4669 \text{ lb} \cdot \text{in}$$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$$c = 0.791 \text{ in.} \qquad d = 2c$$

$$d = 1.582 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *DEF*:

$$T = r_D F_{CD} = (3.5)(583.6) = 2043 \text{ lb} \cdot \text{in}$$

Bending moment at *E*:

$$M_E = (4)(583.6) = 2334 \text{ lb} \cdot \text{in}$$

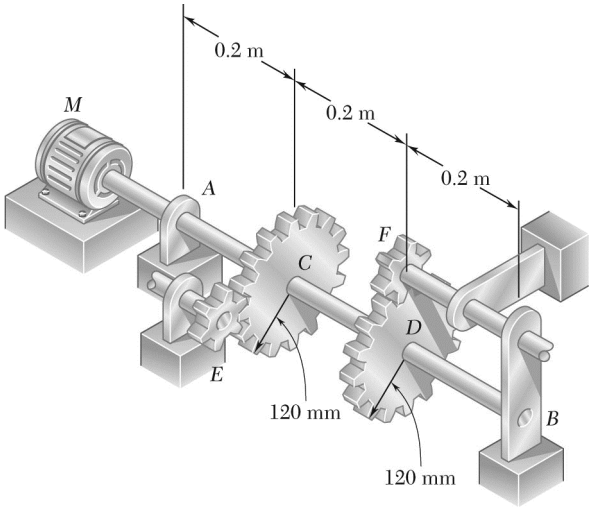
$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{2334^2 + 2043^3}}{7500} = 0.41362 \text{ in}^3$$

$$c = 0.6410 \text{ in.} \qquad d = 2c$$

$$d = 1.282 \text{ in.} \blacktriangleleft$$

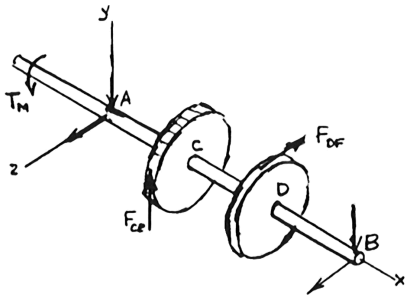
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

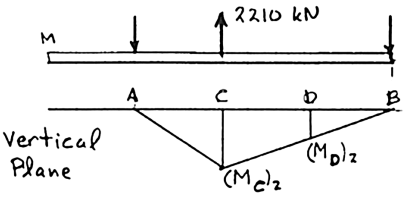


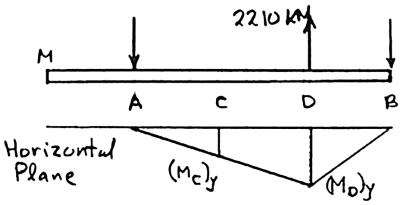
PROBLEM 8.25

The solid shaft AB rotates at 360 rpm and transmits 20 kW from the motor M to machine tools connected to gears E and F. Knowing that $\tau_{all} = 45$ MPa and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft AB.

SOLUTION







$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{P_M}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$T_D = \frac{P_D}{2\pi f} = \frac{10\times10^3}{2\pi(6)} = 265.26 \text{ N}\cdot\text{m}$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = \frac{265.26}{120\times10^{-3}} = 2.210\times10^3 \text{ N}$$

$$(M_C)_z = \frac{(0.2)(0.4)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

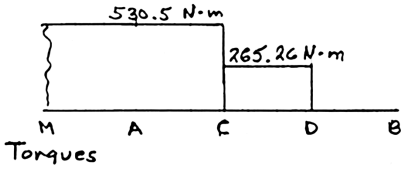
$$(M_D)_z = \frac{1}{2}(M_B)_z = 147.37 \text{ N}\cdot\text{m}$$

$$(M_C)_y = \frac{(0.4)(0.2)(2.210\times10^3)}{0.6} = 294.7 \text{ N}\cdot\text{m}$$

$$(M_D)_y = \frac{1}{2}(M_C)_y = 147.37 \text{ N}\cdot\text{m}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.25 *(Continued)*



Torques in shaft:

$$T_{MAC} = 530.5 \text{ N} \cdot \text{m}$$
$$T_{CD} = 265.26 \text{ N} \cdot \text{m}$$
$$T_{DB} = 0$$

Just to the left of gear *C*:

$$\max \sqrt{M_y^2 + M_z^2 + T^2} = 624.5 \text{ N} \cdot \text{m}$$

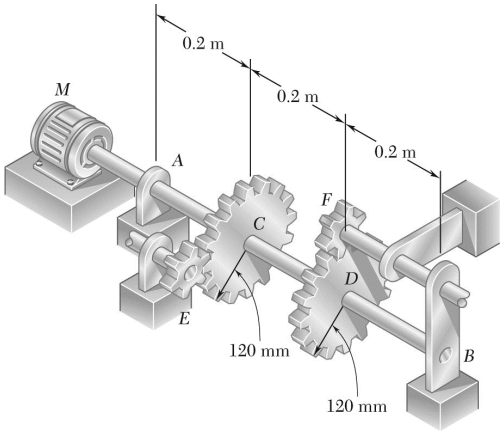
$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\max \sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\max}}$$

$$= \frac{624.5}{45 \times 10^6} = 13.878 \times 10^{-6} \text{ m}^3$$

$$c = 20.67 \times 10^{-3} \text{ m} = 20.67 \text{ mm}$$

$$d = 2c \qquad d = 41.3 \text{ mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

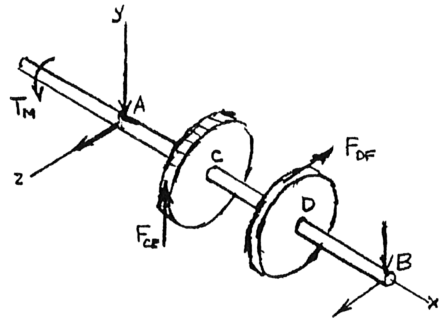


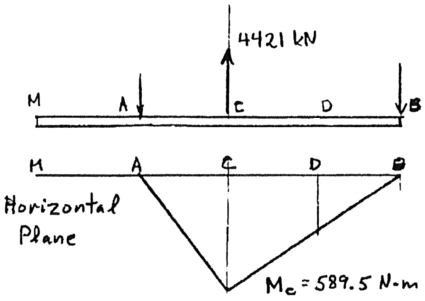
PROBLEM 8.26

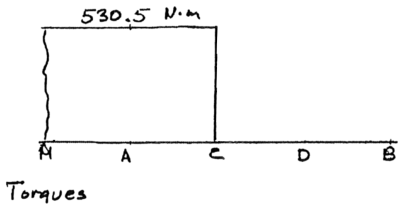
Solve Prob. 8.25, assuming that the entire 20 kW is taken off at gear *E*.

PROBLEM 8.25 The solid shaft *AB* rotates at 360 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *E* and *F*. Knowing that $\tau_{\text{all}} = 45$ MPa and assuming that 10 kW is taken off at each gear, determine the smallest permissible diameter of shaft *AB*.

SOLUTION







$$f = 360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

$$T_M = \frac{P_M}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{2\pi f} = \frac{20\times10^3}{2\pi(6)} = 530.5 \text{ N}\cdot\text{m}$$

$$T_D = \frac{P_E}{2\pi f} = 0$$

$$F_{CE} = \frac{T_C}{r_C} = \frac{530.5}{120\times10^{-3}} = 4.421\times10^3 \text{ N}$$

$$F_{DF} = \frac{T_D}{r_D} = 0$$

Since $F_{DF} = 0$, there is no bending in the horizontal plane.

$$M_C = \frac{(0.2)(0.4)(4.421\times10^3)}{0.6} = 589.5 \text{ N}\cdot\text{m}$$

Torques in shaft. $T_{MAC} = 530.5 \text{ N}\cdot\text{m}$

$T_{CDE} = 0$

Just to the left of gear *C*:

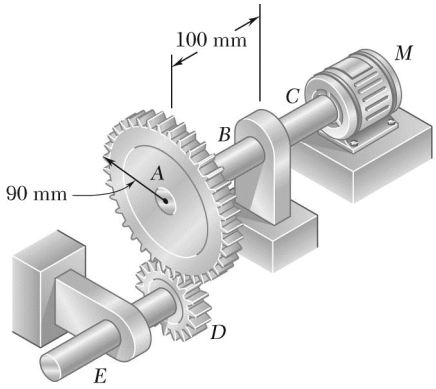
$$\max\sqrt{M_y^2 + M_z^2 + T^2} = \sqrt{0 + 589.5^2 + 530.5^2}$$

$$= 793.1 \text{ N}\cdot\text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\max\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{\max}} = \frac{793.1}{45\times10^6} = 17.624\times10^{-6} \text{ m}^3$$

$c = 22.39\times10^{-3} = 22.39$ mm $\quad d = 2c \quad\quad d = 44.8$ mm ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



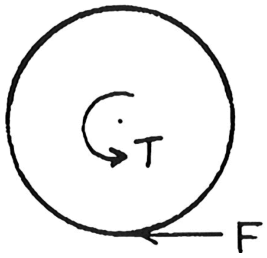
PROBLEM 8.27

The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/ min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N} \cdot \text{m}$$



Gear *A*: $Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at *B*:

$$M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N} \cdot \text{m}$$

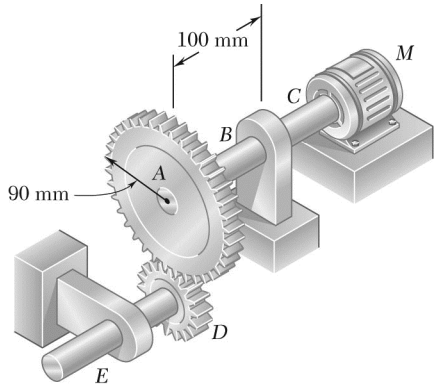
$$\tau_{\text{all}} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi(60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$$c = 18.479 \times 10^{-3} \text{ m} \qquad d = 2c = 37.0 \times 10^{-3} \text{ m} \qquad d = 37.0 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.28

Assuming that shaft *ABC* of Prob. 8.27 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.

PROBLEM 8.27 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that $\tau_{\text{all}} = 60$ MPa, determine the smallest permissible diameter of shaft *ABC*.

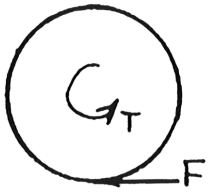
SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10\times10^{3}}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear *A*: $Fr_A - T = 0$

$$F = \frac{T}{r_A} = \frac{397.89}{90\times10^{-3}} = 4421 \text{ N}$$



Bending moment at *B*:

$$M_B = L_{AB}F = (100\times10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{c_o}{J}\sqrt{M^2+T^2} \qquad c_o = \frac{1}{2}d_o = 25\times10^{-3}\text{m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2+T^2}}{\tau_{\text{all}}}$$

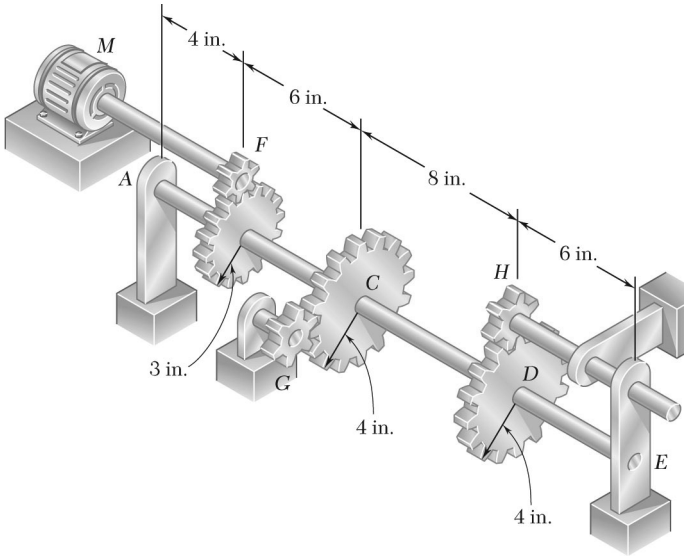
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2+T^2}}{\pi\tau_{\text{all}}}$$

$$= (25\times10^{-3})^4 - \frac{(2)(25\times10^{-3})\sqrt{442.1^2+397.89^2}}{\pi(60\times10^{6})}$$

$$= 390.625\times10^{-9} - 157.772\times10^{-9} = 232.85\times10^{-9}$$

$$c_i = 21.967\times10^{-3}\text{m} \qquad d_i = 2c_i = 43.93\times10^{-3}\text{m} \qquad d = 43.9 \text{ mm} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.29

The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ in} \cdot \text{lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

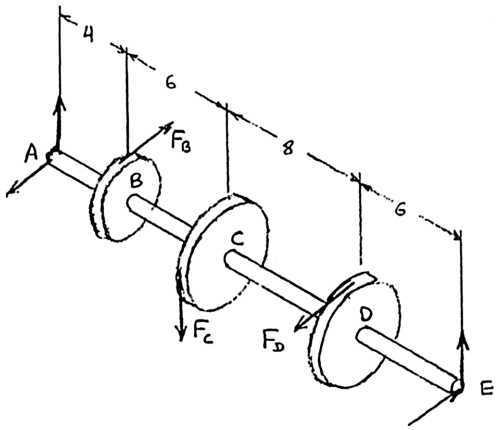
Torque on gear *B*:

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5 \text{ lb} \cdot \text{in}$$

Torques on gears *C* and *D*:

$$T_C = \frac{40}{60} T_B = 4201.7 \text{ lb} \cdot \text{in}$$

$$T_D = \frac{20}{60} T_B = 2100.8 \text{ lb} \cdot \text{in}$$



Shaft torques:

$$AB: \ T_{AB} = 0$$
$$BC: \ T_{BC} = 6302.5 \text{ lb} \cdot \text{in}$$
$$CD: \ T_{CD} = 2100.8 \text{ lb} \cdot \text{in}$$
$$DE: \ T_{DE} = 0$$

Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb}$$

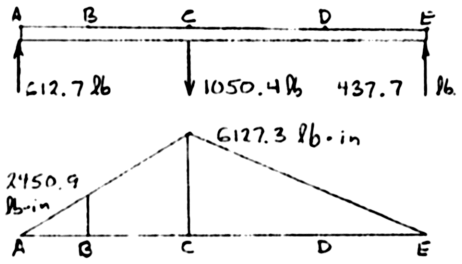
$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb}$$

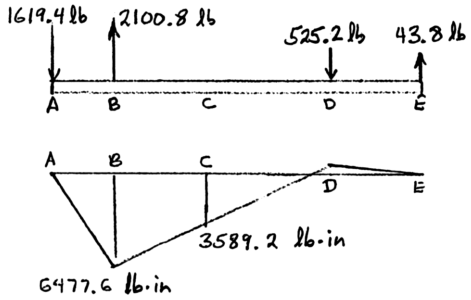
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.29 *(Continued)*

Forces in vertical plane:



Forces in horizontal plane:



$$\text{At } B^+,\ \sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2}$$

$$= 9364 \text{ lb} \cdot \text{in}$$

$$\text{At } C^-,\ \sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2}$$

$$= 9495 \text{ lb} \cdot \text{in} \quad \text{(maximum)}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\max}$$

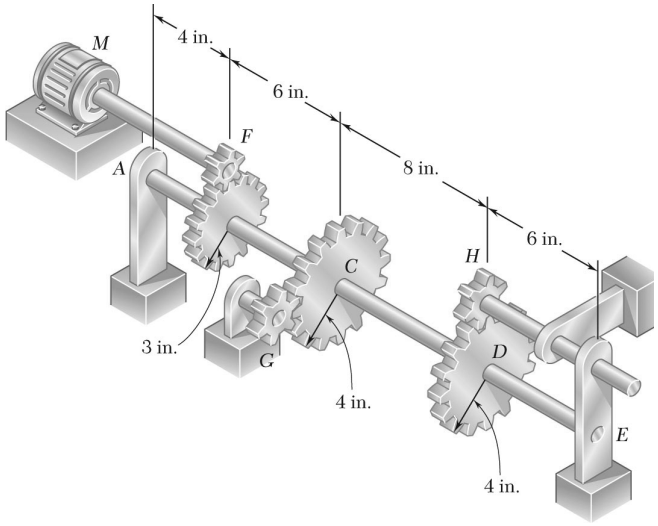
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}}$$

$$= \frac{9495}{8 \times 10^3} = 1.1868 \text{ in}^3$$

$$c = 0.911 \text{ in.} \qquad d = 2c$$

$$d = 1.822 \text{ in.} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.30

Solve Prob. 8.29, assuming that 30 hp is taken off at gear *G* and 30 hp is taken off at gear *H*.

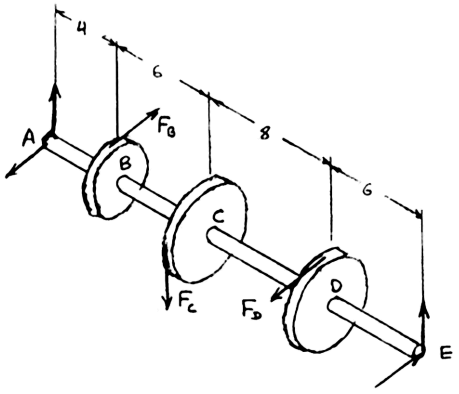
PROBLEM 8.29 The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that $\tau_{\text{all}} = 8$ ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.

SOLUTION

$$60 \text{ hp} = (60)(6600)$$
$$= 396\times10^3 \text{ in}\cdot\text{lb/sec}$$
$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear *B*:

$$T_B = \frac{P}{2\pi f} = \frac{396\times10^3}{2\pi(10)} = 6302.5 \text{ lb}\cdot\text{in}$$



Torques on gears *C* and *D*:

$$T_C = \frac{30}{60}T_B = 3151.3 \text{ lb}\cdot\text{in}$$
$$T_D = \frac{30}{60}T_B = 3151.3 \text{ lb}\cdot\text{in}$$

Shaft torques:

$$AB: \quad T_{AB} = 0$$
$$BC: \quad T_{BC} = 6302.5 \text{ lb}\cdot\text{in}$$
$$CD: \quad T_{CD} = 3151.3 \text{ lb}\cdot\text{in}$$
$$DE: \quad T_{DE} = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.30 *(Continued)*

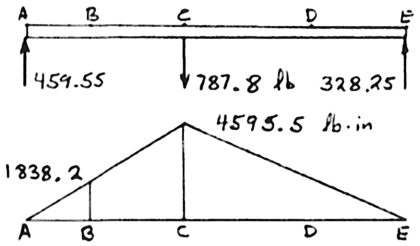
Gear forces:

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb}$$

$$F_C = \frac{T_C}{r_C} = \frac{3151.3}{4} = 787.8 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{3151.3}{4} = 787.8 \text{ lb}$$

Forces in vertical plane:



At B^+, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{1838.2^2 + 6214.9^2 + 6302.5^2}$

$= 9040.2 \text{ lb} \cdot \text{in}$ (maximum)

At C^-, $\sqrt{M_z^2 + M_y^2 + T^2} = \sqrt{4595.5^2 + 2932.4^2 + 6302.5^2}$

$= 8333.0 \text{ lb} \cdot \text{in}$

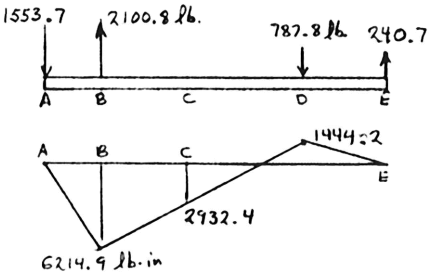
$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{\text{all}}}$$

$$= \frac{9040.3}{8 \times 10^3} = 1.1300 \text{ in}^3$$

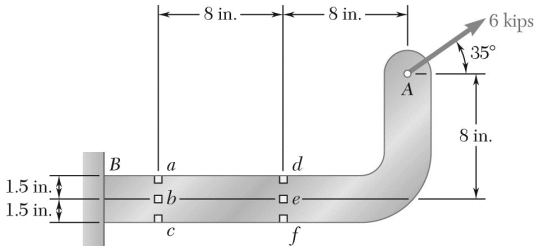
$c = 0.8960$ in. $\quad d = 2c$

Forces in horizontal plane:



$d = 1.792$ in. ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.31

A 6-kip force is applied to the machine element AB as shown. Knowing that the uniform thickness of the element is 0.8 in., determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Thickness = 0.8 in.

At the section containing points *a*, *b*, and *c*,

$$P = 6\cos 35° = 4.9149 \text{ kips} \qquad V = 6\sin 35° = 3.4415 \text{ kips}$$

$$M = (6\sin 35°)(16) - (6\cos 35°)(8) = 15.744 \text{ kip} \cdot \text{in}$$

$$A = (0.8)(3.0) = 2.4 \text{ in}^2$$

$$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$$

(*a*) At point *a*:

$$\sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(15.744)(1.5)}{1.80} \qquad \sigma_x = -11.07 \text{ ksi} \blacktriangleleft$$

$$\tau_{xy} = 0 \blacktriangleleft$$

(*b*) At point *b*:

$$\sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} \qquad \sigma_x = 2.05 \text{ ksi} \blacktriangleleft$$

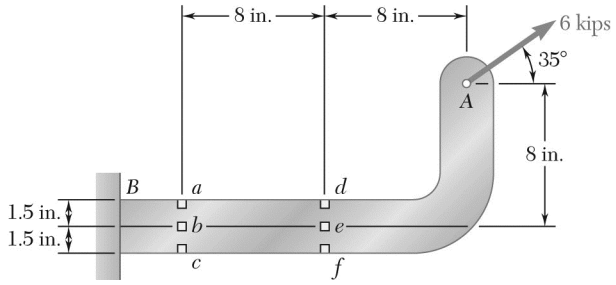
$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2} \cdot \frac{3.4415}{2.4} \qquad \tau_{xy} = 2.15 \text{ ksi} \blacktriangleleft$$

(*c*) At point *c*:

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(15.744)(1.5)}{1.80} \qquad \sigma_x = 15.17 \text{ ksi} \blacktriangleleft$$

$$\tau_{xy} = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.32

A 6-kip force is applied to the machine element *AB* as shown. Knowing that the uniform thickness of the element is 0.8 in., determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*.

SOLUTION

Thickness = 0.8 in.

At the section containing points *d*, *e*, and *f*,

$$P = 6\cos 35° = 4.9149 \text{ kips} \qquad V = 6\sin 35° = 3.4415 \text{ kips}$$

$$M = (6\sin 35°)(8) - (6\cos 35°)(8) = -11.788 \text{ kip} \cdot \text{in}$$

$$A = (0.8)(3.0) = 2.4 \text{ in}^2$$

$$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$$

(*a*) At point *d*:

$$\sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(11.788)(1.5)}{1.8}$$

$\sigma_x = 11.87$ ksi ◀

$\tau_{xy} = 0$ ◀

(*b*) At point *e*:

$$\sigma_x = \frac{P}{A} = \frac{4.9149}{2.4}$$

$\sigma_x = 2.05$ ksi ◀

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2} \cdot \frac{3.4415}{2.4}$$

$\tau_{xy} = 2.15$ ksi ◀

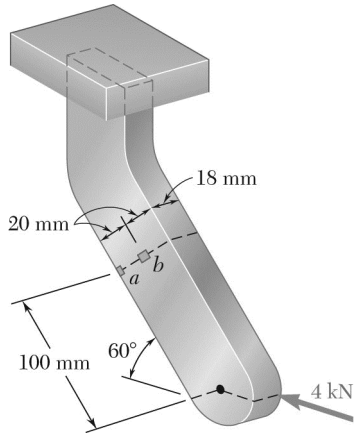
(*c*) At point *f*:

$$\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(11.788)(1.5)}{1.8}$$

$\sigma_x = -7.78$ ksi ◀

$\tau_{xy} = 0$ ◀

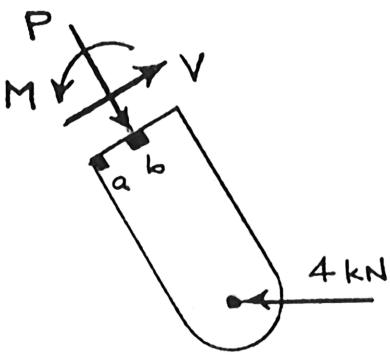
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.33

For the bracket and loading shown, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

SOLUTION



Draw free body diagram of portion below section *ab*.

From statics:

$$P = 4000\cos 60° = 2000 \text{ N}$$

$$V = 4000\sin 60° = 3464.1 \text{ N}$$

$$M = (0.1)(4000)\sin 60° = 346.41 \text{ N}\cdot\text{m}$$

Section properties:

$$A = (0.018)(0.040) = 720\times10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(0.018)(0.040)^3 = 96\times10^{-9} \text{ m}^4$$

(*a*) Point *a*:

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{2000}{720\times10^{-6}} - \frac{(346.41)(0.02)}{96\times10^{-9}}$$

$$= -74.9\times10^{6} \text{ Pa}$$

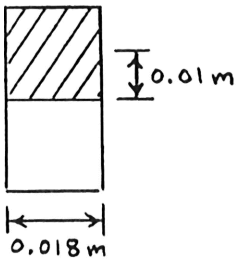
$\sigma = -74.9$ MPa ◀

$\tau = 0$ ◀

(*b*) Point *b*:

$$\sigma = -\frac{P}{A} = -\frac{2000}{720\times10^{-6}}$$

$\sigma = -2.78$ MPa ◀



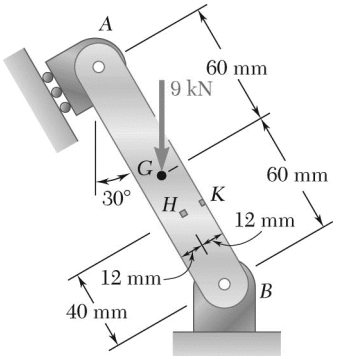
$$A = (0.018)(0.02) = 360\times10^{-6} \text{ m}^3 \qquad \bar{y} = 0.01 \text{ m}$$

$$Q = A\bar{y} = 3.6\times10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(3464.1)(3.6\times10^{-6})}{(96\times10^{-9})(0.018)} = 7.22\times10^{6} \text{ Pa}$$

$\tau = 7.22$ MPa ◀

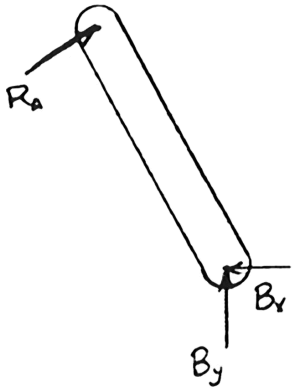
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.34

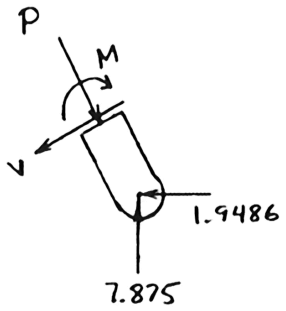
Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (9)(60\sin 30°) - 120R_A = 0$$
$$R_A = 2.25 \text{ kN}$$
$$\rightarrow + \Sigma F_x = 0: \quad 2.25\cos 30° - B_x = 0$$
$$B_x = 1.9486 \text{ kN} \leftarrow$$
$$+\uparrow \Sigma F_y = 0: \quad 2.25\sin 30° - 9 + B_y = 0$$
$$B_y = 7.875 \text{ kN} \uparrow$$

At the section containing points H and K,



$$P = 7.875\cos 30° + 1.9486\sin 30° = 7.794 \text{ kN}$$
$$V = 7.875\sin 30° - 1.9486\cos 30° = 2.25 \text{ kN}$$
$$M = (7.875\times10^3)(40\times10^{-3}\sin 30°) - (1.9486\times10^3)(40\times10^{-3}\cos 30°)$$
$$= 90 \text{ N}\cdot\text{m}$$
$$A = 10\times24 = 240 \text{ mm}^2 = 240\times10^{-6} \text{ m}^2$$
$$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^3 \text{ mm}^4 = 11.52\times10^{-9} \text{ m}^4$$

(*a*) At point H,

$$\sigma_x = -\frac{P}{A} = -\frac{7.794\times10^3}{240\times10^{-6}} \qquad \sigma_x = -32.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times10^3}{240\times10^{-6}} \qquad \tau_{xy} = 14.06 \text{ MPa} \blacktriangleleft$$

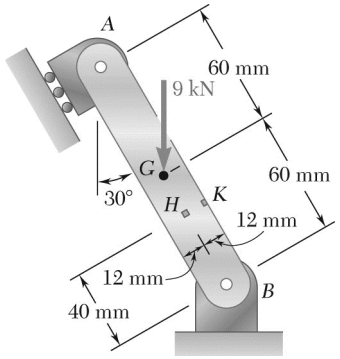
(*b*) At point K,

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794\times10^3}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$$

$$\sigma_x = -126.2 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = 0 \blacktriangleleft$$

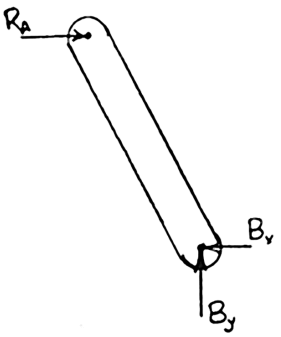
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.35

Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

SOLUTION



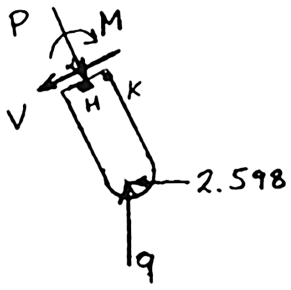
$$+\circlearrowleft \Sigma M_B = 0: \quad (120\cos 30°)R_A - (60\sin 30°)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9 = 0 \qquad B_y = 9 \text{ kN} \uparrow$$

$$\rightarrow + \Sigma F_x = 0: \quad 2.598 - B_x = 0 \quad B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points H and K,



$$P = 9\cos 30° + 2.598\sin 30° = 9.093 \text{ kN}$$

$$V = 9\sin 30° - 2.598\cos 30° = 2.25 \text{ kN}$$

$$M = (9\times 10^3)(40\times 10^{-3}\sin 30°) - (2.598\times 10^3)(40\times 10^{-3}\cos 30°)$$

$$= 90 \text{ N}\cdot\text{m}$$

$$A = 10\times 240 = 240 \text{ mm}^2 = 240\times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52\times 10^3 \text{ mm}^4 = 11.52\times 10^{-9} \text{ m}^4$$

(*a*) At point H:

$$\sigma_x = -\frac{P}{A} = -\frac{9.093\times 10^3}{240\times 10^{-6}} \qquad \sigma = -37.9 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times 10^3}{240\times 10^{-6}} \qquad \tau = 14.06 \text{ MPa} \blacktriangleleft$$

(*b*) At point K:

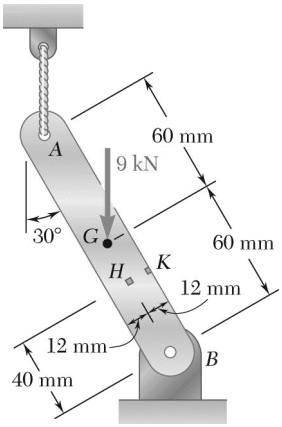
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I}$$

$$= -\frac{9.093\times 10^3}{240\times 10^{-6}} - \frac{(90)(12\times 10^{-3})}{11.52\times 10^{-9}}$$

$$\sigma = -131.6 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.36

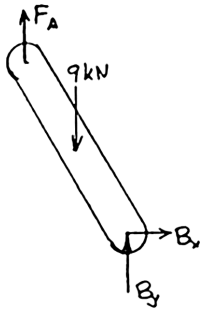
Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stress at (*a*) point H, (*b*) point K.

SOLUTION

$$\xrightarrow{+}\Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(120\sin 30^\circ) - 9(60\sin 30^\circ) = 0$$

$$B_y = 4.5 \text{ kN}$$



At the section containing points H and K,

$$P = 4.5\cos 30^\circ = 3.897 \text{ kN}$$

$$V = 4.5\sin 30^\circ = 2.25 \text{ kN}$$

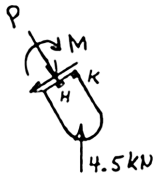
$$M = (4.5\times10^3)(40\times10^{-3}\sin 30^\circ)$$

$$= 90 \text{ N}\cdot\text{m}$$

$$A = 10\times24 = 240 \text{ mm}^2 = 240\times10^{-6}\text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52\times10^3 \text{ mm}^4$$

$$= 11.52\times10^{-9}\text{ m}^4$$



(*a*) At point H,

$$\sigma_x = -\frac{P}{A} = -\frac{3.897\times10^3}{240\times10^{-6}}$$

$$\sigma = -16.24 \text{ MPa} \blacktriangleleft$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25\times10^3}{240\times10^{-6}}$$

$$\tau = 14.06 \text{ MPa} \blacktriangleleft$$

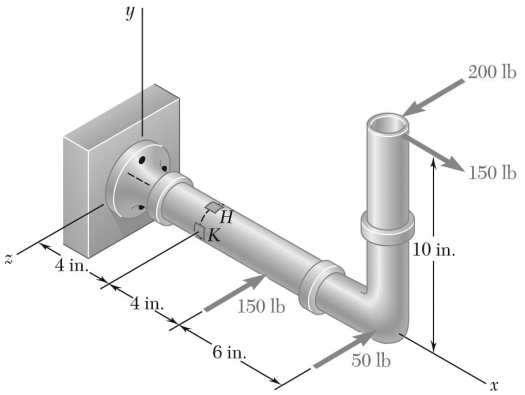
(*b*) At point K,

$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897\times10^3}{240\times10^{-6}} - \frac{(90)(12\times10^{-3})}{11.52\times10^{-9}}$$

$$\sigma = 110.0 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.37

Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 1.61 and 1.90 in., respectively, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION

Section properties:

$$P = 150 \text{ lb}$$

$$T = (200 \text{ lb})(10 \text{ in.}) = 2000 \text{ lb} \cdot \text{in}$$

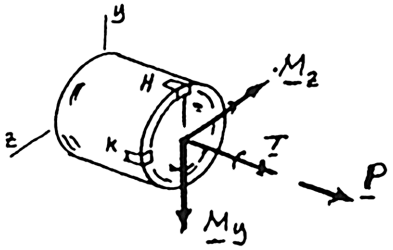
$$M_z = (150 \text{ lb})(10 \text{ in.}) = 1500 \text{ lb} \cdot \text{in}$$

$$M_y = (200 \text{ lb} - 50 \text{ lb})(10 \text{ in.}) - (150 \text{ lb})(4 \text{ in.})$$

$$= 900 \text{ lb} \cdot \text{in}$$

$$V_z = 200 - 150 - 50 = 0$$

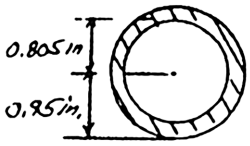
$$V_y = 0$$



$$A = \pi(0.95^2 - 0.805^2) = 0.79946 \text{ in}^2$$

$$I = \frac{\pi}{4}(0.95^4 - 0.805^4) = 0.30989 \text{ in}^4$$

$$J = 2I = 0.61979 \text{ in}^4$$



(*a*) Point *H*:

$$\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} + \frac{(1500 \text{ lb} \cdot \text{in})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} + 4593 \text{ psi} \qquad \sigma_H = 4.79 \text{ ksi} \blacktriangleleft$$

$$\tau_H = \frac{Tc}{J} = \frac{(2000 \text{ lb} \cdot \text{in})(0.95 \text{ in.})}{0.61979 \text{ in}^4} = 3065.6 \text{ psi} \qquad \tau_H = 3.07 \text{ ksi} \blacktriangleleft$$

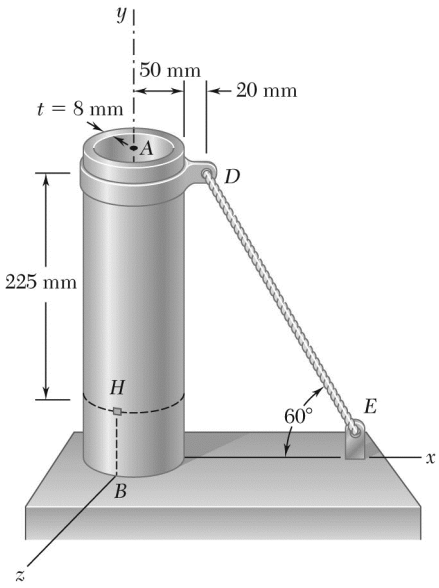
(*b*) Point *K*:

$$\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} - \frac{(900 \text{ lb} \cdot \text{in})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} - 2759 \text{ psi} \qquad \sigma_K = -2.57 \text{ ksi} \blacktriangleleft$$

$$\tau_K = \frac{Tc}{J} = \text{ same as for } \tau_H \qquad \tau_K = 3.07 \text{ ksi} \blacktriangleleft$$

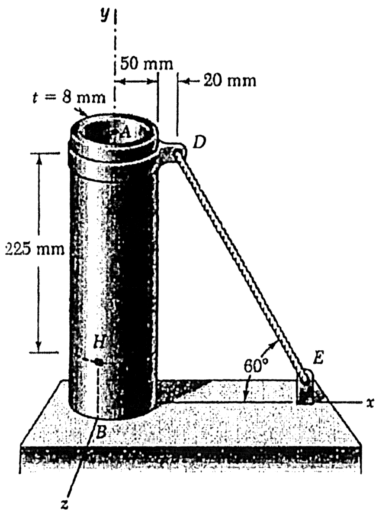
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

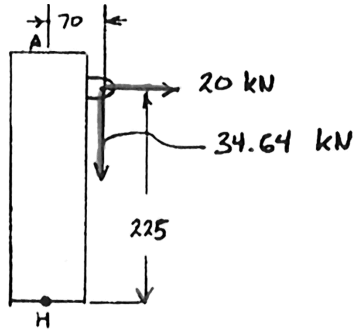


PROBLEM 8.38

The steel pipe *AB* has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point *H*.

SOLUTION





Vertical force:

$40\cos 30° = 34.64$ kN

Horizontal force:

$40\sin 30° = 20$ kN

Point *H* lies on neutral axis of bending.

Section properties:

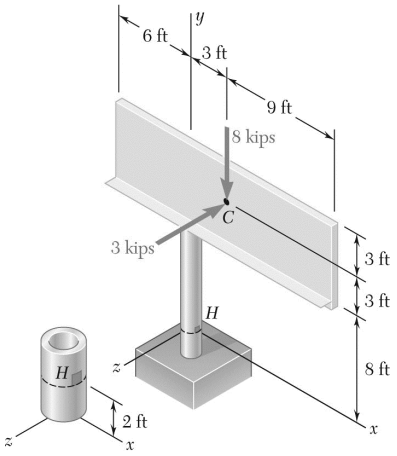
$$d_o = 100 \text{ mm}, \qquad c_o = \frac{1}{2}d_o = 50 \text{ mm}, \qquad c_i = c_o - t = 42 \text{ mm},$$

$$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-6}} \qquad \sigma = -14.98 \text{ MPa} \blacktriangleleft$$

For thin pipe, $\tau = 2\dfrac{V}{A} = \dfrac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}}$ $\qquad \tau = 17.29 \text{ MPa} \blacktriangleleft$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



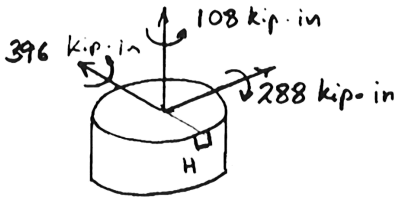
PROBLEM 8.39

The billboard shown weighs 8000 lb and is supported by a structural tube that has a 15-in. outer diameter and a 0.5-in. wall thickness. At a time when the resultant of the wind pressure is 3 kips, located at the center C of the billboard, determine the normal and shearing stresses at point H.

SOLUTION

At section containing point H,

$$P = 8 \text{ kips (compression)}$$
$$T = (3)(3) = 9 \text{ kip} \cdot \text{ft} = 108 \text{ kip} \cdot \text{in}$$
$$M_x = -(11)(3) = -33 \text{ kip} \cdot \text{ft} = -396 \text{ kip} \cdot \text{in}$$
$$M_z = -(3)(8) = -24 \text{ kip} \cdot \text{ft} = -288 \text{ kip} \cdot \text{in}$$
$$V = 3 \text{ kips}$$



Couples

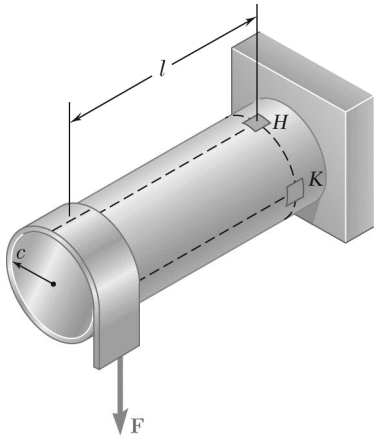
Section properties.

$$d_o = 15 \text{ in.} \quad c_o = \frac{1}{2} d_o = 7.5 \text{ in.} \quad c_i = c_o - t = 7.0 \text{ in.}$$
$$A = \pi \left(c_o^2 - c_i^2 \right) = 22.777 \text{ in}^2$$
$$I = \frac{\pi}{4} \left(c_o^4 - c_i^4 \right) = 599.31 \text{ in}^4$$
$$J = 2I = 1198.62 \text{ in}^4$$
$$Q = \frac{2}{3} \left(c_o^3 - c_i^3 \right) = 52.583 \text{ in}^3$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8}{22.777} - \frac{(288)(7.5)}{599.31} = -0.351 - 3.604 \qquad \sigma = -3.96 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(108)(7.5)}{1198.62} + \frac{(3)(52.583)}{(599.31)(1.0)} = 0.675 + 0.268 \qquad \tau = 0.938 \text{ ksi} \blacktriangleleft$$

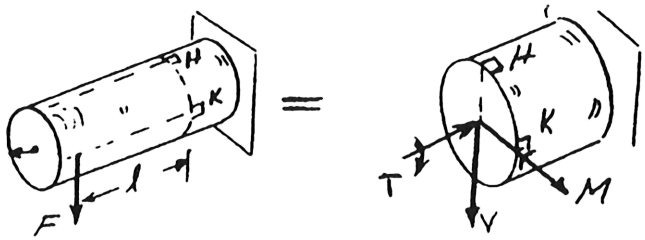
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.40

A thin strap is wrapped around a solid rod of radius $c = 20$ mm as shown. Knowing that $l = 100$ mm and $F = 5$ kN, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION



At the section containing points *H* and *K*,

$$T = Fc \qquad M = Fl \qquad V = F$$

$$J = \frac{\pi}{2}c^4 \qquad I = \frac{\pi}{4}c^4$$

Point *H*:

$$\sigma = \frac{Mc}{I} = \frac{Flc}{\frac{\pi}{4}c^4} \qquad \sigma = \frac{4Fl}{\pi c^2}$$

$$\tau = \frac{Tc}{J} = \frac{Fc^2}{\frac{\pi}{4}c^4} \qquad \tau = \frac{2F}{\pi c^2}$$

Point *K*: Point *K* lies on the neutral axis. $\sigma = 0$

Due to torque:

$$\tau = \frac{Tc}{J} = \frac{2F}{\pi c^2}$$

Due to shear: For a semicircle, $Q = \frac{2}{3}c^3, \quad t = d = 2c$

$$\tau = \frac{VQ}{It} = \frac{F\frac{2}{3}c^3}{\frac{\pi}{4}c^4(2c)} = \frac{4F}{3\pi c^2}$$

Combined.

$$\tau = \frac{2F}{\pi c^2} + \frac{4F}{3\pi c^2} \qquad \tau = \frac{10F}{3\pi c^2}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.40 *(Continued)*

Data: $F = 5 \text{ kN} = 5\times10^3 \text{ N}, \quad l = 100 \text{ mm} = 0.100 \text{ m}$

$c = 20 \text{ mm} = 0.020 \text{ m}$

(*a*) Point *H*:

$$\sigma = \frac{(4)(5\times10^3)(0.100)}{\pi(0.020)^3} \qquad \sigma = 79.6 \text{ MPa} \blacktriangleleft$$

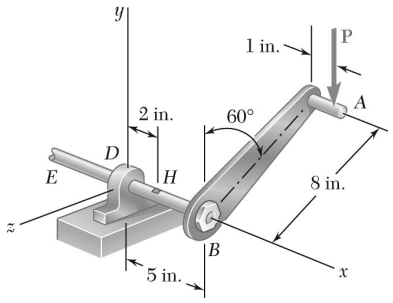
$$\tau = \frac{(2)(5\times10^3)}{\pi(0.020)^2} \qquad \tau = 7.96 \text{ MPa} \blacktriangleleft$$

(*b*) Point *K*:

$$\sigma = 0 \blacktriangleleft$$

$$\tau = \frac{(10)(5\times10^3)}{3\pi(0.020)^2} \qquad \tau = 13.26 \text{ MPa} \blacktriangleleft$$

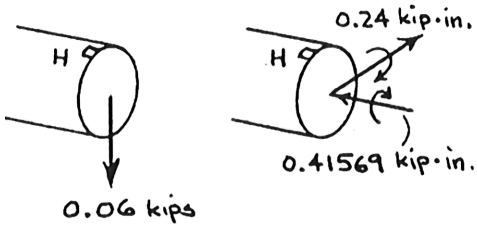
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.41

A vertical force **P** of magnitude 60 lb is applied to the crank at point *A*. Knowing that the shaft *BDE* has a diameter of 0.75 in., determine the principal stresses and the maximum shearing stress at point *H* located at the top of the shaft, 2 in. to the right of support *D*.

SOLUTION



Force-couple system at the centroid of the section containing point *H*:

$$F_x = 0, \qquad V_y = -0.06 \text{ kips}, \qquad V_z = 0$$

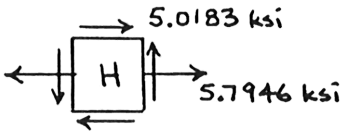
$$M_z = -(5 - 2 + 1)(0.06) = -0.24 \text{ kip} \cdot \text{in}$$

$$M_x = -(8 \sin 60°)(0.06) = -0.41569 \text{ kip} \cdot \text{in}$$

$$d = 0.75 \text{ in.} \qquad c = \frac{1}{2} d = 0.375 \text{ in.}$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (0.375)^4 = 15.5316 \times 10^{-3} \text{ in}^4$$

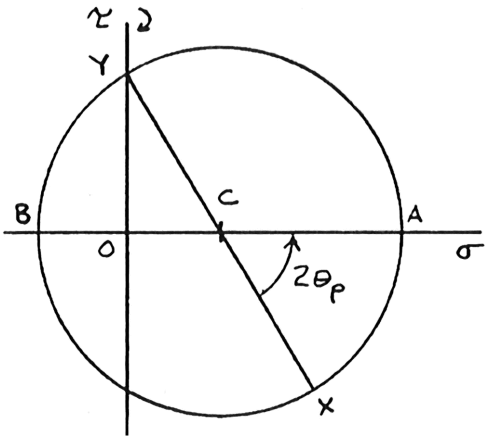
$$J = 2I = 31.063 \times 10^{-3} \text{ in}^4$$



At point *H*:

$$\sigma_H = -\frac{M_z y}{I_z} = -\frac{(-0.24)(0.375)}{15.5316 \times 10^{-3}} = 5.7946 \text{ ksi}$$

$$\tau_H = \frac{Tc}{J} = \frac{(0.41569)(0.375)}{31.063 \times 10^{-3}} = 5.0183 \text{ ksi}$$



Use Mohr's circle: $\sigma_{\text{ave}} = \frac{1}{2} \sigma_H = 2.8973 \text{ ksi}$

$$R = \sqrt{\left(\frac{\sigma_H}{2} \right)^2 + \tau_H^2} = 5.7946 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 8.69 \text{ ksi} \blacktriangleleft$$

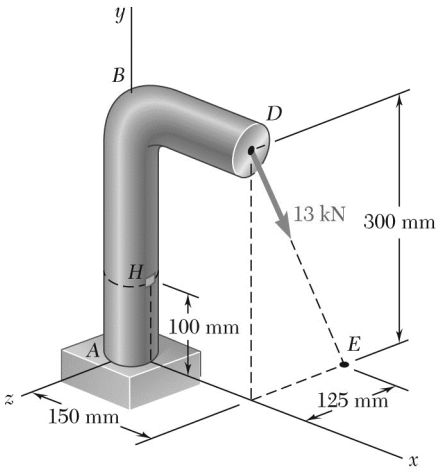
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -2.90 \text{ ksi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{2(5.0183)}{5.7946} = 1.7321$$

$$\theta_a = 30.0° \qquad \theta_b = 120.0°$$

$$\tau_{\max} = R \qquad \tau_{\max} = 5.79 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.42

A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

At point *D*,

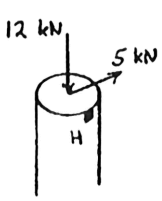
$$F_x = 0$$

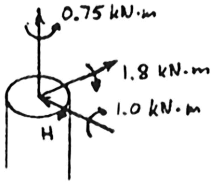
$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system at *C*, the centroid of the section containing point *H*:

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\vec{i} + 0.75\vec{j} - 1.8\vec{k} \text{ kN} \cdot \text{m}$$





Section properties:

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$$

$$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$$

For a semicircle,

$$Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$$

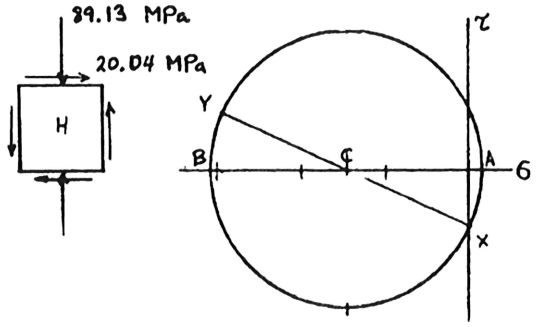
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.42 *(Continued)*

At point H,

$$\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12\times10^3}{2.8274\times10^{-3}} - \frac{(1.8\times10^3)(30\times10^{-3})}{636.17\times10^{-9}} = -89.13 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75\times10^3)(30\times10^{-3})}{1.2723\times10^{-6}} + \frac{(5\times10^3)(18.00\times10^{-6})}{(636.17\times10^{-9})(60\times10^{-3})} = 20.04 \text{ MPa}$$



(*a*)

$$\sigma_{\text{ave}} = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 4.3 \text{ MPa} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -93.4 \text{ MPa} \blacktriangleleft$$

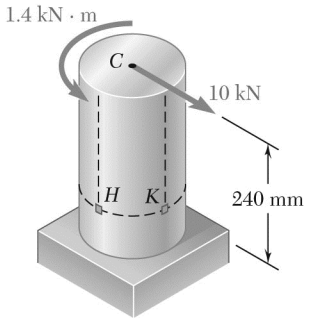
$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$$

$$\theta_a = 12.1°, \quad \theta_b = 102.1° \blacktriangleleft$$

(*b*)

$$\tau_{\max} = R = 48.9 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.43

A 10-kN force and a 1.4-kN · m couple are applied at the top of the 65-mm diameter brass post shown. Determine the principal stresses and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$V = 10 \text{ kN} = 10\times10^3 \text{ N}$$

$$M = (10\times10^3)(240\times10^{-3}) = 2.4\times10^3 \text{ N}\cdot\text{m}$$

$$T = 1.4\times10^3 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 32.5 \text{ mm} = 0.0325 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = 1.75248\times10^{-6} \text{ m}^4$$

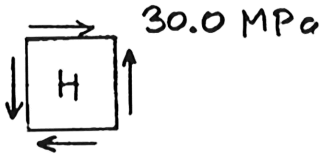
$$I = \frac{1}{2}J = 0.87624\times10^{-6} \text{ m}^4$$

For a semicircle, $$Q = \frac{2}{3}c^3 = \frac{2}{3}(0.0325)^3 = 22.885\times10^{-6} \text{ m}^3$$

(*a*) Stresses at point *H*. *H* lies on the neutral axis: $\sigma = 0$

Due to torque:
$$\tau = \frac{Tc}{J} = \frac{(1.4\times10^3)(0.0325)}{1.75248\times10^{-6}} = 25.963 \text{ MPa}$$

Due to shear:
$$\tau = \frac{VQ}{It} = \frac{(10\times10^3)(22.885\times10^{-6})}{(0.87624\times10^{-6})(0.065)} = 4.018 \text{ MPa}$$



Total at *H*:
$$\tau = 30.0 \text{ MPa}$$

$$\sigma_{ave} = 0, \quad R = 30.0 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 30.0 \text{ MPa} \blacktriangleleft$$

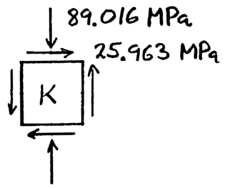
$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -30.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 30.0 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.43 *(Continued)*

(*b*) Stresses at point *K*.

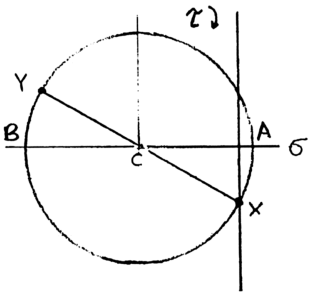


Due to shear: $\tau = 0$

Due to torque: $\tau = \dfrac{Tc}{J} = 25.963 \text{ MPa}$

Due to bending: $\sigma = -\dfrac{Mc}{I} = -\dfrac{(2.4\times10^{3})(0.0325)}{(0.87624\times10^{-6})}$

$= -89.016 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{-89.016}{2} = -44.508 \text{ MPa}$$

$$R = \sqrt{\left(\frac{89.016}{2}\right)^2 + (25.963)^2}$$

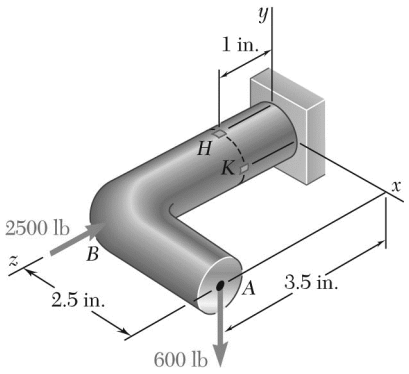
$$= 51.527 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 7.02 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -96.0 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 51.5 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



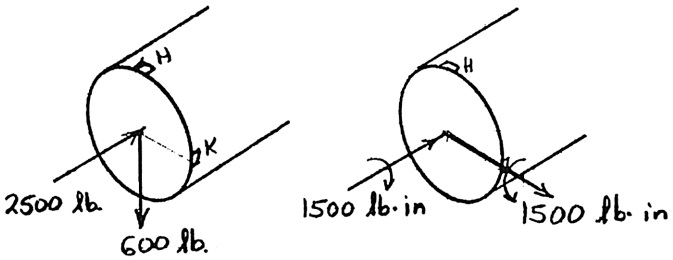
PROBLEM 8.44

Forces are applied at points *A* and *B* of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

SOLUTION

At the section containing points *H* and *K*,

$$P = 2500 \text{ lb} \quad \text{(compression)}$$
$$V_y = -600 \text{ lb}$$
$$V_x = 0$$
$$M_x = (3.5-1)(600) = 1500 \text{ lb}\cdot\text{in}$$
$$M_y = 0$$
$$M_z = -(2.5)(600) = -1500 \text{ lb}\cdot\text{in}$$



Forces Couples

$$c = \frac{1}{2}d = 0.4 \text{ in.}$$
$$A = \pi c^2 = 0.50265 \text{ in}^2$$
$$I = \frac{\pi}{4}c^4 = 20.106\times10^{-3} \text{ in}^4$$
$$J = 2I = 40.212\times10^{-3} \text{ in}^4$$

For semicircle,

$$Q = \frac{2}{3}c^3$$
$$= 42.667\times10^{-3} \text{ in}^3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

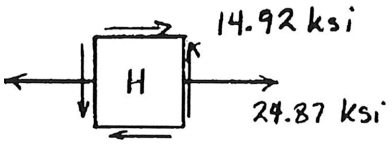
PROBLEM 8.44 *(Continued)*

(*a*) At point *H*:

$$\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106\times10^{-3}}$$

$$= 24.87\times10^{3} \text{ psi}$$

$$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212\times10^{-3}} = 14.92\times10^{3} \text{ psi}$$



$$\sigma_{\text{ave}} = \frac{24.87}{2} = 12.435 \text{ ksi}$$

$$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 31.9 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -6.99 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 19.42 \text{ ksi} \blacktriangleleft$$

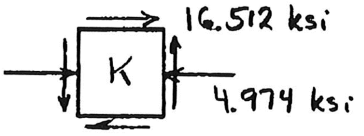
(*b*) At point *K*:

$$\sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974\times10^{3} \text{ psi}$$

$$\tau_K = \frac{Tc}{J} + \frac{VQ}{It}$$

$$= \frac{(1500)(0.4)}{40.212\times10^{-3}} + \frac{(600)(42.667\times10^{-3})}{(20.106\times10^{-3})(0.8)}$$

$$= 16.512\times10^{3} \text{ psi}$$



$$\sigma_{\text{ave}} = -\frac{4.974}{2} = -2.487 \text{ ksi}$$

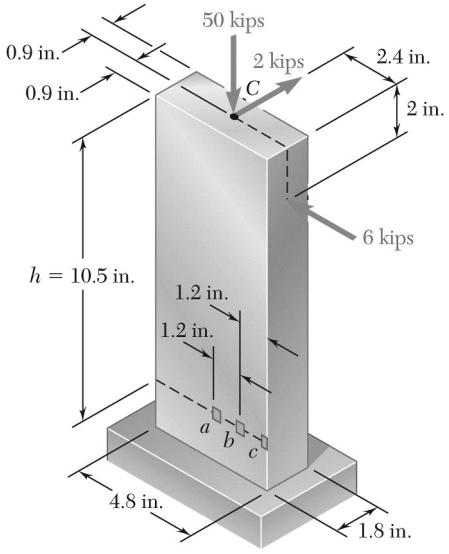
$$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 14.21 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -19.18 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \qquad \tau_{\max} = 16.70 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

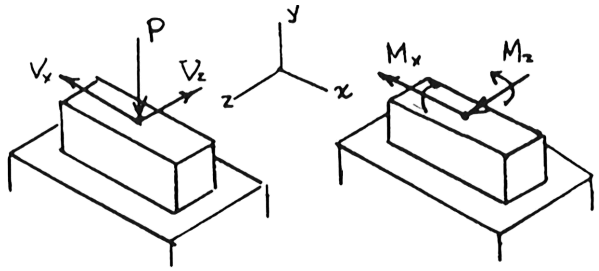


PROBLEM 8.45

Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.



Forces Couples

$$h = 10.5 \text{ in.}$$

$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (10.5 - 2)(6) = 51 \text{ kip} \cdot \text{in}$$

$$M_x = (10.5)(2) = 21 \text{ kip} \cdot \text{in}$$

Section properties.

$$A = (1.8)(4.8) = 8.64 \text{ in}^2$$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4$$

$$I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

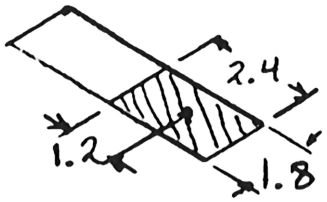
Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.45 *(Continued)*

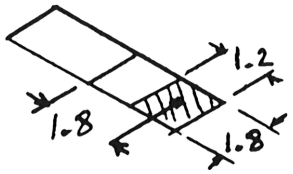
(*a*) Point *a*: $x = 0, \quad z = 0.9$ in., $\quad Q = (1.8)(2.4)(1.2) = 5.184$ in^3



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 2.31 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} \qquad \tau = 1.042 \text{ ksi} \blacktriangleleft$$

(*b*) Point *b*: $x = 1.2$ in., $\quad z = 0.9$ in., $\quad Q = (1.8)(1.2)(1.8) = 3.888$ in^3



$$\sigma = -\frac{50}{8.64} + \frac{(51)(1.2)}{16.5888} + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 6.00 \text{ ksi} \blacktriangleleft$$

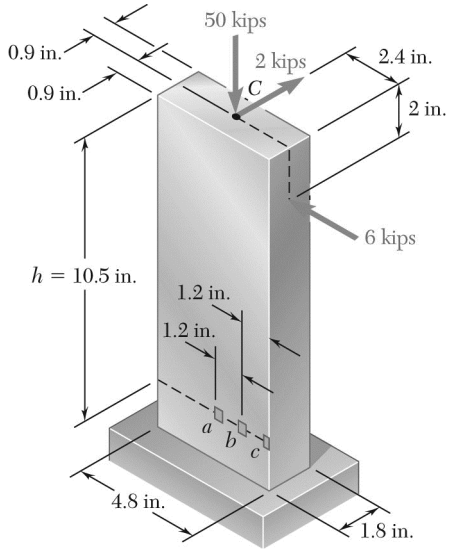
$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} = 0.781 \text{ ksi} \qquad \tau = 0.781 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*: $x = 2.4$ in., $\quad z = 0.9$ in., $\quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(51)(2.4)}{16.5888} + \frac{(21)(0.9)}{2.3328} \qquad \sigma = 9.69 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



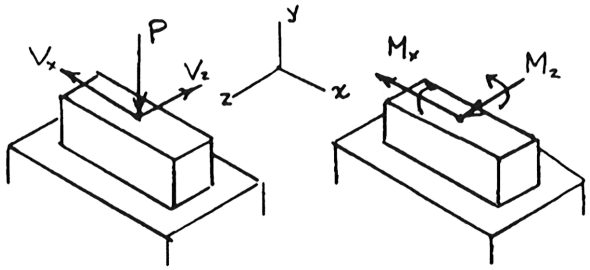
PROBLEM 8.46

Solve Prob. 8.45, assuming that $h = 12$ in.

PROBLEM 8.45 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Calculate forces and couples at section containing points *a*, *b*, and *c*.



Forces Couples

$$h = 12 \text{ in.}$$

$$P = 50 \text{ kips} \quad V_x = 6 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_z = (12 - 2)(6) = 60 \text{ kip} \cdot \text{in}$$

$$M_x = (12)(2) = 24 \text{ kip} \cdot \text{in}$$

Section properties.

$$A = (1.8)(4.8) = 8.64 \text{ in}^2$$

$$I_x = \frac{1}{12}(4.8)(1.8)^3 = 2.3328 \text{ in}^4$$

$$I_z = \frac{1}{12}(1.8)(4.8)^3 = 16.5888 \text{ in}^4$$

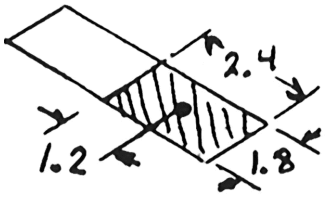
Stresses.

$$\sigma = -\frac{P}{A} + \frac{M_z x}{I_z} + \frac{M_x z}{I_x} \qquad \tau = \frac{V_x Q}{I_z t}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.46 *(Continued)*

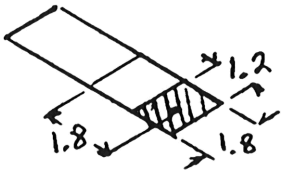
(*a*) Point *a*: $x = 0, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(2.4)(1.2) = 5.184 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + 0 + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 3.47 \text{ ksi} \blacktriangleleft$$

$$\tau = \frac{(6)(5.184)}{(16.5888)(1.8)} \qquad \tau = 1.042 \text{ ksi} \blacktriangleleft$$

(*b*) Point *b*: $x = 1.2 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = (1.8)(1.2)(1.8) = 3.888 \text{ in}^3$



$$\sigma = -\frac{50}{8.64} + \frac{(60)(1.2)}{16.5888} + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 7.81 \text{ ksi} \blacktriangleleft$$

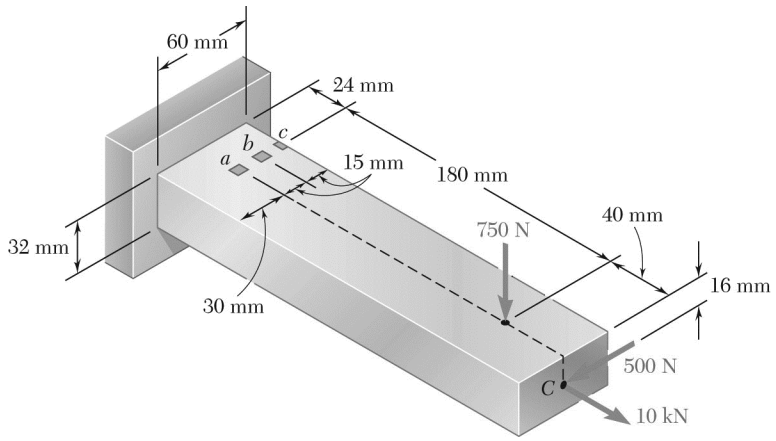
$$\tau = \frac{(6)(3.888)}{(16.5888)(1.8)} \qquad \tau = 0.781 \text{ ksi} \blacktriangleleft$$

(*c*) Point *c*: $x = 2.4 \text{ in.}, \quad z = 0.9 \text{ in.}, \quad Q = 0$

$$\sigma = -\frac{50}{8.64} + \frac{(60)(2.4)}{16.5888} + \frac{(24)(0.9)}{2.3328} \qquad \sigma = 12.15 \text{ ksi} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.47

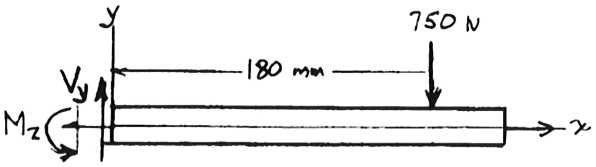
Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920 \times 10^{-6} \text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3 = 163.84 \times 10^3 \text{ mm}^4$$
$$= 163.84 \times 10^{-9} \text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 579 \times 10^3 \text{ mm}^4$$
$$= 576 \times 10^{-9} \text{ m}^4$$

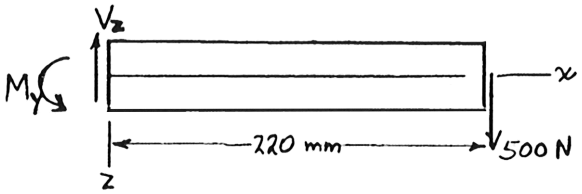
At the section containing points *a*, *b*, and *c*,

$$P = 10 \text{ kN}$$
$$V_y = 750 \text{ N},$$
$$V_z = 500 \text{ N}$$
$$M_z = (180 \times 10^{-3})(750)$$
$$= 135 \text{ N} \cdot \text{m}$$



Side View

$$M_y = (220 \times 10^{-3})(500)$$
$$= 110 \text{ N} \cdot \text{m}$$
$$T = 0$$
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$
$$\tau = \frac{V_z Q}{I_y t}$$

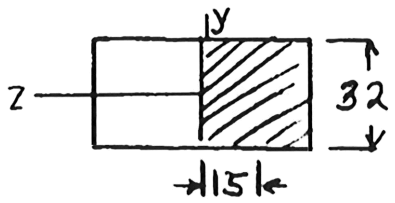


Top View

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.47 *(Continued)*

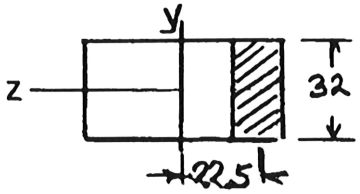
(*a*) Point *a*: $y = 16$ mm, $z = 0$, $Q = A\bar{z} = (32)(30)(15) = 14.4\times10^{3}\ \text{mm}^3$



$$\sigma = \frac{10\times10^{3}}{1920\times10^{-6}} + \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - 0 \qquad \sigma = 18.39\ \text{MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(14.4\times10^{-6})}{(576\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.391\ \text{MPa} \blacktriangleleft$$

(*b*) Point *b*: $y = 16$ mm, $z = -15$ mm, $Q = A\bar{z} = (32)(15)(22.5) = 10.8\times10^{3}\ \text{mm}^3$



$$\sigma = \frac{10\times10^{3}}{1920\times10^{-6}} + \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-15\times10^{-3})}{576\times10^{-9}} \qquad \sigma = 21.3\ \text{MPa} \blacktriangleleft$$

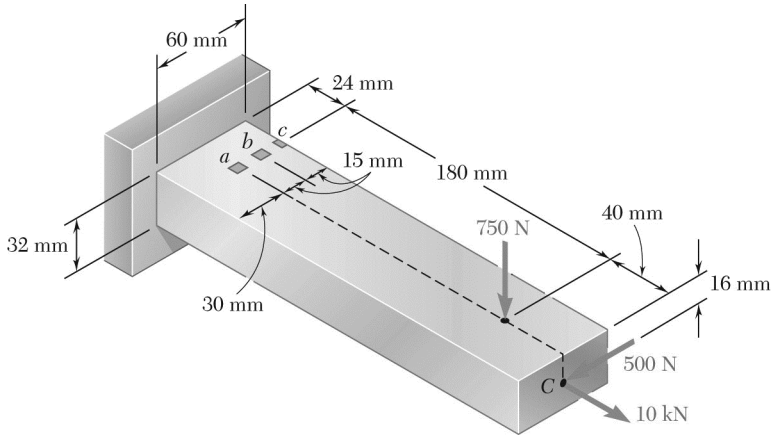
$$\tau = \frac{(500)(10.8\times10^{-6})}{(576\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.293\ \text{MPa} \blacktriangleleft$$

(*c*) Point *c*: $y = 16$ mm, $z = -30$ mm, $Q = 0$

$$\sigma = \frac{10\times10^{3}}{1920\times10^{-6}} + \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-30\times10^{-6})}{576\times10^{-9}} \qquad \sigma = 24.1\ \text{MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.48

Solve Prob. 8.47, assuming that the 750-N force is directed vertically upward.

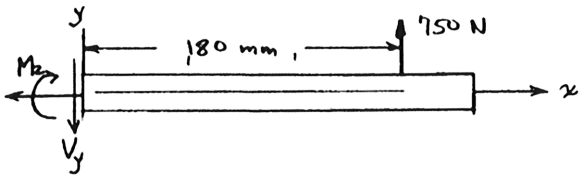
PROBLEM 8.47 Three forces are applied to the bar shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

$$A = (60)(32) = 1920 \text{ mm}^2$$
$$= 1920 \times 10^{-6} \text{ m}^2$$
$$I_z = \frac{1}{12}(60)(32)^3$$
$$= 163.84 \times 10^3 \text{ mm}^4$$
$$= 163.84 \times 10^{-9} \text{ m}^4$$
$$I_y = \frac{1}{12}(32)(60)^3$$
$$= 576 \times 10^3 \text{ mm}^4$$
$$= 576 \times 10^{-9} \text{ m}^4$$

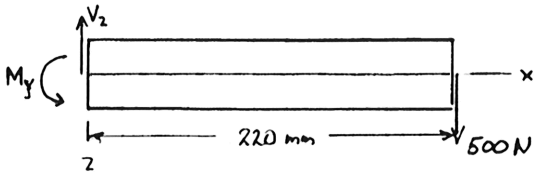
At the section containing points *a*, *b*, and *c*,

$$P = 10 \text{ kN} \qquad T = 0$$
$$V_y = 750 \text{ N}$$
$$V_z = 500 \text{ N}$$
$$M_z = (180 \times 10^{-3})(750)$$
$$= 135 \text{ N} \cdot \text{m}$$



Side View

$$M_y = (220 \times 10^{-3})(500)$$
$$= 110 \text{ N} \cdot \text{m}$$



Top View

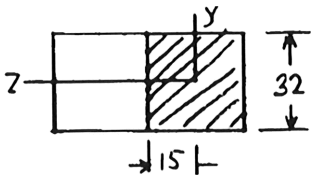
$$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} - \frac{M_y z}{I_y}$$

$$\tau = \frac{V_z Q}{I_y t}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.48 *(Continued)*

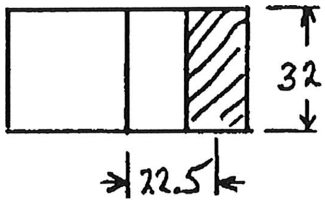
(*a*) Point *a*: $y = 16$ mm, $z = 0$, $Q = A\bar{z} = (32)(30)(15) = 14.4\times10^{3}\ \text{mm}^3$



$$\sigma = \frac{10\times10^{3}}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} = 0 \qquad \sigma = -7.98\ \text{MPa} \blacktriangleleft$$

$$\tau = \frac{(500)(14.4\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-3})} \qquad \tau = 0.391\ \text{MPa} \blacktriangleleft$$

(*b*) Point *b*: $y = 16$ mm, $z = -15$ mm, $Q = A\bar{z} = (32)(15)(22.5) = 10.8\times10^{3}\ \text{mm}^3$



$$\sigma = \frac{10\times10^{3}}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-15\times10^{-3})}{576\times10^{-9}} \qquad \sigma = -5.11\ \text{MPa} \blacktriangleleft$$

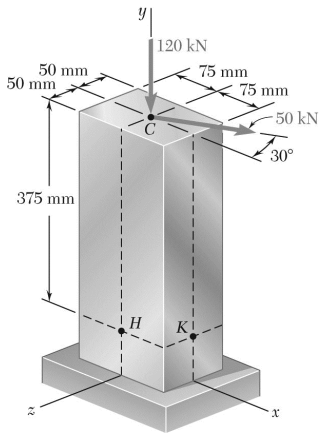
$$\tau = \frac{(500)(10.8\times10^{-6})}{(163.84\times10^{-9})(32\times10^{-9})} \qquad \tau = 0.293\ \text{MPa} \blacktriangleleft$$

(*c*) Point *c*: $y = 16$ mm, $z = -30$ mm, $Q = 0$

$$\sigma = \frac{10\times10^{3}}{1920\times10^{-6}} - \frac{(135)(16\times10^{-3})}{163.84\times10^{-9}} - \frac{(110)(-30\times10^{-3})}{576\times10^{-9}} \qquad \sigma = -2.25\ \text{MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.49

For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

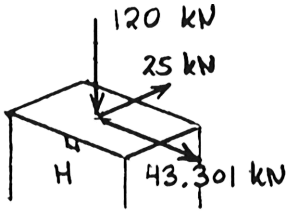
SOLUTION

Components of force at point *C*:

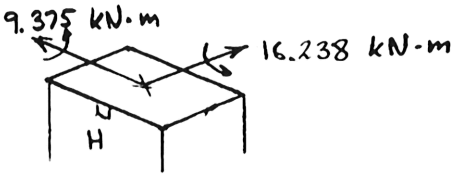
$$F_x = 50 \cos 30° = 43.301 \text{ kN}$$
$$F_z = -50 \sin 30° = -25 \text{ kN}$$
$$F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points *H* and *K*:

$$P = 120 \text{ kN} \qquad \text{(compression)}$$
$$V_x = 43.301 \text{ kN}$$
$$V_z = -25 \text{ kN}$$
$$M_x = -(25)(0.375) = -9.375 \text{ kN} \cdot \text{m}$$
$$M_y = 0$$
$$M_z = -(43.301)(0.375) = -16.238 \text{ kN} \cdot \text{m}$$
$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$
$$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$$



Forces



Couples

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

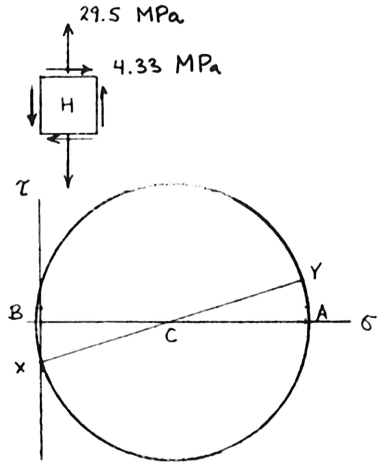
PROBLEM 8.49 *(Continued)*

Stresses at point H:

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120\times10^3)}{15\times10^{-3}} - \frac{(-9.375\times10^3)(50\times10^{-3})}{12.5\times10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301\times10^3}{15\times10^{-3}} = 4.33 \text{ MPa}$$

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 30.1 \text{ MPa} \blacktriangleleft$$

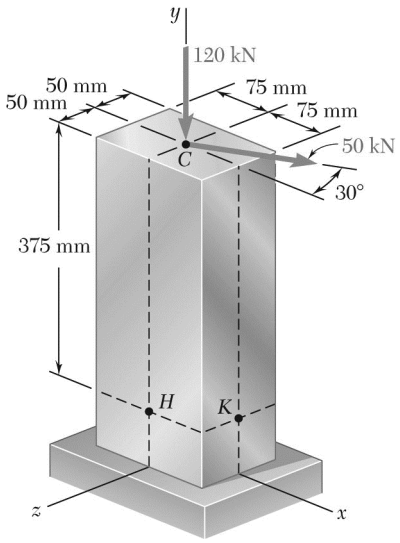
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.62 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

$$\theta_a = -8.2^\circ \quad \theta_b = 81.8^\circ \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 15.37 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.50

For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *K*.

