

A TEXT BOOK
OF

SURVEYING

M. Shahjahan

M. A. Aziz

Hafiz Book Center

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OF
SURVEYING

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FACULTY OF ENGINEERING

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Published by:
Kazi Mahfuzur Rahman
Hafiz Book Center
167, Dhaka New Market
Dhaka. Cell: 01715279410

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For and on behalf of the Authors and their Successors.

First Edition : November 1965

Second Edition : February 1974

Third Edition : July 1982

Revised Reprint : April 2010

Reprint : 2013

Cover Design & Compose:

Asterisk Trading

Distributor :

Brothers Publication

Islamia Market

Nilkhet, Dhaka.

Cell: 01819440121

Price: Tk. 220.00 (Two Hundred Twenty only)

FOREWORD

The progress of education in this country has suffered for want of text books, particularly in the field of engineering. Hardly any text book on engineering subjects has been published in this country. Foreign text books are so expensive that most of our students can not afford to buy them. Attempts to supply students with foreign text books through Kinnaird Library or Reading Libraries have not been unqualified success because students can read the text books only for a period of a session when the course is offered in the class and can not refer back to them later. Low priced editions have been brought out for many books written long ago which are not considered pertinent to the present day.

*Dedicated to the sweet memory
of our beloved
MOTHERS*

Writing and publication of engineering textbooks may not be financially rewarding in this country at present. However, it is always a challenge to a teacher in a form better than that presented over before. It is in the spirit more of our teachers should endeavor to write text books, one or more in every course offered. The University will make every endeavor to assist them.

Dated Dhaka
14/11/82
M. A. Rashid, B.Sc. (Engg.)
Vice-Chancellor
Bangladesh University of
Engineering & Technology
Dhaka

FOREWORD

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For these reasons, it is very necessary that text books are written and published locally and I am very happy that two teachers of this institution have written 'A Text Book of Surveying'. I have no doubt that our students will be immensely benefited by the publication of this book.

Writing and publication of engineering textbooks may not be financially rewarding in this country at present. However, it is always a challenge to a teacher in a form better than that presented ever before. It is in this spirit more of our teachers should endeavour to write text books- one or more in every course offered. The University will make every endeavour to assist them.

Dated, Dhaka
14.11.65

M.A. Rashid, D.Sc.
Vice-Chancellor
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Dhaka.

PREFACE TO THE THIRD EDITION

In this edition, the subject matter has been revised partially and a few chapters have been enlarged. Chapter 14 which is on Hydrographic Surveying has been added new. The scope and importance of this surveying has increased considerably in Bangladesh due to hundreds of small and medium scale irrigation, navigation, siltation and hydraulic schemes. Account has been taken throughout of suggestions offered by the many users of the book and grateful acknowledgement is made to them. Further suggestions will be greatly appreciated.

Bangladesh University of
Engineering and Technology
Dhaka
July 1982

Shahjahan & Aziz

PREFACE TO THE FIRST EDITION

The book is maiden venture in the field of Engineering. It has been written primarily as a text to fill the need of the students for a simple but complete coverage of the principles of surveying. The book is designed in such a lucid way that it provides solutions to great many problems which the students are facing now a days. The book has been composed in such an elaborated and simple way that the readers do not need any previous knowledge in this field.

The authors express their heartiest thanks to Dr. M. A. Rashid, Vice-Chancellor, Bangladesh University of Engineering & Technology, Dhaka, who has kindly written the foreword of this book. Professor and Head, Civil Engineering Department for his constant inspiration, guidance and help in writing this book.

The authors also express their thanks to their reverend professors M. Kabiruddin and S. M. Nazmul Haq who inspired them to write this book by their inspirational teaching and professional guidance. Sincere thanks are also due to professor M. A. Jabbar, B. R. Biswas and M. Ali for going through and correcting some of the chapters.

The authors' thanks are also given to Dr. A. Hasmat, Dr. Zahoorul Huq, Prof. Mazharul Huq, Dr. S. H. K. Eusufzai, D. Jahidul Alam and Prof. M. A. Rauf for their inspirations and kind suggestions. The authors also express their thanks to their students A. Aiz Uddin Ahmed and Saleh Mustafa Kamal for proof reading and Mr. Md. Shaihid Udding of C. E. Department for drawing the diagrams of this book.

Bangladesh University of
Engineering and Technology

Dhaka

July 1982

Shahjahan & Aziz

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CHAPTER 1**INTRODUCTION**

1-1 Definition : Surveying is the technique of finding the relative position of different features on the surface of the earth by taking measurements and finally representing them on a sheet of paper known as plan or map. The plotting of plans, sections and maps are made by selecting a suitable scale. From these prepared plans, sections and maps, the area and volume of a particular plot of land can be calculated. A map represents the horizontal projection of the area surveyed and not the actual area. But the vertical distances can be represented more correctly by drawing of sections.

1--2 Classification : Surveying can be mainly classified into two groups, viz., (1) Plane surveying and (2) Geodetic or Trigonometrical surveying. Plane surveying deals with small areas on the earth's surface assuming the surface of the land to be plane. So the curvature of the earth is neglected. But in geodetic surveying the curvature is considered because it deals with vast areas.

Plane surveying may be again sub-divided in the following ways ;

(1) *Chain Surveying* : It is the simplest type of surveying in which the area to be surveyed is divided into a number of triangles. The lengths of the sides are measured and the interior details are recorded. The whole area is then plotted on a drawing sheet to a suitable scale to prepare a map.

(2) *Traverse Surveying (Compass and Theodolite Surveying)* : It is a type of surveying in which the plot of land to be surveyed is enclosed by a series of straight lines making

angles with one another. The length of the lines and the angles are measured and plotted with all interior details on a drawing sheet to a suitable scale to produce a map.

(3) *Plane table Surveying* : It is a method of surveying in which observations and plottings are done simultaneously.

(4) *Ordinary levelling* : It is a type of surveying in which the relative elevations of different points on the earth's surface are determined.

Geodetic surveying may be again sub-divided in the following ways :

(1) *Triangulation* : In this type of surveying a network of well-defined triangles are formed on the plot of land to be surveyed. Only one line known as base line and all other angles are measured very carefully.

(2) *Reciprocal levelling* : This type of surveying is required to obtain the difference in level between two points which are separated by obstacles.

(3) *Stadia surveying* : It is a type of surveying in which vertical and horizontal distances are computed from stadia readings.

(4) *Astronomical surveying* : It is a special branch of surveying in which the meridian, azimuth, latitude, longitude, time, etc. of a place on the surface of the earth are determined by observation of some heavenly bodies.

(5) *Photographic surveying* : This is a method of surveying in which maps are prepared from photographs.

Besides these classifications there are also the following branches of surveying :

(1) *Archaeological surveying* : To find out the relics of ancient time by unearthing.

(2) *Geological surveying* : To determine the different types of strata of rocks in the earth's crust.

(3) *Exploratory surveying* : To explore hidden minerals from below the surface of the earth's crust.

(4) *Military surveying* : To determine places of strategic importance.

(5) *Cadastral surveying* : To determine boundaries of fields, houses etc. for fixing up revenues on them.

(6) *Marine or Navigation surveying* : To determine the positions of the course of ships and harbours, etc.

(7) *Hydrographic surveying* : To determine the shore lines, soundings, navigational depth and silting of rivers and lakes, etc.

(8) *Aerial Surveying* : Aerial Surveying is comprised of taking aerial photographs and using them to obtain the qualitative information and quantitative data required. Photographic interpretation and analyses are the means by which the qualitative information is obtained. Photogrammetry, supplemented by control surveys on the ground, when necessary, is used to obtain the quantitative data by measurement procedure.

(9) *Electronic Surveying* : Electronic surveying is the precision use of light waves and microwaves to measure slope distances accurately from which, combined with elevations or accurately measured vertical angles, or both, precise horizontal distances are computed. The electronic surveying instruments are especially effective and efficient for precision measuring of long distances of less than a mile to many miles.

(10) *Topographical surveying* : To determine the positions both in plan and elevation of the different natural and artificial features of a terrain and representing them by measure of conventional symbols upon map.

(11) *Project surveying* : This includes the surveying required for roads, railways, irrigation canals, water and sewer lines, dams, reservoirs and other engineering structures.

1-3 Importance of Surveying : Surveying is of vital importance in any engineering project. The first necessity in surveying is to prepare a plan and a section of the area to be covered by the project and from these prepared maps and sections the best possible alignment, amount of

earth work and other necessary details depending upon the nature of the project can be calculated. Nobody can think of a project like railways, highways, tunneling, irrigation, dams, reservoirs, water works, sewerage works, airfields, ports, massive buildings, etc. without proper surveying. Even the measurement of land and the fixation of its boundaries cannot be done without surveying. Also the economic feasibility or the engineering feasibility of a project cannot be properly ascertained without undertaking a survey work.

1-4 Useful data and formulae for computation of survey works :

The following are the data and formulae which are generally used for computation of different types of survey works.

- 1 Furlong = 660 ft = 10 Gunter's chains = 40 poles of rods
- 1 Mile = 5280 ft. = 8 furlongs = 80 Gunter's chains = 1,609 kilometers.
- 1 Nautical Mile = 6080 ft.
- 1 Knot = 1 Nautical mile per hour.
- 1 Engineer's chain = 100 ft. = 100 links.
- 1 Gunter's chain = 66 ft. = 100 links.
- 1 Inch = 2.54 centimeters.
- 1 Meter = 100 centimeters = 3.281 ft. = 39.37 inches.
- 1 Yard = 0.9144 meter.
- 1 Fathom = 6 ft.
- Diameter of the Earth = 7916 miles
- 1 Acre = 4840 sq. yds. = 43560 sq. ft. = 10 sq. Gunter's chains
= 160 sq. poles = 3 bighas 8 chataks = 100 decimals.
= 0.4068 hectare.
- 1 sq. mile = 640 acres.
- 1 Ganda = 1 sq. hath = 2.25 sq. ft.
- 1 Chatak = 20 gandas.
- 1 Katha = 16 chataks.
- 1 Bigha = 20 kathas = 1600 sq. yards = 33.33 decimals
- 1 lb. = 16 oz.

1 Stone = 14 lbs.

1 Cwt. = 112 lbs.

1 Ton = 20 cwt. = 2240 lbs.

1 Cft. of water weights 62.4 lbs.

1 Litre of water = 1000 c. c., weighs 1 kg.

1 Gallon of water = 0.1605 cu. ft. = 4.546 litres ; weighs 10 lbs.

$$\pi = \frac{22}{7} = 3.14159$$

$$e = 2.71828$$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57^{\circ}18' \text{ (nearly)} = 3438 \text{ minutes} \\ = 206265 \text{ seconds.}$$

Trigonometrical formulae :

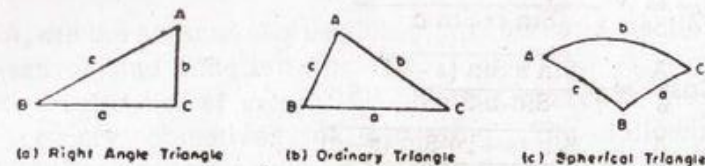


FIG. 1.1

Fig. 1.1 (a)

$$\sin B = \frac{b}{c}, \quad \cos B = \frac{a}{c}, \quad \tan B = \frac{b}{a}, \quad \cot B = \frac{a}{b},$$

$$\operatorname{cosec} B = \frac{c}{b}, \quad \operatorname{sec} B = \frac{c}{a}, \quad \sin^2 B + \cos^2 B = 1$$

$$\operatorname{sec}^2 B = 1 + \tan^2 B, \quad \operatorname{cosec}^2 B = 1 + \cot^2 B$$

Fig. 1.1 (b)

$$A + B + C = 180^{\circ}$$

$$2s = a + b + c, \quad \text{where } s = \frac{1}{2}(a + b + c) = \text{half perimeter,}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \dots \dots \text{ (Sine Rule)}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \dots \dots \text{ (Cosine Rule)}$$

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} = 2 \sqrt{\frac{s(s-a)(s-b)(s-c)}{bc}}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}; \quad \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}; \quad \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

Fig. 1.1 (c)

The sides a, b and c are expressed in angular measurements.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \dots \dots \text{(Sine Rule)}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \dots \dots \text{(Cosine Rule)}$$

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} = -\cos B \cos C$$

$$+ \sin B \sin C \cos a$$

$$\sin \frac{A}{2} = \sqrt{\frac{\sin s (s-b) \sin (s-c)}{\sin b \sin c}}$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}}$$

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}$$

Area: Area of a Rectangle = Product of two adjacent sides = Base \times Height.

Area of a Square = Base \times Height = (Base)² = (Side)².

Area of a Triangle = $\frac{1}{2}$ base \times altitude = $\sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a+b+c)$, a, b and c are the sides of a triangle.

Area of Parallelogram = Base \times Height = Product of adjacent sides \times Sine of the included angle.

Area of a Trapezium = $\frac{1}{2}$ sum of the two parallel sides \times perpendicular distance between them.

Area of Circle = $\pi r^2 = \frac{\pi d^2}{4}$ where, $d = 2r =$ diameter

Surface area of a Cylinder = $2\pi rh$ where, $h =$ height

Surface area of a Sphere = $4\pi r^2$

Area of an Ellipse = $\frac{\pi}{4}$ (Major axis \times Minor axis)

Perimeter of an Ellipse = $\frac{\pi}{2}$ (Major axis + Minor axis)

Volume:

Volume of a Rectangular Solid = Length \times Breadth \times Height

Volume of Cube = (Length)³

Volume of a Cylinder = $\pi r^2 \times h$

Volume of a Sphere = $\frac{4}{3} \pi r^3$

Volume of a Cone = $\frac{1}{3} \pi r^2 h$

Volume of a Pyramid = $\frac{1}{3}$ (Area of base \times height)

Frustum of a Cone = $\frac{h}{3} (A_1 + A_2 + \sqrt{A_1 A_2})$ where, A_1, A_2 are the area at the ends and h is the height.

Volume of Prismoid = $\frac{h}{6} (A_1 + 4A_2 + A_3)$, where A_1, A_2 and A_3 are the areas at the beginning, mid and end sections respectively and h the length.

1-5 Calculation of areas: Calculation of area is one of the primary objectives of surveying. The following methods are generally applied to compute areas.

A. For figures bounded by straight lines:

(1) By dividing the area into a number of triangles: In this method, the area is divided into a number of triangles.

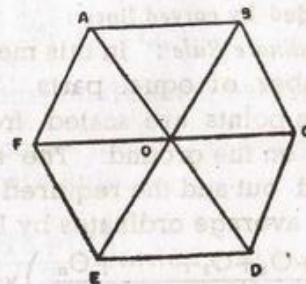


FIG.- P2

Area of each triangle is calculated. Total area will be equal to the sum of areas of individual triangles. The

lengths and directions of each lines OA, OB,.....etc. are measured in the field and area of each triangle is computed. This method is suitable for small works where closing error is not important. The accuracy of field work may be determined by measuring the diagonal in the field.

(2) By co-ordinate methods : If $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$etc are the co ordinates of A, B, C, D.....etc. respectively, the area of the traverse is calculated from the formula :

$$\text{Area} = \frac{1}{2} \{ y_1(x_2 - x_n) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + \dots + y_n(x_1 - x_{n-1}) \}$$

where n = number of stations.

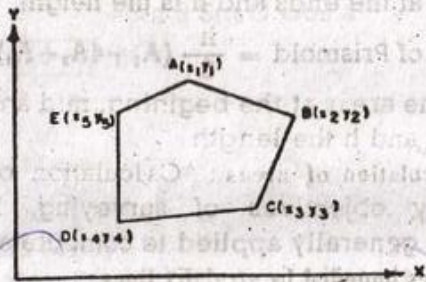


FIG. 1.3

(3) By latitude and departure : See Chapter 3.

B. For figures bounded by curved lines :

(1) Average Ordinate Rule : In this method the area is divided into a number of equal parts. The ordinates at each of the division points are scaled from the plan or measured directly on the ground. The average of these ordinates are found out and the required area is calculated by multiplying the average ordinates by length.

$$\text{Area} = \left(\frac{O_0 + O_1 + O_2 + \dots + O_n}{n+1} \right) \times l$$

where, O_n, O_1, O_2 , etc. the ordinates at each point of division, l = length of the base line.

n = number of equal division into which the base has been divided.

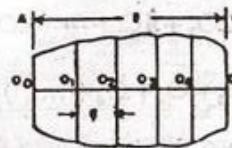


FIG. 1.2

Fig. 1.4

Example : The following offsets were taken at an interval of 20 ft. from a line to an irregular fencing. Calculate the area bounded between the chain line and fencing.

15.2, 18.7, 20.8, 16.4, 14.3, 17.5, 16.2.

Solution :

$$l = 6 \times 20 = 120 \text{ ft.}, n = 6$$

$$\text{Area} = \left(\frac{15.2 + 18.7 + 20.8 + 16.4 + 14.3 + 17.5 + 16.2}{6+1} \right) \times 120$$

$$= 2041.714 \text{ sq. ft.}$$

(2) Trapezoidal Rule : In this method the area is divided into a number of trapezoids i. e., the boundaries between the extremities of the ordinates are assumed to be straight lines. This method is more accurate than the first one.

$$\text{Area} = \left(\frac{O_0 + 2O_1 + 2O_2 + \dots + 2O_{n-1} + O_n}{2} \right) \times \frac{l}{n}$$

$$= \frac{d}{2} (O_0 + 2O_1 + 2O_2 + \dots + 2O_{n-1} + O_n)$$

$$= d \left(\frac{O_0 + O_n}{2} + O_1 + O_2 + \dots + O_{n-1} \right)$$

where d = Common distance between the ordinates.

Example : Calculate the area of the problem given in the Average Ordinate Rule.

Solution :

$$\text{Area} = 20 \left(\frac{15.2 + 16.2}{2} + 18.7 + 20.8 + 16.4 + 14.3 + 17.5 \right)$$

$$= 2068 \text{ sq. ft.}$$

(3) Simpson's Rule : According to Simpson's Rule, area is given by the formula,

$$\begin{aligned} \text{Area} &= \frac{d}{3} (O_0 + 4O_1 + 2O_2 + 4O_3 + \dots + 2O_{n-2} + 4O_{n-1} + O_n) \\ &= \frac{d}{3} \{ O_0 + O_n + 2(O_2 + O_4 + O_6 + \dots) + 4(O_1 + O_3 + \dots) \} \\ &= \frac{\text{Common distance}}{3} \{ \text{Ist. ordinate} + \text{Last ordinate} + 2 \\ &\quad (\text{sum of even ordinates}) + 4(\text{Sum of odd ordinates.}) \} \end{aligned}$$

The rule is applicable to a figure which is divided into even number of strips having odd number of ordinates. If there be an odd number of strips having even number of ordinates, the area of the result obtained by applying Simpson's Rule to the remaining even number of strips. Simpson's Rule is more accurate than the other two methods.

Example : Calculate the area of the problem given in the Average Ordinate Rule.

Solution :

$$\begin{aligned} \text{Area} &= \frac{20}{3} \{ (15.2 + 16.2) + 2(20.8 + 14.3) + 4(18.7 + 16.4 + 17.4) \} \\ &= 2080 \text{ sq. ft.} \end{aligned}$$

Example : Calculate the area by the Simpson's Rule from the following data :

Chainage in ft. :	0	10	20	30	40	50	60	70
Offset in ft. :	8.0	9.5	10.2	9.8	10.6	11.4	8.7	7.0

Solution :

There are odd number of strips (7 in number) and even of ordinates (8 in number). So the area of the first six strips should be calculated by the Simpson's Rule and that of the last one separately.

$$\text{Area of the last strip} = \frac{1}{2}(8.7 + 7.0) 10 = 78.5 \text{ sq. ft.}$$

$$\begin{aligned} \text{Area of the first six strips} &= \frac{10}{3} \{ (8 + 8.7 + 2(10.2 + 12.8) \\ &\quad + 4(9.5 + 9.8 + 11.4)) \} = 617 \text{ sq. ft.} \end{aligned}$$

Total area = 78.5 + 617 = 695.5 sq. ft.

(4) **Mid-Ordinate Rule :** The boundaries between the extremities of the ordinates are assumed to be straight lines. The base line is divided into a number of divisions and the ordinates are measured at the mid-points of each division.

Area = Average ordinate X Length of base

$$= \frac{O_1 + O_2 + O_3 + \dots + O_n}{n} \times L$$

$$= (O_1 + O_2 + O_3 + \dots + O_n) \times d$$

$$= d \sum O$$

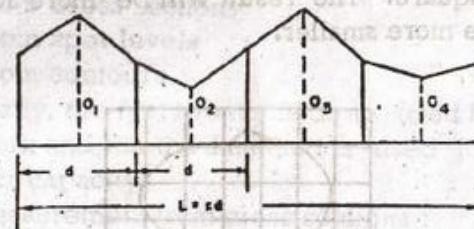


FIG. 1-5

(5) **Offsets at irregular interval :** The total area is calculated by adding together the area of individual trapezoids.

$$\text{Area} = \frac{d_1}{2} (O_1 + O_2) + \frac{d_2}{2} (O_2 + O_3) + \frac{d_3}{2} (O_3 + O_4) + \dots$$

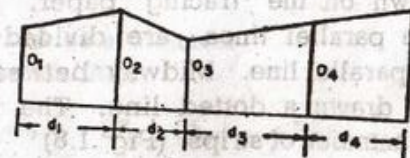


FIG. 1-6

C. **Computation of area from map :**

(1) **By division of the area into geometric figures :** The area on the map is divided into a common geometric figures like triangle, rectangle, square, trapezoid etc. The required lengths are sealed off from the map and area of the individual figures are calculated by using the usual

formulae. Total area is equal to the summation of areas of individual figures.

(2) *By sub-division into squares* : Small squares of known area are drawn on a tracing paper to the same scale as that of the map. This tracing paper is placed on the map and the number of squares enclosed in the figure is counted. The portions of the fractional squares at curved boundary are estimated. The total area of the figure will be equal to the number of squares multiplied by the factor represented by each square. The result will be more accurate if the squares are more smaller.

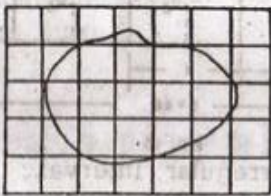


FIG. 1.7

(3) *By division into trapezoids* : A tracing paper is placed on the map. Two parallel lines, enclosing the figure on the map, are drawn on the tracing paper. The distance between these parallel lines are divided by a number of equidistant parallel line. Midway between each pair of lines there is drawn a dotted line. The figure is thus divided into a number of strips. (Fig 1.8)

Area = summation of area of individual strips.

= summation of (mid ordinate X breadth).

= (sum of the length of dotted lines intercepted within the map) X breadth.

(4) *Area by planimeter* : See Chapter 6.

1-6 Calculation of Volumes

The following are the general methods of calculating

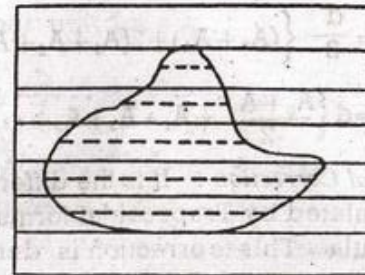


FIG. 1.8

volumes—

- (i) From cross-sections
- (ii) From spot levels
- (iii) From contours.

Generally, the first two methods are used for calculation of earth work and the third method is used for calculation of reservoir capacity.

(i) *Measurement from cross-sections* :

The total volume is divided into a number of solids by planes of cross-sections. The spacing of the sections depends upon the character of the ground and the accuracy required. The cross-sectional areas are calculated from the standard formulae discussed in Art 1-7, and the volumes are calculated by either prismoidal rule or Trapezoidal rule.

1-6 Calculation of Volumes

(1) *Prismoidal Rule*

$$= \frac{d}{3} \left\{ A_0 + 4A_1 + 2A_2 + 4A_3 + \dots + 2A_{n-2} + 4A_{n-1} + A_n \right\}$$

$$= \frac{d}{3} \left\{ A_0 + A_n + 4(A_1 + A_3 + A_5 + \dots) + 2(A_2 + A_4 + \dots) \right\}$$

where d = the distance between the cross sectional areas.

In the Prismoidal Rule, the number of sections should be odd. In case of even number of sections, the volume of the last section is calculated separately by Trapezoidal Rule and the remaining sections by Prismoidal Rule.

(2) Trapezoidal Rule

$$\text{Volume} = \frac{d}{2} \left\{ (A_0 + A_n) + 2(A_1 + A_2 + A_3 + \dots + A_{n-1}) \right\}$$

$$= d \left\{ \frac{A_0 + A_n}{2} + A_1 + A_2 + A_3 + \dots + A_{n-1} \right\}$$

Prismoidal Correction: It is the difference between the volume calculated by Trapezoidal formula and that by Prismoidal formula. This correction is denoted by C_p . The volume by Prismoidal formula is less than that by Trapezoidal formula. This correction should be subtracted from the volume determined by Trapezoidal formula. In special cases it is added.

Curvature correction (C_c): In the Prismoidal and the Trapezoidal Rules, the end sections are assumed to be in parallel planes. If the centre line of cutting or an embankment is curved in plan, the volume is calculated assuming the end sections in parallel planes and then the correction for curvature is applied. Sometimes the correction is applied to the cross-sectional areas and then Prismoidal and Trapezoidal rules are used.

(a) Level Section: No correction is necessary.

(b) Two level section and three level section:

$$C_c = \frac{1}{6R} (w_1^2 - w_2^2) \left(d + \frac{b}{2s} \right)$$

where R = radius of curvature

L = distance between two cross-sections.

(For other notions See Fig. 1-10)

(c) For a two level section, the curvature correction to

the area is $= \frac{Ae}{R}$ per unit length

where e = horizontal distance from the centre line to the centroid of the area

$$= \frac{w_1 w_2 (w_1 + w_2)}{3AS} \quad (\text{For notations See Fig. 1-10})$$

(d) For side hill two level section

$$C_c = \frac{Ae}{R} \text{ per unit length}$$

where $e = \frac{1}{3} (w_1 + \frac{b}{2} - sd)$ For larger area

and $e = \frac{1}{3} (w_2 + \frac{b}{2} + sd)$ For smaller area

(For notations See Fig. 1-11)

1-7 Calculation of Cross-sectional Areas: The cross sections are calculated from the following different conditions:

(1) Level Section: When the ground is levelled in the transverse direction or laterally.

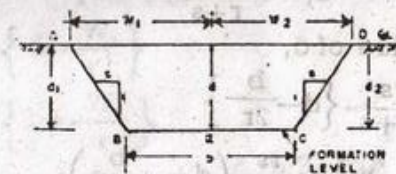


Fig. 1.9

In Fig. 1.9.

b = width at formation level.

d = depth of cut or fill at the centre E .

d_1 and d_2 = depths of cut or fill at the edges or toe points or side heights.

W_1 and W_2 = side widths or half breadths or horizontal distances from the central line to the intersection of the side slope with the original ground level.

$s:1$ = Side slope where s horizontal and 1 vertical.

$r:1$ = Transverse slope of the original ground.

Area of the section = Area of ABCD.

$$= \left\{ \frac{b + (b + 2sd)}{2} \right\} d$$

$$= d(b + sd) \quad (\because d = d_1 = d_2 \text{ and } W_1 = W_2)$$

(2) Two level Section: When the ground slopes laterally but does not cut the formation level.

INTRODUCTION

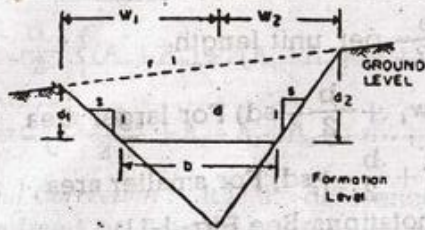


Fig. 1.10

$$W_1 = b/2 + sd_1; \text{ Now } \theta = \frac{1}{r} = \frac{d-d_1}{b/2 + sd_1}$$

$$\therefore d_1 = \frac{dr - b/2}{r+s}$$

Putting the value of d_1

$$W_1 = b/2 + \frac{rs}{r+s} \left\{ d - \frac{b}{2r} \right\}$$

$$\text{Similarly, } W_2 = b/2 + \frac{rs}{r-s} \left(d + \frac{b}{2r} \right)$$

$$\text{Again, } d_1 = \frac{dr - b/2}{r+s}$$

$$rd_1 + sd_1 = dr - b/2$$

$$\therefore rd_1 = dr - (b/2 + sd_1) \\ = dr - W_1$$

$$\therefore d_1 = d - \frac{W_1}{r}$$

$$\text{Similarly, } d_2 = \left(d + \frac{W_2}{r} \right)$$

$$\text{Area} = \frac{1}{2} \left\{ (W_1 + W_2) \left(d + \frac{b}{2s} \right) - \frac{b_2}{2s} \right\}$$

$$= \frac{s(b/2)^2 + r^2bd + r^2sd^2}{(r^2 - s^2)}$$

(3) Side hill two level section—When the ground slopes laterally and cuts the formation level in such a way that one portion is in cut while the other in fill.

INTRODUCTION

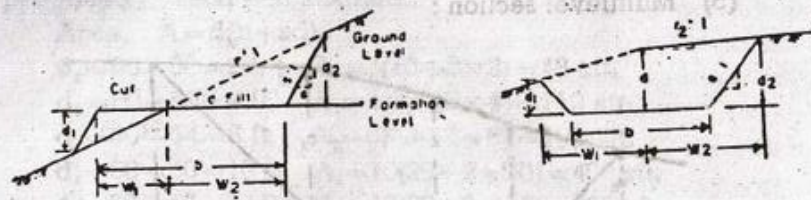


FIG. 1.11

In Fig. 1.11.

$$W_1 = \frac{b}{2} + \frac{rs}{(r-s)} \left\{ \frac{b}{2r} - d \right\}$$

$$W_2 = b/2 + \frac{rs}{r-s} \left\{ \frac{b}{2r} + b \right\}$$

$$d_1 = \left\{ d - \frac{W_1}{r} \right\}$$

$$d_2 = \left\{ b + \frac{W_2}{r} \right\}$$

$$\therefore \text{Area of cutting} = \frac{1}{2} \left\{ \frac{(b/2 - rd)^2}{(r-s)} \right\}$$

$$\text{Area of filling} = \frac{1}{2} \left\{ \frac{(b/2 + rd)^2}{(r-s)} \right\}$$

(4) Three level section: When the lateral or transverse slope of the ground is not uniform. The expressions for W_1 or W_2 can be applied to both side widths whether the ground falls or rises from the centre to both sides.

In Fig. 1.12

$$W_1 = \frac{r_1 s}{r_1 + s} \left\{ d + \frac{b}{2s} \right\}$$

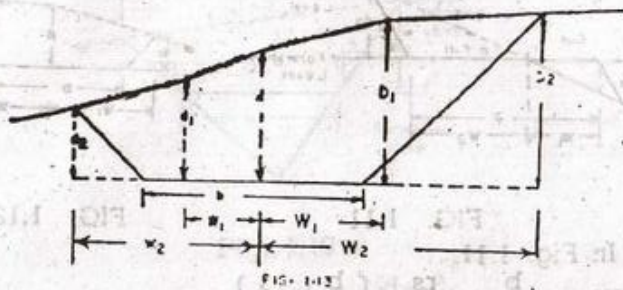
$$W_2 = \frac{r_2 s}{r_2 - s} \left\{ d + \frac{b}{2s} \right\}$$

$$\therefore \text{Area} = \left\{ \frac{1}{2} d (W_1 + W_2) + \frac{b}{4} (d_1 + d_2) \right\}$$

$$d_1 = \left\{ d - \frac{W_1}{r_1} \right\}$$

$$d_2 = \left\{ d + \frac{W_2}{r_2} \right\}$$

(5) Multilevel section :



$$\text{Area} = \frac{1}{2} [d_2 (b/2 - w_1) + d_1 w_2 + d (w_1 + w_2) - D_1 W_2 + D_2 (b/2 - W_1)]$$

Example : Calculate the volume of earth work necessary for a portion of an irrigation canal from the following data :

Chainage : 0 100 200 300 400 500 in. ft.

Area : 850 875 860 855 860 865 in. sq. ft.

Solution :

Volume by Trapezoidal Rule :

$$V = 100 \left\{ \frac{850+865}{2} + 875 + 860 + 855 + 860 \right\} = 430750 \text{ cft.}$$

Volume by Prismoidal Rule :

Volume for the first five cross-sections

$$= \frac{100}{3} \left\{ (850+860) + 4(875+855) + 2(860) \right\} = 345000 \text{ cft.}$$

Volume of the last cross-section = $\frac{1}{2}(860+855)100 = 86250$ cubic ft. Total Volume = $345000 + 86250 = 431250$ cft.

Example : A coastal embankment at a constant reduced level of 60.00 is to be constructed. The transverse ground is levelled. The following are the levels of the ground surface along the alignment at 50 ft. interval. The width of the formation level is 20 ft with a side slope of 2 : 1.

Chainage : 0 50 100 150 200 250 300 350 400
Surface Level : 58 56 54 50 48 52 57 58 54

Calculate the amount of earth work to construct the proposed coastal embankment.

Area, $A = d(b + sd)$

$$d_0 = 60 - 58 = 2 \text{ ft. ; } A_0 = 2(20 + 2 \times 2) = 48 \text{ sft.}$$

$$d_1 = 60 - 56 = 4 \text{ ft. , } A_1 = 4(20 + 2 \times 4) = 112 \text{ sft.}$$

$$d_2 = 60 - 54 = 6 \text{ ft. , } A_2 = 6(20 + 2 \times 6) = 192 \text{ sft.}$$

$$d_3 = 60 - 50 = 10 \text{ ft. , } A_3 = 10(20 + 2 \times 10) = 400 \text{ sft.}$$

$$d_4 = 60 - 48 = 12 \text{ ft. , } A_4 = 12(20 + 2 \times 12) = 528 \text{ sft.}$$

$$d_5 = 60 - 52 = 8 \text{ ft. , } A_5 = 8(20 + 2 \times 8) = 288 \text{ sft.}$$

$$d_6 = 60 - 57 = 3 \text{ ft. , } A_6 = 3(20 + 2 \times 3) = 78 \text{ sft.}$$

$$d_7 = 60 - 58 = 2 \text{ ft. , } A_7 = 2(20 + 2 \times 2) = 43 \text{ sft.}$$

$$d_8 = 60 - 54 = 6 \text{ ft. , } A_8 = 6(20 + 2 \times 6) = 192 \text{ sft.}$$

Volume by Trapezoidal Rule

$$V = 50 \left\{ \frac{48+182}{2} + 112 + 192 + 400 + 528 + 288 + 78 + 43 \right\} = 88300 \text{ cubic ft.}$$

Volume by Prismoidal Rule

$$V = \frac{50}{3} \left\{ 48 + 192 + 4(112 + 400 + 288 + 48) + 2(192 + 528 + 78) \right\} = 87133.50 \text{ cubic ft.}$$

Prismoidal correction $C_p = 88300 - 87133.54 = 1166.50$ cu. ft.

Correct volume = 87133.50 cu. ft.

Example : An irrigation canal of 2000 ft. long with 20 ft. formation level is to be constructed. The canal has a side slope of 1 : 1 and the land has an uniform slope of 1 in 5. The average depth of cutting is 8 ft. Calculate the volume of cutting.

$d = 8$ ft., $b = 20$ ft., $s : 1 = 1 : 1$, $r : 1 = 5 : 1$

$$\text{Area} = \frac{s(b/2)^2 + r^2bd + r^2sd^2}{r^2 - s^2} = \frac{1(10)^2 + 5^2 \times 20 \times 8 + 5^2 \times 1 \times 8^2}{5^2 - 1^2} = \frac{100 + 4000 + 1600}{24} = 237.5 \text{ sq. ft.}$$

Volume = 237.5×2000

$$= 475000 \text{ cubic ft.}$$

Example : A road 1 mile long having a width of 20 ft. at the formation level is to be constructed. The road has a side slope at 1:1 and the land has an uniform slope of 1.8. The average depth is two ft. with one portion of the road under cut and the other in fill (Fig. 1.5). If the material in cutting is utilised in filling, find the net volume.

$$b=20 \text{ ft.}, r=8, s=1, d=2 \text{ ft.}$$

$$\text{Area of cutting} = \frac{1}{2} \left\{ \frac{b/2 - rd)^2}{r-s} \right\} = \frac{1}{2} \left\{ \frac{(10-8 \times 2)^2}{8-1} \right\} = 2.57 \text{ sft}$$

$$\text{Area of filling} = \left\{ \frac{b/2 + rd)^2}{r-s} \right\} = \frac{1}{2} \left\{ \frac{(10+8 \times 2)^2}{8-1} \right\} = 48.2 \text{ sft.}$$

$$\text{Net area in filling} = 48.2 - 2.55 = 45.65 \text{ sft.}$$

$$\begin{aligned} \text{Net Volume of filling} &= 45.65 \times 5280 \\ &= 241500 \text{ cft.} \end{aligned}$$

Example : The following data refer to a certain portion of a proposed road. Calculate the volume of filling.

Chainage Height of filling Transverse slope of land.

0	6 ft.	1:10
50	8 ft.	1:7
100	10 ft.	1:8
150	9 ft.	1:6
200	5 ft.	1:5

Width of formation = 20 ft. Side slope = 1:1

$$A_0 = \frac{1(10)^2 + 10^2 \times 20 \times 6 + 10^2 \times 1 \times 6^2}{10^2 - 1^2} = 158.5 \text{ sft}$$

$$A_{50} = \frac{1(10)^2 + 7^2 \times 20 \times 8 + 7^2 \times 1 \times 8^2}{7^2 - 1^2} = 230.5 \text{ fsft.}$$

$$A_{100} = \frac{1(10)^2 + 8^2 \times 20 \times 10 + 8^2 \times 1 \times 10^2}{8^2 - 1^2} = 306.2 \text{ sft.}$$

$$A_{150} = \frac{1(12)^2 + 6^2 \times 20 \times 9 + 6^2 \times 1 \times 9^2}{6^2 - 1^2} = 271.0 \text{ sft}$$

$$A_{200} = \frac{1(10)^2 + 5^2 \times 20 \times 5 + 5^2 \times 1 \times 5^2}{5^2 - 1^2} = 134.5 \text{ sft}$$

Using Prismoidal formula :

$$\begin{aligned} \text{Volume} &= \frac{50}{3} \left\{ 153.5 + 134.5 + 4(230.5 + 271.0) + (306.0) \right\} \\ &= 48516.5 \text{ cft.} \end{aligned}$$

1-8 Calculation of Volume of Earth work from Spot Levels :
In this method the whole irregular figure is divided into regular figures of triangles and parallelograms. The spot levels at each corner will be known. The volume is obtained as follows :

(a) when the area is a triangle;

Volume = Area of triangle $\times \frac{1}{3}$ of the sum of depths

(b) when the area is a parallelogram;

Volume = Area of parallelogram $\times \frac{1}{4}$ of the sum of depths.

Example : Calculate the amount of earth with respect to the lowest R.L. of the plot of land shown in Fig. 1.14.

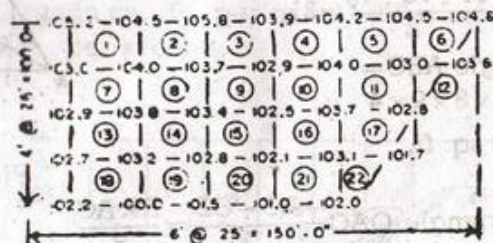


Fig. 1.14

Taking the average of the reduced levels at the corners for different plot and subtracting 100 (lowest R.L.) from these, the following heights are obtained.

$d_1=3.625 \text{ ft.}, d_2=3.95 \text{ ft.}, d_3=3.05 \text{ ft.}, d_4=37.5 \text{ ft.}, d_5=3.925 \text{ ft.},$
 $d_6=4.025 \text{ ft.}, d_7=3.675 \text{ ft.}, d_8=3.725 \text{ ft.}, d_9=3.125 \text{ ft.},$
 $d_{10}=3.275 \text{ ft.}, d_{11}=3.375 \text{ ft.}, d_{12}=3.20 \text{ ft.}, d_{13}=3.15 \text{ ft.}, d_{14}=$
 $3.08 \text{ ft.}, d_{15}=2.70 \text{ ft.}, d_{16}=2.85 \text{ ft.}, d_{17}=2.825 \text{ ft.}, d_{18}=2.025$
 $\text{ft.}, d_{19}=1.875 \text{ ft.}, d_{20}=1.85 \text{ ft.}, d_{21}=1.55 \text{ ft.}, d_{22}=2.27 \text{ ft.}$

Total Volume = $(25 \times 25) (3.625 + 3.95 + 3.05 + 3.75 + 3.92 + 3.675$
 $+ 3.725 + 3.125 + 3.275 + 3.375 + 3.15 + 3.05 + 2.70 + 2.85 +$

$$2.025 + 1.875 + 1.85 + 1.55 + \left\{ \frac{25 + 12.5}{2} \times 25 \right\}$$

$$(4.025 + 2.825) \times \left\{ \frac{32.5}{2} \times 25 \right\} (3.20 + 2.27)$$

$$= 34100 + 3430 + 856 = 38386 \text{ cft.}$$

Additional Examples :

1. A triangular plot of ground has sides 12, 13 and 17 ft. The area is to be paved except for a circular area touching each side. Calculate the area to be paved.

Solution :

In the triangle ABC, the lines bisecting the angles meet at O, the centre of the inscribed circle. OD is the radius.

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b and c are the sides and

$$s = \frac{a+b+c}{2} = \frac{17+13+12}{2} = 21$$

$$\text{Area of triangle ABC} =$$

$$= \sqrt{21 \times 8 \times 9 \times 4}$$

$$= 77.76 \text{ sq. ft.}$$

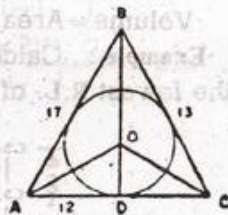


Fig. 1.15

$$\text{Area of triangle OAC} = \frac{AC \times OD}{2} = \frac{R \times AC}{2}$$

Similarly, areas of triangles OAB and OBC are

$$\frac{R \times AB}{2} \text{ and } \frac{R \times BC}{2}$$

$$\therefore \text{total area of triangle ABC} = \frac{R}{2} (AC+AB+BC)$$

$$= R \left\{ \frac{AC+AB+BC}{2} \right\} = Rs, \text{ where } R = \text{radius of the circle.}$$

$$R = \frac{\text{area of triangle}}{s} = \frac{77.76}{21}$$

$$R = 3.703 \text{ ft.}$$

$$\text{Area of circle} = 3.142 \times (3.703)^2$$

$$= 43.10 \text{ sq. ft.}$$

$$\text{Difference of areas} = 77.76 - 43.10$$

$$= 34.66 \text{ sq. ft.}$$

2. A cutting has been formed to provide a watercourse. Readings from a level on to staff positions across the cutting are as follows :

Position of staff	Readings	Remarks
1	2.50	proposed water level
2	2.90	
3	3.56	
4	4.20	
5	4.00	
6	3.10	
7	2.50	proposed water level

The readings were taken at 6 ft. intervals.

Calculate the cross-sectional area of the watercourse and the discharge in cu. ft. per minute if the mean velocity is to be 0.75 ft. per second.

Solution :

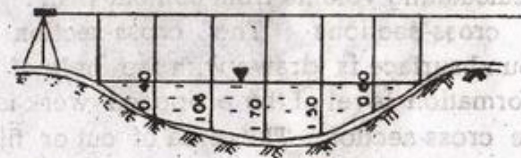


Fig. 1.16

By Simpson's Rule

Ordinates : 0, 0.40, 1.06, 1.70, 1.50, 0.30, 0.

$$\text{Area} = \frac{8}{3} \left\{ 0 + 0 + 2(1.06 + 1.50) + 4(0.4 + 1.70 + 0.30) \right\}$$

$$= 2(5.12 + 10.80)$$

$$= 31.84 \text{ ft}^2$$

$$\text{Discharge} = AV = 31.84 \times 0.75 \times 60$$

$$= 1432.8 \text{ cu. ft. per min.}$$

3. A cutting is to be made for a straight lane 400 ft. long. The depths at 50 ft. intervals are : 0, 3, 3.5, 4.5, 4.0, 2.8, 0. The width of the road is 8 ft. and the slope of the sides is 2 vertical to horizontal.

How many cubic feet must be excavated ?

Solution :

Area No.	Calculation	Area ft ²	Multiplier	Product ft ³
1	0	0	0	0
2	3 × 9.4	28.5	4	114.0
4	3.5 × 9.75	34.125	2	68.25
4	4.5 × 10.25	46.125	4	184.50
5	4 × 10	40.0	2	80.00
6	2.6 × 9.3	24.18	4	96.72
7	0	0	1	0

543.47 ft³

$$\text{Volume} = \frac{50}{3} \times 543.47$$

$$= 9057 \text{ cu. ft.}$$

1-9 Volume from contour plan: The following are the methods of calculating volume from contour plan :

(1) By cross-sections The cross-section of the existing ground surface is drawn with the help of contour plan. The formation level of the proposed work is drawn on the same cross-section. The area of cut or fill is estimated by methods discussed before. The volume of earth work between adjacent cross sections is calculated by any suitable method.

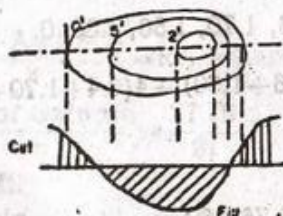


Fig. 1.17

(2) By equal depth contours; Contours of finished surface are drawn on the contour map at the same interval as that of the contours. The cut or fill at the points where

the contours of finished surface intersect or contour of existing surface is the difference in elevation between these two contours. By joining the points of equal cut or fill, a set of lines (equal depth contours) are obtained. The irregular area bounded by each of these lines are determined by multiplying the average of the two areas by the depth between them or by prismoidal rule. The sum of the volumes of all the layers is the total volume.

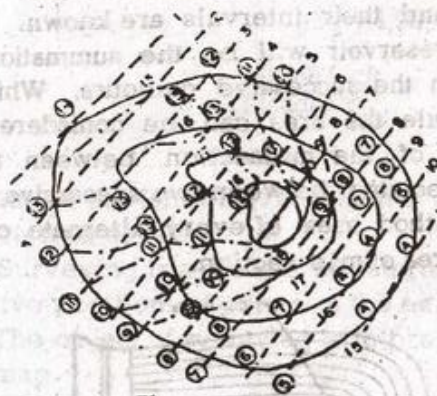


Fig. 1.18

In the figure the continuous lines are the ground contours and dotted lines are finished plane surfaces, both are at the interval of 1 ft. At each point where these two sets of lines meet, the amount of cutting is written (the enclosed figures). broken lines denote the equal depth contours.

Let A_1, A_2, A_3, \dots etc. be the areas enclosed in each of the broken lines. (This is the whole area lying within an equal-depth contour and that of the strip between the adjacent contours)

h = contour interval

Then total volume

$$v = \sum \frac{h}{2} (A_1 + A_2) \text{ by trapezoidal rule}$$

$$\text{or } v = \sum \frac{h}{3} (A_1 + 4A_2 + A_3) \text{ by prismoidal rule.}$$

3. **Capacity of Reservoirs:** The capacity of a reservoir can be obtained from the contour map of the reservoir site. The area enclosed by each contour can be obtained by the use of planimeter. The volume is thus calculated by the use of Prismoidal or Trapezoidal Rule when the contour areas and their intervals are known. The total capacity of the reservoir will be the summation of these volume between the successive contours. While using the Prismoidal Rule the area can be considered in two ways. The area of the midsection between two contours can be interpolated between two successive contours. In the second method area of every alternate contour is assumed as the area of mid-section.

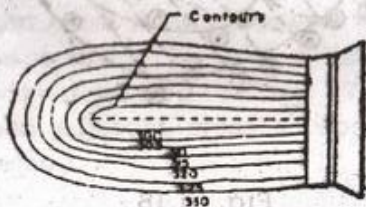


FIG. 1.19

Example: Calculate the capacity of the proposed reservoir in Fig. 1.19 from the following data:

Contour	Area in sq. at.
300	2,000
305	30,000
310	80,000
315	170,000
320	2,80,000
325	4,90,000
330	10,70,000

Volume by Prismoidal Rule:

$$= \frac{5}{3} \{ (2,000 + 10,70,000 + 4(30,000 + 1,70,000 + 4,90,000) + 2(80,000 + 2,80,000)) \}$$

$$= 7930000 \text{ cft.}$$

$$= 182 \text{ acre ft.}$$

$$\text{Volume by Trapezoidal Rule} = \left\{ \frac{2000 + 10,70,000}{2} + 30,000 + 80,000 + 1,70,000 + 2,80,000 + 4,90,000 \right\} \times 5$$

$$= 7586667 \text{ cft.}$$

$$= 174 \text{ acre ft.}$$

Exercise

- Examine the following statements and write whether they are true or false.
 - Surveying is a process of determining the relative positions of points on the earth's surface.
 - The object of surveying is to prepare a plan or map.
 - Geological survey is conducted for exploring mineral wealth.
 - Surveying is also important in military engineering services.
 - 5280 ft. make one nautical mile.
 - Area computed by the Trapezoidal Rule is more accurate than that by the Prismoidal Rule.
 - The area of an irregular figure can be determined by a planimeter.
 - Prismoidal correction is the difference between the volume calculated by the Trapezoidal formula and that by the Prismoidal formula.
 - Engineer's Chains are generally 66 ft. long containing 100 links.
 - The lengths of the links in a chain are not uniform.
 - With an engineer's chain the minimum distance that can be accurately measured is 100 ft.

- (1) With a gunter's chain the distance of 56 ft. can be accurately measured.
2. Explain the term "Surveying" Why it is important in engineering projects?
 3. Discuss briefly the different types of surveying and their specific uses.
 4. Explain the various methods of determining the area of an irregular figure. Which method do you think best and why?
 5. Discuss the methods of calculating the volume of earth works.
 6. A series of offsets were taken from a chain line to a curved fencing at interval of 30 ft. and offsets were 0, 7.8, 5.2, 6.4, 7.0, 7.8, 8.4 and 0 ft.

Calculated the area of the plot of land between the fencing and the chain line by (a) Trapezoidal Rule and (b) Simpson's Rule.

Ans. (a) 1278.4 sq. ft. 1334.2 sq. ft.

7. It is proposed to set up a public hall on a street side bounded by a curved fencing behind. After surveying, the following data were recorded:

Chainage in ft. 0 25 50 75 100 175 250 300 350
 Offsets in ft. 10.12 9.62 7.32 14.09 16.98 15.80 10.12 6.04 12.33

Calculated the area in sq. ft. bounded by the road side and the fencing by (a) Simpson's Rule, (b) Trapezoidal Rule.

Ans (a) 4169.16 sq. ft. (b) 4166.72 sq. ft.

8. Calculated the volume of earthwork in cutting for a road by both Trapezoidal and Prismoidal formula at a constant R.L. of 180.00 which runs East to West. The ground is levelled in North South direction. The surface levels along the centre line of the road are as follows:

Chainage in ft. 0 50 100 150 200 250 300

Level in ft. : 190 188 187 185 183 186 182

The width of the formation level is 24 ft. and the side

1:1
 Ans. (a) 21874.98 cu. ft. (b) 21400.04 cu. ft.

9. A coastal embankment 20 ft. wide at the formation level with side slopes 2:1 and with an average height of bank of 8.00 ft. is to be constructed with an average gradient of 1 in 50 from 200 ft. contour to 680 ft. contour.

Calculate the length of the road in miles and the volume of earth-work in filling in cu. yds.

Ans. 4.545 miles, 256,000 cu. yds.

10. Calculate the capacity of the proposed reservoir shown in Fig : 1.7 from the following data.

Contour	Area in sq. ft.
650	24,000
670	32,250
675	34,750
680	36,800
635	39,500
690	41,000
695	42,250

Ans. 20.8 Acre ft.

11. A railway embankment, 800 ft. long and having the width of the formation level of 30 ft. is to be constructed. The ground on which it is to be constructed has a transverse slope of 1 in 34 in the direction of the railway and side slopes are 2:1. Calculate the volume of earthwork in embankment in cu. ft.

Ans. 4,83,333.3 cu. ft.

(Prismoidal Rule)

12. What is the area of a triangle having sides 42, 28 and 30 ft.

Ans. 542.22 sq. ft.

13. In a triangle ABC, angles B and C are 48° and 85° respectively. Side AB measures 65 ft. What are the lengths of the other sides?

Ans. BC=47.72 ft, AC=48.50 ft.

14. In a triangle ABC, AB and BC are 120 ft. and 105 ft. respectively and the included angle B is $38^\circ 30'$. Solve the triangle.

Ans. $AC=75.56$ ft. Angle $BAC=59^{\circ}32'$. Angle $ABC=80^{\circ}38'$.

15. Find the area of a triangle in which two adjacent sides are 130 ft. and 96 ft. and the included angle is $42^{\circ}18'$.

ft Ans. 4200 sq. ft.

16. Cross sections at 10' intervals along an embankment contained the following areas :

64.2, 43.8, 34.5, 15.28, 3.45, 9.75, 5.81, and 71.6 ft²
Find the volume in cu. ft. Ans. 3850.6 cu. ft.

17. A basin formation in land is to be used for storing water. A contour plan is prepared and the areas at the contour levels are found to be as follows :

Contour (ft) 0 3 6 9 12 15 18

Area (ft)² 49 97 188 365 672 1051 1297.

Calculate the volume of water in cu.ft. that can be stored up to contour level 18 ft.

18. The following details were obtained during a survey operation alongside the boundary wall of a field :

Distance along chain line Offsets

in meters	in meters
0	1
50	2
100	3
150	2
200	4
250	5
300	3

Calculate the area in sq. meters contained between the chain line and the wall.

Ans. 900 sq. meters by Trapezoidal Rule.

19. A short road is to be constructed entirely in cutting at a uniformly rising gradient of 1 in 100. The natural surface level are as follows :

Disnce 0 100 200 300 400 500

in ft

Surface 70.0 71.0 80.0 77.0 77.0 73.0

level above datum on ft

If the formation level at 0 distance is 63.00 ft above the assumed datum, estimate the volume of cutting in cu ft for a formation.

width of 320 ft, the side slopes are 1 vertical and 4 horizontal and the original surface level in transverse direction.

Ans. 369600 cu ft. by Trapezoidal Rule.

and 346467 cu ft. by Prismoidal Rule.

20. An irrigation canal of 200 ft long with 20 ft formation width is to be constructed. The canal has a side slope of 1 vertical 1 and 2 horizontal and the land has an uniform slopes of 1 in 5 in transverse direction. The average depth of cutting is 8 ft. Calculate the volume of earthwork in cutting in cu yds. Ans. 24600 cu yds.

21. A road one mile long having a width of 20 ft at the formation level is to be constructed. The road has side slope of 1 vertical and 2 horizontal and the land has an uniform slope of in transverse direction. The average depth is 2 ft with one portion of the road under cut the other in fill. If the material in cutting is utilized in filling find out the net volume of earthwork cu yds.

Ans. 3580 cu yds.

2-1 Definition: Chain survey is the simplest type of survey in which the area to be surveyed is divided into a number of triangles. Because of all the geometrical figures triangle is the only one whose shape and size are determined when the lengths of the sides are known. The perpendicular distance, called offsets, of various objects in the field from the line are measured and recorded in a book called field book. From these records in the field book the whole area can be plotted on a drawing sheet to a reduced scale. The following instruments are used in chain survey:

Chain, Tape, Arrows, Ranging Rods, Offset Staff and Optical Square.

Chain: It is a steel wire with links connected by steel rings. It has brass handles at both ends. There are many types of chains of which the Engineer's chain and Gunter's chain are commonly used in our country. The Engineer's chain is 100 ft. long and the Gunter's chain 66 ft. The Engineer's chain consists of 100 links each one a foot long and at every 10 links a tag or tally is attached to facilitate reading. The Gunter's chain is also divided into 100 links, each link is 7.92 inches.

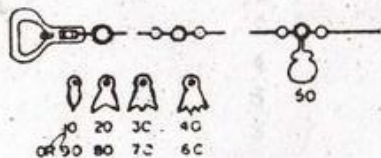


FIG. 2.1 CHAIN AND BRASS TELLERS

FIG. 2.1-CHAIN AND BRASS TELLERS

Tapes: They may be either of steel or linen to measure generally short lengths. They are generally of 100 ft., 66 ft., or 33 ft., and are graduated in feet and inches.

Pegs: These are wooden blocks of conical shape used in fixing stations.

Arrows: They are of steel wire 15" long pointed at one end and the other end is looped for convenience of handling. They are used for making chain lengths on the ground



FIG. 2.2 ARROW

Ranging Rods: They are about 10 ft. long, 1 1/2 inches diameter round or hexagonal wooden poles painted with



FIG. 2.3 RANGING ROD

black and white alternate bands to make it suitable for marking station points. Each band is of one foot length.



FIG. 2.4 OFFSET STAFF

Offset Staff: They are wooden rods 10 feet long. Each foot is painted black and white alternately. They are used for measuring short lengths.

Optical Square: It is used to find the foot of the perpendicular from a given object in the field to a given chain



(a)

OPTICAL SQUARE

FIG. 2.5 (a)

line, i.e., to take offsets. It consists of a wedge shaped hollow brass box of about 2" sides and 1 1/2" depth with a brass

handle about 3' long fixed at the bottom. Two plane mirrors, set at 45° are fixed to the inclined sides of the box. There are two slits above these mirrors. In using it, a ranging rod is held at object for which an offset is to be taken. A man holding the optical square in his right hand stands on the chain line. He looks towards the front ranging rod on the chain line with the open face of the optical square towards the ranging rod at the object. Now the man looks through one mirror as shown in Fig. 2.5, while the other mirror is turned towards the object. Then he walks slowly along the chain line forwards and backward till he sights the image of the ranging rod at the object in mirror and the front ranging rod in the slit in the same line. The position

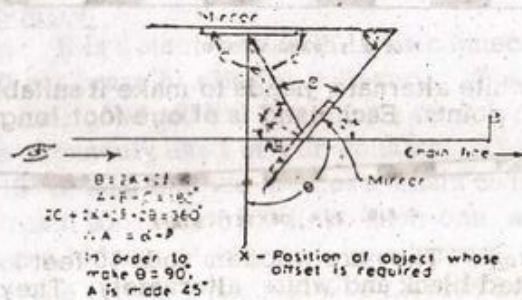


FIG. 2.5 (b)

of the man on the chain line gives the exact point at which the perpendicular from the object meets the chain line. The working principle is explained in Fig. 2.5(b).

2-2 Procedure: The entire operation of chain survey can be divided into three major groups namely, field work, keeping of records in the field book and plotting of data to prepare maps.

✓ **Field work:** It includes reconnaissance, selection of station, measurement of lines and taking offsets of different objects in the field.

✓ **Reconnaissance:** This is the preliminary survey in which the survey party will examine the plot to be surveyed in

order to know as to how the works can be executed in the best possible ways. The party will note all details like roads, buildings, canals, ditches, culverts and the difficulties and obstacles that may arise during the carrying out of the work. The party should locate the suitable points for stations by driving pegs, sometimes a small triangle or a circle is made around the stations and the pegs are inserted at the centres. The party should then make a rough sketch of the plot showing the possible stations and from there the arrangement of different lines. It is important to give a north line on the rough sketch and though the sketch is not prepared according to the scale, it should represent the approximate positions of the different things in the plot and hence to be a good guide for further work.

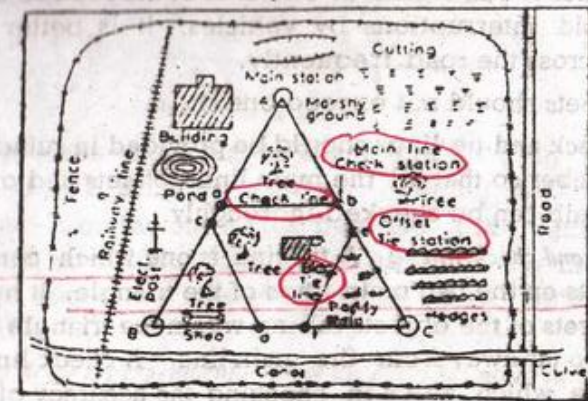


Fig. 2.6

✓ **Stations:** These are points on the ground fixed by driving pegs. Every station should be located with respect to three permanent objects i.e., the distances from these objects to the stations should be measured very accurately and recorded in the field book. The advantage of taking this measurement is that if in future the peg at the station is lost, then it can be located again by knowing descriptions and distances of these objects. The selection of a particular station depends upon the following important considera-

* **tions:**

of short base, permanent
arranging the line
marks the line straight

- (1) The triangle should be a well-defined one, i.e., nearly equilateral triangle.
- (2) Every main station should be visible from the other two.
- (3) There should be minimum number of obstacles in ranging and chaining.
- (4) The chain line should run near the boundary of the plot.
- (5) The chain line should be as few as possible.
- (6) The chain line should be over an approximately levelled ground.
- (7) In case of chaining along the road, it is always better to run chains at one side of the road so as to avoid interruptions by vehicles. It is better not to cross the road frequently.
- (8) Offsets should not exceed one chain.
- (9) Check and tie lines should be provided in sufficient number so that all the main lines, offsets and other details can be checked thoroughly.

* Tie lines and check lines: A tie line is one which connects two points on the two main lines of the triangle. It helps in taking offsets of the objects falling within the triangle and which are too far away from the main line. A check line is also a tie line which helps in checking the accuracy of the work after plotting in a drawing sheet. A check line or tie line is never extended beyond the main lines (See Fig. 2.6).

* Measurement of lines and taking offsets: In Fig. 2.6., the main station, A is located with respect to three permanent objects and a ranging rod is fixed on the station. One ranging rod is fixed at main station B and another at an intermediate point in between A & B. The three rods will be in a straight line when only the intermediate rod is visible if a man looks from A to B. Now measurement of line AB is taken by chain. The chain should be properly stretched so that

Stringing rod rule to make the line straight.

there is no sag in it. As the measurement proceeds, offsets are taken on both sides of the line AB and recorded in the field book (shown in Fig. 2.6). In this way all the lines including tie and check lines are measured and offsets taken and recorded in the field book.

* 2-3 Chaining across obstacles: Sometimes it will be observed that many obstructions like rivers, canals, ponds, thick jungles, ridges, ditches, buildings, etc. lie on the chain line. These obstacles can be avoided in chaining operation by applying some fundamental geometric rules.

- (1) To draw a perpendicular from a point on the chain line:

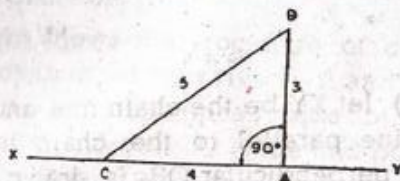


FIG. 2.7

AC is taken 4 units on the chain line XY (Fig. 2.7), AB and BC, 3 and 5 units respectively. Then $\angle BAC$ will be 90° at point A on the chain line because if the sum of the squares on two sides of triangle is equal of the third, the included angle by the two sides is a right angle ($BC^2 = AB^2 + AC^2$).

- (2) To draw a perpendicular from an external point to chain line

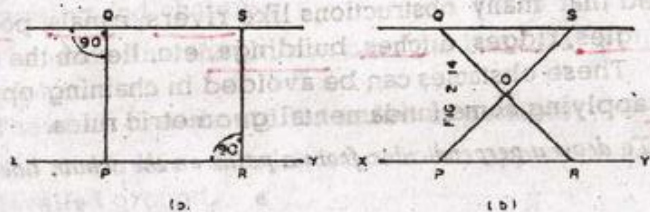


FIG. 2.8

XY is the chain line and P is the external point. Keeping the zero end of the tape at P and swinging the tape along

the chain line the point of minimum tape length on the chain line is noted which should be the foot of the perpendicular. Because the perpendicular is the shortest distance (Fig. 2.8).

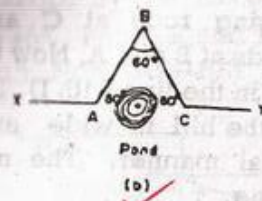
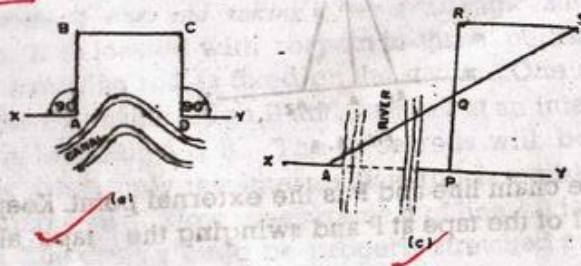
(3) To draw a line parallel to the chain line :



In Fig. 2.9 (a), let XY be the chain line and Q is a point through which a line parallel to the chain line is to be drawn. From Q, perpendicular OP is drawn on XY at P. Point R is now selected on XY and RS is drawn perpendicular to XY at R in such a way that $RS = PQ$; QS is joined which is now parallel to XY.

In Fig. 2.9 (b), point R is selected on XY. QR is joined and bisected at O. Another point P on XY is selected and PO is joined. Now PO is extended to S so that $PO = OS$. QS is joined. QS is parallel to XY.

The following are the geometrical figures by which chaining can be done inspite of obstacles lying on the chain line.



VISION FREE BUT CHAINING OBSTRUCTED

FIG. 2.10

How chaining operation can be done when it is obstructed by a bend of a canal, has been shown in Fig. 2.10 (a) which is self explanatory.

Fig. 2.10 (b) shows the procedure of chaining operation when it is obstructed by a river. A and P are the two points close to the bank on opposite sides of the river. At P, a perpendicular PR is drawn. Q is the mid-point of PR. At R, again a perpendicular RS is drawn. Point S is fixed by extending AQ. From two similar triangles APQ and QRS, $RS = AP$.

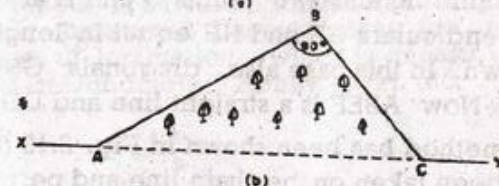
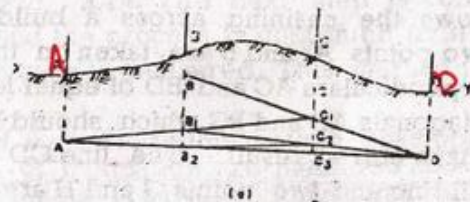


FIG. 2.11 VISION OBSTRUCTED BUT CHAINING FREE

FIG. 2.11 VISION OBSTRUCTED BUT CHAINING FREE

Fig. 2.11 (a) shows the method of chaining when it is obstructed by a hill or ridge. A and D are the foot-hill points, each hidden from view of the other on the either

side of the hill. Points B and C are chosen in such a way that a man at B can easily see ranging rods at C and D, while at C, can see the ranging rods at B and A. Now C puts B in the line with A, and B puts C in the line with D. Hence, A, B, C, D are in the same line. If the hill is wide enough, then it can be chained in the usual manner. The method is also known as reciprocal ranging.

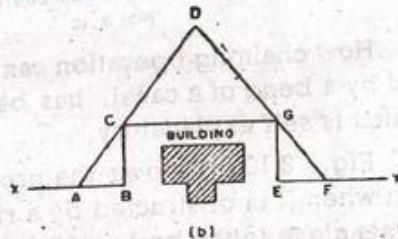
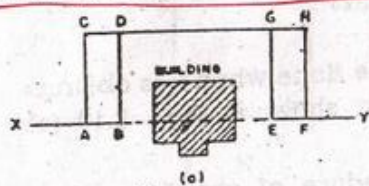


FIG. 2.12 VISION AND CHAINING BOTH OBSTRUCTED

FIG. 2.12 VISION AND CHAINING BOTH OBSTRUCTED
 Fig. 2.12 (b) shows chaining through a thick wood. The figure is self explanatory. From the Fig., $AC = \sqrt{AB^2 + BC^2}$

Fig. 2.12 shows the chaining across a building. In Fig. 2.12 (a) two points A and B are taken on the chain line and two perpendiculars AC and BD of equal length are erected. The diagonals AD and BC which should be equal, are checked to have correct result. The line CD is produced past the building and two points G and H are taken on it. Two perpendiculars GE and HF equal in length to AC or BD are drawn. In this case also, diagonals GF and HE are checked. Now ABEF is a straight line and $DG = BE$.

Another method has been shown in Fig. 2.12 (b), where a point B has been taken on the chain line and perpendicular BC erected. A is another point on the chain line so that $BC = BA$. This makes the angle $BAC = 45^\circ$. AC is joined and extended upto D which is roughly opposite the middle point in length of the building. At D, a perpendicular DF is set to AD, making $DF = DA$. On DF, point G is taken in such a way

that $DG = DC$. By the procedure, explained above, $\angle GFE$ is made 45° . Points E and F lie on the straight line AB produced. Now $CG = BE$.

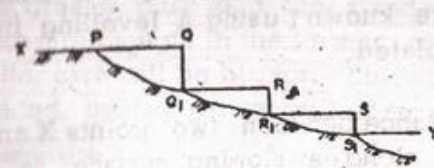


Fig. 2.13

Fig. 2.14

2-4 Chaining along sloping ground :

First method : During chaining along a sloping surface, the horizontal projection or a chain line is found by the process shown in Fig. 2.13. In this method, a portion of the chain, 15 ft. to 30 ft. is generally used. The length of chain, of course, depends upon the steepness of the sloping surface. The chain is held horizontally with one end of it at P on the ground, while the point Q₁ vertically below the other end of the chain at Q is found by means of a drop arrow shown in Fig. 2.14. The next step is commenced from point Q₁ and the process is continued until the whole horizontal distance is measured. This method is also known as stepping.

Second method : In this method, the sloping length and the angle of inclination are measured and the horizontal projected length is calculated mathematically. From Fig. 2.15, $XY = L_1 =$ measured distance along the slope, $ZX = h$, and $\theta =$ angle of inclination, which is measured by instruments such as Clinometer, Abney level, etc.

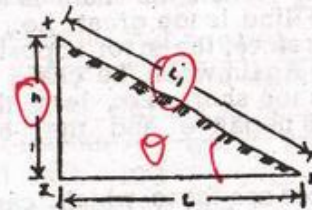


FIG. 2.15

$$\cos \theta = \frac{L}{L_1}$$

$$\therefore L = L_1 \cos \theta$$

Third method: If the difference of height h between the points X and Y are known (using a levelling instrument), L can be calculated,

$$L = \sqrt{L_1^2 - h^2}$$

Example: The distance between two points X and Y (Fig. 2.15) measured along a sloping surface is 12.4 chains. Calculate the horizontal projected distance when the angle of inclination is $10^\circ 30'$. Also find the same when the elevations of X and Y above mean sea-level are 740 and 840 respectively.

$$L = L_1 \cos \theta = 1240 \cos 10^\circ 30' = 1220 \text{ ft.}$$

$$\text{Again, } L = \sqrt{L_1^2 - h^2} = \sqrt{(1240)^2 - (840 - 740)^2} = 1237 \text{ ft.}$$

2-5 Errors in Chaining: It is always very difficult practically to measure length accurately. The permissible error with a steel tape is 1 in 2000 in a flat country and 1 in 3000 for a rough undulating country.

The error in chaining may happen in various ways. Sometimes there may be mistakes or confusion in reading the tallies such as 30 and 70. There may be also omission of chain lengths due to miscounting or when chaining is interrupted by buildings, canals, etc.

The error may also be either cumulative or compensating. Cumulative errors are these which may either go on increasing or decreasing when a chain is shorter or longer than its standard length. When the chain is too short, the measured length of the line is too great, i.e., greater than its true length and therefore, the error is positive and the correction is negative. Again when the chain is too long, the measured length is too short, i.e., less than its true length. So the error is negative and the correction is positive.

Compensating errors are those which cancel one another and finally their total effect remains approximately

same. While stretching a chain one may pull it less than the standard pull of that chain. Again one may stretch it with a greater pull than the standard one. As a result the measured length in the former case will be less and in the latter case will be higher. But when these two lengths are added, the two errors will compensate each other.

Correction:

(a) Correction in Length, Area and Volume:

Let L_e be the incorrect length of the chain and L_c the correct length of the chain.

The correct distance, $L = \left(\frac{L_e}{L_c}\right) \times$ measured length by the incorrect chain or tape.

The correct area, $A = \left(\frac{L_e}{L_c}\right)^2 \times$ calculated incorrect area

The correct volume, $V = \left(\frac{L_e}{L_c}\right)^3 \times$ calculated incorrect volume.

