

## CHAPTER XII

# PHOTO-ELECTRIC EFFECT

### Introduction

When radiations such as  $\gamma$ -rays, x-rays, ultraviolet rays and even visible light falls on a good number of substances, chiefly metals, electrons are ejected from these substances. This phenomenon is called *photo-electric effect*.

The phenomenon was first discovered by a telegraph operator, W. Smith, who using selenium resistors in the apparatus for the measurement of the resistance of transatlantic cables, observed that when sunlight fell upon the resistors the current in the circuit varied considerably. In 1887, Heinrich Hertz observed the same phenomenon accidentally while working with resonance electric circuits in connection with electro-magnetic waves. He noted that when ultraviolet rays fell on a spark gap the sparks passed more easily. In 1888, Hallwachs, Elster and Geitel made the important observation that

- (i) when ultra-violet light falls on a neutral zinc plate, the plate becomes positively charged.
- (ii) when ultra-violet light falls on a negatively charged zinc plate, it loses its negative charge.
- and (iii) when ultra-violet light falls on a positively charged zinc plate, it becomes more positively charged.

Hallwachs *et. al.*, therefore came to the conclusion that only negatively charged particles can be emitted by the zinc plate when it is irradiated with ultra-violet light. [Afterwards, it was discovered that alkali metals like lithium, sodium, potassium, rubidium and caesium eject electrons even when ordinary light *i.e.*, visible light falls on them.]

P. Lenard and J.J. Thomson in 1900 measured the  $e/m$  (ratio of electric charge to mass) of the ejected ions and found them to be electrons. Although these electrons are no different from all other electrons, it is customary to refer to them as *photo-electrons*.

## 12.1 Experimental study of photo-electric effect

The phenomenon of photo-electric emission can be studied in detail with the apparatus shown in Fig. 12.1. In this figure, S is a source of radiation of variable and known frequency  $\nu$  and intensity I. E is an emitting electrode of the material being studied and C is the

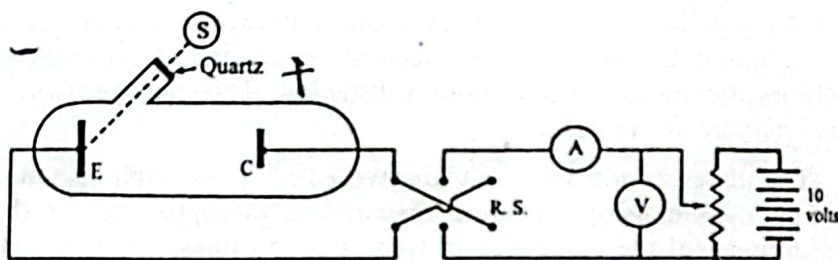


Fig. 12.1

collecting electrode. Both the electrodes are enclosed in an evacuated glass envelope with a quartz window that permits the passage of ultra-violet and visible light. As shown in the figure any potential difference can be established between E and C. A reversible switch R.S. enables the polarities of the two electrodes to be reversed. If the tube is in the dark, *i.e.*, no light falls on the electrode, there are no flow of current in the circuit and the microammeter reads zero. If ultra-violet light is allowed to fall on the emitting electrode, and if the collecting electrode is made positive with respect to the emitting electrode, current starts flowing as indicated by the microammeter. The explanation lies in the fact that when E is irradiated with ultra-violet light, electrons are liberated. As C is positive with respect to E, these photo-electrons will be quickly swept away from the emitter and attracted by the collector thereby starting the current flow. However, if the polarities between the plates are reversed with the help of the reversing switch R.S., no current flow is indicated by the ammeter. Although electrons are still being ejected by the emitter E, they cannot reach the plate C

(i) because of the pulling effect of the plate E which is now positively charged

and (ii) due to repulsion from the plate C which is now negatively charged.

*This confirms the negative nature of the charge of the ejected particles.*

## Characteristics (laws) of photo-electric emission

From the experimental data collected by Richardson and Compton, the following fundamental laws regarding the emission of photo-electrons were established.

(i) By holding the frequency of radiation ( $\nu$ ) that falls on the photosensitive surface and the accelerating potential V constant, the intensity of the radiation is gradually increased and the corresponding photo-electric current is measured. When the photo-electric current is plotted against the corresponding intensity of radiation, a graph similar to one in Fig. 12.2 is obtained. As can be seen

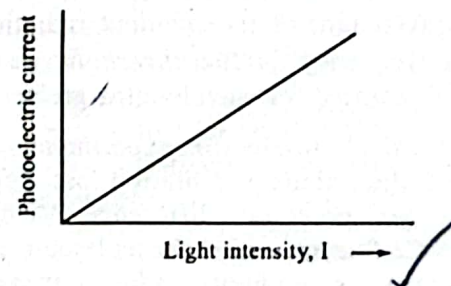


Fig. 12.2

from the graph *the strength of the photo-electric current (i.e., the number of electrons emitted per second) is directly proportional to the intensity of light or radiation used, provided the frequency of the radiation is kept constant.*

(ii) Next the intensity of light or radiation was kept constant. Starting from zero, the frequency of the radiation was gradually increased. At first there was no photo-electric emission and hence no current was indicated in the ammeter. This continued until a certain value of the frequency was reached when photo-electrons began to be emitted, registering a current in the ammeter. With further increase in frequency, the photo-electric current is found to increase although non-linearly. When the result is represented graphically, curves similar to those shown in Fig. 12.3 were obtained. A and B in Fig. 12.3, represent two different irradiated materials. The significant thing about these curves is that for every substance

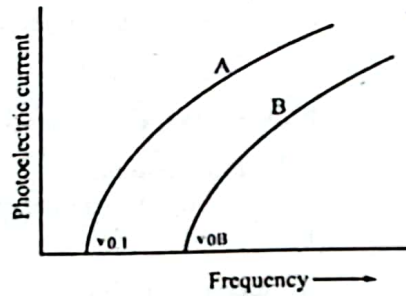


Fig. 12.3

irradiated there is a limiting or critical frequency below which no photo-electrons are emitted. This limiting frequency is called the *threshold frequency*,  $\nu_0$ , and is a characteristic of the material irradiated *i.e.*, its value depends on the nature of the material irradiated. The wavelength of the incident radiation corresponding to the threshold frequency is the *threshold wavelength*,  $\lambda_0$ . No photo-electrons are emitted for wavelengths greater than this.

(iii) In the third photo-electric experiment both the intensity and frequency of the incident radiation are kept constant. The variable now is the potential difference  $V$  between the two electrodes E and C. Starting with the collector at about 10 volts positive, the potential is gradually reduced to zero and then run negative until the photo-electric current stops entirely. Curve  $I_1$  in Fig. 12.4 shows the type of curve expected for a particular substance. The curve requires careful interpretation.

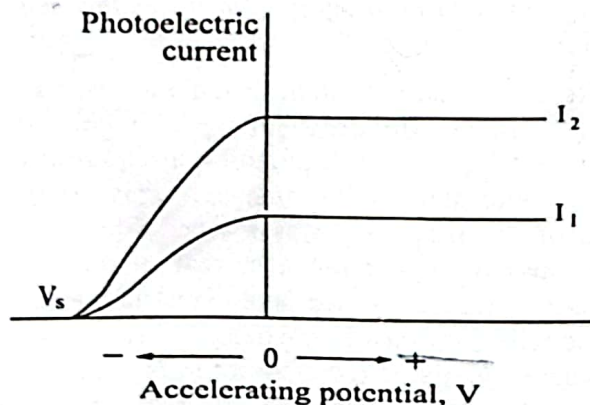


Fig. 12.4

When the potential difference between E and C is about 10 V or more, all the emitted electrons are immediately collected by C. This is the limiting or *saturation* value of the current. Further increase in  $V$  hardly produces any appreciable increase in current as shown by the flat portion of the curve. As the accelerating potential is now reduced from positive values through zero to negative values, the photo-electric current does not immediately drop to zero. This proves that electrons are emitted from E with some definite velocity-sufficient enough to give kinetic energy to the electrons so as to surmount (overcome) the retarding (opposing) electric field between the electrodes. Hence, some electrons still manage to reach C despite the fact that the electric field oppose their motion.

Eventually when the negative potential is made large enough, a value  $V_s$  is reached when the current is reduced to zero.  $V_s$  is known as the *stopping potential* and is defined as that value of the retarding potential difference between the two electrodes which is just sufficient to halt the most energetic photo-electron emitted. Therefore, the product of the stopping potential and the electronic charge,  $V_s \cdot e$ , is equal to the maximum kinetic energy that an emitted electron can have. Since the stopping potential has a definite value, it indicates that the emitted electrons have a definite upper limit to their kinetic energy.

Doubling the intensity of light merely doubles the current at each potential as shown by curve  $I_2$ , but does not affect the value of  $V_s$ . Or, *the stopping potential,  $V_s$ , is independent of the intensity of incident radiation.*

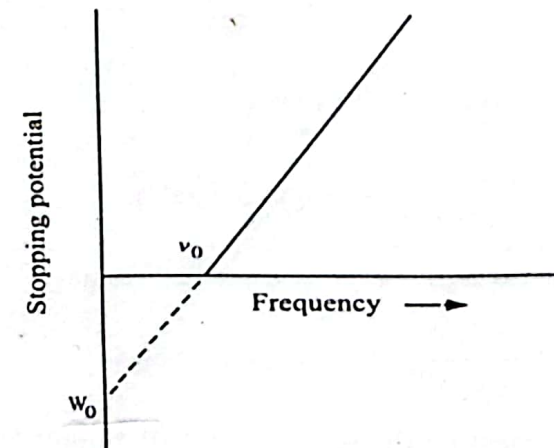


Fig. 12.5

If, however, the experiment is repeated by varying the frequency of light, it is found that the stopping potential varies linearly with the frequency as shown in Fig. 12.5. As no electrons are emitted below the threshold frequency, the stopping potential is zero for that region. But as the frequency is increased above the threshold frequency, the stopping potential varies linearly with the frequency of the incident light.

The maximum velocity,  $v_{\max}$ , of emission of a photo-electron can be determined from the value of the stopping potential,  $V_s$ , as follows:

$$\frac{1}{2}mv_{\max}^2 = e.V_s$$

where  $m$  and  $e$  are the mass and charge of an electron.

$$\begin{aligned} \text{or, } v_{\max} &= \sqrt{\frac{2eV_s}{m}} \\ &= \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times V_s}{9.109 \times 10^{-31}}} \\ &= \sqrt{2 \times 1.602 \times 10^{11} \times V_s} \\ &= 5.93 \times 10^5 \sqrt{V_s} \text{ metre/second.} \end{aligned}$$

Obviously,

$$\begin{aligned} E_{\max} &= \frac{1}{2}mv_{\max}^2 = eV_s \text{ joules.} \\ &= \frac{e.V_s}{1.6 \times 10^{-19}} \text{ electron volts (eV).} \end{aligned}$$

since  $1 \text{ eV} = 1.6 \times 10^{-12} \text{ ergs.} = 1.6 \times 10^{-19} \text{ joules.}$

$$\therefore E_{\max} = \frac{1.6 \times 10^{-19}}{1.6 \times 10^{-19}} V_s \text{ volts} = V_s \text{ volts.}$$

If the experiment on the measurement of stopping potential is repeated first by varying the intensity of light while keeping the

frequency of emission constant and then by varying the frequency while keeping the intensity constant, the following important conclusion emerges.

*"The maximum velocity of an electron emitted (and hence its kinetic energy) varies linearly with the frequency of the incident light but is independent of its intensity."*

From the measurements for stopping potential it was further established that the velocities of emitted electrons have values between zero and a definite maximum. The proportion of the electrons having a particular velocity is independent of the intensity of radiation.

(iv) It was also established experimentally that the photo-electric emission is an instantaneous process. If there is any time lag between the arrival of light at a metal surface and the emission of photo-electrons, it should be less than  $2 \times 10^{-9}$  seconds which is the limit of experimental accuracy.

### 12 Failure of classical electro-magnetic theory to explain photo-electric effect

Although at the first glance the electro-magnetic theory of light seems to be capable of explaining the photo-electric effect, it soon becomes obvious that the main features of photo-electric emission leads to contradictions if electro-magnetic theory is applied to the problem.

The electric and magnetic fields which are attributed to light waves in the electro-magnetic theory, may well be able to exert sufficient force on the electrons in the metal and release them from its surface. But the energy carried by an electro-magnetic wave is described by the intensity of the wave, i.e., the square of the amplitude of the wave. The photo-electrons ejected from the metal should then have energies which are dependent on the intensity, but the experimental results described above show that this is not so.

It would further be expected that light of low frequency (infrared) if sufficiently intense would be as much effective as the light of high frequency (ultra-violet), as long as the intensity and time of exposure is great enough. But this is again in contradiction

to the experimental results which show that light of frequency below a sharply defined threshold is absolutely incapable of producing any effect.

(10) Equally difficult to explain was the fact that there is no time lag between the arrival of the radiation on the metal surface and the emission of photo-electrons. For example, let us consider an experiment as depicted in Fig. 12.1 where violet light falls on a sodium surface. A detectable photo-electric current will result when  $10^{-6}$  watt/m<sup>2</sup> of electromagnetic energy is absorbed by the surface. Since sodium is a good reflector of light, a more intense beam will be required in practice. Now there are about  $10^{19}$  atoms in a layer of sodium one atom thick and 1 m<sup>2</sup> in area. If it is assumed that the incident light is absorbed in the 10 uppermost layers of sodium atoms, the  $10^{-6}$  watt/m<sup>2</sup> is distributed among  $10^{20}$  atoms. Hence each atom receives energy at the average rate of  $10^{-26}$  watt, which is less than  $10^{-7}$  eV/sec. It should therefore take more than  $10^7$  seconds, or almost a year, for any single electron to accumulate the 1 eV or so of energy that the photo-electrons are found to possess. In the maximum possible time of  $3 \times 10^{-9}$  second, an average electron according to electro-magnetic theory, will have gained only  $3 \times 10^{-16}$  eV. Even if some kind of resonance process is called upon to explain why some electrons acquire more energy than others, the fortunate electrons will still have no more than  $10^{-10}$  of the observed energy.