SOLUTION

Components of force at point *C*:

$$F_x = 50\cos 30° = 43.301 \text{ kN}$$

$$F_z = -50\sin 30° = -25 \text{ kN} \qquad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points *H* and *K*:

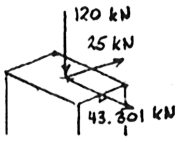
$$P = 120 \text{ kN (compression)}$$

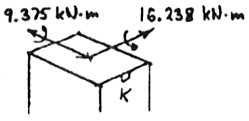
$$V_x = 43.301 \text{ kN}, \qquad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN} \cdot \text{m}$$

$$M_y = 0$$

$$M_z = -(43.301)(0.375) = -16.238 \text{ kN} \cdot \text{m}$$





$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2$$

$$= 15 \times 10^{-3} \text{ m}^2$$

$$I_z = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

$$= 28.125 \times 10^{-6} \text{ m}^4$$

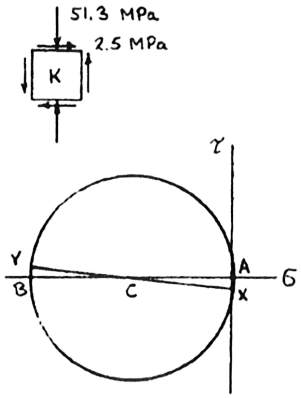
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.50 *(Continued)*

Stresses at point K:

$$\sigma_K = -\frac{P}{A} + \frac{M_z x}{I_z} = -\frac{120 \times 10^3}{15 \times 10^{-3}} + \frac{(-16.238 \times 10^3)(75 \times 10^{-3})}{28.125 \times 10^{-6}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{25 \times 10^3}{15 \times 10^{-3}} = 2.5 \text{ MPa}$$



Use Mohr's circle.

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_K = -25.65 \text{ MPa}$$

$$R = \sqrt{\left(\frac{51.3}{2}\right)^2 + (2.5)^2} = 25.77 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 0.12 \text{ MPa} \blacktriangleleft$$

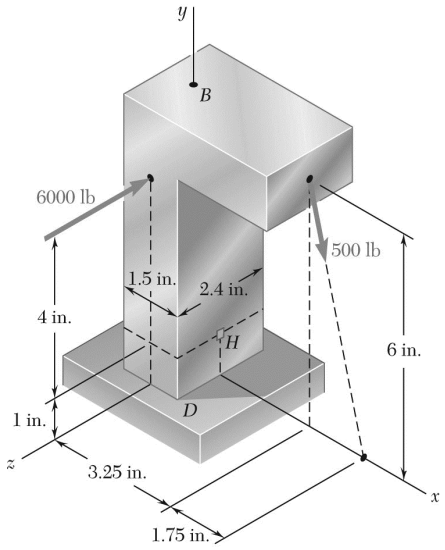
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -51.4 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$$

$$\theta_a = 2.8° \qquad \theta_b = 92.8° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 25.8 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.51

Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 500-lb force:

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500-lb force:

$$\vec{r} = 3.25\vec{i} + (6 - 1)\vec{j}$$

Moment of 500-lb force:

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\vec{k} \text{ lb} \cdot \text{in}$$

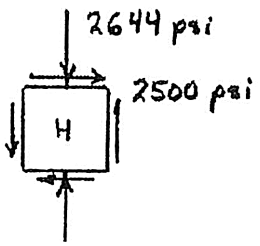
At the section containing point *H*, $P = -480$ lb $V_x = 140$ lb

$V_z = -6000$ lb, $M_z = -2260$ lb · in, $M_x = -(4)(6000) = -24{,}000$ lb · in

$A = (1.5)(2.4) = 3.6 \text{ in}^2$ $I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

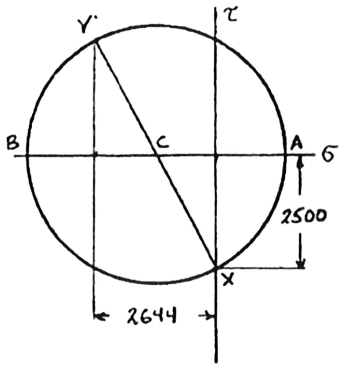
$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6000}{3.6} = 2500 \text{ psi}$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.51 *(Continued)*



Use Mohr's circle.

$$\sigma_{\text{ave}} = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 1506 \text{ psi} \blacktriangleleft$$

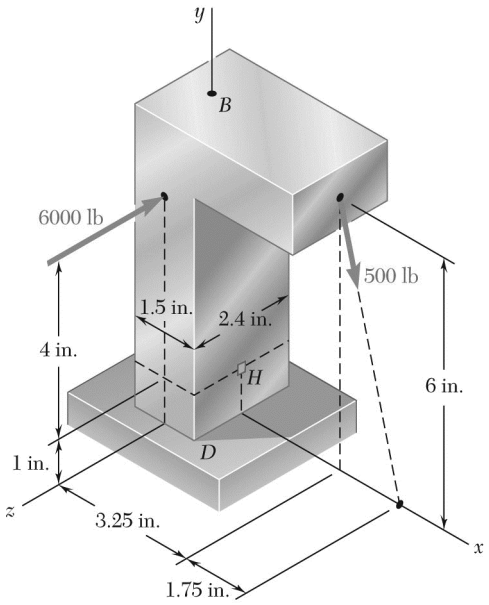
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -4150 \text{ psi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(2500)}{2644} = 1.891$$

$$\theta_a = 31.1°, \qquad \theta_b = 121.1° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 2828 \text{ psi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.52

Solve Prob. 8.51, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

PROBLEM 8.51 Two forces are applied to the small post *BD* as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point *H*.

SOLUTION

Components of 500-lb force:

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500-lb force:

$$\vec{r} = 3.25\vec{i} + (6-1)\vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\vec{k} \text{ lb} \cdot \text{in}$$

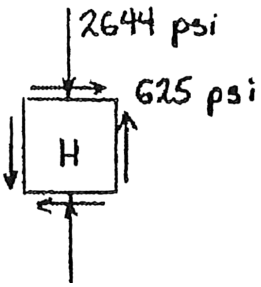
At the section containing point *H*, $P = -480$ lb $V_x = 140$ lb

$V_z = -1500$ lb, $M_z = -2260$ lb · in, $M_x = -(4)(1500) = -6000$ lb · in

$A = (1.5)(2.4) = 3.6 \text{ in}^2$ $I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

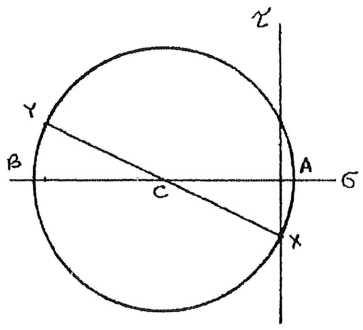
$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{1500}{3.6} = 625 \text{ psi}$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.52 *(Continued)*



Use Mohr's circle.

$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462 \text{ psi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 140 \text{ psi} \blacktriangleleft$$

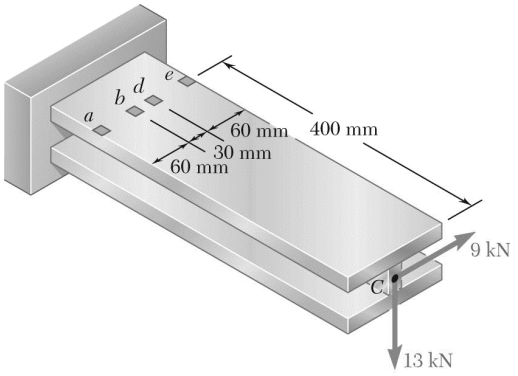
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -2784 \text{ psi} \blacktriangleleft$$

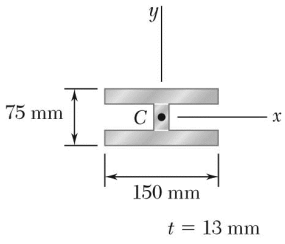
$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$$

$$\theta_a = 12.7° \qquad \theta_b = 102.7° \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 1462 \text{ psi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 8.53

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

SOLUTION

Equivalent force-couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13\times10^3) = 5200 \text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9\times10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75-26)$$

$$= 4537 \text{ mm}^2$$

$$= 4537\times10^{-6}\text{ m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3$$

$$= 3.9303\times10^6 \text{ mm}^4$$

$$= 3.9303\times10^{-6}\text{ m}^4$$

$$I_y = 2\cdot\frac{1}{12}(13)(150)^3 + \frac{1}{12}(75-26)(13)^3$$

$$= 7.3215\times10^6 \text{ mm}^4 = 7.3215\times10^{-6}\text{ mm}^4$$

For point *a*,

$$Q_x = 0, \quad Q_y = 0$$

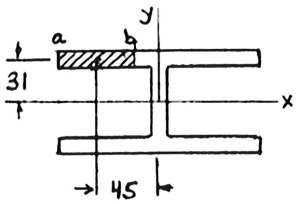
For point *b*,

$$A^* = (60)(13) = 780 \text{ mm}^2$$

$$\bar{x} = -45 \text{ mm} \quad \bar{y} = 31 \text{ mm}$$

$$Q_x = A^*\bar{y} = 24.18\times10^3 \text{ mm}^3 = 24.18\times10^{-6} \text{ m}^3$$

$$Q_y = A^*\bar{x} = -35.1\times10^{-3} \text{ mm}^3 = -35.1\times10^{-6} \text{ m}^3$$



***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.53 ***(Continued)***

Direction of shearing stress for horizontal and for vertical components of shear.

At point *a*:

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 86.5 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

At point *b*:

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

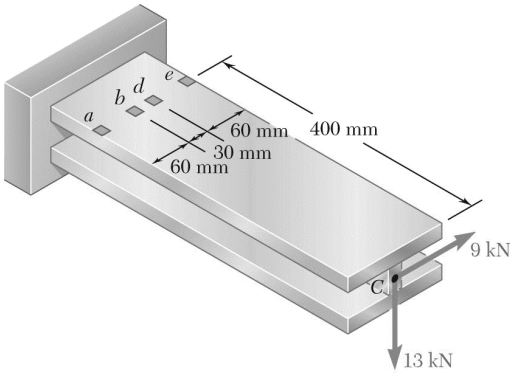
$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 57.0 \text{ MPa} \blacktriangleleft$$

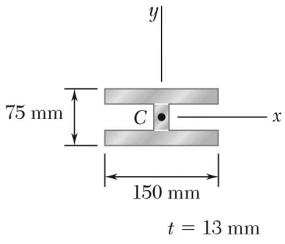
$$\tau = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t}$$

$$= \frac{(9\times10^{3})(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^{3})(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} \qquad \tau = 9.47 \text{ MPa} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 8.54

Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.

SOLUTION

Equivalent force-couple system at section containing points *d* and *e*.

$$F_x = 9\text{ kN}, \quad F_y = -13\text{ kN}, \quad F_z = 0$$

$$M_x = (0.400)(13\times10^3) = 5200\text{ N}\cdot\text{m}$$

$$M_y = (0.400)(9\times10^3) = 3600\text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75-26)$$

$$= 4537\,\text{mm}^2$$

$$= 4537\times10^{-6}\,\text{m}^2$$

$$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3$$

$$= 3.9303\times10^6\,\text{mm}^4$$

$$= 3.9303\times10^{-6}\,\text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75-26)(13)^3$$

$$= 7.3215\times10^6\,\text{mm}^4 = 7.3215\times10^{-6}\,\text{m}^4$$

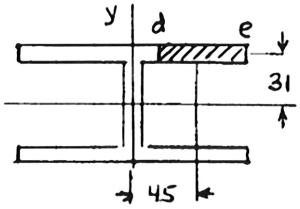
For point *d*,

$$A^* = (60)(13) = 780\text{ mm}^2$$

$$\bar{x} = 45\text{ mm} \quad \bar{y} = 31\text{ mm}$$

$$Q_x = A^*\bar{y} = 24.18\times10^3\,\text{mm}^3 = 24.18\times10^{-6}\,\text{m}^3$$

$$Q_y = A^*\bar{x} = 35.1\times10^3\,\text{mm}^3 = 35.1\times10^{-6}\,\text{m}^3$$



For point *e*, $Q_x = 0, \quad Q_y = 0$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.54 *(Continued)*

At point *d*:

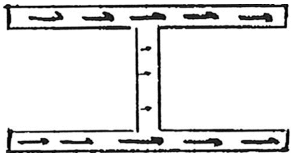
$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(15\times10^{-3})}{7.3215\times10^{-6}} \qquad \sigma = 42.2 \text{ MPa} \blacktriangleleft$$

Due to V_x:

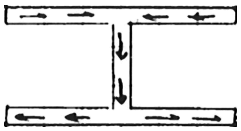
$$\tau = \frac{|V_x||Q_y|}{I_y t} = \frac{(9000)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})}$$

$$= 3.32 \text{ MPa} \rightarrow$$



Due to V_y:

$$\tau = \frac{|V_y||Q_x|}{I_x\ t} = \frac{(13000)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})} = 6.15 \text{ MPa} \leftarrow$$



By superposition, the net value is $\tau = 2.83$ MPa ◀

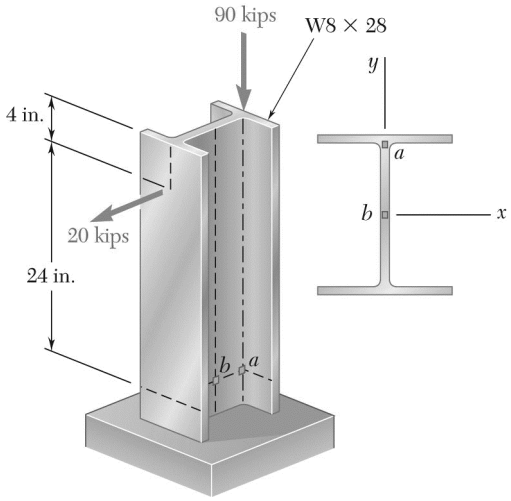
At point *e*:

$$\sigma = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(75\times10^{-3})}{7.3215\times10^{-6}}$$

$$\sigma = 12.74 \text{ MPa} \blacktriangleleft$$

$$\tau = 0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.55

Two forces are applied to a $W8 \times 28$ rolled-steel beam as shown. Determine the principal stresses and maximum shearing stress at point *a*.

SOLUTION

For $W8 \times 28$ rolled steel section,

$$A = 8.24 \text{ in}^2, \quad d = 8.06 \text{ in.}, \quad b_f = 6.54 \text{ in.}$$

$$t_f = 0.465 \text{ in.}, \quad t_w = 0.285 \text{ in.}, \quad I_x = 98.0 \text{ in}^4$$

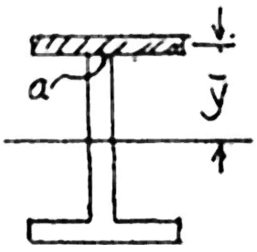
At the section containing points *a* and *b,*

$$P = -90 \text{ kips}, \quad V = 20 \text{ kips}$$

$$M = (20)(24) - (4.03)(90) = 117.3 \text{ kip} \cdot \text{in}$$

At point *a*, $y = \frac{1}{2}d - t_f = 4.03 - 0.465 = 3.565$ in.

$$\sigma = \frac{P}{A} + \frac{My}{I} = -\frac{90}{8.24} - \frac{(-117.3)(3.565)}{98.0} = -6.6552 \text{ ksi}$$



$$\bar{y} = \frac{1}{2}d - \frac{1}{2}t_f = 4.03 - 0.2325 = 3.7975 \text{ in.}$$

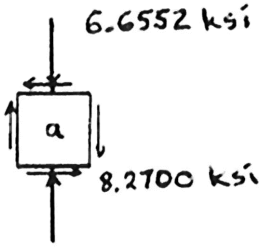
$$A_f = b_f t_f = (6.54)(0.465) = 3.0411 \text{ in}^2$$

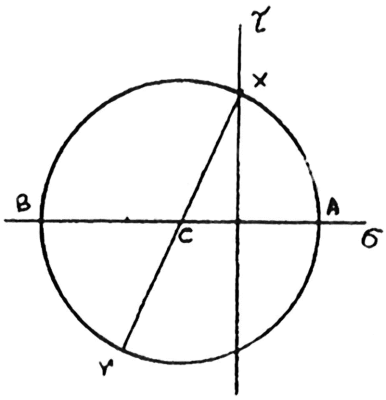
$$Q_a = A_f \bar{y} = 11.549 \text{ in}^3$$

$$\tau = \frac{VQ_A}{It_w} = \frac{(20)(11.549)}{(98.0)(0.285)} = 8.2700 \text{ ksi}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.55 *(Continued)*





$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-8.270)}{0 + 6.6552} = -2.4853$$

$$\theta_a = -34.0^\circ, \qquad \theta_b = 56.0^\circ$$

$$\sigma_{\text{ave}} = -\frac{6.6552}{2} = -3.3276 \text{ ksi}$$

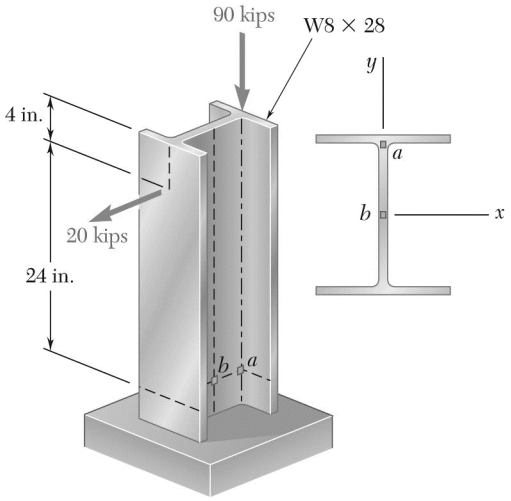
$$R = \sqrt{\left(-\frac{6.6552}{2}\right)^2 + (8.2700)^2} = 8.9144 \text{ ksi}$$

$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 5.59 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -12.24 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 8.91 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.56

Two forces are applied to a $W8 \times 28$ rolled-steel beam as shown. Determine the principal stresses and maximum shearing stress at point *b*.

SOLUTION

For W8 × 28 rolled steel section,

$$A = 8.24 \text{ in}^2, \quad d = 8.06 \text{ in.}, \quad b_f = 6.54 \text{ in.}$$

$$t_f = 0.465 \text{ in.}, \quad t_w = 0.285 \text{ in.}, \quad I_x = 98.0 \text{ in}^4$$

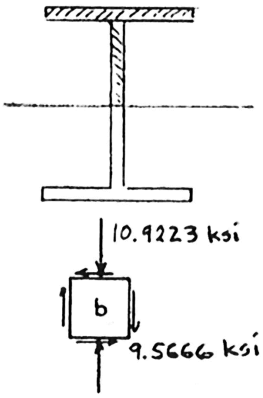
At the section containing points *a* and *b*,

$$P = -90 \text{ kips}, \quad V = 20 \text{ kips}$$

$$M = (20)(24) - (4.03)(90) = -117.3 \text{ kip} \cdot \text{in}$$

Point *b* lies on the neutral axis of bending.

At point *b*, $\sigma = \dfrac{P}{A} = \dfrac{-90}{8.24} = -10.9223 \text{ ksi}$



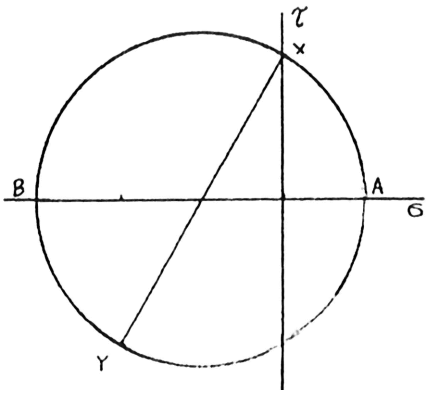
Part	A (in^2)	$\bar{y}$ (in.)	$A\bar{y}$ (in^3)
Flange	3.0411	3.7975	11.5486
Half-web	1.01603	1.7825	1.81107
Σ			13.3597

$Q_b = 13.3597 \text{ in}^3$

$$\tau = \frac{VQ_b}{It_w} = \frac{(20)(13.3597)}{(98.0)(0.285)} = 9.5666 \text{ ksi}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.56 *(Continued)*



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-9.5666)}{0 + 10.9223} = -1.75176$$

$$\theta_a = -30.1^\circ \qquad \theta_b = 59.9^\circ$$

$$\sigma_{\text{ave}} = -\frac{10.9223}{2} = -5.4612 \text{ ksi}$$

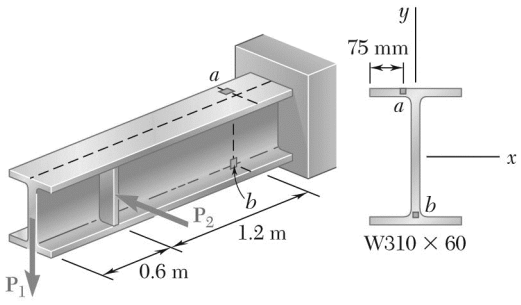
$$R = \sqrt{\left(-\frac{10.9223}{2}\right)^2 + (9.5666)^2} = 11.0156 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R \qquad \sigma_{\max} = 5.55 \text{ ksi} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R \qquad \sigma_{\min} = -16.48 \text{ ksi} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 11.02 \text{ ksi} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.57

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $\mathbf{P}_1 = 25$ kN and $\mathbf{P}_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *a*.

SOLUTION

At the section containing points *a* and *b*,

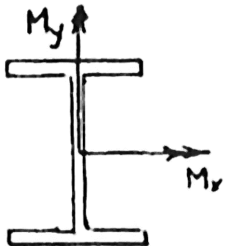
$$M_x = (1.8)(25) = 45 \text{ kN} \cdot \text{m}$$
$$M_y = -(1.2)(24) = -28.8 \text{ kN} \cdot \text{m}$$
$$V_x = -24 \text{ kN} \qquad V_y = -25 \text{ kN}$$

For W310 × 60 rolled steel section,

$$d = 302 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.49 \text{ mm}$$
$$I_x = 128 \times 10^6 \text{ mm}^4 = 128 \times 10^{-6} \text{ m}^4, \quad I_y = 18.4 \times 10^6 \text{ mm}^4 = 18.4 \times 10^{-6} \text{ m}^4$$

Normal stress at point *a*: $x = -\dfrac{b_f}{2} + 75 = -26.5 \text{ mm}$

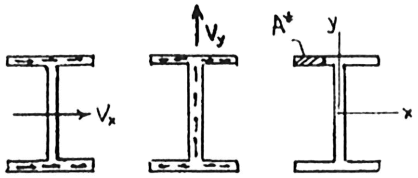
$$y = \frac{1}{2}d = 151 \text{ mm}$$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(151 \times 10^{-3})}{128 \times 10^{-6}} - \frac{(-28.8 \times 10^3)(-26.5 \times 10^{-3})}{18.4 \times 10^{-6}}$$
$$= 53.086 \text{ MPa} - 41.478 \text{ MPa} = 11.608 \text{ MPa}$$

Shearing stress at point *a*:

$$\tau_{xz} = -\frac{V_x A^* \overline{x}}{I_y t_f} - \frac{V_y A^* \overline{y}}{I_x t_f}$$

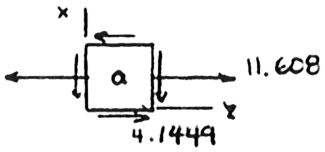


$$A^* = (75 \times 10^{-3})(13.1 \times 10^{-3}) = 982.5 \times 10^{-6} \text{ m}^2$$
$$\overline{x} = -\frac{b_f}{2} + \frac{75}{2} = -64 \text{ mm}$$
$$\overline{y} = \frac{d}{2} - \frac{t_f}{2} = 144.45 \text{ mm}$$

$$\tau_{xz} = -\frac{(-24 \times 10^3)(982.5 \times 10^{-6})(-64 \times 10^{-3})}{(18.4 \times 10^{-6})(13.1 \times 10^{-3})} - \frac{(-25 \times 10^3)(982.5 \times 10^{-6})(144.45 \times 10^{-3})}{(128 \times 10^{-6})(13.1 \times 10^{-3})}$$
$$= -6.2609 \text{ MPa} + 2.1160 \text{ MPa} = -4.1449 \text{ MPa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.57 *(Continued)*



$$\sigma_{ave} = \frac{11.608}{2} = 5.804 \text{ MPa}$$

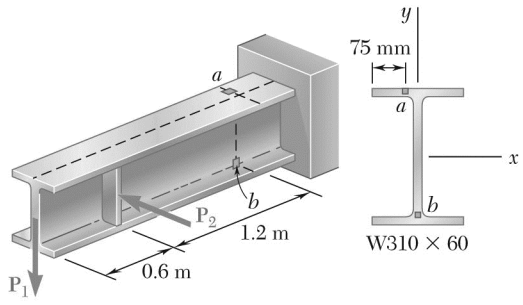
$$R = \sqrt{\left(\frac{11.608}{2}\right)^2 + (4.1449)^2} = 7.1321 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 12.94 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -1.33 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 7.13 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.58

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $\mathbf{P}_1 = 25$ kN and $\mathbf{P}_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point *b*.

SOLUTION

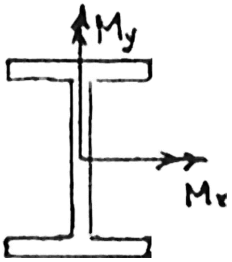
At the section containing points *a* and *b*,

$$M_x = (1.8)(25) = 45 \text{ kN} \cdot \text{m}$$
$$M_y = -(1.2)(24) = -28.8 \text{ kN} \cdot \text{m}$$
$$V_x = -24 \text{ kN}, \qquad V_y = -25 \text{ kN}$$

For W310 × 60 rolled steel section,

$$d = 302 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.49 \text{ mm}$$
$$I_x = 128 \times 10^6 \text{ mm}^4 = 128 \times 10^{-6} \text{ m}^4, \quad I_y = 18.4 \times 10^6 \text{ mm}^4 = 18.4 \times 10^{-6} \text{ m}^4$$

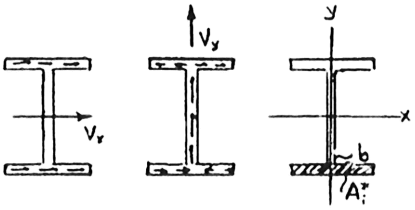
Normal stress at point *b*: $x \approx 0, \quad y = -\frac{1}{2}d + t_f = -137.9 \text{ mm}$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(-137.9 \times 10^{-3})}{128 \times 10^{-6}} - 0$$
$$= -48.480 \text{ MPa}$$

Shearing stress at point *b*:

$$\tau_{yz} = -\frac{V_y A^* \bar{y}}{I_x t_w}$$

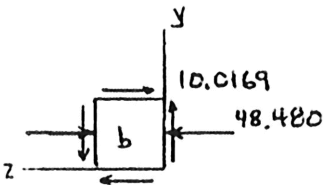


$$A^* = A_f = b_f t_f = 2659.3 \text{ mm}^2$$
$$= 2659.3 \times 10^{-6} \text{ m}^2$$
$$\bar{x} = 0, \qquad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.45 \text{ mm}$$

$$\tau_{yz} = -\frac{(-25 \times 10^3)(2659.3 \times 10^{-6})(-144.45 \times 10^{-3})}{(128 \times 10^{-6})(7.49 \times 10^{-3})} = -10.0169 \text{ MPa}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.58 *(Continued)*



$$\sigma_{ave} = -\frac{48.480}{2} = -24.240 \text{ MPa}$$

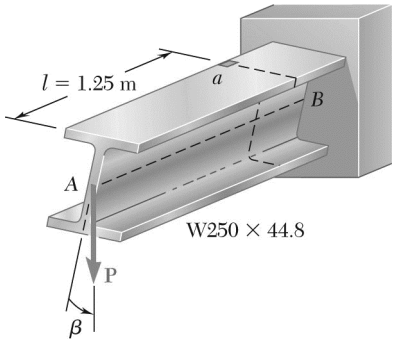
$$R = \sqrt{\left(\frac{48.48}{2}\right)^2 + (10.0169)^2} = 26.228 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 1.99 \text{ MPa} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -50.5 \text{ MPa} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 26.2 \text{ MPa} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.59

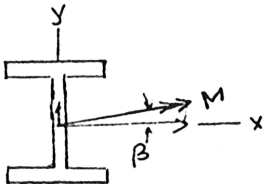
A vertical force **P** is applied at the center of the free end of cantilever beam *AB*. (*a*) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis *AB* horizontal, determine the magnitude of the force **P** for which the normal stress at point *a* is +120 MPa. (*b*) Solve part *a*, assuming that the beam is installed with $\beta = 3°$.

SOLUTION

For W250 × 44.8 rolled steel section,

$$S_x = 531 \times 10^3 \text{ mm}^3 = 531 \times 10^{-6} \text{ m}^3$$
$$S_y = 94.2 \times 10^3 \text{ mm}^3 = 94.2 \times 10^{-6} \text{ m}^3$$

At the section containing point *a*,



$$M_x = Pl\cos\beta, \qquad M_y = Pl\sin\beta$$

Stress at *a*:

$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

Allowable load. $$P_{\text{all}} = \frac{\sigma_{\text{all}}}{l}\left[\frac{\cos\beta}{S_x} + \frac{\sin\beta}{S_y}\right]^{-1}$$

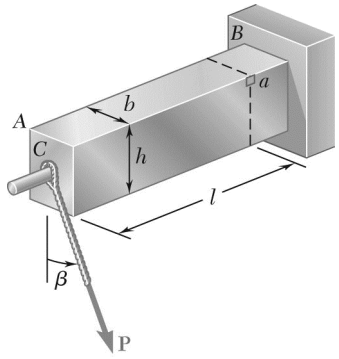
(*a*) $\beta = 0$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{1}{531 \times 10^{-6}} + 0\right]^{-1} = 51.0 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 51.0 \text{ kN} \blacktriangleleft$$

(*b*) $\beta = 3°$: $$P_{\text{all}} = \frac{120 \times 10^6}{1.25}\left[\frac{\cos 3°}{531 \times 10^{-6}} + \frac{\sin 3°}{94.2 \times 10^{-6}}\right]^{-1} = 39.4 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 39.4 \text{ kN} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.60

A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and β, (*b*) the values of β for which the normal stress at *a* is zero.

SOLUTION

$$I_x = \frac{1}{12}bh^3 \quad I_y = \frac{1}{12}hb^3$$

$$\sigma = \frac{M_x(h/2)}{I_x} - \frac{M_y(b/2)}{I_y}$$

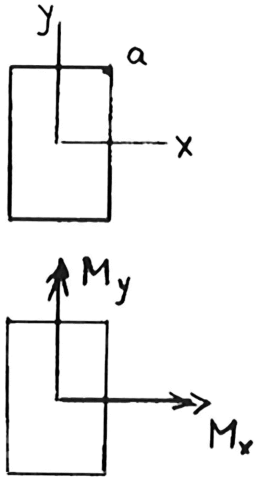
$$= \frac{6M_x}{bh^2} - \frac{6M_y}{hb^2}$$

$$\boldsymbol{P} = P\sin\beta\,\boldsymbol{i} - P\cos\beta\,\boldsymbol{j} \quad \boldsymbol{r} = l\,\boldsymbol{k}$$

$$\boldsymbol{M} = \boldsymbol{r}\times\boldsymbol{P} = l\,\boldsymbol{k}\times(P\sin\beta\boldsymbol{i} - P\cos\beta\,\boldsymbol{j})$$

$$= Pl\cos\beta\,\boldsymbol{i} + Pl\sin\beta\,\boldsymbol{j}$$

$$M_x = Pl\cos\beta \qquad M_y = Pl\sin\beta$$

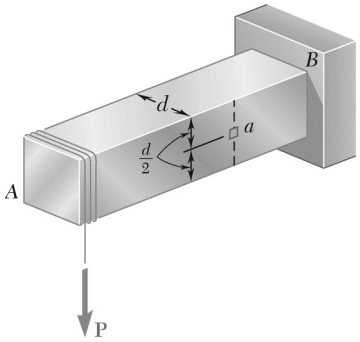


(*a*) $$\sigma = \frac{6\,Pl\cos\beta}{bh^2} - \frac{6\,Pl\sin\beta}{hb^2} = \frac{6\,Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b}\right]$$ ◀

(*b*) $$\sigma = 0 \qquad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \qquad \tan\beta = \frac{b}{h}$$

$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.61*

A 5-kN force **P** is applied to a wire that is wrapped around bar *AB* as shown. Knowing that the cross section of the bar is a square of side $d = 40$ mm, determine the principal stresses and the maximum shearing stress at point *a*.

SOLUTION

Bending: Point *a* lies on the neutral axis.

$$\sigma = 0$$

Torsion:
$$\tau = \frac{T}{c_1 a b^2} \quad \text{where} \quad a = b = d$$

and $c_1 = 0.208$ for a square section.

Since
$$T = \frac{Pd}{2}, \quad \tau_T = \frac{P}{0.416\, d^2} = 2.404\frac{P}{d^2}.$$

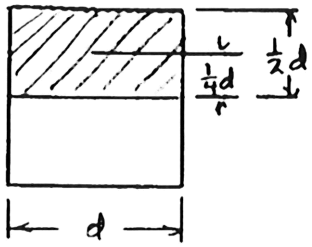
Transverse shear:

$$V = P \qquad I = \frac{1}{12}d^4$$

$$A = \frac{1}{2}d^2 \qquad \bar{y} = \frac{1}{4}d \qquad Q = A\bar{y} = \frac{1}{8}d^3$$

$$t = d$$

$$\tau_V = \frac{VQ}{It} = 1.5\frac{P}{d^2}$$

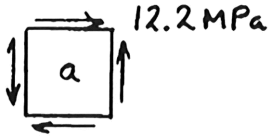


By superposition,

$$\tau = \tau_T + \tau_V = 3.904\frac{P}{d^2}$$

$$\tau = \frac{(3.904)(5\times10^3)}{(40\times10^{-3})^2}$$

$$= 12.2\times10^6 \text{ Pa} \qquad 12.2 \text{ MPa}.$$



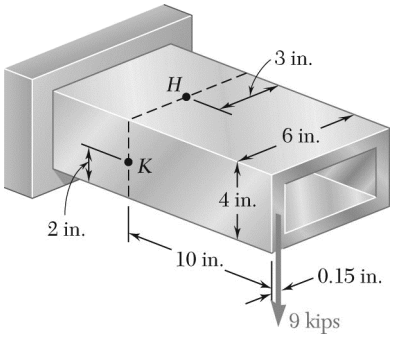
By Mohr's circle,

$\sigma_{max} = 12.2$ MPa ◀

$\sigma_{min} = -12.2$ MPa ◀

$\tau_{max} = 12.2$ MPa ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.62*

Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (*a*) point *H*, (*b*) point *K*.

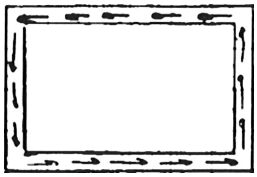
SOLUTION

At the section containing points *H* and *K*,

$$V = 9 \text{ kips} \quad M = (9)(10) = 90 \text{ kip} \cdot \text{in}$$

$$T = (9)(3 - 0.15) = 25.65 \text{ kip} \cdot \text{in}$$

Torsion:



$$\mathcal{A} = (5.7)(3.7) = 21.09 \text{ in}^2$$

$$\tau = \frac{T}{2t\mathcal{A}} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$

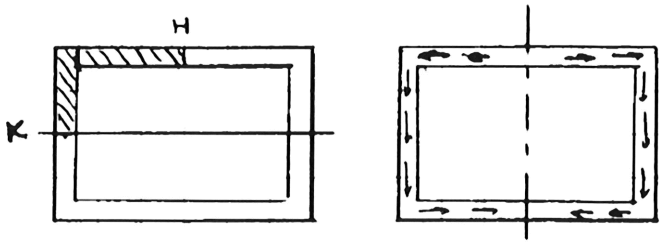
Transverse shear:

$$Q_H = 0$$

$$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85) = 2.0985 \text{ in}^3$$

$$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132 \text{ in}^4$$

$$\tau_H = 0 \qquad \tau_K = \frac{VQ_Q}{It} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398 \text{ ksi}$$



Bending:

$$\sigma_H = \frac{Mc}{I} = \frac{(90)(2)}{14.3132} = 12.576 \text{ ksi}, \qquad \sigma_K = 0$$

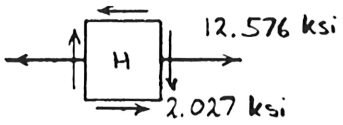
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.62* *(Continued)*

(*a*) Point *H*:

$$\sigma_{ave} = \frac{12.576}{2} = 6.288 \text{ ksi}$$

$$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607 \text{ ksi}$$



$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 12.90 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = -0.32 \text{ ksi} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma} = -0.3224 \qquad \theta_p = -8.9°,\ 81.1° \blacktriangleleft$$

$$\tau_{max} = R = 6.61 \text{ ksi} \qquad 6.61 \text{ ksi} \blacktriangleleft$$

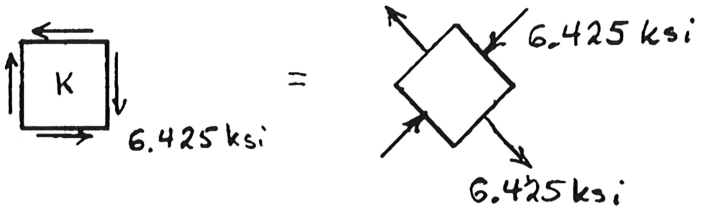
(*b*) Point *K*: $\sigma = 0 \quad \tau = 2.027 + 4.398 = 6.425$ ksi

$\sigma_{max} = 6.43$ ksi ◀

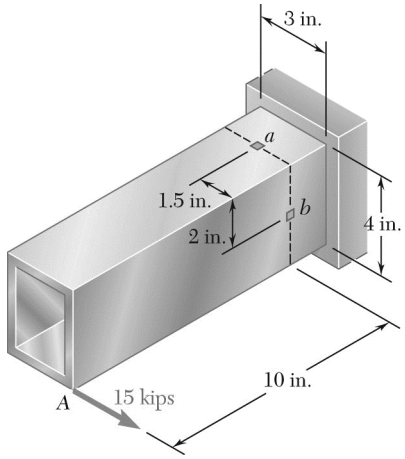
$\sigma_{min} = -6.43$ ksi ◀

$\theta = \pm 45°$ ◀

$\tau_{max} = 6.43$ ksi ◀



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



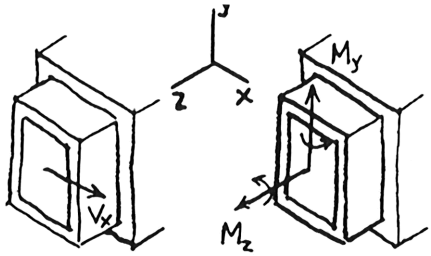
PROBLEM 8.63*

The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Calculate forces and couples at section containing points *a* and *b*.

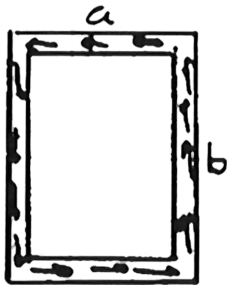
$$V_x = 15 \text{ kips}$$
$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip} \cdot \text{in}$$
$$M_y = (10)(15) = 150 \text{ kip} \cdot \text{in}$$



Forces Couples

Shearing stresses due to torque $T = M_z$:

$$\mathcal{A} = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$
$$q = \frac{M_z}{2\mathcal{A}} = \frac{27.75}{(2)(9.99)} = 1.3889 \text{ kip/in}$$



At point *a*, $t = 0.3$ in. $\tau_a = \dfrac{q}{t} = \dfrac{1.3889}{0.3} = 4.630 \text{ ksi} \leftarrow$

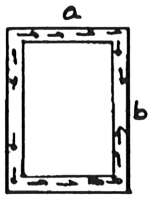
At point *b*, $t = 0.3$ in. $\tau_b = \dfrac{q}{t} = \dfrac{1.3889}{0.3} = 4.630 \text{ ksi} \uparrow$

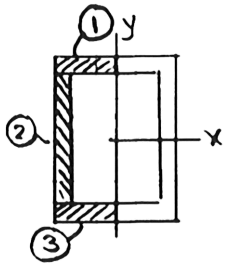
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.63* *(Continued)*

Shearing stresses due to V_x :

At point *a*,





Part	$A(\text{in}^2)$	$\bar{x}(\text{in.})$	$A\bar{x}(\text{in}^3)$
①	0.45	–0.75	–0.3375
②	1.02	–1.35	–1.377
③	0.45	–0.75	–0.3375
Σ			–2.052

$$Q = \left|\Sigma A\bar{x}\right| = 2.052 \text{ in}^3$$

$$t = (2)(0.3) = 0.6 \text{ in.}$$

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832$$

$$\tau = \frac{V_x Q}{I_y t} = \frac{(15)(2.052)}{(5.0832)(0.6)} = 10.092 \text{ ksi}$$

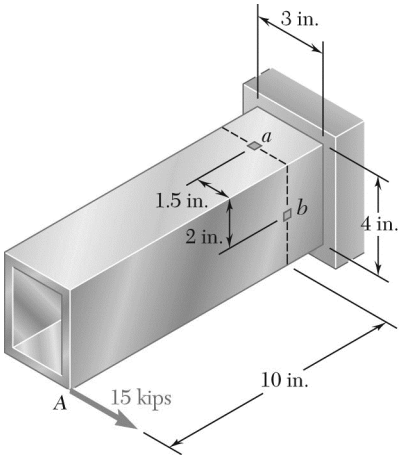
At point *b*, $\tau_b = 0$

Combined shearing stresses.

(*a*) At point *a*. $\tau_a = 4.630 \leftarrow +10.092 \rightarrow = 5.46 \text{ ksi} \rightarrow$ $\tau_a = 5.46 \text{ ksi}$ ◀

(*b*) At point *b*. $\tau_b = 4.630 \uparrow + 0 = 4.63 \text{ ksi} \uparrow$ $\tau_b = 4.63 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.64*

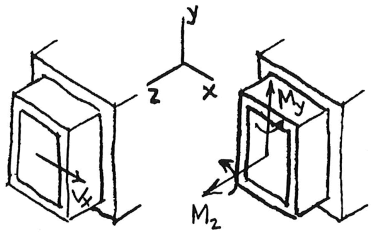
For the tube and loading of Prob. 8.63, determine the principal stresses and the maximum shearing stress at point *b*.

PROBLEM 8.63* The structural tube shown has a uniform wall thickness of 0.3 in. Knowing that the 15-kip load is applied 0.15 in. above the base of the tube, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

SOLUTION

Calculate forces and couples at section containing point *b*.

$$V_x = 15 \text{ kips}$$
$$M_z = (2 - 0.15)(15) = 27.75 \text{ kip} \cdot \text{in}$$
$$M_y = (10)(15) = 150 \text{ kip} \cdot \text{in}$$

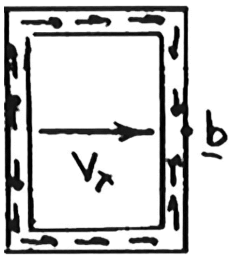


Forces Couples

$$I_y = \frac{1}{12}(4)(3)^3 - \frac{1}{12}(3.4)(2.4)^3 = 5.0832 \text{ in}^4$$
$$\sigma_b = -\frac{M_y x_b}{I_y} = -\frac{(150)(1.5)}{5.0832} = -44.26 \text{ ksi}$$

Shearing stress at point *b* due to torque M_z:

$$\mathcal{A} = [3 - (2)(0.15)][4 - (2)(0.15)] = 9.99 \text{ in}^2$$
$$q = \frac{M_z}{2\mathcal{A}} = \frac{27.75}{(2)(9.99)} = 1.38889 \text{ kip/in}$$
$$\tau = \frac{q}{t} = \frac{1.38889}{0.3} = 4.630 \text{ ksi}$$



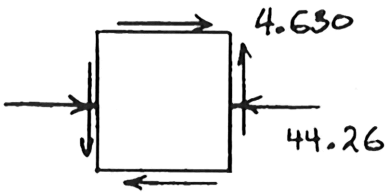
Shearing at point *b* due to V_x:

$$\tau = 0$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.64* *(Continued)*

Calculation of principal stresses and maximum shearing stress.



$$\sigma_{ave} = \frac{-44.26 + 0}{2} = -22.13 \text{ ksi}$$

$$R = \sqrt{\left(-\frac{44.26 - 0}{2}\right)^2 + (4.630)^2} = 22.61 \text{ ksi}$$

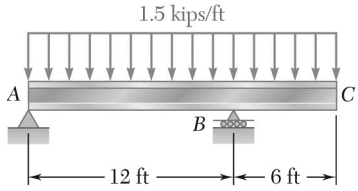
$$\sigma_{max} = \sigma_{ave} + R \qquad \sigma_{max} = 0.48 \text{ ksi} \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R \qquad \sigma_{min} = 44.7 \text{ ksi} \blacktriangleleft$$

$$\tau_{max} = R \qquad \tau_{max} = 22.6 \text{ ksi} \blacktriangleleft$$

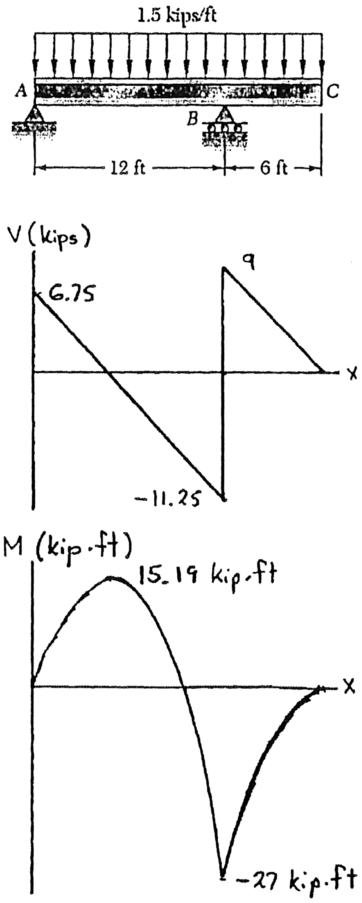
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.65



(*a*) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad -12R_A + (1.5)(18)(3) = 0 \quad R_A = 6.75 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 12R_B + (1.5)(18)(9) = 0 \quad R_B = 20.25 \text{ kips} \uparrow$$

$$|V|_{max} = 11.25 \text{ kips}$$

$$|M|_{max} = 27 \text{ kip} \cdot \text{ft} = 324 \text{ kip} \cdot \text{in}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

	Shape	$S(\text{in}^3)$
	$W12 \times 16$	17.1
$\rightarrow$	$W10 \times 15$	13.8
	$W8 \times 18$	15.2
	$W6 \times 20$	13.4

(*a*) Use $W10 \times 15$. ◀

$$d = 10.0 \text{ in.}$$

$$t_f = 0.270 \text{ in.}$$

$$t_w = 0.230 \text{ in.}$$

(*b*) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.478 \text{ ksi}$$

$$\sigma_m = 23.5 \text{ ksi} \blacktriangleleft$$

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{11.25}{(10.0)(0.230)} = 4.8913 \text{ ksi}$$

$$\tau_m = 4.89 \text{ ksi} \blacktriangleleft$$

$$c = \frac{1}{2}d = \frac{10.0}{2} = 5.00 \text{ in.}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.65 *(Continued)*

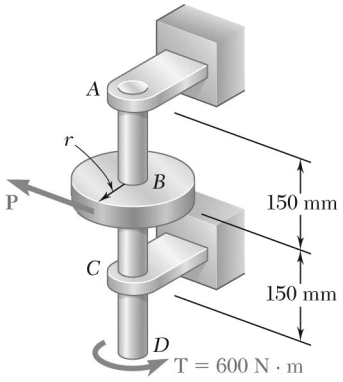
$$y_b = c - t_f = 5.00 - 0.270 = 4.73 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.73}{5.00}\right)(23.478) = 22.210 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.210}{2}\right)^2 + (4.8913)^2} = 12.1345 \text{ ksi}$$

$$\sigma_{\max} = \frac{\sigma_b}{2} + R = \frac{22.210}{2} + 12.1345 \qquad \sigma_{\max} = 23.2 \text{ ksi} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.66

Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{all} = 60$ MPa and that the radius of disk *B* is $r = 80$ mm.

SOLUTION

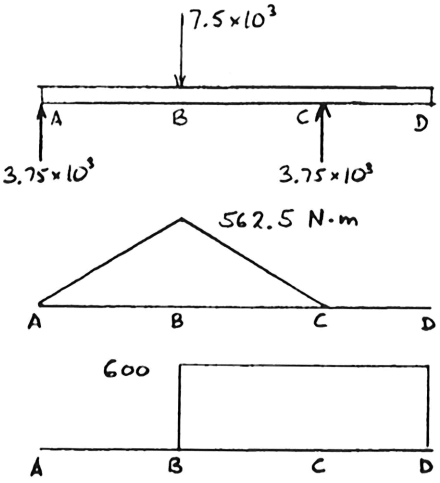
$$\Sigma M_{axis} = 0: \quad T - Pr = 0 \quad P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^{3}$$

$$R_A = R_C = \frac{1}{2}P$$

$$= 3.75 \times 10^{3}\,\text{N}$$

$$M_B = (3.75 \times 10^{3})(150 \times 10^{-3})$$

$$= 562.5\ \text{N} \cdot \text{m}$$



Bending moment: (See sketch).

Torque: (See sketch).

Critical section lies at point *B*.

$$M = 562.5\ \text{N} \cdot \text{m}, \qquad T = 600\ \text{N} \cdot \text{m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^{6}}$$

$$= 8.726 \times 10^{-6}\,\text{m}^3$$

$$c = 20.58 \times 10^{-3}\,\text{m} \qquad d = 2c = 41.2 \times 10^{-3}\,\text{m}$$

$$d = 41.2\ \text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.67

Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a circular shaft can be expressed as follows:

$$\sigma_{\max} = \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right]_{\max}$$

SOLUTION

Maximum bending stress:

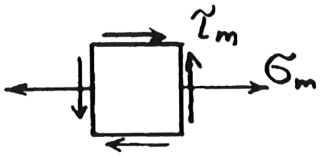
$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{I}$$

Maximum torsional stress:

$$\tau_m = \frac{Tc}{J}$$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$

Using Mohr's circle,



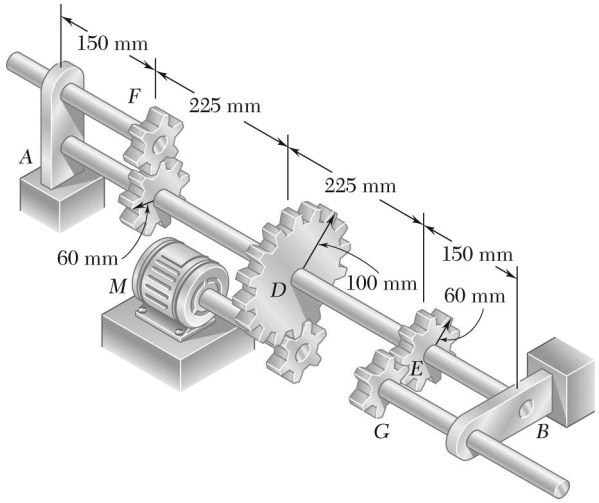
$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}\left(M_y^2 + M_z^2\right) + \frac{T^2c^2}{J^2}}$$

$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$= \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{1/2} + \left(M_y^2 + M_z^2 + T^2\right)^{1/2}\right] \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

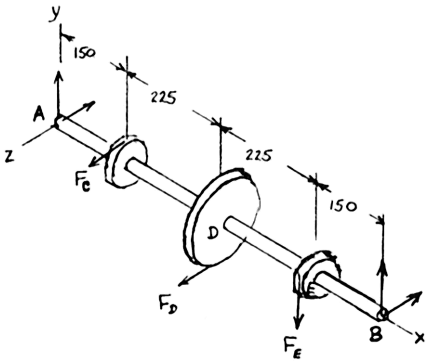


PROBLEM 8.68

The solid shaft *AB* rotates at 450 rpm and transmits 20 kW from the motor *M* to machine tools connected to gears *F* and *G*. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear *F* and 12 kW is taken off at gear *G*, determine the smallest permissible diameter of shaft *AB*.

SOLUTION

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$



Torque applied at *D*:

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)}$$
$$= 424.41 \text{ N} \cdot \text{m}$$

Torques on gears *C* and *E*:

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N} \cdot \text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N} \cdot \text{m}$$

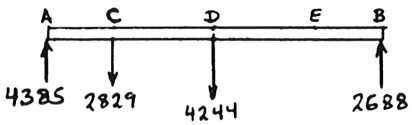
Forces on gears:

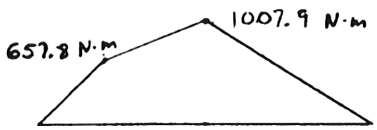
$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

Forces in horizontal plane:





Torques in various parts:

AC: $T = 0$

CD: $T = 169.76 \text{ N} \cdot \text{m}$

DE: $T = 254.65 \text{ N} \cdot \text{m}$

EB: $T = 0$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.68 *(Continued)*

Critical point lies just to the right of D.

$$T = 254.65 \text{ N} \cdot \text{m}$$

$$M_y = 1007.9 \text{ N} \cdot \text{m}$$

$$M_z = 318.3 \text{ N} \cdot \text{m}$$

$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max} = 1087.2 \text{ N} \cdot \text{m}$$

$$\tau_{\text{all}} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{\max}}{\tau_{\text{all}}}$$

$$= \frac{1087.2}{55 \times 10^6}$$

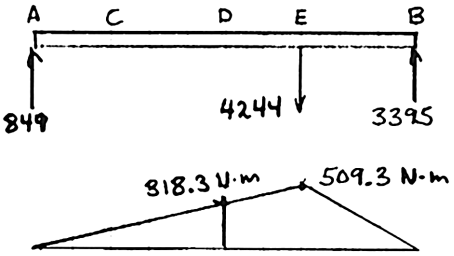
$$= 19.767 \times 10^{-3} \text{m}^3$$

$$c = 23.26 \times 10^{-3} \text{m}$$

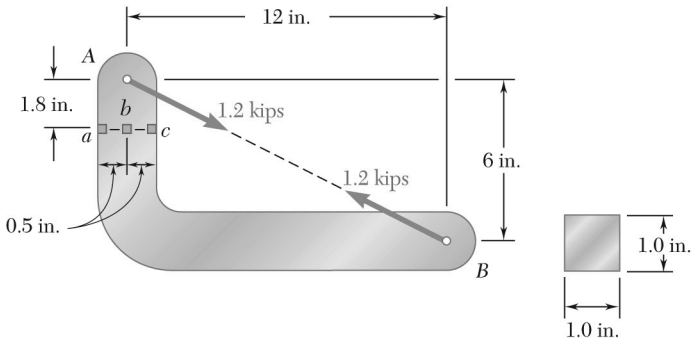
$$d = 2c = 46.5 \times 10^{-3} \text{m}$$

$$d = 46.5 \text{ mm} \blacktriangleleft$$

Forces in vertical plane:



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.69

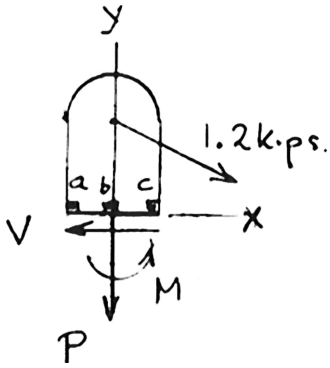
Two 1.2-kip forces are applied to an L-shaped machine element AB as shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.

SOLUTION

Let B be the slope angle of line AB.

$$\tan\beta = \frac{6}{12} \qquad \beta = 26.565°$$

Draw a free body sketch of the portion of the machine element lying above section *abc*.



$$P = -(1.2)\sin\beta$$
$$= -0.53666 \text{ kips}$$
$$V = 1.2\cos\beta = 1.07331 \text{ kips}$$
$$M = (1.8)(1.2\cos\beta) = 1.93196 \text{ kip}\cdot\text{in}$$

Section properties:

$$A = (1.0)^2 = 1.0 \text{ in}^2$$
$$I = \frac{1}{12}(1.0)(1.0)^3 = 0.083333 \text{ in}^4 \qquad c = 0.5 \text{ in.}$$

(*a*) Point *a*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = \frac{-0.53666}{1.0} - \frac{(1.93196)(-0.5)}{0.083333}$$

$\sigma = 11.06$ ksi ◀

$\tau = 0$ ◀

(*b*) Point *b*:

$$\sigma = \frac{P}{A} = \frac{-0.53666}{1.0}$$

$\sigma = -0.537$ ksi ◀

$$Q = (0.5)(1.0)(0.25) = 0.125 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(1.07331)(0.125)}{(0.083333)(1.0)}$$

$\tau = 1.610$ ksi ◀

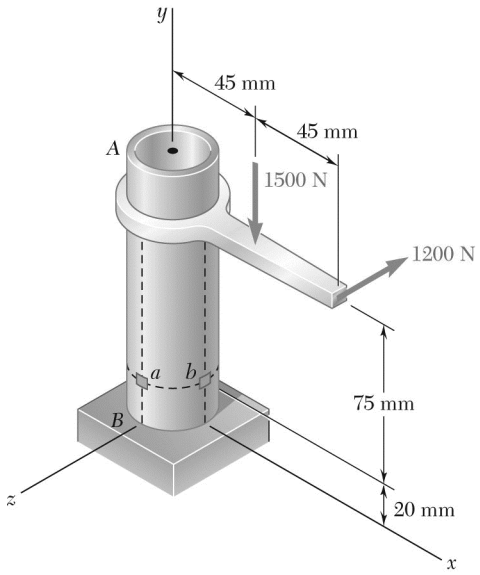
(*c*) Point *c*:

$$\sigma = \frac{P}{A} - \frac{Mx}{I} = \frac{-0.53666}{1.0} - \frac{(1.93196)(0.5)}{0.083333}$$

$\sigma = -12.13$ ksi ◀

$\tau = 0$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.70

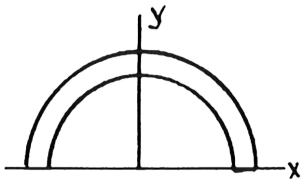
Two forces are applied to the pipe *AB* as shown. Knowing that the pipe has inner and outer diameters equal to 35 and 42 mm, respectively, determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*.

SOLUTION

$$c_0 = \frac{d_o}{2} = 21 \text{ mm}, \quad c_i = \frac{d_i}{2} = 17.5 \text{ mm} \quad A = \pi\left(c_o^2 - c_i^2\right) = 423.33 \text{ mm}^2$$

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right) = 158.166 \times 10^3 \text{mm}^4 \quad I = \frac{1}{2}J = 79.083 \times 10^3 \text{mm}^4$$

For semicircle with semicircular cutout,



$$Q = \frac{2}{3}\left(c_o^3 - c_i^3\right) = 2.6011 \times 10^3 \text{mm}^3$$

At the section containing points *a* and *b*,

$$P = -1500 \text{ N} \quad V_z = -1200 \text{ N} \quad V_x = 0$$

$$M_z = -(45 \times 10^{-3})(1500) = -67.5 \text{ N} \cdot \text{m}$$

$$M_x = -(75 \times 10^{-3})(1200) = -90 \text{ N} \cdot \text{m}$$

$$T = (90 \times 10^{-3})(1200) = 108 \text{ N} \cdot \text{m}$$

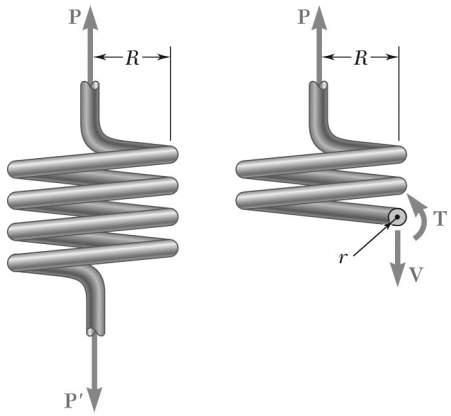
(*a*) $$\sigma = \frac{P}{A} - \frac{M_x c}{I} = \frac{-1500}{423.33 \times 10^{-6}} - \frac{(-90)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = 20.4$ MPa ◀

$$\tau = \frac{Tc}{J} + \frac{V_x Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + 0$$ $\tau = 14.34$ MPa ◀

(*b*) $$\sigma = \frac{P}{A} + \frac{M_z c}{I} = \frac{-1500}{423.33 \times 10^{-6}} + \frac{(-67.5)(21 \times 10^{-3})}{79.083 \times 10^{-9}}$$ $\sigma = -21.5$ MPa ◀

$$\tau = \frac{Tc}{J} + \frac{|V_z| Q}{It} = \frac{(108)(21 \times 10^{-3})}{158.166 \times 10^{-9}} + \frac{(1200)(2.6011 \times 10^{-6})}{(79.083 \times 10^{-9})(7 \times 10^{-3})}$$ $\tau = 19.98$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.71

A close-coiled spring is made of a circular wire of radius r that is formed into a helix of radius R. Determine the maximum shearing stress produced by the two equal and opposite forces **P** and **P′**. (*Hint:* First determine the shear **V** and the torque **T** in a transverse cross section.)

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad P - V = 0 \qquad V = P$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad T - PR = 0 \qquad T = PR$$

Shearing stress due to T.

$$\tau_T = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2PR}{\pi r^3}$$

Shearing stress due to V.

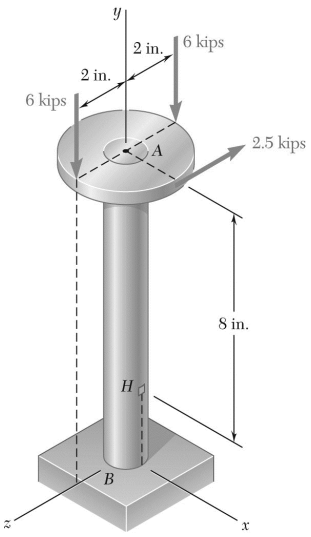
For semicircle, $$Q = \frac{2}{3}r^3, \quad t = d = 2r$$

For solid circular section, $$I = \frac{1}{2}J = \frac{\pi}{4}r^3$$

$$\tau_V = \frac{VQ}{It} = \frac{V\left(\frac{2}{3}r^3\right)}{\frac{\pi}{4}r^4(2r)} = \frac{4V}{3\pi r^2} = \frac{4P}{3\pi r^2}$$

By superposition, $$\tau_{\max} = \tau_T + \tau_V \qquad \tau_{\max} = P(2R + 4r/3)/\pi r^3 \blacktriangleleft$$

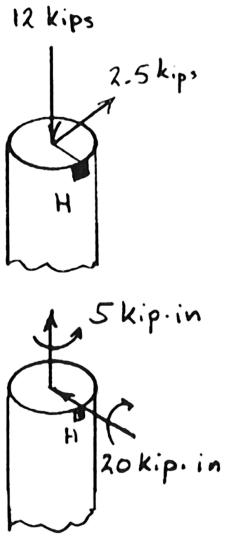
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.72

Three forces are applied to a 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft *AB*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION



At the section containing point *H*,

$$P = 12 \text{ kips} \quad \text{(compression)}$$
$$V = 2.5 \text{ kips}$$
$$T = (2)(2.5) = 5 \text{ kip} \cdot \text{in}$$
$$M = (8)(2.5) = 20 \text{ kip} \cdot \text{in}$$
$$d = 1.8 \text{ in.} \quad c = \frac{1}{2}d = 0.9 \text{ in.}$$
$$A = \pi c^2 = 2.545 \text{ in}^2$$
$$I = \frac{\pi}{4}c^4 = 0.5153 \text{ in}^4$$
$$J = 2I = 1.0306 \text{ in}^4$$

For a semicircle,

$$Q = \frac{2}{3}c^3 = 0.486 \text{ in}^3$$

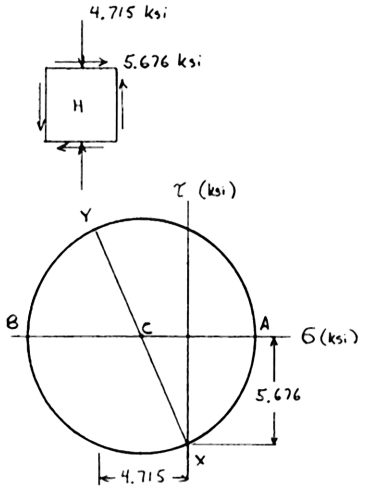
Point *H* lies on neutral axis of bending.

$$\sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715 \text{ ksi}$$
$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)}$$
$$= 5.676 \text{ ksi}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.72 *(Continued)*

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}(-4.715)$$
$$= -2.3575 \text{ ksi}$$
$$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2}$$
$$= 6.1461 \text{ ksi}$$

(*a*) $\sigma_a = \sigma_{\text{ave}} + R$ $\quad \sigma_a = 3.79 \text{ ksi}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R$ $\quad \sigma_b = -8.50 \text{ ksi}$ ◀

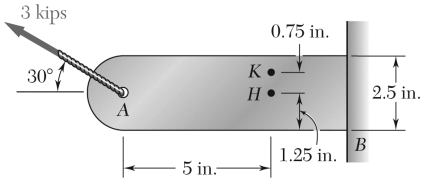
$$\tan 2\theta_p = \frac{(2)(5.676)}{4.715} = 2.408$$

$\theta_a = 33.7°$ ◀

$\theta_b = 123.7°$ ◀

(*b*) $\tau_{\max} = R$ $\quad \tau_{\max} = 6.15 \text{ ksi}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.73

Knowing that the bracket AB has a uniform thickness of $\frac{5}{8}$ in, determine (*a*) the principal planes and principal stresses at point K, (*b*) the maximum shearing stress at point K.

SOLUTION

Resolve the 3 kip force **F** at point A into x and y components.

$$F_x = -F\cos 30° = -(3)\cos 30° = -2.598 \text{ kips}$$

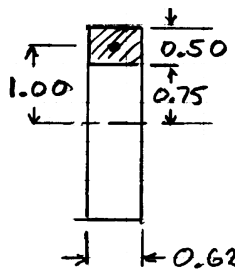
$$F_y = F\sin 30° = (3)\sin 30° = 1.5 \text{ kips}$$

At the section containing points H and K,

$$P = -F_x = 2.598 \text{ kips}, \quad V = F_y = 1.5 \text{ kips}$$

$$M = (5)(1.5) = 7.5 \text{ kip} \cdot \text{in}$$

Section properties. $t = \frac{5}{8}$ in. $= 0.625$ in., $A = \left(\frac{5}{8}\right)(2.5) = 1.5625 \text{ in}^2$,



$$I = \frac{1}{12}(0.625)(2.5)^3 = 0.8138 \text{ in}^4, \qquad c = 1.25 \text{ in.}$$

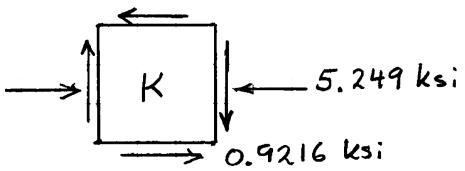
At point k, $y = 0.75$ in.

$$Q = (0.625)(0.50)(1.00) = 0.3125 \text{ in}^3$$

Stresses at point K.

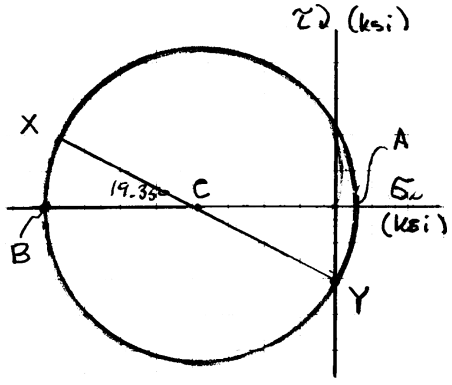
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{2.598}{1.5625} - \frac{(7.5)(0.75)}{0.8138} = -5.249 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(1.5)(0.3125)}{(0.8138)(0.625)} = 0.9216 \text{ ksi}$$



$$\sigma_x = -5.249 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = -0.9216 \text{ ksi}$$

Mohr's circle



$$X: (\sigma_x, -\tau_{xy}) = (-5.249 \text{ ksi}, 0.9216 \text{ ksi})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, -0.9216 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (-2.6245 \text{ ksi}, 0)$$

$$\frac{\sigma_x - \sigma_y}{2} = -2.6245 \text{ ksi}$$

$$R = \sqrt{(2.6245)^2 + (0.9216)^2} = 2.7816 \text{ ksi}$$

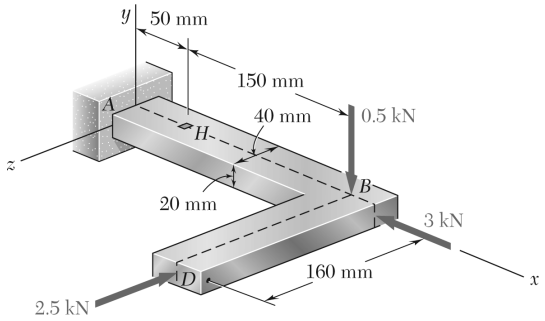
$$\tan 2\theta_p = \frac{0.9216}{2.6245} = 0.35112 \quad 2\theta_p = 19.35°$$

(*a*) $\sigma_{max} = \sigma_{ave} + R = -2.6245 + 2.7816$ $\qquad \sigma_{max} = 0.157 \text{ ksi at } 80.3°$ ◀

(*b*) $\sigma_{min} = \sigma_{ave} - R = -2.6245 - 2.7816$ $\qquad \sigma_{min} = -5.41 \text{ ksi at } 9.7°$ ◀

$\tau_{max} = R = 2.7816$ ksi $\qquad \tau_{max} = 2.78 \text{ ksi}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 8.74

Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20×40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

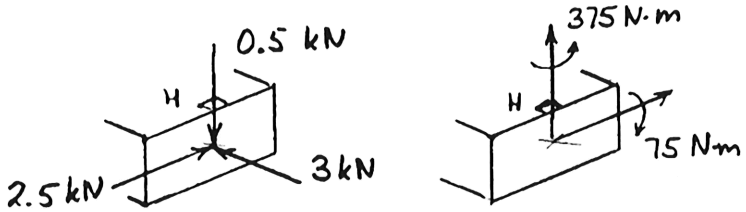
SOLUTION

Equivalent force-couple system at section containing point *H*:

$$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800\text{ mm}^2$$
$$= 800 \times 10^{-6}\text{ m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3$$
$$= 26.667 \times 10^{3}\text{ mm}^4$$
$$= 26.667 \times 10^{-9}\text{ m}^4$$

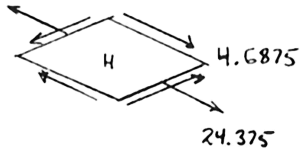
$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z}$$
$$= \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}}$$
$$= 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{2500}{800 \times 10^{-6}}$$
$$= 4.6875\text{ MPa}$$

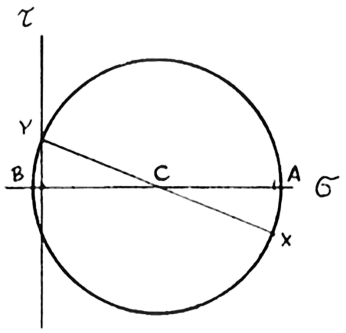
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.74 *(Continued)*

Use Mohr's circle.



$$\sigma_{\text{ave}} = \frac{1}{2}\sigma_H$$
$$= 12.1875 \text{ MPa}$$
$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2}$$
$$= 13.0579 \text{ MPa}$$



$$\sigma_a = \sigma_{\text{ave}} + R \qquad \sigma_a = 25.2 \text{ MPa} \blacktriangleleft$$

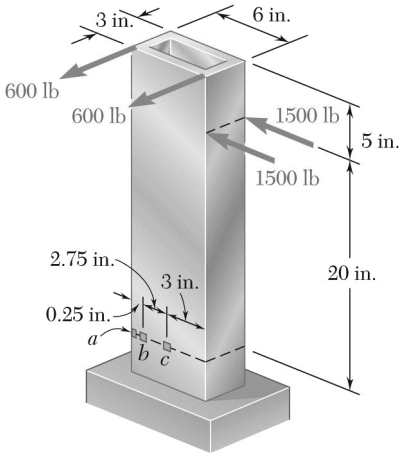
$$\sigma_b = \sigma_{\text{ave}} - R \qquad \sigma_b = -0.87 \text{ MPa} \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$

$$\theta_a = 10.5°, \quad \theta_b = 100.5°$$

$$\tau_{\max} = R \qquad \tau_{\max} = 13.06 \text{ MPa} \blacktriangleleft$$

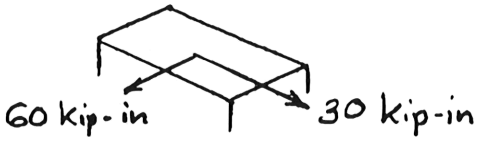
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



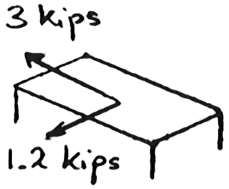
PROBLEM 8.75

Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.