Example: The road from Dacca to Mirpur is actually 25320 ft. long. This distance was measured by an Engineer's defective chain and was found to be 25270 ft. How much correction does the chain need?

$$L = \frac{L_e}{L_c} \times \text{measured incorrect length}$$

$$L_e = \frac{L \times L_c}{\text{measured length}} = \frac{25320 \times 100}{25270} = 1000.197 \text{ ft.}$$

So the chain should be shortened by 0.197 ft.

Example: The length and breadth of a plot of land were measured by an Engineer's chain exactly 100 ft. in length at the beginning. But it was found to be 100.3 ft. long at the end of the survey work. The area of the plot drawn to a scale 1 inch = 100 ft. was 25.60 sq. inches. What was the true area of the plot?

$$\text{True area} = \left(\frac{L_e}{L_c}\right)^2 \times \text{calculated incorrect area}$$

$$= \left(\frac{100.3}{100}\right)^2 \times 25.6 \text{ sq. in.} = 25.70 \text{ sq. in.}$$

From the scale on the map $1 \text{ in.}^2 = 100^2 = 10,000 \text{ sq. ft.}$

$$\text{Area of the plot} = \frac{25.70 \times 10,000}{43560} \text{ (1 acre} = 43560 \text{ sq. ft.)}$$

$$= 5.89 \text{ Acres.}$$

Example: The length, breadth and depth of a pond were measured by an incorrect Gunter's chain. The volume of the pond was calculated to be 1,60,000 cft. The chain was tested at the end of the measurement of the tank.

$$\text{True volume} = \left(\frac{L_c}{L_c}\right)^3 \times \text{Incorrect volume}$$

$$= \left(\frac{65.8}{66}\right)^3 \times 160000$$

$$= 159200 \text{ cft.}$$

(b) Correction for Pull: Sometimes, a steel tape is pulled in excess of the pull at standardization, then the correction to be made is as follows:

$$\text{Correction } C_p = \frac{L(F_f - F_s)}{AE}$$

Where L = length of tape, A = cross-sectional area of tape, F_f = pull applied in the field, F_s = pull at standardization, and E = Young's Modulus of Elasticity (for steel, $E = 30 \times 10^6$ p.s.i.). Since the effect of pull on tape is to make the measured length too short, the correction is always positive.

Example: A steel tape of 100 ft. length, standardized at 25 lb. pull, was used in the field with a pull of 35 lbs. The cross-sectional area of the tape is 0.025 sq. inch. Take the value of Young's Modulus of Elasticity for steel, 30×10^6 p.s.i. Calculate the correction for excess pull.

$$\text{Correction} = \frac{100(35 - 25)}{0.025 \times 30 \times 10^6}$$

$$= 0.00134 \text{ ft. (positive)}$$

(c) Correction for Sag:

$$\text{Correction } C_s = \frac{W^2 L}{24 F_r^3}$$

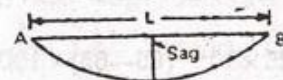


FIG. 2.16

Where, W = wt. of the tape in lb., L = length of the tape in ft. and F_r = pull applied in the field in lb. Since the effect of sag on tape is to make the measured length too large the correction is always negative.

Example: A steel tape of 100 ft. length weighing 1.2 lbs. was pulled with a force of 20 lbs. in the field to measure a certain distance. Calculate the correction for sag.

$$\text{Correction} = \frac{W^2 L}{24 F_r^3} = \frac{(1.2)^2 \times 100}{24 \times 20^3} = 0.15 \text{ ft. (negative)}$$

(d) Temperature Correction: Since the length of the tape is increased as temperature is raised, where measured distance is too small, it is therefore essential to apply this correction.

$$\text{Correction } L = a(t_r - t_s) \times L$$

Where, L = measured length in ft., t_s = temperature at which the tape was standardized, t_r = temperature at which the tape is used in the field, and a = co-efficient of thermal expansion of the tape per degree °F per foot length. The co-efficient of thermal expansion of steel varies from 5.5×10^{-6} to 6.85×10^{-6} per degree °F. The sign of the correction is plus or minus according as t_r is greater or less than t_s . The steel tapes are generally standardized at 65°F.

Example: A distance of 1840 ft. was measured with a steel tape which was exactly 100 ft. long at 65°F. The temperature during measurement in the field was 85°F.

Calculate the actual length of the line. Take the co-efficient of thermal expansion of tape = 6.25×10^{-6} per 1°F .

First Method :

Temperature correction for each tape length = $a(t_2 - t_1) \times L$

$$= 6.25 \times 10^{-6} (85 - 65) \times 100 \\ = 0.012 \text{ ft. (positive)}$$

Length tape at $85^\circ\text{F} = 100 + 0.012 = 100.012 \text{ ft.}$

True length of the line = $\frac{100.012}{100} \times 1840 = 1840.230 \text{ ft.}$

Second Method :

Total correction = $6.25 \times 10^{-6} (85 - 65) \times 1840 = 0.230 \text{ ft.}$

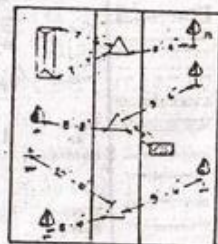
True length of the line = $1840 + 0.230 = 1840.230 \text{ ft.}$

Annex
2-6 Record Keeping : All the details including a rough sketch of different types of stations, offsets, etc. in the field are recorded in a book called Field Book. It is an important book or document which should be maintained carefully. It is $9" \times 5"$ in size with two parallel lines ruled longitudinally in the centre of every page. These two parallel lines are imaginary lines representing the chain line and the space in between has no existence in the field. The record keeping starts from the bottom of the end page of the field book. A rough sketch of the plot is drawn beforehand on the last page for reference. Recording of stations, chain lines and other details are shown in Fig. 2.17 and 2.18. Neat figures and sketches with clearness in representing points to which offsets are taken should be properly maintained.

2-7 Plotting of details : Before plotting the details of chain survey on a drawing paper a suitable scale should be chosen first. Because drawings are prepared to a reduced scale.

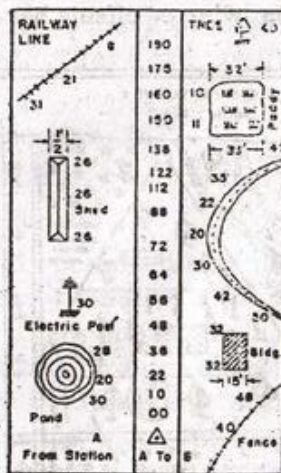
A scale is the ratio between the actual length on the ground and the corresponding length of the line on the map. For an example, when the scale of a map is 1 inch

= 100 ft., it indicates a length of 100 ft. on ground being equal to 1 inch on the map. A scale may also be denoted by the term Representative Fraction (R F) which is the ratio of 1 inch to the distance on the ground both reduced to inches. As for an example, the Representative Fraction of 1 inch = 100 ft. scale is $\frac{1}{1200}$ i.e., 1 inch to 1200 inches.



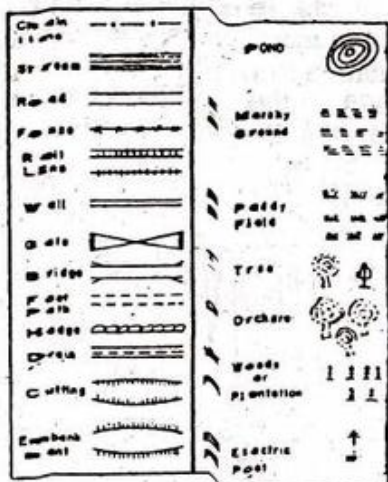
RECORDING OF STATION
FIG. 2.17

The triangle is first plotted from its known sides according to a suitable reduce scale. Then tie lines and check lines are drawn and checked the accuracy of the work. Now to plot offsets like railroads, trees, building, electric posts, etc. lines are taken up one by one



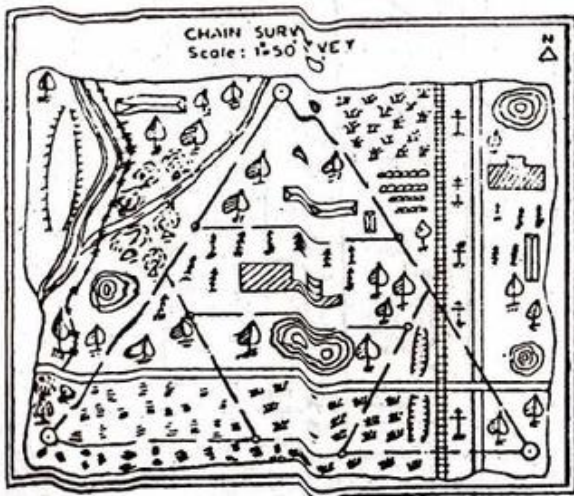
RECORDING IN A FIELD BOOK
FIG. 2.18

CHAIN SURVEY



CONVENTIONAL SIGNS FOR DIFFERENT OBJECTS
FIG. 2

and on it marked, the distances where offsets have been taken are



AN EXAMPLE OF PLAN PLOTTED FROM THE DATA RECORDED IN A FIELD BOOK OF A SIMPLE CHAIN SURVEY

Then the offsets are laid down on either side of the lines. When the offsets are plotted the conventional signs for offsets shown in Fig. 2 19 are put in.

2-8 Advantages and disadvantages of chain Survey: This type of survey work is suited for a small plain ground. It requires simple instruments. Plotting of maps is very simple and easy. But this type of surveying is not suitable for undulation land where chaining operation is tedious and subject to errors. This method is not generally recommended for a crowded city with large number of buildings and obstacles because it cannot be divided into well conditioned triangles. In case of route surveying i.e. the survey work of a road, irrigation canal, railways, water and sewer lines, tunneling, etc, this method is not recommended at all.

2-9. Linear Measurements;

There are various methods of making linear measurements and their relative merit depends upon the degree of precision required. They can be mainly divided into three heads;

1. Direct measurements.
2. Measurements by optical means.
3. Electronic methods.

In the case of direct measurements, distances are actually measured on the ground with the help of a chain or a tape or any other instrument. In the optical methods, observations are taken through a telescope and calculations are done for the distances, such as in tacheometry or triangulation. In the electronic methods, distances are measured with instruments that rely on propagation, reflection and subsequent reception of either radio or light waves. The various instruments that are used under the electronic methods are;

- (i) geodimeter,
- (ii) tellurometer,
- (iii) the decca navigator, and
- (iv) the lambda position fixing system.

The method of measurement in the case of geodimeter is based on the propagation of modulated light waves. The other three instruments use radio waves for distance measurements.

The various methods of measuring the distances directly are as follows :

1. Pacing
2. Measurement with passometer
3. Measurement with pedometer
4. Measurement by odometer and speedometer

(1) *Pacing*. Measurements of distances by pacing is chiefly confined to the preliminary surveys and explorations where a surveyor is called upon to make a rough survey as quickly as possible. It may also be used to roughly check the distances measured by other means.

The method consists in counting the number of paces between the two points of a line. The length of the line can then be computed by knowing the average length of the pace. The length of the pace varies with the individual, and also with the nature of the ground, the slope of the country and the speed of packing. A length of pace more nearly that of one's natural step is preferable. The length of one's natural step may be determined by walking on fairly level ground over various lines of known lengths. One can soon learn to pace distances along level, unobstructed ground with a degree of accuracy equivalent approximately to 1 in 100. However, pacing over rough ground or on slopes may be difficult.

(2) *Passometer*. Passometer is an instrument shaped like a watch and is carried in pocket or attached to one leg. The mechanism of the instrument is operated by motion of the body and it automatically registers the number of

paces, thus avoiding the monotony and strain of counting the paces, by the surveyor. The number of paces registered by the passometer can then be multiplied by the average length of the pace to get the distance.

(3) *Pedometer* : Pedometer is a device similar to the passometer except that, adjusted to the length of the pace of the person carrying it, it registers the total distance covered by any number of paces.

(4) *Odometer and Speedometer* : The odometer is an instrument for registering the number of revolutions of a wheel. The well known speedometer works on this principle. The odometer is fitted to a wheel which is rolled along the line whose length is required. The number of revolutions registered by the odometer can then be multiplied by the circumference of the wheel to get the distance. Since the instrument registers the length of the surface actually passed over, its readings obtained on undulating ground are inaccurate. If the route is smooth, the speedometer of an automobile can be used to measure the distance approximately.

Exercise

1. Examine the following statements very carefully and write whether they are true or false :
 - (a) Chain survey is a type of survey in which the survey is done with the help of a chain and tape only.
 - (b) Chain lines can be measured with the help of a chain only.
 - (c) Tie lines are generally extended outside the main triangle.
 - (d) In an optical square the mirrors are placed at an angle of 60° to each other.
 - (e) Right angles can also be set with the help of a chain and a tape.

- (f) The space between two parallel lines in the field book has no existence in the field.
- (g) Check lines are also tie lines.
- (h) Chain surveying is suitable for crowded cities.
- (i) Gunter chain is most suitable for measuring areas.
- (j) A chain survey is a class of survey in which a triangle is formed in the field whose sides are used as reference lines.
- (k) In Chain Survey angles are measured with considerable accuracy.
- (l) The stations are the points from which measurements are taken upto any object in the field.
- (m) If the triangle is too big then it may be necessary to run few more straight lines within the triangle.
- (n) It is not necessary for the tie lines to touch any other tie line or main line.
- (o) The main lines and the tie lines have a common name known as chain lines.
- (p) The offsets are also chain lines measured upto different objects in the field.
- (q) Comparing the plotted length of the tie lines on the map with the measured length in the field the accuracy of the survey may be determined.
- (r) The start with as big a triangle as possible is formed and subsequently the area inside is subdivided by tie lines.
- (s) Chain is used for measuring the chain lines and tape is used for measuring the offsets.
- (t) The offsets should not be more than the full length of the tape measuring them.
- (u) The ranging rods which are painted in red and white are used for fixing stations also.
- (v) Arrows are used for aligning chain lines.
2. Fill up the blanks in the following statements, each with one word.

- (a) Going around and examining the field to be sur-

- veyed and preparing the sketch map showing the positions of stations fixed etc. is known as..... survey.
- (b) The main stations in chain survey are so fixed that each is..... from the other two.
- (c) In order to make the points prominent a circle or a triangle is cut on the ground encircling it.
- (d) Each station is..... by taking accurate measurements from three permanent objects.
- (e) Recording of offsets and measurements of lines are entered in the field book starting from the of the page of it.
- (f) The gap between the two red lines in the field book is meant for writing..... along the chain line.
3. What is the minimum distance that you can measure with a chain ?
4. What are the sources of errors in chaining ? What correction do you apply to measure chain lines due to the errors in chain ?
- (5) What is a ranging rod ? Why it is painted in different colours ?
6. What is an optical square ? Explain its working principle with neat sketch. What are the sources of error in an optical square ? How can you detect them ?
7. Discuss the procedures of chaining when it is obstructed by a bend of a canal, a building, a pond and a bush. Show with sketches.
8. What is reconnaissance survey ? Why a sketch map of the plot to be surveyed is prepared ?
9. Write notes on :
- (a) Offset (b) Field Book (c) Cross-staff (d) Reciprocal Ranging (e) Stepping Method of Chaining (f) Well-conditioned Triangle (g) Sag Correction.
10. The true length of a line was known to be 1500 ft. The line was measured with a 100 ft. tape and found to be 1505 ft. Calculate the correct length of the tape. Ans. 99.66 ft.

11. The distance between two sub-stations was found to be 5305 ft. when measured by an Engineers' chain and 7946 links by Gunter's chain. Both the chains were incorrect. What correction is needed in the Engineers' chain if the Gunter's chain is 0.4 link too long? Ans. 0.748 too short.

12. A draftsman measured a line on a map prepared by chain survey by 1 inch=80 ft. scale and found it to be 3200 ft. long on the ground. Later he discovered that he made a mistake and the scale was 1 inch=60 ft. Find the correct distance in his calculation. Ans. 2400.0 ft.

13. On a map covering an area of 2.5 sq. miles, the scale is by mistake recorded to be 12 inch=1 mile with the result that the area comes to be 4 sq. miles and 2844 $\frac{1}{2}$ sq. chains. Determine the correct scale of the map. Ans. 16 inches=1 mile

14. A 100 ft. tape is suspended between its ends under a pull of 20 lbs. The wt. of the tape is 1.5 lbs. Calculate the connected length of the tape between its ends. Ans. 99.976 ft.

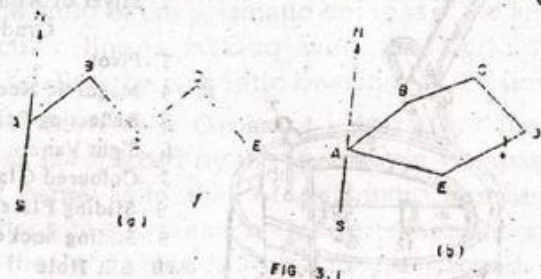
15. A survey was commenced at 7-30 A.M. when the temp. was 65°F with a 100 ft steel tape which was correct. After chaining a distance of 12650 ft. the work was suspended at 1-30 P.M., when the temp. was 102°F. Find the correct distance. Take the coeff. of expansion of steel= 6.25×10^{-6} per deg. °F. Ans. 12,652.92 ft.

16. A line was measured with a 20 m chain and found to be 98.4 m long. The chain was subsequently found to be 0.02 m too short. What was the correct measurement of the line? Ans. 98.3 m.

17. Chaining along a slope the first 30 ft. were on a slope of 8° and the next 20 ft were on a slope of 5°. What is the true horizontal distance? Ans. 49.633 ft.

18. A chain line terminates at an inaccessible point D. Chaining is continued up to point B and a perpendicular BC is laid out 20 ft. in length. At C a right angle DCA is erected; A being on the chain line. BA is measured and found to be 8.62 ft. What is the distance from B to the inaccessible point D? Ans. 46.4 ft.

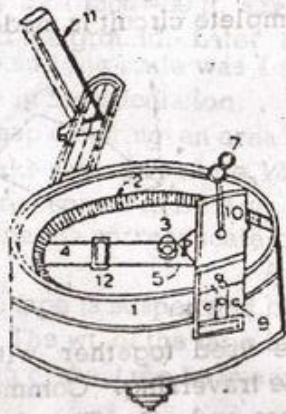
3-1 Definition: A traverse is a number of straight lines of known lengths and making known angles with each other. Traverses are of two types. (1) open traverse, and (2) closed traverse. In an open traverse the end point of the last line will not meet the starting point of the first line. But in closed traverse a complete circuit is made (Fig 3.1)



Angular instruments are used together with those in chain survey to carry out the traversing. Common angular instruments are (1) Compass and (2) Theodolite. Theodolites generally give more accurate and reliable results than the compass. There are two types of compass, (a) Prismatic compass and (b) Surveyor's compass.

3-2 Prismatic Compass; It consists of a cylindrical metallic box having a thin magnetic needle pivoted in the centre. There is a circular disc around which the needle swings freely. The graduation of the disc starts from south end indicating 0° and ends at north with 180° (Fig. 3.2) the figures being written inverted. The magnetic needle carries a sliding collar at its north end to balance the dip (See definition). The sighting vane and the reflecting prism with a sighting slit at the top are placed diametrically opposite to the box. The sight vane has a fine thread or horse hair stressed

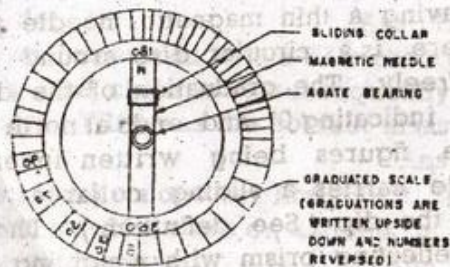
along its opening in the direction of its length and made to bisect any object by turning round the box horizontally. A plane mirror is also attached to the sight vane. The mirror can be tilted to any angle so as to reflect the object which are below or above the level of the instrument. For making observation a plate with a narrow slit is fitted above the prism. Coloured glasses are attached



(a) Prismatic Compass
FIG. 3.2

- 1 Brass Compass Box
- 2 Silver or Aluminium Graduated Ring
- 3 Pivot
- 4 Magnetic Needle
- 5 Reflecting Prism
- 6 Sight Vane
- 7 Coloured Glass
- 8 Sliding Plate
- 9 Sliding Socket
- 10 Slit Hole
- 11 Mirror
- 12 Sliding Collar

to the prism so as to cut the glaring sun rays. The compass can be screwed on a tripod stand provided with



(b) GRADUATED CIRCULAR DIAL OF PRISMATIC COMPASS
FIG. 3.2

a ball socket arrangement at the top so that the compass can be made level.

3-3 Procedure of using Prismatic Compass : The compass is levelled first on the tripod stand. Now by turning the body of the compass the object is intersected by means of the slit and the sight vane. When the object is at the north end of the compass, the reading will be zero as indicated by 0° at the south. When the object lies between North and East, the reading will be in between 0° and 90° . When the object on the East, the reading will be 90° . Hence with the help of the prismatic compass the angle at which a particular line is making with the North, is read. This angle is called the magnetic bearing of the line.

3-4 Surveyor's Compass : Nowadays the Surveyor's Compass is replaced by the prismatic compass, its principle is same as to that of prismatic compass. It is not very accurate and takes a longer time for survey work. Where the land value is less and the ground, is wooded, this instrument is generally used.

Since there is no prism, so the readings are taken by naked eyes. The graduated ring is fixed on the bottom of the box and moves with the box. The needle remains steady between North and South.

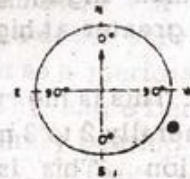


FIG. 3.3

The graduated circle is divided into four quadrants and the graduations run in anti-clockwise direction and these are numbered from 0° to 90° in each quadrant as shown in Fig. 3.3. The 0° degree is marked by N and S while East is marked E and West is marked W.

The following definitions will clarify some useful terms in traverse survey :

(1) *Inclination or Dip* : It is the angle of the axis of the magnetic needle with the horizontal caused by the tendency of the north end of the magnetic needle to point towards the magnetic north pole in the Northern Hemisphere and south end towards the South pole in the Southern Hemisphere.

(2) *Declination* : Magnetic north varies from the true geographical north by a certain magnitude which varies from place to place and also at the same place at different times. This direction of the needle is known as magnetic meridian. The angle between the magnetic meridian and the geographical meridian is known as the declination.

The changes in declination are of four different types :

(a) *Secular Variation* : The geographical poles are fixed but the magnetic poles are continually changing their positions with respect to the geographical poles. This variation observed after a number of years is known as secular variation.

(b) *Durnal Variation* : This is variation of the declination in 24 hours. This is affected by the locality, season of the year, altitudes etc. It is generally 2 to 10 minutes. It is greater during day than night, greater in summer than in other seasons and greater at high altitudes than at the equator.

(c) *Annual Variation* : This is the variation of declination in a year which is generally 2 to 3 minutes.

(d) *Irregular Variation* : This is due to magnetic storms, sun spots and earthquakes which displace the magnetic needle through an amount in the extent of 1° to 2°

(3) *Isogonic Lines*—These are lines of equal magnetic declination.

(4) *Agonic Lines*—These are lines of zero declination.

(5) *Isoclinic lines* : These are lines of equal inclination or dip,

(6) *Aclinic Line* : The line which has no dip is known as *acclinic line or Magnetic Equator*.

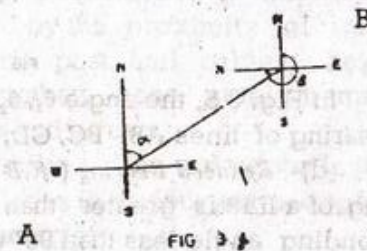
(7) *Bearing* : The bearing of a line is the angle which the line makes with a reference direction or meridian in the clock-wise direction. The meridian may be (a) true meridian, (b) magnetic meridian or (c) arbitrary meridian.

(8) *True Meridian* : The geographical meridian which is also known as the true meridian passing through a point on the surface of the earth is the line in which the plane passing through north and south poles and the given point intersect the surface of the earth. They converge at the poles. But for small surveys, they are assumed to be parallel lines. The angle between the true meridian and a line is called true bearing or azimuth of the line.

(9) *Magnetic Meridian* : The direction indicated by a freely suspended magnetic needle is called the magnetic meridian. The angle which a line makes with the magnetic meridian is called the magnetic bearing or simply bearing of the line. In plane surveys magnetic bearing is taken while in geodetic survey true bearing is considered.

(10) *Arbitrary Meridian* : For a small survey work sometimes the first line of the survey or any convenient direction may be taken as a meridian which is known as arbitrary meridian. The angle between this meridian and a line is known as arbitrary bearing of the line.

3-5 *Types of Bearings* : The bearing of a line is the angle which the line makes with the fixed reference line (Magnetic Meridian) and is always measured in clockwise direction. In Fig. 34, N-S indicates the North-South line which is the reference line. The angle α measured in clockwise direction from North line upto AB is known as the bearing of the line AB



Similarly, the bearing of the line BA is β . In surveying the line AB is not same as BA, because the direction are different.

(a) *Forward Bearing (F.B.)*: The angle α is called the forward bearing of the line AB and the angle β is the forward bearing BA. So the forward bearing of a line is the bearing in the direction of progress of survey.

(b) *Backward Bearing (B.B.)*: The angle β is the backward bearing of AB and the angle α is the backward bearing of BA. So the backward bearing of a line is the bearing in the reverse direction of the progress of survey.

The forward and backward bearings of a line differ by 180° .

Whole Circle Bearing (W.C.B.): The angle between the magnetic meridian and the line is called the whole circle bearing. It may have any value between 0° . The bearing of a line observed in the field is the whole circle bearing. The whole circle bearing of a line which coincides with the magnetic meridian is 0° .

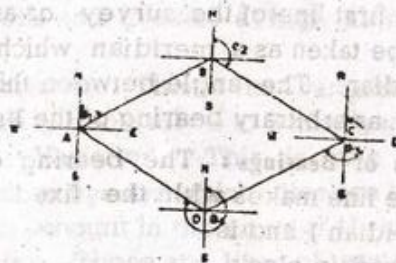


FIG. 3.5

In Fig. 3.5, the angle θ_1 , θ_2 , θ_3 and θ_4 are the whole circle bearing of lines AB, BC, CD, and DA respectively.

(d) *Reduced Bearing (R.B.)*: If the whole circle bearing of a line is greater than 90° , it is reduced to a corresponding angle less than 90° which has same value of trigonometrical functions. The corresponding reduced angle is known as reduced bearing. The procedure to obtain

reduced bearing from whole circle bearing is shown in Table 3.1.

Table 3.1.

Whole circle bearing (W.C.B.) between	Reduced bearing (R.B.)	Cardinal direction or Quadrant
$0^\circ - 90^\circ$	Same as W.C.B.	N.E.
$90^\circ - 180^\circ$	$180^\circ - \text{W.C.B.}$	S.E.
$180^\circ - 270^\circ$	$\text{W.C.B.} - 180^\circ$	S.W.
$270^\circ - 360^\circ$	$360^\circ - \text{W.C.B.}$	N.W.

When the forward bearing of a line is known, the backward bearing of the same can be obtained by the following relation:

$$\text{Backward Bearing} = \text{Forward Bearing} \pm 180^\circ$$

If the forward bearing is less than 180° , plus sign is to be used while if it exceeds 180° , minus sign should be used. The difference of the bearings of two lines at a station is the angle between them (included or excluded angle).

If the difference between forward and backward bearing of a line is not 180° , then there are errors in the measured angles. The errors may be due to (a) faulty instrument, (b) observation and (c) local attraction.

(e) *Local Attraction*: The magnetic needle will be deviated from the magnetic meridian due to the influence of magnetic substances. This phenomenon is known as local attraction. This is caused by the proximity of iron, steel structures, rails, electric post and cables, keys, knives, iron buttons, wrist watches, pens, steel framed spectacles, chains, steel tapes and arrows.

(f) *Correction for local attraction*: The corrections in the bearing of different lines are made in various ways, depending upon the magnitude of the error:

(i) When the error is small i.e., the difference of back bearing and fore bearing is nearly 180° , the back bearing

should be increased or decreased by 180° in order to obtain the corresponding fore bearing. Then the mean is taken between this and the fore bearing.

Example : Let the observed fore and back bearings of a line are $82^\circ 18'$ and $262^\circ 30'$ respectively. The difference between the bearings is $180^\circ 12'$. Hence $12'$ is the error which needs to be adjusted. Half of this is added to fore bearing and the other half is subtracted from the back bearing. New corrected bearing are $82^\circ 24'$ and $262^\circ 24'$ respectively and difference is 180° exactly.

(2) When the error is considerable then the following procedure is adopted.

In this method the included angles of affected stations are calculated from the observed bearings. Starting from the unaffected line and using the calculated included angle, the correct bearing of each line is then computed.

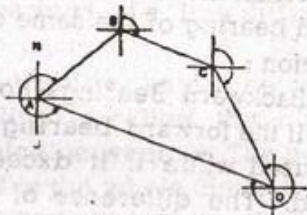


FIG. 3.6

Example : (Fig. 3.6)

Line.	F.B.	B.B.
AB	$52^\circ 15'$	$232^\circ 15'$
BC	$140^\circ 40'$	$320^\circ 55'$
CD	$170^\circ 30'$	$350^\circ 10'$
DA	$290^\circ 10'$	$110^\circ 15'$

From the above data it is found that stations A & B are free from local attraction as the difference between the fore and back bearings of AB is 180° .

Included angles	Calculation
$\angle A$	$= \text{B.B. of DA} - \text{F.B. of AB} = 58^\circ 0'$
$\angle B$	$= \text{B.B. of AB} - \text{F.B. of BC} = 91^\circ 35'$
$\angle C$	$= \text{B.B. of BC} - \text{F.B. of CD} = 150^\circ 25'$
$\angle D$	$= \text{B.B. of CD} - \text{F.B. of DA} = 60^\circ 0'$
	Sum = 360° (Checked)

Corrected bearings :

$$\text{F.B. of BC} = \text{B.B. of AB} - \angle B = 140^\circ 40'$$

$$\text{B.B. of BC} = \text{F.B. of BC} + 180^\circ = 320^\circ 40'$$

$$\text{F.B. of CD} = \text{B.B. of BC} - \angle C = 170^\circ 15'$$

$$\text{B.B. of CD} = \text{F.B. of CD} + 180^\circ = 350^\circ 15'$$

$$\text{F.B. of D.A} = \text{B.B. of CD} - \angle D = 290^\circ 15'$$

$$\text{B.B. of DA} = \text{F.B. of DA} + 180^\circ = 110^\circ 15'$$

3-6 Field Procedure : First the reconnaissance of the plot to be traversed and the fixation of stations on the same should be done as in the case of chain survey. When stations are fixed, the compass is placed over the station A (Fig. 3.5) and levelled by properly setting the ball and socket joints and the legs of the tripod stand. The centre of the compass and that of the station should be on the same vertical line and this is checked by dropping a plumb bob from the hook attached to the bottom centre of the compass. Now the sight vane of the compass is turned towards the ranging rod at stations B and the forward bearing of the line AB is taken and the length of AB is measured. Simultaneously offsets are taken on both sides with the help of chains, tapes and optical squares as in case of chain survey. The compass is now shifted to station B and levelled. Now turning the sight vane towards station A, the bearing of BA is taken. This angle is the forward bearing of BA and back bearing of AB. In this way the F.B. and B.B. and lengths taking offsets of all the lines are measured. To minimise the errors in readings of the angles, at least three readings of a line from the same station should be taken and their mean will be the correct bearing.

The field book to record all the details of the plot to be traversed, is the same as in chain survey except that the forward and backward bearings of each line should be recorded on the central column at start and end respectively. To avoid any confusion afterwards the backward bearing of the previous line and the forward bearing of the front line should be recorded at each station.

3-7 Plotting of a Compass Traverse: The following steps should be considered before plotting a traverse :

- Selection of a suitable scale.
- A rough sketch of the traverse to an approximate scale to see as to how best way the sides can be arranged on drawing sheets.
- Correction of observed bearings.

Method of Plotting: The following are the different methods :

(a) **By parallel meridians through each station:** Station A is first fixed on the drawing sheet and then a line NS representing magnetic the meridian is drawn through it. The bearing of the line AB is drawn with a protractor and its length is taken from the chosen scale. The station B is now located. Now through B again magnetic meridian NS is drawn. The bearing and the length of the line BC are drawn in the same way as AB. The same procedure is followed for the stations C & D (Fig 3.7.)

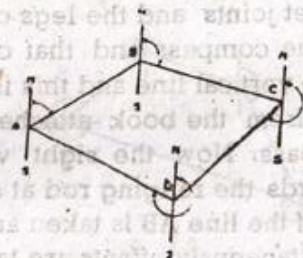


FIG 3.7

(b) **By Included Angles:** This method consists in drawing a magnetic meridian NS through the starting station A. The bearing of the line AB is drawn by a protractor and its length is measured from the chosen Scale. The station B is now fixed. Now the included angle ABC at B which was calculated from bearings of AB and BC is drawn. The length BC is taken from the chosen scale and the station C is fixed. At C, D and E the same procedure is followed (Fig. 3.8)

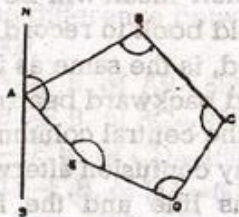


FIG 3.8

(c) **By Rectangular Co-ordinates:** In this method every station is plotted with reference to two lines drawn at right

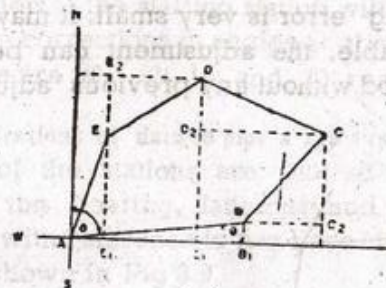


FIG 3.9

angles through some convenient point or the starting station A (Fig 3.9)

The reference axes are generally the magnetic meridian NS and the East-West line EW perpendicular to it at A. If the bearing and length of the line AB are known, its projection on the line parallel to the magnetic meridian NS and on the line perpendicular to it i.e., EW may be obtained. From Fig 3.9, $AB_1 = AB \sin \theta$, where θ is the bearing of the line AB, and $EB_1 = AB \cos \theta$. Knowing AB_1 and EB_1 , the station can be easily located. Similarly, stations C, D and E can be located.

3-8 Closing Error: In a closed traverse, the end of last line should meet the starting point. But while plotting, it will be observed that the last line does not generally end at the starting point. This discrepancy is termed as closing error. This error is due to incorrect measurements of angles and sides in the field. In compass survey the permissible error per bearing should not be more than 15 minutes because the least value that can be read in the graduated scale is 15 minutes. So the total angular error of closure

should not exceed $15\sqrt{N}$, where N is the number of sides of a traverse. In the case of lengths of the sides, the error of closure should not exceed 1 in 600.

If the closing error is very small, it may be neglected. If it is considerable, the adjustment can be done by the following method without any previous adjustment of the angles.



F-6. 3 10

In Fig. 3.10, $ABCDEA_1$ is a polygon which has been plotted from the measured field data. According to definition of a closed polygon points A_1 and A should coincide. The distance AA_1 is the closing error which needs to be adjusted. AA_2 line is drawn to a suitable scale, in such a way that AA_2 equals the sum of the sides of the traverse. A perpendicular A_2a equals to the closing error AA_1 is drawn at A_2 . Aa is joined and perpendiculars Bb, Cc, Dd and Ee are erected to cut Aa . These perpendiculars are the respective corrections or distances through which the stations B, C, D and E should be moved upwards in this case.

At stations D and E of the traverse, lines parallel to AA_1 are drawn. Now the stations B, C, D and E are shifted upwards by the amounts Bb, Cc, Dd and Ee respectively. $abcde$ is the adjusted or corrected traverse.

3-9 Adjustment of Angular Error: In a closed traverse the sum of included angles must be equal to $(2N-4) \times \text{right}$

angles, where N is the number of sides. But in practical cases there is always angular error. This error can be corrected by the following method.

If the total angular error in a traverse of 5 sides is $20'$ then the correction at the starting station will be $20'$ divided by 5, i.e. $4'$. The corrections in angles at the 2nd, 3rd, 4th and 5th stations are $8', 12', 16'$ and $20'$ respectively.

3-10 Computations of data to plot a Traverse: The different locations of the stations are plotted on a drawing sheet knowing the bearing, latitudes and departures of different sides with reference to two lines NS and EW with the origin A as shown in Fig 3.9.

Latitude: It is the distance measured parallel to the magnetic meridian (NS line). It is positive when measured northward from origin. This positive latitude is termed as Northing. The latitude is negative when measured southward from origin and is termed as Southing.

$$\text{Latitude} = \text{Length of the side} \times \text{Cosine of its bearing}$$

$$(BB_1 = AB \cdot \cos \theta)$$

Departure: It is the distance measured parallel to the line which is perpendicular to the magnetic meridian (EW line). The departure may be positive or negative. Eastward departure is positive and is termed as Easting while westward departure is negative and is termed as Westing.

$$\text{Departure} = \text{Length of the side} \times \text{Sine of its bearing}$$

$$(AB_1 = AB \cdot \sin \theta)$$

Consecutive Co-ordinates: The latitude and departure of any station with reference to the proceeding station are termed as consecutive co-ordinates of the station. The consecutive co-ordinates of the station B are AB_1 and B_1B and both are positive. Similarly the consecutive co-ordinates

of the station C are BC_2 and C_2C and both are positive. And those of station D are CD_2 and D_2D and are negative and positive respectively.

Independent Co-ordinates: The latitude and departure of any station with respect to a common origin are termed as independent co-ordinates of that station. They are also termed as total latitude and total departure of a station. The independent co-ordinates of any station are calculated by taking the algebraic sum of the latitudes and departures of the sides between that station and the origin.

The independent co-ordinates of the station D as for an example (Fig. 3.9) are departure AD_1 and latitude D_1D , where $AD_1 = AB_1 + BC_1 - CD_1$ and $D_1D = B_1B + C_1C + D_1D$.

3-11 Characteristics of a Closed Traverse :

- Sum of included angles $= (2N - 4) \times 90^\circ$, where $N =$ number of sides
- Sum of excluded angles $= (2N + 4) \times 90^\circ$
- Forward bearing and backward bearing of a line must differ by 180° .
- Sum of the Northings = Sum of the Southings
- Sum of the Eastings = Sum of the Westings
- The difference between the bearings of two lines at a station is the included or excluded angle.

3-12 Correction for balancing a Traverse: After calculations of the consecutive co-ordinates of the different stations of a closed traverse, it may be found that the sum of the Northings is not equal to the sum of the Southings, and the sum of the Easting is not equal to the sum of the Westings. This difference is distributed to the latitudes and departures of different stations by applying the following rules.

(i) First Rule :

- Correction to Northing of any side $= \frac{1}{2} \times \text{Total error in latitude} \times \frac{\text{Northing of that side}}{\text{Sum of Northings}}$
- Correction to Southing of any side $= \frac{1}{2} \times \text{Total error in latitude} \times \frac{\text{Southing of that side}}{\text{Sum of Southings}}$
- Correction to Easting of any side $= \frac{1}{2} \times \text{Total error in departure} \times \frac{\text{Easting of that side}}{\text{Sum of Eastings}}$
- Correction to Westing of any side $= \frac{1}{2} \times \text{Total error in departure} \times \frac{\text{Westing of that side}}{\text{Sum of Westings}}$

(ii) Second Rule (Bowditch Rule) :

- Correction to Latitude of any side $= \text{Total error in latitude} \times \frac{\text{Length of the side}}{\text{Perimeter of the traverse}}$
- Correction to Departure of any side $= \text{Total error in departure} \times \frac{\text{Length of the side}}{\text{Perimeter of the traverse}}$

(iii) Third Rule (Transit Rule) :

- Correction to Latitude of any side $= \text{Total error in latitudes} \times \frac{\text{Latitude of the side}}{\text{Arithmetic sum of all latitudes}}$
- Correction to Departure of any side $= \text{Total error in departures} \times \frac{\text{Departure of the side}}{\text{Arithmetic sum of all departure}}$

3-13 Traverse Computation Chart: The computation including necessary corrections of a closed travers is arranged systematically on a sheet shown in Travers chart...1.

But it is less precise and very often subject to errors due to the fact that the needle is not perfectly straight or needle is not sensitive. Sometimes the plane of sight is not vertical or the vertical hair being too thick or loose. Errors may also arise due to inaccurate levelling of the compass, imperfect sighting of the ranging rods and mistakes in recording bearings and lengths. There may be errors due to local attraction, variation of magnetic meridian and magnetic changes in the atmosphere.

Example: The bearing of a line AB was observed by a prismatic compass and found to be $240^{\circ}15'$. If the true bearing of the same line is $239^{\circ}45'$, what is the variation?

$$\begin{aligned} \text{Variation} &= \text{Magnetic bearing} - \text{True bearing} \\ &= 240^{\circ}15' - 239^{\circ}45' = 0^{\circ}30' \end{aligned}$$

Example: The following bearings were observed in a closed traverse ABCDEFA. Calculate the whole circle bearings (W.B.) reduced bearings (R.B.) and the included angles.

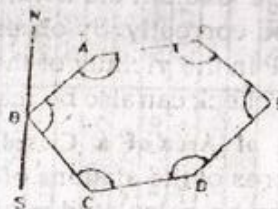


FIG. 3.12

Line	F.B.	B.B.
AB	203°S.W.	23°N.E.
BC	$133^{\circ}37' \text{ S.E.}$	$313^{\circ}37' \text{ N.W.}$
CD	$89^{\circ}41' \text{ N.E.}$	$269^{\circ}41' \text{ S.W.}$
DE	$24^{\circ}25' \text{ N.E.}$	$204^{\circ}25' \text{ S.W.}$
EF	$314^{\circ}20' \text{ N.W.}$	$134^{\circ}20' \text{ S.E.}$
FA	$256^{\circ}56' \text{ S.W.}$	$76^{\circ}56' \text{ N.E.}$

From Fig. 3.12.

whole circle bearings are the same as the forward bearings.

Reduced bearing of AB	$= 203^{\circ} - 180^{\circ}$	$= 23^{\circ} \text{S.W.}$
" " BC	$= 180^{\circ} - 133^{\circ}37'$	$= 46^{\circ}23' \text{S.E.}$
" " CD	$= 89^{\circ}41' \text{ N.E.}$	
" " DE	$= 24^{\circ}25' \text{ N.E.}$	
" " EF	$= 360^{\circ} - 314^{\circ}20'$	$= 45^{\circ}40' \text{N.W.}$
" " FA	$= 256^{\circ}56' - 180^{\circ}$	$= 76^{\circ}56' \text{S.W.}$
Included angles, A	$= 203^{\circ} - 76^{\circ}56'$	$= 126^{\circ}4'$
B	$= 133^{\circ}37' - 23^{\circ}$	$= 110^{\circ}37'$
C	$= 89^{\circ}41' + (180^{\circ} - 133^{\circ}37')$	$= 136^{\circ}4'$
D	$= 24^{\circ}25' + (180^{\circ} - 89^{\circ}41')$	$= 115^{\circ}44'$
E	$= 314^{\circ}20' - (180^{\circ} + 24^{\circ}25')$	$= 109^{\circ}55'$
F	$= 256^{\circ}56' - (314^{\circ}20' - 180^{\circ})$	$= 122^{\circ}39'$

Example: The bearings of the different sides of a closed traverse ABCDEFA (Fig. 3.13) were taken by a prismatic compass and the included angles were calculated from the observed bearings. The lengths of the different sides and the included angles are shown on traverse Chart-1. Applying necessary corrections, calculate the consecutive and the independent co-ordinates.

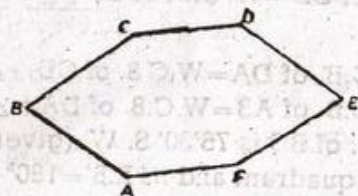


FIG. 3.13

Complete calculation is shown in Chart-1 page-70.

Example: Calculate the area of the traverse from the calculated data on Chart-1. and the corresponding Fig 3.13.

$$\begin{aligned} \text{Area, } A &= \Sigma \frac{(D_2 + D_1)}{2} (L_2 - L_1) \\ &= \frac{68.755 + (-84.930)}{2} \times (482.940 - 273.954) = -1690.00 \\ &= \frac{287.227 + 68.755}{2} \times (508.256 - 482.940) = +4520.00 \end{aligned}$$

$$\frac{431.795 + 287.227}{2} \times (263.295 - 503.253) = -86300.00$$

$$\frac{299.113 + 431.795}{2} \times (-23.165 - 263.295) = -106000.00$$

$$\frac{0.00 + 299.113}{2} \times \{0.00 - (-23.165)\} = +3450.00$$

$$\frac{-84.930 + 0.00}{2} \times (273.954 - 0.00) = -11,600.00$$

$$A = 197.610 \text{ sq. ft.}$$

$$= 4.52 \text{ acres.}$$

Example: From the observed bearings of a traverse ABCDA the following included angles were calculated.

$\angle DAB = 65^\circ 20'$, $\angle ABC = 95^\circ 45'$, $\angle BCD = 95^\circ 25'$, and $\angle CDA = 113^\circ 30'$. If the reduced bearing of the side BC is $75^\circ 30'$ S.W., calculate the reduced bearing of the other sides.

R.B. of BC = $75^\circ 30'$, $\angle W.C.B.$ of BC = $180^\circ + 75^\circ 30' = 255^\circ 30'$
 $\angle W.C.B.$ of CD = $\angle W.C.B.$ of BC + $\angle BCD - 180^\circ = 255^\circ 30' + 95^\circ 25' - 180^\circ = 170^\circ 55'$.

$\angle W.C.B.$ of DA = $\angle W.C.B.$ of CD + $\angle CDA - 180^\circ = 104^\circ 25'$

$\angle W.C.B.$ of AB = $\angle W.C.B.$ of DA + $\angle DAB + 180^\circ = 349^\circ 45'$

R.B. of BC is $75^\circ 30'$ S.W. (given). So line CD lies in the second quadrant and its R.B. = $180^\circ - 170^\circ 55' = 9^\circ 5'$ S.E.

The line DA also lies in the second quadrant and its R.B. = $180^\circ - 104^\circ 25' = 75^\circ 35'$ S.E. The line AB lies in the fourth quadrant and its R.B. = $360^\circ - 349^\circ 45' = 10^\circ 15'$ N.W.

Example: Calculate the length and bearing of the closing side DA of a closed traverse ABCDA from the following data:

side	Length	W.C.B.
AB	300'	$260^\circ 15'$
BC	900'	$190^\circ 30'$
CD	600'	$80^\circ 45'$

Calculation is shown below:

Side	Length in ft.	R. B.	Cardinal direction	Latitude		Departure	
				N	S	E	W
AB	300	$80^\circ 15'$	S. W.		51.00		296.00
BC	900	$10^\circ 30'$	S. W.		884.70		164.00
CD	600	$80^\circ 45'$	N. E.	97.20		592.00	
DA							
				97.20	935.70	592.00	460.00

Side	Latitude	Departure
AB = 300'	$300 \times \cos 80^\circ 15' = 51.00$	$300 \times \sin 80^\circ 15' = 296.00$
BC = 900'	$900 \times \cos 10^\circ 30' = 884.70$	$900 \times \sin 10^\circ 30' = 164.00$
CD = 600'	$600 \times \cos 80^\circ 45' = 97.20$	$600 \times \sin 80^\circ 45' = 592.99$
	Difference of N-S	Difference of E-W
	838.50 (N)	132.00 (W)

The co-ordinates of the side DA are 838.50 (N) and 132.00 (W)

$$DA = \sqrt{(838.50)^2 + (132)^2} = 849.50 \text{ (approx.)}$$

If θ be the reduced bearing of DA, $\theta = \tan^{-1} \frac{132}{838.5} = 9^\circ 0'$ (4th quadrant)

$$\angle W.C.B. \text{ of DA} = 360^\circ - 9^\circ = 351^\circ$$

Example: Calculate the bearings of the sides DE and FA of the closed traverse shown in Fig. 3.14 from the following table.

Side	Length	Bearing	Calculations are shown below.
AB	400'	$135^\circ 30'$	R.B. of AB = $44^\circ 30'$ S.E.
BC	430'	$85^\circ 45'$	R.B. of BC = $85^\circ 45'$ N.E.
CD	450'	$46^\circ 15'$	R.B. of CD = $46^\circ 15'$ N.E.
DE	500'	—	R.B. of DE = $85^\circ 30'$ N.W.
EF	350'	$274^\circ 30'$	
FA	450'	—	

Side	Latitude	Departure
AB	$400 \times \cos 44^\circ 30' = 288.00$	$400 \times \sin 44^\circ 30' = 280.00$
BC	$430 \times \cos 85^\circ 45' = 31.80$	$430 \times \sin 85^\circ 45' = 429.00$
CD	$450 \times \cos 46^\circ 15' = 311.60$	$450 \times \sin 46^\circ 15' = 324.50$
EF	$350 \times \cos 85^\circ 30' = 27.40$	$350 \times \sin 85^\circ 30' = 349.50$

Side	Length in ft.	R. B.	Cardinal Direction	Latitude		Departure	
				N	S	E	W
AB	400	44°30'	S. E.		283.00	230.00	
BC	430	85°45'	N. E.	31.80		429.00	
CD	450	46°15'	N. E.	311.60		324.50	
DE	500						
EF	330	45°30'	N. W.	27.40			349.50
FA	450						
				370.80	288.00	103.350	349.50

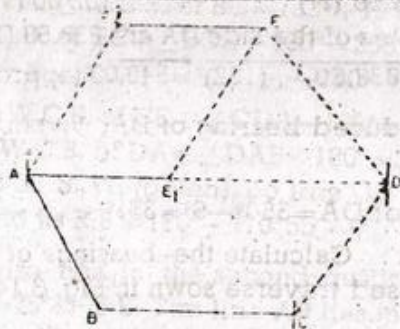


FIG. 3.14

The line EF is shifted to its new position AE parallel to itself. By doing so the length and bearing of the line FE are not altered. Now ABCDE₁A is a closed traverse. Latitude of DE₁ = (370.80 - 288.00) = 82.80' (southing) and departure of DE₁ = 103.350 - 349.50 = 684.00' (westing).

$$\therefore \text{Length } DE_1 = \sqrt{(82.80)^2 + (684)^2} = 686.00'$$

Now in $\triangle DE_1E$, all the sides are known.

$$\text{Reduced Bearing of } DE_1 = \tan^{-1} \frac{614}{82.80} = 93^\circ 0'$$

$$\text{W.C.B. of } DE_1 = 180^\circ + 83^\circ = 263^\circ 0'$$

$$\text{By trigonometry, } ED^2 = EE_1^2 + E_1D^2 - 2 \times EE_1 \times E_1D \cos \angle EE_1D$$

$$(500)^2 = (450)^2 + (686)^2 - 2 \times 450 \times 686 \cos \angle EE_1D$$

$$250000 = 202500 + 472880 - 617400 \cos \angle EE_1D$$

$$\therefore \cos \angle EE_1D = \frac{425380}{617400} = 0.69, \therefore \angle EE_1D = 46^\circ 18'$$

$$\text{W.C.B. of } E_1E = 83^\circ 0' - 46^\circ 18' = 36^\circ 42'$$

$$\text{W.C.B. of } FA = 180^\circ + 36^\circ 42' = 216^\circ 42'$$

$$\text{Again, } (EE_1)^2 = (ED)^2 + (E_1D)^2 - 2 \times ED \times E_1D \cos \angle E_1DE$$

$$(450)^2 = (500)^2 + (686)^2 - 2 \times 500 \times 686 \cos \angle E_1DE$$

$$202500 = 250000 + 472880 - 686000 \cos \angle E_1DE$$

$$\cos \angle E_1DE = \frac{520380}{686000} = 0.759, \therefore \angle E_1DE = 40^\circ 36'$$

$$\therefore \text{W.C.B. of } DE = 263^\circ + 40^\circ 36' = 303^\circ 36'$$

3-17 Theodolite It is the most complicated but accurate instrument with which both horizontal and vertical angles can be measured. This is also used for locating points, establishing slope, extending lines, finding difference in elevation, ranging curves and traversing.

Theodolites are of three types, namely (i) Transit (ii) Wye and (iii) Everest. The last two types are obsolete nowadays. The first one is used in most of the works. A transit theodolite is one whose telescope can be transited or revolved through a complete circle in the vertical plane about its horizontal axis. Wye and Everest types are non-transit.

There are different sizes of theodolites. The diameter of the graduated circle on the lower plate defines its size. In engineering works, generally, 4 inches to 6 inches theodolites are used while in triangulation, 8 inches to 12 inches are used.

3-18 Parts of a Transit Theodolite :

It consists of the following main parts (Fig. 3.15).

(1) *The Telescope* : The telescope is fitted centrally and at right angles to the horizontal axis. It consists of an eye-

piece glass, diaphragm with cross-hairs, an object glass and a focussing screw.

(2) *The Levelling Head*: It is the bottom part which is screwed to the tripod stand. It consists of two circular plates, fixed at a certain distance apart known as parallel plates and three foot-screws known as levelling screws or a tribrach plate with three arms, each of those carrying a foot-screw. There is a central aperture in the lower plate. The plumb bob is suspended through this aperture to check whether the vertical axis of the theodolite is exactly on the station or not. The upper plate is supported by four or three foot-screws.

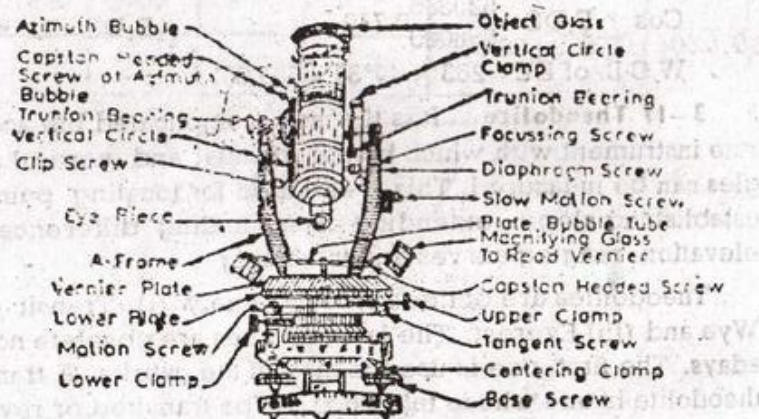


Fig. 3.15

(3) *The Lower Circular Plate*: This is a circular graduated plate also known as lower plate or scale plate. It has two attached screws. The first one is known as lower clamp and the second one tangent screw. The upper plate is fixed by tightening the clamp screw. The lower plate and the upper part of the theodolite can be rotated slightly by turning the tangent screw.

(4) *The Upper Circular Plate*: This rests on the lower plate. It is also known as vernier plate. It has also a clamp

screw and a tangent screw. These are provided to fix the vernier plate to the lower plate very accurately. The upper plate has two or three verniers with magnifying glasses placed 180° or 120° apart for reading horizontal angle.

(5) *The Vertical Circle*: It is a graduated circle by which vertical angles are read. It is provided with two or three vernier scales with or without magnifying glasses. This vertical circle can be set with the telescope at any desired position in the vertical plane by means of a clamp and a tangent screw.

(6) *The A-Frames (or standards)*: These frames resemble the letter A in shape and support the horizontal axis of theodolite.

(7) *The Azimuth and Plate Bubble Tubes*: The azimuth bubble tube is fixed on the upper surface of the vertical circle while the plate bubble tube is fixed on the upper surface on the upper circular plate. They are placed at right angles to each other. They are used to level the theodolite. Bubble tubes are sealed glass tube placed in a brass tube. The tubes are nearly filled up with alcohol, ether or a mixture of both. The upper space of the tube is occupied by an air bubble. The outer shape of the glass tube is cylindrical but the upper part of the inner surface is an arc of a circle when cut longitudinally. The outer surface is graduated on both directions with zero at the centre. The axis of the bubble is the tangent to the circular arc when the bubble is at the centre of its run. This axis is also known as the bubble line. The length of the bubble varies with the variation of temperature.

The sensitiveness or sensitivity of a bubble is its fast moving properties and is measured by the angle through which the bubble tube is tilted to cause the bubble to move through one division of the scale. It may also be expressed in terms of the radius of curvature of the tube. Sensitivity is inversely proportional to the number of

seconds. It varies from 8 to 45 seconds for different instruments.

(8) *The Compass* : The theodolites are fitted with a circular trough or tubular type of compass mounted on one of the A frames to read the magnetic meridian.

(9) *The Plumb Bob* : A plumb bob (a conical shaped weight) is suspended by a string from the hook attached to the bottom of the central vertical axis of the theodolite to check whether the vertical axis is exactly over the station or not.

(10) *Tripod Stand* : It is three legged support on which the bottom most part of the theodolite is screwed up. The legs are made of seasoned timber with pointed steel shoes at the lower ends so as to fix them firmly on the ground.

3-19 *Definitions* : The following definitions should be minutely studied in order to understand clearly and thoroughly the principle of handling a theodolite.

(1) *Horizontal Axis (Transverse or Trunnion Axis)* : This is the axis about which the telescope is rotated in a vertical plane.

(2) *Vertical Axis* : This is the axis about which the telescope is rotated in a horizontal plane.

(3) *Telescope Axis* : This is the line joining the optical centre of the objective and the centre of the eyepiece.

(4) *Line of Collimation (Line of Sight)* : The imaginary line passing through the intersection of the cross-hairs of the diaphragm and the optical centre of the objective and its extension upto infinity is known as the line of collimation.

(5) *Diaphragm* : It is a ground glass ring consisting of cross-hairs. This is fitted in the telescope tube. The cross-hairs may be of spider webs, lines on glass and platinum wires. The diaphragm may be moved vertically or horizontally by means of screws.

(6) *Face Left and Face Right* : If the vertical circle of the theodolite is on the left side of the observer when taking a reading, the position is termed as face left. When the vertical circle lies on the right of the observer, the position is face right. And corresponding observations are called face left and face right observations.

(7) *Objective and Eye-piece* : These are two lenses. The object glass is fixed at the fore end of the telescope while the eye-piece is fixed at the rear end. The objective is a double convex lens while the eye piece is a plano-convex lens.

(8) *Centering* : It is the operation of setting the theodolite exactly over a station with the help of a plumb bob.

(9) *Parallax* : When the image of an object is not exactly on the plane of the diaphragm, there is relative movement of the image with respect to the cross-hairs while moving the eye up and down. This phenomenon is known as parallax. And the error that will arise due to this is known as parallax error.

(10) *Chromatic Aberration* : The dispersion of white light into its component colours by a lens is known as chromatic aberration. Achromatic lenses are used to minimise this defect.

(11) *Spherical Aberration* : It is a defect in the image formed by a lens of spherical surface.

(12) *Changing Face* : It is the art of bringing the vertical circle to the left of the observer if at the beginning it is to the right and vice-versa. In case of a transit theodolite it is done by turning the telescope through 180° about its horizontal axis. This can also be done by turning through 180° about its vertical axis. But in non-transit case it is achieved by lifting the telescope from its supports, reversed and replaced on its supports.

(13) *Telescope Normal and reversed* : When the position of the bubble tube is on the top of the telescope, it is called

telescope normal. When the telescope is transited, the position of the bubble tube is at the bottom which is called telescope reversed or inverted.

3-20 Adjustment of a Transit Theodolite: If the theodolite is in perfect adjustment, then the following conditions will be satisfied.

(a) The line of collimation should be at right angles to the horizontal axis. (b) The horizontal axis should be perpendicular to the vertical axis. (c) The axis of the telescope bubble should be parallel to the line of collimation. (d) The axis of the plate levels should be perpendicular to the vertical axis. (e) When the line of collimation is horizontal the vertical circle vernier should read zero.

If any one of the above conditions is not satisfied, the theodolite needs adjustment.

There are mainly two types of adjustment:

(1) Temporary and (2) Permanent. Before taking any observation, temporary adjustments are made at every set up of the instrument. While permanent adjustments are made for accuracy of observations.

Temporary adjustments: (a) *Setting*: This consists of centering and approximately levelling of the instruments by tripod stands.

(b) *Levelling*: After setting, the instrument is levelled with respect to plate levels by means of levelling screws in such a way that the vertical axis is truly vertical. This is done by turning the telescope until it is parallel to the line joining any two levelling screws of foot screws. By turning both the screws simultaneously in opposite direction the bubble is brought to the centre of its run. The other bubble is brought to the central position by turning the third screw. The process is repeated until both the bubbles are brought to centre. Now if the instrument is rotated through any degree and if it is found that the bubbles remain centrally

then the vertical axis is truly vertical. In case of instruments having four levelling screws one of the bubble should be made parallel to diagonal joining the opposite screws. Now the above procedure is followed.

(c) *Focussing of Eye-piece and Objective*: Focussing of the eyepiece is done to see the cross-hairs distinctly. This is obtained by focussing the telescope towards a distinct object (white wall or white sheet of paper) and moving the eye-piece in and out.

The object glass is focussed to bring the image of the object in the plane of the cross-hairs. To obtain it, the telescope is directed towards the object and the focussing screw is turned until a sharp and clear image is formed. This is same as the elimination of the parallax error.

Permanent adjustments: There are six permanent adjustments in a transit theodolite.

- Adjustment of parallel plate bubble tubes.
- Adjustment of horizontal (transverse) axis of the telescope.
- Adjustment of the line of collimation vertically.
- Adjustment of the line of collimation laterally.
- Adjustment of the bubble tube on the telescope or T-frame.
- Determination of the index error of the vertical circle.

Adjustment of parallel plate bubble tubes: The object of this adjustment is to set the bubble tube in such a way that when it is horizontal, the vertical axis becomes truly vertical. If it is not in adjustment then, when the bubble tube is horizontal, the vertical axis will make an angle α with truly vertical line (Fig 3 16). In this condition the graduated scale is inclined and any horizontal angle observed will have error. First the bubble is brought to the centre of its run by using foot screws. Now the instrument is turned

through 180° in azimuth. If the bubble moves towards any side of the tube, then there is error of this nature. If the movement of the bubble is $2n$, then $2n/2$ division i.e. n divisions should be brought back by Capstan headed screw and the other half i.e. n divisions by the levelling screws. It

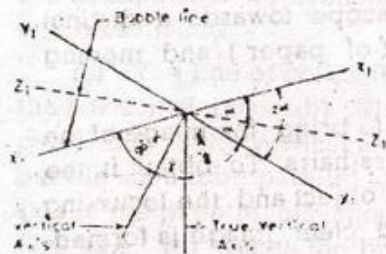


FIG. 3.6

should be remembered clearly that when the bubble is reversed end to end, the deviation of the bubble is twice the actual error in the axis of the bubble. That is why the correction by the levelling screws is only half the amount of the error. The instrument is rotated through 360°

and if it is found that the bubble remains at the centre then the instrument is ready for use. After this adjustment the Capstan headed screw should not be handled.

Adjustment of the horizontal (transverse) axis of the telescope :

The object of this adjustment is to make the horizontal axis of the telescope at right angles to the vertical axis. There will be error in the measurement of horizontal angles if the instrument is not adjusted. The adjustment is done by spire test. The theodolite is set up first near a spire or an elevated object and the ends of the horizontal axis are unclamped. The telescope is now focussed to an elevated object O making exact coincidence of the cross-hairs with the object. The telescope is lowered keeping both the lower and upper clamps tightened. The foot of the object is marked as F_1 on the ground by means of an arrow where the cross-hairs cut. The telescope is lifted from its supports and the instrument is rotated through 180° in azimuth by loosening the lower clamp. In doing so the position of A-frame is reversed. The telescope is now placed again on these already reversed supports. The

telescope is focussed at the object O and exact coincidence with the cross-hairs is made. The telescope is depressed and the cross-hairs must cut the previously located foot of the object F_1 if the instrument is in adjustment. If not it will locate another point F_2 where the cross-hairs now cut the foot of the object.

To adjust this error, one end of the support is lowered by screw attached to A-frame until the cross-hairs seem to have been moved by $\frac{1}{4}$ th of the distance F_1F_2 . By doing this, the line of collimation is thrown from position OF_1 to O_1F_2 (Fig. 3.17). Again the telescope is directed to O by adjusting the lower clamp and tangent screw. The telescope is now depressed and if the error is adjusted by the above method then the cross-hairs should coincide with F which is vertically below O . F is the midpoint of F_1F_2 .

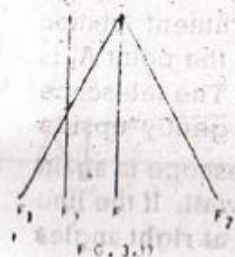


FIG. 3.17

If the cross-hairs do not coincide with F , the above procedure is repeated till exact coincidence with F is attained. There will be error also in the measurement of vertical angles if the horizontal axis of the telescope is not at right angles to the vertical axis. But this error cannot be corrected by the above method. To correct the same, the axis needs adjustment.

Adjustment of line of collimation vertically : The purpose of this adjustment is to coincide the line of collimation with the longitudinal axis of the telescope and also to put the intersection of the cross-hairs on the horizontal diameter of the telescope. For measuring horizontal angles this adjustment is not essential. But this will affect the measurement of vertical angles. To adjust this, the theodolite is levelled first. A levelling staff is held erect in front of the telescope and extreme readings are noted. Now the intersection of the cross-hairs is put at the middle of the extreme readings

by means of the vertical diaphragm screw. Now it is on the horizontal diameter of the telescope.

Adjustment of the line of collimation laterally: The purpose of this adjustment is to make the line of collimation at right angle to the horizontal axis of the telescope. If the line of collimation is not perpendicular to the horizontal axis, it is said to be horizontally out. The horizontal angles measured with such an instrument give error specially when the two points are at different levels. This error comes into picture because the line of collimation traces a curve very close to a hyperbola on the ground instead of a plane surface which is described by the revolution of the telescope when the line of collimation is in adjustment laterally.

To test this adjustment, the instrument is set up on a convenient station O (Fig. 3.18) and levelled. The telescope is focussed to a point A_1 on a wall, situated at a convenient distance (generally 50 ft.) away from the instrument station and exact coincidence of the cross-hairs with the point A_1 is made by means of one of the tangent screws. The telescope is lifted from its supports and replaced very gently upside

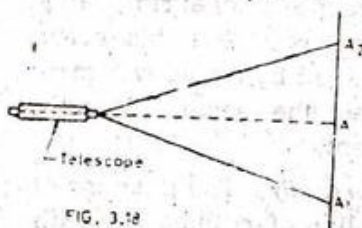


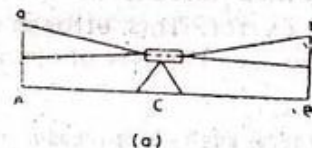
FIG. 3.18

down. The telescope is again focussed to the wall. If the line of collimation is at right angles to the horizontal axis of the telescope, the cross-hairs should cut again the point A_1 on the wall. If not, another point A_2 will be cut by the intersection

of the cross-hairs on the wall. Now, the point A which is the mid-point of A_1A_2 , will give the position on the wall where the line of collimation will be exactly perpendicular to the horizontal axis. To adjust it, diaphragm is moved laterally by the Capstan headed screw till the cross-hairs coincide with A. The procedure is repeated until the error is eliminated.

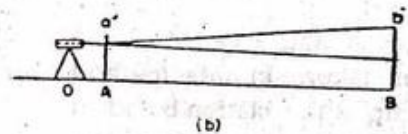
Adjustment of the bubble tube on the telescope or T-frame:

The purpose of this adjustment is to know that the line of collimation or the longitudinal axis of the telescope is parallel to the axis of the bubble. To test this condition, two stations A & B are fixed at a convenient distance apart on an approximately levelled ground. The instrument is placed at the station C & levelled. The vernier is set at zero if possible, otherwise the vernier reading is noted as index error. Staff readings are taken at A & B. The difference of these two readings will give the true difference of elevation between A & B, because the error due to the line of collimation being inclined will be equal on both the stations



(a)

A & B. The instrument is then placed to another station D very close to station A (Fig. 3.19) and levelled. Staff readings are taken again at A & B and their difference is found out. If this difference is same as in the previous case, then the line of collimation is parallel to the axis of the bubble. If they are not equal



(b)

FIG. 3.19

then by trial the telescope is continued turning till a set of readings on A & B gives the same difference as in the case of the instrument placed at the midpoint of A & B. Now the bubble moves to one side of the tube. And this is brought to the centre of its run by adjusting the Capstan headed screw of the bubble tube. This test is also known as two peg test.

Determination of the index error of the vertical circle: The object of this determination is to see that when the telescope bubble is at its central run i. e., the line of collimation is horizontal and parallel to the longitudinal axis of the telescope, the vertical circle should read zero. This is

determined only in those cases where the altitude bubble is attached to the top of the telescope. But in transit theodolite there is no index error because the vernier can be clamped at zero and the telescope is then brought into horizontal position by the help of the screws provided for the purpose.

If the vernier circle does not read zero then the magnitude of the initial reading is known as index error. This is determined by setting up the instrument and levelling it properly. The telescope bubble is then brought to its central run by vertical tangent screw. The reading on the vernier circle is noted. If it is zero then there is no index error. If not, the value is the index error. This error may be added to or subtracted from observed angles of elevation or depression:

3-21 Methods of measuring horizontal angles by a theodolite : There are two methods by which horizontal angles can be measured.

(a) *By Repetition* : Here the same angle is measured several times and their average is taken. In this method (Fig. 3.20) the instrument is set up at the station B and it is required to measure the horizontal angle θ between two stations A and C. The instrument is centered and levelled. The telescope is now focussed at the foot of the ranging rod at A by the lower clamp and the tangent screw. The vernier arrow should coincide with zero of the main scale.

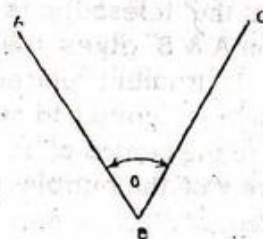


FIG. 3.20

The upper plate is unclamped and the telescope is turned clockwise to the foot of the ranging rod at C and the exact coincidence of the cross-hairs with it is obtained by the upper clamp and tangent screw. The angle is now

read and it is found to be 0. Leaving the vernier unchanged, the lower plate is unclamped and the telescope is turned clockwise until the foot of the ranging rod at A is again bisected accurately by the cross-hairs with the help of lower clamp and tangent screw. The vernier is now checked. It should indicate the same reading as before. Keeping the lower plate fixed, the upper plate is unclamped and the telescope is turned clockwise and focussed to the foot of the ranging rod at C and exact coincidence of the cross hairs with it is obtained.

The vernier should now read 2θ . The procedure is repeated any number of times and final reading after n repetitions should be $n\theta$. 360° is added for each complete revolution to get the value of $n\theta$. This sum divided by the number of repetitions n will give the value of angle θ . The average of all the values of angles read by different vernier should be taken. By this methods any error arising due to eccentric centering, imperfect bisection of the object, imperfect graduation of scales, and any personal error in reading the verniers, is eliminated.

(b) *By reiteration* ; This is easier and quicker than repetition. This is less tedious and is always preferred when a large number of angles are to be measured from the same station. In Fig 3.21 S is the instrument station and it is required to measure the angles $\theta_1, \theta_2, \theta_3, \theta_4,$ and θ_5 subtended at S. The theodolite is set up accurately on station S.

The leading vernier is clamped so that its arrow reads

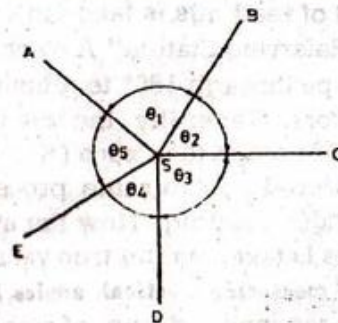


FIG. 3.21

zero with the main scale by the help of upper plate clamp and tangent screw. The telescope is now directed to the foot of the ranging rod at station A, which is taken as the "Referring Station" and exact coincidence is obtained by the help of lower clamp and tangent screw. The vernier readings are noted. The upper plate is loosened and the telescope is turned clockwise until the foot of the ranging rod at B is exactly bisected by the cross-hairs by turning upper tangent screw. The vernier readings now noted will give the value of the angle θ_1 . The upper plate is then unclamped and the telescope is turned clockwise to the foot of the ranging rod at C and exact coincidence of cross-hairs with it is obtained. The vernier readings are noted.