## 12.2 The quantum theory of light

Thus the electromagnetic theory which could so successfully explain other phenomena exhibited by light such as interference, diffraction and polarization, completely fails to explain the photo-electric effect. In 1905, Albert Einstein found that the paradox presented by the photo-electric effect could be understood only by taking seriously a notion proposed five years earlier by the German theoretical physicist Max Planck. Planck was seeking to explain the characteristics of radiations emitted by black bodies – bodies hot enough to be luminous. The different formulae for energy distribution in black body radiation, derived on the basis of classical theory, not only led to wrong and even absurd conclusions, but also could not stand the test of observation. Planck realized that the failure of the above formulae was due to the assumption that the energy changes of radiators take place continuously. He therefore

proposed a new theory – the *quantum theory* according to which the radiation of energy from a system or the exchange of radiant energy between different systems occurs not in continuous fashion permitting all possible values as demanded by the wave theory but *discontinuously* as little bursts (or bundles) of energy, which are integral multiples of an elementary quantum of energy. These bursts of energy are called quanta. Planck found that the quanta associated with a particular frequency of light all have the same energy and that this energy  $E$  is directly proportional to the frequency  $\nu$  of the incident radiation. That is

$$E = h\nu$$

where  $h$  is a constant, known today as Planck's constant, and has the value

$$h = 6.63 \times 10^{-34} \text{ joule-sec.}$$

[Note: Since the frequency  $\nu$  is different for different radiations, the quantum unit  $h\nu$  is not the same for all kinds of radiation. In the original quantum theory the energy of the photon was always considered to be an integral multiple of the unit  $h\nu$ , as  $h\nu$ ,  $2h\nu$ ,  $3h\nu$ , ..... But in the new quantum mechanics it is further refined to a value of  $\left(n + \frac{1}{2}\right) h\nu$ ,  $n$  being an integer, which means that the

limiting value of the energy of the photon is not zero, but  $\frac{1}{2}h\nu$ ]

## 12.3 Einstein's photo-electric equation

While Planck had to assume that a hot body radiates energy intermittently in the form of a bundle of energy (quanta), he did not doubt that it propagates continuously through space as electromagnetic waves. According to wave theory of light pulsating electromagnetic fields spread out from their source. Einstein proposed that light not only is emitted a quantum at a time, but also propagates as individual quanta – radiation is not only emitted or absorbed in discrete amounts of quanta, but also the same quantum structure is retained by radiation while travelling through space, very much like a shot fired from a gun, without spreading out as waves in any supposed ether.

Einstein applied this theory to the photo-electric process and obtained a consistent and satisfactory explanation of all the experimental facts of photo-electric emission. In his theory, radiation is regarded as a shower of photons each of energy  $h\nu$  moving in space with the velocity of light. When a single photon is incident on a metal surface, it is completely absorbed by an atom. The energy is subsequently imparted to one of the electrons of the atom. This energy is utilized for two purposes:

(i) partly for getting the electron free from the atom and away from the metal surface. This energy is known as the *photo-electric work function* of the metal and is represented by either  $\phi$  or  $w_0$ .

(ii) the balance of the photon energy is used up in imparting to the freed electron a kinetic energy of  $\frac{1}{2}mv^2$ .

Einstein expressed this assumption in the form

$$\checkmark h\nu = w_0 + \frac{1}{2}mv^2 \quad (12.1)$$

where

$h\nu$  = energy content of each quantum of the incident light.

$w_0$  = photo-electric work function of the metal surface being irradiated.

$\frac{1}{2}mv^2$  = kinetic energy of the ejected photo-electron.

Expression 12.1 is known as *Einstein's photo-electric equation*.

### Explanation of the characteristics of photo-electric equation

Einstein's photo-electric equation shows clearly why

(i) *the maximum velocity of the ejected photo-electron is directly proportional to the frequency of the incident radiation.*

Eqn. 12.1 can be written as

$$\frac{1}{2}mv^2 = h\nu - w_0$$

Since  $w_0$  is constant for a given emitter,  $\frac{1}{2}mv^2$  and hence  $v$ , the velocity of the ejected electron, increases as  $\nu$  increases. ✓

(ii) *The velocity of the photo-electron is independent of the intensity of the radiation.*

If  $\nu$  remains constant, increasing intensity merely increases the number of photons striking the emitter. This results in greater number of collisions resulting in the emission of greater number of photo-electrons. But each photo-electron will receive exactly the same energy ( $h\nu - w_0$ ) and hence would acquire the same velocity irrespective of the intensity. ✓

However, increase in the number of photo-electrons emitted results in the increase in the strength of photo-electric current. This explains why photo-electric current varies directly as the intensity of radiation. ✓

(i) *Existence of a threshold frequency that varies with the nature of the emitter.*

A electron is electro-statically bound to the emitter. The amount of energy required to free this electron from the emitter is known as the photo-electric work function,  $w_0$ , and this comes from the energy of the incident radiation  $h\nu$ . As  $\nu$  is reduced the energy of the incident radiation is reduced also. At a value, say  $\nu_0$ , let  $h\nu_0 = w_0$ . The photon energy is then just sufficient to liberate the electron only and no energy will be available for imparting kinetic energy to the electron. Hence eqn. 12.1 would reduce to

$$h\nu_0 = w_0$$

$\nu_0$  is called the threshold frequency and is defined as the minimum frequency which can cause photo-electric emission. For frequencies lower than  $\nu_0$  the energy of the incident photon is less than the work function of the material and hence there would be no emission of electrons. But for frequencies greater than  $\nu_0$ , electrons would be emitted with a certain definite velocity (and hence kinetic energy). This explains why there is a threshold frequency  $\nu_0$  connected with photo-electric emission. ✓

(iv) There is no time-lag in the process of photo-lectric emission.

As the phenomenon of photo-electric emission is considered as a collision process between two particles – the photon and the atom, there is no time-lag between the incidence of the photons on the metal surface and the emission of electrons therefrom.

(v) Emission of photo-electrons with all possible velocities upto a certain maximum.

$w_0$  is the work that must be done to take an electron through the metal surface from just beneath it; more work is required when the electron originates deeper in the metal. Thus electrons from various depths below the metal surface would emerge with different energies *i.e.*, velocities. The greater the depth the less is the velocity. It is therefore easy to see why not all the photo-electrons have the same energy, but emerge with all possible velocities upto a certain maximum. For an electron just beneath the metal surface the kinetic energy will be maximum. Eqn. 12.1 then becomes

$$h\nu = w_0 + \frac{1}{2}mv_{\max}^2$$

$$= w_0 + (\text{K.E.})_{\max}$$

### Long Wavelength Limit ( $\lambda_0$ )

The wavelength corresponding to the threshold frequency  $\nu_0$  is referred to as the long wavelength limit  $\lambda_0$ . Radiations having wavelength longer than  $\lambda_0$  would not be able to eject electron from a given material whereas those having  $\lambda < \lambda_0$  will. In other words, it represents the upper limit of wavelength for photo-electric emission. By analogy, it is also referred to as *threshold wavelength*.

$$\text{Now } c = \nu_0 \lambda_0 \quad \text{or, } \lambda_0 = \frac{c}{\nu_0}$$

$$\text{Also } w_0 = h\nu_0 \quad \text{or, } \frac{1}{\nu_0} = \frac{h}{w_0}$$

$$\therefore \lambda_0 = \frac{c}{\nu_0} = \frac{ch}{w_0}$$

(i) when  $w_0$  is in joules

$$\lambda_0 = \frac{3 \times 10^8 \times 6.625 \times 10^{-34}}{w_0}$$

$$= \frac{19.875 \times 10^{-26}}{w_0} \text{ metre}$$

(ii) when  $w_0$  is in electron-volts (eV)

$$\lambda_0 = \frac{3 \times 10^8 \times 6.625 \times 10^{-34}}{1.602 \times 10^{-19} \times w_0}$$

$$= \frac{12.4 \times 10^{-7}}{w_0} \text{ metre}$$

$$= \frac{12,400}{w_0} \text{ \AA.}$$

### Kinetic energy of photo-electrons

From Einstein's photo-electric equation we get

$$h\nu = w_0 + \frac{1}{2}mv^2$$

$$\text{or, } \frac{1}{2}mv^2 = h\nu - w_0$$

No  $w_0$  is the energy necessary to free the electron from the atom. If  $\nu_0$  is the threshold frequency for a particular metal, then all the energy of the incident photon is just sufficient to liberate the electron only and no energy is available to impart kinetic energy to the electron.

In this case

$$\nu_0 = h\nu_0$$

for the particular metal

$$\text{Thus } \frac{1}{2}mv^2 = hv - hv_0$$

$$\text{or, K. E.} = hv - hv_0$$

$$= h(v - v_0)$$

$$= ch \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \text{since } v = \frac{c}{\lambda}$$

$$= 3 \times 10^8 \times 6.625 \times 10^{-34} \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

where  $c = 3 \times 10^8$  m/sec

and  $h = \text{Planck's constant}$

$$= 6.625 \times 10^{-34} \text{ joules-sec.}$$

$$\therefore \text{K. E.} = 19.875 \times 10^{-26} \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ joules ; } \lambda \text{ and } \lambda_0 \text{ in metres.}$$

$$= \frac{19.875 \times 10^{-26}}{10^{-10}} \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ joules ; } \lambda \text{ and } \lambda_0 \text{ in angstrom}$$

unit (1 A.U. =  $10^{-10}$  m)

$$= 19.875 \times 10^{-16} \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ joules ; } \lambda \text{ and } \lambda_0 \text{ in A.U.}$$

$$= \frac{19.875 \times 10^{-16}}{1.602 \times 10^{-19}} \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV ; } \lambda \text{ and } \lambda_0 \text{ in } \text{\AA}.$$

$$\therefore \text{K. E.} = 12,400 \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV} \quad (12.2)$$

where  $\lambda$  and  $\lambda_0$  are expressed in Angstrom unit ( $\text{\AA}$ ).

The kinetic energy referred to in eqn. 12.2 above represents the maximum kinetic energy that an electron may possess.

$E_{\text{max}}$  can also be obtained from the expression given below.

$$E_{\text{max}} = hv - hv_0 = h(v - v_0) = h \Delta v \text{ joules.}$$

### Photo-electric work function

As already explained, the photo-electric work function is defined as the energy which is just sufficient to liberate electrons from a body with zero velocity. Its value is given by

$$w_0 = hv_0 = \frac{ch}{\lambda_0}$$

$$= \frac{3 \times 10^8 \times 6.625 \times 10^{-34}}{\lambda_0} \text{ joules}$$

$$= \frac{19.875 \times 10^{-26}}{\lambda_0} \text{ joules ; } \lambda_0 \text{ in metres}$$

$$= \frac{19.875 \times 10^{-16}}{\lambda_0} \text{ joules ; } \lambda_0 \text{ in } \text{\AA}$$

$$= \frac{19.875 \times 10^{-16}}{1.602 \times 10^{-19} \lambda_0} \text{ eV ; } \lambda_0 \text{ in } \text{\AA}$$

$$\therefore w_0 = \frac{12,400}{\lambda_0} \text{ eV ; } \lambda_0 \text{ in } \text{\AA}. \quad (12.3)$$

### 12.4 Millikan's experiment

To verify Einstein's photo-electric equation and to determine the value of the Planck's constant  $h$  and the photo-electric work function  $w_0$  Millikan performed an experiment in 1916. The experiment is based on the determination of stopping potential. Millikan's experimental set up is shown in Fig. 12.6. The apparatus consists of an evacuated chamber C. At the centre of the chamber is kept a drum D which can rotate freely about a vertical axis. Four

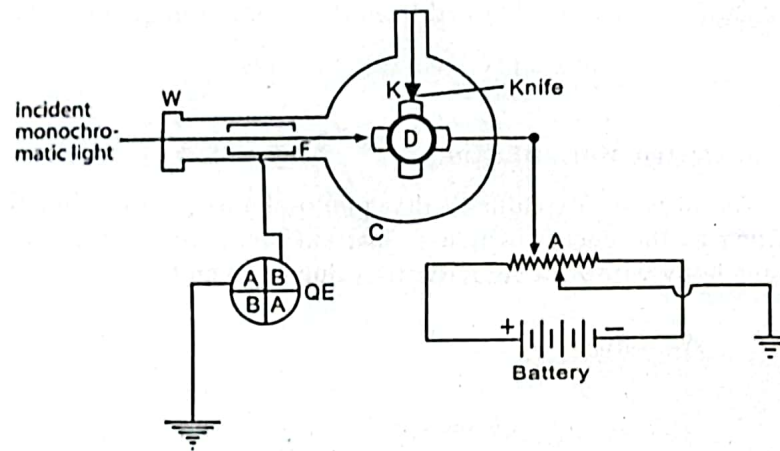


Fig. 12.6

cylindrical blocks of alkali metals like sodium, potassium, lithium, etc. are fixed on the periphery of the drum. Alkali metals were preferred as they readily exhibit photo-electric emission even with visible light. By rotating the drum, any one of the metal blocks could be turned towards a quartz window W through which monochromatic light of known frequency is allowed to pass. As these metals get oxidized easily, the experiment was carried out in vacuum. Further, fresh surfaces of the metals were obtained by scraping the metals with a sharp knife-edge K which could be brought opposite the metal block by means of an electromagnet placed outside the chamber. F is a Faraday cylinder connected to a quadrant electrometer QE. With the help of a sensitive potential divider arrangement suitable positive or negative potential can be applied to the drum with respect to the cylinder F.

To start with, the surface of any one of the metal blocks is scraped with the knife-edge K and then by rotating the drum, the block is turned towards the window W. Monochromatic radiation of known frequency  $\nu$  is then allowed to be incident on the surface of the metal block. If the drum is kept at a negative potential with respect to the cylinder F, the photo-electrons ejected from the surface are accelerated towards the cylinder thereby recording a deflection in the quadrant electrometer. When a small positive potential is applied to the drum to prevent the

photo-electrons from leaving the drum, only those fast moving photo-electrons *i.e.*, photo-electrons having sufficient kinetic energy to overcome this opposing retarding potential reach the cylinder F. The deflection in the electrometer therefore decreases. The positive potential of the drum is gradually increased till the deflection in the electrometer becomes zero. At this stage no photo-electron can reach the cylinder. This particular potential  $V_s$  for which no electron can reach the cylinder is called the stopping potential. The stopping potential under these conditions is the positive potential applied to the drum which corresponds to zero current in the electrometer.

Even when the intensity of light was increased by keeping the frequency constant, the deflection becomes zero at the same potential. *This means that at a constant frequency of incident radiation the stopping potential  $V_s$  is the same and is independent of the intensity of the incident radiation.* The experiment was repeated with different lights *i.e.*, radiation with different frequencies. It was found that the stopping potential  $V_s$  increases with increase in frequency for the same metal. According to Einstein's photo-electric equation

$$h\nu - w_0 = \frac{1}{2}mv^2$$

$$\checkmark \text{ or, } h\nu - h\nu_0 = \frac{1}{2}mv^2 \quad [w_0 = h\nu_0]$$

$$\checkmark \text{ Also } eV_s = \frac{1}{2}mv^2$$

$$\therefore eV_s = h\nu - h\nu_0 \\ = h(\nu - \nu_0)$$

$$\checkmark \therefore V_s = \frac{h}{e}(\nu - \nu_0)$$

The experiment was repeated for different metals and graphs were drawn with  $\nu$  along the X-axis and  $V_s$  along the Y-axis. The resulting graph was a straight line in all cases and this is in agreement with Einstein's theory.