SOLUTION



Bending moment



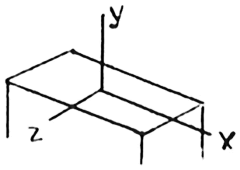
Shear

$b_o = 6$ in. $\quad b_i = b_o - 2t = 5.5$ in.

$h_o = 3$ in. $\quad h_i = h_0 - 2t = 2.5$ in.

$$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4$$

$$I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$$



Normal stresses.

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

At a: $\dfrac{(60)(-3)}{19.3385} - \dfrac{(30)(1.5)}{6.3385}$ $\qquad \sigma_a = -16.41$ ksi ◀

At b: $\dfrac{(60)(-2.75)}{19.3385} - \dfrac{(30)(1.5)}{6.3385}$ $\qquad \sigma_b = -15.63$ ksi ◀

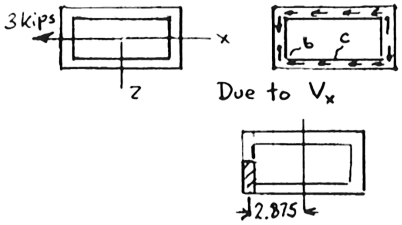
At c: $\dfrac{(60)(0)}{19.3385} - \dfrac{(30)(1.5)}{6.3385}$ $\qquad \sigma_c = -7.10$ ksi ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 8.75 *(Continued)*

Shearing stresses.

Point *a* is an outside corner. $\tau_a = 0$ ◀



At point *b*,

$$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$$

$$\tau_{b,v_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.669 \text{ ksi}$$

At point *c*,

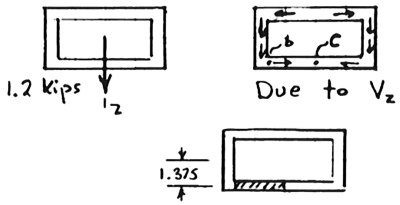
$$Q_{zc} = Q_{zb} + (2.75)(0.25)\left(\frac{2.75}{2}\right) = 2.0234 \text{ in.}^3$$

$$\tau_{c,v_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$$

At point *b*,

$$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in.}^3$$

$$\tau_{b,v_z} = \frac{V_z Q_y}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.716 \text{ ksi}$$



At point *c*, (symmetry axis)

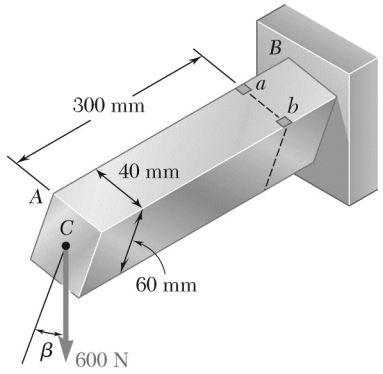
$$\tau_{c,v_z} = 0$$

Net shearing stress at points *b* and *c*:

$$\tau_b = 0.716 - 0.669 \qquad \tau_b = 0.047 \text{ ksi} \blacktriangleleft$$

$$\tau_c = 1.256 \qquad \tau_c = 1.256 \text{ ksi} \blacktriangleleft$$

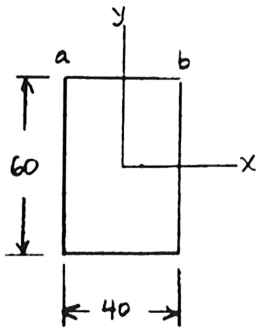
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



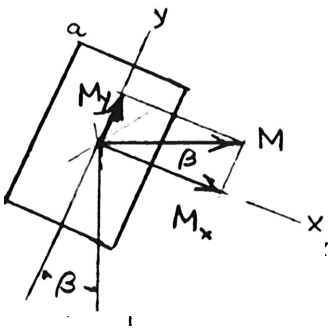
PROBLEM 8.76

The cantilever beam AB will be installed so that the 60-mm side forms an angle β between 0 and 90° with the vertical. Knowing that the 600-N vertical force is applied at the center of the free end of the beam, determine the normal stress at point a when (a) $\beta = 0$, (b) $\beta = 90°$. (c) Also, determine the value of β for which the normal stress at point a is a maximum and the corresponding value of that stress.

SOLUTION



$$S_x = \frac{1}{6}(40)(60)^2 = 24\times10^3\ \text{mm}^3$$
$$= 24\times10^{-6}\ \text{m}^3$$
$$S_y = \frac{1}{6}(60)(40)^2 = 16\times10^3\ \text{mm}^3$$
$$= 16\times10^{-3}\ \text{m}^3$$
$$M = Pl = (600)(300\times10^{-3}) = 180\ \text{N}\cdot\text{m}$$
$$M_x = M\cos\beta = 180\cos\beta$$
$$M_y = M\sin\beta = 180\sin\beta$$



$$\sigma_a = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{180\cos\beta}{24\times10^{-6}} + \frac{180\sin\beta}{16\times10^{-6}}$$
$$= (7.5\times10^6)\left(\cos\beta + \frac{3}{2}\sin\beta\right)\text{Pa}$$
$$= 7.5\left(\cos\beta + \frac{3}{2}\sin\beta\right)\text{MPa}$$

(a) $\beta = 0$. $\sigma_a = 7.50$ MPa $\sigma = 7.50$ MPa ◀

(b) $\beta = 90°$. $\sigma_a = 11.25$ MPa $\sigma = 11.25$ MPa ◀

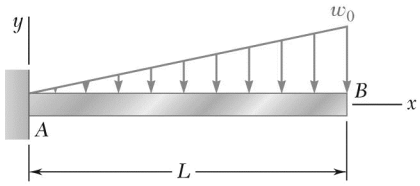
(c) Maximum. $\dfrac{d\sigma_a}{d\beta} = 7.5\left(-\sin\beta + \dfrac{3}{2}\cos\beta\right) = 0$

$\sin\beta = \dfrac{3}{2}\cos\beta \qquad \tan\beta = \dfrac{3}{2}$ $\beta = 56.3°$ ◀

$\sigma_a = 7.5\left(\cos 56.3° + \dfrac{3}{2}\sin 56.3°\right)$ $\sigma = 13.52$ MPa ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

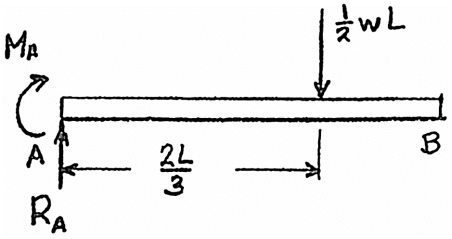
CHAPTER 9



PROBLEM 9.1

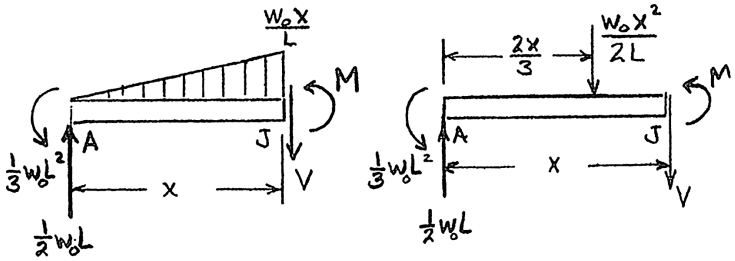
For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \qquad R_A = \frac{1}{2}wL = 0$$

$$R_A = \frac{1}{2}w_0 L$$



$$+\circlearrowleft \Sigma M_A = 0: \qquad -M_A = \frac{2L}{3}\cdot\frac{wL}{2} = 0$$

$$M_A = -\frac{1}{3}w_0L^2$$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{1}{3}w_0L^2 - \frac{1}{2}w_0Lx + \frac{w_0x^2}{2L}\cdot\frac{x}{3} + M = 0$$

$$M = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{dy}{dx} = -\frac{1}{3}w_0L^2x + \frac{1}{4}w_0Lx^2 - \frac{w_0x^4}{24L} + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \qquad 0 = -0 + 0 - 0 + C_1 \qquad C_1 = 0$$

$$EIy = -\frac{1}{6}w_0L^2x^2 + \frac{1}{12}w_0Lx^3 - \frac{w_0x^5}{120L} + C_2$$

$$[x = 0, y = 0]: \qquad 0 = -0 + 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

(*a*) Elastic curve:

$$y = -\frac{w_0}{EIL}\left(\frac{1}{6}L^3x^2 - \frac{1}{12}Lx^4 + \frac{1}{120}x^5\right) \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.1 *(Continued)*

(*b*) y at $x = L$

$$y_B = -\frac{w_0 L^4}{EI}\left(\frac{1}{6} - \frac{1}{12} + \frac{1}{120}\right) = -\frac{11}{120}\frac{w_0 L^4}{EI}$$

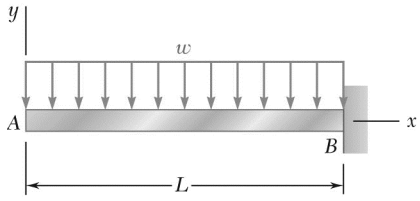
$$y_B = \frac{11}{120}\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$

(*c*) $\dfrac{dy}{dx}$ at $x = L$

$$\left.\frac{dy}{dx}\right|_B = -\frac{w_0 L^3}{EI}\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{24}\right) = -\frac{1}{8}\frac{w_0 L^4}{EI}$$

$$\theta_B = \frac{1}{8}\frac{w_0 L^3}{EI} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

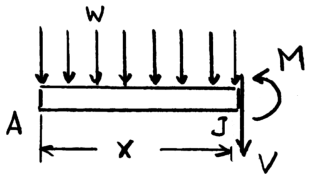


PROBLEM 9.2

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$



$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{6}wL^3 + C_1 \quad C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x = L,\ y = 0] \quad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

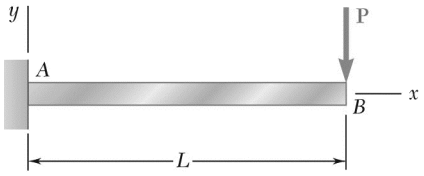
$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

(*a*) Elastic curve. $y = -\dfrac{w}{24EI}(x^4 - 4L^3x + 3L^4)$ ◀

(*b*) y at $x = 0$. $y_A = -\dfrac{3wL^4}{24EI} = -\dfrac{wL^4}{8EI}$ $y_A = \dfrac{wL^4}{8EI} \downarrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = 0$. $\left.\dfrac{dy}{dx}\right|_A = \dfrac{wL^3}{6EI}$ $\theta_A = \dfrac{wL^3}{6EI}$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



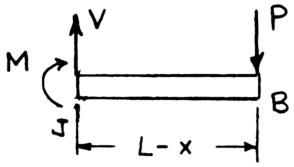
PROBLEM 9.3

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$



$$EI\frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI\frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \qquad 0 = -0 + 0 + C_1 \qquad C_1 = 0$$

$$EIy = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$$

$$[x = 0,\ y = 0]: \qquad 0 = -0 + 0 + 0 + C_2 \qquad C_2 = 0$$

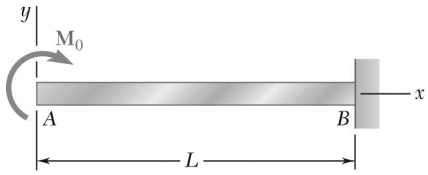
(*a*) Elastic curve. $$y = -\frac{Px^2}{6EI}(3L - x) \blacktriangleleft$$

$$\frac{dy}{dx} = -\frac{Px}{2EI}(2L - x)$$

(*b*) y at $x = L$. $$y_B = -\frac{PL^2}{6EI}(3L - L) = -\frac{PL^3}{3EI} \qquad y_B = \frac{PL^3}{3EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = L$. $$\left.\frac{dy}{dx}\right|_B = -\frac{PL}{2EI}(2L - L) = -\frac{PL^2}{2EI} \qquad \theta_B = \frac{PL^2}{2EI} \blacktriangleleft$$

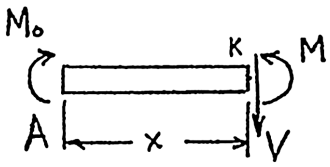
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.4

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION



$$+\circlearrowleft \sum M_K = 0: \quad -M_0 + M = 0$$

$$M = M_0$$

$$EI\frac{d^2y}{dx} = M = M_0$$

$$EI\frac{dy}{dx} = M_0x + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \qquad 0 = M_0L + C_1 \qquad C_1 = -M_0L$$

$$EIy = \frac{1}{2}M_0x^2 + C_1x + C_2$$

$$[x = L,\ y = 0] \qquad 0 = \frac{1}{2}M_0L^2 - M_0L^2 + C_2 \qquad C_2 = \frac{1}{2}M_0L^2$$

(*a*) Elastic curve: $y = \dfrac{M_0}{2EI}(x^2 - 2Lx + L^2)$ ◀

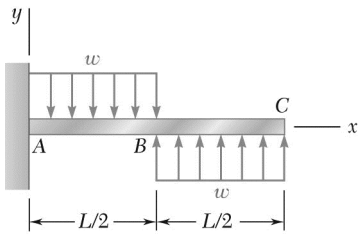
$$y = \frac{M_0}{2EI}(L - x)^2 \blacktriangleleft$$

(*b*) y at $x = 0$: $\quad y_A = \dfrac{M_0}{2EI}(L - 0)^2$ $\qquad y_A = \dfrac{M_0L^2}{2EI} \uparrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = 0$: $\quad \dfrac{dy}{dx} = -\dfrac{M_0}{EI}(L - x) = -\dfrac{M_0}{EI}(L - 0) = -\dfrac{M_0L}{EI}$

$$\theta_A = \frac{w_0L}{EI} \blacktriangleleft$$

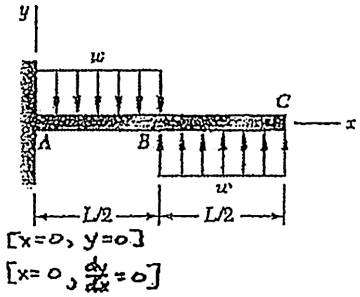
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

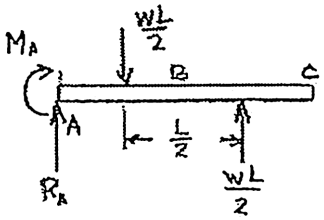


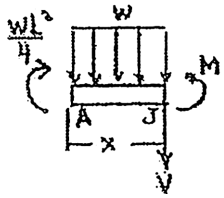
PROBLEM 9.5

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION







Using *ABC* as a free body,

$$+\uparrow \Sigma F_y = 0: \quad R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \quad R_A = 0$$

$$+\circlearrowleft M_A = 0: \quad -M_A + \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$$

Using *AJ* as a free body (*J* between *A* and *B*),

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL^2}{4} + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right]: \quad 0 = 0 - 0 + C_1 \qquad C_1 = 0$$

$$EIy = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$$

$$[x = 0, \ y = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

(*a*) Elastic curve.

$$y = \frac{w}{EI}\left(\frac{1}{8}L^2x^2 - \frac{1}{24}x^4\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{1}{4}L^2x - \frac{1}{6}x^3\right)$$

(*b*) y at $x = \frac{L}{2}$.

$$y_B = \frac{w}{EI}\left\{\frac{1}{8}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{24}\left(\frac{L}{2}\right)^4\right\} = \frac{wL^4}{EI}\left\{\frac{1}{32} - \frac{1}{384}\right\}$$

$$= \frac{11wL^4}{384EI}$$

$$y_B = \frac{11wL^4}{384EI} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.5 ***(Continued)***

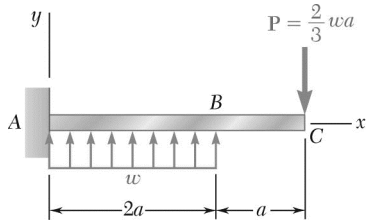
(*c*) $\dfrac{dy}{dx}$ at $x = \dfrac{L}{2}$.

$$\theta_B = \frac{w}{EI}\left\{\frac{1}{4}L^2\left(\frac{L}{2}\right) - \frac{1}{6}\left(\frac{L}{2}\right)^3\right\} = \frac{wL^3}{EI}\left\{\frac{1}{8} - \frac{1}{48}\right\}$$

$$= \frac{5wL^3}{48EI}$$

$$\theta_B = \frac{5wL^3}{48EI} \measuredangle x \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



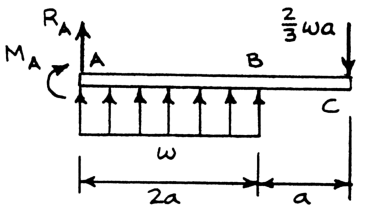
PROBLEM 9.6

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION

Using *ABC* as a free body,

FBD *ABC*:



$$+\uparrow \Sigma F_y = 0: \quad R_A + 2wa - \frac{2}{3}wa = 0$$

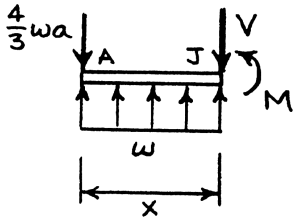
$$R_A = -\frac{4}{3}wa = \frac{4}{3}wa \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + (2wa)(a) - \left(\frac{2}{3}wa\right)(3a) = 0$$

$$M_A = 0$$

Using *AJ* as a free body,

FBD *AJ*:



$$+\circlearrowleft \Sigma M_J = 0: \quad M + \left(\frac{4}{3}wa\right)(x) - (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{2}{3}wax^2 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \quad 0 = 0 - 0 + C_1 \quad \therefore \quad C_1 = 0$$

$$EIy = \frac{1}{24}wx^4 - \frac{2}{9}wax^3 + C_2$$

$$[x = 0, y = 0]: \quad 0 = 0 - 0 + C_2 \quad \therefore \quad C_2 = 0$$

(*a*) Elastic curve over *AB*.

$$\frac{dy}{dx} = \frac{w}{6EI}(x^3 - 4ax^2)$$

$$y = \frac{w}{72EI}(3x^4 - 16ax^3) \blacktriangleleft$$

(*b*) y at $x = 2a$.

$$y_B = -\frac{10wa^4}{9EI}$$

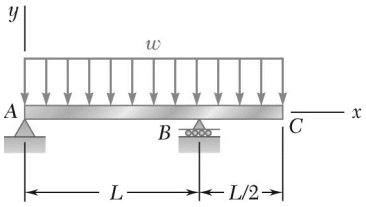
$$y_B = \frac{10wa^4}{9EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = 2a$.

$$\left(\frac{dy}{dx}\right)_B = -\frac{4wa^3}{3EI}$$

$$\theta_B = \frac{4wa^3}{3EI} \blacktriangleleft$$

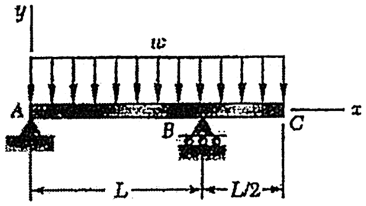
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

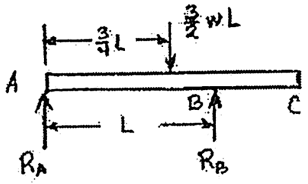


PROBLEM 9.7

For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion A*B* of the beam, (*b*) the slope at *A*, (*c*) the slope at *B*.

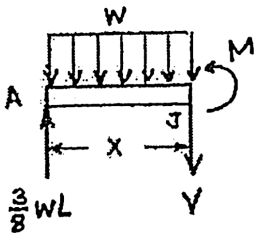
SOLUTION





$$+\circlearrowleft \Sigma M_B = 0: \qquad -R_A L + \left(\frac{3}{2}wL\right)\left(\frac{1}{4}L\right) = 0$$

$$R_A = \frac{3}{8}wL$$



For portion *AB* only, $(0 \le x < L)$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{3}{8}wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{8}wLx - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = \frac{3}{8}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{3}{16}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{16}wLx^3 - \frac{1}{24}wx^4 + C_1x + C_2$$

$[x = 0,\ y = 0]$ $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\qquad 0 = \frac{1}{16}wL^3 - \frac{1}{24}wL^4 + C_1L \qquad C_1 = -\frac{1}{48}wL^3$

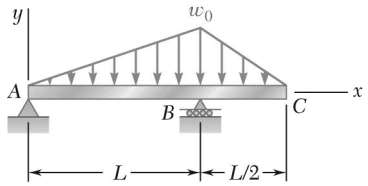
(*a*) Elastic curve. $\qquad y = \frac{w}{EI}\left(\frac{1}{16}Lx^3 - \frac{1}{24}x^4 - \frac{1}{48}L^3x\right)$ ◀

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{3}{16}Lx^2 - \frac{1}{6}x^3 - \frac{1}{48}L^3\right)$$

(*b*) $\frac{dy}{dx}$ at $x = 0$. $\qquad \left.\frac{dy}{dx}\right|_A = \frac{w}{EI}\left(0 - 0 - \frac{1}{48}L^3\right) = -\frac{wL^3}{48EI}$ $\qquad \theta_A = \frac{wL^3}{48EI}$ ↘ ◀

(*c*) $\frac{dy}{dx}$ at $x = L$. $\qquad \left.\frac{dy}{dx}\right|_B = \frac{w}{EI}\left(\frac{3}{16}L^3 - \frac{1}{6}L^3 - \frac{1}{48}L^3\right) = 0$ $\qquad \theta_B = 0$ ◀

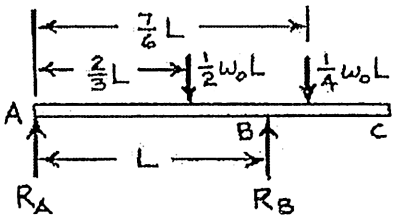
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

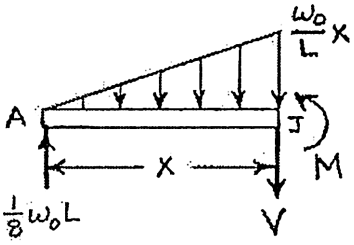


PROBLEM 9.8

For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.

SOLUTION





Reactions:

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{1}{2}w_0 L\right)\left(\frac{1}{3}L\right) - \left(\frac{1}{4}w_0 L\right)\left(\frac{1}{6}L\right) = 0$$

$$R_A = \frac{1}{8}w_0 L$$

Boundary conditions: $[x = 0, y = 0]$ $[x = L, y = 0]$

For portion *AB* only, $(0 \le x < L)$

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{8}w_0 L x + \frac{1}{2}\left(\frac{w_0}{L}x\right)(x)\left(\frac{x}{3}\right) + M = 0$$

$$M = \frac{1}{8}w_0 L x - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{8}w_0 L x - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{16}w_0 L x^2 - \frac{1}{24}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{48}w_0 L x^3 - \frac{1}{120}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$[x = 0, \ y = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L, \ y = 0]: \quad 0 = \frac{1}{48}w_0 L^4 - \frac{1}{120}w_0 L^4 + C_1 L \quad C_1 = -\frac{1}{80}w_0 L^3$

(*a*) Elastic curve.

$$y = \frac{w_0}{EIL}\left(\frac{1}{48}L^2x^3 - \frac{1}{120}x^5 - \frac{1}{80}L^4 x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{16}L^2x^2 - \frac{1}{24}x^4 - \frac{1}{80}L^4\right)$$

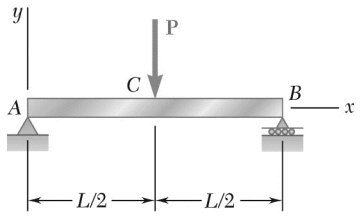
(*b*) y at $x = \frac{L}{2}$.

$$y_{L/2} = \frac{w_0}{EIL}\left(\frac{L^5}{384} - \frac{L^5}{3840} - \frac{L^5}{160}\right) = -\frac{15w_0L^4}{3840EI} \qquad y_{L/2} = \frac{w_0L^4}{256EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = L$.

$$\left.\frac{dy}{dx}\right|_B = \frac{w_0}{EIL}\left(\frac{L^4}{16} - \frac{L^4}{24} - \frac{L^4}{80}\right) = +\frac{2w_0L^3}{240EI} \qquad \theta_B = \frac{w_0L^3}{120EI} \measuredangle \blacktriangleleft$$

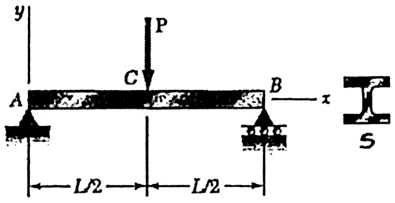
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.9

Knowing that beam AB is an $S200 \times 34$ rolled shape and that $P = 60$ kN, $L = 2$ m, and $E = 200$ GPa, determine (*a*) the slope at A, (*b*) the deflection at C.

SOLUTION



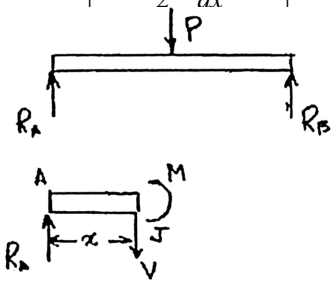
Use symmetry boundary conditions at C.

By symmetry, $\quad R_A = R_B = \frac{1}{2}P$

Using free body AJ, $\quad \left(0 \le x \le \frac{L}{2}\right)$

$[x = 0, y = 0] \qquad [x = L, y = 0]$

$\left[x = \frac{L}{2}, \frac{dy}{dx} = 0\right]$



$$+\circlearrowleft \sum M_J = 0: \quad M - R_A x = 0$$

$$M = R_A x = \frac{1}{2}Px$$

$$EI\frac{d^2y}{dy^2} = \frac{1}{2}Px$$

$$EI\frac{dy}{dx} = \frac{1}{4}Px^2 + C_1$$

$$EIy = \frac{1}{12}Px^3 + C_1 x + C_2$$

$[x = 0, \ y = 0] \qquad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = \frac{L}{2}, \frac{dy}{dx} = 0\right] \qquad 0 = \frac{1}{4}P\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{1}{16}PL^2$

Elastic curve. $\qquad y = \frac{PL}{48EI}(4x^3 - 3L^2x)$

$$\frac{dy}{dx} = \frac{PL}{16EI}(4x^2 - L^2)$$

Slope at $x = 0$. $\qquad \left.\frac{dy}{dx}\right|_A = -\frac{PL^2}{16EI} \qquad \theta_A = \frac{PL^2}{16EI}$ ↘

Deflection at $x = \frac{L}{2}$. $\qquad y_C = -\frac{PL^3}{48EI} \qquad y_C = \frac{PL^3}{48EI} \downarrow$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.9 *(Continued)*

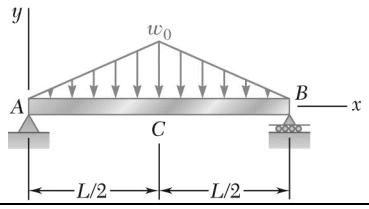
Data: $P = 60 \times 10^3$ N, $I = 26.9 \times 10^6$ mm^4 $= 26.9 \times 10^{-6}$ m^4

$E = 200 \times 10^9$ Pa $EI = 5.38 \times 10^6$ N · m^2 $L = 2$ m

(*a*) $$\theta_A = \frac{(60 \times 10^3)(2)^2}{(16)(5.38 \times 10^6)}$$ $\theta_A = 2.79 \times 10^{-3}$ rad ↘ ◀

(*b*) $$y_C = \frac{(60 \times 10^3)(2)^3}{(48)(5.38 \times 10^6)} = 1.859 \times 10^{-3} \text{ m}$$ $y_C = 1.859$ mm ↓ ◀

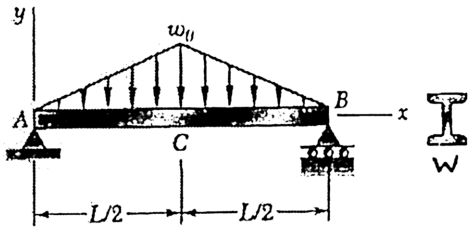
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.10

Knowing that beam AB is a $W10 \times 33$ rolled shape and that $w_0 = 3$ kips/ft, $L = 12$ ft, and $E = 29 \times 10^6$ psi, determine (*a*) the slope at A, (*b*) the deflection at C.

SOLUTION



Use symmetry boundary conditions at C.

Using free body ACB and symmetry,

$$R_A = R_B = \frac{1}{4} w_0 L$$

For $0 < x < \frac{L}{2}$, $\quad w = \frac{2w_0 x}{L}$

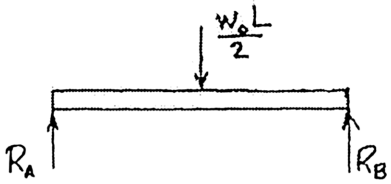
$$\frac{dV}{dx} = -w = -\frac{2w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{L} + R_A = \frac{w_0}{L}\left(\frac{1}{4}L^2 - x^2\right)$$

$$M = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right) + C_M$$

$[x = 0, \ y = 0] \quad [x = L, \ y = 0]$

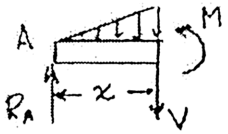
$\left[x = \frac{L}{2}, \ \frac{dy}{dx} = 0\right]$



But $M = 0$ at $x = 0$; hence $C_M = 0$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L}\left(\frac{1}{4}L^2 x - \frac{1}{3}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4\right) + C_1$$



$\left[x = \frac{L}{2}, \ \frac{dy}{dx} = 0\right]$

$$0 = \frac{w_0}{L}\left(\frac{1}{32}L^4 - \frac{1}{192}L^4\right) + C_1 = 0 \qquad C_1 = -\frac{5}{192}w_0 L^3$$

$$EIy = \frac{w_0}{L}\left(\frac{1}{24}L^2 x^3 - \frac{1}{120}x^5\right) - \frac{5}{192}w_0 L^3 x + C_2$$

$[x = 0, \ y = 0]$ $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

Elastic curve.

$$y = \frac{w_0}{EIL}\left(\frac{1}{24}L^2 x^3 - \frac{1}{60}x^5 - \frac{5}{192}L^4 x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{8}L^2 x^2 - \frac{1}{12}x^4 - \frac{5}{192}L^4\right)$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.10 *(Continued)*

Data: $w_0 = 3$ kips/ft, $E = 29 \times 10^6$ psi, $I = 171 \text{ in}^4$

$$EI = (29 \times 10^6)(171) = 4.959 \times 10^9 \text{ lb} \cdot \text{in}^2 = 34.438 \times 10^3 \text{ kip} \cdot \text{ft},$$

$$L = 12 \text{ ft}$$

(*a*) Slope at $x = 0$.

$$\frac{dy}{dx} = \frac{3}{(34.438 \times 10^3)(12)}\left[-\left(\frac{5}{192}\right)(12)^4\right] = -3.92 \times 10^3$$

$$\theta_A = 3.92 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

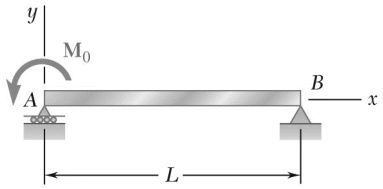
(*b*) Deflection at $x = 6$ ft.

$$y_C = \frac{3}{(34.438 \times 10^3)(12)}\left[\left(\frac{1}{24}\right)(12)^2(6)^3 - \frac{1}{60}(6)^5 - \frac{5}{192}(12)^4(6)\right] = -15.0531 \times 10^{-3} \text{ ft}$$

$$y_C = 0.1806 \text{ in.} \downarrow \blacktriangleleft$$

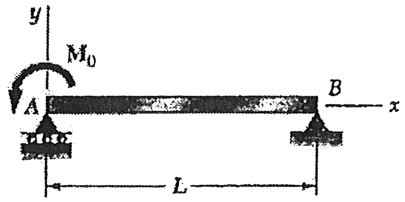
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.11



(*a*) Determine the location and magnitude of the maximum deflection of beam *AB*. (*b*) Assuming that beam *AB* is a W360 × 64, $L = 3.5$ m, and $E = 200$ GPa, calculate the maximum allowable value of the applied moment $\mathbf{M}_0$ if the maximum deflection is not to exceed 1 mm.

SOLUTION

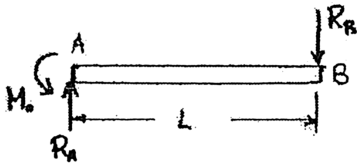


Using entire beam as a free body,

$$+\circlearrowleft \Sigma M_B = 0: \quad M_0 - R_A L = 0 \quad R_A = \frac{M_0}{L}$$

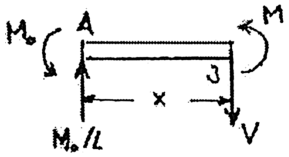
Using portion AJ,

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$ $+\circlearrowleft \Sigma M_J = 0: \quad M_0 - \dfrac{M_0}{L}x + M = 0$



$$M = \frac{M_0}{L}(x - L)$$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$



$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2 - Lx\right) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2\right) + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \dfrac{M_0}{L}\left(\dfrac{1}{6}L^3 - \dfrac{1}{2}L^3\right) + C_1 L + 0 \quad C_1 = \dfrac{1}{3}M_0 L$

$$y = \frac{M_0}{EIL}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2 + \frac{1}{3}L^2 x\right) \qquad \frac{dy}{dx} = \frac{M_0}{EIL}\left(\frac{1}{2}x^2 - Lx + \frac{1}{3}L^2\right)$$

(*a*) To find location maximum deflection, set $\dfrac{dy}{dx} = 0$.

$$\frac{1}{2}x_m^2 - Lx_m + \frac{1}{3}L^2 = 0 \qquad x_m = L - \sqrt{L^2 - (4)\left(\frac{1}{2}\right)\left(\frac{1}{3}L^2\right)} = \left(1 - \sqrt{\frac{1}{3}}\right)L$$

$$= 0.42265L \qquad x_m = 0.423L \blacktriangleleft$$

$$y_m = \frac{M_0 L^2}{EI}\left\{\left(\frac{1}{6}\right)(0.42265)^3 - \left(\frac{1}{2}\right)(0.42265)^2 + \left(\frac{1}{3}\right)(0.42265)\right\} \qquad y_m = 0.06415\frac{M_0 L^2}{EI} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.11 *(Continued)*

Solving for M_0, $$M_0 = \frac{EIy_m}{0.06415L^2}$$

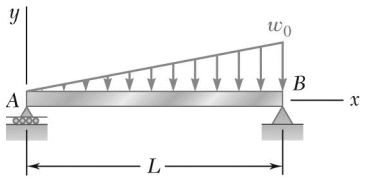
(*b*) Data: $$E = 200 \times 10^9 \text{ Pa}, \quad I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$$

$$L = 3.5 \text{ m} \quad y_m = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$M_0 = \frac{(200 \times 10^9)(178 \times 10^{-6})(10^{-3})}{(0.06415)(3.5)^2} = 45.3 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_0 = 45.3 \text{ kN} \cdot \text{m} \blacktriangleleft$$

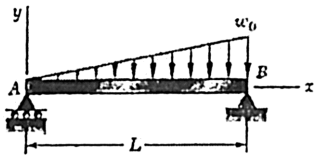
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.12

For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W18 × 50 rolled shape and that $w_0 = 4.5$ kips/ft, $L = 18$ ft, and $E = 29 \times 10^6$ psi.

SOLUTION

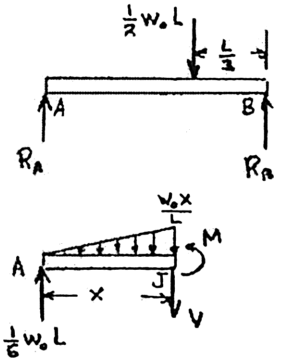


Using entire beam as a free body,

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{1}{2} w_0 L\right)\left(\frac{L}{3}\right) = 0$$

$$R_A = \frac{1}{6} w_0 L$$

$[x = 0, y = 0]$ $[x = L, y = 0]$



Using AJ as a free body,

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{6} w_0 L x + \left(\frac{1}{2}\frac{w_0 x^2}{L}\right)\left(\frac{x}{3}\right) + M = 0$$

$$M = \frac{1}{6} w_0 L x - \frac{1}{6}\frac{w_0}{L} x^3$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{6} w_0 L x - \frac{1}{6}\frac{w_0}{L} x^3$$

$$EI\frac{dy}{dx} = \frac{1}{12} w_0 L x^2 - \frac{1}{24}\frac{w_0}{L} x^4 + C_1$$

$$EIy = \frac{1}{36} w_0 L x^3 - \frac{1}{120}\frac{w_0}{L} x^5 + C_1 x + C_2$$

$[x = 0, y = 0]$ $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L, y = 0]$ $\quad 0 = \frac{1}{36} w_0 L^4 - \frac{1}{120} w_0 L^4 + C_1 L + 0 \qquad C_1 = -\frac{7 w_0 L^3}{360}$

Elastic curve.

$$y = \frac{w_0}{EI}\left\{\frac{1}{36} L x^3 - \frac{1}{120}\frac{x^5}{L} - \frac{7}{360} L^3 x\right\}$$

$$\frac{dy}{dx} = \frac{w_0}{EI}\left\{\frac{1}{12} L x^2 - \frac{1}{24}\frac{x^4}{L} - \frac{7}{360} L^3\right\}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.12 *(Continued)*

(*a*) To find location of maximum deflection, set $\dfrac{dy}{dx} = 0$.

$$15x_m^4 - 30L^2x_m^2 + 7L^4 = 0 \qquad x_m^2 = \frac{30L^2 - \sqrt{900L^4 - 420L^4}}{30}$$

$$x_m^2 = \left(1 - \sqrt{\frac{8}{15}}\right)L^2 = 0.2697L^2 \qquad x_m = 0.5193L \blacktriangleleft$$

$$y_m = \frac{w_0}{EI}\left\{\frac{1}{36}L(0.5193L)^3 - \frac{1}{120}\frac{(0.5193L^5)}{L} - \frac{7}{360}L^3(0.5193L)\right\}$$

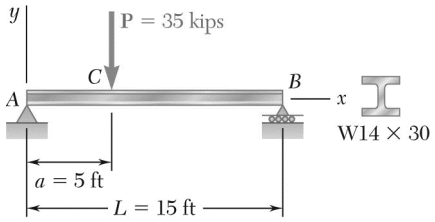
$$= -0.00652\frac{w_0L^4}{EI} \qquad y_m = 0.00652\frac{w_0L^4}{EI}\downarrow \blacktriangleleft$$

(*b*) Data: $w_0 = 4.5 \text{ kips/ft} = \dfrac{4500}{12} = 375 \text{ lb/in}, \quad L = 18 \text{ ft} = 216 \text{ in.},$

$I = 800 \text{ in}^4$ for $\text{W}18 \times 50.$

$$y_m = \frac{(0.00652)(375)(216)^4}{(29 \times 10^6)(800)} \qquad y_m = 0.229 \text{ in.}\downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.13

For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

$[x = a,\ y = y]$

$\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$

SOLUTION

Let $\quad b = L - a.$

Reactions: $\quad R_A = \dfrac{Pb}{L} \uparrow,$

$\qquad\qquad R_B = \dfrac{Pa}{L} \uparrow$

Bending moments:

$$0 < x < a: \quad M = \frac{Pb}{L}x$$

$$a < x < L: \quad M = \frac{P}{L}[bx - L(x - a)]$$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EIy = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x - a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x - a)^2\right] + C_3 \qquad (3)$$

$$EIy = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x - a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x = 0,\ y = 0]$ Eq. (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$ Eqs. (1) and (3) $\quad \dfrac{P}{L}\left(\cancel{\frac{1}{2}ba^2}\right) + C_1 = \dfrac{P}{L}\left[\cancel{\frac{1}{2}ba^2} + 0\right] + C_3 \quad \therefore \quad C_3 = C_1$

$[x = a,\ y = y]$ Eqs. (2) and (4) $\quad \dfrac{P}{L}\left(\cancel{\frac{1}{6}ba^3}\right) + \cancel{C_1a} + C_2$

$$= \frac{P}{L}\left[\cancel{\frac{1}{6}ba^3} + 0\right] + \cancel{C_1a} + C_4 \qquad C_4 = C_2 = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.13 *(Continued)*

$$[x = L,\ y = 0] \quad \text{Eq. (4):} \qquad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3 L = 0$$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq. (2).

$$y_C = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 35$ kips, $E = 29 \times 10^6$ psi

$= 29 \times 10^3$ kips/in^2

$L = 15$ ft, $a = 5$ ft, $b = 10$ ft

$I = 291$ in^4, $EI = 8.439 \times 10^6$ kip $\cdot$ in^2

$= 58.604 \times 10^3$ kip $\cdot$ ft^2

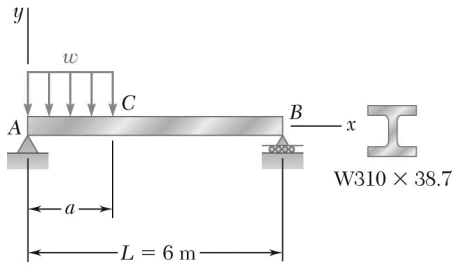
$$y_C = \frac{35}{(6)(58.604 \times 10^3)(15)}[(10)(5)^3 + (10^3)(5) - (15)^2(5)(10)]$$

$= -33.179 \times 10^{-3}$ ft $= -0.398$ in.

$y_C = 0.398$ in. $\downarrow$ ◀

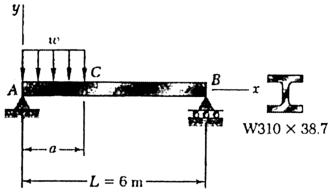
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.14



For the beam and loading shown, knowing that $a = 2$ m, $w = 50$ kN/m, and $E = 200$ GPa, determine (*a*) the slope at support *A*, (*b*) the deflection at point *C*.

SOLUTION



Using ACB as a free body and noting that $L = 3a$,

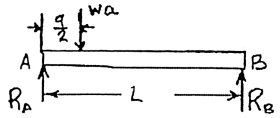
$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - (wa)\left(\frac{a}{2}\right) = 0$$

$$R_B = (wa)\frac{a}{2L} = \frac{1}{6}wa$$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

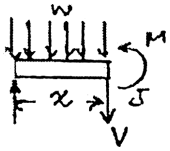
$[x = a,\ y = y]$

$\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wa = 0 \qquad R_A = \frac{5}{6}wa$$

$0 \le x \le a$



$$+\circlearrowleft \Sigma M_J = 0:$$

$$M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$$

$$M = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

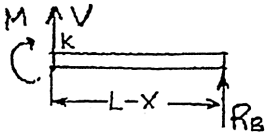
$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$a \le x \le L$



$$+\circlearrowleft \Sigma M_K = 0:$$

$$-M + R_B(L - x) = 0$$

$$M = R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = R_B(L - x)$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + C_3$$

$$EIy = \frac{1}{6}R_B(L - x)^3 + C_3 x + C_4$$

$$[x = L,\ y = 0] \quad 0 = 0 + C_3 L + C_4 \qquad C_4 = -C_3 L$$

$$EIy = \frac{1}{6}R_B(L - x)^2 - C_3(L - x)$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + C_3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.14 *(Continued)*

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right] \qquad \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + C_1 = -\frac{1}{2}R_B(2a)^2 + C_3$$

$$C_3 = C_1 + \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + \frac{1}{2}R_B(2a)^2 = C_1 + \frac{7}{12}wa^3$$

$$[x = a,\ y = y] \qquad \frac{1}{6}R_A a^3 - \frac{1}{24}wa^4 + C_1 a = \frac{1}{6}R_B(2a)^3 - \left(C_1 + \frac{7}{12}wa^3\right)(2a)$$

$$3C_1 a = -\frac{1}{6}R_A a^3 + \frac{1}{24}wa^4 + \frac{1}{6}R_B(2a)^3 - \frac{7}{12}wa^2(2a) = -\frac{25}{24}wa^4$$

$$C_1 = -\frac{25}{72}wa^3$$

For $0 \le x \le a$, $\quad EIy = \frac{5}{36}wax^3 - \frac{1}{24}wx^4 - \frac{25}{72}wa^3x$

$$EI\frac{dy}{dx} = \frac{5}{12}wax^2 - \frac{1}{6}wx^3 - \frac{25}{72}wa^3$$

Data: $\quad w = 50 \times 10^3$ N/m, $\quad a = 2$ m, $\quad E = 200 \times 10^9$ Pa

$$I = 84.9 \times 10^6\ \text{mm}^4 = 84.9 \times 10^{-6}\ \text{m}^4, \quad EI = 16.98 \times 10^6\ \text{N} \cdot \text{m}^2$$

(*a*) Slope at $x = 0$.

$$16.98 \times 10^6 \left.\frac{dy}{dx}\right|_A = 0 - 0 - \frac{25}{72}(50 \times 10^3)(2)^3$$

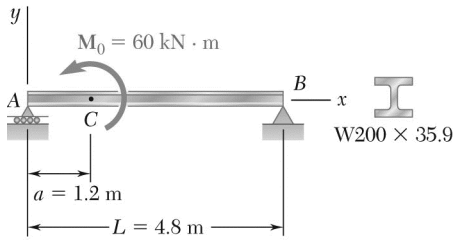
$$\left.\frac{dy}{dx}\right|_A = \theta_A = -8.18 \times 10^{-3} \qquad \theta_A = 8.18 \times 10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at $x = 2$ m.

$$EIy_C = \frac{5}{36}wa^4 - \frac{1}{24}wa^4 - \frac{25}{72}wa^4 = -\frac{1}{4}wa^4$$

$$16.98 \times 10^6 y_C = -\frac{1}{4}(50 \times 10^3)(2)^4 \quad y_C = -11.78 \times 10^{-3}\ \text{m} \qquad y_C = 11.78\ \text{mm} \downarrow \blacktriangleleft$$

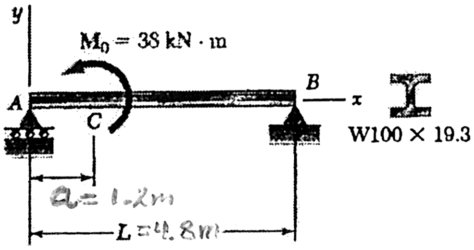
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.15

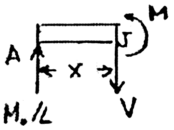
For the beam and loading shown, determine the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION



Reactions: $R_A = M_0/L \uparrow, \quad R_B = M_0/L \downarrow$

$0 < x < a$: $+\circlearrowleft \Sigma M_J = 0$:



$$-\frac{M_0}{L}x + M = 0$$

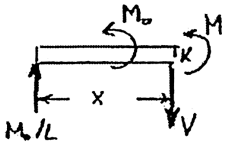
$$M = \frac{M_0}{L}x$$

$[x = 0, y = 0]$ $[x = L, y = 0]$

$[x = a, y = y]$

$\left[x = a, \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$

$a < x < L$: $+\circlearrowleft M_K = 0$:



$$-\frac{M_0}{L}x + M_0 + M = 0$$

$$M = \frac{M_0}{L}(x - L)$$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}x$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2\right) + C_1 \qquad (1)$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2 - Lx\right) + C_3 \qquad (3)$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2\right) + C_3x + C_4 \qquad (4)$$

$[x = 0, \ y = 0]$ Eq. (2): $0 = 0 + 0 + C_2$ $\qquad C_2 = 0$

$\left[x = a, \ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$ Eqs. (1) and (3): $\dfrac{M_0}{L}\left(\dfrac{1}{2}a^2\right) + C_1 = \dfrac{M_0}{L}\left(\dfrac{1}{2}a^2 - La\right) + C_3$

$$C_3 = C_1 + M_0a$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.15 *(Continued)*

$[x = a,\ y = y]$ Eqs. (2) and (4): $\dfrac{M_0}{L}\left(\dfrac{1}{6}a^3\right) + C_1 a = \dfrac{M_0}{L}\left(\dfrac{1}{6}a^3 - \dfrac{1}{2}La^2\right) + (C_1 + M_0 a)a + C_4$

$$C_4 = -\frac{1}{2}M_0 a^2$$

$[x = L,\ y = 0]$ Eq. (4): $\dfrac{M_0}{L}\left(\dfrac{1}{6}L^3 - \dfrac{1}{2}L^3\right) + (C_1 + M_0 a)L - \dfrac{1}{2}M_0 a^2 = 0$

$$C_1 = \frac{M_0}{L}\left(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL\right)$$

Elastic curve for $0 < x < a$. $\quad y = \dfrac{M_0}{EIL}\left[\dfrac{1}{6}x^3 + \left(\dfrac{1}{3}L^2 + \dfrac{1}{2}a^2 - aL\right)x\right]$

Make $x = a$. $\quad y_C = \dfrac{M_0}{EIL}\left[\dfrac{1}{6}a^3 + \dfrac{1}{3}L^2 a + \dfrac{1}{2}a^3 - a^2 L\right] = \dfrac{M_0}{EIL}\left[\dfrac{2}{3}a^3 + \dfrac{1}{3}L^2 a - La^2\right]$

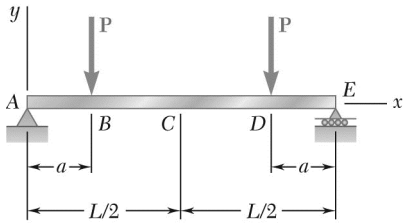
Data: $\quad E = 200 \times 10^9\ \text{Pa},\ I = 34.4 \times 10^6\ \text{mm}^4 = 34.4 \times 10^{-6}\ \text{m}^4,\ M_0 = 60 \times 10^3\ \text{N} \cdot \text{m}$

$a = 1.2\ \text{m},\ L = 4.8\ \text{m}$

$$y_C = \frac{(60 \times 10^3)\left[(2)(1.2)^3/3 + (4.8)^2(1.2)/3 - (4.8)(1.2)^2\right]}{(200 \times 10^9)(34.4 \times 10^{-6})(4.8)} = 6.28 \times 10^{-3}\ \text{m}$$

$$y_C = 6.28\ \text{mm} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.16

Knowing that beam AE is an $S200 \times 27.4$ rolled shape and that $P = 17.5$ kN, $L = 2.5$ m, $a = 0.8$ m and $E = 200$ GPa, determine (*a*) the equation of the elastic curve for portion BD, (*b*) the deflection at the center C of the beam.

SOLUTION

Consider portion ABC only. Apply symmetry about C.

Reactions: $R_A = R_E = P$

Boundary conditions: $[x = 0,\ y = 0]$, $[x = a,\ y = y]$, $\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$, $\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = 0\right]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = M = Px$$

$$EI\frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \qquad (1)$$

$$EIy = \frac{1}{6}Px^3 + C_1x + C_2 \qquad (2)$$

$$[x = 0,\ y = 0] \rightarrow C_2 = 0$$

$a < x < L - a$

$$EI\frac{d^2y}{dx^2} = M = Pa$$

$$EI\frac{dy}{dx} = Pax + C_3$$

$$EIy = \frac{1}{2}Pax^2 + C_3x + C_4$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \rightarrow C_3 = -\frac{1}{2}PaL$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right] \qquad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \qquad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$$

$$\left[x = \frac{L}{2},\ y = y\right] \qquad \frac{1}{6}Pa^3 + \left(\frac{1}{2}Pa^2 - \frac{1}{2}PaL\right)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$$

$$C_4 = \frac{1}{6}Pa^3$$

(*a*) Elastic curve for portion BD.

$$y = \frac{1}{EI}\left(\frac{1}{2}Pax^2 + C_3x + C_4\right)$$

$$y = \frac{P}{EI}\left(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3\right) \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.16 *(Continued)*

For deflection at C,

set $$x = \frac{L}{2}.$$

$$y_C = \frac{P}{EI}\left(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3\right)$$

$$= -\frac{Pa}{EI}\left(\frac{1}{8}L^2 - \frac{1}{6}a^2\right)$$

Data: $I = 23.9 \times 10^6 \text{mm}^4 = 23.9 \times 10^{-6} \text{m}^4$,

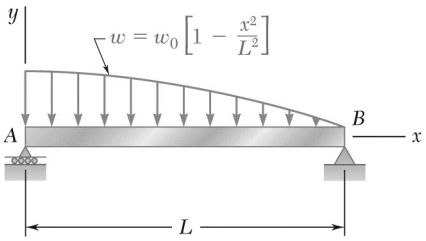
$E = 200 \times 10^9 \text{Pa}$

$P = 17.5 \times 10^3 \text{N}$,

$L = 2.5 \text{ m}, \quad a = 0.8 \text{ m}$

(*b*) $$y_C = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^6)}\left\{\frac{2.5^2}{8} - \frac{0.8^2}{6}\right\} = -1.976 \times 10^{-3} \text{m} \qquad y_C = 1.976 \text{ mm} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.17

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.

SOLUTION

$$w = w_0\left[1 - \frac{x^2}{L^2}\right] = \frac{w_0}{L^2}[L^2 - x^2]$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}[x^2 - L^2]$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left[\frac{x^3}{3} - L^2x\right] + C_1$$

$$M = \frac{w_0}{L^2}\left[\frac{1}{12}x^4 - \frac{1}{2}L^2x^2\right] + C_1x + C_2$$

$$[x = 0,\ M = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad \therefore \quad C_2 = 0$$

$$[x = L,\ M = 0]: \quad 0 = \frac{w_0}{L^2}\left[\frac{1}{12}L^4 - \frac{1}{2}L^4\right] + C_1L \qquad \therefore \quad C_1 = \frac{5}{12}w_0L$$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left[\frac{1}{12}x^4 - \frac{1}{2}L^2x^2 + \frac{5}{12}L^3x\right]$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left[\frac{1}{60}x^5 - \frac{1}{6}L^2x^3 + \frac{5}{24}L^3x^2\right] + C_3$$

$$EIy = \frac{w_0}{L^2}\left[\frac{1}{360}x^6 - \frac{1}{24}L^2x^4 + \frac{5}{72}L^3x^3\right] + C_3x + C_4$$

$$[x = 0,\ y = 0]: \quad 0 = 0 - 0 + 0 + 0 + C_4 \qquad \therefore \quad C_4 = 0$$

$$[x = L,\ y = 0]: \quad 0 = \frac{w_0}{L^2}\left[\frac{1}{360}L^6 - \frac{1}{24}L^6 + \frac{5}{72}L^6\right] + C_3L \quad \therefore \quad C_3 = -\frac{11}{360}w_0L^3$$

(*a*) Elastic curve. $y = w_0(x^6 - 15L^2x^4 + 25L^3x^3 - 11L^5x)/360\ EIL^2$ ◀

$$\frac{dy}{dx} = w_0(6x^5 - 60L^2x^3 + 75L^3x^2 - 11L^5)/360\ EIL^2$$

(*b*) Slope at end *A*. Set $x = 0$ in $\frac{dy}{dx}$. $\left.\frac{dy}{dx}\right|_A = -\frac{11}{360}\frac{w_0L^3}{EI}$ $\theta_A = \frac{11}{360}\frac{w_0L^3}{EI}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.17 *(Continued)*

(*c*) Deflection at midpoint (say, point *C*). Set $x = \dfrac{L}{2}$ in. *y*.

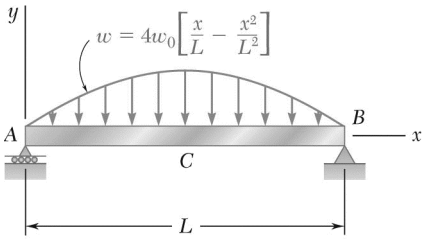
$$y_C = w_0\left(\frac{1}{64}L^6 - \frac{15}{16}L^6 + \frac{25}{8}L^6 - \frac{11}{2}L^6\right)\Big/360\,EIL^2$$

$$= w_0\left(\frac{1}{64}L^6 - \frac{60}{64}L^6 + \frac{200}{64}L^6 - \frac{352}{64}L^6\right)\Big/360\,EIL^2$$

$$= -\frac{211}{23040}\frac{w_0L^4}{EI}$$

$$y_C = 0.00916\frac{w_0L^4}{EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.18

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.

$[x = 0,\ M = 0]$ $[x = L,\ M = 0]$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

SOLUTION

Boundary conditions at *A* and *B* are noted.

$$w = \frac{w_0}{L^2}(4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}(4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left(\frac{4}{3}x^3 - 2Lx^2\right) + C_1$$

$$M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3\right) + C_1x + C_2$$

$[x = 0,\ M = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ M = 0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{3}L^4 - \frac{2}{3}L^4\right) + C_1L + 0 \qquad C_1 = \frac{1}{3}w_0L$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2\right) + C_3$$

$$EIy = \frac{w_0}{L^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3\right) + C_3x + C_4$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + 0 + C_4 \qquad C_4 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{90}L^6 - \frac{1}{30}L^6 + \frac{1}{18}L^6\right) + C_3L + 0 \qquad C_3 = -\frac{1}{30}w_0L^3$

(*a*) Elastic curve.

$$y = \frac{w_0}{EIL^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 - \frac{1}{30}L^5x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 - \frac{1}{30}L^5\right)$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.18 *(Continued)*

(*b*) Slope at end *A*. Set $x = 0$ in $\dfrac{dy}{dx}$. $\left.\dfrac{dy}{dx}\right|_A = -\dfrac{1}{30}\dfrac{w_0 L^3}{EI}$

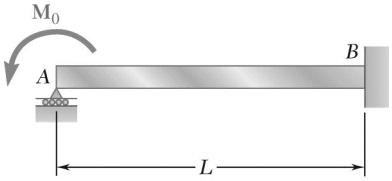
$$\theta_A = \frac{1}{30}\frac{w_0 L^3}{EI} \quad \blacktriangleleft$$

(*c*) Deflection at midpoint. Set $x = \dfrac{L}{2}$ in y.

$$y_C = \frac{w_0 L^4}{EI}\left\{\left(\frac{1}{90}\right)\left(\frac{1}{2}\right)^6 - \left(\frac{1}{30}\right)\left(\frac{1}{2}\right)^5 + \frac{1}{18}\left(\frac{1}{2}\right)^3 - \frac{1}{30}\left(\frac{1}{2}\right)\right\}$$

$$= \frac{w_0 L^4}{EI}\left\{\frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60}\right\} = -\frac{61}{5760}\frac{w_0 L^4}{EI} \qquad y_C = \frac{61}{5760}\frac{w_0 L^4}{EI} \downarrow \quad \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.19

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

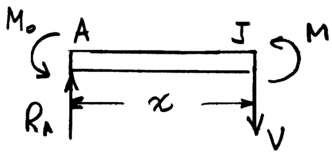
$$\left[x = L,\ \frac{dy}{dx} = 0\right]$$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *AJ*,



$$+\circlearrowleft \Sigma M_J = 0: \quad M_0 - R_A x + M = 0$$

$$M = R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0 x + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad 0 = \frac{1}{2}R_A L^2 - M_0 L + C_1$$

$$C_1 = M_0 L - \frac{1}{2}R_A L^2$$

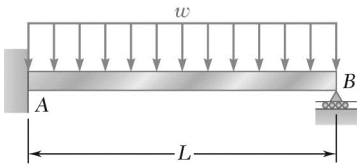
$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^3 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \qquad C_2 = 0$$

$$[x = L,\ y = 0] \quad 0 = \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + \left(M_0 L - \frac{1}{2}R_A L^2\right)L + 0$$

$$R_A = \frac{3}{2}\frac{M_0}{L} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.20

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

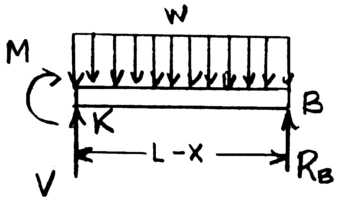
Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *KB*,

$$+\circlearrowleft M_K = 0: \quad R_B(L - x) - w(L - x)\left(\frac{L - x}{2}\right) - M = 0$$

$$M = R_B(L - x) - \frac{1}{2}w(L - x)^2$$



$$EI\frac{d^2y}{dx^2} = R_B(L - x) - \frac{1}{2}w(L - x)^2$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + \frac{1}{6}w(L - x)^3 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{2}R_BL^2 + \frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{2}R_BL^2 - \frac{1}{6}wL^3$$

$$EI_y = \frac{1}{6}R_B(L - x)^3 - \frac{1}{24}w(L - x)^4 + C_1x + C_2$$

$$[x = 0,\ y = 0]: \quad 0 = \frac{1}{6}R_BL^3 - \frac{1}{24}wL^3 + C_2$$

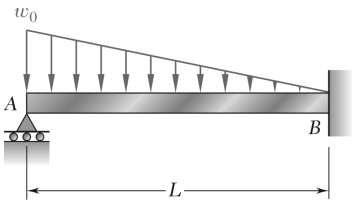
$$C_2 = -\frac{1}{6}R_BL^3 + \frac{1}{24}wL^4$$

$$[x = L,\ y = 0]: \quad 0 = 0 - 0 + C_1L + C_2$$

$$\frac{1}{2}R_BL^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_BL^3 + \frac{1}{24}wL^4 = 0$$

$$R_B = \frac{3}{8}wL \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.21

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]$$

SOLUTION

Reactions are statically indeterminate.

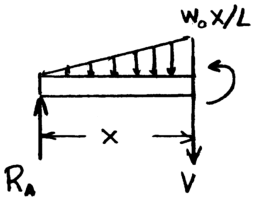
Boundary conditions are shown above.

$$w = \frac{w_0}{L}(L - x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L - x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$



$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\frac{1}{6}Lx^3 - \frac{1}{24}x^4\right) + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = -\frac{w_0}{L}\left(\frac{1}{24}Lx^4 - \frac{1}{120}x^5\right) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad -\frac{w_0}{L}\left(\frac{1}{6}L^4 - \frac{1}{24}L^4\right) + \frac{1}{2}R_A L^2 + C_1 = 0$$

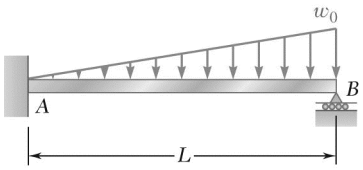
$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$$[x = L,\ y = 0] \quad -\frac{w_0}{L}\left(\frac{1}{24}L^5 - \frac{1}{120}L^5\right) + \frac{1}{6}R_A L^3 + \left(\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{8} - \frac{1}{24} + \frac{1}{120}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L \qquad R_A = \frac{11}{40}w_0 L \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.22

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $[x =$

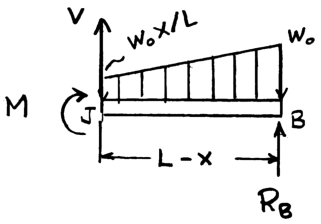
$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *JB*,



$$+\circlearrowleft \Sigma M_J = 0: \quad -M + R_B(L-x) + \frac{1}{2}w_0(L-x)\frac{2}{3}(L-x)$$

$$+\frac{1}{2}\frac{w_0x}{L}(L-x)\frac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}[2L(L-x)^2 + x(L-x)^2]$$

$$= R_B(L-x) - \frac{w_0}{6L}[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3]$$

$$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) - \frac{w_0}{6L}\left(\frac{1}{4}x^4 - \frac{3}{2}L^2x^2 + 2L^3x\right) + C_1$$

$$EIy = R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) - \frac{w_0}{6L}\left(\frac{1}{20}x^5 - \frac{1}{2}L^2x^3 + L^3x^2\right) + C_1x + C_2$$

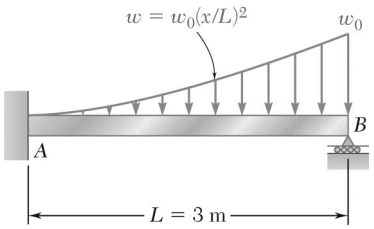
$$[x = 0,\ y = 0] \quad \rightarrow C_2 = 0$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \rightarrow C_1 = 0$$

$$[x = L,\ y = 0] \quad 0 = R_BL^3\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{w_0L^4}{6}\left(\frac{1}{20} - \frac{1}{2} + 1\right)$$

$$\frac{1}{3}R_B = \left(\frac{1}{6}\right)\left(\frac{11}{20}\right)w_0L \qquad R_B = \frac{11}{40}w_0L \uparrow \blacktriangleleft$$

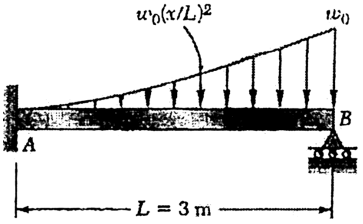
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.23

For the beam shown, determine the reaction at the roller support when $w_0 = 15$ kN/m.

SOLUTION



Reactions are statically indeterminate.

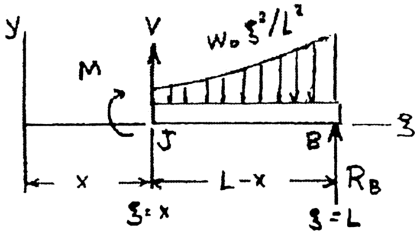
Boundary conditions are shown at left.

Using free body *JB*,

$+\circlearrowleft \Sigma M_J = 0$:

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$



$$-M + \int_x^L \frac{w_0}{L^2}\xi^2(\xi - x)d\xi + R_B(L - x) = 0$$

$$M = \frac{w_0}{L^2}\int_x^L \xi^2(\xi - x)d\xi - R_B(L - x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}\xi^4 - \frac{1}{3}x\xi^3\right)\Bigg|_x^L - R_B(L - x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L - x)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4x - \frac{1}{6}L^3x^2 + \frac{1}{60}x^5\right) - R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EIy = \frac{w_0}{L^2}\left(\frac{1}{8}L^4x^2 - \frac{1}{18}L^3x^3 + \frac{1}{360}x^6\right) - R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1x + C_2$$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$ $\quad 0 = 0 + 0 + C_1 \qquad C_1 = 0$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\quad \left(\dfrac{1}{8} - \dfrac{1}{18} + \dfrac{1}{360}\right)w_0L^4 - \left(\dfrac{1}{2} - \dfrac{1}{6}\right)R_BL^3 = 0$

$$\frac{13}{180}w_0L^4 - \frac{1}{3}R_BL^3 = 0 \qquad R_B = \frac{13}{60}w_0L$$

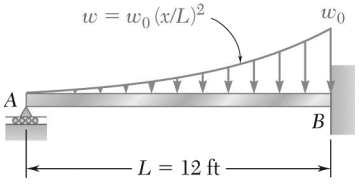
Data: $w_0 = 15$ kN/m $\qquad L = 3$ m

$$R_B = \frac{13}{60}(15)(3) = 9.75 \text{ kN}$$

$R_B = 9.75$ kN ↑ ◀

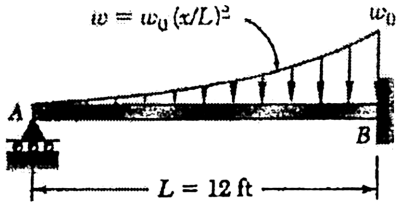
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.24



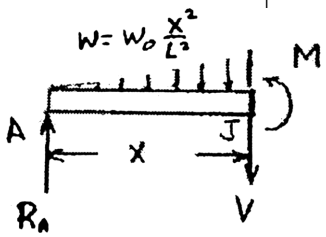
For the beam shown, determine the reaction at the roller support when $w_0 = 6$ kips/ft.