This will give the value of the angle $\angle ASC = \theta_1 + \theta_2$. So the angle $\angle BSC = \angle ASC - \angle ASB = \theta_2$. In this way stations D and E are bisected and angles $\angle CSD = \theta_3$ and $\angle DSE = \theta_4$ are calculated. Finally, the telescope is turned to station A, the "Referring Station" and the leading vernier should now read 360° , because the lower clamp and the tangent screw have not been disturbed during the complete revolution of the telescope. The angle $\angle ESA = \theta_5$ is calculated. If the leading vernier does not read 360° , the reading is noted. This is the error due to slip, etc. If the error is small, it is equally distributed among the observed angles. If it is considerable, the readings should be disregarded and a new set will have to be taken in lieu of those.

The second set of readings is taken in anticlockwise direction from the "Referring Station" A by changing face and turning the telescope through 180° to eliminate the effect of instrumental errors. Generally the leading vernier is clamped at any where other than zero (45° , 60° or 90°) and the angles are measured by the same procedure as in first set, with the new index reading. Now the average of these two sets of readings is taken as the true value of the angles.

3-22 Method of measuring vertical angles by a theodolite :
The angle between the inclined line of sight and the hori-

zontal is known as the vertical angle and it is positive when the angle is above the horizontal plane and negative when below the plane. In Fig. 3.22, the vertical angle BOA is required to be measured. The theodolite is placed at S and properly levelled with respect to both parallel plate and

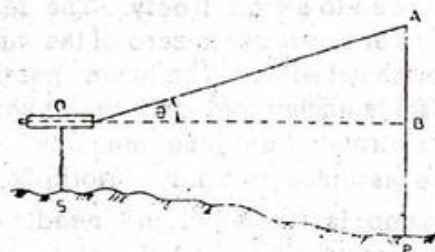


FIG. 3.22

altitude bubbles. The arrow of the vernier is set to the zero of the vertical circle by means of vertical circle clamp and tangent screw. Since the altitude bubble is at the centre of

its run, the vernier should read zero because the line of collimation is horizontal. Now the vertical circle clamp is loosened and the telescope is focussed towards A and exact coincidence of cross-hairs with it is obtained by turning the tangent screw. Both the verniers in the vertical circle are now read and their mean is taken. This gives the value of the vertical angle BOA. For greater accuracy the face of the instrument is changed and the same procedure is repeated and another value of the angle θ is obtained. The mean of this latter angle and the previous one will give the more accurate angle BOA.

This angle θ is the angle of elevation because A lies above the horizontal plane OB. If A lies below B, then the angle will be termed as depression angle.

3-23 Permissible error in angle measurement : In case of single observation the angular error should not exceed $30''$. For greater accuracy the maximum permissible error should not exceed $15''\sqrt{N}$, where N is the number of angular measurements.

3-24 To obtain the magnetic bearing of a line with a

theodolite : The bearing of the line AB (Fig. 3.23) is to be

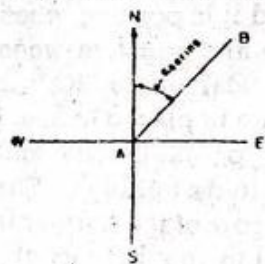


FIG. 3.23

measured. The theodolite is set at A and properly levelled. The trough compass is screwed to the bottom of the lower plate and the needle is allowed to swing freely. The leading vernier is set to zero of the main horizontal scale. The lower parallel plate is unclamped and the theodolite is rotated until the magnetic needle assumes roughly North-South

direction. The lower clamp is fixed and the needle is brought in its exact N-S direction by the help of tangent screw. Now the telescope is pointing towards the magnetic north and the vernier reads zero. The telescope is rotated towards the station B by unclamping upper plate. Exact coincidence of the cross-hairs with it is obtained by the tangent screw. Now the two verniers are read and the mean of the two angles gives the bearing of the line AB. When a trough compass is not available, the bearing is taken by a prismatic compass.

3-25 Traversing with a theodolite : In case of a closed traverse (Fig. 3.24) the bearing of the line AB from the starting station A is taken. By reconnaissance the stations A, B, C, D, and E on the plot to be traversed have been already fixed.

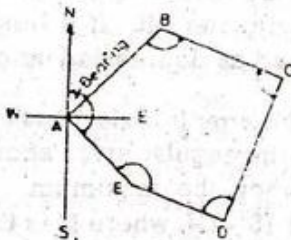


FIG. 3.24

The included angle EAB is measured either by the repetition or by the reiteration method. The theodolite is now shifted to station B and the included angle ABC is measured in the same way. Similarly by shifting the theodolite at C, D and E, the other included angles are measured. Instead of measuring the interior angles, exterior angles can also be measured.

In case of an open traverse the same procedure of measuring angles can be adopted as shown in Fig. 3.25.



FIG. 3.25

3-26 Traverse computation : The calculation is exactly similar to that explained in case of a compass traverse.

3-27 Advantages and disadvantages of using compass and theodolite in surveying : In both cases linear and angular measurements are taken. The angular measurements are taken either by a compass or by a theodolite and the linear measurements are taken by the tape and chain. Theodolite surveying is more accurate than the compass surveying. Moreover, the range of vision is more in theodolite than in compass. Compass measurements are faster than theodolite. In compass survey there is a possibility of local attraction but theodolite is free from this phenomenon. The handling of a theodolite is more complicated than that of a compass.

EXERCISE

Examine carefully the following statements and write whether true or false.

- (a) The graduation in a prismatic compass starts from north.
- (b) The body of the prismatic compass remain fixed but the scale moves in taking the bearing of a line.
- (c) In prismatic compass angles can be read upto 1'.
- (d) The bearing of a line AB and that of BA is same.
- (e) The F. B. and B. B. of a line differ by 180° .
- (f) The difference between the bearings of two lines at a station is the included angle.
- (g) The error in measuring the included angles of a closed traverse is distributed proportionally on all sides.
- (h) If the W. C. B. of a line is $305^\circ 15'$ then its reduced bearing is $35^\circ 15'$.
- (i) The W. C. B. and the R. B. of a line is same in the second quadrant.
- (j) A 6" theodolite means the length of the telescope is 6".
- (k) A theodolite can be used as a levelling instrument.
- (l) The bubble tube is filled up with distilled water.
- (m) In theodolite traversing no bearing is required to be measured.
- (n) The bubble tube is cylindrical in shape.
- (o) In a transit theodolite the telescope cannot be rotated in the vertical plane.
- (p) For each theodolite there are two vernier constants for the two verniers.
- (q) In a theodolite the upper parallel plate contains the main scale.
- (r) Dip is the angle of axis of the magnetic needle with the true geographical north.
- (s) The variation in the dip in 24 hours is known as diurnal variation.
- (t) The R. B. of a line in the third quadrant is the W. C. B. of line minus 180° and its cardinal direction is N. W.

- (u) In a closed traverse of six sides the sum of the included angles is 540° .
- (v) The sum of the exterior angles of the closed travers of four sides is 1080° .
- (w) The line of collimation does not generally coincide with the longitudinal axis of the telescope tube.
- (x) If the transverse axis of the theodolite is not in adjustment, there will be error in the measured vertical angles.
- (y) The latitude of a line is equal to the length of the line multiplied by sine of its R.B.
- (z) The independent co-ordinates of the last point in a closed traverse are 0,0.
- (a') The area of a closed traverse can be calculated by knowing the independent co-ordinates of different stations only.
- (b') The line of collimation is the imaginary line joining the intersection of the cross-hairs and the optical centre of the eye-piece.
- (c') Telescope axis is the line joining the optical centre of the objective and the intersection point of the cross-hairs on the diaphragm.
- (d') Effect of local attraction may be eliminated by taking forward and backward bearing of a line.
- (e) There will be errors in the measurements of both horizontal and vertical angles if the plate levels are not in adjustment.
- (f) There will be error in horizontal angles when measured between points at widely different elevations due to line of collimation not being perpendicular to the horizontal axis.
- (g) There will be errors in both horizontal and vertical angles due to horizontal axis not being perpendicular to the vertical axis.
- (h) Due to non-parallelism of the axis of the telescope level and the line of collimation, there will be

error in the measurement of vertical angles.

- (i) Bearing of a line is the acute angle that the line makes with the reference line.
 - (j) The bearing of a line is measured in a clockwise direction from the reference line.
 - (k) In surveying, direction of a line is not so important as its bearing.
 - (l) The forward bearing of a line AB is the same as the backward bearing of line BA.
 - (m) The magnetic needle in a prismatic compass is rigidly fixed with the circular graduated scale.
 - (n) The direction of the magnetic needle is taken as the direction of the reference line in a prismatic compass survey.
 - (o) The effect of local attraction of a station may be detected by taking forward and backward bearing of the lines meeting at the station, and also of the adjacent sides, provided the stations of one of which is free from such attraction.
 - (p) Printing of a north line on the map and also the scale in which it is drawn are essential.
 - (q) The lines are not usually taken in a prismatic compass survey.
2. Fill up the following blanks each with one word.
- (a) The forward and.....bearings of a line differ by
 - (b) The included angle between two lines measuring at a point is the of their bearings.
 - (c) Local at a station is due to the presence of a substance near the station.
 - (d) In a closed polygon the sum of all the included angles plus right angles is equal to as many right angles as the No. of sides of the figure.
 - (e) The error in the included angles is distributed amongst the angles in a prismatic compass survey.
 - (f) The closing error occurs due to the errors in measuring included angles and the sides.

(g) The closing error at the end of each side is to the total length of the perimeter up to the end of that side from the starting point.

(h) If the bearing of AB is $70^{\circ}-30'$ and the bearing of BC is $280^{\circ}-20'$ then the angle ABC is

(i) If the forward bearing of AB is $70^{\circ}-30'$ and the backward bearing of BC is $280^{\circ}-20'$ then the angle ABC is

(j) If the backward bearing of the line BA is $70^{\circ}-30'$ and the forward bearing of the line BC is $280^{\circ}-20'$ then the angle ABC is

(k) If the backward bearing of BA is $70^{\circ}-30'$ and the backward bearing of CB is $230^{\circ}-20'$ then the angle ABC is

(l) If the forward bearing of BA is $70^{\circ}-30'$ and the backward bearing of CB is $230^{\circ}-20'$ then the angle ABC is

(m) If the forward bearing of BA is $70^{\circ}-30'$ and the forward bearing of CB is $280^{\circ}-20'$ then the angle ABC is

(n) The telescope in a theodolite is fitted on two frames through a horizontal about which it can in a vertical plane.

(o) Theodolites can measure angles correct up to

(p) The telescope of a theodolite has two one through which the surveyor sees is known as the eye piece and the other is known as the glass.

(q) The point in the glass through which a ray passes undeviated is known as the centre.

(r) Two thin lines cut a right angle on the are known hairs,

(s) A theodolite should be before measuring any angle in order to effect correct measurements.

(t) A transit is one whose telescope makes a complete in a plane.

3. What is meant by traverse surveying? List the points on which it differs from chain surveying. Compare between a closed traverse and an open traverse. Suppose

you are asked to conduct a traverse surveying for an irrigation canal, which type of traversing do you prefer and why?

4. Write explanatory notes on the following :

Surveyor's compass, Declination, Annual variation, Isogonic lines, True meridian, Local attraction, Closing error, Consecutive co-ordinates, Traverse chart, Cardinal direction and Adjustment of angular errors.

5. Name the factors that are to be considered before plotting a compass traverse. Explain briefly the methods of plotting a compass traverse.

6. What are the different types of correction for balancing a traverse? Explain.

Discuss the merits and demerits of theodolite surveying over compass surveying.

8. Explain clearly the following in a theodolite ; Foot screws, Tangent screws, Vertical circle, Levelling head, Vertical and transverse axis, Bubble axis, Sensitivity of the bubble, parallex error, Index error, Transiting, Centering, Face left and face right, Diaphragm, Changing face, Telescope axis, Chromatic and Spherical aberration, Capstan headed screw, Altitude bubble, Line of collimation, Azimuth.

9. What are the necessities of adjusting a theodolite? What are the temporary and permanent adjustments of a transit theodolite? When and how are they performed?

10. In a closed traverse ABCDA the following bearings were observed by a prismatic compass :

Side	F.B.	B.B.
AB	140°30'	32°30'
BC	84°15'	262°00'
CD	326°15'	151°20'
DA	224°30'	42°30'

Is there any local attraction at any station? Detect the same and eliminate it.

11. The magnetic bearing of a line AB is 130°15'. Calculate its true bearing if the direction of the magnetic north at A is 10°15' West of true North and 7°30' East of true North.

[Hints ; To solve this problem, see additional informations on page 102]

Ans. 120°, 137°45'.

12. The following lengths and bearings of different sides of a closed traverse ABCDA are observed :

Side	Length	Bearing
AB	248'	30°
BC	320'	140°
CD	180'	210°
DA

Calculate the length and bearing of DA.

Ans. 3435 ft., 307°50'

13. Find the bearings of line BC and DE of the closed traverse ABCDEFA from the following data :

Side	Length	Bearing
AB	872'	41°35' N.W.
BC	322'	...
CD	770'	63°12' S.W.
DE	406'	...
EF	1079'	26°39' S.E.
FA	1480'	53°30' N.E.

Ans. 300°57', 197°55'.

14. In a closed traverse ABCDEFA the following length of sides and included angles were observed :

Side	Length	Angle
AB	164'	A=96°12'
BC	125'	B=160°34'
CD	152'	C=129°50'
DE	157'	D=117°45'
EF	185'	E=93°.7'
FA	283'	F=121°51'

Calculate the corrected included angles, whole circle bearings, consecutive and independent co-ordinates. Show

all the necessary corrections and tabulate the calculated data in a traverse chart. Also find the area of the traverse.

15. Compute the area of the closed traverse ABCDA from the following data :

Side	Latitude	Departure
AB	-116.1	-44.4
BC	+6.8	+58.2
CD	+80.5	+17.2
DA	+28.8	-31.0

Ans. 4723 sq. ft.

16. The magnetic bearing of a line by a prismatic compass at a station was found to be $90^{\circ}30'$. If the local attraction at this station is known to be $6^{\circ}45'$ E and the declination $4^{\circ}15'$ E, calculate the true bearing of the line.

Ans. $101^{\circ}30'$

17. A compass traverse ABCD was surveyed round a building site and the following readings were taken ;

Line	Length (ft.)	Whole circle bearing
AB	1481.7	0°
BC	121.58	$261^{\circ}15'$
CD	133.5	$153^{\circ}32'$

Calculate length and W.C.B. of DA.

Ans : Length of DA = 71.65 ft.

W.C.B. of DA = $94^{\circ}18'$

18. Given the following data find the length and bearing of line AF :

Line	Distance (ft)	Bearing
AB	155.0	N 66° E
BC	203.5	S 85° E
CD	112.0	S 53° E
DE	151.0	N 69° E
EF	83.0	N 12° E

Ans : Length of AF = 603 ft, Bearing of AF = N $79^{\circ}10'$ E

19. In a four-sided traverse PQRS the whole circle bearing of line PQ is $42^{\circ}20'$. From the internal angles shown below calculate the quadrant bear-

ings of line QR, RS and SP. Angles are, at P : $210^{\circ}30'$, at Q, $22^{\circ}20'$, at R, $61^{\circ}30'$ and at S, $65^{\circ}40'$.

Ans : QR = $20^{\circ}10'$ NW, RS = $41^{\circ}30'$ NE, SP = $72^{\circ}50'$ NE,

20. The distance between two points A and B opposite banks of a river is required. Compass bearings taken from A are AB 42° , AD 68° . From C along AD the bearing of CB is 22° . If the distance AC is 20 ft, calculate the distance AB.

Ans : 42.06 ft.

21. Convert the following Whole Circle Bearing to Quadrant Bearings : 213° , 14° , $103^{\circ}40'$, 252° , 270° .

Ans : 77° NW, 14° NE, $76^{\circ}20'$ SE, 72° SW, 90° W.

22. In a traverse ABCDE, AB = 267 ft, DE = 341 ft, EA = 411 ft. Lengths BC and CD are unattainable. Included angle A = $287^{\circ}51'$, B = $294^{\circ}09'$, C = not known, D = 267° , E = 308° .

Find the lengths and bearing of BC and CD.

Ans. 57.9 ft 131° , 54°

23. The following readings were obtained on a theodolite traverse ABCDE :

Theodolite station	Bank station	Readings Forward station
A	E $00^{\circ}00'$	B $259^{\circ}32'$
B	A $259^{\circ}32'$	C $161^{\circ}17'$
C	B $161^{\circ}17'$	D $52^{\circ}48'$
D	C $52^{\circ}48'$	E $297^{\circ}18'$
E	D $297^{\circ}18'$	A $179^{\circ}59'$

Calculate and correct the observed angles and determine the bearing of BC if the bearing of AB is $48^{\circ}52'$.

Ans : Corrected angles :

$259^{\circ}32'12''$

$261^{\circ}45'12''$

$251^{\circ}31'12''$

$244^{\circ}30'12''$

$242^{\circ}41'12''$

Bearing of BC = $130^{\circ}37'12''$.

“ADDITIONAL INFORMATIONS REGARDING DETERMINATION OF BEARINGS”

Determination of True Bearings: All revenue survey maps are plotted with reference to the true meridian. If a survey is made with a compass, the readings observed are the magnetic bearings. Knowing the magnetic declination at a place, the true bearings may be deduced by the following rule.

Rule 1: True bearing of a line = magnetic bearing of the line \pm declination.

Use plus sign, when the declination is *east*, and minus sign, when it is *west*.

If a line of old survey is to be relaid on the ground with a compass, the magnetic declination now must be known to obtain the magnetic bearing of the line, which may be deduced by the following rule.

Rule 2: Magnetic bearing of a line = true bearing of the line \mp magnetic declination.

Use minus sign, when the declination is *east*, and plus sign, when it is *west*.

Note: These rules are applicable in the case of whole circle bearings only.

Example: The magnetic bearing of a line AB is $134^{\circ}45'$. Find its true bearing, if the magnetic declination is $13^{\circ}15'$ W.

Solution:

Since the magnetic meridian is to the west of the true meridian, true bearing of AB = magnetic bearing of AB - declination
 $= 134^{\circ}45' - 10^{\circ}15' = 134^{\circ}30'$.

Example: The magnetic bearing of a line AB is $S32^{\circ}E$ and the magnetic declination is $8^{\circ}16'$ E. What is the true bearing of the line?

Solution:

True bearing of AB = magnetic bearing of AB - declination
 $= 32^{\circ} - 8^{\circ}16' = 23^{\circ}44'$ S.E.

Check: Convert the given quadrantal bearing to the whole circle bearing and then apply the Rule (1).

W. C. B. of AB = $180^{\circ} - 32^{\circ} = 148^{\circ}$.

True bearing of AB = magnetic bearing of AB + magnetic declination
 $= 148^{\circ} + 8^{\circ}16' = 156^{\circ}16'$
 $= 23^{\circ}44'$ S.E.

Example: The true bearing of a line is $225^{\circ}38'$ and the magnetic declination is $12^{\circ}14'$ W. Find the magnetic bearing of the line.

Solution:

Since the north end of the needle points to the west of the true meridian,

magnetic bearing of the line = true bearing of the line + declination.

$= 225^{\circ}38' + 12^{\circ}14' = 237^{\circ}52'$.

Example: A line was drawn to a magnetic bearing of $234^{\circ}40'$ on an old map when the magnetic declination was $4^{\circ}16'$ E. To what bearing should it be set now, if the present magnetic declination is $2^{\circ}20'$ W.?

Solution:

True bearing of the line = magnetic bearing of the line + declination
 $= 234^{\circ}40' - 4^{\circ}16' = 238^{\circ}56'$.

Present declination = $2^{\circ}20'$ W.

\therefore Magnetic bearing of the line = true bearing of the line + declination.

$= 238^{\circ}56' + 2^{\circ}20' = 241^{\circ}16'$.

The line should be set now to the bearing of $241^{\circ}16'$.

Example: The true bearing of a tower as observed from a station A is $348^{\circ}38'12''$ and the magnetic bearing of the tower as observed by a theodolite is $2^{\circ}15'44''$. The magnetic bearing of the line AB is also observed with the same instrument and found to be $148^{\circ}26'10''$. What is the true bearing of the line AB?

Solution :

Since the true bearing of the tower is $348^{\circ}38'12''$, it is to the west of the true meridian. Also, its magnetic bearing $2^{\circ}15'44''$. The magnetic meridian is, therefore to the west of the tower.

$$\begin{aligned}\therefore \text{Magnetic declination} &= 2^{\circ}15'44'' + (360^{\circ} - 348^{\circ}38'12'') \\ &= 2^{\circ}15'44'' + 11^{\circ}21'48'' \\ &= 13^{\circ}37'32''\text{W}\end{aligned}$$

Now the magnetic bearing of $AB = 148^{\circ}26'10''$.

Since the declination is west,

$$\begin{aligned}\text{true bearing of the line } AB &= 148^{\circ}26'10'' - 13^{\circ}37'32'' \\ &= 134^{\circ}48'38''.\end{aligned}$$

Example : Find the magnetic declination, if the magnetic bearing of the sun at noon is (a) $186^{\circ}30'$ and (b) $356^{\circ}42'$.

Solution :

(a) At noon the sun is exactly on the geographical meridian. Since the magnetic bearing of the sun is $186^{\circ}30'$, it is at the south pole. The magnetic bearing of the south pole is, therefore, $186^{\circ}30'$. Hence the magnetic bearing of the north pole is $6^{\circ}30'$. It, therefore, follows that the magnetic meridian is $6^{\circ}30'$ to the west of the true or geographical meridian.

$$\therefore \text{Magnetic declination} = 6^{\circ}30' \text{ W.}$$

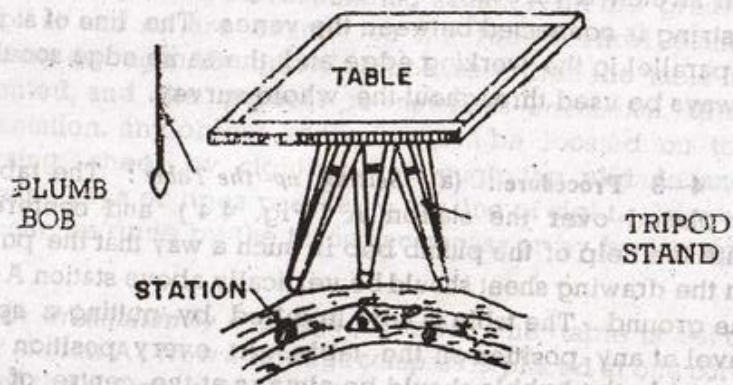
(b) The magnetic bearing of the sun at noon being $356^{\circ}42'$, the magnetic bearing of the north pole is $356^{\circ}42'$. The magnetic meridian is, therefore, $360^{\circ} - 356^{\circ}42' = 3^{\circ}18'$ to the east of the meridian.

$$\therefore \text{Magnetic declination} = 3^{\circ}18' \text{ E.}$$

PLANE TABLE SURVEYING

4-1 Definition : It is a method of surveying in which observations and plotting are done simultaneously. This type of surveying is very suitable in plotting the interior details like buildings, trees, roads, electric posts or any other permanent objects. It should be remembered that in plane tabling the main stations and lines are fixed by traverse survey. This is particularly used in small scale mapping and allows the work to be done very quickly. Since the observations and plottings are done simultaneously, so there is less possibility of any mistake. The handling of the instrument is very simple. The main disadvantage is this that during rainy season it cannot be performed.

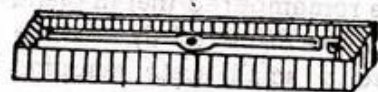
4-2 Instruments : Instruments required for this type of surveying are (a) a drawing board, (b) tripod stand, (c) alidade, (d) trough compass, (e) plumb bob, (f) plumbing fork or U fork, (g) spirit level, (h) tape or chain, (i) drawing sheet with board pins or clips (Figs. 4.1, 4.2, 4.3).



PLANE TABLE

FIG. 4.1

The drawing board or table is made of finer variety of wood. Its sizes are generally $16'' \times 12''$, $24'' \times 18''$, and $30'' \times 24''$. The table is fitted to a tripod stand by a ball and socket arrangement attached to the central bottom of the table.



TROUGH COMPASS

FIG. 4.2



ALIDADE

FIG. 4.3

The alidade or sight rule is a piece of flat wood $2''$ wide and $\frac{1}{4}''$ thick and length of $20''$. It is fitted with sight vanes of brass at its ends. In one vane there is a narrow slit through which the objects are sighted while in the other one there is a broad opening having at its centre a horse hair stretched vertically. To sight objects at higher levels a string is connected between the vanes. The line of sight is parallel to the working edge and the same edge should always be used throughout the whole survey.

4-3 Procedure: (a) *Setting up the Table:* The table is placed over the station A (Fig. 4.4) and centered with the help of the plumb bob in such a way that the point on the drawing sheet should be vertically above station A on the ground. The table is now levelled by putting a spirit level at any position on the table. At every position on the table the bubble should be always at the centre of its run. If not, then this is done by adjusting the legs of the tripod. In case the spirit level is not available, levelling of the table may be done approximately by a round shaped wooden pencil. The pencil should remain fixed at any position on the table if it is levelled.

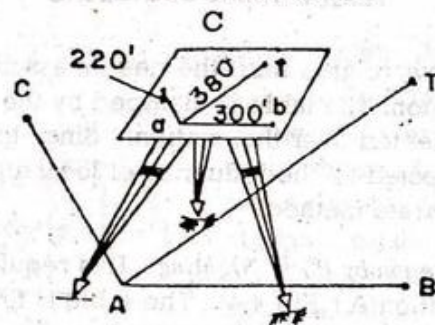


FIG. 4.4

(b) *Orientation:* A plane table is said to be oriented when a line drawn on the drawing sheet to represent a corresponding line on the ground in such a way that the line lies in the same vertical plane with the line on the ground. All the plotted lines on the drawing sheet will then be parallel and proportional to the corresponding lines on the ground. From Fig. 4.4 if the point a is exactly vertically over the corresponding station A on the ground and if ab and ac are exactly on the two vertical planes passing through AB and AC respectively, then the table is oriented, and the process is known as *orientation*. After orientation, any object in the field can be located on the drawing sheet by sighting it through the alidade and drawing rays or lines parallel to the line of sight. Orientation can be done by the trough compass or by back sighting.

(i) *Orientation by a Trough Compass:* The table is set up over station A. Now a trough compass is placed at one corner of the drawing sheet and moved in such a way that the needle assumes its normal North-South position. A line drawn along the longer edge of the compass and an arrow is put at the north end. The table is now oriented with respect to the magnetic meridian. When the table is placed over any other station, the trough compass is placed with its longer edge in coincidence with the previously drawn N-S line

The table is now rotated until the needle assumes its normal N-S direction. The table is clamped by the screw. Now the table is oriented over that station. Since the magnetic needle is subjected to the influence of local attraction, it is not a very accurate method.

(ii) *Orientation by Back Sighting*: It is required to orient the table at station A (Fig 4.4). The table is first set up and centered over station A. A line ab is drawn in proportionate scale on the drawing sheet after sighting the ranging rod at B through the alidade and measuring the distance AB on the ground. The table is now shifted and placed over another station B and centered so that the point b is exactly vertically over the corresponding station B on the ground. The alidade is placed with its edge in coincidence with the line ab and the table is turned till the ranging rod at A is sighted through the alidade. The table is clamped. The line ba is parallel one over the line BA on the ground. The table is now said to be oriented at the station B. The same procedure is followed for all other stations.

After orientation over any station, any object on the field can be located on drawing sheet by sighting it through the alidade and drawing rays or lines parallel to the line of sight.

4-4 Methods of plane Tabling: There are four methods:

(a) Radiation, (b) Intersection, (c) Traversing and (d) Resection.

(a) *Radiation*: This method is very suitable for a small plot of land which is visible from a centrally located point. Rays or lines are drawn from this point on the drawing sheet along the direction of the objects or stations and distances are plotted on the sheet by measuring them with tape or chain.

ABCDEF (Fig. 4.5) is the plot of land to be surveyed.

P is the centrally located point from which all the stations

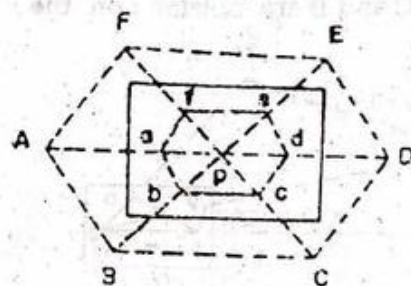


FIG 4.5

are visible. The plane table is placed over P, levelled and centered so that the point p is exactly over the station P on the ground. A pin inserted at the point P and the alidade is placed with its edge touching the pin. Now the ranging rod at A is sighted through the alidade.

The distance PA is mea-

sured. A line pa is drawn in the direction of a and the distance pa is cut according to a suitable scale from it. The same procedure is followed for other stations. Finally, $abcdef$ will be the plot of land on the drawing sheet. All the interior details can be located by this method.

(b) *Intersection*: In this method an object is located on the drawing sheet by the intersection of the rays or lines drawn from two stations. This is the swiftest method of locating an object which is inaccessible. This is also used to locate details like rivers, ponds, canals, broken boundaries, roads, mountainous terrain and also distant objects. It is also used to check objects, stations, etc. In this method except the base line, no linear measurement is required.

The base line connecting two points P and Q on the ground is measured (Fig. 4.6). The line is selected in such a way that maximum number of objects in the field are visible from both P and Q. The plane table is set over the station P, levelled, centered and oriented. From point P on the drawing sheet rays of the object A, C and D are drawn by the alidade. Now the table is placed over station Q and oriented with respect to QP. From this point q again

rays of the same object are drawn with the alidade. The positions of the objects A, B, C and D are obtained on the

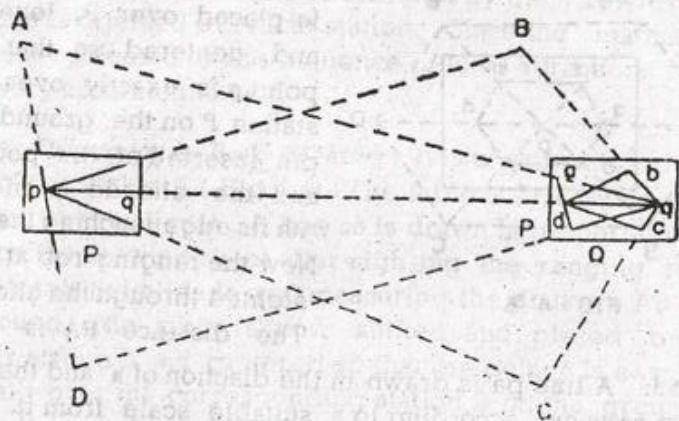


FIG. 4.6

drawing sheet as a, b, c and d respectively when the rays from q cut the corresponding rays from p.

(c) *Traversing*: In this method the plane table is shifted from one station to next station fixing all details by radiation. This method is used to run a survey where main stations have been fixed previously by theodolite or compass. This is very useful for the survey of roads, canals, rivers, etc. This method also checks the accuracy of a survey work.

Station A, B, C and D are selected on the ground in such a way that from each station the preceding and the forwarding stations should be visible. The table is placed at A and oriented with respect to a. The North-South line also drawn with the help of a trough compass on one corner of the drawing sheet (Fig. 4.7). A ray ab is drawn from point a by sighting station B.

The length of ab is plotted from the measured length of AB in the field. All the details are fixed by radiation or

Intersection method. Now the table is placed over station

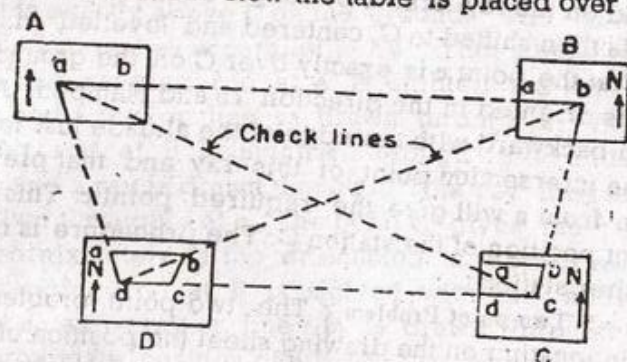


FIG. 4.7

B and oriented by back sighting on A and clamped. With the alidade at b, the station C is sighted and a line or ray is drawn. The line bc is scaled off from the measured distance of BC. The details are located as before. The same procedure is followed for stations C and D.

(d) *Resection*: In this method only the stations are fixed. It locates the same station over which the instrument is placed. This method will have small errors if the scale of the map is very small.

A base line AB (Fig. 4.3) is measured very accurately on the ground and plotted to a suitable scale on the drawing sheet in a suitable position as ab. The table is placed over station A, centred and levelled in such a way that the point on the drawing sheet is exactly vertically over station A. By the help of a trough compass, a magnetic

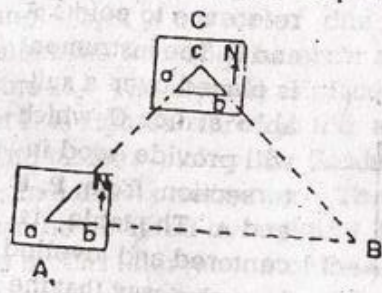


FIG. 4.8

north line is drawn on one side of the drawing sheet. The alidade is placed just touching the point a and the ranging rod at C is sighted through it. Also with the edge of the alidade touching a ranging rod at C is sighted and a ray is drawn in that direction. The position of c is

marked on the ray drawn by rough estimation. The plane table is then shifted to C, centered and levelled in such a way that the point c is exactly over C on the ground. The table is oriented in the direction cb and clamped. A ray is drawn backward with the edge of the alidade just touching b. The intersection point of this ray and that previously drawn from a will give the required point c. This is the correct position of the station C. The procedure is repeated for other stations.

4-5 Two-point Problem ; The two point problem consists in locating on the drawing sheet the position of a point which is occupied by the plane table on the ground by means of observations to two known points which are visible from the instrument station and whose positions are already plotted on the drawing sheet.

This method is generally applied to verify an old map prepared by any method of surveying in which there are two known points drawn. Now it is required to locate a few more details on the same old map. This problem gives the solution as to how the position of the plane table should be fixed with reference to these two known points so that the new objects to be surveyed now will fit nicely in the old map.

A and B are two known points on the field (Fig. 4.9) and a and b are their positions on the drawing. P is the new instrument station the position of which is to be located

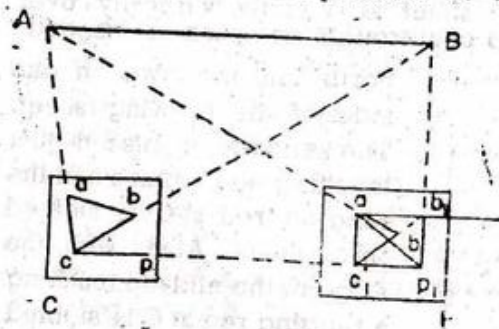


FIG. 4.9

on the drawing with reference to points A and B. The instrument is placed over a suitable station C which will provide good intersection from P, B and A. The table is centered and levelled in such a way that the point c on the draw-

ing sheet is exactly above C on the ground. The table is then oriented so that ab is parallel to AB. The table is now clamped. The ranging rod at A is sighted through the alidade touching a and a ray or line is drawn through a. Again the ranging rod at B is sighted through the alidade touching b and a ray is drawn through b. This ray intersects the ray drawn through a at c_1 . The point c_1 gives the position of c approximately as the orientation is approximate. Now the ranging rod at P is sighted with the alidade centered at c_1 , and a ray or line c_1p_1 is drawn through c_1 , giving approximate position of p_1 .

The table is now shifted over the station P. It is levelled and centered in such a way that p_1 is exactly over P. The alidade is placed along c_1p_1 and the table is rotated gently until c is disected. The table is now clamped. The ranging rod at A is sighted with the alidade touching against a and a ray or line is drawn through a, intersecting the line c_1p_1 at p_1 . With the alidade touching p the ranging rod at B is sighted and a ray is drawn through p_1 . If the previous orientation is correct, this line should pass through b. The orientation of the table at C and P might be incorrect. This ray then will pass through b_1 but not through b. The point b_1 which is the intersection point of p_1B and c_1b is marked. Now the point b_1 represents B and hence $a_1p_1b_1$ represents ACPB. Since ab is the true representation of AB, so the angle b_1ab is the error due to inaccurate orientation. To eliminate this angular error a station D is fixed along the line ab_1 . The alidade is placed along ab. The ranging rod at D is sighted through the alidade by turning the table. The table is clamped. So ab is parallel to AB and the table is in correct orientation. The true position of the point p can be obtained by sighting through a to A and through b to B. This intersection of these two rays will give the true position of the station P.

4-6 Three-point Problem: The three-point problem consists in locating on the drawing sheet the position of a point which is occupied by the plane table on the ground by means of observation to three well-defined points, whose position are already plotted on the drawing sheet. The three points should be visible from the instrument station.

A, B and C are the three well-defined points on the ground (Fig. 4.10) and they have been plotted as a, b and c respectively on the drawing sheet. P is the new instrument station which is to be located on the drawing with reference to points A, B and C. This problem can be solved by (1) Tracing paper or Mechanical method, (2) Graphical method and (3) Trial and Error method.

Tracing Paper or Mechanical Method: The table is placed over the station P (Fig. 4.10) centered, oriented and clamped. A piece of tracing paper is laid over the drawing sheet. A point P' is chosen on the tracing paper in such a way that it approximately lies over P. Now rays are drawn towards A, B and C from P'. These rays will not generally pass through a, b and c. The tracing paper is now unfastened and rotated over the plane table until three rays

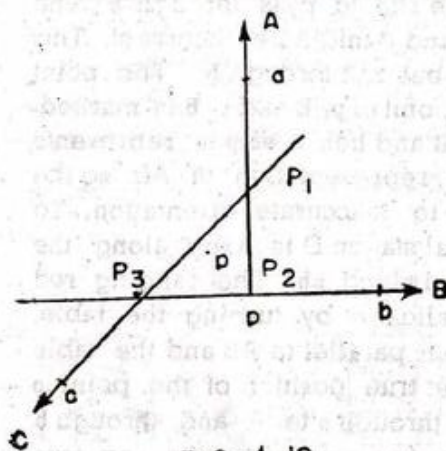


FIG. 4.10

pass simultaneously through a, b and c. Now it is pricked through P' and this will fix the point P on the drawing sheet. The table is turned with the alidade placed along pa until A is sighted. The table is oriented along PA. If this orientation is correct, the rays Bb and Cc should now meet at P, if not, there will be an error in the form of small triangle P₁P₂P₃. This triangle of error can be eliminated by repeated trials.

Graphical Method: This is also known as Bessel's Method. The plane table is placed over the station P, centered and levelled. The alidade is placed along ca and by turning the table, A is sighted. The table is clamped. Now

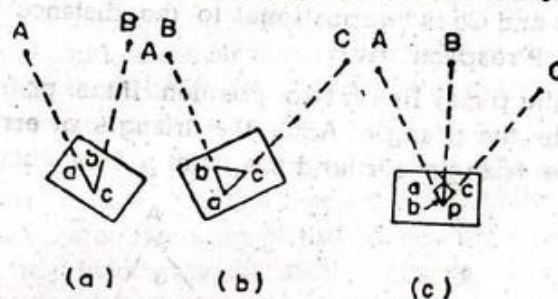


FIG. 4.11

a ray cB is drawn by placing the alidade at c (Fig. 4.11 a). The alidade is now placed along ac (Fig. 4.11 b) and the table is turned until C is sighted. The instrument is clamped. B is sighted by putting the alidade at a and a ray aB is drawn through a. This ray aB intersects the ray cB at point t. Now with the alidade along bt, the table is turned (Fig. 4.11 c) until B is sighted. The instrument is clamped. By doing this the table has been oriented and P should lie on tb and also on Aa and Cc. A is sighted by putting the alidade at a and a ray is drawn through a which intersects the ray bt at P. This P is the instrument station. If the work is correct then the ray Cc drawn with the alidade at c must pass through p, if not, there is error and to eliminate the error, the procedure is repeated.

Trial and Error Method: This is also known as Lehmann's method or triangle of error method. The instrument is set over the station P, levelled, centered and oriented approximately. Rays towards A, B and C are drawn through points a, b and c respectively (Fig. 4.10). If the table is correctly oriented then all these three rays will meet at p. But it will not be possible to orient the table correctly in first

trial. So there will be an error in the form of a triangle which is known as the *triangle of error*. This error depends upon how much the table is out of azimuth. The distance to the point p (in case of triangle of error) from each of the rays Aa , Bb and Cc is proportional to the distance of A , B and C from P respectively.

The point p may lie on two position. If the position of P is outside the triangle ABC , the triangle of error lies outside the triangle abc and the point p falls outside the

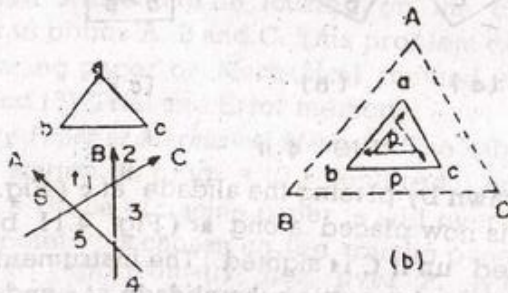


FIG. 4.12

triangle of error (Fig. 4.12 a). When the position of P is within the triangle ABC , the triangle of error falls inside the triangle abc and the point p also falls inside the triangle of error (Fig. 4.12b).

If the point sought is outside the triangle of error, then it can be eliminated by applying *Lehmann's Rules* which state that the point P will be found on the same side, either right or left, of all the rays when looking in the direction of each of the distant objects and the distance of the point p from the three rays Aa , Bb and Cc will be proportional to the distances of A , B and C from the station P . So according to the above conditions, sector 3 is the only place where P can be located. The exact position of P will be decided by trials. If the point p is within the triangle of error, its exact

position is located by trial and error method. The rays are rubbed out and alidade placed along pa , pb and pc , the station A , B and C are intersected respectively. Then the intersection of these rays will locate the exact position of the point p . If the rays do not pass through the same point, then a second and near approximation is made and so forth. Generally three or four trials will be required to find the exact position of the point p .

4-7 Errors in Plane Table Surveying : The common sources of errors are : (a) The plane table not being horizontal, (b) Inaccurate centering of the table, (c) Incorrect orientation, (d) Inaccurate sighting through alidade, (e) The alidade not being correctly centered on the station point on the drawing sheet, (f) The expansion and the contraction of the drawing sheet and (g) Inaccuracy in plotting.

4-8 Advantages and Disadvantages in Plane Table Surveying : The plotting and observations are done simultaneously in the field with the different objects to be surveyed before the eyes of the surveyor. The surveyor can, therefore, compare the plotted work with the actual features of the area surveyed. So there is less possibility of overlooking of any important feature. It is very rapid and suitable for preparing small-scale maps. Measurement of lines and angles are almost avoided as most of the plotting is done by graphical method. In this type of surveying no field book is required and hence the error arising due to faulty writing is eliminated. This is very suitable for surveying a magnetic area where compass survey is not reliable and also in hilly terrains and forests where strict accuracy is not required. When the scale of the map is very small and great number of details are to be filled in, plane table surveying is considered to be the most speedy, less costly and fairly accurate than theodolite surveying.

This type of survey is not suitable in wet weather. When the scale of the map is large, this type of survey is

not preferred. If the area to be surveyed is very large, frequent change in drawing sheets is necessary to carry out the survey work as the size of the table is limited.

Exercise

1. Examine the following statements very carefully and write whether true or false :
 - (a) A base line must be the north-south line in plane table survey.
 - (b) In plane table survey, no distance is required to be measured.
 - (c) In plane table survey, at least two stations should be located on the table.
 - (d) In plane table survey, the rays or lines are drawn along the line of sight.
 - (e) In plane table survey, no bearing is required to be measured.
 - (f) Orientation is the process of levelling the plane table over a station.
 - (g) The plane tabling by resection method locates the same station over which the instrument is placed.
 - (h) A base line must be measured in plane tabling by traversing method.
 - (i) Intersection method of plane tabling requires no measurement in the field.
 - (j) Plane table survey is not suitable when the scale of the map is large.
 - (k) The surface of the table should be a perfect plane.
 - (l) The surface of the table should be perpendicular to the vertical axis of the instrument.
 - (m) The ruling edge of the alidade is a straight line.
2. Explain with a neat sketch the fundamental princi-

ple of plane table surveying. What are the merits and demerits of plane tabling over theodolite surveying?

3. Discuss with neat sketches the various methods of plane tabling. Which method do you recommend for a hilly terrain and why?
4. Suppose you have got an old map of a plot of land and there are three known points on the map; it is now required to locate a few more details of the land on the same old map. Explain how the position of the plane table should be fixed on the ground with reference to these three known points.
5. Explain with a neat sketch how can you fix your position on a survey of a plot of land by means of plane table when two well defined points which have already been plotted on the drawing sheet are visible.



5-1 Definitions : A level is an instrument by which the relative heights of different points on the surface of the earth are determined. And leveling is the process by means of which the difference in elevation of various points on the earth's surface is calculated. The main instruments that are essential in carrying out the levelling operation are (1) a level and (2) a staff.

Levels : There are different types of levels, such as Dumpy level, Wye or Y-level, Cooke's Reversible level, Cushing's level, and Zeiss, Wild's and Watt's modern levels. Among these the Dumpy and Y-levels are very commonly used for their simplicity and compactness.

Dumpy Level : In this type of level the telescope, the frame and the vertical axis are rigidly fixed. Because of its rigidity it retains its two adjustments for a long time, moreover, it has greater optical power than the Wye level (Fig 5.1).

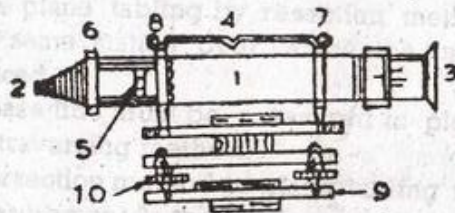


FIG. 5.1 Dumpy Level

1. Telescope 2. Eye Piece
3. Ray Shade 4. Level Tube
5. Focussing Screw 6. Diaphragm Screw
7. Cross Bubble Tube
8. Level Tube Nuts 9. Levelling Head
10. Levelling Screw

Yor Wye Level : In this type of level the telescope is fitted on two Y-supports. The heights of these Y-supports can be adjusted. But this adjustment doesnot last long (Fig. 5.2).



FIG. 5.2 Y-Level

1. Telescope 2. Eye Piece 3. Ray Shade
4. Clipse 5. Diaphragm 6. Focussing Screw
7. Wyes 8. Levelling Screw 9. Level Tube

Cooke's Reversible Level : In this type of level, the combined advantages of Dumpy and Y-levels are provided. The telescope is supported on two collars and these collars are connected by a rigid socket. The telescope can be rotated around its longitudinal axis within the socket.

Cushing's level : In this type, the telescope can not be removed from its socket and also the telescope can not be rotated about its longitudinal axis. But the eye-piece and objective are interchangeable to make the telescope reverse.

Zeiss, Wild's and Watt's Modern Levels : These are for precise levelling works and are self adjusting. They are also known as *tilting levels* because the telescope has a small motion about its horizontal axis.

Levelling Staff : The levelling staff which is most commonly used is shown in Fig. 5.3. It is made of best variety of seasoned timber and is in two pieces, each 7 ft. long when pulled out to full length, the staff measures 14 ft. and is held in position by a brass spring catch. It is graduated into feet, tenths and hundredths of a foot. The hundredths

are coloured in black and white alternately. The line showing the first place of decimal are longer than others. The figures indicating tenths of a foot are painted black while the figures indicating foot are painted red.



FIG. 5.3 Levelling Staff

5-2 Adjustment of Levels : There are mainly two types of adjustments : (1) Temporary adjustments and (2) permanent adjustments.

Temporary Adjustments : These are performed at each set up of the level. These include planting the tripod and levelling the instrument by foot screws and focussing the eye-piece and objective to eliminate parallax.

Permanent Adjustments : In spite of all the care of the instruments, the relative positions of the principal parts are very often disturbed and hence the level does not give correct results. So, it is required to test all these parts and set them in their proper position. This operation is termed as the *Permanent adjustment* of the levelling instruments. The

number, nature, order and the procedure of adjustments are different for different instruments.

5-3 Permanent Adjustments of the Dumpy Level : It has got two adjustments ; (1) Adjustment of the bubble tube and (2) Adjustment of the line of collimation.

Adjustment of the Bubble tube : The purpose of this adjustment is to set the vertical axis of the telescope at right angles to the bubble tube so that when the bubble tube is horizontal, the vertical axis is truly vertical and when the telescope is turned horizontally it always remains in a horizontal plane. It is adjusted in the same way as the parallel plate bubble tube of a transit theodolite.

The level is set up on a fairly level ground and temporary adjustments are performed. Then, the telescope is turned in such a way that it is over two opposite foot screws

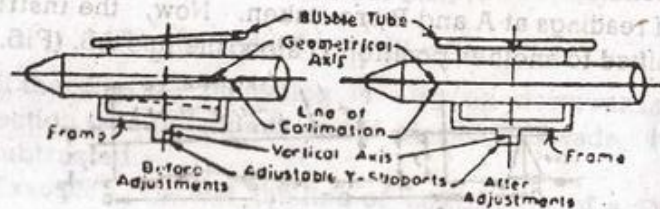


FIG. 5.4 Dumpy Level

(levelling screws). The bubble is brought to the centre of its run by turning both the screws inwards or outwards. The telescope is turned now through 90° and the bubble is brought to the centre of its run by means of the second pair levelling screws or the third screw. The process is repeated for two or three times. The telescope is now turned through 180° in azimuth, if the bubble remains in the centre of its run, the level is in adjustment. If not, the number of divisions through which the bubble has moved from its central position, are noted. Half of this error is corrected by means of foot screws and the other half by

means of Capstan headed screws attached to the bubble tube. The process is repeated until adjusted.

Adjustment of the line of collimation: The purpose of this adjustment is to set the line of collimation at right angles to the vertical axis. It is tested and adjusted by two peg test.

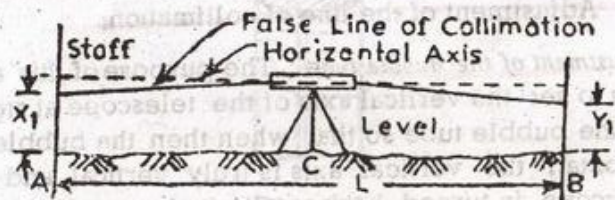


FIG. 5.5 (a)

Two ranging rods A and B are placed about 200 ft. apart on an approximately levelled ground. The level is placed at C which is the middle point of the line AB (Fig. 5.5a). The instrument is levelled and the bubble tube is made perpendicular with the vertical axis. From this mid position, staff readings at A and B are taken. Now, the instrument is shifted to another position D along the line AB. (Fig. 5.5b).

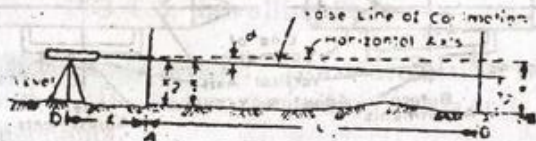


FIG. 5.5 (b)

Again staff readings from this new position are taken. If the instrument is in adjustment, the difference between the first set and second set of readings should be the same. If there is a difference, that is due to inclination of the line of collimation to the horizontal axis. It is adjusted by moving the cross-hairs with the help of diaphragm screws until the difference is same as it was when the instrument was at C.

Let x_1 and y_1 be the staff readings at A and B respectively when the instrument was at the mid-point C. If $x_1 > y_1$,

then the station A is at lower level than B and vice-versa or in other words, there is a rise from A to B; if $x_1 < y_1$, there is a fall from A to B. While the instrument was at D, the readings are x_2 and y_2 at A and B respectively.

If the line of collimation is parallel to the horizontal axis, let then the readings at A and B be x and y respectively. The correction is applied in the following way.

(a) $y = x_2 \pm$ True difference.

Plus sign (+) is to be used when the true difference is a fall and minus (-) when it is a rise.

(b) If $y_2 > y$, the line of collimation is inclined upwards and if $y_2 < y$, it is inclined downwards. The line of collimation error in the distance L is either $y - y_2$ or $y_2 - y$.

(c) Correction on the far staff B

$$= \frac{L+l}{L}(y-y_2) \text{ or } \frac{L+l}{L}(y_2-y)$$

(d) Correction on the near staff A

$$= \frac{l}{L}(y-y_2) \text{ or } \frac{l}{L}(y_2-y)$$

If the line of collimation is sloping downwards, the correction is to be added, and if sloping upwards, it is to be subtracted.

Example: To test the line of collimation of a Dumpy level, the instrument was placed exactly mid-way between two points A and B, 200 ft. apart, on a fairly level ground. The staff readings at A and B were 6.78 and 3.92 respectively. The instrument was then placed at D, 45 ft. behind A in the same straight line and staff reading at A and B were 5.25 and 2.20 ft respectively. (a) Is the line of collimation in adjustment? (b) If not, is it inclined upwards or downwards? (c) What should be the staff reading at A and B when the level is adjusted?

Solution:

True difference of level between A and B = $6.78 - 3.92 = 2.86$ ft... (i)

Since the staff reading at B is smaller, it is at higher level. The apparent difference of level when the instrument was shifted = $5.25 - 2.20 = 3.05$ ft.

(a) As (i) and (ii) differs, so the line of collimation is not in adjustment.

(b) In order to have B at the same level as A, the reading on B should be y , which equals to $5.25 - 2.86 = 2.39$.

Since $2.39 > 2.20$, the line of collimation is inclined downwards.

(c) Correction on staff reading at A

$$= \frac{l}{L}(y - y_2) = \frac{45}{200}(2.39 - 2.20) = 0.043 \text{ ft}$$

Correct reading on A = $5.25 + 0.043 = 5.297$ ft.

Correction on staff reading at B

$$= \frac{L+l}{L}(y - y_2) = \frac{200+45}{200}(2.39 - 2.20) = 0.233 \text{ ft.}$$

Correct reading on B = $2.20 + 0.233 = 2.433$ ft.

Example: A Dumpy level was placed at the mid-point C of A and B which were 200 ft. apart. The readings at A and B were 2.5 ft and 4.6 ft respectively. The instrument was then shifted and placed midway between A and C. The reading at A and B were 3.2 ft, and 5.6 ft. respectively. Find the correct readings on A and B.

Solution.

True difference between A and B = $4.6 - 2.5 = 2.10$ ft.

When the reading in the second case at A is 3.2 the reading at B should be $3.2 + 2.1 = 5.3$ ft. But the reading at B is 5.6 ft. Which indicates that the line of collimation is inclined upwards.

Let x be the error in l ft. which is same as $\tan a = \frac{x}{l}$

(Fig. 5 5b)

$$\text{then } \frac{x}{50} = \frac{5.6 - 5.3}{(150 - 50)} \therefore x = 0.15 \text{ ft}$$

Staff reading at A = $3.2 - 0.15 = 3.05$ ft.

Staff reading at B = $5.6 - 0.3 - 0.15 = 5.15$ ft.

Check: $5.15 - 3.05 = 2.10$ ft.

Example: Two ranging rods were placed 200 ft. apart on a fairly level ground. A Dumpy level was placed in the midway and the readings on A and B were 3.92 ft. and 5.48 ft. respectively. The instrument was then shifted very near to A and the readings on A and B were found to be 5.23 ft. and 6.44 ft. respectively. Calculate the correct readings on A and B.

Solution

True difference between A & B = $5.48 - 3.92 = 1.56$ ft. As the reading at B is higher, so there is a fall from A to B. Apparent difference from second position of the instrument = $6.44 - 5.23 = 1.21$ ft. Since the instrument was placed very near to A, so the second reading on A i.e., 5.23 ft is taken to be correct. The reading on B should be $5.23 + 1.56 = 6.79$ ft.

Example: In the two peg test of a Dumpy level, the following observations were recorded

Instrument station	Staff reading	Staff reading
A	A	B
B	5.39	8.33
	5.9	7.32

When the instrument is at B, what staff reading on A should be obtained so that line of collimation be adjusted?

Solution

When the instrument is at A, the apparent difference of level between A and B = $8.33 - 6.42 = 2.11$ ft.

When the instrument is at B, the apparent difference of level between A and B = $7.32 - 5.39 = 1.93$ ft.

True difference of levels = $\frac{2.11 + 1.93}{2} = 2.02$ ft.

For a staff reading on B of 7.32 ft., the staff reading on A should be $7.32 - 2.02 = 5.30$ ft. but the observed reading on A = 5.39 ft.

Hence, the line of collimation is not in adjustment and it is inclined upwards and should be tilted down to read a staff reading of 5.30 ft.

5-4 Permanent Adjustments of Y-Levels: There are three adjustments in this type of instruments: (1) Adjustment of the line of collimation, (2) Adjustment of the bubble tube and (3) Adjustment of the height of Y-supports.

Adjustment of the line of collimation: The purpose of this adjustment is to set the line of collimation along the longitudinal axis of the telescope tube. The instrument is placed firmly on a firm ground. The telescope after opening the Y-clips is then directed towards a staff placed against a wall at such a distance that the graduations of the staff can be read distinctly. After recording the staff reading the telescope is lifted up and placed upside down on the Y-supports, and another reading on the same staff is taken. If the two readings are the same, the line of collimation is in adjustment, if not, the horizontal cross-wire is raised or lowered by the vertical diaphragm screws till the mean reading is obtained. The instrument is then turned back to its original position and the reading on the staff is checked. If it does not indicate the same reading, the process is repeated till the same reading is obtained in both positions of the telescope.

Adjustment of the bubble tube: The purpose of the adjustment is to bring the axis of the bubble tube in parallel with the line of collimation. This is done in the same way as described in the case of Dumpy level.

Adjustment of the height of Y-Supports: The purpose of this adjustment is to set both the line of collimation and the bubble axis at right angles to the vertical axis. The procedure is same as the adjustment of the bubble tube in the case of Dumpy level, except that the bubble is brought half way back by the Y-nuts, and the other half by the foot screw between the telescope.

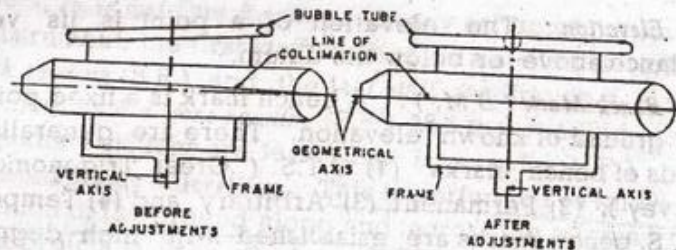


FIG. 5.6 Y-Level

5-5 Definition: The following terms should be clearly understood before proceeding with levelling operation.

A level surface: A level surface may be defined as the surface which coincides with the shape adopted by the surface of a free liquid. The surface of a still lake may be considered as a level surface. The tangent plane at every point of a level surface is perpendicular to the direction of gravity.

Mean Sea-level (M.S.L): It is the average elevation of the surface of the sea. In Bangladesh, the mean sea-level at Cox's Bazar is taken as zero.

Geoid: The surface of the earth at mean sea-level is termed as geoid.

Datum: It is an imaginary surface with respect to which the heights of different points on the earth's surface are determined. In almost every country the mean sea-level is considered most suitable for datum. The M.S.L. at Cox's Bazar is the datum in Bangladesh. In levelling operations, a certain datum is assumed (also known as assumed datum), and the relative heights of different points are calculated with respect to this datum.

The elevation of any point with reference to the assumed datum is termed as the Reduced level (R.L.) of that point.

Elevation: The elevation of a point is its vertical distance above or below the datum.

Bench Mark (B.M.): A bench mark is a fixed point on the ground of known elevation. There are generally four kinds of bench marks (1) G.T.S. (Great Trigonometrical Survey), (2) Permanent, (3) Arbitrary and (4) Temporary. G.T.S. bench marks are established with high degree of precision at regular intervals throughout the country and their elevations above the mean sea level at Cox's Bazar are given by the Survey Department of Bangladesh. Permanent bench marks are the fixed points of reference of known elevations between the G.T.S. bench marks given by the Bangladesh P.W.D. Arbitrary bench marks are the reference points whose elevations are assumed arbitrarily for small levelling works. Temporary bench marks are the reference points which are generally established at the break of any levelling work on some permanent objects.

Level line, Horizontal line and Vertical line: A level line is a line lying wholly in the geoidal or corresponding spheroidal surface, while a horizontal line is that lying in a plane tangential to level surface. So, a level line is a curved line while a horizontal one is a straight one. A vertical line at any point is a line normal to the level surface through that point and the plane containing this line is termed as vertical plane.

Height of the Instrument (H.I.): The elevation of the line of collimation above datum is termed as the height of the instrument. This is also known as the R.L. of the line of collimation.

Station: A station is a point whose elevation is to be determined. It is a point where staff reading is taken but not the point where the level is set up.

Change point: It is an intermediate station on which two readings are taken while the position of the instrument is shifted.

Back, Inter and Fore Readings: In any set up of the levelling instrument, the first staff reading on a station is termed as back reading (B.R.) and the last staff reading on a station is termed as the fore reading (F.R.) And the reading on the intermediate stations are termed as inter reading (I.R.)

5-6 Purpose of levelling: This is performed to know the undulation of the ground along and across the alignment of a route project such as roads, railways, irrigation canals, water and sewer lines, etc. so as to determine the amount of earth work in cutting and in embankment. It also determines the area from contour maps (such as reservoir area catchment area, etc.) This helps in setting out the formation levels of roads, canals, sewer lines, etc.

5-7 Procedure of Levelling Operation: We know that by levelling the relative heights of different points on the earth's surface are determined with respect to a datum. In Fig. 5.7, it is required to determine the heights of the points P_1, P_2, P_3 , etc. These heights can be determined with reference to a bench mark P whose R.L. (reduced level)

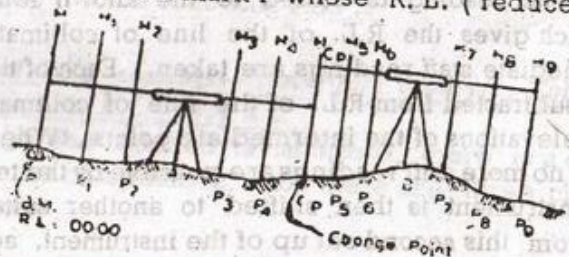


Fig. 5.7

is known (say 100.00). The level is placed at A. After performing proper levelling and adjustments of the level, staff readings are taken at P, P_1, P_2, P_3, P_4 and Cp . The instrument is now shifted to B and staff readings are taken at Cp, P_5, P_6, P_7, P_8 and P_9 . In taking readings on these different points, care should be taken so that the staff is always perpendicular to the line of collimation. This is achieved by swinging the staff backward and forward and recording the

smallest reading. Sometimes negative staff readings are taken when the ground is above the line of collimation. And this reading is taken with the staff upside down. In this case, the image of the staff is erect and not inverted. This negative reading is added to the R.L. of the line of collimation to obtain the R.L. of the required point.

5-8 Methods of calculating Reduced Levels: There are two methods, (1) Line of collimation method and (2) Rise and Fall method.

Line of Collimation Method (or Height of Instrument Method): In this method the elevation of the line of collimation for every set up of the instrument is found out and the reduced levels of different points with reference to the line of collimation are obtained. After setting the instrument at a point on the ground, staff reading is taken on a bench mark. This staff reading is added to the known R.L. of the B.M. which gives the R.L. of the line of collimation. Now, intermediate staff readings are taken. Each of these readings is subtracted from R.L. of the line of collimation giving the elevations of the intermediate points. When it is found that no more staff readings are possible by the telescope, the instrument is then shifted to another suitable position. From this second set up of the instrument, again a reading on the last station of the preceding set up is taken. This reading is added to the R.L. of this last point (change point) which will now give the R.L. of the line of collimation for the second set up of the instrument. The reduced levels of the intermediate stations are now calculated by taking reading as before. The process is continued till the work finished.

All these readings for different positions or set up of the instrument are recorded in a book termed as "Level

Book" (Table 5.1). The accuracy of the work can be checked by applying the following rule:

The difference of the sum of fore and back readings is equal to the difference of the R.L. of the first point and the last point. This rule only verifies the calculation of the R.L. of the line of collimation and change points. But this does not check the R.L. of the intermediate points.

Rise and Fall Method: In this method the difference of level between consecutive points is found out by comparing each reading with the preceding one. If the staff reading at a point is greater than the preceding one, it indicates a fall. And if the staff reading at a point is smaller than the preceding one, it indicates a rise. The R.L. of different points are then calculated by adding the rise or subtracting the fall, to or from the R.L. of the preceding point (Table 5.2). The accuracy of the work can be checked by applying the the following rule:

$$\frac{\text{Sum of back readings} - \text{Sum of fore readings} = \text{Total Rise} - \text{Total fall}}{\text{Total fall} = \text{Last R.L.} - \text{first R.L.}}$$

Though the method is laborious, there is a complete check on the computation, since there are three checks as shown above.

Example: In Fig 5.7. there are two set-ups of the instrument at A and B. From the position A staff readings on p_1, p_2, p_3, p_4 and C_p are 5.42, 7.24, 6.46, 5.38, 6.55, and 8.92 respectively. From position B staff readings on C_p, P_1, P_2, P_3, P_4 and P_5 are .78, 8.52, 6.24, 5.96, 6.35 and 7.54 respectively. If the R.L. of the bench mark P is 100.00, calculate the reduced levels of the above points by both the methods and apply necessary checks. Draw also the profile of the earth surface through these points.

Line of Collimation Method
Table 5.1

Station	Bearing	Distance	Staff Reading			Ht. of Instrument	R. L.	Remarks
			Back	Inter	Fore			
A	46°15'	00	5.42			105.42	100.00	B.M.
		50		7.24			98.18	
		100		6.46			98.98	
		150		5.38			100.04	
		200		6.55			98.87	
B	75°45'	250	8.78		6.92	107.78	98.50	Change point
		300		8.52			98.78	
		350		6.24			101.04	
		400		5.96			101.32	
		450		6.35			100.93	
		500			7.54		99.74	
Sum			14.20		14.46			

Check : Sum of fore readings \sim Sum of back readings = 14.20
 \sim 14.46 = 0.26

First R.L. \sim Last R.L. = 100 \sim 99.74 = 0.26

Rise and Fall Method
Table 5.2

Station	Bearing	Distance	Staff Readings			Difference		R. L.	Remarks
			Back	Inter	Fore	Rise	Fall		
A	46°15'	00	5.42					100.00	B. M.
		50		7.24			1.82	98.18	
		100		6.46		0.78		98.96	
		150		5.38		1.08		100.04	
		200		6.55			1.17	98.87	
B	75°45'	250	8.78		6.92		0.37	98.50	Change point
		300		8.52		0.26		98.76	
		350		6.24		2.28		101.04	
		400		5.96		0.28		101.32	
		450		6.35			0.39	100.93	
		500			7.54		1.19	99.74	
Sum			14.20		14.46	4.68	4.96		

Check : Sum of back readings \sim Sum of fore readings = 14.20
 \sim 14.46 = 0.26

Total Rise \sim Total Fall = 4.68 \sim 4.96 = 0.26

Last R.L. \sim First R.L. = 99.74 \sim 100 = 0.26

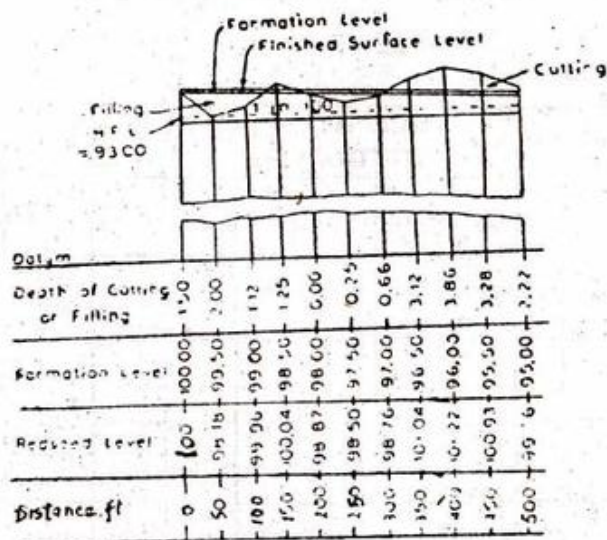


Fig. 5.8 Longitudinal Section or Profile

Example: The following is the page of a Level Book. Calculate the missing data and apply necessary checks.

Distance	Back	Inter	Fore	Rise	Fall	R. L.
0	4.25					
50		7.65				
100		9.86				
150			11.78			108.6
200		5.84				
250		4.96				
300	7.24		13.21			105.60
350		6.88				
400						103.75

Solution :

In the first set-up of the instrument between 0 and 150 by applying usual check,

$$4.25 - 11.78 = 108.6 - x$$

where $x = R.L.$ of the first point.

$$\therefore x = 116.13$$

In the second set-up between 150 and 300

$$x - 13.21 = 105.60 - 108.60$$

$$\therefore x = 10.21$$

where $x =$ Back reading of the second set-up

In the third set up between 300 and 400, $7.24 - x = 103.75 - 105.60$

$$\therefore x = 9.09$$

where $x =$ Fore reading of the third set up.

Results of all Calculation are shown in the following Table.

Distance	Back	Inter	Fore	Rise	Fall	R. L.
0	4.25					116.13
50		7.65			3.40	112.73
100		9.86			2.21	110.52
150	10.21		11.78		1.82	108.60
200		5.84		4.37		112.97
250		4.96		0.88		112.85
300	7.24		13.96		8.25	105.60
350		6.38		0.86		108.46
400			3.09		2.71	103.75
Sum	21.70		34.08	6.11	18.49	

Check : Sum of B.R. - Sum of F.R. = Total Rise - Total Fall = Last R.L. - First R.L.,
 $21.70 - 34.08 = 12.38$
 $6.11 - 18.49 = 12.38$
 $103.75 - 116.13 = 12.38$

Example : In a levelling operation the reduced levels of the first and last bench marks were 108.78 and 114.95 respectively. The sum of the back and fore readings were 24.88 and 18.64 respectively. Determine the closing error.

Solution :

$$\begin{aligned} \text{Sum of B.R.} - \text{Sum of F.R.} &= 24.88 - 18.64 = 6.24 \\ \text{The R.L. of the last B.M. should be} &= 108.78 + 6.24 = 115.02 \\ \text{But it is} &= 114.95 \\ \text{Closing error} &= 115.02 - 114.95 \\ &= 0.07 \text{ (-ve)} \end{aligned}$$

5-8 Effect of Curvature on Levelling : The effect of curvature is to increase the staff reading because the line of collimation is not a level line but a horizontal one. For ordinary small survey works this effect is not generally considered but for precise levelling and longer sights, correction for curvature is to be applied. As the distance AB increases, the difference BC between the level line and collimation line (horizontal line) goes on increasing (Fig. 5.9b).

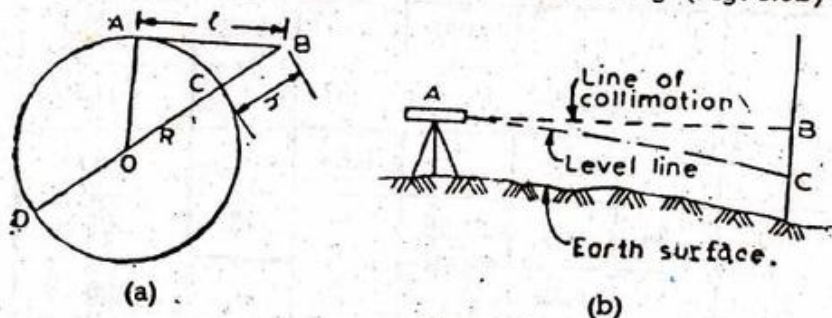


Fig. 5.9

From Fig. 5.9 (a),

$$(OB)^2 = (OA)^2 + (AB)^2$$

$$\text{Or, } (OC + CB)^2 = (OA)^2 + (AB)^2, \text{ for small distance, } AC = AB.$$

$$\text{Or, } (OC)^2 + 2OC \cdot CB + (CB)^2 = (OA)^2 + (AC)^2$$

$$\text{Or, } (R^2 + 2R \cdot h + h^2) = R^2 + l^2$$

$$\therefore h = \frac{l^2}{2R+h} = \frac{l^2}{2R}, \text{ h is very small in comparison to } 2R.$$

$$= \frac{l^2}{D}, \text{ where } D = \text{Diameter of the earth} = 7916 \text{ miles.}$$

and h is the correction which is to be subtracted from the staff reading.