Determination of Plank's constant

The stopping potentials for a particular metal were determined for different frequencies in the manner mentioned above. A graph is then plotted with  $\nu$  along the X-axis and  $V_s$  along the Y-axis. The resulting graph was a straight line as shown in Fig. 12.7(a). Similar graphs were obtained for cesium, and zinc as shown in Fig. 12.7(b). The point at which the straight line AB intercepts the X-axis gives the threshold or cut off or critical frequency  $\nu_0$ , and varies from

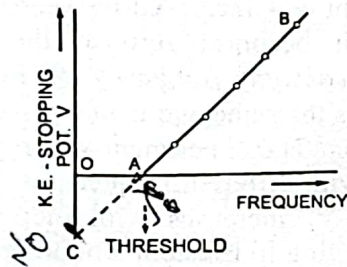


Fig. 12.7(a)

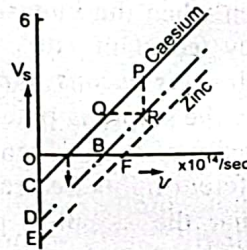


Fig. 12.7(b)

metal to metal as B and F in Fig. 12.7(b). For this frequency the kinetic energy of the emitted photoelectron is zero ( $\frac{1}{2}mv^2 = 0$ ) and the energy of the photon  $h\nu_0 = w_0$ . At a frequency less than  $\nu_0$  no electrons are emitted from the surface. The slope of the graph

$$= \frac{PR}{QR} = \frac{V_s}{\nu - \nu_0}$$

But the equation of the graph may be expressed as

$$e.V_s = h\nu - h\nu_0 \tag{12.4}$$

or,  $\frac{1}{2}mv^2 = h\nu - h\nu_0$

from eqn. 12.4

$$\frac{V_s}{\nu - \nu_0} = \frac{h}{e}$$

Therefore, the slope  $\frac{PR}{QR} = \frac{h}{e}$  on the experimental graph; then

$\frac{h}{e} = \tan \theta$ . Thus knowing the value of  $e$ ,  $h$  can be calculated from the relation

$$h = e \cdot [\text{slope of the graph}]$$

By experimental determination of the stopping potential for a metal, and the frequency of the incident light and also the threshold frequency, it was found that the value of  $h$  turns out to be  $6.62 \times 10^{-34}$  J-sec when  $e = 1.6 \times 10^{-19}$  coulomb. This value is in agreement with the value of the Planck's constant as determined by other methods. Millikan's experiment is therefore a confirmation of Einstein's photo-electric equation and hence quantum theory of radiation on which it is based. The intercept  $OC$  of the line with the vertical axis gives the value of  $\frac{w_0}{e}$  where  $w_0 = h\nu_0$  is the work function of the metal.

Hence

$$w_0 = e \times OC$$

Experimentally it is found that the work function  $w_0$  for barium = 2.5 eV, for sodium = 2.46 eV, for potassium = 2.24 eV and for cesium = 1.92 eV.

From what has been discussed above it follows that the graphs in Fig. 12.7 can be used to determine the value of (i) the Planck's constant  $h$ , (ii) threshold frequency  $\nu_0$  of a metal and (iii) the work function  $w_0$  for the metal.

12.5 Photo-electric cell

Photoelectric cell is an arrangement to convert light energy into electrical energy. The following three main types of photo-electric cells are worth considering :

(i) **Photoemissive cell** : it depends on the emission of electrons from a metal cathode when it is exposed to light or other radiation.

(ii) **Photovoltaic cell** : the sensitive element here is a semiconductor - not a metal. It generates voltage in proportion to the light or any radiant energy incident on it.

(iii) **Photoconductive cell** : it depends on the change of resistance of a semiconductor material in accordance with the radiant energy received.

### Photo-emissive cell or Phototube:

#### construction

it consists of two *metallic* electrodes – a cathode and an anode,

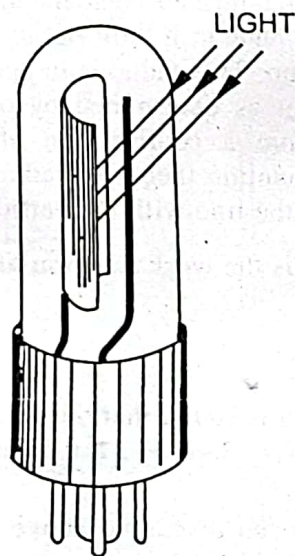


Fig. 12.8

enclosed in an evacuated glass bulb fitted with a base like a thermionic valve as shown in Fig. 12.8. The cathode is either V-shaped or semi-cylindrical metal plate coated with an emissive material and is known as the emitter. The anode is in the form of a thin wire of platinum or nickel fixed along the axis of the cylinder or it can be a small button so placed as not to obscure the light source from the cathode. This wire is known as the collector.

Various types of photocathodes in use are

- (i) pure metals like Ni, Zn, Al etc. for ultra-violet radiations.
- (ii) alkali metals for visible light.

(iii) composite cathodes for infra-red radiations such as a silver plate coated with caesium or potassium.

(iv) alloy cathodes made up of thin films of antimony caesium.

#### working

when radiation falls on the cathode, electrons are emitted and are attracted by the positive anode. Hence, a current is produced whose magnitude, for a given cathode, depends on

(i) intensity of incident radiation

and (ii) anode-to-cathode voltage.

The variations are shown in Fig. 12.9. As can be seen, the current produced by such a cell is of the order of a few microamperes. The

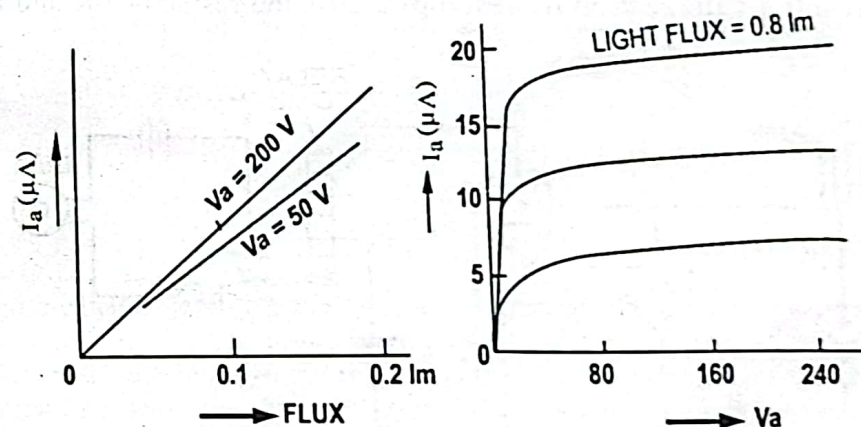


Fig. 12.9

current, therefore, has to be further amplified before being put to use.

As the current in a vacuum type photo-cell is very weak, gas filled type of photo-cells are used. The gas used is either argon or helium. When bombarded by photo-electrons, the gas molecules are ionized; resulting in a larger current flow in the circuit.

uses

photo-electric cells find wide use in the field of photometry – for accurate comparison of light intensities, in calorimetry, television, burglar alarms, fire alarms, counting machines, complexion metres and traffic regulators. They are used for colour identification, for reproduction of sound from a motion-picture film and for automatic switching on and off of street lights. They are also used for *on and off* circuits and in other circuits concerning the counting or sorting of objects on a conveyer belt, the automatic opening of a door as it is approached, etc.

### Automatic Bell Alarm Circuit

Fig. 12.10 shows the circuit of an automatic bell alarm circuit in which a bell rings up whenever light beam between the lamp and the phototube is interrupted. Whenever light falls on the phototube, it causes a small current to flow from battery through resistance  $R$ . As a result a voltage drop  $iR$  develops across the resistance  $R$  and is

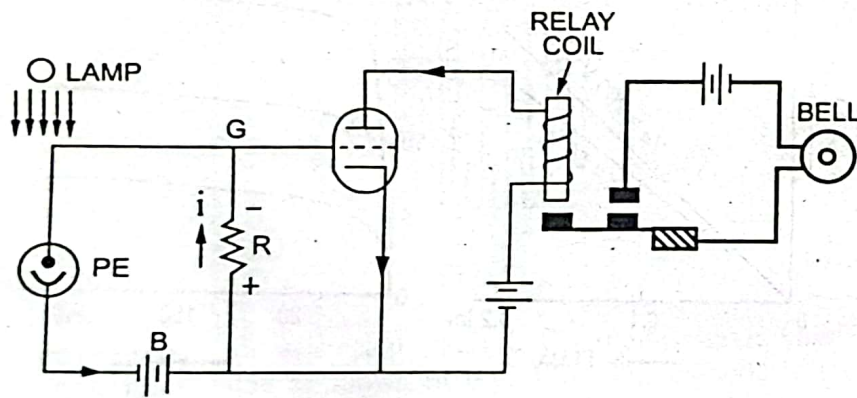


Fig. 12.10

applied to the grid  $G$  of a triode valve. The grid  $G$  is maintained negative with respect to the cathode. Due to this negative bias to the grid, very little or no anode current flows. Should the light weaken or be cut off, the grid bias reduces and allows more current to flow. The anode current flows through the coil of a sensitive relay. Because of this current flow, the armature of the relay is attracted

thus closing the contacts of a circuit-consisting of a battery and an electric bell or buzzer. If the photocathode is made sensitive to the infra-red or ultra-violet light, then there is no need to use visible light.

### Photo-electric relay

One of the most important uses of the photoelectric cell is to operate a relay which automatically opens or closes a local circuit. This can be employed for a variety of useful purposes.

Fig. 12.11 shows the circuit. With no light incident on the phototube, the grid  $G$ , of the triode is maintained sufficiently negative by the battery to cut off the plate current. However, when light is incident on the photoelectric cell, photoelectrons are ejected out of the cylinder  $B$ . The resulting

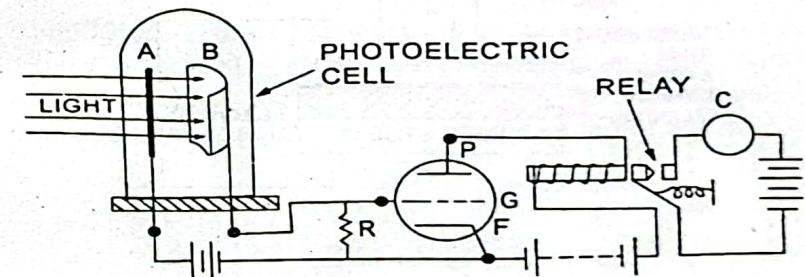


Fig. 12.11

photo-electric current flows through  $R$  making the grid of the triode valve less negative, thereby allowing the triode valve to pass current. The current flowing through the relay circuit keeps the armature of the relay attracted towards the electro-magnet. When the light is cut off, no photoelectrons are ejected and hence no photo-electric current. The grid becomes more negative in the absence of the photo-electric current and hence no current is allowed to flow in the plate circuit. The armature is no longer attracted by the electro-magnet and it springs back to make contact with  $D$ . The local circuit closes and the current flows through it. If  $C$  is a bell it starts ringing and if  $C$  is a bulb, it glows. It is therefore obvious that so long as the photo tube is illuminated, the relay remains inoperative but it starts operating immediately on interruption of the light. A number of industrial operations are performed in this way.

## Photo-voltaic cell

In the case of photo-electric cells, a positive potential is to be applied to the collector to attract the electrons and therefore an external battery is a necessity. But the photo-voltaic cell is a self-generating cell where the electrons ejected by light, themselves produce a potential difference between the two plates which causes the current to flow through the external circuit even though no external battery is applied.

The most commonly used photo voltaic cells are of the barrier layer type like iron-selenium cells or Cu - CuO<sub>2</sub> cells. In the iron-selenium cell, selenium layer is placed on an iron disc (Fig. 12.12).

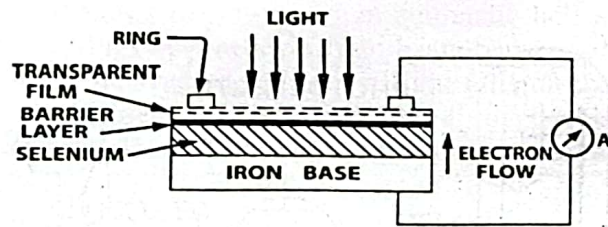


Fig. 12.12

An extremely thin semi-transparent layer of gold or silver is formed on the selenium to act as a front electrode. The barrier layer is formed by cathode-sputtering the semi-transparent film on the selenium. A contact ring on the silver layer acts as one electrode and the iron base as the other.

When light or radiation falls on the semiconductor *i.e.*, selenium, it ejects electrons which travel from selenium to the front silver electrode as shown in Fig. 12.12. As the (boundary) barrier layer acts as a rectifier, it does not permit the flow of electrons in the opposite direction. The e.m.f. generated internally between silver electrode and selenium is almost directly proportional to the incident flux.

The main advantage of a photo-voltaic cell is that *no external battery is required for its operation i.e.*, it is self-generating. Moreover, the internal e.m.f. and hence current generated by it are

large enough to be measured on a pointer galvanometer. Hence such cells can be calibrated and used in devices like *light meters* in photography and direct-reading illumination metres. With low resistance relays, photo-voltaic cells can also be used for on/off operations and other monitoring operations in industry.

## Photoconductive cell

Photoconductive cell is based on the principle that resistivity of semiconductor materials like selenium (Se), cadmium sulphide (CdS), lead sulphide (PbS) and thallium sulphide (TlS) decreases when irradiated. In other words, such materials have high *dark* resistance and low *irradiated* resistance. In some cases the resistance drops from dark values of 10 to 20 million ohms to approximately one million ohms when exposed to light.

Fig. 12.13(a) shows a simple form of photo-conductive cell using selenium. It consists of the semi-conductor material with two electrodes attached to it. When the cell is unilluminated, its resistance is so high that current through the circuit of Fig. 12.13(a) is

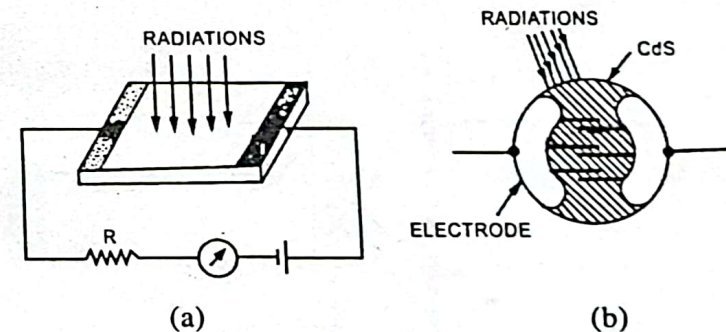


Fig. 12.13

very low. When radiation is allowed to fall on the cell, its resistance decreases and the current through the circuit becomes large. The shape of the semiconductor material is so made as to obtain a large ratio of *dark to light* resistance.

Fig. 12.13(b) shows a commonly used CdS cell. It has a very high *dark to light* ratio and gives maximum response at 5000 Å. As shown in the figure, the two electrodes are extended in an inter digital pattern in order to increase the contact area with the sensitive material.

Photoconductive cells using TIS and PbS have been used for detection of ships and aircrafts by the radiations given out by their exhausts or tunnels. They have also been used for telephony by modulated infra-red light.

### Photo-multiplier tube

Photomultiplier tube is a device in which current is amplified by electron multiplication through secondary emission. When a high energy electron strikes the surface of a metal plate, preferably an alkali metal, it knocks free additional electrons from the surface. This process is known as *secondary emission* and the electrons thus emitted are known as *secondary electrons*. In case of cesium and potassium, one primary electron can knock out about eight to ten electrons from the surface.