SOLUTION



$[x = 0, y = 0]$ $[x = L, y = 0]$

$\left[x = L, \dfrac{dy}{dx} = 0\right]$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2}\frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L^2}\frac{x^5}{60} + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = -\frac{w_0}{L^2}\frac{x^6}{360} + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = L,\ \dfrac{dy}{dx} = 0\right]$ $\quad -\dfrac{1}{60}w_0 L^3 + \dfrac{1}{2}R_A L^2 + C_1 = 0 \qquad C_1 = \dfrac{1}{60}w_0 L^3 - \dfrac{1}{2}R_A L^2$

$[x = L,\ y = 0]$ $\quad -\dfrac{1}{360}w_0 L^4 + \dfrac{1}{6}R_A L^3 + \left(\dfrac{1}{60}w_0 L^4 - \dfrac{1}{2}R_A L^3\right)L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{60} - \frac{1}{360}\right)w_0 L$$

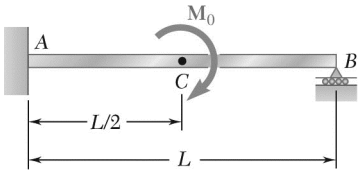
$$\frac{1}{3}R_A = \frac{1}{72}w_0 L \qquad R_A = \frac{1}{18}w_0 L$$

Data: $w_0 = 6$ kips/ft, $\quad L = 12$ ft

$$R_A = \frac{1}{18}(6)(12)$$

$R_A = 4.00$ kips ↑ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.25

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$ $[x =$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \qquad R_A + R_B = 0 \qquad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \qquad -M_A - M_0 + R_B L = 0 \qquad M_A = R_B L - M_0$$

$$\underline{0 < x < \frac{L}{2}}$$

$$M = R_B x + M_A = -M_0 + R_B L - R_B x$$

$$EI\frac{d^2y}{dx^2} = -M_0 + R_B(L - x)$$

$$EI\frac{dy}{dx} = -M_0 x + R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EIy = -\frac{1}{2}M_0 x^2 + R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1 x + C_2$$

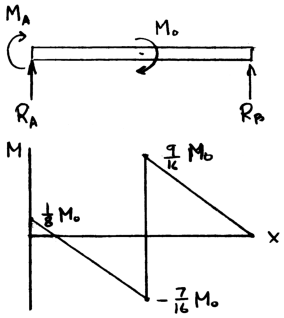
$$\underline{\frac{L}{2} < x < L}$$

$$M = R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = R_B(L - x)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) + C_3$$

$$EIy = R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_3 x + C_4$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.25 *(Continued)*

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$-M_0\frac{L}{2} + \cancel{R_B\left(\frac{1}{2}L^2 - \frac{1}{6}L^2\right)} = \cancel{R_B\left(\frac{1}{2}L^2 - \frac{1}{6}L^2\right)} + C_3 \qquad C_3 = -\frac{M_0 L}{2}$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$-\frac{1}{2}M_0\left(\frac{L}{2}\right)^2 + \cancel{R_B\left(\frac{1}{8}L^3 - \frac{1}{48}L^3\right)} = \cancel{R_B\left(\frac{1}{8}L^3 - \frac{1}{48}L^3\right)} + C_3\frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{8}M_0L^2 - \frac{1}{2}C_3L$$

$$= \left(-\frac{1}{8} + \frac{1}{4}\right)M_0L^2 = \frac{1}{8}M_0L^2$$

$$[x = L,\ y = 0]$$

$$R_B\left(\frac{1}{2}L^3 - \frac{1}{6}L^3\right) + \frac{M_0L}{2}L + \frac{1}{8}M_0L^2 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = \left(\frac{1}{2} - \frac{1}{8}\right)M_0L^2 \quad \frac{1}{3}R_B = \frac{3}{8}\frac{M_0}{L}$$

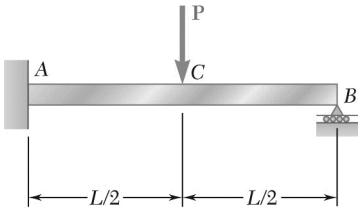
$$R_B = \frac{9}{8}\frac{M_0}{L} \uparrow \blacktriangleleft$$

$$M_A = \frac{9}{8}M_0 - M_0 = \frac{1}{8}M_0 \qquad M_A = \frac{1}{8}M_0 \blacktriangleleft$$

$$M_{C^-} = -M_0 + \frac{9}{8}\frac{M_0}{L} = -\frac{7}{16}M_0 \qquad M_{C^-} = -\frac{7}{16}M_0 \blacktriangleleft$$

$$M_{C^+} = R_B\left(L - \frac{L}{2}\right) = \frac{9}{8}\frac{M_0}{L}\left(\frac{L}{2}\right) = \frac{9}{16}M_0 \qquad M_{C^+} = \frac{9}{16}M_0 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.26

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

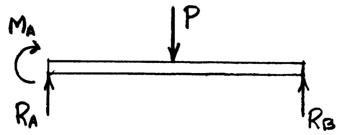
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + \frac{1}{2}PL - R_B L = 0$$

$$M_A = R_B L - \frac{1}{2}PL$$



$$\underline{0 < x < \frac{1}{2}L:}$$

$$M = M_A + R_A x$$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$\underline{\frac{1}{2}L < x < L:}$$

$$M = M_A + R_A x - P\left(x - \frac{1}{2}L\right)$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left(x - \frac{1}{2}L\right)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left(x - \frac{1}{2}L\right)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left(x - \frac{L}{2}\right)^3 + C_3 x + C_4$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.26 *(Continued)*

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$\frac{1}{2}M_A L + \frac{1}{8}R_A L^2 + 0 = \frac{1}{2}M_A L + \frac{1}{8}R_A L^2 - 0 + C_3 \qquad C_3 = 0$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 + 0 + 0 = \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x = L,\ y = 0]$$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{48}PL^3 + 0 + 0 = 0$$

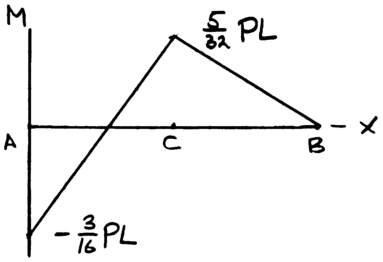
$$\frac{1}{2}\left(R_B L - \frac{1}{2}P\right)L^3 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \qquad R_B = \frac{5}{16}P \uparrow \blacktriangleleft$$

$$R_A = P - \frac{5}{16}P \qquad R_A = \frac{7}{16}P \uparrow$$

$$M_A = \frac{5}{16}PL - \frac{1}{2}PL \qquad M_A = -\frac{3}{16}PL \blacktriangleleft$$

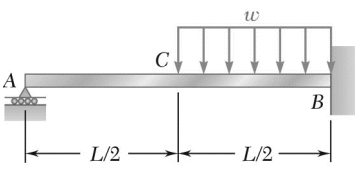
$$M_C = R_B\left(\frac{L}{2}\right) = \left(\frac{5}{16}P\right)\left(\frac{L}{2}\right) \qquad M_C = \frac{5}{32}PL \blacktriangleleft$$

$$M_B = 0 \blacktriangleleft$$



Bending moment diagram

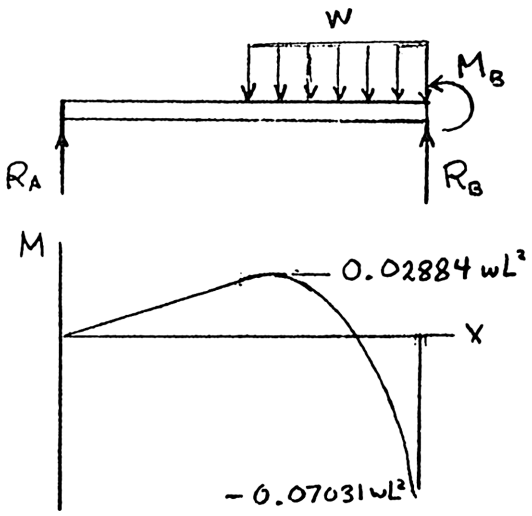
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.27

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x \tag{1}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \tag{2}$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2 \tag{3}$$

$$\frac{L}{2} < x < L$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2 \tag{4}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3 \tag{5}$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4 \tag{6}$$

$\left[x = 0,\ y = 0\right]$ $\quad 0 = 0 + 0 + C_2$ $\quad C_2 = 0$

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$ $\quad \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 + C_1 = \frac{1}{2}R_A\left(\frac{L}{2}\right)^2 + 0 + C_3$ $\quad C_1 = C_3$

$\left[x = \frac{L}{2},\ y = y\right]$ $\quad \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + C_1\frac{L}{2} + C_2 = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 - 0 + C_3\frac{L}{2} + C_4$ $\quad C_2 = C_4 = 0$

$\left[x = L,\ \frac{dy}{dx} = 0\right]$ $\quad \frac{1}{2}R_x L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_3 = 0$ $\quad C_3 = \frac{1}{48}wL^3 - \frac{1}{2}R_A L^2$

$\left[x = L,\ y = 0\right]$ $\quad \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\frac{1}{48}wL^3 - \frac{1}{2}R_A L^2\right)L + 0 = 0$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.27 *(Continued)*

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3=\left(\frac{1}{48}-\frac{1}{384}\right)wL^4 \qquad \frac{1}{3}R_A=\frac{7}{384}wL \qquad R_A=\frac{7}{128}wL \uparrow \blacktriangleleft$$

From (1), with $x=\frac{L}{2}$, $\quad M_C=R_A\left(\frac{L}{2}\right)=\frac{7}{256}wL^2 \qquad M_C=0.02734\,wL^2 \blacktriangleleft$

From (4), with $x=L$, $\quad M_B=R_A L-\frac{1}{2}w\left(\frac{L}{2}\right)^2=\left(\frac{7}{128}-\frac{1}{8}\right)wL-\frac{9}{128}wL^2$

$$M_B=-0.07031wL \blacktriangleleft$$

Location of maximum positive M:

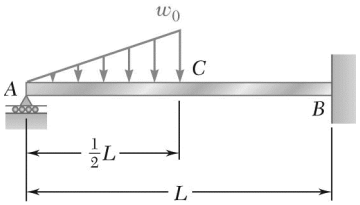
$$\frac{L}{2}<x<L \qquad V_m=R_A-w\left(x_m-\frac{L}{2}\right)=0 \qquad x_m-\frac{L}{2}=\frac{R_A}{w}=\frac{7}{128}L$$

$$x_m=\frac{L}{2}+\frac{7}{128}L=\frac{71}{128}L$$

From (4), with $x=x_m$, $\quad M_m=R_A x_m-\frac{1}{2}w\left(x_m-\frac{L}{2}\right)^2$

$$=\left(\frac{7}{128}wL\right)\left(\frac{71}{128}L\right)-\frac{1}{2}w\left(\frac{7}{128}L\right)^2 \qquad M_m=0.02884\,wL^2 \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

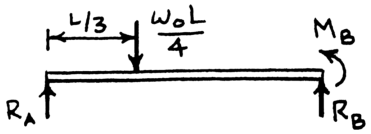


PROBLEM 9.28

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Reactions are statically indeterminate.



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - \frac{w_0 L}{4} = 0 \qquad R_B = \frac{w_0 L}{4} - R_A$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{w_0 L}{4}\right)\left(\frac{2L}{3}\right) + M_B = 0 \qquad M_B = R_A L - \frac{w_0 L^2}{6}$$

$0 \le x \le \frac{L}{2}$:

$$w = \frac{2w_0}{L}x$$

$$V = R_A - \frac{w_0}{L}x^2$$

$$M = R_A x - \frac{w_0}{3L}x^3$$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w_0}{3L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$\frac{L}{2} \le x \le L$:

$$M = R_A x - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right)$$

$$EI\frac{d^2y}{dx^2} = R_A x - w_0 L\left(\frac{1}{4}x - \frac{1}{12}L\right)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - w_0 L\left(\frac{1}{8}x^2 - \frac{1}{12}Lx\right) + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - w_0 L\left(\frac{1}{24}x^3 - \frac{1}{24}Lx^2\right) + C_3 x + C_4$$

$[x = 0,\ y = 0]$: $0 = 0 - 0 + 0 + C_2 \qquad \therefore \quad C_2 = 0$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]: \ \cancel{\frac{1}{8}R_A L^2} - \frac{1}{192}w_0 L^3 + C_1 = \cancel{\frac{1}{8}R_A L^2} + \frac{1}{96}w_0 L^3 + C_3 \ \therefore \ C_3 = C_1 - \frac{1}{64}w_0 L$$

$$\left[x = \frac{L}{2},\ y = y\right]: \ \cancel{\frac{1}{48}R_A L^3} - \frac{1}{1920}w_0 L^4 + \cancel{\frac{1}{2}C_1 L} + 0$$

$$= \cancel{\frac{1}{48}R_A L^3} - w_0 L\left(\frac{1}{192}L^3 - \frac{1}{96}L^3\right)$$

$$+\left(\cancel{C_1} - \frac{1}{64}w_0 L^3\right)\left(\frac{L}{2}\right) + C_4 \qquad \therefore \quad C_4 = \frac{1}{480}w_0 L^4$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.28 *(Continued)*

$$\left[x = L,\ \frac{dy}{dx} = 0\right] : \frac{1}{2}R_A L^2 - w_0 L\left(\frac{1}{8}L^2 - \frac{1}{12}L^2\right) + C_3 = 0 \quad \therefore \quad C_3 = -\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3$$

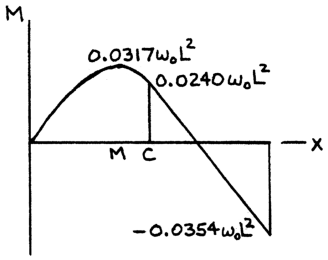
$$[x = L,\ y = 0] : \frac{1}{6}R_A L^3 - w_0 L\left(\frac{1}{24}L^3 - \frac{1}{24}L^3\right) + \left(-\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3\right)(L) + \frac{1}{480}w_0 L^4 = 0$$

$$R_A = \frac{21}{160}w_0 L \uparrow \blacktriangleleft$$

$$R_B = \frac{w_0 L}{4} - \frac{21}{160}w_0 L \qquad R_B = \frac{19}{160}w_0 L \uparrow$$

$$M_B = \frac{21}{160}w_0 L^2 - \frac{w_0 L^2}{6} \qquad M_B = -\frac{17}{480}w_0 L^2 = -0.0354 w_0 L^2 \blacktriangleleft$$

Over $0 < x < \dfrac{L}{2}$, $V = R_A - \dfrac{w_0}{L}x^2 = \dfrac{21}{160}w_0 L - \dfrac{w_0}{L}x^2$

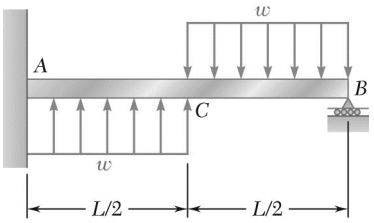


$V = 0$ at $x = x_m = 0.36228L$

$$M = \frac{21}{160}w_0 L x - \frac{w_0}{3L}x^3 \quad M_A = M(x = 0) = 0$$

$$M_C = M\left(x = \frac{L}{2}\right) = 0.0240 w_0 L^2 \qquad M_m = M(x_m = 0.36228L) = 0.0317 w_0 L^2 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.29

Determine the reaction at the roller support and the deflection at point *C*.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

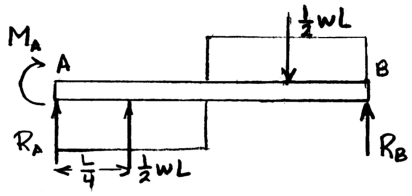
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{1}{2}wL\right)\frac{L}{2} + R_B L = 0$$

$$M_A = R_B L - \frac{1}{4}wL^2$$



From *A* to *C*: $0 < x \le \frac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.29 *(Continued)*

From C to B: $\dfrac{L}{2} \le x < L$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wL\left(x - \frac{L}{4}\right) - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}wL\left(x - \frac{L}{4}\right)^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{12}wL\left(x - \frac{L}{4}\right)^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4$$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$ $\quad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x = 0,\ y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$

$$\cancel{M_A\frac{L}{2}} - \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + \frac{1}{6}w\left(\frac{L}{2}\right)^3 = \cancel{M_A\frac{L}{2}} + \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + \frac{1}{4}wL\left(\frac{L}{4}\right)^2 - 0 + C_3$$

$$C_3 = \left(\frac{1}{48} - \frac{1}{64}\right)wL^3 = \frac{1}{192}wL^3$$

$\left[x = \dfrac{L}{2},\ y = y\right]$

$$\cancel{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2} + \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \frac{1}{24}w\left(\frac{L}{2}\right)^4 = \cancel{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2} + \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \frac{1}{12}wL\left(\frac{L}{4}\right)^3 - 0 + \frac{1}{192}wL^3\left(\frac{L}{2}\right) + C_4$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.29 ***(Continued)***

$$C_4 = \left(\frac{1}{384} - \frac{1}{768} - \frac{1}{384}\right)wL^4 = -\frac{1}{768}wL^4$$

$$[x = L,\ y = 0]$$

$$\frac{1}{2}M_AL^2 + \frac{1}{6}R_AL^3 + \frac{1}{12}wL\left(\frac{3L}{4}\right)^3 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$$

$$\frac{1}{2}\left(R_BL - \frac{1}{4}wL^2\right)L^2 + \frac{1}{6}(-R_B)L^3 + \left(\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768}\right)wL^4 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = \left(\frac{1}{8} - \frac{7}{192}\right)wL^4 \quad \frac{1}{3}R_B = \frac{17}{192}wL \qquad R_B = \frac{17}{64}wL \uparrow \blacktriangleleft$$

$$R_A = -R_B = -\frac{17}{64}wL$$

$$M_A = R_BL - \frac{1}{4}wL^2 = \left(\frac{17}{64} - \frac{1}{4}\right)wL^2 = \frac{1}{64}wL^2$$

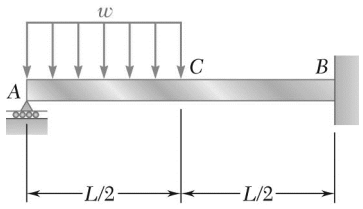
Deflection at C. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$EIy_C = \frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \frac{1}{2}\left(\frac{1}{64}wL^2\right)\left(\frac{L}{2}\right)^2 + \frac{1}{6}\left(-\frac{17}{64}wL\right)\left(\frac{L}{2}\right)^3 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \left(\frac{1}{512} - \frac{17}{3072} + \frac{1}{384}\right)wL^4 = -\frac{1}{1024}wL^4 \qquad y_C = \frac{1}{1024}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

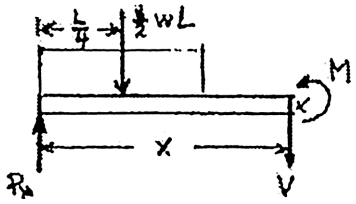


PROBLEM 9.30

Determine the reaction at the roller support and the deflection at point C.

SOLUTION

Reactions are statically indeterminate.



$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$\frac{L}{2} < x < L \qquad \text{(See free body diagram.)}$$

$$+\circlearrowleft \sum M_K = 0: \quad -R_A x + \frac{1}{2}wL\left(x - \frac{L}{4}\right) + M = 0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}wL\left(x - \frac{1}{4}L\right)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{4}wL\left(x - \frac{L}{4}\right)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{12}wL\left(x - \frac{L}{4}\right)^3 + C_3 x + C_4$$

$$[x = 0,\ y = 0]: \qquad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]: \quad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} - \frac{1}{4}wL\left(\frac{L}{4}\right)^2 + C_3$$

$$C_1 = C_3 + \frac{1}{48}wL^3 - \frac{1}{64}wL^3 = C_3 + \frac{1}{192}wL^3$$

$$\left[x = \frac{L}{2},\ y = y\right]: \quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\cancel{C_3} + \frac{1}{192}wL^3\right)\frac{L}{2} = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 - \frac{1}{12}wL\left(\frac{L}{4}\right)^3 + \cancel{C_3\frac{L}{2}} + C_4$$

$$C_4 = -\frac{1}{384}wL^4 + \frac{1}{384}wL^4 + \frac{1}{768}wL^4 = \frac{1}{768}wL^4$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.30 *(Continued)*

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \qquad \frac{1}{2}R_A L^2 - \frac{1}{4}wL\left(\frac{3L}{4}\right)^2 + C_3 = 0 \qquad C_3 = \frac{9}{64}wL^3 - \frac{1}{2}R_A L^2$$

$$[x = L,\ y = 0]: \qquad \frac{1}{6}R_A L^3 - \frac{1}{12}wL\left(\frac{3L}{4}\right)^3 + \left(\frac{9}{64}wL^3 - \frac{1}{2}R_A L^2\right)L + \frac{1}{768}wL^4 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A L^3 = \left(\frac{9}{64} - \frac{27}{768} + \frac{1}{768}\right)wL^4 \qquad \frac{1}{3}R_A = \frac{41}{384}wL \qquad R_A = \frac{41}{128}wL \uparrow \blacktriangleleft$$

$$C_3 = \frac{9}{64}wL^3 - \frac{1}{2}\frac{41}{128}wL^3 = -\frac{5}{256}wL^3$$

$$C_1 = -\frac{5}{256}wL^3 + \frac{1}{192}wL^3 = -\frac{11}{768}wL^3$$

Deflection at *C*. $\left(y \text{ at } x = \dfrac{L}{2}\right)$

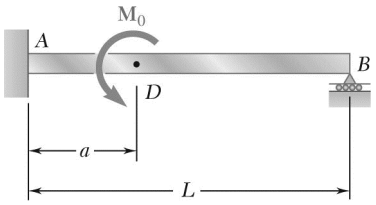
$$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\cdot\left(\frac{1}{2}\right)^3 - \frac{1}{24}\cdot\left(\frac{1}{2}\right)^4 - \frac{11}{768}\cdot\frac{1}{2} + 0\right\}$$

$$= \left(\frac{41}{6144} - \frac{1}{384} - \frac{11}{1536}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI} \qquad y_C = \frac{19}{6144}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

or $$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\left(\frac{1}{2}\right)^3 - \frac{1}{12}\cdot\left(\frac{1}{4}\right)^3 + \frac{5}{256}\cdot\frac{1}{2} + \frac{1}{768}\right\}$$

$$= \left(\frac{41}{6144} - \frac{\cancel{1}}{\cancel{768}} - \frac{5}{512} + \frac{\cancel{1}}{\cancel{768}}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.31

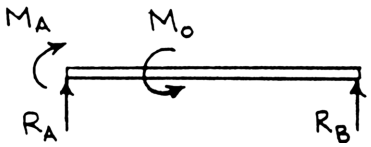
Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_0 - M_A + R_B L = 0 \quad M_A = R_B L + M_0$$

$\underline{0 \le x \le a}$:



$$M = M_A + R_A x$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\underline{a \le x \le L}$:

$$M = M_A + R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - M_0 x + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^2 + C_3 x + C_4$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right]: \quad 0 + 0 + C_1 = 0 \qquad \therefore \quad C_1 = 0$$

$$[x = 0, \ y = 0]: \quad 0 + 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$$

$$\left[x = a, \ \frac{dy}{dx} = \frac{dy}{dx}\right]:$$

$$M_A a + \frac{1}{2}R_A a^2 = M_A a + \frac{1}{2}R_A a^2 - M_0 a + C_3 \qquad \therefore \quad C_3 = M_0 a$$

$$[x = a, \ y = y]:$$

$$\frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 = \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 - \frac{1}{2}M_0 a^2 + (M_0 a)(a) + C_4 \ \therefore \quad C_4 = -\frac{1}{2}M_0 a^2$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.31 *(Continued)*

$$[x = L,\ y = 0]:$$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + (M_0 a)(L) - \frac{1}{2}M_0 a^2 = 0$$

$$\frac{1}{2}(R_B L + M_0)L^2 + \frac{1}{6}(-R_B)L^3 - \frac{1}{2}M_0 L^2 + M_0 aL - \frac{1}{2}M_0 a^2 = 0$$

$$R_B = \frac{3M_0 a}{2L^3}(a - 2L) = \frac{3M_0}{2L^3}\left(\frac{L}{3}\right)\left(\frac{L}{3} - 2L\right) = -\frac{5M_0}{6L} \qquad R_B = \frac{5M_0}{6L} \downarrow \blacktriangleleft$$

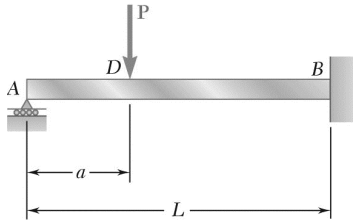
Deflection at *D*. $\left(y \text{ at } x = a = \dfrac{L}{3}\right)$

$$y_D = \frac{1}{EI}\left\{\frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{2}\left(-\frac{5M_0}{6L}L + M_0\right)\left(\frac{L}{3}\right)^2 + \frac{1}{6}\left(+\frac{5M_0}{6L}\right)\left(\frac{L}{3}\right)^3\right\}$$

$$= -\frac{7M_0 L^2}{486EI} \qquad y_D = \frac{7M_0 L^2}{486EI} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



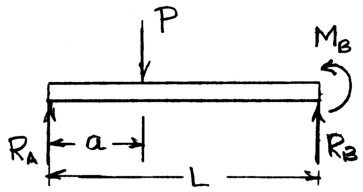
PROBLEM 9.32

Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

SOLUTION

$\underline{0 \le x \le a}$:

$$M = R_A x$$



$$EI\frac{d^2y}{dx^2} = M = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\underline{a \le x \le L}$:

$$M = R_A x - P(x-a)$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$[x = 0, y = 0]$: $\quad 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$

$\left[x = a, \ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$:

$$\frac{1}{2}R_A a^2 + C_1 = \frac{1}{2}R_A a^2 - 0 + C_3 \qquad \therefore \quad C_1 = C_3$$

$[x = a, \ y = y]$:

$$\frac{1}{6}R_A a^3 + C_1 a + 0 = \frac{1}{6}R_A a^3 - 0 + C_1 a + C_4 \qquad \therefore \quad C_4 = 0$$

$\left[x = L, \ \dfrac{dy}{dx} = 0\right]$:

$$\frac{1}{2}R_A L^2 - \frac{1}{2}P(L-a)^2 + C_3 = 0 \qquad \therefore \quad C_3 = \frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.32 *(Continued)*

$$[x = L,\ y = 0]:$$

$$\frac{1}{6}R_A L^3 - \frac{1}{6}P(L-a)^3 + \left[\frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2\right](L) + 0 = 0$$

$$R_A = \frac{P}{2L^3}(2L^3 - 3aL^2 + a^3) = \frac{P}{2L^3}\left(2L^3 - L^3 + \frac{L^3}{9}\right) \qquad R_A = \frac{14}{27}P \uparrow \blacktriangleleft$$

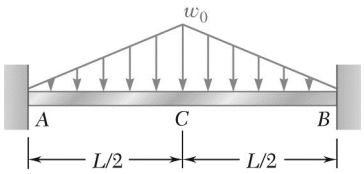
Deflection at *D*. $\left(y \text{ at } x = a = \dfrac{L}{3}\right)$

$$y_D = \frac{1}{EI}\left\{\frac{1}{6}R_A\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right)\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{6}\left(\frac{14}{27}P\right)\left(\frac{L}{3}\right)^3 + \left[\frac{1}{2}P\left(L - \frac{L}{3}\right)^2 - \frac{1}{2}\left(\frac{14}{27}P\right)L^2\right]\left(\frac{L}{3}\right)\right\}$$

$$= -\frac{20}{2187}\frac{PL^3}{EI} \qquad y_D = \frac{20}{2187}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

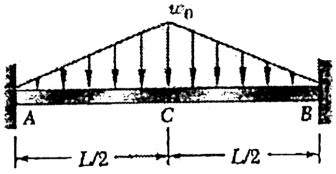
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.33

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

Because of symmetry, $\dfrac{dy}{dx} = 0$ and $V = 0$ at $x = \dfrac{L}{2}$.

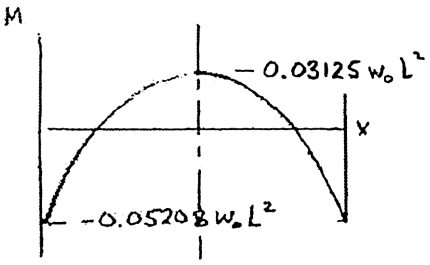
$[x = 0,\ y = 0]$ | $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$ | $\left[x = L,\ \dfrac{dy}{dx} = 0\right]$

$\left[x = \dfrac{L}{2},\ V = 0\right]$

$\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = 0\right]$

Use portion AC of beam. $\left(0 < x \le \dfrac{L}{2}\right)$



$$\frac{dV}{dx} = -w = -2\frac{w_0}{L}x$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}x^2 + R_A \tag{1}$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{3}\frac{w_0}{L}x^3 + R_A x + M_A \tag{2}$$

$$EI\frac{dy}{dx} = -\frac{1}{12}\frac{w_0}{L}x^4 + \frac{1}{2}R_A x^2 + M_A x + C_1 \tag{3}$$

$$EIy = -\frac{1}{60}\frac{w_0}{L}x^2 + \frac{1}{6}R_A x^3 + \frac{1}{2}M_A x^2 + C_1 x + C_2 \tag{4}$$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$: $\quad 0 = 0 + 0 + 0 + C_1 \quad C_1 = 0$

$[x = 0,\ y = 0]$: $\quad 0 = 0 + 0 + 0 + 0 + C_2 \quad C_2 = 0$

$\left[x = \dfrac{L}{2},\ V = 0\right]$: $\quad -\dfrac{w_0}{L}\left(\dfrac{L}{2}\right)^2 + R_A = 0 \quad R_A = \dfrac{wL}{4}$ ◀

$\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = 0\right]$: $\quad -\dfrac{1}{12}\dfrac{w_0}{L}\left(\dfrac{L}{2}\right)^4 + \dfrac{1}{2}\left(\dfrac{1}{4}w_0 L\right)\left(\dfrac{L}{2}\right)^2 + M_A\dfrac{L}{2} + 0 = 0$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.33 ***(Continued)***

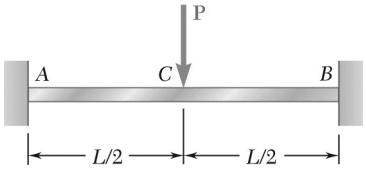
$$M_A = -2\left(\frac{1}{32} - \frac{1}{192}\right)w_0L^2 = -\frac{5}{96}w_0L^2 \qquad M_A = -0.05208w_0L^2 \blacktriangleleft$$

From (2), with $x = \dfrac{L}{2}$,

$$M_C = -\frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 + \left(\frac{1}{4}w_0L\right)\left(\frac{L}{2}\right) - \frac{5}{96}w_0L^{12}$$

$$= \left(-\frac{1}{24} + \frac{1}{8} - \frac{5}{96}\right)w_0L^2 = \frac{1}{32}w_0L^2 \qquad M_C = 0.03125w_0L^2 \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.34

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

$\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = 0\right]$

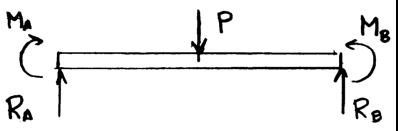
SOLUTION

By symmetry, $\quad R_A = R_B \quad \text{and} \quad \dfrac{dy}{dx} = 0 \quad \text{at} \quad x = \dfrac{L}{2}.$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = R_B = \frac{1}{2}P \blacktriangleleft$$

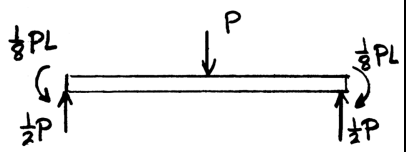
Moment reaction is statically indeterminate.

$$\underline{0 < x < \frac{L}{2}}:$$



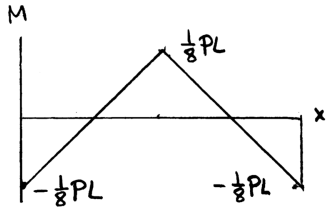
$$M = M_A + R_A x = M_A + \frac{1}{2}Px$$

$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$$



$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$$

$$y_B = \frac{P}{EI}\left\{-0 + 0 + \frac{1}{12}a^3\right\} = \frac{Pa^3}{12EI}$$



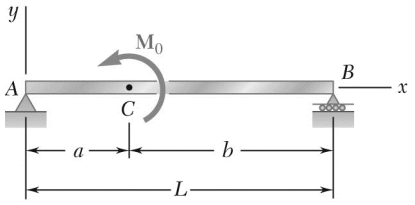
$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \quad M_A\frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = \upsilon$$

$$M_A = -\frac{1}{8}PL \qquad M_A = \frac{1}{8}PL\ \text{)} \blacktriangleleft$$

By symmetry, $\quad M_B = M_A \qquad M_B = \dfrac{1}{8}PL\ \text{)} \blacktriangleleft$

$$M_C = M_A + \frac{1}{2}P\frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL \qquad M_C = \frac{1}{8}PL \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



[$x = 0,\ y = 0$] [$x = L,\ y = 0$]

PROBLEM 9.35

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION

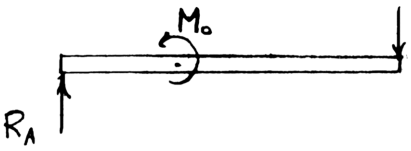
Reactions:

$$R_A = \frac{M_0}{L} \uparrow, \quad R_B = \frac{M_0}{L} \downarrow$$

$$0 < x < a \qquad M = R_A x$$

$$a < x < L \qquad M = R_A x - M_0$$

Using singularity functions,



$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x - a\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x - a\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x - a\rangle^2 + C_1 x + C_2$$

$$[x = 0, y = 0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x = L, y = 0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L - a)^2 + C_1 L + 0 = 0$$

$$C_1 L = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0 b^2$$

$$C_1 = \frac{M_0}{6L}(3b^2 - L^2)$$

(*a*) Elastic curve. $$y = \frac{1}{EI}\left\{\frac{1}{6}\frac{M_0}{L}x^3 - \frac{1}{2}M_0\langle x - a\rangle^2 + \frac{M_0}{6L}(3b^2 - L^2)x\right\}$$

$$y = \frac{M_0}{6EIL}\left\{x^3 - 3L\langle x - a\rangle^2 + (3b^2 - L^2)x\right\} \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL}\left\{3x^2 - 6L\langle x - a\rangle^1 + (3b^2 - L^2)\right\}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.35 *(Continued)*

(*b*) Slope at *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$\theta_A = \frac{M_0}{6EIL}\{0 - 0 + 3Lb^2 - L^3\} \qquad \theta_A = \frac{M_0}{6EIL}(3b^2 - L^2) \curvearrowright \blacktriangleleft$$

(*c*) Deflection at *C*. (y at $x = a$)

$$y_C = \frac{M_0}{6EIL}\{a^3 - 0 + (3b^2 - L^2)a\}$$

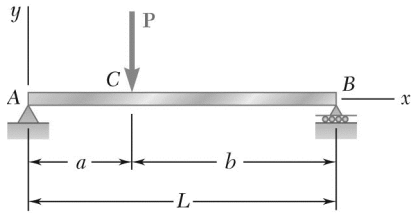
$$= \frac{M_0 a}{6EIL}\{a^2 + 3b^2 - (a + b)^2\}$$

$$= \frac{M_0 a}{6EIL}\{\cancel{a^2} + 3b^2 - \cancel{a^2} - 2ab - b^2\}$$

$$= \frac{M_0 a}{6EIL}\{2b^2 - 2ab\}$$

$$y_C = \frac{M_0 ab}{3EIL}(b - a) \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$[x = 0,\ M = 0]$ $[x = L,\ M = 0]$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

PROBLEM 9.36

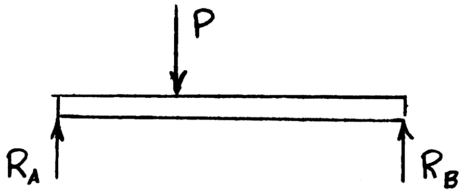
For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + Pb = 0 \qquad R_A = \frac{Pb}{L}$$

$$\frac{dM}{dx} = V = R_A - P\langle x - a\rangle^0 = \frac{Pb}{L} - P\langle x - a\rangle^0$$

$$M = \frac{Pb}{L}x - P\langle x - a\rangle^1 + \cancel{M_A}$$



$$EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x - P\langle x - a\rangle$$

$$EI\frac{dy}{dx} = \frac{Pb}{2L}x^2 - \frac{1}{2}P\langle x - a\rangle^2 + C_1$$

$$EIy = \frac{Pb}{6L}x^3 - \frac{1}{6}P\langle x - a\rangle^3 + C_1 x + C_2$$

$[x = 0, y = 0]$ $\quad C_2 = 0$

$[x = L,\ y = 0]$ $\quad \frac{Pb}{6L}L^3 - \frac{1}{6}P(L - a)^3 + C_1 L = 0$

$$C_1 = -\frac{1}{6}\frac{R}{L}(bL^2 - b^3) = -\frac{1}{6}\frac{Pb}{L}(L^2 - b^2)$$

(*a*) Elastic curve.

$$y = \frac{P}{EI}\left\{\frac{b}{6L}x^3 - \frac{1}{6}\langle x - a\rangle^3 - \frac{1}{6}\frac{b}{L}(L^2 - b^2)x\right\}$$

$$y = \frac{P}{6EIL}\{bx^3 - L\langle x - a\rangle^3 - b(L^2 - b^2)x\} \blacktriangleleft$$

(*b*) Slope at end *A*.

$$EI\left.\frac{dy}{dx}\right|_{x=0} = C_1 = -\frac{Pb}{6L}(L^2 - b^2)$$

$$\theta_A = -\frac{Pb}{6EIL}(L^2 - b^2) \qquad \theta_A = \frac{Pb}{6EIL}(L^2 - b^2)\ \searrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.36 *(Continued)*

(*c*) Deflection at *C*.

$$EIy_C = \frac{Pb}{6L}a^3 + C_1 a = \frac{Pba^3}{6L} - \frac{Pb}{6L}(L^2 - b^2)a$$

$$= \frac{Pba}{6L}(a^2 - L^2 - b^2)$$

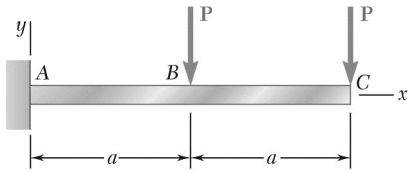
$$y_C = -\frac{Pab}{6EIL}(L^2 - a^2 - b^2)$$

$$= -\frac{Pab}{6EIL}\left\{\cancel{a^2} + 2ab + \cancel{b^2} - \cancel{a^2} - \cancel{b^2}\right\}$$

$$= -\frac{Pa^2b^2}{3EIL}$$

$$y_C = \frac{Pa^2b^2}{3EIL} \downarrow \blacktriangleleft$$

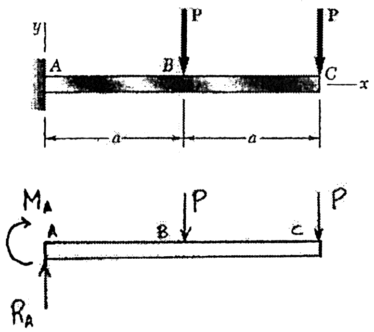
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.37

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at at the free end, (*c*) the deflection of the free end.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P = 0 \qquad R_A = 2P$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - Pa - P(2a) = 0 \qquad M_A = -3Pa$$

$$\frac{dM}{dx} = V = 2P - P\langle x - a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = 2Px - P\langle x - a\rangle^1 - 3Pa$$

$$EI\frac{dy}{dx} = Px^2 - \frac{1}{2}P\langle x - a\rangle^2 - 3Pax + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$EIy = \frac{1}{3}Px^3 - \frac{1}{6}P\langle x - a\rangle^3 - \frac{3}{2}Pax^2 + C_2$$

$$[x = 0,\ y = 0]: \quad 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$$

$$EIy = \frac{1}{3}Px^3 - \frac{1}{6}P\langle x - a\rangle^3 - \frac{3}{2}Pax^2$$

(*a*) Elastic curve.

$$y = \frac{P}{EI}\left\{\frac{1}{3}x^3 - \frac{1}{6}\langle x - a\rangle^3 - \frac{3}{2}ax^2\right\} \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{x^2 - \frac{1}{2}\langle x - a\rangle^2 - 3ax\right\}$$

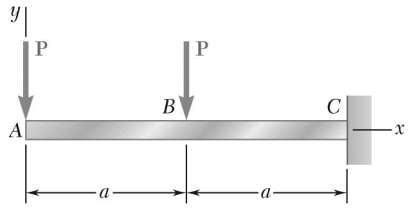
(*b*) Slope at end *C*. $\left(\frac{dy}{dt} \text{ at } x = 2a\right)$

$$\left.\frac{dy}{dx}\right|_A = \frac{P}{EI}\left\{(2a)^2 - \frac{1}{2}a^2 - (3a)(2a)\right\} = -\frac{5Pa^2}{2EI} \qquad \theta_C = \frac{5Pa^2}{2EI} \searrow \blacktriangleleft$$

(*c*) Deflection at end *C*. (y at $x = 2a$)

$$y_A = \frac{P}{EI}\left\{\frac{1}{3}(2a)^3 - \frac{1}{6}a^3 - \left(\frac{3}{2}a\right)(2a)^2\right\} = -\frac{7Pa^3}{2EI} \qquad y_C = \frac{7Pa^3}{2EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.38

For the beam and loading shown, determine (*a*) the equation curve, (*b*) the slope at the free end, (*c*) the deflection of the free end.

SOLUTION

$$\frac{dM}{dx} = V = -P - P\langle x-a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -Px - P\langle x-a\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{2}Px^2 - \frac{1}{2}P\langle x-a\rangle^2 + C_1$$

$$EIy = -\frac{1}{6}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 + C_1x + C_2$$

$$\left[x = 2a,\ \frac{dy}{dx} = 0\right] \quad -\frac{1}{2}P(2a)^2 - \frac{1}{2}P(a)^2 + C_1 = 0 \qquad C_1 = \frac{5}{2}Pa^2$$

$$\left[x = 2a,\ y = 0\right] \quad -\frac{1}{6}P(2a)^3 - \frac{1}{6}P(a)^3 + \frac{5}{2}Pa^2(2a) + C_2 = 0 \qquad C_2 = -\frac{7}{2}Pa^3$$

$$EIy = -\frac{1}{6}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 + \frac{3}{2}Pa^2x - \frac{3}{2}Pa^3$$

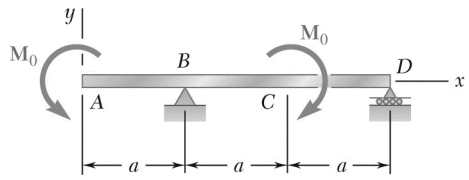
(*a*) Elastic curve. $$y = \frac{P}{EI}\left\{-\frac{1}{6}x^3 - \frac{1}{6}\langle x-a\rangle^3 + \frac{5}{2}a^2x - \frac{7}{2}a^3\right\}$$ ◀

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\frac{1}{2}x^2 - \frac{1}{2}\langle x-a\rangle^2 + \frac{5}{2}a^2\right\}$$

(*b*) Slope at *A*. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$ $$\left.\frac{dy}{dx}\right|_A = \frac{5Pa^2}{2EI} \qquad \theta_A = \frac{5Pa^2}{2EI}\ \measuredangle$$ ◀

(*c*) Deflection at *A*. (y at $x = 0$) $$y_A = -\frac{7Pa^3}{2EI} \qquad y_A = \frac{7Pa^3}{2EI}\downarrow$$ ◀

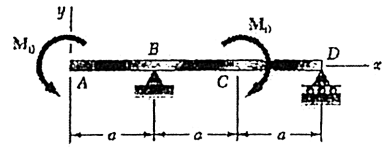
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.39

For the beam and loading shown, determine (*a*) the deflection end *A*, (*b*) the deflection at point *C*, (*c*) the slope at end *D*.

SOLUTION



$[x = a,\ y = 0]$ $[x = 3a,\ y = 0]$

Since loads self-equilibrate,

$R_B = 0, \qquad R_D = 0$

$(0 < x < 2a)$: $\quad M = -M_0$

$(2a < x < 3a)$: $\quad M = -M_0 + M_0 = 0$

Using singularity functions,

$$EI\frac{d^2y}{dx^2} = M = -M_0 + M_0\langle x - 2a\rangle^0$$

$$EI\frac{dy}{dx} = -M_0x + M_0\langle x - 2a\rangle^1 + C_1$$

$$EIy = -\frac{1}{2}M_0x^2 + \frac{1}{2}M_0\langle x - 2a\rangle^2 + C_1x + C_2$$

$[x = 3a,\ y = 0]$ $\quad -\frac{1}{2}M_0(3a)^2 + \frac{1}{2}M_0a^2 + C_1(3a) + C_2 = 0 \quad 3aC_1 + C_2 = 4M_0a^2$

$[x = a,\ y = 0]$ $\quad -\frac{1}{2}M_0a^2 + 0 + C_1a + C_2 = 0 \quad aC_1 + C_2 = \frac{1}{2}M_0a^2$

Subtracting, $\quad 2aC_1 = \frac{7}{2}M_0a^2 \qquad C_1 = \frac{7}{4}M_0a$

$$C_2 = \frac{1}{2}M_0a^2 - aC_1 = -\frac{5}{4}M_0a^2$$

$$y = \frac{M_0}{EI}\left\{-\frac{1}{2}x^2 + \frac{1}{2}\langle x - 2a\rangle^2 + \frac{7}{4}ax - \frac{5}{4}a^2\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{EI}\left\{-x + \langle x - a\rangle^1 + \frac{7}{4}a\right\}$$

(*a*) Deflection at *A*. $\quad (y \text{ at } x = 0)$

$$y_A = \frac{M_0a^2}{EI}\left\{-0 + 0 + 0 - \frac{5}{4}\right\} = -\frac{5}{4}\frac{M_0a^2}{EI}, \qquad y_A = \frac{5}{4}\frac{M_0a^2}{EI}\downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.39 *(Continued)*

(*b*) Deflection at *C*. (y at $x = 2a$)

$$y_C = \frac{M_0 a^2}{EI}\left\{-\frac{1}{2}(2)^2 + 0 + \frac{7}{4}(2) - \frac{5}{4}\right\} = \frac{1}{4}\frac{M_0 a^2}{EI}$$

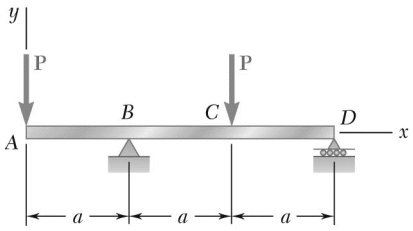
$$y_C = \frac{1}{4}\frac{M_0 a^2}{EI} \uparrow \blacktriangleleft$$

(*c*) Slope at *D*. $\left(\frac{dy}{dx} \text{ at } x = 3a\right)$

$$\theta_D = \frac{M_0 a}{EI}\left\{-3 + 1 + \frac{7}{4}\right\} = -\frac{1}{4}\frac{M_0 a}{EI},$$

$$\theta_D = \frac{1}{4}\frac{M_0 a}{EI} \curvearrowright \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

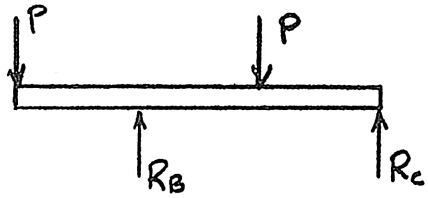


PROBLEM 9.40

For the beam and loading shown, determine (*a*) the deflection at end *A*, (*b*) the deflection at point *C*, (*c*) the slope at end *D*.

$[x = a,\ y = 0]$ $[x = 3a,\ y = 0]$

SOLUTION



Reactions: $R_B = 2P \uparrow, \quad R_D = 0$

$(0 < x < a)$: $V = -P$

$(a < x < 2a)$: $V = -P + 2P$

$(2a < x < 3a)$: $V = -P + 2P - P$

Using singularity functions,

$$\frac{dM}{dx} = V = -P + 2P\langle x - a\rangle^0 - P\langle x - 2a\rangle^0$$

$$M = -Px + 2P\langle x - a\rangle^1 - P\langle x - 2a\rangle^1 + M_A$$

But $M = 0$ at $x = 0$ $\quad M_A = 0$

$$EI\frac{d^2y}{dx^2} = M = -Px + 2P\langle x - a\rangle^1 - P\langle x - 2a\rangle^1 \tag{1}$$

$$EI\frac{dy}{dx} = -\frac{1}{2}Px^2 + P\langle x - a\rangle^2 - \frac{1}{2}P\langle x - 2a\rangle^2 + C_1 \tag{2}$$

$$EIy = -\frac{1}{6}Px^3 + \frac{1}{3}P\langle x - a\rangle^3 - \frac{1}{6}P\langle x - 2a\rangle^3 + C_1x + C_2 \tag{3}$$

$[x = a,\ y = 0]$ $\quad -\frac{1}{6}Pa^3 + 0 - 0 + C_1a + C_2 = 0 \quad aC_1 + C_2 = \frac{1}{6}Pa^3$ (4)

$[x = 3a,\ y = 0]$ $\quad -\frac{1}{6}P(3a)^3 + \frac{1}{3}P(2a)^3 - \frac{1}{6}Pa^3 + C_1(3a) + C_2 = 0 \quad 3aC_1 + C_2 = 2Pa^2$ (5)

Eq (5) – Eq (4) $\quad 2C_1a + \frac{11}{6}Pa^2 \quad C_1 + \frac{11}{12}Pa^2$

$$C_2 = \frac{1}{6}Pa^2 - aC_1 = -\frac{3}{4}Pa^3$$

$$y = \frac{P}{EI}\left\{-\frac{1}{6}x^3 + \frac{1}{3}\langle x - a\rangle^3 - \frac{1}{6}\langle x - 2a\rangle^3 + \frac{11}{12}a^2x - \frac{3}{4}a^3\right\}$$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\frac{1}{2}x^2 + \langle x - a\rangle^2 - \frac{1}{2}\langle x - 2a\rangle^2 + \frac{11}{12}a^2\right\}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.40 *(Continued)*

(*a*) Deflection at *A*. (y at $x = 0$)

$$y_A = \frac{Pa^3}{EI}\left\{-0 + 0 - 0 + 0 - \frac{3}{4}\right\} = -\frac{3}{4}\frac{Pa^3}{EI} \qquad y_A = \frac{3}{4}\frac{Pa^3}{EI} \downarrow \blacktriangleleft$$

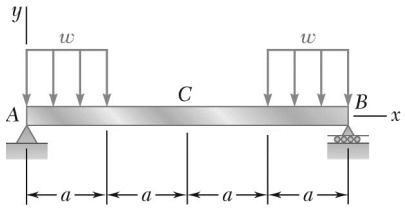
(*b*) Deflection at *C*. (y at $x = 2a$)

$$y_C = \frac{Pa^3}{EI}\left\{-\frac{1}{6}(2)^3 + \frac{1}{3}(1)^3 - 0 + \frac{11}{12}(2) - \frac{3}{4}\right\} \qquad y_C = \frac{1}{12}\frac{Pa^3}{EI} \uparrow \blacktriangleleft$$

(*c*) Slope at D. $\left(\frac{dy}{dx} \text{ at } x = 3a\right)$

$$\theta_D = \frac{Pa^2}{EI}\left\{-\frac{1}{2}(3)^2 + (2)^2 - \frac{1}{2}(1)^2 + \frac{11}{12}\right\} = -\frac{1}{12}\frac{Pa^2}{EI} \qquad \theta_D = \frac{1}{12}\frac{Pa^2}{EI} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.41

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

SOLUTION

By symmetry,

$$R_A = R_B = wa$$

$$w(x) = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2 \quad (\text{with } M_A = 0)$$

$$EI\frac{d^2y}{dx^2} = M = wax - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}wx^3 + \frac{1}{6}w\langle x-a\rangle^3 - \frac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EIy = \frac{1}{6}wax^3 - \frac{1}{24}wx^4 + \frac{1}{24}w\langle x-a\rangle^4 - \frac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$$[x=0,\ y=0]: \quad 0 - 0 + 0 - 0 + 0 + C_2 = 0 \quad \therefore \quad C_2 = 0$$

$$[x=4a,\ y=0]: \quad \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(4a)^4 + \frac{1}{24}w(3a)^4 - \frac{1}{24}w(a)^4 + C_1(4a) = 0$$

$$\therefore \quad C_1 = -\frac{5}{6}wa^3$$

(*a*) Equation of elastic curve.

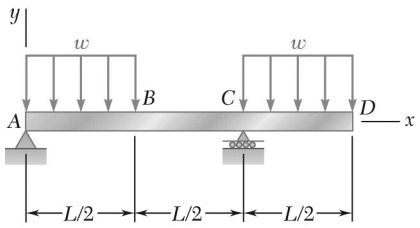
$$y = \frac{w}{EI}\left[\frac{1}{6}ax^3 - \frac{1}{24}x^4 + \frac{1}{24}\langle x-a\rangle^4 - \frac{1}{24}\langle x-3a\rangle^4 - \frac{5}{6}a^3x\right] \blacktriangleleft$$

(*b*) Deflection at *C*. (y at $x = 2a$)

$$y_C = \frac{wa^4}{EI}\left[\frac{1}{6}(2)^3 - \frac{1}{24}(2)^4 + \frac{1}{24}(1)^4 - 0 - \frac{5}{6}(2)\right]$$

$$= -\frac{23}{24}\frac{wa^4}{EI} \qquad y_C = \frac{23}{24}\frac{wa^4}{EI}\downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.42

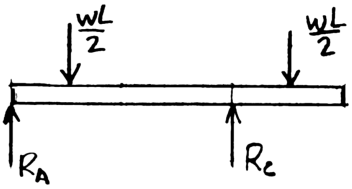
For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *D*.

SOLUTION

Use free body *ABCD* with the distributed loads replaced by equivalent concentrated loads.

$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$



$$+\circlearrowleft \Sigma M_A = 0: \quad R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4}wL$$

$$\frac{dV}{dx} = -w = -w + w\left\langle x - \frac{L}{2}\right\rangle^0 - w\langle x - L\rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\left\langle x - \frac{L}{2}\right\rangle^1 - w\langle x - L\rangle^1 + R_A + R_C\langle x - L\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2 + \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{2}w\langle x - L\rangle^2 + R_A x + R_C\langle x - L\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{1}{6}w\langle x - L\rangle^3 + \frac{1}{2}R_A x^2 + \frac{1}{2}R_C\langle x - L\rangle^2 + C_1$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}R_A x^3 + \frac{1}{6}R_C\langle x - L\rangle^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\quad -\cancel{\frac{1}{24}wL^4} + \frac{1}{24}w\left(\frac{L}{2}\right)^4$

$$-0 + \cancel{\frac{1}{6}\left(\frac{wL}{4}\right)L^3} + 0 + C_1 L + 0 = 0 \qquad C_1 = -\frac{1}{384}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}\left(\frac{wL}{4}\right)x^3 + \frac{1}{6}\left(\frac{3wL}{4}\right)\langle x - L\rangle^3 - \frac{1}{384}wL^3 x$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.42 *(Continued)*

(*a*) Elastic curve.

$$y = \frac{w}{24EI}\left\{-x^4 + \left\langle x - \frac{L}{2}\right\rangle^4 - \langle x - L\rangle^4 + Lx^3 + 3L\langle x - L\rangle^3 - \frac{1}{16}L^3x\right\} \blacktriangleleft$$

(*b*) Deflection at *B*. $\left(y \text{ at } x = \dfrac{L}{2}\right)$

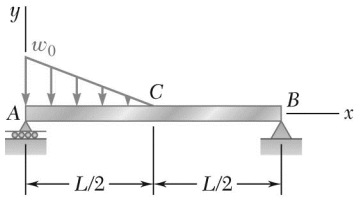
$$y_B = \frac{w}{24EI}\left\{-\left(\frac{L}{2}\right)^4 + 0 - 0 + (L)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right)\right\} \qquad y_B = \frac{wL^4}{768EI} \uparrow \blacktriangleleft$$

(*c*) Deflection at *D*. $\left(y \text{ at } x = \dfrac{3L}{2}\right)$

$$y_D = \frac{w}{24EI}\left\{-\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + (L)\left(\frac{3L}{2}\right)^3 + (3L)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right)\right\}$$

$$= -\frac{5wL^4}{256EI} \qquad y_D = \frac{5wL^4}{256EI} \downarrow \blacktriangleleft$$

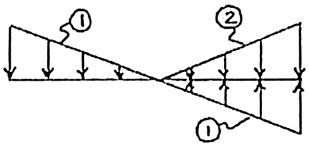
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.43

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

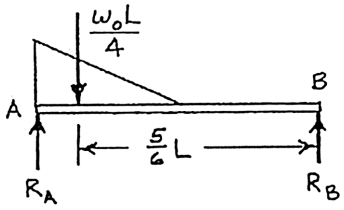
SOLUTION



Distributed loads: $\quad k = \dfrac{2w_0}{L}$

(1) $\quad w_1(x) = w_0 - kx \qquad$ (2) $\quad w_2(x) = k\left\langle x - \dfrac{L}{2}\right\rangle^1$

$$+\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{w_0 L}{4}\right)\left(\frac{5}{6}L\right) - R_A L = 0 \qquad R_A = \frac{5}{24}w_0 L \uparrow$$



$$w(x) = w_0 - kx + k\left\langle x - \frac{L}{2}\right\rangle^1 = w_0 - \frac{2w_0}{L}x + \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{2w_0}{L}x - \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{5}{24}w_0 L - w_0 x + \frac{w_0}{L}x^2 - \frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = \frac{5}{24}w_0 L x - \frac{1}{2}w_0 x^2 + \frac{1}{3}\frac{w_0}{L}x^3 - \frac{1}{3}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$EI\frac{dy}{dx} = \frac{5}{48}w_0 L x^2 - \frac{1}{6}w_0 x^3 + \frac{1}{12}\frac{w_0}{L}x^4 - \frac{1}{12}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1$$

$$EIy = \frac{5}{144}w_0 L x^3 - \frac{1}{24}w_0 x^4 + \frac{1}{60}\frac{w_0}{L}x^5 - \frac{1}{60}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^5 + C_1 x + C_2$$

$[x = 0,\ y = 0]$: $\qquad C_2 = 0$

$$[x = L,\ y = 0]: \quad \frac{5}{144}w_0 L^4 - \frac{1}{24}w_0 L^4 + \frac{1}{60}w_0 L^4 - \frac{1}{60}\frac{w_0}{L}\left(\frac{L}{2}\right)^5 + C_1 L = 0 \qquad C_1 = -\frac{53}{5760}w_0 L^3$$

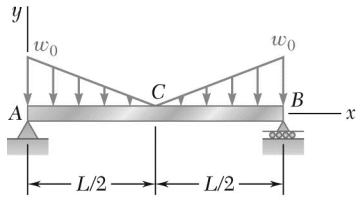
(*a*) Equation of elastic curve.

$$y = w_0\left[96x^5 - 96\left\langle x - \frac{L}{2}\right\rangle^5 - 240Lx^4 + 200L^2x^3 - 53L^4x\right]/5760\,EIL \quad \blacktriangleleft$$

(*b*) Deflection at *C*. $\qquad \left(y \quad \text{at} \quad x = \dfrac{L}{2}\right)$

$$y_C = \frac{w_0 L^4}{5760EI}\left(\frac{96}{32} - 0 - \frac{240}{16} + \frac{200}{8} - \frac{53}{2}\right) = -\frac{3w_0 L^4}{1280EI} \qquad y_C = \frac{3w_0 L^4}{1280EI} \downarrow \quad \blacktriangleleft$$

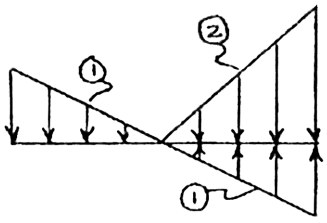
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

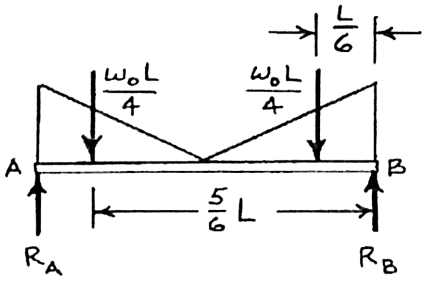


PROBLEM 9.44

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

SOLUTION





Distributed loads: $k_1 = \dfrac{2w_0}{L}$ $k_2 = \dfrac{4w_0}{L}$

(1) $w_1(x) = w_0 - k_1 x$

(2) $w_2(x) = k_2 \left\langle x - \dfrac{L}{2} \right\rangle^1$

$+\circlearrowleft \Sigma M_B = 0$: $\left(\dfrac{w_0 L}{4}\right)\left(\dfrac{5}{6}L\right) + \left(\dfrac{w_0 L}{4}\right)\left(\dfrac{L}{6}\right) + R_A L = 0$

$R_A = \dfrac{w_0 L}{4} \uparrow$

$$w(x) = w_0 - k_1 x + k_2 \left\langle x - \frac{L}{2} \right\rangle^1$$

$$= w_0 - \frac{2w_0}{L} x + \frac{4w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^1$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{2w_0}{L} x - \frac{4w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{4} - w_0 x + \frac{w_0}{L} x^2 - \frac{2w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^2$$

$$EI \frac{d^2 y}{dx^2} = M = \frac{1}{4} w_0 L x - \frac{1}{2} w_0 x^2 + \frac{1}{3} \frac{w_0}{L} x^3 - \frac{2}{3} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^3$$

$$EI \frac{dy}{dx} = \frac{1}{8} w_0 L x^2 - \frac{1}{6} w_0 x^3 + \frac{1}{12} \frac{w_0}{L} x^4 - \frac{1}{6} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^4 + C_1$$

$$EIy = \frac{1}{24} w_0 L x^3 - \frac{1}{24} w_0 x^4 + \frac{1}{60} \frac{w_0}{L} x^5 - \frac{1}{30} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^5 + C_1 x + C_2$$

$[x = 0,\ y = 0]$: $C_2 = 0$

$$[x = L,\ y = 0]: \quad \frac{1}{24} w_0 L^4 - \frac{1}{24} w_0 L^4 + \frac{1}{60} w_0 L^4 - \frac{1}{30} \frac{w_0}{L} \left(\frac{L}{2}\right)^5 + C_1 L = 0$$

$$C_1 = -\frac{1}{64} w_0 L^3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.44 *(Continued)*

(*a*) Equation of elastic curve.

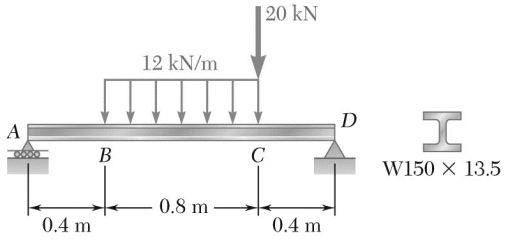
$$y = w_0\left[16x^5 - 32\left\langle x - \frac{L}{2}\right\rangle^5 - 40Lx^4 + 40L^2x^3 - 15L^4x\right]\Big/ 960\,EIL \quad \blacktriangleleft$$

(*b*) Deflection at *C*. $\left(y \quad \text{at} \quad x = \frac{L}{2}\right)$

$$y_C = \frac{w_0L^4}{960EI}\left(\frac{1}{2} - 0 - \frac{5}{2} + 5 - \frac{15}{2}\right) = -\frac{3w_0L^4}{640EI}$$

$$y_C = \frac{3w_0L^4}{640EI} \downarrow \quad \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

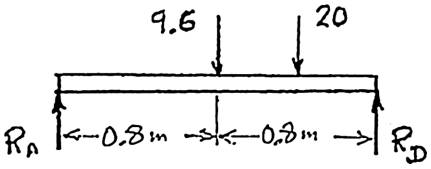


PROBLEM 9.45

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN, lengths in m



$$+\circlearrowleft M_D = 0:$$

$$-1.6\,R_A + (9.6)(0.8) + (20)(0.4) = 0$$

$$R_A = 9.8 \text{ kN}$$

$$w(x) = 12\langle x - 0.4\rangle^0 - 12\langle x - 1.2\rangle^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w(x) = -12\langle x - 0.4\rangle^0 + 12\langle x - 1.2\rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = 9.8 - 12\langle x - 0.4\rangle^1 + 12\langle x - 1.2\rangle^1 - 20\langle x - 1.2\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 9.8x - 6\langle x - 0.4\rangle^2 + 6\langle x - 1.2\rangle^2 - 20\langle x - 1.2\rangle^1 \quad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x - 0.4\rangle^3 + 2\langle x - 1.2\rangle^3 - 10\langle x - 1.2\rangle^2 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EIy = 1.63333x^3 - \frac{1}{2}\langle x - 0.4\rangle^4 + \frac{1}{2}\langle x - 1.2\rangle^4 - \frac{10}{3}\langle x - 1.2\rangle^3 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x = 0,\ y = 0]:\ 0 - 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 1.6,\ y = 0]:\ (1.63333)(1.6)^3 - \frac{1}{2}(1.2)^4 + \frac{1}{2}(0.4)^4 - \frac{10}{3}(0.4)^3 + C_1(1.6) + 0 = 0$$

$$C_1 = -3.4080 \text{ kN} \cdot \text{m}^2$$

Data: $E = 200 \times 10^9$ Pa, $I = 6.83 \times 10^6$ mm^4 $= 6.83 \times 10^{-6}$ mm^4

$$EI = (200 \times 10^4)(6.83 \times 10^{-6}) = 1.366 \times 10^6 \text{ N} \cdot \text{m}^2 = 1366 \text{ kN} \cdot \text{m}^2$$

(*a*) Slope at *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\frac{dy}{dx} = 0 - 0 + 0 - 0 - 3.4080 \quad \text{kN} \cdot \text{m}^2$$

$$\theta_A = -\frac{3.4080}{1366} = -2.49 \times 10^{-3} \text{ rad} \qquad \theta_A = 2.49 \times 10^{-3} \text{ rad} \ \searrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.45 *(Continued)*

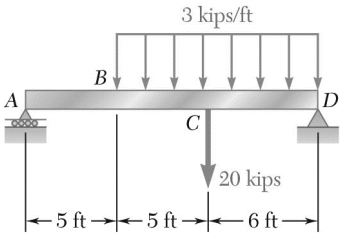
(*b*) Deflection at *C*. (y at $x = 1.2$ m)

$$EIy_C = (1.63333)(1.2)^3 - \frac{1}{2}(0.8)^4 + 0 - 0 - (3.4080)(1.2) + 0$$

$$= -1.4720 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{1.4720}{1366} = -1.078 \times 10^{-3} \text{m} \qquad y_C = 1.078 \text{ mm} \downarrow \blacktriangleleft$$

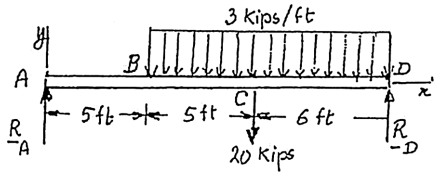
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.46

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the point *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips, lengths in feet.

$$E = 29 \times 10^3 \text{ ksi}, \quad I = 758 \text{ in}^4$$

$$EI = (29 \times 10^3)(758) = 21.982 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 152{,}650 \text{ kip} \cdot \text{ft}^2$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20)(6) + (3 \times 11)(5.5) - R_A(16) = 0 \qquad R_A = 18.844 \text{ kips} \uparrow$$

Express the loading as a singularity function.

$$w(x) = 3\langle x - 5\rangle^0$$

$$V(x) = R_A - \int w\,dx - 20\langle x - 10\rangle^0 = 18.844 - 3\langle x - 5\rangle^1 - 20\langle x - 10\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M(x) = 18.844x - 1.5\langle x - 5\rangle^2 - 20\langle x - 10\rangle^1$$

$$EI\frac{dy}{dx} = 9.422x^2 - 0.5\langle x - 5\rangle^3 - 10\langle x - 10\rangle^2 + C_1$$

$$EIy = 3.141x^3 - 0.125\langle x - 5\rangle^4 - 3.333\langle x - 10\rangle^3 + C_1x + C_2$$

Boundary conditions:

$$[x = 0,\ y = 0]: \quad C_2 = 0$$

$$[x = 16,\ y = 0]: 3.141(16)^3 - (0.125)(11)^4 - (3.3333)(6)^3 + 16\,C_1 = 0 \quad C_1 = -644.7 \text{ kip} \cdot \text{ft}^2$$

$$EI\frac{dy}{dx} = 9.442x^2 - 0.5\langle x - 5\rangle^3 - 10\langle x - 10\rangle^2 - 644.7$$

$$EIy = 3.141x^3 - 0.125\langle x - 5\rangle^4 - 3.333\langle x - 10\rangle^3 - 644.7x$$

(*a*) Slope at *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

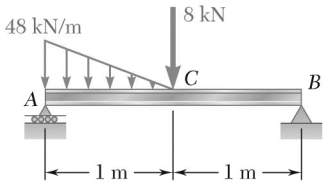
$$152{,}650\,\theta_A = 0 + 0 + 0 - 644.7 \quad \theta_A = -4.22 \times 10^{-3} \text{ rad} \qquad \theta_A = 4.22 \times 10^{-3} \text{ rad} \searrow \blacktriangleleft$$

(*b*) Deflection at *C*. $(y \text{ at } x = 10 \text{ ft})$

$$152{,}650\,y_C = (3.141)(10)^3 - (0.125)(5)^4 + 0 - (644.7)(10) = -3384$$

$$y_C = -0.02217 \text{ ft} \qquad y_C = 0.266 \text{ in.} \downarrow \blacktriangleleft$$

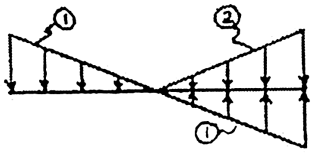
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.47

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

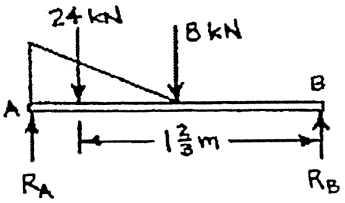
SOLUTION



Distributed loads: (1) $w_1(x) = w_0 - kx$ (2) $w_2 = k\langle x-1\rangle^1$

$$w_0 = 48 \text{ kN/m}, \quad k = 48 \text{ kN/m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: -2R_A + (24)\left(1\frac{2}{3}\right) + (8)(1) = 0 \qquad R_A = 24 \text{ kN} \uparrow$$



$$w(x) = w_0 - kx + k\langle x-1\rangle^1 = 48 - 48x + 48\langle x-1\rangle^1$$

$$\frac{dV}{dx} = -w = -48 + 48x - 48\langle x-1\rangle^1 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = 24 - 48x + 24x^2 - 24\langle x-1\rangle^2 - 8\langle x-1\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 24x - 24x^2 + 8x^3 - 8\langle x-1\rangle^3 - 8\langle x-1\rangle^1 \quad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EIy = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$$[x = 0,\ y = 0]: \ 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$$

$$[x = 2,\ y = 0]: \ 4(2)^3 - 2(2)^4 + \frac{2}{5}(2)^5 - \frac{2}{5}(1)^5 - \frac{4}{3}(1)^3 + C_1(2) = 0 \quad \therefore \quad C_1 = -\frac{83}{15} \text{ kN} \cdot \text{m}^2$$

Data: $E = 200(10^6) \text{ kN/m}^2 \quad I = 5.12(10^6) \text{ mm}^4 = 5.12(10^{-6}) \text{ m}^4$

$$EI = (200 \times 10^6)(5.12 \times 10^{-6}) = 1024 \text{ kN} \cdot \text{m}^2$$

(*a*) Slope at *A*. $\left(\frac{dy}{dx} \text{ at } x = 0\right)$

$$EI\theta_A = 0 - 0 + 0 - 0 - 0 - \frac{83}{15} \text{ kN} \cdot \text{m}^2$$

$$\theta_A = -\frac{83/15}{1024} = -5.4036 \times 10^{-3} \text{ rad} \qquad \theta_A = -5.40 \times 10^{-3} \text{ rad} \ \searrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

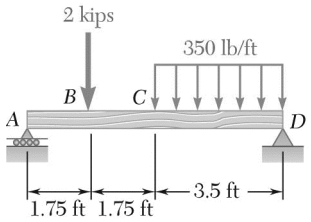
PROBLEM 9.47 ***(Continued)***

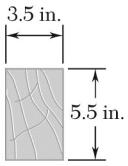
(*b*) Deflection at *C*. (y at $x = 1\,\text{m}$)

$$EIy_C = 4(1)^3 - 2(1)^4 + \frac{2}{5}(1)^4 - 0 - 0 - \frac{83}{15}(1) = -3.1333\,\text{kN}\cdot\text{m}^3$$

$$y_C = -\frac{3.1333}{1024} = -3.0599 \times 10^{-3}\,\text{m} \qquad y_C = 3.06\,\text{mm}\downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 9.48

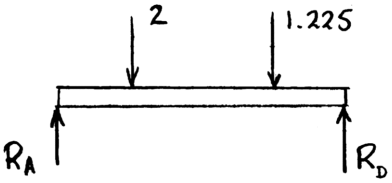
For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 1.6 \times 10^6$ psi.