When l is in miles

$$h = \frac{l^2 \times 5280}{7916} = \frac{2}{3} l^2 \text{ (approximately).}$$

So the error due to curvature in ft is equal to two-third of the square of the distance in miles. This is approximately 8 inches for first mile.

$$\therefore \text{Correct staff reading} = \text{Observed staff reading} - \frac{2}{3} l^2$$

5.9 Effect of Refraction on Levelling : In case of the effect of curvature it is assumed that the rays of light follow a horizontal path from the staff but actually they trace a curved path due to refraction (Fig. 5.10). The effect varies with atmospheric condition,

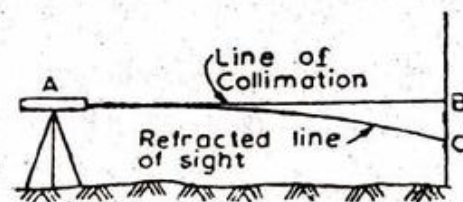


Fig. 5.10

This correction should be added to the staff reading. It is generally taken as 1/7th of the correction due to curvature.

$$\text{Correction due to refraction} = \frac{1}{7} \times \frac{2}{3} l^2 = \frac{2}{21} l^2$$

$$\text{Correct staff reading} = \text{Observed staff reading} - \frac{2}{3} l^2 + \frac{2}{21} l^2$$

$$\text{Observed staff reading} - \frac{4}{7} l^2.$$

Example : A Dumpy level was placed at C on a line AB, 2200 ft from A and 4500 ft from B. The back reading on

A was 4.98 and the fore reading on B is 12.76. Calculate the true difference of level between A and B.

Solution :

$$\begin{aligned} \text{Combined correction for 2200 ft} &= 4/7 l^2 = 4/7 \times \left(\frac{2200}{5280}\right)^2 \\ &= 0.0995 = 0.10 \text{ ft.} \end{aligned}$$

$$\text{Corrected staff reading on A} = 4.98 - 0.10 = 4.88 \text{ ft.}$$

$$\begin{aligned} \text{Combined correction for 4500 ft} &= \frac{4}{7} l^2 = \frac{4}{7} \times \left(\frac{4500}{5280}\right)^2 \\ &= 0.417 \text{ ft.} \end{aligned}$$

$$\text{Corrected staff reading on B} = 12.76 - 0.417 = 12.343 \text{ ft.}$$

$$\begin{aligned} \text{Hence the difference of level between A and B} &= 12.343 \\ &- 4.88 = 7.463 \text{ ft.} \end{aligned}$$

Example : The top of the Kutubdia light house is visible just above the horizon from a certain place in the Bay of Bengal. The distance of the light house from the observer is 20 miles. Calculate the height of the light house.

Solution :

Let the height of the light house be h ft

$$\therefore h = \frac{4}{7} l^2 = \frac{4}{7} (20)^2 = 228.5 \text{ ft.}$$

Example : In the above problem if the height of the observer is 50 ft from the sea-level, what is the height of the light house ?

Solution :

$$h_1 = \frac{4}{7} l_1^2, h_2 = \frac{4}{7} l_2^2, l = l_1 + l_2$$

$$h_1 = 50 \text{ ft and } l = 20 \text{ miles}$$

$$l_1 = \sqrt{\frac{7}{4} \times h_1} = \sqrt{\frac{7}{4} \times 50} = 9.35 \text{ miles.}$$

$$\therefore l_2 = 20 - 9.35 = 10.65 \text{ miles.}$$

$$h_2 = \frac{4}{7} (10.65)^2 = 65.2 \text{ ft.}$$

$$\therefore \text{Height of the light house} = 65.2 \text{ ft.}$$

Example : A ship having a mast 100 ft was leaving the Chittagong Port sailing at a speed of 15 miles per hour. How long the ship will be visible from a jetty 10 ft above the sea level. Assume the height of the observer to be 5'-6".

Solution :

$$h_1 = 10' + 5'6" = 15.5 \text{ ft.}$$

$$l_1 = \sqrt{\frac{7}{4} \times 15.5} = 5.21 \text{ miles.}$$

$$l_2 = \sqrt{\frac{7}{4} \times 100} = 13.2 \text{ miles.}$$

$$l = l_1 + l_2 = 5.21 + 13.2 = 18.41 \text{ miles.}$$

$$\text{Time} = \frac{18.41}{15} = 1.228 \text{ hrs}$$

5-10 Different Types of Levelling : The following are the most common types of levelling :

- Fly or Differential Levelling
- Check Levelling.
- Profile or Longitudinal Levelling.
- Cross-sectional Levelling or Cross Sectioning
- Reciprocal Levelling
- Contouring.
- Trigonometrical Levelling.
- Precise Levelling.
- Barometric Levelling.
- hypsometry.

Fly or Differential Levelling : This consists of taking back and fore readings without measuring distances between the stations. It is a very quick method in which the difference in elevation of two distant places is found out. This method is also needed when it is required to carry the reduced level from a B.M. to a point near the proposed work.

Check Levelling : This operation is similar to that of fly levelling. But the object is different. Its purpose is to

check a series of points whose levels have already been fixed by a surveyor.

Profile or Longitudinal Levelling: The object of this type of levelling is to determine the undulations of the ground surface along a predetermined line. This line is generally determined during reconnaissance of the whole land through which a road or a railway or a canal or a pipe line will have to be aligned. This line may be a single straight line or a series of lines connected by curves and it passes along the centre line of project for which the survey is to be conducted. From the longitudinal profile the land and also a few sections perpendicular to this line when plotted enable to calculate the quantity of earth work that is to be filled (in case of embankment) or cut (in case of excavation).

In Fig. 5.8. a longitudinal profile is shown. It shows the ground level, formation level, the finished surface level, highest flood level, the depths of cutting and the heights of embankments, the proposed gradient and the type of soil. On the profile two parallel lines are drawn. The lower line in red indicates the formation level i.e., the line upto which the earth work will have to be carried out. The upper line in blue indicates the surface framing top of road, railway, etc. The ground level, depths of cutting and heights of embankment are generally shown in black. The horizontal scale for plotting the profile is generally 100 ft or 200 ft to an inch.

Cross-Sectional Levelling or Cross Sectioning: These are sections run perpendicular to the longitudinal section on either side of it so as to determine the undulation on both sides of the centre line. Generally for highways and railways cross sections are taken at 100 ft interval along the centre line. The length of the cross-section is 150 ft on either side in case of highway and 300 ft. for railways. Generally, staff readings are taken at 25 ft interval when

the land is more or less flat. This interval is shortened if the land is undulated.

Reciprocal Levelling: This type of levelling is used to obtain the difference in level between two points which are separated by obstacles such as river, canal, marshy land, lake, etc., where the levelling instrument can not be set up midway between them.

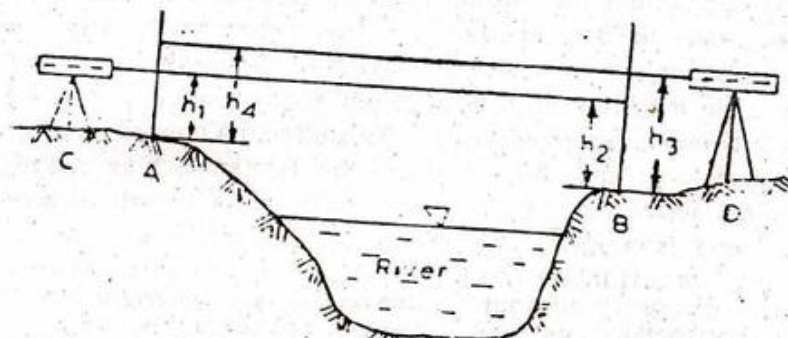


Fig. 5.11

In Fig. 5.11. A and B are the points on either sides of the river. The difference of level between A and B is to be found out. The instrument is placed at C at a certain distance from A. Staff reading h_1 and h_2 at A and B respectively are taken. The instrument is then placed at D at a certain distance from B and staff readings h_3 and h_4 at A and B respectively are taken.

True difference of elevation between A and B = $\frac{d_1 + d_2}{2}$

where d_1 = difference between h_1 and h_2

and d_2 = difference between h_3 and h_4

Example: In levelling across an irrigation canal the following observations were made (Fig 5.11);

Instrument Station	Staff readings	
	A	B
C	6.72	5.86
D	4.96	4.18

If the reduced level of B is 108.92, calculate R.L. of A.

Solution :

$$\begin{aligned} \text{True difference of level between A and B} \\ = \frac{(6.72 - 5.86) + (4.96 - 4.18)}{2} = 0.82 \text{ ft.} \end{aligned}$$

$$\therefore \text{R.L. of A} = 108.92 - 0.82 = 108.10 \text{ ft.}$$

Contouring : Contours are imaginary lines joining points of equal altitudes upon the earth's surface with reference to a fixed datum. The process by which a contour map is prepared is known as *contouring*. And the map showing the altitudes of all these points is called *contour map or topographic map*. By studying a contour map it is easy to predict the nature of the terrain such as ridge, valley, depression, etc. It also indicates as to which way the land is sloping. Contours are generally plotted at equal vertical intervals. The vertical distance between any two consecutive contours is known as *contour interval* while the horizontal distance between two consecutive contours is known as *horizontal equivalent*. Contour intervals depend upon the nature of the terrain, purpose of the survey, scale of the map, expenditure and extent of time in its determination. For engineering purposes as canals, reservoirs, town planning and building sites, contour interval is generally 5 ft to 10 ft. For greater accuracy as in the case of earth work computation it may be 1 ft. For hilly regions it is generally 50 ft to 100 ft.

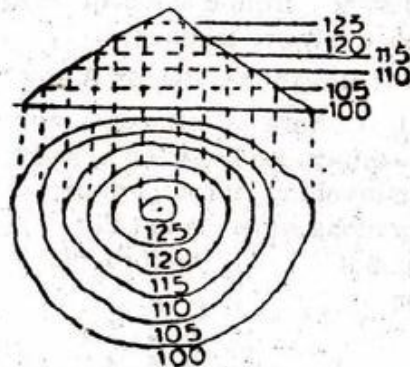


Fig. 5.12 Plan and Elevation of a Hill with Contours.

Characteristics of Contours : (1) Two contour lines of different elevations do not cross each other. (2) Contours never split or run into one except on an overhanging cliff. (3) All points on a contour line have the same elevation. (4) Contour lines run close together near the top of a hill which represents very steep ground, and wide apart at the foot of the hill indicating flat ground. (5) If the contours are uniformly spaced it indicates a uniform slope while a plane surface is represented when the contours are straight, parallel and equal spaced. (6) Contours cross ridge or valley lines at right angles. (7) Each contour line must be a continuous one until it either goes off the map or closes itself. (8) A series of closed contours on the map indicates a depression or a hill according to the lower or higher values inside them. (9) A series of contours indicate either a drainage or a watershed. If the centre of the curve is towards the low side it is a drainage while if the centre of the curve is towards the high ground, it is a watershed.

Uses of contour map : (1) A contour map indicates whether the country is hilly, flat or undulating. (2) It helps to locate suitable strategic position for placing guns, amunitions and reserves for military purposes. It also enables to locate sites for taking shelters. (3) It enables to select an economical and suitable alignment for important engineering works like irrigation canals, drainage canals, water and sewer lines, highways and railways. (4) It enables to determine the area of a drainage basin and reservoir capacity. (5) It enables to calculate the amount of earth work in cutting and in embankment.

Methods of contouring : There are two methods, viz.

(1) Direct method and (2) Indirect method.

Direct Method : In this method the different contour lines are first assumed (say 100, 105, 110 etc.) and the points on each contour line are located on the ground by

a level. These points are fixed on the ground and their positions on the map are plotted by plane tabling or by any other method. As for an example, it is required to locate a contour of 100 ft. The height of the instrument (say) is 104.56. The required staff readings on each point located at 100 ft contour line should be 4.56. Now the staff is held on different points and the required staff reading (here 4.56 ft) are obtained within the range of the instrument. These points of required staff readings are marked with pegs and are plotted on the map by plane tabling. The instrument is then shifted to a new position, the height of the instrument taken and the required staff readings calculated in the similar manner. The same process is repeated for every set up of the instrument. This is a slow and laborious process but very accurate for small areas.

Direct contouring can also be done by the method of radial lines.

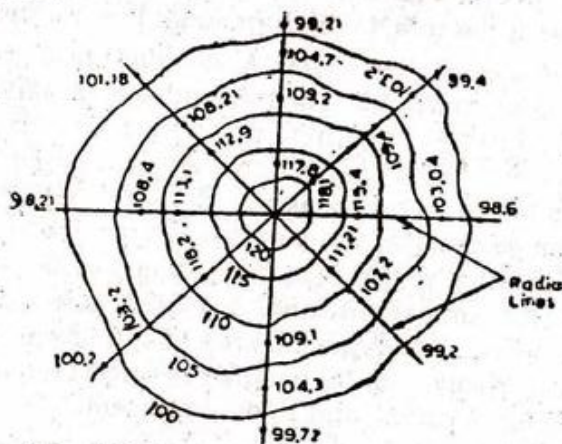


Fig. 5.13 Contouring by Radial Line

In this method radial lines from a common and suitable point are ranged out by a theodolite or a prismatic compass by measuring the angles between them. Now the required staff readings are taken along the radial lines and pegs are placed. These points are now plotted on the map by plane

tabling and the contours are drawn by joining the points of equal altitudes.

Indirect Method: This method is very fast and less laborious. The following are the different systems of indirect contouring.

(a) *Square or Grid System*: In this method the area to be contoured is divided into a series of squares and the corners of the squares are fixed by pegs (Fig. 5.14). Staff readings are taken at the corners of the squares with a levelling instrument. The calculated reduced level with respect to a bench mark are written at the corners of the squares. The required contour lines are then interpolated.

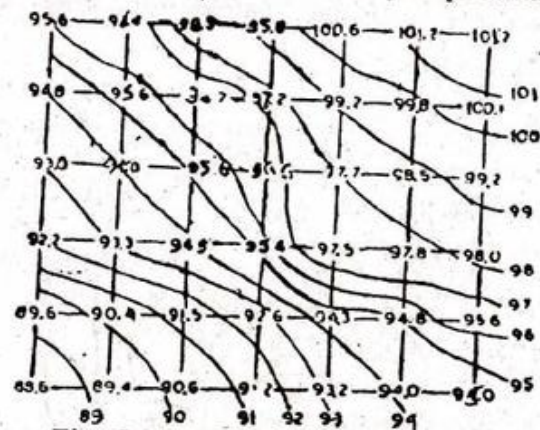


Fig. 5.14 contouring by Squares.

(b) *Radial System or Tacheometric Method*: This is suitable for a hilly region. A number of radial lines from a common and suitable point are set out by the help of a theodolite or a prismatic compass. From a suitable instrument station, staff readings are taken on the radial lines at a regular interval. The calculated reduced levels with respect to a bench mark are written at the corresponding points and the required contours are then interpolated.

(c) *Cross-section method*: This method is generally used in highway and railway projects. Cross-sections are

taken at right angles to the longitudinal section. In hilly region the spacing of cross-sections is generally 100 ft. while in flat country, it is 200 ft to 300 ft. Staff readings are taken at regular intervals along the cross-section on either side of the longitudinal section. The interpolation of the contours is done exactly in the same way as discussed above.

To determine the catchment area and storage capacity of a reservoir, longitudinal sections along the main river and its tributaries are taken with the help of a level, compass and chain. Cross-sections are also taken at suitable intervals (Fig. 5.15). From the plotted contours the storage capacity and the catchment area can be determined.

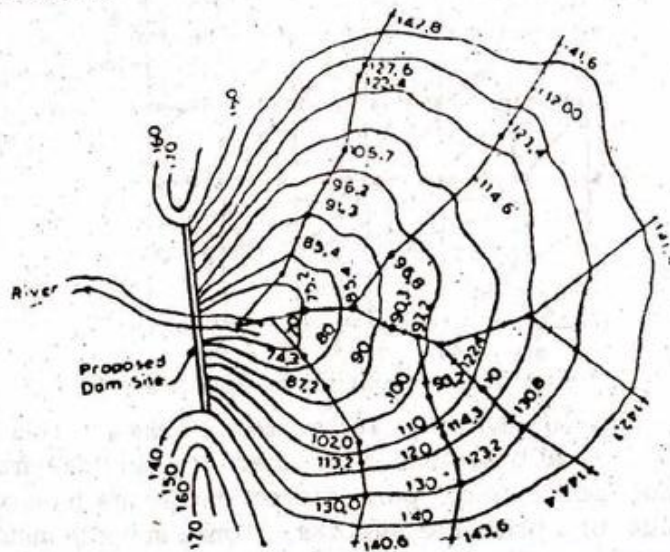


Fig. 5.15 Contouring in a Valley to determine the Catchment Area and Storage Capacity for a Proposed Dam.

Interpolation of Contours; This means the process of drawing contours proportionally between the plotted points on the map. It is always assumed that the land in between the contours is uniformly sloping while spacing the con-

ours. Contours are generally interpolated by (1) Estimation Method, (2) Graphical Method and (3) Arithmetic Method.

Estimation Method: This is the quickest method but not very accurate. In this method the locations of contour points between plotted points are roughly estimated and the contours are then drawn through these points.

Graphical Method (Kennedy's Method): This method is quick and gives accurate result. On a drawing sheet (Fig. 5.16), a line XY is drawn and it is divided into a number of equal parts. Z is the mid-point of XY. A perpendicular ZO is drawn at Z where O is any suitable point. From O, rays are drawn to the points of division of XY. In joining from O, every fifth line is made thicker than the rest. Now ZO is divided into 4 or 5 small divisions by drawing lines parallel to XY. This diagram is traced on a tracing paper. As for an example, it is required to interpolate the contours of 100, 105 and 110 in between two points a and b whose reduced levels are 98 and 112 respectively. It is assumed that the lowest radial line OX will represent an elevation of 95 ft. Now the other thick lines will represent the elevation of 100, 105, 110, etc. respectively. The tracing paper is now placed on a map in such a way that the third radial line falls on a and the vertical line XY is parallel to ab. Now by pricking through the thick lines between a and b, the exact positions of 100 and 105 contours are obtained.

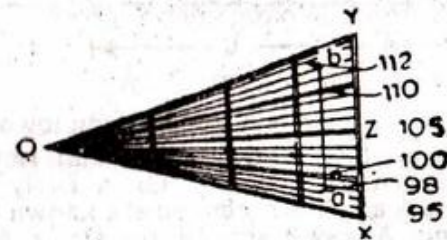


Fig. 5.16 Graphical Method of Interpolation of contours.

Arithmetic Method: This is a very accurate method. X and Y are two ground points and their reduced levels are 103.6 and 108.8 respectively. The distance between X and Y is 30 ft and the contour interval is 2 ft. So between X and Y, 104 ft, 106 ft. and 108 ft contours can be spaced. Now the total difference of level between X and Y is $108.8 - 103.6 = 5.2$ ft.

The difference of level between X and 104 ft contour is $104 - 103.6 = 0.4$ ft. Hence the distance of the 104 ft contour from point X is the $\frac{0.4}{5.2} \times 30 = 2.31$ ft. In the same way, the distance of the 106 ft and 108 ft contour points from X are 13.87 ft and 25.4 ft respectively and they can be plotted on the map.

Trigonometrical Levelling: This is also known as *indirect levelling*. In this method the vertical angles are measured by a theodolite and horizontal distances are either measured or calculated. This method is generally applied in hilly regions where ordinary levelling is very laborious

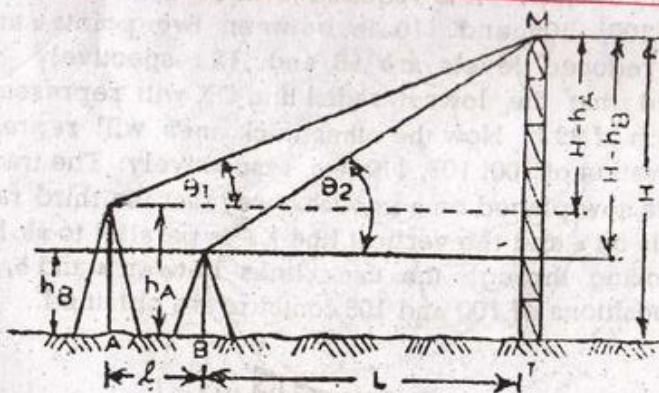


Fig. 5.17

Case 1: In Fig. 5.17 TM is a transmission tower whose foot is inaccessible. It is required to determine the height of the transmission tower. On a fairly levelled ground, two stations A and B are chosen at a known distance l apart. The points A and B should be along AT. It is assumed that the elevations of A, B and T are approximately same. By placing the theodolite at A and B, the vertical

angles θ_1 and θ_2 are measured and staff reading h_A and h_B are also taken.

$$\tan \theta_1 = \frac{H - h_A}{L + l} \quad (1)$$

$$\tan \theta_2 = \frac{H - h_B}{L} \quad (2)$$

In Equations (1) and (2) h_A , h_B , l , θ_1 , and θ_2 are known and H and L can be found out by solving these two simultaneous equations.

Example: Find the height of the transmission tower in Fig. 5.17 from the following observations.

Inst. station	Staff reading	Vertical angle
A	8.98	$20^\circ 30'$
B	5.36	$27^\circ 54'$

The distance between A and B is 100 ft.

Solution:

$$\tan \theta_1 = \frac{H - h_A}{L + l} \text{ or } \tan 20^\circ 30' = \frac{H - 8.98}{L + 100}$$

$$\therefore H = 0.3733L + 46.37 \quad (1)$$

$$\tan \theta_2 = \frac{H - h_B}{L} \text{ or } \tan 27^\circ 54' = \frac{H - 5.36}{L}$$

$$\therefore L = 1.88H - 10.15 \quad (2)$$

Putting the value of L in equation (1), we get
 $H = 155$ ft.

Case 2—Fig 5.18 shows an elevated water tank whose diameter is to be found out. Two stations A and B are fixed at a known distance apart on a fairly levelled ground.

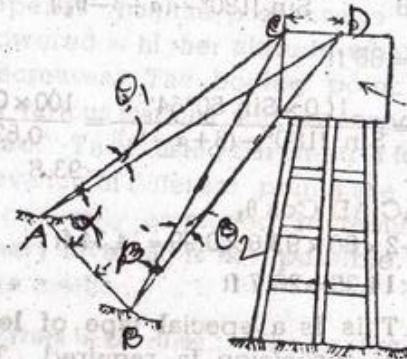


Fig. 5.18

The theodolite is first placed at A and any point C of the edge of the tank is focussed. The telescope is now rotated horizontally and the central point D is focussed. The angle θ_1 is recorded. Similarly the angle α is obtained by focussing the ranging rod at B. The instrument is now placed over B and the angles θ_2 and β are obtained. CD is the radius of the tank as denoted by r .

$$\text{Now in } \triangle ABC, \frac{AC}{\sin(\beta - \theta_2)} = \frac{AB}{\sin \angle ACB}$$

$$\therefore AC = \frac{AB \sin(\beta - \theta_2)}{\sin \angle ACB} \quad (1)$$

$$\text{and in } \triangle ABD, \frac{AD}{\sin \beta} = \frac{AB}{\sin \angle ADB}$$

$$\therefore AD = \frac{AB \sin \beta}{\sin \angle ADB} \quad (2)$$

$$\text{Again in } \triangle ACD, r^2 = AC^2 + AD^2 - 2AC \cdot AD \cos \theta_1 \quad (3)$$

Putting the value of AC and AD in equation (3), we get the value of r .

Diameter of the tank = $2r$

Example: In determining the diameter of a water tank (Fig. 5.13), the following observations were made.

$$\theta_1 = 7^\circ 30' \quad \theta_2 = 6^\circ 18' \quad \alpha = 80^\circ 42' \quad \beta = 50^\circ 54' \quad AB = 100 \text{ ft}$$

Calculate the diameter of tank.

Solution:

$$AC = \frac{AB \sin(\beta - \theta_2)}{\sin \angle ACB} = \frac{100 \sin(50^\circ 54' - 6^\circ 18')}{\sin(180^\circ - (\alpha + \beta - \theta_2))}$$

$$= \frac{100 \times 0.702}{0.815} = 86 \text{ ft}$$

$$AD = \frac{AB \sin \beta}{\sin \angle ADB} = \frac{100 \times \sin 50^\circ 54'}{\sin(180^\circ - (\beta + \alpha - \theta_1))} = \frac{100 \times 0.776}{0.828}$$

$$= 93.8$$

$$r^2 = AC^2 + AD^2 - 2AC \cdot AD \cos \theta_1$$

$$r = \sqrt{(86^2 + 93.8^2 - 2 \times 86 \times 93.8 \times 0.99)} = 14.34 \text{ ft}$$

$$\therefore \text{Diameter} = 2 \times 14.35 = 28.7 \text{ ft}$$

Precise Levelling: This is a special type of levelling where high degree of precision is required. This is

generally employed to establish bench marks at widely distributed points throughout the whole country. This is mainly conducted by the Govt. Agency like Geodetic Survey of Bangladesh.

Barometric Levelling: In this method, barometer is used to determine the differences in heights of points in a hilly or mountainous country where ordinary levelling is not possible. This method is based on the relation that the atmospheric pressure varies inversely with the height. This is generally employed on exploratory or reconnaissance surveys. This is not a very accurate method as the atmospheric pressure at any point is constantly changing and the readings are affected by the temperature of the air.

Hypsometry: In this method, the altitudes of different points are determined by the help of a hypsometer which is nothing but a thermo-barometer. The working principle of a hypsometer is based on the fact that a liquid boils when its pressure is equal to the atmospheric pressure. It consists of a graduated thermometer and a small boiler filled up with water.

The water is boiled by a spirit lamp and steam temperature is recorded by the thermometer. The boiling point of water depends upon the pressure to which it is subjected. It is lowered at higher altitude where the atmospheric pressure decreases. The boiling point temperatures are recorded at various stations and corresponding pressures are calculated. Then, using barometric formula the difference in elevation of different points are calculated. This method is generally employed in mountainous regions where ordinary levelling is not possible. But this is not a very accurate method.

5-11 Errors in Levelling: The errors in levelling may occur due to (a) imperfect adjustment of the instrument, (b)

defective focussing and bubble tube, (c) faulty sighting, (d) settlement of the level and the staff (e) curvature, refraction and the effects of wind and temperature, (f) faulty recording of field data such as recording fore and back readings, reading the wrong number in the staff, omitting an entry, ect.,

5-12 Accuracy Required in Levelling Operation: The allowable error in different types of levelling operations is as follows:

- (a) Ordinary Levelling; $e = \pm 0.1\sqrt{d}$, where e is the allowable error in ft. and d is the distance in miles.
- (b) Accurate Levelling, $e = \pm 0.05\sqrt{d}$
- (c) Levelling of High Precision, $e = \pm 0.004\sqrt{d}$,

EXERCISE

1. Examine the following statements carefully and write whether the statement is true or false.

- (a) A level line on the surface of the earth is a curved line equidistant from the centre of the earth at every point.
- (b) A horizontal line at a point on the surface of the earth is a straight line perpendicular to the line of gravity at the point and extends infinitely in both directions.
- (c) The level is an instrument for determining relative distances of different point on the earth's surface.
- (d) Bench mark is a point of known R.L. on a permanent object.
- (e) Reduced level of Datum is always 100.00
- (f) It is possible to determine the R.L. of a point, underneath an overhead electric wire by holding the levelling staff upside down.
- (g) It is possible to determine the difference of

height between two points with a level without bringing the bubble in the centre of its run.

- (h) The R.L. of the line of sight is higher than that of the datum.
- (i) A levelling staff is graduated to feet and inches only.
- (j) Line of collimation should coincide with the longitudinal axis of the telescope tube for correct staff reading.
- (k) The best position of the staff is when it is held perpendicular to the surface of the earth.
- (l) The horizontal distance between any two contour lines is called the contour interval.

readings are known as fore readings and back readings respectively.

- (v) The higher staff reading indicates elevation and lower staff reading indicates depression.
 - (w) The error due to curvature in ft is equal to two-third of the square of the distance in miles.
 - (x) The effect of refraction is to make observed staff reading higher.
 - (y) Formation level is the finished surface level of a road.
 - (z) In levelling, a station is a point on which the instrument is set up.
 - (a) Stagnant water-surface maintains equal elevation at every point.
 - (b) Small vertical angles can be measured by a levelling instrument.
2. Enumerate the objectives of levelling
 3. Explain the essential differences between a Dumpy level and a Y-level.
 4. Discuss briefly the temporary and permanent adjustments of Y-Levels.
 5. Narrate the conditions for perfect adjustment of a Dumpy Level.
 6. What is two peg test? Why do you go for this test? Explain.
 7. Discuss the methods of reducing levels. What are the merits and demerits of rise and fall method over the line of collimation method?
 8. Write explanatory notes on the following with neat sketch wherever possible.
 - (a) Levelling Staff, (b) Check Levelling, (c) Profile Levelling (d) Cross-sectioning, (e) Reciprocal Levelling, (f) Trigonometrical Levelling, (g) Differential Levelling.
 9. Explain the term "Contouring". Discuss briefly the various methods of contouring with their merits and demerits.

10. What are the uses of contour maps? Name the characteristics of contours.

11. A Dumpy Level was placed midway between A and B, 200 ft apart on a fairly level ground and the staff readings on A and B were 7.78 and 4.92 respectively. The level was then placed at C, 40 ft behind A on the line BA produced and the staff readings on A and B were 6.24 and 3.20 respectively. (a) Is the instrument in perfect adjustment?

(b) If not, is the line of collimation inclined upwards?
 (c) What should be the correct readings after the adjustment of the line of collimation?

Ans. (a) No, Inclined downwards, (c) 6.273 ft, 3.416 ft

12. In a levelling operation, the bench mark whose R.L. is 100.50 ft is 3.20 ft. below the height of the instrument in the first setting of the instrument. From the first set up of the instrument the staff readings 4.72, 9.60, 7.52 & 8.78 were taken on points A, B, C, and D respectively. From the second set up of the instrument, staff readings 6.72, 3.58, 4.82, and 7.78 were taken on D, E, F and G respectively. Calculate the reduced levels of A, B, C, D, E, F and G which are 50 ft apart from one another, after entering them in a level book form. Apply necessary arithmetic checks.

13. Rule out of a level book and enter the following staff readings which were taken at an interval of 50 ft.

Ist set up of the instrument :	5.72, 5.08, 3.42, 9.45
2nd "	" " " 6.54, 7.68, 5.12, 6.57
3rd "	2.64, 4.58, 5.24 (B.M.), 4.22 and 1.72

The B. M. occurs in the 3rd set up 200 ft right of the chain line on the top of a mile-store. The R. L. of the bench mark is 98.50. Calculate the difference in level between the starting point and last point. Apply necessary checks.

14. In levelling across a wide river, the line of collimation of the telescope placed on one bank cut an electric post 12.45 ft above the level of the ground on the other bank. The distance between the instrument position and the electric post was 4250 ft. The back reading to a bench mark close by was 6.26 ft. and the R.L. of the bench mark was 100.64 ft. Calculate the R.L. of the ground at the electric post. Apply combined correction for refraction and curvature.

Ans: 94.82 ft. above datum.

15. The light house on the Kutubdia Island near Cox's Bazar is just visible above the horizon from a distance of 50 miles in the Bay of Bengal. Calculate the height of the light house.

Ans. 1425 ft.

16. A ship having a mast 110 ft was leaving the Mongla Port. An observer having a height of 5'9" standing over the jetty of 12 ft high above the sea-level reported that the mast of the ship was visible for 1 hour and 15 minutes after leaving the port. What was the speed of the ship?

Ans: 15.5 miles per hour.

17. P and Q are two points 250 ft apart of the ground. A theodolite was placed on P and directed towards Q and another point R and the angle $-3^{\circ}2'$ and $15^{\circ}12'$ were recorded respectively. The height of the instrument at P was 4.15 ft. The instrument was then shifted to Q where the height of the instrument was 5.12 ft and angle of elevation of R from Q is $20^{\circ}26'$. If the R.L. of P is 725.60, calculate the R.L. of the point R.

Ans: 1019.76 ft

18. A theodolite was set up at P at a distance of 500 ft from a tower at Q and the following observations were recorded.

- (a) Vertical angle measured to the top of the tower = $+9^{\circ}42'$

- (b) Vertical angle measured to the foot of the tower = $-13^{\circ}24'$

(c) Height of the instrument at p = 4.76 ft

Calculate the height of the tower if the R.L. of P = 2448.75 ft
Ans. 266.47 ft

19. A tower stands on the ground the level of which is 40.722 ft above datum. From a theodolite 58 ft away horizontally an angle of elevation of $12^{\circ}42'$ is obtained. If the height of the instrument is 1.43 ft and the R.L. of the instrument station is 44.502 ft, how high is the tower?
Ans: 18.28 ft.

CHAPTER 6

SMALL INSTRUMENTS

6-1 Scales: After surveying a particular plot of land, it is required to draw it on a sheet of paper to produce plan or map. Since the size of the paper is limited, it is not possible to draw the area surveyed to its full size. It is generally drawn to a reduced size. The proportion of the drawing to the actual area is known as its *scale*. This is expressed as:

(1) 1 inch = 10 ft i.e., 1 inch length on the drawing represents 10 ft length of the land surveyed.

(2) 1 inch = 8 miles i.e., 1 inch length on the drawing represents 8 miles length of the land.

(3) $\frac{1}{2}$ full size i.e., if the length of the object is 1 inch then its length on the drawing is $\frac{1}{2}$ inch.

(4) **Representative Fraction (R.F.):** The scale may also be expressed by a fraction whose numerator is unity. This fraction is known as *representative fraction*. It is therefore the ratio of the length on the object with the numerator reduced to unity.

$$(a) \text{ R. F. of a } \frac{3}{4} \text{ full size drawing} = \frac{3 \div 3}{4 \div 3} = \frac{1}{1.33}$$

$$(b) \text{ R. F. of a } \frac{1}{2} \text{ full size drawing} = \frac{1}{2}$$

$$(c) \text{ R. F. of a 40 ft to an inch scale} = \frac{1}{40 \times 12} = \frac{1}{480}$$

$$(d) \text{ R. F. of 6 miles to an inch scale} = \frac{1}{8 \times 5280 \times 12} = \frac{1}{506880}$$

Types of Scale: Scales are of three types, viz., (1) Plain,

(2) Diagonal and (3) Vernier.

Plain Scale: Plain scales show two units, a major and a minor units, viz., feet and inches, miles and furlongs, inches and fraction of an inch.

Example: Construct a plan scale, R. F. = $\frac{1}{60}$ to show yards and feet. Show also 6 yards 2 feet on this scale.

Solution:

1 inch represents 60 inches i.e. 5 ft or $\frac{5}{3}$ yards

So, 6 inches represent $6 \times \frac{5}{3} = 10$ yards

Take a length of 6 inches and divide it into 10 equal parts to show yards and the left hand division again into 3 equal parts to show feet (Fig. 6.1)



Fig 6.1

Diagonal Scale: A diagonal scale shows three units, one major and two minor units, viz., yards, feet and inches; miles, furlongs and yards etc.

Example: Construct a diagonal scale having a R. F. = $\frac{1}{24}$ to show yards, feet and inches. Show also 2 yards 1 foot and 5 inches on this scale.

Solution:

1 inch represents 24 inches or 2 feet or $\frac{2}{3}$ yard

So, 6 inches represent $6 \times \frac{2}{3} = 4$ yards

Take a length AB of 6 inches and divide it into 4 equal divisions, each division representing 1 yard. Divide the extreme left hand division into 3 equal sub-divisions, each sub-division representing a foot. The second minor unit is

is $\frac{1}{12}$ th of the first minor unit which is a foot.

Draw a perpendicular at A. Choose a suitable height (generally 1 inch) and divide it into 12 equal parts. Draw lines parallel to AB through each point. Join 3-2, 2-1 and 1-0 (Fig. 6.2)

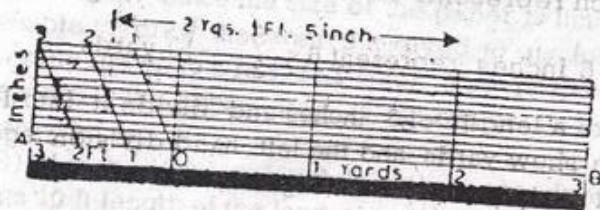


Fig. 6.2

Verniers: Vernier is a device used for measuring fractional part of the smallest division of the main or the primary scale. It is used both for linear and angular fraction measurement. It is placed parallel to the main scale and moves along the edge of the latter. The vernier carries an index mark which forms the zero of the vernier divisions and it is denoted by an arrow. Each division of the vernier scale is either shorter or longer than each division of the main scale by the required fraction. There are mainly two types of verniers, viz., (1) Direct Vernier and (2) Retrograde Vernier.

Direct Vernier: In a direct vernier both the scales are graduated in the same direction either from the right towards the left or from the left to the right and each division of the vernier is shorter than each division of the main scale. Fig 6.3 (a) shows the construction of a direct vernier where the lower scale is the main scale and the upper one is the vernier scale. It is seen that the space occupied by 9 divisions of the main scale is divided into 10 divisions of the vernier scale. If d is the value of the smallest division of the main scale, v , the length of one division of the vernier and n , the no. of divisions on the vernier, then.

$$(n-1)d = nv \therefore v = \frac{n-1}{n} \times d$$

$$\text{Again, } d - v = d - \frac{n-1}{n} \times d = \frac{1}{n} d$$

This difference, $(d-v) = \frac{1}{n} d$ is known as the least count

of the vernier. The value of the least count of the vernier may therefore be obtained by dividing the value of the smallest division on the main scale by the number of divisions on the vernier.

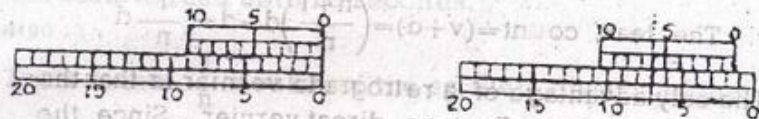


Fig. 6.3

The first division of the vernier scale falls short of the first division of the main scale by $\frac{1}{n}$ of one division of the main scale. The second division of the vernier falls short of the second of the main by $\frac{2}{n}d$ and so on. For example, in Fig. 6.3 (b), the 6th division of the vernier coincides with a division of the main scale. So, the index arrow of the vernier should be moved to the left by $\frac{d}{n}$ of a division of the main scale. Therefore, the reading will be $(1 + \frac{6}{10}) = 1.6$. In the above case, the least count is $\frac{1}{10}$. But if the 29 divisions of the main scale are divided into 30 divisions of the vernier, the least count would be $\frac{1}{30}$.

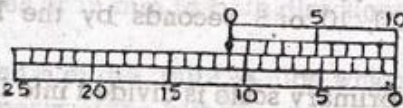


Fig 6.4

Retrograde Vernier : The retrograde vernier is one in which the graduations of the main scale are marked in the direction opposite to that of the vernier scale and each division of the vernier is longer than each of the main scale. Fig. 6.4 shows the construction of a retrograde vernier. The space occupied by 11 divisions of the main scale is divided into 10 divisions of the vernier.

$$nv = (n+1)d; \therefore v = \left(\frac{n+1}{n}\right)d$$

$$\therefore \text{The least count} = (v+d) = \left(\frac{n+1}{n}\right)d - d = \frac{1}{n}d$$

The only advantage of a retrograde vernier is that the graduations are not so fine as a direct vernier. Since, the direct vernier is simpler to read, it is generally preferred.

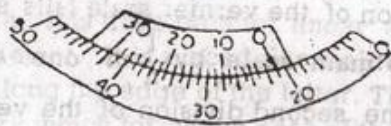


Fig. 6.5 shows the vernier attached to the circular scale of a theodolite. The lower one is the main scale and the upper one is the vernier. The graduations on the main scale are made to read 30' minutes or $\frac{1}{2}$ degree, and the space of 29 divisions of the main scale is divided into 30 divisions of the vernier. Sometimes, 20 divisions of the vernier coincide with 19 smallest divisions of the main scale. The circular scale on a theodolite is graduated in degree and then each of these graduations is sub-divided sometimes in 30 minutes and sometimes in 20 minutes. Depending upon the type of instrument, the readings may be taken upto 30, 20, 10 or 5 seconds by the help of a vernier.

Example : The primary scale is divided into inches and tenths of inchs. Construct a vernier to read upto hundredth of an inch.

Solution :

Least count = $\frac{1}{n}$ of the primary scale division

$$\therefore \frac{1}{100} \text{ inch} = \frac{1}{n} \times \frac{1}{10} \text{ inch or } n=10$$

A length equal to 9 smaller divisions on the primary scale is taken and divided into 10 equal parts.

Example : The horizontal circle of a theodolite is divided into degrees and one-third degrees. Construct a vernier scale to read upto 20 seconds.

Solution :

Least count = $\frac{1}{n} \times$ value of the smallest division of the main scale.

Now, the smallest division on the main scale = $\frac{1}{3}$ degree = 20'

$$\therefore 20' = \frac{1}{n} \times 20 \times 60 \text{ or } n=60.$$

A length equal to fifty nine one-third degree divisions is taken as the length of the vernier and it is divided into 60 equal parts.

6-2 Bubble Tube : The bubble tube, cylindrical shaped small instrument, is curved to a strictly uniform radius which varies from 50 to 1200 ft. The sensitivity will be high if the radius is longer. It is partially filled with alcohol or ether leaving an air bubble inside which occupies the highest position of the tube. When the bubble is at the centre of its run, the imaginary tangent line, (bubble line) becomes horizontal. The top surface of the tube is graduated either in inches or in mm in both directions with zero at the centre.

The sensitiveness of the bubble tube which denotes the quick acting property of the bubble as relating to the rapidity and amount of its movement, depends upon the radius

of the curvature of the tube and is directly proportional to it. It is measured by the angle through which the line of sight moves in order to cause the bubble to move over one division of the scale. The following is the procedure to determine the sensitivity and the radius of curvature of the bubble tube.

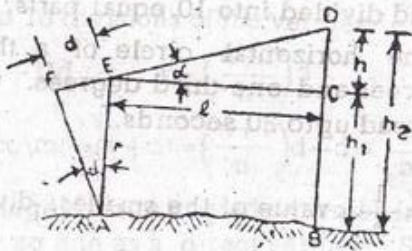


Fig. 6.6

In Fig. 6.6., A and B are two points on the ground at a distance of 100 ft apart. The instrument is set up at A and correctly levelled. The staff reading h_1 at B is taken with the bubble at the centre of its run. By turning the foot screws beneath the telescope, the forward end of the telescope is either raised or lowered so that the bubble moves through the arc $EF=d$, the distance measured in divisions on the top of the tube. In Fig. 6.6., the forward end of the telescope has been raised. Now, the staff reading h_2 is again taken at B. Then, $h_2 - h_1 = h$. From the similarity of the triangles AEF and CED,

$$\frac{AF}{EF} = \frac{EC}{DC} \text{ or } \frac{r}{d} = \frac{l}{h}$$

$$\therefore r = \frac{dl}{h}$$

$$\text{Sensitivity of the bubble tube} = \frac{d}{\alpha(\text{seconds})} = \frac{r}{206265}$$

where α = angle between the line of sight in radians and 1 radian = 206265 seconds.

Example: The bubble of a bubble tube moves through

5 mm for a change of 30 seconds inclination. Calculate the radius of curvature of the bubble tube and the sensitivity of the bubble.

Solution:

$$\text{Sensitivity per second} = \frac{5}{30} = 0.167$$

$$\text{Sensitivity of the bubble tube} = \frac{r}{206265}$$

$$\therefore \frac{r}{206265} = \frac{5}{30} \therefore r = \frac{206265 \times 5}{30} = 34377 \text{ mm}$$

$$= 34.377 \text{ metres}$$

6-3 Hand Level: This is a small and compact instrument generally 4 to 6 inches long used by hand. This hand instrument is used for rough survey works like reconnaissance and preliminary surveys, locating contours in topo-

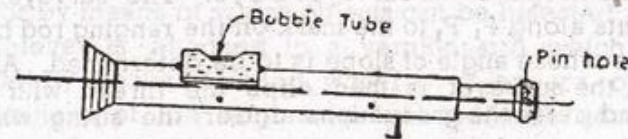


Fig. 6.7 Hand Level

graphic surveying and taking cross-sections in case of height of embankment and depth of cutting. There are different types, the simplest and commonly used one has been shown in Fig 6.7. It consists of a cylindrical sighting tube, a pin-hole at the end, cross-wire at the other end, a reflecting mirror in the middle set at 45° and a small bubble tube at the top. The lower end of the mirror coincides with the diameter of the tube. The line joining the pin-hole and the intersection of the cross-wire gives the line of sight. The horizontal plane of the tube and the bubble tube have been adjusted in such a way that the central line becomes horizontal when the bubble is seen by the eye, through the pinhole. To use the level, it is held in hand against a ranging rod at a known height above the ground. The staff at a known distance apart is then sighted. The forward end of

the level is raised or lowered until the image of the bubble as seen in the mirror is bisected by the cross-wires. The reading of the staff at which the cross-wires appear to cut is recorded.

6-4 Clinometers: These are light and compact hand instruments. They are used for rough but rapid type of survey works for measuring vertical angles, taking cross sections, observing ground slopes and locating points on a grade. There are various forms of clinometers, the simplest and the most commonly used one has been shown in Fig. 6.8 It consists of a graduated semi-circle resembling a protractor, two pins P_1 and P_2 for sighting and a light plumb bob on a long thread suspended from the centre. In using this instrument, a point is marked on a ranging rod at the height of the surveyor's eye. The surveyor is then sights along $P_1 P_2$ to the mark on the ranging rod held at a place whose angle of slope is to be determined. After sighting, the surveyor is then clips the thread with his thumb and notes the graduations under the string which gives the angle of slope.

Fig. 6.9 shows another common type of clinometer, known as Abney's level. It is a very light, compact and hand instrument and suitable for rapid survey works. It is used for measuring the angles of elevations and depressions measuring the slope of the ground when chaining operation is performed along uneven ground and tracing a point on a rising or a falling contour. It consists of a telescope tube

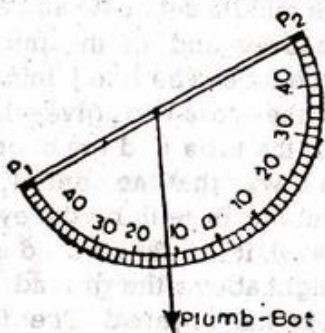


Fig. 6.8 Simple Clinometer

fitted with an eye-piece at one end and at the other end, a mirror at an angle of 45° and occupies half the width of the tube. A wire is fixed across the tube behind the

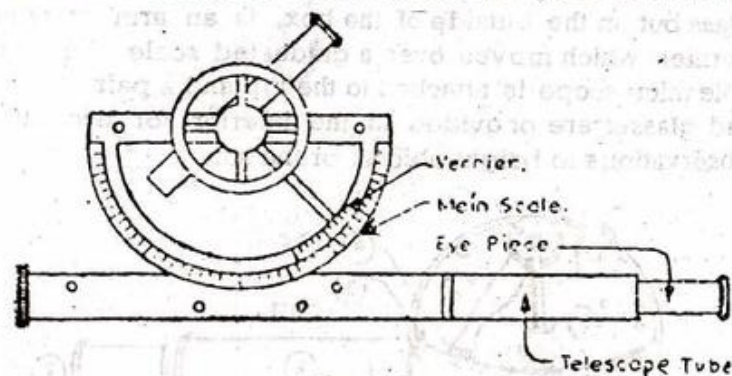


Fig. 6.9 Abney's Level

mirror by means of which objects can be bisected. A small spirit level is attached to a vernier arm which can be rotated by means of milled wheel over an arc on which graduations of the angles are printed with zero at the centre and up to 90° on both sides. By the help of the vernier, vertical angles can be read up to 10 minutes.

To use it, the telescope is sighted on to a vane and the bubble is brought at the centre of its run and the object is intersected by the line of collimation. The bubble tube should always be adjusted to the horizontal position while the telescope is parallel to the slope of the ground. The inclination is then read from the scale by the help of the vernier and the magnifying glass.

6-5 Sextants: These are small and compact instruments for measuring both horizontal and vertical angles. There are different types of sextants of which box sextants and nautical sextants are commonly used.

Box Sextant: It consists of a cylindrical box of about 3 inches diameter and about $1\frac{1}{2}$ inches thick. It is just like the optical square having a half-silvered and half plane horizon

glass and fully silvered index glass inside. The index glass can be moved by a screw so that it can be used to measure any angle. Attached to the same axis as the index glass but on the outside of the box, is an arm carrying a vernier which moves over a graduated scale. An adjustable microscope is attached to the top and a pair of coloured glasses are provided in the interior for use during observations to bright objects or the sun.

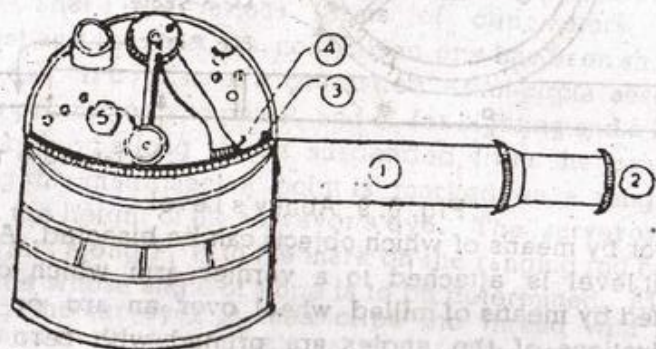


Fig. 6.10 A Box Sextant.

1. Telescope Tube
2. Eye Piece
3. Main Scale
4. Vernier
5. Magnifying Glass.

In using the instrument, it is held in right hand exactly over the station P at which the angle is to be measured. The left hand object A is sighted by direct vision through the lower portion of the horizon glass and the milled head B is turned until at the same time the image of the object F is viewed in the silvered upper portion of the glass. When the two objects A and F apparently coincide, the reading indicated by the vernier on the main scale gives the value of the angle APF.

The angle APF between two objects A and F is double the angle between the mirrors i.e. $\theta = 2\alpha$ (Fig. 6.11).

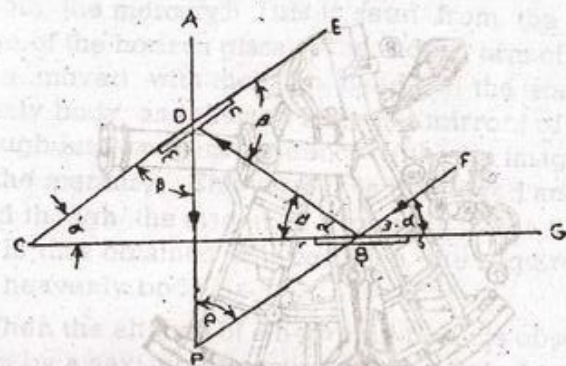


FIG. 6.11

In the Fig. 6.11, mn is the horizontal glass and rs is the index mirror. At the point C, the lines of the two mirrors when produced, meet. ADP is the ray of light coming directly through the plane glass and the ray FB, reflected from the index mirror goes to the silvered part of the horizon glass in the direction BD and reflected back as DP to the eye at P, marking the angle θ at P.

$$\angle BDE = \angle BCD + \angle CBD \text{ or } \beta = \alpha + \angle CBD, \text{ (In } \triangle BDC \text{)}$$

$$\text{But } \angle CBA = (\beta - \alpha) \text{ and also } \angle FBG = (\beta - \alpha)$$

$$\therefore \angle DBF = 180^\circ - (\angle CBD - \angle FBG) = 180^\circ - 2\beta + 2\alpha$$

$$\text{Again, } \angle PDC = BDE = \beta$$

$$\therefore \angle DBP = 180^\circ - \angle PDC - \angle BDE = 180^\circ - 2\beta$$

$$\text{Now, in the } \triangle BDP, \angle DBP = \angle BDP + \angle BPD$$

$$\therefore 180^\circ - 2\beta + 2\alpha = 180^\circ - 2\beta + 0$$

$$\text{or } 2\alpha = 0$$

$$\therefore \alpha = \frac{\theta}{2}$$

Nautical Sextant; This is a very simple and handy instrument. It is specially adopted for use in a boat or a ship on water where the motion renders the use of fixed instruments impracticable, Fig. 6.12 shows a nautical

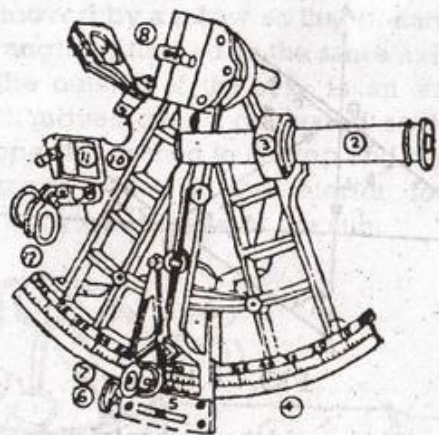


Fig. 6.12 Nautical Sextant

1. Index Arm 2. Telescope 3. Clamp
4. Graduated Arc 5. Vernier
6. Tangent Screw 7, 9 & 12. Coloured Glass 8. Adjusting Screw 10. Arm or Limb 11. Horizon Glass

sextant with its different parts. It is used for measuring angles and also the altitudes of celestial bodies.

To measure the horizontal angle between two objects at different altitudes, the plane of the limb is held in left hand in the plane containing the two objects and then looked through the telescope towards the less distinct object. The index arm is moved by the right hand until the reflected image of the brighter object comes in contact with the direct image of the less distinct object. The index bar is now clamped and exact coincidence of the two images are obtained with the tangent screw. Now the vernier reading is noted which gives the desired angle. Correction for index error is to be applied if there is any index error.

To measure the altitude of a heavenly body on land, an artificial horizon is required. The sextant is held in the left hand and the telescope is focussed towards

the image of the heavenly body (star, sun, etc.) as reflected from the mercury. This is seen from the unsilvered portion of the horizon glass. The index arm of the sextant is then moved with the right hand till the image of the heavenly body as reflected from the mirrors of the sextant is brought into exact coincidence with the image reflected from the mercury. The vernier is then fixed and the angle is read through the magnifying glasses. The value of the angle is thus obtained will be twice the required altitude of the heavenly body.

When the altitude of a heavenly body is observed from the sea by a sextant, correction for dip is to be applied.

The nautical sextant is specially used for coastal surveys, marine and navigation surveys, and making observations on heavenly bodies for latitude, longitude, time, meridian and altitude.

6-6 Proportional Compass: It is a small instrument used to copy plans to any scale, to reduce lines to any ratio, to

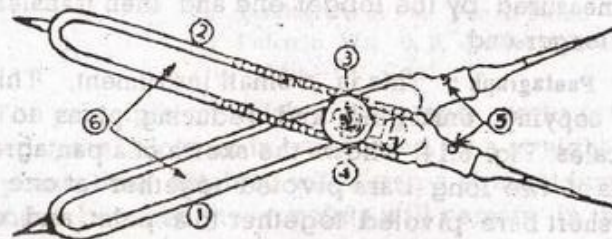


Fig 6.13 Proportional Compass.

1. & 2. Brass Arms. 3. Milled headed Screw. 4. Pivot. 5. Projecting Pins, 6. Slots.

inscribe a regular polygon inside a circle of given radius and to reduce or enlarge given areas or volumes to any desired ratio.

This consists of two brass arms having a slot down the middle of each, along which a pivot can be moved and

locked in any desired position by means of a milled-headed screw. The arms have four scales, generally two on each arm, marked as "lines," "plans," "solids" and "circles" respectively. To use the instrument the arms are first folded one upon the other. After loosening the milled-headed screw, the slider is moved by means of the screw until the index mark on it accurately registers with the line of the scale of ratios (say 3). The screw is then tightened and the arms are pulled apart. The distance between the points at one end will be exactly three times that of the other. If it is required to enlarge a figure, then two points are marked on the boundary of the original figure, with the help of the shorter end as a divider. Let us suppose, it is $\frac{1}{2}$ inch. The points at the other end will be 1.5 inches apart. Now the longer end is used as a divider to enlarge the figure three times on the copy. Now, to reduce a figure to a certain ratio, just the opposite is done. The distance on the original figure is to be measured by the longer end and then transferred it by the longer end.

6-7 Pantagraph; This is a small instrument, This is used for copying, enlarging and reducing plans to any desired scales. Fig. 6.14, shows the sketch of a pantagraph. It consists of two long bars pivoted together at one end and two short bars pivoted together to a point, and each again pivoted to the centre of the long bars. The instrument is supported by small roller wheels. A weight, pivoted at one of the long bars holds the instrument in position. The other long bar carries a tracing point for tracing lines of original figure, one of the short bars again carries a pencil point for copying, enlarging or reducing plans to the desired scale. The position of the tracer, pencil and weight is interchangeable. The pencil is connected by a cord to the tracing point where a loop is placed for the finger in order that the pencil may be raised when passing over the space between two lines. There

are fixed sharp points on the under side of the weight to keep the weight firm and steady in position. The shorter bars are pivoted with the longer bars in such a way that the four sides form a parallelogram. Short bars and one of the long bars have figures engraved in the ratio in which the plans are to be enlarged or reduced,

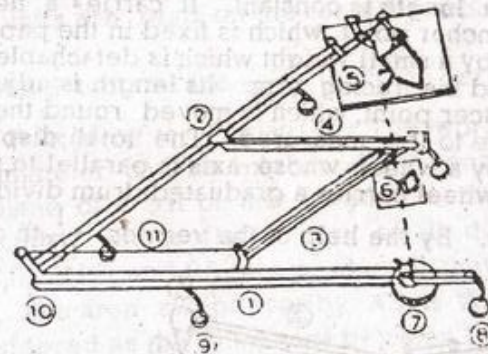


Fig. 6.14 Pantagraph

- 1, 2. Long Bars 3, 4. Short Bars
5. Tracing Point 6. Pencil Point
7. Fulcrum Wt. 8, 9. Roller Wheels
10. Pivot 11. Cord.

The principle on which pantagraph works is that if any jointed parallelogram is intersceted by a straight line, the four points thus obtained will lie in a straight line, and the distances between these points will remain in the relative constant proportion to each other, even though the angles of the parallelogram are altered. When the pencil and the tracer are fixed to the same figure which are engraved on the short and the long bars respectively, the instrument is adjusted in such a way that the weight, the tracer and the pencil lie in the same straight line. Now, it is required to reduce a given figure in a proportion 1:4. The index of the fulcrum weight which slides along graduated scale bar to read 1:4. This is now clamped in that position. The sliding index carrying a pencil is also set to read 1:4. on the small graduated bar and by doing so, the fulcrum weight, pencil

and the tracer shall be now in a straight line. The given figure is now traced by the tracer and the pencil itself will trace the same figure in the reduced proportion of 1:4. By interchanging the positions of the tracer and the pencil, the figure can be enlarged in the ratio 4:1.

6-8 Planimeter: It is an instrument which can measure areas of all regular or irregular figure very accurately. This consists of two arms pivoted at a point. One is called anchor arm and its length is constant. It carries a needle point known as anchor point, which is fixed in the paper and held in position by a small weight which is detachable. The other arm is called the tracing arm. Its length is adjustable and carries a tracer point, which is moved round the boundary of the figure to be measured. The total displacement is measured by a wheel whose axis is parallel to the tracing arm. The wheel carries a graduated drum divided into 100 equal parts. By the help of the vernier, $\frac{1}{10}$ th part may be

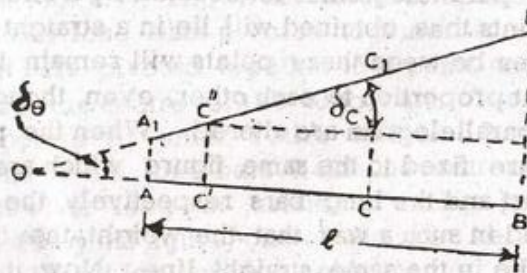
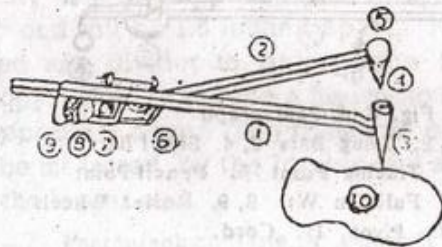


Fig. 6.15 Planimeter

1. Tracing Arm 2. Anchor Arm
3. Tracing Point 4. Anchor Point
5. Fulcrum Wt. 6. Pivote
7. Counting Disc 8. Roller Wheel
9. Vernier 10. Area to be Measured.

read. The complete revolutions of the roller wheel are read on the counting disc to which the roller wheel is geared. The counting disc is divided into 10 equal parts and advances one line at each such turn of the roller wheel and thereby performs one revolution at every 10 turns of the roller wheel. Each complete reading is a figure of four digits, the units are read on the counting disc $\frac{1}{10}$ th and

$\frac{1}{100}$ th on the drum and the $\frac{1}{1000}$ th on the vernier.

Working Principle of the Planimeter: The working principle of a planimeter can be explained by considering the motion on plane of a bar of any length. In Fig. 6.6, AB is a bar of length l which after a very small displacement moves to A₁B₁. If BA and B₁A₁ are produced, they will meet at a point O. The area swept over by AB is AA₁BB₁. This area is considered as the difference between two triangles BB₁O and AA₁O.

$$\begin{aligned} \text{Area Swept over} &= \frac{1}{2} OB \cdot BB_1 - \frac{1}{2} OA \cdot AA_1 \\ &= \frac{1}{2} OB \cdot OB_1 \sin \delta\theta - \frac{1}{2} OA \cdot OA_1 \sin \delta\theta \\ &= \frac{1}{2} \sin \delta\theta (OB \cdot OB_1 - OA \cdot OA_1) \end{aligned}$$

Since the movement is very small, $\sin \delta\theta = \delta\theta$, $OB_1 = OB$ and $OA_1 = OA$.

$$\begin{aligned} \therefore \text{The area swept over} &= \frac{1}{2} \delta\theta (OB^2 - OA^2) \\ &= \frac{1}{2} \delta\theta (OB + OA)(OB - OA) \\ &= \delta\theta \frac{OB + OA}{2} \times (OB - OA) \end{aligned}$$

but $\frac{OB + OA}{2} = OC$, and $OB - OA = l$ where C = mid-point of AB.

\therefore Area = $OC \cdot \delta\theta \cdot l$, but $OC \cdot \delta\theta$ is the small displacement of C at right angles to AB.

Therefore, the area = $l \times \delta C$, where $\delta C = OC \cdot \delta\theta$. Then for a considerable movement of the bar AB, the area swept

over $= \int \delta C$. This means the area swept over is equal to the length of the bar multiplied by the summation of the small displacement of its middle point at right angle to it.

$$\text{Now, } \delta C = \delta C' + C' C \cdot \delta \theta \quad \therefore \int dC = \int dC' + C' C \int d\theta$$

Again if the bar moves out and again comes back to its original position without any of its end making a complete turn about the other end then,

$$d\theta = \int 0, \text{ and } \int dC = dC'$$

Therefore, area swept over $= \int dC'$, where $\int dC'$ is the sum of all the small movements of any other point on the bar at right angle to its length recorded by the graduated drum

Use of the planimeter: The index mark on the levelled edge of the side is set to the scale to which the figure is drawn. The anchor point is fixed firmly on the paper inside or outside the figure according as the figure big or small. The tracing point is always moved in the clockwise direction around the figure so as to see that all points in the boundary can be reached without any difficulty and also the dial is observed in order to see whether the total rotation of the wheel is a forward or a backward direction. Marking a definite point on the outline of the figure, the tracing point is set exactly on it. The initial readings on the dial and wheel are recorded. The tracing point is then moved exactly around the outline always in a clockwise direction until it again reaches the starting point exactly. Again, the dial and wheel readings are recorded as final readings. The number of times the zero mark of the dial passes the fixed index mark in a clockwise direction or anti clockwise direction, during the movement of the tracing point along the outline of the figure, are noted.

The area of the figure obtained by a planimeter.

$$A = M(F.R. - I.R. \pm 10N + C)$$

where A = area obtained by the planimeter, M = Multiplier whose value is engraved on the tracing arm next to the scale division.

F.R. = Final reading on the dial and wheel.

I.R. = Initial reading on the dial and wheel.

N = Number of times the zero mark of the dial passes over the fixed index mark. Plus sign is to be used when the zero mark of the dial passes over the fixed index mark in a clockwise direction and minus sign for anti-clockwise direction.

C = Value of the constant engraved on the top of the tracing arm. This is to be used when the anchor point is inside the figure.

If the zero of the roller wheel is set at the index mark of the vernier and the anchor point is outside, the expression for area becomes as follows:

$$\text{Area, } A = M(F.R. \pm 10N)$$

Example: The perimeter of an irregular figure was traversed by a planimeter in clockwise direction with the anchor point outside and the tracing arm was so set that one revolution of the roller measured 8 sq. inch on the paper. The initial and final readings were 10.925 and 3.245 respectively. The zero mark of the disc passed the fixed index mark once in the clockwise direction. Calculate the area of the figure.

$$M = 8, F.R. = 3.245, I.R. = 10.925, N = 1, C = 0$$

$$\text{Area, } A = M(F.R. - I.R. + 10N + C)$$

$$= 8(3.245 - 10.925 + 10 \times 1 + 0)$$

$$= 18.56 \text{ sq. inches.}$$

Example: Calculate the area of an irregular figure measured by a planimeter in clockwise direction with anchor point inside and with the tracing arm set to the natural scale from the following:

$$\text{Initial reading} = 2.425, \text{ Final reading} = 9.236, C = 20.215$$

The zero mark of the dial passed the fixed index mark twice in the reverse direction and one revolution of the roller measured 8 sq. inches.

Initial reading = 1.054, Final reading = 6.459, $C = 20.425$

Area = $M(F.R. - I.R. - 10N + C)$

$$= 8(9.326 - 2.425 - 10 \times 2 + 20.215)$$

$$= 56.928 \text{ sq. inches.}$$

Example: An irregular figure was measured by a planimeter in clockwise direction with anchor point inside and with tracing arm set to natural scale. The horizontal and the vertical scale for the figure were 50 ft to 1 inch and 5 ft to 1 inch. The zero of the disc passed the fixed index mark once in the reverse direction and one revolution of the roller measured 10 sq. inches.

Calculate the area of the figure from the following readings: Initial reading = 1.054, Final reading = 6.450, $C = 20.425$

Area = $M(F.R. - 10N + C)$

$$= 10(6.450 - 1.054 - 10 \times 1 + 20.425)$$

$$= 15.821 \text{ sq. inches.}$$

Now, one sq. inch on the paper represents $50 \times 5 = 250$ sq. ft.

\therefore Area of the figure = $15.821 \times 250 = 3960$ sq. ft.

EXERCISE

1. Examine the following statements very carefully and write whether they are true or false :

- The ratio which the lengths on a drawing bear to the actual lengths on the objects is called the scale of the drawing.
- Representative fraction is actually the ratio of lengths on the drawings to the lengths on the objects with the numerator reduced to unity.
- Plain scales show only two units, one major and one minor.
- A diagonal scale is used to measure distances in one major and two other minor units.
- A vernier is used for angular measurements only.

- In a direct vernier, both the scales are graduated in the same direction.
 - The least count of a vernier is obtained by dividing the value of the smallest division on the main scale by the number of divisions on the vernier.
 - Each division of a retrograde vernier is smaller than each of the main scale.
 - A hand level is a small instrument by which vertical angles can be measured.
 - Ground slopes can be measured by a clinometer.
 - Only horizontal angles can be measured by a sextant.
 - Altitude of the sun can be determined by a box sextant.
 - A pantagraph is used for measuring the areas of irregular figures.
 - A figure can be enlarged or reduced to a suitable scale by a planimeter.
2. (a) Define a scale. Discuss the types of scales with their individual uses,
- Construct a plain scale of R.F. = $\frac{1}{32}$ to show feet and inches.
 - On a drawing, 1" represents $1\frac{1}{5}$ miles. Draw a plain scale to show miles and furlongs. On this scale, show 3 miles and 6 furlongs.
3. (a) Construct a full size diagonal scale $\frac{3}{4}$ " high to show inches, $\frac{1}{10}$ th of an inch and $\frac{1}{100}$ th of an inch.
- On a survey map, 16" represent one mile. Draw a diagonal scale to show furlongs and yards. On this scale, show a distance of one furlong and one hundred and eighty six yards.

- (c) Draw a scale of $1/48$ to show yards, feet and inches and on it show a distance of 2 yards, 2 feet and 6 inches.
4. What is a vernier? Name the different types of verniers and discuss their merits and demerits.
5. (a) A scale 1 ft long is divided to 100 equal parts. Construct a vernier for the scale by means of which it can be read to the thousandths of a foot.
 (b) A theodolite circle is divided into degrees and half degrees. Construct a vernier to read upto one minute.
6. (a) What is the sensitivity of a bubble tube? How is it measured?
 (b) The bubble of a bubble tube moves through 4 millimeters for a change of inclination of 30 seconds. Calculate the radius of the curvature of the bubble tube and the sensitivity of the bubble. Ans, 27502 m.m, 0.13 m.m.
7. Write explanatory notes on the followings:
 (a) Hand level (b) Box sextant (c) Pantagraph (d) Simple clinometer (e) Proportional compass.
8. What is a Nautical Sextant? Explain its working principles with its special use.
9. Explain the working principles of a planimeter. Deduce an expression for the area swept over by a planimeter in anticlockwise direction with anchor point fixed outside the area.
10. The area of an irregular figure is 19 sq. inches. It was measured by a planimeter with anchor point fixed outside it and the difference between the final and initial readings was 0.786. Keeping the final reading on the planimeter, another area was measured in clockwise direction with anchor point outside. The difference between final and initial readings was 1862 this time. Compute the area of the second figure.
 Ans. 28.42 sq. inches.

TACHEOMETRY OR STADIA SURVEYING

7-1 Definition: Tacheometry is the type of surveying in which vertical and horizontal distances are computed from stadia readings without using chain or tape. This is done by the help of a special type of transit theodolite known as *Tacheometer* and a staff known as *stadia rod*.

The object of this type of survey is to prepare a contour map or plan as in cases of location surveys for roads, railways, reservoirs, canals and sewers, etc. This can also be used to check accurate measurements in locating contours and completing topographic surveys with interior details. Though the method is not very accurate still it is used in places where the ground is hilly with steep gorges and where there is a swamp or a marshy land.

7-2 Description of Instruments:

Tacheometer: It is a special type of transit theodolite. Its telescope contains two horizontal hairs called *stadia hairs* in addition to the regular cross hairs. The extra

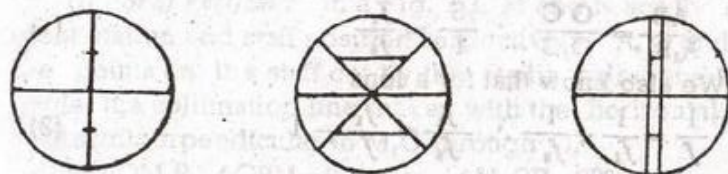


Fig. 7.1

stadia hairs are equidistant from the central cross-hairs and they are specially termed as *stadia lines* or *stadia webs*. The common types of stadia diaphragms are shown in fig. 7.1.

Stadia Rod: This is a special type of levelling staff. It is graduated in feet and decimals of a foot and the smallest sub-division being 0.01 ft. It is usually of one piece but for convenience in carrying the rods are sometimes hinged at the middle.

7-3 Theory of Tacheometry :

(a) *Horizontal Sight* : In Fig. 7.2, C is the optical centre of the object glass. A₁, B₁ are the stadia webs placed at a distance *i* apart and O₁ is the axial web of the diaphragm. A₁B₁ is the image of the staff AB when the telescope is focussed correctly. In this case, A₁CA and B₁CB are the two straight lines passing through the point C and O₁C is the

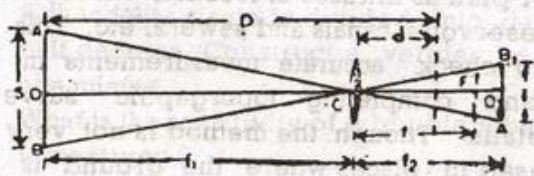


Fig. 7.2

line of collimation. *f*₁ and *f*₂ are the conjugate focal lengths of the lens and *f* is the length from C to the principal focus of the lens F. The distance from the point C to the vertical axis of the instrument is denoted by *d*.