When light falls on the cathode or the emitter A (Fig. 12.14), photo-electrons are ejected from it. Instead of being collected by the

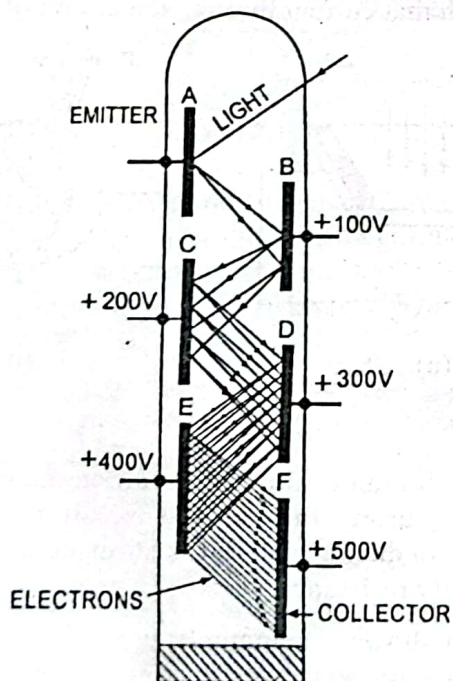


Fig. 12.14

anode, these electrons are directed towards a secondary electron emitting electrode B, usually called a *dynode*, which is more positive with respect to A by 100 volts and more electrons are ejected from it. The electrons ejected from B are then directed towards another dynode C and more electrons are ejected from it. C should be at a positive potential with respect to B. This amplification or multiplication of electrons can be repeated many times within the tube by making the electrons strike on a succession of dynodes which are maintained at increasingly positive potential as shown in the figure. As a result of this multiplication, an avalanche of electrons reaches the collector plate F resulting in a strong current flow in the outer circuit.

A photo-multiplier tube is capable of detecting a light beam of very small intensity and is mostly used in detecting faint light signals of visible, infra-red, ultra-violet and gamma radiation. The main advantage of photo-multiplier tube over the ordinary photo-electric cells lies in the fact that if each electron releases  $n$  secondary electrons, then in  $m$  stages,  $n^m$  electrons reach the collector. For example, if  $n = 5$  and  $m = 6$ , then  $n^m = 15625$  electrons. Thus there is an enormous gain over the signal obtained from an ordinary photo-electric cell.

**Example 12.1.** The work function of potassium is 2.0 eV. When ultraviolet light of wavelength  $3500 \text{ \AA}$  ( $1 \text{ \AA} = 1 \text{ Angstrom unit} = 10^{-10} \text{ m}$ ) falls on a potassium surface, what is the maximum energy in electron volts of the photoelectrons?

**Soln.**

$$T_{\max} (\text{K.E.}_{\max}) = h\nu - h\nu_0$$

Since  $h\nu_0$  is already expressed in electron volts, we need to compute the quantum energy  $h\nu$  of 3500 A.U. light.

$$\begin{aligned} h\nu &= \frac{hc}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \text{ joules-sec} \times 3 \times 10^8 \text{ m/sec}}{3500 \times 10^{-10} \text{ m}} \end{aligned}$$

$$\begin{aligned}
 &= 5.7 \times 10^{-19} \text{ joules} \\
 &= \frac{5.7 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \quad [1\text{eV} = 1.6 \times 10^{-19} \text{ joules}] \\
 &= 3.6 \text{ eV.}
 \end{aligned}$$

Hence the maximum photo-electron energy

$$\begin{aligned}
 T_{\max} &= hv - hv_0 \\
 &= 3.6 \text{ eV} - 2.0 \text{ eV} \\
 &= 1.6 \text{ eV.}
 \end{aligned}$$

**Example 12.2.** Light having a wavelength of  $5000 \text{ \AA}$  falls on a material having a photo-electric work function of  $1.90 \text{ eV}$ . Find (a) the energy of the photon in eV, (b) the kinetic energy of the most energetic photoelectron in eV and in joules, and (c) the stopping potential.

**Soln.**

$$\begin{aligned}
 \text{(a) } hv &= \frac{hc}{\lambda} \\
 &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} \text{ joules} \\
 &= 3.96 \times 10^{-19} \text{ joules} \\
 &= \frac{3.96 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \quad [1\text{eV} = 1.6 \times 10^{-19} \text{ joules}] \\
 &= 2.47 \text{ eV.}
 \end{aligned}$$

alternately,

$$\begin{aligned}
 hv &= \frac{12,400}{\lambda} \text{ eV} \\
 &= \frac{12,400}{5,000} \text{ eV} \\
 &= 2.47 \text{ eV.}
 \end{aligned}$$

$$\begin{aligned}
 T_{\max} &= hv - w_0 \\
 &= 2.47 \text{ eV} - 1.90 \text{ eV} \\
 &= 0.57 \text{ eV} \\
 &= 0.57 \times 1.6 \times 10^{-19} \text{ joules} \\
 &= 0.912 \times 10^{-19} \text{ joules.}
 \end{aligned}$$

$$T_{\max} = V_s \cdot e$$

where  $V_s$  = stopping potential

$e$  = electronic charge.

$$\therefore V_s = \frac{T_{\max}}{e} = \frac{0.57 \text{ eV}}{1 \text{ electronic charge}} = 0.57 \text{ V.}$$

**Example 12.3.** What is the threshold wavelength for a tungsten surface whose work function is  $4.5 \text{ eV}$ .

**Soln.**

$$\text{Here } w_0 = 4.5 \text{ eV} = 4.5 \times 1.6 \times 10^{-19} \text{ joules.}$$

$$\text{Again } w_0 = hv_0 = h \frac{c}{\lambda_0}$$

$$\begin{aligned}
 \text{or, } \lambda_0 &= \frac{hc}{w_0} \\
 &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{4.5 \times 1.6 \times 10^{-19}} \\
 &= 2760 \times 10^{-10} \text{ m} \\
 &= 2760 \text{ \AA.}
 \end{aligned}$$

$$\text{Alternately } w_0 = \frac{12,400}{\lambda_0}$$

$$\begin{aligned} \text{or, } \lambda_0 &= \frac{12,400}{w_0} \\ &= \frac{12,400}{4.5} \\ &= 2755 \text{ \AA}. \end{aligned}$$

**Example 12.4.** The photo-electric threshold of copper is  $3200 \text{ \AA}$ .

If ultra-violet light of wavelength  $2500 \text{ \AA}$  falls on it, find (a) the maximum kinetic energy of the photo-electrons ejected, (b) maximum velocity of the photo-electrons and (c) the value of the work function.

**Soln.**

(a) Maximum kinetic energy of photo-electrons is

$$\begin{aligned} T_{\max} &= h\nu - h\nu_0 \\ &= hc \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \\ &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} \left( \frac{1}{2500} - \frac{1}{3200} \right) \\ &= \frac{6.625 \times 3 \times 7}{25 \times 32} \times 10^{-18} \text{ joules} \\ &= \frac{6.625 \times 3 \times 7 \times 10^{-18}}{25 \times 32 \times 1.6 \times 10^{-19}} \text{ eV} \\ &= 1.087 \text{ eV}. \end{aligned}$$

Alternately

$$\begin{aligned} T_{\max} &= 12,400 \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV} \quad \text{where } \lambda \text{ and } \lambda_0 \text{ are in A.U.} \\ &= 12,400 \left( \frac{1}{2500} - \frac{1}{3200} \right) \text{ eV} \end{aligned}$$

$$= 1.087 \text{ eV}.$$

(b) Maximum velocity of the photo-electrons is given by

$$\begin{aligned} \frac{1}{2} m v_{\max}^2 &= T_{\max} \\ \therefore v_{\max} &= \sqrt{\frac{2T_{\max}}{m}} \\ &= \sqrt{\frac{2 \times 6.625 \times 3 \times 7 \times 10^{-18}}{25 \times 32 \times 9.1 \times 10^{-31}}} \\ &= 6.18 \times 10^5 \text{ m/sec}. \end{aligned}$$

(c) Work function  $w_0 = h\nu_0 = \frac{hc}{\lambda_0}$

$$\begin{aligned} &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{3200 \times 10^{-10}} \text{ joules} \\ &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{3200 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV} \\ &= 3.88 \text{ eV}. \end{aligned}$$

Alternately,  $w_0 = \frac{12,400}{\lambda_0} \text{ eV}$   $\lambda_0$  in A.U.

$$\begin{aligned} &= \frac{12,400}{3,200} \text{ eV} \\ &= 3.88 \text{ eV}. \end{aligned}$$

**Example 12.5.** A photo-electric surface has a work function of 4 eV. What is the maximum velocity of the photoelectrons emitted by light of frequency  $10^{15}$  Heriz incident on the surface.  $h = 6.6 \times 10^{-34}$  joule-sec.;  $e = 1.6 \times 10^{-19}$  coulomb;  $m = 9 \times 10^{-31}$  kg.

**Soln.**

$$w_0 = 4 \text{ eV} = 4 \times 1.6 \times 10^{-19} \text{ joules}$$

$$= 6.4 \times 10^{-19} \text{ joules}$$

$$\frac{1}{2}mv^2 = hv - w_0$$

$$= 6.6 \times 10^{-34} \times 10^{15} - 6.4 \times 10^{-19}$$

$$= 0.2 \times 10^{-19} \text{ joules}$$

$$\therefore v = \sqrt{\frac{2 \times 0.2 \times 10^{-19}}{m}}$$

$$= \sqrt{\frac{0.4 \times 10^{-19}}{9 \times 10^{-31}}}$$

$$= 2.107 \times 10^5 \text{ m/sec.}$$

**Example 12.6.** Calculate the threshold frequency and the corresponding wavelength of radiation incident on a certain metal whose work function is  $3.31 \times 10^{-19}$  J. Given, Planck's constant =  $6.62 \times 10^{-34}$  J-s.

**Soln.**

$$\text{Work function, } w_0 = hv_0$$

$$\text{or, } v_0 = \frac{w_0}{h}$$

$$\text{Here, } w_0 = 3.31 \times 10^{-19} \text{ J.}$$

$$h = 6.62 \times 10^{-34} \text{ J-s}$$

$$\therefore v_0 = \frac{3.31 \times 10^{-19} \text{ J}}{6.62 \times 10^{-34} \text{ J-s}}$$

$$= 5 \times 10^{14} \text{ Hertz}$$

$$\therefore \lambda_0 = \frac{c}{v_0} = \frac{3 \times 10^8 \text{ m/sec}}{5 \times 10^{14}}$$

$$= 6 \times 10^{-7} \text{ m} = 6000 \text{ \AA}.$$

**Example 12.7.** Photo-electrons are emitted with a maximum speed of  $7 \times 10^5$  m/sec from a metal surface when light of frequency  $8 \times 10^{14}$  kHz falls on it. What is the threshold frequency of the metal.

**Soln.**

$$T_{\text{max}} = hv - hv_0$$

$$\frac{1}{2}mv^2 = h(v - v_0)$$

$$\text{or, } \frac{1}{2} \times 9.1 \times 10^{-31} \times (7 \times 10^5)^2 = 6.6 \times 10^{-34} (8 \times 10^{14} - v_0)$$

$$\text{or, } 6.6 \times 10^{-34} v_0 = (52.8 - 22.3) \times 10^{-20}$$

$$\text{or, } v_0 = \frac{30.5 \times 10^{-20}}{6.6 \times 10^{-34}} = 4.62 \times 10^{14} \text{ Hz.}$$

**Example 12.8.** A tungsten cathode whose threshold wavelength is  $2300 \text{ \AA}$  is irradiated by ultraviolet light of wavelength  $1800 \text{ \AA}$ . Calculate (i) the work function for tungsten and (ii) the maximum energy of the photoelectrons emitted, both in electron-volts.

**Soln.**

$$(i) \text{ the work function, } w_0 = \frac{12,400}{\lambda_0} \text{ eV.}$$

where  $\lambda_0$  is expressed in Angstrom unit.

$$\therefore w_0 = \frac{12,400}{3,200} = 5.4 \text{ eV.}$$

(ii) Similarly

$$T_{\text{max}} = 12400 \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV}$$

where  $\lambda$  and  $\lambda_0$  are in Angstrom units,

$$\therefore T_{\text{max}} = 12400 \left( \frac{1}{1800} - \frac{1}{2300} \right) = 1.5 \text{ eV.}$$

**Example 12.9.** Light of wavelength  $4300 \text{ \AA}$  is incident on (a) nickel surface of work function 5 electron volts and (b) potassium surface of work function 2.3 electron volts. Find out, if electrons will be emitted, and if so, the maximum velocity of the emitted electrons in each case.

**Soln.**

(i) For the nickel surface

$$w_0 = hv_0 = 5 \text{ eV}$$

$$= 5 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore v_0 = \frac{w_0}{h} = \frac{5 \times 1.6 \times 10^{-16}}{6.625 \times 10^{-34}} \text{ Hz}$$

$$\begin{aligned} \therefore \lambda_0 &= \frac{c}{v_0} = \frac{3 \times 10^8 \times 6.624 \times 10^{-34}}{5 \times 1.6 \times 10^{-19}} \text{ m} \\ &= 2484 \times 10^{-10} \text{ m} \\ &= 2484 \text{ \AA}. \end{aligned}$$

$2484 \text{ \AA}$  is the longest wavelength at which photo-electric emission will occur at nickel surface. Since  $\lambda_0$  is less than the wavelength of the incident radiation ( $\lambda = 4300 \text{ \AA}$ ), electrons will not be emitted from nickel surface.

(ii) For the potassium surface

$$w_0 = 2.3 \text{ electron volts}$$

$$= 2.3 \times 1.6 \times 10^{-19} \text{ J.}$$

$$\begin{aligned} \therefore \lambda_0 &= \frac{c}{v_0} = \frac{ch}{w_0} = \frac{3 \times 10^8 \times 6.624 \times 10^{-34}}{2.3 \times 1.6 \times 10^{-19}} \text{ m.} \\ &= 4389 \times 10^{-10} \text{ m.} \\ &= 4389 \text{ \AA}. \end{aligned}$$

As  $\lambda_0$  is greater than  $\lambda (= 4300 \text{ \AA})$ , electrons will be emitted.

The maximum energy of the emitted electrons

$$\frac{1}{2} mv_{\max}^2 = hv - hv_0$$

The energy  $hv$  corresponding to  $\lambda = 4300 \text{ \AA}$

$$= \frac{12400}{\lambda} \text{ eV} = \frac{12400}{4300} \text{ eV} = 2.9 \text{ eV.}$$

$$\begin{aligned} \therefore \frac{1}{2} mv_{\max}^2 &= (2.9 - 2.3) \text{ eV} \\ &= 0.6 \text{ eV} = 0.6 \times 1.6 \times 10^{-19} \text{ J.} \end{aligned}$$

$$\begin{aligned} \text{or, } v_{\max} &= \sqrt{\frac{2 \times 0.6 \times 1.6 \times 10^{-19}}{m}} \\ &= \sqrt{\frac{2 \times 0.6 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \\ &= \sqrt{0.21 \times 10^{12}} = 4.6 \times 10^5 \text{ m/s.} \end{aligned}$$

**Example 12.10.** Radiant energy of wavelength  $6600 \text{ \AA}$  is falling on a layer of barium oxide at a rate of  $3 \times 10^{-8}$  watt. Calculate the number of photo-electrons yielded, assuming that every photon incident on the barium oxide layer produces photo-electric effect. Calculate also the maximum velocity of photo-electrons released by the radiation (work function of barium = 1.4 eV).