SOLUTION

Units: Forces in kips; lengths in ft.

$$+\circlearrowleft \Sigma M_D = 0: \quad -7R_A + (2)(5.25) + (1.225)(1.75) = 0$$

$$R_A = 1.80625 \text{ kips}$$



$$w(x) = 0.350\langle x - 3.5\rangle^0$$

$$\frac{dV}{dx} = -w = -0.35\langle x - 3.5\rangle^0$$

$$\frac{dM}{dx} = V = 1.80625 - 2\langle x - 1.75\rangle^0 - 0.35\langle x - 3.5\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = 1.80625x - 2\langle x - 1.75\rangle^1 - 0.175\langle x - 3.5\rangle^2 \qquad \text{kip} \cdot \text{ft}$$

$$EI\frac{dy}{dx} = 0.903125x^2 - 1\langle x - 1.75\rangle^2 - 0.05833\langle x - 3.5\rangle^3 + C_1 \qquad \text{kip} \cdot \text{ft}^2$$

$$EIy = 0.301042x^3 - \frac{1}{3}\langle x - 1.75\rangle^3 - 0.014583\langle x - 3.5\rangle^4 + C_1x + C_2 \qquad \text{kip} \cdot \text{ft}^3$$

$$[x = 0,\ y = 0] \qquad C_2 = 0$$

$$[x = 7,\ y = 0] \quad (0.301042)(7)^3 - \frac{1}{3}(5.25)^3 - 0.014583(3.5)^4 + C_1(7) + 0 = 0$$

$$C_1 = -7.54779 \text{ kip} \cdot \text{ft}^2$$

Data:

$$E = 1.6 \times 10^6 \text{ psi} = 1.6 \times 10^3 \text{ ksi}$$

$$I = \frac{1}{12}(3.5)(5.5)^3 = 48.526 \text{ in}^4$$

$$EI = (1.6 \times 10^3)(48.526) = 77.6417 \text{ kip} \cdot \text{in}^2 = 539.18 \text{ kip} \cdot \text{ft}^2$$

(*a*) Slope at *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\frac{dy}{dx} = 0 - 0 - 0 - 7.54779 \text{ kip} \cdot \text{ft}^2$$

$$\theta_A = -\frac{7.54779}{539.18} = -14.00 \times 10^{-3} \text{rad} \qquad \theta_A = 14.00 \times 10^{-3} \text{rad} \ \searrow \ \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.48 *(Continued)*

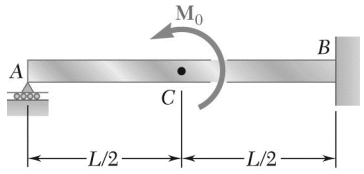
(*b*) Deflection at *C*. (y at $x = 3.5$ ft)

$$EIy_C = (0.301042)(3.5)^3 - \frac{1}{3}(1.75)^3 - 0 - (7.54779)(3.5) + 0$$

$$= -15.297 \text{ kip} \cdot \text{ft}^3$$

$$y_C = -\frac{15.297}{539.18} = -28.37 \times 10^{-3} \text{ft} \qquad y_C = 0.340 \text{ in.}\downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.49

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

$[x = 0,\ y = 0]$ $\left[x = L,\ \dfrac{dy}{dx} = 0\right]$

$[x = L,\ y = 0]$

SOLUTION

For $0 \le x \le \dfrac{1}{2}$, $M = R_A x$

For $\dfrac{L}{2} \le x \le L$, $M = R_A x - M_0$

Then

$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\left\langle x - \frac{L}{2}\right\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\left\langle x - \frac{L}{2}\right\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{2}\right\rangle^2 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$\left[x = L, \dfrac{dy}{dx} = 0\right]$ $\quad \dfrac{1}{2}R_A L^2 - M_0\left(\dfrac{L}{2}\right) + C_1 = 0 \qquad C_1 = \dfrac{1}{2}(M_0 L - R_A L^2)$

$[x = L,\ y = 0]$ $\quad \dfrac{1}{6}R_A L^3 - \dfrac{1}{2}M_0\left(\dfrac{L}{2}\right)^2 + \dfrac{1}{2}(M_0 L - R_A L^2)L + 0 = 0$

$$-\frac{1}{3}R_A L^3 + \frac{3}{8}M_0 L^2 = 0$$

(*a*) Reaction at *A*. $\qquad M_A = 0, \qquad R_A = \dfrac{9M_0}{8L} \uparrow$ ◀

$$C_1 = \frac{1}{2}\left[M_0 L - \left(\frac{9M_0}{8L}\right)(L^2)\right] = -\frac{1}{16}M_0 L$$

$$EIy = \frac{1}{6}\left(\frac{9M_0}{8L}\right)x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{16}M_0 L x + 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.49 *(Continued)*

Elastic curve. $$y = \frac{M_0}{EIL}\left\{\frac{9}{8}x^3 - \frac{1}{2}L\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{16}L^2x\right\}$$

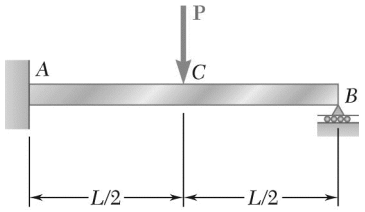
(*b*) Deflection at point *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{M_0}{EIL}\left\{\left(\frac{1}{6}\right)\left(\frac{9}{8}\right)\left(\frac{L}{2}\right)^3 - 0 - \left(\frac{1}{16}L^2\right)\left(\frac{L}{2}\right)\right\}$$

$$= -\frac{M_0L^2}{128EI}$$

$$y_C = \frac{M_0L^2}{128\,EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.50

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - P\frac{L}{2} + R_B L = 0 \qquad M_A = R_B L - \frac{1}{2}PL$$

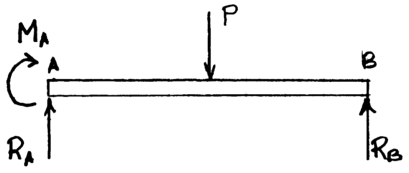
Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{2}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{2}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{2}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 x + C_2$$



$\left[x = 0, \dfrac{dy}{dx} = 0\right]$ $\quad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$

$[x = 0, y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$$

$$\frac{1}{2}\left(R_B L - \frac{1}{2}PL\right)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_B L^3 = \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{48}\right)PL^3 \qquad \frac{1}{3}R_B = \frac{5}{48}P$$

$$(a)\ R_B = \frac{5}{16}P \uparrow \blacktriangleleft$$

$$R_A = P - \frac{5}{16}P = \frac{11}{16}P$$

$$M_A = \frac{5}{16}PL - \frac{1}{2}PL = -\frac{3}{16}PL$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

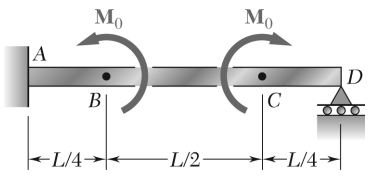
PROBLEM 9.50 *(Continued)*

(*b*) Deflection at *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{1}{EI}\left\{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + 0 + 0 + 0\right\}$$

$$= \frac{PL^3}{EI}\left\{\left(\frac{1}{2}\right)\left(-\frac{3}{16}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{11}{16}\right)\left(\frac{1}{8}\right)\right\} = -\frac{7}{168}\frac{PL^3}{EI} \qquad y_C = \frac{7}{168}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

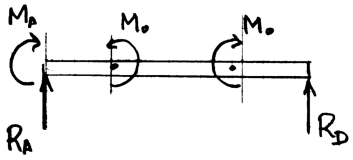
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.51

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_D = 0 \qquad R_A = -R_D$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + M_0 - M_0 + RL = 0$$

$$M_A = R_D L$$

$$M(x) = M_A + R_A x - M_0\left\langle x - \frac{L}{4}\right\rangle^0 + M_0\left\langle x - \frac{3L}{4}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = R_D L - R_D x - M_0\left\langle x - \frac{L}{4}\right\rangle^0 + M_0\left\langle x - \frac{3L}{4}\right\rangle^0$$

$$EI\frac{dy}{dx} = R_D L x - \frac{1}{2}R_D x^2 - M_0\left\langle x - \frac{L}{4}\right\rangle^1 + M_0\left\langle x - \frac{3L}{4}\right\rangle^1 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \quad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EIy = \frac{1}{2}R_D L x^2 - \frac{1}{6}R_D x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{4}\right\rangle^1 + \frac{1}{2}M_0\left\langle x - \frac{3L}{4}\right\rangle^1 + C_2$$

$$[x = 0, y = 0] \qquad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = L, y = 0] \qquad \frac{1}{2}R_D L^3 - \frac{1}{6}R_D L^3 - \frac{1}{2}M_0\left(\frac{3L}{4}\right)^2 + \frac{1}{2}M_0\left(\frac{L}{4}\right)^2 = 0$$

(*a*) Reaction at *D*.

$$R_D = \frac{3M_0}{4L} \uparrow \blacktriangleleft$$

$$EIy = \frac{1}{2}\left(\frac{3M_0}{4L}\right)Lx^2 - \frac{1}{6}\left(\frac{3M_0}{4L}\right)x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{4}\right\rangle^2 + \frac{1}{2}M_0\left\langle x - \frac{3L}{4}\right\rangle^2$$

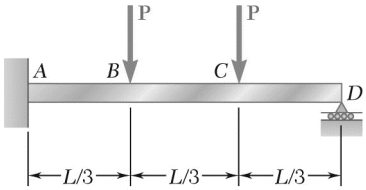
Elastic curve.

$$y = \frac{M_0}{EIL}\left\{\frac{3}{8}Lx^2 - \frac{1}{8}x^3 - \frac{1}{2}L\left\langle x - \frac{L}{4}\right\rangle^2 + \frac{1}{2}L\left\langle x - \frac{3L}{8}\right\rangle^2\right\}$$

(*b*) Deflection at point *B*. $\left(y \text{ at } x = \frac{L}{4}\right)$

$$y_B = \frac{M_0}{EIL}\left\{\left(\frac{3}{8}L\right)\left(\frac{L}{4}\right)^2 - \left(\frac{1}{8}\right)\left(\frac{L}{4}\right)^3 - 0 + 0\right\} \qquad y_B = \frac{11M_0L^2}{512EI} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

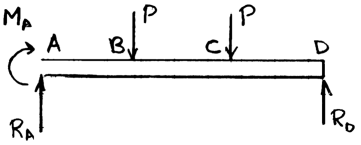


PROBLEM 9.52

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad R_A - P - P + R_D = 0 \qquad R_A = 2P - R_D$$



$$+\circlearrowleft \Sigma M_A = 0: \qquad -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$$

$$M_A = R_D L - PL$$

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0 - P\left\langle x - \frac{2L}{3}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{3}\right\rangle^1 - P\left\langle x - \frac{2L}{3}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{1}{2}P\left\langle x - \frac{2L}{3}\right\rangle^2 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}P\left\langle x - \frac{2L}{3}\right\rangle^3 + C_2$$

$$[x = 0,\ y = 0]\ 0 + 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}P\left\langle x - \frac{2L}{3}\right\rangle^3$$

$$[x = L,\ y = 0] \quad \frac{1}{2}(R_D L - PL)L^2 + \frac{1}{6}(2P - R_D)L^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 - \frac{1}{6}P\left(\frac{L}{3}\right)^3 = 0$$

$$\frac{1}{3}R_D L^3 - \frac{2}{9}PL^3 = 0$$

(*a*) Reaction at *D*. $\qquad R_D = \frac{2}{3}P\uparrow$ ◀

$$M_A = \frac{2}{3}PL - PL = -\frac{1}{3}PL \qquad R_A = 2P - \frac{2}{3}P = \frac{4}{3}P$$

$$EIy = \frac{1}{2}\left(-\frac{1}{3}PL\right)x^2 + \frac{1}{6}\left(\frac{4}{3}P\right)x^2 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}\left\langle x - \frac{2L}{3}\right\rangle^3$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.52 *(Continued)*

Elastic curve.

$$y = \frac{P}{EI}\left\{-\frac{1}{6}Lx^2 + \frac{2}{9}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}\left\langle x - \frac{2L}{3}\right\rangle^3\right\}$$

(*b*) Deflection at *B*.

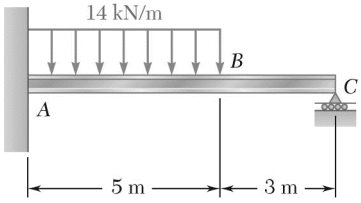
$$\left(y \text{ at } x = \frac{L}{3}\right)$$

$$y_B = \frac{P}{EI}\left\{-\left(\frac{1}{6}L\right)\left(\frac{L}{3}\right)^2 + \frac{2}{9}\left(\frac{L}{3}\right)^3 - 0 - 0\right\}$$

$$= -\frac{5PL^3}{486EI}$$

$$y_B = \frac{5PL^3}{486EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.53

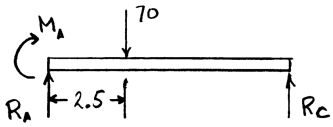
For the beam and loading shown, determine (*a*) the reaction at point *C*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.

$[x = 0,\ y = 0]$ $[x = 8,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

Units: Forces in kN; lengths in m.



$$+\uparrow \Sigma F_y = 0: \quad R_A - 70 + R_C = 0$$

$$R_A = 70 - R_C \quad \text{kN}$$

$$+\circlearrowleft M_A = 0: \quad -M_A - (70)(2.5) + 8R_C = 0$$

$$M_A = 8R_C - 175 \quad \text{kN} \cdot \text{m}$$

Reactions are statically indeterminate.

$$w(x) = 14 - 14\langle x - 5\rangle^0 \qquad \text{kN/m}$$

$$\frac{dV}{dx} = -w = -14 + 14\langle x - 5\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = R_A - 14x + 14\langle x - 5\rangle^1 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x - 5\rangle^2 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{7}{3}x^3 + \frac{7}{3}\langle x - 5\rangle^3 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{7}{12}x^4 + \frac{7}{12}\langle x - 5\rangle^4 + C_1 x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$\left[x = 0, \dfrac{dy}{dx} = 0\right]$ $\quad 0 + 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$

$[x = 0, y = 0]$ $\quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x = 8, y = 0]$ $\quad \dfrac{1}{2}M_A(8)^2 + \dfrac{1}{6}R_A(8)^3 - \dfrac{7}{12}(8)^4 + \dfrac{7}{12}(3)^4 + 0 + 0 = 0$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.53 *(Continued)*

$$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28105}{12} = 0$$

$$170.667R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \qquad R_C = 11.536 \text{ kN} \uparrow$$

(*a*) Reaction at *C*. $R_C = 11.546 \text{ kN} \uparrow$ ◀

$$M_A = (8)(11.536) - 175 = -82.715 \text{ kN} \cdot \text{m}$$
$$R_A = 70 - 11.536 = 58.464 \text{ kN}$$

Data: $E = 200 \times 10^9 \text{ Pa} \qquad I = 216 \times 10^6 \text{ mm}^4 = 216 \times 10^{-6} \text{ m}^4$

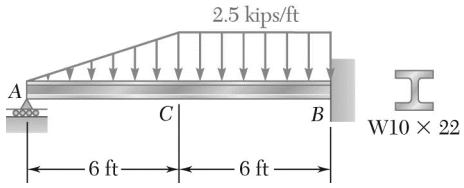
$$EI = (200 \times 10^9)(216 \times 10^{-6}) = 43.2 \times 10^6 \text{ N} \cdot \text{m}^2 = 43200 \text{ kN} \cdot \text{m}^2$$

(*b*) Deflection at *B*. (y at $x = 5$ m)

$$EIy_B = \frac{1}{2}(-82.715)(5)^2 + \frac{1}{6}(58.464)(5)^3 - \frac{7}{12}(5)^4 = -180.52 \text{ kN} \cdot \text{m}^3$$

$$y_B = -\frac{180.52}{43200} = -4.18 \times 10^{-3} \text{ m} \qquad y_B = 4.18 \text{ mm} \downarrow$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

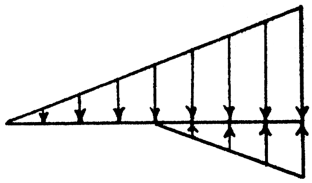


PROBLEM 9.54

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$k = \frac{2.5 \text{ kips/ft}}{6\text{ft}} = \frac{5}{12} \text{kips/ft}^2$$



$$w(x) = \frac{5}{12}x - \frac{5}{12}\langle x-6\rangle^1 \text{ kip/ft}$$

$$\frac{dV}{dx} = -w(x) = -\frac{5}{12}x + \frac{5}{12}\langle x-6\rangle^1 \text{ kip/ft}$$

$$\frac{dM}{dx} = V = R_A - \frac{5}{24}x^2 + \frac{5}{24}\langle x-6\rangle^2 \text{ kip}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{5}{72}x^3 + \frac{5}{72}\langle x-6\rangle^3 \text{ kip} \cdot \text{ft}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{5}{288}x^4 + \frac{5}{288}\langle x-6\rangle^4 + C_1 \text{ kip} \cdot \text{ft}^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{288}x^5 + \frac{1}{288}\langle x-6\rangle^5 + C_1 x + C_2 \text{ kip} \cdot \text{ft}^3$$

$$[x = 0, y = 0] \quad 0 - 0 + 0 + 0 + C_2 = 0 \quad \therefore \quad C_2 = 0$$

$$\left[x = 12\text{ft}, \frac{dy}{dx} = 0\right] \quad \frac{1}{2}R_A(12)^2 - \frac{5}{288}(12)^4 + \frac{5}{288}(6)^4 + C_1 = 0$$

$$\therefore \quad C_1 = 337.5 - 72R_A \text{ kip} \cdot \text{ft}^2$$

$$[x = 12 \text{ ft}, y = 0] \quad \frac{1}{6}R_A(12)^3 - \frac{1}{288}(12)^5 + \frac{1}{288}(6)^5 + (337.5 - 72R_A)(12) = 0$$

$$(864 - 288)R_A = 3213$$

$$R_A = 5.5781 \text{ kips}$$

(*a*) Reaction at *A*. $R_A = 5.58 \text{ kips} \uparrow$ ◀

$$C_1 = 337.5 - 72(5.5781) = -64.123 \text{ kip} \cdot \text{ft}^2$$

Data: $E = 29 \times 10^3$ ksi $I = 118 \text{ in}^4$

$$EI = (29 \times 10^3)(118) = 3.422 \times 10^6 \text{ kip} \cdot \text{in}^2 = 23764 \text{ kip} \cdot \text{ft}^2$$

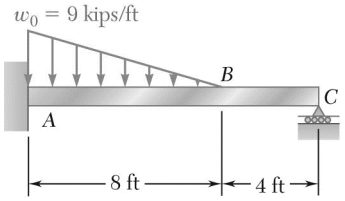
(*b*) Deflection at *C*. (y at $x = 6$ ft)

$$EIy_C = \frac{1}{6}(5.5781)(6)^3 - \frac{1}{288}(6)^5 + 0 - 64.123(6)$$

$$= -210.93 \text{ kip} \cdot \text{ft}^3$$

$$y_C = -\frac{210.93}{23764} = -8.8760 \times 10^{-3}\text{ft} \qquad y_C = 0.1065 \text{ in.} \downarrow$$ ◀

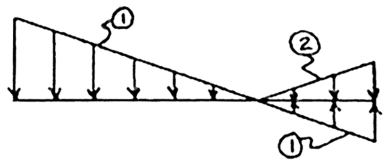
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.55

For the beam and loading shown, determine (*a*) the reaction at point *C*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.

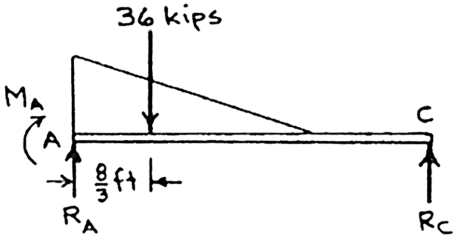
SOLUTION



Distributed loads: $k = \dfrac{9}{8}$

(1) $w_1(x) = w_0 - kx$ (2) $w_2(x) = k\langle x - 8\rangle^1$

$w_0 = 9$ kips/ft, $k = \dfrac{9}{8}$ kips/ft^2



$+\uparrow \sum F_y = 0$: $R_A - 36 + R_C = 0$ $R_A = (36 - R_C)$ kips

$+\circlearrowleft \sum M_A = 0$: $12R_C - M_A - \left(\dfrac{8}{3}\right)(36) = 0$

$M_A = (12R_C - 96)$ kip · ft

$w(x) = w_0 - kx + k\langle x - 8\rangle^1 = 9 - \dfrac{9}{8}x + \dfrac{9}{8}\langle x - 8\rangle^1$ kips/ft

$\dfrac{dV}{dx} = -w = -9 + \dfrac{9}{8}x - \dfrac{9}{8}\langle x - 8\rangle^1$ kips/ft

$\dfrac{dM}{dx} = V = R_A - 9x + \dfrac{9}{16}x^2 - \dfrac{9}{16}\langle x - 8\rangle^2$ kips

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{9}{2}x^2 + \frac{3}{16}x^3 - \frac{3}{16}\langle x - 8\rangle^3 \quad \text{kip} \cdot \text{ft}$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{3}{2}x^3 + \frac{3}{64}x^4 - \frac{3}{64}\langle x - 8\rangle^4 + C_1 \quad \text{kip} \cdot \text{ft}^2$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{3}{8}x^4 + \frac{3}{320}x^5 - \frac{3}{320}\langle x - 8\rangle^5 + C_1 x + C_2 \quad \text{kip} \cdot \text{ft}^3$$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$: $C_1 = 0$ $[x = 0,\ y = 0]$: $C_2 = 0$

$$[x = 12,\ y = 0]: \frac{1}{2}M_A(12)^2 + \frac{1}{6}R_A(12)^3 - \frac{3}{8}(12)^4 + \frac{3}{320}(12)^5 - \frac{3}{320}(4)^5 + 0 + 0 = 0$$

$$72\,(12R_C - 96) + 288\,(36 - R_C) - 5452.8 = 0$$

$R_C = 3.4667$ $R_C = 3.47$ kips ↑ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.55 *(Continued)*

(*a*) Reaction at *C*: $R_C = 3.47 \text{ kips} \uparrow$ ◀

$$M_A = 12(3.4667) - 96 = -54.400 \text{ kip} \cdot \text{ft} \qquad R_A = 36 - 3.4667 = 32.533 \text{ kips}$$

Data: $E = 29 \times 10^3 \text{ ksi} \quad I = 307 \text{ in}^4 \quad EI = (29 \times 10^3)(307) = 8.903 \times 10^6 \text{ kip} \cdot \text{in}^2$

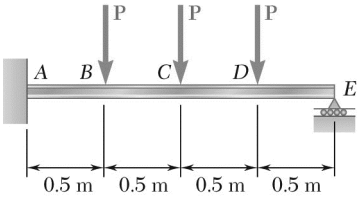
$$= 61{,}826 \text{ kip} \cdot \text{ft}^2$$

(*b*) Deflection at *B*. (y at $x = 8$ ft)

$$EIy_B = \frac{1}{2}(-54.400)(8)^2 + \frac{1}{6}(32.533)(8)^3 - \frac{3}{8}(8)^4 + \frac{3}{320}(8)^5 - 0 = -193.451 \quad \text{kip} \cdot \text{ft}^3$$

$$y_B = \frac{-193.451}{61{,}826} = -3.1290 \times 10^{-3} \text{ ft} \qquad y_B = 0.0376 \text{ in.} \downarrow$$ ◀

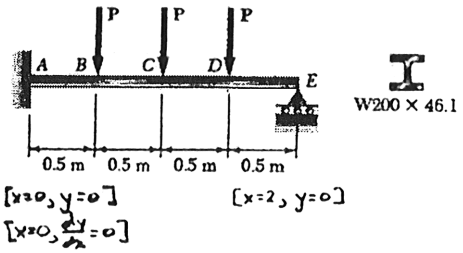
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

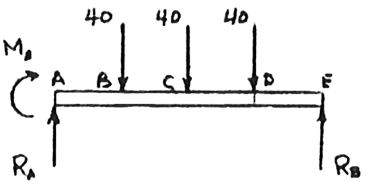


PROBLEM 9.56

For the beam shown and knowing that $P = 40$ kN, determine (*a*) the reaction at point *E*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION





Units: Forces in kN; lengths in m.

$$+\uparrow \sum F_y = 0: \quad R_A - 40 - 40 - 40 + R_E = 0$$

$$R_A = 120 - R_E \quad \text{kN}$$

$$+\circlearrowleft M_A = 0: \quad -M_A - 20 - 40 - 60 + 2R_B = 0$$

$$M_A = 2R_E - 120 \quad \text{kN} \cdot \text{m}$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - 40\langle x - 0.5\rangle^0 - 40\langle x - 1\rangle^0 - 40\langle x - 1.5\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 40\langle x - 0.5\rangle^1 - 40\langle x - 1\rangle^1 - 40\langle x - 1.5\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - 20\langle x - 0.5\rangle^2 - 20\langle x - 1\rangle^2 - 20\langle x - 1.5\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{20}{3}\langle x - 0.5\rangle^3 - \frac{20}{3}\langle x - 1\rangle^3 - \frac{20}{3}\langle x - 1.5\rangle^3 + C_1 x + C_2$$

$$[x = 0, \ \frac{dy}{dx} = 0] \qquad 0 + 0 + 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0, \ y = 0] \qquad 0 + 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 2, \ y = 0] \qquad \frac{1}{2}M_A(2)^2 + \frac{1}{6}R_A(2)^3 - \frac{20}{3}(1.5)^3 - \frac{20}{3}(1)^3 - \frac{20}{3}(0.5)^3 + 0 + 0 = 0$$

(*a*) Reaction at *E*.

$$\frac{1}{2}(2R_E - 120)(2)^2 + \frac{1}{6}(120 - R_E)(2)^3 = 30$$

$$2.66667\,R_E = 30 + 240 - 160 = 110 \qquad R_E = 41.25 \text{ kN} \uparrow \blacktriangleleft$$

$$M_A = (2)(41.25) - 120 = -37.5 \text{ kN} \cdot \text{m}$$

$$R_A = 120 - 41.25 = 78.25 \text{ kN}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.56 *(Continued)*

Data: $E = 200 \times 10^9$ Pa, $I = 45.8 \times 10^6$ mm^4 $= 45.8 \times 10^{-6}$ m^4

$$EI = (200 \times 10^9)(45.8 \times 10^{-6}) = 9.16 \times 10^6 \text{ N} \cdot \text{m}^2 = 9160 \text{ kN} \cdot \text{m}^2$$

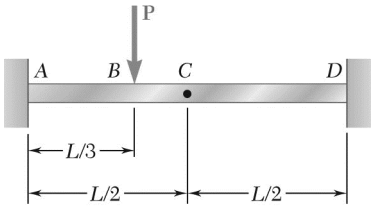
(*b*) Deflection at *C*. (y at $x = 1$ m)

$$EIy_C = \frac{1}{2}(-37.5)(1)^2 + \frac{1}{6}(78.75)(1)^3 - \frac{20}{3}(0.5)^3 - 0 - 0 + 0 + 0$$

$$= -6.4583 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{6.4583}{9160} = -0.705 \times 10^{-3} \text{ m} \qquad y_C = 0.705 \text{ mm} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

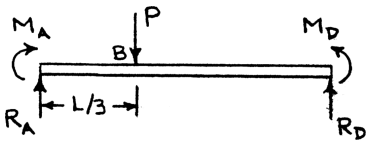


PROBLEM 9.57

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.

SOLUTION

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0$$



$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{3}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 + C_1 x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 + C_1 = 0 \qquad \therefore\ C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 - 0 + C_2 = 0 \qquad \therefore\ C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{2}P\left(\frac{2L}{3}\right)^2 = 0 \tag{1}$$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 = 0 \tag{2}$$

(*a*) Solving Eqs. (1) and (2) simultaneously,

$$R_A = \frac{20}{27}P \qquad M_A = -\frac{4}{27}PL$$

$$R_A = \frac{20}{27}P \uparrow \blacktriangleleft$$

$$M_A = \frac{4}{27}PL \circlearrowleft \blacktriangleleft$$

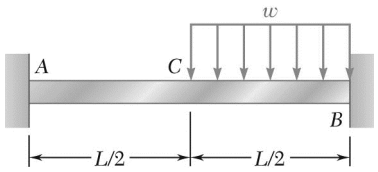
Elastic curve. $$y = \frac{P}{EI}\left[-\frac{2}{27}Lx^2 + \frac{10}{81}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3\right]$$

(*b*) Deflection at midpoint *C*. $\left(y \text{ at } x = \dfrac{L}{2}\right)$

$$y_C = \frac{P}{EI}\left[-\frac{2}{27}L\left(\frac{L}{2}\right)^2 + \frac{10}{81}\left(\frac{L}{2}\right)^3 - \frac{1}{6}\left(\frac{L}{6}\right)^3\right] = -\frac{5PL^3}{1296EI}$$

$$y_C = \frac{5PL^3}{1296EI} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.58

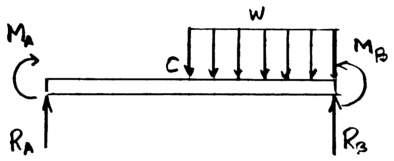
For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.

SOLUTION

$$w(x) = w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\left\langle x - \frac{L}{2}\right\rangle^1$$



$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + C_1 x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \qquad 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \qquad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 = 0 \tag{1}$$

$$[x = L,\ y = 0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 = 0 \tag{2}$$

Solving Eqs. (1) and (2) simultaneously,

(*a*) $$R_A = \frac{3wL}{32} \qquad M_A = -\frac{5wL^2}{192}$$

$$R_A = \frac{3wL}{32} \uparrow \blacktriangleleft$$

$$M_A = \frac{5wL^2}{192} \circlearrowright \blacktriangleleft$$

$$EIy = -\frac{5}{384}wL^2x^2 + \frac{3}{192}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.58 *(Continued)*

Elastic curve.

$$y = \frac{w}{EI}\left\{-\frac{5}{384}L^2x^2 + \frac{3}{192}Lx^3 - \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4\right\}$$

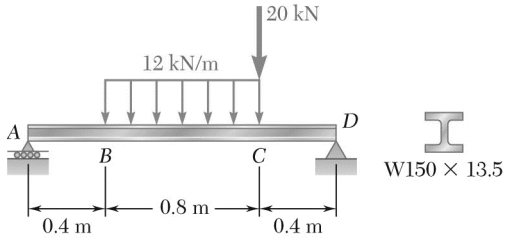
(*b*) Deflection at midpoint *C*. $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{w}{EI}\left\{\left(-\frac{5}{384}L^2\right)\left(\frac{L}{2}\right)^2 + \left(\frac{3}{192}L\right)\left(\frac{L}{2}\right)^3 - 0\right\}$$

$$= -\frac{wL^4}{768EI}$$

$$y_B = \frac{wL^4}{768EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.59

For the beam and loading of Prob. 9.45, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.45 for the derivation of the equations used in the following:

$$EI = EI = 1366 \text{ kN} \cdot \text{m}^2$$

$$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x - 0.4\rangle^3 + 2\langle x - 1.2\rangle^3 - 10\langle x - 1.2\rangle^2 - 3.4080 \text{ kN} \cdot \text{m}^2$$

$$EIy = 1.63333x^3 - \frac{1}{2}\langle x - 0.4\rangle^4 + \frac{1}{2}\langle x - 1.2\rangle^4 - \frac{10}{3}\langle x - 1.2\rangle^3 - 3.4080x \text{ kN} \cdot \text{m}^3$$

To find the location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0.4 < x < 1.2$.

$$4.9x^2 - 2(x - 0.4)^3 - 3.4080 = f(x) = 0$$

Solve by iteration:

$x =$	0.8	0.858	0.857	0.8570
$df/dx =$	6.88	7.123	7.145	

$x_m = 0.8570$ m ◀

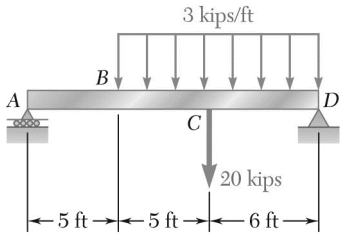
$$EIy_m = (1.63333)(0.8570)^3 - \frac{1}{2}(0.8570 - 0.4)^4 - (3.4080)(0.8570)$$

$$= -1.9144 \text{ kN} \cdot \text{m}$$

$$y_m = -\frac{1.9144}{1366} = -1.401 \times 10^{-3} \text{m}$$

$y_m = 1.401$ mm ↓ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.60

For the beam and loading of Prob. 9.46, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.46 for the derivation of following:

$$EI = 152{,}650 \text{ kip} \cdot \text{ft}^2$$

$$EI\frac{dy}{dx} = 9.422x^2 - 0.5\langle x-5\rangle^3 - 10\langle x-10\rangle^2 - 644.7 \qquad \text{kip} \cdot \text{ft}^2$$

$$EIy = 3.141x^3 - 0.125\langle x-5\rangle^4 - 3.333\langle x-10\rangle^3 - 644.7x \qquad \text{kip} \cdot \text{ft}^3$$

To find the location of maximum $|y|$, set $\dfrac{dy}{dx} = 0$.

Assume $5 \le x < 10$.

$$EI\frac{dy}{dx} = 9.422x^2 - 0.5(x-5)^3 - 644.7 = f(x) = 0 \tag{1}$$

$$\frac{df}{dx} = 18.844x - 1.5(x-5)^2$$

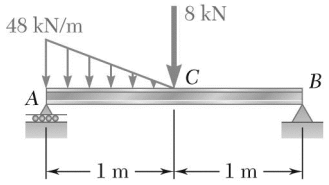
Solve Eq. (1) by iteration: $\quad x_{i+1} = x_i - \dfrac{f}{df/dx}$

x =	9	8.406	8.397	$x_m = 8.40$ ft ◀
f =	86.48	1.310	0.0415	
df/dx =	145.6	141.0		

$$EIy_m = (3.141)(8.397)^3 - (0.125)(8.397-5)^4 - (644.7)(8.397) = -3570.5 \text{ kip} \cdot \text{ft}^3$$

$$152{,}650\, y_m = -3570.5 \qquad y_m = -0.02339 \text{ ft} \qquad y_m = 0.281 \text{ in.} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



S130 × 15

PROBLEM 9.61

For the beam and loading of Prob.9.47, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.47 for the derivation of the equations used in the following:

$$EI = 1024 \text{ kN} \cdot \text{m}^2$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 - \frac{83}{15} \quad \text{kN} \cdot \text{m}^2$$

$$EIy = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 - \frac{83}{15}x \quad \text{kN} \cdot \text{m}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0 < x < 1$ m.

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - \frac{83}{15} = 0$$

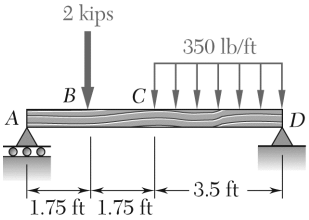
Solving: $x = 0.94166$ m $\qquad x_m = 0.942$ m ◀

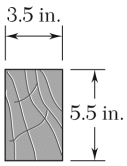
$$EIy_m = 4(0.94166)^3 - 2(0.94166)^4 + \frac{2}{5}(0.94166)^5 - \frac{83}{15}(0.94166)$$

$$= -3.1469 \text{ kN} \cdot \text{m}^3$$

$$y_m = -\frac{3.1469}{1024} = -3.0731 \times 10^{-3} \text{ m} \qquad y_m = 3.07 \text{ mm} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 9.62

For the beam and loading of Prob. 9.48, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.48 for the derivation of the equations used in the following:

$$EI = 539.18 \text{ kip} \cdot \text{ft}^2$$

$$EI\frac{dy}{dx} = 0.903125x^2 - 1\langle x - 1.75\rangle^2 - 0.05833\langle x - 3.5\rangle^2 - 7.54779 \quad \text{kip} \cdot \text{ft}^2$$

$$EIy = 0.301042x^3 - \frac{1}{3}\langle x - 1.75\rangle^3 - 0.014583\langle x - 3.5\rangle^4 - 7.54779x \quad \text{kip} \cdot \text{ft}^3$$

To find the location of maximum $|\ y\ |$, set $\dfrac{dy}{dx} = 0$. Assume $1.75 < x_m < 3.5$.

$$0.903125x_m^2 - 1(x_m - 1.75)^2 - 7.54779$$

$$0.096875x_m^2 - 8.5x_m + 10.61029 = 0$$

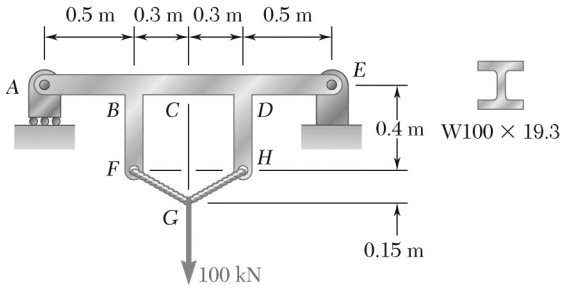
$$x_m = \frac{3.5 - \sqrt{(3.5)^2 - (4)(0.096875)(10.61029)}}{(2)(0.096875)} \qquad x_m = 3.34 \text{ ft} \blacktriangleleft$$

$$EIy_m = (0.301042)(3.340)^3 - \frac{1}{3}(3.340 - 1.73)^3 - (7.54779)(3.340)$$

$$= -15.3328 \text{ kip} \cdot \text{ft}^3$$

$$y_m = -\frac{15.3328}{539.18} = -12.44 \times 10^{-3} \text{ft} \qquad y_m = 0.341 \text{ in.} \downarrow \blacktriangleleft$$

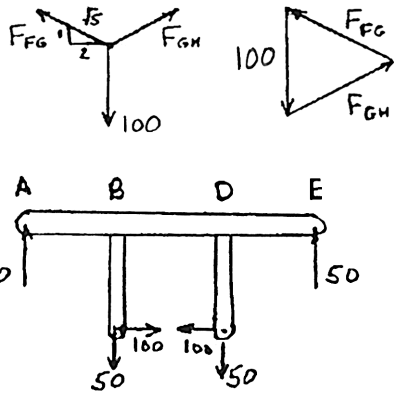
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.63

The rigid bars *BF* and *DH* are welded to the rolled-steel beam *AE* as shown. Determine for the loading shown (*a*) the deflection at point *B*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION



Use joint *G* as a free body,

By symmetry, $F_{GH} = F_{FG}$

$$+\uparrow \sum F_y = 0: \quad 2F_{GHy} - 100 = 0 \qquad F_{GHy} = 50 \text{ kN}$$

$$F_{GHx} = 2\,F_{GHy} = 100 \text{ kN}.$$

Forces in kN; lengths in m.

$$V = 50 - 50\langle x - 0.5\rangle^0 - 50\langle x - 1.1\rangle^0 \quad \text{kN}$$

$$M = 50x - 50\langle x - 0.5\rangle^1 - 50\langle x - 1.1\rangle^0 + 40\langle x - 0.5\rangle^0 - 40\langle x - 1.1\rangle^0 \quad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 25x^2 - 25\langle x - 0.5\rangle^2 - 25\langle x - 1.1\rangle^2 - 40\langle x - 0.5\rangle^1 + 40\langle x - 1.1\rangle^1 + C_1 \quad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{25}{3}x^3 - \frac{25}{3}\langle x - 0.5\rangle^3 - \frac{25}{3}\langle x - 1.1\rangle^3 - 20\langle x - 0.5\rangle^2 + 20\langle x - 1.1\rangle^2 + C_1x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$[x = 0,\ y = 0] \qquad C_2 = 0$

$[x = 1.6,\ y = 0]$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \text{ kN} \cdot \text{m}^3$$

For EIy_B, $x = 0.5\,\text{m}$

$$EIy_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667 \text{ kN} \cdot \text{m}^3$$

For EIy_C, $x = 0.8$ m

$$EIy_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0$$

$$= -0.8417 \text{ kN} \cdot \text{m}^3$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.63 *(Continued)*

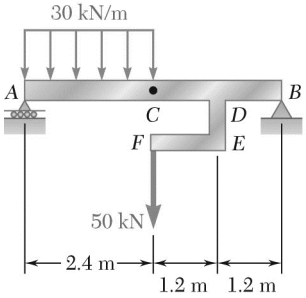
For $W\,100 \times 19.3$ rolled-steel shape, $I = 4.70 \times 10^{6}\,\text{mm}^4 = 4.70 \times 10^{-6}\,\text{m}^4$

$$EI = (200 \times 10^{9})(4.70 \times 10^{-6}) = 940 \times 10^{3}\,\text{N} \cdot \text{m}^2 = 940\,\text{kN} \cdot \text{m}^2$$

(*a*) $y_B = \dfrac{0.1667}{940} = 0.177 \times 10^{-3}\,\text{m}$ $\qquad y_B = 0.177\,\text{mm} \uparrow$ ◀

(*b*) $y_C = \dfrac{0.8417}{940} = 0.895 \times 10^{-3}\,\text{m}$ $\qquad y_C = 0.895\,\text{mm} \uparrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.64

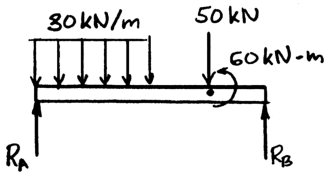
The rigid bar *DEF* is welded at point *D* to the rolled-steel beam *AB*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

$$+\circlearrowleft M_B = 0: \quad -4.8R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$$

$$R_A = 79 \text{ kN} \uparrow$$



$$I = 212 \times 10^6 \text{mm}^4 = 212 \times 10^{-6} \text{m}^4$$

$$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{N} \cdot \text{m}^2$$

$$= 42400 \text{ kN} \cdot \text{m}^2$$

$$w(x) = 30 - 30\langle x - 2.4\rangle^0$$

$$\frac{dV}{dx} = -w = -30 + 30\langle x - 2.4\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = 79 - 30x + 30\langle x - 2.4\rangle^1 - 50\langle x - 3.6\rangle^0 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx} = M = 79x - 15x^2 + 15\langle x - 2.4\rangle^2 - 50\langle x - 3.6\rangle^1 - 60\langle x - 3.6\rangle^0 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5\langle x - 2.4\rangle^3 - 25\langle x - 3.6\rangle^2 - 60\langle x - 3.6\rangle^1 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x - 2.4\rangle^4 - \frac{25}{3}\langle x - 3.6\rangle^3 - 30\langle x - 3.6\rangle^2 + C_1x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$[x = 0, y = 0]$ $\quad 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 4.8, y = 0]$

$$\left(\frac{79}{6}\right)(4.8)^3 - \left(\frac{5}{4}\right)(4.8)^4 + \left(\frac{5}{4}\right)(2.4)^4 - \left(\frac{25}{3}\right)(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$$

$$C_1 = -161.76 \text{ kN} \cdot \text{m}^2$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.64 *(Continued)*

(*a*) Slope at point *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = 0 - 0 + 0 - 0 - 0 - 161.76$$

$$= -161.76 \text{ kN} \cdot \text{m}^2$$

$$\left(\frac{dy}{dx}\right)_A = \frac{-161.76}{42400} = -3.82 \times 10^{-3} \qquad \theta_A = 3.82 \times 10^{-3} \text{rad.} \curvearrowright \blacktriangleleft$$

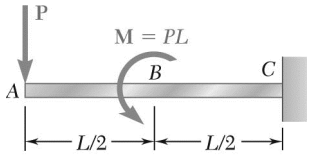
(*b*) Deflection at midpoint *C*. (y at $x = 2.4$)

$$EIy_C = \left(\frac{79}{6}\right)(2.4)^3 - \left(\frac{5}{4}\right)(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0$$

$$= -247.68 \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-247.68}{42400} = -5.84 \times 10^{-3} \text{m} \qquad y_C = 5.84 \text{ mm} \downarrow \blacktriangleleft$$

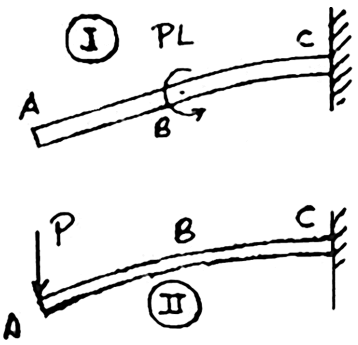
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.65

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION



Loading I: Counterclockwise couple PL at B.

Case 3 of Appendix D applied to portion BC.

$$\theta'_B = -\frac{(PL)(L/2)}{EI} = \frac{1}{2}\frac{PL^2}{EI}$$

$$y'_B = \frac{(PL)(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^3}{EI}$$

AB remains straight.

$$\theta'_A = \theta'_B = \frac{1}{2}\frac{PL^2}{EI}$$

$$y'_A = y'_B - \left(\frac{L}{2}\right)\theta'_B = -\frac{1}{8}\frac{PL^3}{EI} - \frac{1}{4}\frac{PL^3}{EI} = -\frac{3}{8}\frac{PL^3}{EI}$$

Loading II: Case 1 of Appendix D.

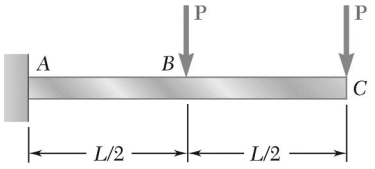
$$\theta''_A = \frac{PL^2}{2EL}, \quad y''_A - \frac{PL^3}{3EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = \frac{1}{2}\frac{PL^2}{EI} + \frac{1}{2}\frac{PL^2}{EI} = \frac{PL^2}{EI} \qquad \theta_A = \frac{PL^2}{EI} \measuredangle \blacktriangleleft$$

$$y_A = y'_A + y''_A = -\frac{3}{8}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{17}{24}\frac{PL^3}{EI} \qquad y_A = \frac{17}{24}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



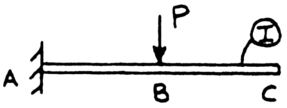
PROBLEM 9.66

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION

Loading I: P downward at B.

Case 1 of Appendix D applied to portion AB.



$$\theta'_B = -\frac{P(L/2)^2}{2EI} = -\frac{1}{8}\frac{PL^2}{EI}$$

$$y'_B = -\frac{P(L/2)^3}{3EI} = -\frac{1}{24}\frac{PL^3}{EI}$$

BC remains straight.

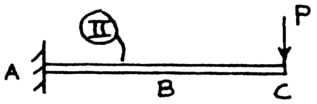
$$\theta'_C = \theta'_B = -\frac{1}{8}\frac{PL^2}{EI}$$

$$y'_C = y'_B - \left(\frac{L}{2}\right)\theta'_B = -\frac{1}{24}\frac{PL^3}{EI} - \frac{1}{16}\frac{PL^3}{EI}$$

$$= -\frac{5}{48}\frac{PL^3}{EI}$$

Loading II: P downward at C.

Case 1 of Appendix D.



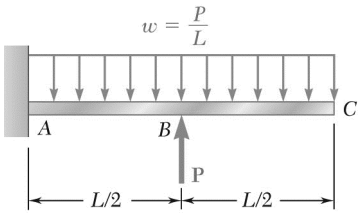
$$\theta''_C = -\frac{PL^2}{2EI} \qquad y''_C = -\frac{PL^3}{3EI}$$

By superposition,

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{8}\frac{PL^2}{EI} - \frac{1}{2}\frac{PL^2}{EI} = -\frac{5}{8}\frac{PL^2}{EI} \qquad \frac{5PL^2}{8EI} \curvearrowright \blacktriangleleft$$

$$y_C = y'_C + y''_C = -\frac{5}{48}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{21}{48}\frac{PL^3}{EI} \qquad \frac{7PL^3}{16EI} \downarrow \blacktriangleleft$$

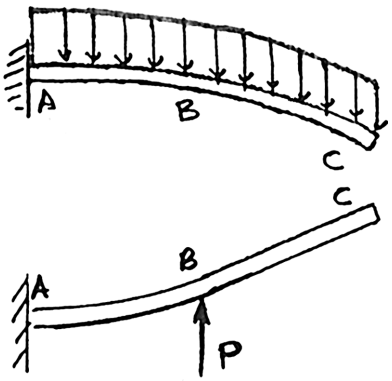
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.67

For the cantilever beam and loading shown, determine the slope and deflection at the end.

SOLUTION



Loading I: Uniformly distributed downward loading with $w = P/L$.

Case 2 of Appendix *D*.

$$\theta'_C = -\frac{(P/L)L^3}{6EI} = -\frac{1}{6}\frac{PL^2}{EI}$$

$$y'_C = -\frac{(P/L)L^4}{8EI} = -\frac{1}{8}\frac{PL^3}{EI}$$

Loading II: Upward concentrated load at *P* at point *B*.

Case 1 of Appendix *D* applied to portion *AB*.

$$\theta''_B = \frac{P(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^2}{EI}$$

$$y''_B = \frac{P(L/2)^3}{3EI} = \frac{1}{24}\frac{PL^3}{EI}$$

Portion *BC* remains straight.

$$\theta''_C = \theta''_B = \frac{1}{8}\frac{PL^2}{EI}$$

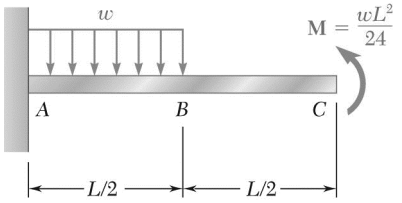
$$y''_C = y''_B + \frac{L}{2}\theta''_B = \frac{1}{24}\frac{PL^3}{EI} + \frac{1}{16}\frac{PL^3}{EI} = \frac{5}{48}\frac{PL^3}{EI}$$

By superposition,

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{6}\frac{PL^2}{EI} + \frac{1}{8}\frac{PL^2}{EI} = -\frac{1}{24}\frac{PL^2}{EI} \qquad \theta_C = \frac{PL^2}{24EI} \measuredangle \blacktriangleleft$$

$$y_C = y'_C + y''_C = -\frac{1}{8}\frac{PL^3}{EI} + \frac{5}{48}\frac{PL^2}{EI} = -\frac{1}{48}\frac{PL^3}{EI} \qquad y_C = \frac{PL^3}{48EI} \downarrow \blacktriangleleft$$

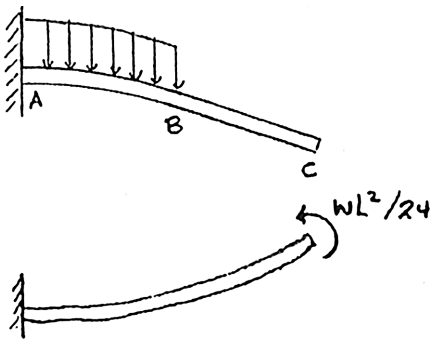
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.68

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION



Loading I: Downward distributed load w applied to portion AB.

Case 2 of Appendix D applied to portion AB.

$$\theta'_B = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^2}{EI}$$

$$y'_B = -\frac{w(L/2)^4}{8EI} = -\frac{1}{128}\frac{wL^4}{EI}$$

Portion BC remains straight.

$$\theta'_C = \theta'_B = -\frac{1}{48}\frac{wL^3}{EI}$$

$$y'_C = y'_B + \left(\frac{L}{2}\right)\theta'_B = -\frac{1}{128}\frac{wL^4}{EI} - \frac{1}{96}\frac{wL^4}{EI} = -\frac{7}{384}\frac{wL^4}{EI}$$

Loading II: Counterclockwise couple $\dfrac{wL^2}{24}$ applied at C.

Case 3 of Appendix D.

$$\theta''_C = \frac{(wL^2/24)L}{EI} = \frac{1}{24}\frac{wL^3}{EI}$$

$$y''_C = \frac{(wL^2/24)L^2}{2EI} = \frac{1}{48}\frac{wL^4}{EI}$$

By superposition,

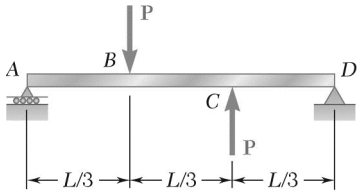
$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{48}\frac{wL^3}{EI} + \frac{1}{24}\frac{wL^3}{EI}$$

$$\theta_C = \frac{1}{48}\frac{wL^3}{EI} \curvearrowright \blacktriangleleft$$

$$y_C = y'_C + y''_C = -\frac{7}{384}\frac{wL^4}{EI} + \frac{1}{48}\frac{wL^4}{EI}$$

$$y_C = \frac{1}{384}\frac{wL^4}{EI} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.69

For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION

Loading I: Downward load *P* at *B*.

Use Case 5 of Appendix *D* with

$$P = P, \quad a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad L = L, \quad x = \frac{2L}{3}$$

For $x < a$, given elastic curve is $\quad y = \dfrac{Pb}{EIL}[x^3 - (L^2 - b^2)x]$

To obtain elastic curve for $x > a$, replace x by $L - x$ and interchange a and b to get

$$y = \frac{Pa}{6EIL}[(L - x)^3 - (L^2 - a^2)(L - x)] \text{ with } x = \frac{2L}{3} \text{ at point } C.$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left(L^2 - \left(\frac{L}{3}\right)^2\right)\left(\frac{L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)[L^2 - (2L/3)^2]}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Upward load at *C*. Use Case 5 of Appendix *D* with

$$P = -P, \quad a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad L = L, \quad x = a = \frac{2L}{3}$$

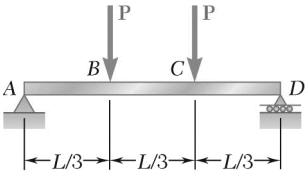
$$y_C = -\frac{(-P)(2L/3)^2(L/3)^2}{3EIL} = \frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{(-P)(L/3)(L^2 - (L/3)^2)}{6EIL} = \frac{4}{81}\frac{PL^2}{EI}$$

(*a*) Deflection at *C*. $\quad y_C = -\dfrac{7}{486}\dfrac{PL^3}{EI} + \dfrac{4}{243}\dfrac{PL^3}{EI}$ $\qquad y_C = \dfrac{1}{486}\dfrac{PL^3}{EI} \uparrow$ ◀

(*b*) Slope at *A*. $\quad \theta_A = -\dfrac{5}{81}\dfrac{PL^2}{EI} + \dfrac{4}{81}\dfrac{PL^2}{EI}$ $\qquad \theta_A = \dfrac{1}{81}\dfrac{PL^2}{EI}$ ↘ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.70

For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION

Loading I: Load at *B*. Case 5 of Appendix *D*.

$$a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad x = \frac{2L}{3}$$

For $x > a$, replace x by $L - x$ and interchange a and b.

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(L - \frac{2L}{3}\right)^3 - \left(L^2 - \frac{L^2}{9}\right)\left(L - \frac{2L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)(L^2 - 4L^2/9)}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Load at *C*. Case 5 of Appendix *D*.

$$a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad x = \frac{2L}{3}$$

$$y_C = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x] = \frac{P(L/3)}{6EIL}\left[\frac{8L^3}{27} - \left(L^2 - \frac{L^2}{9}\right)\left(\frac{2L}{3}\right)\right]$$

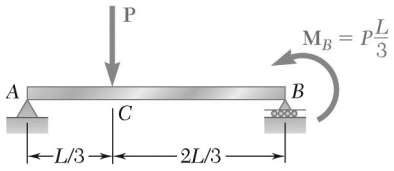
$$= -\frac{8}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(L/3)(L^2 - L^2/9)}{6EIL} = -\frac{4}{81}\frac{PL^2}{EI}$$

(*a*) Deflection at *C*. $y_C = -\frac{7}{486}\frac{PL^3}{EI} - \frac{8}{486}\frac{PL^3}{EI} = -\frac{15}{486}\frac{PL^3}{EI}$ $\qquad y_C = \frac{5}{162}\frac{PL^3}{EI} \downarrow$ ◀

(*b*) Slope at *A*. $\theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{4}{81}\frac{PL^2}{EI} = -\frac{1}{9}\frac{PL^2}{EI}$ $\qquad \theta_A = \frac{1}{9}\frac{PL^2}{EI}$ ↻ ◀

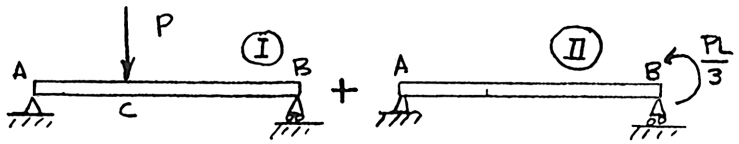
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.71

For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION



Loading I: Case 5. $a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad P = P, \quad x = a$

$$y_C = -\frac{Pa^2b^2}{6EIL} = -\frac{P}{6EIL}\left(\frac{L}{3}\right)^2\left(\frac{2L}{3}\right)^2 = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P}{6EIL}\left(\frac{2L}{3}\right)\left[L^2 - \left(\frac{2L}{3}\right)^2\right] = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Case 7. $M = -\frac{PL}{3}, \qquad x = \frac{L}{3}$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = +\frac{PL/3}{6EIL}\left\{\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right\} = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = +\frac{ML}{6EI} = \frac{-(PL/3)L}{6EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

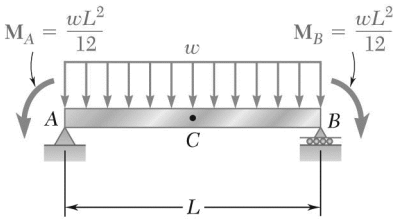
(*a*) Deflection at *C*. $$y_C = -\frac{4}{243}\frac{PL^3}{EI} - \frac{4}{243}\frac{PL^3}{EI} = -\frac{8}{243}\frac{PL^3}{EI}$$

$$y_C = \frac{8}{243}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*. $$\theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{1}{18}\frac{PL^2}{EI} = -\frac{19}{162}\frac{PL^2}{EI}$$

$$\theta_A = \frac{19}{162}\frac{PL^2}{EI} \blacktriangleleft$$

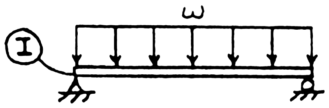
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.72

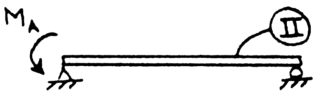
For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION



Loading I: Case 6 in Appendix *D*.

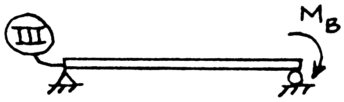
$$y_C = -\frac{5wL^4}{384EI} \qquad \theta_A = -\frac{wL^3}{24EI}$$



Loading II: Case 7 in Appendix *D*.

$$y_C = -\frac{M_A}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right] = \frac{1}{16}\frac{M_A L^2}{EI} \quad \theta_A = \frac{M_A L}{3EI}$$

with $M_A = \dfrac{wL^2}{12}$ $y_C = \dfrac{1}{192}\dfrac{wL^4}{EI}$ $\theta_A = \dfrac{1}{36}\dfrac{wL^3}{EI}$



Loading III: Case 7 in Appendix *D*.

$$y_C = \frac{1}{16}\frac{M_B L^3}{EI} \quad \text{(using Loading II result)}$$

$$\theta_A = \frac{M_B L}{6EI}$$

with $M_B = \dfrac{wL^2}{12}$ $y_C = \dfrac{1}{192}\dfrac{wL^4}{EI}$ $\theta_A = \dfrac{1}{72}\dfrac{wL^3}{EI}$

(*a*) Deflection at *C*.

$$y_C = -\frac{5}{384}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} = -\frac{1}{384}\frac{wL^4}{EI}$$

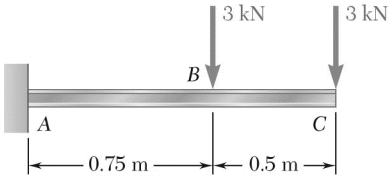
$$y_C = \frac{1}{384}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*.

$$\theta_A = -\frac{1}{24}\frac{wL^3}{EI} + \frac{1}{36}\frac{wL^3}{EI} + \frac{1}{72}\frac{wL^3}{EI} = 0$$

$$\theta_A = 0 \blacktriangleleft$$

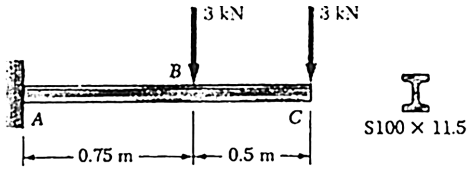
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

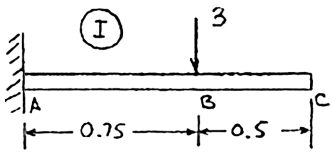


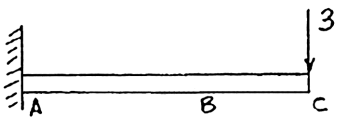
PROBLEM 9.73

For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

Loading I: Concentrated load at B

Case 1 of Appendix D applied to portion AB.

$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight.

$$\theta'_C = \theta'_B = -\frac{0.84375}{EI}$$

$$y'_C = y'_B - (0.5)\theta'_B = -\frac{0.84375}{EI}$$

Loading II: Concentrated load at C. Case 1 of Appendix D.

$$\theta''_A = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y''_A = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = -\frac{3.1875}{EI}$$

$$y_A = y'_A + y''_A = -\frac{2.796875}{EI}$$

Data:

$$E = 200 \times 10^9 \text{ Pa}, \quad I = 2.52 \times 10^6 \text{ mm}^4 = 2.52 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^4)(2.52 \times 10^{-6}) = 504 \times 10^3 \text{ N} \cdot \text{m}^2 = 504 \text{ kN} \cdot \text{m}^2$$

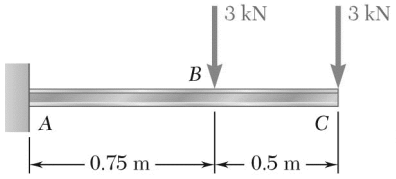
Slope at C.

$$\theta_C = -\frac{3.1875}{504} = -6.32 \times 10^{-3} \text{ rad} \qquad \theta_C = 6.32 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

Deflection at C.

$$y_C = -\frac{2.796875}{504} = -5.55 \times 10^{-3} \text{ m} \qquad y_C = 5.55 \text{ mm} \downarrow \blacktriangleleft$$

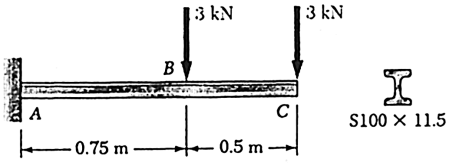
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

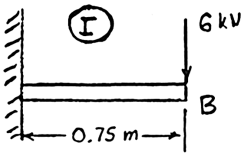


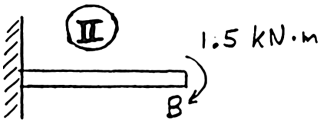
PROBLEM 9.74

For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.

Reduce the force at C to an equivalent force-couple system at B and add the force already at B to obtain the loadings I and II shown.

Loading I: Case 1 of Appendix D.

$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D.

$$\theta''_B = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y''_B = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta'_B + \theta''_B = -\frac{2.8125}{EI}$$

$$y_B = y'_B + y''_B = -\frac{1.265625}{EI}$$

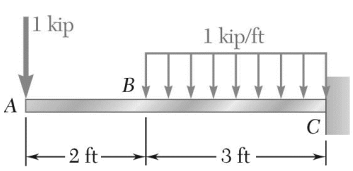
Data: $E = 200 \times 10^9$ Pa, $I = 2.52 \times 10^6$ mm^4 $= 2.52 \times 10^{-6}$ m^4

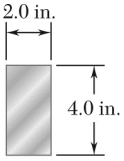
$$EI = (200 \times 10^9)(2.52 \times 10^{-6}) = 504 \times 10^3 \text{ N} \cdot \text{m}^2 = 504 \text{ kN} \cdot \text{m}^2$$

Slope at B. $\theta_B = -\dfrac{2.8125}{504} = -5.58 \times 10^{-3}$ rad $\qquad \theta_B = 5.58 \times 10^{-3}$ rad ↘◀

Deflection at B. $y_B = -\dfrac{1.265625}{504} = -2.51 \times 10^{-3}$ m $\qquad y_B = 2.51$ mm ↓◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

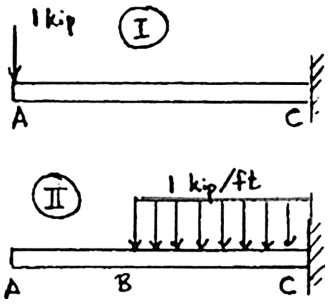




PROBLEM 9.75

For the cantilever beam and loading shown, determine the slope and deflection at end A. Use $E = 29 \times 10^6$ psi.

SOLUTION



Units: Forces in kips; lengths in ft.

Loading I: Concentrated load at A.

Case 1 of Appendix D.

$$\theta'_A = \frac{PL^2}{2EI} = \frac{(1)(5)^2}{2EI} = \frac{12.5}{EI}$$

$$y'_A = -\frac{PL^3}{3EI} = -\frac{(1)(5)^3}{3EI} = -\frac{41.667}{EI}$$

Loading II: Uniformly distributed load over portion BC.

Case 2 of Appendix D applied to portion BC.

$$\theta''_B = \frac{wL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

$$y''_B = -\frac{wL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI}$$

Portion AB remains straight. $\theta''_A = \theta''_B = \dfrac{4.5}{EI}$

$$y''_A = y''_B - a\theta''_B = -\frac{10.125}{EI} - (2)\left(\frac{4.5}{EI}\right) = -\frac{19.125}{EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = \frac{12.5}{EI} + \frac{4.5}{EI} = \frac{17}{EI}$$

$$y_A = y'_A + y''_A = -\frac{41.667}{EI} - \frac{19.125}{EI} = -\frac{60.792}{EI}$$

Data:

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

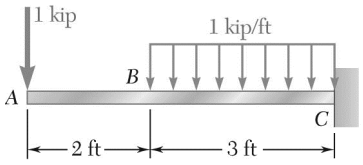
$$I = \frac{1}{12}(2.0)(4.0)^3 = 10.667 \text{ in}^4$$

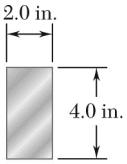
$$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3 \text{ kip} \cdot \text{in}^2 = 2148 \text{ kip} \cdot \text{ft}^2$$

Slope at A. $\theta_A = \dfrac{17}{2148}$ $\theta_A = 7.91 \times 10^{-3}$ rad ◀

Deflection at A. $y_A = -\dfrac{60.792}{2148} = -28.30 \times 10^{-3}$ ft $y_A = 0.340$ in. ↓ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 9.76

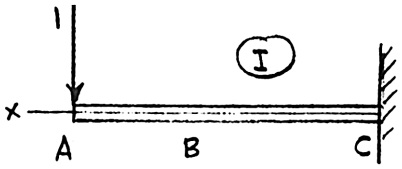
For the cantilever beam and loading shown, determine the slope and deflection at point *B*. Use $E = 29 \times 10^6$ psi.

SOLUTION

Units: Forces in kips; lengths in ft.

Loading I: Concentrated load at *A*.

Case 1 of Appendix *D*.



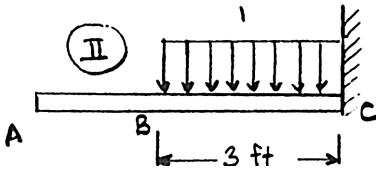
$$y = \frac{P}{6EI}[x^3 - 3Lx^2]$$

$$\frac{dy}{dx} = \frac{P}{6EI}[3x^2 - 6Lx]$$

with $P = 1\,\text{kip}, \quad L = 5\,\text{ft}, \quad x = 3\,\text{ft}$

$$y'_B = \frac{1}{6EI}[(3)^3 - (3)(5)(3)^2] = -\frac{18}{EI}$$

$$\left.\frac{dy}{dx}\right|_B = \frac{1}{6EI}[(3)(3)^2 - (6)(5)(3)] = -\frac{10.5}{EI}$$



Adjusting the sign, $\theta'_B = \dfrac{10.5}{EI}$

Loading II: Uniformly distributed load over portion *BC*.

Case 2 of Appendix *D* applied to portion *BC*.

$$y''_B = -\frac{wL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI} \qquad \theta''_B = \frac{wL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

By superposition,

$$\theta_B = \theta'_B + \theta''_B = \frac{10.5}{EI} + \frac{4.5}{EI} = \frac{15}{EI} \qquad y_B = y'_B + y''_B = -\frac{18}{EI} - \frac{10.125}{EI} = -\frac{28.125}{EI}$$

Data:

$$E = 29 \times 10^6\,\text{psi} = 29 \times 10^3\,\text{ksi} \qquad I = \frac{1}{12}(2.0)(4.0)^3 = 10.667\,\text{in}^4$$

$$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3\,\text{kip} \cdot \text{in}^2 = 2148\,\text{kip} \cdot \text{ft}^2$$

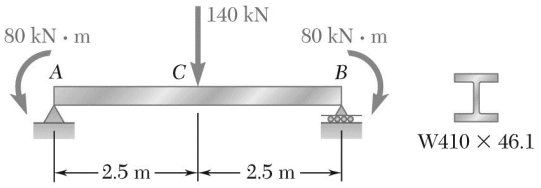
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.76 *(Continued)*

Slope at B. $\theta_B = \dfrac{15}{2148} = 6.98 \times 10^{-3}$ $\theta_B = 6.98 \times 10^{-3}$ rad ◀

Deflection at B. $y_B = -\dfrac{28.125}{2148} = -13.09 \times 10^{-3}$ ft $y_B = 0.1571$ in. ↓ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.77

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

Loading I: Moment at *B*.

Case 7 of Appendix *D*. $M = 80 \text{ kN} \cdot \text{m}, \quad L = 5.0 \text{ m}, \quad x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2 x) = -\frac{80}{6EI(5.0)}[2.5^3 - (5.0)^2(2.5)] = \frac{125}{EI}$$

Loading II: Moment at *A*. Case 7 of Appendix *D*.

$M = 80 \text{ kN} \cdot \text{m}, \quad L = 5.0 \text{ m}, \quad x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \qquad \text{(Same as loading I.)}$$

Loading III: 140 kN concentrated load at *C*. $P = 140$ kN

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}, \quad I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$

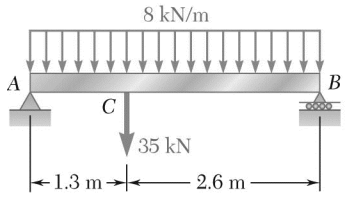
$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N} \cdot \text{m}^2 = 31200 \text{ kN} \cdot \text{m}^2$$

(*a*) Slope at *A*. $\theta_A = \dfrac{67.667 + 133.333 - 218.75}{31200} = -0.601 \times 10^{-3} \text{ rad}$

$\theta_A = 0.601 \times 10^{-3} \text{ rad}$ ↻ ◀

(*b*) Deflection at *C*. $y_C = \dfrac{125 + 125 - 364.583}{31200} = -3.67 \times 10^{-3} \text{ m}$ $\qquad y_C = 3.67 \text{ mm} \downarrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



W360 × 39

PROBLEM 9.78

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

Loading I: 8 kN/m uniformly distributed.

Case 6: $w = 8$ kN/m, $L = 3.9$ m, $x = 1.3$ m

$$\theta_A = -\frac{WL^3}{24EI} = -\frac{(8)(3.9)^3}{24EI} = -\frac{19.773}{EI}$$

$$y_C = -\frac{w}{24EI}[x^4 - 2Lx^3 + L^3x] = -\frac{8}{24EI}[(1.3)^4 - (2)(3.9)(1.3)^3 + (3.9)^3(1.3)]$$

$$= -\frac{20.945}{EI}$$

Loading II: 35 kN concentrated load at *C*. Case 5 of Appendix *D*.

$P = 35$ kN, $L = 3.9$ m, $a = 1.3$ m, $b = 2.6$ m, $x = a = 1.3$ m

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(2.6)(3.9^2 - 2.6)^2}{6EI\,(3.9)} = -\frac{32.861}{EI}$$

$$y_C = -\frac{Pa^2b^2}{3EIL} = -\frac{(35)(1.3)^2(2.6)^2}{3EI(3.9)} = -\frac{34.176}{EI}$$

Data: $E = 200 \times 10^9$, $I = 102 \times 10^6\ \text{mm}^4 = 102 \times 10^{-6}\ \text{m}^4$

$$EI = (200 \times 10^9)(102 \times 10^{-6}) = 20.4 \times 10^6\ \text{N} \cdot \text{m}^2 = 20{,}400\ \text{kN} \cdot \text{m}^2$$

(*a*) Slope at *A*.

$$\theta_A = -\frac{19.773 + 32.861}{20{,}400} = -2.58 \times 10^{-3}\ \text{rad}$$

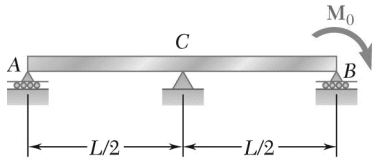
$$\theta_A = 2.58 \times 10^{-3}\ \text{rad} \searrow \blacktriangleleft$$

(*b*) Deflection at *C*.

$$y_C = -\frac{20.934 + 34.176}{20{,}400} = -2.70 \times 10^{-3}\ \text{m}$$

$$y_C = 2.70\ \text{mm} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

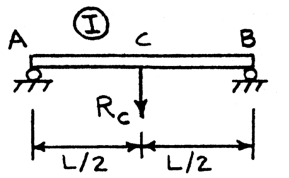


PROBLEM 9.79

For the uniform beam shown, determine the reaction at each of the three supports.

SOLUTION

Consider R_C as redundant and replace loading system by I and II.

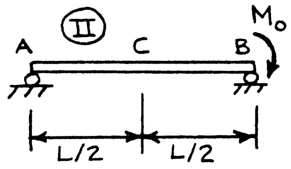


Loading I: (Case 4 of Appendix *D*.) $y'_C = -\dfrac{R_C L^3}{48EI}$

Loading II: (Case 7 of Appendix *D*.)

$$y''_C = -\frac{M_0}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right]$$

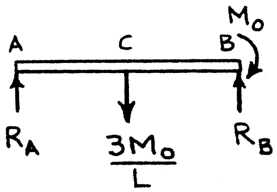
$$= \frac{M_0 L^2}{16EI}$$



Superposition and constraint:

$$y_C = y'_C + y''_C = 0$$

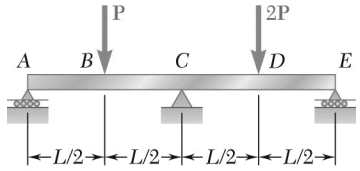
$$-\frac{R_C L^3}{48EI} + \frac{M_0 L^2}{16EI} = 0 \qquad R_C = \frac{3M_0}{L} \downarrow \blacktriangleleft$$



$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{3M_0}{L}\right)\left(\frac{L}{2}\right) - M_0 = 0 \qquad R_A = \frac{M_0}{2L} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_Y = 0: \quad \frac{M_0}{2L} - \frac{3M_0}{L} + R_B = 0 \qquad R_B = \frac{5M_0}{2L} \uparrow \blacktriangleleft$$

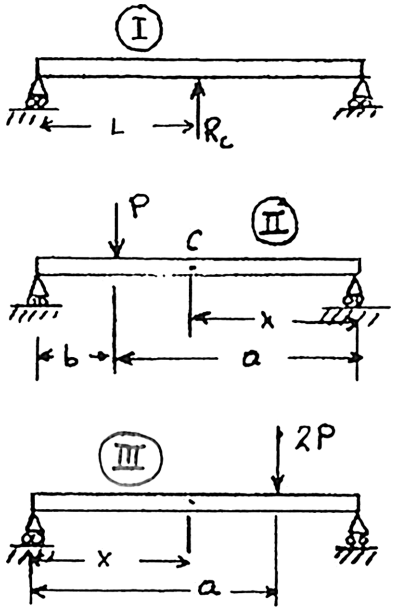
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.80

For the uniform beam shown, determine the reaction at each of the three supports.

SOLUTION



Beam is indeterminate to first degree. Consider R_C to be the redundant reaction, and replace the loading by loadings I, II, and III.

Loading I: Case 4 of Appendix *D*.

$$(y_C)_I = \frac{R_C(2L)^3}{48EI} = \frac{1}{6}\frac{R_C L^2}{EI}$$

Loading II: Case 5 of Appendix *D*.

$$(y_C)_{II} = \frac{Pb}{6EI(2L)}[x^3 - \{(2L)^2 - b^2\}x]$$

$$= \frac{P(L/2)}{12EIL}\left[L^3 - \left\{4L^2 - \left(\frac{L}{2}\right)^2\right\}L\right] = -\frac{11}{48}\frac{PL^3}{EI}$$

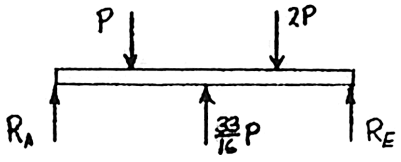
Loading III: Case 5 of Appendix *D*.

$$(y_C)_{III} = 2(y_C)_{II} = -\frac{11}{24}\frac{PL^3}{EI}$$

Superposition and constraint: $y_C = (y_C)_I + (y_C)_{II} + (y_C)_{III} = 0$

$$\frac{1}{6}\frac{R_C L^3}{EI} - \frac{11}{48}\frac{PL^3}{EI} - \frac{11}{24}\frac{PL^3}{EI} = \frac{1}{6}\frac{R_C L^3}{EI} - \frac{11}{16}\frac{PL^3}{EI} = 0$$

$$R_C = \frac{33}{16}P \uparrow \blacktriangleleft$$



Statics: $+\circlearrowleft \sum M_E = 0$:

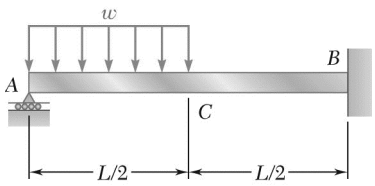
$$-R_A(2L) + P\left(\frac{3L}{2}\right) - \left(\frac{33}{16}P\right)L + (2P)\left(\frac{L}{2}\right) = 0$$

$$R_A = \frac{7}{32}P \uparrow \blacktriangleleft$$

$$+\uparrow \sum F_y = 0: \quad \frac{7}{32}P - P + \frac{33}{16}P - 2P + R_E = 0$$

$$R_E = \frac{23}{32}P \uparrow \blacktriangleleft$$

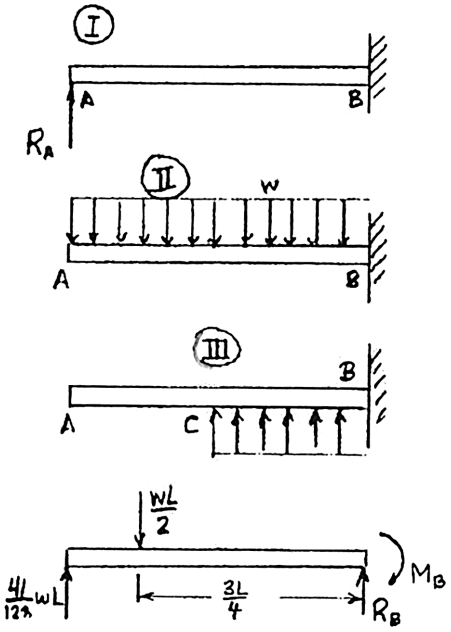
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.81

For the uniform beam shown, determine (*a*) the reaction at *A,* (*b*) the reaction at *B*.

SOLUTION



Beam is indeterminate to first degree. Consider R_A as redundant and replace the given loading by loadings I, II, and III.