From the similar triangles, ABC and A₁B₁C

$$\frac{AB}{A_1B_1} = \frac{OC}{O_1C} \therefore \frac{S}{i} = \frac{f_1}{f_2} \dots \dots (1)$$

We also know that for a lens

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \therefore \frac{f_1}{f_2} = \frac{f_1}{f} - 1 \dots \dots (2)$$

Putting the value of $\frac{f_1}{f_2}$ in Eqn. (1), -

$$\frac{S}{i} = \frac{f_1}{f} - 1 \therefore f_1 = S \frac{f}{i} + f$$

Adding *d* on both sides

$$f_1 + d = S \frac{f}{i} + f + d$$

$$\therefore D = S \frac{f}{i} + (f + d) \dots \dots (3)$$

This formula is to be used when the stadia rod is perpendicular to the line of collimation keeping the telescope horizontal.

(b) *Inclined Sight* : Due to the great difference in altitudes between the instrument station and staff position it will be seen that the observation is impossible when the telescope is horizontal. So an inclined sight must be taken. For this inclined sight the staff may be held either vertically or at right angles to the line of collimation.

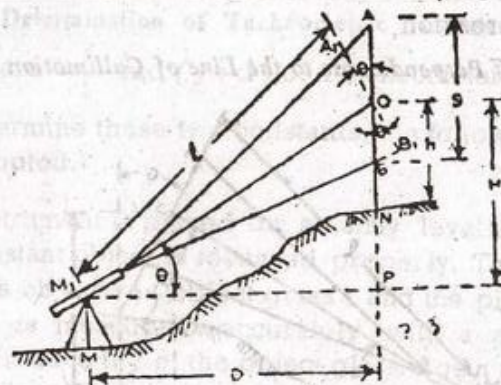


Fig. 7.3

(i) *Staff Vertical* : In a Fig. 7.3, M and N are the instrument station and staff position respectively. A, O and B are the points on the staff cut by the stadia hairs and θ is the angle, the collimation line makes with the horizontal. A₁B₁ is drawn perpendicular to M₁O through O.

In $\Delta O M_1 P$, $\angle O P M = 90^\circ \therefore \angle M_1 O P = 90^\circ - \theta$
 $\therefore \angle B O B_1 = 90^\circ - \angle M_1 O P = 90^\circ - (90^\circ - \theta) = \theta$

Now, $A_1 B_1 = AB \cdot \cos \theta = S \cos \theta$

We know from Eqn. (3),

$$D = S \frac{f}{i} + (f + d), \text{ when the sight is horizontal.}$$

In this case, $l = A_1 B_1 \times \frac{f}{i} + (f + d)$
 $= S \cos \theta \frac{f}{i} + (f + d) \dots \dots (4)$

Now, $D = l \cos \theta = S \cos^2 \theta \frac{f}{i} + (f + d) \cos \theta \dots \dots (5)$

and $\frac{H}{l} = \sin \theta$

$$\therefore H = l \sin \theta = S \cos \theta. \sin \theta / i + (f+d) \sin \theta \dots \dots (6)$$

$$= S \frac{\sin 2\theta}{2} / i + (f+d) \sin \theta$$

The elevation of the staff station $N = R.L.$ of instrument axis $\pm H - h$.

When the observed angle is an angle of elevation, the sign of H is Positive and it is negative when the angle is an angle of depression,

(ii) *Staff Perpendicular to the Line of Collimation :*

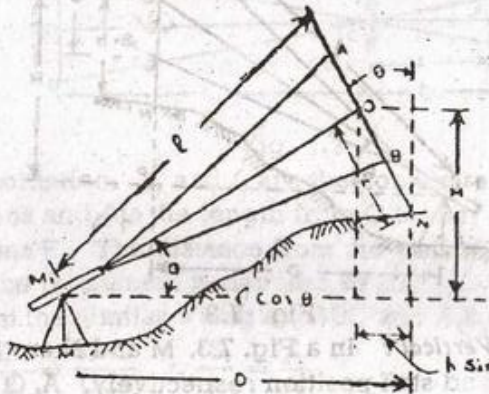


FIG. 7.4

In Fig. 7.4, the stadia rod is placed perpendicular to the line of collimation and all other notations are as before.

The distance along the line of collimation,

$$l = S \frac{f}{i} + (f+d)$$

$$\text{Now, } D = l \cos \theta + h \sin \theta = S \frac{f}{i} \cos \theta + (f+d) \cos \theta + h \sin \theta \quad (7)$$

$$\text{and } H = l \sin \theta = \frac{f}{i} \sin \theta + (f+d) \sin \theta \dots \dots (8)$$

The elevation of the staff station when the observed angle θ in the angle of elevation

$$= R.L. \text{ of the instrument axis} + H - h \cos \theta.$$

When θ is the angle of depression,

$$D = S \frac{f}{i} \cos \theta = (f+d) \cos \theta - h \sin \theta \dots \dots (9)$$

And the elevation of the staff station

$$= R.L. \text{ of the instrument axis} - H - h \cos \theta \dots \dots (10)$$

7-4 Determination of Tacheometric Constants: These constants are $\frac{f}{i}$ and $(f+d)$ of a tacheometer.

To determine these two constants, the following procedure is adopted.

The instrument is placed on a fairly levelled ground and a far distant object is focussed properly. The distance between the objective (object glass) and the plane of the cross hairs is measured accurately with a scale. This measured distance is f of the object glass. Again the distance from the object glass to the vertical axis of the instrument is measured carefully. This distance is d . Now several lengths of D_1, D_2, D_3 , etc. along a line from the instrument station are measured and the stadia intercepts S_1, S_2, S_3 , etc. respectively are taken. By applying formula (3) several values of $\frac{f}{i}$ are calculated from known values of $(f+d)$.

The average of these values will give the value of f/i is also known as *multiplying constant* and its value is different for different instruments. Generally, this value varies from 50 to 200. The constant $(f+d)$ is also known as *additive constant* and its value varies from 1 to 2 ft.

7-5 Anallatic Lens: An additional lens is sometimes placed between the object glass and the eye piece of the telescope in order to eliminate the additive constant $(f+d)$. This is done to make the expression for the distance D between instrument station and staff position more simplified. The *anallatic lens* is provided only in an external focussing telescope but not in the internal focussing one which

is virtually anallatic since the value of $(f+d)$ is only a few inches. This arrangement simplifies the calculation of heights and distances from field book, specially for inclined sights. But the drawback is that it reduces the brilliancy of the image due to absorption of light.

The expression for distance is

$D = S \frac{f}{i} + (f+d)$, term $S \frac{f}{i}$ indicates the distance of a point from the staff and it is less by an amount $(f+d)$ from the vertical axis of the instrument. If this point can be made to coincide with this axis, the expression for D is $D = S \cdot \frac{f}{i} = m S$, where $m = \frac{f}{i}$

By choosing a suitable value of m which is also termed as *multiplying factor*, the distance D is easily calculated by knowing the stadia reading S . The value of m is generally selected as 50, 100 or 200.

7-6 Field Procedure: Suitable station points are selected on the land to be surveyed. A base line AB is set out and measured accurately. The instrument is set up over the station A , centered and levelled. The height of the instrument axis is measured very accurately. The telescope is then directed to a B.M. and the bearing, the angle of elevation or depression and the stadia readings are observed. The position and the R.L. of the station A can be easily calculated from these observed readings. From station A , a number of side shots or observations are taken to prominent objects within the range of telescope. All these data are recorded in a field book. Completing all observations from the station A , a final complete set of observations is made to the next station B and the instrument is shifted to B keeping the vernier of the horizontal scale clamped. The telescope is transitted and the staff at A is observed. The vertical angle, stadia reading and the height of the instrument above B are observed. From

these readings the distance between A & B and R.L. of B are calculated and checked. After transitting the telescope, side shots are again taken on prominent objects within the range of the telescope and recorded in the field book. The instrument is then shifted to the third station and the same procedure is followed.

7-8 Errors and Accuracy: The errors in tacheometric surveying may occur due to (1) imperfect adjustment of the tacheometer, (2) incorrect values of $\frac{f}{i}$ and $(f+d)$, (3) manipulation and sighting such as non-verticality of the stadia rod, erroneous stadia readings etc. (4) natural causes such as high velocity of wind, refraction, bad visibility, etc.

Average error in distance should vary 1 in 500 to 1 in 900. Closing error in a stadia traverse should not be greater than $0.1\sqrt{P}$ ft where P is the perimeter of the traverse in ft.

B	10.30	3.75	1.46	3.89	1.38
A	10.30	3.75	1.46	3.89	1.38

2-7 Field Book

The following table shows the form of a tacheometric field book.

Table 7.1—Tacheometric Field Book : $f/i=100, f+d=1$

Instrument station	Staff station	Ht. of instrument	Azimuth	Vertical angle	Stadia reading		Distance along the line of collimation	Horizontal distance D	Vertical component H	Axial reading	Rise	Fall	R.L.		Remarks
					Instrument axis	Staff station							Instrument axis	Staff station	
A	K	4.80	70°30'	+4°12'	6.42 3.22	319.0	318.70	23.30	4.82	18.48			104.80	123.28	R.L. of the instrument station A is 100.
B	I		5°30'	+2°18'	5.85 3.15	269.8	269.70	12.75	4.50	8.25				113.05	
B	B		204°18'	-3°24'	6.36 3.16	319.5	319.20	-18.90	4.76			23.66		81.14	
B	A	3.70	24°18'	+3°30'	7.48 4.28	319.5	319.20	18.90	5.88	13.02			84.84	97.86	

Example : A tacheometer was placed over a station whose R.L. is 100.00, Stadia readings were taken on another point with the staff vertical and the telescope horizontal. The readings were 4.28, 5.88 and 7.48. The height of the instrument axis over the station was 5.30 and the values of $(f+d)$ and f/i were 1 and 100 respectively. Calculate the R.L. of the staff station and the distance between the staff position and the instrument station.

Solution :

$$D = S \frac{f}{i} + (f+d)$$

$$= (7.483 - 4.28) 100 + 1$$

$$= 321 \text{ ft.}$$

$$\text{R.L. of staff station} = (100 + 5.30) - 5.88 = 99.42$$

Example : Determine the tacheometric constants from the following observed readings :

Distance measured	Stadia readings
400	3.980, 5.975, 7.970
600	5.760, 8.755, 11.750

Solution :

$$400 = 3.99 \frac{f}{i} + (f+d) \quad (1)$$

$$600 = 5.99 \frac{f}{i} + (f+d) \quad (2)$$

$$\text{Solving (1) and (2), } \frac{f}{i} = 100 \text{ and } f+d=1$$

Example : A tacheometer was set up at a station S and the readings on a stadia rod held upon a bench mark A whose R.L. is 110.00, were 3.22, 4.82, and 6.42 and the vertical angle was 4°12'. Again the stadia readings at a station B were 2.00, 4.12 and 6.24 and the angle of depression was 7°36'. Calculate the horizontal distance between A and B and the R.L. of the station B. Take $f/i=100$ and $f+d=1$. Draw the Figure first.

Solution :

Considering left side of the Fig. $S = 5.42 - 3.22 = 3.20$

$$H_1 = S \frac{f}{i} \frac{\sin 2\theta}{2} + (f+d) \sin \theta$$

$$= 3.20 \times 100 \sin \frac{8^\circ 24'}{2} + 1 \times \sin 4^\circ 12'$$

$$= 22.8 + 0.073$$

$$= 22.873 \text{ ft.}$$

$$\text{R.L. of the instrument axis} = \text{R.L. of A} - H_1 + h_2$$

$$= 110 - 22.873 + 4.86$$

$$= 91.947 \text{ ft.}$$

Considering the right side, $S = 5.24 - 2.00 = 4.24$

$$H_2 = 4.24 \times 100 \times \sin \frac{15^\circ 10'}{2} + 1 \times \sin 7^\circ 39'$$

$$= 55.60 + 0.132$$

$$= 55.732 \text{ ft.}$$

$$\text{R.L. of the station B} = \text{R.L. of instrument axis} - H_2 - h_1$$

$$= 91.947 - 55.732 - 4.120$$

$$= 32.095 \text{ ft.}$$

$$D_1 = S \times \frac{f}{i} \cos^2 \theta + (f+d) \cos \theta$$

$$= 3.20 \times 100 \cos^2 4^\circ 12' + 1 \times \cos 4^\circ 12'$$

$$= 318 + 0.997$$

$$= 318.997 \text{ ft.}$$

$$= 319 \text{ ft.}$$

$$\text{Again } \frac{H_2}{D_2} = \tan \theta \therefore D_2 = \frac{H_2}{\tan 7^\circ 36'} = \frac{55.732}{0.1383} = 419 \text{ ft.}$$

\therefore Distance between A and B $= D_1 + D_2 = 319 + 419 = 738 \text{ ft.}$

Examples : The following observations were made with an anallatic tacheometer with the staff vertical.

Instrument station	Ht. axis	Inst. station	Staff Vertical angle	Stadia readings
A	5.50	on B.M.	$7^\circ 12'$	3.20, 5.20, 7.20
B	5.50	B	$-5^\circ 18'$	2.50, 5.00, 7.50
B	6.24	C	$14^\circ 30'$	6.40, 7.40, 8.40

Calculate the horizontal distances AB and BC Also find the R.L. of A, B and C.

Take R.L. of B.M. = 1000 and $\frac{f}{i} = 100$.

Solution :

$$D = \frac{f}{i} S \cos^2 \theta$$

$$\therefore AB = 100 (7.5 - 2.5) \cos^2 5^\circ 18' = 495 \text{ ft.}$$

$$BC = 100 (8.4 - 6.4) \cos^2 14^\circ 30' = 188 \text{ ft.}$$

$$H_1 = \frac{f}{i} S_1 \frac{\sin 2\theta}{2} = 100 (7.20 - 3.20) \frac{\sin 14^\circ 24'}{2} = 50.13 \text{ ft.}$$

$$H_2 = \frac{f}{i} S_2 \frac{\sin 2\theta}{2} = 100 \times \frac{5 \sin 10^\circ 36'}{2} = 45.75 \text{ ft.}$$

$$H_3 = 100 \times 2 \frac{\sin 29^\circ 0'}{2} = 50.07 \text{ ft.}$$

$$\text{R.L. of the instrument axis at A} = 1000 + 5.20 - 50.13 = 955.07 \text{ ft.}$$

$$\text{R.L. of B} = 955.07 - 5.5 = 949.57 \text{ ft.}$$

$$\text{R.L. of C} = 949.57 - 45.75 - 5.00 = 904.32 \text{ ft.}$$

$$\text{R.L. of instrument axis at C} = 904.32 + 6.24 = 910.56 \text{ ft.}$$

$$\text{R.L. of C} = 910.56 + 50.07 - 7.4 = 953.23 \text{ ft.}$$

Exercise

1. Examine the following statements and write whether they are true or false.

- Tacheometer is a transit theodolite.
- The stadia hairs are equidistant from the central cross-hairs.
- Stadia rod has similar graduation to that of levelling staff.
- Anallatic lens is a plano-convex one.
- Anallatic lens is placed between the object glass and the eye-piece.
- The function of the anallatic lens is to eliminate the additive constant.
- Different tacheometers have different constants.

CURVES AND CURVE RANGING

8-1 Definition: A curve which is a circular arc required to connect two straight lengths and these must be tangential to the curve in order that there shall be no abrupt break at the junctions.

In general, curves may be classified in two main groups: (1) Circular Curves and (2) Parabolic Curves. Again there are three types of circular curves;

(a) Simple, (b) Compound and (c) Reverse, and

There are two types of parabolic curves:

(a) Transition and (b) Vertical.

Simple Curve: It is a circular curve connecting two lengths of straight lines meeting at an angle, shown in Fig. 8.1.

Compound Curve: A compound curve consists of two arcs of different radii bending in the same direction and lie on the same side of their common tangent, their centres being on the same side of the curve, shown in Fig. 8.2.

Reverse Curve: A reverse curve is composed of two arcs of equal or different radii bending in opposite directions with a common tangent at their junction, their centres being on opposite sides of the curve, shown in Fig. 8.3.

Transition Curve: Whenever it is intended to change the track of a train from a straight to a circular curve as easy and free from shocks as possible. Circular curves are always connected with their tangents by transition curves, also known as *easement curve*. It is in fact a means of toning down the abrupt track from a tangent to a circular curve. Transition curves are of three different types: (i) Cubic parabola, (ii) Lemniscate of Bernolli and (iii) Spiral, shown in Fig. 8.4,

Vertical Curve: They are generally arcs and parabolas.

- (h) In stadia surveying no bearing is taken.
 (i) A base line is accurately measured in a stadia surveying.

2. Explain the fundamental principle of stadia surveying with its merits and demerits.

3. Deduce an expression of distance for inclined sight when the staff is held vertically.

4. What are the tacheometric constants? How are they determined?

5. What is an anallatic lens? Explain its purpose.

6. Discuss briefly the field procedure of stadia surveying including necessary details.

7. A tacheometer was placed with its axis horizontal 5.10 ft. above a station A whose R.L. is 320. A staff was held at B vertically and the stadia readings were 4.50, 6.50 and 8.50. Calculate the distance between A & B and the R.L. of B. Take $f/i=100$ and $f+d=1$. Ans 401, 318.60.

8. Calculate the tacheometric constants from the following observations;

Distance measured	Staff intercepts.
200	1.988
300	2.991

Ans. 100.1.

9. The following observations were made with an anallatic tacheometer with the staff vertical;

Instrument station	Height of the instrument axis	Staff position	Vertical angle	Stadia readings.
P	4.80	B.M.	$-5^{\circ}30'$	3.02, 5.76, 8.50
"	4.80	R	$+3^{\circ}24'$	3.12, 5.58, 8.04
R	4.60	S	$+6^{\circ}12'$	2.94, 6.46, 9.98

If the R.L. of the B.M. was 685.40, calculate the horizontal distances PR and RS and also determine the elevations of P, R & S.

Ans. 490.3 ft, 696 ft., 738.65.
 767.01 and 840.75

They are used where an ascending grade is followed by a descending grade or vice versa as in highways and railways, shown in Fig. 8.5.

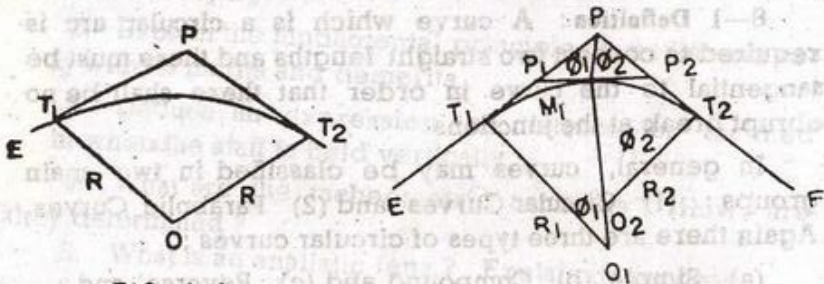


FIG. 8.1

FIG. 8.2

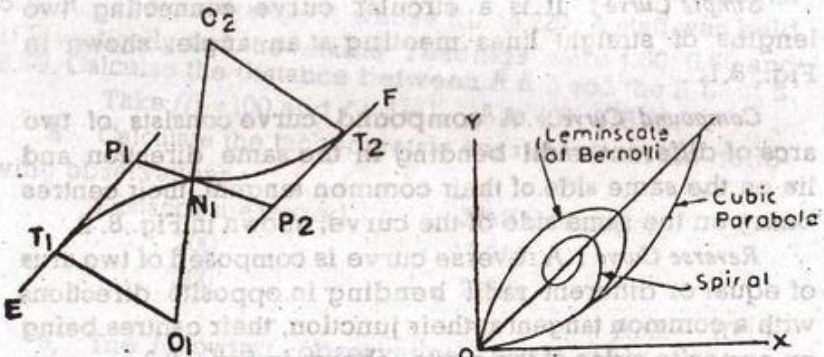
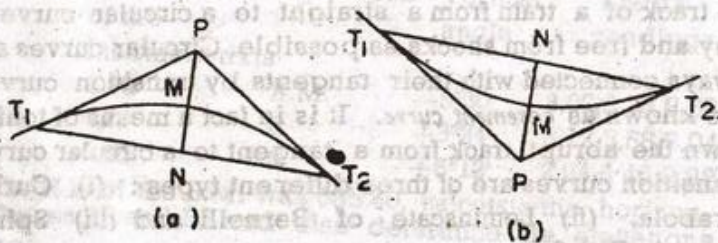


FIG. 8.3

FIG. 8.4



(a)

(b)

FIG. 8.5

8-2 Notations for Circular Curves: Referring to Fig. 8.6, EP and FP are two straights or tangents meeting at P, the point of intersection or the apex. T₁ and T₂ are tangent points. The angles EPF and P₁PF are called the intersection and deflection angles respectively and are denoted by θ and ϕ respectively.

By geometry, angle ϕ is equal to the angle T₁OT₂, subtended at the centre by the tangent points. The horizontal line T₁NT₂, joining the central points is called the long chord (L) and the arc T₁MT₂ is called the length of the curve, denoted by l . Its mid-point M is the summit of the curve and PM is known as apex distance. The distance MN is called the versed sine or middle ordinate. The line OP bisects

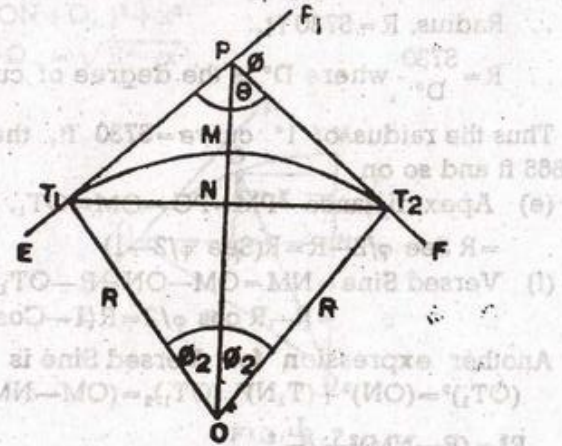


FIG. 8.6

the angle φ at the centre and θ at the vertex and also bisects the long chord and the length of the curve (due to symmetry).

$$\angle P_1PF = 2\angle PT_1T_2 \therefore \angle PT_1T_2 = \angle PT_2T_1 = \varphi/2$$

8-3 Elements of Circular Curve :

- (a) Tangent Length = $PT_1 = PT_2 = OT_1 \tan \varphi/2 = R \tan \varphi/2$
where R = radius of the curve.
- (b) Length of the long chord, $L = T_1T_2 = 2OT_1 \sin \varphi/2$
 $= 2R \sin \varphi/2$

$$(c) \text{ Length of circular curve, } l = \frac{\pi R \phi}{180^\circ}$$

$$\text{If it subtends } D^\circ \text{ at the centre, } l = \frac{100 \times \phi}{D^\circ}$$

(d) Length of radius R of the curve when degree of curvature is known.

The degree of curvature is the angle subtended by a 100 ft chord at the centre. If 1° at the centre subtends a chord of 100 ft at the circumference, for 360° at the centre, length of circumference will be 360×100 ft.

$$\therefore \text{ Diameter} = \frac{360 \times 100}{3.1416} = 2 \times 5730 \text{ ft.}$$

$$\therefore \text{ Radius, } R = 5730 \text{ ft.}$$

$$\therefore R = \frac{5730}{D^\circ} \text{ where } D^\circ \text{ is the degree of curvature.}$$

Thus the radius of 1° curve = 5730 ft. that of 2° curve = 2865 ft and so on.

$$(e) \text{ Apex distance} = PM = PO - OM = OT_1, \text{ See } \phi/2 - OM.$$

$$= R \text{ See } \phi/2 - R = R(\text{See } \phi/2 - 1)$$

$$(f) \text{ Versed Sine} = NM = OM - ON = R - OT_1 \cos \phi/2 \\ = R - R \cos \phi/2 = R(1 - \cos \phi/2)$$

Another expression for Versed Sine is

$$(OT_1)^2 = (ON)^2 + (T_1N)^2, (OT_1)^2 = (OM - NM)^2 + (T_1N)^2$$

$$R^2 = (R - NM)^2 + \left(\frac{L}{2}\right)^2$$

$$(R - NM)^2 = R^2 - \left(\frac{L}{2}\right)^2$$

$$\therefore NM = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

8-4 Methods of Ranging Curve : There two methods :

- (1) Linear methods where only a tape and chain are used.
- (2) Angular methods where a tape, a chain and a theodolite are used.

Linear methods :

- (i) Ordinates from long chord
- (ii) Offsets from tangents
- (iii) Offsets from chords

Angular methods :

- (i) By one theodolite
- (ii) By two theodolites.

Ordinates from Long Chord : Let ET_1 = first tangent, T_1N = half of long chord, MN = Versed Sine denoted by V_s , O the centre of the curve, and O_x the ordinate at a distance x from OM .

Now $GH = QN = O_x$, an $GQ = HN = x$, and $MN = V_s$.
From Fig. 8.7, $OG^2 = OQ^2 + GQ^2 = (ON + NQ)^2 + GQ^2$
or $R^2 = (ON + O_x)^2 + x^2$
or $ON + O_x = \sqrt{R^2 - x^2}$

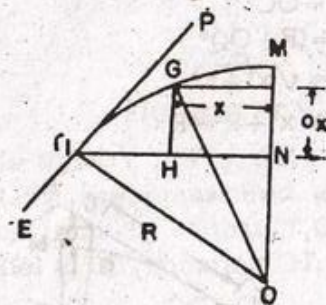


FIG. 8.7

$$\therefore O_x = \sqrt{R^2 - x^2} - ON$$

$$\text{But } ON = OM - NM = R - V_s$$

$$O_x = \sqrt{R^2 - x^2} - (R - V_s)$$

Example : Calculate the ordinates at 25 ft. distances for a circular curve having a long chord of 200 ft and a versed sine of 10 ft.

Solution :

From Fig. 8.7.,

$$OT_1^2 = ON^2 + T_1N^2 = (OM - NM)^2 + T_1N^2$$

$$\text{or } R^2 = (R - NM)^2 + (L/2)^2$$

$$\text{or } R - NM = \sqrt{R^2 - (L/2)^2}$$

$$\therefore NM = V_s = R - \sqrt{R^2 - (L/2)^2}$$

$$\text{Hence, } 10 = R - \sqrt{R^2 - (100)^2}, R = 505 \text{ ft.}$$

$$\text{and } R - V_s = 505 - 10 = 495 \text{ ft.}$$

$$O_o = 10.00 \text{ ft.}$$

$$O_{25} = \sqrt{505^2 - 25^2} - 495 = 9.37 \text{ ft}$$

$$O_{50} = \sqrt{505^2 - 50^2} - 495 = 7.53 \text{ ft,}$$

$$O_{75} = \sqrt{505^2 - 75^2} - 495 = 4.42 \text{ ft.}$$

Offsets from Tangents :

(a) *Radial Offsets :* Let O_x be the value of radial offset at a distance x from T_1 , the tangent point of the curve along T_1P , (Fig. 8.8).

In the right angled triangle CT_1O ,

$$CT_1^2 + OT_1^2 = OC^2$$

$$\text{or } x^2 + R^2 = (R + O_x)^2$$

$$\text{or } R + O_x = \sqrt{x^2 + R^2}$$

$$\therefore O_x = \sqrt{x^2 + R^2} - R$$

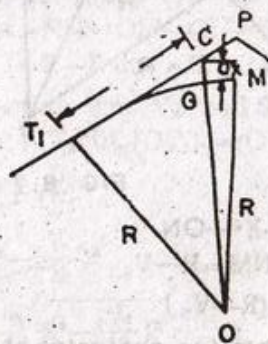


FIG. 8.8

When the radius of the curve is large as compare to the length of the long chord, the offsets may be calculated from the approximate formula which may be deduced as follows :

Expanding the factor $\sqrt{R^2 + x^2}$, we get

$$O_x = R \left(1 + \frac{x^2}{2R^2} - \frac{x^4}{8R^4} \dots \right) - R$$

Neglecting other terms except the first two, we get

$$O_x = R + \frac{Rx^2}{2R^2} - R$$

$$= \frac{x^2}{2R}$$

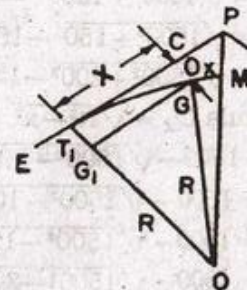


FIG. 8.9

(b) *Perpendicular Offsets :* In Fig. 8.9, let CG be the perpendicular offset at distance x measured along the tangent EP from the tangent point T_1 so that $T_1C = x$. Through G , GG_1 is drawn parallel to CT_1 meeting OT_1 at G_1 .

Then $CT_1 = GG_1 = x$, $T_1G_1 = CG = O_x$

$$OG_1 = OT_1 - T_1G_1 = (R - O_x)$$

$$\text{From triangle } OGG_1, OG^2 = GG_1^2 + OG_1^2$$

$$\text{or } R^2 = x^2 + (R - O_x)^2$$

$$\therefore O_x = R - \sqrt{R^2 - x^2}$$

Example : Calculate the offsets at 50 ft interval along tangents to locate a curve having a radius of 1500 ft.

Solution :

$$(1) \text{ Using formula } O_x = \frac{x^2}{2R}$$

$$O_{50} = \frac{50^2}{2 \times 1500} = 0.833 \text{ ft.}$$

$$O_{100} = \frac{100^2}{2 \times 1500} = 3.33 \text{ ft.}$$

$$O_{150} = \frac{150^2}{2 \times 1500} = 7.50 \text{ ft.}$$

$$O_{200} = \frac{200^2}{2 \times 1500} = 13.33 \text{ ft.}$$

(2) Using formula $O_x = \sqrt{R^2 + x^2} - R$

$$O_{50} = \sqrt{1500^2 + 50^2} - 1500 = 0.833 \text{ ft.}$$

$$O_{100} = \sqrt{1500^2 + 100^2} - 1500 = 3.33 \text{ ft.}$$

$$O_{150} = \sqrt{1500^2 + 150^2} - 1500 = 7.48 \text{ ft.}$$

$$O_{200} = \sqrt{1500^2 + 200^2} - 1500 = 13.30 \text{ ft.}$$

(3) Using formula $O_x = \sqrt{R^2 - x^2}$

$$O_{50} = 1500 - \sqrt{1500^2 - 50^2} = 0.833 \text{ ft.}$$

$$O_{100} = 1500 - \sqrt{1500^2 - 100^2} = 3.337 \text{ ft.}$$

$$O_{150} = 1500 - \sqrt{1500^2 - 150^2} = 7.52 \text{ ft.}$$

$$O_{200} = 1500 - \sqrt{1500^2 - 200^2} = 13.39 \text{ ft.}$$

Offsets from Chords: The point T_1, G_1, G_2 and G are on the curve. T_1 is the starting tangent point. It is assumed that $T_1C_1 = T_1G_1 =$ the first chord of length c_1, G_1C_2, G_2G_3 , etc..

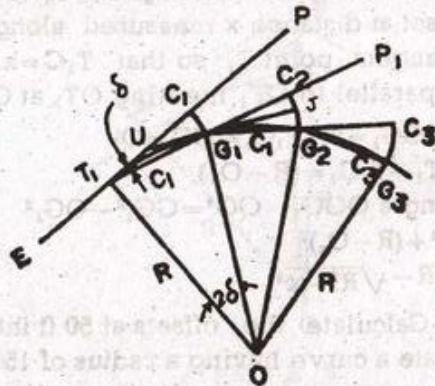


FIG. 8.10

be the successive chord lengths c_1, c_2, c_3 , etc. $G_1G_1 = O_1 =$ the offset from the tangent PT_1 . δ is the angle in radian between the tangent PT_1 and the chord T_1G_1 . The tangent

at G_1 on the curve meets PT_1 at U . T_1G_1 is produced to C_1 in such way that it equals chord c_2 .

Since chord T_1G_1 is assumed equal to arc T_1G_1

$$\text{so } T_1G_1 = R \times 2\delta \quad \therefore \delta = \frac{T_1G_1}{2R}$$

Similarly, chord $C_1G_1 =$ arc C_1G_1

$$\therefore C_1G_1 = T_1G_1 \delta.$$

Putting the value of δ

$$C_1G_1 = \frac{(T_1G_1)^2}{2R}$$

$$O_1 = \frac{c_1^2}{2R}$$

Again $UT_1 = UG_1$ because both are tangents to the circle from U .

$\therefore \angle UT_1G = \angle UG_1T_1$ and $\angle UG_1T_1 = \angle C_2G_1J$ (\because opposite angle). The triangles $C_1T_1G_1$ and C_1G_1J are similar

$$\therefore \frac{C_2J}{G_1C_2} = \frac{C_1G_1}{T_1C_1} \text{ or } C_2J = \frac{C_1G_1 \times G_1C_2}{T_1C_1}$$

$$\therefore G_2J = O_1 \times \frac{c_2}{c_1} = \frac{c_1^2}{2R} \times \frac{c_2}{c_1} = \frac{c_1 c_2}{2R}$$

As JG_2 being the offset from the tangent G_1J , hence

$$JG_2 = \frac{c_2^2}{2R}$$

Now the second offset $C_2G_2 = C_2J + JG_2$

$$\therefore O_2 = \frac{c_1 c_2}{2R} + \frac{c_2^2}{2R} = c_2 \frac{(c_1 + c_2)}{2R}$$

Similarly, $O_3 = \frac{c_3 (c_2 + c_3)}{2R}$ if c_2 & c_3 are equal then

$$O_3 = \frac{c_3^2}{R}$$

The last chord length $O_n = \frac{c_n (c_{n-1} + c_n)}{2R}$

Example: The new diversion rail road meets the existing one at Tejgaon at an angle of 130° . The chainage of the point of intersection 156. Calculate the necessary data for setting out the 4° circular curve to connect two straights of the road.

Deflection angle $\varphi = 180^\circ - 130^\circ = 50^\circ$; $R = \frac{5730}{4}$ ft. = 1432.5 ft.

Tangent length $PT_1 = PT_2 = R \tan \frac{50^\circ}{2} = 1432.5 \tan 25^\circ$
= 668 ft.

Length of the curve = $\frac{100 \times 50}{4}$ = 1250 ft.

Chainage at $T_1 = (15600 - 668)$ ft. = 14932 ft.

Chainage at $T_2 = (15600 + 668)$ ft. = 16268 ft.

Taking the length of the first chord $c_1 = 70$ ft. and

$c_2 = c_3 = c_4 = c_5 = c_6 = 100$ ft and $c_7 = 55$ ft.

$$O_1 = \frac{c_1^2}{2R} = \frac{70^2}{2 \times 1432.5} = 1.71 \text{ ft.}$$

$$O_2 = \frac{c_2(c_1 + c_2)}{2R} = \frac{100(70 + 100)}{2 \times 1432.5} \text{ ft.} = 5.93 \text{ ft.}$$

$$O_3 = O_4 = O_5 = O_6 = \frac{c_3^2}{R} = \frac{100^2}{1432.5} \text{ ft.} = 6.98 \text{ ft.}$$

$$O_7 = \frac{c_7(c_6 + c_7)}{2R} = \frac{55(100 + 55)}{2 \times 1432.5} \text{ ft.} = 2.78 \text{ ft.}$$

Angular Method by One Theodolite : From Fig. 8.11, the

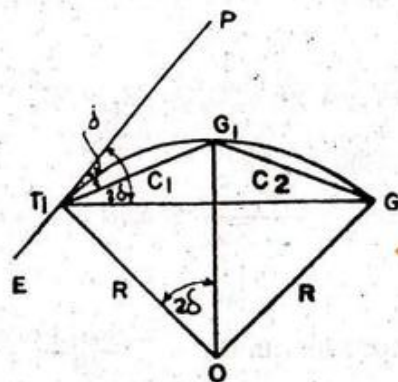


FIG. 8.11

angle PT_1G_1 is angle δ . Subsequently the angle T_1OG_1 is 2δ . If the second chord G_1G_2 is drawn equal to the first

chord T_1G_1 , then the angle PT_1G_2 will be 2δ . Similarly if three chords of same length are taken, their combined tangential angle will be 3δ and so on.

In this method the calculation of chord length and tangential angle is as follows :

$\frac{\delta^\circ}{360^\circ} = \frac{\text{Length of arc}}{2\pi \times \text{radius}}$ (\therefore Length of arc = Length of chord)

$$\text{or } \delta = \frac{180}{\pi} \times \frac{c_1}{R}$$

$$= 1718.9 \frac{c_1}{R} \text{ in minutes.}$$

Procedure of setting the curve in the field : First the theodolite is set up at T_1 and the telescope is directed towards P. The vernier of the horizontal scale is set to zero and then the lower plate is clamped. Now the telescope is turned round through an angle of δ which will be shown by the vernier. At this stage other vernier is clamped again. Keeping the zero of the chain at T_1 , the other end is pulled towards G_1 in such a way that the man looking through the telescope sights the other end in the line of angle δ . An arrow is put at G_1 . Now the vernier is unclamped and telescope is turned in the same direction at an angle 2δ . The vernier is clamped. Keeping the zero end of the chain at G_1 , the other end is pulled towards G_2 , and by swinging the chain, G_2 is fixed when the man on the telescope sights the other end of the chain. An arrow is now fixed at G_2 . For subsequent points the tangential angle is to be added to the previous vernier readings and arrows are fixed accordingly.

Angular Method by Two Theodolite : From Fig. 8.12, the angle $PT_1G_1 =$ the angle $G_1T_2T_1$ (the tangential angle made by a chord is equal to the angle subtended by the same chord in the opposite segment).

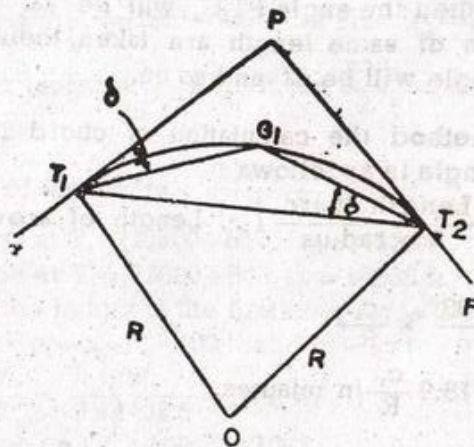


FIG. 8 12

Procedure of setting the curve in the field: One theodolite at T_1 and the other at T_2 . The theodolites are leveled and the two verniers of the horizontal scales are brought to zero. The telescope is turned towards P from T_1 and other one from T_2 is turned towards T_1 so as to intersect it. The telescope at T_1 is turned through an angle and the vernier clamped. Similarly from T_2 the telescope is turned in the same direction through an angle δ and the vernier is clamped. A man with a ranging rod in hand should move in the vicinity of G_1 until his rod is exactly sighted by both the instruments. An arrow is then fixed at G_1 which will be first point on the curve. To get the subsequent points, the verniers are set to the corresponding angles explained above and the procedure is repeated.

Example: Calculate the necessary data to set out a 5° curve by one theodolite between two straight roads intersecting at an angle of 160° . The chainage at the point of intersection is 3.540.00 ft.

Solution:

$$R = \frac{5730}{5} \text{ ft} = 1146 \text{ ft}$$

$$\text{Tangent length, } PT_1 = R \tan \frac{(180^\circ - 160^\circ)}{2} = 1146 \times \tan 10^\circ = 212 \text{ ft.}$$

$$\text{Length of the curve, } l = \frac{100 \times 20}{5} = 400 \text{ ft}$$

$$\text{Chainage at } T_1 = 3540 - 212 = 3,328 \text{ ft.}$$

$$\text{Chainage at } T_2 = 3328 + 400 = 3,729 \text{ ft.}$$

Assuming major chords of 50' length, then there will be 7 major chords and two minor chords at the ends.

$$\text{First minor chord} = 3350' - 3328' = 22'$$

$$\text{Last minor chord} = 3728' - 3700' = 28'$$

$$\delta_1 = 1718.9 \times \frac{22}{1146} = 33' - 0''$$

$$\delta_2 = 1718.9 \times \frac{50}{1146} = 1015' - 0''$$

$$\delta_3 = 1718.9 \times \frac{28}{1146} = 42' - 0''$$

$$\Delta_1 = \delta_1 = 33' - 0''$$

$$\Delta_2 = \Delta_1 + \delta_2 = 33' - 0'' + 1^\circ 15' = 1^\circ 48'$$

$$\Delta_3 = \Delta_2 + \delta_3 = 1^\circ 48' + 1^\circ 15' = 3^\circ 3'$$

$$\Delta_4 = \Delta_3 + \delta_4 = 3^\circ 3' + 1^\circ 15' = 4^\circ 18'$$

$$\Delta_5 = \Delta_4 + \delta_5 = 4^\circ 18' + 1^\circ 15' = 5^\circ 33'$$

$$\Delta_6 = \Delta_5 + \delta_6 = 5^\circ 33' + 1^\circ 15' = 6^\circ 48'$$

$$\Delta_7 = \Delta_6 + \delta_7 = 6^\circ 48' + 1^\circ 15' = 8^\circ 3'$$

$$\Delta_8 = \Delta_7 + \delta_8 = 8^\circ 3' + 1^\circ 15' = 9^\circ 18'$$

$$\Delta_9 = \Delta_8 + \delta_9 = 9^\circ 18' + 0^\circ 42' = 10^\circ$$

$$\text{Check: } \Delta_9 = \phi/2 = \frac{20^\circ}{2} = 10^\circ$$

Example: Two straight EP and EF meet in an inaccessible point P and are connected by a circular curve of 15 chain radius. A straight line XY intersects them making $\angle XEY = 110^\circ 30'$ and $\angle FYX = 140^\circ 20'$. The length of XY = 4

chains. Chainage at X is 40.50 chains. Calculate the necessary data to set out the curve.

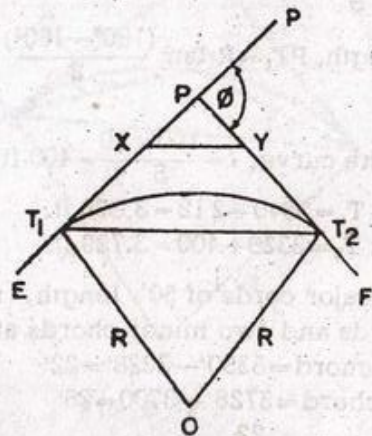


FIG. 8.13

Solution :

$$\angle PXY = 180^\circ - 110^\circ 30' = 69^\circ 30'$$

$$\angle PYX = 180^\circ - 140^\circ 20' = 39^\circ 40'$$

$$\angle XPY = 180^\circ - (69^\circ 30' + 39^\circ 40') = 70^\circ 50'; \quad \varphi = 180^\circ - 70^\circ 50' = 109^\circ 10'$$

$$\frac{PX}{XY} = \frac{\sin \angle PYX}{\sin \angle XPY} = \frac{\sin 39^\circ 40'}{\sin 70^\circ 50'}$$

$$\therefore PX = 400 \times \frac{0.638}{0.944} = 270 \text{ ft.}$$

$$\text{Similarly, } PY = 400 \times \frac{\sin 69^\circ 30'}{\sin 70^\circ 50'} = 400 \times \frac{0.937}{0.944} = 397 \text{ ft.}$$

$$\text{Tangent length, } PT_1 = R \tan \frac{\varphi}{2} = 1500 \tan \frac{109^\circ 10'}{2} = 1500 \times 1.407 = 2,110 \text{ ft.}$$

$$XT_1 = PT_1 - PX = 2,110 - 270 = 1,840 \text{ ft.}$$

$$YT_2 = PT_2 - PY = 2,110 - 397 = 1,713 \text{ ft.}$$

$$\text{Length of the curve} = \frac{\pi R \varphi}{180} = \pi \times 1500 \times \frac{109.167}{180} = 2,858 \text{ ft.}$$

$$\text{Chainage at } T_1 = 4050 + 270 - 2100 = 2,222 \text{ ft.}$$

$$\text{Chainage at } T_2 = 2220 + 2862 = 5,082 \text{ ft.}$$

Assuming major chord length of 100 ft, there will be 28 major chords and two minor chords at the ends.

$$\text{First minor chord} = 2,000 \text{ ft} - 1,950 \text{ ft} = 50 \text{ ft.}$$

$$\text{Last minor chord} = 4,812 \text{ ft} - 4,800 \text{ ft} = 12 \text{ ft.}$$

The rest follows the previous example.

Example : Determine the chainages of the tangent points and the point of compound curvature from the following data of the compound curve :

$$\text{Deflection angle, } \varphi = 60^\circ$$

$$\text{Chainage of the point of intersection} = 80.5 \text{ chains}$$

$$\text{First arc radius} = 10 \text{ chains}$$

$$\text{Second arc radius} = 15 \text{ chains}$$

The angle between the first tangent and the line joining the intersection point and the point of compound curvature measured in clockwise direction from the first tangent is 310° ; the distance of the point of compound curvature from the intersection point is 400 ft.

Solution :

From Fig. 8.2

$$\text{Chord } T_1N_1 = 2R \sin \frac{\varphi_1}{2} \times 1000 \sin \frac{\varphi_1}{2}$$

$$\angle T_1PN_1 = 360^\circ - 310^\circ = 50^\circ$$

$$\text{Now } \frac{PN_1}{T_1N_1} = \frac{\sin \varphi/2}{\sin 50^\circ} \text{ or } \sin \varphi_1/2 = \frac{PN_1}{T_1N_1} \sin 50^\circ = \frac{400 \sin 50^\circ}{2000 \sin \varphi_1/2}$$

$$\therefore \sin \frac{\varphi_1}{2} = \frac{400 \sin 50^\circ}{2000}$$

$$\therefore \varphi_1/2 = 23^\circ, \therefore \varphi_1 = 46^\circ$$

$$\varphi_2 = \varphi - \varphi_1 = 60^\circ - 46^\circ = 14^\circ$$

$$\text{Length of first tangent } PT_1 = T_1P_1 + P_1P = R_1 \tan \varphi_1/2 + (R_1 \tan \varphi_1/2 + R_2 \tan \varphi_2/2) \times \frac{\sin \varphi_2}{\sin \varphi}$$

$$\begin{aligned}
 &= 1000 \tan 23^\circ + (1000 \tan 23^\circ + 1500 \tan 7^\circ) \times \frac{\sin 14^\circ}{\sin 60^\circ} \\
 &= 1000 + 0.424 + (1000 \times 0.424 + 1500 \times 0.123) \times \frac{0.242}{0.866} \\
 &= 424 + (424 + 184.6) \times 0.279 \\
 &= 424 + 608.6 \times 0.279 = 424 + 170 \\
 &= 594 \text{ ft.}
 \end{aligned}$$

$$\text{Length of the first arc} = \frac{\pi \times 1000 \times 46}{180} = 790 \text{ ft.}$$

$$\text{Length of second arc} = \frac{\pi \times 1500 \times 14}{180} = 366 \text{ ft.}$$

$$\text{Chainage at } T_1 = 8,050 - 595 = 7456 \text{ ft.}$$

$$\text{Chainage at } N_1 = 7,456 + 790 = 8246 \text{ ft.}$$

$$\text{Chainage at } T_2 = 8,246 + 366 = 8,612 \text{ ft.}$$

Compound Curve Setting: The curve is set out by the method of deflection angle from two points T_1 and N_1 , the first arc from T_1 and the second one from N_1 as in the case of setting simple curve.

Example: Two straight EP_1 and FP_2 intersect at X . The common tangent P_1P_2 intersects EX_1 at P_1 and FX_2 at P_2 respectively. It is proposed to introduce a reverse curve of radius R between them. The angles EP_1P_2 and P_1P_2F are $146^\circ 30'$ and $126^\circ 48'$ respectively. The length of the common tangent P_1P_2 is 1200 ft. Calculate the common radius R and the chainages of the tangent points and the point of reverse curvature. The chainage at P is 2,000 ft.

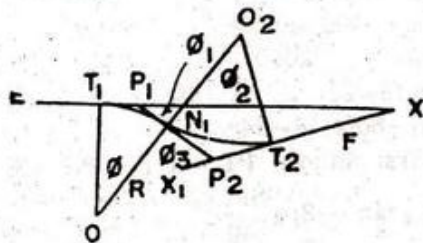


FIG. 8.14

Solution:

$$\varphi_1 = 180^\circ - 146^\circ 30' = 33^\circ 30'$$

$$\varphi_2 = 180^\circ - 126^\circ 48' = 53^\circ 12'$$

$$\begin{aligned}
 P_1P_2 &= P_1N_1 + P_2F_1 = R(\tan \varphi_1/2 + \tan \varphi_2/2) \\
 &= R(\tan 16^\circ 45' + \tan 26^\circ 36') = 1200
 \end{aligned}$$

$$\begin{aligned}
 \therefore R &= \frac{1200}{\tan 16^\circ 45' + \tan 26^\circ 36'} \\
 &= 1500 \text{ ft.}
 \end{aligned}$$

$$\text{Length of tangent, } T_1P_1 = R \tan \varphi_1/2 = 1500 \times 0.301 = 451 \text{ ft.}$$

$$\text{Length of tangent, } T_2P_2 = R \tan \varphi_2/2 = 1500 \times 0.501 = 752 \text{ ft.}$$

$$\text{Length of the first arc} = \frac{\pi \times 1500 \times 33.50}{180} = 878 \text{ ft.}$$

$$\text{Length of the second arc} = \frac{\pi \times 1500 \times 53.2}{180} = 1390 \text{ ft.}$$

$$\text{Chainage at } T_1 = 2000' - 451' = 1549 \text{ ft.}$$

$$\text{Chainage at } N_1 = 1549' + 878' = 2427 \text{ ft.}$$

$$\text{Chainage at } T_2 = 2427' - 1390' = 381 \text{ ft.}$$

8-5 Transition Curves (or Easement Curve): This may be defined as the process of easing the abrupt passage from a tangent to a circular curve.

Objects; Transition curve obtains the transition from the tangent to the circular curve and from the circular curve to the tangent. It also obtains a gradual increase of curvature from zero at the tangent point to the required magnitude at the meeting point of the transition curve with the circular one. It also provides a means of obtaining a gradual increase of super-elevation from zero on the tangent point to the required magnitude on the main circular curve so as to attain the full super-elevation simultaneously with the curvature of the circular curve at the meeting point of the transition curve with the circular one.

Advantages: It lessens the danger of derailment, wear and tear of the running gears and discomfort to the passengers. It also enables the centrifugal force to be applied gradually to avoid lateral shock.

Characteristics: It should meet the circular curve and the original straight tangentially and at the meeting points its radius is the same to that of the circular one. The rate of increase of curvature and the of superelevation should be equal along the transition curve.

Types: There are three common types of transition curves, cubic parabola, spiral or clothoid and lemniscate of Bernoulli as shown in Fig. 8.4. The first two are used in Railways and the last one in Highways.

Superelevation: If a vehicle moves from a straight path to a curved one, there will be two forces acting on it, viz. the weight of the vehicle and the centrifugal force. The effect of the centrifugal force is to push the vehicle off the rails or track. In order to balance this action the outer rail or the outer edge of the road is super-elevated or raised above the inner one. This raising is called superelevation or cant. The speed of the vehicle and the radius of the curve will govern the magnitude of superelevation.

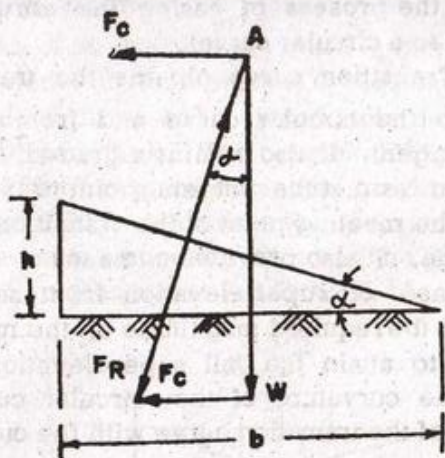


FIG. 8.15

In fig. 8.15

 $W = Wt.$ of the vehicle $F_c =$ Centrifugal force $F_r =$ Resultant force $h =$ Super elevation $b =$ Width of the road in ft. $G =$ Distance between the centres of the rails in ft. $R =$ Radius of the curve in ft. $v =$ Speed of the vehicle in ft/sec. $g =$ Acceleration due to gravity, ft/sec². $\alpha =$ Inclination of the road of rail surface.

$$F_c = \text{mass} \times \text{acceleration} = \frac{W}{g} \times \frac{v^2}{R} \therefore \frac{F_c}{W} = \frac{v^2}{gR}$$

$$\tan \alpha = \frac{F_c}{W} = \frac{v^2}{gR}$$

$$\text{Again, } \tan \alpha = \frac{h}{G} \text{ or } \tan \alpha = \frac{h}{b}$$

$$\therefore \frac{h}{G} = \frac{v^2}{gR} \therefore h = \frac{Gv^2}{gR} \text{ for railways.}$$

$$\text{Again, } h = \frac{bv^2}{gR} \text{ for highways.}$$

For broad gauge (5'-9"), $h = 6\frac{1}{2}"$ and for meter gauge (3'-3 $\frac{3}{8}"$).

$$h = 4", \text{ Centrifugal Ratio} = \frac{F_c}{W} = \frac{Wv^2}{gRW} = \frac{v^2}{gR}$$

The maximum value of centrifugal ratio is 1/8 for railways and 1/4 for highways.

Length of Transition curve $L = \frac{v^3}{aR}$ where a is the rate of change of radial acceleration, generally assumed to be unity.

8-6 Properties of a Transition Curve: In Fig. 8.16, let X and Y be any two points on the transition curve at a distance l and $l+d$ from tangent point T_1 measured along the curve. The tangents from X and Y meet the tangent T_1M at P and Q respectively, making angles α and $\alpha+d$. R is the

radius of the curve at X. For a small strip dl , R may be

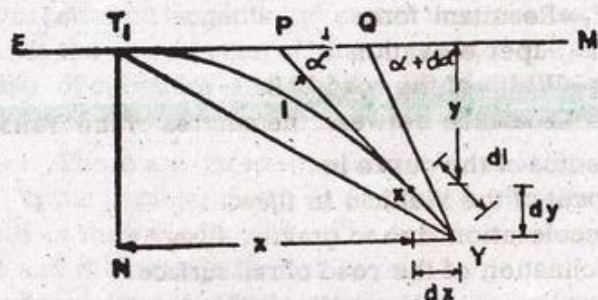


FIG. 8.16

assumed same at Y.

It is assumed that the radius of curvature at any point on the curve varies inversely as the distance l from the beginning of the curve.

Hence $\frac{1}{R} \propto l$ or $\frac{1}{R} = Kl$, where K is a constant.

Now $dl = Rd\alpha$

$$\frac{1}{R} = \frac{d\alpha}{dl}$$

$$\therefore kl = \frac{d\alpha}{dl}$$

$$dl = k.l.l \, d\alpha$$

$$\text{Integrating, } \alpha = K \frac{l^2}{2} + K_1$$

when $l=0$ and $\alpha=0$, then $K_1=0$

$$\alpha = Kl^2/2$$

$$\therefore l = K_2 \sqrt{\alpha}, \text{ where } K_2 = \sqrt{\frac{2}{K}}$$

So the true equation of an ideal transition curve is of the form $l = K_2 \sqrt{\alpha}$.

Now the above equation is identical to the known cubic parabola, $y = cx^3$ for small deflection angle, where x is the distance along the tangent and y is the offset.

$$dx = dl \cos \alpha = dl \left(1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} \dots \right)$$

$$\therefore l = K_2 \sqrt{\alpha} \therefore l^2 = K_2^2 \alpha \therefore \alpha = \frac{l^2}{K_2^2} \text{ and } \alpha^2 = \frac{l^4}{K_2^4}$$

$$dx = dl \left(1 - \frac{l^4}{2!K_2^4} + \dots \right)$$

Integrating,

$$x = l \left(1 - \frac{l^4}{10K_2^4} + \dots \right)$$

But as the angle α is in radians and very small, and k_2 is very large, so the second and the following terms are neglected for all ordinary cases.

Hence, $x = l$ (approximately)

Again, $dy = dl \sin \alpha$

$$= dl \left(\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \dots \right)$$

$$= dl \left(\frac{l^2}{K_2^2} - \frac{l^6}{6K_2^6} + \dots \right)$$

$$\text{Integrating, } y = \left(\frac{l^3}{3K_2^2} - \frac{l^7}{42K_2^6} + \dots \right)$$

$$y = \frac{l^3}{3K_2^2} \text{ (approximately)}$$

$$\text{Since, } x = l, \therefore y = \frac{x^3}{3K_2^2}$$

$$\therefore y = cx^3, \text{ where } c = \frac{1}{3K_2^2}$$

This is equation of a cubic parabola. This may be written also in the form

$$y = \frac{x^3}{6LR}, \text{ where, } \frac{1}{6LR} = \frac{1}{3K_2^2} \text{ (} \therefore K_2 = \sqrt{2RL} \text{)}$$

Now $\text{loc} = \frac{1}{r}$, where r = radius of the curve

$\therefore lr = \text{const.} = LR$, where L = length of transition curve at each end of circular curve.

$$\text{Again } \frac{d\alpha}{dl} = \frac{1}{r} = \frac{l}{LR}$$

$$\text{or } dx = \frac{ldl}{LR}$$

$$\therefore \alpha = \frac{l^2}{2LR} \text{ where } l=L, \alpha = \frac{L}{2R}$$

Shift: To fit the transition curve between the straight and the circular curve, the circular one is shifted parallel to itself by a magnitude S called the shift.

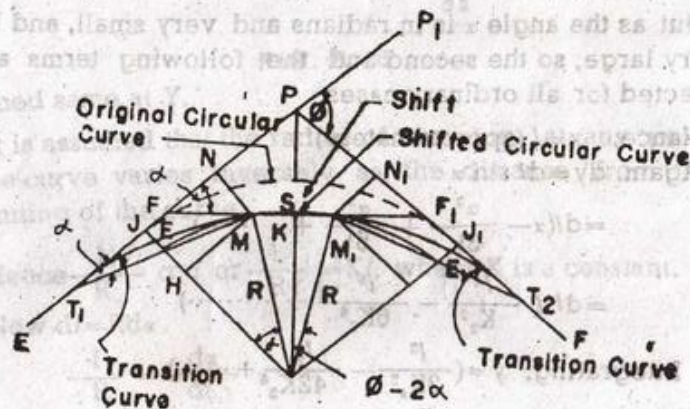


FIG. 8.17

In Fig. 8.17, Shift $= S = JE = JH = EH = NM = NO$

Equation of the transition curve,

$$y = \frac{X^3}{6LR} \text{ where } X=L \text{ and } y=MN$$

$$= \frac{L^3}{6LR} = \frac{L^2}{6R} = MN$$

From circular curve we know

$$O_1 = \frac{c_1^2}{2R}$$

$$\therefore MN = \frac{c_1^2}{2R}, \text{ where } c_1 = L/2$$

$$\therefore MN = \frac{(L/2)^2}{2R} = \frac{L^2}{8R}$$

$$\text{and } S = \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24R}$$

$$\text{Now, } y = \frac{x^3}{6LR} = \frac{4S}{L^3} x^3$$

Elements of a transition curve

$$1. \text{ Equation, } y = \frac{x^3}{6LR} = \frac{4S}{L^3} x^3$$

$$2. \text{ Shift } S = \frac{L^2}{24R}$$

$$3. T_1 J = \frac{1}{3}L$$

$$4. T_1 F = \frac{2}{3}L$$

$$5. y = S/2, \text{ when } x = L/2$$

$$6. y = 4S, \text{ when } x = L$$

7. Tangent distance, $T_1 P = (R+S) \tan \phi/2 + \frac{L}{2}$ (when ϕ is very small)

$$T_1 P = (R+S) \tan \phi/2 + \frac{L}{2} \left(1 - \frac{S}{5R}\right)$$

(when ϕ large)

8. Apex distance, $PK = (R+S) \sec \phi/2 - R$

9. Length of the combined curve, $= 2L + \frac{100 \times \phi^3}{D^3}$

10. Length of the transition curve, $T_1 M = 2 \times \text{circular arc ME}$.

11. Length of the circular curve $= \frac{\pi R(\phi - 2L)}{180}$ or $\frac{\pi R \phi}{180} - L$

12. $\alpha_1 = 1/3\alpha$

13. $\pi = \frac{L}{2R}$

8-8 Procedure to set out the transition curve in the field.

1. By means of offsets from tangent

$$y = \frac{4S}{L^3} x^3$$

2. Deflection angles from $T_1 P$,

$$y = \frac{X^3}{6LR}$$

$$\text{or } \frac{y}{x} = \frac{x^2}{6LR}$$

but $\frac{y}{x} = \tan \alpha = \alpha_1$ when α_1 is very small

$$\begin{aligned} \therefore \alpha_1 &= \frac{x^2}{6LR} \text{ radians} \\ &= \frac{180}{\pi} \times \frac{x^2}{6LR} \times 60 \text{ minutes} \\ &= \frac{1800}{\pi RL} x^2 \text{ minutes.} \end{aligned}$$

Example: Two broad gauge lines meet at an angle $112^\circ 0'$. It is proposed to insert a circular curve of 8 chain radius with transition curve at each end. The speed of the train is 60 M.P.H., super-elevation, $6''$, and the gradient is $1/300$. Calculate the necessary data to set out the combined curve (chainage at the intersection point is 12.30 chain).

Solution:

In Fig. 8.17, Length of the transition curve $= 300 \times \frac{1}{2} = 150$ ft.

$$\text{Shift, } S = \frac{L^2}{24R} = \frac{150^2}{24 \times 800} = 1.18 \text{ ft.}$$

Deflection angle, $\Phi = 180^\circ - 112^\circ 30' = 67^\circ 30'$.

$$\begin{aligned} \text{Tangent length, } T_1P &= (R+S) \tan \frac{\Phi}{2} + L/2 \\ &= (80 + 1.18) \tan \frac{67^\circ 30'}{2} + \frac{150}{2} \\ &= 610 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{Length of the circular curve} &= \frac{\pi R \Phi}{180} L \\ &= \frac{\pi \times 800 \times 67.5}{180} = 790 \text{ ft.} \end{aligned}$$

Chainage at $T_1 = 1230' - 610' = 620$ ft.

Chainage at $M = 620 + 150 = 770$ ft.

Chainage at $M_1 = 770' + 790' = 1560$ ft.

Chainage at $T_2 = 1560' + 150' = 1710$ ft.

Deflection angles for the transition curve assuming 50 feet intervals,

$$\alpha_1 = \frac{1800}{\pi RL} x^2 = \frac{1800 \times 30^2}{\pi \times 800 \times 150} = 4'18''$$

$$\alpha_2 = \frac{1800 \times 80^2}{\pi \times 800 \times 150} = 30'36''$$

$$\alpha_3 = \frac{1800 \times 130^2}{\pi \times 800 \times 150} = 1^\circ 20'30''$$

$$\alpha_4 = \frac{1800 \times 150^2}{\pi \times 800 \times 150} = 1^\circ 47'30''$$

Deflection angles for the circular curve assuming 100 ft interval, there will be 7 major chords, each 100 feet long and two minor chords 30 feet and 60 feet at the beginning and at the end respectively.

$$\delta_1 = \frac{17.89 \times 30}{800} = 1^\circ 4'24''$$

$$\delta_2 = \frac{1718.9 \times 100}{800} = 3^\circ 34'54''$$

$$\delta_3 = \frac{1718.9 \times 60}{800} = 2^\circ 8'48''$$

$$\Delta_1 = \delta_1 = 1^\circ 4'24''$$

$$\Delta_2 = \Delta_1 + \delta_2 = 4^\circ 39'18''$$

$$\Delta_3 = \Delta_2 + \delta_3 = 8^\circ 14'12''$$

$$\Delta_4 = \Delta_3 + \delta_4 = 11^\circ 59'6''$$

$$\Delta_5 = \Delta_4 + \delta_5 = 15^\circ 24'0''$$

$$\Delta_6 = \Delta_5 + \delta_6 = 18^\circ 58'54''$$

$$\Delta_7 = \Delta_6 + \delta_7 = 22^\circ 23'48''$$

$$\Delta_8 = \Delta_7 + \delta_8 = 25^\circ 58'42''$$

$$\Delta_9 = \Delta_8 + \delta_9 = 28^\circ 7'23''$$

For the transition curve on other end, the same procedure is followed.

Example: Two highways intersecting at a deflection angle of $40^\circ 30'$ are to be connected by a 5° circular curve by means of two equal transition curve to each end. The

width of the highway is 12 feet. Maximum speed of the car is 30 M.P.H. rate of superelevation, 1.5. inches per second. Calculate (a) Tangent length, (b) Length of transition curve, (c) Length of circular curve, and (d) Shift.

Solution :

$$R = \frac{5730}{5} = 1146 \text{ ft. } 30 \text{ M.P.H.} = 44. \text{ ft/sec.}$$

$$h = \frac{bv^2}{gR} = \frac{12 \times 44^2}{32.2 \times 1146} = 0.63 \text{ ft} = 7.65''$$

Since 1.5 inches of superelevation is gained in second,

∴ Time required to gain 7.65'' of superelevation

$$= \frac{7.65}{1.5} = 5.03 \text{ sec}$$

(b) Length of the transition curve = $44 \times 5.03 = 221 \text{ ft}$ (approx.).

$$(c) \text{ Shift} = \frac{(221)^2}{24 \times 1146} = 1.775 \text{ ft.}$$

$$(d) \text{ Tangent length} = (R+S) \tan 20^\circ 15' + \frac{221}{2}$$

$$= (1146 + 1.775) \times 0.368 + 110.5$$

$$= 532.5 \text{ ft.}$$

$$(e) \text{ Length of circular curve} = \frac{\pi \times 1146 \times 40.5}{180} - 221$$

$$= 591 \text{ ft}$$

8-8 Vertical Curve : It is either the arc of a parabola or the arc of a circle. Generally, the arc of a parabola is used. It is set in railways and highways where horizontal portion meets an ascending or descending grade and also where the ascending and descending grade meet on a *summit* or a *sag*. There are different types of vertical curves according to different ground formations as shown in the Fig. 8.18.

Gradient can be expressed in two ways.

(1) As percentage, i.e., 1%, 2%, etc.

(2) As 1 in 80, 1 in 80 where 60 and 80 are the horizontal distances in feet corresponding to 1 foot rise or fall.

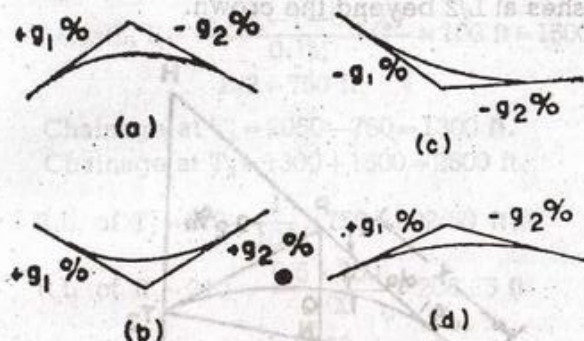


FIG. 8.18

Ascending or upgrade and descending or downgrade are expressed by + sign and - sign respectively. Generally in our country for first class railways, the gradient is 0.1% at summit and 0.05% at sag per 100 feet station. For second class railways, twice of these values are to be taken.

Length of Vertical Curves : It is obtained by dividing the algebraic difference of the two grades by the rate of change of grade.

L = Length of vertical curve (projected length)

$$= \frac{\text{Algebraic difference of the two grades}}{\text{Rate of change of grade}}$$

An ascending gradient of 1 in 60 is joined to a descending gradient of 1 in 80 in a track with a rate of change of 0.1%, then

$$L = \frac{\frac{1 \times 100}{60} + \frac{1 \times 100}{80}}{1} \times 100$$

$$= \frac{(1.667 + 1.250)10}{1} \times 100 = 2.917 \times 1000$$

$$= 2917 \text{ ft}$$

Notes: L is expressed in nearest complete chains and the vertical curve is to start at L/2 chains from the crown and it finishes at L/2 beyond the crown.

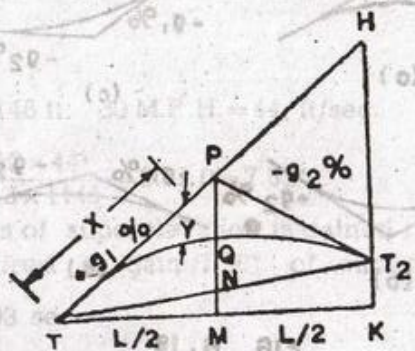


FIG. 8.19

Procedure to locate a vertical curve: Assuming the equation of the vertical curve as a parabola (Fig. 8.19), the offsets from the tangent are proportional to the square of the distance along the tangent.

$$\frac{XY}{PQ} = \frac{T_1X^2}{T_1P^2} \quad \therefore XY = \frac{PQ \cdot T_1X^2}{T_1P^2}$$

The ordinates can be calculated by applying the above equation. The ordinates must be parallel to the main axis of the parabola to obtain a true curve but for simplicity of calculation, they are made vertical.

N is the middle point of T_1T_2 line and $PN \parallel HK$

$$\therefore PT_1 = PH, \text{ and } PQ = \frac{1}{4}HT_2$$

Example: Calculate the necessary data to set out a vertical curve which connects an upgrade of 1% with a downgrade of 0.5%. Chainage at the point of intersection is 20.50 chains and Reduced Level is 210 feet. Take the rate of change in gradient, 0.1% per chain of 100 ft length.

Solution:

In Fig. 8.19,

$$\text{Length, } L = \frac{+1\% - (-0.5\%)}{0.1\%} \times 100 \text{ ft} = 1500 \text{ ft.}$$

$$\therefore L/2 = 750 \text{ ft.}$$

$$\text{Chainage at } T_1 = 2050 - 750 = 1300 \text{ ft.}$$

$$\text{Chainage at } T_2 = 1300 + 1500 = 2800 \text{ ft.}$$

$$\text{R.L. of } T_1 = 210 - \frac{1}{100} \times 750 = 202.50 \text{ ft.}$$

$$\text{R.L. of } T_2 = 210 - \frac{0.5 \times 750}{100} = 206.25 \text{ ft.}$$

$$\text{R.L. of } H = 210 + \frac{1 \times 750}{100} = 217.50 \text{ ft.}$$

$$\begin{aligned} HT_2 &= \text{R.L. of } H - \text{R.L. of } T_2 = 217.50 - 206.25 \\ &= 11.25 \text{ ft.} \end{aligned}$$

$$PQ = \frac{1}{4}HT_2 = \frac{1}{4} \times 11.25 = 2.81 \text{ ft.}$$

Taking 100 ft intervals,

$$y = \frac{PQx^2}{T_1P^2}$$

$$\therefore y_1 = 2.81 \times \frac{2^2}{7.5^2} = 0.05 \text{ ft.}$$

$$y_2 = 2.81 \times \frac{2^2}{7.5^2} = 0.20 \text{ ft.}$$

$$y_3 = 2.81 \times \frac{3^2}{7.5^2} = 0.45 \text{ ft.}$$

$$y_4 = 2.81 \times \frac{4^2}{7.5^2} = 0.80 \text{ ft.}$$

$$y_5 = \frac{2.81 \times 5^2}{7.5^2} = 1.25 \text{ ft.}$$

$$y_6 = \frac{2.81 \times 6^2}{7.5^2} = 1.80 \text{ ft.}$$

$$y_7 = \frac{2.81 \times 7^2}{7.5^2} = 2.45 \text{ ft.}$$

$$y_8 = \frac{2.81 \times 7.5^2}{7.5^2} = 2.81 \text{ ft.}$$

Chart

Chainage	R.L. on the Tangent	Offset	R.L. on the Curve
1300 ft.	202.50	0.00 ft.	202.50
1400 ft.	203.50	0.05	203.45
1500 ft.	204.50	0.20	204.30
1600 ft.	205.50	0.45	205.05
1700 ft.	206.50	0.80	205.70
1800 ft.	207.50	1.25	206.25
1900 ft.	208.50	1.80	206.70
2000 ft.	209.50	2.45	207.05
2150 ft.	210.60	2.91	207.77

Calculation for the other half is similar.