**Soln.**

Rate at which the energy is falling on the barium layer

$$= 3 \times 10^{-8} \text{ watt}$$

$$= 0.3 \times 10^{-7} \text{ joules/sec.}$$

Suppose the number of electrons emitted per second is  $n$  when an energy at the rate of  $0.3 \times 10^{-7}$  joules/sec falls on the barium layer.

$\therefore$  energy necessary for ejection of one electron =  $\frac{0.3 \times 10^{-7}}{n}$  joule

Now  $E = hv = hc/\lambda$

$$\therefore \frac{0.3}{n} \times 10^{-7} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{6600 \times 10^{-10}}$$

$$\text{or, } n = \frac{0.3 \times 66}{6.625 \times 3} \times 10^{11} \\ = 10^{11} \text{ electrons.}$$

Number of electrons yielded per sec =  $10^{11}$

The maximum velocity of the photo-electrons is given by

$$\frac{1}{2} m v_{\max}^2 = hv - w_0 \\ = \frac{hc}{\lambda} - w_0 \quad \text{where } w_0 = 1.4 \text{ eV} \\ = 1.4 \times 1.6 \times 10^{-19} \text{ J}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{6600 \times 10^{-10}} - 1.4 \times 1.6 \times 10^{-19} \\ = \frac{6.625 \times 3}{66} \times 10^{-18} - 1.4 \times 0.16 \times 10^{-18} \\ = 0.077 \times 10^{-18} \text{ Joules.}$$

$$\therefore v_{\max} = \sqrt{\frac{2 \times 0.077 \times 10^{-18}}{m}} \\ = \sqrt{\frac{2 \times 0.077 \times 10^{-18}}{9.1 \times 10^{-31}}} \\ = \left[ \frac{2 \times 0.77}{9.1} \right]^{1/2} \times 10^6 \\ = 4.1 \times 10^5 \text{ m/sec.}$$

**Example 12.11.** A surface having work function 1.51 eV is illuminated by light of wavelength 4000 Å. Calculate (i) the maximum kinetic energy of the ejected electrons and (ii) the stopping potential.

**Soln.**

The energy content of a photon having a wavelength of 4000 Å

$$= \frac{12,400}{4,000} \text{ eV} \\ = 3.1 \text{ eV.}$$

Maximum kinetic energy of electrons ejected by photons of wavelength 4000 Å

$$= hv - w_0 \\ = 3.1 - 1.51 \quad (\text{here } w_0 = 1.51 \text{ eV}) \\ = 1.59 \text{ eV} \\ = 1.59 \times 1.6 \times 10^{-19} \text{ joules} \\ = 2.544 \times 10^{-19} \text{ joules.}$$

Also,

$$K. E._{\max} = hv - w_0 = e.V_s$$

where  $V_s$  is the stopping potential in volts for these electrons

$$\therefore e.V_s = 1.59 \text{ eV}$$

$$\text{or, } V_s = \frac{1.59 \times 1.6 \times 10^{-19} \text{ J}}{e} \\ = \frac{1.59 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ V} \\ = 1.59 \text{ V.}$$

$$[\text{or, } V_s = \frac{1.59 \text{ eV}}{e} = 1.59 \text{ V}]$$

**Example 12.12.** The stopping potential for electrons emitted from a metal due to photo-electric effect is found to be 1 V for light of  $2500 \text{ \AA}$ . Calculate the work function of the metal in eV.

**Soln.**

Maximum energy of the ejected photo-electron

$$E_{\max} = e.V_s \text{ joules} \\ = V_s \text{ electron-volt}$$

where  $V_s$  is the stopping potential in volt and  $e$  is the electronic charge ( $= 1.6 \times 10^{-19} \text{ C}$ )

Hence,  $E_{\max}$  in this case

$$= V_s \\ = 1 \text{ eV.}$$

Energy content of photon of wavelength  $2500 \text{ \AA}$ .

$$= \frac{12,400}{2,500} \text{ eV} \\ = 4.96 \text{ eV.}$$

Hence, from  $E_{\max} = h\nu - w_0$ , we have

$$w_0 = h\nu - E_{\max}$$

where  $w_0$  is the required work function.

$$\therefore w_0 = 4.96 - 1 = 3.96 \text{ eV.}$$

**Example 12.13.** A photon of wavelength  $3310 \text{ \AA}$  falling on a photo-cathode ejects an electron of energy  $3 \times 10^{-19} \text{ J}$  and one of wavelength  $5000 \text{ \AA}$  ejects an electron of energy  $0.972 \times 10^{-19} \text{ J}$ . Calculate the value of Planck's constant and the threshold wavelength for the photo cathode.

**Soln.**

$$h\nu = w_0 + T$$

$$\text{or, } \frac{hc}{\lambda} = w_0 + T$$

In the first case,

$$\frac{h \times 3 \times 10^8}{3310 \times 10^{-10}} = w_0 + 3 \times 10^{-19} \quad \text{(i)}$$

In the second case,

$$\frac{h \times 3 \times 10^8}{5000 \times 10^{-10}} = w_0 + 0.972 \times 10^{-19} \quad \text{(ii)}$$

Subtracting (ii) from (i),

$$h \times 3 \times 10^{18} \left[ \frac{1}{3310} - \frac{1}{5000} \right] = [3.0 - 0.972] \times 10^{-19}$$

$$\text{or, } \frac{h \times 3 \times 10^{18} \times 1690}{1655 \times 10^4} = 2.028 \times 10^{-19}$$

$$\text{or, } h \times 0.306 \times 10^{15} = 2.028 \times 10^{-19}$$

$$\text{or, } h = \frac{2.028 \times 10^{-19}}{0.306 \times 10^{15}} = 6.627 \times 10^{-34} \text{ J-sec.}$$

Substituting this value of  $h$  in (ii), we get

$$\frac{6.627 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} = w_0 + 0.972 \times 10^{-19}$$

$$\text{or, } 3.976 \times 10^{-19} = w_0 + 0.972 \times 10^{-19}$$

$$\text{or, } w_0 = [3.976 - 0.972] \times 10^{-19} \\ = 3.002 \times 10^{-19}$$

From  $w_0 = h\nu_0 = \frac{ch}{\lambda_0}$ , we have

$$\lambda_0 = \frac{ch}{w_0} = \frac{3 \times 10^8 \times 6.627 \times 10^{-34}}{3.002 \times 10^{-19}}$$

$$\begin{aligned}
 &= 6.620 \times 10^{-7} \text{ m} \\
 &= 6620 \times 10^{-10} \text{ m} \\
 &= 6620 \text{ \AA}.
 \end{aligned}$$

**Example 12.14.** The stopping potential is 4.6 V for light of frequency  $2 \times 10^{15}$  Hz. When light of frequency  $4 \times 10^{15}$  Hz is used, the stopping potential is 12.9 V. Calculate the value of Planck's constant.

**Soln.**

$$\begin{aligned}
 h\nu &= w_0 + T_{\max} \\
 &= h\nu_0 + e.V_s
 \end{aligned}$$

where  $\nu_0$  is the threshold frequency and  $V_s$  is the stopping potential.

$$\text{or, } e.V_s = h(\nu - \nu_0)$$

In the first case,

$$4.6 e = h(2 \times 10^{15} - \nu_0) \quad \text{(i)}$$

In the second case,

$$12.9 e = h(4 \times 10^{15} - \nu_0) \quad \text{(ii)}$$

Subtracting (i) from (ii)

$$8.3 e = h \cdot 2 \times 10^{15}$$

Substituting  $e = 1.6 \times 10^{-19}$  C, we get

$$h \cdot 2 \times 10^{15} = 8.3 \times 1.6 \times 10^{-19}$$

$$\text{or, } h = \frac{8.3 \times 1.6 \times 10^{-19}}{2 \times 10^{15}}$$

$$= 6.64 \times 10^{-34} \text{ J-s.}$$

**Example 12.15.** A certain metallic surface is illuminated by monochromatic light of variable wavelength. No photoelectrons are emitted above a wavelength of  $5000 \text{ \AA}$ . With an unknown wavelength, a stopping potential of 3.1 V is necessary to stop photoelectric current. Find the unknown wavelength.

**Soln.**

Kinetic energy of the emitted electrons,

$$T = 12,400 \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV where } \lambda \text{ and } \lambda_0 \text{ are in } \text{\AA}.$$

If  $V_s$  is the stopping potential in volts, then the kinetic energy of the emitted electrons is  $V_s$  electron volt.

$$\begin{aligned}
 3.1 &= 12,400 \left( \frac{1}{\lambda} - \frac{1}{5000} \right) \\
 &= \frac{12400}{\lambda} - 2.48
 \end{aligned}$$

$$\text{or, } \frac{12400}{\lambda} = 3.1 + 2.48$$

$$\text{or, } 5.58 \lambda = 12400$$

$$\text{or, } \lambda = \frac{12400}{5.58} = 2222 \text{ \AA}.$$

**Example 12.16.** If the photo-electric threshold of metallic silver is  $3800 \text{ \AA}$  and ultraviolet light of  $\lambda = 2600 \text{ \AA}$  falls on it, find (a) maximum kinetic energy of photo-electron ejected (b) maximum velocity of photo-electrons and (c) value of work function in joules.

$$\begin{aligned}
 \text{(c) } w_0 &= \frac{12400}{3800} \\
 &= 3.26 \text{ eV.}
 \end{aligned}$$

(a) Energy of photon of wavelength  $2600 \text{ \AA}$

$$= \frac{12400}{2600} = 4.77 \text{ eV}$$

$$\begin{aligned} \therefore T_{\max} &= hv - w_0 \\ &= 4.77 - 3.26 \\ &= 1.51 \text{ eV} \\ &= 1.51 \times 1.6 \times 10^{-19} \text{ J.} \end{aligned}$$

(b)  $\frac{1}{2}mv_{\max}^2 = T_{\max}$

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times (v_{\max})^2 = 1.51 \times 1.6 \times 10^{-19}$$

$$\begin{aligned} \text{or, } v_{\max} &= \sqrt{\frac{2 \times 1.51 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \\ &= 7.28 \times 10^5 \text{ m/sec.} \end{aligned}$$

**Example 12.17.** A certain metal has a threshold wavelength of  $6000 \text{ \AA}$ . Find the stopping potential when the metal is irradiated with,

- monochromatic light of wavelength  $4000 \text{ \AA}$
- light having twice the intensity and frequency and
- if a material having double the work function were used, what would be the answers to (i) and (ii) above?

**Soln.**

(i) The maximum kinetic energy of photo-electrons is given by

$$T_{\max} = e \cdot V_s = hv - hv_0$$

where  $V_s$  is the stopping potential

$$eV_s = hv \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

$$= 6.625 \times 10^{-34} \times 3 \times 10^8 \left( \frac{1}{4000} - \frac{1}{6000} \right) \times 10^{10}$$

$$= \frac{6.625 \times 3 \times 2000 \times 10^{-16}}{4000 \times 6000} \text{ joule}$$

$$= \frac{6.625 \times 3 \times 2 \times 10^{-19}}{24 \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.03 \text{ eV.}$$

$\therefore$  stopping potential,  $V_s = 1.03$  volts.

(ii) Increase of intensity will have no effect on the stopping potential.

When the frequency of light is twice, the wavelength of light becomes half of that in (i) i.e.,  $\lambda = 4000/2 = 2000 \text{ \AA}$ . So the stopping potential is given by

$$eV_s = 6.625 \times 10^{-34} \times 3 \times 10^8 \left( \frac{1}{2000} - \frac{1}{6000} \right) \times 10^{10}$$

$$= \frac{6.625 \times 3 \times 10^{-16} \times 4000}{2000 \times 6000} \text{ joules}$$

$$= \frac{6.625 \times 12 \times 10^{-16}}{12 \times 1.69 \times 10^{-19}} \text{ eV}$$

$$= 4.14 \text{ eV}$$

$\therefore V_s = 4.14$  volts.

(iii) If the work function is doubled, then  $\lambda_0$  is reduced to half i.e.,  $\lambda_0 = 6000/2 = 3000 \text{ \AA}$ . Since the wavelength of the incident light is  $4000 \text{ \AA}$ , it would not be able to produce photo-emission.

In the second case,  $\lambda = 2000 \text{ \AA}$ .

Hence

$$\begin{aligned} e \cdot V_s &= 12400 \left( \frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV} \\ &= 12400 \left( \frac{1}{2000} - \frac{1}{3000} \right) \\ &= \frac{12400 \times 1000}{2000 \times 3000} \text{ eV} \\ &= 2.06 \text{ eV} \end{aligned}$$

Hence the stopping potential,

$$V_s = 2.06 \text{ volts.}$$

*Example 12.18.* It takes 4.2 eV to remove one of the least tightly bound electrons from a metal surface. When ultra-violet photons of a single frequency strike a metal, electrons with kinetic energies from zero to 2.6 eV are ejected. What are the energy and wavelength of the incident photons?

Soln.

$$w_0 = 4.2 \text{ eV.} \quad T_{\text{max}} = 2.6 \text{ eV.}$$

We have

$$\begin{aligned} E_{\text{photon}} = h\nu &= T_{\text{max}} + w_0 \\ &= 2.6 + 4.2 = 6.8 \text{ eV.} \end{aligned}$$

Also

$$E_{\text{photon}} = \frac{12400}{\lambda} \text{ eV}$$

$$\begin{aligned} \text{or, } \lambda &= \frac{12400}{6.8} \text{ \AA} \\ &= 1823 \text{ \AA.} \end{aligned}$$

*Example 12.19.* When violet light of  $\lambda = 4000 \text{ \AA}$  strikes the cathode of a photo-cell, a retarding potential of 0.4 eV is required to stop the emission of electrons. Find (a) the frequency of the light, (b) the energy of the wavelength, (c) work function of the surface, threshold frequency and its wavelength, the net energy and velocity with which the electron leaves the surface.