Loading I: Case 1 of Appendix *D*.

$$(y_A)_I = \frac{R_A L^3}{3EI}$$

Loading II: Case 2 of Appendix *D*.

$$(y_A)_{II} = -\frac{wL^4}{8EI}$$

Loading III: Case 2 of Appendix *D* (portion *CB*).

$$(\theta_C)_{III} = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$(y_C)_{III} = \frac{w(L/2)^4}{8EI} = \frac{1}{128}\frac{wL^4}{EI}$$

Portion *AC* remains straight.

$$(y_A)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{7}{384}\frac{wL^4}{EI}$$

Superposition and constraint: $y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III} = 0$

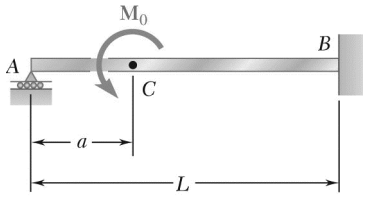
(*a*) $$\frac{1}{3}\frac{R_A L^3}{3EI} - \frac{1}{8}\frac{wL^4}{EI} + \frac{7}{384}\frac{wL^4}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{41}{384}\frac{wL^4}{EI} = 0$$ $$R_A = \frac{41}{128}wL \uparrow \blacktriangleleft$$

Statics:

(*b*) $$+\uparrow \Sigma F_y = 0: \quad \frac{41}{128}wL - \frac{1}{2}wL + R_B = 0$$ $$R_B = \frac{23}{128}wL \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -\left(\frac{41}{128}wL\right)L - \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_B = 0$$ $$M_B = \frac{7}{128}wL^2 \circlearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

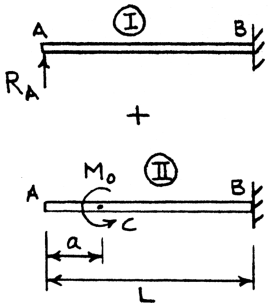


PROBLEM 9.82

For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION

Consider R_A as redundant and replace loading system by I and II.



Loading I: (Case 1 of Appendix *D*.) $y'_A = \dfrac{R_A L^3}{3EI}$

Loading II: Portion *BC*. (Case 3 of Appendix *D*.)

$$y''_C = -\frac{M_0(L-a)^2}{2EI} \qquad \theta''_C = \frac{M_0(L-a)}{EI}$$

Portion *AC* is straight. $y''_A = y''_C - (a)\theta_C$

$$= -\frac{M_0(L-a)^2}{2EI} - \frac{aM_0(L-a)}{EI}$$

(*a*) Superposition and constraint: $y_A = y'_A + y''_A = 0$

$$\frac{R_A L^3}{3EI} - \frac{M_0(L-a)^2}{2EI} - \frac{aM_0(L-a)}{EI} = 0$$

$$\frac{2}{3}R_A L^3 - M_0(L-a)(L-a+2a) = 0$$

$$\frac{2}{3}R_A L^3 = M_0(L^2 - a^2)$$

$$R_A = \frac{3M_0}{2L^3}(L^2 - a^2)\uparrow \blacktriangleleft$$

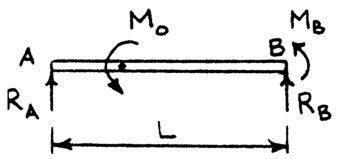
(*b*) $+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \qquad R_B = -R_A$

$$R_B = \frac{3M_0}{2L^3}(L^2 - a^2)\downarrow \blacktriangleleft$$

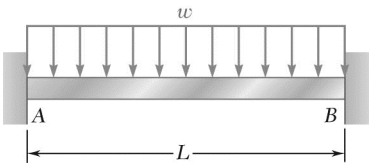
$+\circlearrowleft \Sigma M_B = 0: \quad M_B + M_0 - R_A L = 0 \qquad M_B + M_0 - \dfrac{3}{2}\dfrac{M_0}{L^2}(L^2 - a^2) = 0$

$$M_B = \frac{3M_0}{2L^2}\left(L^2 - a^2 - \frac{2}{3}L^2\right)$$

$$M_B = \frac{M_0}{2L^2}(L^2 - 3a^2)\circlearrowleft \blacktriangleleft$$



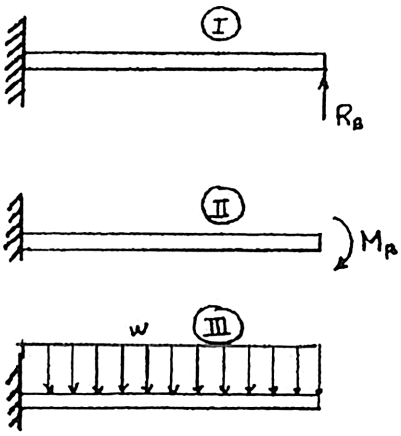
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.83

For the beam shown, determine the reaction at B.

SOLUTION



Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

Loading I: Case 1 of Appendix *D*.

$$(y_B)_I = \frac{R_B L^3}{3EI} \qquad (\theta_B)_I = \frac{R_B L^2}{2EI}$$

Loading II: Case 3 of Appendix *D*.

$$(y_B)_{II} = -\frac{M_B L^2}{2EI} \qquad (\theta_B)_{II} = -\frac{M_B L}{EI}$$

Loading III: Case 2 of Appendix *D*.

$$(y_B)_{III} = -\frac{wL^4}{8EI} \qquad (\theta_B)_{III} = -\frac{wL^2}{6EI}$$

Superposition and constraint:

$$y_B = (y_B)_{\mathrm{I}} + (y_B)_{\mathrm{II}} + (y_B)_{\mathrm{III}} = 0$$

$$\frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B - \frac{wL^4}{8EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_{\mathrm{I}} + (\theta_B)_{\mathrm{II}} + (\theta_B)_{\mathrm{III}} = 0$$

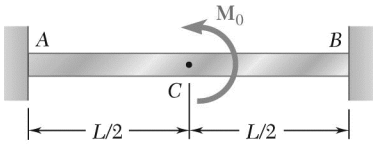
$$\frac{L^2}{2EI} R_B - \frac{L}{EI} M_B - \frac{wL^3}{6EI} = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$R_B = \frac{1}{2} wL \uparrow \blacktriangleleft$$

$$M_B = \frac{1}{12} wL^2 \circlearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.84

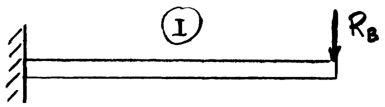
For the beam shown, determine the reaction at *B*.

SOLUTION

Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

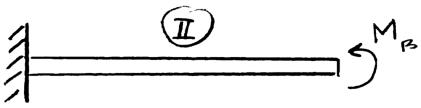
Loading I: Case 1 of Appendix *D*.

$$(y_B)_{\text{I}} = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_{\text{I}} = -\frac{R_B L^2}{2EI}$$



Loading II: Case 3 of Appendix *D*.

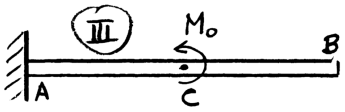
$$(y_B)_{\text{II}} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{\text{II}} = \frac{M_B L}{EI}$$



Loading III: Case 3 applied to portion *AC*.

$$(y_C)_{\text{III}} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{\text{III}} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$



Portion *CB* remains straight.

$$(y_B)_{\text{III}} = (y_C)_{\text{III}} + \frac{L}{2}(\theta_C)_{\text{III}} = \frac{3}{8}\frac{M_0 L^2}{EI}$$

$$(\theta_B)_{\text{III}} = (\theta_C)_{\text{III}} = \frac{1}{2}\frac{M_0 L^2}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_{\text{I}} + (y_B)_{\text{II}} + (y_B)_{\text{III}} = 0$$

$$-\frac{L^3}{3EI}R_B + \frac{L^2}{2EI}M_B + \frac{3}{8}\frac{M_0 L^2}{EI} = 0 \tag{1}$$

$$\theta_B = (\theta_B)_{\text{I}} + (\theta_B)_{\text{II}} + (\theta_B)_{\text{III}} = 0$$

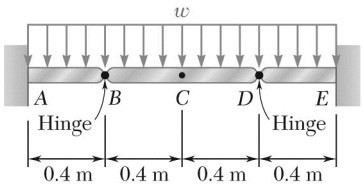
$$-\frac{L^2}{2EI}R_B + \frac{L}{EI}M_B + \frac{1}{2}\frac{M_0 L}{EI} = 0 \tag{2}$$

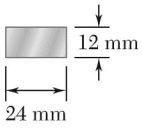
Solving Eqs. (1) and (2) simultaneously,

$$R_B = \frac{3}{2}\frac{M_0}{L} \downarrow \blacktriangleleft$$

$$M_B = \frac{1}{4}M_0 \circlearrowleft \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.





PROBLEM 9.85

A central beam *BD* is joined at hinges to two cantilever beams *AB* and *DE*. All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at *C* does not exceed 3 mm. Use $E = 200$ GPa.

SOLUTION

Let $a = 0.4$ m.

Cantilever beams *AB* and *CD*.

Cases 1 and 2 of Appendix *D*. $$y_B = y_D = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{11}{24}\frac{wa^4}{EI}$$

Beam *BCD*, with $L = 0.8$ m, assuming that points *B* and *D* do not move.

Case 6 of Appendix *D*.

$$y'_C = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points *B* and *D*.

$$y''_C = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

Total deflection at *C*. $$y_C = y'_C + y''_C$$

$$y_C = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

Data: $E = 200\times10^9$ Pa,

$$I = \frac{1}{12}(24)(12)^3 = 3.456\times10^{-3}\text{ mm}^4 = 3.456\times10^{-9}\text{ m}^4$$

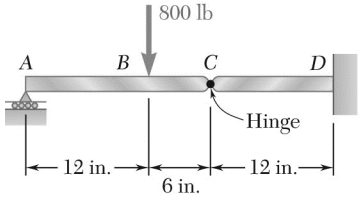
$$EI = (200\times10^9)(3.456\times10^{-9}) = 691.2\text{ N}\cdot\text{m}^2$$

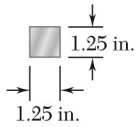
$$y_C = -3\times10^{-3}\text{ m}$$

$$-3\times10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69^{-6}\,w$$

$$w = 121.5\text{ N/m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

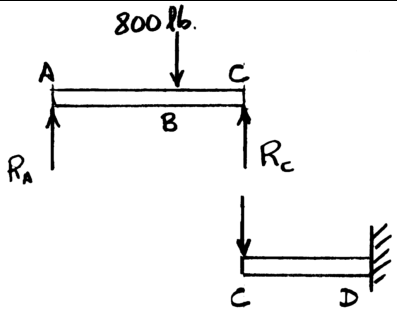




PROBLEM 9.86

The two beams shown have the same cross section and are joined by a hinge at *C*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 29\times10^6$ psi.

SOLUTION



Using free body *ABC*,

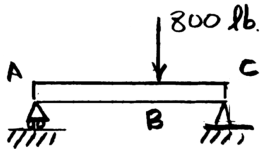
$$+\circlearrowleft \Sigma M_A = 0: \quad 18R_C - (12)(800) = 0 \qquad R_C = 533.33 \text{ lb}$$

$$E = 29\times10^6 \text{ psi}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1.25)(1.25)^3 = 0.20345 \text{ in}^4$$

$$EI = (29\times10^6)(0.20345) = 5.900\times10^6 \text{ lb}\cdot\text{in}^2$$

Using cantilever beam *CD* with load R_C,



Case 1 of Appendix *D*.

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(533.33)(12)^3}{(3)(5.900\times10^6)} = -52.067\times10^{-3} \text{ in.}$$

Calculation of θ'_A and y'_B assuming that point *C* does not move.

Case 5 of Appendix *D*.

$$P = 800 \text{ lb}, \quad L = 18 \text{ in.}, \quad a = 12 \text{ in.}, \quad b = 6 \text{ in.}$$

$$\theta'_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(800)(6)(18^2 - 6^2)}{(6)(5.900\times10^6)(18)} = -2.1695\times10^{-3} \text{ rad}$$

$$y'_B = -\frac{Pb^2a^2}{3EIL} = -\frac{(800)(6)^2(12)^2}{(3)(5.900\times10^6)(18)} = -13.017\times10^{-3} \text{ in.}$$

Additional slope and deflection due to movement of point *C*.

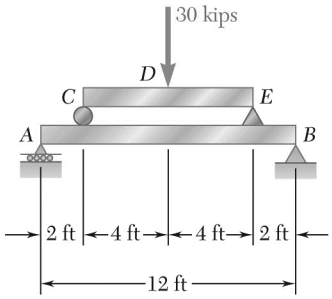
$$\theta''_A = \frac{y_C}{L_{AC}} = -\frac{52.067\times10^{-3}}{18} = -2.8926\times10^{-3} \text{ rad}$$

$$y''_B = \frac{a}{L}y_C = -\frac{(12)(52.067\times10^{-3})}{18} = -34.711\times10^{-3} \text{ in.}$$

(*a*) Slope at *A*. $\theta_A = \theta'_A + \theta''_A = -2.1695\times10^{-3} - 2.8926\times10^{-3}$ $\theta_A = 5.06\times10^{-3}$ rad ◀

(*b*) Deflection at *B*. $y_B = y'_B + y''_B = -13.017\times10^{-3} - 34.711\times10^{-3}$ $y_B = 0.0477$ in. ↓ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.87

Beam *CE* rests on beam *AB* as shown. Knowing that a $W10 \times 30$ rolled-steel shape is used for each beam, determine for the loading shown the deflection at point *D*. Use $E = 29 \times 10^6$ psi.

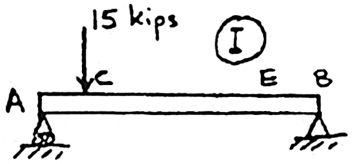
SOLUTION

For $W10 \times 30$, $I = 170 \text{ in}^4$

$$EI = (29 \times 10^6)(170) = 4.93 \times 10^9 \text{ lb} \cdot \text{in}^2 = 34{,}236 \text{ kip} \cdot \text{ft}^2$$

Beam *AB*: 15 kip downward loads at *C* and *E*.

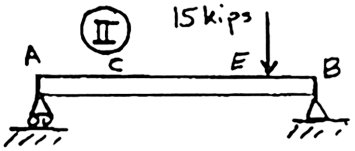
Refer to Case 5 of Appendix *D*.



Loading I: $(y_C)_1 = -\dfrac{Pa^2b^2}{3EIL}$

with $a = 2$ ft, $b = 10$ ft, $L = 12$ ft

$$(y_C)_1 = -\frac{(15)(2)^2(10)^2}{(3)(34236)(12)} = -4.8682 \times 10^{-3} \text{ ft}$$



Loading II: $(y_C)_2 = \dfrac{Pb\left[x^2 - (L^2 - b^2)x\right]}{6EIL}$

with $b = 2$ ft, $x = 2$ ft, $L = 12$ ft

$$(y_C)_2 = \frac{(15)(2)[2^3 - (12^2 - 2^2)(2)]}{(6)(34{,}236)(12)} = -3.3104 \times 10^{-3} \text{ ft}$$

$$y_C = (y_C)_1 + (y_C)_2 = -8.1786 \times 10^{-3} \text{ ft}$$

By symmetry, $y_E = y_C$

Beam *CDE*: 30 kip downward loads at *D*.

Refer to Case 4 of Appendix *D*.

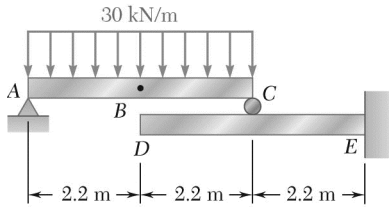
$$y_{D/C} = -\frac{PL^3}{48EI}$$

with $P = 30$ kips and $L = 8$ ft

$$y_{D/C} = -\frac{(30)(8)^3}{(48)(34{,}236)} = -9.3469 \times 10^{-3} \text{ ft}$$

Total deflection at *D*. $y_D = y_C + y_{D/C} = 17.5255 \times 10^{-3}$ ft $\qquad y_D = 0.210$ in. ↓ ◀

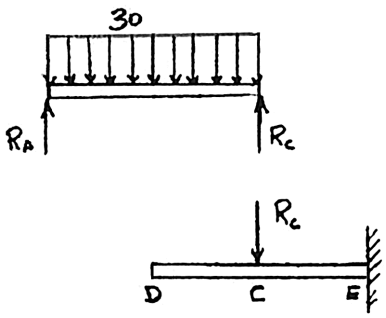
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.88

Beam AC rests on the cantilever beam DE as shown. Knowing that a $W410 \times 38.8$ rolled-steel shape is used for each beam, determine for the loading shown (*a*) the deflection at point *B,* (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION



Units: Forces in kN; lengths in m.

Using free body ABC,

$$+\circlearrowleft \sum M_A = 0: \; 4.4R_C - (4.4)(30)(2.2) = 0$$

$$R_C = 66.0 \text{ kN}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 125 \times 10^6 \text{ mm}^4 = 125 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(125 \times 10^{-6}) = 25.0 \times 10^{-6} \text{ N} \cdot \text{m}^2$$

$$= 25{,}000 \text{ kN} \cdot \text{m}^2$$

For slope and defection at C, use Case 1 of Appendix D applied to portion CE of beam DCE.

$$\theta_C = \frac{R_C L^2}{2EI} = \frac{(66.0)(2.2)^2}{(2)(25{,}000)} = 6.3888 \times 10^{-3} \text{ rad}$$

$$y_C = -\frac{R_C L^3}{3EI} = \frac{(66.0)(2.2)^3}{(3)(25{,}000)} = -9.3702 \times 10^{-3} \text{ m}$$

Defection at B, assuming that point C does not move.

Use Case 6 of Appendix D. $\quad (y_B)_1 = -\dfrac{5WL^4}{384EI} = -\dfrac{(5)(30)(4.4)^4}{(384)(25{,}000)} = -5.8564 \times 10^{-3}$

Additional defection at B due to movement of point C: $\quad (y_B)_2 = \dfrac{1}{2} y_C = -4.6851 \times 10^{-3} \text{ m}$

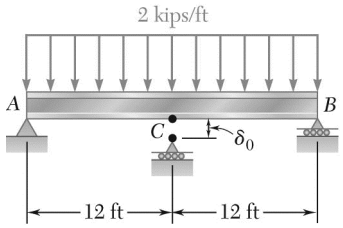
(*a*) Total deflection at *B*. $\quad y_B = (y_B)_1 + (y_B)_2 = -10.54 \times 10^{-3} \text{ m} \quad y_B = 10.54 \text{ mm} \downarrow \blacktriangleleft$

Portion DC of beam DCB remains straight.

(*b*) Deflection at *D*. $\quad y_D = y_C - a\theta_C = -9.3702 \times 10^{-3} - (2.2)(6.3888 \times 10^{-3}) = -23.4 \times 10^{-3} \text{ m}$

$$y_D = 23.4 \text{ mm} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.89

Before the 2-kip/ft load is applied, a gap, $\delta_0 = 0.8$ in., exists between the W16×40 beam and the support at *C*. Knowing that $E = 29\times10^6$ psi, determine the reaction at each support after the uniformly distributed load is applied.

SOLUTION

Data:

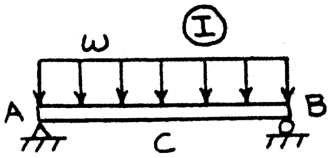
$$\delta_0 = 0.8 \text{ in.} = 66.667\times10^{-3} \text{ ft}$$

$$E = 29\times10^6 \text{ psi} = 29\times10^3 \text{ ksi}$$

$$I = 518 \text{ in}^4$$

$$EI = 15.022\times10^6 \text{ kip}\cdot\text{in}^2$$

$$= 104.319\times10^3 \text{ kip}\cdot\text{ft}^2$$

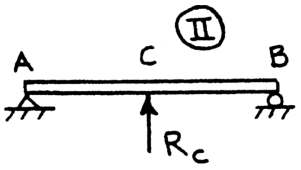


Loading I: Case 6 of Appendix *D*.

$$y_C' = -\frac{5wL^4}{384EI}$$

$$= -\frac{5(2)(24)^4}{384(104.319\times10^3)}$$

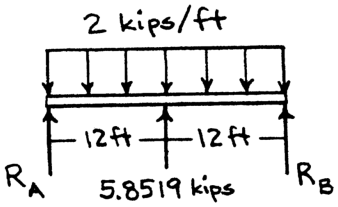
$$= -82.823\times10^{-3} \text{ ft}$$



Loading II: Case 4 of Appendix *D*.

$$y_C'' = \frac{R_C L^3}{48EI} = \frac{R_C(24)^3}{48(104.319\times10^3)}$$

$$= 2.7608\times10^{-3} R_C$$



Deflection at *C*.

$$y_C = y_C' + y_C'' = -\delta_0$$

$$-82.823\times10^{-3} + 2.7608\times10^{-3} R_C = -66.667\times10^{-3}$$

$$R_C = 5.8519 \text{ kips} \qquad R_C = 5.85 \text{ kips} \uparrow \blacktriangleleft$$

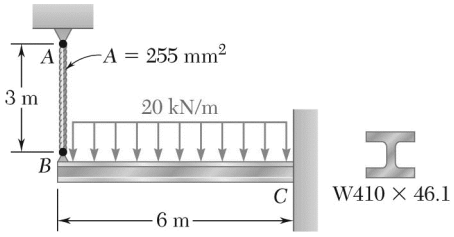
$$+\circlearrowleft \Sigma M_B = 0: \quad (2)(24)(12) - R_A(24) - (5.8519)(12) = 0$$

$$R_A = 21.074 \text{ kips} \qquad R_A = 21.1 \text{ kips} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 21.074 - 2(24) + 5.8519 + R_B = 0$$

$$R_B = 21.1 \text{ kips} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.90

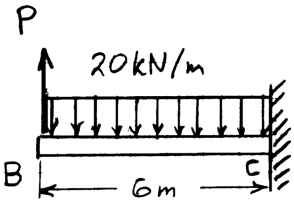
The cantilever beam *BC* is attached to the steel cable *AB* as shown. Knowing that the cable is initially taut, determine the tension in the cable caused by the distributed load shown. Use $E = 200$ GPa.

SOLUTION

Let P be the tension developed in member AB and δ_B be the elongation of that member.

Cable *AB*: $A = 255 \text{ mm}^2 = 255\times10^{-6} \text{ m}^2$

$$\delta_B = \frac{PL}{EA} = \frac{(P)(3)}{(200\times10^9)(255\times10^{-6})}$$
$$= 58.82\times10^{-9}\,\text{P}$$



Beam *BC*: $I = 156\times10^6 \text{ mm}^4 = 156\times10^{-6}\,\text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6})$$
$$= 31.2\times10^6\,\text{N}\cdot\text{m}^2$$

Loading I: 20 kN/m downward.

Refer to Case 2 of Appendix *D*.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(20\times10^3)(6)^4}{(8)(31.2\times10^6)}$$
$$= -103.846\times10^{-3}\,\text{m}$$

Loading II: Upward force P at Point B.

Refer to Case 1 of Appendix *D*.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2\times10^6)} = 2.3077\times10^{-6}\,P$$

By superposition, $y_B = (y_B)_1 + (y_B)_2$

Also, matching the deflection at B,

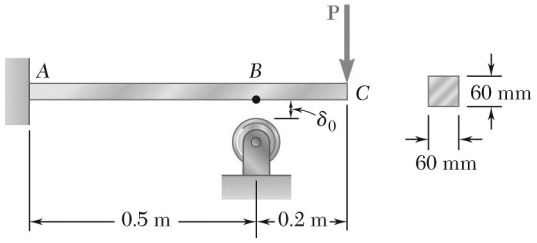
$$y_B = -\delta_B$$

$$-103.846\times10^{-3} + 2.3077\times10^{-6}\,P = -58.82\times10^{-9}\,P$$

$$2.3666\times10^{-6}\,P = 103.846\times10^{-3} \qquad P = 43.9\times10^3 \text{ N}$$

$$P = 43.9 \text{ kN} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.91

Before the load **P** was applied, a gap, $\delta_0 = 0.5\,\text{mm}$, existed between the cantilever beam AC and the support at B. Knowing that $E = 200$ GPa, determine the magnitude of **P** for which the deflection at C is 1 mm.

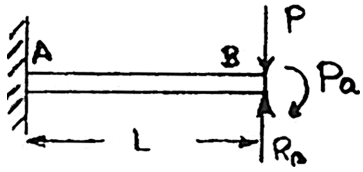
SOLUTION

Let length $AB = L = 0.5$ m

length $BC = a = 0.2$ m

Consider portion AB of beam ABC.

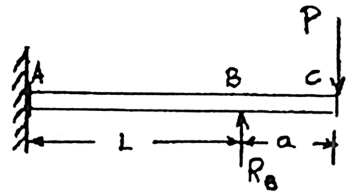
The loading becomes forces P and R_B at B plus the couple Pa. The deflection at B is δ_0. Using Cases 1 and 3 of Appendix D,



$$\delta_0 = \frac{(P - R_B)L^3}{3EI} + \frac{PaL^2}{2EI}$$

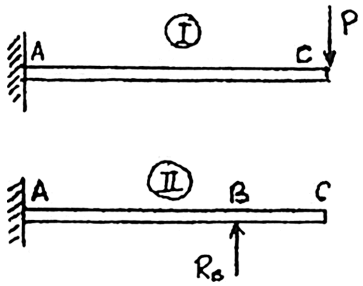
$$\left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)P - \frac{L^3}{3}R_B = EI\delta_0 \qquad (1)$$

The deflection at C depends on the deformation of beam ABC subjected to loads P and R_B. For loading I, using Case 1 of Appendix D,



$$(\delta_C)_1 = \frac{P(L + a)^3}{3EI}$$

For loading II, using Case 1 of Appendix D,



$$y_B = \frac{R_B L^3}{3EI} \quad \theta_B = \frac{R_B L^2}{2EI}$$

Portion BC remains straight.

$$y_C = y_B + a\theta_B = \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

By superposition, the downward deflection at C is

$$\delta_C = \frac{P(L + a)^3}{3EI} - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

$$\frac{(L + a)^3}{3}P - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)R_B = EI\delta_C \qquad (2)$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.91 *(Continued)*

Data: $E = 200 \times 10^9$ Pa $\qquad I = \frac{1}{12}(60)(60)^3 = 1.08 \times 10^6 \text{ mm}^4 = 1.08 \times 10^{-6} \text{ m}^4$

$$EI = 216 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$\delta_0 = 0.5 \times 10^{-3} \text{ m} \qquad \delta_C = 1.0 \times 10^{-3} \text{ m}$$

Using the data, eqs (1) and (2) become

$$0.06667\,P - 0.04167\,R_B = 108 \tag{1}'$$

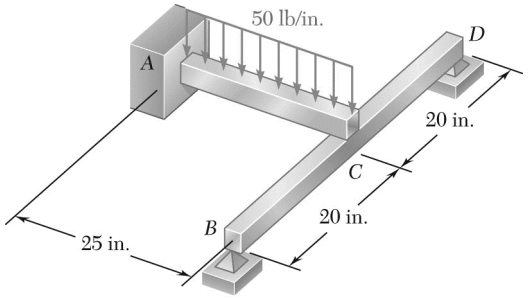
$$0.11433\,P - 0.06667\,R_B = 216 \tag{2}'$$

Solving simultaneously,

$$P = 5.63 \times 10^3 \text{ N} \qquad P = 5.63 \text{ kN} \downarrow \blacktriangleleft$$

$$R_B = 6.42 \times 10^3 \text{ N}$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

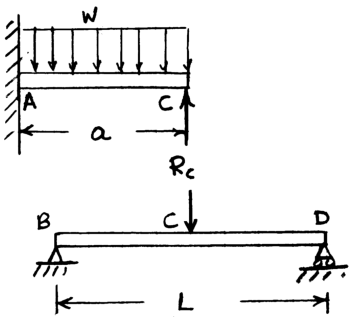


PROBLEM 9.92

For the loading shown, knowing that beams AC and BD have the same flexural rigidity, determine the reaction at B.

SOLUTION

Consider the two beams shown at the right.



Let R_C be the contact force between beams AC and BCD.

Applying Cases 1 and 2 of Appendix D to cantilever beam AC,

$$y_C = \frac{R_C a^3}{3EI} - \frac{wa^4}{8EI}$$

Applying Case 4 of Appendix D to simply supported beam BCD,

$$y_C = -\frac{R_C L^3}{48EI}$$

Equating expressions for y_C,

$$\frac{R_C a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{R_C L^3}{48EI}$$

$$(16a^3 + L^3)R_C = 6wa^4$$

$$R_C = \frac{6wa}{16 + L^3/a^3}$$

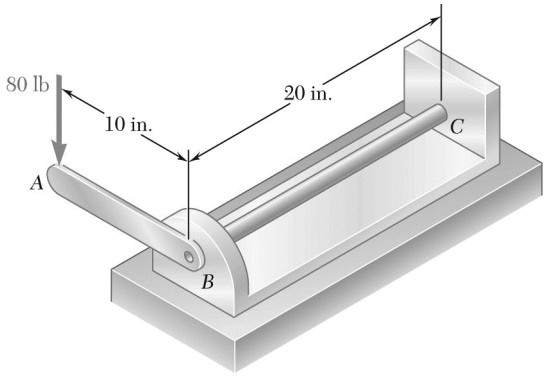
Data: $w = 50$ lb/in, $a = 25$ in., $L = 20 + 20 = 40$ in.

$$R_C = \frac{(6)(50)(25)}{16 + (40/25)^3} = 373.21 \text{ lb}$$

Using beam BCD as a free body,

$$+\circlearrowleft \Sigma M_D = 0: \quad -R_B L + R_C \frac{L}{2} = 0 \qquad R_B = \frac{1}{2} R_C = 186.6 \text{ lb} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.93

A $\frac{7}{8}$-in.-diameter rod BC is attached to the lever AB and to the fixed support at C. Lever AB has a uniform cross section $\frac{3}{8}$ in. thick and 1 in. deep. For the loading shown, determine the deflection of point A. Use $E = 29\times10^6$ psi and $G = 11.2\times10^6$ psi.

SOLUTION

Deformation of rod BC. (Torsion)

$$c = \frac{1}{2}d = \frac{1}{2}\left(\frac{7}{8}\right) = 0.4375 \text{ in.}$$

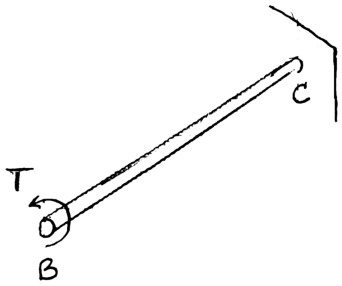
$$J = \frac{\pi}{2}C^4 = 57.548\times10^{-3}\text{ in}^4$$

$$T = Pa = (80)(10) = 800 \text{ lb}\cdot\text{in}$$

$$L = 20 \text{ in.}$$

$$\varphi_B = \frac{TL}{GJ} = \frac{(800)(20)}{(11.2\times10^6)(57.548\times10^{-3})}$$

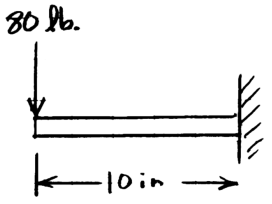
$$= 24.824\times10^{-3}\text{ rad}$$



Deflection of point A assuming lever AB to be rigid.

$$(y_A)_1 = a\varphi_B = (10)(24.824\times10^{-3})$$

$$= 0.24824 \text{ in.}\downarrow$$



Additional deflection due to bending of lever AB.

Refer to Case 1 of Appendix D.

$$I = \frac{1}{12}\left(\frac{3}{8}\right)(1)^3 = 31.25\times10^{-3}\text{ in}^4$$

$$(y_A) = \frac{PL^3}{3EI} = \frac{(80)(10)^3}{(3)(29\times10^6)(31.25\times10^{-3})}$$

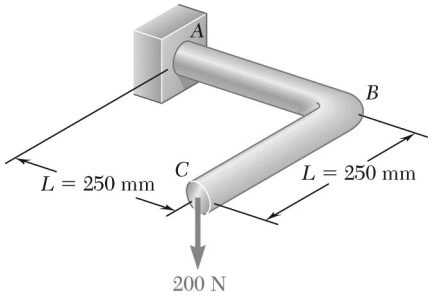
$$= 0.02943 \text{ in.}\downarrow$$

Total deflection at point A.

$$y_A = (y_A)_1 + (y_A)_2$$

$$y_A = 0.278 \text{ in.}\downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.94

A 16-mm-diameter rod has been bent into the shape shown. Determine the deflection of end C after the 200-N force is applied. Use $E = 200$ GPa and $G = 80$ GPa.

SOLUTION

Let 200 N = P.

Consider torsion of rod AB.

$$\phi_B = \frac{TL}{JG} = \frac{(PL)L}{JG} = \frac{PL^2}{JG}$$

$$y'_C = -L\phi_B = -\frac{PL^3}{JG}$$

Consider bending of AB. (Case 1 of Appendix D.)

$$y''_C = y_B = -\frac{PL^3}{3EI}$$

Consider bending of BC. (Case 1 of Appendix D.)

$$y'''_C = -\frac{PL^3}{3EI}$$

Superposition:

$$y_C = y'_C + y''_C + y'''_C$$

$$= -\frac{PL^3}{JG} - \frac{PL^3}{3EI} - \frac{PL^3}{3EI} = -\frac{PL^3}{EI}\left(\frac{EI}{JG} + \frac{2}{3}\right)$$

Data:

$$G = 80(10^9)\ \text{Pa} \qquad J = \frac{1}{2}\pi(0.008)^4 = 6.4340(10^{-9})\ \text{m}^4$$

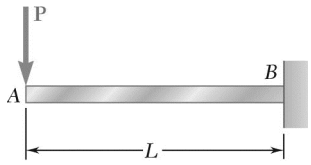
$$E = 200(10^9)\ \text{Pa} \qquad I = \frac{1}{2}J = 3.2170(10^{-9})\ \text{m}^4$$

$$EI = 643.40\ \text{N}\cdot\text{m}^2 \quad JG = 514.72\ \text{N}\cdot\text{m}^2$$

$$y_C = -\frac{(200)(0.25)^3}{643.40}\left(\frac{643.40}{514.72} + \frac{2}{3}\right) = -9.3093(10^{-3})\ \text{m}$$

$$y_C = 9.31\ \text{mm}\downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.95

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

Place reference tangent at *B*.

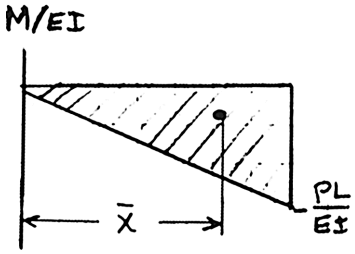
Draw *M*/*EI* diagram.

$$A = \frac{1}{2}\left(-\frac{PL}{EI}\right)L = -\frac{PL^2}{2EI}$$

$$\bar{x} = \frac{2}{3}L$$

$$\theta_{B/A} = A = -\frac{PL^2}{2EI}$$

$$t_{A/B} = A\bar{x} = \left(-\frac{PL^3}{2EI}\right)\left(\frac{2}{3}L\right) = -\frac{PL^3}{3EI}$$



(*a*) Slope at end *A*.

$$\theta_B = \theta_A + A$$

$$0 = \theta_A - \frac{PL^2}{2EI}$$

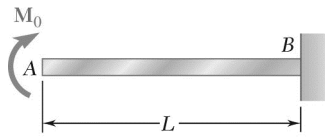
$$\theta_A = \frac{PL^2}{2EI}$$ ◀

(*b*) Deflection at *A*.

$$y_A = t_{A/B} = -\frac{PL^3}{3EI}$$

$$y_A = \frac{PL^3}{3EI}\downarrow$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



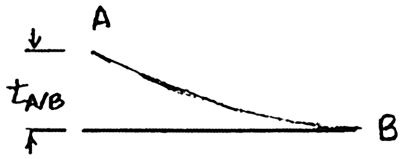
PROBLEM 9.96

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

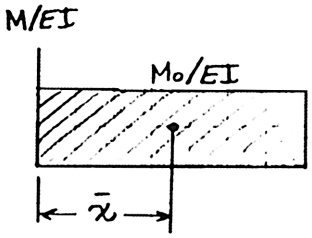
Place reference tangent at *B*.

Draw *M*/*EI* diagram.



$$A = \left(\frac{M_0}{EI}\right)L = \frac{M_0 L}{EI}$$

$$\bar{x} = \frac{1}{2}L$$



$$\theta_{B/A} = A = \frac{M_0 L}{EI}$$

$$t_{B/A} = A\bar{x} = \left(\frac{M_0 L}{EI}\right)\left(\frac{1}{2}L\right)$$

$$= \frac{M_0 L^2}{2EI}$$

(*a*) Slope at end *A*.

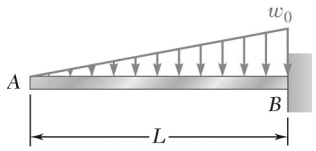
$$\theta_B = \theta_A + A \qquad 0 = \theta_A + \frac{M_0 L}{EI}$$

$$\theta_A = -\frac{M_0 L}{EI} \qquad \theta_A = \frac{M_0 L}{EI} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *A*.

$$y_A = t_{A/B} = \frac{M_0 L^2}{2EI} \qquad y_A = \frac{M_0 L^2}{2EI} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.97

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

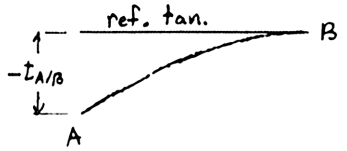
SOLUTION

Place reference tangent at *B*.

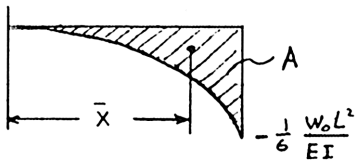
$$\theta_B = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{1}{2} w_0 L\right)\frac{L}{3} + M_B = 0$$

$$M_B = -\frac{1}{6} w_0 L^2$$



Draw M/EI curve as cubic parabola.



$$A = -\frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0 L^3}{EI}$$

$$\bar{x} = L - \frac{1}{5}L = \frac{4}{5}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{24}\frac{w_0 L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{24}\frac{w_0 L^3}{EI} = \frac{1}{24}\frac{w_0 L^3}{EI}$$

By second moment-area theorem,

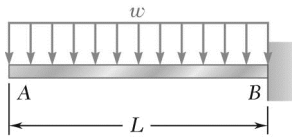
$$t_{A/B} = \bar{x}A = \left(\frac{4}{5}L\right)\left(-\frac{1}{24}\frac{w_0 L^3}{EI}\right) = -\frac{1}{30}\frac{w_0 L^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{30}\frac{w_0 L^4}{EI}$$

(*a*) $\theta_A = \dfrac{w_0 L^3}{24EI}$ ◀

(*b*) $y_A = \dfrac{w_0 L^4}{30EI} \downarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.98

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

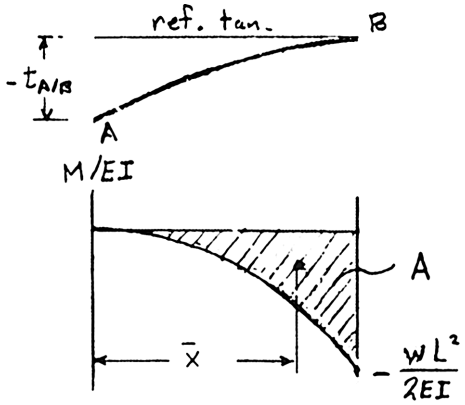
Place reference tangent at *B*.

$$\theta_B = 0$$

Draw *M*/*EI* curve as parabola.

$$A = -\frac{1}{3}\left(\frac{wL^2}{2EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$



By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6}\frac{wL^3}{EI} = \frac{1}{6}\frac{wL^3}{EI}$$

By second moment-area theorem,

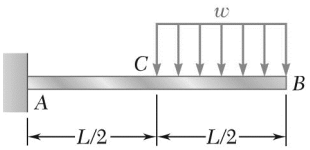
$$t_{A/B} = \bar{x}A = \left(\frac{3}{4}L\right)\left(-\frac{1}{6}\frac{wL^3}{EI}\right) = -\frac{1}{8}\frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8}\frac{wL^4}{EI}$$

(*a*) $\theta_A = \dfrac{wL^3}{6EI}$ ◀

(*b*) $y_A = \dfrac{wL^4}{8EI} \downarrow$ ◀

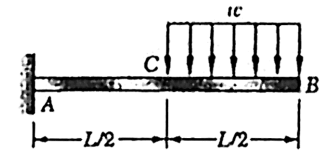
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

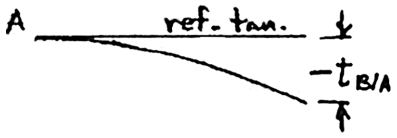


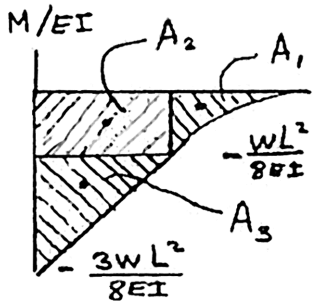
PROBLEM 9.99

For the uniform cantilever beam and loading shown, determine the slope and deflection at (*a*) point *B*, (*b*) point *C*.

SOLUTION







Place reference tangent at *A*.

Draw $\dfrac{M}{EI}$ diagram.

$$A_1 = \frac{1}{3}\left(-\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{wL^3}{48EI}$$

$$A_2 = \left(-\frac{wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{2wL^2}{8EI}\right)\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$$

$$\theta_A = 0, \quad y_A = 0$$

(*a*) Slope at *B*.

$$\theta_B = \theta_A + A_1 + A_2 + A_3$$

$$= -\frac{7wL^3}{48EI} \qquad \theta_B = \frac{7wL^3}{48EI} \curvearrowleft \blacktriangleleft$$

Deflection at *B*.

$$y_B = t_{B/A} = A_1\left(\frac{3}{4}\cdot\frac{L}{2}\right) + A_2\left(\frac{L}{2} + \frac{1}{2}\cdot\frac{L}{2}\right) + A_3\left(\frac{L}{2} + \frac{2}{3}\cdot\frac{L}{2}\right)$$

$$= -\frac{wL^4}{128EI} - \frac{3wL^4}{64EI} - \frac{5wL^4}{96EI} = -\frac{41wL^4}{384\,EI}$$

$$y_B = \frac{41wL^4}{384EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *C*.

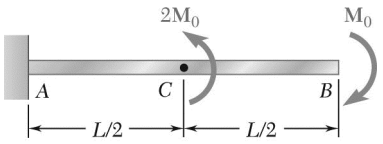
$$\theta_C = \theta_A + A_2 + A_3 = -\frac{wL^3}{8EI} \qquad \theta_C = \frac{wL^3}{8EI} \curvearrowleft \blacktriangleleft$$

Deflection at *C*.

$$y_C = t_{C/A} = A_2\left(\frac{1}{2}\cdot\frac{L}{2}\right) + A_3\left(\frac{R}{3}\cdot\frac{L}{2}\right)$$

$$= -\frac{7wL^4}{192EI} \qquad y_C = \frac{7wL^4}{192EI} \downarrow \blacktriangleleft$$

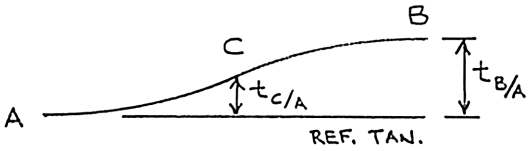
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.100

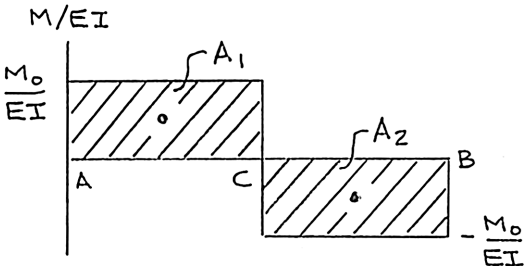
For the uniform cantilever beam and loading shown, determine the slope and deflection at (*a*) point *B*, (*b*) point *C*.

SOLUTION



Place reference tangent at A. $\theta_A = 0$

Draw $\dfrac{M}{EI}$ diagram.



$$A_1 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{2}\frac{M_0 L}{EI}$$

$$A_2 = \left(-\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{2}\frac{M_0 L}{EI}$$

(*a*) Slope at *B*.

$$\theta_{B/A} = A_1 + A_2 = \frac{1}{2}\frac{M_0 L}{EI} - \frac{1}{2}\frac{M_0 L}{EI} = 0$$

$$\theta_B = \theta_A + \theta_{B/A} = 0 \qquad \theta_B = 0 \blacktriangleleft$$

Deflection at *B*.

$$y_B = t_{B/A} = A_1\left(\frac{L}{2} + \frac{1}{2}\cdot\frac{L}{2}\right) + A_2\left(\frac{1}{2}\cdot\frac{L}{2}\right)$$

$$= \frac{3}{8}\frac{M_0 L^2}{EI} - \frac{1}{8}\frac{M_0 L^2}{EI} = \frac{1}{4}\frac{M_0 L^2}{EI} \qquad y_B = \frac{1}{4}\frac{M_0 L^2}{EI} \uparrow \blacktriangleleft$$

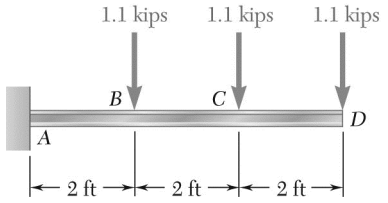
(*b*) Slope at *C*.

$$\theta_{C/A} = A_1 = \frac{1}{2}\frac{M_0 L}{EI} \quad \theta_C = \theta_A + \theta_{C/A} \qquad \theta_C = \frac{1}{2}\frac{M_0 L}{EI} \measuredangle \blacktriangleleft$$

Deflection at *C*.

$$y_C = t_{C/A} = A_1\left(\frac{1}{2}\cdot\frac{L}{2}\right) = \frac{1}{8}\frac{M_0 L^2}{EI} \qquad y_C = \frac{1}{8}\frac{M_0 L^2}{EI} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.101

Two C6×8.2 channels are welded back to back and loaded as shown. Knowing that $E = 29\times10^6$ psi, determine (*a*) the slope at point *D*, (*b*) the deflection at point *D*.

SOLUTION

Units: Forces in kips; lengths in ft.

$$E = 29\times10^6 \text{ psi} = 29\times10^3 \text{ ksi} \qquad I = (2)(13.1) = 26.2 \text{ in}^4$$

$$EI = (29.10^3)(26.2) = 759.8\times10^3 \text{ kip}\cdot\text{in}^2 = 5276 \text{ kip}\cdot\text{ft}^2$$

Draw *M*/*EI* diagram by parts.

$$\frac{M_1}{EI} = \frac{(1.1)(6)}{EI} = -\frac{6.6}{EI} \text{ ft}^{-1}$$

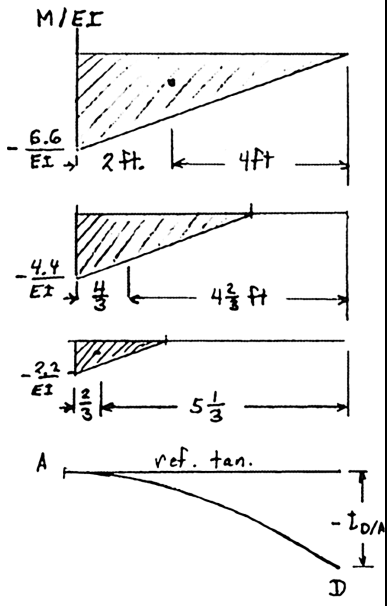
$$A_1 = \frac{1}{2}\left(\frac{6.6}{EI}\right)(6) = -\frac{19.8}{EI} \qquad \bar{x}_1 = \frac{1}{3}(6) = 2 \text{ ft}$$

$$\frac{M_2}{EI} = -\frac{(1.1)(4)}{EI} = -\frac{4.4}{EI} \text{ ft}^{-1}$$

$$A_2 = \frac{1}{2}\left(-\frac{4.4}{EI}\right)(4) = -\frac{8.8}{EI} \qquad \bar{x}_2 = \frac{1}{3}(4) = \frac{4}{3} \text{ ft}$$

$$\frac{M_3}{EI} = -\frac{(1.1)(2)}{EI} = -\frac{2.2}{EI} \text{ ft}^{-1}$$

$$A_3 = \frac{1}{2}\left(-\frac{2.2}{EI}\right)(2) = -\frac{2.2}{EI} \qquad \bar{x}_3 = \frac{1}{3}(2) = \frac{2}{3} \text{ ft}$$



Place reference tangent at *A*. $\theta_A = 0$

(*a*) Slope at *D*. $\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{30.8}{EI} = -\frac{30.8}{5276} = -5.84\times10^{-3} \text{ rad}$

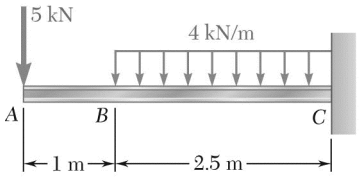
$$\theta_D = \theta_A + \theta_{D/A} \qquad \theta_D = 5.89\times10^{-3} \text{ rad.} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *D*. $t_{D/A} = \left(-\frac{19.8}{EI}\right)(4) + \left(-\frac{8.8}{EI}\right)\left(4\frac{2}{3}\right) + \left(-\frac{2.2}{EI}\right)\left(5\frac{1}{3}\right)$

$$= -\frac{132.0}{EI} = -\frac{132.0}{5276} = 25.02\times10^{-3} \text{ ft}$$

$$y_D = t_{D/A} = 25.02\times10^{-3} \text{ ft} \qquad y_D = 0.300 \text{ in.} \downarrow \blacktriangleleft$$

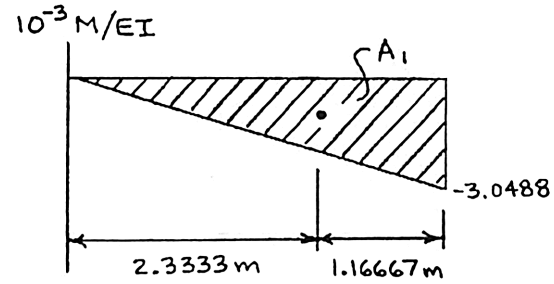
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

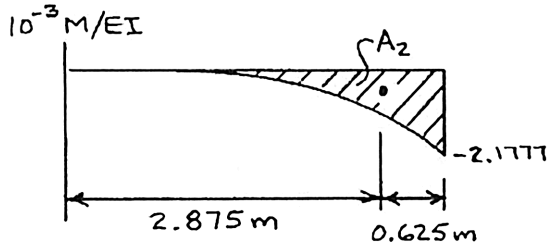


PROBLEM 9.102

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION





Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 28.7 \times 10^6 \text{ mm}^4 = 28.7 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(28.7 \times 10^6)$$

$$= 5.74 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 5740 \text{ kN} \cdot \text{m}^2$$

Draw M/EI diagram by parts:

$$\frac{M_1}{EI} = -\frac{(5)(3.5)}{5740} = -3.0488 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(-3.0488 \times 10^{-3})(3.5) = -5.3354 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(3.5) = 1.16667 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(4)(2.5)^2}{(2)(5740)} = -2.1777 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-2.1777 \times 10^{-3})(2.5) = -1.81475 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.5) = 0.625 \text{ m}$$

Place reference tangent at *C*. $\theta_C = 0$

$$\theta_{C/A} = A_1 + A_2 = -7.1502 \times 10^{-3}$$

(*a*) Slope at *A*. $\theta_A = \theta_C - \theta_{C/A} = 7.1502 \times 10^{-3}$

$$\theta_A = 7.15 \times 10^{-3} \text{ rad} \measuredangle \blacktriangleleft$$

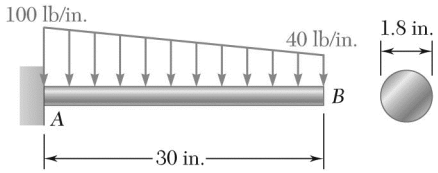
$$t_{A/C} = (2.3333)(-5.3354 \times 10^{-3}) + (2.875)(-1.81475 \times 10^{-3})$$

$$= -17.6665 \times 10^{-3} \text{ m}$$

(*b*) Deflection at *A*.

$$y_A = t_{AC} = -17.67 \times 10^{-3} \text{ m} \qquad y_A = 17.67 \text{ mm} \downarrow \blacktriangleleft$$

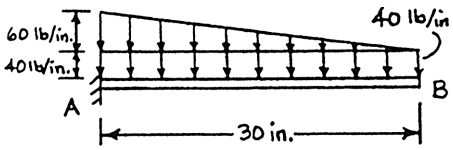
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

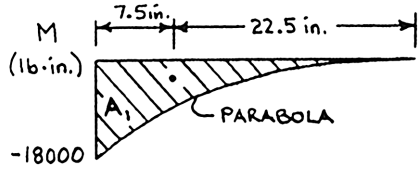


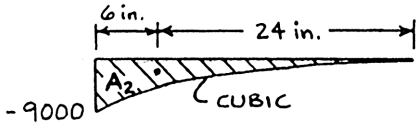
PROBLEM 9.103

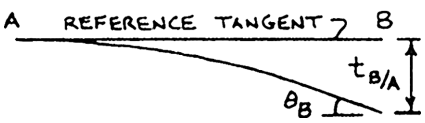
For the cantilever beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *B*. Use $E = 29\times10^6$ psi.

SOLUTION









$$I = \frac{\pi}{4}\left(\frac{1.8}{2}\right)^4 = 0.51530 \text{ in}^4$$

$$EI = (29\times10^6)(0.51530) = 14.9437\times10^6 \text{ lb}\cdot\text{in}^2$$

$$M_A = -\frac{wL^2}{2} = -\frac{(40)(30)^2}{2} = -18000 \text{ lb}\cdot\text{in}$$

$$A_1 = \frac{1}{3}(-18000)(30) = -180\times10^3 \text{ lb}\cdot\text{in}^2$$

$$M_A = -\frac{wL^2}{6} = -\frac{(60)(30)^2}{6} = -9000 \text{ lb}\cdot\text{in}$$

$$A_2 = \frac{1}{4}(-9000)(30) = -67.5\times10^3 \text{ lb}\cdot\text{in}^2$$

(*a*) Slope at *B*.

$$EI\theta_{B/A} = A_1 + A_2 = -180\times10^3 - 67.5\times10^3$$
$$= -247.5\times10^3 \text{ lb}\cdot\text{in}^2$$

$$\theta_B = \theta_{B/A} = \frac{-247.5\times10^3}{14.9437\times10^6} = -16.5622\times10^{-3} \text{ rad}$$

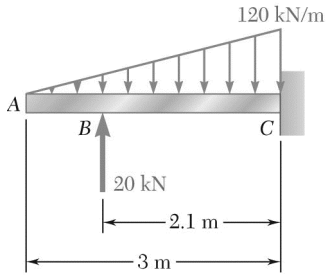
$$\theta_B = 16.56\times10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *B*.

$$EIy_B = EIt_{B/A}$$
$$= (-180\times10^3)(22.5) + (-67.5\times10^3)(24)$$
$$= -5.67\times10^6 \text{ lb}\cdot\text{in}^3$$

$$y_B = \frac{-5.67\times10^6}{14.9437\times10^6}$$
$$= -0.37942 \text{ in.} \qquad y_B = 0.379 \text{ in.} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.104

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

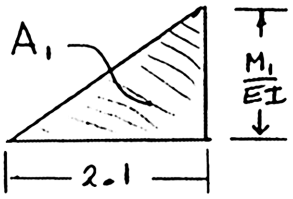
$$I = 178\times10^{6}\ \text{mm}^4 = 178\times10^{-6}$$

$$EI = (200\times10^{9})(178\times10^{-6}) = 35600\ \text{kN}\cdot\text{m}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(20)(2.1)}{35600} = 1.17978\times10^{-3}\ \text{m}^{-1}$$

$$A_1 = \left(\frac{1}{2}\right)(2.1)(1.17978\times10^{-3}) = 1.23876\times10^{-3}$$



$$M_2 = -\frac{\left(\frac{1}{2}\right)(120)(3)(1)}{35600} = -5.0562\times10^{-3}\ \text{m}^{-1}$$

$$A_2 = \left(\frac{1}{4}\right)(3)(-5.0562\times10^{-3}) = -3.7921\times10^{-3}$$

Place reference tangent at *C*.

$$\theta_C = 0$$

(*a*) Slope at *A*.

$$\theta_A = -\theta_{C/A} = -A_1 - A_2$$

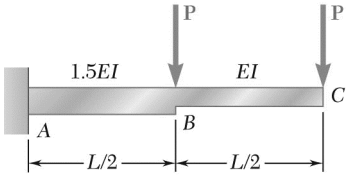
$$\theta_A = 2.55\times10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *A*.

$$y_A = t_{A/C}$$

$$y_C = A_1(3 - 0.7) + A_2\left(3 - \frac{3}{5}\right) = -6.25\times10^{-3}\ \text{m} \qquad y_C = 6.25\ \text{mm}\downarrow \blacktriangleleft$$

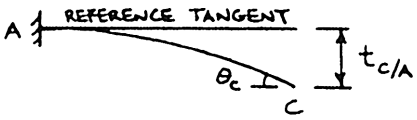
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.105

For the cantilever beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*.

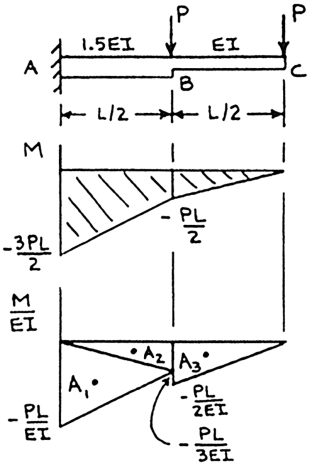
SOLUTION



$$A_1 = \frac{1}{2}\left(-\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = -\frac{PL^2}{4EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{PL}{3EI}\right)\left(\frac{L}{2}\right) = -\frac{PL^2}{12EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{PL^2}{8EI}$$



(*a*) Slope at *C*.

$$\theta_C = A_1 + A_2 + A_3$$

$$= -\frac{PL^2}{EI}\left(\frac{1}{4} + \frac{1}{12} + \frac{1}{8}\right) = -\frac{11PL^2}{24EI}$$

$$\theta_C = \frac{11PL^2}{24EI} \measuredangle \blacktriangleleft$$

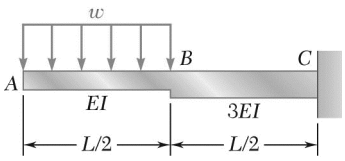
(*b*) Deflection at *C*.

$$y_C = t_{C/A} = A_1\left(\frac{5}{6}L\right) + A_2\left(\frac{2}{3}L\right) + A_3\left(\frac{1}{3}L\right)$$

$$= \left(-\frac{PL^2}{4EI}\right)\left(\frac{5}{6}L\right) + \left(-\frac{PL^2}{12EI}\right)\left(\frac{2}{3}L\right) + \left(-\frac{PL^2}{8EI}\right)\left(\frac{1}{3}L\right) = -\frac{22PL^3}{72EI}$$

$$y_C = \frac{11PL^3}{36EI} \downarrow \blacktriangleleft$$

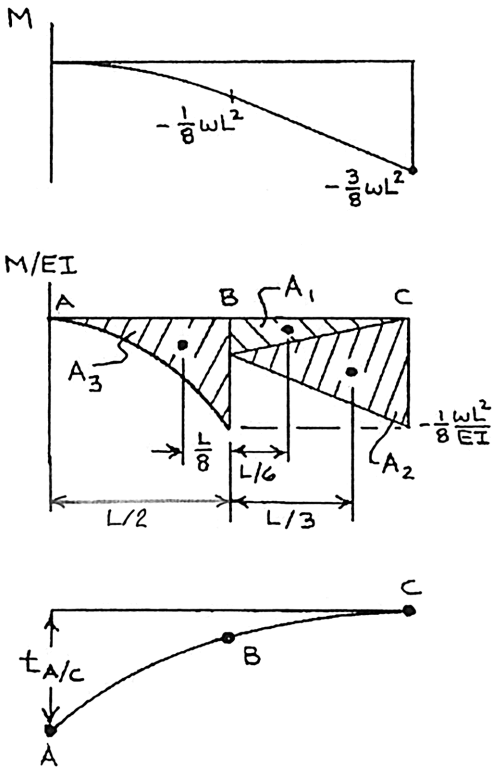
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.106

For the cantilever beam and loading shown, determine (*a)* the slope at point *A*, (*b*) the deflection at point *A*.

SOLUTION



Draw the $\frac{M}{EI}$ diagram using the M diagram.

$$A_1 = \frac{1}{2}\left(-\frac{1}{24}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{96}\frac{wL^3}{EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{32}\frac{wL^3}{EI}$$

$$A_3 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

Place reference tangent at *C*.

(*a*) Slope at *A*. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 - A_3 = \frac{1}{16}\frac{wL^3}{EI}$$

$$\theta_A = \frac{1}{16}\frac{wL^3}{EI} \measuredangle \blacktriangleleft$$

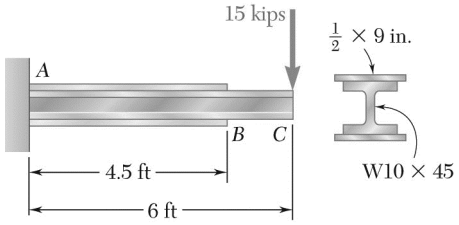
(*b*) Deflection at *A*. $y_A = t_{A/C}$

$$y_A = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{5}{6}L\right) + A_3\left(\frac{3}{8}L\right)$$

$$= -\frac{47}{1152}\frac{wL^4}{EI}$$

$$y_A = \frac{47}{1152}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

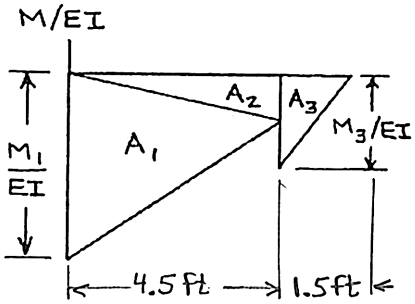
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.107

Two cover plates are welded to the rolled-steel beam as shown. Using $E = 29\times10^6$ psi, determine (*a*) the slope at end *C*, (*b*) the deflection at end *C*.

SOLUTION



Portion *BC*: $I = 248 \text{ in}^4$

$EI = (29\times10^6)(248) = 7.192\times10^9 \text{ lb}\cdot\text{in}^2 = 49{,}944 \text{ kip}\cdot\text{ft}^2$

Portion *AB*:

	$A(\text{in}^2)$	$d(\text{in.})$	$Ad^2(\text{in}^4)$	$\bar{I}(\text{in}^4)$
Top plate	4.5	5.3	126.405	0.09375
W12 × 45				248
Bot. plate	4.5	5.3	126.405	0.09375
Σ			252.81	248.19

$I = 252.81 + 248.19 = 501.00 \text{ in}^4 \quad EI = (29\times10^6)(501) = 14.529\times10^9 \text{lb}\cdot\text{in}^2 = 100{,}896 \text{ kip}\cdot\text{ft}^2$

Draw $\frac{M}{EI}$ diagram:

$$\frac{M_1}{EI} = -\frac{(15)(6)}{100{,}896} = -892.01\times10^{-6}\text{ft}^{-1}$$

$$\frac{M_3}{EI} = -\frac{(15)(1.5)}{49{,}944} = -450.50\times10^{-6}\text{ft}^{-1}$$

$$A_1 = \frac{1}{2}(4.5)(-892.01\times10^{-6}) = -2.0070\times10^{-3}$$

$$A_2 = \frac{1.5}{6}A_1 = -0.50175\times10^{-3}$$

$$A_3 = \frac{1}{2}(1.5)(-450.50\times10^{-6}) = -0.33788\times10^{-3}$$

Place reference tangent at *A*.

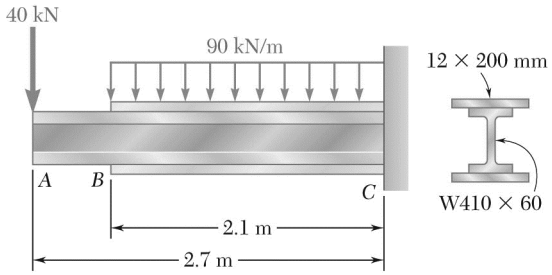
(*a*) Slope at *C*. $\theta_C = \theta_{C/A} = A_1 + A_2 + A_3$ $\theta_C = 2.85\times10^{-3}\text{rad}$ ◀

(*b*) Deflection at *C*. $y_C = t_{C/A}$

$$y_C = (4.5)(A_1) + (3)(A_2) + (1)(A_3) = -10.8746\times10^{-3}\text{ft}$$

$y_C = 0.1305\text{in.}\downarrow$ ◀

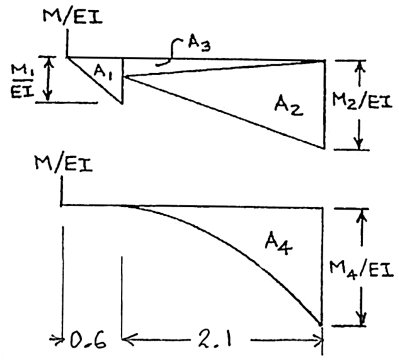
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.108

Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*.

SOLUTION



Portion *AB*: $I = 216 \times 10^6 \text{ mm}^4$

$$EI = (200 \times 10^6 \text{ kPa})(216 \times 10^{-6} \text{ m}^4) = 43{,}200 \text{ kN} \cdot \text{m}^2$$

Portion *BC*:

	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
Top plate	2400	209	104.834×10^6	28,800
W410 × 60				216×10^6
Bot. plate	2400	209	104.834×10^6	28,800
Σ			209.67×10^6	216.06×10^6

$$I = 209.67 \times 10^6 + 216.06 \times 10^6 = 425.73 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^6 \text{ kPa})(425.73 \times 10^{-6} \text{ m}^4) = 85{,}146 \text{ kN} \cdot \text{m}^2$$

Draw $\dfrac{M}{EI}$ diagram:

$$\frac{M_1}{EI} = -\frac{(40)(0.6)}{43200} = -0.55556 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(40)(2.7)}{85146} = -1.26841 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_4}{EI} = -\frac{(90)(2.1)(1.05)}{85146} = -2.3307 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(0.6)(-0.55556 \times 10^{-3}) = -0.166668 \times 10^{-3}$$

$$A_2 = \frac{1}{2}(2.1)(-1.26841 \times 10^{-3}) = -1.33183 \times 10^{-3}$$

$$A_3 = \frac{0.6}{2.7} A_2 = -0.29596 \times 10^{-3}$$

$$A_4 = \frac{1}{3}(2.1)(-2.3307 \times 10^{-3}) = -1.63149 \times 10^{-3}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.108 *(Continued)*

Place reference tangent at C. $\theta_C = 0$

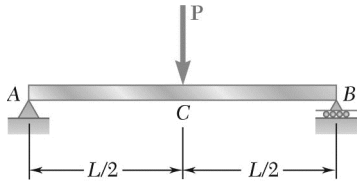
(a) Slope at A. $\theta_A = \theta_C - \theta_{A/C} = 0 - (A_1 + A_2 + A_3 + A_4)$ $\theta_A = 3.43 \times 10^{-3}\text{rad}\,\measuredangle$ ◀

(b) Deflection at A. $y_A = t_{A/C}$

$$y_A = (0.4)(A_1) + (2)(A_2) + (1.3)(A_3) + (2.175)(A_4) = -6.66 \times 10^{-3}\text{ m}$$

$y_A = 6.66\text{ mm} \downarrow$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



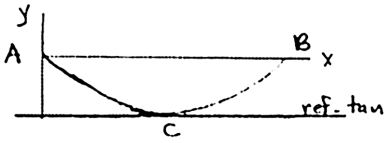
PROBLEM 9.109

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

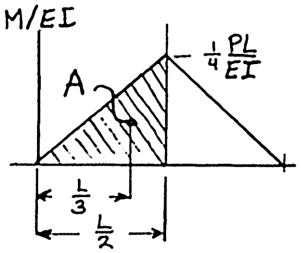
Symmetric beam and loading.

Place reference tangent at *C*.



$$\theta_C = 0, \quad y_C = -t_{A/C}$$

Reactions: $R_A = R_B = \frac{1}{2}P$



Bending moment at *C*. $M_C = \frac{1}{4}PL$

$$A = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$$

(*a*) Slope at *A*. $\theta_A = \theta_C - \theta_{C/A}$

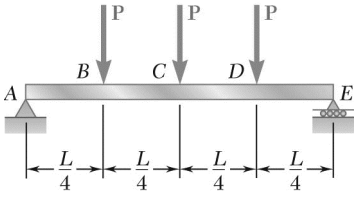
$$\theta_A = 0 - \frac{1}{16}\frac{PL^2}{EI}$$

$$\theta_A = -\frac{1}{16}\frac{PL^2}{EI} \measuredangle \blacktriangleleft$$

(*b*) Deflection at *C*. $y_C = -t_{A/C} = -A\left(\frac{L}{3}\right) = -\left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{L}{3}\right)$

$$y_C = \frac{1}{48}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

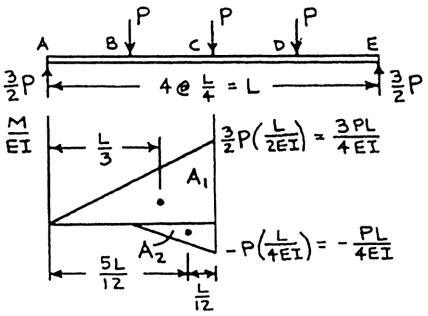
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

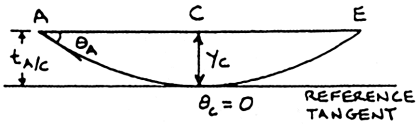


PROBLEM 9.110

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION





$$A_1 = \frac{1}{2}\left(\frac{3PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{3PL^2}{16EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{PL}{4EI}\right)\left(\frac{L}{4}\right) = -\frac{PL^2}{32EI}$$

(*a*) Slope at *A*. $\theta_C = \theta_A + \theta_{C/A}; \quad \theta_A = 0 - \theta_{C/A}$

$$\theta_A = -\theta_{C/A} = -(A_1 + A_2)$$

$$= -\left[\frac{3PL^2}{16EI} - \frac{PL^2}{32EI}\right]$$

$$= -\frac{5PL^2}{32EI}$$

$$\theta_A = \frac{5PL^2}{32EI} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *C*.

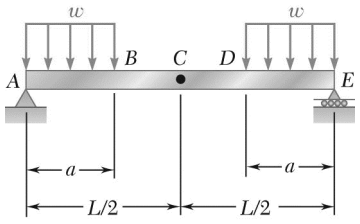
$$t_{A/C} = A_1\left(\frac{L}{3}\right) + A_2\left(\frac{5L}{12}\right)$$

$$= \left(\frac{3PL^2}{16EI}\right)\left(\frac{L}{3}\right) + \left(-\frac{PL^2}{32EI}\right)\left(\frac{5L}{12}\right)$$

$$= \frac{19PL^3}{384EI}$$

$$y_C = -t_{A/C} = -\frac{19PL^3}{384EI} \qquad y_C = \frac{19PL^3}{384EI} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.111

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

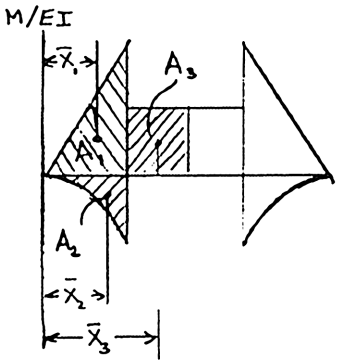
Place reference tangent at *C*. $\theta_C = 0$

Reactions: $R_A = R_E = wa$

Bending moment.