EXERCISE

1. Examine the following statements and write whether they are true or false :
 - (a) A curve may be defined as an arc of a circle or a parabola.
 - (b) The radius of a 1° circular curve is 5730 ft.
 - (c) The purpose of a transition curve is to minimise the speed of the vehicle on the track.
 - (d) The difference in level between two rails is known as superelevation.
 - (e) Superelevation varies directly as the square of the speed of the vehicle.
 - (f) The superelevation is maximum at the tangent point.
 - (g) For a cubic parabola, the curvature is proportional to the length.
 - (h) The shift is directly proportional to the radius of the curve.
 - (i) A compound curve is one where the curve bends in two opposite directions.
 - (j) The rate of change of grade is the algebraic difference of the grades at two points on the curve divided by the length of the curve.

- (k) When a horizontal grade meets a down grade, there will be a sag curve.
2. What are the different types of curves that are generally used in Railways and Highways? Explain with sketches.
3. What are the methods of curve ranging? Explain the angular method by two theodolites.
4. What is a transition curve? What are its uses? Derive an expression for the ideal easement curve.
5. What is shift? Deduce an expression for the same.
6. What is a vertical curve? What are its uses?
7. Calculate the ordinates at 50 ft. intervals for a circular curve having a long chord of 400 ft. and versed sine of 16 feet.
Ans. 16 ft., 15 ft., 12 ft., 7 ft.
8. Calculate the ordinates at 25 ft intervals for a circular curve, given the length of the chord 200 ft. and the radius 600 ft.
Ans. 7.8 ft., 6.3 ft., 3.8 ft., 0 ft.
9. Determine the offsets to be set out at half chain intervals along the tangents to locate a 20 chain curve.
Ans. 0.26 ft., 2.5 ft., 5.62 ft., 10.03 ft., 2.63 ft., 40.41 ft., 63.51 ft.
10. In aligning an irrigation canal a turn is taken to the right after reaching a certain point through an angle of $32^\circ 40'$. The tangent point is at 29625 ft. It is proposed to join the tangents by a 2° curve. Calculate, (a) chord length, (b) middle ordinate, and (c) depth of the curve showing a table of angles and distances upto half ways.
Ans. 1611.4 ft., 114.66 ft., 120.5 ft.
11. Two roads intersecting at deflection angle $33^\circ 40'$ are to be connected with a 4° circular curve by means of

equal transition curves at each end. The speed of the train is 40 MPH and the rate of superelevation, 1.6" per sec and gauge length, 4.71 ft. Calculate (a) tangent distance (b) length of transition curve. (c) shift (d) deflection angles.

Ans. 513.6 ft., 160 ft., 0.73 ft.

12. A circular railway curve is to be set out with a radius of 1000 ft connecting two straights which intersect at 150° . The chainage at the first tangent point is 8642 ft. Calculate the chainage at the intersection point and the second tangent point.

Ans. 3909.9 ft., 9165.33 ft.

13. The straight EF and FG intersect at an inaccessible point. A straight line AB intersects them making $\angle FAB = 107^\circ 35'$ and $\angle GBA = 135^\circ 5'$. The length of AB is 404 ft. If the radius of the curve is 2000 ft and the chainage at A is 3558 ft, calculate necessary data to set out the curve with 100 ft chord length.

Ans. AF = 314 ft, BF = 430.2 ft., Tangent length = 1700.8 ft. length of curve = 4064.4 ft.

14. Calculate the reduced levels of the various points on a vertical curve connecting two uniform grades of 1.5% and 0.7%. The chainage and the reduced level of the point of intersection are 1400 ft, and 850.75 ft respectively. Take the rate of change of grade as 0.1%

Ans. Length of the curve = 1200 ft, chainage at the beginning = 800 ft. and at the end = 2000 ft R.L. of the 1st tangent point = 847.75 ft. and at last tangent point = 848.55 ft.

15. Two straight roads are to be connected by a vertical curve. Calculate the values of offsets to the curve for setting out the curve if chainage at the intersection point is 1540 ft. A descending of 0.6% will meet an ascending grade of 0.8%. Take R.L. of intersection point as 152.50 and the rate of change of grade 0.05% per chain.

Ans. Length = 2300 ft. Chainage at first and last tangent points = 1.4 chains, 29.40 chains.

16. Two roads meet at an angle of 150° . It is decided to join them by 4° circular curve. The chainage at the starting tangent point is 154.00 chains. It is required to locate a point P on the curve at a chainage of 157.30 chains. Calculate the necessary data and explain the procedure with a neat sketch to locate P only on the curve in the field by the help of a chain, a tape, ranging rods and a theodolite (Rankin's method of tangential angles is not permitted).

17. Two broad gauge rail roads intersecting at an angle of $144^\circ 00'$ were connected with a 5° circular curve by means of two equal transition curves at each end. The combined length of the curve was 1010 ft, the rate of superelevation, 1.2 inches per sec., gauge length, 5'-6". What were the length of the transition curve, and superelevation? Ans. 290 ft., 6 inches.

18. A horizontal grade meets a down grade of 1 in 84. It is required to insert a vertical curve between them for a highway for a maximum speed of vehicles 30 miles per hour. The R.L. of the point of intersection is 760.50 ft at a chainage of 232.50 chains and the rate of change grade is 0.1% per 100 ft station. Calculate the reduced levels of different points on the curve taking 150 ft station along the tangent. Ans. L = 11.9 chains,

19. In a road project, the alignment was deflected from its original direction due to an obstruction at a station A and the measured length of the deflected section AC was 8 chains. It was deflected at C at an angle of $52^\circ 30'$ so as to reach the original deflection of the alignment at B. The measured length of BC was again 8 chains. It is intended to connect AC and BC by a simple curve with tangent points at A and B. Calculate the radius and the degree of the curve. Ans. 1620 ft, 3.54°

CHAPTER 9
GEODETIC SURVEYING

9-1 Definition : Geodesy is the science which deals with the investigations of the shape and dimensions of the earth's surface. And geodetic surveying is that type of surveying which determines precisely the relative positions of a system of widely separated points with their lengths and directions on the surface of the earth. It differs from plane surveying in which the curvature of the earth is not considered. This is also known as *trigonometrical* or *triangulation* surveying. For a large and extensive area, this type of surveying is preferred. The relative positions of different points are determined in terms of azimuth and the lengths of the lines joining them, and their absolute positions in terms of latitude, longitude and the elevation above the mean sea-level. Since this type of surveying is very extensive and expensive, this is generally conducted by the state agency such as the Geodetic Survey of Bangladesh.

The methods generally applied are either triangulation or precise traversing. Triangulation is the common and accurate method while precise traversing is followed when the country is densely wooded.

Triangulation : In triangulation, a net work of well defined triangles are formed. The vertices of the triangles are known as *triangulation stations* and the whole figure is called a *triangulation system*. It involves a minimum distance measurement and a maximum of angular measurements. The measured distances are called *base lines*. In a triangulation system, generally, one line known as *base* is measured very accurately and all the angles are measured carefully. While the lengths of the remaining sides are computed on the basis of trigonometrical rules that if three angles and one side of a triangle are known then by

applying sine rule, the length of the remaining sides can be computed.

9-2 Forms of Triangulation : The main forms triangulation are as follows :

Simple Chain of Triangles : This is very rapid but less accurate system generally applied for a narrow strip of land for less important works. This is a less expensive method.

Dual Triangles ; This is very reliable and more accurate and is generally used for important works.

Quadrilaterals : This system costs much but very accurate. This is generally used where a very high degree of accuracy is needed.

Polygon System : This system is more economical and covers, a greater area. This is generally used for a topographical survey.

Gridiron System : This system is used for extensive areas

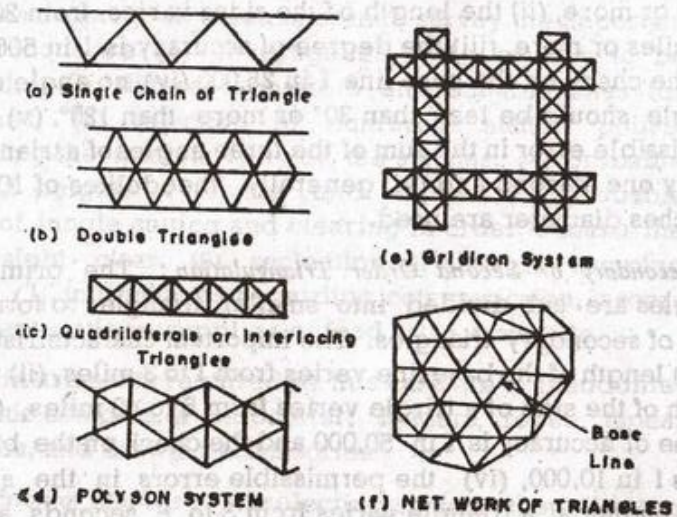


FIG. 9.1

where two series of chains of triangles are formed, One series is laid out along the north-south direction while the other cuts it at right angles and the enclosed area is field up with a net-work of smaller triangles. This is very accurate but very expensive.

Net-work of Triangles: This is generally employed for the survey of a whole country. In this system the whole area is covered with a net-work of triangles extending outwards in all possible directions from the base line.

9-3 Triangulation System: Triangulation system may be classified into (a) primary triangulation, (b) secondary triangulation and (c) tertiary triangulation. This classification is based on the degree of accuracy required and the volume of the work.

Primary or First Order Triangulation: The first chain of triangles which covers the whole country is known as primary triangulation. The important characteristics in this class are (i) the length of the base line varies from 3 to 10 miles or more, (ii) the length of the sides varies from 20 to 100 miles or more, (iii) the degree of accuracy is 1 in 500.00 and the check on the base line 1 in 25,000 (iv) no angle of a triangle should be less than 30° or more than 120° , (v) the permissible error in the sum of the three angles of a triangle is only one second and (vi) generally, theodolites of 10 to 12 inches diameter are used.

Secondary or Second Order Triangulation: The primary triangles are sub-divided into smaller triangles to form a chain of secondary triangles. The important characteristics are (i) length of the base line varies from 1 to 3 miles, (ii) the length of the side of a triangle varies from 5 to 40 miles, (iii) degree of accuracy is 1 in 50,000 and the check on the base line is 1 in 10,000, (iv) the permissible errors in the sum of the angles of a triangle varies from 3 to 8 seconds and (v) the angles are measured with 8 to 10 inches theodolites.

Tertiary or Third Order Triangulation: The secondary triangles are again subdivided to form tertiary triangles. The essential characteristics are (i) the length of the base line varies from $\frac{1}{2}$ to $1\frac{1}{2}$ miles, (ii) the length of the sides varies from less than a mile to 6 miles, (iii) the degree of accuracy is more than 1 in 5,000 and the check on the base line 1 in 5,000, (iv) the permissible error in the sum of the angles of a triangle is 10 to 15 seconds and (v) the theodolite is generally 5 to 8 inches.

9-4 Field Procedure and Office Works: The following steps are generally considered to conduct a triangulation survey: (i) Reconnaissance, (ii) Selection of base lines, check lines and station points, (iii) Erection of towers and signals, (iv) Measurement of horizontal angles, (v) Computations of the observed angles, including adjustments, (vi) Computation of the length of the sides of each triangle, (vii) Computation of latitudes and longitudes of the stations.

Reconnaissance: In reconnaissance survey the important considerations are (1) investigation of the country to be surveyed, (2) different possible and suitable sites for base lines, (3) selection of numerous station points considering the intervisibility of the stations, (4) probable sites and heights of towers and signals, (5) probable volume of jungle cutting and clearing in order to make the line of sight clear, (6) selection of suitable camping ground, (7) informations regarding communication, access to different stations, supplies of food, staffs, water etc.

The instruments required in this case are a theodolite, a prismatic compass, a barometer, ladders, ropes, tapes, binoculars, and drawing accessories.

Selection of Stations: In selecting stations, the following points should be considered carefully:

- (1) Each station should be visible from the adjacent

stations. For this purpose the most elevated places such as hill tops or mountaing are selected.

(2) The triangles formed by the stations should be well-conditioned.

(3) They should be easily approachable.

(4) The length of sight should not be too large or too small.

(5) Permanency of stations.

(6) Cost of cutting and clearing of obstructions and the construction of towers should be minimum.

(7) *Marking of Station*: The station are marked with copper or bronze tablet on which the name and the year in which it is set, are mentioned, It is set with reference to two reference marks. Somtimes, an iron post or a masonry pillar is used as station-mark.

Height or station or towers or scaffolds: When the distance between two stations is large and the difference in altitude between them is small, it is essential to raise both the instruments and the signals to overcome the curvature of the earth and to clear all the intervening obstructions. in such a case, a structure known as a scaffold or tower is erected over a station for the support of the instrument and the observing party. The height of the towers or scaffolds or the signals depends upon the distance between the stations, their altitudes and the profile of the intervening ground.

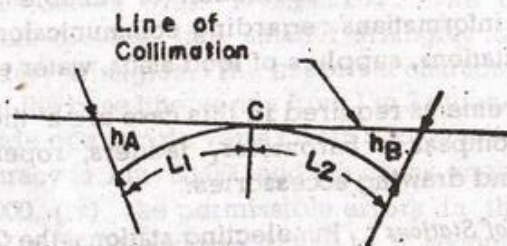


FIG. 9.2

In Fig. 9.2, A and B are the two stations on which towers h_A and h_B are to be erected. The distance between A and B is $L=L_1+L_2$. The height of the tower which is just visible at a given distance L_1 from the tangent point C, is calculated from the expression,

$$h_A(1-2m) \times \frac{L_1^2}{2R}$$

where R = Mean radius of the earth.

m = coefficient of refraction

= 0.07 on land and 0.08 on water

When L_1 and R are in miles and m is 0.07, the result of h_A will be in ft.

$$\therefore h_A = 0.574L_1^2$$

$$\text{Similarly, } h_B = 0.574L_2^2$$

Example: Two stations P and Q in a triangulation survey have elevations of 970 and 900 respectively. The distance between them is 30 miles. Assuming the intervening ground to be of uniform level of 810 ft, find the height of the scaffold at Q so that the line of sight may be 10 ft clear of earth's surface.

Solution:

Minimum ht. of the line of sight = $810 + 10 = 820$ ft.

$$h_P = 970 - 800 = 150 \text{ ft.}$$

$$L_1 = \sqrt{\frac{h}{0.574}} = \sqrt{\frac{150}{0.574}} = 14.50 \text{ miles.}$$

$$L_2 = L - L_1 = 30 - 14.52 = 15.48 \text{ miles.}$$

$$h_Q = 0.574 \times (15.48)^2 = 137.8 \text{ ft.}$$

The elevation of the line of sight at Q = $820 + 137.8 = 957.8$ ft.

The ht. of the scaffold = $957.8 - 900 = 57.8$ ft.

Example: Two stations P and Q 60 miles apart, are 600 ft and 2,000 ft above datum respectively. The two peaks R and S lying in between them have elevations of 600 ft and 1000 ft respectively. The distances PR and PS are 20

and 40 miles respectively. Is Q visible from P? If not calculate the height of the tower at Q, assuming P as the station on the ground.

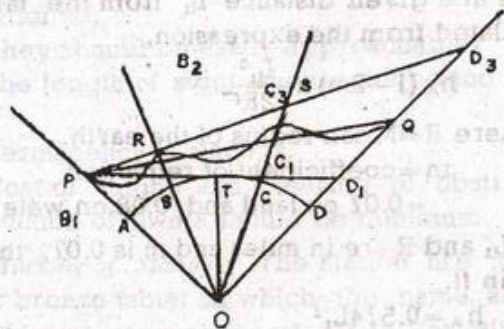


FIG. 9.3

Solution :

$$PT = \sqrt{\frac{h}{0.574}} = \sqrt{\frac{600}{0.574}} = 32.4 \text{ miles}$$

$$BT = 32.4 - 20 = 1.24 \text{ miles.}$$

$$TC = 40 - 32.4 = 7.6 \text{ miles.}$$

$$TD = 60 - 32.4 = 27.6 \text{ miles.}$$

The corresponding heights BB_1 , CC_1 , and DD_1 are

$$BB_1 = 0.574 \times (1.24)^2 = 88.2 \text{ ft.}$$

$$CC_1 = 0.574 \times (7.6)^2 = 33.2 \text{ ft.}$$

$$DD_1 = 0.574 \times (27.6)^2 = 44 \text{ ft.}$$

To test whether the line of sight PQ will clear peaks R and S

$$\frac{B_1B_2}{D_1Q} = \frac{PB_1}{PD_1} = \frac{20}{60} = \frac{C_1C_2}{D_1Q} = \frac{AC_1}{PD_1} = \frac{40}{60}$$

$$\text{Again } D_1Q = DQ - DD_1 = 2000 - 440 = 1560 \text{ ft.}$$

$$B_1B_2 = \frac{20}{60} \times 1560 = 520 \text{ ft and } C_1C_2 = \frac{40}{60} \times 1560 =$$

$$1040 \text{ ft.}$$

The elevation of the line of sight at $BB_1 + B_1B_2$

$$= 88.2 + 520 = 608.2 \text{ ft}$$

$$\begin{aligned} \text{The elevation of the line of sight at } S &= CC_1 + C_1C_2 \\ &= 33.2 + 1040 = 1073.2 \text{ ft.} \end{aligned}$$

The given elevation of R = 600 ft and that of S = 1200 ft. So, Q is not visible from B. S fails to clear by $1200 - 1073.2 = 126.8 \text{ ft.}$

The line of sight should be raised at Q by the amount:

$$QD = \frac{PQ}{PS} \times c_2c_3 = \frac{60}{40} \times 126.8 = 190 \text{ ft.}$$

So, the minimum ht. of the tower at Q = 190 ft.

Example : P and Q are the two stations on the surface of earth and their elevations are 8,500 ft and 6,100 ft respectively. The horizontal distance of Q from P at the level of Q is 10 miles. Calculate the horizontal distance from Q to P at the level of P. Take the diameter of the earth at Q to be 7,920 miles.

Solution :

$$\text{Height of station P over Q} = 8,500 - 6,100 = 2,400 \text{ ft.}$$

Two arcs are drawn through P and Q with the centre of the earth as a common centre. R_1 and R_2 are the radii of the arcs passing through Q and P respectively.

$$R_1 = 3,960 \text{ miles.}$$

$$R_2 = \left(3960 \times \frac{2400}{3208}\right) \text{ miles.}$$

Since the concentric arcs are proportional to their radii

$$\frac{\text{Distance PQ at the level of P}}{\text{Distance PQ at the level of Q}} = \frac{R_2}{R_1} = \frac{3960 \times \frac{2400}{5280}}{3960}$$

$$\text{Distance PQ at the level of P} = 10 \times \frac{3960 \times \frac{2400}{5280}}{3960}$$

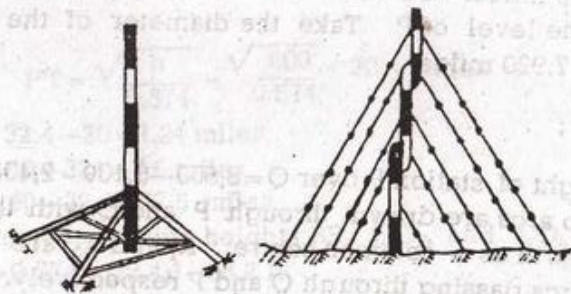
$$= 10 \left(1 + \frac{2400}{5280 \times 3960}\right)$$

$$= 10 + \frac{24000}{5280 \times 3930}$$

$$= 10 \text{ miles} + 6.05 \text{ ft.}$$

Signals: These are devices erected on stations to locate the exact position of the stations. Signals may be of different types such as daylight signals, sunlight signals and night signals.

Daylight Signals: They are also known as *non-luminous* or *opaque* signals. Their various forms may be of poles, targets and tin cones. These are used for direct lights of less than 4 miles.

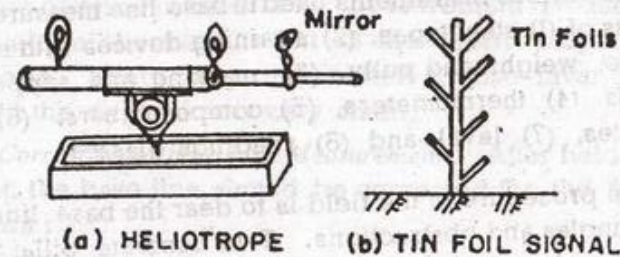


(a) TRIPOID SIGNAL

(b) GUYED MAST SIGNAL

FIG. 9.4

Sum Signalas: These are mechanical devices by which sun rays are reflected to the observer. They are generally used when the distance between two stations is more than 20 miles. The common types are *heliotropes*, *heliographs* and *tin-foils*. A heliotope is a plane mirror with some device for pointing it in such a manner that the reflected sun light will reach the observer's eye of distant station.



(a) HELIOTROPE

(b) TIN FOIL SIGNAL

FIG. 9.5

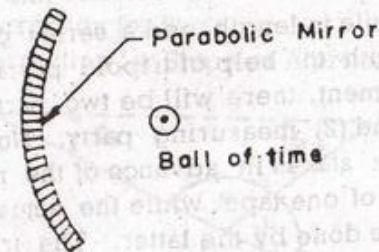


FIG. 9.6

Night Signals: They are used for night observation. The common types are *large kerosine lamp*, *acetylene gas lamp*, *electric lamp* and *Drummond's light*. A Drummond's light consists of small ball of lime placed in the focus of a parabolic reflector which emits white light when it gets heated by oxyacetylene flame. Even stations at a distance of 60 miles are visible in foggy nights.

Base Line Measurement: Base line should be measured with utmost care because the accuracy of the other sides of the triangulation system depends upon it. The main considerations to select a base line are (1) the ground should be fairly levelled and free from sudden undulations, (2) the whole length of the base line should be free from obstruction, (3) the ends of the line should be so located that the base may be connected with the main scheme by a few well-conditioned triangles, (4) the ends must be visible from the neighbouring stations.

The various instruments used in base line measurement consists of (1) steel tapes, (2) straining devices with spring balance, weight and pulley, (3) marking and supporting tripods, (4) thermometers, (5) compound bars, (6) theodolites, (7) levels and (8) readings glasses.

The procedure in the field is to clear the base line site of all jungles and obstructions. Two concrete pillars are built at each end of the base line for marking the ends permanently. The line is then divided into small section of about $\frac{1}{2}$ to $\frac{3}{4}$ mile in length and a series of line—marks are established with the help of tripod posts. In conducting this measurement, there will be two parties: (1) setting out party and (2) measuring party. Now the setting out party will put stakes in advance of the measurement at correct intervals of one tape, while the actual measurement of the line will be done by the latter. The tripods are set:

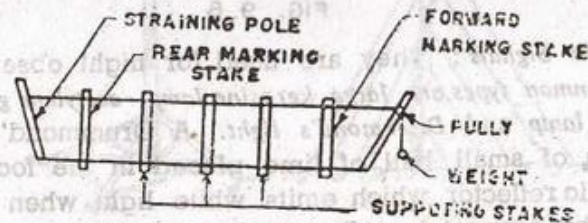


FIG. 9.7

at correct intervals by means of tapes and at the same time the pull should be by a spiring balance. Each tripod has its head levelled by a levelling instrument and adjusted into alignment at the proper distances. Setting of tripods and measurement of base line go on simultaneously and tripods from the rear finished length are carried forward and the process is repeated until the end of the section is reached. The section is again measured in the reverse direction as final check, otherwise, the mean length is taken. The

source of error in base line measurement is due to the difficulty of the measurement of the actual temperature of the tape. That is why, it is better to use invar tapes in which the expansion is very small.

Correction to Base Line Measurement: After field measurement, the base line should be corrected for the following factors:

- (1) Correction for Temperature
- (2) Correction for Pull
- (3) Correction for Sag
- (4) Correction for Slope or Vertical Alignment

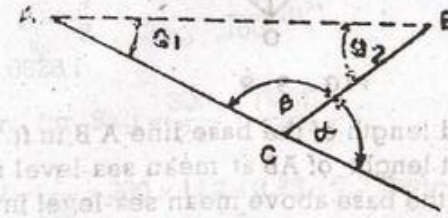


FIG. 9.8

- (5) Correction for Horizontal Alignment
- (6) Reduction to Mean Sea level.

Correction for temperature, Pull, Sag and Slope have been discussed in chapter 2.

Correction for Horizontal Alignment: When a base line is not possible to set out in one continuous straight line, it necessary to deviate it. It is then called a *broken base*.

In Fig. 9.8, $AB = AC \cos \theta_1 + BC \cos \theta_2$

In this case, θ_1 and θ_2 are measured by a theodolite.

The correction to be subtracted from the apparent length of line is $AC(1 - \cos \theta_1) + BC(1 - \cos \theta_2)$.

When θ_1 and θ_2 are not possible to measure due to mutual invisibility of A and B, then the angle β can be measured.

$$AB = \sqrt{AC^2 + BC^2 - 2AC \cdot BC \cdot \cos \beta}$$

Reduction to Mean Sea-level : If the length of the base line be reduced to its equivalent length at mean sea-level, the computed lengths of all other lines of the triangulation system will correspond to this level. And it helps in the comparison of all bases. In Fig. 9.9.

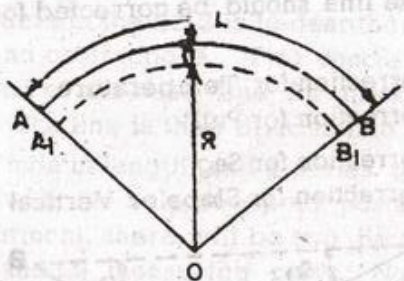


FIG. 9.9

L = Measured length of the base line AB in ft.
 l = Equivalent length of AB at mean sea-level in ft.
 h = Height of the base above mean sea-level in ft.
 R = Radius of the earth in ft.

$$\frac{l}{L} = \frac{R}{R+h}$$

$$\frac{l}{L} = \frac{1}{1 + \frac{h}{R}} = \left(1 + \frac{h}{R}\right)^{-1} = \left(1 - \frac{h}{R} + \frac{h^2}{R^2} - \frac{h^3}{R^3} + \dots\right)$$

Since h is very small in comparison to R

$$\therefore \frac{l}{L} = \left(1 - \frac{h}{R}\right), \text{ or } l = L - \frac{Lh}{R}$$

$$\therefore \text{Correction} = L - l = \frac{Lh}{R} \text{ (-ve)}$$

Corrected base line

= Measured base line + Temp. Corr. + Pull Corr. - Sag
 Corr. - slope Corr. = Horiz. algin. corr. - Red. to MSL
 Corr.

Example : A base line 2 miles long was measured with a tape of 300 ft in length which was standardized under no

pull at 60°F. The tape is $\frac{1}{8}$ " wide and $\frac{1}{20}$ " thick. If the line was measured at a temperature of 80°F and the tape being stressed with a pull of 50 lbs, calculate the corrected length of the base line. Take co-efficient of thermal expansion of steel = 6.5×10^{-6} per deg F, $E = 29 \times 10^6$ psi, Wt. of steel = 0.28 lb. per in³, $R = 4000$ miles, Ht. of base line above mean sea-level = 400 ft.

Solution :

$$\text{Correction for pull} = \frac{50 \times 300 \times 160}{1 \times 29 \times 10^6}$$

$$\begin{aligned} \text{(Area, } A &= \frac{1}{8} \times \frac{1}{20} = \frac{1}{160} \text{ in}^2) \\ &= 0.0028 \text{ ft.} \end{aligned}$$

$$\text{Correction for Sag} = \frac{300}{24} \times \frac{(6.3)^2}{50^2}$$

$$(W = \frac{1}{160} \times 300 \times 112 \times 0.28 = 6.3 \text{ lbs}) = 0.1984 \text{ ft}$$

$$\text{Correction for Temperature} = 300 \times 6.5 \times 10^{-6} \times 20 = 0.039 \text{ ft.}$$

$$\begin{aligned} \text{Reduction to mean sea-level correction} &= 300 \times \frac{400}{4000 \times 5280} \\ &= 0.0057 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{Corrected length for 300 ft.} &= 300 + 0.0828 - 0.1984 + 0.039 \\ &\quad - 0.0057 = 299.9177 \text{ ft.} \end{aligned}$$

$$\text{Correction for 300 ft} = 300 - 299.9177 = 0.0823 \text{ ft.}$$

$$\text{Correction for 2 miles} = \frac{0.0823 \times 2 \times 5280}{300} = 2.898 \text{ ft}$$

$$\text{Corrected length} = 2 \text{ miles} - 2.898 \text{ ft.} = 1 \text{ mile} + 5277.102 \text{ ft.}$$

Example : In measuring a base line a steel tape of 400 ft. long was used. It was standardized under no pull at 60°F. The cross-section of the tape was $\frac{1}{8} \times \frac{1}{20}$ " and the measurement was taken at a mean temperature of 85°F. The measured length was 1000 ft with the following slopes.

Distance	Slope
0—400 ft	2°30'
400—800 ft	3°18'
800—1000 ft	1°42'

Calculate the actual length of the line if the coefficient of expansion is 6.5×10^{-6} per deg F.

Solution :

$$\begin{aligned} \text{Correction for temperature} &= 1000 \times 6.5 \times 10^{-6} \times 25 \\ &= 0.1625 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{Correction for slope} &= 400(1 - \cos 2^\circ 30') + 400(1 - \cos 3^\circ 18') \\ &\quad + 200(1 - \cos 1^\circ 42') \\ &= 0.380 + 0.664 + 0.088 = 1.032 \text{ ft} \end{aligned}$$

$$\text{Actual length} = 1000 - 0.1625 - 1.032 = 998.8055 \text{ ft}$$

Measurement of Horizontal and Vertical Angles : In measuring angles generally two types of theodolites are used in triangulation, viz. (1) the repeating theodolite and (2) the direct theodolite. The former one has two vertical axes and is provided with two or more verniers which can read upto 5 seconds. The latter one has one vertical axes and is provided with or these micrometer microscopes instead of verniers.

The horizontal angles are measured by two methods, viz. (1) repetition method and (2) the direct or reiteration method. Both the methods have already been described in chapter 3. For the first case, repeating theodolite is used and for the second case, the direct one is used.

The vertical angles are measured face left and face right as discussed in chapter 3.

Sometimes it is impossible to set up the instrument exactly over a station. In that case a subsidiary station is chosen very near the main station. This subsidiary station is called a satellite station and the distance between the main station and the satellite station is known as eccentric distance which is determined by trigonometrical levelling or

by triangulation. By placing the instrument over the satellite station, all the angles are measured with the same accuracy as would have been taken from the main station. These measured angles however, will not be the same as those when measured from the main station. Therefore, they are reduced to the centre. This operation is known as reduction to centre,

Extension of a Base Line : In primary or first order triangulation the sides of the main triangles are 30 to 60 miles or more in length but it is not often possible to measure directly a base not longer than 6 to 10 miles. The common practice is to measure a short base line and then extend it by means of well conditioned triangles. There are various methods of which the following two are more practical.

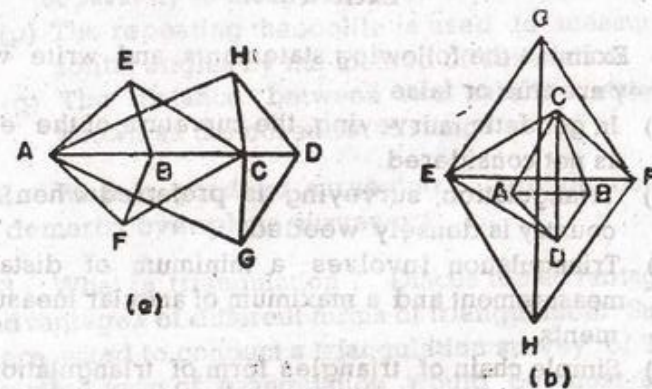


FIG. 9.10

(1) Base AB is to be extended upto C (Fig. 9.10 a). Two points E & F are chosen on either side of AB in such a way that they are clearly visible from A & B and from well-conditioned triangles. The station is now fixed on AB prolonged by a theodolite placed on A or B in such a way that E & F are visible from C and the triangle ACE and ACF are well-conditioned. By setting theodolite on A, B, C, E

and F the angles ABE, ABF, BCE, BCF, ACE and ACF are measured. From these observed angles, the length BC is calculated in four different ways and the mean is taken. By following the same procedure the base may be extended upto D.

(2) In Fig. 9.10 (b), the base AB is to be extended. Two suitable stations C & D are selected on either side of AB. Two values of CD can be computed from the known length of AB and the observed angles of the triangles ABC, ABD, and BCD. The mean value is taken. The new base, CD is then enlarged to EF by selecting two suitable stations E and F with the help of triangles ECD and ECD. By repeating the same procedure, the base EF can be extended to GH.

Exercise

1. Examine the following statements and write whether they are true or false :

- In geodetic surveying, the curvature of the earth is not considered.
- Triangulation surveying is preferred when the country is densely wooded.
- Triangulation involves a minimum of distance measurement and a maximum of angular measurements.
- Simple chain of triangles form of triangulation is generally adopted for an extensive area.
- When the survey of the whole country is needed, double triangles form of triangulation is most suitable.
- The accuracy in the first order triangulation is 1 in 50,000.
- The permissible error in the sum of the angles of a triangle in the third order triangulation is 10 to 15 seconds.

- Each station in a triangulation should be visible from the adjacent stations:
 - Scaffolds are needed in triangulation to overcome the curvature of the earth and to clear all the intervening obstructions between stations.
 - The height of the scaffold will be large if the value of the co-efficient of refraction is high.
 - A signal is a device erected on stations to locate the exact position of stations.
 - The entire length of the base line should be free from obstructions.
 - The correction for pull is negative.
 - The correction for reduction to mean sea-level is additive.
 - It is necessary to reduce each triangulation line separately to mean sea-level.
 - The repeating theodolite is used to measure horizontal angles by the method of reiteration.
 - The distance between two satellite stations is known as the eccentric distance.
- What is geodetic surveying? Explain its merits and demerits over plane surveying.
- What is triangulation? Discuss the advantages and disadvantages of different forms of triangulation. Suppose, you are asked to conduct a triangulation survey of Bangladesh, what form of triangulation would be most suitable and why?
- Enumerate the field works and office works necessary for a triangulation survey with a complete list of equipment required.
- Discuss different orders of triangulation with respect to (a) length of the base line, (b) length of the side of the triangles, (c) degree of accuracy on base line and also on

checklines, (d) the permissible error in the sum of the angles of the triangles.

6. What are the points to be noted in the selection of stations for triangulation survey?

7. What is a signal? Why is it used? Describe with neat sketches the different types of signals that are used in a trigonometrical survey.

8. (a) Explain the procedure of measuring a base line with a neat sketch in a triangulation survey.

(b) Discuss the corrections that are to be applied to the measurement of the length of a base line.

9. (a) Discuss briefly the operation of measuring vertical and horizontal angles in a triangulation.

(b) Why a base line is prolonged? How will you extent it?

10. In measuring a base line, two station P and Q, 40 miles apart were selected. The elevations of P and Q were 900 ft, and 1500 ft. respectively. The intervening ground has an average elevation of 800 ft. What should be the height of the scaffold at Q so that the line of sight may be 10 ft clear of the earth's surface? Ans. 105 ft.

11. The stations P and Q 75 miles apart have elevation of 420 ft and 1530 ft respectively. There is a peak of 530 ft high at R, 25 miles from Q. This peak R obstructs the line of sight from Q. Calculate the height of the scaffold at Q so that the line of sight will clear the peak at R by 40 ft.

12. Three peaks P, Q and R are in a straight line. The distances PQ and QR are 9 miles and 14.5 miles respectively. The elevations of P, Q and R are 650 ft, 600 ft and 700 ft respectively. Calculate the height of the tower to be erected on R, the top of which will be just visible from P.

13. Four hill P, Q, R and S are in a straight line and their elevations are 900 ft, 870 ft, 1100 ft and respectively. The distances PQ, QR and RS are 10 miles, 20 miles and 12 miles respectively. Calculate the height of the scaffolds on P and Q to sight over R and S with a clearance of 8 ft. (Two scaffolds are to be of same height).

14. A line 2 miles long, is measured with a tape of length 300 ft which is standardized under no pull at 60°F. and the tape is stressed with a pull of 50 lbs. find the corrections on the total length. Co-efficient of expansion $= 6.5 \times 10^{-6}$, wt. of 1 cubic inch of steel of steel = 0.28 lb; $E = 29 \times 10^6$ p.s.i.

Ans. Sag = -5.985 ft, Pull = + 2.914 ft, Temp = + 1.37 ft.
Total = -3.041 ft.

15. A base line $1\frac{1}{2}$ miles long was measured with a tape of length 300 ft. This tape was suspended in three equal spans of 100 ft each, to measure the line. The tape was stressed with a pull of 25 lbs. The tape was standardized under a pull of 10 lbs at 54°F. The cross sectional area of the tape was 0.006 sq. in. and the mean temperature during measurement was 80°F. Calculate the correct length of the base line, given that the co. eff. of expansion $= 6.5 \times 10^{-6}$, $E = 30 \times 10^6$ p.s.i.

Ans. 7919.338 ft.

16. A base was deflected from the base line proper at a station P and the measured length of the deflected section PR was found to be 3000 ft. It was again deflected at R at an angle of $3^\circ 15'$ so as to reach the original direction of the base at Q. The measured length of RQ was 3665 ft. The tape, 300 ft long, was standardized on the flat and was correct under a pull of 20 lbs. at 64°F. The mean temperature during measurement was 80°F. The tape was used in eatenary, in three equal spans of 100 ft each during measurement and was stressed with a pull of 30 lbs.

The supports of the tape were at the same level. The sectional area of the tape was 0.004 sq. inch, and the wt. of one cu. inch of steel, 0.28 lb. Calculate the length of the broken base PQ when $E=23 \times 10^6$ p.s.i. and co-eff. of expansion $=6.2 \times 10^{-6}$.

Ans PQ=6663.177 ft.

17. The length of a base line AB 1746.58 meters measured at an average elevation of 34.5 meters from the mean sea level. The latitude of the middle point is $38^\circ 30'$. The azimuth of the base is $16^\circ 48'$. What is the length of the case reduced to the mean sea-level? Assume R in meter = Antilog 6.80355

Ans, 1746,57064 meters.

CHAPTER 10

ASTRONOMICAL SURVEYING

10-1 Definitions: Astronomical Surveying may be defined as that branch of surveying in which the meridian, azimuth, latitude, longitude, time, etc. of a place on the surface of the earth are determined by observation of some heavenly bodies. To understand the principles of this branch of surveying, the following terms should be carefully studied.

Sphere: It is a solid formed by the revolution of a circle or a semicircle about its diameter and every point on its surface is equidistant from the centre. The section of a sphere by any plane is a circle. If the cutting plane passes through the centre of the sphere then the section of the sphere is called a *great circle* otherwise it forms *small circle*. The shortest distance between any two points on the surface of a sphere is along an arc of a great circle passing through them. Great circles which pass through the poles of another great circle are called *secondaries* to that great circle. A great circle and its secondaries cut at right angle.

poles: If at the center of a circle on a sphere a perpen-

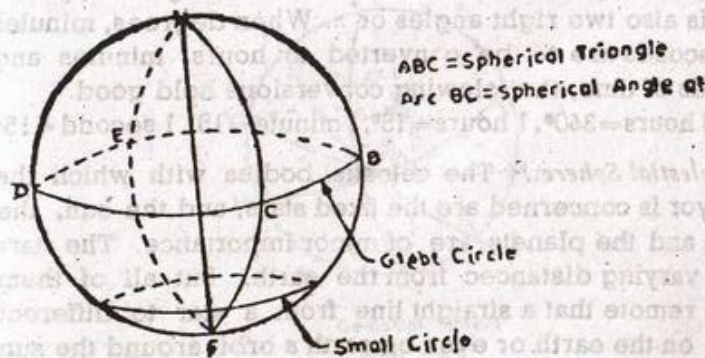


FIG. 10.1

dicular be erected to its plane and produced bothways, the two points in which it cuts the sphere are called the *poles* of that circle. In Fig. 10.1, A and F are the two poles.

Spherical Triangle : A spherical triangle is that triangle which is formed upon the surface of a sphere by the intersection of three great circles. Any two sides of a spherical triangle is greater than the third one. The greater angle is opposite the greater side and vice-versa. The sum of the three angles of a spherical triangle is not exactly equal to 180° as in case of a plane triangle. Their sum exceeds two right angles by an amount known as *spherical excess*. The magnitude of the spherical excess is expressed in degrees by the expression

$$S_e = \frac{\text{Area of triangle}}{\pi R^2} \times 180^\circ, \text{ where } R = \text{Radius of the Sphere}$$

But the sum is less than 3π . The area of a spherical triangle is ;

$$\text{Area} = \pi R^2 \frac{(A + B + C - 180^\circ)}{180^\circ} = \frac{\pi R^2 S_e}{180}$$

where A, B, & C are the spherical angles. The sides of a spherical triangle are expressed in angles and hours. If the sum of any two sides of a spherical triangle is equal to π or two right angles, the sum of the angles opposite them is also two right angles or π . When degrees, minutes and seconds are to be converted to hours, minutes and seconds of time, the following conversions hold good.

$$24 \text{ hours} = 360^\circ, 1 \text{ hour} = 15^\circ, 1 \text{ minute} = 15', 1 \text{ second} = 15''$$

Celestial Sphere : The celestial bodies with which the surveyor is concerned are the fixed stars, and the sun, the moon, and the planets are of minor importance. The stars are at varying distances from the earth. But all of them are so remote that a straight line from a star to different points on the earth or even on earth's orbit around the sun may be considered parallel for all practical purposes. In

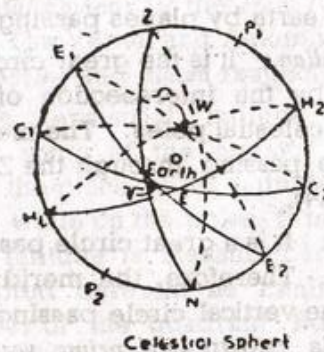
other words, the orbit of the earth may be treated as a point in comparison with the radius of the celestial sphere. Since the surveyor is concerned with the angular positions of the stars and not with their actual distances, it is very convenient to consider all these heavenly bodies studded upon the surface of an imaginary sphere called the *celestial sphere*.

Torrestrial Equator : It is the great circle of the earth the plane of which is perpendicular to the axis of rotation. The extremities of an axis of rotation (polar axis) of the earth are known as the poles. They are called North and South poles.

Celestial Equator : It is the great circle traced upon the celestial sphere by a plane which passes through the centre of the earth and is perpendicular to the polar axis. In Fig. 10.2, $E_1 E E_2$ is the *celestial equator*.

Celestial Poles : These are points at which the polar axis when produced intersects the celestial sphere. P_1 and P_2 are the celestial poles (Fig. 10.2).

Zenith and Nadir : Zenith is the point on the celestial sphere above the surveyor's station while Nadir lies on the celestial sphere vertically below the surveyor's station. They are denoted by Z and N respectively. (Fig. 10.2)



Celestial Sphere

FIG. 10.2

Horizon: The celestial horizon (or True or Rational Horizon) is the great circle in which a plane at right angles to Zenith and Nadir line and passing through the centre of the earth intersects the celestial sphere. The Zenith and Nadir are the poles of the celestial horizon (Fig. 10.2).

The *sensible horizon* is a plane passing through the eye of the surveyor at right angles to the direction of gravity at the point of observation. It is a great circle in case of a celestial sphere (Fig. 10.3). But the True or Rational Horizon is a plane parallel to the sensible horizon passing through the centre of the earth.

The *visible horizon* is the great circle in which the more distant visible features of the earth's surface appear to cut the celestial sphere (Fig. 10.3).

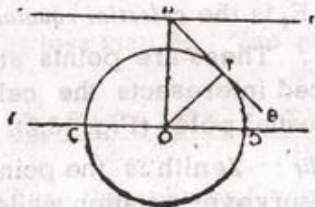


FIG. 10.3

Terrestrial Meridian: These are the great circles of intersection of the earth by planes passing through its axis.

Celestial Meridian: It is the great circle formed on the celestial sphere by the intersection of a plane passing through the two celestial poles. The meridian of a place is the great circle passing through the Zenith, Nadir and the poles (Fig. 10.4).

Vertical Circle: It is a great circle passing through the Zenith and Nadir. Therefore, the meridian of a place is a vertical circle. The vertical circle passing through the east and west points is known as a *prime vertical*. The plane which traces the prime vertical on the celestial sphere thus

passes Z and N and at right angles to the meridian of a place.

Ecliptic: The ecliptic is the great circle which the sun

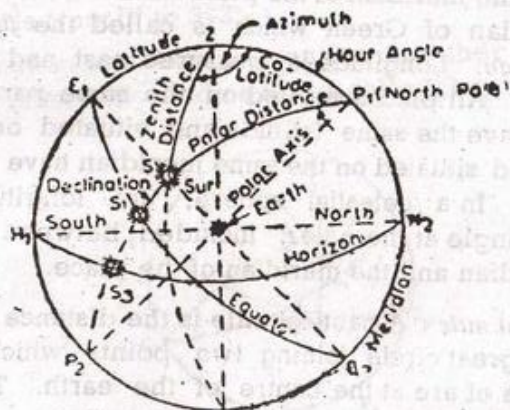


FIG. 10.4

appears to describe on the celestial sphere with the earth as a centre in year. The plane of the ecliptic makes an angle of $23^{\circ}27'$ with the plane of the equator and is known as *obliquity of the ecliptic*. The points of intersection of the ecliptic with the equator are known as *equinoxes*. The sun is at the *Vernal Equinox* or the *First Point of Aries* γ on the 21st March when it crosses the equator from south to north and it is on the *Autumnal Equinox* or the *First Point of Libra* U on 23rd September. The Vernal and Autumnal Equinoxes mark the beginning of *Spring* and *Autum* respectively (Fig. 10.4).

Latitude: The terrestrial latitude of a place is its distance north or south of the equator measured on the meridian through the place. The latitude of a point upon the equator is 0° while on the poles it is 90° . In a celestial sphere, the latitude is measured by the arc of the meridian intercepted between the Zenith and the equator. The remainder of the quadrant between the Zenith and the pole is known as *co-latitude* = 90° latitude. The latitude is also the *declination* of the Zenith.

Longitude : The longitude of a place is its distance East or West of the *first meridian* and is measured by the number of degrees in the arc intercepted on the equator between the meridian of the place and the first meridian. The meridian of Green which is called the *first or standard meridian*. Longitude is measured east and west from 0° to 180° . All places situated on the same parallel to the equator have the same latitude and situated on the same latitude and situated on the same meridian have the same longitude. In a celestial sphere, the longitude is the spherical angle at the poles, included between the standard meridian and the meridian of the place.

Nautical mile : A nautical mile is the distance measured along the great circle joining two points which subtend one minute of arc at the centre of the earth. Taking the radius of the earth to be equal to 3960 miles,

$$\begin{aligned} \text{One nautical mile} &= \frac{\text{Circumference of the great circle}}{360^\circ \times 60} \\ &= \frac{2\pi \times 3960 \times 5280}{360 \times 60} \text{ 6080 ft.} \end{aligned}$$

The distance between two points in nautical miles measured along the parallel latitude (a parallel latitude is a small circle of which P_1 is the pole, and points on the parallel latitude having the same latitude) is called the *departure*.

$$\therefore \text{Departure} = \text{Difference of longitude in minutes} \times \text{Cos (Latitude)}$$

Declination : The *declination* of a heavenly body is the angular distance from the Equator to the heavenly body, measured along the celestial meridian and the *co-declination* is the remainder at the quadrant; i.e. it is the arc of the meridian intercepted between the heavenly body and the nearer or the elevated pole. Therefore, *co-declination* = $90^\circ - \text{declination}$.

Right Ascension : It is the angular distance between the meridian of the heavenly body and the meridian through the Vernal Equinox. It is measured eastward from the First Point of Arise and is expressed in hours, minutes and seconds from 0° to 363° .

Declination and Right Ascension together define the position of heavenly bodies on the celestial sphere.

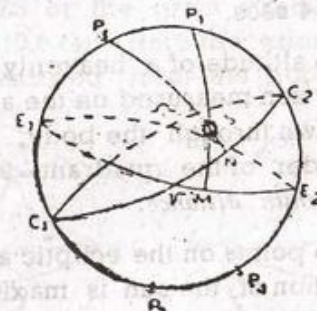


FIG. 10.5

Azimuth : The Azimuth of a point is the angle between the plane of the meridian and the vertical through the point, i.e. it is the spherical angle at the zenith, between the plane which passes through the zenith, Nadir and poles and the plane which passes through the Zenith, Nadir and the point in question. The Azimuth is therefore, the arc intercepted on the horizon between the foot of the vertical drawn through the body and the meridian. The celestial pole which is above the observer's horizon is known as the *elevated pole* and the value of the azimuth computed from the astronomical observations is the nearest angle from the elevated pole to the heavenly body measured either to East or West and is not greater than 180° . The computed value is then transformed into a whole circle azimuth measured from 0° to 360° in the clock wise direction.

Hour Angle : It is the spherical angle of a heavenly body at the pole, between the meridian of the observer

and the declination circle of the heavenly body. The arc of the equator intercepted between these two planes is a measure of the *hour angle*. By knowing hour angle, it is possible to calculate the time that must be elapsed before the star crosses the meridian or the time which has elapsed since it last crossed it from the fact that the star completes a revolution of 360 round the celestial pole in 23 hrs. 55 mins. 4 secs.

Altitude : The altitude of a heavenly body is its distance from the horizon measured on the arc perpendicular to the horizon drawn through the body. And the co-altitude is the remainder of the quadrant = $90^\circ - \text{altitude}$. This is also known as *zenith distance*.

Solstices : The points on the ecliptic at which the north and south declination of the sun is maximum are known as *solstices* (Fig. 10.5). The point C_2 at which the north declination of the sun is maximum is called the *summer solstice* while the *winter solstice* at C_1 , the south declination of the sun is maximum. In southern hemisphere the reverse is the case. The declination and the right ascension are each equal to zero at the Vernal Equinox on 21st March. On 21st June the sun is at C_2 on the ecliptic and 90° from π and its declination is maximum and equals $23^\circ 27' N$ and its right ascension is 6 hours (or 90°). When the sun is at Autumnal Equinox on 22nd September its declination is zero and the right ascensions 12 hours (or 180°). When the sun is at C_1 on 22nd December, its right ascension is 18 hours or (270°) and the declination is again maximum and equals $23^\circ 27' S$. On March 21 and September 22, the days and nights are equal all over the world.

10-2 System of co-ordinates of a Heavenly body

The position of a heavenly body on the celestial sphere can be located by means of three different systems of co-ordinates. They are :

- (1) The Altitude and Azimuth System.
- (2) The Declination and Hour Angle System.
- (3) The Declination and Right Ascension System.

The Altitude and Azimuth System ; In this system, the reference plane is the horizon H_1WH_2 and co-ordinates of the heavenly body S are the altitude α (or the great circle arc SS_1 or the angle SoS_1) and the azimuth A (or the spherical angle P_1ZS or the great circle arc H_1S_1). This is shown in Fig. 10.6 (a). Here the azimuth of the heavenly body S is measured from the north point towards

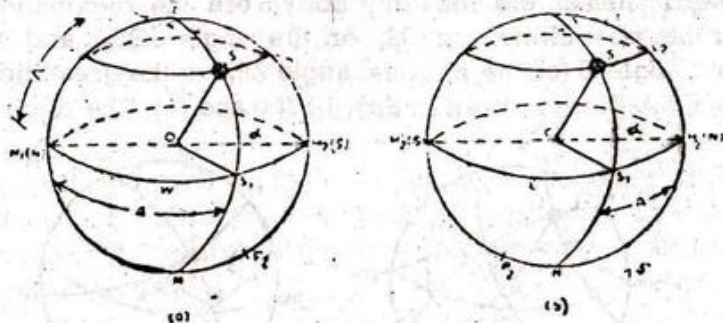


FIG. 10.6

west and its value lies between 0° and 180° . The great circle arc ZS is called the *zenith distance* or *co-altitude*.

$$\begin{aligned} \text{Zenith distance} &= ZS = ZS_1 = SS_1 = 90^\circ - \alpha \\ &= 90^\circ - \text{altitude} \end{aligned}$$

In the Fig. 10.6 (a), the star S lies in the western part of the celestial sphere through which the small circle L_1SL_2 passes parallel to the horizon H_1WH_2 . If the star lies in the eastern part of the celestial sphere then the azimuth is measured from the north point towards east i.e., from N_2 to E (Fig. 10.6b.). As before, the spherical angle P_1ZS or the great circle arc H_1S_1 or the angle H_1OS_1 is the azimuth giving the cardinal direction E .

$$\begin{aligned} \text{Zenith distance} &= ZS_1 = SS_1 = 90^\circ - \alpha \\ &= 90^\circ - \text{altitude.} \end{aligned}$$

When the azimuth of the star is 90°E or 90°W, the star is on the prime vertical. It may be noted here that this co-ordinates are variable due to the diurnal motion of the star.

Now, if θ be the latitude of the observer, then co-latitude i.e., $ZP_1 = 90^\circ - \theta$. And the altitude of the $P_1 = P_1$
 $H_1 = ZH_1 - ZP_1 = 90^\circ - (90 - \theta) = \theta$. Hence, the altitude of the pole is equal to the latitude of the observer.

The Declination and the Hour Angle System : In this system, the reference plane is the equator E_1WE_2 and the co-ordinates of the heavenly body S are the declination δ (or the great circle arc SS_1 or the angle SOS_1) and the hour angle H (or the spherical angle ZP_1S or the great circle arc E_2S_1). This is shown in Fig. 10.7(a) and (b). The declina-

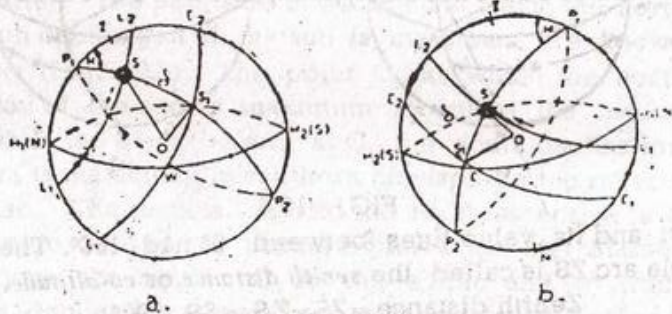


FIG. 10.7

tion (δ) of the heavenly body S is measured along the great circle $P_1S_1P_2$ which is termed as the *declination circle*, when the heavenly body S is north of the equator i.e. between the celestial equator and the north pole P_1 , its declination is north and positive ($+\delta N$), while it is south or negative ($-\delta S$) when the body is south of the celestial equator and the south pole P_2 . The arc P_1S is known as *co-declination* or *north polar distance*.

$$P_1S = PS_1 - SS_1 = 90^\circ - \delta = 90^\circ - \text{declination} = \text{co-declination}.$$

When the declination is south, the north polar distance

or the co-declination $= 90^\circ - (-\delta) = 90^\circ + \delta = 90^\circ + \text{declination}$ and the declination is north, the south polar distance $= 90^\circ - \delta = 90^\circ - \text{declination}$.

The Declination and Right Ascension System : In this system, the reference plane is the celestial equator E_1WE_2 and the Vernal Equinox or the First point of Aries, γ is chosen as a reference point. (Fig. 10.8). The co-ordinates of the heavenly body S are the declination δ (or the great circle arc YS_1 , or the spherical angle γOS_1). The right ascension of a heavenly body is measured eastwards from γ along the equator from 0° to 369° or in time units from 0 hours. Now, the spherical angle $E_2P_1S_1$ or the arc E_2S_1 is the hour angle (H) of the heavenly body S .

Here, it is to be noted that the direction of measuring the right ascension is opposite to that of the hour angle. The declination and the right ascension of a heavenly body are constant. Hence, this system is the most convenient for specifying the relative positions of the heavenly bodies on the celestial sphere.

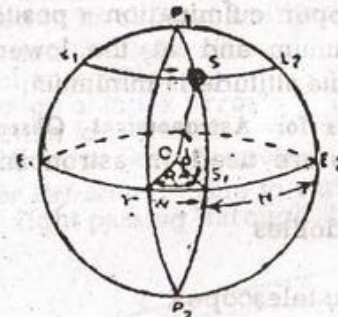


FIG. 20.8

The most important points to be noted regarding the azimuth and the hour angle are (1) when the observer is in northern hemisphere, the azimuth of a heavenly body is measured from the north point to the east or to the west, (2) when the observer is in the southern hemisphere, the azimuth is measured from the south point to the east or to

the west, (3) when the heavenly body is in the western hemisphere, its azimuth is west and its hour angle is in between 0 hour and 12 hours, (5) when the heavenly body in the eastern hemisphers, its azimuth is east and its hour angle is in between 12 hours to 24 hours.

Circumpolar Stars: The stars which are above the horizon are always visible and they do not set. These stars are called *circumpolar stars*. They will always appear to the surveyor to describe a small circle about the pole P_1 (Fig. 10.7 a). It is to be noted that for all circumpolar stars the distance from the pole is less than the latitude of the place of observation i.e., the declination of a circumpolar star must be greater than the co-latitude $P_1E_1 \angle P_1L_2$ for the circumpolar star S.

Culmination: When a heavenly body crosses the meridian it is said to be *culminated or transited*. In one revolution round the pole each star crosses the meridian twice. As a result, two culminations or transits (upper culmination and lower culmination) are obtained. A star is said to be in the upper culmination (position L_2) when its altitude is maximum and at the lower culmination (position L_1) when its altitude is minimum.

10-3 Instruments for Astronomical Observations: The following instruments are used in astronomical observations:

- (1) Transit theodolites
- (2) Sextants
- (3) Photographic telescopes
- (4) Microscopes
- (5) Micrometers
- (6) Chronographs
- (7) Alt-Azimuth instruments
- (8) Solar Attachments.

The transit theodolite and the Sextant have been discussed in chapter 3 and chapter 6 respectively.

Photographic Telescope: Most of the observations on heavenly bodies are made photographically by a Photographic telescope. This is nothing but a telescope fitted with a camera.

Microscopes and Micrometers: The microscopes are generally used for reading the observation on heavenly bodies. Every transit theodolite or photographic telescope is fitted with a micrometer for measuring small angular distances such as the angle subtended at the observer by the two neighbouring stars in the field of view of the telescope.

Chronograph: A chronograph is a small instrument for observing the transits of heavenly bodies.

Alt-Azimuth Instrument: This is a particular type of instrument in which the axis points towards the zenith. It admits a double motion in latitude and azimuth. It is generally used in ex meridian observations.

Solar Attachment: It is a special apparatus fitted to the telescope of an ordinary theodolite for determining the direction of the meridian, the altitude and local time.

10-4 Astronomical Corrections: In determining the true altitude of a heavenly body the following corrections are to be applied in its observed altitude.

- (1) Correction for Refraction
- (2) Correction for Dip
- (3) Correction of Index-Error
- (4) Correction for Parallax
- (5) Correction for Semi-diameter.

Correction for Refraction: Due to variation of density of air the rays of light passing through the atmosphere are

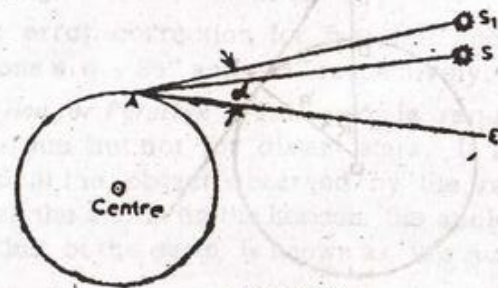


FIG. 10.9

refracted or bent. As a result, the heavenly body S appears to be located at S_1 (Fig. 10.9). Therefore, due to refraction the observed altitude of S appears greater. So the correction is to be subtracted from the observed altitude. The refraction correction does not depend upon the distance of the object but is dependent on the altitude, barometric pressure and temperature. It is 0 when the heavenly body is in the zenith and 33' when on the horizon. The following formula is generally used for rough calculation.

Refraction in seconds = $53'' \times \cot \alpha = 53'' \times \tan z$, where α and z are the apparent altitude and the zenith distance respectively of the heavenly body and the correction is at a pressure of 30" of mercury and temperature of 50°F. For more accurate results, Chamber's or Bessel's Refraction Tables should be consulted.

Correction for Dip: The angle between the sensible horizon and the visible horizon is called the angle of dip. The

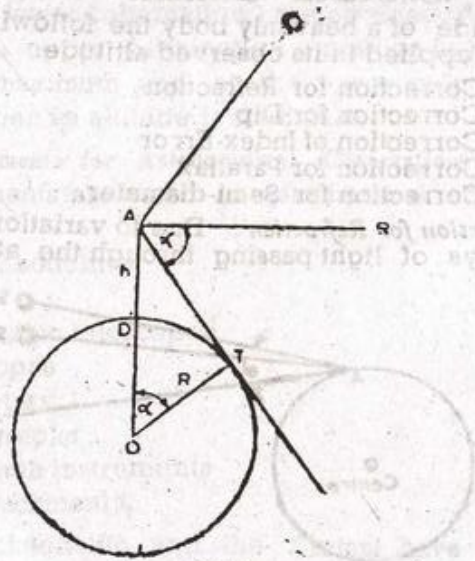


Fig. 10.10

angle $BAC = \alpha$ (Fig. 10.10) is the angle of dip. AB is sensible horizon, AC is the visible horizon and h , the height of the observer above sea level in ft. The correction for dip i.e., angle α is applied when observations of the heavenly S is taken with a box sextant only at sea.

$$\tan \alpha = \frac{AT}{OT} = \sqrt{\frac{(R+h)^2 - R^2}{R}} = \sqrt{\frac{2h}{R}} \text{ (approximately)}$$

The altitude corrected for dip = $SAB - SAT - BAT =$ Observed altitude corrected for refraction - correction for dip.

It is seen that the correction for dip is negative. The various values of corrections different heights are obtained from Chamber's or Molesworth's Table.

Correction for Index Error: When the altitude is zero, the vernier reading of the instrument should also be zero, if not, there will be a small angular error e which is known as *index error*. Now the reading of any vertical angle observed with the instrument either will be too large or too small by this amount α_1 . The correction is either $+e$ or $-e$ according as e is above or below zero. As for an example, let $\alpha_1 = 6^\circ 25' 30''$ and $\alpha_2 = 6^\circ 26' 20''$, where α_1 and α_2 are the face left and face right observations by the theodolite on a top of an electric post.

$$\begin{aligned} \text{Mean vertical angle} &= \frac{1}{2}(6^\circ 25' 30'' + 6^\circ 26' 20'') \\ &= 6^\circ 25' 55'' \end{aligned}$$

Index error correction for face left and face right observations are $+25''$ and $-25''$ respectively.

Correction for Parallax: This error is required only in case of the sun but not for distant stars. It is the angle subtended at the object observed by the radius of the earth, when the sun is on the horizon, the angle subtended by the radius of the earth is known as the *sun's horizontal parallax*.

The angle OS_1A is the horizontal parallax and the angle OSA is the parallax for any position of the sun.

(Fig. 10.11), OB is the true horizon, SOB the true altitude. SAS₁ the apparent altitude.

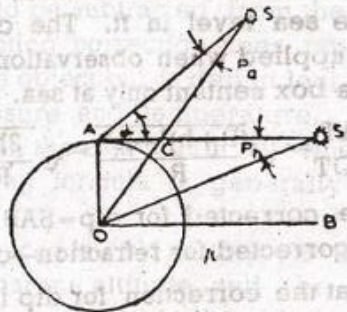


Fig. 10.11

When the sun is on the horizon at S₁,

Horizontal parallax error = $P_h = \angle OS_1A = \sin \angle OS_1A$

(∵ small angle)

$$= \frac{OA}{OS_1} = \frac{4000}{92 \times 10^6} \text{ radians.}$$

$$= \frac{4000}{92 \times 10^6} \times 206265''$$

(∵ 1 rad = 206265'') = 8.9''

The average value is taken to be 8.8''

When the sun is at the position S,

Parallax error = $P_2 = \angle OSA = \sin \angle OSA$

$$\text{Now in } \Delta OAS, \frac{\sin \angle OSA}{\sin \angle OAS} = \frac{OA}{OS}$$

$$\sin \angle OSA = \frac{OA}{OS} \sin \angle OAS = \frac{OA}{OS} \sin (90^\circ + \alpha)$$

$$= \frac{OA}{OS} \cos \alpha = \frac{OA}{OS_1} \cos \alpha$$

(Assuming $OS = OS_1$)

$$= 8.8'' \times \cos \alpha$$

This error is always additive i. e.,

$$\text{Parallax error} = +8.8'' \times \cos \alpha$$

Correction for Semi-diameter: In taking the altitude of the sun the centre of it should be observed. But practi-

cally, it is not possible due to the diameter of the sun considerable. As a result, either the upper or lower

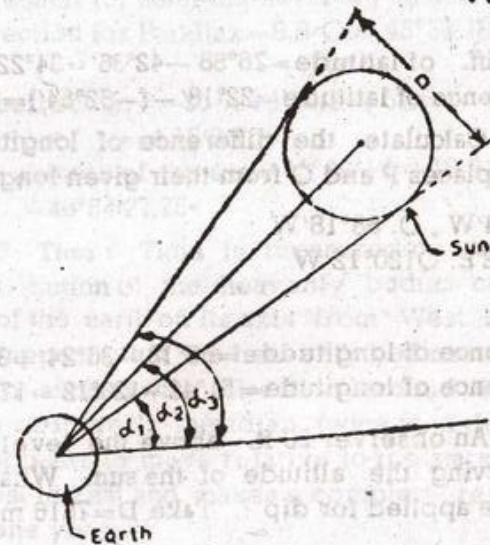


Fig. 10.12

edge is focussed with the telescope. The altitude of the sun's centre is obtained by applying the correction of semi-diameter. The half of the angle D subtended at the centre of the earth by the diameter of the sun is known as *semi-diameter*. Due to the variation of the sun's distance from the earth, the semi-diameter varies throughout the year and its value can be obtained from the Nautical Almanac. In Fig. 10.12, α is the altitude of the sun's centre, α_1 and α_2 are the observed altitudes at the lower and upper edge respectively.

$$\text{Then, } \alpha = \alpha_1 + \frac{D}{2} = \alpha_2 - \frac{D}{2}$$

The average value of $D/2$ is 15 30''. This correction is not taken into consideration while observing the stars because observations can be made upon their centres as they appear as points of light.

Example: Calculate the difference of latitude between two places P and Q from their individual given latitudes:

(1) P, $42^{\circ}36'N$; Q, $76^{\circ}58'N$ (2) P, $22^{\circ}18'N$; Q, $52^{\circ}54'S$ **Solution :**(1) The diff. of latitude = $76^{\circ}58' - 42^{\circ}36' = 34^{\circ}22'$ (The difference of latitude = $22^{\circ}18' - (-52^{\circ}54') = 75^{\circ}12'$)**Example :** Calculate the difference of longitude between the two places P and Q from their given longitudes :(1) P, $36^{\circ}24'W$, Q, $68^{\circ}18'W$ (2) P, $51^{\circ}42'E$; Q $120^{\circ}12'W$ **Solution :**(1) Difference of longitude = $68^{\circ}18' - 36^{\circ}24' = 31^{\circ}54'$ (2) Difference of longitude = $51^{\circ}42' + 120^{\circ}12' = 171^{\circ}54'$ **Example :** An observer 30 ft. above the level of the sea was observing the altitude of the sun. What is the correction to be applied for dip? Take $D = 7616$ miles**Solution :**

$$\tan \alpha = \sqrt{\frac{2h}{R}} = \sqrt{\frac{2 \times 30}{2069000}} = 0.001691$$

$$\therefore \alpha = 5' - 42'' \text{ (approximately)}$$

Example : An observation was made on the sun using the lower limb and the reading was $46^{\circ}40'25''$ with face left. The semi-diameter of the sun at the time of observation was found to be $15'58.7''$. The face left and the face right observations of the theodolite on the top of an electric post were $12^{\circ}15'56''$ and $12^{\circ}13'42''$ respectively. Calculate the true altitude of the sun.**Solution :**

$$\text{Correction for index error} = \frac{1}{2}(12^{\circ}15'56'' - 12^{\circ}13'42'')$$

$$= 1'7'' \text{ (-ve)}$$

$$\text{Corrected altitude} = 46^{\circ}40'25'' - 1'7''$$

$$= 46^{\circ}39'18''$$

$$\text{Correction for refraction} = -58'' \cos 46^{\circ}39'18''$$

$$= -55''$$

$$\text{Correction for semi-diameter} = +15'58.7''$$

$$\text{Correction for Parallax} = 8.8'' \cos 46^{\circ}39'18''$$

$$= +6.05''$$

$$\text{Net correction} = -55'' + 15'58.7'' + 6.05''$$

$$= +15'9.75''$$

$$\text{True altitude of the sun} = 46^{\circ}39'18'' + 15'9.75''$$

$$= 46^{\circ}54'27.75''$$

10-5 Time : Time is measured on the basis of the apparent motion of the heavenly bodies caused by the rotation of the earth on its axis from West to East. As a result, it appears that the heavenly bodies move from East to West in a clockwise direction around the earth and cross the observer's meridian twice in a day. Similarly, the sun appears to move relative to the stars in a direction from West to East and makes a complete revolution of the stars in one year.

10-5 Systems of Time : There are four different systems of measuring time. They are : (1) Sidereal Time, (2) Apparent Solar Time, (3) Mean Solar Time, and (4) Standard Time. The former two systems are convenient for the astronomers and latter two are useful for everyday life.

Sidereal Time : This system of time measurement is based upon the complete revolution of the fixed stars around the celestial pole. The time interval between two successive upper transits of the First Point of Aries over the same meridian is known as the *sidereal day* and the instant of crossing is termed as *sidereal noon*. The sidereal day is divided into 24 hours, the hour into 60 minutes and the minute into 60 seconds. The sidereal time is measured from the instant when the First Point of Aries just crosses the meridian of the place of observation. Therefore, it is 0 hour when γ is on the meridian and 24 hours when γ is

again on the same meridian in the same direction of the next transit. The sidereal time at any instant can be measured by the interval that has elapsed since the First Point of Aries was on the meridian. So the sidereal time at any instant is the Hour Angle of the γ expressed in time at the rate of 15° to one hour. The Right Ascension (R.A.) of the meridian of a place is known as *local sidereal time* (L.S.T.)

L.S.T. = R.A. of a star + Hour angle of a star,

Also L.S.T. = R.A. of mean sun + 12 hours + mean time at the place.

Apparent Solar Time : This system of time measurement is based on the daily motion of the sun. Time interval between two successive lower transits of the sun across the same meridian is called an *apparent solar day*. It is divided into 24 hours, each hour into 60 minutes and each minute into 60 seconds. The Sun Dial gives the apparent solar time. The apparent solar day is not of uniform length as the sun does not move at an uniform rate along the ecliptic.

Mean Solar Time : This system of time measurement is based on an imaginary point called the *mean sun* which moves along the equator in such a way that its Right Ascension increases uniformly. This system is adopted to overcome the disadvantages in the sidereal and apparent solar times due to their fluctuations. The mean solar day is the time interval between two successive lower transits of the mean sun. Therefore, the mean solar time of the mean time at any instant is the westward Hour Angle of the mean sun expressed in time plus 12 hours. The mean time is reckoned 0 hour at midnight. This time is recorded by ordinary clocks. The mean solar time is also known as *civil time*. A mean day is divided into 24 hours, each hour into 60 minutes and each minute into 60 seconds. There are two ways by which mean solar time is reckoned viz., (1) Civil time and (2) Astronomical time. Both of them begin at 0

hour midnight. The civil day is divided into two periods. The one which occurs from midnight to noon is denoted by A.M. (Anti Meridian) and the other from noon to midnight by P.M. (Post Meridian). The astronomical day is continuous from 0 hour to 24 hours. In railways, airways and navigation, the astronomical time is used.

Greenwich Mean Time (G.M.T.) : It is measured from the lower transit of the Greenwich meridian by the mean sun i.e. from Greenwich Mean Midnight, 0 hour to 24 hours. This is same as Universal Time (U.T.) as recommended by the International Astronomical Union.

Local Time or Local Mean Time L.M.T. : It is measured from the lower transit of the meridian of the place of observation.

In an interval of 24 hours, the mean sun describes an arc of 360° . So a difference in the longitude of 1° corresponds to a difference of 4 minutes in local time. So,

$$\begin{aligned} \text{L.M.T.} &= \text{G.M.T.} - \text{West longitude} \\ &= \text{G.M.T.} + \text{East longitude.} \end{aligned}$$

Standard Time : In order to overcome the difficulties arising from the use of different local times at different places, the country as a whole adopts the local mean time of a place corresponding to a particular meridian. In most cases it differs from Greenwich Mean Time by a whole number of hours. A country sometimes may have more than one standard time. In Bangladesh the standard time is the Madras time. $82\frac{1}{2}$ th meridian (E) which corresponds to 5 hours 30 minutes ahead of G.M.T. In Bangladesh there is only one local time viz Bangladesh Local Time (B.L.T.)