Soln.

$$(a) \quad \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{4000 \times 10^{-10}} = 7.5 \times 10^{14} \text{ Hz.}$$

$$(b) \quad E = h\nu = 6.6925 \times 10^{-34} \times 7.5 \times 10^{14} \text{ joules.}$$

$$= \frac{6.625 \times 10^{-34} \times 7.5 \times 10^{14}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 3.1 \text{ eV.}$$

$$\text{Work function, } w_0 = h\nu_0 = h\nu - \frac{1}{2} mv^2$$

$$= h\nu - eV_s \quad \left( \frac{1}{2} mv^2 = e \cdot V_s \right)$$

$$= 3.1 - 0.4 = 2.7 \text{ eV.}$$

$$\text{Threshold frequency, } \nu_0 = \frac{w_0}{h} = \frac{2.7 \times 1.6 \times 10^{-19}}{6.624 \times 10^{-34}}$$

$$= 6.53 \times 10^{14} \text{ c/s (Hz)}$$

$$\text{Corresponding wavelength, } \lambda = \frac{c}{\nu_0} = \frac{3 \times 10^8}{6.53 \times 10^{14}}$$

$$= 4590 \times 10^{-10} \text{ m}$$

$$= 4590 \text{ \AA.}$$

Energy of the emitted electron,

$$E_{\max} = h\nu - h\nu_0 \\ = 3.1 - 2.7 = 0.4 \text{ eV.}$$

Velocity of the emitted electron,

$$v = \sqrt{\frac{2E}{m}} = 1.2 \times 10^8 \text{ m/sec.}$$

*Example 12.20. Compute the number of photons of yellow light of wavelength  $6000 \text{ \AA}$  required to make an erg of energy.*

**Soln.**

Energy content of a photon,

$$E = h\nu = h \cdot \frac{c}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}} \text{ J} \\ = \frac{6.625 \times 10^{-34} \times 3 \times 10^8 \times 10^7}{6000 \times 10^{-10}} \text{ ergs} \\ = \frac{6.625 \times 3 \times 10^{-19}}{6 \times 10^{-7}}$$

$$\text{No. of photons} = \frac{1 \times 6 \times 10^{-7}}{6.625 \times 3 \times 10^{-19}} \\ = \frac{6 \times 10^{-7}}{20 \times 10^{-19}} \\ \approx 3 \times 10^{-7} \times 10^{18} \\ \approx 3 \times 10^{11}.$$

### EXERCISE

1. What is photo-electric effect? Describe an experiment for studying the phenomenon of photo-electric emission and discuss the results obtained therefrom. How has this phenomenon been explained by Einstein?
2. What is photo-electric effect? Define photo-electric work function and threshold frequency.
3. Give an account of the observed facts about photo-electric emission and indicate in a general way the difficulties encountered by the classical wave theory of light in explaining these facts.
4. What was the hypothesis on which Einstein based his photo-electric equation? Discuss how this equation provides a satisfactory explanation of the observed facts regarding photo-electric emission.
5. State and explain the laws governing photo-electric emission. Establish Einstein's photo-electric equation and show how it explains these laws.
6. Establish Einstein's photo-electric equation and show that the maximum velocity of the emitted electrons depends upon the frequency of the incident radiation and not on its intensity.
7. What is meant by stopping potential in connection with photo-electric effect? Show that the stopping potential varies linearly with the frequency of the incident radiation but is independent of its intensity.
8. Establish Einstein's photo-electric equation. Describe Millikan's method of verifying this equation.
9. Describe Millikan's experiment and show how Einstein's photo-electric equation can be verified. How would you determine the value of (i) Planck's constant, (ii) the threshold frequency and (iii) the work function by Millikan's experiment?
10. Establish Einstein's photo-electric equation. Describe an experiment to verify this equation.
11. Describe the construction, working and applications of various types of photo-cells.
12. Describe the construction and working of a photo-voltaic cell.
13. Discuss the characteristics of photo-electric emission. Establish Einstein's photo-electric equation and show how it can explain these characteristics.
14. What is photo-electric effect? State and explain the laws of photo-electricity.
15. State the laws of photo-electricity and explain these laws with the help of Einstein's theory of photo-electricity.

16. Give quantum interpretation of photo-electric emission indicating briefly the failure of classical theory and the success of the quantum theory to explain the results of photo-electric emission.
17. Find the energy content of ultra-violet light photons of wavelength 3000 Å. in (i) joules and (ii) electron-volt. [(i)  $6.62 \times 10^{-19}$  J, (ii) 4.13 eV]
18. Calculate the frequency and amount of energy (in eV) associated with a quantum of electro-magnetic radiation at each of the following wavelengths (i) 1 m (ii) 1 mm (iii) 5000 Å and (iv) 1 Å. [(i)  $3 \times 10^8$  Hz;  $1.25 \times 10^{-16}$  eV, (ii)  $3 \times 10^{11}$  Hz;  $1.25 \times 10^{-3}$  eV, (iii)  $6 \times 10^{14}$  Hz; 1.25 eV, (iv)  $3 \times 10^{18}$  Hz;  $1.25 \times 10^4$  eV]
19. Evaluate the threshold wavelength of photo-electric material whose work function is 2 eV. [ $6.21 \times 10^{-7}$  m]
20. The photo-electric threshold of copper is 3200 Å. If the ultra-violet light of wavelength 2500 Å falls on it, find (a) value of work function of copper (b) maximum kinetic energy of the ejected photo-electrons. [(a)  $6.21 \times 10^{-19}$  J, (b)  $1.739 \times 10^{-19}$  J]
21. Silver has a photo-electric function of 4.5 eV. Find (i) its threshold wavelength, (ii) maximum velocity of ejected photo-electrons with ultra-violet light of wavelength 2000 Å, (iii) stopping potential for the most energetic photo-electrons. [(i) 2760 Å, (ii)  $7.71 \times 10^5$  m/sec, (iii) 1.7 V]
22. The wavelength of the photo-electric threshold for silver is 3250 Å. Determine the maximum energy of the electrons ejected by light of wavelength 2537 Å. [ $1.625 \times 10^{-19}$  J]
23. The long wavelength limit for photo-electric effect for a certain metal is 2750 Å. Find (i) work function  $w_0$  (ii) the maximum velocity of electrons ejected from the metal by radiation of  $\lambda = 1800$  Å (iii) maximum kinetic energy of these electrons [(i) 4.5 eV (ii)  $9.1 \times 10^5$  m/s (iii)  $3.8 \times 10^{-19}$  J]
24. Calculate the frequency of the incident radiation which eject from a metal surface electrons fully retarded by a reverse potential of 3 V. The threshold frequency for this metal is  $6 \times 10^{14}$  Hz. Also find the work function of this metal. [ $13.2 \times 10^{14}$  Hz; 2.48 eV]

25. With what velocity must an electron travel so that its kinetic energy is equal to the energy of a photon of wavelength  $\lambda = 5200$  Å? [ $9.2 \times 10^5$  m/s]
26. Light of  $\lambda = 5000$  Å falls on a photo-metal having work function of 1.9 eV. Find (a) energy of the photon in eV (b) K.E. of the most energetic photo-electron in joule and eV (c) the stopping potential. [(a) 2.47 eV (b) 0.57 eV (c) 0.57 V]
27. If the photo-electric threshold of a given metal lies in the green of the visible spectrum  $\lambda = 5000$  Å, and ultra-violet light of wavelength  $\lambda = 4000$  Å falls on it, find (a) K.E. of ejected photo-electrons (b) velocity of the photo-electrons and (c) the value of the work function. [(a)  $9.8 \times 10^{-20}$  joules (b)  $4.69 \times 10^5$  m/sec (c)  $0.2 \times 10^{-19}$  joule]
28. When light of frequency  $1.3 \times 10^{15}$  c/s falls on a metal, photo-electrons are emitted with a maximum energy of 1.8 eV. Find the work function of the metal in ergs and eV. Also find the threshold frequency for the metal. [ $w_0 = 3.58$  eV,  $5.728 \times 10^{-12}$  ergs,  $v_0 = 8.05 \times 10^{14}$  cycles]
29. A metal surface when illuminated by light of frequency  $0.90 \times 10^{15}$  cycles/sec emits electrons which can be stopped by a retarding potential of 0.60 volt. When the same surface is illuminated by light of frequency  $1.26 \times 10^{15}$  cycles/sec, the required retarding potential is 2.1 volts. Deduce the value of Planck's constant and work function of the metal. [ $6.67 \times 10^{-34}$  Joule-sec,  $3.31 \times 10^{-19}$  J]
30. A certain metal has a threshold wavelength of 6526 Å. Find the stopping potential when the metal is irradiated with  
(a) monochromatic light of wavelength 3263 Å  
(b) monochromatic light having twice the frequency and three times the intensity of that in (a).  
[(a) 1.9 volt (b) 5.7 volt; no change with intensity]
31. A radiation of frequency  $10^{15}$  Hz falls on a photo-cathode and eject electrons with maximum energy of  $4.2 \times 10^{-19}$  joule. If the frequency of radiation is changed to  $5 \times 10^{14}$  Hz, the maximum energy of ejected electrons becomes  $0.9 \times 10^{-19}$  Joule. Calculate the value of Planck's constant threshold frequency and work function of photo-cathode material. [ $6.6 \times 10^{-34}$  J-sec;  $3.64 \times 10^{14}$  Hz;  $2.4 \times 10^{-19}$  J]

32. The photo-electric threshold for tungsten is 2300 Å. Find the energy of the emitted electrons from the surface by ultra-violet light of  $\lambda = 180 \text{ Å}$ . [63.58 eV]
33. When a tungsten surface is illuminated by light of wavelength 1800 Å. the maximum energy of the electrons liberated is 1.50 eV. Photo-electric emission for tungsten ceases when the wavelength of the incident light exceed 2300 Å. What is the magnitude of Planck's constant? [6.624 x 10<sup>-34</sup> J-sec]
34. Ultra-violet light of wavelength  $3000 \times 10^{-8} \text{ cm}$  falls on the surface of a metal of work function 2.28 eV and ejects an electron. What will be the velocity of the ejected electron? (Mass of electron =  $9.1 \times 10^{-28} \text{ gm}$ ,  $1 \text{ eV} = 1.6 \times 10^{-12} \text{ erg}$  and  $h = 6.6 \times 10^{-27} \text{ erg-sec}$ ). [2.55 x 10<sup>8</sup> cm/sec]
35. Find the maximum velocity of the photo-electrons when light of  $\lambda = 2.537 \times 10^{-7} \text{ m}$  falls on a metal whose work function is 4.51 eV. [3.66 x 10<sup>5</sup> m/sec]
36. When light of frequency  $5 \times 10^{14} \text{ Hz}$  falls on a metal surface, electrons of energy  $2.31 \times 10^{-19} \text{ J}$  are emitted. Find the wavelength of ultra-violet light which falling on the same surface would liberate electrons of energy  $8.93 \times 10^{-19} \text{ J}$ . ( $c = 3 \times 10^8 \text{ m/sec}$ ,  $h = 6.62 \times 10^{-34} \text{ J-s}$ ) [2 x 10<sup>-7</sup> m]
37. Radiation of wavelength 5000 Å and intensity  $2 \times 10^{-2} \text{ watt/sq.cm}$  falls on a photo-sensitive surface. Assuming that every absorbed photon results in the ejection of a photo-electron, find how many photo-electrons are produced per square centimeter per second. [5 x 10<sup>16</sup> per sq.cm]
38. A retarding potential of 0.4 eV is required to block the movement of electrons from the cathode of a photo-cell when violet light of  $\lambda = 4000 \text{ Å}$  strikes the surface. Find (a) the frequency of this light, (b) the energy of this  $\lambda$ , (c) the work function of this surface, (d) the threshold frequency, (e)  $\lambda$  for this frequency, (f) energy for  $\lambda = 3000 \text{ Å}$ , (g) net energy after the electron leaves the metal surface, (h) energy in joule, (i) the velocity of this electron after it leaves the surface. ( $h = 6.63 \times 10^{-34} \text{ J-sec} = 4.13 \times 10^{-15} \text{ eV}$ )  
 [(a)  $7.5 \times 10^{14} \text{ Hz}$  (b) 3.09 eV (c) 2.7 eV (d)  $6.53 \times 10^{14} \text{ Hz}$  (e) 4593 Å (f) 4.13 eV (g) 1.43 eV (h)  $2.29 \times 10^{-12} \text{ J}$  (i)  $7.1 \times 10^5 \text{ m/sec}$ ]

39. Find (a) the longest  $\lambda$  that can emit photo-electrons from a caesium surface for which  $w_0 = 1.35 \text{ eV}$ , (b) the maximum velocity of the emitted photo-electron when illuminated with light of  $\lambda = 4 \times 10^{-7} \text{ m}$ , and (c) the potential difference just sufficient to prevent a current from passing through a caesium photo-cell when illuminated with  $\lambda = 4 \times 10^{-7} \text{ m}$ . [(a)  $9 \times 10^{-7} \text{ m}$ , (b)  $7.9 \times 10^5 \text{ m/sec}$  (c) 1.7 volts]
40. Calculate the longest wavelength of the incident radiation which will eject electrons from a metal whose work function as 6 eV. Planck's constant  $h = 6.62 \times 10^{-34} \text{ J-s}$ . [2070 Å]
41. Calculate in electron-volt, the energy of a quantum of light of wavelength =  $5.3 \times 10^{-7} \text{ m}$ . [2.34 eV]
42. Photo-electrons with a velocity of  $4 \times 10^5 \text{ m/sec}$  are emitted from a metal surface when light of wavelength 6000 Å falls on it. What is the photo-electric threshold wavelength for the metal? [7631 Å]
43. A photon of wavelength 3210 Å falls on a photo-cathode and an electron of energy  $3 \times 10^{-19} \text{ joule}$  is ejected; if the wavelength of the incident photon is changed to 5000 Å, the energy of the ejected electron is  $0.972 \times 10^{-19} \text{ joule}$ . Calculate the value of the Planck's constant and the threshold wavelength of the photo-cathode ( $c = 3 \times 10^8 \text{ m/s}$ ) [6.62 x 10<sup>-34</sup> J.s; 6620 Å]
44. When light of wavelength  $\lambda = 5500 \text{ Å}$  falls on a surface, photoelectrons are ejected for which the stopping potential is  $V_s = 0.19 \text{ volt}$ . If the wavelength of the incident photon is changed to 1900 Å, calculate (a) the stopping potential (b) the work junction of the surface. [4.47 v; 2.07 eV;  $5 \times 10^{14} \text{ Hz}$ ]
45. The work function of molybdenum is 4.15 volts. If ultra-violet light of wavelength 1000 Å is incident upon molybdenum, find maximum velocity of the ejected photo-electrons. [1.72 x 10<sup>6</sup> m/sec.]

### 13.13 Compton Effect

In 1923 A.H. Compton, while making a spectroscopic study of X-rays scattered by matter, discovered that, when a beam of X-rays of well defined wavelength  $\lambda$  is scattered through an angle  $\theta$  by sending the radiation through the metallic foil, the scattered radiation, in addition to the expected incident wavelength  $\lambda$  contains a component of well-defined wavelength  $\lambda'$  which is longer than  $\lambda$ . This phenomenon is called the *Compton effect*.

The principle of the experiment performed by Compton is shown in Fig. 13.22 X-rays from a molybdenum target is allowed to pass through a filter which absorbs all wavelengths except  $\text{MoK}\alpha$  line. The  $\text{MoK}\alpha$  line of wavelength  $\lambda = 0.71 \text{ \AA}$  is then allowed to fall on a carbon target. The radiation scattered from the target at various

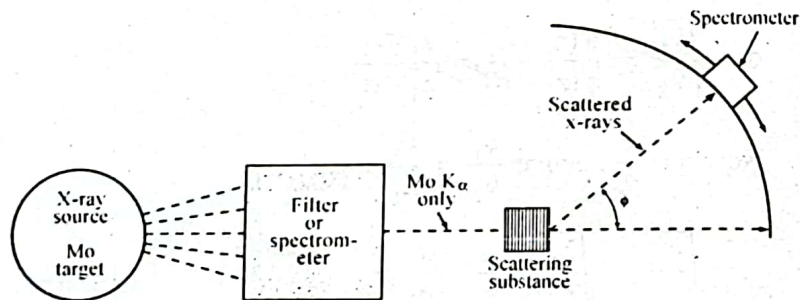


Fig. 13.22

scattering angles  $\phi$  was analyzed by a spectrometer in which the grating used was a crystal (Bragg spectrometer). The results are shown in Fig. 13.23.