Over *AB*: $M = wax - \frac{1}{2}wa^2$

Over *BD*: $M = \frac{1}{2}wa^2$

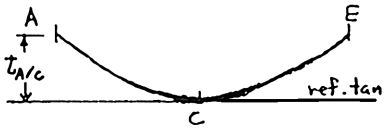


Draw *M*/*EI* diagram by parts.

$$\frac{M_1}{EI} = \frac{wa^2}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2}\frac{wa^2}{EI}$$



$$A_1 = \frac{1}{2}\frac{M_1}{EI}a = \frac{1}{2}\frac{wa^3}{EI}$$

$$A_2 = -\frac{1}{3}\frac{M_2}{EI}a = -\frac{1}{6}\frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{1}{2}\frac{wa^3}{EI} + \frac{1}{6}\frac{wa^3}{EI} - \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12}\frac{wa^2}{EI}(3L - 2a)$$

$$\theta_A = \frac{wa^2}{12EI}(3L - 2a) \quad \searrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.111 *(Continued)*

(*b*) Deflection at *C*.

$$y_C = -t_{C/A}$$

$$\bar{x}_1 = \frac{2}{3}a,$$

$$\bar{x}_2 = \frac{3}{4}a,$$

$$\bar{x}_3 = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_C = -t_{C/A} = -A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3$$

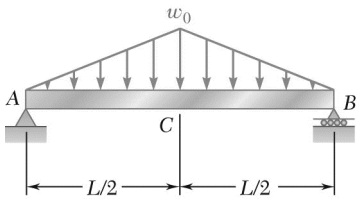
$$= -\left(\frac{1}{2}\frac{wa^3}{EI}\right)\left(\frac{2}{3}a\right) + \left(\frac{1}{6}\frac{wa^3}{EI}\right)\left(\frac{3}{4}a\right) - \frac{1}{4}\left(\frac{wa^2}{EI}\right)(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{3}\frac{wa^3}{EI} + \frac{1}{8}\frac{wa^3}{EI} - \frac{1}{16}\frac{wa^2}{EI}(L^2 - 4a^2)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48}\frac{wa^2}{EI}(3L^2 - 2a^2)$$

$$y_A = \frac{wa^2}{48EI}(3L^2 - 2a^2) \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.112

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

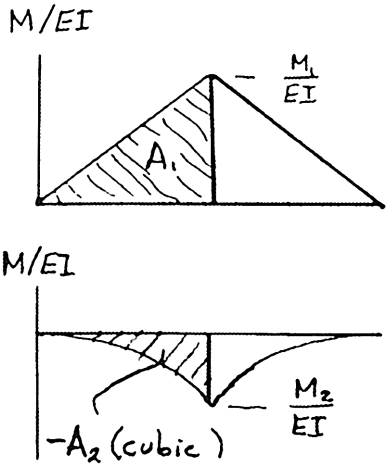
SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0$

Reactions: $R_A = R_B = \dfrac{w_0 L}{4}$

Draw $\dfrac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8EI}$$

$$A_1 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{M_1}{EI}\right) = \frac{w_0 L^3}{32EI}$$

$$\frac{M_2}{EI} = \frac{1}{EI}\cdot\frac{1}{2}\left(-\frac{w_0 L}{2}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{w_0 L^2}{24EI}$$

$$A_2 = \frac{1}{4}\left(\frac{L}{2}\right)\left(-\frac{w_0 L^2}{24EI}\right) = -\frac{w_0 L^3}{192EI}$$

(*a*) Slope at *A*. $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192}\right)\frac{w_0 L^3}{EI} \qquad \theta_A = \frac{5 w_0 L^3}{192EI} \curvearrowright \blacktriangleleft$$

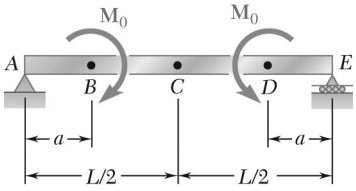
(*b*) Deflection at *C*. $y_C = t_{A/C}$

$$t_{A/C} = A_1\left[\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)\right] + A_2\left[\left(\frac{4}{5}\right)\left(\frac{L}{2}\right)\right] = \left[\left(\frac{1}{3}\right)\left(\frac{1}{32}\right) - \left(\frac{2}{5}\right)\left(\frac{1}{192}\right)\right]\frac{w_0 L^4}{EI}$$

$$= \frac{w_0 L^4}{120EI}$$

$$y_c = \frac{w_0 L^4}{120EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.113

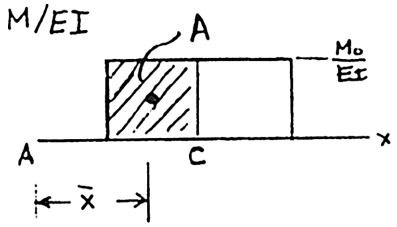
For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0.$

Draw $\dfrac{M}{EI}$ diagram.



(*a*) Slope at *A*. $\theta_A = 0$

$$A = \frac{M_0}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{2}\frac{M_0}{EI}(L - 2a)$$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - A = -\frac{1}{2}\frac{M}{EI}(L - 2a)$$

$$\theta_A = \frac{1}{2}\frac{M_0}{EI}(L - 2a) \curvearrowright \blacktriangleleft$$

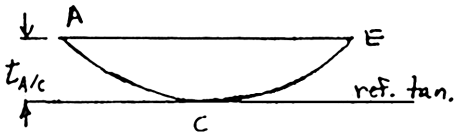
(*b*) Deflection at *C*.

$$\bar{x} = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_C = -t_{C/A} = A\bar{x}$$

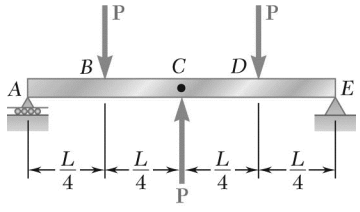
$$= -\frac{1}{2}\frac{M_0}{EI}(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2)$$



$$y_C = \frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2) \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.114

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

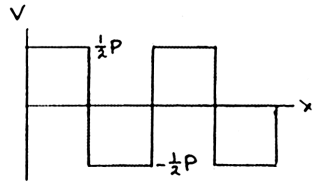
SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*.

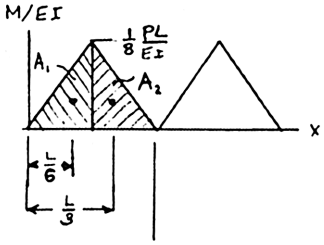
$$\theta_C = 0$$

Reactions: $$R_A = R_E = \frac{1}{2}P$$



Draw *V* (shear) and *M/EI* diagrams.

$$A_1 = A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\frac{L}{4} = \frac{1}{64}\frac{PL^2}{EI}$$

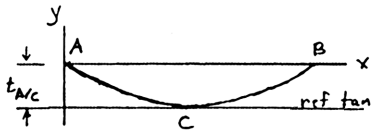


(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{A/C} = 0 - A_1 - A_2$$

$$= -\frac{1}{32}\frac{PL^2}{EI}$$

$$\theta_A = \frac{PL^2}{32EI} \measuredangle \blacktriangleleft$$



(*b*) Deflection at *C*.

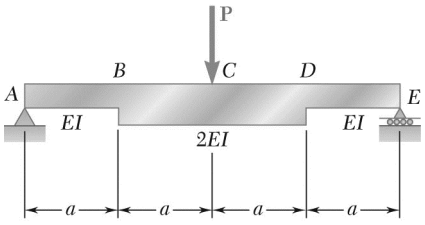
$$y_C = -t_{A/C} = -\left(A_1\frac{L}{6} + A_2\frac{L}{3}\right)$$

$$= -\left(\frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{6} + \frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{3}\right)$$

$$= -\frac{1}{128}\frac{PL^3}{EI}$$

$$y_C = \frac{PL^3}{128EI}\downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.115

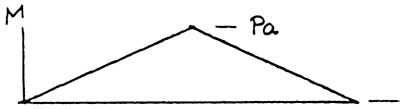
For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

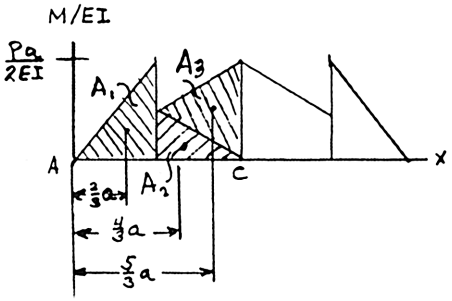
Symmetric beam and loading.

$$R_A = R_E = \frac{1}{2}P$$

$$M_{\max} = \left(\frac{1}{2}P\right)(2a) = Pa$$



Draw *M* and *M*/*EI* diagrams.



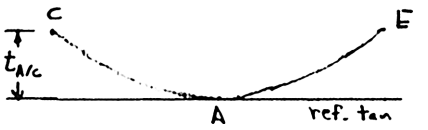
$$A_1 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

Place reference tangent at *C*.

$$\theta_C = 0$$



(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8}\frac{Pa^2}{EI}$$

$$\theta_A = \frac{5Pa^2}{8EI} \curvearrowright \blacktriangleleft$$

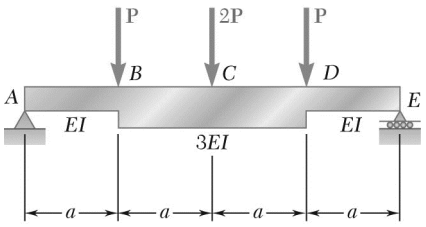
(*b*) Deflection at *C*.

$$|y_C| = t_{A/C} = A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right)$$

$$= \frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI} = \frac{3}{4}\frac{Pa^3}{EI}$$

$$y_C = \frac{3Pa^3}{4EI} \downarrow \blacktriangleleft$$

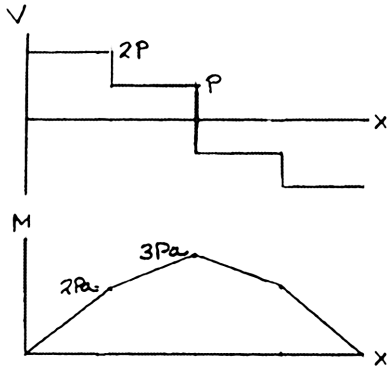
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.116

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION



Symmetric beam and loading.

$$R_A = R_E = 2P.$$

Draw V, M, and M/EI diagrams.

$$A_1 = \frac{1}{2}\left(\frac{2Pa}{EI}\right)a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{2}{3}\frac{Pa}{EI}\right)a = \frac{1}{3}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{EI}\right)a = \frac{1}{2}\frac{Pa^2}{EI}$$

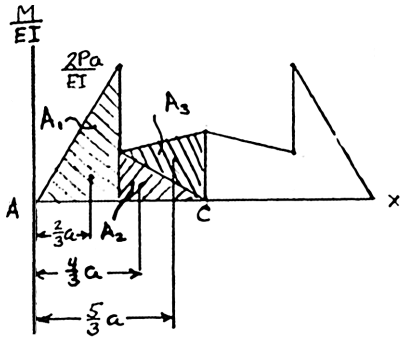
Place reference tangent at *C*.

$$\theta_C = 0$$

(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6}\frac{Pa^2}{EI} \qquad \theta_A = \frac{11Pa^2}{6EI} \curvearrowright \blacktriangleleft$$

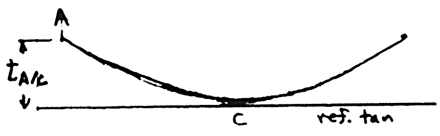


(*b*) Deflection at *C*.

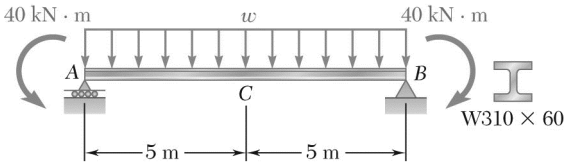
$$|y_C| = t_{A/C}$$

$$= A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right)$$

$$= \frac{35}{18}\frac{Pa^3}{EI} \qquad y_C = \frac{35Pa^3}{18EI} \downarrow \blacktriangleleft$$



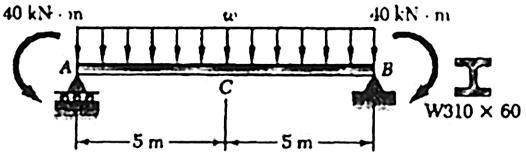
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.117

For the beam and loading shown and knowing that $w = 8$ kN/m, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.

SOLUTION



$$E = 200\times10^{9}\ \text{Pa}$$

$$I = 128\times10^{6}\ \text{mm}^4 = 128\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^{9})(128\times10^{-6}) = 25.6\times10^{6}\ \text{N}\cdot\text{m}^2$$

$$= 25{,}600\ \text{kN}\cdot\text{m}^2$$

Symmetrical beam and loading.

$$R_A = R_B = \frac{1}{2}(8)(10) = 40\ \text{kN}$$

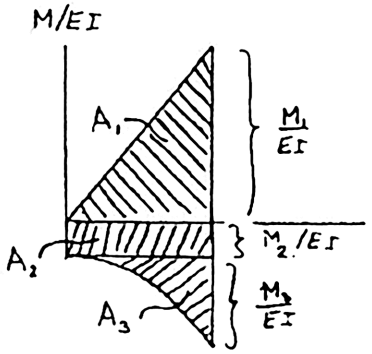
Bending moment:

$$M = 40x - 40 - \frac{1}{2}(8)x^2$$

At $x = 5$,

$$M = 200 - 40 - 100$$

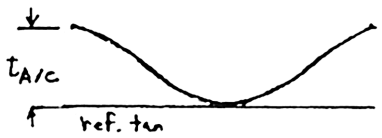
Draw $\dfrac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = \frac{200}{25{,}600} = 7.8125\times10^{-3}\ \text{m}^{-1}$$

$$\frac{M_2}{EI} = \frac{-40}{25{,}600} = -1.5625\times10^{-3}\ \text{m}^{-1}$$

$$\frac{M_3}{EI} = \frac{-100}{25{,}600} = -3.9063\times10^{-3}\ \text{m}^{-1}$$



$$A_1 = \frac{1}{2}(7.8125\times10^{-3})(5) = 19.5313\times10^{-3} \qquad \bar{x}_1 = \left(\frac{2}{3}\right)(5) = 3.3333\ \text{m}$$

$$A_2 = -(1.5625)(5) = -7.8125\times10^{-3} \qquad \bar{x}_2 = \left(\frac{1}{2}\right)(5) = 2.5\ \text{m}$$

$$A_3 = -\frac{1}{3}(3.9063)(5) = -6.5105\times10^{-3} \qquad \bar{x}_3 = \left(\frac{3}{4}\right)(5) = 3.75\ \text{m}$$

Place reference tangent at *C*. $\theta_C = 0$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.117 *(Continued)*

(*a*) Slope at *A*.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$\theta_A = -(19.5313\times10^{-3} - 7.8125\times10^{-3} - 6.5105\times10^{-3}) = -5.21\times10^{-3}$$

$$\theta_A = 5.21\times10^{-3}\ \text{rad} \curvearrowright \blacktriangleleft$$

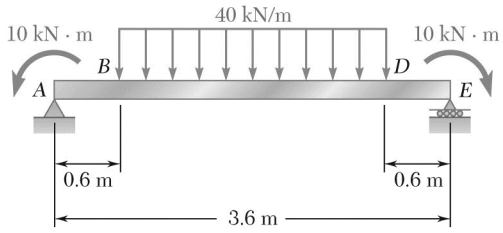
(*b*) Deflection at *C*.

$$|y_C| = t_{A/C}$$

$$= (19.5313\times10^{-3})(3.3333) - (7.8125\times10^{-3})(2.5) - (6.5105\times10^{-3})(3.75)$$

$$= 21.2\times10^{-3}\ \text{m} \qquad y_C = 21.2\ \text{mm}\downarrow \blacktriangleleft$$

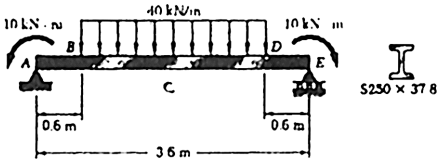
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.118

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

SOLUTION



Use units of kN and m.

For $S250 \times 37.8$

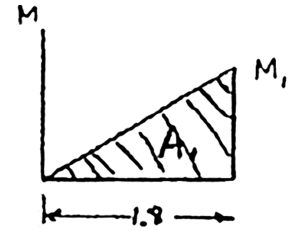
$$I = 51.2 \times 10^6 \text{ mm}^4 = 51.2 \times 10^{-6} \text{ m}^4$$

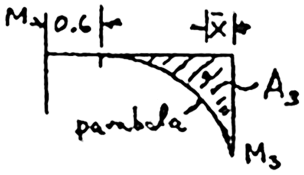
$$EI = (200 \times 10^9)(51.2 \times 10^{-6})$$

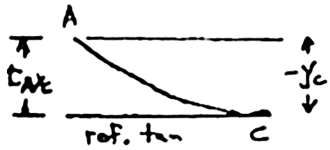
$$= 10.24 \times 10^6 \text{ N} \cdot \text{m}^2 = 10{,}240 \text{ kN} \cdot \text{m}^2$$

Place reference tangent at midpoint *C*.

Reactions: $R_A = R_E = \frac{1}{2}(40)(3.6 - 1.2) = 48 \text{ kN} \uparrow$







Draw bending moment diagram of left half of beam by parts.

$$M_1 = (48)(1.8) = 86.4 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(1.8)(86.4) = 77.76 \text{ kN} \cdot \text{m}^2$$

$$A_2 = (1.8)(-10) = -18 \text{ kN} \cdot \text{m}^2$$

$$M_3 = \frac{1}{2}(40)(1.8 - 0.6)^2 = -28.8 \text{ kN} \cdot \text{m}$$

$$A_3 = \frac{1}{3}(1.2)(-28.8) = -11.52 \text{ kN} \cdot \text{m}^2$$

$$\bar{x} = \frac{1}{4}(1.2) = 0.30 \text{ m}$$

(*a*) Slope at end *A*. $\theta_A = -\theta_{A/C}$

$$\theta_A = \frac{1}{EI}\{-A_1 - A_2 - A_3\} = \frac{-77.76 + 18 + 11.52}{10{,}240}$$

$$= -4.71 \times 10^{-3} \text{rad}$$

$$\theta_A = 4.71 \times 10^{-3} \text{rad} \curvearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.118 ***(Continued)***

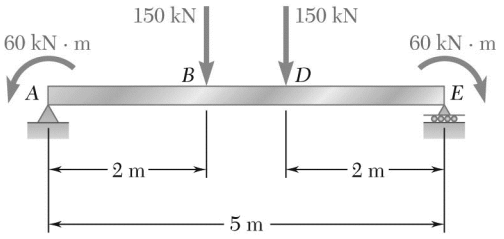
(*b*) Deflection at midpoint *C*. $y_C = -t_{A/C}$

$$t_{A/C} = \frac{1}{EI}\{1.2A_1 + 0.9A_2 + (1.8 - 0.3)A_3\}$$

$$= \frac{(1.2)(77.76) - (0.9)(18) - (1.5)(11.52)}{10,240} = 5.84 \times 10^{-3} \text{ m}$$

$$y_C = -5.84 \times 10^{-3} \text{ m} \qquad y_C = 5.84 \text{ mm} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.119

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

SOLUTION

Use units of kN and m.

For W460×74,

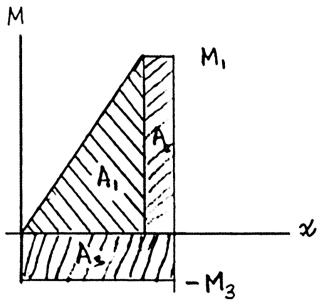
$$I = 333\times10^{6}\,\text{mm}^4 = 333\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^{9})(333\times10^{-6})$$

$$= 66.6\times10^{6}\,\text{N}\cdot\text{m}^2 = 66600\ \text{kN}\cdot\text{m}^2$$

Symmetric beam and loading. Place reference tangent at midpoint *C* where $\theta_C = 0$.

Reactions: $R_A = R_E = 150\ \text{kN}\uparrow$



Draw bending moment diagram of left half of beam by parts.

$$M_1 = (2)(150) = 300\ \text{kN}\cdot\text{m}$$

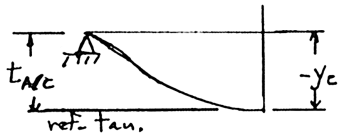
$$A_1 = \left(\frac{1}{2}\right)(2)(300) = 300\ \text{kN}\cdot\text{m}^2$$

$$A_2 = (0.5)(300) = 150\ \text{kN}\cdot\text{m}^2$$

$$M_3 = -60\ \text{kN}\cdot\text{m}$$

$$A_3 = (2.5)(-60) = -150\ \text{kN}\cdot\text{m}^2$$

(*a*) Slope at end *A*. $\theta_A = -\theta_{C/A}$



$$\theta_A = \frac{1}{EI}\{-A_1 - A_2 - A_3\}$$

$$= \frac{-300-150+150}{66600}$$

$$= -4.50\times10^{-3}\,\text{rad}$$

$\theta_A = 4.50\times10^{-3}\,\text{rad}$ ◀

(*b*) Deflection at midpoint *C*. $y_C = -t_{A/C}$

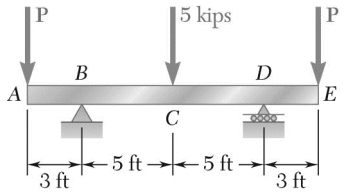
$$t_{A/C} = \frac{1}{EI}\left\{\left(\frac{2}{3}\cdot 2\right)A_1 + \left(2+\frac{0.5}{2}\right)A_2 + \left(\frac{2.5}{2}\right)A_3\right\}$$

$$= \frac{400+337.5-187.5}{66600} = 8.26\times10^{-3}\,\text{m}$$

$$y_C = -8.26\times10^{-3}\,\text{m}$$

$y_C = 8.26\ \text{mm}\downarrow$ ◀

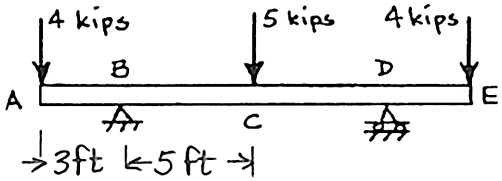
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

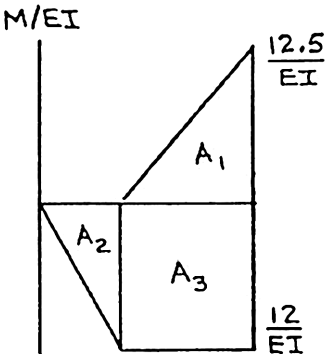


PROBLEM 9.120

Knowing that $P = 4$ kips, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 29 \times 10^6$ psi.

SOLUTION





$$E = 29 \times 10^3 \text{ ksi} \qquad I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6) = 1.1484 \times 10^6 \text{ kip} \cdot \text{in}^2$$

$$= 7975 \text{ kip} \cdot \text{ft}^2$$

Symmetric beam and loading:

$$R_A = R_B = P + 2.5 = 4 + 2.5 = 6.5 \text{ kips}$$

Bending moment:

Over *AB*: $M = -Px = -4x$

Over *BC*: $M = -4x + 6.5(x - 3)$

$$= 2.5\,(x - 3) - 12$$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{12.5}{EI}\right)(5) = \frac{31.25}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{12}{EI}\right)(3) = -\frac{18}{EI}$$

$$A_3 = -\left(\frac{12}{EI}\right)(5) = -\frac{60}{EI}$$

Place reference tangent at *C*. $\theta_C = 0$

(*a*) Slope at *A*. $\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$$\theta_A = -\left(\frac{31.25}{EI} - \frac{18}{EI} - \frac{60}{EI}\right) = \frac{46.75}{EI} = \frac{46.75}{7975}$$

$$\theta_A = 5.86 \times 10^{-3} \text{rad} \measuredangle \blacktriangleleft$$

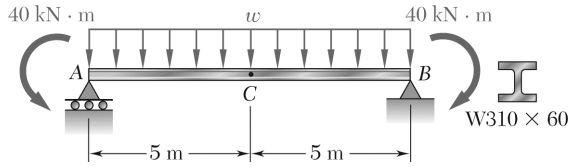
(*b*) Deflection at *C*. $y_C = -t_{B/C}$

$$= -(A_1\bar{x}_1 + A_3\bar{x}_3)$$

$$= -\left[\left(\frac{31.25}{EI}\right)\left(\frac{2}{3}(5)\right) - \left(\frac{60}{EI}\right)\left(\frac{1}{2}(5)\right)\right] = \frac{45.833}{EI} = \frac{45.833}{7975} = 5.7471 \times 10^{-3} \text{ft}$$

$$y_C = 0.0690 \text{ in.} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.121

For the beam and loading of Prob. 9.117, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.

SOLUTION

Symmetric beam and loading:

$$R_A = R_B = 5w \qquad (w \text{ in kN/m})$$

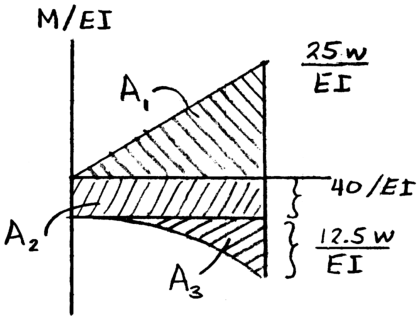
Bending moment in kN·m:

$$M = 5wx - 40 - \frac{1}{2}wx^2$$

At $x = 5$ m,

$$M = 25w - 40 - 12.5w$$

Draw M/EI diagram by parts.



$$A_1 = \frac{1}{2}\left(\frac{25w}{EI}\right)(5) = \frac{62.5w}{EI}$$

$$A_2 = -\frac{(40)(5)}{EI} = -\frac{200}{EI}$$

$$A_3 = -\frac{1}{3}\left(\frac{12.5w}{EI}\right)(5) = -\frac{20.833w}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(5) = 3.3333 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2}(5) = 2.5 \text{ m}$$

$$\bar{x}_3 = \frac{3}{4}(5) = 3.75 \text{ m}$$

Place reference tangent at C.

Deflection at C is zero.

$$t_{A/C} = y_A - y_C = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = 0$$

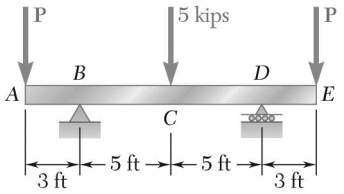
$$\left(\frac{62.5\,w}{EI}\right)(3.3333) - \left(\frac{200}{EI}\right)(2.5) - \left(\frac{20.833\,w}{EI}\right)(3.75) = 0$$

$$\frac{130.21\,w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$

$$w = 3.84 \text{ kN/m} \blacktriangleleft$$

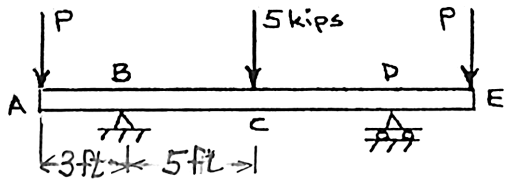
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.122

For the beam and loading of Prob 9.120, determine the magnitude of the forces **P** for which the deflection is zero at end A of the beam. Use $E = 29 \times 10^6$ psi.

SOLUTION



Symmetric beam and loading:

$$R_A = R_B = P + 2.5 \qquad (P \text{ in kips})$$

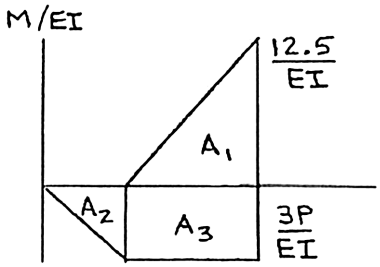
Bending moment:

Over AB: $M = -Px$

Over BC: $M = -Px + (P + 2.5)(x - 3)$

$= 2.5(x - 3) - P(3)$

At $x = 8$ ft, $M = 12.5 - P(3)$

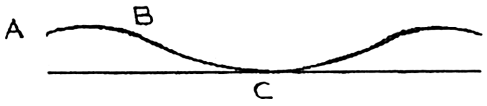


Draw $\dfrac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{12.5}{EI}\right)(5) = \frac{31.25}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{3P}{EI}\right)(3) = -\frac{4.5P}{EI}$$

$$A_3 = -\left(\frac{3P}{EI}\right)(5) = -\frac{15P}{EI}$$



Place reference tangent at C.

$$y_A = y_B = 0 \quad y_A - y_B = 0 \quad t_{A/C} - t_{B/C} = 0$$

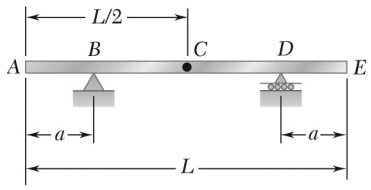
$$\left[A_1\left(3 + \frac{2}{3}(5)\right) + A_3\left(3 + \frac{1}{2}(5)\right) + A_2\left(\frac{2}{3}(3)\right)\right] - \left[A_1\left(\frac{2}{3}(5)\right) + A_3\left(\frac{1}{2}(5)\right)\right] = 0$$

$$A_1(3) + A_3(3) + A_2(2) = 0$$

$$\frac{93.75}{EI} - \frac{45P}{EI} - \frac{9P}{EI} = 0$$

$$P = 1.736 \text{ kips} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.123*

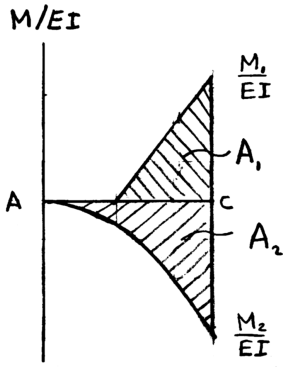
A uniform rod AE is to be supported at two points B and D. Determine the distance a for which the slope at ends A and E is zero.

SOLUTION

Let w = weight per unit length of rod.

Symmetric beam and loading:

$$R_B = R_D = \frac{1}{2}wL$$



Bending moment:

Over AB: $\quad M = -\frac{1}{2}wx^2$

Over BCD: $\quad M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$

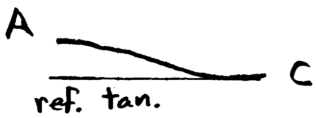
Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\frac{L}{2} = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at C. $\quad \theta_C = 0$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16}\frac{wL(L-2a)^2}{EI} + \frac{1}{48}\frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$.

$$1 - 3(1-2u)^2 = 0$$

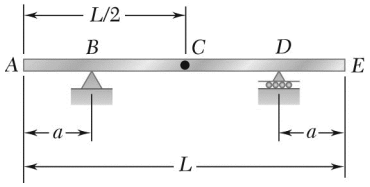
$$1 - 2u = \frac{\sqrt{3}}{3}$$

$$u = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right) = 0.21132$$

$$\frac{a}{L} = 0.211$$

$$a = 0.211L \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.124*

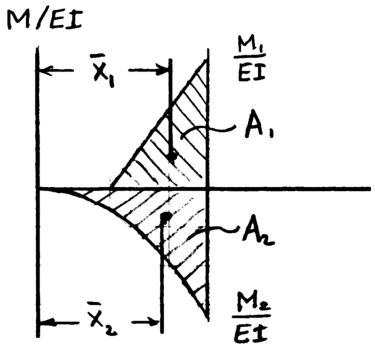
A uniform rod AE is to be supported at two points B and D. Determine the distance a from the ends of the rod to the points of support, if the downward deflections of points A, C, and E are to be equal.

SOLUTION

Let w = weight per unit length of rod.

Symmetric beam and loading:

$$R_B = R_D = \frac{1}{2}wL$$



Bending moment:

Over AB: $$M = -\frac{1}{2}wx^2$$

Over BCD: $$M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$$

Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3}\left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}\left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_A - y_c = t_{A/C} = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = 0$$

$$\frac{1}{16}\frac{wL(L-2a)^2}{EI}\frac{1}{3}(L+a) - \frac{1}{48}\frac{wL^3}{EI}\frac{3}{8}L = 0$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.124* *(Continued)*

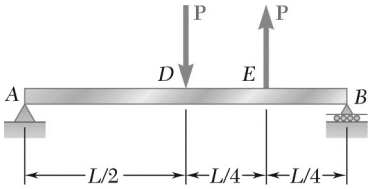
Let $u = \dfrac{a}{L}$. Divide by $\dfrac{wL^4}{48EI}$.

$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for u, $u = 0.22315$ $\dfrac{a}{L} = 0.223$ $a = 0.223\,\text{L}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



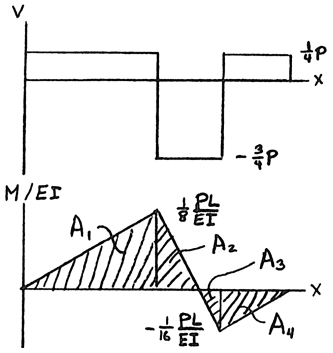
PROBLEM 9.125

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \qquad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \qquad R_A = \frac{1}{4}P \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \qquad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \qquad R_B = \frac{1}{4}P \downarrow$$



Draw V (shear) diagram and $\dfrac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{6}\right) = \frac{1}{96}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{12}\right) = -\frac{1}{384}\frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{4}\right) = -\frac{1}{128}\frac{PL^3}{EI}$$

Place reference tangent at *A*.

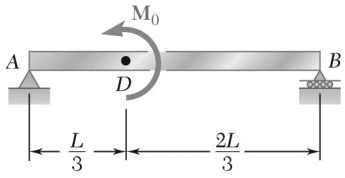
$$t_{B/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{2L}{3}\right) + \left(\frac{1}{96}\frac{PL^2}{EI}\right)\left(\frac{L}{2} - \frac{1}{3}\cdot\frac{L}{6}\right)$$
$$+\left(-\frac{1}{384}\frac{PL^2}{EI}\right)\left(\frac{L}{4} + \frac{1}{3}\cdot\frac{L}{12}\right) + \left(-\frac{1}{128}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{L}{4}\right)$$
$$= \frac{1}{48}\frac{PL^3}{EI} + \frac{1}{216}\frac{PL^3}{EI} - \frac{5}{6912}\frac{PL^3}{EI} - \frac{1}{768}\frac{PL^3}{EI} = \frac{3}{128}\frac{PL^3}{EI}$$

$$t_{D/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = \frac{1}{192}\frac{PL^3}{EI}$$

(*a*) Deflection at *D*. $\qquad y_D = t_{D/A} - \dfrac{x_D}{L}t_{B/A} = \dfrac{1}{192}\dfrac{PL^3}{EI} - \dfrac{1}{2}\left(\dfrac{3}{128}\dfrac{PL^3}{EI}\right) \qquad y_D = -\dfrac{5}{768}\dfrac{PL^3}{EI}$ ◀

(*b*) Slope at *A*. $\qquad \theta_A = -\dfrac{t_{B/A}}{L} \qquad \theta_A = -\dfrac{3}{128}\dfrac{PL^2}{EI}$ ◀

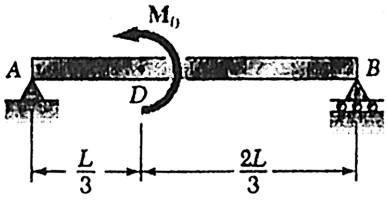
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

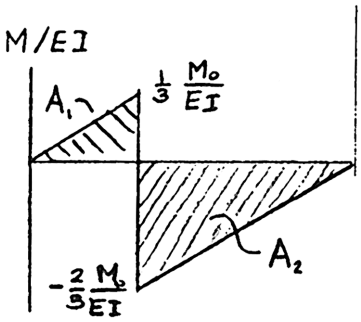


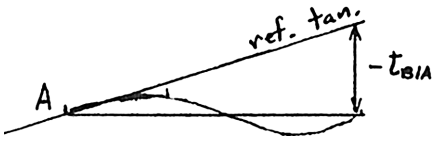
PROBLEM 9.126

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION







Reactions: $R_A = \dfrac{M_0}{L} \uparrow, \qquad R_B = \dfrac{M_0}{L} \downarrow$

Draw $\dfrac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(\frac{1}{3}\frac{M_0}{EI}\right)\frac{L}{3} = \frac{1}{18}\frac{M_0 L}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{2}{3}\frac{M_0}{EI}\right)\frac{2L}{3} = -\frac{2}{9}\frac{M_0 L}{EI}$$

Place reference tangent at *A*.

$$t_{B/A} = A_1\left(\frac{L}{9} + \frac{2L}{3}\right) + A_2\left(\frac{2}{3}\cdot\frac{2L}{3}\right)$$

$$= \frac{7}{162}\frac{M_0 L^2}{EI} - \frac{8}{81}\frac{M_0 L^2}{EI} = -\frac{1}{18}\frac{M_0 L^2}{EI}$$

$$t_{D/A} = A_1\frac{L}{9} = \frac{1}{162}\frac{M_0 L^2}{EI}$$

(*a*) Deflection at *D*.

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

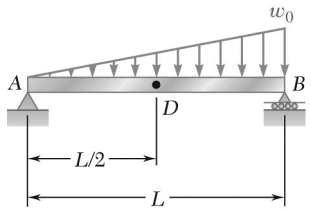
$$= \frac{1}{162}\frac{M_0 L^2}{EI} - \frac{1}{3}\left(-\frac{1}{18}\frac{M_0 L^2}{EI}\right) = \frac{2}{81}\frac{M_0 L^2}{EI}$$

$$y_D = \frac{2}{81}\frac{M_0 L^2}{EI} \uparrow \blacktriangleleft$$

(*b*) Slope at end *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{1}{18}\frac{M_0 L}{EI} \qquad \theta_A = \frac{1}{18}\frac{M_0 L}{EI} \blacktriangleleft$$

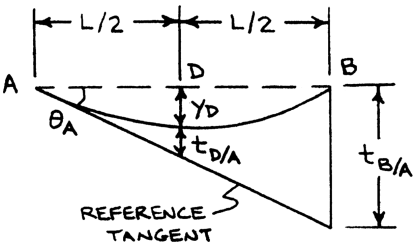
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.127

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

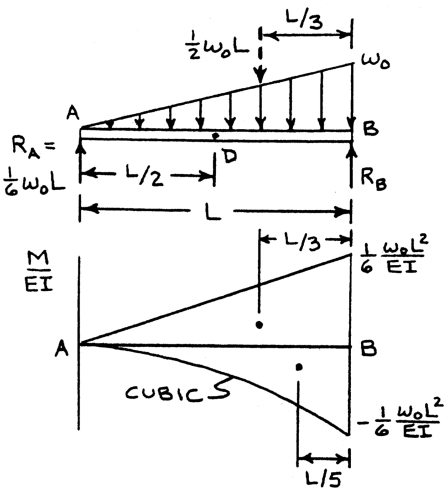


$$t_{B/A} = \frac{1}{2}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{3}\right) + \frac{1}{4}\left(-\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{5}\right)$$

$$= \frac{7w_0L^4}{360EI}$$

$$t_{D/A} = \frac{1}{2}\left(\frac{1}{12}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) + \frac{1}{4}\left(-\frac{1}{48}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{10}\right)$$

$$= \frac{37w_0L^4}{11520EI}$$



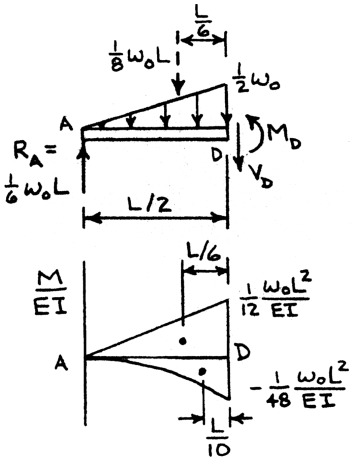
(*a*) Deflection at *D*.

$$y_D = \frac{1}{2}t_{B/A} - t_{D/A}$$

$$= \frac{1}{2}\left(\frac{7w_0L^4}{360EI}\right) - \frac{37w_0L^4}{11520EI}$$

$$= \frac{75w_0L^4}{11520EI}$$

$$y_D = \frac{5w_0L^4}{768EI} \downarrow \blacktriangleleft$$

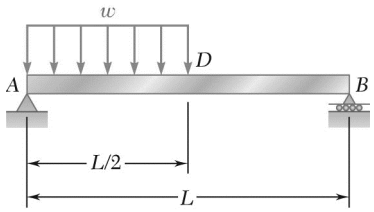


(*b*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7w_0L^3}{360EI}$$

$$\theta_A = \frac{7w_0L^3}{360EI} \curvearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



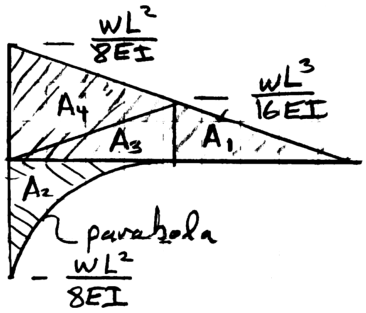
PROBLEM 9.128

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$R_B = \frac{1}{8}wL \uparrow$$



Draw *M*/*EI* diagram by parts.

$$\frac{M_1}{EI} = \frac{R_B L}{EI} = \frac{wL^2}{8EI}$$

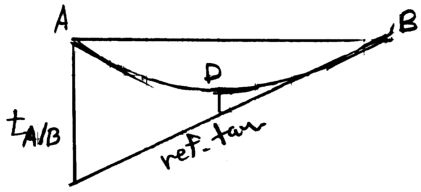
$$\frac{M_2}{EI} = -\frac{wL^3}{8EI}$$

$$A_1 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_2 = \left(\frac{1}{3}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

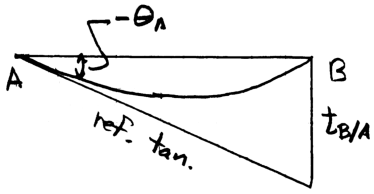
$$A_3 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_4 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$



(*a*) Deflection at *D*.

Place reference tangent at *B*.



$$y_D = t_{D/B} - \frac{L}{2}t_{A/B}$$

$$t_{D/A} = \left(\frac{1}{3}\cdot\frac{L}{2}\right)A_1 = \frac{wL^4}{384EI}$$

$$t_{B/A} = \frac{L}{3}(A_1 + A_3 + A_4) + \left(\frac{1}{4}\cdot\frac{L}{2}\right)A_4 = \frac{7wL^4}{384EI}$$

$$y_D = \frac{wL^4}{384EI} - \frac{1}{2}\cdot\frac{7wL^4}{384EI} = -\frac{5wL^4}{768EI}$$

$$y_D = \frac{5wL^4}{768EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.128 *(Continued)*

(*b*) Slope at *A*. Place reference tangent at *A*.

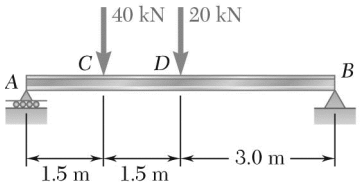
$$\theta_A = -\frac{1}{L} t_{B/A}$$

$$= -\left(\frac{1}{L}\right)\left\{\left(\frac{2L}{3}\right)(A_1 + A_3 + A_4) + \left(L - \frac{1}{4}\cdot\frac{L}{2}\right)A_2\right\}$$

$$= -\frac{3wL^3}{128EI}$$

$$\theta_A = \frac{3wL^3}{128EI} \curvearrowright \blacktriangleleft$$

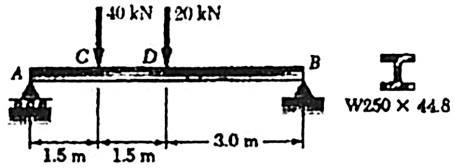
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.129

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION



$$E = 200 \times 10^9 \text{ Pa}$$

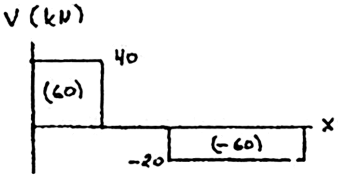
$$I = 70.8 \times 10^6 \text{ mm}^4 = 70.8 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(70.8 \times 10^{-6}) = 19.16 \times 10^6 \text{ N} \cdot \text{m}^2$$

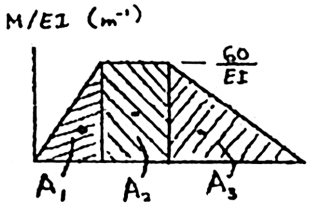
$$= 14{,}160 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -6R_A + (4.5)(40) + (3)(20) = 0$$

$$R_A = 40 \text{ kN}$$



Draw shear and $\dfrac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2}\left(\frac{60}{EI}\right)(1.5) = \frac{45}{EI}$$

$$A_2 = \left(\frac{60}{EI}\right)(1.5) = \frac{90}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{60}{EI}\right)(3) = \frac{90}{EI}$$

Place reference tangent at *A*.

$$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) + A_3(2.0)$$

$$= \frac{742.5}{EI} \text{ m}$$

$$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75)$$

$$= \frac{157.5}{EI} \text{ m}$$

(*a*) Slope at *A*.

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14{,}160}$$

$$= -8.74 \times 10^{-3}$$

$$\theta_A = 8.74 \times 10^{-3} \text{ rad} \measuredangle \blacktriangleleft$$

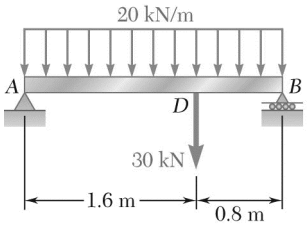
(*b*) Deflection at *D*.

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6}\right)\left(\frac{742.5}{EI}\right) = -\frac{213.75}{EI}$$

$$= -\frac{213.75}{14{,}160} = -15.10 \times 10^{-3} \text{ m}$$

$$y_D = 15.10 \text{ mm} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.130

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

For W150×24,

$$I = 13.4\times10^{6}\ \text{mm}^4 = 13.4\times10^{-6}\ \text{m}^4$$

$$EI = (200\times10^{9})(13.4\times10^{-6}) = 2.68\times10^{6}\ \text{N}\cdot\text{m}^2 = 2680\ \text{kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -2.4R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$$

$$R_A = 34\ \text{kN}\uparrow$$

Draw bending moment diagram by parts.

$$M_1 = (1.6)(34) = 54.4\ \text{kN}\cdot\text{m}$$

$$M_2 = (2.4)(34) = 81.6\ \text{kN}\cdot\text{m}$$

$$M_3 = -\frac{1}{2}(20)(1.6)^2 = -25.6\ \text{kN}\cdot\text{m}$$

$$M_4 = -\frac{1}{2}(20)(2.4)^2 = -57.6\ \text{kN}\cdot\text{m}$$

$$M_5 = -(0.8)(30) = -24\ \text{kN}\cdot\text{m}$$

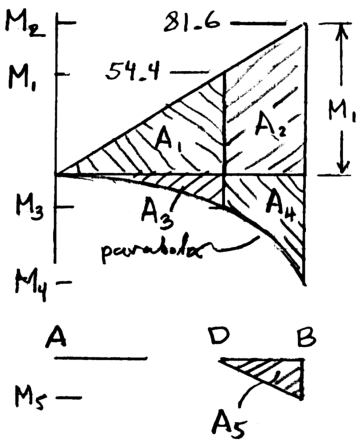
$$A_1 = \frac{1}{2}(1.6)(54.4) = 43.52\ \text{kN}\cdot\text{m}^2$$

$$A_1 + A_2 = \frac{1}{2}(2.4)(81.6) = 97.92\ \text{kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{3}(1.6)(-25.6) = -13.6533\ \text{kN}\cdot\text{m}^2$$

$$A_3 + A_4 = \frac{1}{3}(2.4)(-57.6) = -46.08\ \text{kN}\cdot\text{m}^2$$

$$A_5 = \frac{1}{2}(0.8)(-24) = -9.6\ \text{kN}\cdot\text{m}^2$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.130 *(Continued)*

(*a*) Slope at *A*. Place reference tangent at *A*.

$$\theta_A = -\frac{1}{L} t_{B/A}$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)\left(\frac{1}{3}\right)(2.4) + (A_3 + A_4)\left(\frac{1}{4}\right)(2.4) + A_5\left(\frac{1}{3}\right)(0.8)\right\}$$

$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3}\,\text{m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3}$$

$$\theta_A = 7.48 \times 10^{-3}\,\text{rad.} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at point *D*.

$$y_D = t_{D/A} + \theta_A x_D$$

$$t_{D/A} = \frac{1}{EI}\left\{A_1\left(\frac{1}{3}\right)(1.6) + A_2\left(\frac{1}{4}\right)(1.6)\right\}$$

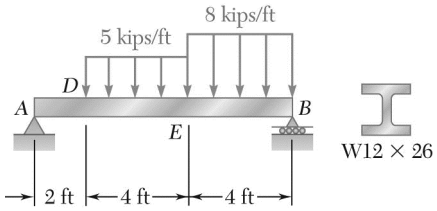
$$= \frac{17.7493}{2680} = 6.62289 \times 10^{-3}\,\text{m}$$

$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6)$$

$$= -5.3492 \times 10^{-3}\,\text{m}$$

$$y_D = 5.35\ \text{mm} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.131

For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *E*. Use $E = 29\times10^6$ psi.

SOLUTION

Units: Forces in kips; lengths in ft.

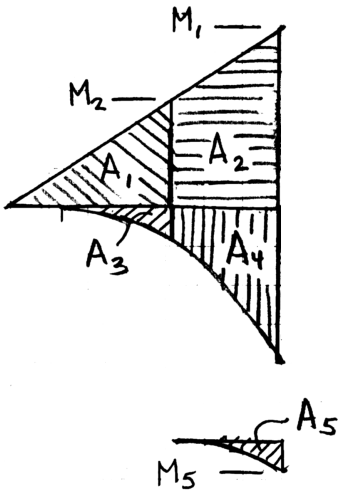
For W12×26, $I = 204 \text{ in}^4$

$$EI = (29\times10^6)(204) = 5.916\times10^9 \text{ lb}\cdot\text{in}^2$$
$$= 41083 \text{ kip}\cdot\text{ft}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -10R_A + (6)(4)(5) + (2)(4)(8) = 0 \qquad R_A = 18.4 \text{ kips} \uparrow$$

Consider loading as 5 kips/ft from *D* to *B* plus 3 kips/ft from *E* to *B*. Draw bending moment diagram by parts.

$$M_1 = 10R_A = 184 \text{ kip}\cdot\text{ft}$$
$$M_2 = 6R_A = 110.4 \text{ kip}\cdot\text{ft}$$
$$M_3 = -\frac{1}{2}(5)(8)^2 = -160 \text{ kip}\cdot\text{ft}$$
$$M_4 = -\frac{1}{2}(5)(4)^2 = -40 \text{ kip}\cdot\text{ft}$$
$$M_5 = -\frac{1}{2}(3)(4)^2 = -24 \text{ kip}\cdot\text{ft}$$
$$A_1 + A_2 = \frac{1}{2}(10)(184) = 920 \text{ kip}\cdot\text{ft}^2$$
$$A_1 = \frac{1}{2}(6)(110.4) = 331.2 \text{ kip}\cdot\text{ft}^2$$
$$A_3 + A_4 = \frac{1}{3}(8)(-160) = -426.667 \text{ kip}\cdot\text{ft}^2$$
$$A_3 = \frac{1}{3}(4)(-40) = -53.333 \text{ kip}\cdot\text{ft}^2$$
$$A_5 = \frac{1}{3}(4)(-24) = -32 \text{ kip}\cdot\text{ft}^2$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.131 *(Continued)*

(*a*) Slope at *A*. $y_B = y_A + \theta_A L + t_{B/A} \qquad y_A = y_B = 0$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)\left(\frac{1}{3}\right)(10) + (A_3 + A_4)\left(\frac{1}{4}\right)(8) + (A_5)\left(\frac{1}{4}\right)(4)\right\}$$

$$= \frac{2181.33}{41083} = 53.096\times10^{-3}\text{ ft}$$

$$\theta_A = -\frac{53.096\times10^{-3}}{10} = -5.3096\times10^{-3}$$

$\theta_A = 5.31\times10^{-3}$ rad ↻ ◀

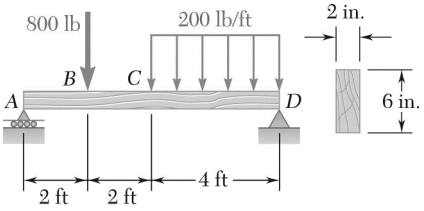
(*b*) Deflection at *E*. $y_E = x_E\theta_A + t_{E/A}$

$$t_{E/A} = \frac{1}{EI}\left\{(A_1)\left(\frac{1}{3}\right)(6) + (A_3)\left(\frac{1}{4}\right)(4)\right\} = \frac{609.067}{41083} = 14.8253\times10^{-3}\text{ ft}$$

$$y_E = (6)(-5.3096\times10^{-3}) + 14.8253\times10^{-3} = -17.0323\times10^{-3}\text{ ft}$$

$y_E = 0.204$ in. ↓ ◀

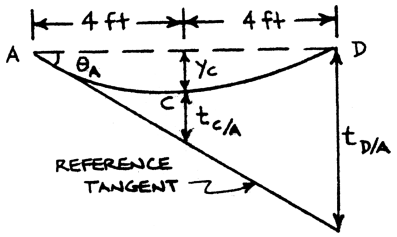
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.132

For the timber beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *C*. Use $E = 1.7\times10^6$ psi.

SOLUTION



$$I = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$EI = (1.7\times10^3 \text{ ksi})(36 \text{ in}^4) = 61.2\times10^3 \text{ kip}\cdot\text{in}^2$$

$$A_1 = \frac{1}{2}(6.4)(8) = 25.6 \text{ kip}\cdot\text{ft}^2$$

$$A_2 = \frac{1}{2}(-4.8)(6) = -14.4 \text{ kip}\cdot\text{ft}^2$$

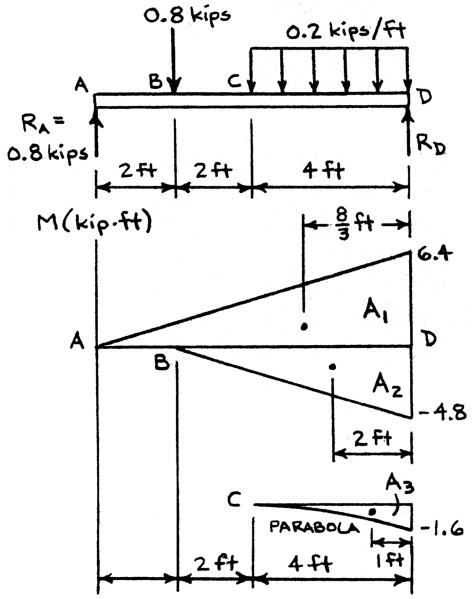
$$A_3 = \frac{1}{3}(-1.6)(4) = -2.1333 \text{ kip}\cdot\text{ft}^2$$

$$EIt_{D/A} = A_1\left(\frac{8}{3}\text{ft}\right) + A_2(2\,\text{ft}) + A_3(1\text{ ft})$$

$$= (25.6)\left(\frac{8}{3}\right) + (-14.4)(2) + (-2.1333)(1)$$

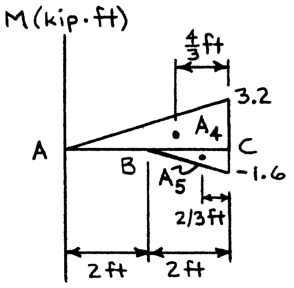
$$= 37.333 \text{ kip}\cdot\text{ft}^3 = 64512 \text{ kip}\cdot\text{in}^3$$

$$t_{D/A} = \frac{64512}{61.2\times10^3} = 1.05412 \text{ in.}$$



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.132 *(Continued)*



$$A_4 = \frac{1}{2}(3.2)(4) = 6.4 \text{ kip} \cdot \text{ft}^2$$

$$A_5 = \frac{1}{2}(-1.6)(2) = -1.6 \text{ kip} \cdot \text{ft}^2$$

$$EIt_{C/A} = A_4\left(\frac{4}{3}\text{ft}\right) + A_5\left(\frac{2}{3}\text{ft}\right) = (6.4)\left(\frac{4}{3}\right) + (-1.6)\left(\frac{2}{3}\right)$$

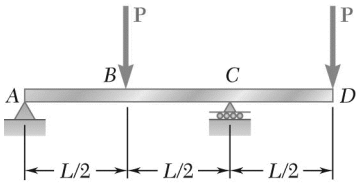
$$= 7.4667 \text{ kip} \cdot \text{ft}^3 = 12902.4 \text{ kip} \cdot \text{in}^3$$

$$t_{C/A} = \frac{12902.4}{61.2 \times 10^3} = 0.21082 \text{ in.}$$

(*a*) $\theta_A = -\dfrac{t_{D/A}}{L} = -\dfrac{1.05412}{96} = -0.0109804 \text{ rad}$ $\qquad \theta_A = 10.98 \times 10^{-3} \text{ rad} \curvearrowright$ ◀

(*b*) $y_C = t_{C/A} - \dfrac{1}{2}t_{D/A} = 0.21082 \text{ in.} - \dfrac{1}{2}(1.05412 \text{ in.})$ $\qquad y_C = 0.316 \text{ in.} \downarrow$ ◀

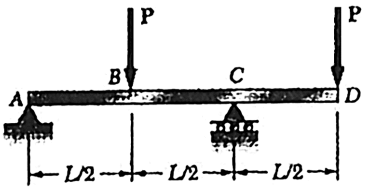
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.133

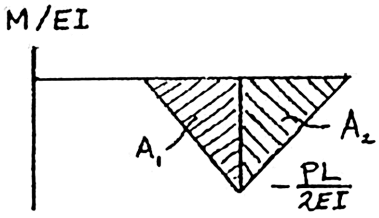
For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.

SOLUTION



$$+\circlearrowleft \sum M_C = 0: \quad -R_A L + P\frac{L}{2} - P\frac{L}{2} = 0 \qquad R_A = 0$$

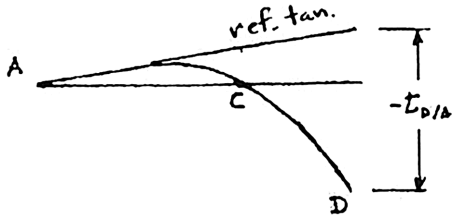
Draw $\dfrac{M}{EI}$ diagram.



$$A_1 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

Place reference tangent at *A*.



$$t_{C/A} = A_1 \cdot \left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{1}{48}\frac{PL^3}{EI}$$

(*a*) Slope at *A*.

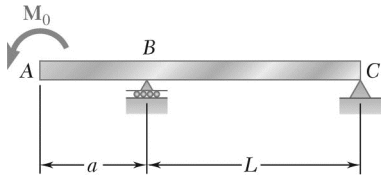
$$\theta_A = -\frac{t_{C/A}}{L} \qquad \theta_A = \frac{1}{48}\frac{PL^2}{EI} \blacktriangleleft$$

(*b*) Deflection at *D*.

$$t_{D/A} = A_1\left(\frac{L}{2}+\frac{L}{6}\right) + A_2\left(\frac{2}{3}\cdot\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^3}{EI}$$

$$y_D = t_{D/A} - \frac{x_D}{L}t_{C/A} = -\frac{1}{8}\frac{PL^3}{EI} - \left(\frac{3}{2}\right)\left(-\frac{1}{48}\frac{PL^3}{EI}\right) \qquad y_D = -\frac{3}{32}\frac{PL^3}{EI} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.134

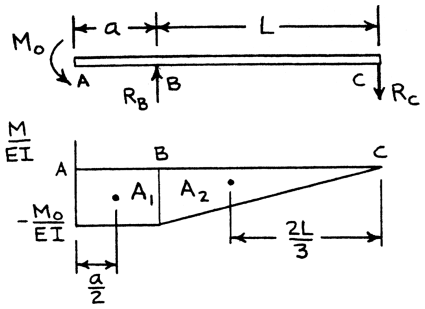
For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*.

SOLUTION

$$A_1 = -\frac{M_0 a}{EI}$$

$$A_2 = -\frac{M_0 L}{2EI}$$

$$t_{C/B} = A_2\left(\frac{2L}{3}\right) = \left(-\frac{M_0 L}{2EI}\right)\left(\frac{2L}{3}\right) = -\frac{M_0 L^2}{3EI}$$

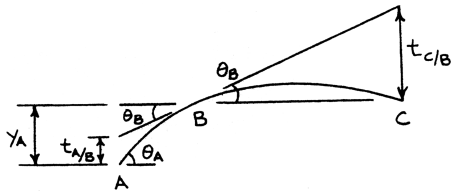


(*a*) Slope at *A*.

$$\theta_B = \frac{t_{C/B}}{L} = \frac{M_0 L}{3EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1$$

$$\frac{M_0 L}{3EI} = \theta_A - \frac{M_0 a}{EI}$$



$$\theta_A = \frac{M_0}{3EI}(L + 3a) \quad \measuredangle \blacktriangleleft$$

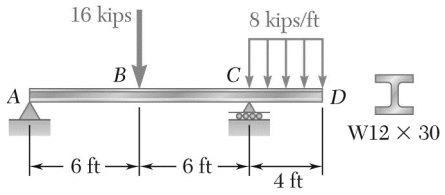
(*b*) Deflection at *A*.

$$t_{A/B} = A_1\left(\frac{a}{2}\right) = -\frac{M_0 a^2}{2EI}$$

$$y_A = \frac{a}{L}t_{C/B} + t_{A/B} = \frac{a}{L}\left(-\frac{M_0 L^2}{3EI}\right) - \frac{M_0 a^2}{2EI}$$

$$y_A = \frac{M_0 a}{6EI}(2L + 3a) \downarrow \blacktriangleleft$$

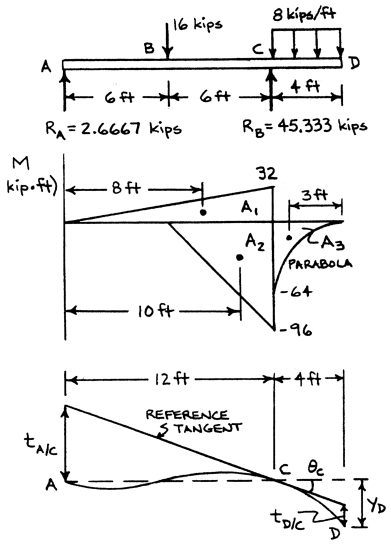
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.135

For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *D*. Use $E = 29\times10^6$ psi.

SOLUTION



Free Body *AD*: $+\circlearrowleft \Sigma M_C = 0$: $(16)(6) - (32)(2) - 12R_A = 0$

$$R_A = 2.6667 \text{ kips} \uparrow$$

$+\uparrow \Sigma F_y = 0$: $2.6667 - 16 + R_B - 32 = 0$

$$R_B = 45.333 \text{ kips} \uparrow$$

For W12×30, $I = 238 \text{ in}^4$

$$EI = (29\times10^3 \text{ ksi})(238 \text{ in}^4)$$
$$= 6.902\times10^6 \text{ kip}\cdot\text{in}^2 = 47931 \text{ kip}\cdot\text{ft}^2$$

(*a*) Slope at *C*.

$$A_1 = \frac{1}{2}(32)(12) = 192 \text{ kip}\cdot\text{ft}^2$$

$$A_2 = \frac{1}{2}(-96)(6) = -288 \text{ kip}\cdot\text{ft}^2$$

$$EIt_{A/C} = A_1(8 \text{ ft}) + A_2(10 \text{ ft})$$
$$= (192)(8) + (-288)(10) = -1344 \text{ kip}\cdot\text{ft}^3$$

$$t_{A/C} = -\frac{1344}{47931} = -28.040\times10^{-3} \text{ ft} = -0.33648 \text{ in.}$$

$$\theta_C = \frac{t_{A/C}}{L} = -\frac{28.040\times10^{-3} \text{ ft}}{12 \text{ ft}}$$

$$\theta_C = 2.34\times10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

(*b*) Deflection at *D*.

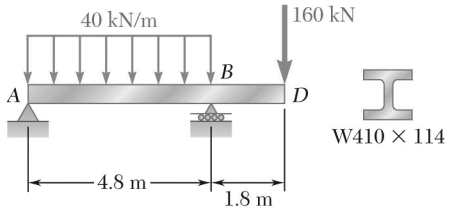
$$EIt_{D/C} = A_1(3 \text{ ft}) = \frac{1}{3}(-64)(4)(3) = -256 \text{ kip}\cdot\text{ft}^3$$

$$t_{D/C} = -\frac{256}{47931} = -5.3410\times10^{-3} \text{ ft} = -0.064092 \text{ in.}$$

$$y_D = t_{D/C} + \frac{4}{12}t_{A/C} = -0.064092 + \frac{1}{3}(-0.33648)$$
$$= -0.176252 \text{ in.}$$

$$y_D = 0.1763 \text{ in.} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.136

For the beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

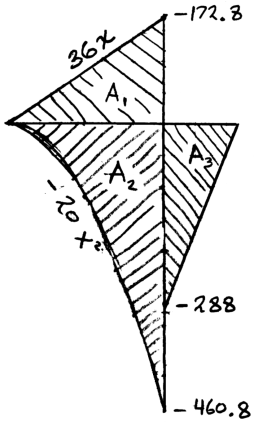
$$I = 462\times10^{6}\,\text{mm}^4 = 462\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^{9})(462\times10^{-6})$$

$$= 92.4\times10^{6}\,\text{N}\cdot\text{m}^2 = 92400\ \text{kN}\cdot\text{m}^2$$

$$+\circlearrowright \Sigma M_B = 0: \quad -4.8R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36\ \text{kN}$$



Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2}(4.8)(172.8) = 414.72\ \text{kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{3}(4.8)(-460.8) = -737.28\ \text{kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{2}(1.8)(-288) = -259.2\ \text{kN}\cdot\text{m}^2$$

Place reference tangent at *B*.

(*a*) Slope at *B*.

$$y_A = y_B - L\theta_B + t_{A/B}$$

$$\theta_B = \frac{t_{B/A}}{L} = \frac{1}{EIL}\left\{A_1\left(\frac{2}{3}\right)(4.8) + A_2\left(\frac{3}{4}\right)(4.8)\right\}$$

$$= \frac{-1327.104}{(92400)(4.8)} = -2.9922\times10^{-3}$$

$\theta_B = 2.99\times10^{-3}$ rad ◀

(*b*) Deflection at *D*.

$$y_D = y_B + a\theta_B + t_{D/B}$$

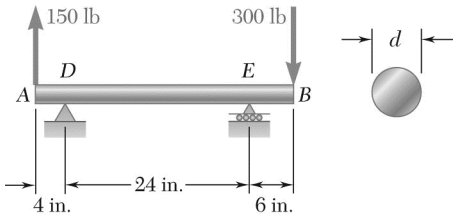
$$= 0 + (1.8)(-2.9922\times10^{-3}) - \frac{1}{EI}\left\{A_3\left(\frac{2}{3}\right)(1.8)\right\}$$

$$= -5.3860\times10^{-3} - \frac{311.04}{92400}$$

$$= -8.75\times10^{-3}\ \text{m}$$

$y_D = 8.75$ mm ↓ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.137

Knowing that the beam AB is made of a solid steel rod of diameter $d = 0.75$ in., determine for the loading shown (*a*) the slope at point *D*, (*b*) the deflection at point *A*. Use $E = 29\times10^6$ psi.

SOLUTION

Units: Forces in lb; lengths in inches.

$$c = \frac{1}{2} = \frac{1}{2}(0.75) = 0.375 \text{ in.}$$

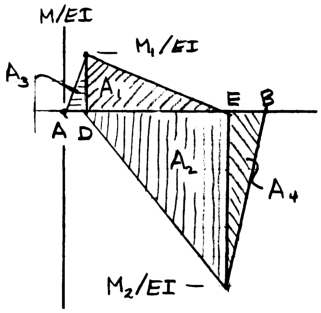
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.375)^4 = 0.0155316 \text{ in}^4$$

$$EI = (29\times10^6)(0.0155316) = 450.4\times10^3 \text{ lb}\cdot\text{in}^2$$

Draw $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

$$\frac{M_1}{EI} = \frac{(150)(4)}{450.4\times10^3} = 1.33215\times10^{-3} \text{ in}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(300)(6)}{450.4\times10^3} = -3.99645\times10^{-3} \text{ in}^{-1}$$



$$A_1 = \frac{1}{2}(24)(1.33215\times10^{-3}) = 15.9858\times10^{-3}$$

$$A_2 = \frac{1}{2}(24)(-3.99645\times10^{-3}) = -47.9574\times10^{-3}$$

$$A_3 = \frac{1}{2}(4)(1.33215\times10^{-3}) = 2.6643\times10^{-3}$$

Place reference tangent at *D*.

(*a*) Slope at point *D*.

$$y_E = y_D + L\theta_D + t_{E/D} \qquad \theta_D = -t_{E/D}/L$$

$$t_{E/A} = 16A_1 + 8A_2 = -127.8864\times10^{-3} \text{ in.}$$

$$\theta_D = \frac{-127.8864\times10^3}{24} = 5.3286\times10^{-3}$$

$\theta_D = 5.33\times10^{-3}$ rad ◀

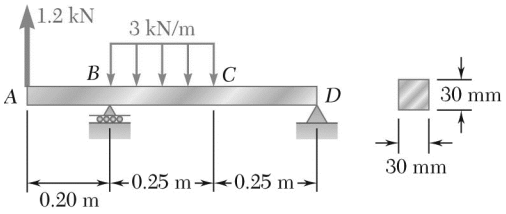
(*b*) Deflection at *A*.

$$y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$$

$$y_A = A_3\left(\frac{2}{3}\right)(4) - (4)(5.3286\times10^{-3}) = -14.21\times10^{-3} \text{ in.}$$

$y_A = 0.01421$ in.↓ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.138

Knowing that the beam AD is made of a solid steel bar, determine the (a) slope at point B, (b) the deflection at point A. Use $E = 200$ GPa.

SOLUTION

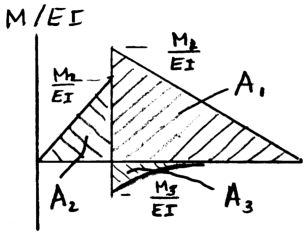
$$E = 200\times10^{9}\text{ Pa} \quad I = \frac{1}{12}(30)(30)^3 = 67.5\times10^{3}\text{ mm}^4 = 67.5\times10^{-9}\text{ m}^4$$

$$EI = (200\times10^{9})(67.5\times10^{-9}) = 13500\text{ N}\cdot\text{m}^2 = 13.5\text{ kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(0.2)(1.2) - (3)(0.25)(0.125) + 5R_D = 0 \quad R_D = 0.6675\text{ kN}$$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375\text{ kN}\cdot\text{m}$$
$$M_2 = (1.2)(0.2) = 0.240\text{ kN}\cdot\text{m}$$
$$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375\text{ kN}\cdot\text{m}$$



$$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$
$$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$
$$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at B.

$$t_{D/B} = A_1\left(\frac{2}{3}\cdot0.5\right) + A_3\left(\frac{3}{4}\cdot(0.25) + 0.25\right) = 0.024395/EI$$

(a) Slope at B. $\quad \theta_B = -\dfrac{t_{D/B}}{L} = -\dfrac{0.024395}{0.5EI} = -\dfrac{0.048789}{EI}$

$$= -3.6140\times10^{-3} \qquad \theta_B = 3.61\times10^{-3}\text{ rad} \curvearrowright \blacktriangleleft$$

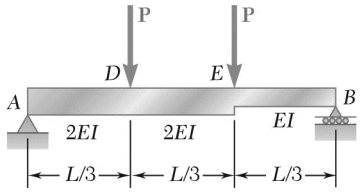
$$t_{A/B} = A_2\left(\frac{2}{3}(0.20)\right) = 0.0032/EI = 0.23704\times10^{-3}\text{ m}$$

(b) Deflection at A. $\quad y_A = t_{A/B} - L_{AB}\theta_B$

$$= 0.23704\times10^{-3} - (0.2)(-3.6140\times10^{-3}) = 0.960\times10^{-3}\text{ m}$$

$$y_A = 0.960\text{ mm} \uparrow \blacktriangleleft$$

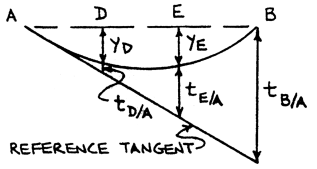
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.139

For the beam and loading shown, determine the deflection (*a*) at point *D*, (*b*) at point *E*.

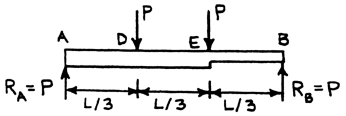
SOLUTION



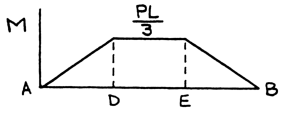
$$A_1 = \frac{1}{2}\left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{36EI}$$

$$A_2 = \left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$

$$A_3 = \frac{1}{2}\left(\frac{PL}{3EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$



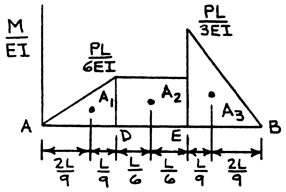
$$t_{D/A} = A_1\left(\frac{L}{9}\right) = \left(\frac{PL^2}{36EI}\right)\left(\frac{L}{9}\right) = \frac{PL^3}{324EI}$$



$$t_{E/A} = A_1\left(\frac{L}{9} + \frac{L}{3}\right) + A_2\left(\frac{L}{6}\right)$$

$$\left(\frac{PL^3}{36EI}\right)\left(\frac{4L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{6}\right)$$

$$= \frac{7PL^3}{324EI}$$



$$t_{B/A} = A_1\left(\frac{7L}{9}\right) + A_2\left(\frac{L}{2}\right) + A_3\left(\frac{2L}{9}\right)$$

$$= \left(\frac{PL^3}{36EI}\right)\left(\frac{7L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{2}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{2L}{9}\right)$$

$$= \frac{5PL^3}{81EI}$$

(*a*) Deflection at *D*.

$$y_D = \frac{1}{3}t_{B/A} - t_{D/A} = \frac{1}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{PL^3}{324EI} = \frac{17PL^3}{972EI}$$

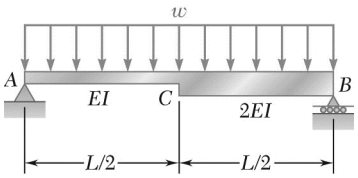
$$y_D = \frac{17PL^3}{972EI} \downarrow \blacktriangleleft$$

(*b*) Deflection at *E*.

$$y_E = \frac{2}{3}t_{B/A} - t_{E/A} = \frac{2}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{7PL^3}{324EI} = \frac{19PL^3}{972EI}$$

$$y_E = \frac{19PL^3}{972EI} \downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.140

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.

SOLUTION

Reactions: $$R_A = R_B = \frac{1}{2}wL$$

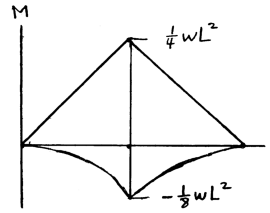
Draw bending moment and *M/EI* diagrams by parts as shown.

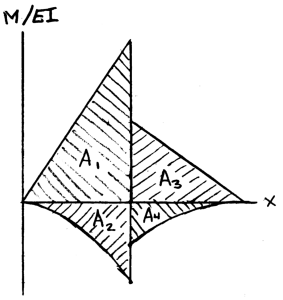
$$A_1 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{4EI} = \frac{wL^3}{16EI}$$

$$A_2 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{32EI}$$

$$A_4 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{16EI} = -\frac{wL^3}{96EI}$$





Place reference tangent at *A*.

(*a*) Slope at end *A*.

$$y_B = y_A + L\theta_A + t_{B/A}$$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{L}{2}+\frac{L}{6}\right)A_1 + \left(\frac{L}{2}+\frac{L}{8}\right)A_2 + \frac{L}{3}A_3 + \frac{3L}{8}A_4$$

$$= \frac{wL^4}{EI}\left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256}\right) = \frac{9wL^4}{256EI}$$

$$\theta_A = -\frac{9wL^4}{256EI}\cdot\frac{1}{L} = -\frac{9wL^3}{256EI}$$

$$\theta_A = \frac{9wL^3}{256EI} \curvearrowright$$

(*b*) Slope at end *B*.