10-7 Equation of Time ; The equation of time is the difference between the mean and the apparent time. It is positive when the mean time is greater than apparent time, and negative when the latter is greater than the former.

$$\text{Equation of Time (E)} = \text{Mean Time} - \text{Apparent Time.}$$

- = Clock Time (C) - Dial Time (D)
 = $\frac{1}{2}$ (Length afternoon - Length morning)
 = Hour angle of the Mean sun - Hour angle of the apparent or true sun.

The cause of the equation of time are : (1) the variable motion of the true sun in the ecliptic owing to the eccentricity of the earth's orbit, and (2) the obliquity of the ecliptic to the equator.

The equation of time is zero four times during the year, i.e., about April 15 June 14, September 21 and December 25. On these dates, the true sun and the mean sun are on the same meridian and as a result, the apparent time and the mean time are equal. In the Nautical Almanac, the equation of time is tabulated for every midnight.

Example : Calculate the G.M.T. of a place (140°E longitude) on 2nd January, 1939, when the time was 4 hours 30 minutes (P.M.)

Solution :

$$15^\circ = 1 \text{ hour}$$

$$140^\circ = \frac{140}{15} = 9 \text{ hours } 20 \text{ minutes}$$

$$\begin{aligned} \text{G.M.T.} &= 4 \text{ hours } 30 \text{ minutes} + 12 \text{ hours} - 9 \text{ hours } 20 \text{ minutes} \\ &= 7 \text{ hours } 10 \text{ minutes (A.M.) on 2nd January 1939.} \end{aligned}$$

Example : Calculate the local apparent time at a place (90°30' E) whose local time is 10 hours 20 minutes. The equation of time at Greenwich Mean Moon (G.M.N.) is 5 minutes 12 seconds (subtractive) and decreasing 0.25 seconds per hour.

Solution :

$$90^\circ 30' = \frac{90.50}{15} = 6 \text{ hours } 2 \text{ minutes.}$$

$$\begin{aligned} \text{Greenwich Mean Time of observation} &= 10 \text{ hours } 20 \text{ minutes} - 6 \text{ hrs.} \\ &= 4 \text{ hours } 14 \text{ minutes } 2 \text{ mins.} = 4 \text{ hours } 18 \text{ minutes.} \\ \text{Mean Time interval before G.M.N.} &= 12 \text{ hours} - 4 \text{ hours } 18 \text{ minutes} \\ &= 7 \text{ hours } 42 \text{ minutes.} \\ &= 7.70 \text{ hours} \end{aligned}$$

Total increase of time = $0.25 \times 7.70 = 1.925$ seconds.

Equation of time of the place of observation.

$$\begin{aligned} &= 5 \text{ minutes } 12 \text{ seconds} + 1.925 \\ &= 5 \text{ minutes } 13.925 \text{ second.} \end{aligned}$$

$$\begin{aligned} \text{Greenwich Apparent Time of observation} &= 4 \text{ hours } 18 \text{ mins.} \\ &+ 5 \text{ mins. } + 13.945 \text{ seconds} \\ &= 4 \text{ hrs. } 23 \text{ mins, } 139.25 \text{ secs} \end{aligned}$$

Local Apparent Time = 4 hours 23 minutes 13.925 secs. + Long. in time.

$$\begin{aligned} &= 4 \text{ hours } 33 \text{ minutes } 13.925 \text{ secs.} + 6 \\ &\quad \text{hrs } 2 \text{ minutes} \\ &= 10 \text{ hours } 25 \text{ minutes } 13.925 \text{ seconds.} \end{aligned}$$

Example : Calculate the Local Mean Time corresponding to 15 hours 18 minutes 22 seconds local sidereal time on January 3, 1965, at a place of longitude 6 hours 34 minutes (E). Take acceleration or retradiation of 9.86 seconds per hour of longitude.

Solution :

$$\begin{aligned} \text{G.S.T. corresponding to L.S.T. of } 15 \text{ hours } 18 \text{ minutes} \\ &22 \text{ seconds.} \\ &= 15 \text{ hours } 18 \text{ minutes } 22 \text{ secs.} - 6 \text{ hrs } 34 \text{ minutes} \\ &= 8 \text{ hours } 44 \text{ minutes } 22 \text{ seconds.} \end{aligned}$$

$$\begin{aligned} \text{Total retardation} &= 6 \text{ hours } 34 \text{ minutes} \times 9.86 \text{ seconds} \\ &\quad \text{per hour} \\ &= 6.5675 \times 9.86 \\ &= 56 \text{ seconds} \end{aligned}$$

L.S.T. of local mean noon = 8 hours 44 minutes 22 secs.

— 53 secs.

= 8 hours 43 minutes 28 seconds

Sidereal interval from L.M.N. = 15 hours 18 minutes 22 seconds.

— 8 hours 43 minutes 23 seconds

= 6 hours 34 minutes 56 seconds

Acceleration = 9.96 × 6 hrs. 34 mins. 56 seconds ÷ 1 minute 7.5 seconds.

Local Mean Time = 6 hours 34 mins. 56 seconds + 1 min. 7.5 secs

= 6 hours 36 minutes 3.5 seconds

Example: On January 2nd, 1939, the sun dial was 16 minutes 21 seconds faster than the clock. If the sunrise time was 6 hours 57 minutes A.M., calculate the time of sunset.

Solution:

$E = \frac{1}{2}(\text{Length afternoon} - \text{Length morning})$

= 16 minutes 21 seconds = $\frac{1}{2} \{ \text{Length afternoon} - (12 \text{ hours} - 6 \text{ hours } 57 \text{ minutes}) \}$

Length afternoon = 4 hours 30 minutes 18 seconds

So, the sun sets at 4 hours 30 minutes 18 seconds P.M.

10-8 Determination of the Azimuth and the Bearing of a

Survey line: X is the station on the survey line CD. The azimuth, A is to be determined. The theodolite is set up at X and properly levelled and adjusted. S is the position of the sun which is to be observed (any other heavenly body can also be observed). The proper adjustment will be indicated if both the horizontal and vertical circles read zero. The telescope is focussed towards the reference point D on the survey line and the vertical and horizontal circles are turned till the sun is sighted. The angle of altitude α and the horizontal angle θ (Fig. 10.13) are obtained.

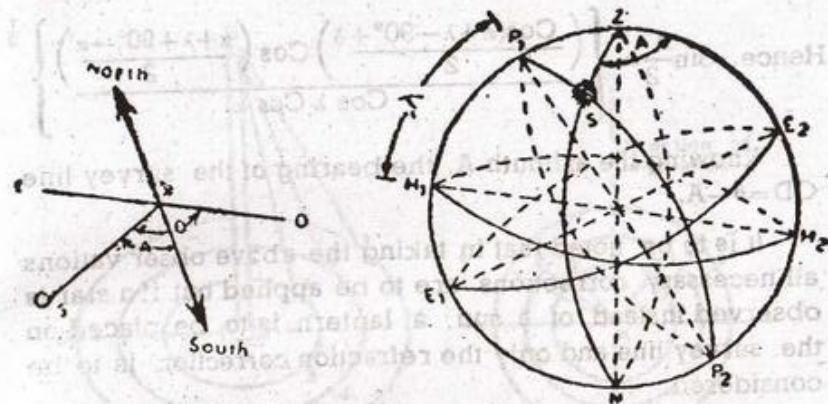


Fig. 10.13

Fig. 10.14

Considering the spherical triangle, $P_1 Z S$ in Fig. 10.14

$P_1 Z = 90^\circ - \text{latitude} = 90^\circ - \lambda = \text{co-latitude}$

$ZS = 90^\circ - \text{altitude} = 90^\circ - \alpha = \text{co-altitude}$

$P_1 S = 90^\circ - \text{declination} = 90^\circ - \delta = \text{co-declination}$

$\angle P_1 Z S = 180^\circ - \angle E_2 Z S = 180^\circ - A$

From the spherical trigonometry, in the spherical triangle $P_1 Z S$.

$$\cos \frac{180^\circ - A}{2} = \cos \frac{\angle P_1 Z S}{2}$$

$$= \sqrt{\frac{\sin S \sin (S - P_1 S)}{\sin P_1 Z \sin ZS}}$$

$$\text{where } 2S = P_1 Z + ZS + P_1 S = (90^\circ - \lambda) + (90^\circ - \alpha) + (90^\circ - \delta)$$

$$= 270^\circ - (\lambda + \alpha + \delta)$$

$$= 180^\circ - (\lambda + \alpha + \delta - 90^\circ)$$

$$\therefore S = 90^\circ - \frac{(\lambda + \alpha + \delta - 90^\circ)}{2}$$

$$\text{Again, } 2(S - P_1 S) = 270^\circ - (\lambda + \alpha + \delta) - 2(90^\circ - \delta) = 90^\circ$$

$$- (\lambda + \alpha - \delta)$$

$$= 180^\circ - (\lambda + \alpha + 90^\circ - \delta)$$

$$S - P_1 S = 90^\circ - \frac{(\lambda + \alpha + 90^\circ - \delta)}{2}$$

$$\text{Hence. } \sin \frac{A}{2} = \left\{ \frac{\left(\frac{\cos(\alpha + \lambda - 90^\circ + \delta)}{2} \right) \cos \left(\frac{\alpha + \lambda + 90^\circ - \alpha}{2} \right)}{\cos \lambda \cdot \cos \alpha} \right\}^{\frac{1}{2}}$$

Knowing the azimuth A , the bearing of the survey line $CD = \theta - A$.

It is to be noted that in taking the above observations all necessary corrections are to be applied but if a star is observed instead of a sun, a lantern is to be placed on the survey line and only the refraction correction is to be considered.

10-9 Determination of True Meridian : In determining the true meridian of the place the latitude of the place of observation must be known. This can be obtained from a map or from the altitude of the Polaris. Polaris or Pole Star is a bright star on the northern hemisphere. The altitude of the Polaris is very nearly equal to the latitude of the place due to its proximity to the pole. There are many methods of determining the true meridian of a place of which the following are very common :

- (1) By the direction of shadow at apparent noon.
- (2) By observation Polaris at elongation.
- (3) By magnetic compass.

By the Direction of Shadow At Apparent Noon : The direction is found out by observing the shadow of a vertical pole AB at apparent noon. With this pole as centre a number of concentric circles are drawn and on these are marked the points where the shadow of the top of the pole intersects them at equal interval of time before noon by means of arrows 1, 2 and 3 (Fig.10.15). Similar points are marked by means of arrows 3, 2 and 1 at the same interval of time after-noon. The points 1 and 1, 2 and 2, and 3 and 3, are joined to the foot of the pole A . The angles $1A1$, $2A2$ and $3A3$ are

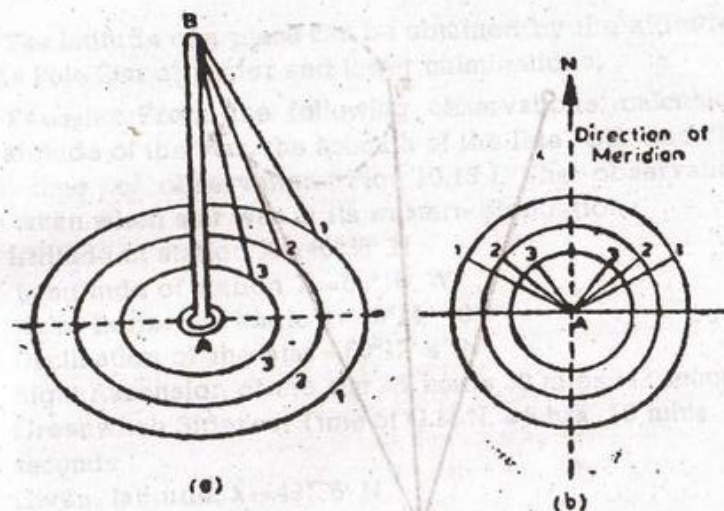


Fig. 10.15

bisected. This bisector NA gives the direction of true meridian.

By Observation on Polaris at Elongation : The time of elongation (i.e., the greatest distance of the Polaris east or west of the meridian) must be known in this method. The Pole Star is at western elongation about 5 hours 55 minutes after upper culmination and it is at eastern elongation about 5 hours 55 minutes before upper culmination. The instrument is placed over a given station M and properly levelled and adjusted. The telescope is directed to the reference object O (Fig. 10.16) and the readings of the verniers of the horizontal scale are noted. The telescope is then directed to the Pole Star about 10 minutes before it is at its greatest elongation to the east. Exact coincidence of the Pole Star with the cross-hairs is obtained with the tangent screw. The vernier readings are noted and the horizontal angle OMS_1 is computed. At this instant, another value of the angle OMS_2 is obtained by changing the face of the instrument in order to eliminate the error due to improper adjustment. When the star is on the opposite side of the

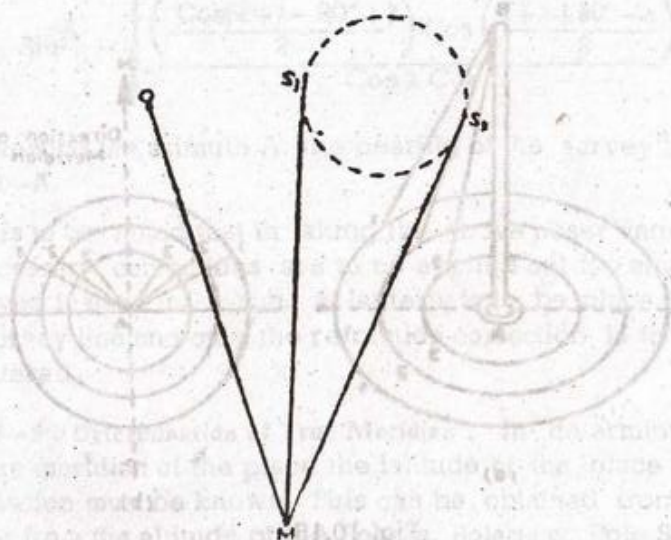


Fig. 10.16

pole about 12 hours, the angle OMS_2 is similarly observed (western elongation). The mean of these two angles gives the value of angle OMP_1 where P_1 is the celestial pole. Therefore, the angle OMP_1 is the azimuth of the line OM. The direction of the true meridian can be obtained from its known azimuth.

By Magnetic Compass: The magnetic meridian of a place can be obtained by a magnetic compass. The true meridian is then calculated by knowing the magnetic declination of the place.

10-10 Determination of Longitude, Time and Latitude: The longitude is calculated by comparing the local time with the time at the standard meridian (Greenwich meridian).

Time is determined by the sun's shadow and by the sun-dial. Local apparent noon is obtained observing the time when the shadow of a vertical pole shows the minimum length. The local time is then calculated from the equation of time. The sun dial gives the apparent solar time from which local time can be calculated.

The latitude of a place can be obtained by the altitudes of the Pole Star at upper and lower culminations.

Example: From the following observations, calculate the altitude of the star, the azimuth of the line CD and the local time of observation (Fig. 10.13). The observation was taken when star was at its western elongation.

Latitude of station $X = 46^{\circ}36' N$

Longitude of station $X = 58^{\circ}18' W$

Mean horizontal angle $\theta = 92^{\circ}24' 48''$

Declination of the star $= 60^{\circ}12' 4'' N$

Right Ascension of the star $= 9$ hours 30 mins. 42 seconds

Greenwich Sidereal Time of G.M.N. $= 5$ hrs. 18 mins. 30 seconds

Given. latitude. $\lambda = 46^{\circ}36' N$

declination. $\delta = 60^{\circ}12' 4'' N$

Solution:

If α is the altitude of the star, then

$$\sin \alpha = \frac{\sin \lambda}{\sin \delta} = \frac{\sin 46^{\circ}36'}{\sin 60^{\circ}12' 4''} = 0.8374 \therefore \alpha = 56^{\circ}54'$$

$$\sin A = \frac{\cos \delta}{\cos \lambda} = \frac{\cos 60^{\circ}12' 4''}{\cos 46^{\circ}36'} = 0.7235 \quad A = 46^{\circ}24' W$$

$$\cos H = \frac{\tan \lambda}{\tan \delta} = \frac{\tan 46^{\circ}36'}{\tan 60^{\circ}12' 4''} = 0.6052$$

$$\therefore H = 52^{\circ}48' = 3 \text{ hours } 31 \text{ minutes } 12 \text{ seconds.}$$

$$\text{Azimuth of the line CD} = \text{Azimuth of the star} + \theta$$

$$= A + \theta = 46^{\circ}24' + 92^{\circ}24' 48''$$

$$= 138^{\circ}48' 48'' \text{ (clockwise from North)}$$

Local Time of Observaion:

$$\text{Longitude} = 53^{\circ}18' = 3 \text{ hrs. } 53 \text{ mins. } 12 \text{ secs.} = 3.887 \text{ hours}$$

We know, acceleration at 9.8296 seconds per hour of S.I.

$$\therefore \text{Total acceleration} = 9.8296 \times 3.887 = 38.2 \text{ second. (+ve)}$$

$$\text{L.S.T. of L.M.N.} = 5 \text{ hrs, } 18 \text{ mins. } 30 \text{ secs} + 38.2 \text{ seconds.}$$

$$= 5 \text{ hour } 19 \text{ minutes } 8.2 \text{ seconds.}$$

$$\text{Local Sidereal Time (L.S.T.)} = \text{R.A. of the star} + \text{H.A. of the star}$$

= 9 hrs. 30 mins. 42 secs + 3

hours 31 minutes 12 seconds

= 13 hours 1 minute 54

second,

Sidereal Interval (S.I.) of L.M.N.

= L.S.T. — L.S.T. of L.M.N.

= 13 hours 1 minute 54 seconds — 5 hours 19 minutes
8.2 secs.

= 6 hours 42 minutes 45.8 secs = 6.7127 hours

Total retardation = 9.8286×6.7127

= 66 seconds = 1 minute 6 seconds

Mean Time Interval from L.M.N.

= 6 hours 42 minutes 45.8 seconds — 1 minute 6 secs.

= 6 hours 41 minutes 39.8 seconds.

Local Time of Observation = 6 hours 41 minutes 39.8
seconds P.M.

Exercise

1. Examine the following statements very carefully and write whether they are true or false :

(a) Any two sides of a spherical triangle is greater than the third one.

(b) The sum of the three angles of a spherical triangle is equal to two right angles.

(c) Sensible horizon is the plane in which the more distant visible features of the earth's surface appear to cut the celestial sphere.

(d) The great circle passing through the Zenith and Nadir is called the vertical circle.

(e) The ecliptic is the great circle which the sun appears to describe on the celestial sphere with the earth as a centre in a year.

(f) The plane of the ecliptic makes an angle of $66^{\circ}33'$ with the plane of the equator.

(g) First Point of Libra is known as Vernal Equinox.

(h) The longitude of a place is its distance east or west of the first meridian.

(i) The Declination of a heavenly body is the angular distance from the Equator to the heavenly body.

(j) Right Ascension is the angular distance between the first meridian and the meridian of the heavenly body.

(k) The Azimuth is the arc intercepted on the Equator between the foot of the vertical drawn through the body and the meridian.

(l) A star completes a revolution of 360° round the celestial pole in 23 hours and 55 minutes.

(m) Zenith distance is also known as co-altitude.

(n) The altitude and the hour angle of the zenith are 90° and 0° respectively.

(o) The latitude of the celestial pole is $66^{\circ}31'$.

(p) Right Ascension of the First Points of Aries is 0.

(q) The latitude of the First points of Aries is 90° .

(r) The sun's right ascension on 21st March is 0° .

(s) The hour angle of the sun at sunrise on 21st March is 90° .

(t) The time of sunrise and sunset at any place during the equinoxes is about 6 A.M. and 6 P.M.

(u) The latitude of a place at which the ecliptic coincides with the horizon is $66^{\circ}33'$.

(v) The latitude of a place at which the celestial equator coincides with horizon is 90° .

(w) The stars which do not set are called the circumpolar stars.

(x) When the heavenly body is in the eastern hemisphere, its azimuth is east and its hour angle is in between 12 to 24 hours.

(y) The declination and the right ascension of a heavenly body are constant.

(z) Due to refraction the observed altitude of the sun appears smaller.

(a₁) The refraction correction depends upon the distance of the object.

- (b₁) Dip is the angle between the sensible and visible horizons.
- (c₁) The correction for index error is always positive.
- (d₁) Error due to parallax is always negative.
- (e₁) The correction for semi-diameter is generally considered while observing a star.
- (f₁) The earth rotates on its axis from West to East in clockwise direction.
- (g₁) Sidereal time measurement is based on the daily motion of the sun.
- (h₁) Apparent time measurement is based on the complete revolution of the fixed stars around the celestial pole.
- (i₁) Sidereal time is generally used in our every day life.
- (j₁) Standard time is used by the astronomers.
- (k₁) In railways, civil time is used.
- (l₁) There is only one standard time for each country.
- (m₁) The equation of time is the difference between sidereal time and apparent time.
- (n₁) The only cause of the equation of time is the obliquity of the ecliptic to the equator.
- (o₁) The Polaris is a bright star in the southern hemisphere.

2. What is a celestial sphere? Draw a neat sketch of a celestial sphere and show the following on it.

(a) Rational Horizon. (b) Ecliptic, (c) First Point of Aries (d) Hour Angle, (e) Right Ascension, (f) Azimuth.

3. Define a spherical triangle and give its characteristics. What is spherical excess? Give its expression.

4. The declination, right ascension, latitude and longitude of the First Point of Aries are zero. Do you agree? Justify your answer.

5. Write explanatory notes on the following with suitable sketch wherever possible:

(a) Great Circle, (b) Terrestrial Equator, (c) Sensible Horizon, (d) Celestial Meridian, (e) Prime Vertical, (f) First Point of Libra, (g) Altitude, (h) Solstices, (i) Zenith Distance, (j) Coaltitude, (k) Circumpolar Stars and (l) Nautical Mile.

3. What are the different systems of co-ordinates to locate the position of a heavenly body on the celestial sphere? Discuss their merits and demerits. Which of the systems, do you think, is the best and why?

7. What are the corrections that are generally applied in determining the true altitude of a heavenly body? Suppose, you are determining the true altitude of a fixed star, what are the corrections you will apply and why?

8. What is time? Discuss the merits and demerits of different systems of time measurement. Which of the systems is used in our everyday life?

9. Write short notes on:

Local Time, Standard Time, Sidereal Time, Solar Time, Equation of Time, Greenwich Mean Time.

10. Explain with a neat sketch how the Azimuth and the Bearing of a survey line are determined.

11. An observer 25 ft above the level of the sea was observing the altitude of the sun. What was the correction for dip? Take $D = 7916$ miles. Ans: $5'19''$

12. P lies to the west of Q and the meridian distance between them is 1-hour 30 minutes. The longitude of Q is $62^\circ - 30' - 40''$ (W). Calculate the longitude of P. Ans: $85^\circ - 00' - 40''$ (W)

13. An observation was made on January 3, 1937, on the sun using the upper limb and the reading was $41^\circ 46' 24''$ with face left. The semi-diameter of the sun at the time of observation was found to be $15' 34''$. The face-left and face-right observation of the theodolite on the top of a flag-mast were $5^\circ 4' 20''$ and $5^\circ 6' 40''$ respectively. Determine the true altitude of the sun.

Ans: $40^\circ 43' 44.84''$

14. Calculate the number of sidereal days in a year. Given the mean time = 4 hours 12 minutes 20 seconds P.M. and the equation of time = +5 minutes 25 seconds. Calculate the apparent time,

Ans. $366\frac{1}{4}$, 5 hours 6 minutes 53 seconds P.M.

15. The Sunrise and Sunset times on a certain day on November 1965 were 6 hours 54 minutes A.M. and 4 hours 53 minutes P.M. respectively. Determine the equation of time.

Ans :—16 minutes 30 seconds.

16. Calculate the G.M.T. to correspond with 2 hours 5 minutes 3 seconds A.M. L.M.T. of a place on January 3, 1937 whose longitude is $56^{\circ}30'15''$ (E)

Ans : 10 hours 19 minutes 2 seconds P.M.

17. Given Sidereal time = 5 hours 32 minutes 37 seconds and the right ascension of the mean noon = 7 hours 37 minutes 32 seconds. Calculate the Mean Time.

Ans : 9 hours 1 minutes 30 seconds A.M.

18. Calculate the time at New York (Long $74^{\circ}10'W$) when it is 8.00 P.M., at Dublin (Long. $6^{\circ}20'W$)

Ans. 10 hours 28 minutes 40 seconds A.M.

19. An observation for determining the azimuth of the sun on May 9, 1965 from a station whose latitude and longitude were $52^{\circ}26'40''N$ and $50^{\circ}22'15''W$ was made by taking readings on the sun's upper limb and the following observations were recorded. Calculate the azimuth of the line applying all necessary corrections.

Horizontal angle $\theta = 48^{\circ}12'13''$

Observed altitude of the sun = $46^{\circ}32'$

L.M.T. of observation = 9 hrs. 30 mins A.M.

Sun's declination at G.M.N. on May 9, 1965 = $17^{\circ}35' - 17.5''N$ increasing at the rate of $31.18''$ per hour

Ans ; $81^{\circ}42'86.73''$

CHAPTER 11 PHOTGRAMMETRY

11-1 Definition: Photogrammetry is a method of surveying in which plans or maps are prepared from photographs taken from suitable camera stations. This is also known as *photographic surveying*. The various purposes of photogrammetry are : (1) Preparation of topographic maps, (2) Preparation of composite pictures of the ground. (3) Acquisition of military intelligence, (4) Soil classification, and (5) Interpretation of geology. Photogrammetry are of two types : (1) Terrestrial or ground photogrammetry and (2) Aerial photogrammetry. The term *terrestrial photogrammetry* denotes that branch of photogrammetry wherein maps are prepared from photographs taken from the camera axis horizontal. The terrestrial photographic surveying is considered as a further development of plane table surveying. The terms *aerial photogrammetry* denotes that branch of photogrammetry wherein maps are prepared from photographs of the terrain in an area taken by a precision camera mounted in an aircraft flying over the area.

At present, ground or terrestrial photograph surveying is not much used except for small scale mapping of open hilly or mountainous countries and reproduction of plan and elevation views of buildings and structures, motion picture photography to make measurements involving transitory phenomena such as wave motion, currents, moving machinery, humans and animals in motion, and for furnishing supplementary ground control for aerial photography. It is not suitable for wooded country, moreover, vagaries of climate have been against its extensive use.

Aerial surveying is economical for quick rapid surveys of difficult and highly developed urban areas though for getting precision of details and for large scale maps, ground photogrammetry is better and cheaper. Aerial surveying is used with great success for the following purposes :

1. Reconnaissance and preliminary surveys
2. Roads and Railways
3. Water supply
4. Power schemes and transmission lines
5. Acquisition of land
6. Town and village planning
7. Flood control, irrigation, drainage and soil conservation.
8. Harbours, navigation channels and coastal defence.
9. Mining prospects
10. Study of geology
11. Soil and agricultural studies
12. Military installations, camping and forbidden zones (tactical use)

In terrestrial photography phototheodolite and favourable atmosphere for photography are the necessary requirements. Air photographs are taken in "strips" or "runs" with camera fixed in the aircrafts, while ground photographs are taken in pairs from the ends of a measured base with camera set on the stand.

11-2 Terrestrial Photogrammetry :

Classification : Terrestrial photographs are taken by a phototheodolite. The camera station is usually fixed in position. Photographs taken from automobiles, moving vehicles, boats or trains are considered to be terrestrial photographs and in all these cases the exposure station is not fixed. Photographs of still object such as building is classified as *still photography* whereas a series of photographs taken in fairly rapid sequence in order to picture the positions of a slow moving object at various circumstances, is termed as *quasi static photography*. The photographs of the movements of ships near a port or traffic flow after a regular interval of time is quasi-static photography. When measurements of an objects which changes its size, shape,

position or orientation from one instant to another are made through a motion-picture camera, it is termed *dynamic photography*.

11-3 Photo-Theodolite : Terrestrial photographs are taken by the help of the photo-theodolite (Fig 11.1) which is a combination of a camera and a theodolite. It does the dual work of recording horizontal angles or bearings of objects. at the same time takes the photograph of the area as viewed from that angle. There are two types of photo-theodolite, one known as the Bridges Lee Photo-theodolite (Fig. 11.1) and the other known as Wild Photo theodolite, the principle being the same.

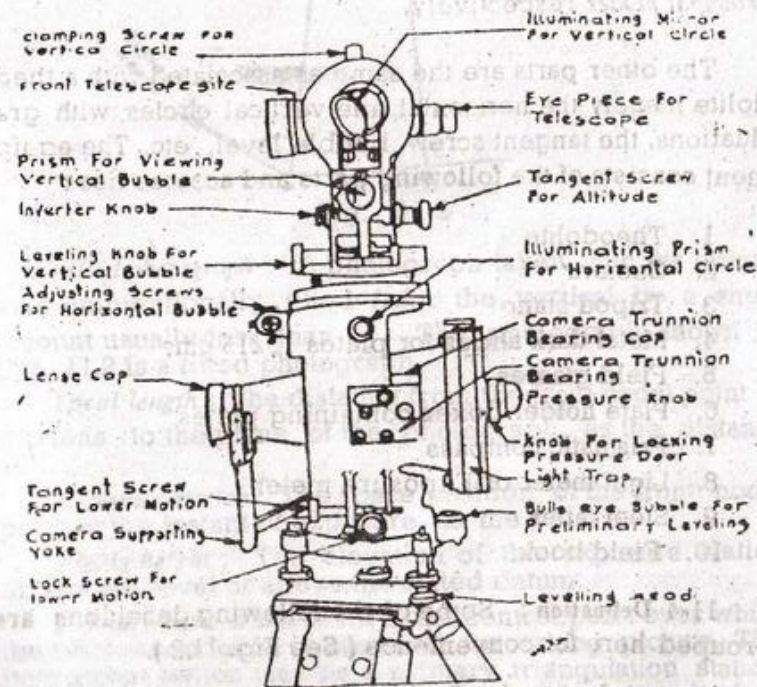


Fig. 11.1

The photo-theodolite consists of a camera of fixed focus type and a lens which is free from any distortion. The camera can be removed from the theodolite. At the back of the camera, immediately in front of the photographic plate, are a vertical and a horizontal hair which intersect at right angles at the centre of the plate, and images of these hairs are formed on the plate when it is exposed and developed. The intersection of the images of the hairs is called the *Principal point* and when the camera is in proper adjustment, this point coincides with the point when the optical axis of the lens meets the plate. The image of the horizontal hair in the plate is called the *horizontal line* and that of the vertical hair the *principal line*, and the horizontal and vertical planes through these lines are called *horizontal and principal planes* respectively.

The other parts are the same as associated with a theodolite namely the horizontal and vertical circles with graduations, the tangent screw, bubble level, etc. The equipment consists of the following parts and accessories :

1. Theodolite
2. Camera
3. Tripod stand
4. Metal dark slides for plates 10x15 cms
5. Field glasses
6. Plate holder boxes containing plates
7. Prismatic compass
8. Light meter or Exposure meter
9. Steel tapes
10. Field book.

11-4 Definition : Some of the following definitions are grouped here for convenience (See Fig. 11.2).

Vertical photograph : A photograph taken with the optical axis coinciding with the direction of gravity.

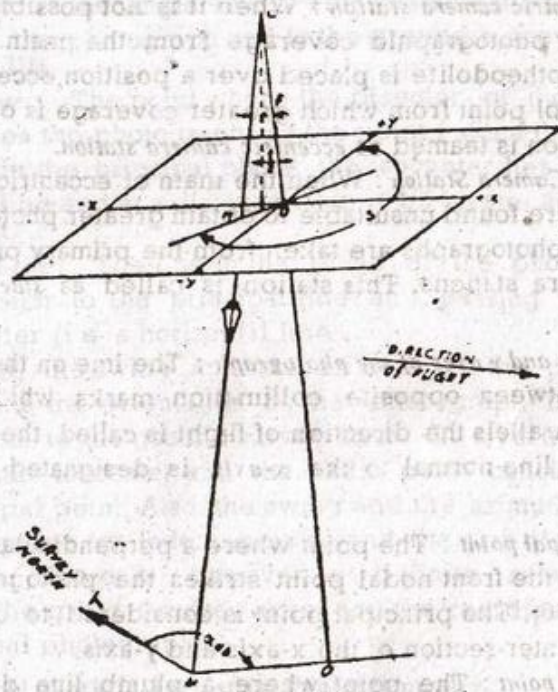


Fig. 11.2

Tilted photograph : A photograph taken with the optical axis unintentionally tilted from the vertical by a small amount usually less than 3° . The photograph shown in Fig. 11.2 is a tilted photograph.

Focal length : The distance from the front nodal point of the lens to the plane of the photograph, as the distance LO_1 .

Exposure station : The space position of the front nodal point at the instant of exposure, as the point L.

Flying height : The elevation of the exposure station above sea level or above the stated datum.

Primary camera station : It is the control point over which the photo-theodolite is centered for taking picture. This instrument station may be a primary triangulation station, or a secondary triangulation station. This can also be a bench mark.

Eccentric camera station : When it is not possible to get a greater photographic coverage from the main station, the phototheodolite is placed over a position eccentric to the control point from which greater coverage is obtained. This station is termed as *eccentric camera station*.

Side Camera Station : When the main or eccentric camera stations are found unsuitable to obtain greater photo coverage, the photographs are taken from the primary or eccentric camera stations. This station is called as *side camera station*.

The x and y axes of the photograph : The line on the photograph between opposite collimation marks which most nearly parallels the direction of flight is called the *x-axis*, while the line normal to the *x-axis* is designated as the *y-axis*.

Principal point : The point where a perpendicular dropped from the front nodal point strikes the photograph is the point O_1 . The principal point is considered to coincide with the intersection of the *x-axis* and *y-axis*.

Nadir point : The point where a plumb line dropped from the photograph, the point N is called the *ground nadir point*.

Tilt : The angle $\angle C_1 L_0 (=t)$ formed between the optical axis and a plumb line.

Principal plane and principal line : The vertical plane $O_1 L_0 N$ (or OLN) containing the optical axis is the principal plane. The line nO_1 which is formed by the intersection of the principal plane with the plane of the photograph is called the principal line.

Swing : The angle S measured in the plane of the photograph from the positive *y-axis* clockwise to a radial line from the principal point to the nadir point.

Azimuth of the principal plane : The clockwise horizontal angle measured about the ground nadir point from the ground survey north meridian to the principal plane of the

photograph, as the angle TNO . This is also called as the azimuth of the photograph and is the ground-survey direction of the tilt.

Isocenter : The point where the bisector of the angle of tilt strikes the photograph, as the point i since the angle of tilt lies in the principal plane, the isocenter lies on the principal line and at a distance $f \tan (t/2)$ from the principal point.

Axis of tilt : The line in the plane of the photograph perpendicular to the principal line and passing through the isocenter (i.e. a horizontal line).

The tilt, the swing and the azimuth of the principal plane define the orientation of the photograph in space with respect to the ground survey axes. On a vertical photograph, the isocenter and the nadir point coincide with the principal point. Also, the swing and the azimuth of the principal plane are indeterminate, and the axis of tilt does not exist. There is, however, a definite relationship between the ground-survey axes and the coordinate axes of a vertical photograph.

11-5 Field Work in Terrestrial Photogrammetry :

The field work of terrestrial photogrammetry consists of (1) reconnaissance, (2) triangulation and (3) camera work.

Reconnaissance : At first, the existing maps of the area to be surveyed should be collected and careful study should be made as it is very helpful in selecting suitable camera stations so that the entire area will be covered with a minimum number of photographs and the work can be done with speed and economy. Next, a careful reconnaissance of the area should be performed with a view to selecting suitable triangulation stations and camera stations.

The following factors are to be considered in selecting camera stations and base line :

1. The stations are chosen so as to give as much coverage of the terrain as possible.

2. The stations should be located on points of higher elevation than the surrounding terrain.
3. The camera station should be selected in such a manner that it will be a good triangulation station.
4. The stations should be so fixed that the object to be plotted on the map can be clearly and easily recognised on at least two photographs taken at different stations.
5. Base to be nearly horizontal or uniformly sloping.
6. Slope of a base line should not exceed 9° .

Triangulation: All camera stations should be connected by a triangulation system. The triangulation stations may sometimes be used as camera stations. The elevations of the camera stations should be determined by direct or trigonometrical levelling.

Camera work: Photographs are taken by photo-theodolite in pairs from the ends of a base line i.e. a line joining the camera stations which is carefully measured. The more important points should appear on three or more photographs and each photograph should contain at least one triangulation station.

Adjacent photographs should sufficiently overlap. The number of photographs to be taken at each station depends upon the area to be surveyed and the field view of the camera.

The following precautions should be taken to achieve accuracy of work:

1. The slide should never be removed or replaced in the direct rays of the sun. While the slide is removed or replaced the camera should be shaded with a focussing cloth.
2. All bubbles on the photo-theodolite should be checked if no bubble has drifted before taking photographs.
3. The horizontal circle of the theodolite to be set to $00^\circ-00'-00''$ and pointed at the initial triangulation station from which all horizontal angles are to be measured.

4. Vertical angles should be read carefully, once direct and once reversed.
5. Displacement of the bubble by one division results of the axis by 30 while the camera is calibrated for 15.
6. Sufficient stereoscope coverage of the area being photographed, which is the essential to the cross-identification between the terrestrial and aerial photographs.

11-6 Principle involved in plotting:

Fig. 11.3 shows A & B two camera stations where the base line AB is accurately measured. Photo-theodolite is kept over station A and camera axis fixed in the direction A and the angle $m \angle AB = \theta_1$, is read with the theodolite attached to the camera. P is a point which needs to be plotted. The direction of camera axis is so set that P is visible. Camera is shifted and set up over B and again camera axis is so turned as to include P again. Angle $m' \angle BA$ is measured on the theodolite $= \theta_2$. The camera is exposed and the left and right photographs are obtained as shown in the figure. The point P is photographed on left and right photographs. Let the photo-coordinates of the image P on the left photograph be (x_p, y_p) and on right photograph be (x_{p_1}, y_{p_1}) .

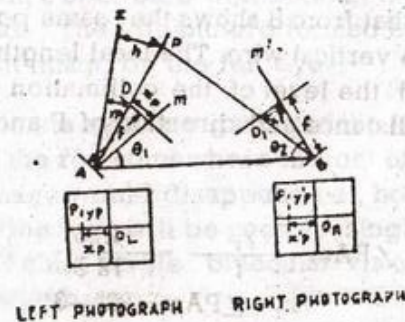


FIG. 11.3

Now plot line AB, a base line to any convenient scale and draw the directions of the camera axis at A and B by

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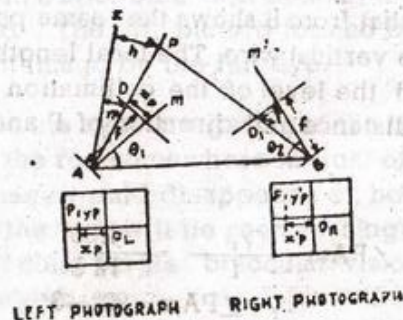


FIG. 11.3

Now plot line AB, a base line to any convenient scale and draw the directions of the camera axis at A and B by

measuring angle θ_1 and θ_2 . At a distance equal to the focal length of camera lens f , draw lines perpendicular to the camera axis and measure along this perpendicular line the photo axis x_1P . Hence P will be along the direction AD and BD_1 . The intersection of these two lines locates the point P on the paper. The distance AP and BP can be measured on the paper or can be analytically computed. In order to find the elevation of the point P we have.

$$\frac{h}{y_p} = \frac{AP}{AD} = \frac{AP}{\sqrt{f^2 + x_p^2}}$$

where AP, f , x_p , y_p are known.

$$h = y_p \frac{AP}{\sqrt{f^2 + x_p^2}}$$

R.L. of P = R.L. of station A + Height of instrument axis + h . Similarly, R.L. of P is also calculated from B and the mean of these two values is considered the R.L. of P.

Problem : In a terrestrial photogrammetric survey, photo-theodolite was set up at A & B of a base line which measured 100 meters. The optical axis of photo-theodolite at A & B are inclined inwards at angles of $52^\circ-32'$ and $40^\circ-38'$ to the base line. The print from A shows a point P 4.56 cms to the left of the vertical wire and 2.50 cms above the horizontal wire ; that from B shows the same point 5.80 cms to the right of the vertical wire. The focal length of the camera is 12.5 cms and the level of the collimation at A is 105 m. Calculate the distance and direction of P and the level of P. (See Fig. 11.3).

Solution :

$$\tan \angle PAm = \frac{x_p}{f} = \frac{4.56}{12.5}$$

$$\therefore \angle PAm = 20^\circ-3'$$

$$\tan \angle PBm' = \frac{x_2P}{f} = \frac{5.80}{12.5}$$

$$\therefore \angle PBm' = 24^\circ-54'$$

$$\angle PAB = 52^\circ-32' + \angle PAm = 72^\circ-35'$$

$$\angle PBA = 40^\circ-38' + \angle PBm' = 65^\circ-32'$$

$$\angle APB = 41^\circ-53'$$

$$\text{and } PA = 100 \times \frac{\sin \angle PAB}{\sin \angle APB} = 143 \text{ metres}$$

$$\text{R.L. of P} = 105 + \frac{2.50}{\sqrt{(12.5)^2 + (4.56)^2}} \times PA = 132 \text{ m}$$

11-7 Stereophotogrammetry

Stereoscopy states that if two overlapping air photographs containing the same objects, taken from different positions, are viewed through a stereoscope, the corresponding objects will fuse and the terrain will appear in three dimensions. Stereoscopic or binocular vision is the facility which makes stereoscopy possible. The perception of depth is made possible in several ways. A person viewing the objects in a room can gain an impression of depth by evaluating the apparent sizes of familiar objects. This appreciation of depth can be obtained by one eye only. Generally, two-eyed vision is required for realizing and measuring depth by stereoscopy. In stereoscopic vision the formation of double image phenomenon is important. If a small rod is placed in front of the eyes and the gaze is fixed on a spot on a wall behind, there will be two images behind. The left picture formed by the right eye while the right image by the left eye.

Now, while the eyes are gazing at the wall, there will be a position on the rod somewhere in front of the wall where the double image would disappear i.e., both the rod and the object on the wall will be seen as single image. This is the depth of clear single binocular vision and is very important in stereoscopy.

11-8 Parallax

The term *parallax* is applied to the movement of the image of one stationary object with respect to the image

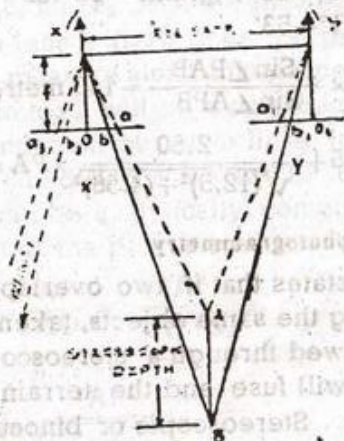


FIG. 11.4

of another stationary object when the eye is moved sideways. When a person sights on a target with a theodolite and the image of the target formed by the telescope objective does not lie in the plane of the cross-hairs, parallax is said to exist. It is detected by moving the eye back and forth slightly behind the eye-piece and if the images of the object and the vertical cross hairs move relative to one another, there is existence of parallax.

Stereoscopic viewing and parallax can be understood from the aid of the Fig. 11.4, where the two photographs containing points a and b representing A and B on the ground are taken. The distance between the lenses (as they are placed under a stereoscope) is the distance between two eyes of a person and O and O_1 are the principal points of respective photographs. Rays from left and right eyes cut the photograph at a and a_1 for A while similarly rays cutting through b and b_1 meet at B . By apparent observing point A above B , the depth impression is obtained and this vertical depth between these two points measured perpendicular to the base is called the *stereoscopic depth*. The image point a on the left photograph is to the right of the principal points O and this has shifted to

the left (at a_1) of the principal point O_1 on the right photograph when photographed from different positions. This apparent movement of the same point due to the photographs being taken from different positions is called *parallax*.

Now if xa_1 line is drawn parallel to ya_1 to cut at a , then the distance aa_1 is called *absolute parallax* of a and is denoted by P_a . Similarly, bb_1 is the absolute parallax of the point b and is denoted by P_b . The difference between the absolute parallaxes of the points A and B namely $P_a - P_b$ is called "difference of absolute parallax" and is a measure of depth perception between those points. The difference of absolute parallax can be measured by an instrument known as *Stereometer* or *Parallax Bar* while the stereoscopic viewing of photographs are taken by stereoscopes and are of two types, namely, the *lens stereoscope* and the *mirror stereoscope*.

A & B are two points whose (Fig. 11.5) heights are h_a & h_b from Datum plane. H is the height of aircraft. f is the focal length of the camera and B is the air base i.e., the distance between positions of aircraft at the instant of next photograph O and O' are the positions of camera buses. The positions of A & B on the two photographs are a_1, a_2 and b_1, b_2 respectively. ca_1 is drawn parallel to $O'a_1$, $a_1a_2 = P_a$ and $b_1b_2 = P_b$ are the absolute parallaxes of A & B respectively.

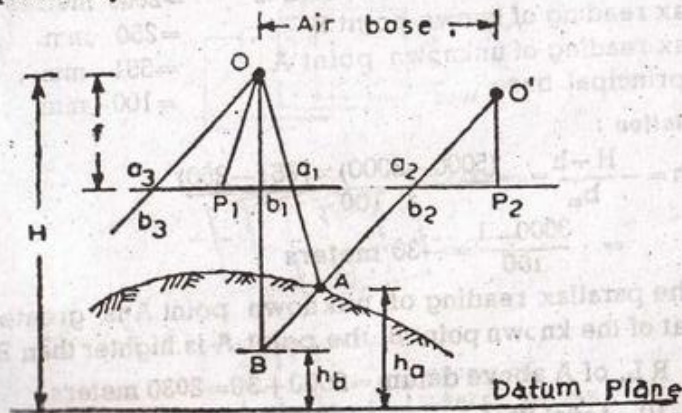


Fig. 11.5

Now considering similar triangles O_3a_1 & O_1BO' or $O_1b_1b_1$ and O_1BO'

$$\frac{a_1 a_1}{f} = \frac{B}{H-h_1} \text{ and } \frac{b_1 b_1}{f} = \frac{B}{H-h_b}$$

$$\text{or } p_a = \frac{fB}{H-h_1} \text{ and } p_b = \frac{B}{H-h_b}$$

$$\text{or the general equation, } p = \frac{fB}{H-h}$$

Differentiating p with respect to h

$$\frac{dp}{dh} = -\frac{fB}{(H-h)^2} \dots \dots \dots (1)$$

If b_m is the mean principal base and equal to the absolute parallax, then

$$\frac{b_m}{f} = \frac{B}{H-h} \therefore fB = b_m (H-h),$$

substituting this in equation (1),

$$\frac{dp}{dh} = \frac{d_m}{H-h} \dots dh = -\frac{H-h}{b_m} \times dp \quad (2)$$

Problem: Compute the reduced level of a point A from the following data.

Height of aircraft above datum plane	= 5000 meters
Height of known point B above datum	= 2000 metres
Parallax reading of known point B	= 250 mm.
Parallax reading of unknown point A	= 551 mm.
Mean principal base	= 100 mm

Solution:

$$\begin{aligned} dh &= -\frac{H-h}{b_m} = -\frac{(5000-2000) \times (251-250)}{100} \\ &= -\frac{3000 \times 1}{100} = -30 \text{ meters} \end{aligned}$$

Since the parallax reading of unknown point A is greater than that of the known point B, the point A is higher than B

\therefore R.L. of A above datum = 2000 + 30 = 2030 meters.

11.-10 **Aerial Photogrammetry:** Aerial photogrammetry consists of four operations, namely (1) flying, (2) photo-

graphy, (3) ground control and (4) compilation or mapping. The equipments required for these operations comprise (i) an aeroplane, (ii) a camera and (iii) accessories required for interpretation and plotting maps, like Zeiss Stereoplathograph, Zeiss Aerocartograph, etc.

Flying: During taking the photographs, it is of utmost importance that the aeroplane should fly at a uniform speed on a straight course in a given direction at a constant height. If there is any variation in the flight altitude, the scale of the photographs will be changed, on the other hand, any tilt of the camera will cause distortion in the photographs.

For good navigation, the course taken by an aircraft and the area covered by a photograph is shown in the Fig. 11.6. If the flight is not properly planned, there will be crabbing and drifting effects on the photograph. Crab is the term given to designate the angle formed between

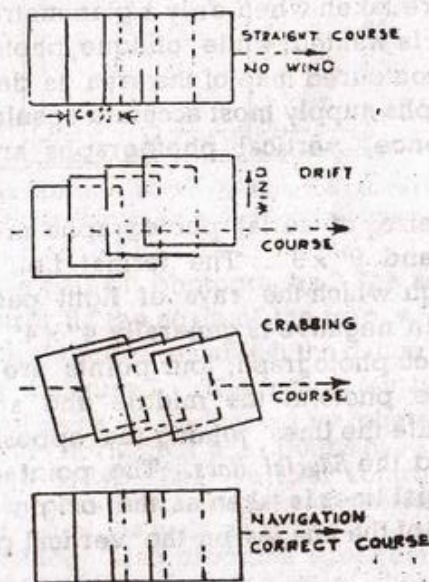


Fig. 11.6

the flight line and the edges of the photograph in the direction of flight. It is caused by not having the focal plane of the camera squared with the direction of flight at the instant of exposure. The effect of crab is to reduce the effective width of coverage of the photograph.

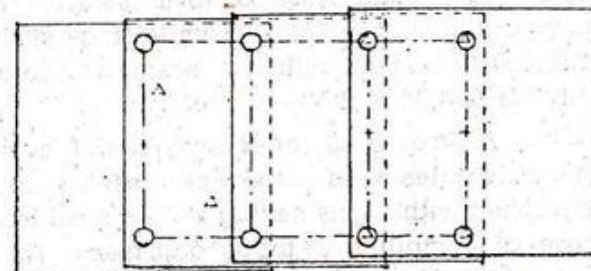
Drift is caused by the failure of the aircraft to stay on the predetermined flight line. If, for an example, the aircraft drifts 400 ft to one side or the other of the flight line and the average scale of the photography is 1 : 9600 the drift amounts to about $\frac{1}{4}$ inch at the photograph scale.

Photography : Photographs are taken automatically with special cameras on parallel strips with generous overlap both in the direction of the flight and at right angles to it. Generally, there are two ways of taking areal photographs : (1) vertical, and (2) oblique. In the former, photographs are taken with the axis of the camera pointing vertically downwards, while in the latter the camera axis is tilted (inclined) by about 30° to forward direction. The vertical photographs are taken when only a planimetre map of the area surveyed is wanted, while oblique photographs are taken when a contoured map of the area is desired. Vertical photographs supply most accurate results than oblique ones. Hence, vertical photographs are preferred always.

The usual sizes of aerial photographs are $4\frac{1}{2}'' \times 4\frac{1}{2}''$, $7'' \times 7''$, $9'' \times 7''$ and $9'' \times 9''$. The format i.e., the size of opening through which the rays of light pass from the lens to strike the negative is generally $4'' \times 4''$, $7'' \times 7''$ and $9'' \times 9''$. On each photograph, four points are marked on each side of the photo at the middle and are known as *fiducial marks* while the lines joining the opposite fiducial marks are called the *fiducial lines*. The point of intersection of the fiducial lines is taken as the origin from which the coordinates of the images on the vertical photographs are referred.

Ground Control : In order to produce an accurate map from aerial photographs, ground control is very essential. It consists in locating the positions of number of points over the area to be surveyed and determining their levels. These control points must be such that they can be easily identified on the photographs. These control points are located by triangulation. Another vital requirement is that each strip of the area must appear on at least two photographs in order to obtain stereoscopic views of the whole area.

Topographic mapping by photogrammetry requires that two points for horizontal control and four points for vertical control (Fig. 11.7) appear in the overlap area of



○ VERTICAL CONTROL POINT Δ HORIZONTAL PASS POINT
 Δ HORIZONTAL CONTROL POINT + PRINCIPAL POINT

Fig. 11.7

each successive pair of photographs. The two points for horizontal control fix the scale of the map, while the four points for vertical control establish the datum above which elevations are measured. The points for horizontal and vertical control have been located by photo control field surveys while the horizontal pass points have been located photogrammetrically. The vertical control points are located at the corners of the area of overlap to control adjacent flight lines, as well as adjacent area of overlap. The corner location also provides strongest determination of the vertical datum. In preparation of the map a photograph should have focal length of the camera lens, altitude of flight of aircraft, longitudinal overlap (usually 60%), lateral overlap (usually 25%), number of exposures required per strip, number of strips and amount of film.

Scale of Vertical photographs: The concept of scale is the ratio of distance measured on a map or drawing to the corresponding distance on the ground. The scale of map may be expressed as a fraction, with the numerator and the denominator in the same units. For example, if 1 inch on map equals 16,000 inches on the ground, the scale may be expressed as 1 in 16,000 in which the scale ratio is 1 ft/16,000 ft. Since the common units can be eliminated, the scale is expressed simply as 1/16,000 which means that 1 unit of length on the map represents 16,000 of the same units of length on the ground. This form of scale expression is called *Representative Fraction (R.F.)* The scale of aerial photographs is expressed as a representative fraction. Small scales vary from 1/30,000 to 1/250,000. Medium scales vary from 1/5,000 to 1/30,000 while large scales vary from 1/250,000 to 1/5,000. Military scales of photography are usually 1/250,000 i.e. 2.5" = 1 mile.

Problem: An area is 20 miles long in the north-south direction and 16 miles wide in the east-west direction is to be photographed with a lens having a 12-in focal length for the purpose of compiling a topographic map. The photograph size is 9 by 9 in. The average scale is to be 1:10,000 effective at an average elevation of 800 ft above sea level. Overlap is to be at least 60% and sidelap 25%. The ground speed of the aircraft will be maintained at 200 mph. The flight lines are to be laid out in a north-south direction on an existing map having a scale of 1:30,000. The two other flight lines are to coincide with the east and west boundaries of the area. Determine the data for the flight plan.

Solution:

- (a) Flying height: $\frac{1 \text{ ft}}{H \text{ ft.}} = \frac{1}{10,000}$ So, $H = 10,000$ ft above the 800 ft elevation, or $H = 10,800$ ft above sea level.

- (b) Ground distance between flight lines: Since the minimum sidelap is 25%, the photographic distance between flight lines is 75% of 9 in or 6.75 in. The ground spacing is

$$\frac{6.75}{12} \times 10,000 = 5625 \text{ ft.}$$

- (c) Number of flight lines: The total width of the area is $16 \times 5280 = 84,480$ ft. So, the required number of flight lines is

$$\frac{84,480}{5625} + 1 = 15.1 + 1 = 16 + 1 = 17$$

- (d) Adjusted ground distance between flight lines: With an integral number of flight lines, the original computed spacing is slightly too large. The actual spacing of flight lines will be

$$\frac{84,480}{16} = 5280 \text{ ft}$$

The sidelap will be greater than 25%.

- (e) Spacing of flight lines on flight map: The distance on the map corresponding to a ground distance of

$$\frac{5280}{50,000} \times 12 = 1.27 \text{ in.}$$

- (f) Ground distance between exposures: Since the overlap is 60% the net gain per photograph is 40% of the width of photograph, or $0.40 \times 9 = 3.60$ in. The corresponding ground distance is

$$\frac{3.6 \times 10,000}{12} = 3,000 \text{ ft.}$$

- (g) *Exposure Interval*: The time interval between exposure is usually an integral number of seconds. The aircraft speed is 200 mph i.e. 293.41 ft/sec. The required exposure interval is

$$\frac{3000}{293.4} = 10.2 \text{ sec. or } 10 \text{ sec.}$$

- (h) Adjusted ground distance between exposures :
For the adjusted exposure interval, the ground distance is $293.4 \times 10 = 2934$ ft.
- (i) number of photographs per flight line : The total length of a flight line is $20 \times 5280 = 105,600$ ft. If allowance is made for two extra exposures at each strip, the number of photographs per flight line is $\frac{105,600}{2934} + 4 = 40$

The entire photography will require $17 \times 40 = 680$ photographs.

Problem : For a flight planning mission, the following data are given :

Focal length of camera lens : 6", altitude of aircraft : 16,000 ft. width : 30 miles, length of flight line : 60 miles. plate size : 7" x 9", longitudinal and lateral over-lap : 60% and 25% respectively, map scale : 1/25,000, aircraft speed : 200 mph.

Find out the scale of photograph, distance between flight line, distance of first flight line from edge of area, total number of lines, total number of photographs per strip and flight time :

Solution :

$$(a) \text{ Scale of photograph} = \frac{f}{H} = \frac{6''}{16,000} = \frac{1}{32,000}$$

(b) Distance between flight lines = $9'' \times 75\% = 6.75''$ on photograph. The corresponding distance on map will be

$$\frac{\text{Map distance}}{\text{photo distance}} = \frac{\text{Map scale}}{\text{Photo scale}}$$

$$\begin{aligned} \therefore \text{Map distance} &= \text{photo distance} \times \frac{\text{Map scale}}{\text{photo scale}} \\ &= 6.75 \times \frac{1}{25,000} \times \frac{32,000}{1} \\ &= 8.65'' \text{ on map,} \end{aligned}$$

So 8.65'', is the distance of flight lines on map whose scale is $\frac{1}{25,000}$.

(c) Distance of first flight line from-edge of area = $4 \text{ } 50'' - 2 \text{ } 25'' = 2.25''$ on photograph

$$\text{Corresponding map-distance} = 2.25 \times \frac{1}{25,000} \times \frac{32,000}{1} = 2.88''$$

So, 2.88'' is the map distance of first line of flight from edge of area

(d) Total number of flight lines

Width of flight = 30 miles = 158,400 ft

This length corresponds to map length

$$\frac{12 \times 158,400}{25,000} = 76.02'' \text{ on map}$$

$$\therefore \text{Number of flight lines} = \frac{76.02}{8.65} = 9 \text{ flights}$$

(e) Total number of photographs per strip length of flight line = 60 miles = 316,800 ft

Each print covers an area of

$$\begin{aligned} &= \frac{7}{12} \times \frac{32,000}{1} \text{ ft by } \frac{9}{12} \times \frac{32,000}{1} \text{ ft.} \\ &= 18,700 \text{ ft by } 24,000 \text{ ft.} \end{aligned}$$

With 60% overlap in line of flight, there is available only 40% each print i.e. $0.4 \times 18,700 = 7,480$ ft

$$\text{No. of prints} = \frac{316,800}{7,480} = 42.5$$

use 43 prints per strip.

(f) Determination of flight time :

Total flight lines = No. of flights \times length of each flight = $9 \times 90 = 810$ miles.

Assume 3 miles as turning for each flight = $9 \times 3 = 27$ miles. Trial run to determine camera position, crab, assuming 2 direction runs = $60 \times 2 + 3 \times 2 = 126$. Total mileage of photo graphing only = $810 + 27 + 126 = 963$ miles.

Coverage of 693 miles @ 200 mph will take

$$\frac{693}{200} = 3.47 \text{ hours.}$$

Adding 0.25 hours for changing of magazine

$$\text{Total} = 4.72 \text{ hours.}$$

Assume that 1 hour will take to reach the place of photography, to and from distance— $2 \times 1 \times 200 = 400$ miles

Mileage for photography = 693

To and from distance = $\frac{400}{1093}$ miles

Total flying hours = $\frac{1093}{200} = 5.47$ hours

Allow contingency of 25% = 0.55 hours

Total flying time = 6.02 hours.

Exercise

1. Examine the following statements and write whether they are true or false.
 - (a) Photographic surveying is most suitable for flat or wooded country.
 - (b) Aerial surveying is not suitable for inaccessible regions.
 - (c) The Field work of terrestrial photographic surveying consists of (1) flying (2) Photography (3) ground control.
 - (d) The photographs are taken in pairs from the end of base line.
 - (e) The base line is the line joining two camera stations.
 - (f) The number of photographs to be taken at each station depends upon the area to be mapped and the field view of the camera.
 - (g) The camera stations are fixed by triangulation survey.
 - (h) In taking aerial photographs, it is utmost important that the aeroplane should fly at a uniform speed on a straight course in a given direction at a constant height with the axis of the camera vertical.

- (i) The scale of the photograph will not be changed if there is any variation in the flight altitude.
 - (j) Ground control is the process of locating the positions of a number of points all over the area to be surveyed and determining their levels.
 - (k) Each portion of the area to be surveyed must appear on at least two photographs.
 - (l) Flight altitude is the true height of the aeroplane above the mean sea level.
 - (m) Maps may be prepared from photographs by the "Three Point Problem"
2. What is meant by photogrammetry? What are the different branches of photogrammetry? Explain their uses.
 3. Discuss briefly the field works that are essential for terrestrial photogrammetry.
 4. Explain with neat sketches the fundamental principle of terrestrial photogrammetry.
 5. Explain the operations in aerial photogrammetry. What is meant by the scale of vertical photographs?
 6. Write notes on :
 - (a) Stereophotogrammetry, and (b) Parallax
 7. Give the different types of photographs you know of with their uses.
 8. The distance between two image points, 200 metres above sea level, is 126.5 mm. The corresponding distance on a $\frac{1}{50,000}$ photo-sheet is 25.30 mm. Find the scale of the photographs.

Ans. $\frac{1}{10,000}$
 9. Explain the following photogrammetric terms :
Principal point, isocentre, nadir point, overlap, crabbing, parallax, fiducial marks, rear vertical photographs.

10. Photographs at a minimum scale of 1:6000 are to be taken for a border road design map of a hilly area having elevation ranges from 160 metres to 2000 metres. If the focal length of the camera lens is 140 mm, what should be the flying height of the aircraft above M.S.L. ? What will be the largest scale ?

$$\text{Ans : } 1060 \text{ m } \frac{1}{4733}$$

11. Derive the parallax equation for determining heights from a pair of vertical photographs.
12. The following data are given for flight planning of the area :
 Photo size : 18x18 cms, focal length 21 cms,
 scale : $\frac{1}{20,000}$ overlap : 60% and 20%, length of terrain East-West side : 100 kilometers and North-South 50 kilometer. The flight is East-West. Image movement allowed 0.02 mm, flying speed : 293 km/hr. Determine (a) flying height above terrain, (b) area covered by one photo, (c) air base, (d) flight line spacing, (e) no. of strips, (f) no. of photos per flight, (g) total no. of exposures, (h) maximum exposure time, (i) exposure interval.

$$\text{Ans : } 4200 \text{ m, } 1296 \text{ sq. km., } 144 \text{ metres, } 51,800, 18$$

$$\text{nos, } 74, 1332 \text{ nos, } \frac{1}{205.5} \text{ secs., } 17.5 \text{ secs.}$$

12-1 Definition : Project Surveying includes all field works and requisite calculations together with maps, profiles and other related drawings involved in the planning and construction of any engineering project like railways, highways, irrigation canals, sewer lines, tunnels, dams, etc. The project should be planned in such a way that it may be constructed and operated with the greatest economy and utility. Every project has its own characteristics and peculiar problems which are to be attacked from different critical angles for economical solution. The accuracy of the project work depends entirely upon the sound knowledge and judgement of the project engineer, the types of instruments used, time at disposal and the fund available.

12-2 Irrigation Project : The surveying that is conducted for the completion of an irrigation project may be classed into three different categories. They are : (1) Preliminary survey, (2) detailed survey, and (3) construction survey.

Preliminary Survey : Before final decision is taken to take up an irrigation project, preliminary survey is essential. It includes the determination of the following :

- Catchment area of the stream proposed to be used as the source of supply.
- Area of land proposed to be irrigated.
- Possible sites of dams in a reservoir project.
- Possible sites of weirs, sluices, headways of canals, and other hydraulic structures.
- Alignment of canals.
- Quality of land and water for irrigation.
- Collection of rainfall, gauge and discharge records of the stream.
- Possible crops and water requirements.
- Drainage requirements.

Items (a) to (e) are determined from the contour maps of the area. The quality of land and water can be determined by taking samples from the field and testing them in the laboratory. Rainfall, gauge and discharge records are obtained from the hydrological department. Possible crops and their water requirements are obtained from agriculture department.

After collecting all the data, a project report is prepared along with a rough cost of the project. Financial aspect is then examined and cost benefit ratio is worked out.

Detailed Survey: After the feasibility of the project has been established, detailed survey of the following is essential:

- (a) Location of headworks, guide bunds, alignment of canals, distributaries and water courses.
- (b) Location of falls, cross drainage works and canal escapes.
- (c) Alignment of drainage canal.
- (d) In case of a reservoir project, detailed geological investigations should be made in addition to the detailed survey of the site, sites of the weirs, outlets and detailed contour survey of the reservoir area.

Construction Survey: The final construction survey includes the following operations:

- (a) Topographical survey of the site, preparing site plan for acquisition of land.
- (b) Setting up of pillars for demarcation and also to serve as reference line.
- (c) Establishment on ground B.M. pegs for carrying on construction and giving alignment of the structures.

- (d) Preparing running bills of the work executed monthly.

12-3 Hydrographic Project: A hydrographic project includes the determination of shore lines, sounding and siltation, navigable depths, velocity and characteristic of the flow of water, location of buoys, light houses, sandbars, etc.

Survey of Shore Lines: This is executed either from a speed boat or by running a traverse along with shore. First the control stations (generally church spires, wind mills, light houses, etc.) are fixed and surveyed by traversing round the coast. All details in the shore located by means of offsets measured with a tape from the traverse line or by stadia or plane table or by prismatic compass. The position of the highwater line may be roughly determined from deposits and marks on the rocks. Since the low water mark is bare for a short time, it is located generally by interpolation from soundings.

Soundings: The measurement of depths below the water surface is known as *sounding*. The object of sounding is to take cross-section or the longitudinal section of the water profile from the bed. The interval of soundings depends upon the nature of the streams. For a small stream where the bed changes abruptly, sounding, should be at 3 ft interval and on a stream upto 300 ft wide, the interval should be 10 to 20 ft and for larger streams the interval may be increased. In case of a shallow stream, soundings may be taken by means of a measuring rod and in case of a deeper stream by log line or sounding rope with a lead weight. If the current is very swift or the depth is very great, an echo-sounder is preferable.

Location of Sounding Points: In Fig. 12.1, A & B are the two stations at a known distance l apart on the bank, C is the position of the boat and θ is the observed angle at B measured by a theodolite.

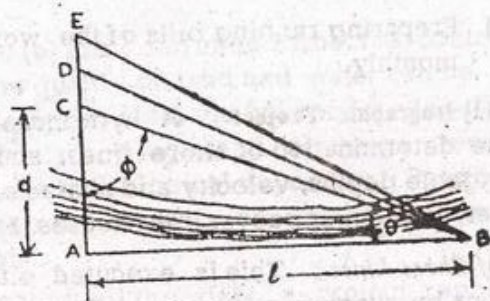


Fig. 12.1

(1) The position of sounding point C can be located by direct tape measurement from A. (2) This can also be located by stadia reading taken from an instrument at A on to the staff held at C. (3) By applying trigonometric functions, distance $AC = d = l \tan \theta$. (4) By angular measurement of ϕ from the boat at C by a sextant.

$$AC = d = l \cot \phi$$

12-4 Sewer pipeline, canal, highways and railway projects : The operations in these type of projects involve the following considerations :

1. **Reconnaissance survey :** It is a quick survey of the whole area through which the different possible routes can be aligned. In these type of survey generally very quick and light instruments like prismatic compass, tape, hand level are used. This type of survey needs the examination and information regarding the following points which are finally submitted by the engineer in the form of a reconnaissance report.

(a) Topography of the country, (b) Probable alignments, (c) Obligatory areas, towns, rivers, ridges, etc., (d) Soil and drainage conditions, (e) Highest flood level, (f) Availability of construction materials and human labour (g) Land value for acquisition of land, (h) Time required for construction, (i) Rise and fall of the country, (j) General slopes of the hills,

Comprehensive notes should be made of all topographical features along the alignment such as directions of streams, water levels, slopes of waterways that may be crossed. The probable quantity of excavation, cost of embankment and bridging per mile should be noted.

2. **Preliminary Survey :** This involves detailed instrumental examination of the strip of land along the selected alignment. The instruments generally used are transit theodolites, levels, tapes, chains, plane tables, tachometers, hand levels, etc. The following is the procedure which is followed in this type of survey.

(a) An open traverse is conducted along the middle of the strip and stakes are set on all stations.

(b) The centre line of the alignment is first marked and levels at 50 ft intervals are taken for longitudinal sectioning.

(c) Cross levels are taken generally at 100 ft along and at right angles to the longitudinal section on either sides.

(d) All the interior details are located by plane tabling.

(e) After finishing the field work, one topographic map and another contour map are to be prepared. The data furnished by these two final maps will serve all the purposes of marking a last paper location of the line which will guide the project engineer for laying out in the field.

(f) In deciding the alignment and giving proper grade for these type of projects, it should be remembered that : changes in alignment should be minimum, the grades should be as small as possible consistent with the nature of land, the proposed alignment should be such that the amount of cut and that of fill balances each other, drainage facilities should be adequate, canals intended for navigation should have less bends and the curves should be of large radius.

(g) The longitudinal section is plotted to a horizontal

scale of 1 inch = 100 ft to 400 ft and vertical scale of 1 inch = 10 ft to 50 ft. The contour interval is usually 5 ft but it may be 1 ft in city survey and 10 ft for steep ground.

3. *Final location survey*: In this survey the alignment of the proposed work which has been finally decided in the office on the preliminary map, is set out on the ground exactly in the same position as it appears on the paper. The procedure adopted is as follows:

- (a) The north direction is located at the starting point where the chainage is assumed to be zero.
- (b) The centre line is staked out first by driving pegs at 25 ft to 100 ft intervals. The entire boundary of the strip of land is marked by constructing pillars so as to secure the right of way with a legal description of the property surveyed.
- (c) The positions of other various points are located on the field with proper measurements by scaling the dimensions from the preliminary map prepared in the office.
- (d) Stakes are driven at the tangent points of the curves and bends. The curves on bends are then laid in the same magnitude as shown on the prepared map.
- (e) Cross sections are taken to calculate the amount of earthwork in cut or fill.
- (f) Data for designing and estimating culverts, bridges and other structures are collected.

(4) *Construction Survey*: In this type of survey the details of the project are set out. It involves the following operations:

- (a) All stakes along the alignment are checked and if some are missing, they are reset from the plan and field data. All levels are checked and additional bench marks are established.

- (b) Borrow pits, bridges, culverts, prismatic compass, out on site.
- (c) Slope stakes marking the extremities are fixed.
- (d) The transition and vertical curves are set out.
- (e) The dimensions of the finished sub-grade are checked by means of tamplets.
- (f) Soundings and bearings are taken for important structures.
- (g) Rivers are surveyed carefully and the waterways for bridges are determined taking careful note of headways.
- (h) The measurements of different works completed are recorded in the Measurement Book (M.B.) and details of materials and number of labourers are noted down at regular intervals to pay running bills during the construction period of the project.

12-5 *Tunnel Project*: Tunnels are underground conduits. They are constructed in railways, highways, big cities, hydro-electric projects, irrigation projects, etc. They shorten the distance between two stations separated by a ridge, reduce grades, avoid excessive cost of maintenance of an open cut subjected to land-slides and avalanches and meet the requirements of rapid movements of traffic in big cities.

Tunnelling involves the following operations:

- (a) *Setting out or surface survey*: This includes: (1) a preliminary survey by transit and stadia for two to three miles on either side of the proposed alignment, (2) a map with a scale of 1 in 1500 ft to 2500 ft. and contour interval of 10 ft to 20 ft is drawn, (3) final alignment is selected from this map, (4) a detail study of the geological formation of strata, (5) gradient to be provided to the proposed tunnel, (6) length of the tunnel with fixing of permanent stations of the alignment.

(b) *Alignment transference from surface to underground*: Shafts (6 ft to 8 ft diameter) are constructed at regular intervals exactly on the alignment. A platform is constructed on top of the shaft and two piano wires with heavy plumb bobs attached at a known distance apart are suspended from the platform. The distance between the wires at the top and at the bottom of the shaft is to be measured carefully and this should be same. The line joining the two wires gives the direction of the alignment underground. The theodolite is placed at the bottom of the shaft in order to prolong the alignment underground. The theodolite is set exactly in the line with both wires by trial and error methods. The excavation is proceeded and the theodolite is shifted by locating intermediate stations.