At  $\phi = 0$ , the spectrum of the scattered radiation is essentially the spectrum of the radiation incident on the carbon target. At other scattering angles, two peaks can be seen in the spectrum of the scattered radiation. One is called the *unmodified* line, having the same wavelength as the incident radiation; the other, which is broader, is called the *modified* line. The wavelength of the modified line is longer than that of the incident radiation and the wavelength increment increases with the increase of the scattering angle  $\phi$ . When the angle between the incident and the scattered radiation was

$90^\circ$ , the difference in wavelength was found to be  $0.0236 \times 10^{-8} \text{ cm}$ , independent of the wavelength of the primary (incident) beam and of the nature of the scattering material.

The unmodified peak shown in Fig. 13.23 can be explained in terms of classical electrodynamics without resorting to the concept of photons as carriers of the electromagnetic radiation. To account for the presence of the shifted component and for the amount of the

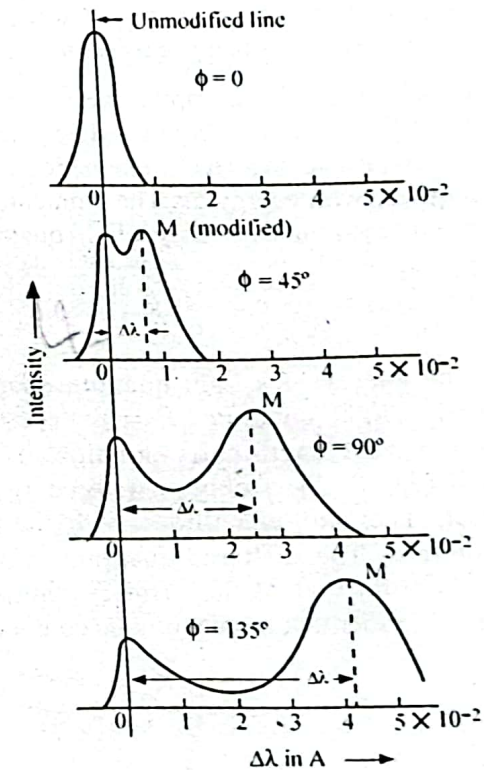


Fig. 13.23

shift, Compton showed that the *corpuscular* theory of X-rays could explain this peak. The scattering of X-rays without any change in wavelength (unmodified line) is referred to as *coherent* scattering while the scattering with change in wavelength (modified line) called *incoherent* scattering. The incoherent scattering is called the *Compton scattering*.

In order to account for change in wavelength of the incident X-rays, Compton assumed that the scattering process could be treated as an elastic collision between a photon and a free electron in which both energy and momentum are conserved. The electron which is alone effective in this act of scattering is treated as free although it is bound to the atomic configuration. This is justified if the binding energy of the electron is negligible compared to the energy of the incident photon. Thus, in Compton scattering, the electrons – especially the more loosely bound electrons can be treated as free in view of the great energy of the X-ray spectrum. The electron is further assumed here as being at rest before the collision.

The collision process in Compton scattering is schematically shown in Fig. 13.24. According to Einstein's theory of the photo-electric effect, the primary X-rays, being electromagnetic radiations like light, are propagated as quanta with energy  $h\nu$ . The momentum associated with this energy (*i.e.*, the momentum carried by a light quantum or photon) is

$$p = mv = \frac{E}{c^2} \cdot v = \frac{h\nu}{c^2} \cdot c = \frac{h\nu}{c} = \frac{h}{\lambda}$$

where  $E = h\nu$  is the energy of a light quantum of frequency  $\nu$ ,  $c$  is the velocity ( $v$ ) of a light quantum,  $E = mc^2$  is the mass energy relation and  $\lambda$  is the wavelength corresponding to the frequency  $\nu$ . When, as shown in Fig. 13.24, a photon with energy  $h\nu$  approaches an electron at rest. The photon collides with the electron and is reflected by an angle  $\phi$ . The scattered quantum or photon moves in different direction from that of the primary photon and carries a different momentum. In order that momentum be conserved, the electron

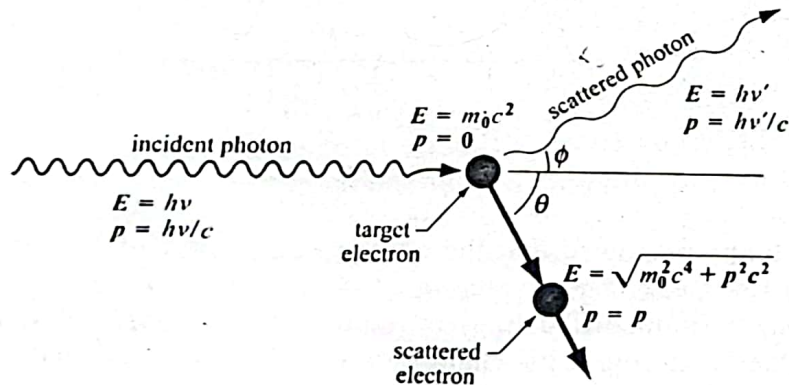


Fig. 13.24

which scatters the photon must recoil with a momentum equal to the vector difference between the momentum of incident photon and the momentum of the scattered photon. The energy of this recoiling electron must come from the primary photon. Thus the scattered photon has less energy – and hence a lower frequency or longer wavelength – than that of the primary photon. Thus

loss in photon energy = gain in electron energy

$$\text{or, } h\nu - h\nu' = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot c^2 - m_0c^2$$

or, the requirement that energy be conserved gives

$$h\nu = h\nu' + m_0c^2 \left[ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right] \quad (i)$$

where  $\nu$  is the frequency of the incident photon,  $\nu'$  that of the scattered photon,  $m_0c^2$  is the rest mass energy of the electron and  $v$  is the recoil velocity of the electron. In eqn. (i) relativistic kinetic energy of the electron is used because the velocity of the electron may be great enough for relativistic effects to be significant.

The requirement for the conservation of momentum gives two equations. In the original photon direction (x-component),

initial momentum = final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos \phi + \frac{m_0v}{\sqrt{1 - \frac{v^2}{c^2}}} \cos \theta \quad (ii)$$

and perpendicular to this direction (y-component)

initial momentum = final momentum

$$0 = \frac{h\nu'}{c} \sin \phi - \frac{m_0v}{\sqrt{1 - \frac{v^2}{c^2}}} \sin \theta \quad (iii)$$

$\phi$  is the angle between the directions of the initial and scattered photons, and  $\theta$  is that between the directions of the initial photon and recoil electron. To solve these equations, let us write  $\beta = \frac{v}{c}$ .

Eqns. i, ii and iii then become

$$h\nu = h\nu' + m_0c^2 \left[ \frac{1}{\sqrt{1-\beta^2}} - 1 \right] \quad (\text{iv})$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \phi + \frac{m_0\beta c}{\sqrt{1-\beta^2}} \cos \theta \quad (\text{v})$$

$$0 = \frac{h\nu'}{c} \sin \phi - \frac{m_0\beta c}{\sqrt{1-\beta^2}} \sin \theta \quad (\text{vi})$$

For a particular angle of scattering  $\phi$ , equations iv, v and vi contain three unknowns, namely  $\nu$ ,  $\beta$  and  $\theta$ , the expressions for which can be found by a straight forward solution of these equations. For comparison with experiment, we are, however, more interested in the shift in the wavelength. Therefore, in terms of wavelength, we write

$$\lambda = \frac{c}{\nu} \quad \text{and} \quad \lambda' = \frac{c}{\nu'}$$

for the incident and scattered radiations respectively. Eqns. iv, v and vi then become

$$\frac{h}{\lambda} - \frac{h}{\lambda'} + m_0c = \frac{m_0c}{\sqrt{1-\beta^2}} \quad (\text{vii})$$

$$\frac{h}{\lambda} - \frac{h}{\lambda'} \cos \phi = \frac{m_0\beta c}{\sqrt{1-\beta^2}} \cos \theta \quad (\text{viii})$$

$$\frac{h}{\lambda'} \sin \phi = \frac{m_0\beta c}{\sqrt{1-\beta^2}} \sin \theta \quad (\text{ix})$$

Squaring equations viii and ix and adding, we have

$$\begin{aligned} \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2 \cos \phi}{\lambda \lambda'} &= \frac{m_0^2 \beta^2 c^2}{1-\beta^2} \\ &= \frac{m_0^2 c^2}{1-\beta^2} - m_0^2 c^2 \end{aligned} \quad (\text{x})$$

Squaring eqn. (vii), we get

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda \lambda'} + 2m_0ch \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) + m_0^2 c^2 = \frac{m_0^2 c^2}{1-\beta^2} \quad (\text{xi})$$

Subtracting (x) from (xi), we get

$$\frac{2h^2}{\lambda \lambda'} (\cos \phi - 1) + 2m_0ch \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) = 0$$

$$\text{or, } 2m_0ch \left( \frac{\lambda' - \lambda}{\lambda \lambda'} \right) = - \frac{2h^2}{\lambda \lambda'} (\cos \phi - 1)$$

$$\begin{aligned} &= \frac{2h^2}{\lambda \lambda'} (1 - \cos \phi) \\ \text{or, } \Delta \lambda = \lambda' - \lambda &= \frac{h}{m_0c} (1 - \cos \phi) \end{aligned} \quad (13.13)$$

Eqn. 13.13 was derived by Arthur H. Compton in the early 1920s. The phenomenon described by this equation was first observed by him and is known as Compton effect. The equation constitutes very strong evidence in support of the quantum theory of radiation.

Eqn. 13.13 gives the change in the wavelength of a photon that is scattered through  $\phi$  by a particle of rest mass  $m_0$ ; it is independent of the wavelength of the incident photon but varies with the scattering angle  $\phi$  as  $(1 - \cos \phi)$ .

The quantity  $h/m_0c$  has the dimension of length. If the values  $h = 6.624 \times 10^{-27}$  erg-s,  $m = 9.107 \times 10^{-28}$  gm and  $c = 2.998 \times 10^{10}$  cm/sec are inserted, then

$$\begin{aligned} \Delta \lambda &= 0.0242 \times 10^{-8} (1 - \cos \phi) \text{ cm} \\ &= 0.0242 (1 - \cos \phi) \text{ \AA} \end{aligned} \quad (13.14)$$

Thus eqn. 13.14 tells us that for a given angle of scattering  $\phi$ , the wavelength of the scattered radiation should be greater than that of the incident radiation by the amount  $0.0242 (1 - \cos\phi) \text{ \AA}$ .

(i) If  $\phi = 90^\circ$ ,  $\cos\phi = 0$  and hence  $\Delta\lambda = 0.0242 \text{ \AA}$  which agrees very well with the observed value of  $0.0236 \text{ \AA}$ . The quantity  $h/m_0c$  ( $= 0.0242 \text{ \AA}$ ) is called the *Compton wavelength*.

(ii) If  $\phi = 0$ ,  $\cos\phi = 1$  and hence  $\Delta\lambda = 0$ .

(iii) When  $\phi = 180^\circ$ ,  $\cos\phi = -1$  and hence  $\Delta\lambda = \frac{2h}{m_0c} = 0.0484 \text{ \AA}$ .

Hence the greatest change in wavelength that can occur takes place when  $\phi = 180^\circ$  when the wavelength change will be twice the Compton wavelength.

The Compton wavelength will be considerably less for other particles owing to their larger rest masses. Hence the maximum wavelength change in Compton effect is  $0.0484 \text{ \AA}$  which takes place for an electron when the scattering angle is  $180^\circ$ . Changes of this magnitude are readily observable only in X-rays since the shift in wavelength for visible light is less than 0.01 per cent of the initial wavelength while for X-rays of  $\lambda = 1 \text{ \AA}$  it is several per cent. The predicted dependence of  $\Delta\lambda$  on the scattering angle  $\phi$  was also verified by experiment.

In the experimental investigation, the scattered X-rays at each angle of scattering was found to include a substantial proportion having the initial wavelength. This is not hard to understand. In deriving equation (13.13), the scattering particle was assumed to be free to move at will, a reasonable assumption since many of the electrons in matter are only loosely bound to their parent atoms. Other electrons, however, are very tightly bound and when struck by a photon, the entire atom recoils instead of the single electron. In this event the value of  $m_0$ , as used in eqn. 13.13, is that of the entire atom, usually tens of thousands of times greater than that of an electron. The resulting Compton shift is accordingly so minute as to be detectable. Further, the broadening of the

modified or Compton peak is the result of the motion of the electron in the atom. Eqn. 13.13 was derived on the basis of an electron initially at rest. If the motion of the electron is taken into account, the broadening of the modified peak can be completely explained.

### Kinetic energy of the recoil electron

The kinetic energy of the recoil electron may be obtained from the relation

$$E_r = hv - hv'$$

where  $hv$  is the energy of the incident photon and  $hv'$  is that of the scattered photon.