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{9wL^3}{256EI} + A_1 + A_2 + A_3 + A_4$$

$$\theta_B = \frac{7wL^3}{256EI}$$

$$\theta_B = \frac{7wL^3}{256EI} \curvearrowleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.140 *(Continued)*

(*c*) Deflection at midpoint *C*.

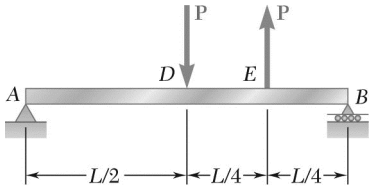
$$y_A = y_C + \frac{L}{2}\theta_A + t_{C/A}$$

$$t_{C/A} = \left(\frac{L}{6}\right)A_1 + \left(\frac{L}{8}\right)A_2 = \frac{wL^4}{128EI}$$

$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9wL^3}{256EI}\right) + \frac{wL^4}{128EI} = -\frac{5wL^4}{512EI}$$

$$y_C = \frac{5wL^4}{512EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.141

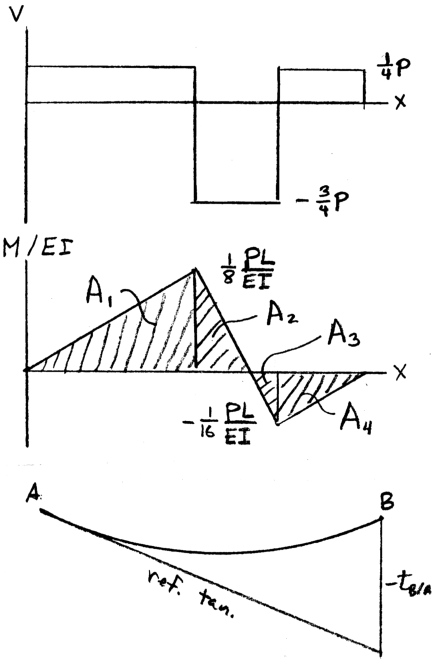
For the beam and loading of Prob. 9.125, determine the magnitude and location of the largest downward deflection.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \qquad R_A = \frac{1}{4}P \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \qquad R_B = \frac{1}{4}P \downarrow$$

Draw V (shear) diagram and M/EI diagram.



$$A_1 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{32}\frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\left(\frac{L}{6}\right) = \frac{1}{96}\frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{12}\right) = -\frac{1}{384}\frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2}\left(-\frac{1}{16}\frac{PL}{EI}\right)\left(\frac{L}{4}\right) = -\frac{1}{128}\frac{PL^3}{EI}$$

Place reference tangent at A.

$$t_{B/A} = \left(\frac{1}{32}\frac{PL^2}{EI}\right)\left(\frac{2L}{3}\right) + \left(\frac{1}{96}\frac{PL^2}{EI}\right)\left(\frac{L}{2} - \frac{1}{3}\frac{L}{6}\right)$$
$$+ \left(-\frac{1}{384}\frac{PL^2}{EI}\right)\left(\frac{L}{4} + \frac{1}{3}\frac{L}{12}\right) + \left(-\frac{1}{128}\frac{PL^2}{EI}\right)\left(\frac{2}{3}\cdot\frac{L}{4}\right)$$
$$= \left(\frac{1}{48} + \frac{1}{216} - \frac{5}{6912} - \frac{1}{768}\right)\frac{PL^3}{EI} = \frac{3}{128}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128}\frac{PL^2}{EI}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.141 *(Continued)*

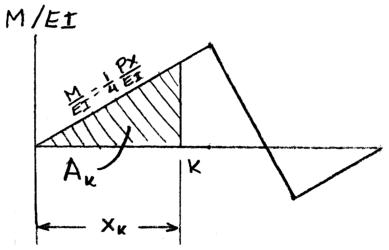
Let point K be the location of $|y_m|$.

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{3}{128}\frac{PL^2}{EI} + A_K$$

$$= -\frac{3}{128}\frac{PL^2}{EI} + \frac{1}{2}\left(\frac{1}{4}\frac{Px_K}{EI}\right)x_K$$

$$= \frac{P}{EI}\left(-\frac{3}{128}L^2 + \frac{1}{8}x_K^2\right) = 0$$



$$x_K = \sqrt{\frac{3}{16}}\,L = \frac{1}{4}\sqrt{3}\,L$$

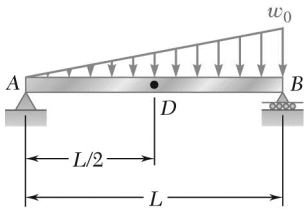
$$x_K = 0.433L \blacktriangleleft$$

$$t_{K/A} = A_K\left(\frac{1}{3}x_K\right) = \frac{1}{2}\left(\frac{1}{4}\frac{Px_K^2}{EI}\right)\frac{x_K}{3} = \frac{1}{24}\frac{Px_K^3}{EI} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A} = \frac{\sqrt{3}}{512}\frac{PL^3}{EI} - \left(\frac{1}{4}\sqrt{3}\right)\frac{3}{128}\frac{PL^3}{EI} = -\frac{\sqrt{3}}{256}\frac{PL^3}{EI}$$

$$y_K = 0.00677\frac{PL^3}{EI}\downarrow \blacktriangleleft$$

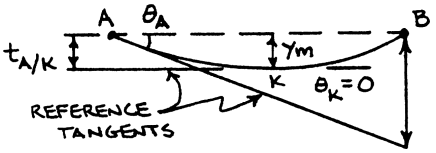
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.142

For the beam and loading of Prob. 9.127, determine the magnitude and location of the largest downward deflection.

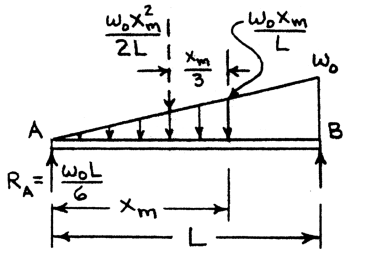
SOLUTION



From Prob. 9.127: $$\theta_A = -\frac{7w_0L^3}{360EI}$$

$$A_1 = \frac{1}{2}\left(\frac{w_0L}{6EI}x_m\right)(x_m) = \frac{w_0Lx_m^2}{12EI}$$

$$A_2 = \frac{1}{4}\left(-\frac{w_0x_m^3}{6EIL}\right)(x_m) = -\frac{w_0x_m^4}{24EIL}$$

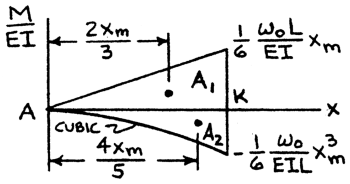


Maximum deflection occurs at *K*, where $\theta_K = 0$.

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1 + A_2$$

$$0 = -\frac{7w_0L^3}{360EI} + \frac{w_0Lx_m^2}{12EI} - \frac{w_0x_m^4}{24EIL}$$

Rearranging: $$0 = \frac{w_0L^2}{360EI}\left[-7 + 30\left(\frac{x_m}{L}\right)^2 - 15\left(\frac{x_m}{L}\right)^4\right]$$



Solving biquadratic: $$\left(\frac{x_m}{L}\right)^2 = 0.26970 \qquad x_m = 0.51933L$$

y_m is $0.519L$ from A. ◀

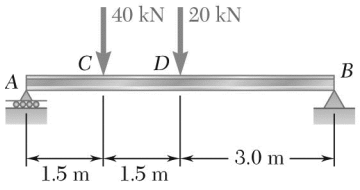
$$t_{A/K} = A_1\frac{2x_m}{3} + A_2\frac{4x_m}{5} = \left(\frac{w_0Lx_m^2}{12EI}\right)\frac{2x_m}{3} + \left(-\frac{w_0x_m^4}{24EIL}\right)\frac{4x_m}{5}$$

$$= \frac{w_0L^4}{90EI}\left[5\left(\frac{x_m}{L}\right)^3 - 3\left(\frac{x_m}{L}\right)^5\right] = \frac{w_0L^4}{90EI}\left[5(0.51933)^3 - 3(0.51933)^5\right]$$

$$= 0.0065222\frac{w_0L^4}{EI} \qquad y_m = |t_{A/K}|$$

$$y_m = 6.52\times10^{-3}\frac{w_0L^4}{EI}$$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



W250 × 44.8

PROBLEM 9.143

For the beam and loading of Prob. 9.129, determine the magnitude and location of the largest downward deflection.

SOLUTION

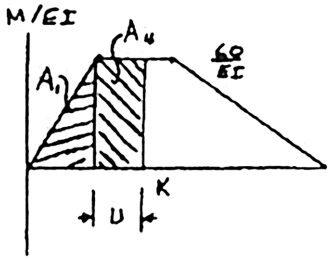
Referring to the solution to Prob. 9.129,

$$EI = 14{,}160 \text{ kN} \cdot \text{m}^2$$

$$R_A = 40 \text{ kN}, \quad A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI} \text{ m}$$

$$\theta_A = -\frac{123.75}{EI}$$



Let K be the location of the maximum deflection. Assume that K lies between C and D.

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{123.75}{EI} + A_1 + A_4$$

$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60u}{EI} = 0$$

$$u = \frac{123.75 - 45}{60} = 1.3125 \text{ m}$$

$$x_K = 1.5 + u = 2.8125 \text{ m}$$

$$t_{K/A} = A_1(u + 0.5) + A_4\left(\frac{1}{2}u\right)$$

$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)\left(\frac{1}{2}\right)(1.3125)}{EI} = \frac{133.242}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A}$$

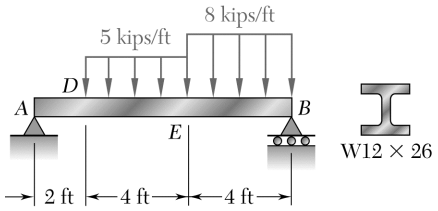
$$= \frac{133.242}{EI} - \frac{2.8125}{6}\left(\frac{742.5}{EI}\right) = -\frac{214.80}{EI} = -\frac{214.80}{14{,}160}$$

$$= -15.17 \times 10^{-3} \text{ m}$$

$$y_K = 15.17 \text{ mm} \downarrow \blacktriangleleft$$

$$x_K = 2.81 \text{ m} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

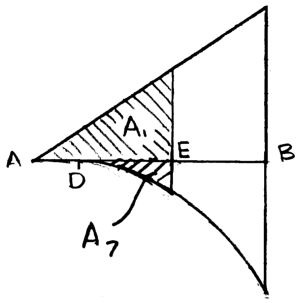


PROBLEM 9.144

For the beam and loading of Prob. 9.131, determine the magnitude and location of the largest downward deflection.

SOLUTION

From the solution to Prob. 9.130,

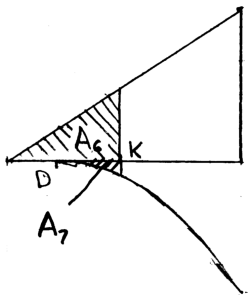


$$EI = 41083 \text{ kip} \cdot \text{ft}^2$$
$$R_A = 18.4 \text{ kips}$$
$$A_1 = 331.2 \text{ kip} \cdot \text{ft}^2$$
$$A_3 = -53.333 \text{ kip} \cdot \text{ft}^3$$
$$\theta_A = -5.3096 \times 10^{-3}$$

Slope at *E*. $\theta_E = \theta_A + \theta_{E/A}$

$$\theta_{E/A} = \frac{1}{EI}\{A_1 + A_3\} = \frac{278.767}{41083} = 6.7855 \times 10^{-3}$$
$$\theta_E = 1.4759 \times 10^{-3}$$

Since $\theta_E > 0,$ the point *K* of zero slope lies to the left of point *E*. Let x_K be the coordinate of point *K*.



$$A_6 = \frac{1}{2} R_A x_K^2 = 9.2 x_K^2$$
$$A_7 = -\frac{1}{6}(5)(x_K - 2)^3$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + \frac{1}{EI}\{A_6 + A_7\} = 0$$
$$A_6 + A_7 + EI\theta_A = 0$$

$$f(x_K) = 9.2x_K^2 - \frac{5}{6}(x_K - 2)^3 - 218.134 = 0$$
$$\frac{df}{dx_K} = 18.4\, x_K - 2.5(x_K - 2)^2$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.144 *(Continued)*

Solve for x_K by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx_K}$$

x_K	5	5.1473	5.1525	$x_K = 5.1525$ ft
f	−10.634	−0.362	0.001	
df/dx_K	72.2	70.131		

$$A_6 = 244.244 \text{ kip} \cdot \text{ft}^2, \quad A_7 = -26.108 \text{ kip} \cdot \text{ft}^2$$

Maximum deflection. $y_A = y_K + t_{A/K} = 0 \qquad y_K = -t_{A/K}$

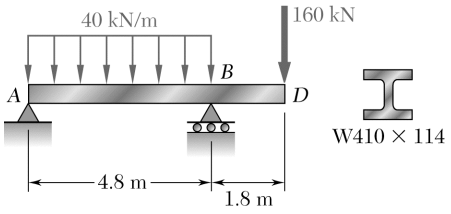
$$\bar{x}_6 = \frac{2}{3}x_K \quad \bar{x}_7 = 2 + \frac{3}{4}(x_K - 2) = \frac{3x_K + 2}{4}$$

$$y_7 = -\frac{1}{EI}\{A_6\bar{x}_6 + A_7\bar{x}_7\} = -\frac{725.033}{41083} = -17.648 \times 10^{-3} \text{ ft}$$

$y_K = 0.212$ in. ↓ ◀

$x_K = 5.15$ ft ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.145

For the beam and loading of Prob. 9.136, determine the largest upward deflection in span *AB*.

SOLUTION

Units: Forces in kN; lengths in meters.

$$I = 462\times10^{6}\,\text{mm}^4 = 462\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^{9})(462\times10^{-6})$$

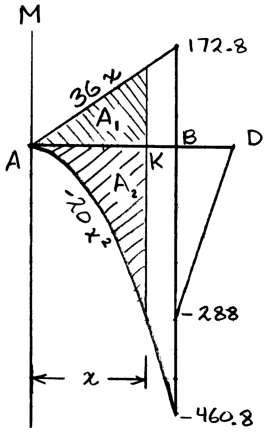
$$= 92.4\times10^{6}\,\text{N}\cdot\text{m}^2 = 92400\ \text{kN}\cdot\text{m}$$

$$+\circlearrowleft M_B = 0: \quad -4.8R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36\ \text{kN}$$

$$A_1 = \frac{1}{2}x(36x) = 18x^2$$

$$A_2 = \frac{1}{3}x(-20x^2) = -\frac{20}{3}x^3$$



Place reference tangent at *A*.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$(A_1)_B = (18)(4.8)^2 = 414.72\ \text{kN}\cdot\text{m}^2$$

$$(A_2)_B = \left(\frac{20}{3}\right)(4.8)^3 = -737.28\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{1}{EIL}\left\{(A_1)_B\left(\frac{1}{3}\right)(4.8) + (A_2)_B\left(\frac{1}{4}\right)(4.8)\right\}$$

$$= -\frac{-221.184}{(92400)(4.8)} = 0.49870\times10^{-3}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.145 *(Continued)*

Locate Point K of maximum deflection.

$$\theta_K = \theta_A + \theta_{K/A} = 0$$

$$EI\theta_A + A_1 + A_2 = 0$$

$$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \qquad \frac{df}{dx} = 36x_K - 20x_K^2$$

Solve by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx}$$

x_K	3	3.39	3.327	3.3251	3.32514 ←
df/dx	−72	−107.8	−101.6	−101.42	
f	28.08	−6.78	−0.188	0.005	

Place reference tangent at K.

$$y_A = y_K + t_{A/K}$$

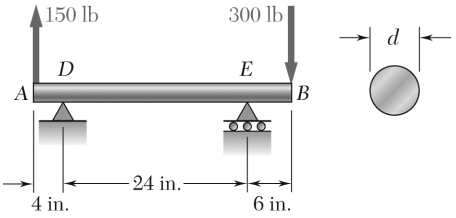
$$y_A - y_K = -t_{A/K}$$

$$= -\frac{1}{EI}\left\{(A_1)\left(\frac{2}{3}x_K\right) + A_2\left(\frac{3}{4}x_K\right)\right\} = -\frac{1}{EI}\left\{12x_K^3 + 5x_K^4\right\}$$

$$= -\frac{170.064}{92400} = -1.841\times10^{-3}\ \text{m}$$

$$y_K = 1.841\ \text{mm} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.146

For the beam and loading of Prob. 9.137, determine the largest upward deflection in span *DE*.

SOLUTION

Units: Forces in lbs; lengths in inches.

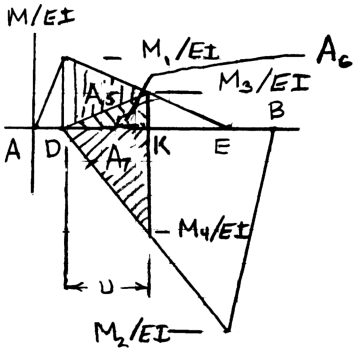
From the solution to Prob. 9.137,

$$EI = 450.4\times10^{3}\ \text{lb}\cdot\text{in}$$

$$\frac{M_1}{EI} = 1.33215\times10^{-3}\ \text{in}^{-1}$$

$$\frac{M_2}{EI} = -3.99645\times10^{-3}\ \text{in}^{-1}$$

$$\theta_D = 5.3286\times10^{-3}$$



Location of maximum deflection.

$$\frac{M_3}{EI} = \frac{M_1}{EI}\left(1-\frac{u}{24}\right)$$

$$\frac{M_4}{EI} = \frac{M_2}{EI}\frac{u}{24}$$

$$A_5 = \frac{1}{2}\frac{M_1}{EI}\cdot u = 0.666075\times10^{-3}u$$

$$A_6 = \frac{1}{2}\frac{M_3}{EI}\left(\frac{u}{24}\right) = 0.666075\times10^{-3}\left(1-\frac{u}{24}\right)u$$

$$A_7 = \frac{1}{2}\frac{M_4}{EI}\frac{u}{24} = -1.998225\times10^{-3}\left(\frac{u}{24}\right)u$$

$$\theta_K = \theta_D + A_5 + A_6 + A_7 = 0$$

Multiply by 10^3.

$$5.3286 + 0.666075u + 0.666075\left(1-\frac{u}{24}\right)u - (1.998225)\frac{u}{24}u = 0$$

$$5.3286 + 1.33215u - 0.1110125u^2 = 0$$

$$u = 15.16515\ \text{in.}$$

$$A_5 = 10.10113\times10^{-3},\quad A_6 = 3.71842\times10^{-3},\quad A_7 = -19.14814\times10^{-3}$$

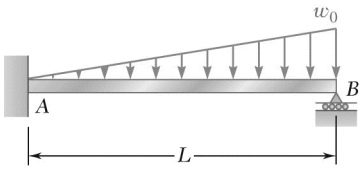
Maximum deflection in portion *DE*. $y_D = y_K + t_{D/K} = 0$

$$y_K = -t_{D/K} = -\left\{A_5\left(\frac{u}{3}\right) + A_6\left(\frac{2u}{3}\right) + A_7\left(\frac{2u}{3}\right)\right\}$$

$$= -\{-0.1049\}$$

$$y_K = 0.1049\ \text{in.} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.147

For the beam and loading shown, determine the reaction at the roller support.

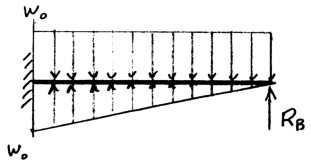
SOLUTION

Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.

Draw M/EI diagram for load w_0 and R_B.

Use parts as shown.



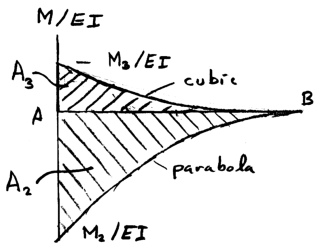
$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2}w_0 L^2$$

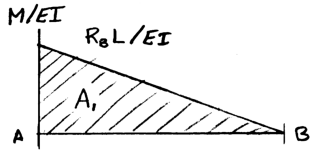
$$A_2 = \frac{1}{3}\left(-\frac{1}{2}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{6}\frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6}\frac{w_0}{L}L^3 = \frac{1}{6}w_0 L^2$$

$$A_3 = \frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{24}\frac{w_0 L^3}{EI}$$



Place reference tangent at A.



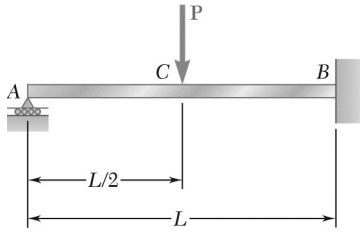
$$t_{B/A} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{3}{4}L\right) + A_3\left(\frac{4}{5}L\right)$$

$$= \frac{1}{3}\frac{R_B L^3}{EI} - \frac{1}{8}\frac{w_0 L^4}{EI} + \frac{1}{30}\frac{w_0 L^4}{EI} = 0$$

$$R_B = \frac{11}{40}w_0 L \uparrow$$

$$R_B = 0.275 w_0 L \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.148

For the beam and loading shown, determine the reaction at the roller support.

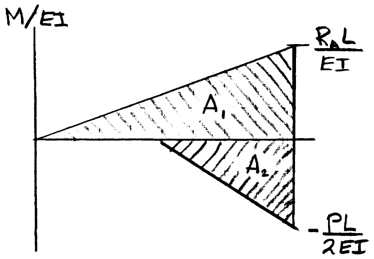
SOLUTION

Remove support A and treat R_A as redundant.

Draw the M/EI diagram by parts.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2}\frac{L}{2}\frac{PL}{2} = -\frac{PL^2}{8EI}$$



Place reference tangent at B.

$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

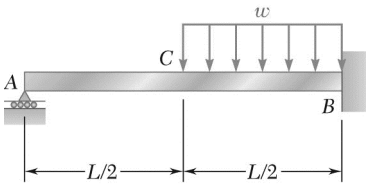
$$t_{A/B} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{3}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0$$

$$R_A = \frac{5}{16} P \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.149

For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

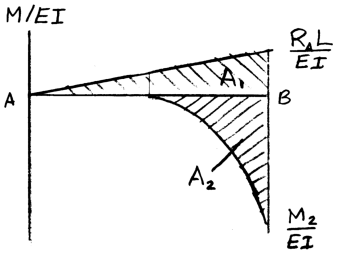
Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w.

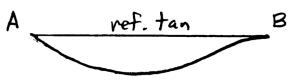
$$M_2 = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 = -\frac{1}{8}wL^2$$

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right)L = \frac{1}{2}\frac{R_A L^2}{EI}$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at B.



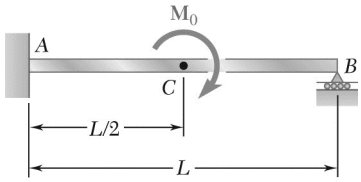
$$t_{A/B} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$

$$R_A = \frac{7}{128}wL$$

$$R_A = \frac{7}{128}wL \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.150

For the beam and loading shown, determine the reaction at the roller support.

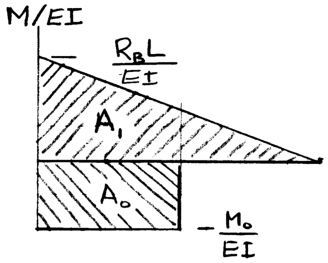
SOLUTION

Remove support B and treat R_B as redundant.

Draw M/EI diagram.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$



Place reference tangent at A.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

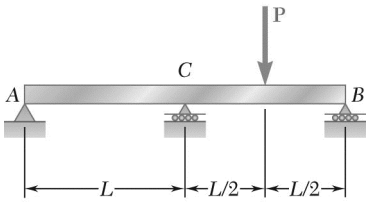
$$t_{B/A} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{4}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0$$

$$R_A = \frac{9}{8}\frac{M_0}{L} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.151

For the beam and loading shown, determine the reaction at each support.

SOLUTION

Remove support C and add reaction R_C.

Draw M/EI due to each of the loads P and R_C.

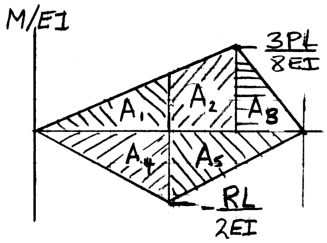
$$A_1 = \frac{1}{2}\cdot L\cdot\frac{2}{3}\cdot\frac{3PL}{8EI} = \frac{PL^2}{8EI}$$

$$A_1 + A_2 = \frac{1}{2}\cdot\frac{3L}{2}\cdot\frac{3PL}{8EI} = \frac{9PL^2}{32EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{3PL}{8EI} = \frac{3PL^2}{32EI}$$

$$A_4 = \frac{1}{2}\cdot L\left(-\frac{R_C L}{2EI}\right) = -\frac{R_C L^2}{4EI}$$

$$A_4 + A_5 = \frac{1}{2}(2L)\left(-\frac{R_C L}{2EI}\right) = -\frac{R_C L^2}{2EI}$$



Place reference tangent at A. $y_A = 0$

$$y_C = L\theta_A + t_{C/A} = 0 \qquad \theta_A = -\frac{t_{C/A}}{L}$$

$$y_B = 2L\theta_A + t_{B/A} = 0 \qquad -2t_{C/A} + t_{B/A} = 0$$

$$-2\left[A_1\frac{L}{3} + A_4\cdot\frac{L}{3}\right] + \left[(A_1 + A_2)\left(\frac{L}{2} + \frac{2}{3}\cdot\frac{3L}{2}\right) + A_3\cdot\frac{2}{3}\cdot\frac{L}{2} + (A_4 + A_5)\cdot L\right] = 0$$

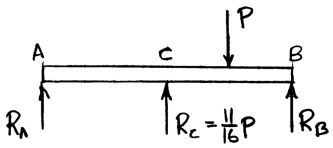
$$-2\left[\frac{PL^3}{24EI} - \frac{R_C L^3}{12EI}\right] + \left[\frac{9PL^3}{32EI} + \frac{PL^3}{32EI} - \frac{R_C L^3}{2EI}\right] = 0$$

$$-\frac{R_C L^3}{3EI} + \frac{11PL^3}{48EI} = 0$$

$$R_C = \frac{11}{16}P\uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.151 *(Continued)*



$$+\circlearrowleft \Sigma M_B = 0: \quad -2LR_A - LR_C + \frac{L}{2}P = 0$$

$$R_A = \frac{P}{4} - \frac{1}{2}R_C = -\frac{3}{32}P$$

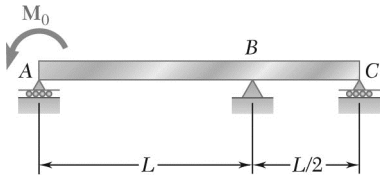
$$R_A = \frac{3}{32}P \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 2LR_B + LR_C - \frac{3L}{2}P = 0$$

$$R_B = \frac{3P}{4} - \frac{1}{2}R_C = \frac{13}{32}P$$

$$R_B = \frac{13}{32}P \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



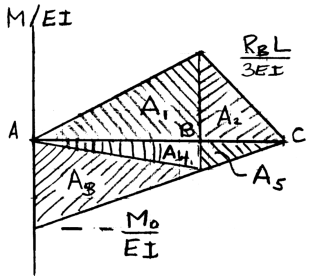
PROBLEM 9.152

For the beam and loading shown, determine the reaction at each support.

SOLUTION

Choose $R_B \downarrow$ as the redundant reaction.

Draw M/EI diagram for the loads R_B and M_0.



$$A_1 = \frac{1}{2}(L)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{6EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{12EI}$$

$$A_3 = \frac{1}{2}(L)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{2EI}$$

$$A_4 = \frac{1}{2}(L)\left(\frac{1}{3}\right)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{6EI}$$

$$A_3 + A_4 + A_5 = \frac{1}{2}\left(\frac{3L}{2}\right)\left(-\frac{M_0}{EI}\right) = -\frac{3M_0 L}{4EI}$$

$$y_B = y_A + L\theta_A + t_{B/A} \qquad \theta_A = -t_{B/A}/L$$

$$y_C = y_A + \frac{3L}{2}\theta_A + t_{C/A} = 0 \qquad -\frac{3}{2}t_{B/A} + t_{C/A} = 0$$

$$t_{B/A} = (A_1)\left(\frac{L}{3}\right) + A_3\left(\frac{2L}{3}\right) + A_4\left(\frac{L}{3}\right) = \frac{R_B L^3}{18EI} - \frac{7M_0 L^2}{18EI}$$

$$t_{C/A} = (A_1)\left(\frac{L}{2} + \frac{L}{3}\right) + A_2\left(\frac{L}{3}\right) + (A_3 + A_4 + A_5)(L) = \frac{R_B L^3}{6EI} - \frac{3M_0 L^2}{4EI}$$

$$-\frac{3}{2}t_{B/A} + t_{C/A} = \frac{R_B L^3}{12EI} - \frac{M_0 L^2}{6EI} = 0 \qquad R_B = \frac{2M_0}{L} \downarrow \blacktriangleleft$$

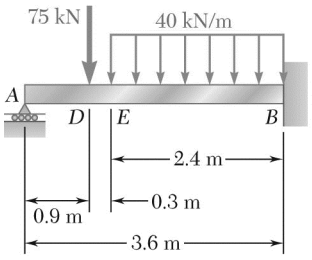
$$+\circlearrowright \Sigma M_C = 0: \quad M_0 + \frac{L}{2}R_B - \frac{3L}{2}R_A = 0$$

$$R_A = \frac{2}{3L}[M_0 + M_0] \qquad R_A = \frac{4M_0}{3L} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B + R_C = 0 \quad \frac{4}{3}\frac{M_0}{L} - \frac{2M_0}{L} + R_C = 0$$

$$R_C = \frac{2M_0}{3L} \uparrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.153

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

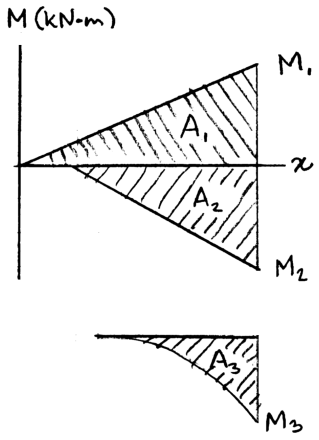
Units: Forces in kN; lengths in meters.

Let R_A be the redundant reaction.

Remove support at A and add reaction $R_A \uparrow$.

Draw bending moment diagram by parts.

$$M_1 = 3.6\ R_A \text{ kN} \cdot \text{m}$$
$$M_2 = -(75)(0.3 + 2.4) = -202.5 \text{ kN} \cdot \text{m}$$
$$M_3 = -\frac{1}{2}(40)(2.4)^2 = -115.2 \text{ kN} \cdot \text{m}$$



$$A_1 = \frac{1}{2}(3.6)(3.6R_A) = 6.48 \text{ kN} \cdot \text{m}^2$$
$$A_2 = \frac{1}{2}(2.7)(-202.5) = -273.375 \text{ kN} \cdot \text{m}^2$$
$$A_3 = \frac{1}{3}(2.4)(-115.2) = -92.16 \text{ kN} \cdot \text{m}^2$$

Place reference tangent at B, where

$$\theta_B = 0 \quad \text{and} \quad y_B = 0.$$

Then

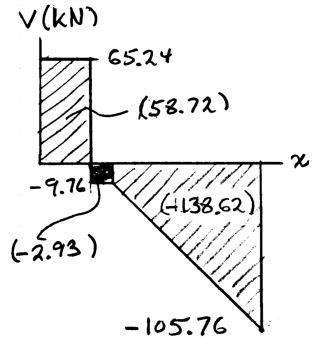
$$y_A = t_{A/B} = 0$$
$$t_{A/B} = \frac{1}{EI}\left\{\left(\frac{2}{3}\cdot 3.6\right)A_1 + \left(0.9 + \frac{2}{3}\cdot 2.7\right)A_2 + \left(0.9 + 0.3 + \frac{3}{4}\cdot 2.4\right)A_3\right\}$$
$$= \frac{1}{EI}\{15.552\ R_A - 1014.5925\} = 0$$

$$R_A = 65.24 \text{ kN} \uparrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.153 *(Continued)*

Draw shear diagram.



A to D: $V = R_A = 65.24$ kN

D to E: $V = 65.24 - 75 = -9.76$ kN

E to B: $V = -9.76 - 40(x - 1.2)$ kN

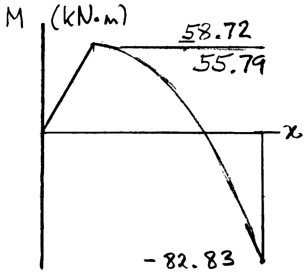
At B, $V_B = -105.76$ kN

Bending moment diagram. $M_A = 0$

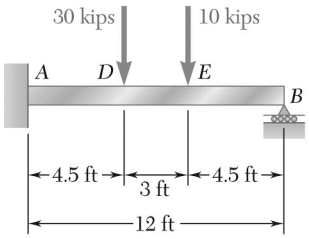
$M_D = M_A + 58.72 = 58.72$ kN · m

$M_E = 58.72 - 2.93 = 55.79$ kN · m

$M_B = 55.79 - 138.62 = -82.83$ kN · m



PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

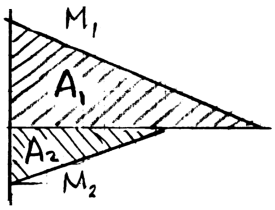


PROBLEM 9.154

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

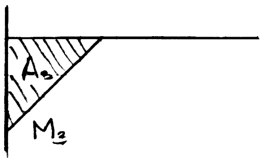
Units: Forces in kips; lengths in feet.



Let R_B be the redundant reaction.

Remove support B and add load R_B.

Draw bending moment diagram by parts.



$$M_1 = 12R_B \text{ kip} \cdot \text{ft}$$

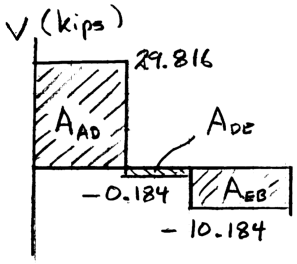
$$M_2 = -(4.5+3)(10) = -75 \text{ kip} \cdot \text{ft}$$

$$M_3 = -(4.5)(30) = -135 \text{ kip} \cdot \text{ft}$$

$$A_1 = \frac{1}{2}(12)(12R_B) = 72R_B \text{ kip} \cdot \text{ft}^2$$

$$A_2 = \frac{1}{2}(7.5)(-75) = -281.25 \text{ kip} \cdot \text{ft}^2$$

$$A_3 = \frac{1}{2}(4.5)(-135) = -303.75 \text{ kip} \cdot \text{ft}^2$$

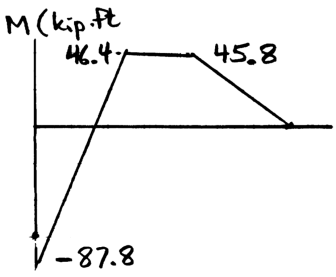


$$y_B = y_A + 12\theta_A + t_{B/A} = 0$$

$$t_{B/A} = 0$$

$$t_{B/A} = \frac{1}{EI}\{(72R_B)(8) + (-281.25)(4.5+5) + (-303.75)(7.5+3)\} = 0$$

$$576R_B - 5861.25 = 0 \qquad R_B = 10.18 \text{ kips} \uparrow \blacktriangleleft$$



Draw shear diagram working from right to left.

B to E: $V = -R_B = -10.176$ kips

E to D: $V = -10.184 + 10 = -0.176$ kips

D to A: $V = -0.176 + 30 = 29.824$ kips

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.154 *(Continued)*

Areas of shear diagram.

$$A_{AD} = (4.5)(29.824) = 134.21 \text{ kip} \cdot \text{ft}$$

$$A_{DE} = (3)(-0.176) = -0.53 \text{ kip} \cdot \text{ft}$$

$$A_{EB} = (4.5)(-10.176) = 45.79 \text{ kip} \cdot \text{ft}$$

Bending moments.

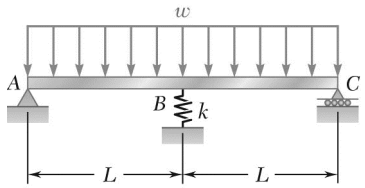
$$M_A = M_1 + M_2 + M_3 = -87.89 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

$$M_D = M_A + A_{AD} = 46.32 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

$$M_E = M_D + A_{DE} = 45.79 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

$$M_B = M_E + A_{EB} = 0 \quad \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.155

For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.

SOLUTION

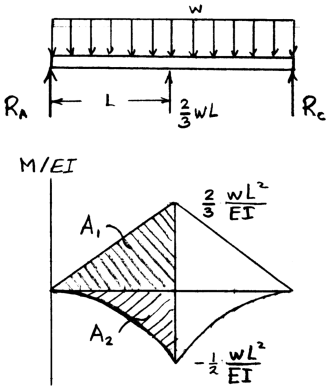
Symmetric beam and loading: $R_C = R_A$

Spring force: $F = \frac{1}{3}(2wL) = \frac{2}{3}wL$

$$+\uparrow \Sigma F_y = 0: \quad R_A + F - 2wL + R_C = 0$$

$$R_A = R_C = \frac{2}{3}wL$$

Draw M/EI diagram by parts.



$$A_1 = \frac{1}{2}\left(\frac{2}{3}\frac{wL^2}{EI}\right)L = \frac{1}{3}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

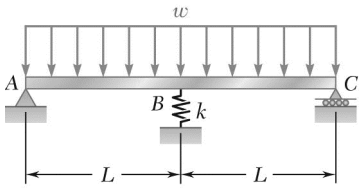
$$= -\frac{7}{72}\frac{wL^4}{EI}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72}\frac{wL^4}{EI}}$$

$$k = \frac{48}{7}\frac{EI}{L^3} \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.156

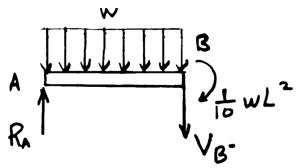
For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.

SOLUTION

Using free body AB,

$$+\circlearrowleft M_B = 0: \quad -R_A L + (wL)\left(\frac{L}{2}\right) - \frac{1}{10}wL^2 = 0$$

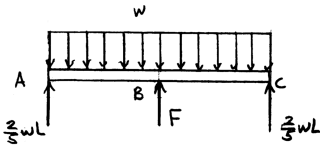
$$R_A = \frac{2}{5}wL \uparrow$$



Symmetric beam and loading: $R_C = R_A$

Using free body ABC, $+\uparrow \Sigma F_y = 0: \quad \frac{2}{5}wL + F + \frac{2}{5}wL - 2wL = 0$

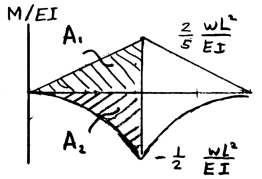
$$F = \frac{6}{5}wL$$



Draw M/EI diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{5}\frac{wL^2}{EI}\right)L = \frac{1}{5}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

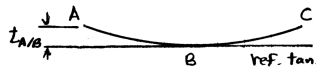


Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

$$= -\frac{1}{120}\frac{wL^4}{EI}$$

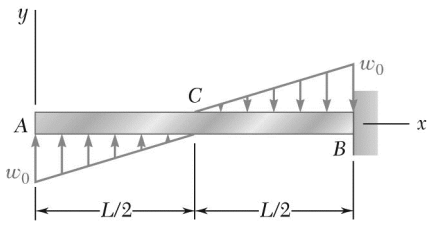


$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120}\frac{wL^4}{EI}}$$

$$k = 144\frac{EI}{L^3} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.157

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$w(x) = \frac{2w_0}{L}x - w_0$$

$$V(x) = -\int w(x)dx = -\int\left(\frac{2w_0}{L}x - w_0\right)dx = -\frac{w_0}{L}x^2 + w_0x + C_1$$

$$[x = 0,\ V = 0] \quad 0 = 0 + 0 + C_1 \quad \therefore \quad C_1 = 0$$

$$M(x) = \int V(x)dx = \int\left(-\frac{w_0}{L}x^2 + w_0x\right)dx = -\frac{w_0}{3L}x^3 + \frac{w_0}{2}x^2 + C_2$$

$$[x = 0,\ M = 0] \quad 0 = 0 + 0 + C_2 \quad \therefore \quad C_2 = 0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{w_0}{3L}x^3 + \frac{w_0}{2}x^2$$

$$EI\frac{dy}{dx} = -\frac{w_0}{12L}x^4 + \frac{w_0}{6}x^3 + C_3$$

$$\left[x = L, \quad \frac{dy}{dx} = 0\right] \quad 0 = -\frac{w_0L^3}{12} + \frac{w_0L^3}{6} + C_3 \quad \therefore \quad C_3 = -\frac{w_0L^3}{12}$$

$$EIy = -\frac{w_0}{60L}x^5 + \frac{w_0}{24}x^4 - \frac{w_0L^3}{12}x + C_4$$

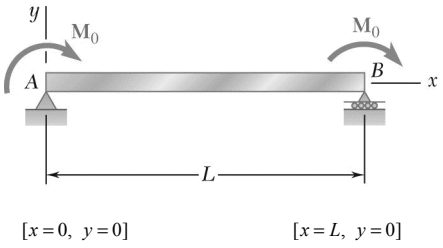
$$[x = L,\ y = 0] \quad 0 = -\frac{w_0L^4}{60} + \frac{w_0L^4}{24} - \frac{w_0L^4}{12} + C_4 \quad \therefore \quad C_4 = \frac{7w_0L^4}{120}$$

(*a*) Elastic curve. $y = -\dfrac{w_0}{120EIL}(2x^5 - 5Lx^4 + 10L^4x - 7L^5)$ ◀

(*b*) y at $x = 0$. $y_A = +\dfrac{7w_0L^4}{120EI}$ $y_A = \dfrac{7w_0L^4}{120EI}\uparrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = 0$. $\left.\dfrac{dy}{dx}\right|_A = -\dfrac{w_0L^3}{12EI}$ $\theta_A = \dfrac{w_0L^3}{12EI}$ ◀

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.158

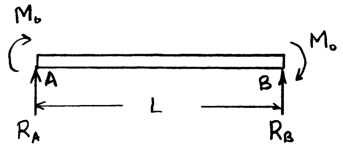
(*a*) Determine the location and magnitude of the maximum absolute deflection in *AB* between *A* and the center of the beam. (*b*) Assuming that beam *AB* is a W18×76 rolled shape, $M_0 = 150 \text{ kip} \cdot \text{ft}$, and $E = 29 \times 10^6$ psi, determine the maximum allowable length *L* so that the maximum deflection does not exceed 0.05 in.

SOLUTION

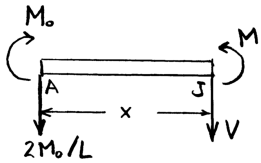
Using *AB* as a free body,

$$\Sigma M_B = 0: \quad -2M_0 - R_A L = 0$$

$$R_A = -\frac{2M_0}{L}$$



Using portion *AJ* as a free body,



$$\Sigma M_J = 0: \quad -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3\right) + C_1 x + C_2$$

$$[x = 0, \ y = 0] \quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x = L, \ y = 0] \quad 0 = \frac{M_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{3}L^3\right) + C_1 L + 0$$

$$C_1 = -\frac{1}{6}M_0 L^2$$

$$y = \frac{M_0}{EIL}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3 - \frac{1}{6}L^2 x\right)$$

$$\frac{dy}{dx} = \frac{M_0}{EIL}\left(Lx - x^2 - \frac{1}{6}L^2\right)$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.158 *(Continued)*

To find location of maximum deflection, set $\dfrac{dy}{dx} = 0.$

$$x_m^2 - Lx_m - \frac{1}{6}L^2 = 0$$

$$x_m = \frac{L - \sqrt{L^2 - (4)\left(\frac{1}{6}L^2\right)}}{2}$$

$$= \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right)L = 0.21132\,L \qquad x_m = 0.211L \blacktriangleleft$$

$$y_m = \frac{M_0 L^2}{EI}\left\{\left(\frac{1}{2}\right)(0.21132)^2 - \left(\frac{1}{3}\right)(0.21132)^3 - \left(\frac{1}{6}\right)(0.21132)\right\}$$

$$= -0.0160375\frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375\frac{M_0 L^2}{EI} \qquad |y_m| = 0.01604\frac{M_0 L^2}{EI} \blacktriangleleft$$

Solving for L, $\quad L = \left\{\dfrac{EI\,|y_m|}{0.0160375\,M_0}\right\}^{1/2}$

Data: $\quad E = 29\times10^6$ psi $\quad$ For W18×76 beam, $\quad I = 1330$ in^4

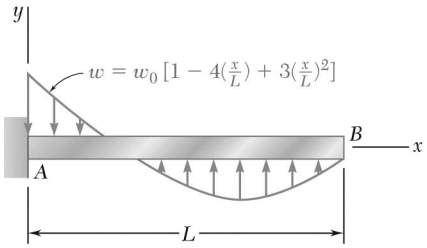
$$|y_m| = 0.05 \text{ in.}$$

$$M_0 = 150 \text{ kip}\cdot\text{ft} = 150\times10^3 \text{ lb}\cdot\text{ft}$$

$$= 1.800\times10^6 \text{ lb}\cdot\text{in}$$

$$L = \left\{\frac{(29\times10^6)(1330)(0.05)}{(0.0160375)(1.800\times10^6)}\right\}^{1/2} = 258.5 \text{ in.} \qquad L = 21.5 \text{ ft} \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.159

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the free end.

SOLUTION

Boundary conditions are shown at right.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ V = 0]$

$[x = 0,\ \dfrac{dy}{dx} = 0]$ $\qquad$ $[x = L,\ M = 0]$

$$\frac{dV}{dx} = -w = -w_0\left[1 - 4\left(\frac{x}{L}\right) + 3\left(\frac{x}{L}\right)^2\right]$$

$$V = -w_0\left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right] + C_V$$

$$[x = L, V = 0]:\ 0 = -w_0[L - 2L + L] + C_V = 0 \qquad C_V = 0$$

$$\frac{dM}{dx} = V = -w_0\left[x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right]$$

$$M = -w_0\left[\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right] + C_M$$

$$[x = L,\ M = 0]:\ 0 = -w_0\left[\frac{1}{2}L^2 - \frac{2}{3}L^2 + \frac{1}{4}L^2\right] + C_M \qquad C_M = \frac{1}{12}w_0L^2$$

$$EI\frac{d^2y}{dx} = M = -w_0\left[\frac{1}{2}x^2 - \frac{2}{3}\frac{x^3}{L} + \frac{1}{4}\frac{x^4}{L^2} - \frac{1}{12}L^2\right]$$

$$EI\frac{dy}{dx} = -w_0\left[\frac{1}{6}x^3 - \frac{1}{6}\frac{x^4}{L} + \frac{1}{20}\frac{x^5}{L^2} - \frac{1}{12}L^2x\right] + C_1$$

$$[x = 0, \frac{dy}{dx} = 0] \qquad C_1 = 0$$

$$EIy = -w_0\left[\frac{1}{24}x^4 - \frac{1}{30}\frac{x^5}{L} + \frac{1}{120}\frac{x^6}{L^2} - \frac{1}{24}L^2x^2\right] + C_2$$

$$[x = 0,\ y = 0] \qquad C_2 = 0$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.159 ***(Continued)***

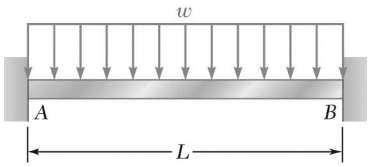
(*a*) Elastic curve.

$$y = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^2x^4 - \frac{1}{30}Lx^5 + \frac{1}{120}x^6 - \frac{1}{24}L^4x^2\right) \blacktriangleleft$$

(*b*) Deflection at $x = L$.

$$y_B = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^6 - \frac{1}{30}L^6 + \frac{1}{120}L^6 - \frac{1}{24}L^6\right) = -\frac{w_0L^4}{40EI} \qquad y_B = \frac{w_0L^4}{40EI}\downarrow \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



$[x = 0,\ y = 0]$ $\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

$[x = L,\ y = 0]$ $\left[x = L,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.160

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

SOLUTION

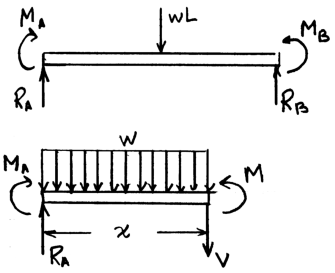
Reactions are statically indeterminate.

By symmetry, $R_B = R_A;\quad M_B = M_A$

$$\frac{dy}{dx} = 0 \quad \text{at} \quad x = \frac{L}{2}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad R_B = R_A = \frac{1}{2}wL \uparrow \ \blacktriangleleft$$

Over entire beam, $M = M_A + R_A x - \dfrac{1}{2}wx^2$



$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}wLx^2 - \frac{1}{6}wx^3 + C_1$$

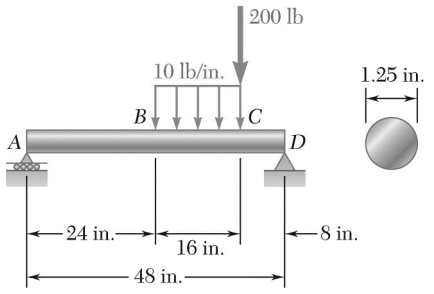
$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \qquad \frac{1}{2}M_A L + \frac{1}{16}wL^3 - \frac{1}{48}wL^3 + 0 = 0$$

$$M_A = -\frac{1}{12}wL^2 \ \blacktriangleleft$$

$$M = -\frac{1}{12}wL^2 + \frac{1}{2}wLx - \frac{1}{2}wx^2 \qquad M = w[6x(L-x) - L^2]/12 \ \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



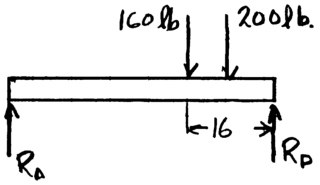
PROBLEM 9.161

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*. Use $E = 29 \times 10^6$ psi.

SOLUTION

Units: Forces in lbs; lengths in inches.

$$c = \frac{1}{2}d = \left(\frac{1}{2}\right)(1.25)$$
$$= 0.625 \text{ in.}$$
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.625)^4$$
$$= 119.84 \times 10^{-3} \text{ in}^4$$
$$EI = (29 \times 10^6)(119.84 \times 10^{-3})$$
$$= 3.4754 \times 10^6 \text{ lb} \cdot \text{in}^2$$



Use entire beam *ABCD* as free body.

$$+\circlearrowleft \Sigma M_D = 0: \quad -48R_A + (16)(160) + (8)(200) = 0 \qquad R_A = 86.667 \text{ lb}\uparrow$$

$$w(x) = 10\langle x-24\rangle^0 - 10\langle x-40\rangle^0 \quad \text{lb/in}$$

$$\frac{dV}{dx} = -w = -10\langle x-24\rangle^0 + 10\langle x-40\rangle^0 \quad \text{lb/in}$$

$$\frac{dM}{dx} = V = -10\langle x-24\rangle^1 + 10\langle x-40\rangle^1 + 86.667 - 200\langle x-40\rangle^0 \quad \text{lb} \cdot \text{in}$$

$$EI\frac{d^2y}{dx^2} = M = -5\langle x-24\rangle^2 + 5\langle x-40\rangle^2 + 86.667x - 200\langle x-40\rangle^1 \quad \text{lb} \cdot \text{in}$$

$$EI\frac{dy}{dx} = -\frac{5}{3}\langle x-24\rangle^3 + \frac{5}{3}\langle x-40\rangle^3 + 43.333x^2 - 100\langle x-40\rangle^2 + C_1 \quad \text{lb} \cdot \text{in}^2$$

$$EIy = -\frac{5}{12}\langle x-24\rangle^4 + \frac{5}{12}\langle x-40\rangle^4 + 14.4444x^3 - \frac{100}{3}\langle x-40\rangle^3 + C_1x + C_2 \quad \text{lb} \cdot \text{in}^3$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.161 *(Continued)*

$$[x = 0,\ y = 0] \qquad -0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 48,\ y = 0] \qquad -\left(\frac{5}{12}\right)(24)^4 + \left(\frac{5}{12}\right)(8)^4 + (14.4444)(48)^3$$
$$-\left(\frac{100}{3}\right)(8)^3 + 48C_1 = 0$$

$$C_1 = -30.08 \times 10^3 \text{ lb} \cdot \text{in}^2$$

(*a*) Slope at end *A*. $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0 + 0 + 0 + C_1$$

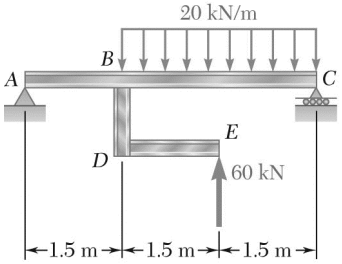
$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-30.08 \times 10^3}{3.4754 \times 10^6} = -8.66 \times 10^{-3} \qquad \theta_A = 8.66 \times 10^{-3} \text{ rad} \ \searrow \ \blacktriangleleft$$

(*b*) Deflection at point *B*. (y at $x = 24$ in.)

$$EIy_B = -0 + 0 + (14.4444)(24)^3 - 0 + (-30.08 \times 10^3)(24)$$
$$= -522.24 \times 10^3 \text{ lb} \cdot \text{in}^3$$

$$y_B = \frac{-522.24 \times 10^3}{3.4754 \times 10^6} = -0.1503 \text{ in.} \qquad y_B = 0.1503 \text{ in.} \downarrow \ \blacktriangleleft$$

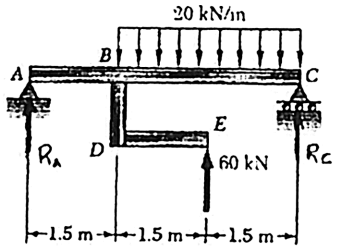
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.162

The rigid bar *BDE* is welded at point *B* to the rolled-steel beam *AC*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.

SOLUTION



$$+\circlearrowleft M_C = 0:$$

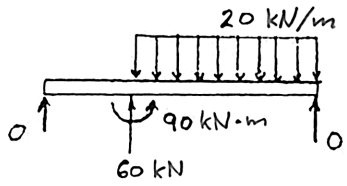
$$-4.5R_A + (20)(3)(1.5) - (60)(1.5) = 0 \qquad R_A = 0$$

Units: Forces in kN; lengths in m.

$$EI\frac{d^2y}{dx^2} = M = 60\langle x-1.5\rangle^1 - 90\langle x-1.5\rangle^0 - \frac{1}{2}(20)\langle x-1.5\rangle^2$$

$$EI\frac{dy}{dx} = 30\langle x-1.5\rangle^2 - 90\langle x-1.5\rangle^1 - \left(\frac{1}{6}\right)(20)\langle x-1.5\rangle^3 + C_1$$

$$EIy = 10\langle x-1.5\rangle^3 - 45\langle x-1.5\rangle^2 - \frac{1}{24}(20)\langle x-1.5\rangle^4 + C_1x + C_2$$



Boundary conditions:

$[x = 0,\ y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 4.5,\ y = 0]$ $\quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5C_1 + 0 = 0 \qquad C_1 = 45\,\text{kN}\cdot\text{m}^2$

Data: $E = 200\times10^9\,\text{Pa},\quad I = 316\times10^6\,\text{mm}^4 = 316\times10^{-6}\,\text{m}^4$

$$EI = (200\times10^9)(316\times10^{-6}) = 63.2\times10^6\,\text{N}\cdot\text{m}^2 = 63{,}200\,\text{kN}\cdot\text{m}^2$$

(*a*) Slope at *A*. $\left(\frac{dy}{dx}\ \text{at}\ x = 0\right) \qquad EI\theta_A = C_1 = 45\,\text{kN}\cdot\text{m}^2$

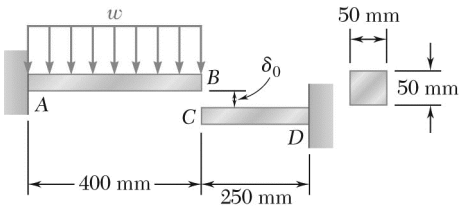
$$\theta_A = \frac{45}{63{,}200} = 0.712\times10^{-3}\,\text{rad} \qquad \theta_A = 0.712\times10^{-3}\,\text{rad}\ \measuredangle\ \blacktriangleleft$$

(*b*) Deflection at *B*. (y at $x = 1.5$)

$$EIy_B = (C_1)(1.5) = (45)(1.5) = 67.5\,\text{kN}\cdot\text{m}^3$$

$$y_B = \frac{67.5}{63{,}200} = 1.068\times10^{-3}\text{m} \qquad y_B = 1.068\,\text{mm}\uparrow\ \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.163

Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2$ mm, exists between the ends of the cantilever bars AB and CD. Knowing that $E = 105$ GPa and $w = 30$ kN/m, determine (*a*) the reaction at A, (*b*) the reaction at D.

SOLUTION

$$I = \frac{1}{12}(50)(50)^3 = 520.833\times10^3\ \text{mm}^3 = 520.833\times10^{-9}\ \text{m}$$

$$EI = (105\times10^9)(520.833\times10^{-6}) = 54.6875\times10^3\ \text{N}\cdot\text{m}^2$$

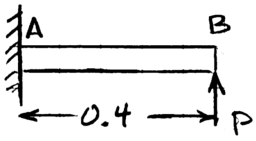
$$= 54.6875\ \text{kN}\cdot\text{m}^2$$

Units: Forces in kN; lengths in meters.

Compute deflection at B due to w. Case 8 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)}$$

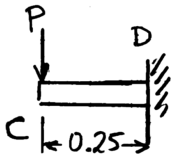
$$= -1.75543\times10^{-3} = -1.7553\ \text{mm}$$



The displacement is more than δ_0, the gap closes.

Let P be the contact force between points B and C.

Compute deflection of B due to P. Use Case 1 of Appendix D.



$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)}$$

$$= 390.095\times10^{-6}\,\text{P} \quad \text{m}$$

Compute deflection of C due to P.

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238\times10^{-6}\,\text{P} \quad \text{m}$$

Displacement condition:

$$y_B + \delta_0 = y_C$$

Using superposition,

$$(y_B)_1 + (y_B)_2 - \delta_0 = y_C$$

$$-1.75543\times10^{-3} + 390.095\times10^{-6}\,\text{P} + 1.2\times10^{-3} = -95.238\times10^{-6}\,\text{P}$$

$$485.333\times10^{-6}\,\text{P} = 0.55543\times10^{-3}$$

$$\text{P} = 1.14443\ \text{kN}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.163 *(Continued)*

(*a*) Reaction at *A*.

$$+\uparrow \Sigma F_y = 0: \quad R_A - 12 + 1.14443 = 0$$

$$R_A = 10.86\,\text{kN} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (0.2)(12) + (0.4)(1.14443) = 0$$

$$M_A = 1.942\,\text{kN} \cdot \text{m} \circlearrowleft \blacktriangleleft$$

(*b*) Reaction at *D*.

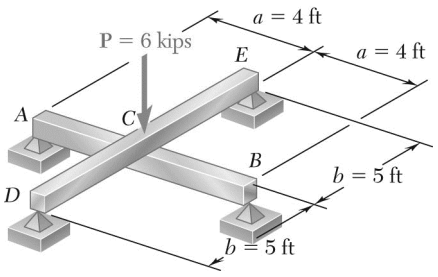
$$+\uparrow \Sigma F_y = 0: \quad R_D - 1.14443 = 0$$

$$R_D = 1.144\,\text{kN} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D + (0.25)(1.14443) = 0$$

$$M_D = 0.286\,\text{kN} \cdot \text{m} \circlearrowright \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



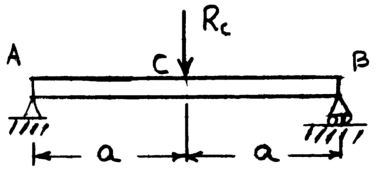
PROBLEM 9.164

For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *B*, (*b*) at *E*.

SOLUTION

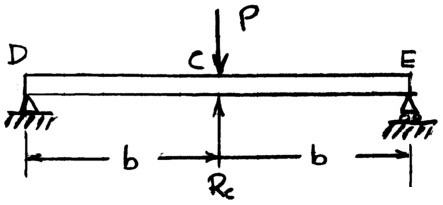
Units: Forces in kips; lengths in ft.

For beam *ACB*, using Case 4 of Appendix *D*:



$$(y_C)_1 = -\frac{R_C(2a)^3}{48EI}$$

For beam *DCE*, using Case 4 of Appendix *D*:



$$(y_C)_2 = \frac{(R_C - P)(2b)^3}{48EI}$$

Matching deflections at *C*,

$$-\frac{R_C(2a)^3}{48EI} = \frac{(R_C - P)(2b)^3}{48EI}$$

$$R_C = \frac{Pb^3}{a^3 + b^3} = \frac{(6)(5)^3}{4^3 + 5^3} = 3.968 \text{ kips}$$

$$P - R_C = 6 - 3.968 = 2.032 \text{ kips}$$

Using free body *ACB*,

$$+\circlearrowleft M_A = 0: \quad 2aR_B - aR_C = 0$$

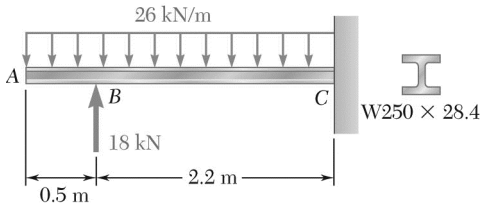
(*a*) Reaction at *B*. $R_B = \frac{1}{2}R_C$ $R_B = 1.984 \text{ kips} \uparrow$ ◀

Using free body *DCE*,

$$+\circlearrowleft M_D = 0: \quad 2bR_E - b(P - R_C) = 0$$

(*b*) Reaction at *E*. $R_E = \frac{1}{2}(P - R_C)$ $R_E = 1.016 \text{ kips} \uparrow$ ◀

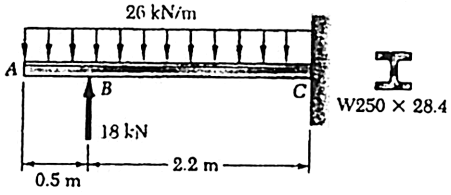
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.165

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION



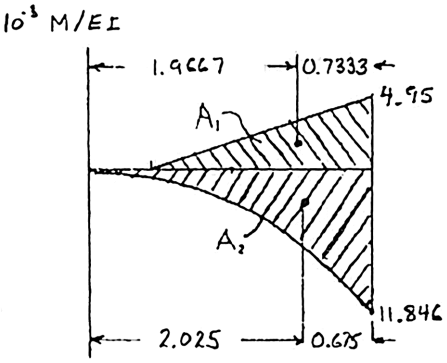
Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 40.1 \times 10^6 \text{ mm}^4 = 40.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(40.1 \times 10^{-6}) = 8.02 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 8020 \text{ kN} \cdot \text{m}^2$$



Draw *M*/*EI* diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8020} = 4.9377 \times 10^{-3} \text{ m}^{-1}$$

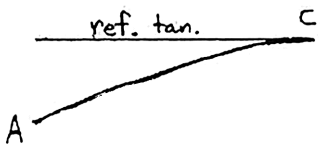
$$A_1 = \frac{1}{2}(4.9377 \times 10^{-3})(2.2) = 5.4315 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8020)} = -11.8167 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-11.8167 \times 10^{-3})(2.7) = -10.6350 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$$



Draw reference tangent at *C*.

$$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$$

(*a*) Slope at *A*.

$$\theta_A = -A_1 - A_2 = -5.4315 \times 10^{-3} + 10.6350 \times 10^{-3}$$

$$= 5.20 \times 10^{-3} \text{ rad}$$

$$\theta_A = 5.20 \times 10^{-3} \text{ rad} \curvearrowright \blacktriangleleft$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

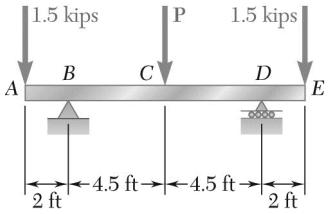
PROBLEM 9.165 ***(Continued)***

(*b*) Deflection at *A*.

$$\begin{aligned} y_A &= y_C - \theta_C L + t_{A/C} \\ &= 0 - 0 + A_1\bar{x}_1 + A_2\bar{x}_2 \\ &= 0 - 0 + (5.4315 \times 10^{-3})(1.9667) - (10.6350 \times 10^{-3})(2.025) \\ &= -10.85 \times 10^{-3}\ \text{m} \end{aligned}$$

$$y_A = 10.85\ \text{mm} \downarrow \blacktriangleleft$$

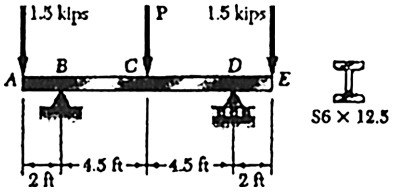
PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.166

Knowing that the magnitude of the load **P** is 7 kips, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*, (*c*) the deflection at midpoint *C* of the beam. Use $E = 29\times10^6$ psi.

SOLUTION



Use units of kips and ft. $P = 7\,\text{kips}$

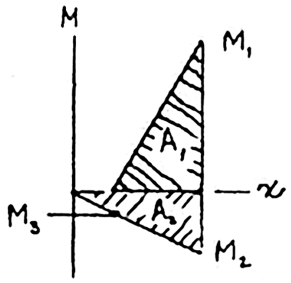
For S6 × 12.5, $I = 22.0\,\text{in}^4$

$$EI = (29\times10^6)(22.0) = 638\times10^6\ \text{lb}\cdot\text{in}^2$$
$$= 4430.6\ \text{kip}\cdot\text{ft}^2$$

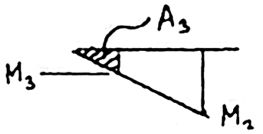
Symmetric beam with symmetric loading. Place reference tangent at midpoint *C* where $\theta_C = 0$.

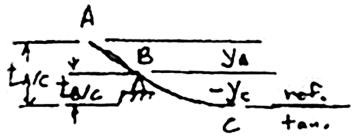
$$R_B = R_D = \frac{1}{2}(1.5 + 7 + 1.5) = 5\ \text{kips}\uparrow$$

Draw the bending moment diagram by parts for the left half of the beam.



$$M_1 = (4.5)(5) = 22.5\ \text{kip}\cdot\text{ft}$$
$$A_1 = \frac{1}{2}(4.5)(22.5) = 50.625\ \text{kip}\cdot\text{ft}^2$$
$$M_2 = -(2 + 4.5)(1.5) = -9.75\ \text{kip}\cdot\text{ft}$$
$$A_2 = \frac{1}{2}(6.5)(-9.75) = -31.6875\ \text{kip}\cdot\text{ft}^2$$
$$M_3 = -(2)(1.5) = -3\ \text{kip}\cdot\text{ft}$$
$$A_3 = \frac{1}{2}(2)(-3) = -3\ \text{kip}\cdot\text{ft}^2$$





Formulas: $\theta_A = -\theta_{C/A}, \quad y_A - y_C = t_{A/C} \quad y_c = y_A - t_{A/C}$

$$y_B = y_A - 2\theta_A + t_{B/A} = 0, \quad y_A = -2\theta_A - t_{B/A}$$

$$\theta_{C/A} = \frac{1}{EI}(A_1 + A_2) = \frac{50.625 - 31.6875}{4430.6} = 4.27425\times10^{-3}$$

$$t_{A/C} = \frac{1}{EI}\left\{(2 + 3)A_1 + \frac{2}{3}(6.5)A_2\right\} = \frac{115.8125}{4430.6} = 26.1392\times10^{-3}\ \text{ft}$$

$$t_{B/A} = \frac{1}{EI}\left\{\frac{1}{3}(2)A_3\right\} = \frac{-2}{4430.6} = -0.45141\times10^{-3}\ \text{ft}$$

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

PROBLEM 9.166 *(Continued)*

(*a*) Slope at end *A*. $\theta_A = -4.27 \times 10^{-3}$ rad $\qquad \theta_A = 4.27 \times 10^{-3}$ rad ↘ ◀

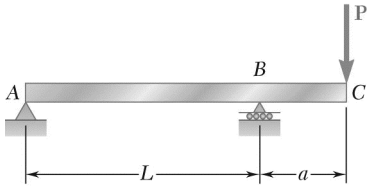
(*b*) Deflection at *A*. $y_A = -(2)(-4.27425 \times 10^{-3}) - (-0.45141 \times 10^{-3})$

$= 8.9999 \times 10^{-3}$ ft $\qquad y_A = 0.1080$ in. ↑ ◀

(*c*) Deflection at *C*. $y_C = 8.9999 \times 10^{-3} - 26.1392 \times 10^{-3} = -17.1393 \times 10^{-3}$ ft

$y_C = -0.206$ in. $\qquad y_C = 0.206$ in. ↓ ◀

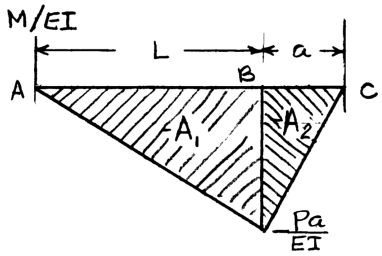
***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.



PROBLEM 9.167

For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*.

SOLUTION



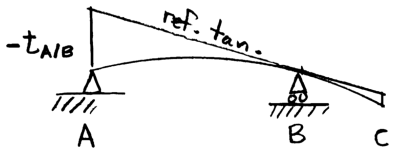
$$A_1 = -\frac{PaL}{2EI},$$

$$A_2 = -\frac{Pa^2}{2EI}$$

$$t_{A/B} = A_1\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$\theta_B = \frac{t_{A/B}}{L} = -\frac{PaL}{3EI}$$

(*a*) Slope at *C*.



$$\theta_C = \theta_B + \theta_{C/B}$$

$$\theta_C = -\frac{PaL}{3EI} - \frac{Pa^2}{2EI}$$

$$= -\frac{Pa(2L+3a)}{6EI}$$

$$\theta_C = \frac{Pa(2L+3a)}{6EI} \measuredangle \blacktriangleleft$$

(*b*) Deflection at point *C*.

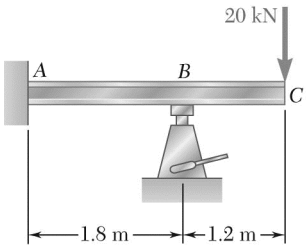
$$y_C = a\theta_B + t_{C/B}$$

$$y_C = -\frac{Pa^2L}{3EI} + \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2}{3}a\right)$$

$$= -\frac{Pa^2(L+a)}{3EI}$$

$$y_C = \frac{Pa^2(L+a)}{3EI} \downarrow \blacktriangleleft$$

***PROPRIETARY MATERIAL.* © 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.

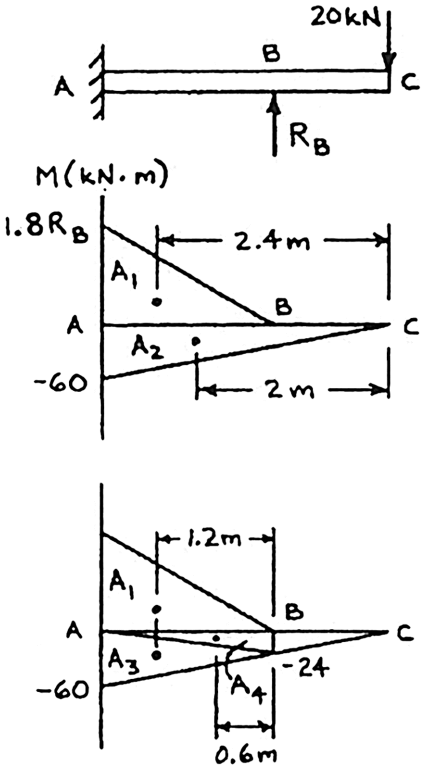


W130 × 23.8

PROBLEM 9.168

A hydraulic jack can be used to raise point B of the cantilever beam ABC. The beam was originally straight, horizontal, and unloaded. A 20-kN load was then applied at point C, causing this point to move down. Determine (*a*) how much point B should be raised to return point C to its original position, (*b*) the final value of the reaction at B. Use $E = 200$ GPa.

SOLUTION



For W130 × 23.8, $I_x = 8.91 \times 10^6 \text{ mm}^4$

$$EI = (200 \times 10^6 \text{ kPa})(8.91 \times 10^{-6} \text{ m}^4) = 1782 \text{ kN} \cdot \text{m}^2$$

Let R_B be the jack force in kN.

$$A_1 = \frac{1}{2}(1.8R_B)(1.8) = 1.62R_B$$

$$A_2 = \frac{1}{2}(-60)(3) = -90 \text{ kN} \cdot \text{m}^2$$

$$EI\,t_{C/A} = (2.4)A_1 + (2)A_2$$

$$0 = (2.4)(1.62R_B) + (2)(-90)$$

$$R_B = 46.296 \text{ kN}$$

$$A_1 = 75 \text{ kN} \cdot \text{m}^2$$

$$A_3 = \frac{1}{2}(-60)(1.8) = -54 \text{ kN} \cdot \text{m}^2$$

$$A_4 = \frac{1}{2}(-24)(1.8) = -21.6 \text{ kN} \cdot \text{m}^2$$

$$EIt_{B/A} = (1.2)A_1 + (1.2)A_3 + (0.6)A_4$$

$$= (1.2)(75) + (1.2)(-54) + (0.6)(-21.6)$$

$$= 12.24 \text{ kN} \cdot \text{m}^2$$

(*a*) Deflection at B. $y_B = t_{B/A} = \dfrac{EI\,t_{B/A}}{EI} = \dfrac{12.24}{1782} = 6.8687 \times 10^{-3} \text{ m}$ $y_B = 6.87 \text{ mm} \uparrow$ ◀

(*b*) Reaction at B. $R_B = 46.3 \text{ kN} \uparrow$ ◀

PROPRIETARY MATERIAL. **© 2012 The McGraw-Hill Companies, Inc.** All rights reserved. No part of this Manual may be displayed, reproduced, or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. A student using this manual is using it without permission.