(c) *Levels transference from surface to underground*: The R.L. of any point on the shaft along the alignment is determined from a bench mark by ordinary levelling. The level is then transferred from this point to the bottom of the shaft deducting the height of the shaft. The height is measured by suspending a chain or a rope or by a specially constructed rod.

12-6 House Setting: It means the setting out on the ground of the outlines of excavation of a building from the plan. The procedure is as follows:

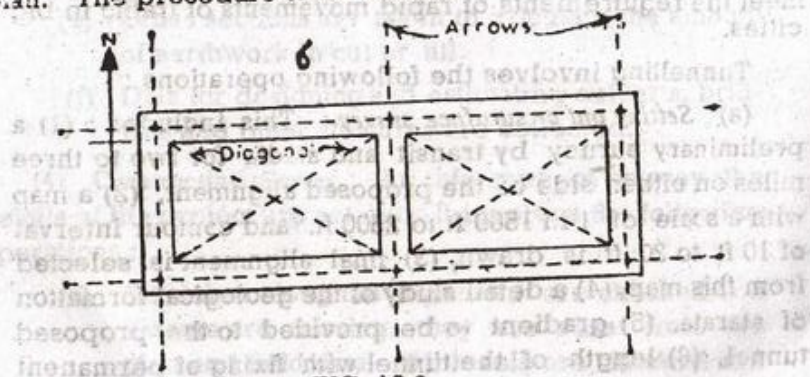


FIG. 12.2

- (a) The north line is fixed by a prismatic compass.
- (b) The centre lines of the outer walls are fixed by arrows and ropes (Fig. 12.2).
- (c) The intersections of the centre lines at the corners are marked by pegs. The centre lines are extended beyond the outer edge of the wall by 3 ft. Here arrows are driven and the ropes are tied.
 - (i) The diagonals are checked for the accuracy of the dimensions of the individual rooms. In calculating the diagonals, slide rule should not be used and the dimensions should be checked upto three place of decimals.
- (e) Foundation trench-width is demarcated in the same way by knowing its width across the centre line from the sectional elevation of the foundation.
- (f) A level is placed near the plot and level pegs are set at various corners of the building. The tops of the pegs will indicate the same elevation. Excavation depths are then measured vertically from the tops of these pegs.
- (g) After excavations, the base of the foundation is checked by using a spirit level. If the bubble of the spirit level remains in the centre of its run, it indicates that the base of the trench is horizontal.

EXERCISE

1. A reservoir is to be constructed by building a dam. In a given position to a given height across a valley. Enumerate the procedures to be carried out in the field and in the office to compute the capacity of the reservoir.
2. Suppose, you are asked to prepare a master plan of sewerage and drainage systems for a city. Discuss in

detail the different types of survey you need to prepare the same.

3. Explain how you would carry out the survey work for an irrigation canal.

4. It is proposed to construct a road from Dacca to Faridpur. What are the different kinds of surveys you must carry out before you can take in hand the construction of the road? Give full details of the works to be done in each of these surveys.

5. Explain the purposes of tunnelling. Discuss the field works necessary for a tunnelling project which is to be used for a railway.

CHAPTER 13

ERRORS IN SURVEYING

13-1 **Definition:** By error in surveying we mean the discrepancy between the observed and the true value of any measured quantity.

Errors in surveying may be classed into the following three groups:

1. **Mistakes:** This type of error can not be avoided. This also does not follow any mathematical rule (Law of Probability). The followings are the few examples:

(a) Erroneous recording such as 945 in place of 954.

(b) The counting of 8 for 3.

(c) Erroneous calculation of observed data.

2. **Cumulative Errors:** This type of error makes the observed reading either too large or too small. These errors may arise due to variation of temperature, humidity, pressure, wind velocity, curvature and refraction. The faulty setting of any instrument or the personal vision of any individual also tends to cause these errors. The followings are the few examples:

(a) When the length of chain is either longer or shorter than the standard one due to variation of field temperature and pull.

(b) Faulty alignment of a line.

(c) When the instrument is not centered properly.

(d) When the instrument is not adjusted properly.

3. **Compensating Errors:** This type of error tends to occur in both directions and thereby compensating each other.

The following are the few examples:

(a) The discrepancy between chain and tape measurements when both are used simultaneously.

(b) Inaccuracy in marking chain lengths on the ground.

- (c) Inaccurate centering.
 (d) Inaccurate bisection of the objects.

Besides these, there are also other types of errors. In order to understand them clearly, the following terms will be helpful.

Observation : In surveying by observation we mean the numerical value of a measured quantity in the field. An observation may be direct or indirect, when the magnitude of any quantity (say measurement of a line) is measured directly it is said to be *direct observation*. And in the case of an indirect observation, the magnitude of the quantity is not measured directly, such as the measurement of an angle by repetition.

Again, an observed quantity may be independent or conditioned. A quantity is said to be independent when its value does not depend upon any other quantity such as R.L. of mean sea levels or bench marks, while a quantity is said to be conditioned when its magnitude depends upon other quantities (the angles of a triangle).

Weight : The weight of an observation is a factor depending on the significance or importance attached to the observation. As for example, if the weight for an angle of magnitude 45° is 3 and that of 44° is 1, then the mean angle is :

$$\frac{3 \times 45^\circ + 1 \times 44^\circ}{3 + 1} = 44^\circ 45'$$

True and observed value of a quantity : The true value of a quantity is absolutely free from any type of error and is never found out while the observed value is obtained from field observation after applying correction for all errors related to the observation.

Most Probable Value of a Quantity : It is the value which is more likely to be the true value than any other value. It is found out from the several measurements on which it is based.

Calculate the most probable value of an angle from the following readings :

Angle	Weight
$60^\circ 40' 30''$	2
$50^\circ 40' 16''$	3
$60^\circ 40' 12''$	4

$$\text{Most probable} = \frac{2 \times 60^\circ 40' 30'' + 3 \times 60^\circ 40' 18'' + 4 \times 60^\circ 40' 12''}{2 + 3 + 4} = 60^\circ 40' 18''$$

4. **True Error :** It is difference between the true and observed values of a quantity.

5. **Residual Error :** It is the difference between the most probable value of quantity and its observed value. This is also known as *apparent error*.

6. **Probable Error :** This is defined by saying that errors are equally likely to be numerically greater or less than the probable error. The probable error of an observation is a mathematical quantity and gives an absolute idea regarding the precision of the results. The precision of different observations may be compared from the known values of their probable errors.

7. **Constant Error :** It is an error which has got the same effect in all the observations. As for an example, if the tape is 0.01° too long, it will give an error of $0.01'$ in the measurement of each tape length.

8. **Systematic Error :** It is the error which occurs systematically. Due to the rise or fall of temperature, there will be expansion or contraction of the measuring appliances such as steel tape. Errors resulting from this are systematic.

9. **Accidental Error :** This error occurs due to the carelessness of the observer, such as reading scales, angles and recording wrong data in the field book.

10. **Average Error :** This is the mean of all the errors without considering the signs. This error is also known as *mean deviation*.

11. **Standard Error**: The standard error or standard deviation of a set of n observations x_1, x_2, \dots, x_n is defined by:

$$\text{Standard error} = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$$

where \bar{x} is the mean value. The standard error is also known as the *root mean square* of the deviation from the mean.

Estimate of Standard Error: This is denoted by the following relation:

$$\text{Estimate of Standard Error} = \frac{\text{Standard Error}}{\sqrt{n}}$$

where, n = number of observations.

12. **Permissible Error**: This is the maximum allowable limit that a measured quantity may vary from its true value. The value of the permissible error of different types of survey works depends upon the scale and the purpose of the works and also on the types of instruments available.

Definition: it is a method of surveying which includes works for projects in rivers, lakes, harbours, bays and coastal areas. Hydrographic surveying is performed for the following purposes:

- Measurement of discharge (or flow of water) and quantity of water in connection with water resources schemes, power schemes, flood control and drainage projects.
- Determinations of bed depths by soundings for navigation, including location of sand bars, rocks, navigation lights, buoys, etc.
- Location of siltation of river beds and bay mouths and their dredging operations.
- Determination of direction of currents for the location of sewer out falls areas subject to scour and silt.
- Preparation of contour maps under water, nautical charts.
- Measurements of tides for marine structures in coastal defence, jetties, harbours and inland ports.

As the water surface level does not remain constant due to the variation of climatic conditions and tides, so contour lines under water or lines of equal depth may be referred to a datum at the mean low water level of spring tides, or to datum at mean sea-level or to any other arbitrary datum. Fig. 14.1 shows the different levels of water as we normally come across. In this country no standard has yet been practiced for water levels to refer to any particular datum as in the case of United Kingdom where datum surface is referred to the **Ordinance Datum Level**.

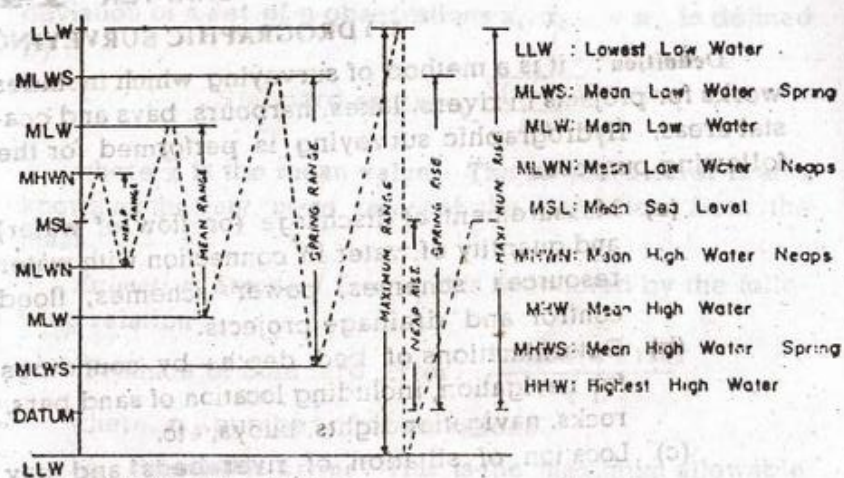


Fig 14.1 Tidal Phenomena

at the entrance to Liverpool Mersey Docks. It should be remembered that datum surface is not a level plane and as a consequence this Ordinance Datum surface is not practically the same as the mean Sea-level round the coast of England.

For terrestrial surveys MSL was determined for the sub-continent at the time of the Great Trigonometrical Survey (GTS) around the change of the century. In Bangladesh the P.W.D. levelling plane is a more widely used datum for reducing height observations. P.W.D. is about 1.5 ft. below GTS. The hydrographers reduce their soundings to a uniform low tidal level i.e., a plane below which the water level but seldom falls and is known as standard low water (SLW). At sea SLW occurs when a low seasonal variation coincides with a great tidal range. On the upper river SLW is the plane relating to an unusually low discharge of all rivers-defined at the exceedance frequency of 95% of the year (which means that on an average daily 18 days of a year will the actual water level be lower). On the tidal rivers SLW occurs by a combination of both.

The reference level used on hydrographic charts is the Chart Datum (CD). All levels, mean heights and predicted tidal levels are referred to Chart Datum. In Bangladesh well established CD exists only at Chittagong and Chalna. CD at Chittagong is 18.70 ft. below a Bench Mark 21.54 ft. near the South East corner of the Old Port and Custom Office while the Chart Datum at Chalna is 11.955 ft. below the BM 14.805 ft on the second step of wireless station at Hiron point.

Though mean low-water level is generally referred to for nautical purpose, but this datum may not be suitable for engineering works because of its considerable variation along different parts of a coast.

For nautical charts the depths are generally required in fathoms (1 fathom = 6 ft), while for engineering works the foot unit, decimally divided, is employed.

One point should be cleared here that the main task in the hydrographic surveying is the preparation of a plan or chart showing physical features below and above water, and involves :

Vertical Control : A chain of bench marks must be established near the shore line and these serve for setting and checking tide gauges to which soundings are referred. Tide or water gauges are kept in operation to establish the common datum and a vertical control must be established to connect these gauges with shore elevations and with each other

Horizontal Control : When making soundings of the depth of a river bed or a sea bed, the location of the moving sounding vessel is made by reference to fixed control points on shore, and the correct establishment of this shore framework is of great importance. The horizontal control may consist of either a triangulation or a traverse. For surveys of large extent, a second or third order triangulation may be used as the main control. For surveys of small

extent, a transit-tape-traverse may be developed. For small detached surveys, a control system may be developed by a combination of stadia and graphical triangulation procedures with plane table. In the case of a long narrow river, the horizontal control is established by running a single traverse line on one shore. If the width of a body of water is more than $\frac{1}{2}$ a mile, traverse may be run on both shores, and may be connected at intervals.

Soundings: Soundings are the vertical distances downwards from the surface of the water to its bed. Whether to determine the contour lines, the cross-sectional areas, longitudinal profiles or the discharges of streams, soundings must be performed to obtain the vertical distances.

For very shallow water and for the interval between high and low water marks, it is generally preferable to take the levels directly with a level and a levelling staff.

For deeper water (15 to 20 ft.) the vertical downward distances are measured by means of a graduated rod, a brass or iron sounding chain subdivided into feet by letters or a rope heavily weighted with a weight.

If water velocities are very high, the rope and weight

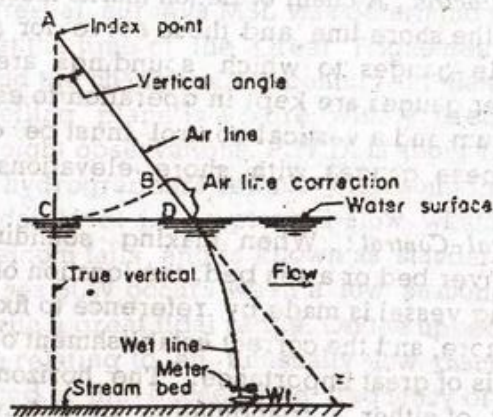


FIG. 14.2 Position of sounding line in swift water.

will not hang vertically below the point of suspension but will be carried downstream by the current (Fig. 14.2). Under these conditions the length of the line paid out is greater than the vertical depth. Very heavy weights are used to minimize this effect, but if the angle between the line and a vertical becomes large it is necessary to apply a correction to the measured depths. The actual correction depends on the relative lengths of line above and below the water surface, but at a vertical angle of 12° the error will be about 2 percent.

Water stage: River stage is the elevation of the water surface at a specified station above some arbitrary zero datum. Because it is difficult to make a direct and continuous measurement of the rate of flow in a stream but relatively simple to obtain a continuous record of water surface elevation, the primary field data gathered at a stream flow measurement station are river stage. Stage data are transformed to flow data by various methods.

Water stages are recorded by gauges. They may be classified as either recording or non-recording gauges. Recording gauges draw a continuous graph of the fluctuations in stage while nonrecording gauges require an observer who reads the gauge and record the readings at regular time intervals—the intervals being 5, 10 or 15 minutes or even more in some cases. A non-recording gauge is usually read by the observer once or twice a day.

A gauge in its simplest form consists of a vertical staff graduated in feet and decimals and placed in such a position that the surface level of the water may be read from the scale at any time (Fig. 14.3-a). If no suitable structure exists in a location which is accessible at all stages a *Sectional Staff gauge* (Fig. 14.3-b) may be used. An alternative to sectional staff is an *inclined staff gauge* (Fig. 14.3-c) which

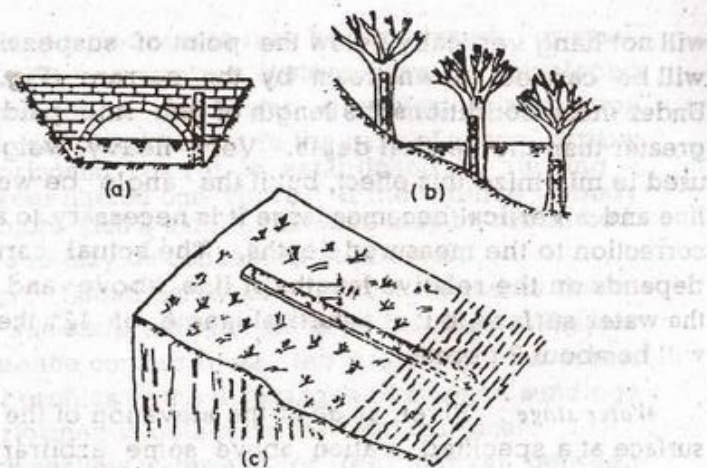


FIG. 14.3 Staff gauges (a) Simple vertical staff (b) Sectional staff (c) Inclined staff

is placed on the slope of the Stream bank and graduated, so that the scale reads directly in vertical depth.

Another type of manual gauge is the *suspended weight gauge* in which a weight is lowered from a bridge or other overhead structure until it reaches the water surface. By subtracting the length of the line drawn out from the elevation of fixed reference point, the water surface elevation can be determined. The wire-weight gauge has a drum such that each revolution unwinds 1 ft of wire. A counter records the number of revolutions of the drum while a fixed reference point indicates hundredths of feet on a scale around the circumference.

In case of tidal rivers or where stages must be read frequently, continuous recorders are used more and more. In this instrument the cylinder is driven by a clock at a constant speed represented on the record sheet (Fig. 14.4) by scales varying from 1.2 in. per day to 864 in. per day. The pen carriage, actuated by a float as the water surface, in-

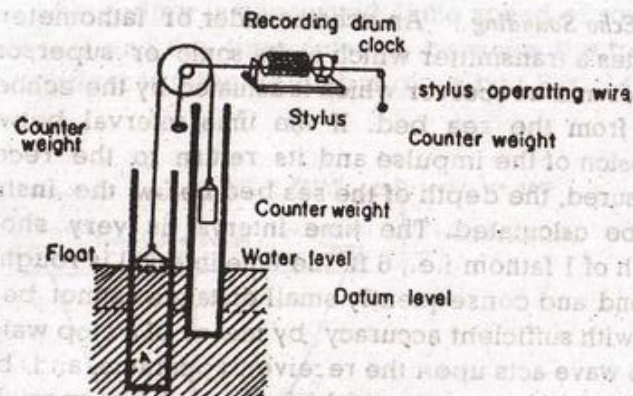


Fig. 14.4 Inlet A is not to be exposed at low water

the well rises and falls, travels back and forth. The rate of travel varies from 0.2 in. to 10 in. for each foot of fluctuation in stage. The chart roll contains enough paper to operate for a year. Clocks are usually weight-driven and will run as long as there is room for the clock weight to drop. Small electric clocks that can operate for 30 days on a flashlight battery have been used.

Water or mercury-filled manometers are often used to indicate reservoir water levels or to actuate recording devices. Remote recorders in which a system of selsyn of motors is used to transmit water level information from stream side to a recorder at a distance are available, as are numerous remote transmitting telephonic or radio gauges. These latter gauges use a coding device which converts stage to a signal which can be transmitted in the form of a series of impulses which can be counted, a change in frequency of oscillation which can be measured, or the time interval required for a sensing element to move from a zero point to water surface at constant speed. Such remote recording devices are used mainly for flood forecasting or reservoir operation.

Echo Sounding: An echo sounder or fathometer incorporates a transmitter which emits sonic or supersonic impulses and a receiver which is actuated by the echoes reflected from the sea bed. If the time interval between the emission of the impulse and its return to the receiver is measured, the depth of the sea bed below the instrument's can be calculated. The time interval is very short for a depth of 1 fathom i.e., 6 ft. the time interval is roughly 1/400 second and consequently small distance cannot be measured with sufficient accuracy by means of a stop watch. The echo wave acts upon the receiver apparatus, and by electronic techniques, a record is produced on specially prepared charts.

The recording of the soundings is produced by the action of a small current passing through chemically impregnated paper from a rotating stylus to an anode plate. The stylus is fixed at one end of radial arm which revolves at constant speed. The stylus makes a record on the paper at the instants when the sound impulse is transmitted and when the echo returns the receiver. The correlation with a tide gauge follows and the positions at which soundings are taken may be determined by sextant observations from the deck.

The speed of sound is not constant but varies with water temperature and degree of salinity, and since a change in the speed of sound will produce an error in the recorded depth the echo sounder is adjusted to meet local conditions by altering the operations rev/min.

The echo sounder normally operated with basic depth scales of 0 to 60 ft. (shallow) and 0 to 120 ft. (deep), the pulse-repetition frequency being greater in the former case. With phasing of scales, effected by electrical switching, a range of 90 ft. to 150 ft. is possible in the first instance and range of 180 ft. to 300 ft. in the second. It thus covers the range of all engineering hydrographic surveying require-

ments so far as sounding is concerned. If the speed of sound in the water is V and the time interval between the transmitter and receiver is t , the depth h is given by (Fig. 14.5)

$$h = \frac{1}{2} vt$$

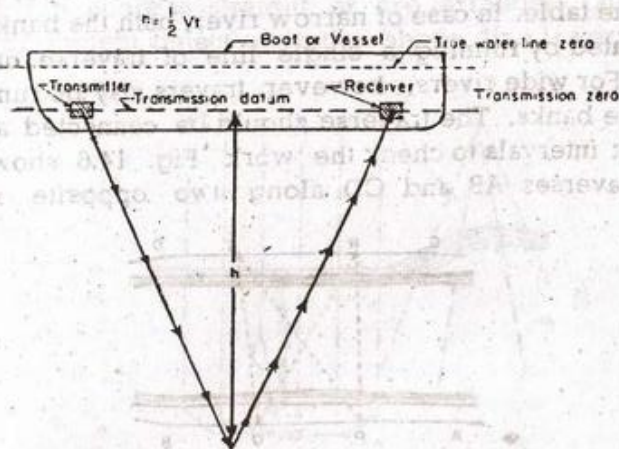


Fig 14.5 Principle of echo sounding

Under favourable conditions, an accuracy ± 3 inches is claimed, but it would be unreasonable to expect this as a general rule. Echo sounding is faster, more accurate and can be taken under strong current and unsuitable weather conditions. Provides a continuous profile of the bed and information for rocks underlying softer materials can be obtained also.

For locating the position of soundings far from land, radar techniques have been applied with success. Radio waves sent out from the ship are returned from two shore stations, enable distances away to be determined. The "Shoran" (short range navigation) method, as developed so extensively by the Canadian Geodetic Survey, is generally used, and within a suitable range, is capable of an accuracy of ± 25 ft. or so at 100 miles or more distance. It is not suitable for short distances of a few miles.

Shore line survey: It consists of (a) determination of

shore lines; (b) location of shore details and prominent features to which soundings may be connected and (c) determination of low and high water lines for average spring tides. Shore lines are located by traversing along the shore and taking offsets to the water edge by tape, stadia or plane table. In case of narrow river, both the banks may be located by running a single line of traverse on one bank. For wide rivers, however, travers may be run along both the banks. The traverse should be connected at convenient intervals to check the work. Fig. 14.6 shows the two traverses AB and CD along two opposite shores.

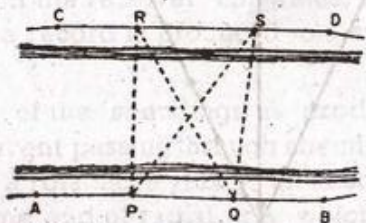


Fig 14.6 Shore line survey.

may be checked by taking observations from P and Q to the points R & S. When the instrument is at Q, angles P O S can be measured. From the measured length of PQ and the four angles, the length RS can be calculated. The work is checked if this length agrees with the measured length of CD. In sea shore survey, buoys anchored off the shore and light houses are used as reference points and are located by triangulation.

For tidal water, the location of high water line may be located roughly from shore deposits and marks on rocks. To determine the HWL accurately, the elevation of mean high water of ordinary spring tide is determined and the points are located on the shore at that elevation as in direct method of contouring. The low water line can also be determined similarly. But the available time for the survey of low water line is limited, so it is usually located by interpolation from soundings.

Range and shore signals: A range or range line is the line on which soundings are taken. They are, in general, laid perpendicular to the shore line and parallel to each other if the shore is straight or are arranged radiating from a prominent object when the shore line is very irregular.

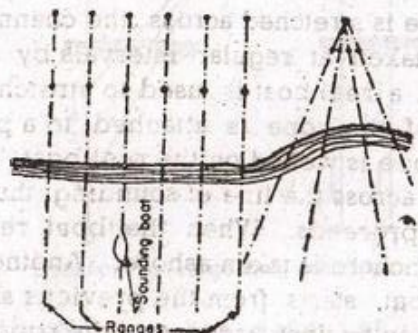


Fig 14.7 Ranges.

Each range line is marked by means of signals erected at two points on it at a considerable distance apart. These signals should be readily seen and easily distinguished from each other. The most satisfactory and economic type of signal is a wooden tripod structure dressed with white and coloured signal of cloth. The position of the signals should be located very accurately since all the soundings are to be located with reference to these signals.

Location of Soundings: Soundings are made with reference to the traverse by observations carried out entirely from the boat or entirely from the shore or from both. The following are the various methods of location.

1. By cross rope
2. .. range and time intervals
3. .. range and one angle from the shore
4. .. range and one angle from the boat
5. .. two angles from the shore
6. .. two angles from the boat

7. By One angle from shore and one from boat
8. .. intersecting ranges
9. .. tacheometry

Location by cross-rope : This method is used for rivers, narrow lakes and for harbours. It is also used to determine the quantity of materials removed by dredging. A single wire or rope is stretched across the channel and the soundings are taken at regular intervals by a pole. In case of harbours, a reel boat is used to stretch the rope. The zero end of the rope is attached to a pole at one shore while the rope is wound on the reel boat. The reel boat is then rowed across the line of sounding, thus unwinding the rope as it proceeds. When the boat reaches the other shore, its anchor is taken ashore. Another boat known as sounding boat, starts from the previous shore and soundings are taken against each tag of the rope.

Location by range and time intervals : This method is used when the channel width is small and when great degree of accuracy is not required. In this method the boat is kept in range with the two signals on the shore and is rowed along it at constant speed. Soundings are taken at different time intervals. Knowing the constant speed and the total time elapsed at the instant of sounding, the distance of the point can be known along the range. This method is used in conjunction with other methods, in which case the first and the last soundings along a range are located by angles from the shore and the intermediate soundings are located by interpolation according to time intervals.

Location by range and one angle from the Shore : The boat is ranged in line with the two shore signals and rowed along the ranges. Any point where soundings taken is fixed on the range by observation of the angle from the shore. As the boat proceeds along the range, other soundings are also fixed by observation of angles from the shore. In Fig 14.8, R is the instrument station, PQ is

the range along which the boat is rowed and a, b, c are the angle measured at R from points 1, 2, 3 respectively. The angle α at sounding point should be greater than 30° and when this becomes smaller than 30° , a new instrument

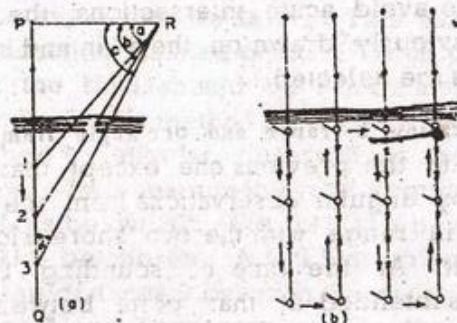


Fig 14.8 Location by range and one angle from shore

station must be chosen. The defect of this method is that the surveyor does not have an immediate control in all the observations. A note keeper is required along with the instrument man if all points are to be fixed by angular observations from the shore. In this case observation and recordings are to be done rapidly. Normally the initial and last soundings and every tenth sounding are fixed by angular observations and the intermediate points. In Fig. 14.8 b, the points with round mark are fixed by angular observations from the shore while the points with cross marks are fixed by time intervals. The arrows show the course of the river of the sounding boat on different sections.

The procedure to fix a point from the shore is that the instrument man at R orients his line of sight towards a shore signal or any prominent point (on the plan) when the reading is zero. The telescope is directed towards the leadman or the bow of the boat, and is kept continually pointing towards the boat as it moves. The surveyor on the boat holds a flag for a few seconds and on the fall

of the flag the sounding and the angle are observed simultaneously. The time at which the flag falls is recorded by the instrument man and the man on the boat. The angles are observed to the nearest 5 minutes. In order to avoid acute intersections, the lines of soundings are previously drawn on the plan and suitable instrument stations are selected.

Location by range and one angle from the boat: This is similar to the previous one except that the angular fix is made by angular observations from the boat. The boat is kept in range with the two shore signals and is towed along it. At the time of sounding being taken the angle subtended at that point between the range and some known point R on the shore, is measured with the help of a sextant. (Fig. 14.9). The telescope is directed on the range signals and the side object is brought into coincidence at the instance the sounding is taken. This method has following advantages over the previous one:

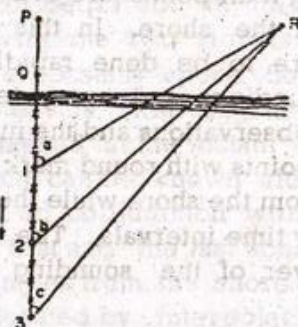


Fig. 14.9 Location by range and one angle from boat.

1. The surveyor has better control over his operations as all observations are taken from the boat.
2. Check may be obtained by measuring a second angle towards some other signal on the shore on important fixes.

3. The mistakes in booking are reduced since the recorder enters the readings directly as they are measured.
4. Various shore objects may be used for reference to measure the angles.

Location by two angles from the Shore: This method is normally used for locating isolated points. When used for extensive survey, the boat should be run on a series of approximate ranges. In this method a point is fixed independent of the range by angular observations from two points on the shore. Two instruments are required and when the angle θ falls below 30° , (Fig. 14.10) a new instrument station should be chosen. A and B are two instrument stations and the distance P between them is accurately measured. Stations A and B are connected to the ground traverse or triangulation and their positions on plan are known. With both the plates clamped to zero A focuses B

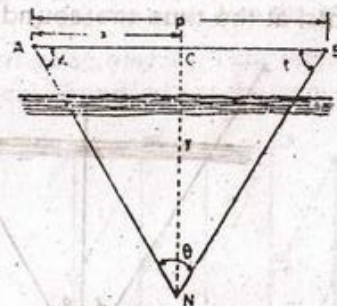


Fig. 14.10 Location by two angles from the shore.

while B bisects A. Both the instrument men then direct the line of sight of the telescope towards the leadsmen and continuously follow it as the boat moves. With the fall of the flag by the surveyor, the soundings and the angles are observed simultaneously. Co-ordinates of P (x, y) of the sounding may be found by the following equations:

$$x = \frac{P \tan \beta}{\tan \alpha + \tan \beta}$$

$$y = \frac{P \tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$$

In this method the preliminary work of setting out and erecting range signals is eliminated. It is useful when there is strong currents due to which it is difficult to row the boat along the range line. But it is laborious.

Location by two angles from the boat: This method is used to take the soundings at isolated points. In this method the exact position of the boat is found out by observing two angles subtended at the boat by three bank or shore objects of known position. This is like the solution of three-point problem either analytically or graphically. Prominent natural objects like mosque top, church spire, light house, flag staff, buoys etc. are selected for this purpose. If such points are not available, range poles or shore signals may be considered. Fig. 14.11 shows P as the position of the boat and A, B, C are the shore objects. Angles α and β are measured at the boat P simultaneously with two sextants, at the time the sounding is taken. In this

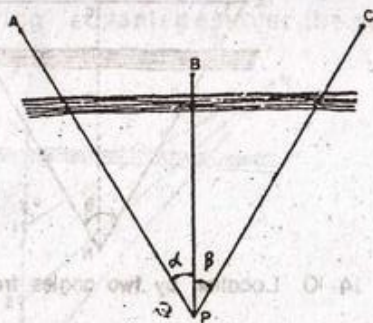


Fig 14.11 Location by two angles from the boat

method the survey party remains in one boat and this gives better control on the operation.

Location by one angle from the Shore and the other from the boat: This method is used to locate the isolated points where soundings are taken. In Fig. 14.12, A and B are selected points on the shore where at one point, say A, a theodolite is set up. When sounding is taken at N, the

angle at A is measured while the angle at the boat N is measured with a sextant. Knowing the distance P, the position of N can be located by calculating the two coordinates x and y .

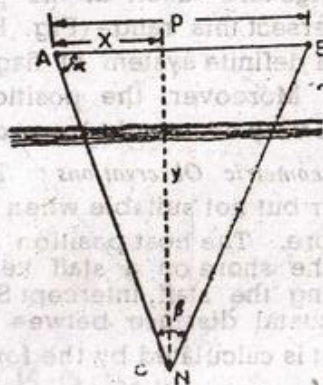


FIG. 14.12 Location by one angle from the shore α and the other from the boat β

Location by intersecting ranges: This technique is applied to find by periodical soundings at the same points, the rate

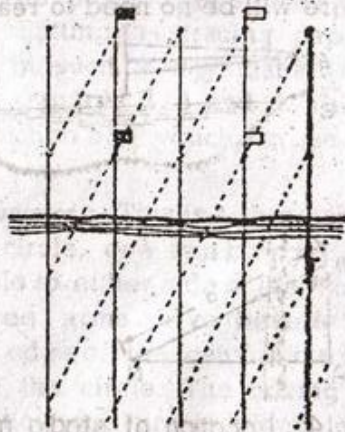


FIG 14.13 Location by intersecting ranges

at which silting or scouring is taking place. The position of sounding is located by the intersection of two ranges. The boat is rowed along a range perpendicular to the shore and soundings are taken at the points in which inclined ranges intersect this range (Fig. 14.13). In order to avoid confusion, a definite system of flagging the range poles is necessary. Moreover, the position of the range poles is determined very accurately by ground survey.

Location by Tacheometric Observations : This method is useful in calm water but not suitable when soundings are taken far from shore. The boat position is located by a tacheometer from the shore on a staff kept vertically on the boat. Observing the staff intercept S at the time of sounding, the horizontal distance between the instrument stations and the boat is calculated by the formula

$$d = \frac{f}{i} s = (f + d) \dots \dots \dots (3)$$

The boat direction N is established by observing the angle α at the instrument station B with reference to any fixed object A . The transit station should be near the water level so that there will be no need to read vertical angle.

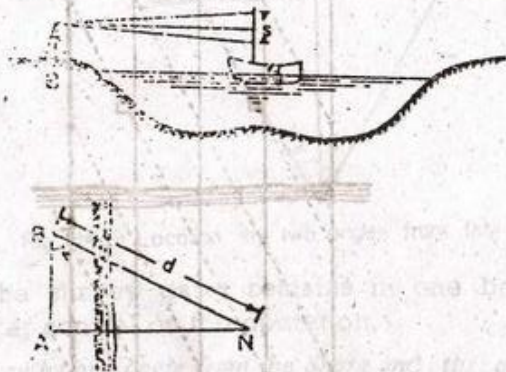


Fig. 14.14 Location of stadia method

The Three point problem : The three shore signals A , B and C and the angles α and β (measured by sextant) subtended by AP , BP and CP at the boat P are given, it is

required to plot the position of P (Fig. 14.15). According to some source, a well conditioned fix by sextant realize accuracies of some order of ± 5 ft when plotting at 1 : 1250 scale. The problem can be solved in various methods as given below :



Fig 14.15 The three point problem

A. Mechanical Solution :

(a) **By tracing paper :** Angles α and β are protracted between three radiating lines from any point on a piece of tracing paper. The positions of signals A , B and C are plotted on the plan. Putting the tracing paper to the plan and moving it over in such a way that all the three rays simultaneously pass through A , B and C . The apex of the angles is then the position of P which can be pricked through a pin.

(b) **By Station pointer :** This is a three armed protractor with a graduated circle, one arm being fixed while the other two are movable to either side of the fixed arm (Fig. 14.16). All the three arms have bevelled or fiducial edges. The fiducial edge of the central fixed arm corresponds to the zero of the circle. The moving arms can be clamped to any position and are provided with verniers and slow motion screws to set the angle very precisely. To plot the position of p , the movable arms are clam-

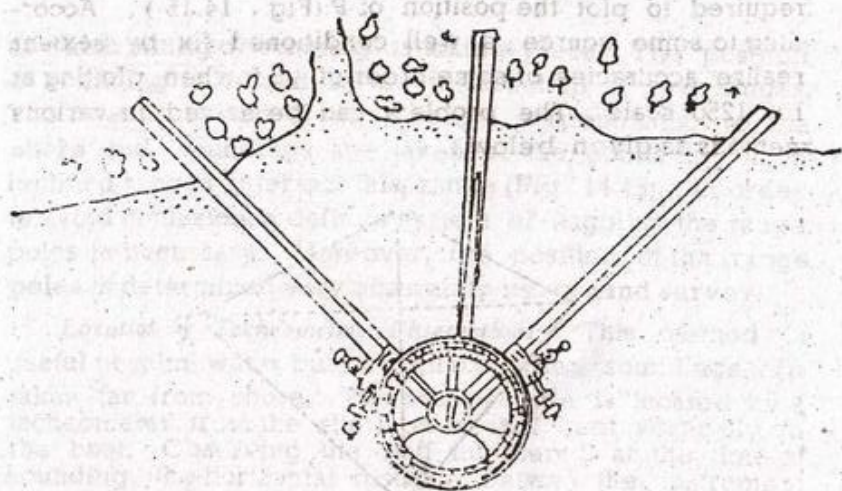


Fig. 14.16

ped to read the angles α and β very precisely. The station pointer is then, moved on the plan till the fiducial edges simultaneously touch A, B and C. The centre of the pointer then represents the position of P which can be recorded by a prick mark.

B. Graphical Solution :

(a) *First Method*: a, b and c are the plotted positions of the shore signals A, B and C respectively (Fig. 14.17) and α and β are the angles subtended at the boat. The point P of the boat position p can be determined by the following procedure. a and c are joined. At a draw ad making an angle β with a c, while at c draw cd making an angle α with ca. Both these lines meet at d. Draw a circle passing through the points a, d, and c. Join d and b and extend it to meet the circle at the point P which is the required position of the boat, because from the properties of a circle-

$$\angle apd = \angle acd = \alpha$$

$$\text{and } \angle cpd = \angle cad = \beta$$

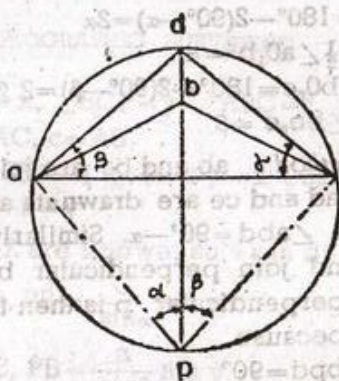


Fig. 14.17

(b) *Second Method (or the method of two intersecting circles)*: Join ab and bc (Fig. 14.18). Draw lines ao_1 and bo_1 , each subtending an angle $(90^\circ - \alpha)$ with ab on the side toward P. They intersect at o_1 . Similarly draw lines bo_2 and co_2 from b and c respectively each subtending an angle $(90^\circ - \beta)$ with ab on the side towards p. They intersect at o_2 . Draw two circles with o_1 and o_2 as centres

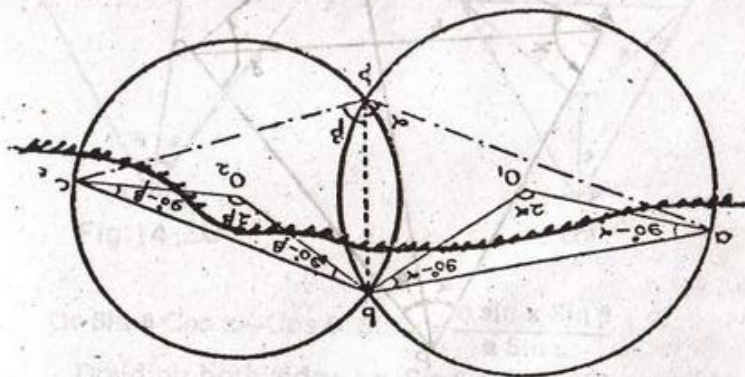


FIG. 14.18

to pass through a and b, and b and c respectively. The circles intersect at p which is the position of the boat.

$$\therefore \angle a_0 b = 180^\circ - 2(90^\circ - \alpha) = 2\alpha$$

$$\therefore \angle apb = \frac{1}{2} \angle a_0 b = \alpha$$

$$\text{Similarly, } \angle b_0 c = 180^\circ - 2(90^\circ - \beta) = 2\beta$$

$$\angle bpc = \frac{1}{2} \angle b_0 c = \beta$$

(c) *Third Method*: ab and bc are joined (Fig. 14.19); perpendiculars ad and ce are drawn at a and c respectively. At b on ab draw $\angle abd = 90^\circ - \alpha$. Similarly draw $\angle cbe = 90^\circ - \beta$. join de and join perpendicular bp on de from b. The foot of the perpendicular p is then the required position of the boat because

$$\angle bad = \angle bpd = 90^\circ$$

\therefore the quadrilateral abpd is cyclic

similarly the quadrilateral bcep is concyclic

$$\text{Hence } \angle adb = \angle apb = \alpha$$

$$\text{and } \angle bpc = \angle bec = \beta$$

The problem is indeterminate if the points ABC and P are concyclic.

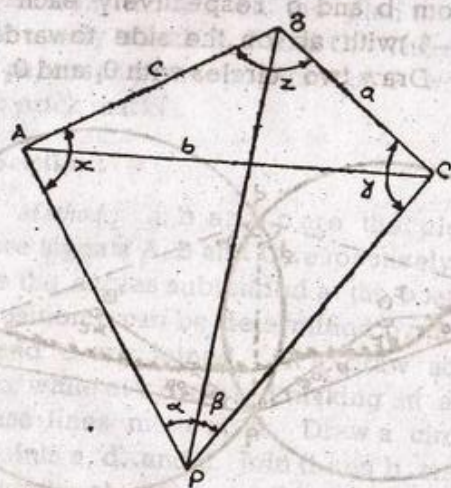


Fig. 14.19

C. *Analytical solution*: α and β are the observed angles (Fig. 14.20) at P and A, B and C are the shore signals whose positions are known.

$$\text{Let } \angle BAP = x, \angle BCP = y, \angle ABC = z$$

$$a = BC, b = AC, c = AB$$

$$\text{Now } x + y = 360^\circ - (\alpha + \beta + z) = \theta \text{ (Say)}$$

$$\therefore \theta - x = y$$

As α, β and z are known, so θ can be calculated

$$\text{From } \triangle PAB, PB = \frac{c}{\sin \alpha} \sin x$$

$$\therefore \text{in } \triangle PBC, PB = \frac{a}{\sin \beta} \sin y$$

$$\text{Equating the two, } \sin y = \frac{c \sin x \sin \beta}{a \sin \alpha}$$

$$\therefore \sin(\theta - x) = \frac{c \sin x \sin \beta}{a \sin \alpha}$$

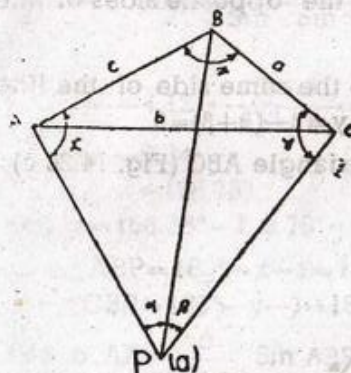
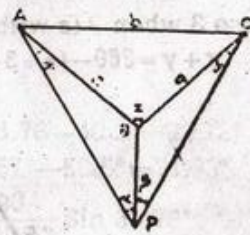


Fig. 14.20



(b)

$$\text{Or } \sin \theta \cos x - \cos \theta \sin x = \frac{c \sin x \sin \beta}{a \sin \alpha}$$

Dividing both sides by $\sin \theta \sin x$

$$\cot x - \cot \theta = \frac{c \sin \beta}{a \sin \alpha \sin \theta}$$

$$\cot x = \cot \theta + \frac{c \sin \beta}{a \sin \alpha \sin \theta}$$

$$= \cot \theta \left(1 + \frac{c \sin \beta \sec \theta}{a \sin \alpha} \right)$$

Thus the value of x can be calculated as all are known in the equation.

Again from ΔABP

$$AP = \frac{c}{\sin \alpha} \sin \angle ABP = \frac{c}{\sin \alpha} \sin (180^\circ - x - \alpha) = \frac{c}{\sin x}$$

$$\sin (x + \alpha) \text{ and } BP = \frac{c}{\sin x} \sin x$$

Similarly, from ΔBPC

$$BP = \frac{a}{\sin \beta} \sin \gamma, \quad CP = \frac{a}{\sin \beta} \sin \beta \quad \angle CBP = \frac{a}{\sin \beta} \sin (\gamma + \beta)$$

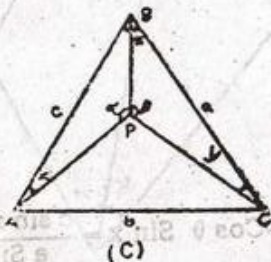
Now calculating AP, BP and CP the position of P can be calculated.

Three cases may arise due to the position of the boat P with respect to the ground signals A, B and C.

Case 1 when B and P are to the opposite sides of the line AC (Fig. 14.20 a)

Case 2 when B and P are to the same side of the line AC (Fig. 14.20 b); $x + y = z - (x + \beta) = 0$

Case 3 when P is within the triangle ABC (Fig. 14.20 c); $x + y = 360 - (x + \beta + z) = 0$



Problem 1 A, B and C are three visible stations in a hydrographic survey. The computed sides AB, BC and CA are 3000 ft, 4000 ft and 5500 ft respectively. Outside this triangle and nearest to AC, a station P is established and its position is to be found by three point resection on A, B and

C, the angles APB and BPC respectively being 40.50° and 50.50° . Determine the distance PA and PC (See Fig. 14.20 a)

Solution : $c = AB = 3000$ ft, $a = BC = 4000$ ft, $b = CA = 5500$ ft.

$$\angle ABC = z$$

Now $b^2 = c^2 + a^2 - 2ac \cos z$

$$\cos z = \frac{c^2 + a^2 - b^2}{2ac} = \frac{(3000)^2 + (4000)^2 - (5500)^2}{2 \times 4000 \times 3000} = \frac{-5250000}{24000000}$$

$$= -0.219$$

$$\cos (180^\circ - z) = .219 \quad \therefore 180^\circ - z = \frac{77.33^\circ}{}$$

$$z = 102.65^\circ$$

$$\begin{aligned} 0 = x + y = 360^\circ - (\alpha + \beta + z) \\ = 360^\circ - (42.5^\circ + 50.5^\circ + 102.65^\circ) \\ = 166.35^\circ \end{aligned}$$

$$\cot x = \cot \theta + \frac{c \sin \beta}{a \sin z \sin \beta} = \cot 166.35^\circ + \frac{3000 \sin 50.5^\circ}{4000 \sin 40.5^\circ \times \sin 166.35^\circ}$$

$$= -4.12 + \frac{2314.87}{613.05} = -4.12 + 3.76$$

$$= -0.34$$

$$\therefore x = 108.78^\circ$$

$$\text{and } y = 166.35^\circ - 108.78^\circ = 57.57^\circ$$

$$\therefore \angle ABP = 180^\circ - x - z = 180^\circ - 108.78^\circ - 40.5^\circ = 30.72^\circ$$

$$\angle CBP = 180^\circ - y - \beta = 180^\circ - 57.57^\circ - 50.5^\circ = 71.93^\circ$$

$$\text{Hence } AP = \frac{c}{\sin \alpha} \sin \angle ABP = \frac{3000}{\sin 40.5^\circ} \sin 30.72^\circ = 2360 \text{ ft.}$$

$$\text{and } CP = \frac{a}{\sin \beta} \sin \angle CBP = \frac{4000}{\sin 50.5^\circ} \sin 71.93^\circ = 4923 \text{ ft.}$$

Tides: All heavenly bodies exert a gravitational force on each other and these forces of attraction between earth and other celestial bodies (moon and sun) cause periodic variations in the level of a water surface, known as tides. Lunar tides are larger than solar tides because solar tides force is 0.458 times lunar tides force. There are two lunar tides at A and B and two low water positions at C and D.

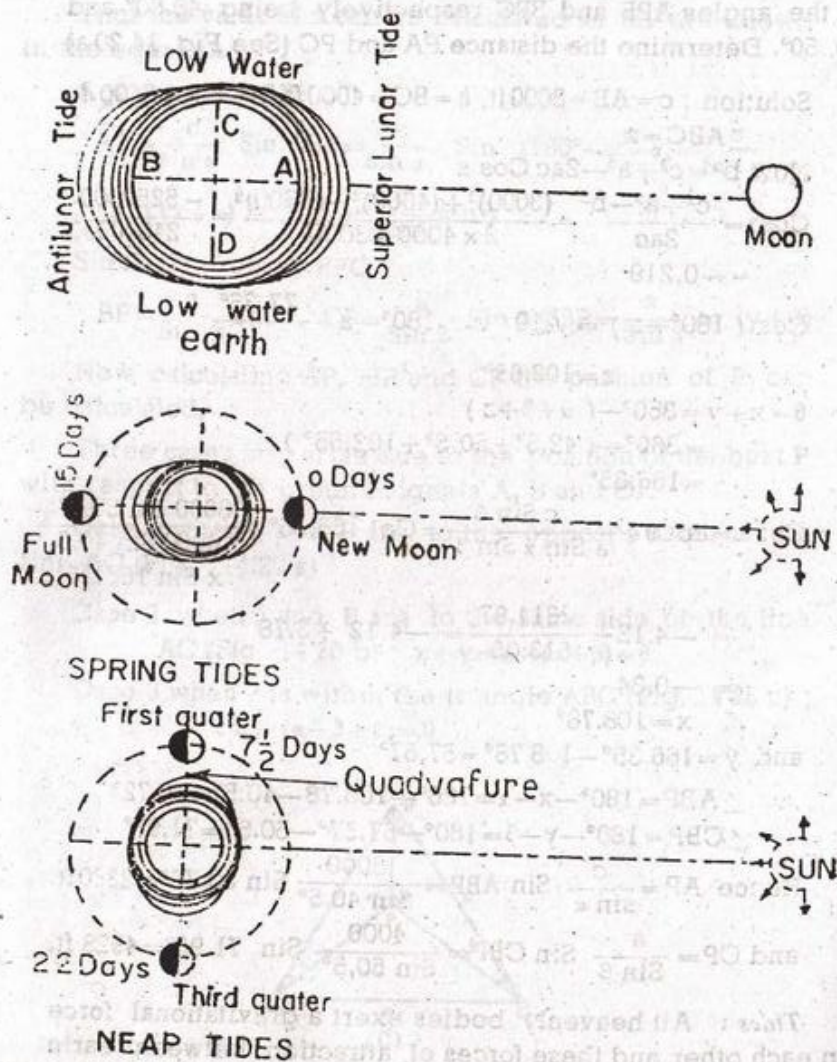


Fig. 14.2'

The tide at A is called superior lunar tide and tide of moon's upper tide, while tide at B is called the inferior or antilunar tide.

Due to rotation of earth about its axis from west to east, once in 24 hrs. point A would occupy successive position C, B and D at intervals of 6 hrs. This it will experience high water (i.e. tide) at intervals of 12 hrs. and low water midway between. This interval of 6 hrs variation is true only if moon is assumed stationary. However, in a lunation of 29.53 days the moon makes one revolution relative to sun from the new moon to new moon. This revolution is in the same direction as the diurnal rotation of earth, and hence there are 29.53 transits of moon across a meridian in 29.53 mean solar days. The interval between successive transits of moon on any meridian will be 24 hrs. 50.5 minutes. Thus the average interval between two successive transits of moon over a meridian is called the tidal day.

However, the combined effect of the earth, moon and sun is important specially at the new moon when both the sun and the moon have the same celestial longitude and cross a meridian at the same instant. Assuming that both the sun and moon lie in the same horizontal plane passing through the equator, the effects of both the tides are added, giving rise to maximum or Spring tide of new moon. The term does not refer to season, but to the springing or waxing of the moon. After the new moon, the moon falls behind the sun and crosses each meridian 50 minutes later each day. In after $7\frac{1}{2}$ days the moon is at quadrature (Fig. 14.21). The crest of moon tide coincides with the trough of the solar tide, giving rise to the neap tide of the first quarter and the high water level is below the average while low water level is above the average. After about 15 days of the start of lunation i.e. when full moon occurs, the difference between the moon's and sun's longitude is 180° and the moon is in opposition. However, the crests of both the tides coincide, giving rise to spring tide of full moon. In about 22 days after the start of lunation, the difference in longitudes of the moon and the sun becomes 270° , and the

neap tide of third quarter is formed. After about $29\frac{1}{2}$ days of previous new moon, both of them (the sun and the moon) have the same celestial longitude and the spring tide of new moon is again formed making the new cycle of spring and neap tides.

The length of the tidal day which is assumed to be 24 hours and 50.5 minutes is not constant due to varying relative positions of the sun and the moon, ellipticity of the orbit of the moon (assumed normally circular) and the earth, declination (i.e. deviation from the plane of the equator) of the sun and the moon, effects of the land masses and deviation of the shape of the earth from the spheroid. Due to the effect of these various factors, the high water at a place may not occur at the moon's upper or lower transit. The effect of varying relative positions of the sun and moon gives rise to what are known as priming of tide and lagging of tide. Due to obstruction of land mass, tide may be heaped up at some places, while equilibrium figure is not achieved instantaneously for inertia and viscosity of sea water.

Discharge Measurement: Discharge is calculated on the basis of the equation Discharge, $Q = \text{Cross-Sectional area} \times \text{velocity} = AV$ where A is the cross-sectional area in ft^2 , V is the average velocity of flow in foot per second and Q is the discharge in ft^3 per second. Sometimes stage record may be transformed to a discharge record by calibration. Since control rarely has a regular shape for which the discharge can be computed, calibration is accomplished by relating field measurements of discharge with the simultaneous river stage. Common methods of discharge measurements are (a) Velocity by floats or current meter and cross-sectional areas (b) Chemical methods (c) Weirs and notches.

Speeds of currents are measured by floats or current meter. (a) Floats: Numerous types of floats are avail-

able, ranging from small surface floats with flags on them to the double float which has a perforated cylinder or a canvas vane suspended at known depth below the surface from a small floating buoy. The fixing of the positions of the floats may be done simultaneous theodolite observations from the shore or better by frequent visits to each float in turn by a surveying vessel, whose position is fixed by sextant observations on the shore-control points. The time of observation is noted in each case, so that the rate of drift can be calculated.

$$\text{velocity or speed} = \frac{\text{distance}}{\text{time}}$$

Surface floats give the velocity of the surface water only, and apart from the sensitivity of such light floats to wind the choice of coefficient to convert surface to mean ranges from about 0.7 to 0.95. The results are thus of doubtful accuracy.

The current meter: the price current meter devised by W. G. Price in 1832 (a civil engineer of the Mississippi River Commission), consists of 6 conical cups rotating about a vertical axis. Electric contacts driven by the cups close a circuit through a battery and the wire of the supporting cable to cause a click for each revolution (or each fifth revolution) in headphones worn by the operator. A pygmy price meter has been used for measuring flow at very shallow depths.

The other type of current meter is the propellor type in which the rotating element is a propellor turning about horizontal axis. The contacting mechanism of a propellor meter is similar to that of a Price meter and similar suspensions are used. A price meter moved vertically in still water will indicate a positive velocity. Hence it tends to overestimate the velocity in a stream. The relations between revolutions per second of the meter cups and water velocity v in fps is given by an equation of the form:

$$v = a + bn$$

Where a is the starting velocity or velocity to overcome mechanical friction. For the price meter a is about 0.1 and b about 2.2. Before use, each meter should be individually calibrated.

Note, by plotting this value of v_{mean} on the velocity/depth diagram for the vertical section concerned, that V_{mean} occurs in each case at about $0.6 \times$ depth. Thus, if time allows only one velocity measurement at each vertical section, this measurement should be made at $0.6 \times$ depth.

The V_{mean} having been obtained for each vertical section, it is possible to obtain mean velocities for the water passing through the trapezoids into which the cross-section is divided, and hence by integration, using planimeter, Simpson's rule, or trapezoidal rule, the discharge is determined from:

$$Q = A.V \\ = \sum a.V_{mean}$$

Alternatively, 'contours' may be drawn on the cross-section, joining up points of equal velocity. The areas of these curves are measured by planimeter, and these are treated as equidistant cross-sectional areas of a solid, the volume of which gives the discharge. Yet another way is to use the 'spot height' analogy assuming the cross-section to be subdivided into a series of areas, the velocities of flow through these being the mean of the velocities at the corners.

(b) Floats

Although the current meter is by far the most widely used of the area-velocity methods for calculating discharges (and normally it is the most accurate method), there are times when floats are used instead—notably when excessive velocities, depth and floating drift prohibit the use of the current meter.

Surface floats give the velocity of the surface water only, and apart from the sensitivity of such light floats to wind, the choice of coefficient to convert $V_{surface}$ to V_{mean} ranges from about 0.7 to 0.95. The results are thus doubtful accuracy.

The double float already mentioned can be used to give velocities at different depths (though allowance must be made for the effect of the surface float), the calculation of discharge thus being exactly as described for the current meter.

Float velocities are measured by releasing the floats at the appropriate point upstream and then along the measured distance to a second station downstream.

(b) Chemical Methods

These methods involve the introduction of a chemical into the stream, and out of many variations, the two most frequently used are the *salt-velocity* and the *salt-dilution* methods.

SALT-VELOCITY METHOD: Salt in solution increases the electrical conductivity of water. Two sets of electrodes a known distance apart in a stream of constant cross-section are connected to a recording galvanometer which records the changes in electrical conductivity of the stream with respect to time. Under normal conditions the graph given is a more or less horizontal straight line, but when a volume of salt solution is injected at the upstream electrodes (which are in the form of pipes), the graph shows a rectangular jump. Later on, when the salt gets down to the second pair of electrodes, a second jump occurs. The time of transit is taken as the time between the centres of area of the two jumps, and dividing this time into the volume of water between the two stations gives the discharge.

SALT-DILUTION METHOD: A salt solution of known concentration is added at a constant rate to the stream to be

gauged and, by analysis, the subsequent dilution of the solution is determined. The samples are taken far enough below the entry point for complete mixing and uniform distribution to have taken place. No measurement of area or distance are necessary, and the method is pre-eminently suitable for use in turbulent mountain streams. The weight of salt that passes in each second at the point where samples are taken must equal the combined weights of salt normally present, and the salt added in solution, i.e.

$$WX + W'X' = (W + W')X''$$

where W is the wt. of water discharged per sec, W' is the wt. of salt solution added per sec, X is the percentage (by weight) of natural salt in the stream, X' is the percentage (by weight) of salt in concentrated solution and X'' is the percentage (by weight) of salt in the sample after mixing.

Therefore

$$W = W' \frac{(X'' - X')}{(X'' - X)}$$

X'' must be uniform at all points in the cross-section and $(X'' - X')$ must be found accurately: the salt used be detectable in small quantities and should be stable in the water. When obtaining samples, bottles can be immersed in the stream at the relevant cross-section or a hand pump can be used for drawing off. Sodium dichromate is a suitable chemical according to Hutton and Spencer, although at concentration of 30 parts per million it is toxic to fish. Five parts per million have been used to achieve an accuracy of $\pm 1-2$ per cent in measurement of flow. Larger quantities of sodium chloride would be needed since it is present in natural waters and in this case X should not exceed $0.15 X''$.

(c) Weirs and Notches

These may be used to measure the discharge of liquids flowing under gravity, i.e. rivers or similar channel flows. A notch, which can be taken as an orifice with

its upper edge free, is usually used to measure flows from reservoirs or tanks, and of streams of modest discharge; flows in sewers are often established by V-notches.

A weir, which may be considered as a large notch, extends across the stream at right angles to the flow (though side weirs set parallel to the flow are often used in storm-water overflows in sewerage practice).

The free water surface is drawn down as it passes over the weir, and the water level before drawdown occurs referred to the top of the weir, is the *head* of water over the weir. Weirs may be *suppressed*—when they extend across the full width of the approach channel, or *contracted*—when they do not extend across the full width. In this latter condition, end contractions are induced.

Weirs may be sharp-crested or broad-crested and this is defined by the nature of the face (crest) over which the water flows. In the latter case, often formed by masonry or concrete, the sheet of water is in contact with the crest over much of its area and the discharge is greater than for a corresponding sharp-crested type, which could have been made of thin stainless steel plate.

The water level is drawn down as it passes over a notch or weir, and the effective head of water, upon which the quantity of flow depends, must be referred to the level before drawdown occurs. Head is best measured by a float and, on connexion to a drum-type recorder, a permanent and continuous record of head, or rate of flow, may be obtained. It is advisable, if possible, to check the discharges as computed by the weir formula by current-meter observations or a similar method, this procedure is frequently adopted to calibrate large weirs.

The following formulae are available and the student is referred to the various textbooks on hydraulics* for their derivation.

RECTANGULAR NOTCH For a simple small rectangular notch, of breadth B , the discharge Q for a head H is given by:

$$Q = \frac{2}{3} Cd. B \sqrt{2g} H^{\frac{3}{2}}$$

The coefficient of discharge, Cd , may be found by experiment. It varies slightly with head but usually for civil engineering purposes a mean value of $\frac{2}{3} Cd \sqrt{2g}$ is determined from experimental results.

V-OR TRIANGULAR NOTCH The wetted length of such a notch depends directly on the head. If the apex angle is θ , it can be shown that:

$$Q = \frac{8}{15} Cd \sqrt{2g} \tan \frac{\theta}{2} H^{\frac{5}{2}}$$

An average value for Cd is 0.6.

RECTANGULAR WEIR The formula above does not apply for large notches or weirs and an empirical formula given by Francis is usually adopted. Neglecting the velocity of approach the discharge can be calculated from

$$Q = 1.83 (B - 0.1 nH) H^{\frac{3}{2}}$$

where
and

B = breadth of weir

n = number of end contractions

n is zero for a suppressed weir. B should be greater than $3H$ and there are certain other limits imposed with regard to the height of the crest above the channel bottom and the position of the weir.

The velocity of approach (V) of the water to the weir may be taken into account by allowing an additional head $= h \frac{V^2}{2g}$ for the kinetic energy of the water. The still water head (H_1) is thus

$$H_1 = H + \frac{V^2}{2g}$$

and this figure can now substituted in the Francis formula to give,

$$Q = 1.83 (B - 0.1 nH_1) (H_1^{\frac{3}{2}} - h^{\frac{3}{2}})$$

EXERCISE

- Examine the following Statements and write whether they are true or false ;
 - Hydrographic Survey deals with the measurement of bodies of water.
 - This Survey is used for determining rocks, sand bars and buoys
 - Hydrographic Survey used for measurements of tall buildings and halls.
 - Horizontal and vertical controls are essential in hydrographic Surveying.
 - Shore line survey can be achieved by hydrographic survey.
 - Sounding is a part of this survey.
 - Sounding lead is a weight attached to the line.
 - Fathometer is needed for sounding.

- In a harbour development scheme at the mouth of the Meghna Estuary, it has been found necessary to take soundings in order to buoy the navigational channel.

Explain clearly how you would determine the levels of points on the river bed and fix the positions of soundings :

- by use of a sextant in a boat.
 - by use of theodolite on the Shore.
- Describe briefly the location of sounding stations by means of ;
 - Cross rope soundings, (b) intersecting ranges.

From a stationary boat off-shore sextant readings are taken to three signals A, B, C on land and the measured angles subtended by AB and BC are $32^{\circ}30'$ and $62^{\circ}30'$ respectively. The positions of the three shore signals are such that $AB = 1200$ ft. and $BC = 2050$ ft. and the angle ABC on the landward side is $233^{\circ}30'$. Determine graphically the distance of the boat from B.

Ans. 2130 ft.

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APPENDIX—3

BIBLIOGRAPHY

1. H. F. Birchall—Modern Surveying for Civil Engineers
2. D. Clark—Plane and Geodetic Surveying for Engineers
3. R. E. Davis & F. S. Foote—Surveying. Theory and Practice
4. N. N. Mitra—A Manual of Surveying
5. H. Ali—A Text Book of Surveying for Engineers
6. W. N. Thomas—Surveying
7. P. B. Shahani & G. I. Bhagia—Practical Surveying
8. R. S. Deshpande—A Text Book of Surveying
9. T. P. Kanetkar—Surveying & Levelling
10. S. M. N. Huq—Notes on Practical Surveying
11. Parker—Astronomy
12. J. M. Kar—Astronomy
13. M. R. Spiegel—Theory and Problems of Statics
14. B. C. Funmia—Surveying

4. In the above problem if the boat is moved inshore and sextant readings again taken, with boat stationary, to A, B and C and it is found that the angles now subtended by AB and BC are $90^{\circ}0'$ and $113^{\circ}30'$ respectively. Determine graphically the distance between the two stationary positions of the boat at which soundings are taken. Use scale of 1 inch to 500 ft.

Ans. 1730 ft.

5. Out line a practical method of measuring the discharge in cfs of (a) a Small rocky creek 10 ft. wide and (b) a river 1 mile wide and 20 ft. deep.

6. (a). Mention the types of tide gauges and describe the various forms of tide gauges in this types.

(b) Mention the considerations which govern the choice of a site for a tide gauge.

7. (a) What is meant by sounding? Mention any six methods of locating soundings.

(b) Explain clearly how would you determine the levels of river bed points and fix the position of sounding.

(i) by use of a sextant in a boat.

(ii) by use of a theodolite on shore.

(c) The cross-section of a stream 30 units wide is measured by means of soundings taken 5 units apart. The depths recorded are :
0, 1.5, 2.0, 3.5, 2.3, 1.0 and 0

The mean velocity is observed to be 3.4 units per second. Compute the discharge of the stream.

APPENDIX—1

SI Units

1. Introduction

Many countries in the world follow different systems of units of measurements. For example, in United States, F.P.S. system of units is followed, while India switched over to the metric system of units. Due to different systems of units followed in different countries, a lot of confusion has been caused, giving a great set-back to industrial production and international trade. In the Eleventh General Conference on weights and Measures held in Paris in 1960, it was decided to have one common system of units which is unified, systematically constituted and coherent for international use. The system of units so evolved and adopted at the conference is called System International, abbreviated as SI units. The SI system rationalises the available units and streamlines them into a coherent logical system, using minimum possible basic units.

2. Basic And Derived Units

(a) *Basic Units*: There are six basic units in the SI system of units, listed below :

TABLE 1

Basic SI units

Quantity	Name of SI units	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Electrical current	ampere	A
5. Thermodynamic temperature	kelvin	K
6. Luminous Intensity	Candela	cd

In the above table, Kelvin is a temperature interval which is equal to temperature interval correspond to 1°C .

Hence $1\text{ K} = 1^\circ\text{C}$. In addition to the basic unit given in the above table, there are two supplementary units in the SI system, given in the table as follows :

TABLE 2
Supplementary Units in SI System

Quantity	Name of SI Unit	
Plane angle	radian	rad
Solid angle	steradian	Sr

(b) Derived Units:

The derived units of SI system are connected to the basic units by some physical law. Some of the common derived units are given below:

1. Area and volume

When a unit of length is multiplied by the unit of length the result is the unit of area which is a derived unit in SI system. The unit of area is expressed by symbol m^2 . Similarly, when a unit of length is multiplied by unit of area, we get the derived unit of volume which is expressed by symbol m^3 .

2. Velocity and Acceleration.

When the unit of length is divided by unit of time, the result of derived unit is the unit of velocity expressed by the symbol m/s . Similarly, in the angular measure, angular velocity (radian per second) is expressed by symbol rad/s . The unit of acceleration is expressed by meter per second square (m/s^2) or by radian per second square (rad/s^2).

3. Force

In the British system, the unit of force is lb (f) while in the metric system, the unit of force is kg (f) . However, in the SI system, the unit of force is Newton expressed by symbol N . It is defined as that force which, when applied

to a body having a mass of one kilogram, gives an acceleration of one metre per second squared. The SI system has therefore discarded the gravitational unit of force. Since 1 newton is small, force is generally expressed in terms of kilonewton and meganewtons. Thus

One kilonewton (KN) = 10^3 newtons

One meganewton (MN) = 10^6 newtons.

4. Stress

Stress is the force per unit area, and can be expressed as newtons per metre squared (N/m^2). However, it is commonly expressed as kilonewton per metre squared (kN/m^2).

5. Work and Energy

When the point of application of a force of one newton is displaced through one metre in the direction of the force one joule (J) or newton-meter (Nm) of work is done. The unit of work or energy is thus joule. Larger magnitude of work or energy is commonly expressed in kilojoule (KJ) where one kilojoule is equal to 10^3 joules.

6. Power

Since power is the rate of doing work, it is expressed in terms of joule per second or newton metre per second. The unit of power is thus watt (W) which is equal to one joule per second (J/s) or one newton metre per second (Nm/s). It should be noted that in the SI system, horsepower as a unit of measurement of power is not used. Power is measured in terms of watts (W), kilowatts (KW) and megawatts (MW) where :

one kilowatt (KW) = 10^3W ,

one megawatt (MW) = 10^6W .

7. Other derived units :

Table 3 gives the summary of various basic and derived units in SI units. The decimal multiples and submulti-

ples of SI units are formed by means of prefixes given below :

Factors by which unit is multiplied	Prefix	Symbol
10^6	mega	M
10^3	kilo	k
10^1	deca	da
10^{-1}	deci	d
10^{-3}	milli	m
10^{-6}	micro	u

Interconversion : In the surveying, the common units are length, area, volume, force and stress. In the MKS system, the unit of force is in the gravitational unit i. e. kilogramme force kgf (commonly expressed as kg only).

Hence $1 \text{ kg (f)} = g \text{ newtons}$

Taking $g = 9.807$

$$1 \text{ kg (f)} = 9.807 \text{ N} = 10 \text{ N}$$

Similarly $1 \text{ tonne (f)} = 1000 \text{ kg (f)} = 10 \text{ kN}$

In the same way, for stress unit

$$1 \text{ kgf/cm}^2 = \frac{10 \text{ N}}{10^{-4} \text{ m}^2} = \frac{1}{10} \frac{10^6 \text{ N}}{\text{m}^2} = \frac{1}{10} \frac{\text{MN}}{\text{m}^2}$$

TABLE 3

Basic and Derived units in the SI system

Quantity	Name of unit	Symbol
1. Length	metre	m
	(kilometre)	km
	(centimetre)	cm
	millimetre	mm
2. Area	square metre	m^2
	(square centimetre)	cm^2
	(square millimetre)	mm^2

3. Volume	cubic metre	m^3
	(cubic centimetre)	cm^3
	(cubic millimetre)	mm^3
4. Velocity	Metre per second	m/s
5. Angular velocity	radian per second	rad/sec.
6. Acceleration	metre per second square	m/s^2
7. Angular acceleration	radian per second square	rad/s^2
8. Frequency	hertz	Hz
9. Discharge	cubic metre per second	m^3/s
10. Mass	Kilogramme	kg
	(tonne)	t
11. Mass density or Specific mass	kilogramme per cubic metre	kg/m^3
	(tonne per cubic metre)	t/m^3
12. Force	newton	N
	(kilonewton)	kN
	(meganewton)	MN
13. Weight density (specific weight)	newton per cubic metre	N/m^3
14. Stress and pressure	newton per metre squared	N/m^2
	(kilonewton per metre squared)	kN/m^2
	(meganewton per metre squared)	MN/m^2
15. Elastic moduli	newton per metre squared	N/m^2
16. Impulse	newton second	Ns
17. Work : Energy	Joule	J
	(kilojoule)	kJ
18. Quantity of heat	joule	J
19. Momentum	kilogramme metre per second	$\text{kg}\cdot\text{m/s}$
20. Moment of Inertia (Dynamic)	kilogramme metre squared	$\text{kg}\cdot\text{m}^2$
21. Moment of Inertia (Area)	—	m^4

22. Angular momentum:	kilogramme metre	kg-m ² /s
moment of momentum squared per second		
23. Moment of force :	Newton metre	Nm
Bending moment :		
Torque	kilonewton metre	kNm
24. Section modulus	—	cm ³
25. Power	watt	w
	(kilowatt)	kw
	(megawatt)	MW
26. Dynamic viscosity	newton second per square metre	Ns/m ²
27. Kinematic viscosity	square metre per second	m ² /s
28. Entropy	joule per kelvin	J/K
29. Specific heat	joule per kilogramme kelvin	J/kg K
30. Thermal conductivity	watt per metre kelvin	W/mK

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