From eqn. 13.13 the change in wavelength in a Compton scattering is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c} (1 - \cos\phi)$$

$$\text{or, } \frac{c}{v'} - \frac{c}{v} = \frac{h}{m_0c} (1 - \cos\phi)$$

$$\frac{1}{v'} - \frac{1}{v} = \frac{h}{m_0c^2} (1 - \cos\phi)$$

$$\frac{1}{hv'} - \frac{1}{hv} = \frac{1}{m_0c^2} (1 - \cos\phi)$$

$$\frac{1}{hv'} = \frac{1}{hv} + \frac{1}{m_0c^2} (1 - \cos\phi)$$

$$= \frac{m_0c^2 + hv(1 - \cos\phi)}{hv.m_0c^2}$$

$$\text{or, } hv' = \frac{.hv.m_0c^2}{m_0c^2 + hv(1 - \cos\phi)}$$

$$= \frac{hv}{1 + \frac{hv}{m_0c^2} (1 - \cos\phi)}$$

$$= \frac{hv}{1 + \alpha(1 - \cos \varphi)}$$

$$\text{where } \alpha = \frac{hv}{m_0 c^2}$$

Hence the kinetic energy of the recoil electron is

$$E_r = hv - hv'$$

$$= hv - \frac{hv}{1 + \alpha(1 - \cos \varphi)}$$

$$= hv \left[ 1 - \frac{1}{1 + \alpha(1 - \cos \varphi)} \right]$$

$$= hv \left[ \frac{1 + \alpha(1 - \cos \varphi) - 1}{1 + \alpha(1 - \cos \varphi)} \right]$$

$$= hv \left[ \frac{\alpha(1 - \cos \varphi)}{1 + \alpha(1 - \cos \varphi)} \right]$$

$$= hv \left[ \frac{\alpha \cdot 2 \sin^2 \frac{\varphi}{2}}{1 + \alpha \cdot 2 \sin^2 \frac{\varphi}{2}} \right]$$

$$= hv \left[ \frac{2\alpha \sin^2 \frac{\varphi}{2}}{1 + 2\alpha \sin^2 \frac{\varphi}{2}} \right] \quad (13.15)$$

#### Direction of recoil electron

From eqns. ii and iii, we have

$$mvc \cos \theta = hv - hv' \cos \varphi \quad (i)$$

$$mvc \sin \theta = hv' \sin \varphi \quad (ii)$$

Dividing ii by i, we get

$$\begin{aligned} \tan \theta &= \frac{hv' \sin \varphi}{hv - hv' \cos \varphi} \\ &= \frac{v' \sin \varphi}{v - v' \cos \varphi} \end{aligned}$$

$$\text{But } v' = \frac{v}{1 + \alpha(1 - \cos \varphi)} = \frac{v}{1 + 2\alpha \sin^2 \frac{\varphi}{2}}$$

$$\therefore \tan \theta = \frac{v \sin \varphi / (1 + 2\alpha \sin^2 \frac{\varphi}{2})}{v - \frac{v \cos \varphi}{1 + 2\alpha \sin^2 \frac{\varphi}{2}}}$$

$$= \frac{\sin \varphi}{1 + 2\alpha \sin^2 \frac{\varphi}{2} - \cos \varphi}$$

$$= \frac{\sin \varphi}{(1 - \cos \varphi) + 2\alpha \sin^2 \frac{\varphi}{2}}$$

$$= \frac{2 \sin \left( \frac{\varphi}{2} \right) \cos \left( \frac{\varphi}{2} \right)}{2 \sin^2 \frac{\varphi}{2} + 2\alpha \sin^2 \frac{\varphi}{2}}$$

$$= \frac{\cos \left( \frac{\varphi}{2} \right)}{\sin \frac{\varphi}{2} + \alpha \sin \frac{\varphi}{2}}$$

$$= \frac{\cos \frac{\varphi}{2}}{(1 + \alpha) \sin \frac{\varphi}{2}}$$

$$= \frac{\cot \frac{\varphi}{2}}{1 + \alpha}$$

$$\text{But } \alpha = \frac{hv}{m_0 c^2}$$

$$\begin{aligned} \therefore \tan \theta &= \frac{\cot \frac{\phi}{2}}{1 + hv/m_0 c^2} \\ &= \frac{\cot \frac{\phi}{2}}{1 + \frac{h}{m_0 c \lambda}} \quad (\because v = \frac{c}{\lambda}) \end{aligned} \quad (13.16)$$

Eqn. 13.16 shows that the recoil angle  $\theta$  depends on the scattering angle  $\phi$ . If  $\phi = 0^\circ$ ,  $\theta = 90^\circ$  and when  $\phi = 180^\circ$ ,  $\theta = 0^\circ$ . Thus as  $\phi$  varies from  $0$  to  $180^\circ$ ,  $\theta$  varies from  $90^\circ$  to  $0^\circ$ . This indicates that electron can get recoiled only in the forward direction at angles less than  $90^\circ$ , whereas a photon can get scattered in all directions.

**Example 13.18.** Find the change in wavelength of an X-ray photon when it is scattered through an angle of  $90^\circ$  by a free electron.

Soln.

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos\phi)$$

$$\text{Here } \phi = 90^\circ \quad \text{or,} \quad \cos 90^\circ = 0$$

$$\therefore h = 6.624 \times 10^{-34} \text{ J-s}$$

$$m_0 = 9.1 \times 10^{-31} \text{ Kg}$$

$$c = 3 \times 10^8 \text{ m/sec.}$$

$$\begin{aligned} \therefore \Delta\lambda &= \frac{h}{m_0 c} = \frac{6.624 \times 10^{-34} \times 10}{9.1 \times 10^{-31} \times 3 \times 10^8} \\ &= 0.0242 \times 10^{-10} \text{ m} \\ &= 0.0242 \text{ \AA.} \end{aligned}$$

**Example 13.19.** Monochromatic X-rays of wavelength  $0.124 \text{ \AA}$  are scattered by a carbon block. Find the wavelength of X-rays scattered through  $180^\circ$ .

Soln.

Change in wavelength

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos\phi)$$

$$\text{Here } \phi = 180^\circ \quad \text{or,} \quad \cos 180^\circ = -1$$

$$\therefore \Delta\lambda = \frac{2h}{m_0 c} = \frac{2 \times 6.624 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 0.0484 \text{ \AA}$$

$$\text{Now } \Delta\lambda = \lambda' - \lambda$$

$$\text{or, } \lambda' = \lambda + \Delta\lambda$$

$$= 0.124 + 0.0484$$

$$= 0.1724 \text{ \AA.}$$

**Example 13.20.** An X-ray photon collides with an electron at rest and is scattered through  $90^\circ$ . What is its frequency after collision? Its initial frequency is  $3 \times 10^{19} \text{ Hz}$ .

Soln.

$$\text{Wavelength of the photon, } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3 \times 10^{19}}$$

$$= 10^{-11} = 0.1 \times 10^{-10} = 0.1 \text{ \AA}$$

Change in wavelength

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos\phi)$$

$$= \frac{h}{m_0 c} (1 - \cos 90^\circ)$$

$$= \frac{h}{m_0 c} = \frac{6.624 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 0.0242 \text{ \AA}$$

Now  $\Delta\lambda = \lambda' - \lambda$

$$\text{or, } \lambda' = \lambda + \Delta\lambda = 0.1 + 0.0242 = 0.1242 \text{ \AA}$$

Hence the frequency of the scattered photon

$$v' = \frac{c}{\lambda'} = \frac{3 \times 10^8}{0.1242 \times 10^{-10}} = 24 \times 10^8 \times 10^{10} = 2.4 \times 10^{19} \text{ Hz.}$$

**Example 13.21.** An X-ray photon of wavelength  $0.3 \text{ \AA}$  undergoes a  $60^\circ$  Compton scattering. Find the wavelength of the scattered photon and the kinetic energy imparted to the recoiling electron.

Soln.

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos \phi)$$

$$\text{Here } \phi = 60^\circ, \quad \therefore \cos 60^\circ = 0.5$$

$$\begin{aligned} \therefore \Delta\lambda &= \frac{6.624 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} (1 - 0.5) \\ &= 0.0121 \times 10^{-10} \text{ m} \\ &= 0.0121 \text{ \AA.} \end{aligned}$$

Now  $\Delta\lambda = \lambda' - \lambda$

$$\text{or, } \lambda' = \lambda + \Delta\lambda = 0.3 + 0.0121 = 0.3121 \text{ \AA}$$

Kinetic energy of the recoil electron

$$\text{K.E.} = kv - hv'$$

$$= h \left[ \frac{c}{\lambda} - \frac{c}{\lambda'} \right]$$

$$= h.c. \left[ \frac{1}{\lambda} - \frac{1}{\lambda'} \right]$$

$$= hc \left[ \frac{\lambda' - \lambda}{\lambda \lambda'} \right]$$

$$= 6.624 \times 10^{-34} \times 3 \times 10^8 \left[ \frac{0.0121 \times 10^{-10}}{0.3 \times 0.3121 \times 10^{-10} \times 10^{-10}} \right]$$

$$= 20 \times 10^{-26} \left[ \frac{0.0121 \times 10^{-10}}{0.3 \times 0.3121 \times 10^{-10} \times 10^{-10}} \right]$$

$$= 20 \times 10^{-26} \times 0.13 \times 10^{10} = 2.6 \times 10^{-16} \text{ J}$$

$$= \frac{2.6 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.625 \times 10^3 \text{ eV.}$$

**Example 13.22.** In Compton scattering the angle of scattering was  $30^\circ$  while the angle of recoil was  $60^\circ$ . Calculate the energy of the incident photon.

Soln.

we have

$$\tan \phi = \frac{\cot \left[ \frac{\theta}{2} \right]}{1 + \alpha} \quad \text{where } \alpha = \frac{hv}{m_0 c^2}$$

$$\text{Here } \phi = 60^\circ, \quad \text{and } \theta = 30^\circ$$

$$\text{or, } \tan 60^\circ = 1.732 \text{ and } \cot \left[ \frac{30}{2} \right] = \cot 15^\circ = 3.732.$$

$$\therefore 1.732 = \frac{3.732}{1 + \alpha}$$

$$\text{or, } 1 + \alpha = \frac{3.732}{1.732} = 2.154$$

$$\text{or, } \alpha = 1.154$$

$$\text{But } \alpha = hv/m_0 c^2$$

Hence, energy of the incident photon

$$hv = \alpha \cdot m_0 c^2$$

$$\begin{aligned}
 &= 1.154 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \\
 &= 9.45 \times 10^{-14} \text{ J} \\
 &= \frac{9.45 \times 10^{-14}}{1.6 \times 10^{-19}} \text{ eV} \\
 &= 5.9 \times 10^5 \text{ eV.}
 \end{aligned}$$

**Example 13.23.** A photon of energy  $5.1 \times 10^5 \text{ eV}$  is incident on aluminium foil. The photon is scattered at an angle of  $90^\circ$ . Calculate

- wavelength of scattered photon
- energy of recoil electron
- direction of the emitted electron

**Soln.**

$$\begin{aligned}
 \text{(i) } E &= 5.1 \times 10^5 \text{ eV} = 5.1 \times 10^5 \times 1.6 \times 10^{-19} \text{ J} \\
 &= 8.16 \times 10^{-14} \text{ J}
 \end{aligned}$$

$$E = hv = h \cdot \frac{c}{\lambda}$$

$$\begin{aligned}
 \text{or, } \lambda &= \frac{hc}{E} = \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{8.16 \times 10^{-14}} \\
 &= 2.435 \times 10^{-12} \text{ m.}
 \end{aligned}$$

$$\text{Now } \Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos \phi)$$

$$\text{Here } \phi = 90^\circ \quad \text{or,} \quad \cos 90^\circ = 0$$

$$\begin{aligned}
 \therefore \lambda' - \lambda &= \frac{h}{m_0 c} = \frac{6.624 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} \\
 &= 2.42 \times 10^{-12} \text{ m.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \lambda' &= 2.42 \times 10^{-12} + \lambda \\
 &= 2.42 \times 10^{-12} + 2.435 \times 10^{-12} \\
 &= 4.855 \times 10^{-12} \text{ m}
 \end{aligned}$$

$$= 0.04855 \text{ \AA}$$

(ii) Energy of the recoil electron

$$= hv - hv'$$

$$= hc \left[ \frac{1}{\lambda} - \frac{1}{\lambda'} \right]$$

$$= hc \left[ \frac{\lambda' - \lambda}{\lambda \lambda'} \right]$$

$$= hc \left[ \frac{\Delta\lambda}{\lambda \lambda'} \right]$$

$$= \frac{6.624 \times 10^{-34} \times 3 \times 10^8 \times 2.42 \times 10^{-12}}{2.435 \times 10^{-12} \times 4.855 \times 10^{-12}}$$

$$= 4.068 \times 10^{-14} \text{ J}$$

$$= \frac{4.068 \times 10^{-14}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 2.54 \times 10^5 \text{ eV.}$$

(iii) Direction of the recoil electron

$$\tan \phi = \frac{\cot \left[ \frac{\theta}{2} \right]}{1 + \left[ \frac{h}{m_0 c \lambda} \right]} = \frac{\cot 45^\circ}{1 + \frac{h}{m_0 c \lambda}}$$

$$= \frac{1}{1 + \left[ \frac{6.624 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8 \times 2.435 \times 10^{-12}} \right]}$$

$$= 0.50$$

$$\therefore \phi = \tan^{-1} (0.50) = 26.6^\circ.$$

**EXERCISE**

1. Give an account of the production of X-rays. Discuss the properties and applications of X-rays.
2. What is the origin of X-rays? Distinguish between continuous X-radiations and characteristic X-ray emission spectra.
3. Describe Moseley's work on X-rays. What is Moseley's law? What is its importance?
4. Describe Laue's experiment and show how it established the wave nature of X-rays.
5. What is meant by absorption co-efficient of X-rays? Obtain an expression for the intensity of X-rays as it passes through an absorbing medium. What is half-value layer?
6. Derive Bragg's law for X-ray diffraction in crystals. How is it verified?
7. Describe and explain the X-ray spectrometer method of determining wavelength of X-rays.
8. Explain how X-ray spectrometer may be used to study the structure of crystals.
9. Describe the powdered crystal method of studying crystal structure.
10. What is Compton effect? Give the theory of Compton effect and obtain an expression for the change in wavelength of a photon undergoing Compton scattering. What is Compton wavelength?
11. Obtain expressions for the Kinetic energy and direction of the recoil electron in Compton scattering.
12. An X-ray tube operates at 12 KV. Find the maximum speed of the electrons striking the anticathode. Mass of electron =  $9.1 \times 10^{-31}$  kg; charge of electron =  $1.602 \times 10^{-19}$  C. [ $6.5 \times 10^7$  ms<sup>-1</sup>]
13. NaCl has its principal planes spaced at 2.820 A.U. The first order of Bragg reflection is located at 10°. Calculate (a) the wavelength of the X-rays and (b) the angle for the second order Bragg reflection. [0.9790 A.U.; 20°19']
14. An X-ray tube is operated at 0.04 Megavolt. The radiations from the tube are analysed by a Bragg spectrometer. Calculate (a) shortest wavelength limit of the spectrum and (b) the glancing angle for the second order of this wavelength, given the crystal grating spacing = 3.036 A.U. [0.309 A.U.; 2°55']

15. The K<sub>α</sub> line from molybdenum has a wavelength of 0.7078 A.U. Calculate the wavelength of the K<sub>α</sub> line of copper. Atomic number of molybdenum = 42. Atomic number of copper = 29. [From Moseley's law,  $(Z - 1)^2 \lambda = \text{const.}$  Let  $\lambda'$  be the wavelength of K<sub>α</sub> line of copper. Then,  $(42 - 1)^2 \times 0.7078 = (29 - 1)^2 \lambda' \therefore \lambda' = 1.517$  A.U.]
16. Monochromatic X-radiation of wavelength 0.124 A.U., undergoes Compton scattering from a carbon block. Calculate the wavelength that is scattered through 180°. [0.1724 A.U.]
17. Photon of energy 1.02 MeV undergoes Compton scattering through 180°. Calculate the energy of the scattered photon.

$$[\lambda = \frac{hc}{E} = \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{1.02 \times 1.6 \times 10^{-13}} = 0.012 \text{ A.U.}]$$

$$d\lambda = 0.048 \text{ A.U.}; \lambda' = \lambda + d\lambda = (0.012 + 0.048) \text{ A.U.} = 0.060 \text{ A.U.}$$

$$\text{Energy of the scattered photon} = \frac{hc}{E} = 0.204 \text{ MeV}]$$

18. Explain why Compton effect is experimentally not observed for visible light rays. [Maximum value of  $\Delta\lambda = 0.048 \text{ \AA}$ . Thus Compton effect can be detected only for those radiations whose wavelength is not greater than a few  $\text{\AA}$ . For visible light ( $\lambda \approx 5000 \text{ \AA}$ ),  $(\Delta\lambda)_{\text{max}}$  is only 0.001% of the initial wavelength which cannot be detected].